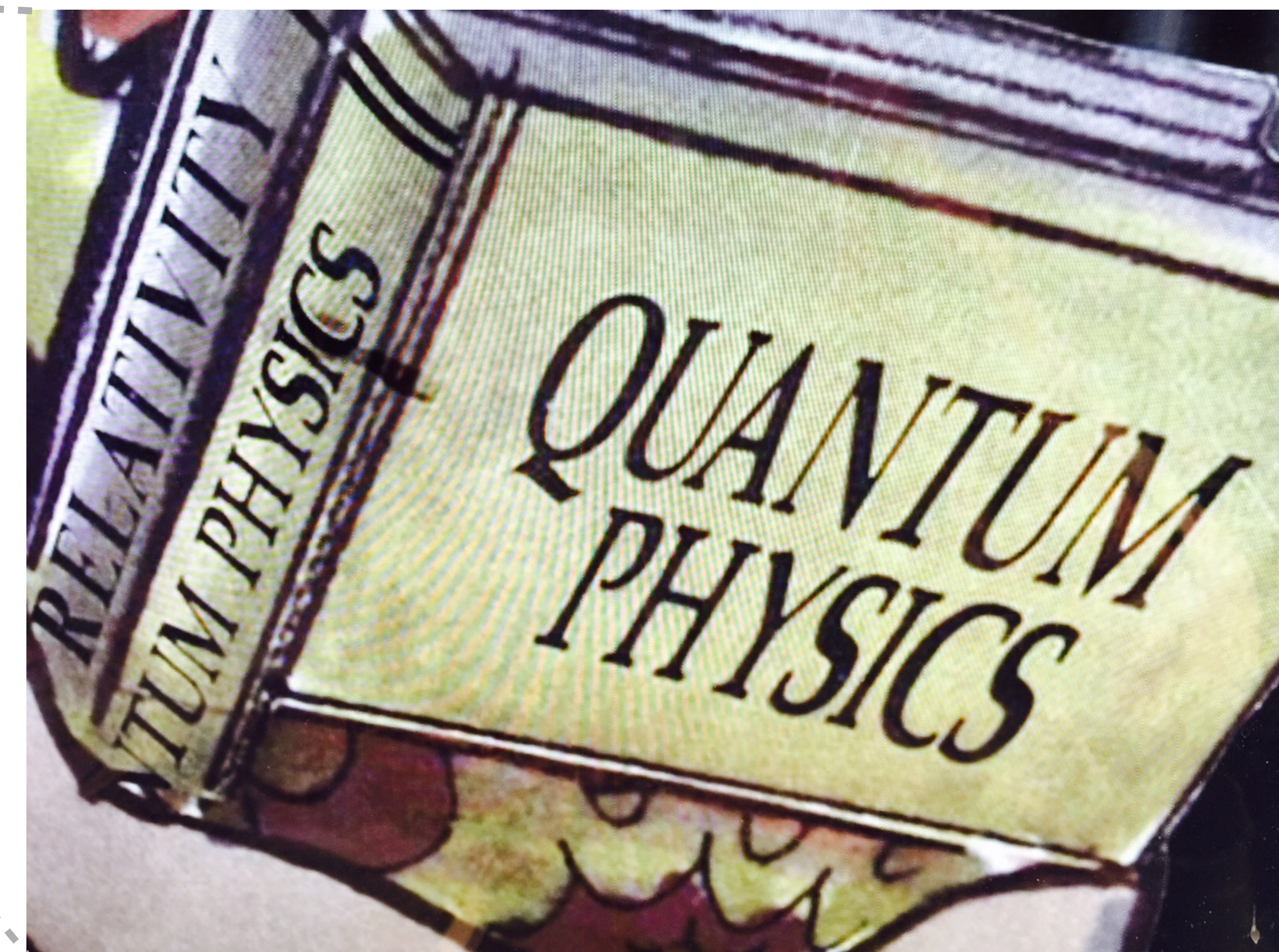
Auxiliary slides for seminar at Rochester Institute of Optics June 19, 2018

Relawavity:

Helping to clarify Quantum Theory and Relativity
by effecting a wavy marriage
between this enigmatic pair

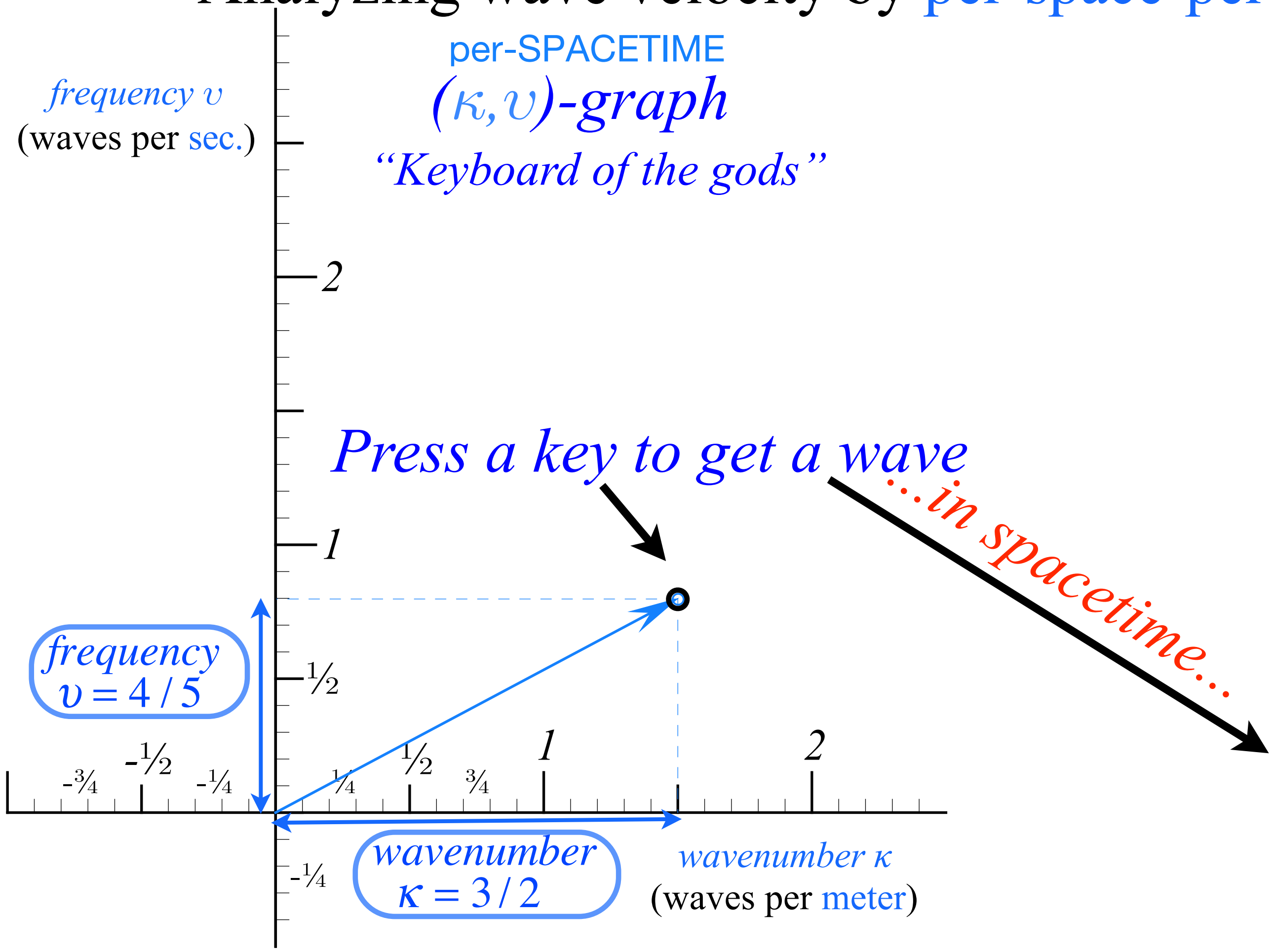
(Why a ***Men In Black*** candidate shot little Suzy)



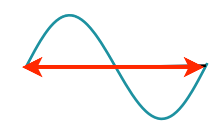
Bad Suzy!
Relativity and Quantum Theory need to be unified in *one* book *half* the size of those old tomes!

We call that a ***Relawavity*** *book.*
(It's a *lot* lighter!)

Analyzing wave velocity by per-space-per-time *and* space-time graphs



SPACETIME
(λ,τ)-graph



"Keyboard of the gods" is known as "Fourier-space"

Jean-Baptiste
Joseph Fourier
1768-1830

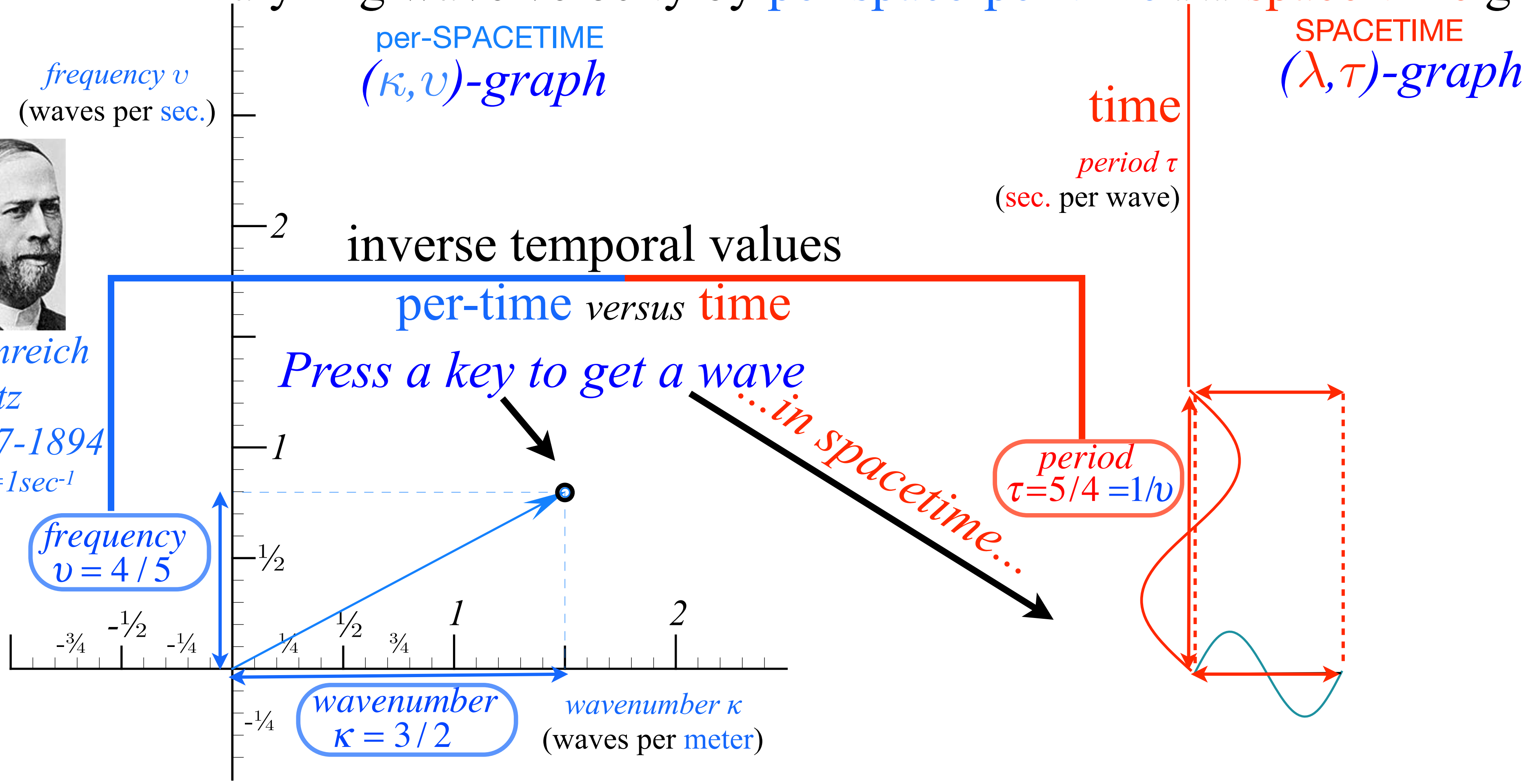
•*How to understand waves*
and
wave velocity V_{wave}

RelaWavity Web Simulation
Keyboard of the Gods
(per-Time *vs* per-Space)

Analyzing wave velocity by per-space-per-time *and* space-time graphs

Heinreich Hertz 1857-1894

1Hz=1sec⁻¹



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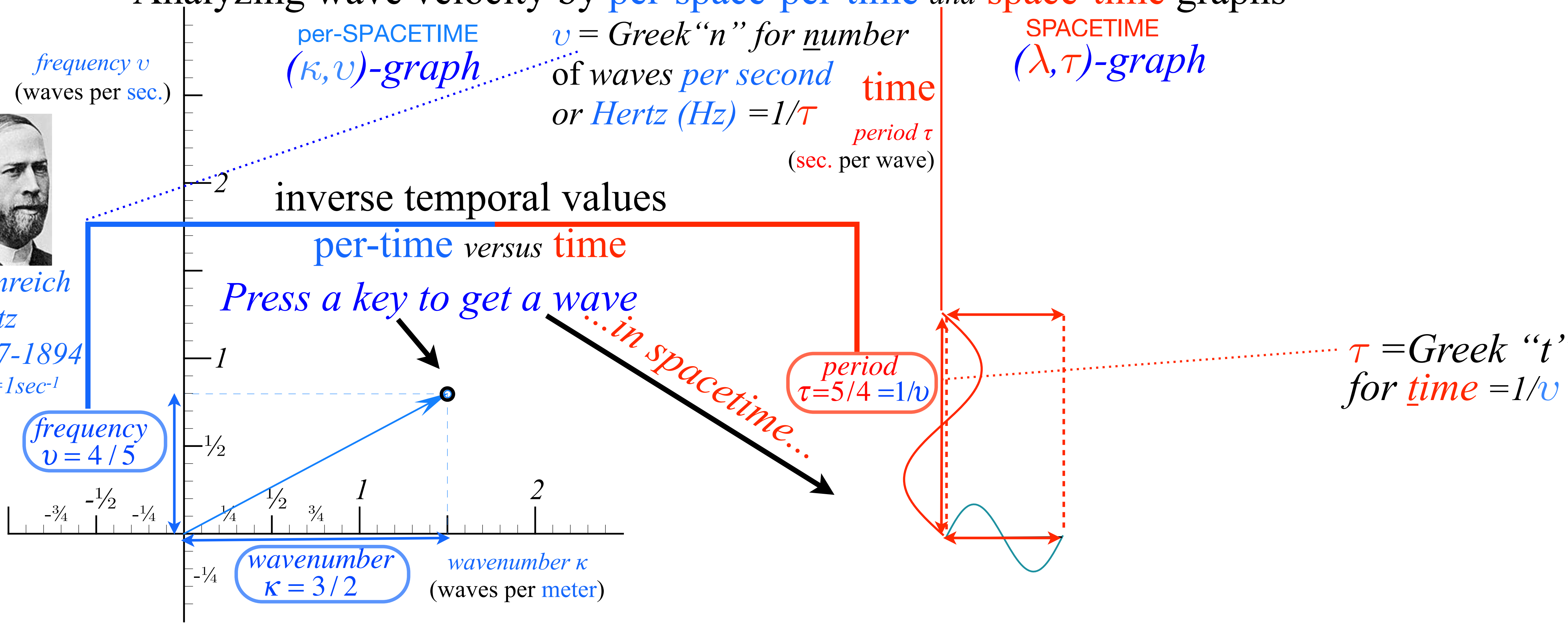

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RelaWavity Web Simulation
Keyboard of the Gods
(Dual Plot)

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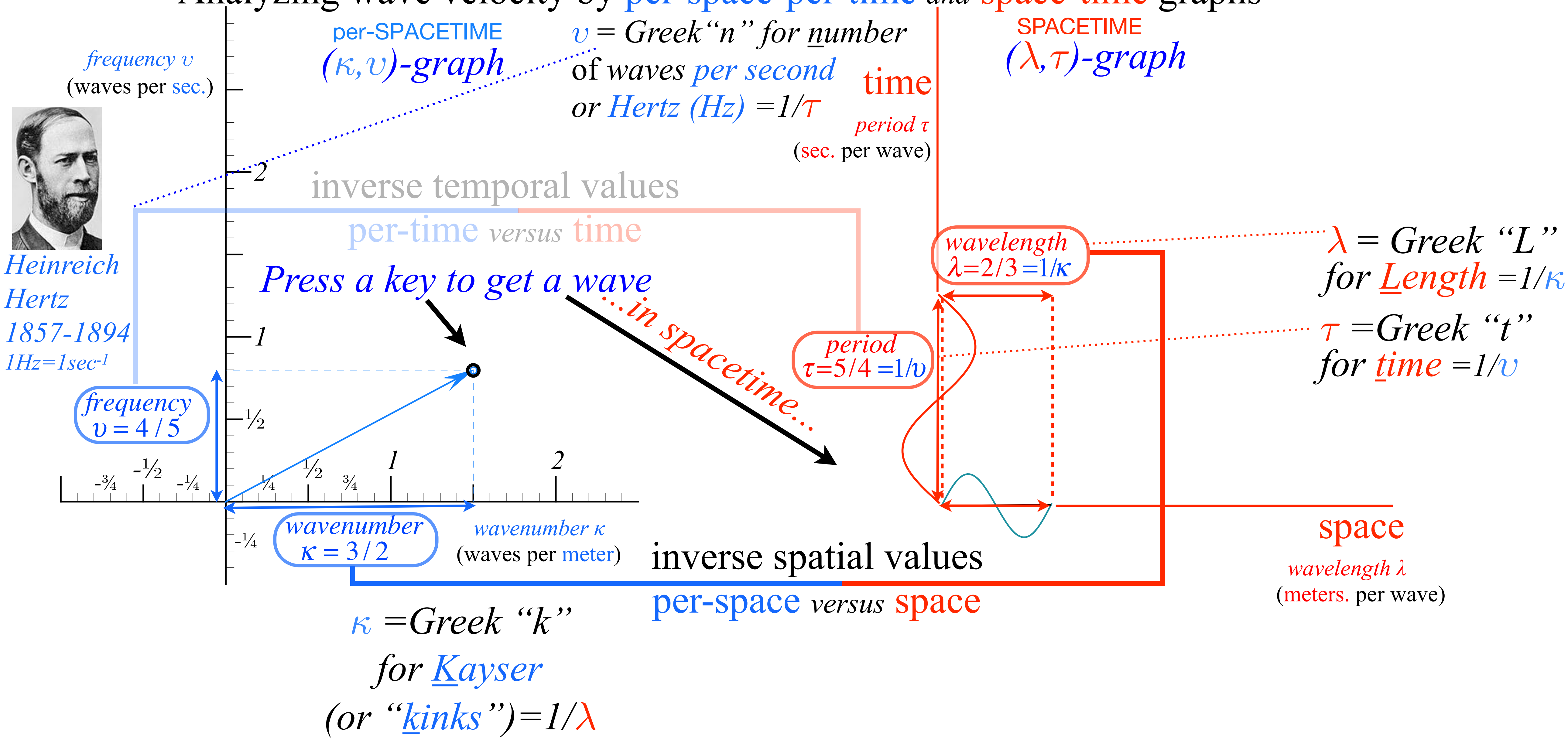

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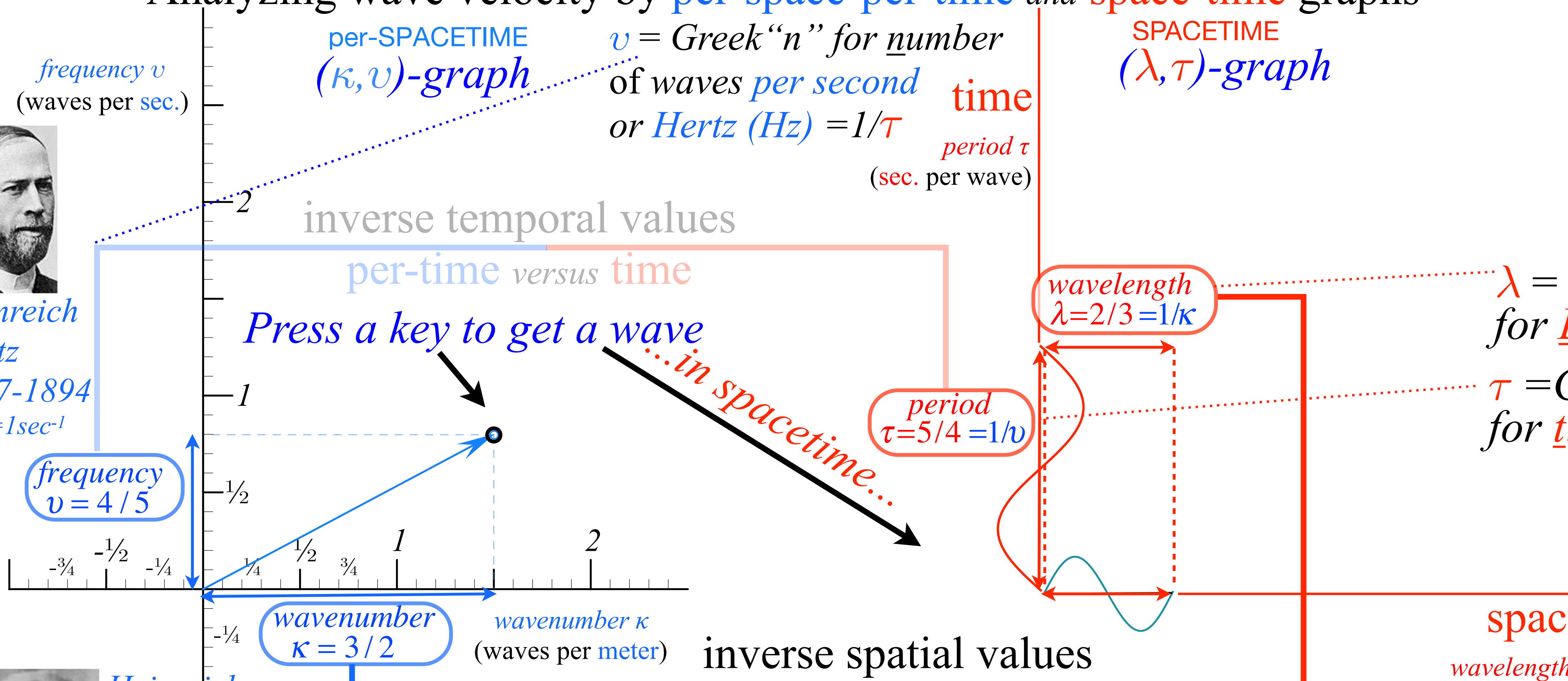
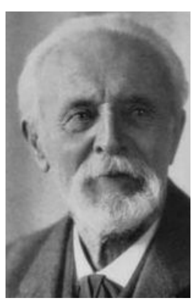


Heinreich Hertz 1857-1894
1Hz=1sec⁻¹

per-space *versus* space



Heinreich Kayser 1853-1940
1Kayser=1cm⁻¹

κ =Greek "k" for Kayser (or "kinks")=1/λ

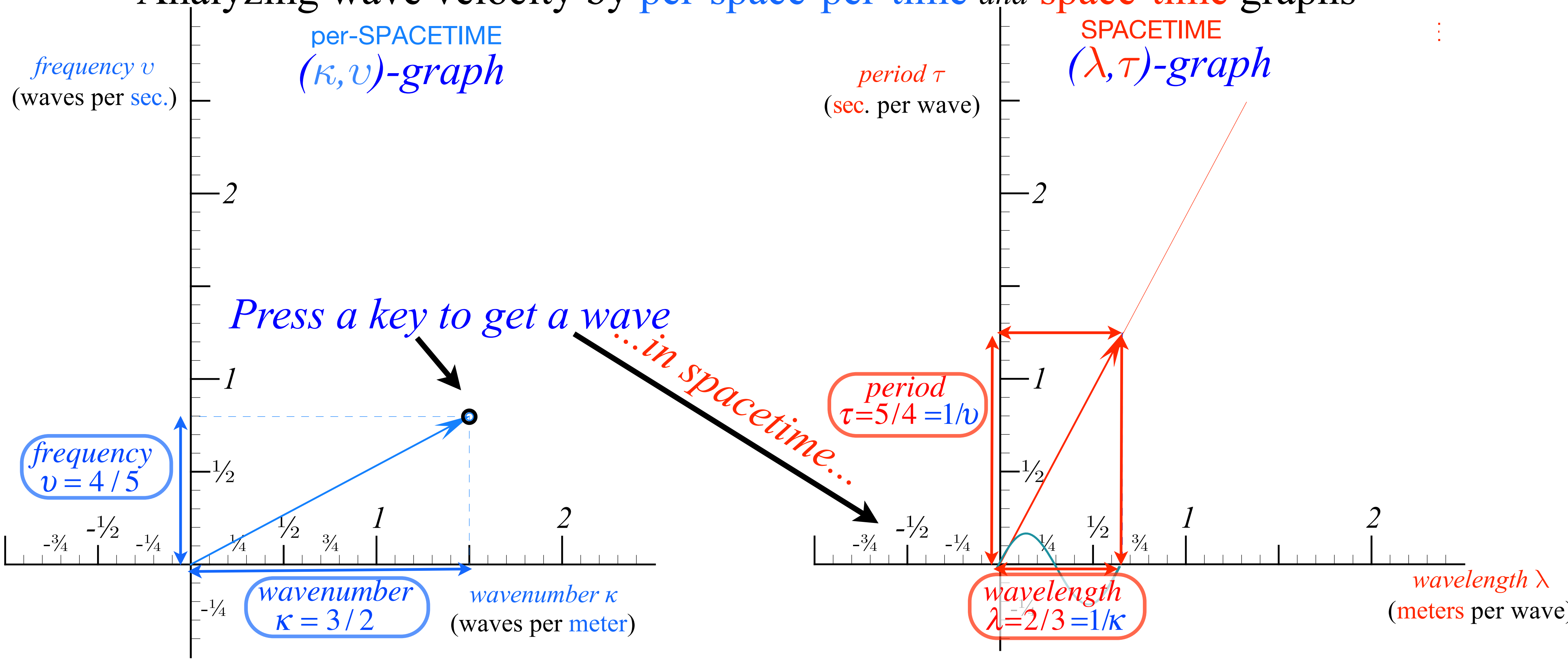
"Keyboard of the gods" is known as "Fourier-space"

Jean-Baptiste Joseph Fourier 1768-1830

•How to understand waves and wave velocity V_{wave}

[RelaWavity Web Simulation](#)
[Keyboard of the Gods](#)
[(Dual Plot)](#)

Analyzing wave velocity by per-space-per-time *and* space-time graphs



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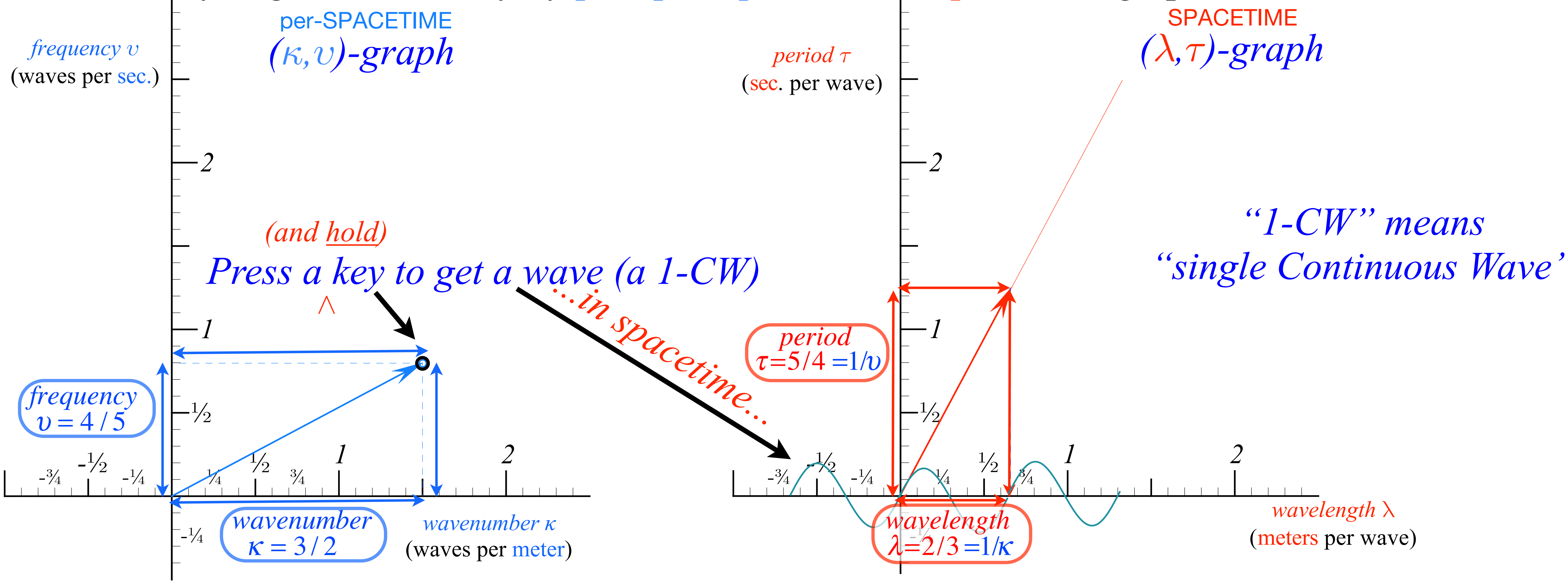

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Keyboard of the Gods
(Dual Plot)

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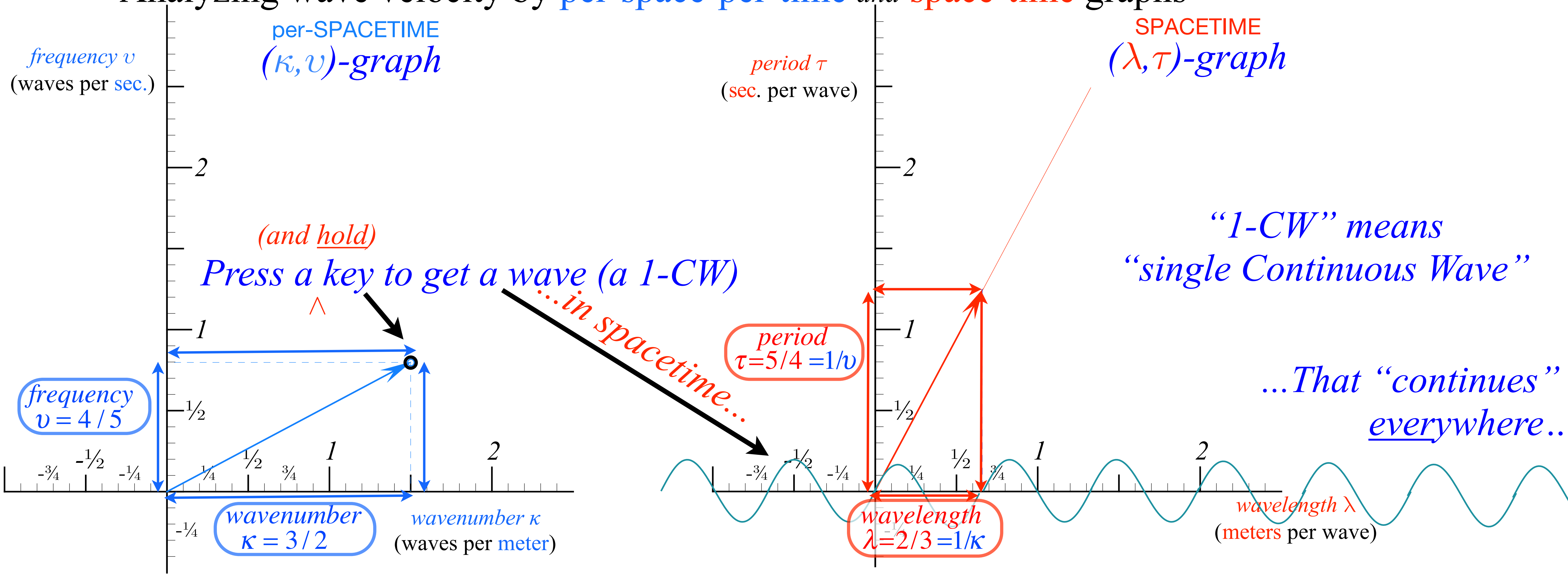

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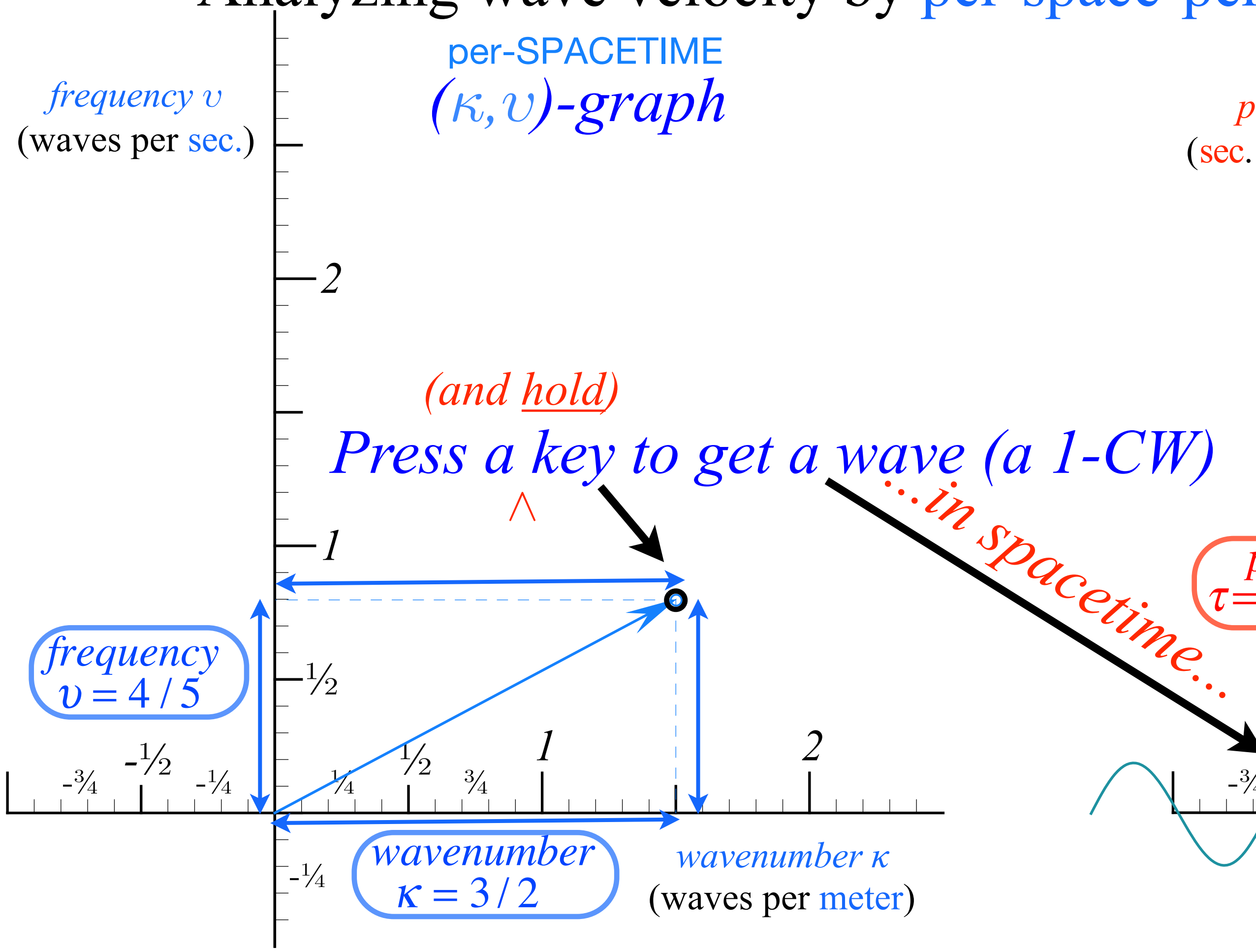
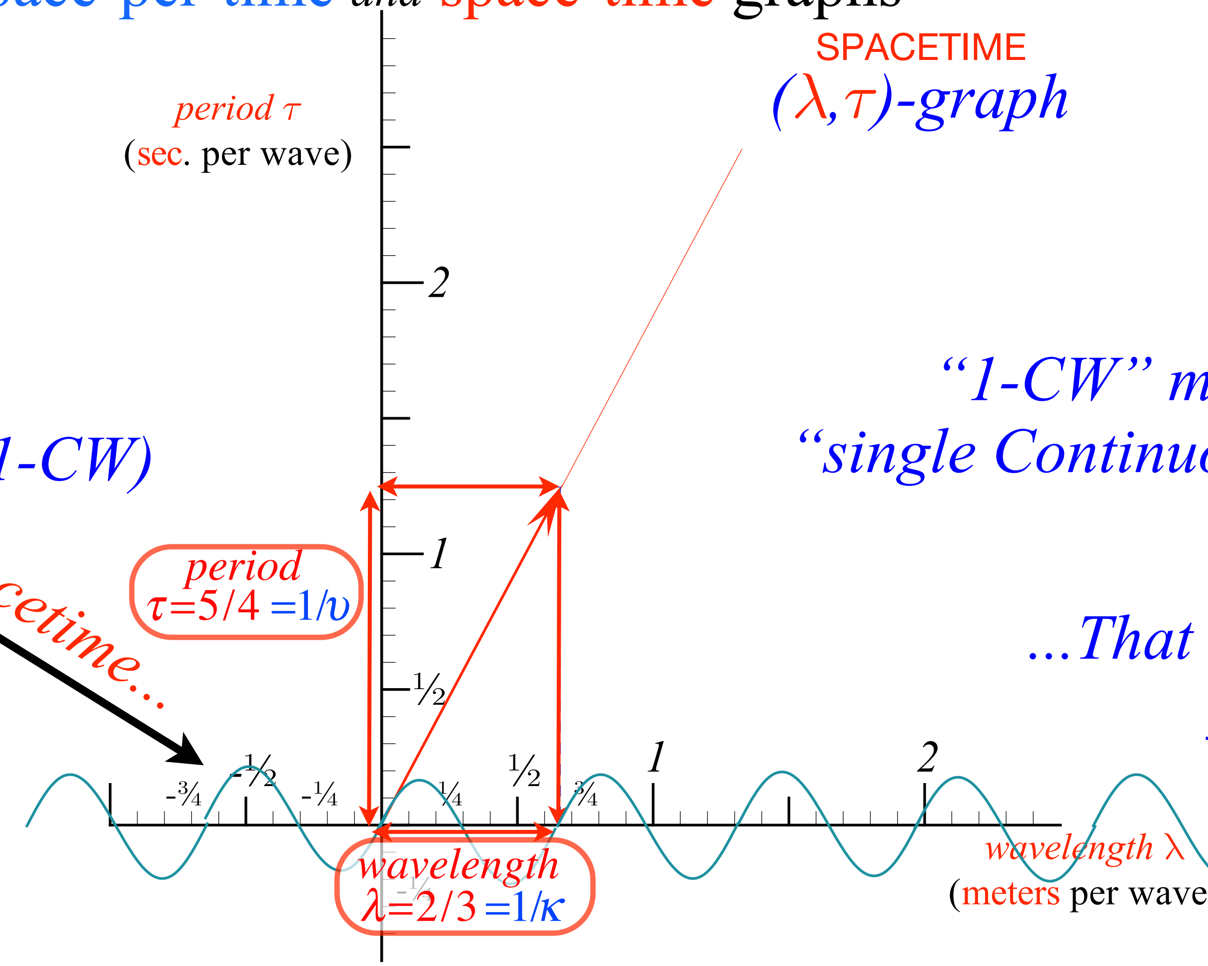
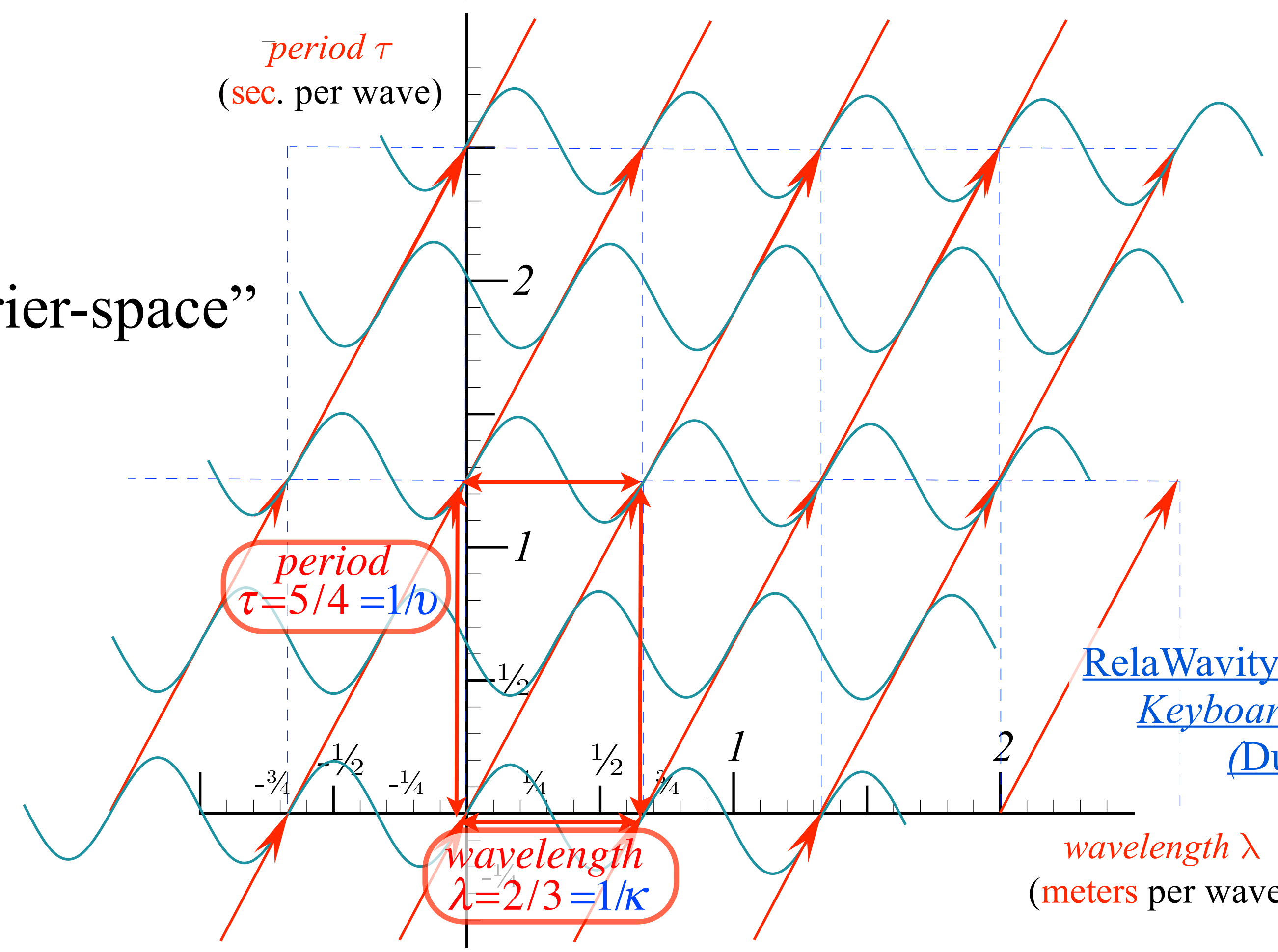




"1-CW" means "single Continuous Wave"

...That "continues" everywhere...

...for all time...



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RelaWavity Web Simulation
Keyboard of the Gods (Dual Plot)

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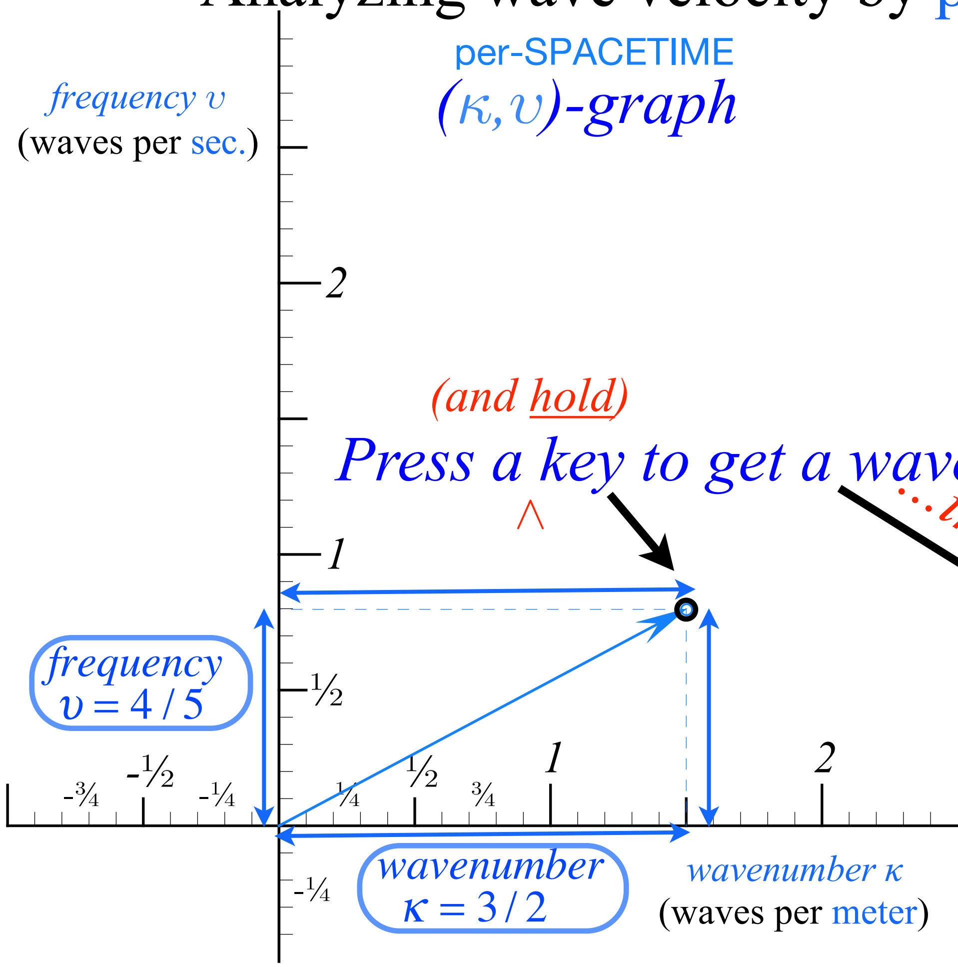
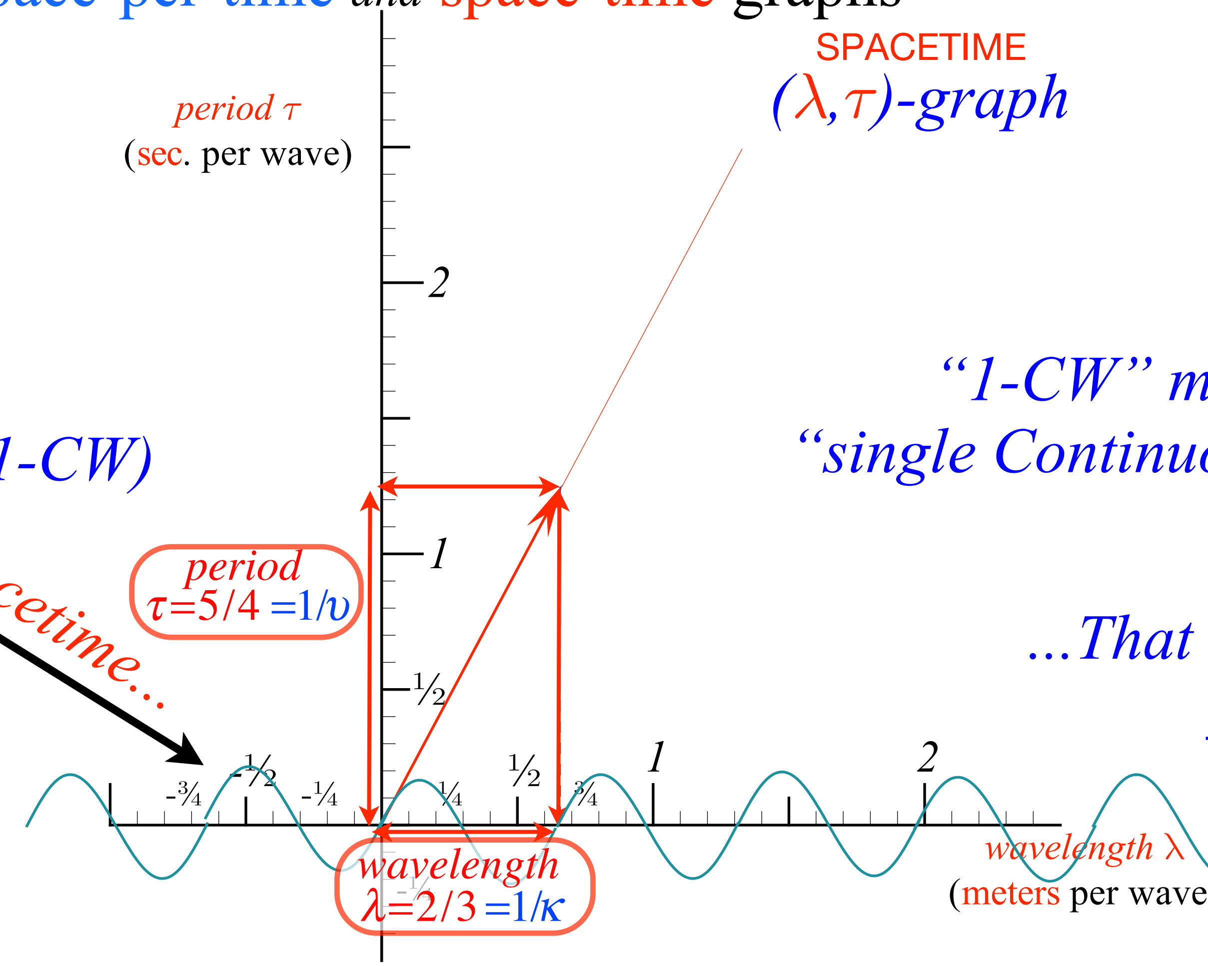
Analyzing wave velocity by per-space-per-time *and* space-time graphs





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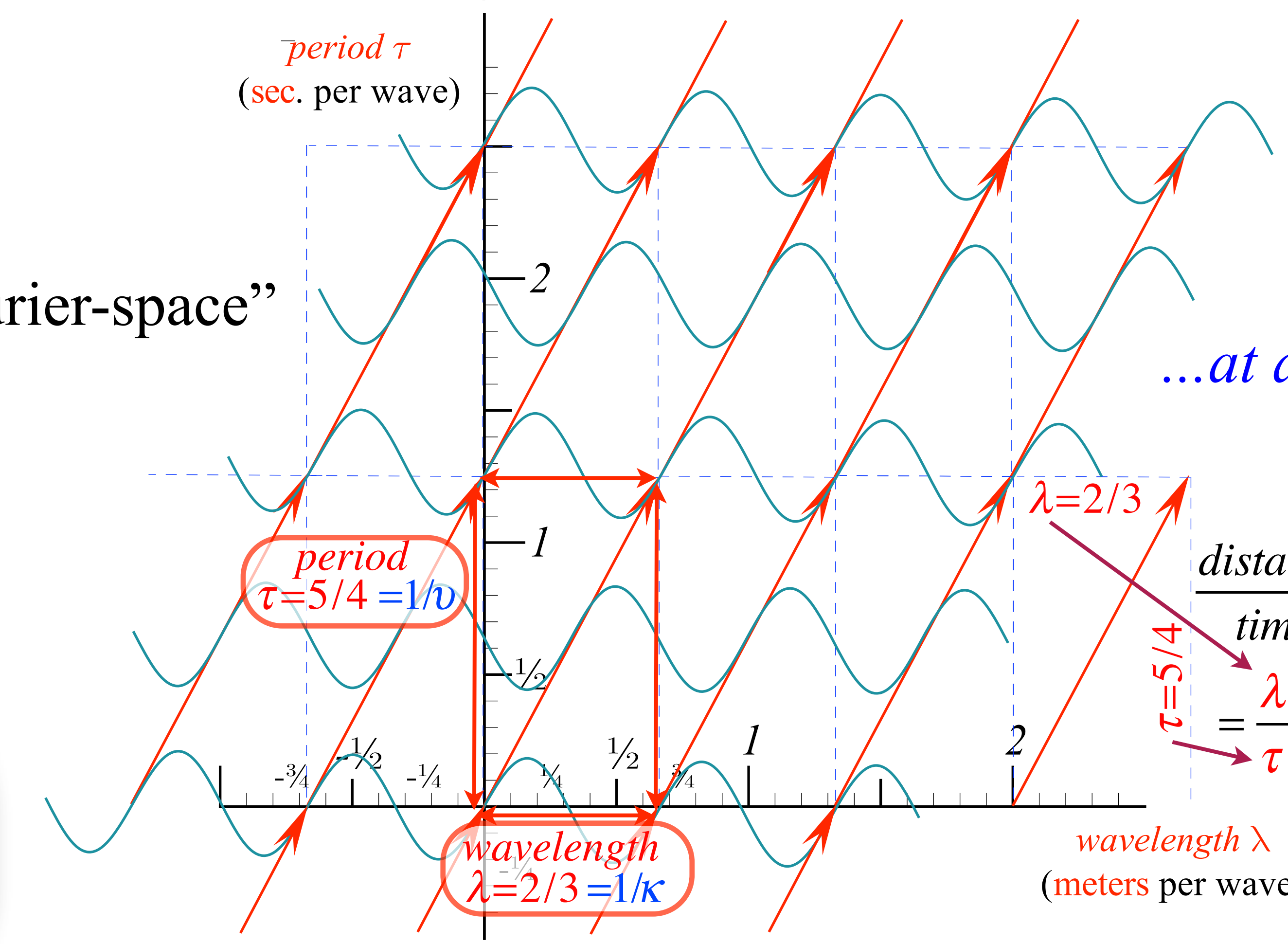

...at a speed of:

$$\frac{distance}{time} = \frac{wavelength}{period} = \frac{\lambda}{\tau}$$

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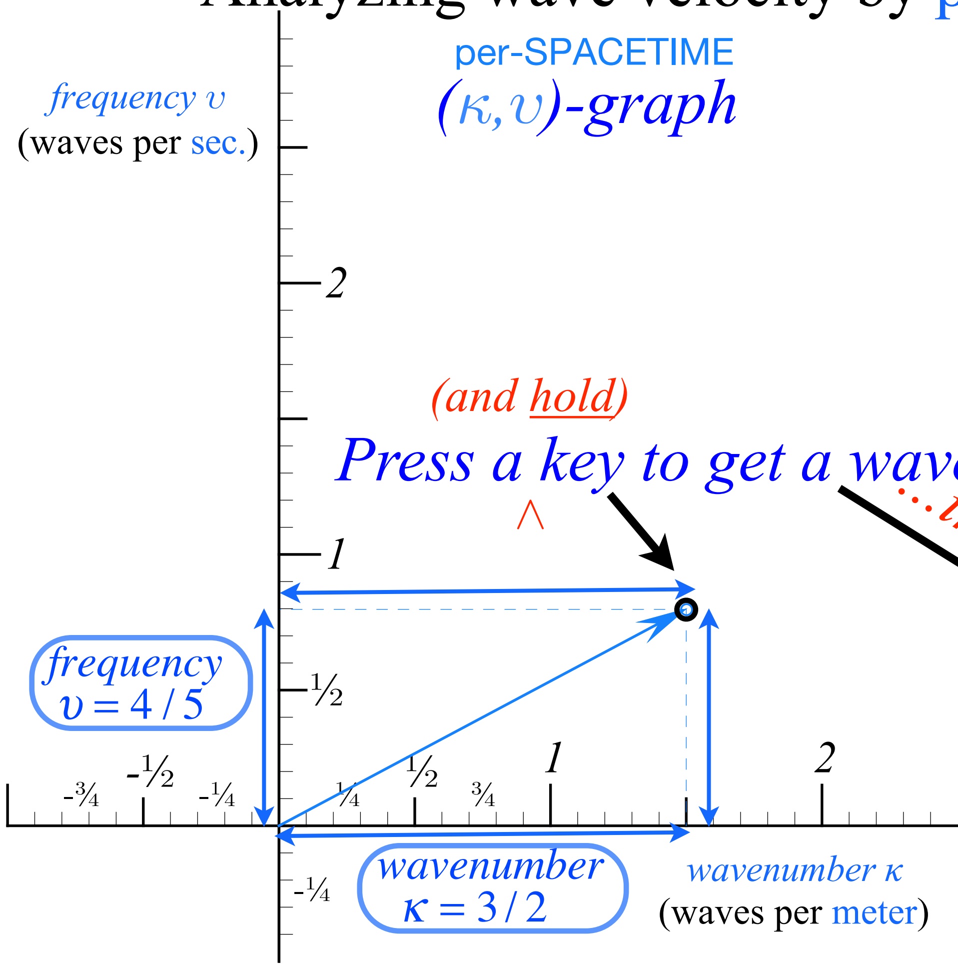
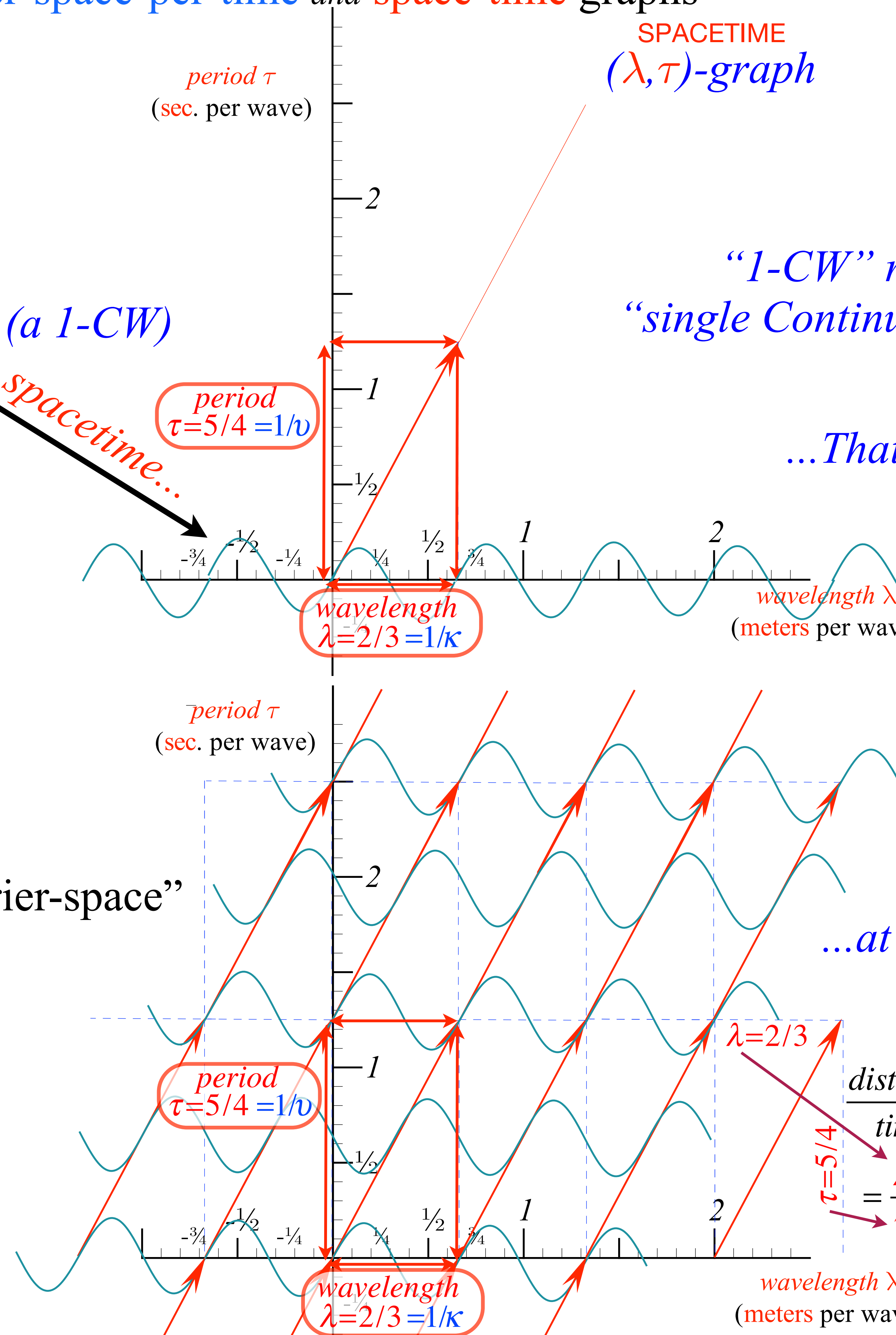
•How to understand waves and wave velocity V_{wave}

Analyzing wave velocity by per-space-per-time *and* space-time graphs



...in spacetime...



"1-CW" means
"single Continuous Wave"

...That "continues" everywhere...

...for all time...

...at a speed of:

$$\frac{distance}{time} = \frac{wavelength}{period} = \frac{\lambda}{\tau} = \frac{2/3}{5/4} = \frac{8}{15}\frac{m.}{s.}$$

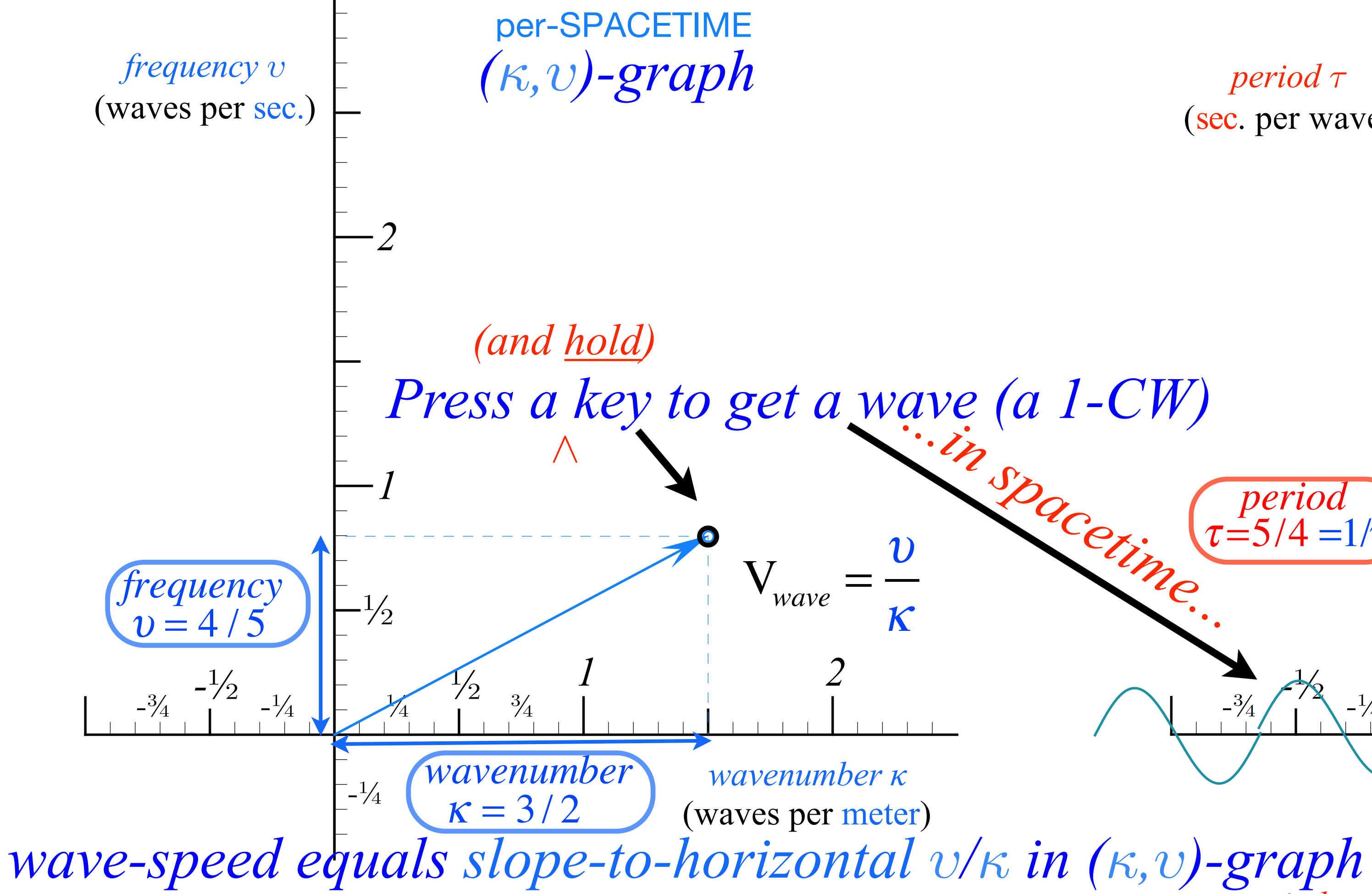
wave-speed equals slope-to-vertical λ/τ in (λ,τ)-graph

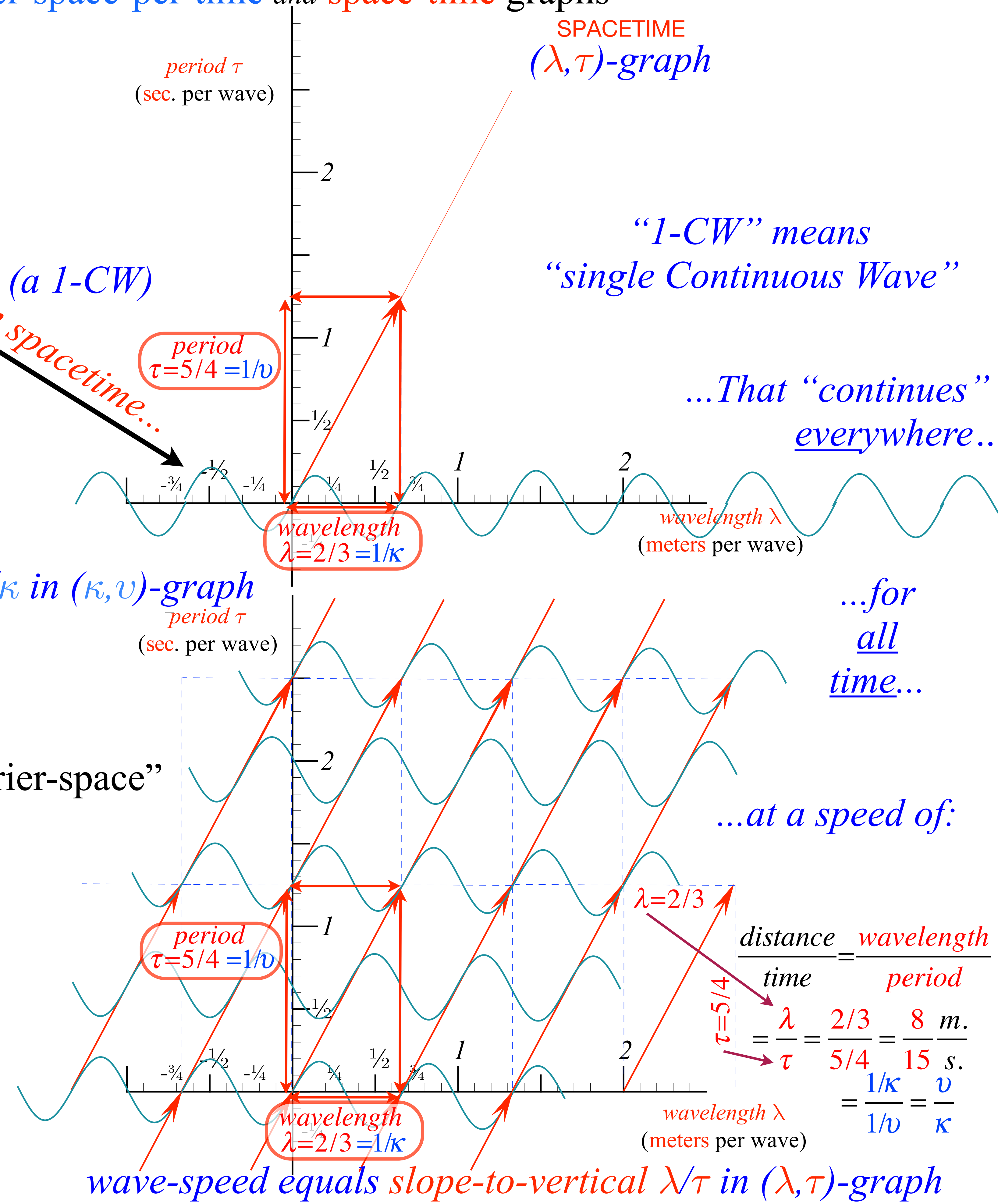
"Keyboard of the gods" is known as "Fourier-space"

Jean-Baptiste Joseph Fourier 1768-1830

•How to understand waves and wave velocity V_{wave}

Analyzing wave velocity by per-space-per-time *and* space-time graphs





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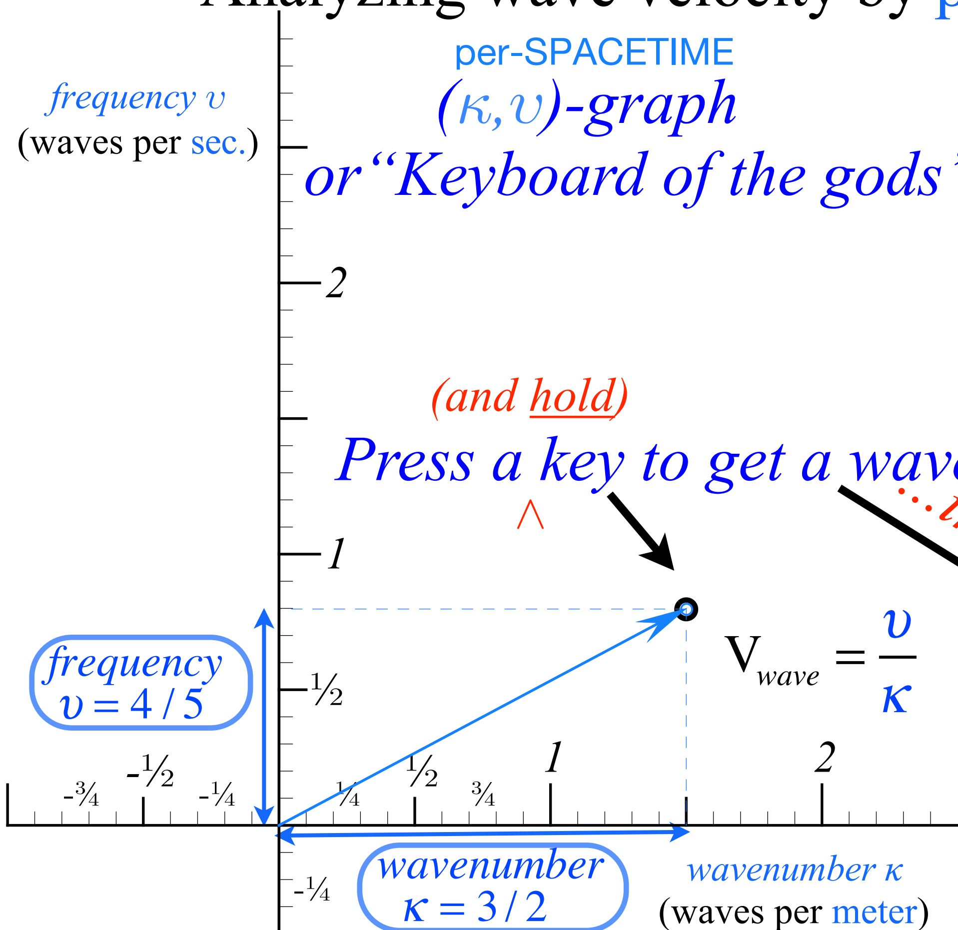
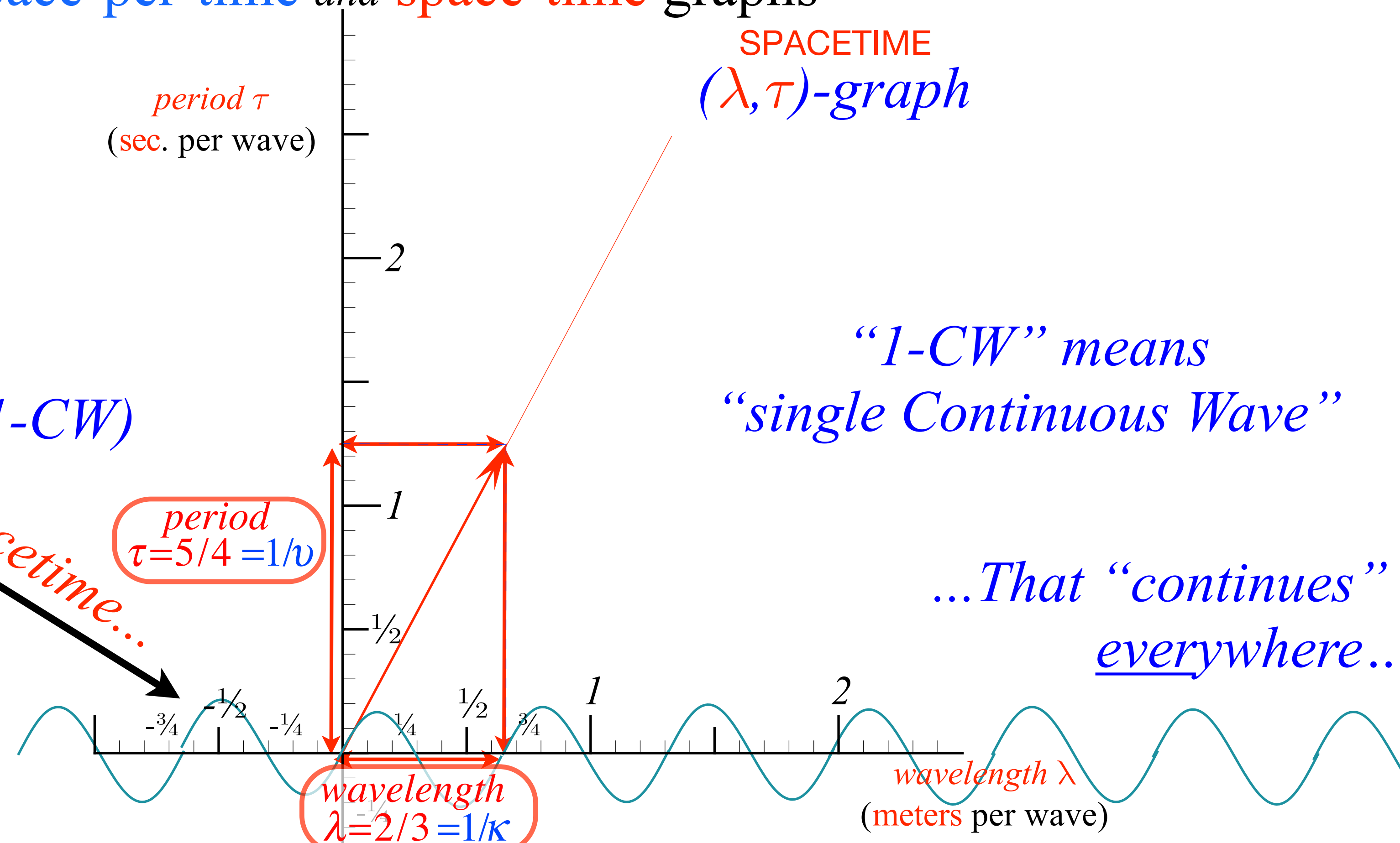


$V_{wave} = \frac{\upsilon}{\kappa}$

...in spacetime...

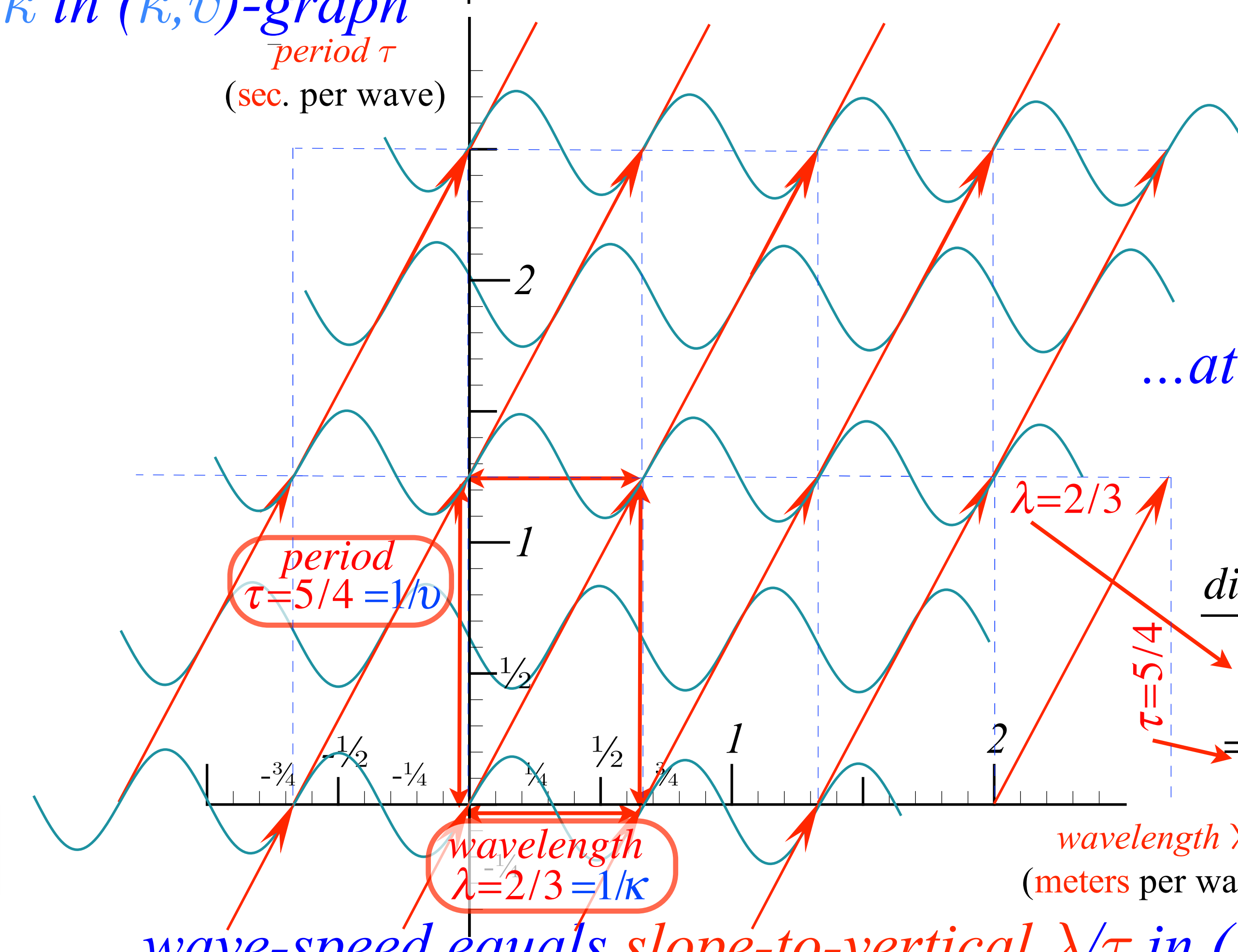


"1-CW" means "single Continuous Wave"

...That "continues" everywhere...

wave-speed equals slope-to-horizontal υ/κ in (κ,υ)-graph

...for all time...



...at a speed of:

$$\frac{distance}{time} = \frac{wavelength}{period} = \frac{\lambda}{\tau} = \frac{2/3}{5/4} = \frac{8}{15}\frac{m.}{s.}$$

wave-velocity formulas

$$\frac{distance}{time} = \frac{wavelength}{period} = \frac{frequency}{wavenumber}$$

$$V_{wave} = \frac{\lambda}{\tau} = \frac{1/\kappa}{1/\upsilon} = \frac{\upsilon}{\kappa} = \frac{1/\tau}{1/\lambda}$$

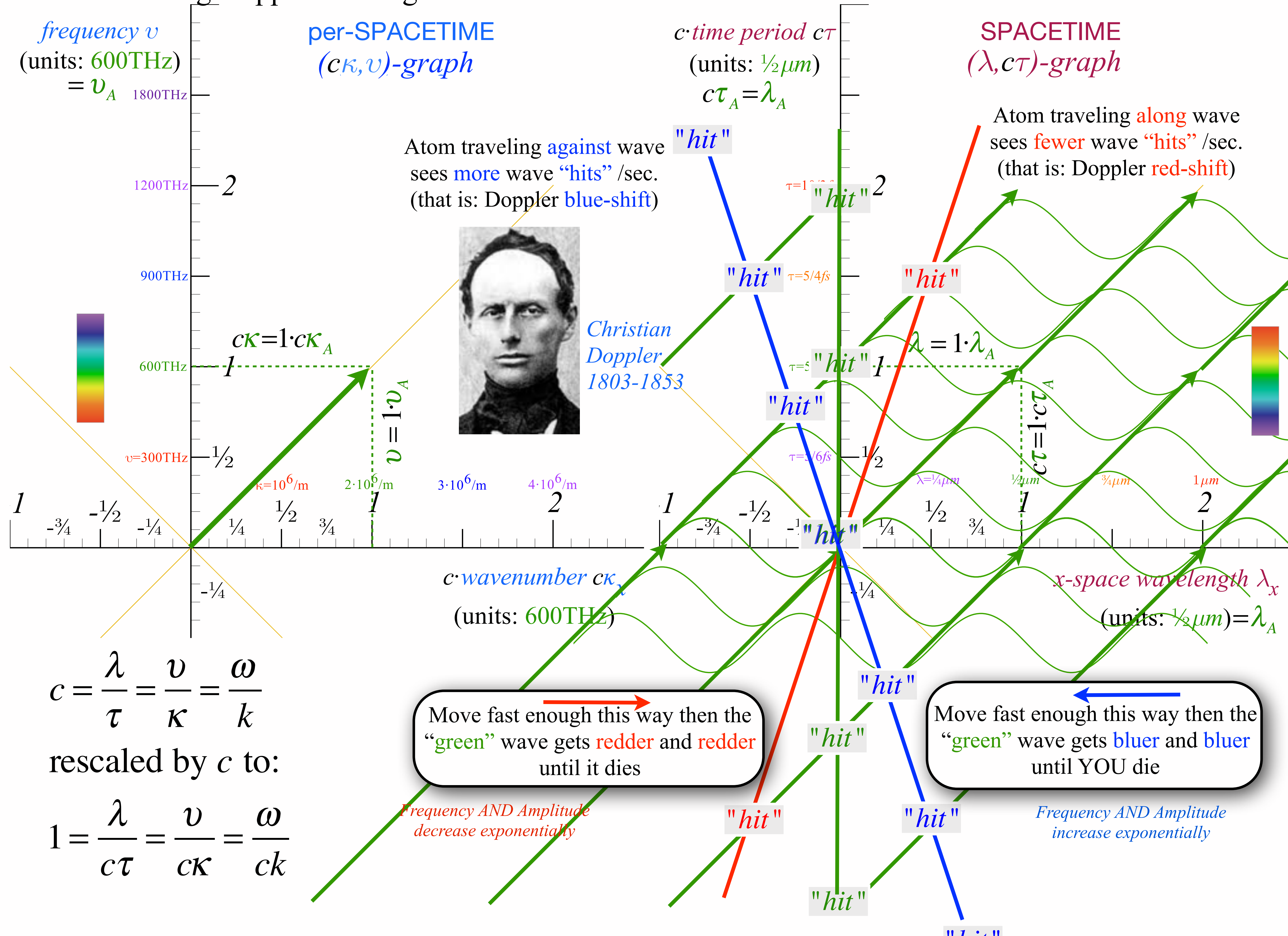
$$= \frac{2/3}{5/4} = \frac{4/5}{3/2} = \frac{8}{15}\frac{m.}{s.}$$

wave arithmetic is simpler to explain using fractions

•How to understand waves and "1st quantization"

wave-speed equals slope-to-vertical λ/τ in (λ,τ)-graph

Introducing Doppler shifting



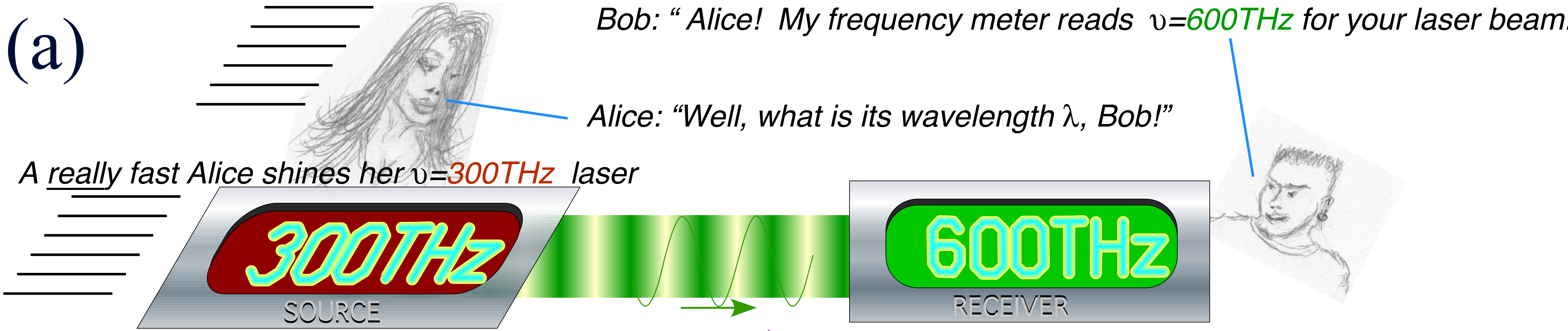
$$c = \frac{\lambda}{\tau} = \frac{\upsilon}{\kappa} = \frac{\omega}{k}$$

rescaled by c to:

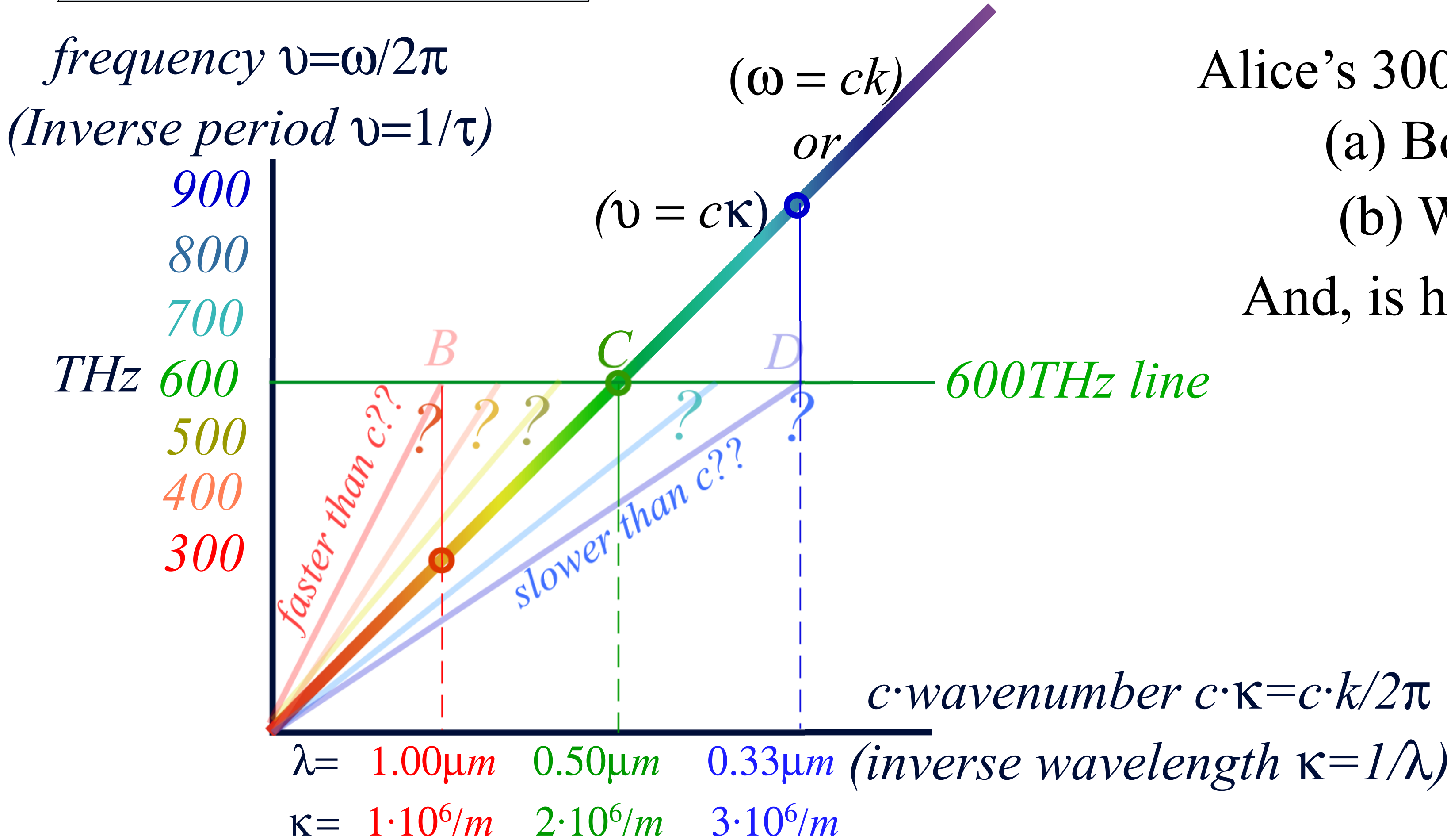
$$1 = \frac{\lambda}{c\tau} = \frac{\upsilon}{c\kappa} = \frac{\omega}{ck}$$

Introducing Doppler shifting and why *c* is *so* constant (and *so slow*)

(a)



(b)



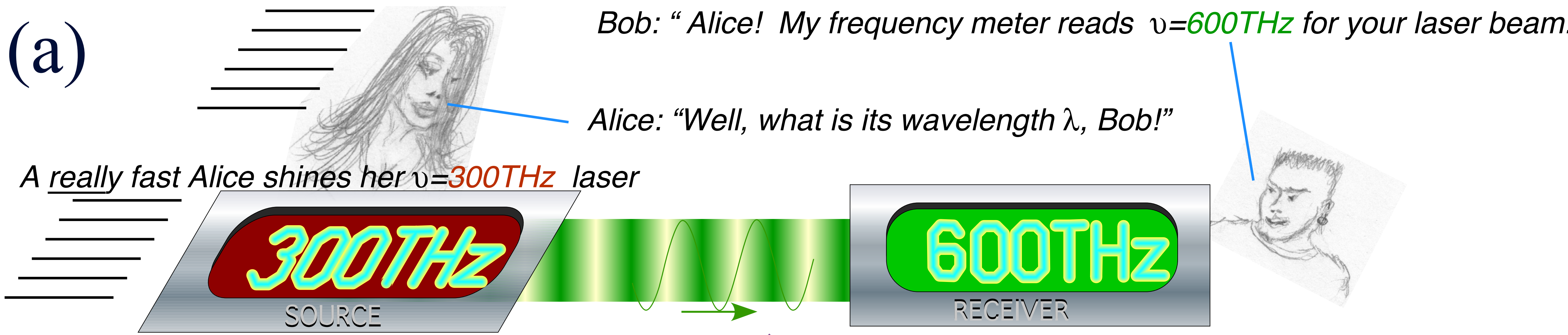
Alice's 300THz laser approaches Bob.

(a) Bob sees υ=600THz.

(b) What $\lambda=1/\kappa$ does Bob measure?

And, is he seeing a '*phony' green*?

Introducing Doppler shifting and why c is constant



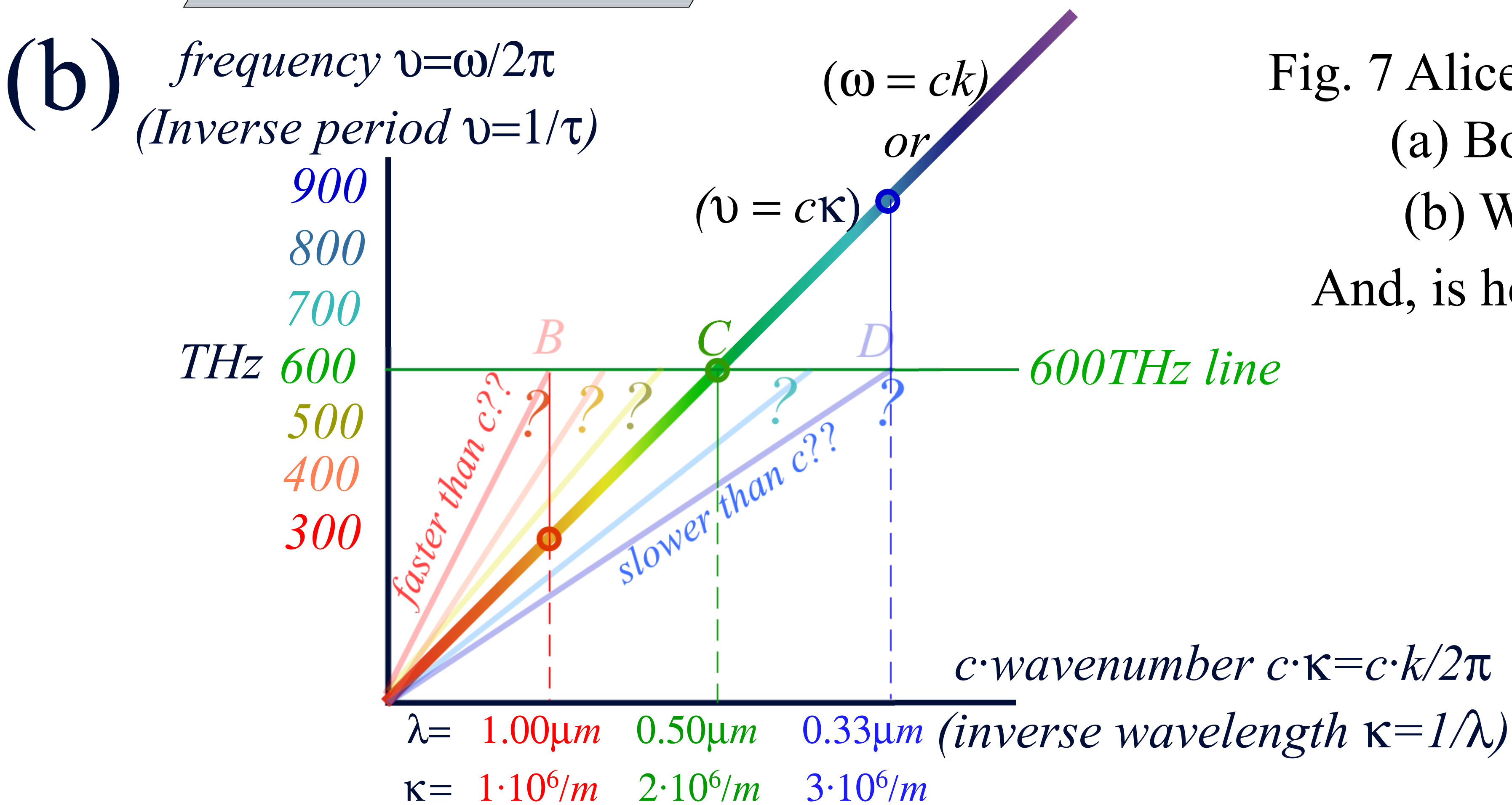


Fig. 7 Alice's 300THz laser approaches Bob.

(a) Bob sees υ=600THz.

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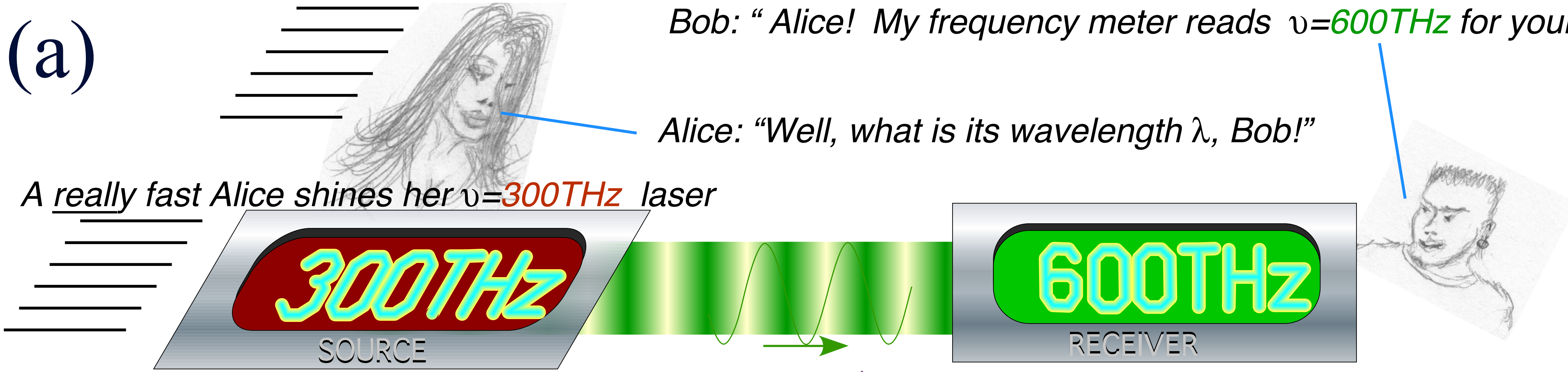
Years of spectroscopy rule out 'phony' 600THz blue-green that do not have wavelength λ=0.5*micron.*

The only choice is C.

Introducing Doppler shifting and why c is constant

(a)



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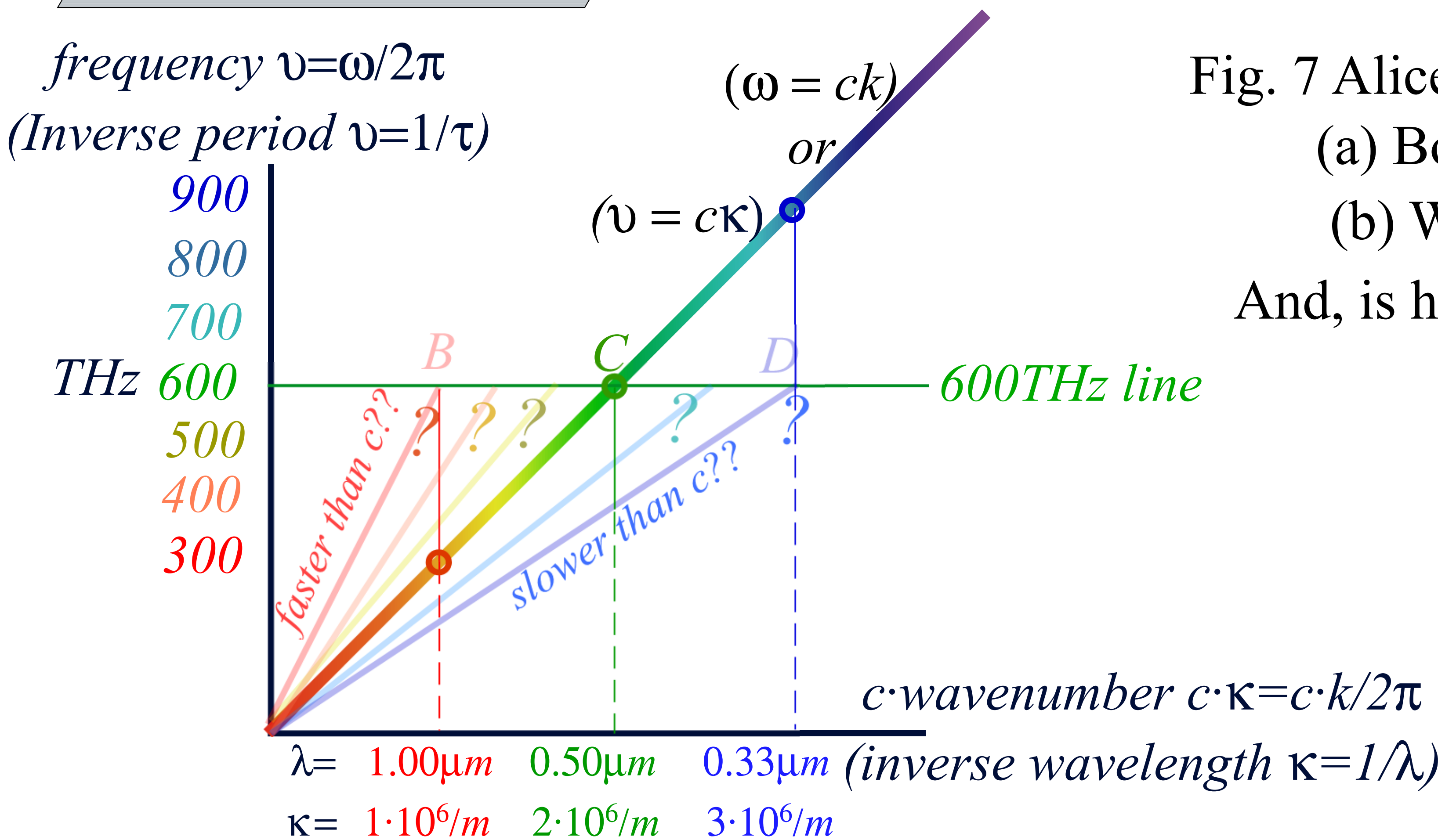

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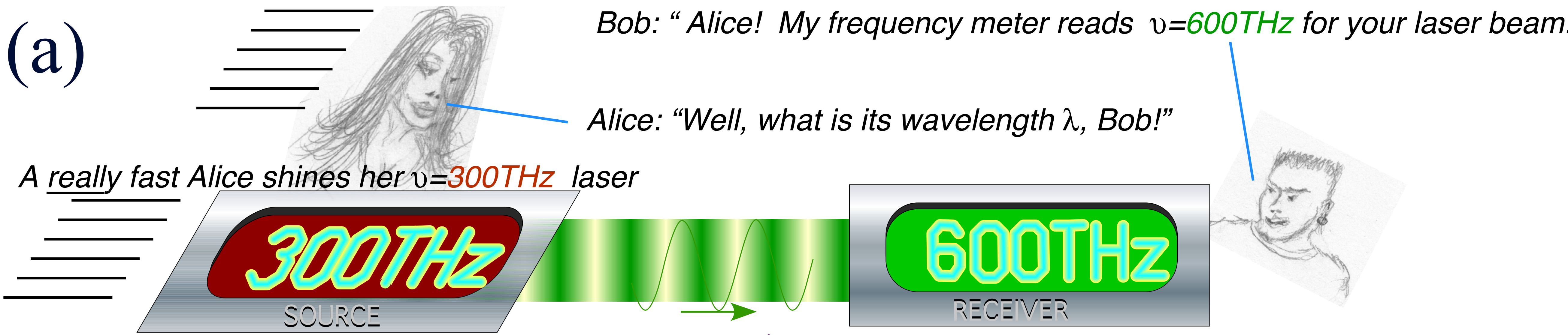
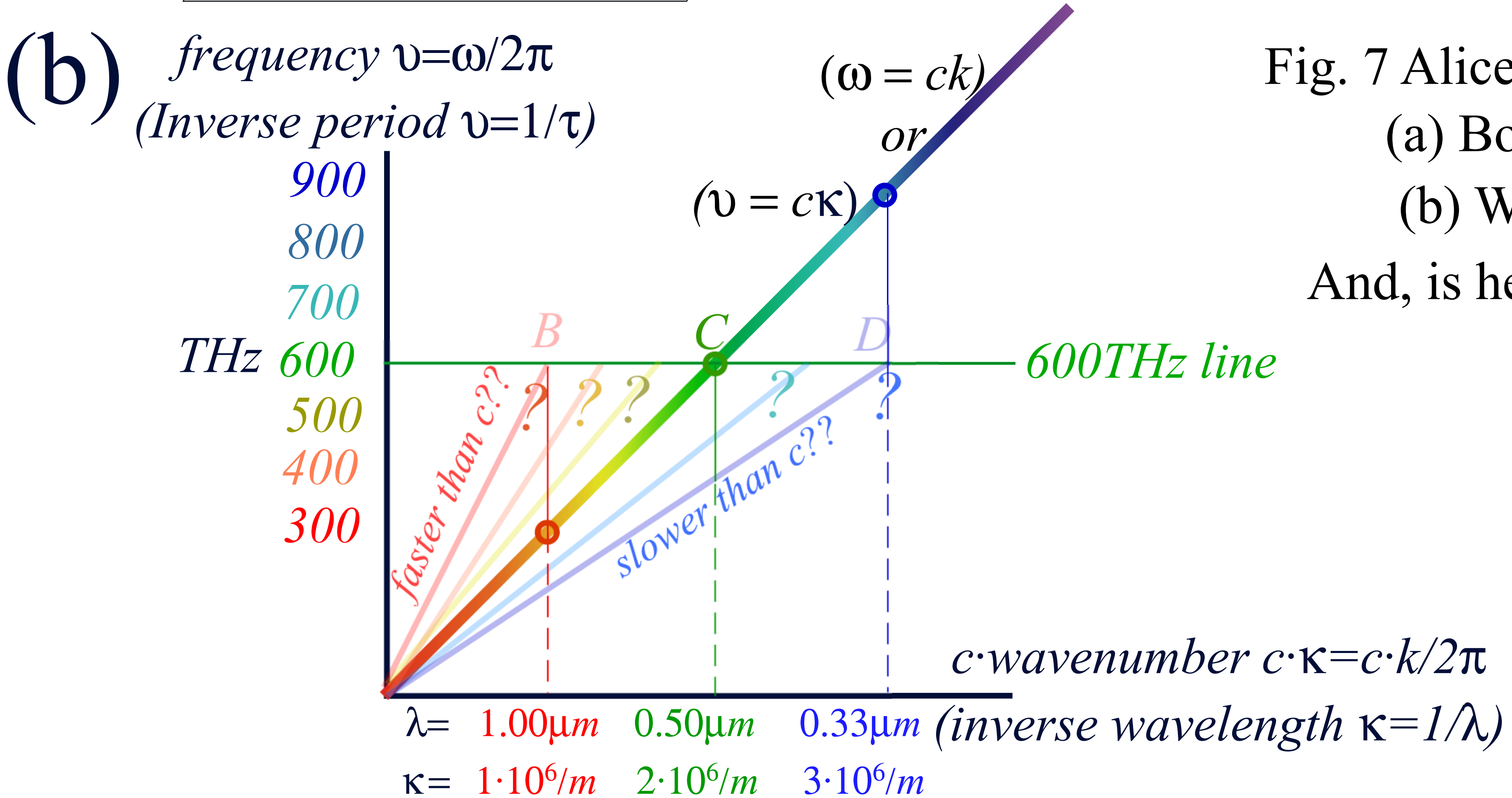
Introducing Doppler shifting and why c is constant





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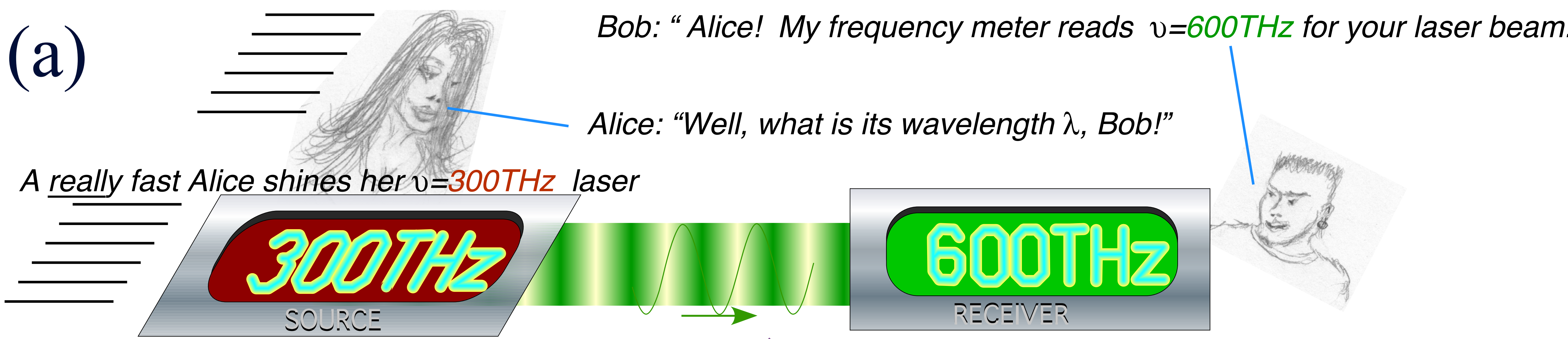
Actually: $2.99792458\cdot 10^{8}\mathrm{m\cdot s^{-1}}$

Introducing Doppler shifting and why c is constant



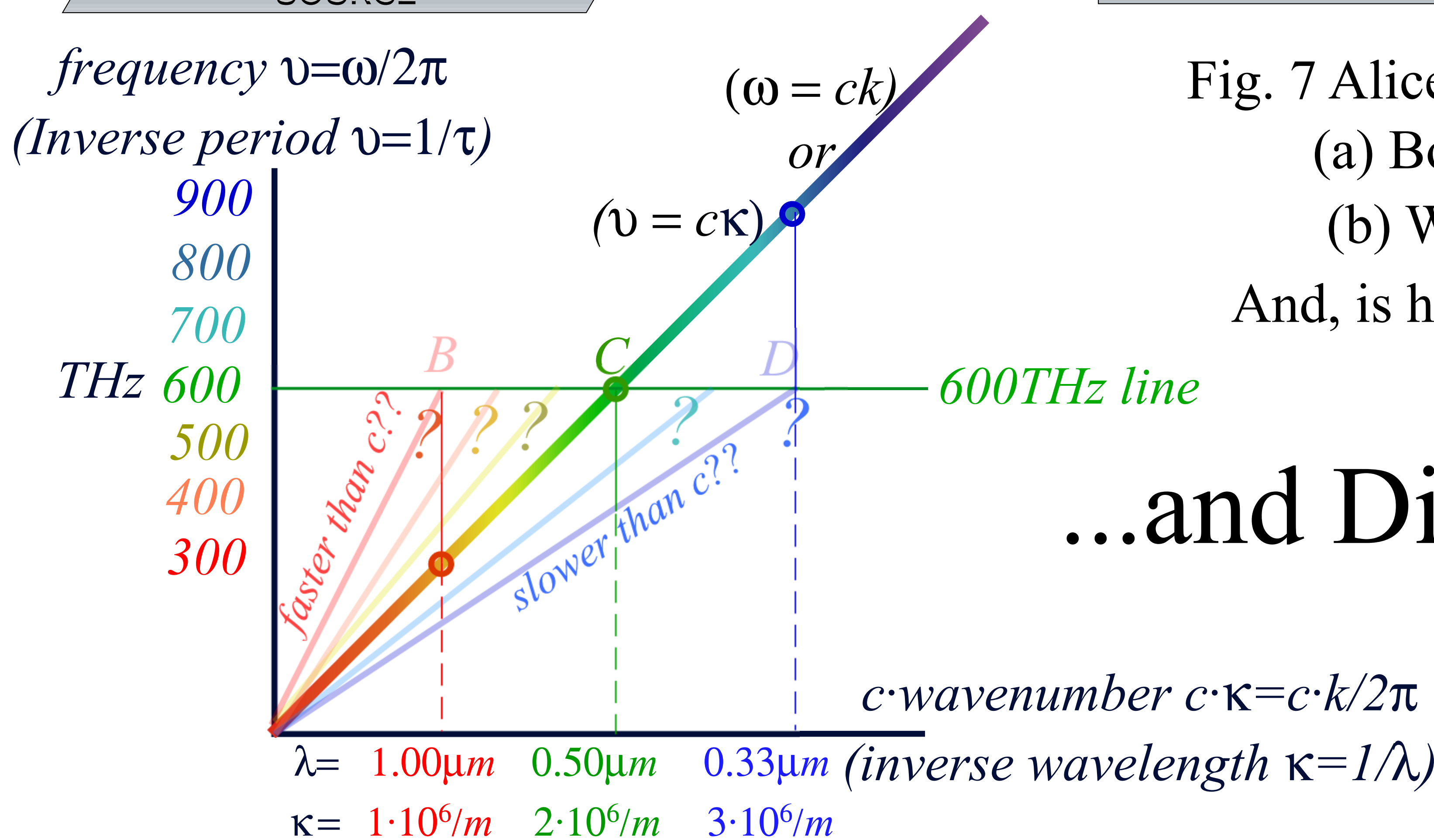


Fig. 7 Alice's 300THz laser approaches Bob.
(a) Bob sees υ=600THz.
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And, is he seeing a '*phony' green*?

...and Dispersion-Free!

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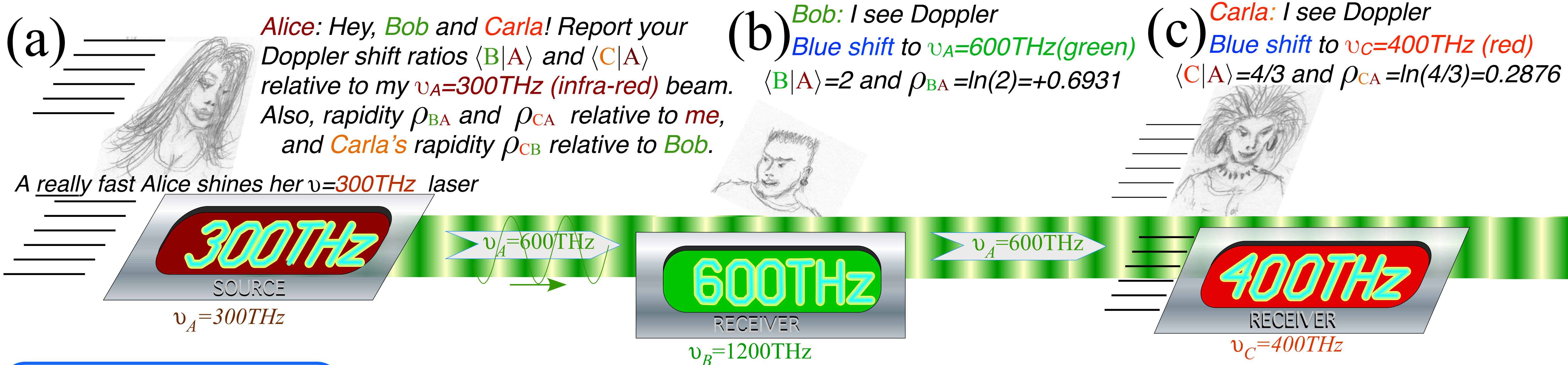


Doppler ratio:

$$\langle R|S\rangle = \frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}$$

$$\rho_{RS} = \ln\langle R|S\rangle$$

or:

$$\langle R|S\rangle = e^{\rho_{RS}} = e^{-\rho_{SR}}$$

Definition of Rapidity ρ_{RS}

Bob-Alice Doppler ratio:

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A} = \frac{600}{300} = \frac{2}{1}$$

Bob-Alice rapidity:

$$\rho_{BA} = \ln\langle B|A\rangle = \ln\frac{2}{1} = 0.6931$$

$$\rho_{AB} = \ln\langle A|B\rangle = \ln\frac{1}{2} = -0.6931 = -\rho_{BA}$$

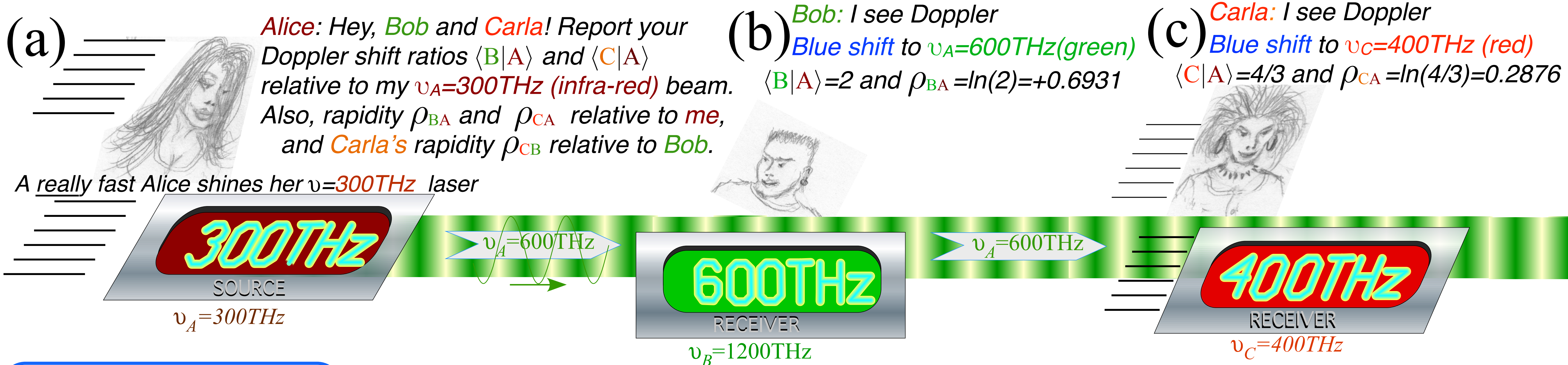
Carla-Alice Doppler ratio:

$$\langle C|A\rangle = \frac{\upsilon_C}{\upsilon_A} = \frac{400}{300} = \frac{4}{3}$$

Carla-Alice rapidity:

$$\rho_{CA} = \ln\langle C|A\rangle = \ln\frac{4}{3} = 0.2876$$

Introducing Doppler Arithmetic and rapidity ρ



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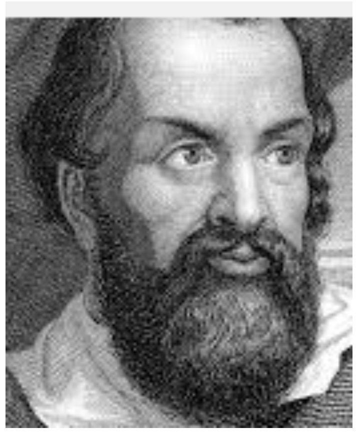
Galileo Galilei



1564-1642

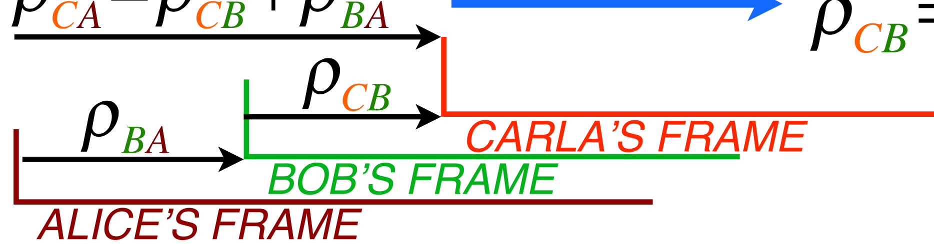
Carla-Bob Doppler ratio:

$$\langle C|B\rangle = \frac{\upsilon_C}{\upsilon_B} = \frac{\upsilon_C}{\upsilon_A}\frac{\upsilon_A}{\upsilon_B} = \langle C|A\rangle\langle A|B\rangle = \frac{4}{3}\frac{1}{2} = \frac{2}{3}$$

Galileo's Revenge (part 1)

Rapidity adds just like Galilean velocity

$$\rho_{CA} = \rho_{CB} + \rho_{BA}$$



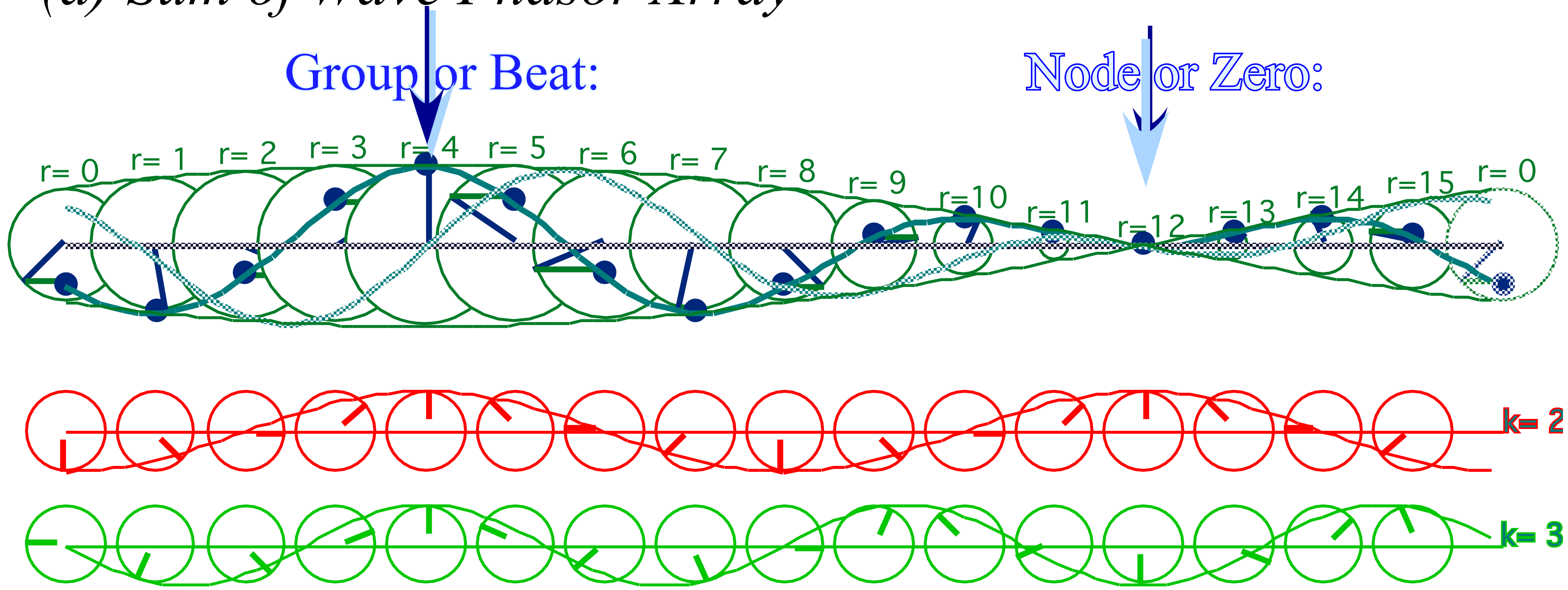
Carla-Bob rapidity:

$$e^{\rho_{CB}} = e^{\rho_{CA}}e^{\rho_{AB}} = e^{\rho_{CA}+\rho_{AB}}$$

$$\rho_{CB} = \rho_{CA} + \rho_{AB} = 0.2876 - 0.6931 = -0.4055$$

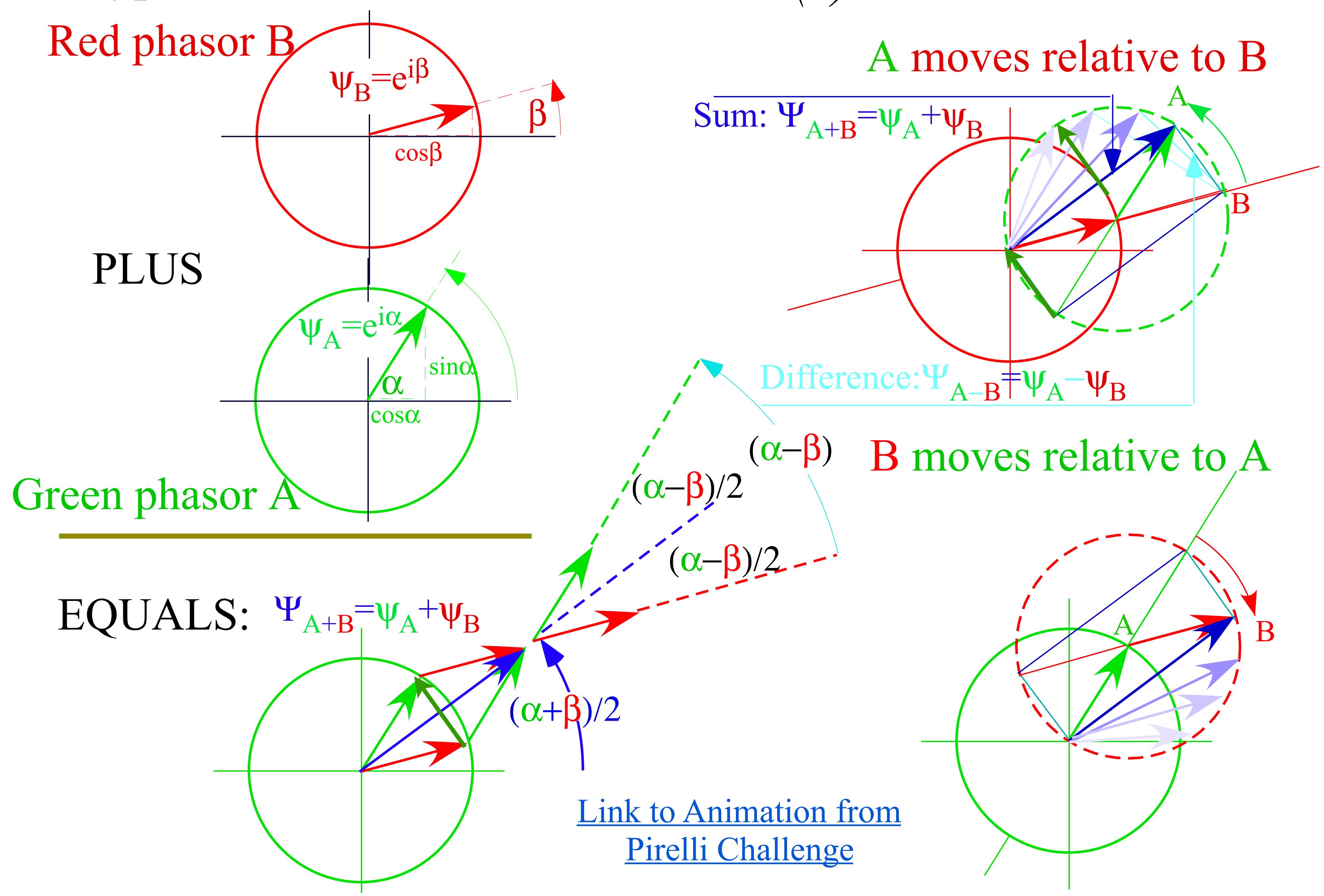
$$= \ln\frac{4}{3} + \ln\frac{1}{2} = \ln\frac{2}{3}$$

(a) Sum of Wave Phasor Array



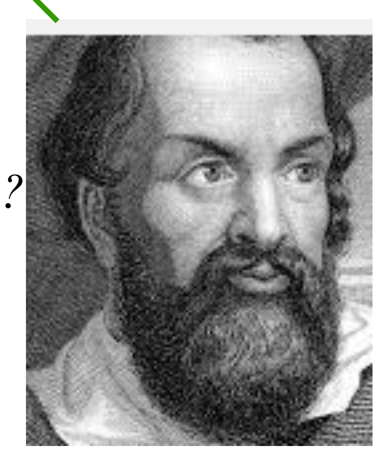
(b) Typical Phasor Sum:

(c) Phasor-relative views

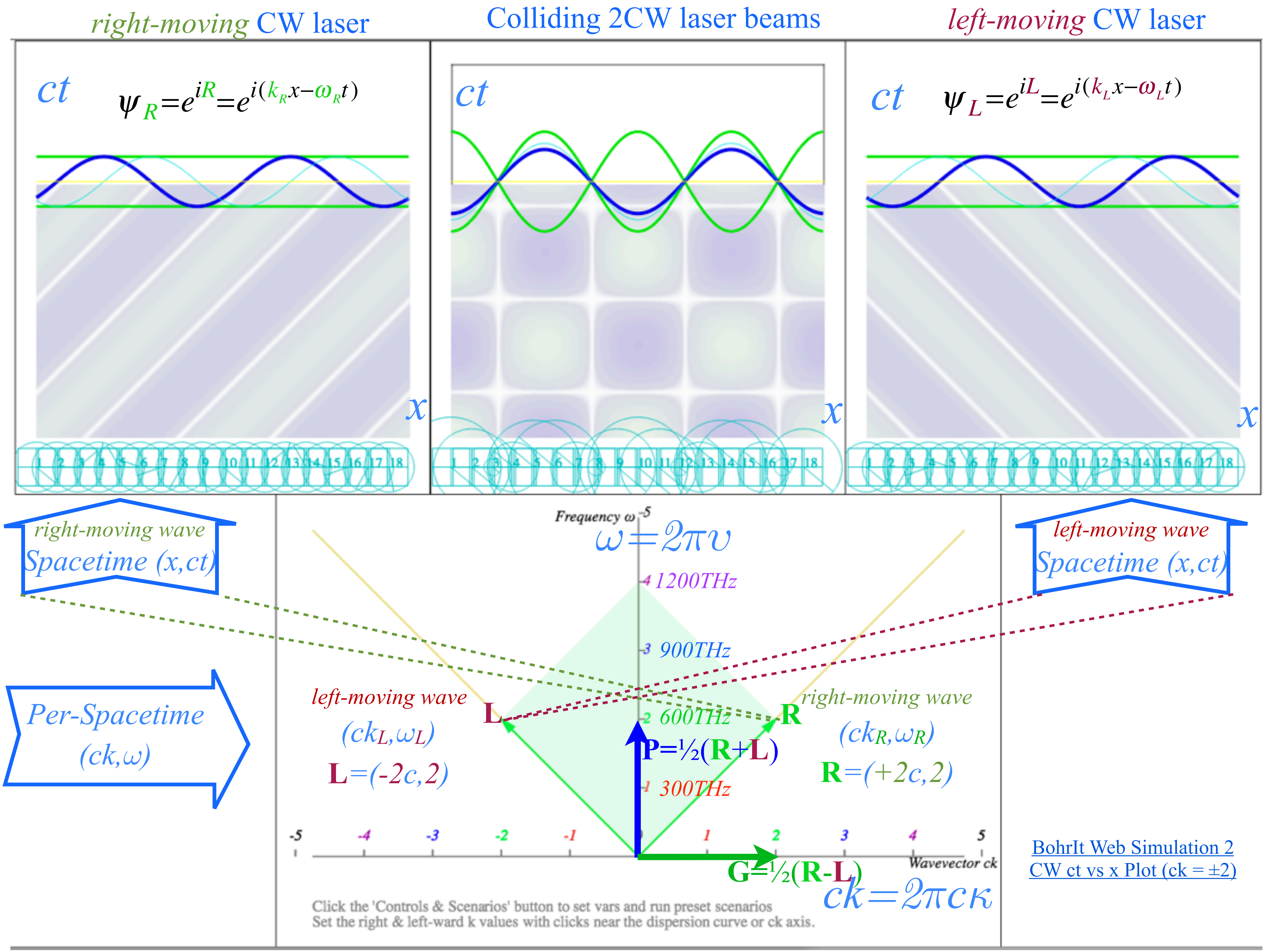


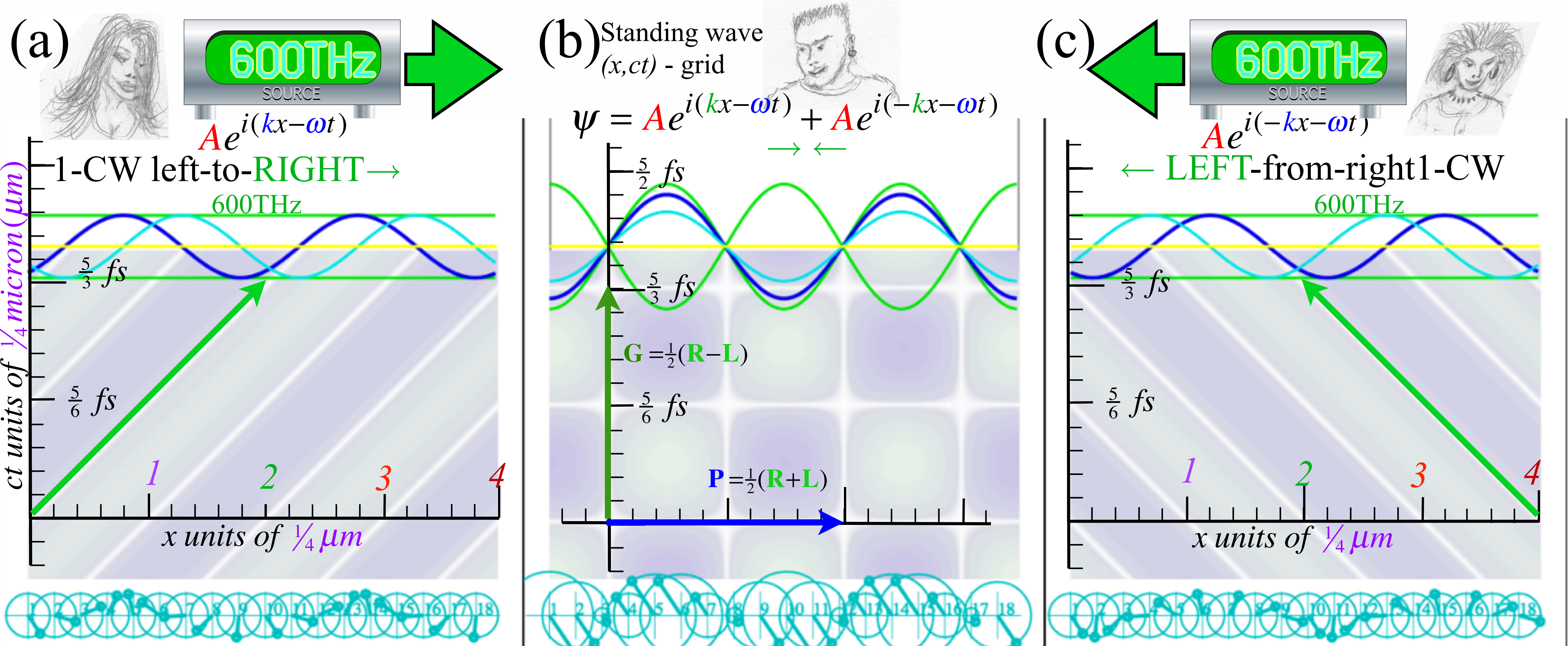
Geometry of the
Half-sum
Phase
and
Half-difference
Group

Happy now?

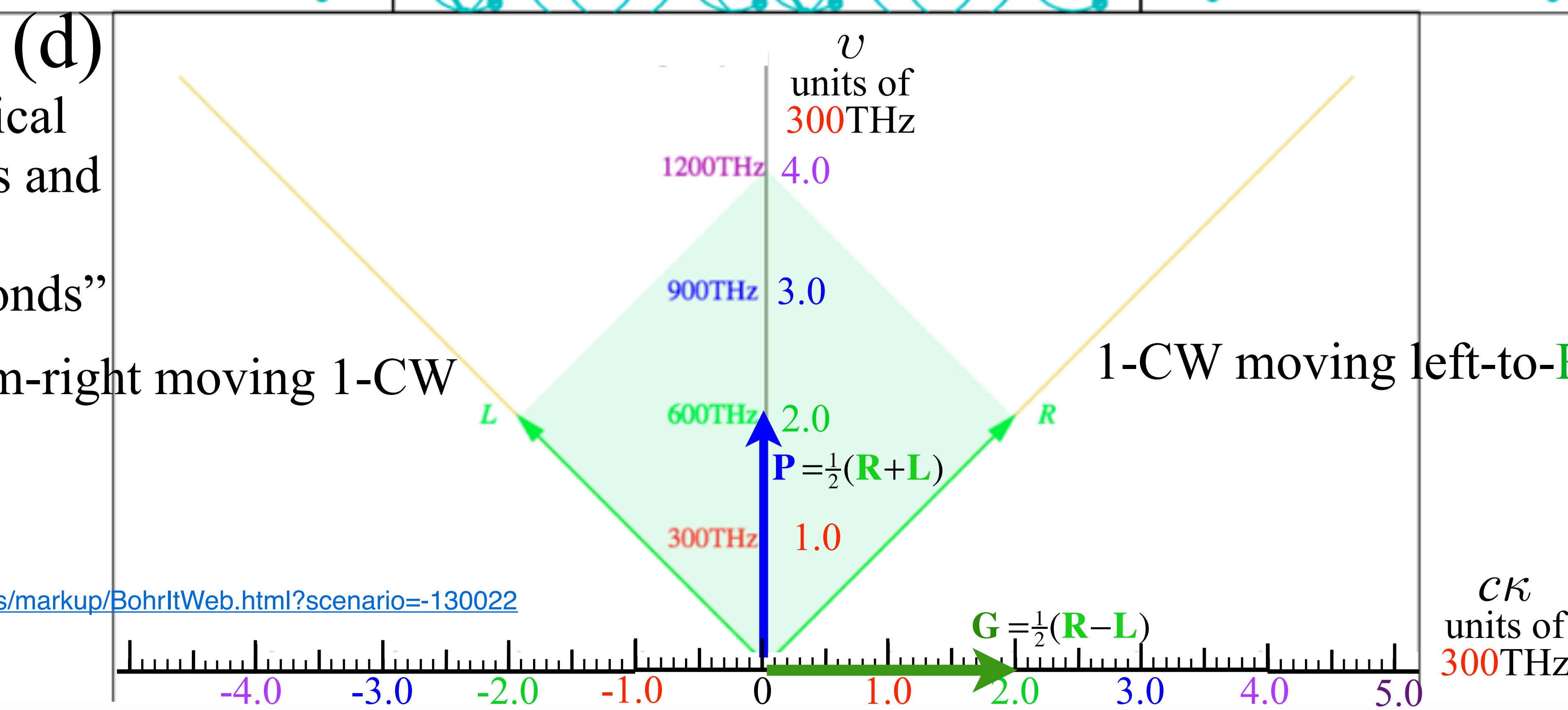


Galileo's Revenge (part 2)
Phasor angular velocity
adds just like
Galilean velocity


right-moving CW laser
Colliding 2CW laser beams
left-moving CW laser
ct
ψR=e^{iR}=e^{i(kRx−ωRt)}
ψL=e^{iL}=e^{i(kLx−ωLt)}
x
right-moving wave
Spacetime (x,ct)
left-moving wave
Spacetime (x,ct)
Per-Spacetime
(ck,ω)
Frequency ω
ω=2πυ
1200THz
900THz
600THz
300THz
left-moving wave
(ckL,ωL)
L=(-2c,2)
right-moving wave
(ckR,ωR)
R=(+2c,2)
P=½(R+L)
G=½(R-L)
Wavevector ck
ck=2πcκ
Click the 'Controls & Scenarios' button to set vars and run preset scenarios
Set the right & left-ward k values with clicks near the dispersion curve or ck axis.
BohrIt Web Simulation 2
CW ct vs x Plot (ck = ±2)

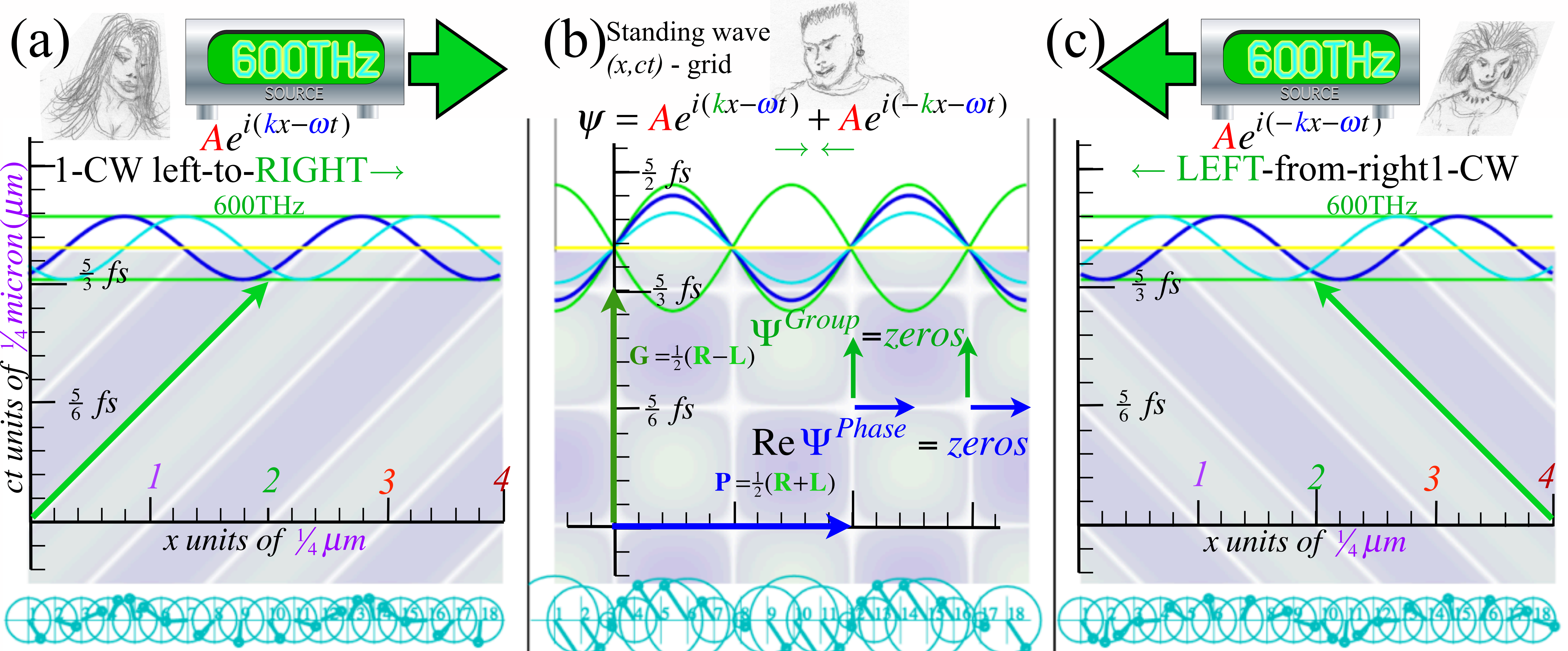


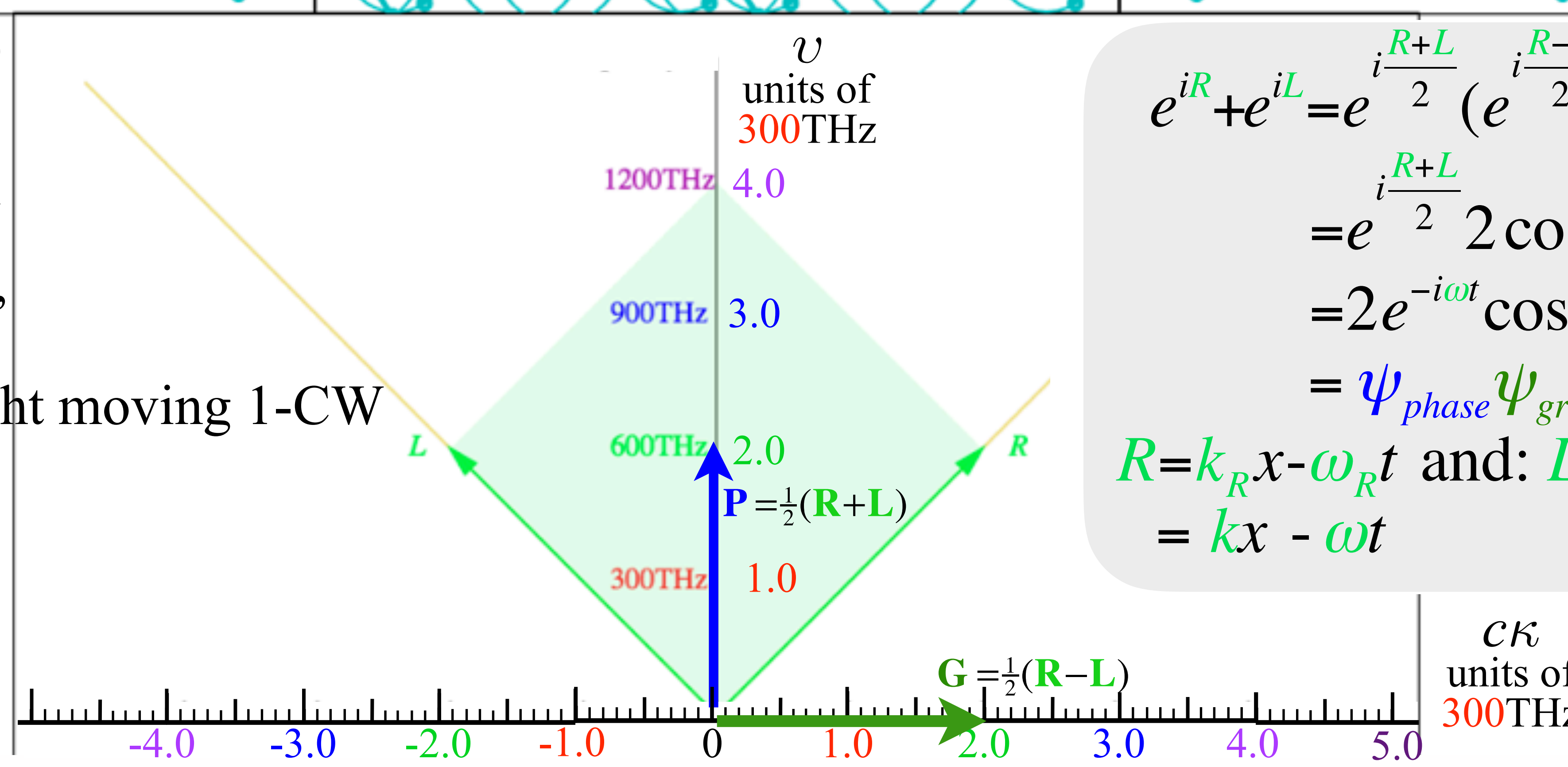
Introducing optical space-time grids and per-space-time "baseball-diamonds"



(ck = ±2)

www.uark.edu/ua/modphys/markup/BohrItWeb.html?scenario=-130022





Introducing optical space-time grids and per-space-time "baseball-diamonds"

← LEFT-from-right moving 1-CW

$$e^{iR}+e^{iL}=e^{i\frac{R+L}{2}}\left(e^{i\frac{R-L}{2}}+e^{-i\frac{R-L}{2}}\right)$$

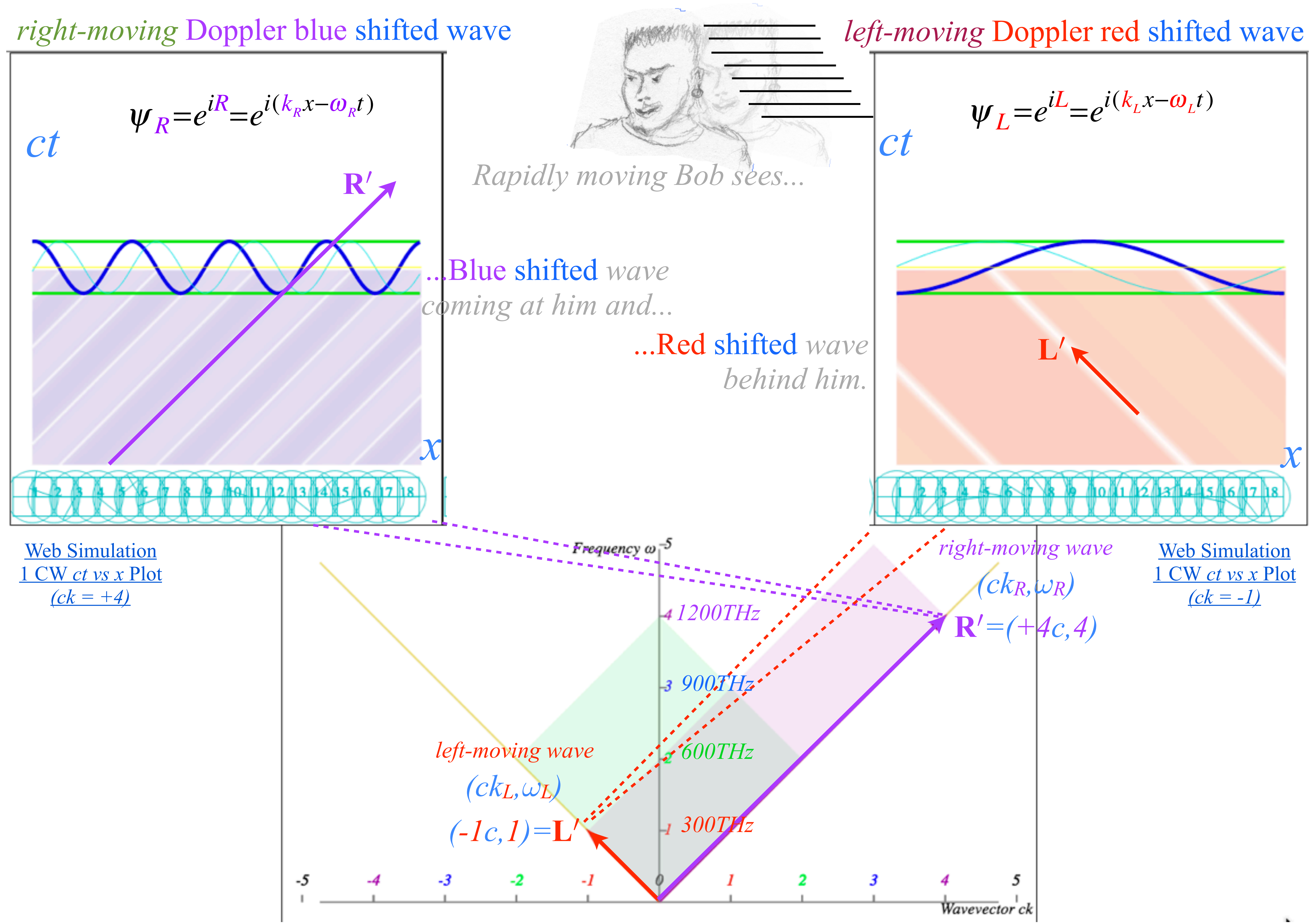
$$=e^{i\frac{R+L}{2}}\,2\cos\frac{R-L}{2}$$

$$=2e^{-i\omega t}\cos kx$$

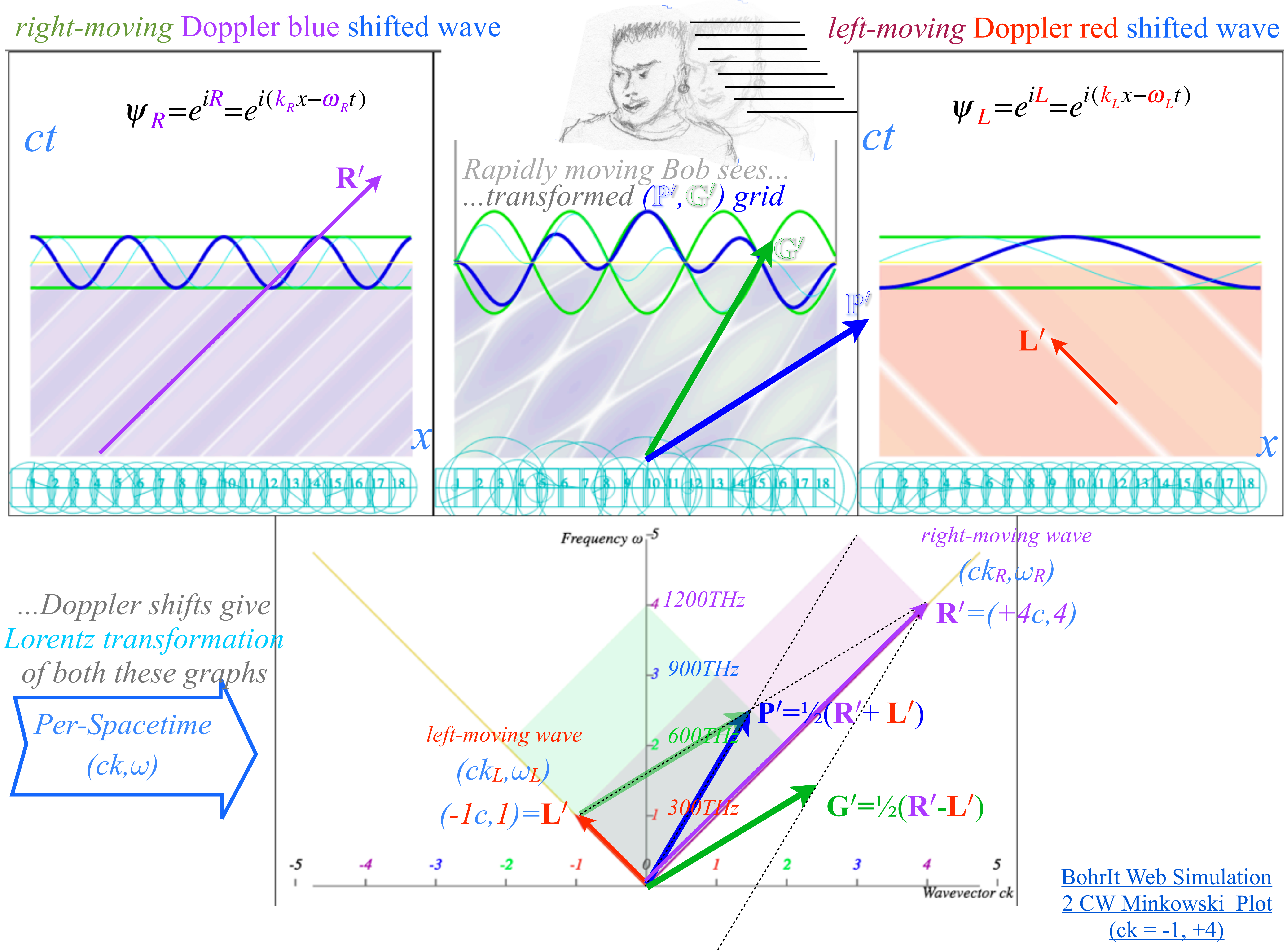
$$=\psi_{phase}\psi_{group}$$

$R=k_R x-\omega_R t$ and: $L=-k_L x-\omega_L t$

$= kx - \omega t$ $\quad\quad = -kx - \omega t$

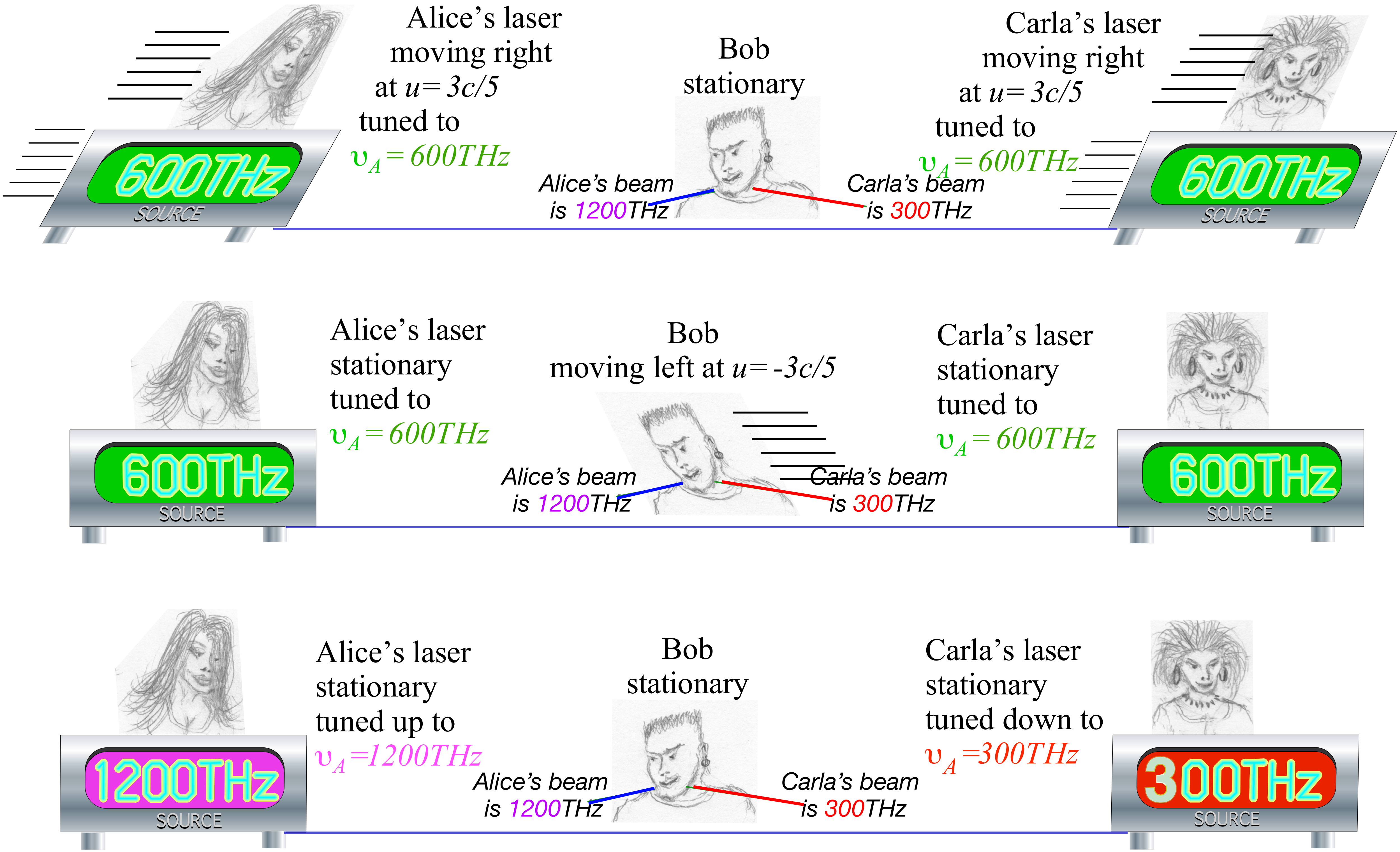


right-moving Doppler blue shifted wave
left-moving Doppler red shifted wave
$\psi_R = e^{iR} = e^{i(k_R x - \omega_R t)}$
$\psi_L = e^{iL} = e^{i(k_L x - \omega_L t)}$
ct
R′
x
ct
L′
x
Rapidly moving Bob sees...
...Blue shifted wave coming at him and...
...Red shifted wave behind him.
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
Web Simulation
1 CW ct vs x Plot
(ck = +4)
Web Simulation
1 CW ct vs x Plot
(ck = -1)
Frequency ω
right-moving wave
(ck_R, ω_R)
$R' = (+4c, 4)$
1200THz
900THz
600THz
300THz
left-moving wave
(ck_L, ω_L)
$(-1c, 1) = L'$
-5 -4 -3 -2 -1 0 1 2 3 4 5
Wavevector ck

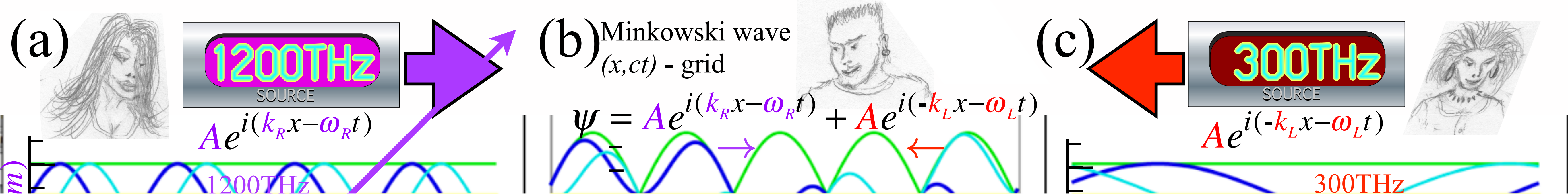


right-moving Doppler blue shifted wave
left-moving Doppler red shifted wave
$\psi_R = e^{iR} = e^{i(k_R x - \omega_R t)}$
$\psi_L = e^{iL} = e^{i(k_L x - \omega_L t)}$
ct
x
R′
L′
Rapidly moving Bob sees...
...transformed (P′,G′) grid
G′
P′
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
...Doppler shifts give
Lorentz transformation
of both these graphs
Per-Spacetime
(ck,ω)
Frequency ω
1200THz
900THz
600THz
300THz
right-moving wave
(ck_R, ω_R)
R′=(+4c,4)
P′=½(R′+ L′)
G′=½(R′-L′)
left-moving wave
(ck_L, ω_L)
(-1c,1)=L′
Wavevector ck
-5 -4 -3 -2 -1 0 1 2 3 4 5
BohrIt Web Simulation
2 CW Minkowski Plot
(ck = -1, +4)

Three scenarios that look the same to Bob



Much cheaper (and safer) to do the 3rd scenario!$!

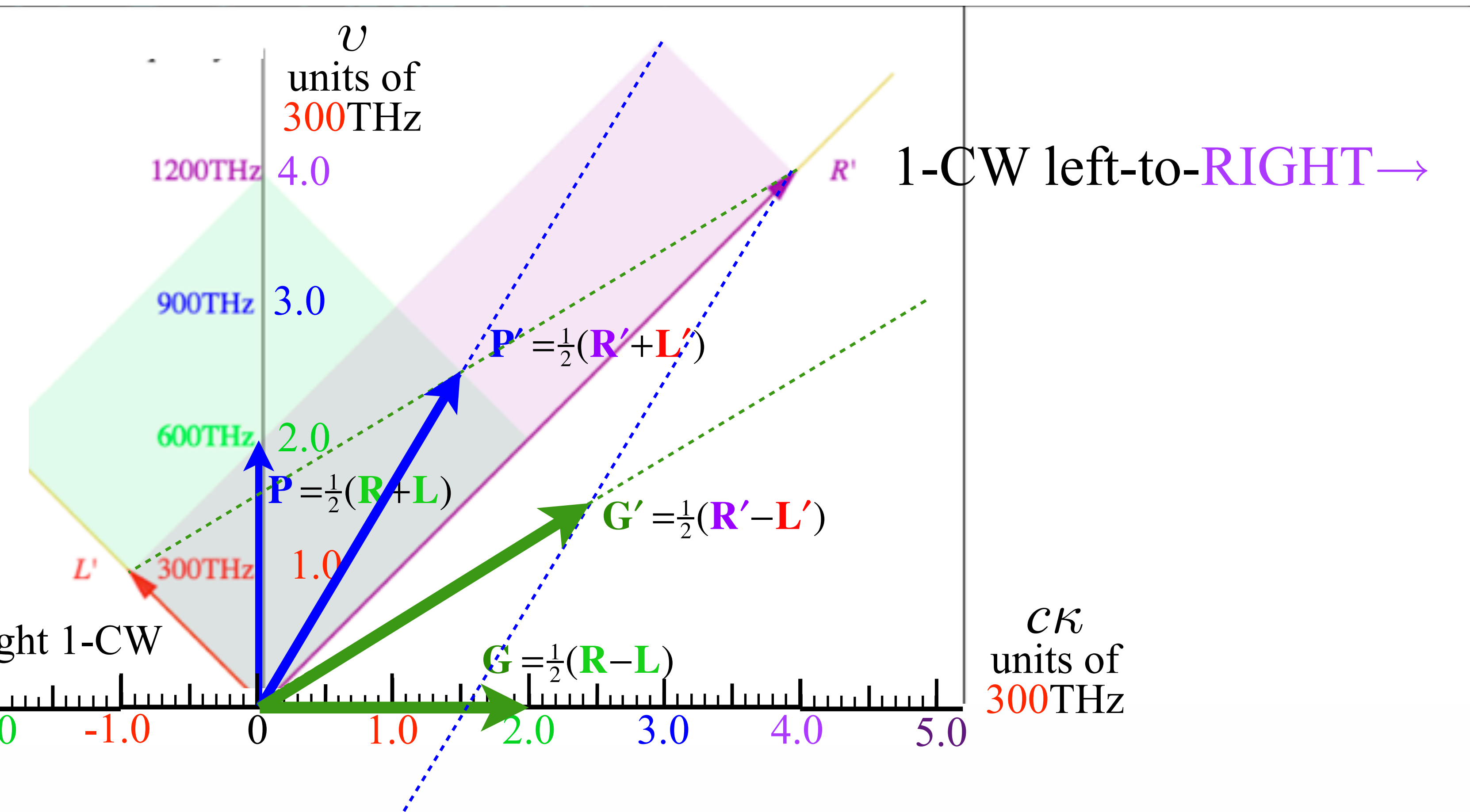


$$\mathbf{P}'=\begin{pmatrix}\upsilon'_{phase}\\ c\kappa'_{phase}\end{pmatrix}=\tfrac{1}{2}(\mathbf{R}'+\mathbf{L}')=\upsilon_A\begin{pmatrix}\tfrac{1}{2}(e^{\rho}+e^{-\rho})\\ \tfrac{1}{2}(e^{\rho}-e^{-\rho})\end{pmatrix}=\upsilon_A\begin{pmatrix}\cosh\rho\\ \sinh\rho\end{pmatrix}=\upsilon_A\begin{pmatrix}\frac{5}{2}\\ \frac{3}{2}\end{pmatrix}_{Bob's\ View}\text{ or: }\upsilon_A\begin{pmatrix}1\\ 0\end{pmatrix}_{Alice's\ View}$$

$$\mathbf{G}'=\begin{pmatrix}\upsilon'_{group}\\ c\kappa'_{group}\end{pmatrix}=\tfrac{1}{2}(\mathbf{R}'-\mathbf{L}')=\upsilon_A\begin{pmatrix}\tfrac{1}{2}(e^{\rho}-e^{-\rho})\\ \tfrac{1}{2}(e^{\rho}+e^{-\rho})\end{pmatrix}=\upsilon_A\begin{pmatrix}\sinh\rho\\ \cosh\rho\end{pmatrix}=\upsilon_A\begin{pmatrix}\frac{3}{2}\\ \frac{5}{2}\end{pmatrix}_{Bob's\ View}\text{ or: }\upsilon_A\begin{pmatrix}0\\ 1\end{pmatrix}_{Alice's\ View}$$

(d)

$$e^{iR'}+e^{iL'}=e^{i\frac{R'+L'}{2}}\left(e^{i\frac{R'-L'}{2}}+e^{-i\frac{R'-L'}{2}}\right)$$
$$=e^{i\frac{R'+L'}{2}}\,2\cos\frac{R'-L'}{2}$$
$$=\psi'_{phase}\psi'_{group}$$
$$R'=k_R x-\omega_R t \text{ and: } L'=-k_L x-\omega_L t$$



Doppler Jeopardy

(1.) To what velocity u_E must Bob accelerate so he sees beams with equal frequency ω_E?

(2.) What is that frequency ω_E ?

Doppler Jeopardy

(1.) To what velocity u_E must Bob accelerate so he sees beams with equal frequency ω_E?

(2.) What is that frequency ω_E ?

Query (1.) has a Jeopardy-style answer-by-question: What is beam group velocity?

$$u_E = V_{group} = \frac{\omega_{group}}{k_{group}} = \frac{\omega_R - \omega_L}{k_R - k_L} = c\frac{\omega_R - \omega_L}{\omega_R + \omega_L}$$

Doppler Jeopardy

(1.) To what velocity u_E must Bob accelerate so he sees beams with equal frequency ω_E?

(2.) What is that frequency ω_E ?

Query (1.) has a Jeopardy-style answer-by-question: What is beam group velocity? $\frac{300}{900}$

$$u_E = V_{group} = \frac{\omega_{group}}{k_{group}} = \frac{\omega_R - \omega_L}{k_R - k_L} = c\frac{\omega_R - \omega_L}{\omega_R + \omega_L} \qquad V_{group} = c\frac{\omega_R - \omega_L}{\omega_R + \omega_L} = c\frac{600 - 300}{600 + 300} = \frac{1}{3}c$$

Doppler Jeopardy

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Query (2.) similarly: What ω_E is blue-shift $b\omega_L$ of ω_L *and* red-shift ω_R/b of ω_R ?

$$\omega_E = b\,\omega_L = \omega_R/b \quad \Rightarrow \quad b = \sqrt{\omega_R / \omega_L} \quad \Rightarrow \quad \omega_E = \sqrt{\omega_R \cdot \omega_L}$$

Doppler Jeopardy

(1.) To what velocity u_E must Bob accelerate so he sees beams with equal frequency ω_E?

(2.) What is that frequency ω_E ?

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$\frac{300}{900}$

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$\sqrt{6 \cdot 3} = 3\sqrt{2} = 4.24$

$$\omega_E = b\,\omega_L = \omega_R/b \quad \Rightarrow \quad b = \sqrt{\omega_R / \omega_L} \quad \Rightarrow \quad \omega_E = \sqrt{\omega_R \cdot \omega_L}$$

Geometric mean

$$\omega_E = \sqrt{\omega_R \cdot \omega_L} = \sqrt{180000} = 424$$

Doppler Jeopardy

(1.) To what velocity u_E must Bob accelerate so he sees beams with equal frequency ω_E?

(2.) What is that frequency ω_E ?

Query (1.) has a Jeopardy-style answer-by-question: What is beam group velocity?

$\frac{300}{900}$

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Query (2.) similarly: What ω_E is blue-shift $b\omega_L$ of ω_L *and* red-shift ω_R/b of ω_R?

$\sqrt{6 \cdot 3} = 3\sqrt{2} = 4.24$

$$\omega_E = b\,\omega_L = \omega_R/b \quad \Rightarrow \quad b = \sqrt{\omega_R/\omega_L} \quad \Rightarrow \quad \omega_E = \sqrt{\omega_R \cdot \omega_L}$$

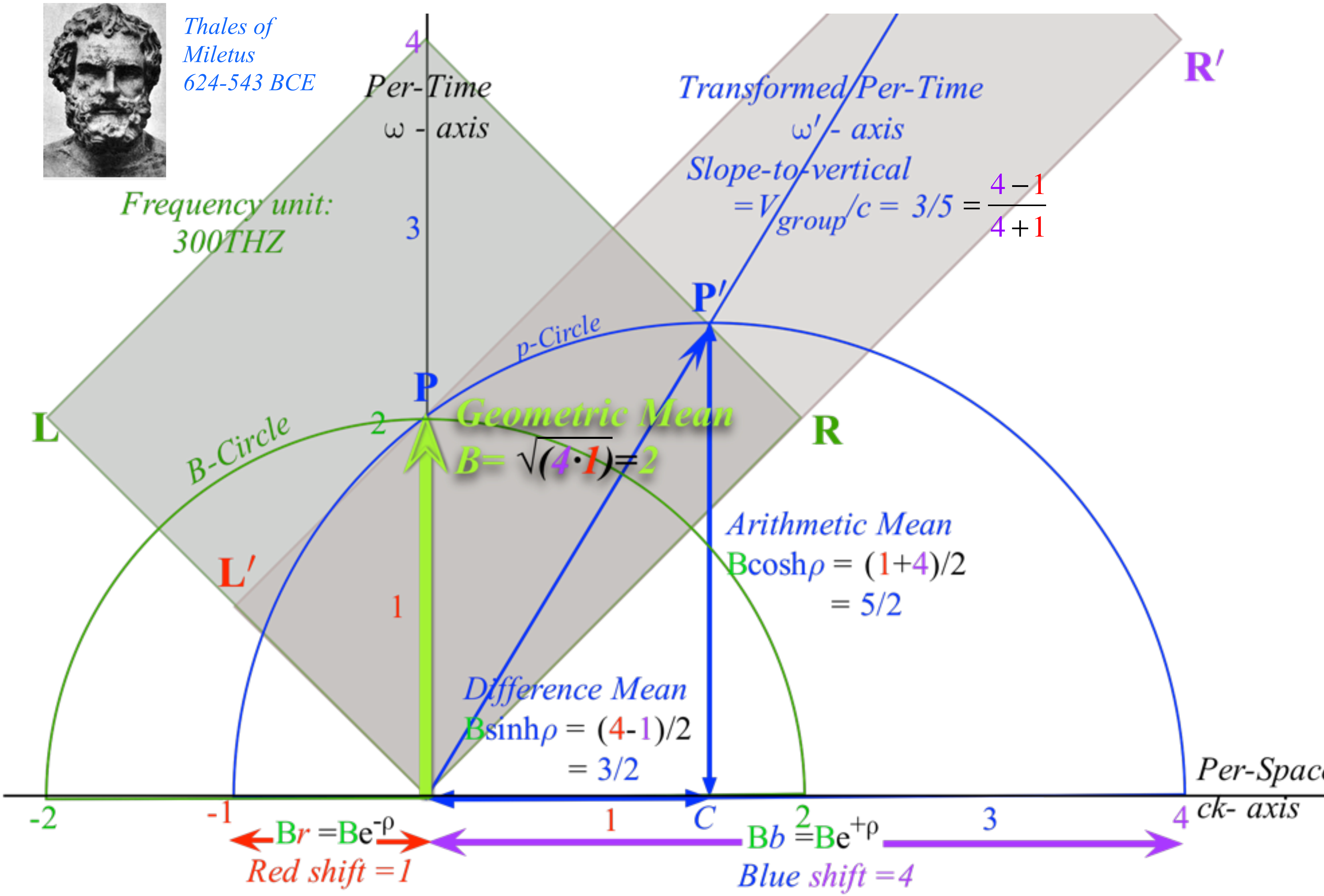
$$\omega_E = \sqrt{\omega_R \cdot \omega_L} = \sqrt{180000} = 424$$

V_{group}/c is ratio of difference mean $\omega_{group} = \frac{\omega_R - \omega_L}{2}$ to arithmetic mean $\omega_{phase} = \frac{\omega_R + \omega_L}{2}$. Frequency $\omega_E = B$ is the geometric mean $\sqrt{\omega_R \cdot \omega_L}$ of left and right-moving frequencies defining the geometry

Geometric mean

Thales Mean Geometry (600BCE)

helps “Relawavity”

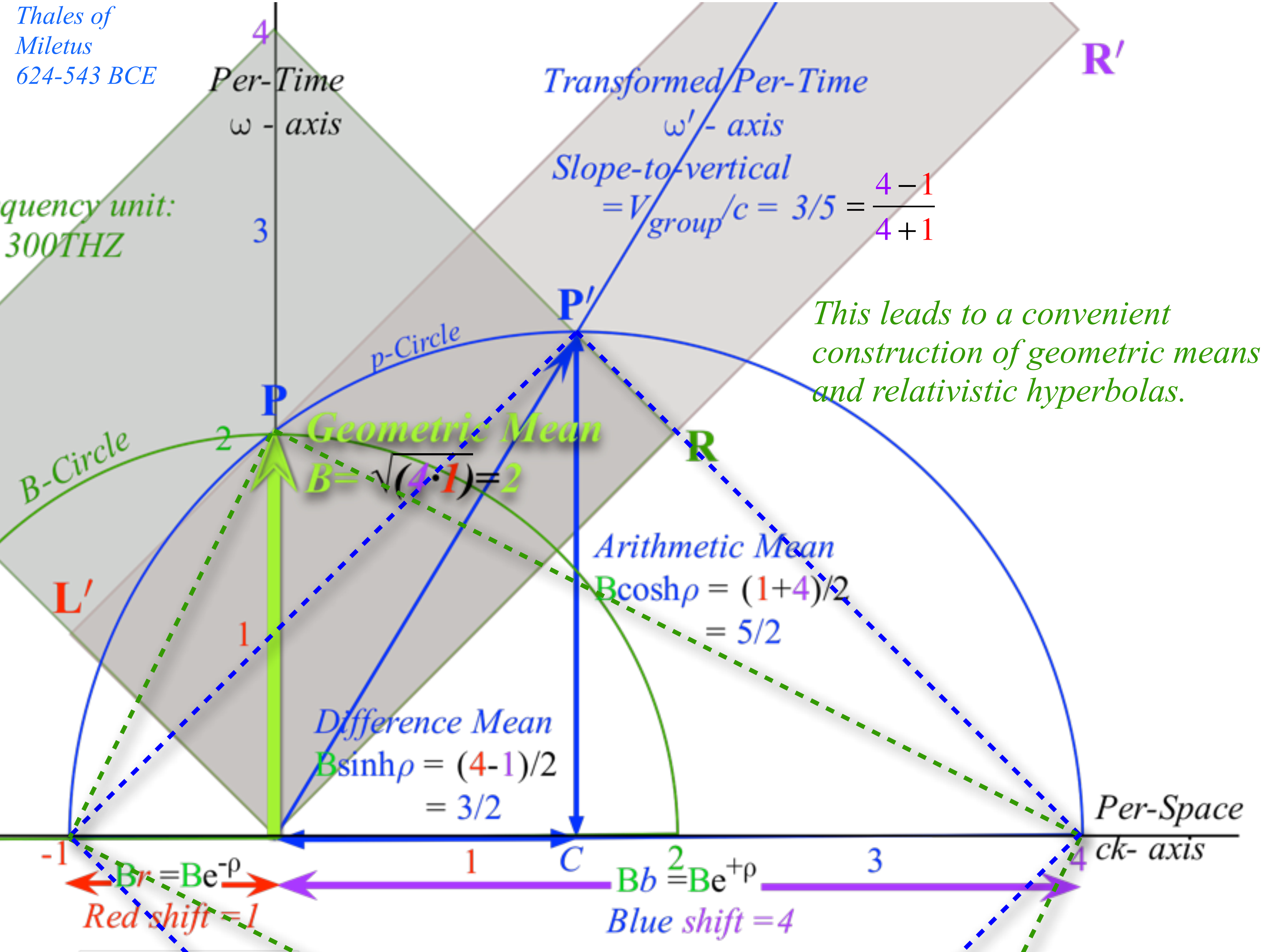


Thales Mean Geometry (600BCE)

helps "Relawavity" *Thales showed a circle diameter subtends a right angle with any circle point P*

Thales of Miletus 624-543 BCE

This leads to a convenient construction of geometric means and relativistic hyperbolas.



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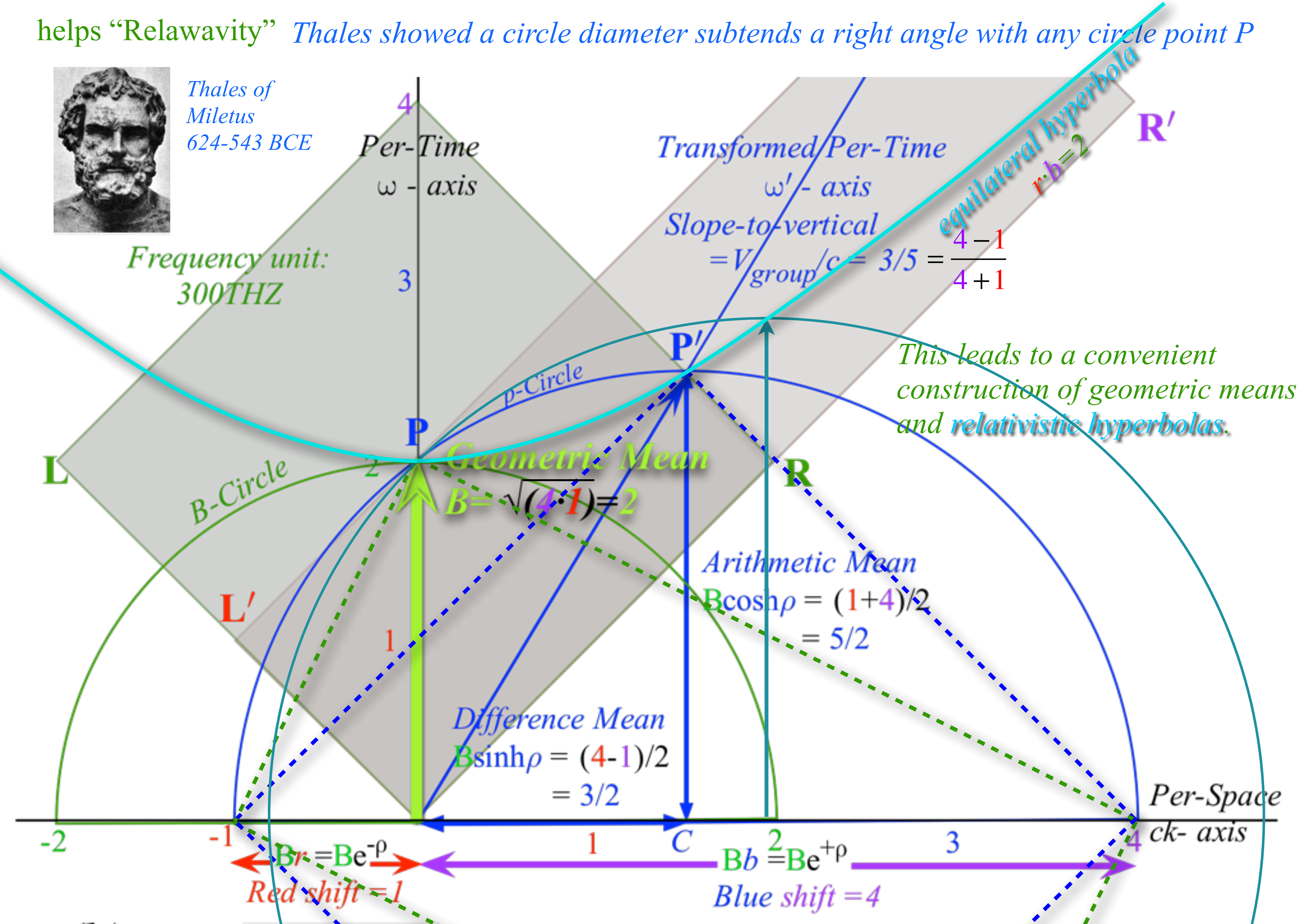

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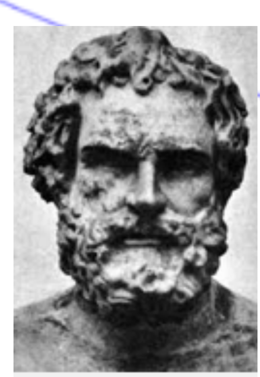
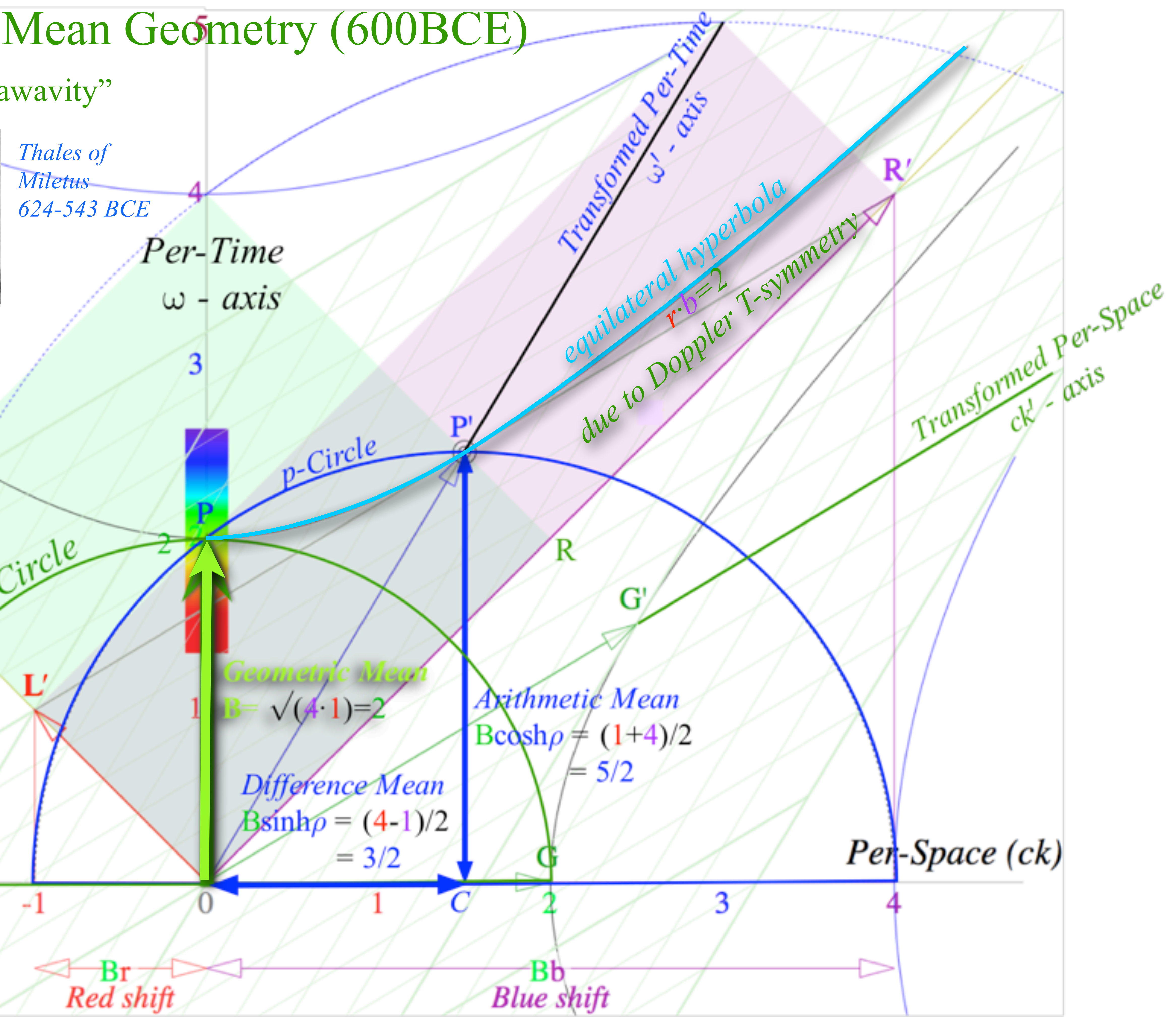
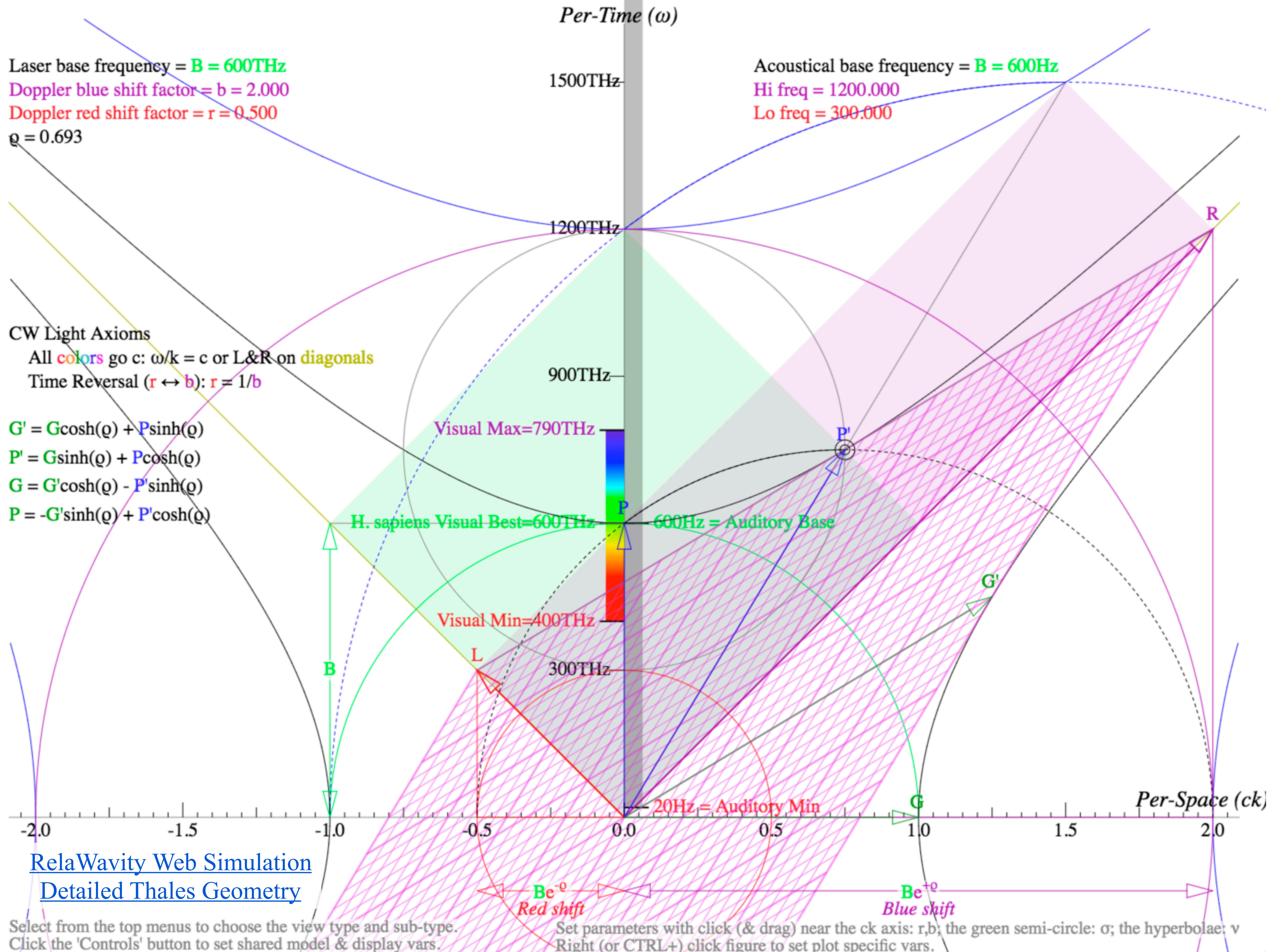
Thales Mean Geometry (600BCE)

helps “Relawavity”



Thales of Miletus 624-543 BCE




Per-Time (ω)
Laser base frequency = B = 600THz
Doppler blue shift factor = b = 2.000
Doppler red shift factor = r = 0.500
ϱ = 0.693
Acoustical base frequency = B = 600Hz
Hi freq = 1200.000
Lo freq = 300.000
1500THz
1200THz
900THz
300THz
CW Light Axioms
All colors go c: ω/k = c or L&R on diagonals
Time Reversal (r ↔ b): r = 1/b
G' = Gcosh(ϱ) + Psinh(ϱ)
P' = Gsinh(ϱ) + Pcosh(ϱ)
G = G'cosh(ϱ) - P'sinh(ϱ)
P = -G'sinh(ϱ) + P'cosh(ϱ)
Visual Max=790THz
H. sapiens Visual Best=600THz
600Hz = Auditory Base
Visual Min=400THz
20Hz = Auditory Min
R
P'
P
G'
G
L
B
Per-Space (ck)
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
RelaWavity Web Simulation
Detailed Thales Geometry
Be-ϱ
Red shift
Be+ϱ
Blue shift
Select from the top menus to choose the view type and sub-type.
Click the 'Controls' button to set shared model & display vars.
Set parameters with click (& drag) near the ck axis: r,b; the green semi-circle: σ; the hyperbolae: v
Right (or CTRL+) click figure to set plot specific vars.

Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

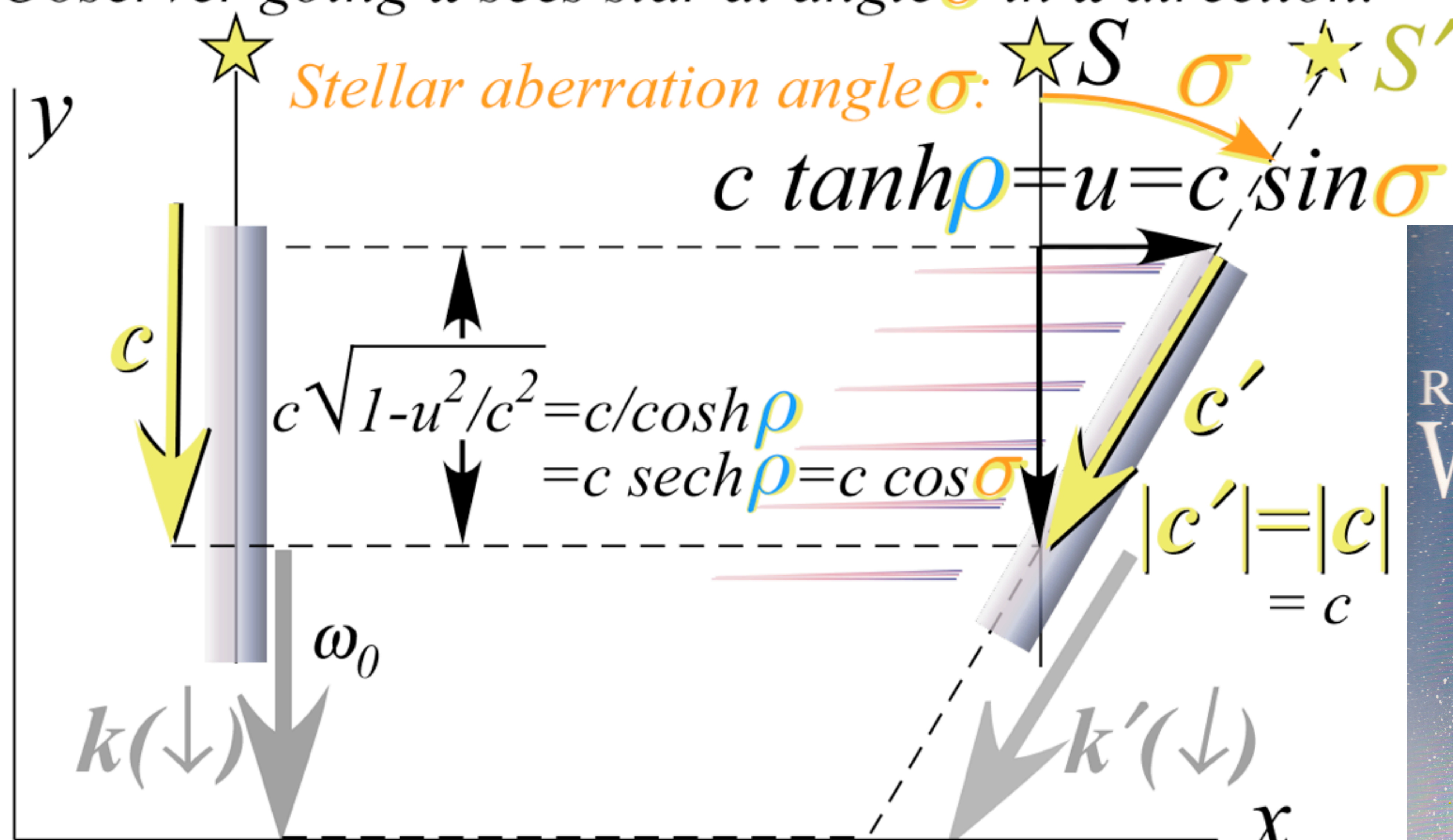
to a Transverse*relativity parameter: Stellar aberration angle σ

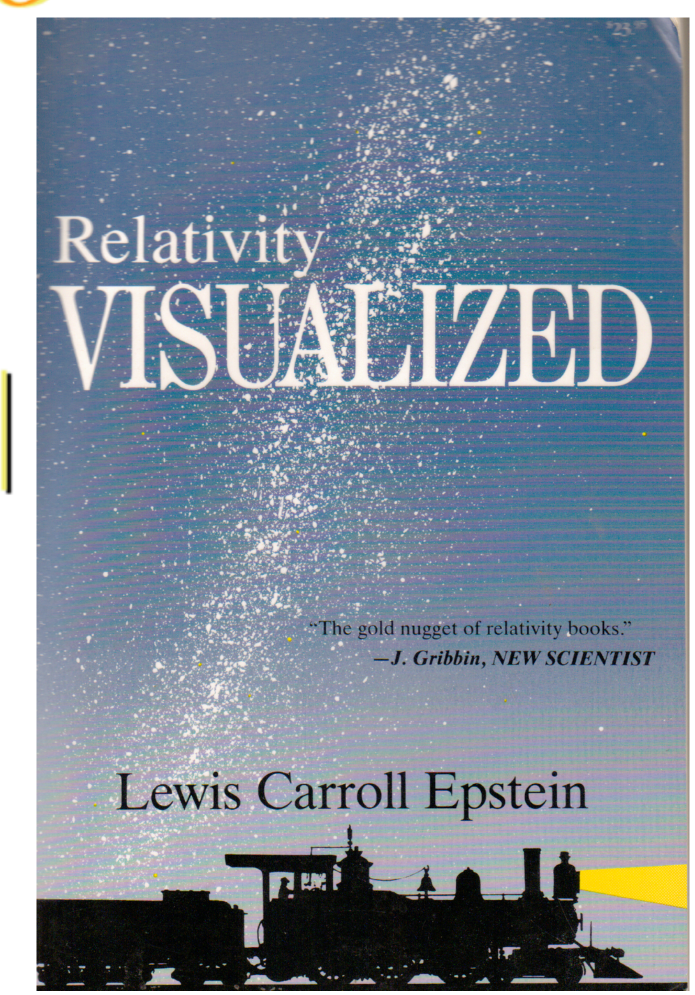
*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-

Observer fixed below star sees it directly overhead.

Observer going ***u*** *sees star at angle* σ *in* ***u*** *direction.*

We used notion σ for stellar-ab-angle, (a "flipped-out" ρ). Epstein not interested in ρ analysis or in relation of σ and ρ.





Purchase at:

Comparing Longitudinal relativity parameter: Rapidity ρ = log$_e$(Doppler Shift)

to a Transverse*relativity parameter: Stellar aberration angle σ

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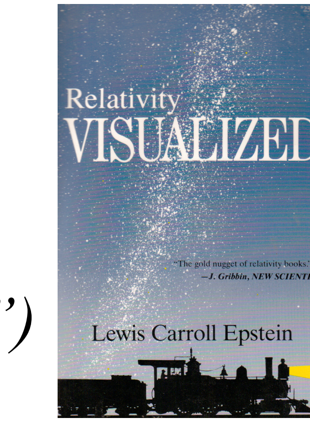
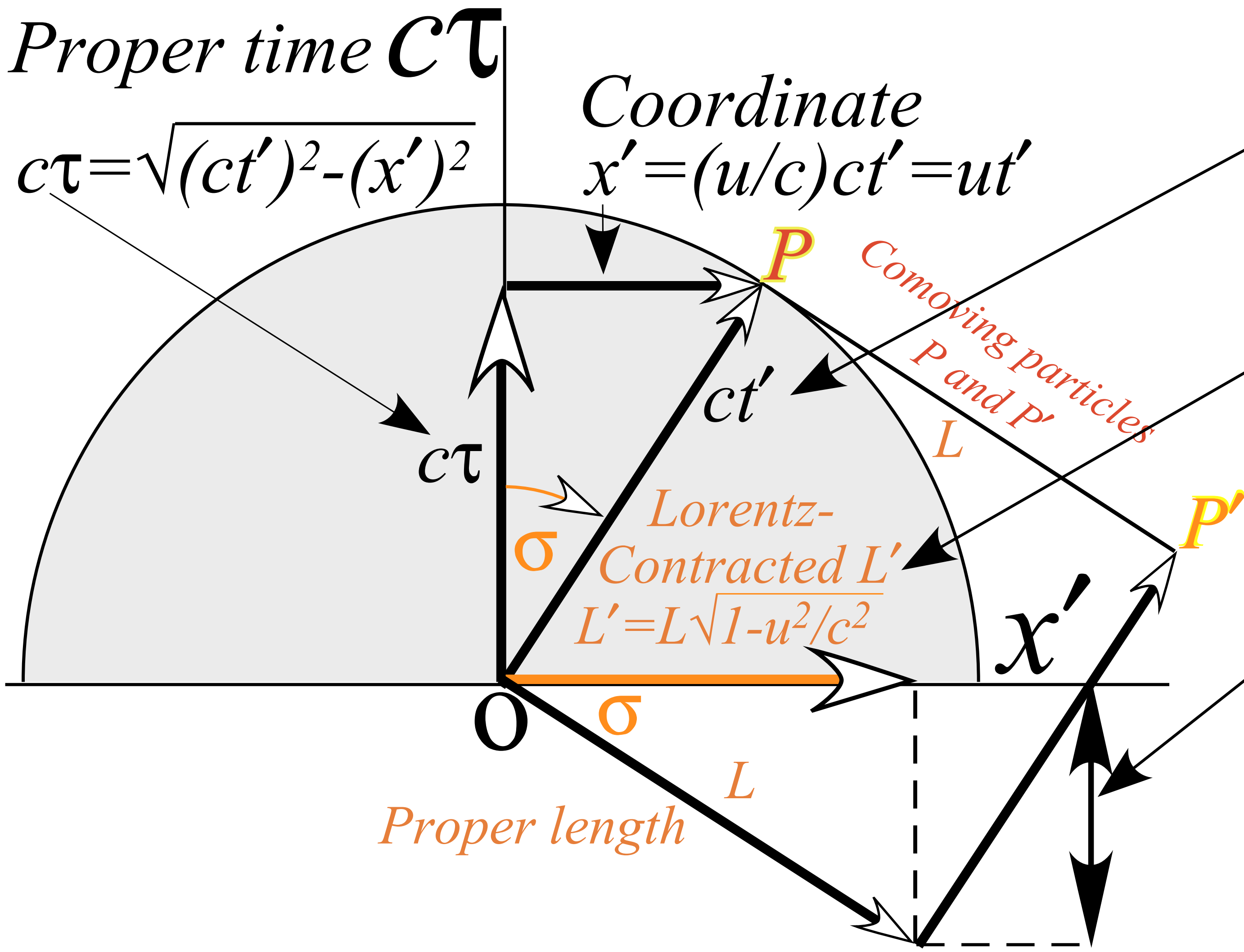


Proper time cτ vs. coordinate space x - (L. C. Epstein's "Cosmic Speedometer")

Particles P and P′ have speed u in (x′,ct′) and speed c in (x, cτ)



Einstein time dilation:

$$ct' = c\tau \sec\sigma = c\tau \cosh\rho = c\tau/\sqrt{1-u^2/c^2}$$

Lorentz length contraction:

$$L' = L\,\mathrm{sech}\rho = L\cos\sigma = L\cdot\sqrt{1-u^2/c^2}$$

Proper Time asimultaneity:

$$c\,\Delta\tau = L'\sinh\rho = L\cos\sigma\sinh\rho$$

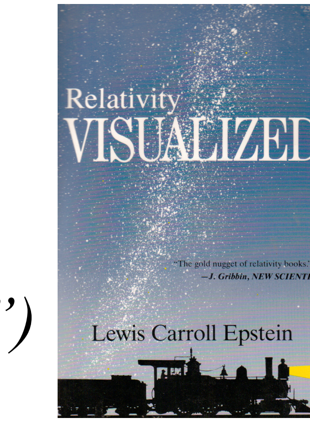
$$= L\cos\sigma\tan\sigma$$

$$= L\sin\sigma \quad = L/\sqrt{c^2/u^2-1} \sim L\,u/c$$

Comparing Longitudinal relativity parameter: Rapidity $\rho = \log_e$(Doppler Shift)

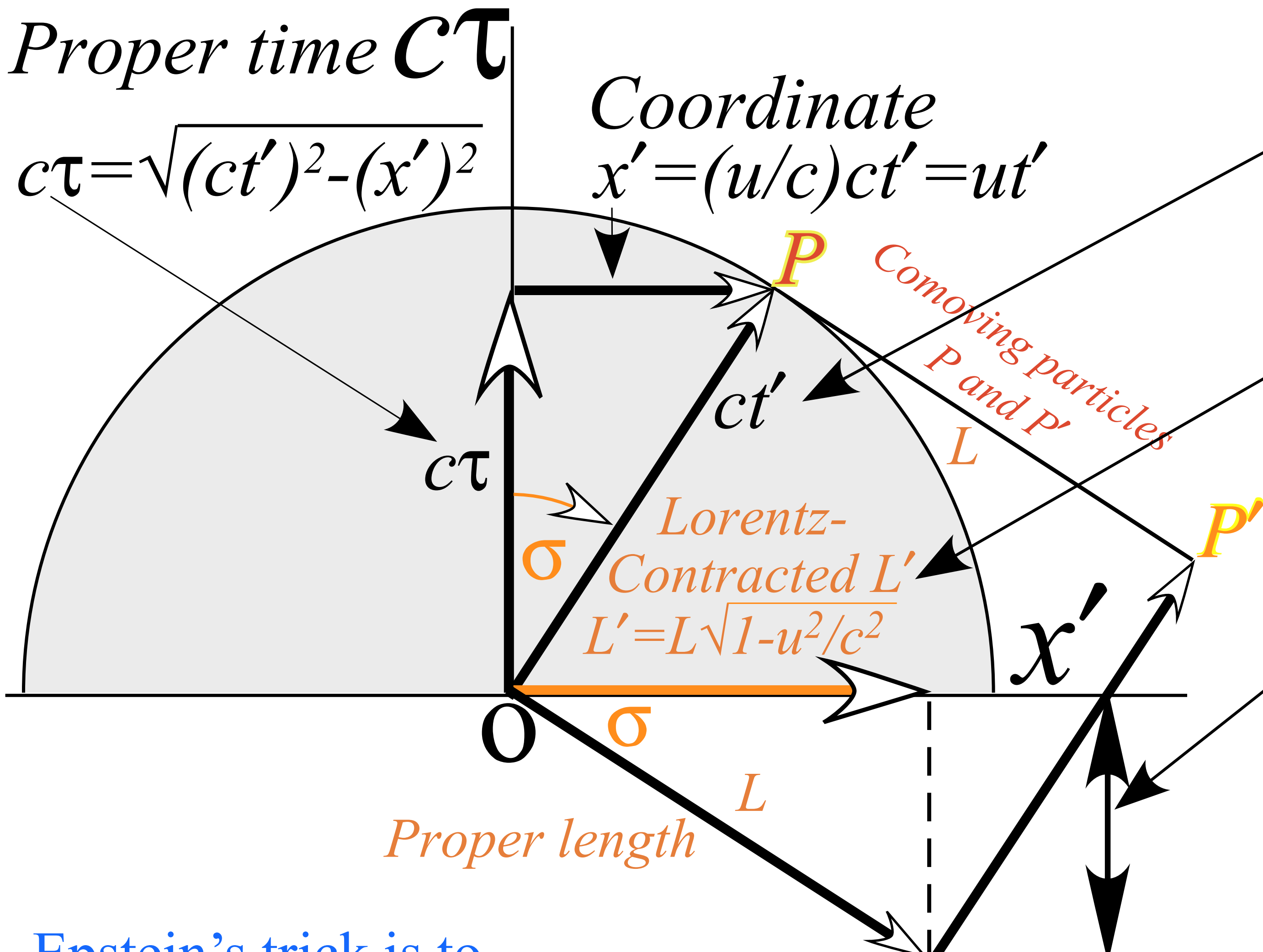
to a Transverse*relativity parameter: Stellar aberration angle σ

*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-



Proper time $c\tau$ vs. coordinate space x - (L. C. Epstein's "Cosmic Speedometer")

Particles P and P′ have speed u in (x',ct') and speed c in $(x, c\tau)$



Einstein time dilation:

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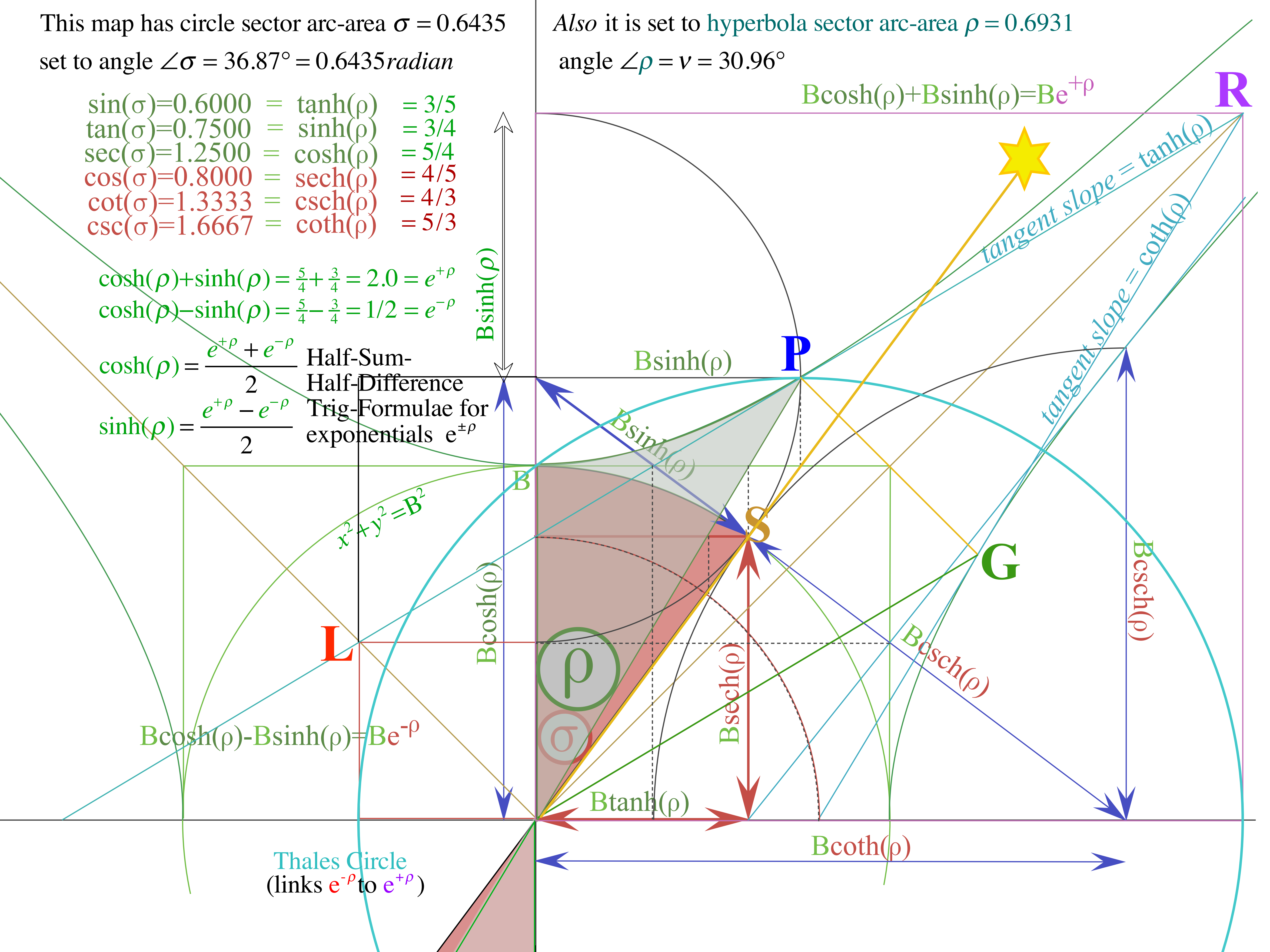
Proper Time asimultaneity:

$$c\,\Delta\tau = L'\sinh\rho = L\cos\sigma\sinh\rho$$
$$= L\cos\sigma\tan\sigma$$
$$= L\sin\sigma \quad = L/\sqrt{c^2/u^2-1} \sim L\,u/c$$

Epstein's trick is to turn a hyperbolic form $c\tau=\sqrt{(ct')^2-(x')^2}$ into a circular form:

$$\sqrt{(c\tau)^2+(x')^2}=(ct')$$

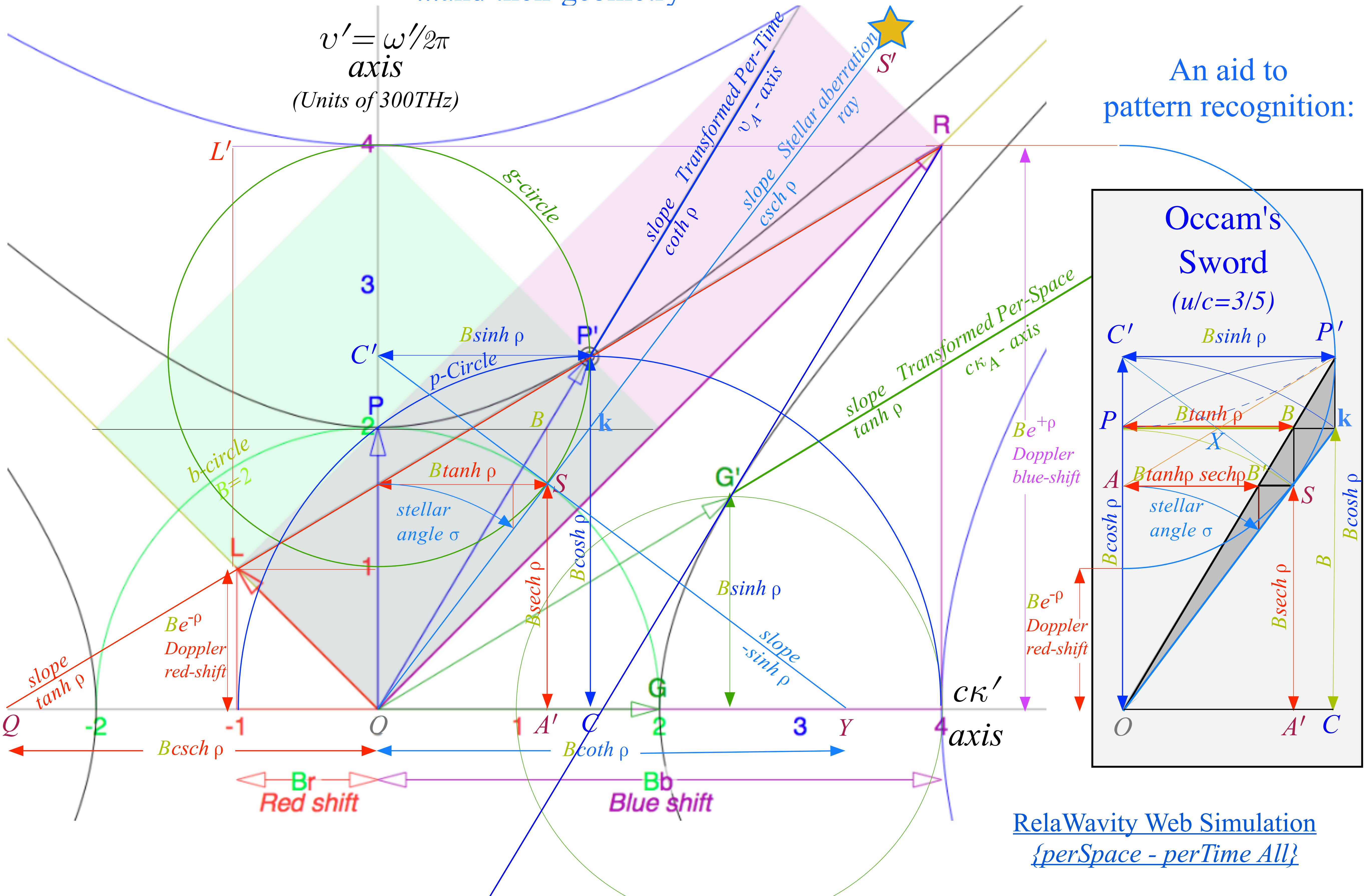
Then everything (and everybody) always goes speed c through $(x',c\tau)$ space!



This map has circle sector arc-area σ = 0.6435
set to angle ∠σ = 36.87° = 0.6435 radian
Also it is set to hyperbola sector arc-area ρ = 0.6931
angle ∠ρ = ν = 30.96°
sin(σ)=0.6000 = tanh(ρ) = 3/5
tan(σ)=0.7500 = sinh(ρ) = 3/4
sec(σ)=1.2500 = cosh(ρ) = 5/4
cos(σ)=0.8000 = sech(ρ) = 4/5
cot(σ)=1.3333 = csch(ρ) = 4/3
csc(σ)=1.6667 = coth(ρ) = 5/3
cosh(ρ)+sinh(ρ) = 5/4 + 3/4 = 2.0 = e^{+ρ}
cosh(ρ)−sinh(ρ) = 5/4 − 3/4 = 1/2 = e^{−ρ}
cosh(ρ) = (e^{+ρ} + e^{−ρ})/2
sinh(ρ) = (e^{+ρ} − e^{−ρ})/2
Half-Sum-
Half-Difference
Trig-Formulae for
exponentials e^{±ρ}
Bcosh(ρ)+Bsinh(ρ)=Be^{+ρ}
R
tangent slope = tanh(ρ)
tangent slope = coth(ρ)
Bsinh(ρ)
P
Bsinh(ρ)
Bsinh(ρ)
B
x²+y²=B²
S
G
Bcsch(ρ)
L
Bcosh(ρ)
ρ
σ
Bsech(ρ)
Bcsch(ρ)
Bcosh(ρ)-Bsinh(ρ)=Be^{-ρ}
Btanh(ρ)
Bcoth(ρ)
Thales Circle
(links e^{-ρ} to e^{+ρ})

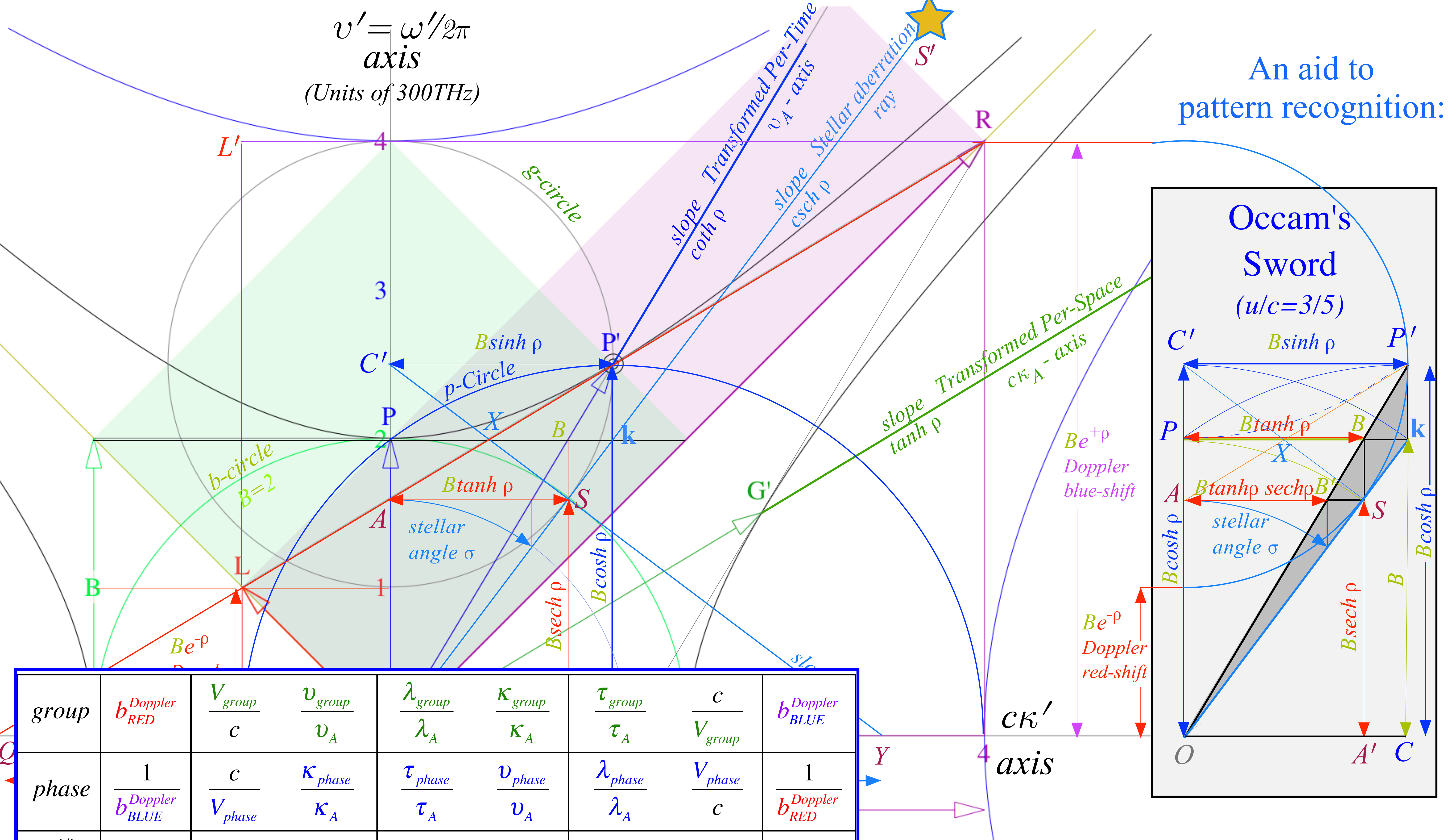
Summary of optical wave parameters for relativity and QM

...and their geometry



RelaWavity Web Simulation
{perSpace - perTime All}

An aid to pattern recognition:



group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Table of 12 wave parameters (includes inverses) for relativity

...and values for *u/c=3/5*

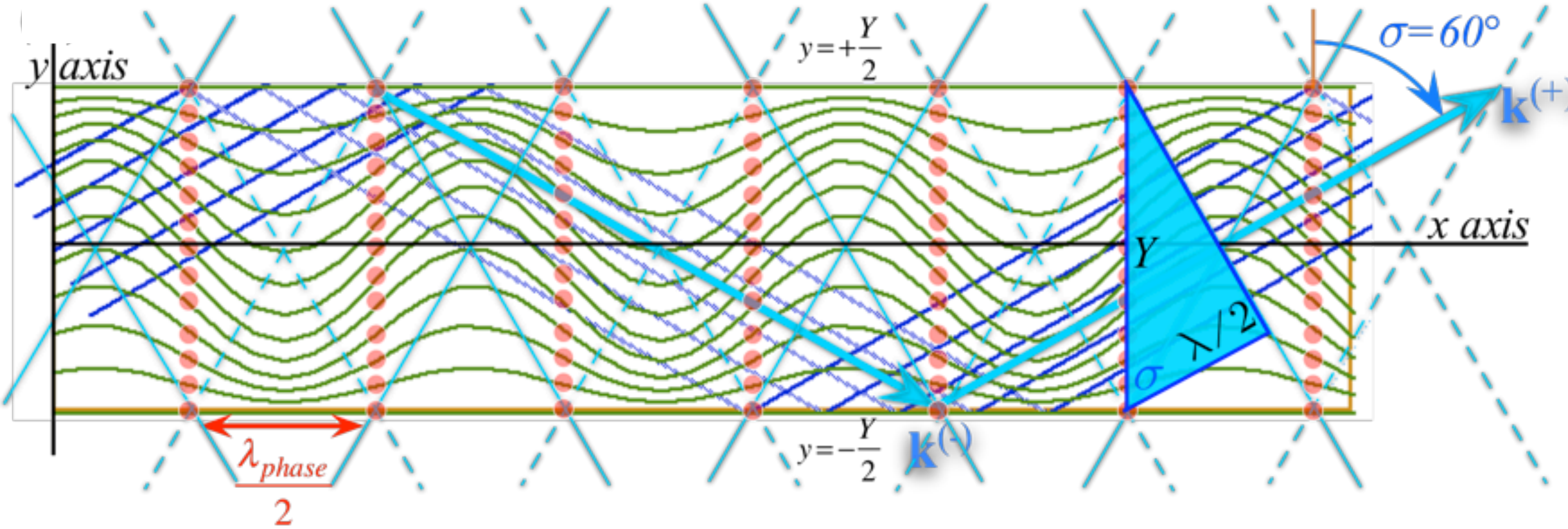
RelaWavity Web Simulation

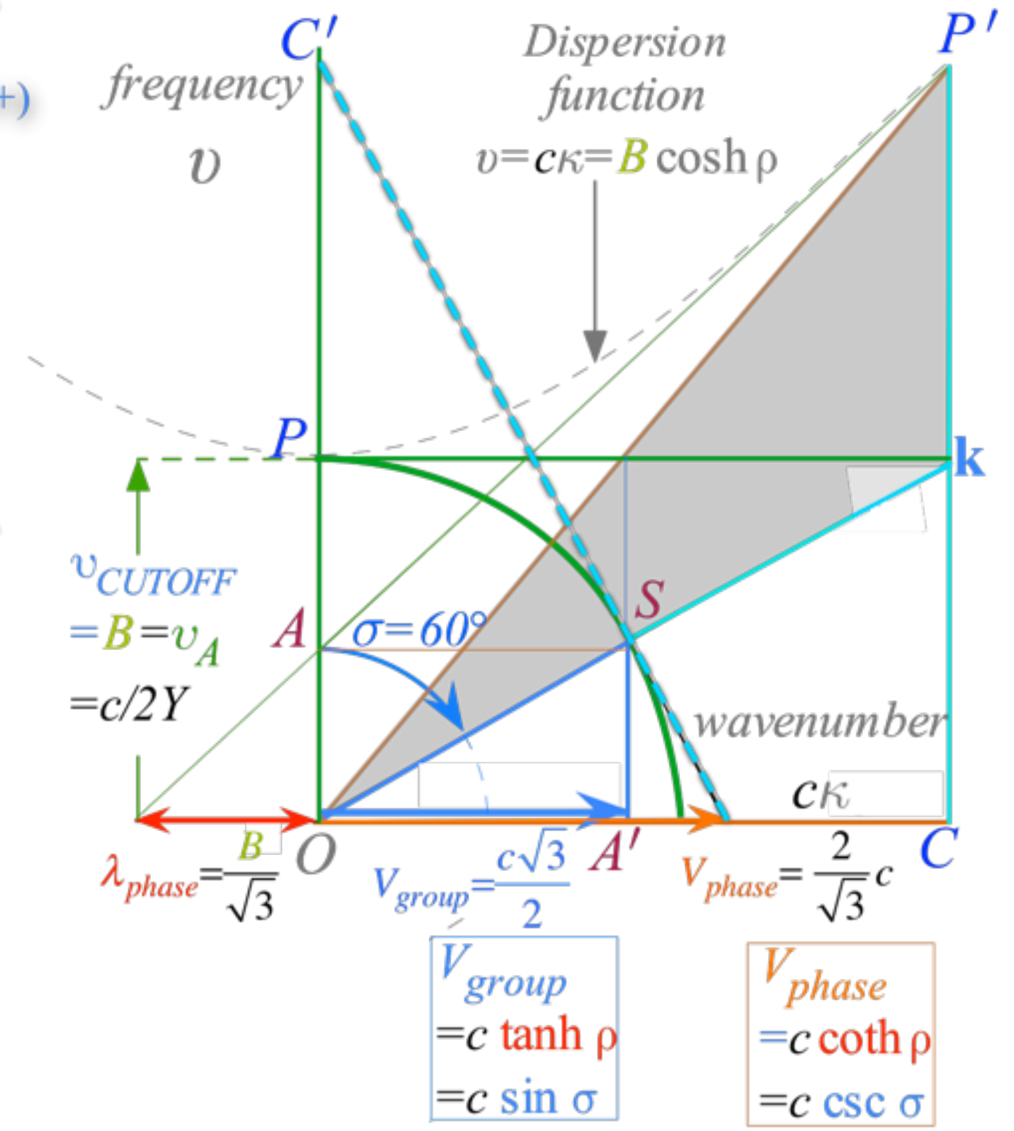
Expanded Relativistic Relations

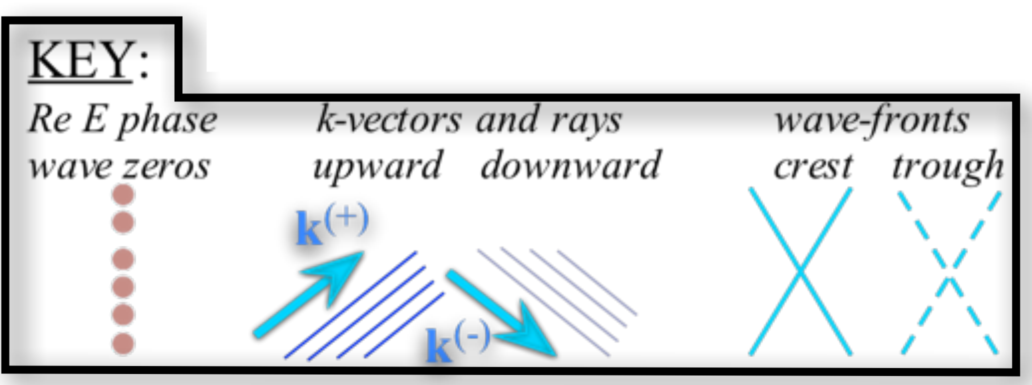
Optical wave guide relativistic geometry aided by Occam's Sword

geometry applies to (x,y) space-space
to (k_x,k_y)per-space-per-space
to (x,ct) space-time

Relativistic mode with near-c $V_{group}=c/2$ and $V_{phase}=2c$. (Low dispersion.)

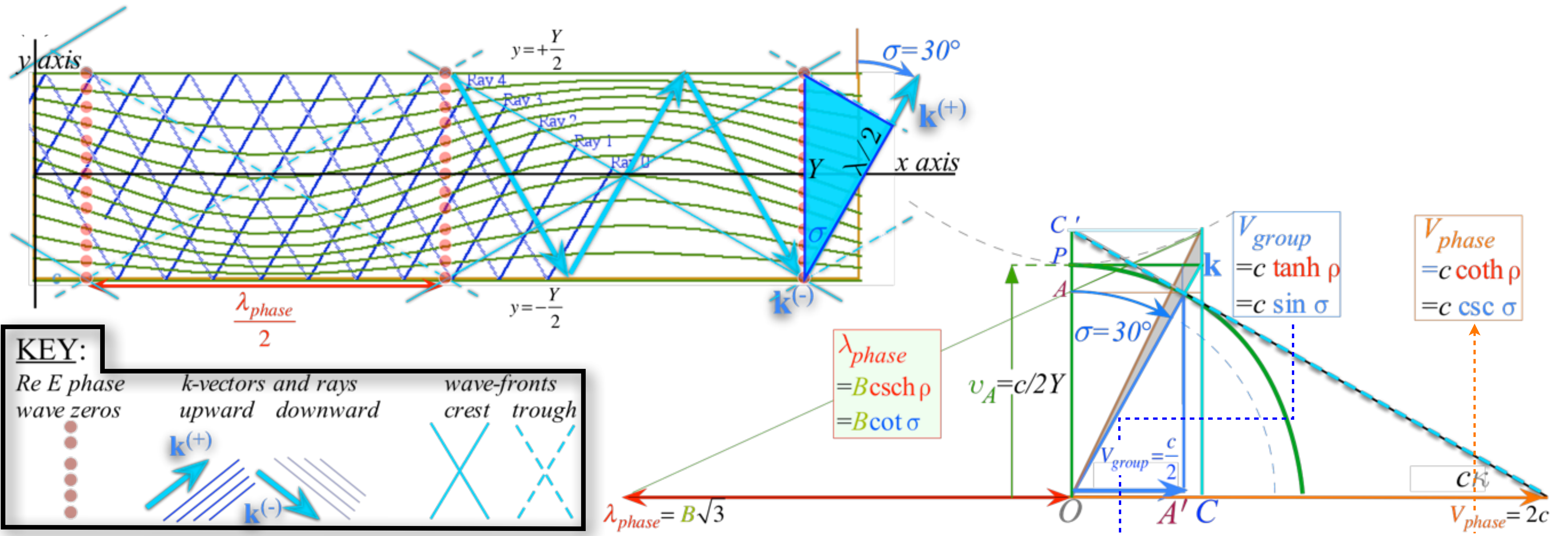






Optical wave guide relativistic geometry aided by Occam's Sword

geometry applies to (x,y) space-space
to (k_x,k_y) per-space-per-space
to (x,ct) space-time

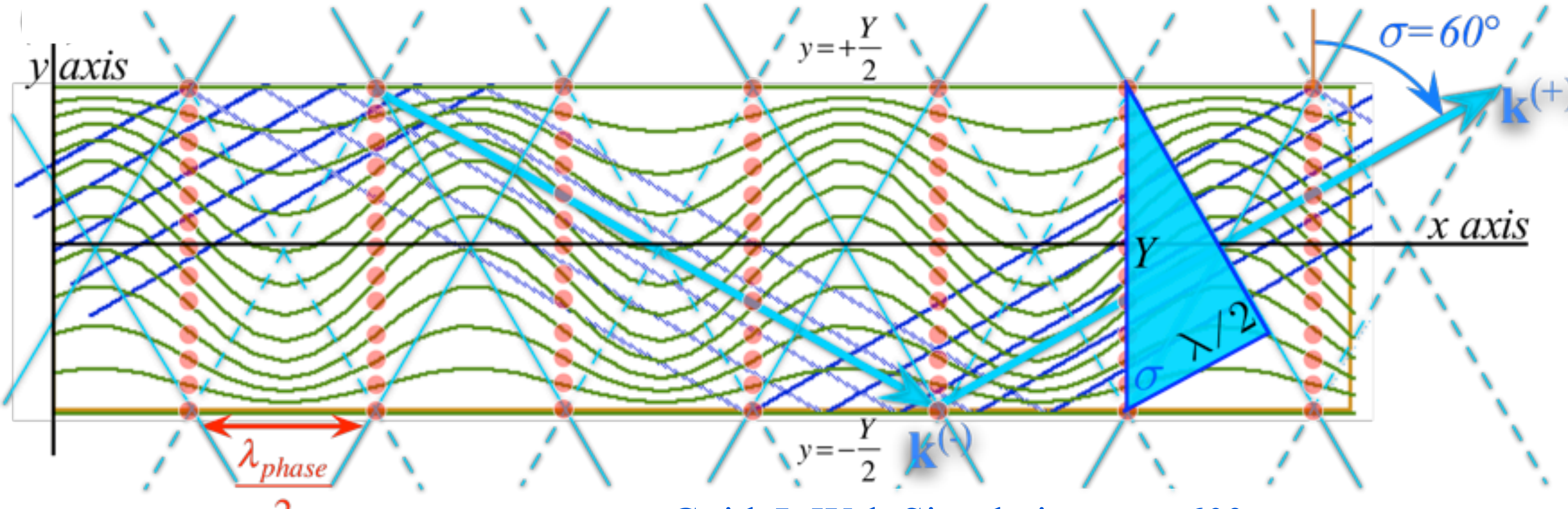


Example of near-cut-off mode with low $V_{group}=c/2$ and high $V_{phase}=2c$. (High dispersion.)

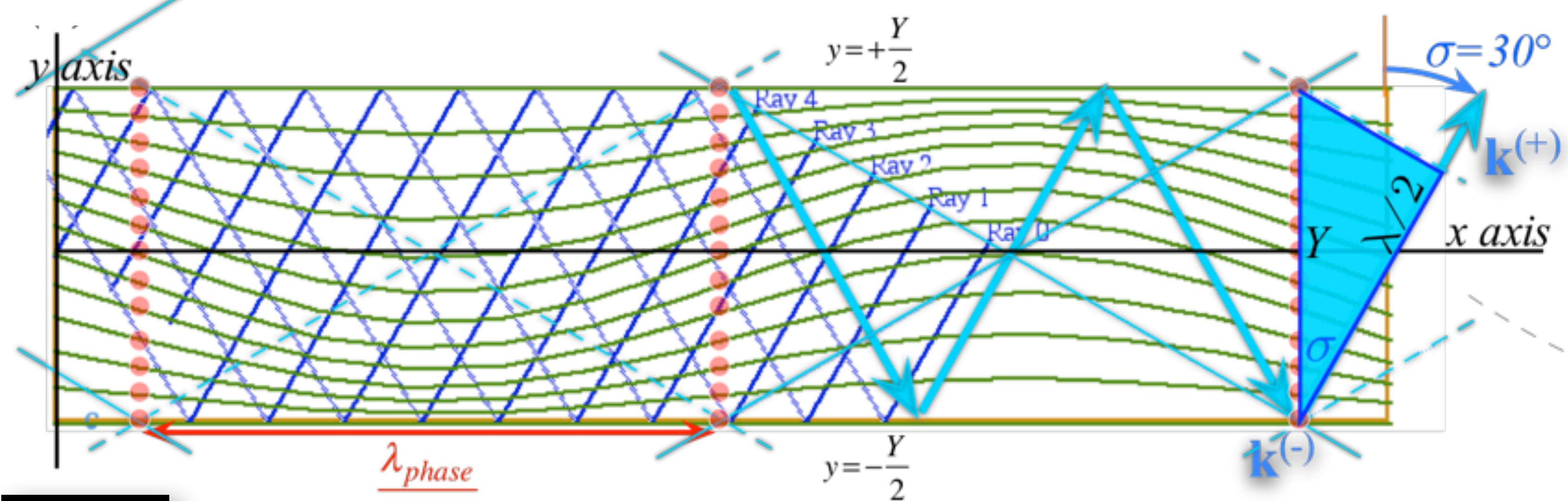
Optical wave guide relativistic geometry aided by Occam's Sword

geometry applies to (x,y) space-space
to (k_x,k_y)per-space-per-space
to (x,ct) space-time

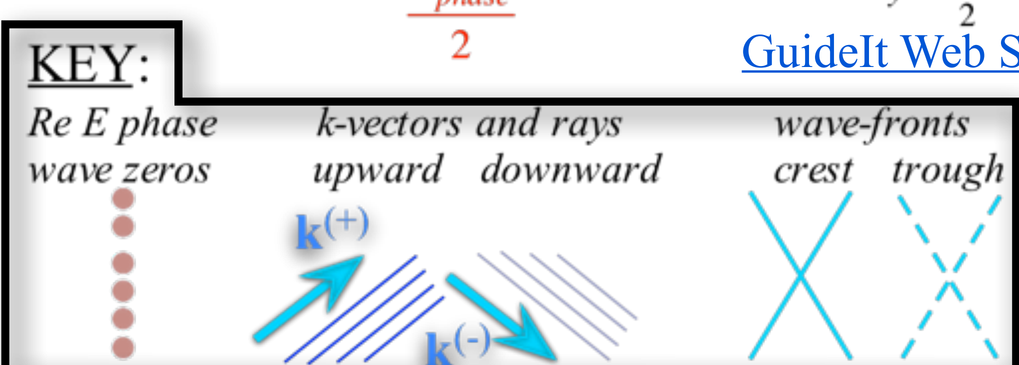
Relativistic mode with near-c $V_{group}=c/2$ and $V_{phase}=2c$. (Low dispersion.)

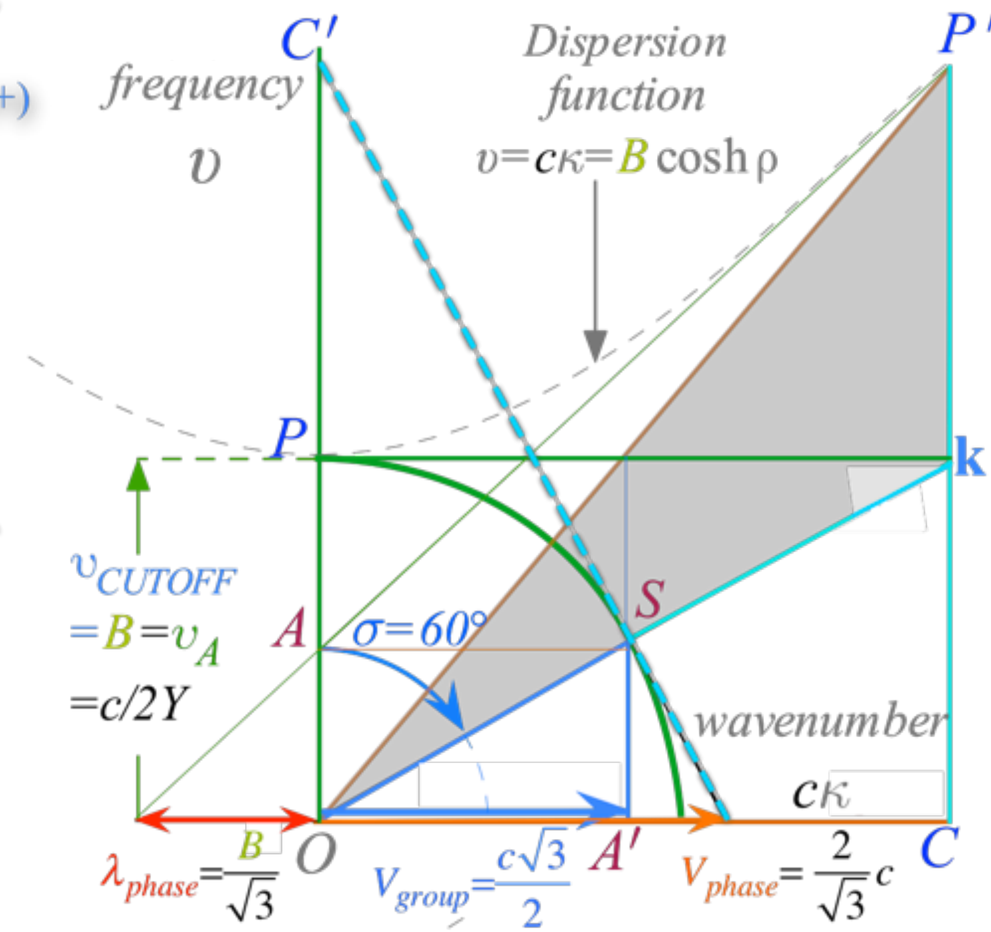


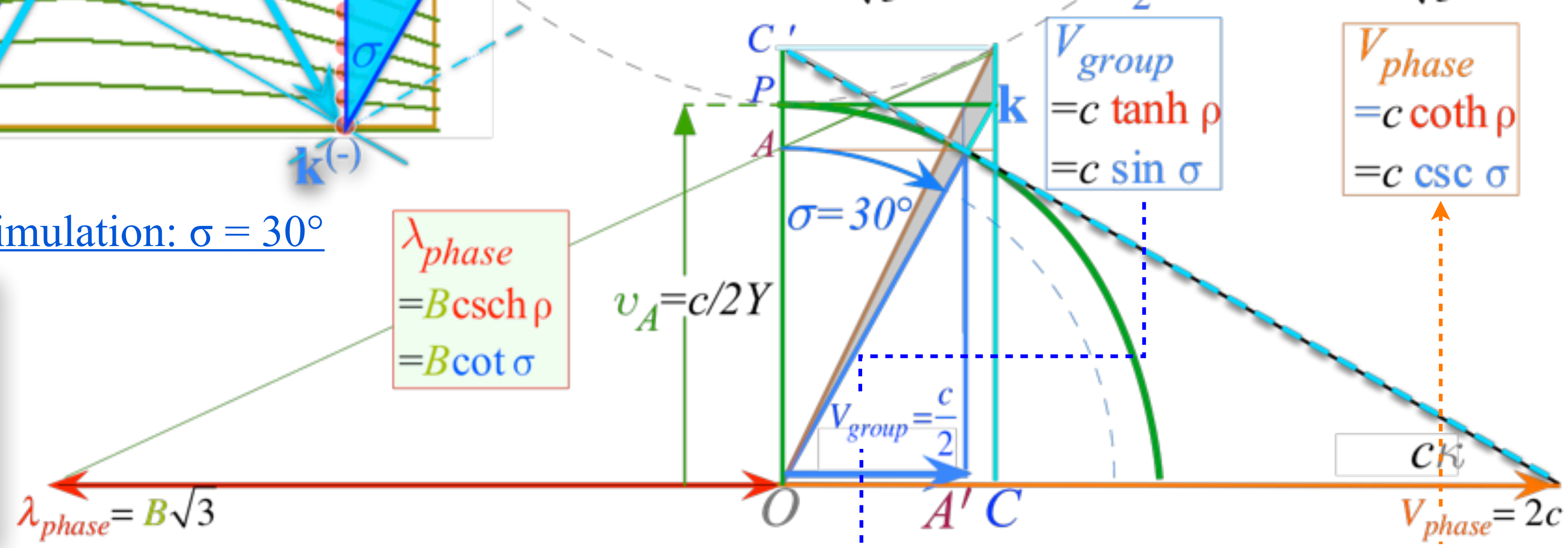
GuideIt Web Simulation: σ = 60°



GuideIt Web Simulation: σ = 30°



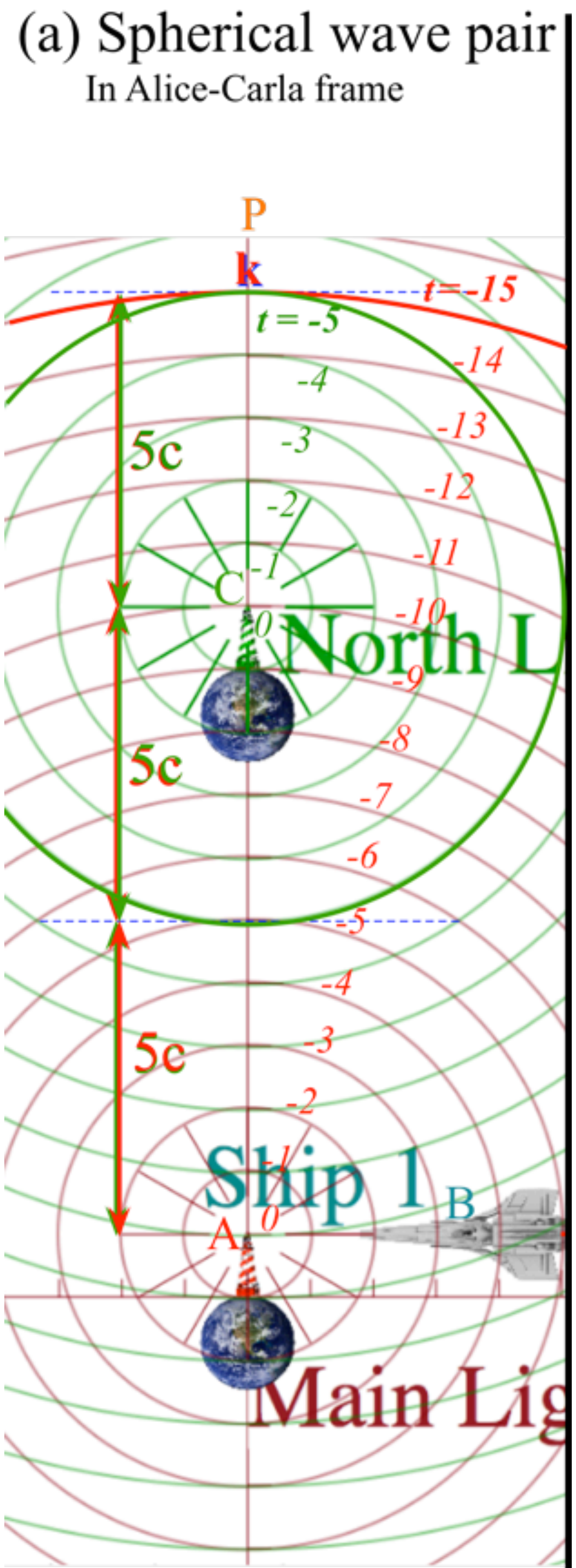


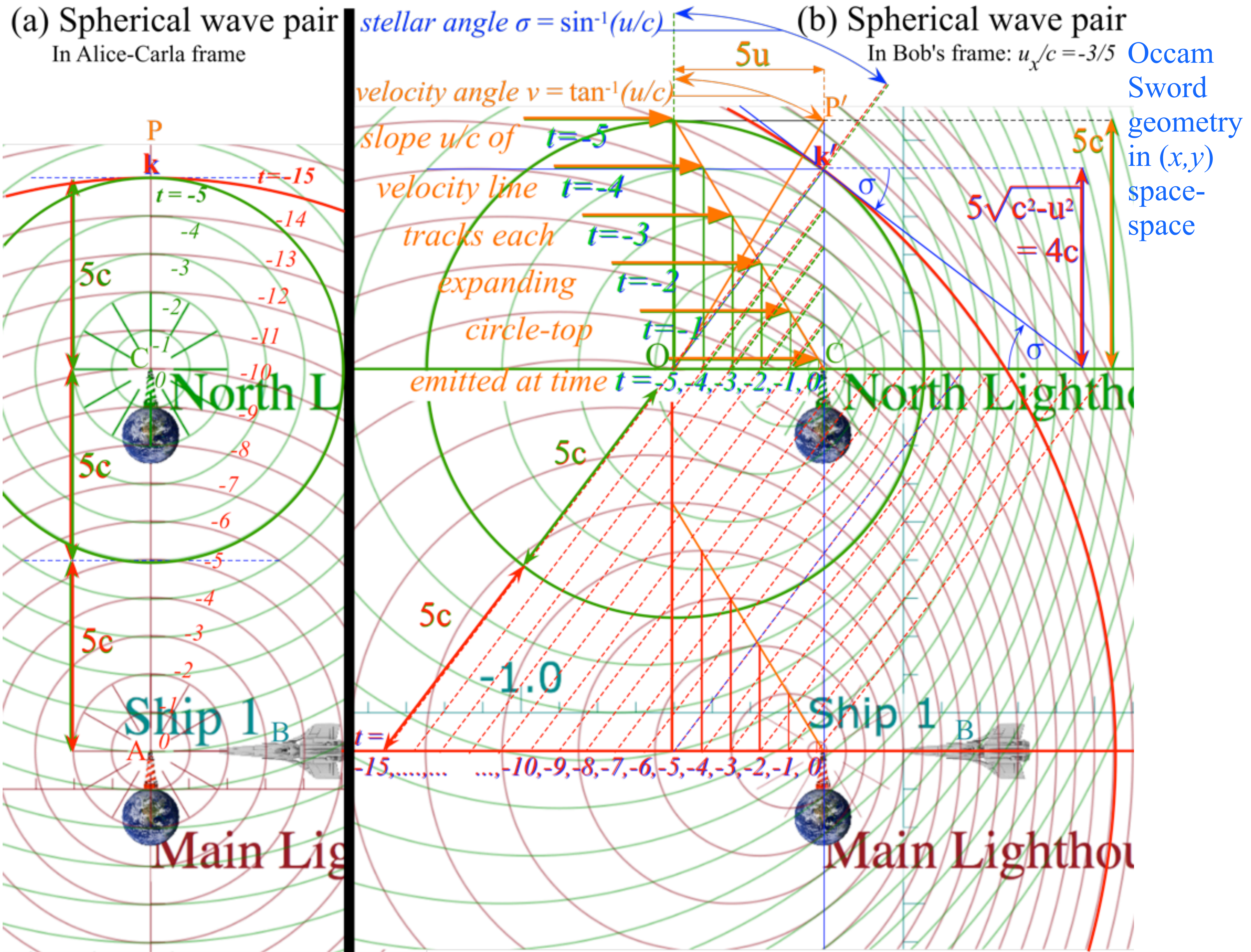


Example of near-cut-off mode with low $V_{group}=c/2$ and high $V_{phase}=2c$. (High dispersion.)

Spherical wave relativistic geometry

Also, aided by Occam's Sword



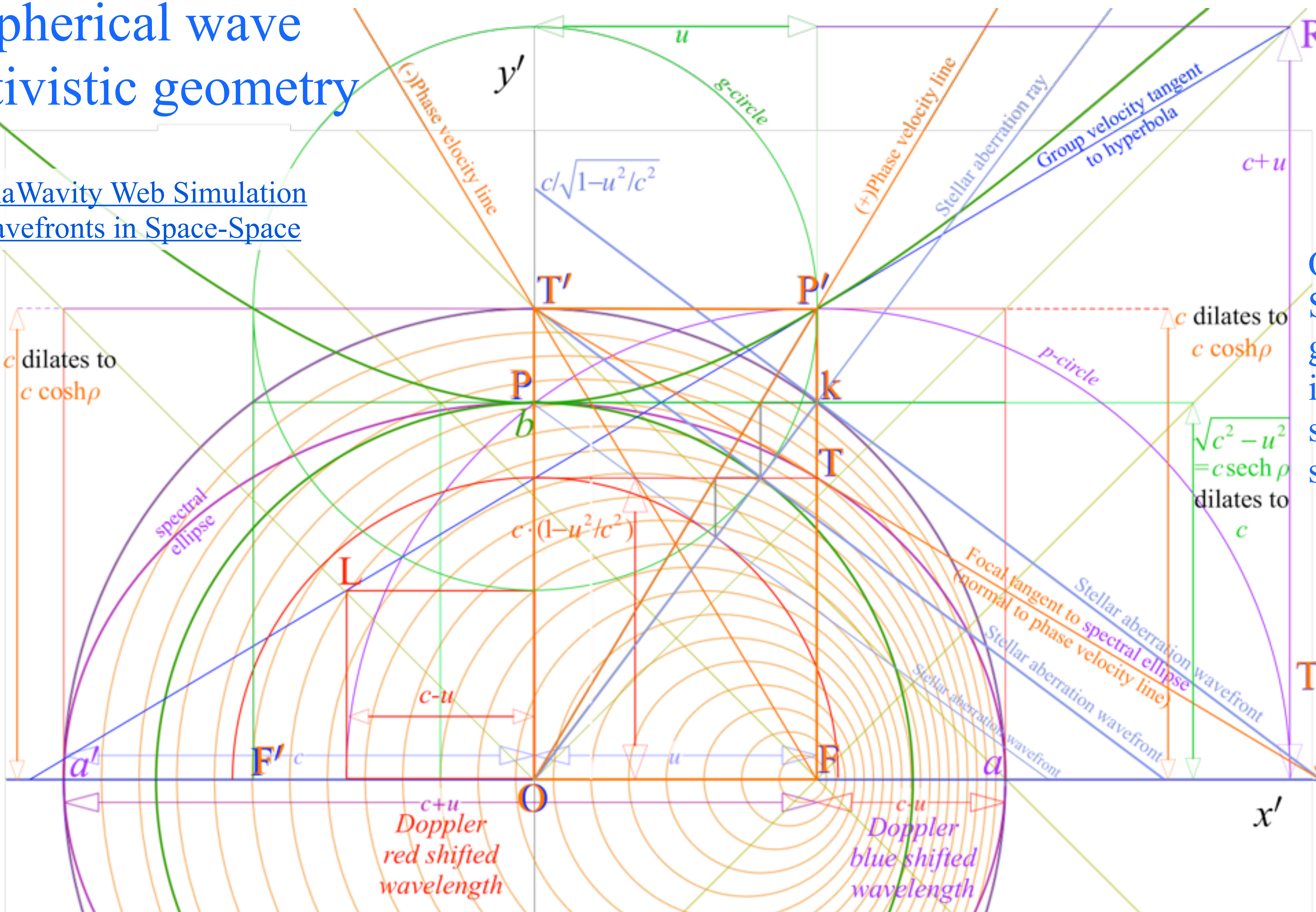


(a) Spherical wave pair
In Alice-Carla frame
(b) Spherical wave pair
In Bob's frame: $u_x/c = -3/5$
stellar angle $\sigma = \sin^{-1}(u/c)$
velocity angle $\nu = \tan^{-1}(u/c)$
slope u/c of velocity line tracks each expanding circle-top emitted at time t = -5,-4,-3,-2,-1,0
Occam Sword geometry in (x,y) space-space
5u
5c
$5\sqrt{c^2-u^2} = 4c$
North Lighthouse
Main Lighthouse
Ship 1
-1.0
t = -15,......... ...,-10,-9,-8,-7,-6,-5,-4,-3,-2,-1, 0

Spherical wave relativistic geometry

[RelaWavity Web Simulation Wavefronts in Space-Space]

Occam Sword geometry in (x,y) space-space



Doppler Red λ=$c+u$
dilates to: $(c+u)\cosh\rho = c\sqrt{\frac{c+u}{c-u}} = ce^{+\rho}$

ellipse major radius a=**OF**a=c
dilates to: $c\cosh\rho = c/\sqrt{1-u^2/c^2}$

Applications of Einstein dilation factor:
$\gamma = \cosh\rho = 1/\sqrt{1-u^2/c^2}$

ellipse focal length **FO**= $u = c\tanh\rho$
dilates to: $u\cosh\rho = c\sinh\rho$

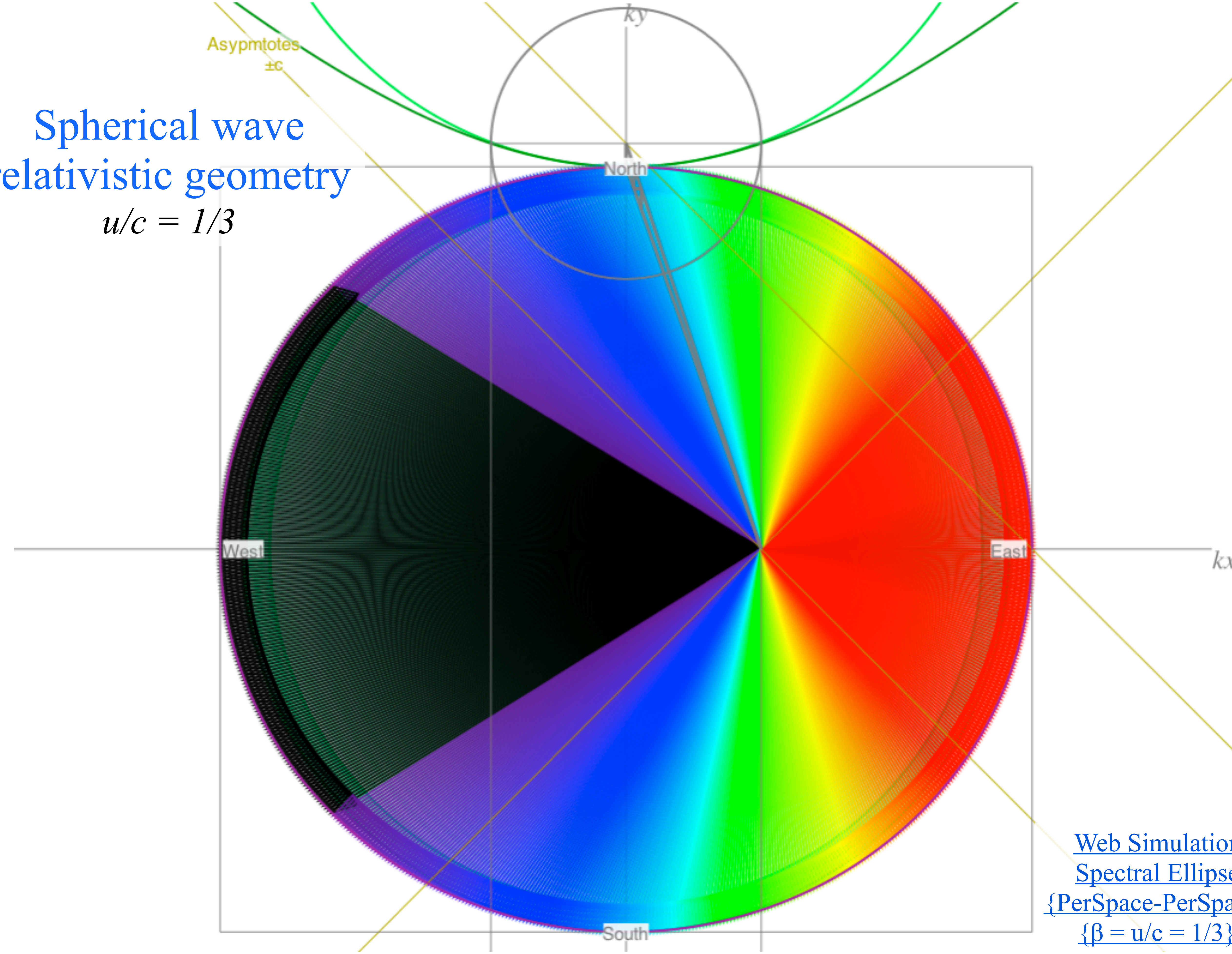
ellipse latus radius **FT**=$c(1-u^2/c^2)$
dilates to: $c(1-u^2/c^2)\cosh\rho$
$= c\sqrt{1-u^2/c^2} = c\ \mathrm{sech}\rho$

Doppler Blue λ=$c-u$
dilates to: $(c-u)\cosh\rho = c\sqrt{\frac{c-u}{c+u}} = ce^{-\rho}$

Base height **FTk**=$\sqrt{c^2-u^2}$
dilates to: $\sqrt{c^2-u^2}\cosh\rho = c$
(equal to ellipse minor radius b)

Spherical wave relativistic geometry

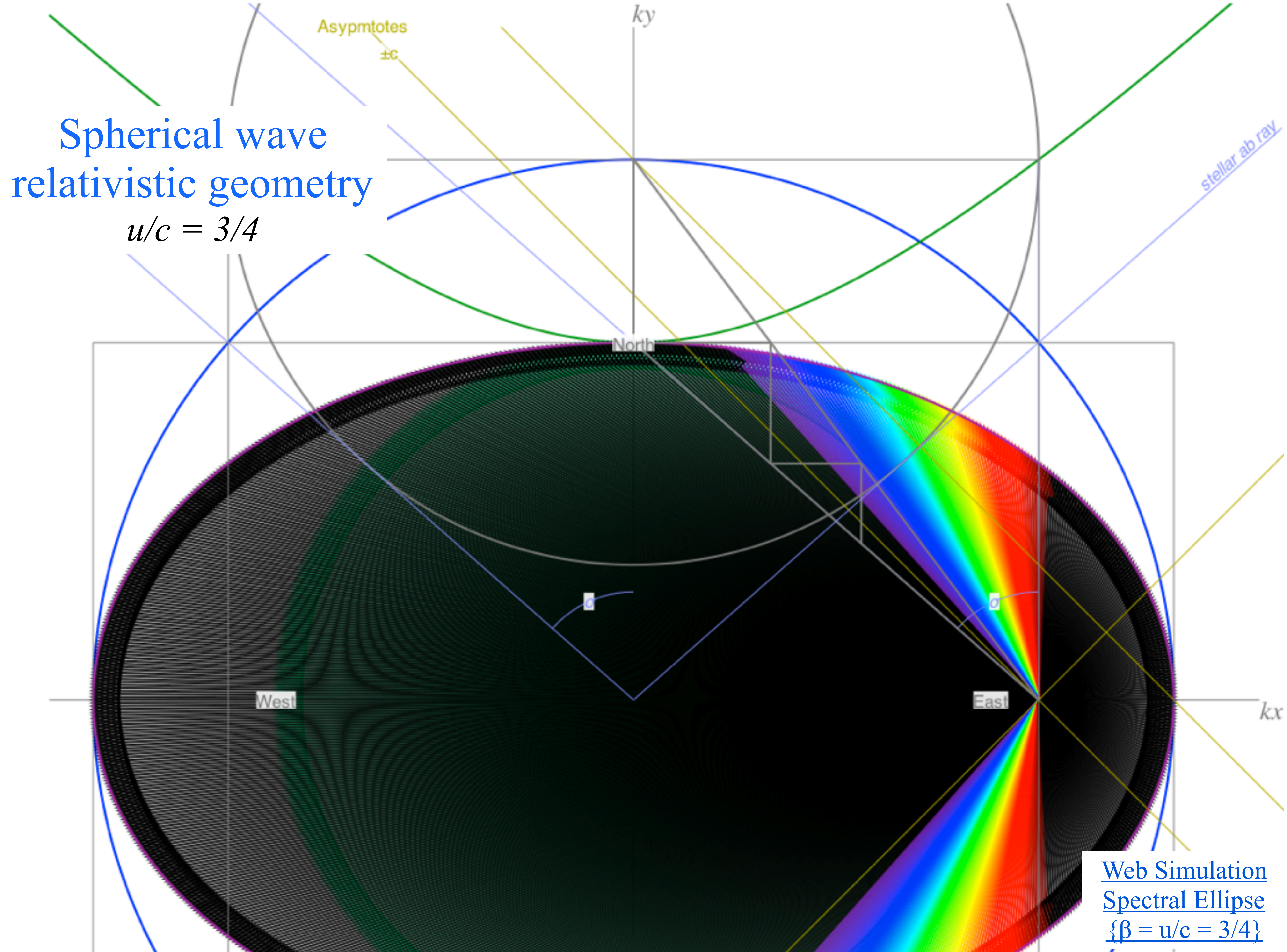
u/c = 1/3



[Web Simulation](#)
[Spectral Ellipse](#)
[{PerSpace-PerSpace}](#)
[{β = u/c = 1/3}](#)

Spherical wave relativistic geometry

u/c = 3/4



Web Simulation
Spectral Ellipse
{β = u/c = 3/4}

Using (some) wave parameters to develop relativistic quantum theory

…and *classical* mechanics

$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \quad (\text{for } u \ll c)$$

$$c\kappa_{phase} = B\sinh\rho \approx B\rho \quad (\text{for } u \ll c)$$

At low speeds:

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2$$

$$\sinh\rho \approx \rho$$

$$B = \upsilon_A$$

$$B = \upsilon_A = c\kappa_A$$

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

RelaWavity Web Simulation - Relativistic *Terms (Expanded Table)*

Using (some) wave parameters to develop relativistic quantum theory

…and *classical* mechanics

$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c\text{)}$$

$$c\kappa_{phase} = B\sinh\rho \approx B\rho \qquad \text{(for } u \ll c\text{)}$$

$$\frac{u}{c} = \tanh\rho \approx \rho \qquad \text{(for } u \ll c\text{)}$$

At low speeds:

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

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group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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$$B = \upsilon_A$$

$$B = \upsilon_A = c\kappa_A$$

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c)$$

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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$$B = \upsilon_A = c\kappa_A$$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

time	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
space	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy and momentum

Resembles: $const. + \frac{1}{2}Mu^2$

Resembles: Mu

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \dfrac{hB}{c^2}$

Resembles: $const. + \frac{1}{2}Mu^2$

Resembles: Mu

time	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
space	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

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At low speeds:

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υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

So attach scale factor h to match units.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$

Resembles: $const. + \frac{1}{2}Mu^2$

Resembles: Mu

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle $\forall\ \sigma$	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

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…and *classical* mechanics

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So attach scale factor h to match units.

Resembles: $const. + \frac{1}{2}Mu^2$

Resembles: Mu

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
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RelaWavity Web Simulation - Relativistic *Terms (Expanded Table)*

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Planck (1900)

$$= \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1 - u^2/c^2}}$$

Einstein (1905)

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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
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$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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Using (some) wave parameters to develop relativistic quantum theory

...and *classical* mechanics

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Quantized amplitude

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Resolution and dirty secret: $\mathbf{E}$, N, and υ_{phase} are all frequencies!

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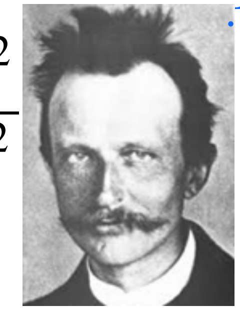

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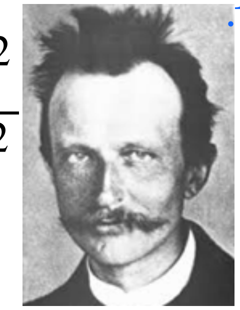

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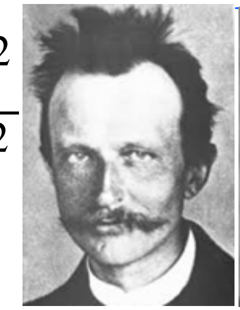


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DeBroglie (1921)

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	[illegible]
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$=0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Using (some) wave parameters to develop relativistic quantum theory

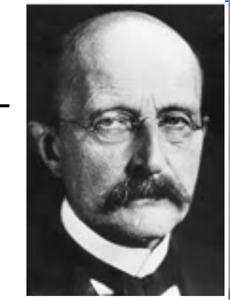
$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \quad (\text{for } u \ll c)$$

$$c\kappa_{phase} = B\sinh\rho \approx B\rho \quad (\text{for } u \ll c)$$

$$\frac{u}{c} = \tanh\rho \approx \rho \quad (\text{for } u \ll c)$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$



Max Planck 1858-1947

Louis DeBroglie 1892-1987

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$ (The famous Mc^2 shows up here!)

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$$

So attach scale factor h (or hN) to match units.

$$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx Mu$$

~~Lucky coincidences??~~ Natural wave conspiracy ~~Cheap~~ Expensive trick??

...Try exact υ_{phase} and κ_{phase}...

Need to replace h with hN to match e.m. energy density $\varepsilon_0 \mathbf{E}^* \bullet \mathbf{E} = hN\upsilon_{phase}$

$$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$$

Planck (1900)

$$= \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1 - u^2/c^2}}$$

Einstein (1905)

This motivates the "particle" normalization $\int \Psi^* \Psi \, dV = N \quad \Psi = \sqrt{\frac{\varepsilon_0}{h\upsilon}}E$

$$hc\kappa_{phase} = hB\sinh\rho = Mc^2\sinh\rho$$

$$cp = \frac{Muc}{\sqrt{1 - u^2/c^2}}$$

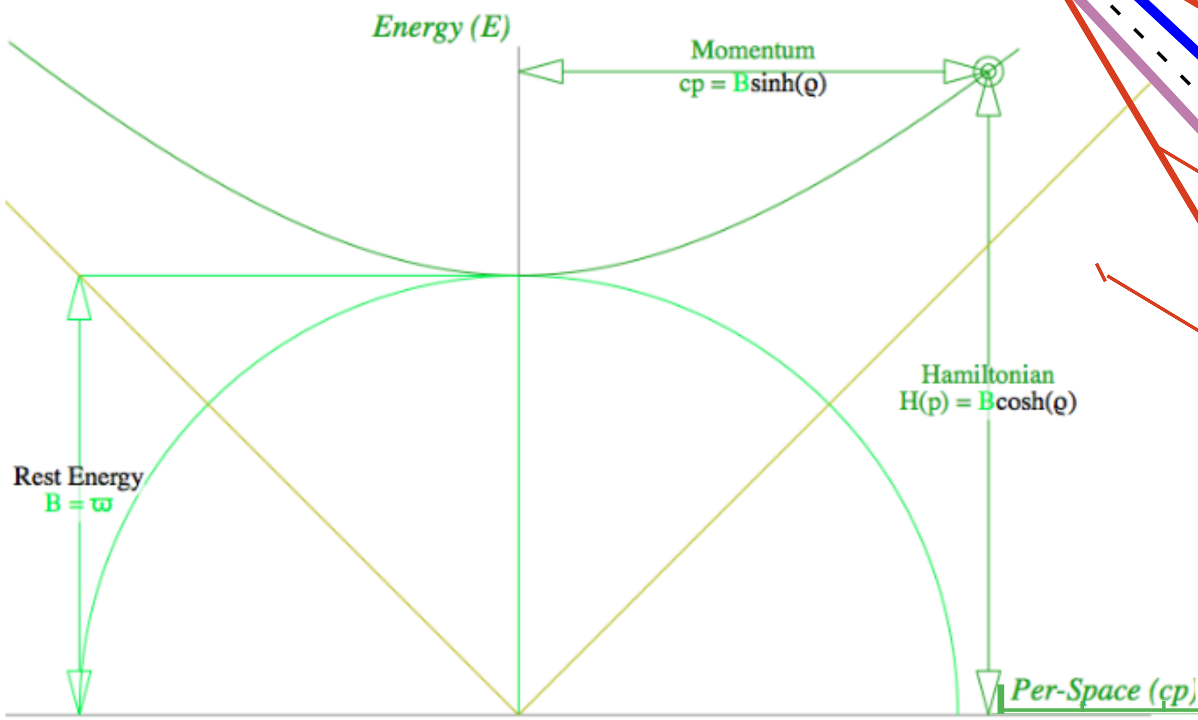
$$\frac{1}{\sqrt{\beta^{-2} - 1}} = \frac{\frac{u}{c}}{\sqrt{1 - \frac{u^2}{c^2}}} \quad \text{(old-fashioned notation)}$$

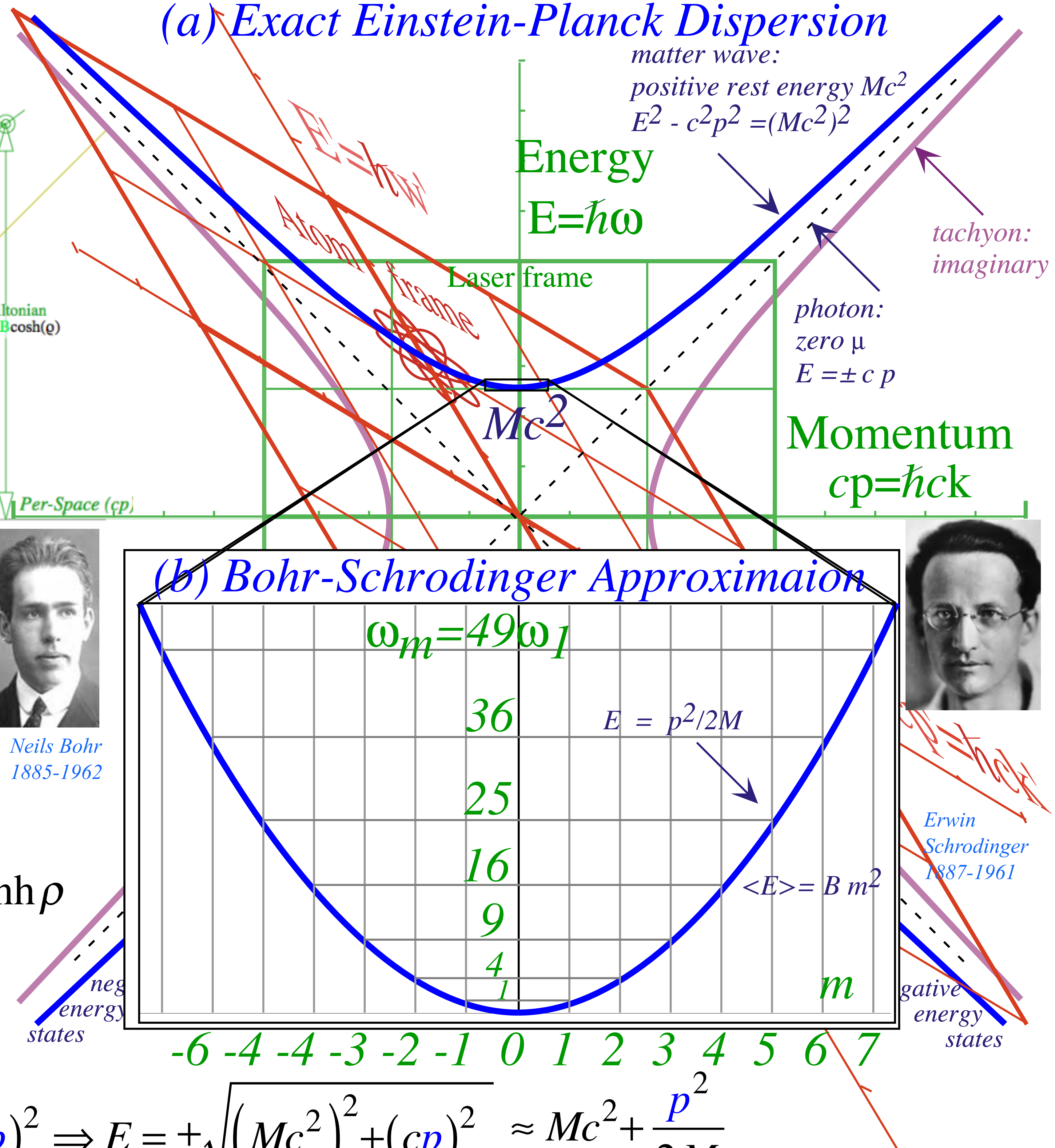
Momentum: $h\kappa_{phase} = p = \dfrac{Mu}{\sqrt{1 - u^2/c^2}}$

DeBroglie (1921)

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$			
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$			
stellar ∀ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Using (some) wave coordinates for relativistic quantum theory





Neils Bohr
1885-1962

Erwin
Schrodinger
1887-1961

Mass (resting)

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Energy

$$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$$

Momentum

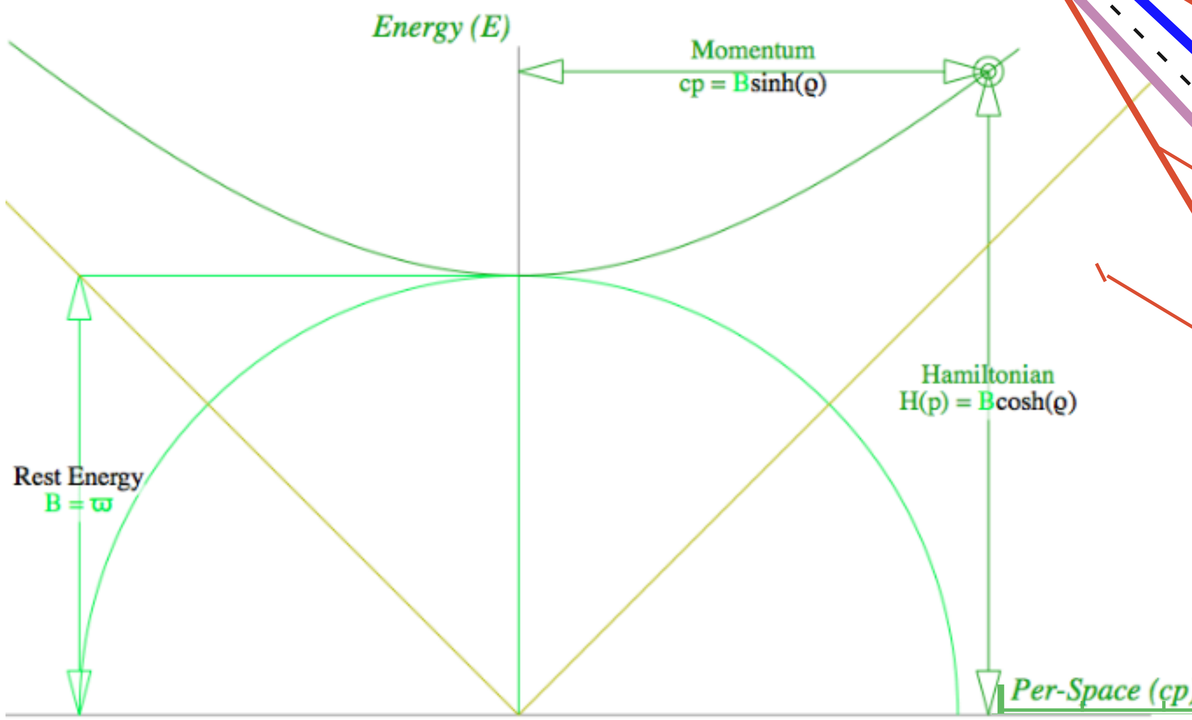
$$hc\kappa_{phase} = cp = hc\kappa_A \sinh\rho = h\upsilon_A \sinh\rho$$

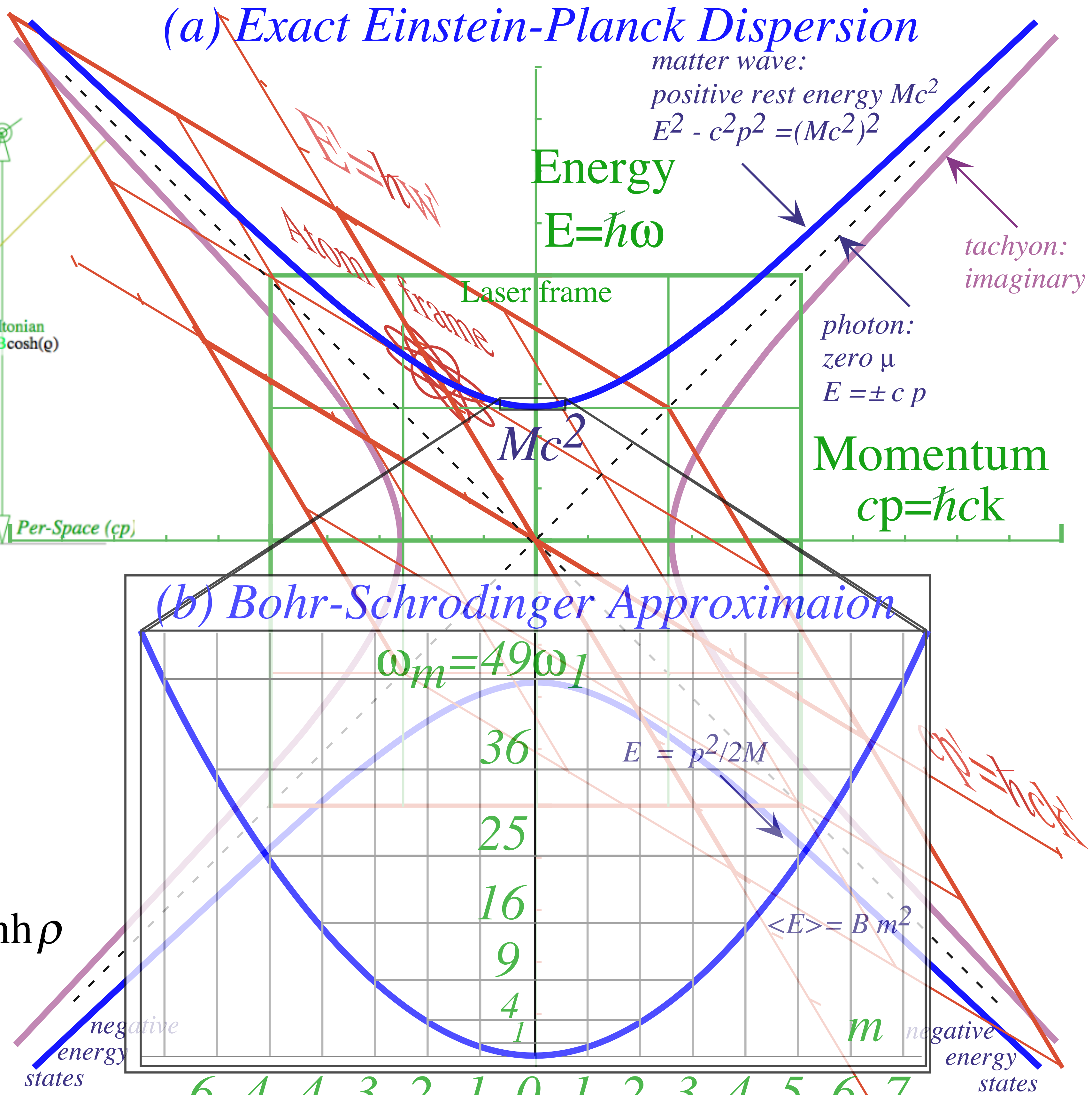
Energy versus Momentum

$$E^2 = \left(Mc^2\right)^2 \cosh^2\rho$$

$$= \left(Mc^2\right)^2\left(1+\sinh^2\rho\right) = \left(Mc^2\right)^2 + (cp)^2 \Rightarrow E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2} \approx Mc^2 + \frac{p^2}{2M}$$

Using (some) wave coordinates for relativistic quantum theory





Mass (resting)

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Energy

$$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$$

Momentum

$$hc\kappa_{phase} = cp = hc\kappa_A \sinh\rho = h\upsilon_A \sinh\rho$$

Energy versus Momentum

$$E^2 = \left(Mc^2\right)^2 \cosh^2\rho$$

$$= \left(Mc^2\right)^2\left(1+\sinh^2\rho\right) = \left(Mc^2\right)^2 + (cp)^2 \Rightarrow E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2} \approx Mc^2 + \frac{p^2}{2M}$$

low speed approximation

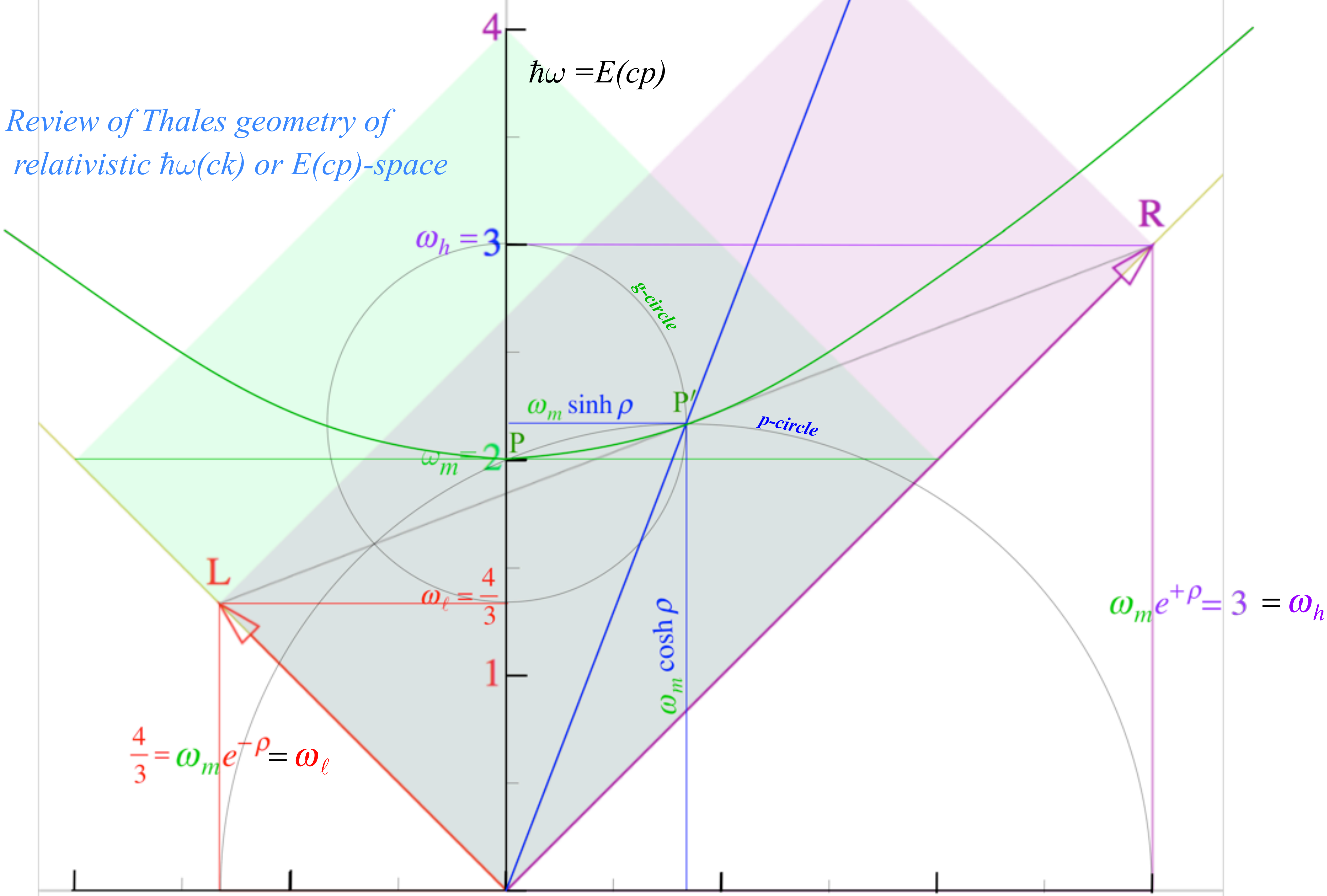
Relawavity variable tables

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$
effects	$b_{RED}^{Doppler}$	V_{group}	*past-future asymmetry* (off-diagonal Lorentz-transform)	*x-contraction*(Lorentz) τ_{phase}*-contraction*	*t-dilation*(Einstein) υ_{phase}*-dilation* (on-diagonal Lorentz-transform)	*inverse asymmetry*	V_{phase}	$b_{BLUE}^{Doppler}$

Relativistic quantum mechanics variable tables

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{\beta}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{1-\beta^2}}{\beta}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$
functions		$V_{group}=c\tanh\rho$	*momentum* $cp=Mc^2\sinh\rho$	*-Lagrangian* $L=-Mc^2\operatorname{sech}\rho$	*Hamiltonian* $H=Mc^2\cosh\rho$	*DeBroglie* $\lambda=\alpha\operatorname{csch}\rho$	$V_{phase}=c\coth\rho$	

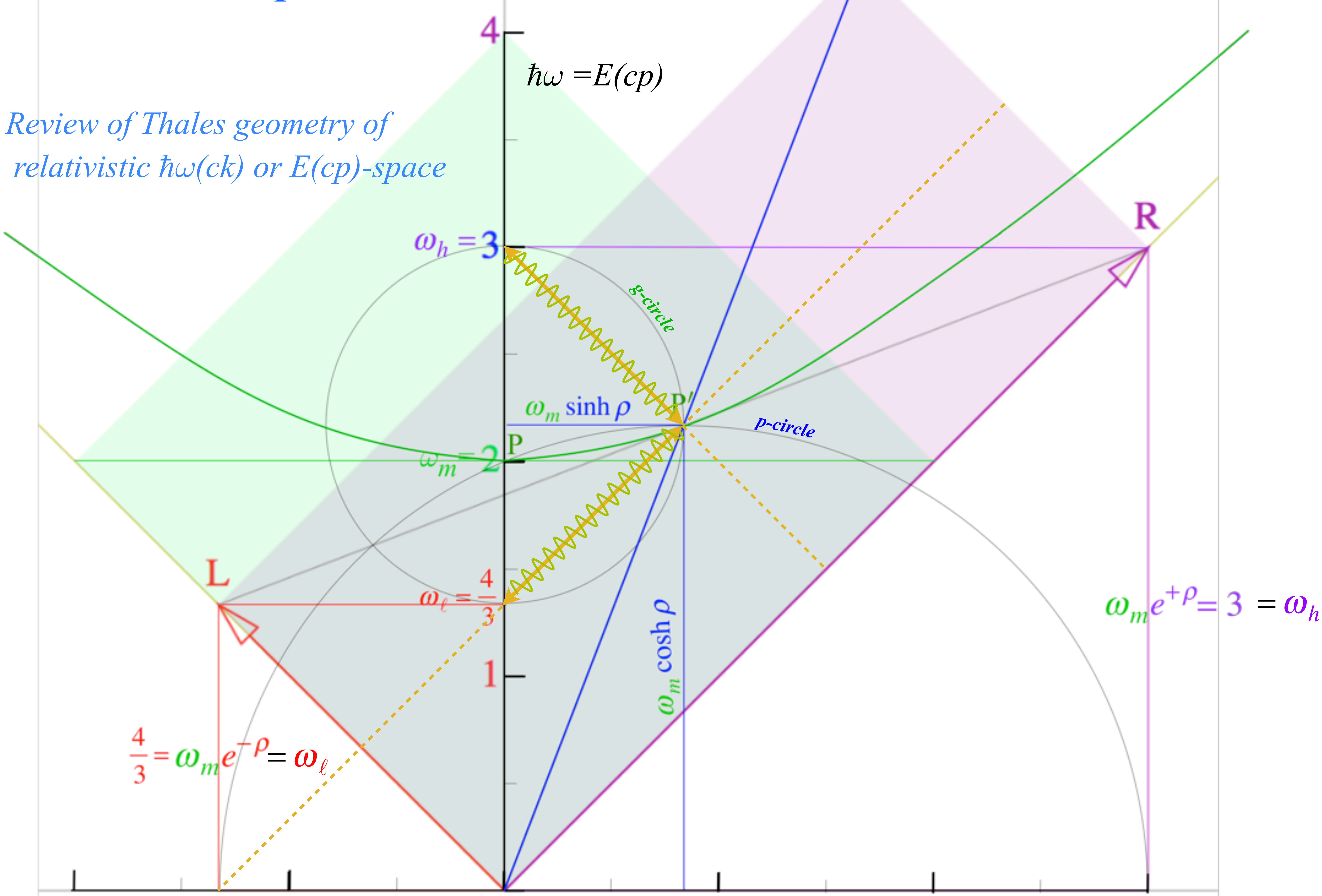
Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$



Doppler RED factor: $\frac{2}{3} = e^{-\rho}$ Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

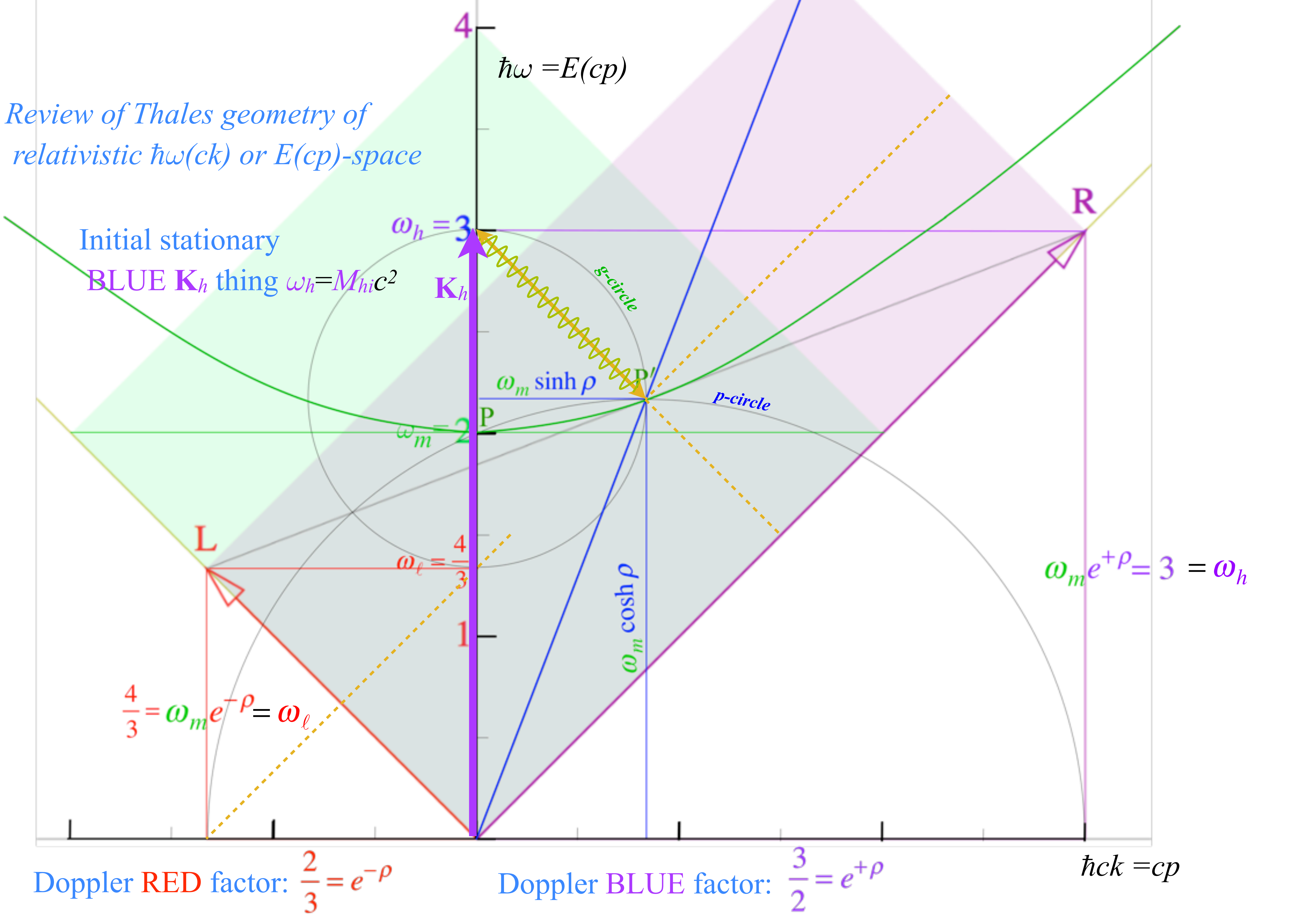
Relativistic optical transitions $|high\rangle=|\omega_h\rangle \rightleftarrows |mid\rangle=|\omega_m\rangle \rightleftarrows |low\rangle=|\omega_\ell\rangle$

Review of Thales geometry of relativistic ħω(ck) or E(cp)-space

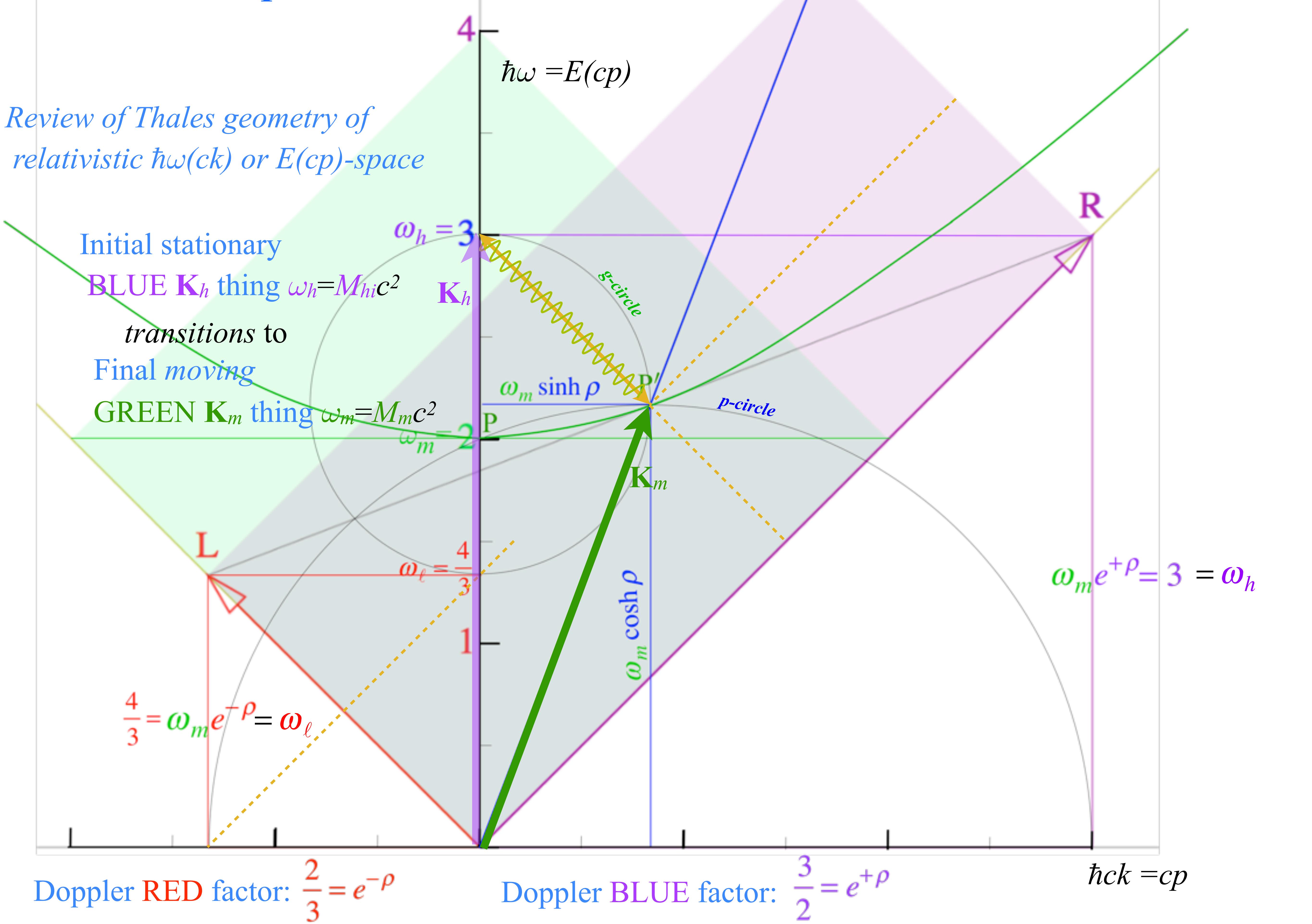


Doppler RED factor: $\frac{2}{3}=e^{-\rho}$ Doppler BLUE factor: $\frac{3}{2}=e^{+\rho}$

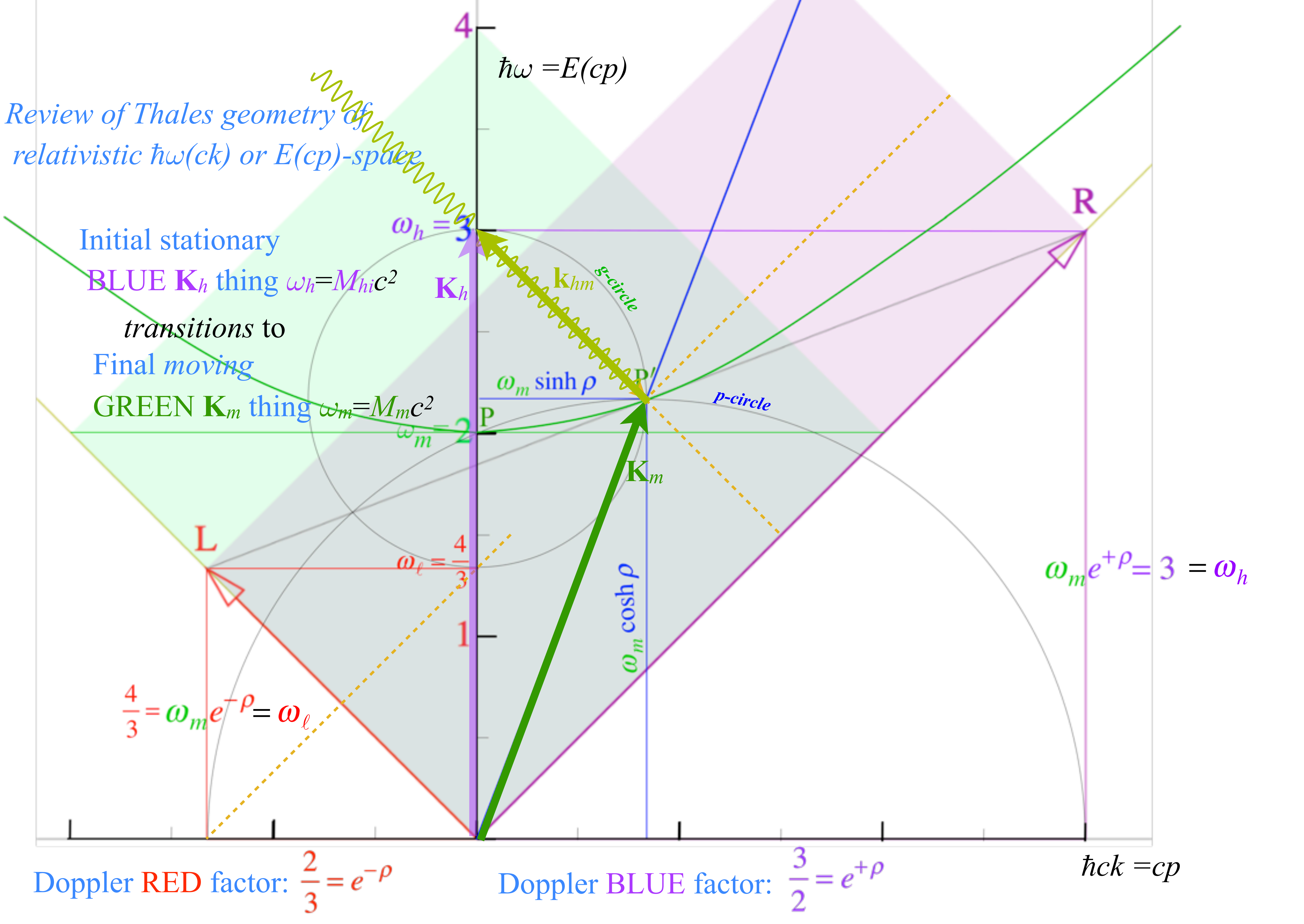
Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$



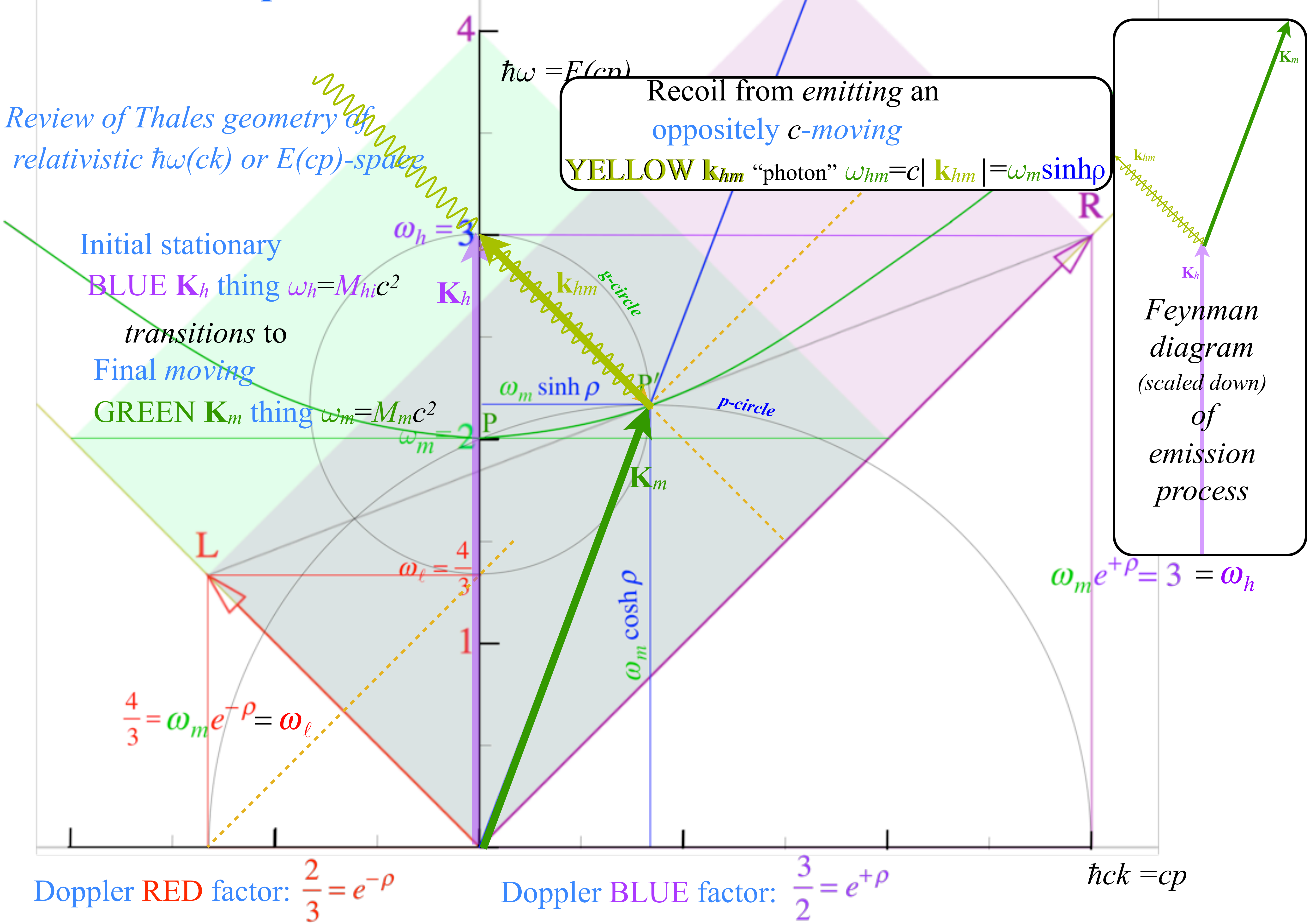
Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$



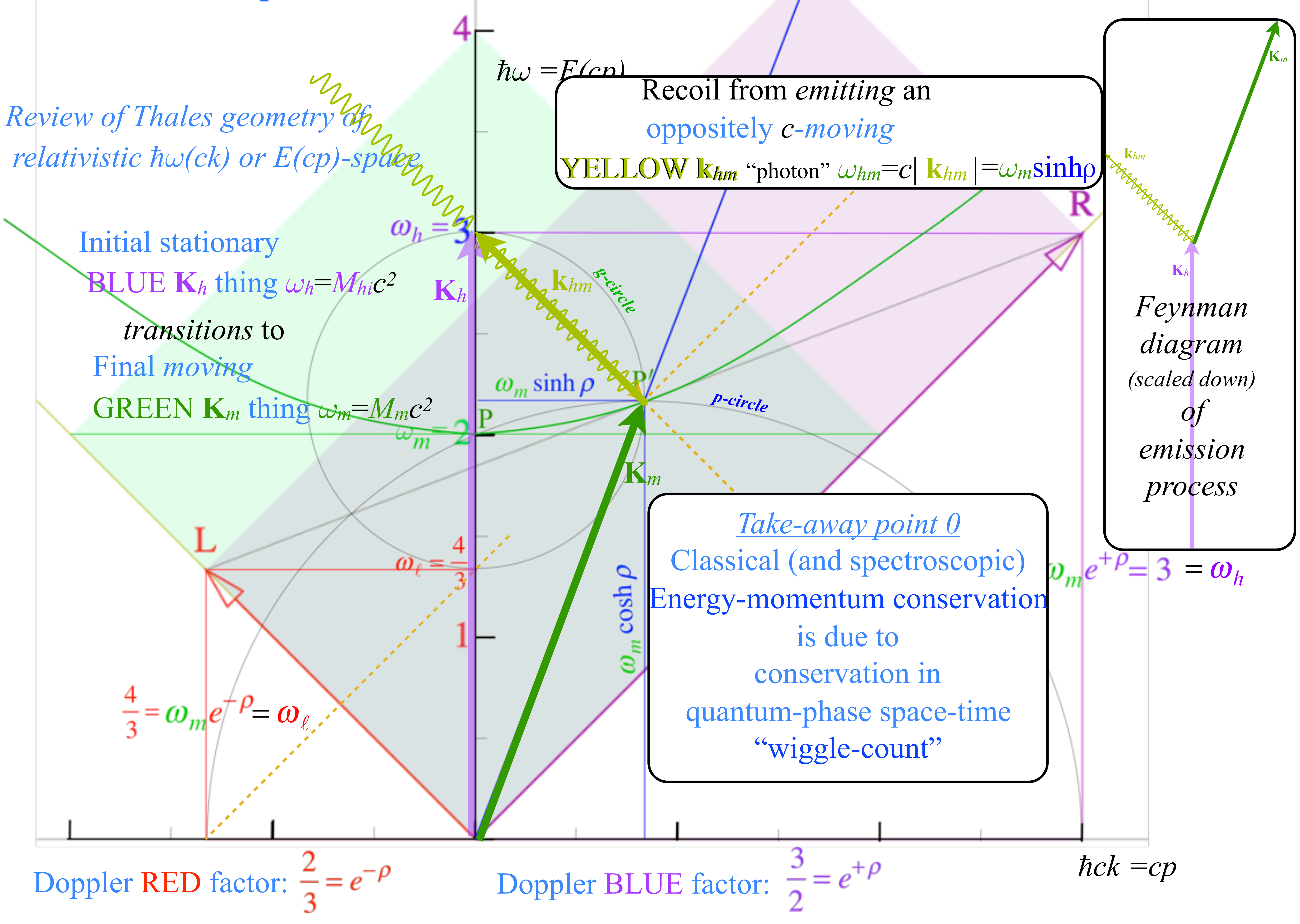
Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$

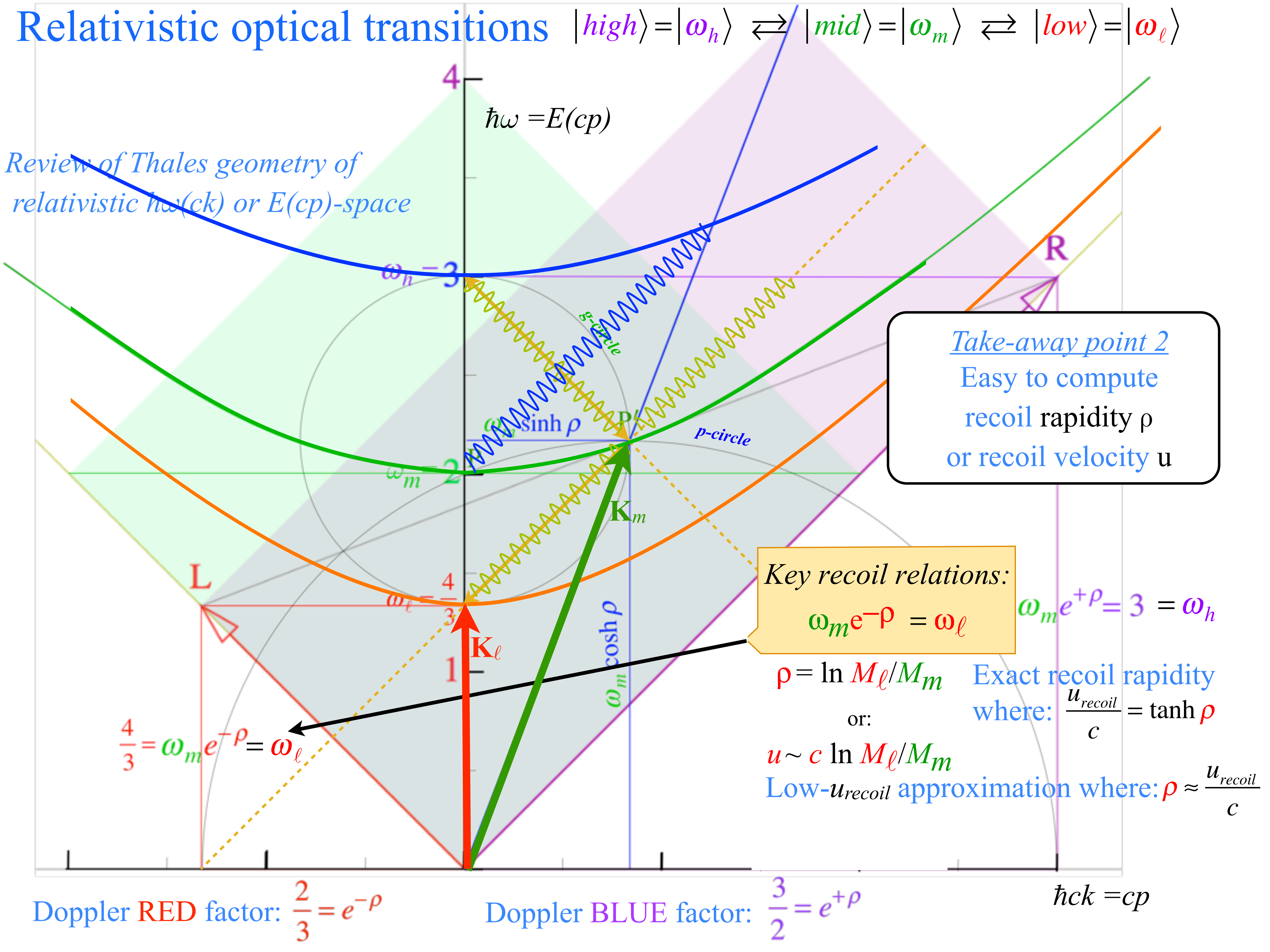


Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$



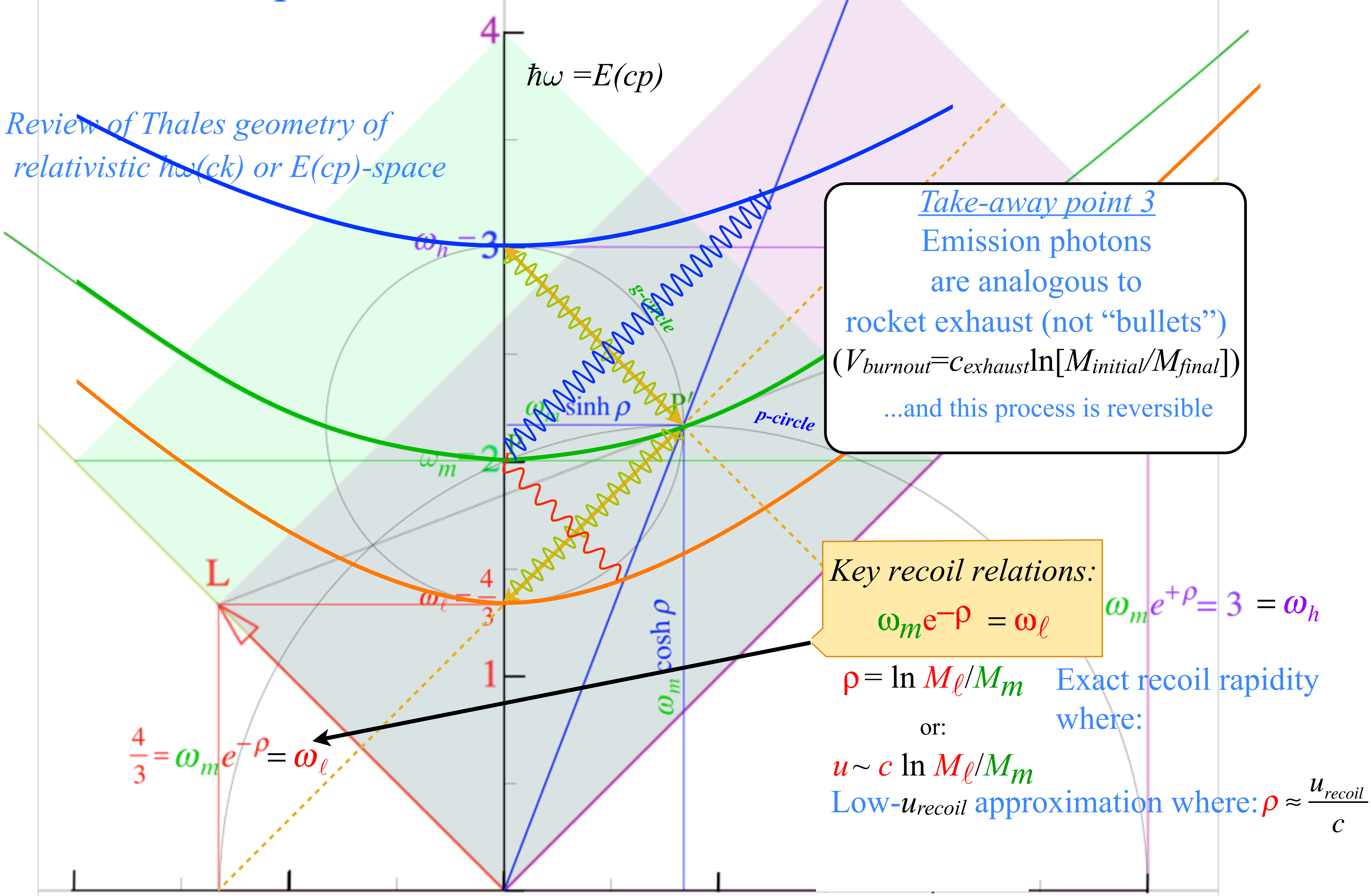
Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$



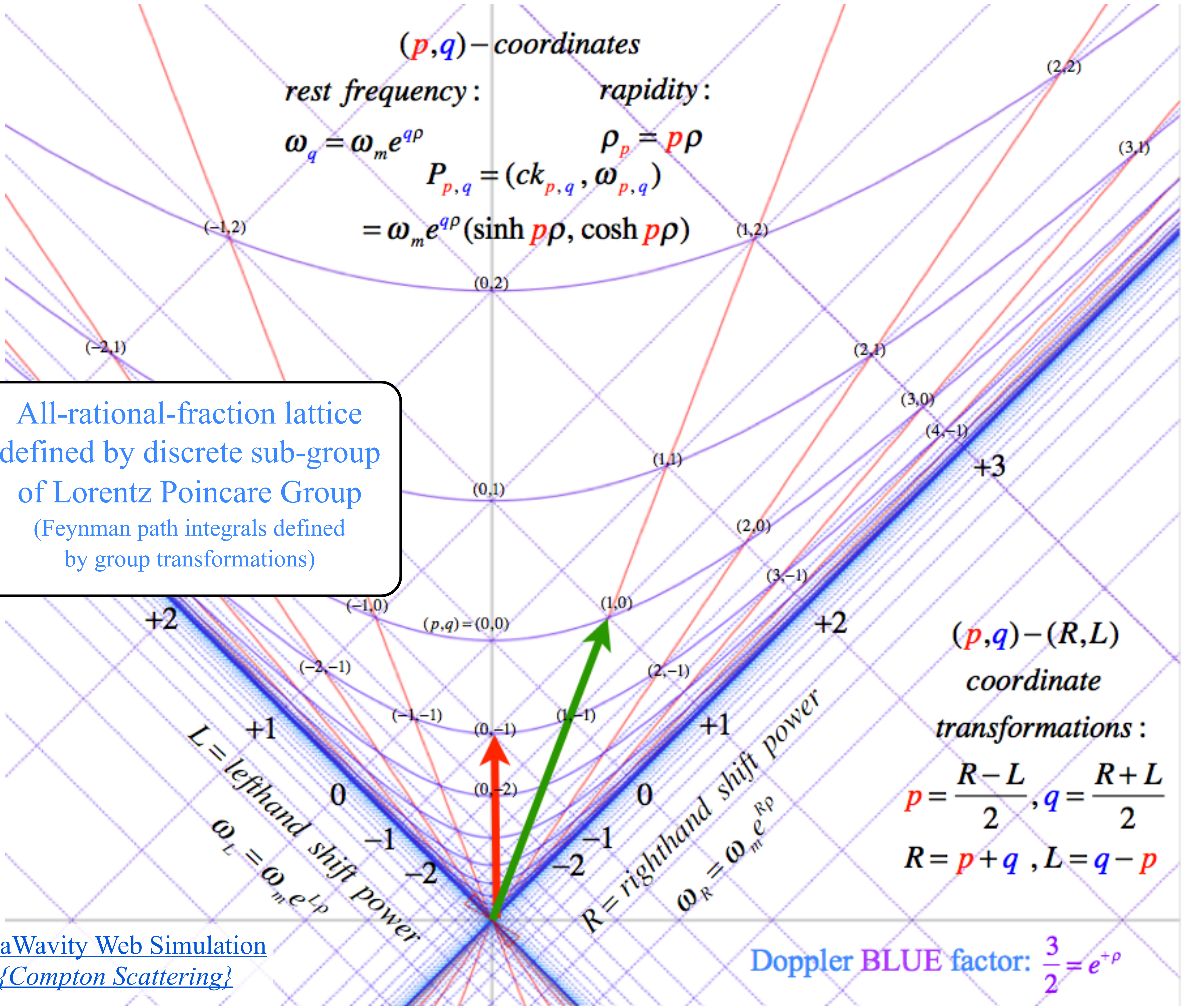


Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$
$\hbar\omega = E(cp)$
Review of Thales geometry of relativistic $\hbar\omega(ck)$ or $E(cp)$-space
$\omega_h = 3$
$\omega_m = 2$
$\omega_\ell = \frac{4}{3}$
4
1
R
L
P
g-circle
p-circle
$\omega_m \sinh\rho$
$\omega_m \cosh\rho$
$\mathbf{K}_m$
$\mathbf{K}_\ell$
Take-away point 2
Easy to compute recoil rapidity ρ or recoil velocity u
Key recoil relations:
$\omega_m e^{-\rho} = \omega_\ell$
$\omega_m e^{+\rho} = 3 = \omega_h$
$\frac{4}{3} = \omega_m e^{-\rho} = \omega_\ell$
$\rho = \ln M_\ell / M_m$
or:
$u \sim c \ln M_\ell / M_m$
Exact recoil rapidity where: $\frac{u_{recoil}}{c} = \tanh\rho$
Low-u_{recoil} approximation where: $\rho \approx \frac{u_{recoil}}{c}$
$\hbar ck = cp$
Doppler RED factor: $\frac{2}{3} = e^{-\rho}$
Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$



Doppler RED factor: $\frac{2}{3} = e^{-\rho}$ Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$



RelaWavity Web Simulation
{Compton Scattering}

Acceleration by chirping laser pairs

Varying acceleration (General case)

From Lect. 35
ModPhys (2012)

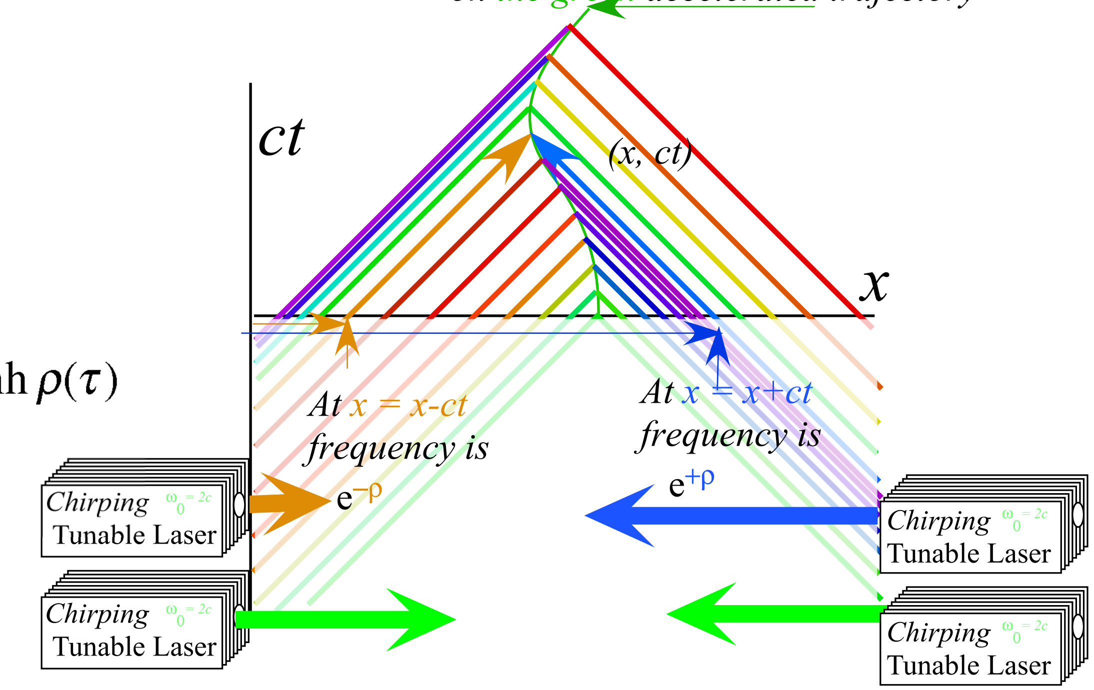
Varying local acceleration $\rho = \rho(\tau)$

$$u = \frac{dx}{dt} = c\tanh(\tau)$$

$$\frac{dt}{d\tau} = \cosh\rho(\tau) \qquad \frac{dx}{d\tau} = \frac{dx}{dt}\frac{dt}{d\tau} = c\tanh\rho(\tau)\cosh\rho(\tau) = c\sinh\rho(\tau)$$

$$ct = c\int \cosh\rho(\tau)\,d\tau \qquad x = c\int \sinh\rho(\tau)\,d\tau$$

Only green-light is seen by observers on the green accelerated trajectory



Constant local acceleration $\rho = \frac{g\tau}{c}$ "*Einstein Elevator*"

$$ct = c\int \cosh\frac{g\tau}{c}\,d\tau \qquad x = c\int \sinh\frac{g\tau}{c}\,d\tau$$

$$= \frac{c^2}{g}\sinh\frac{g\tau}{c} \qquad = \frac{c^2}{g}\cosh\frac{g\tau}{c}$$

Only green-light is seen by observers on the green constant-g hyberbola

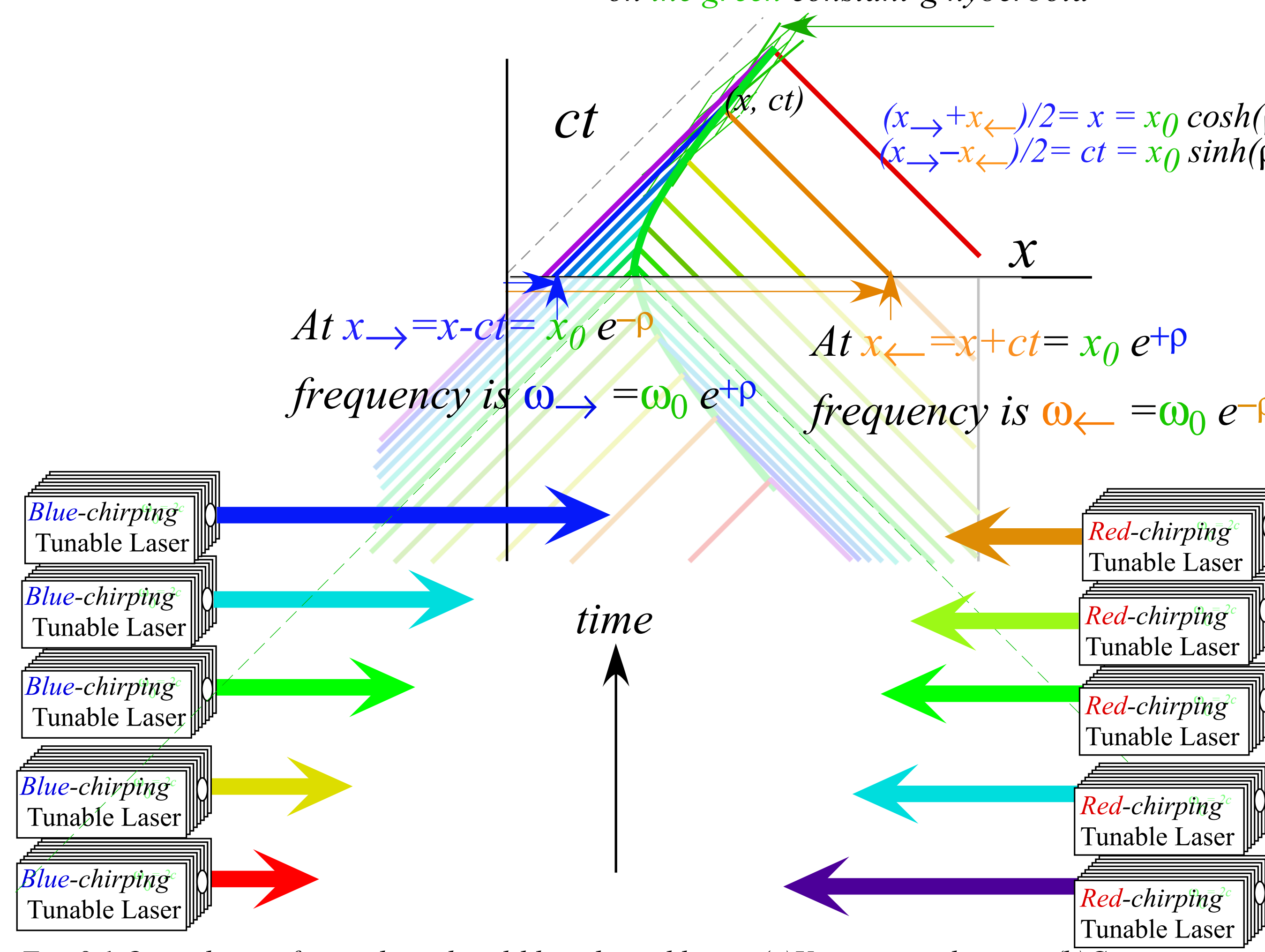


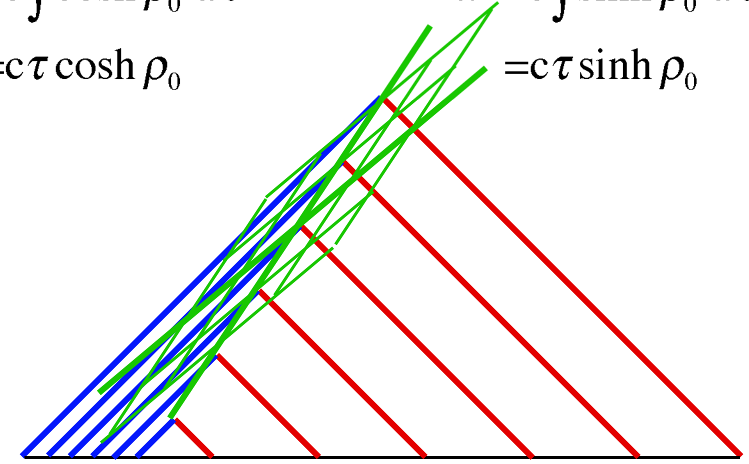
Fig. 8.1 Optical wave frames by red-and-blue-chirped lasers (a)Varying acceleration (b)Constant g

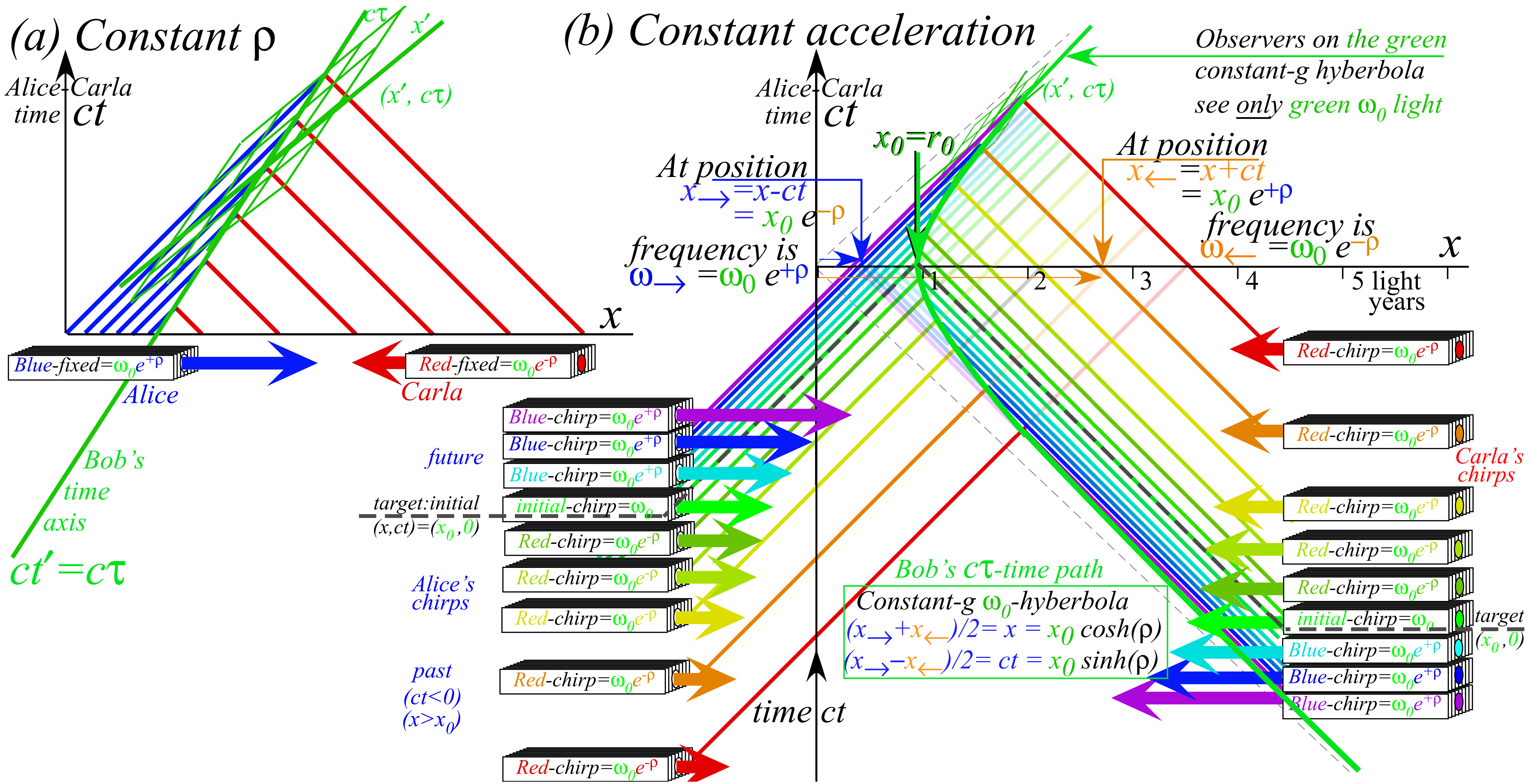
Previous examples involved constant velocity

Constant velocity $\rho = \rho_0 = const.$ "*Lorentztransformation*"

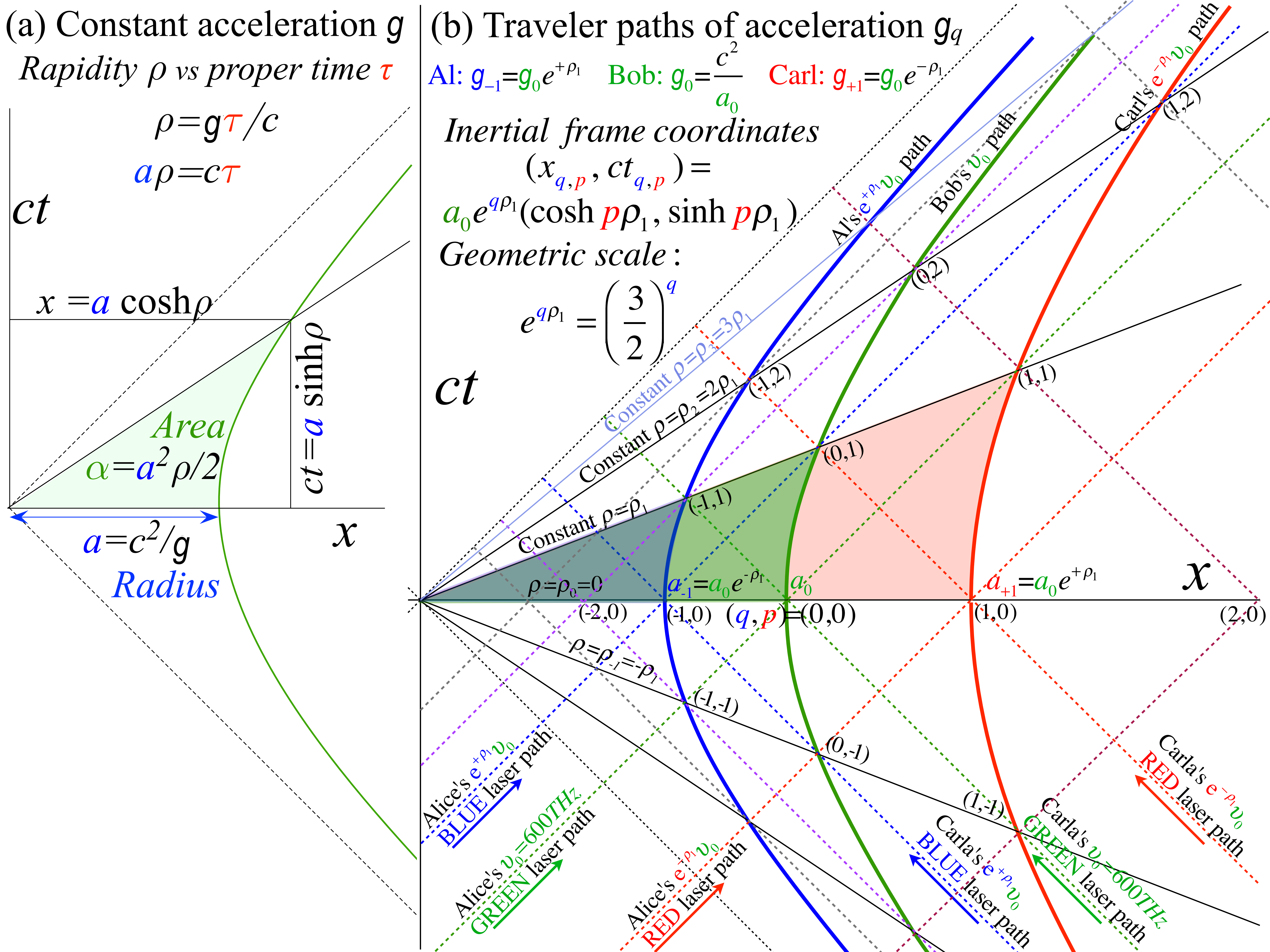
$$ct = c\int \cosh\rho_0\,d\tau \qquad x = c\int \sinh\rho_0\,d\tau$$

$$= c\tau\cosh\rho_0 \qquad = c\tau\sinh\rho_0$$





(a) Constant ρ
Alice-Carla time ct
cτ
x′
(x′, cτ)
x
Blue-fixed=ω0e+ρ
Red-fixed=ω0e-ρ
Alice
Carla
Bob's time axis
ct′=cτ
(b) Constant acceleration
Observers on the green constant-g hyberbola see only green ω0 light
Alice-Carla time ct
(x′, cτ)
x0=r0
At position
x→=x-ct
= x0 e−ρ
frequency is
ω→ =ω0 e+ρ
At position
x←=x+ct
= x0 e+ρ
frequency is
ω← =ω0 e−ρ
x
1
2
3
4
5 light years
Red-chirp=ω0e-ρ
Blue-chirp=ω0e+ρ
initial-chirp=ω0
future
target:initial
(x,ct)=(x0,0)
Alice's chirps
past
(ct<0)
(x>x0)
Carla's chirps
target
(x0,0)
Bob's cτ-time path
Constant-g ω0-hyberbola
(x→+x←)/2= x = x0 cosh(ρ)
(x→−x←)/2= ct = x0 sinh(ρ)
time ct



(a) Constant acceleration g
Rapidity ρ vs proper time τ
ρ=gτ/c
aρ=cτ
ct
x =a coshρ
ct=a sinhρ
Area
α=a²ρ/2
a=c²/g
Radius
x
(b) Traveler paths of acceleration g_q
Al: g_-1=g_0 e^+ρ1
Bob: g_0=c²/a_0
Carl: g_+1=g_0 e^-ρ1
Inertial frame coordinates
(x_q,p , ct_q,p) =
a_0 e^qρ1 (cosh pρ1 , sinh pρ1)
Geometric scale :
e^qρ1 = (3/2)^q
ct
x
Al's e^+ρ1 υ0 path
Bob's υ0 path
Carl's e^-ρ1 υ0 path
Constant ρ=ρ3=3ρ1
Constant ρ=ρ2=2ρ1
Constant ρ=ρ1
ρ=ρ0=0
ρ=ρ-1=-ρ1
a_-1=a_0 e^-ρ1
a_0
a_+1=a_0 e^+ρ1
(q,p)=(0,0)
(-2,0)
(-1,0)
(1,0)
(2,0)
(-1,1)
(0,1)
(1,1)
(-1,2)
(0,2)
(1,2)
(-1,-1)
(0,-1)
(1,-1)
Alice's e^+ρ1 υ0 BLUE laser path
Alice's υ0=600THz GREEN laser path
Alice's e^-ρ1 υ0 RED laser path
Carla's e^+ρ1 υ0 BLUE laser path
Carla's υ0=600THz GREEN laser path
Carla's e^-ρ1 υ0 RED laser path

[RelativIt Web Simulation {Accelerated proper-time frame}]