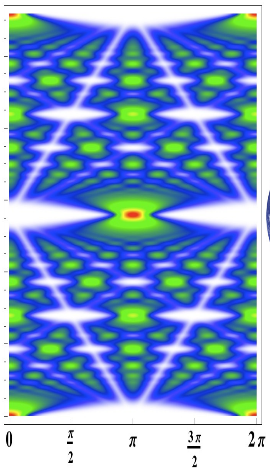
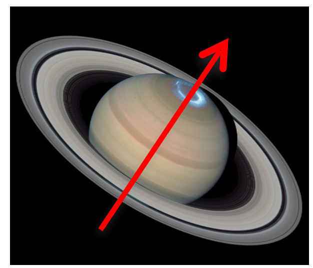
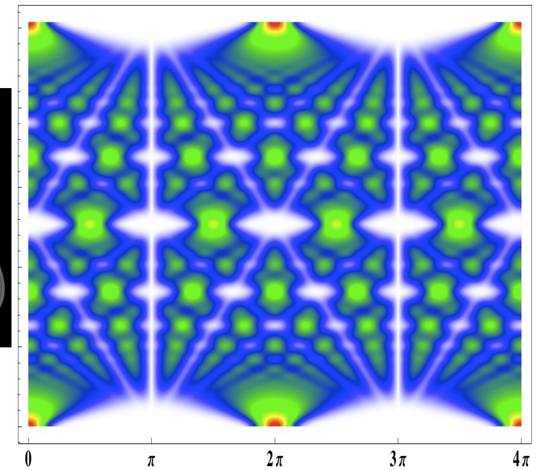
Resonance and Revivals in Quantum Rotors --- Comparing Half-integer Spin and Integer Spin







Alvason Zhenhua Li
William Harter
Microelectronics-Photonics Program, Physics Department
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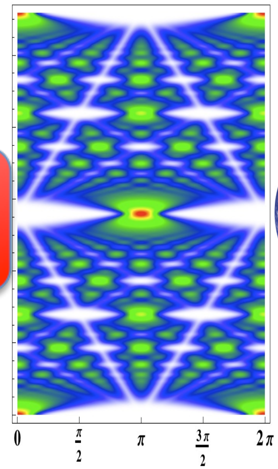
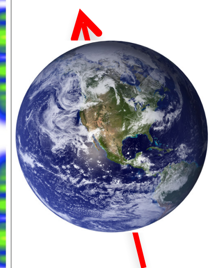
Outline

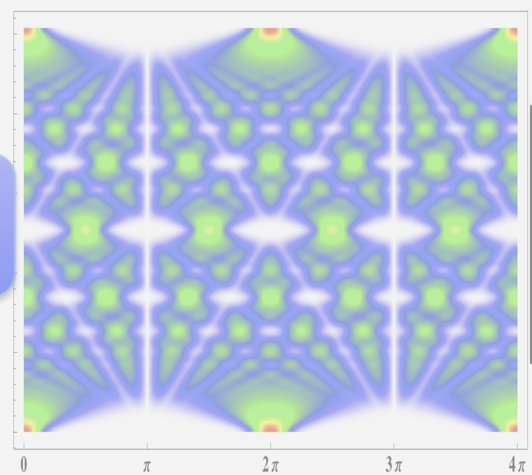
Motivation

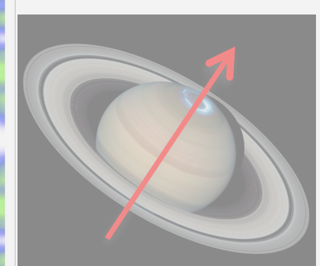
Integer Spin (Symmetric Rotor)





Half-integer Spin (Symmetric Rotor)

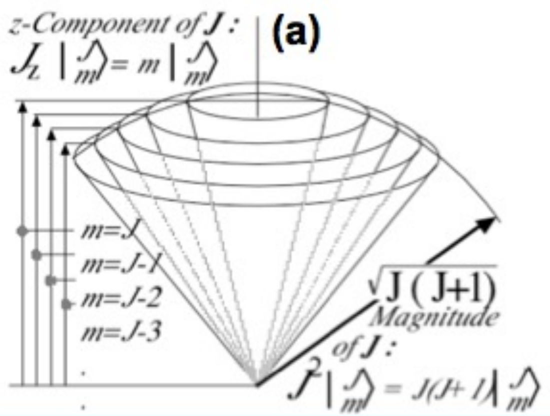




Summary

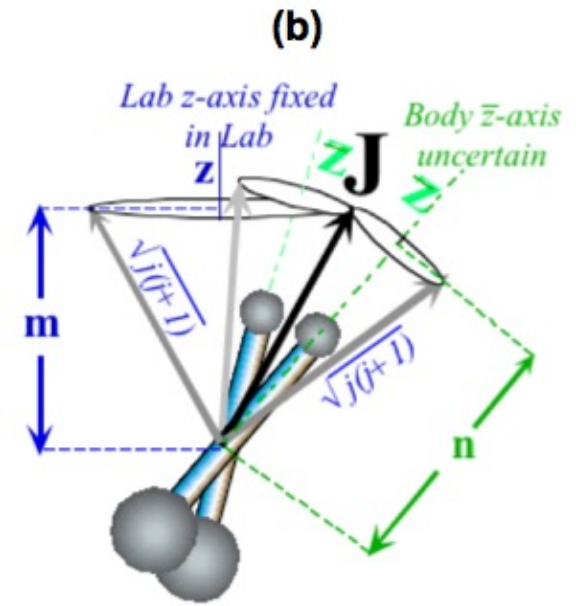
Motivation

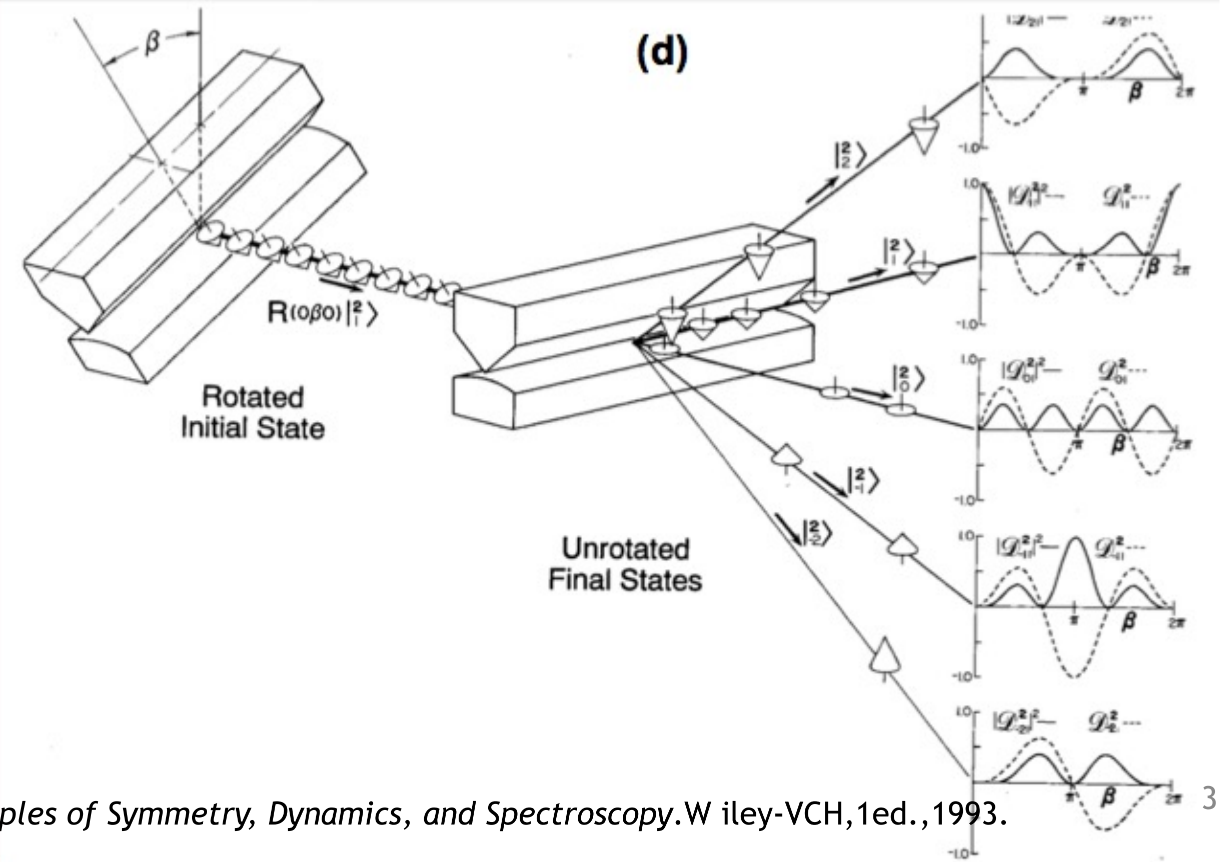
Quantum Rotor



(c)

$$\left\langle {j \atop m} \middle| \mathbf{R}(\alpha\beta\gamma) \middle| {j \atop n} \right\rangle = D^{j}_{m,n}(\alpha\beta\gamma) = \sqrt{(j+n)!(j-n)!}\sqrt{(j+m)!(j-m)!}\,\frac{\sum_k (-1)^k \left(\cos\frac{\beta}{2}\right)^{2j+m-n-2k}\left(\sin\frac{\beta}{2}\right)^{n-m+2k} e^{-i(m\alpha+n\gamma)}}{(j+m-k)!(n-m+k)!k!(j-n-k)!}$$

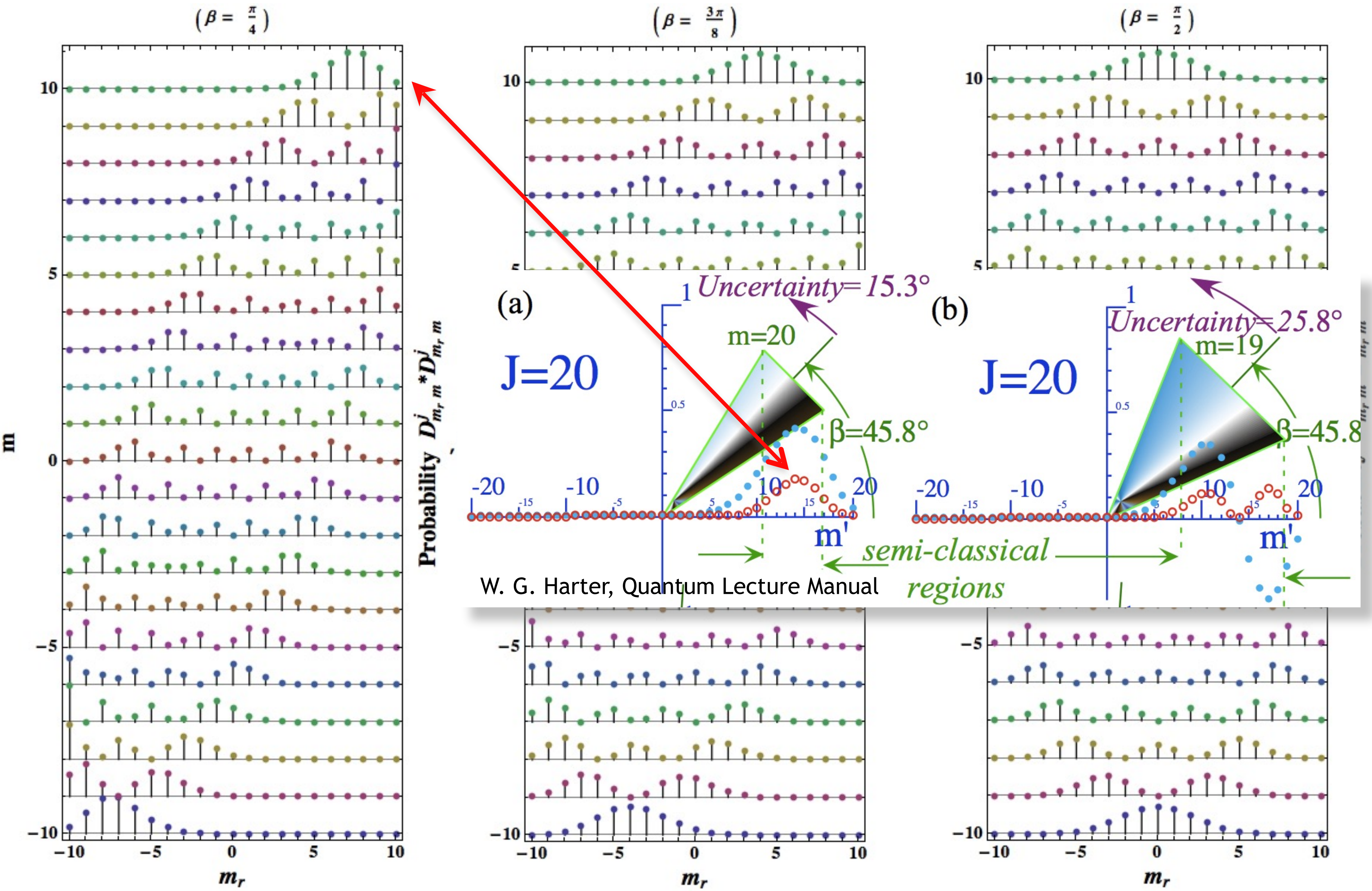




W. G. Harter, *Principles of Symmetry, Dynamics, and Spectroscopy*.W iley-VCH,1ed.,1993.

List-plot

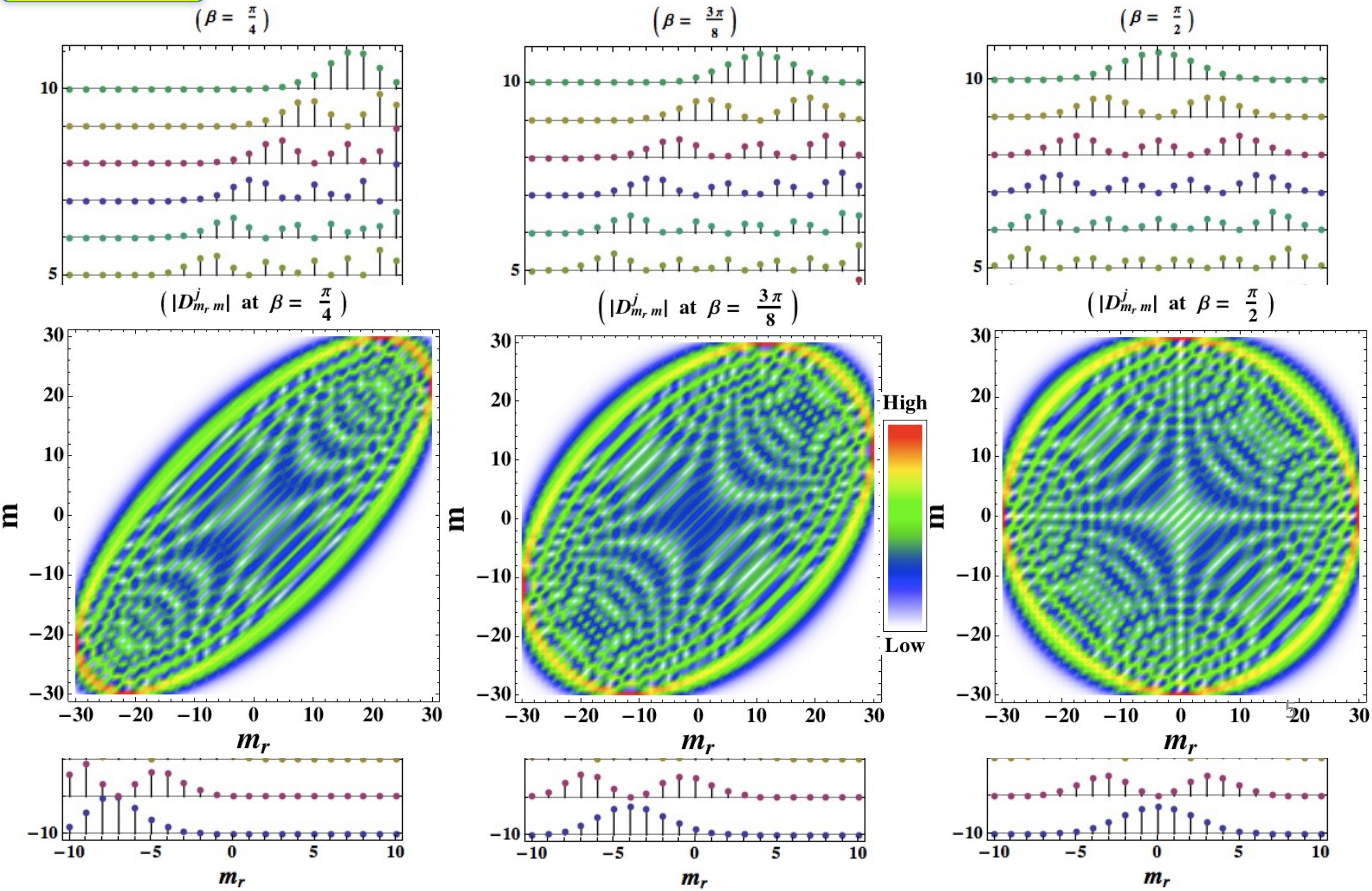
Wigner-D Matrix ---- a Rotational Matrix for Any Spin



A. Z. Li, Quantum Resonant Beats and Revivals in the Morse Oscillators and Rotors, Ph.D. thesis, University of Arkansas (2013).

Density-plot

Wigner-D Matrix ---- a Rotational Matrix for Any Spin



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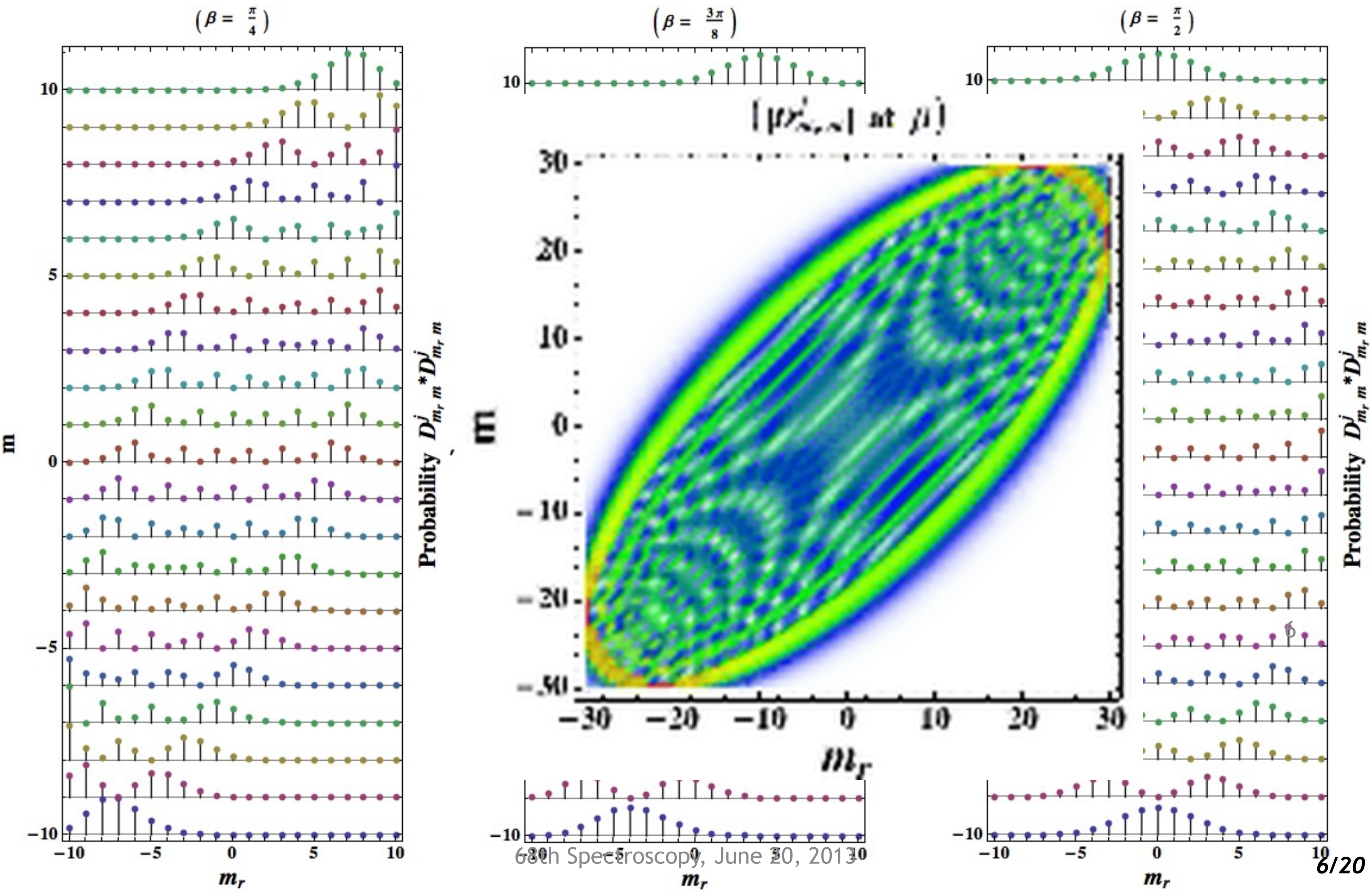
Wigner-D Matrix ---- a Rotational Matrix for Any Spin

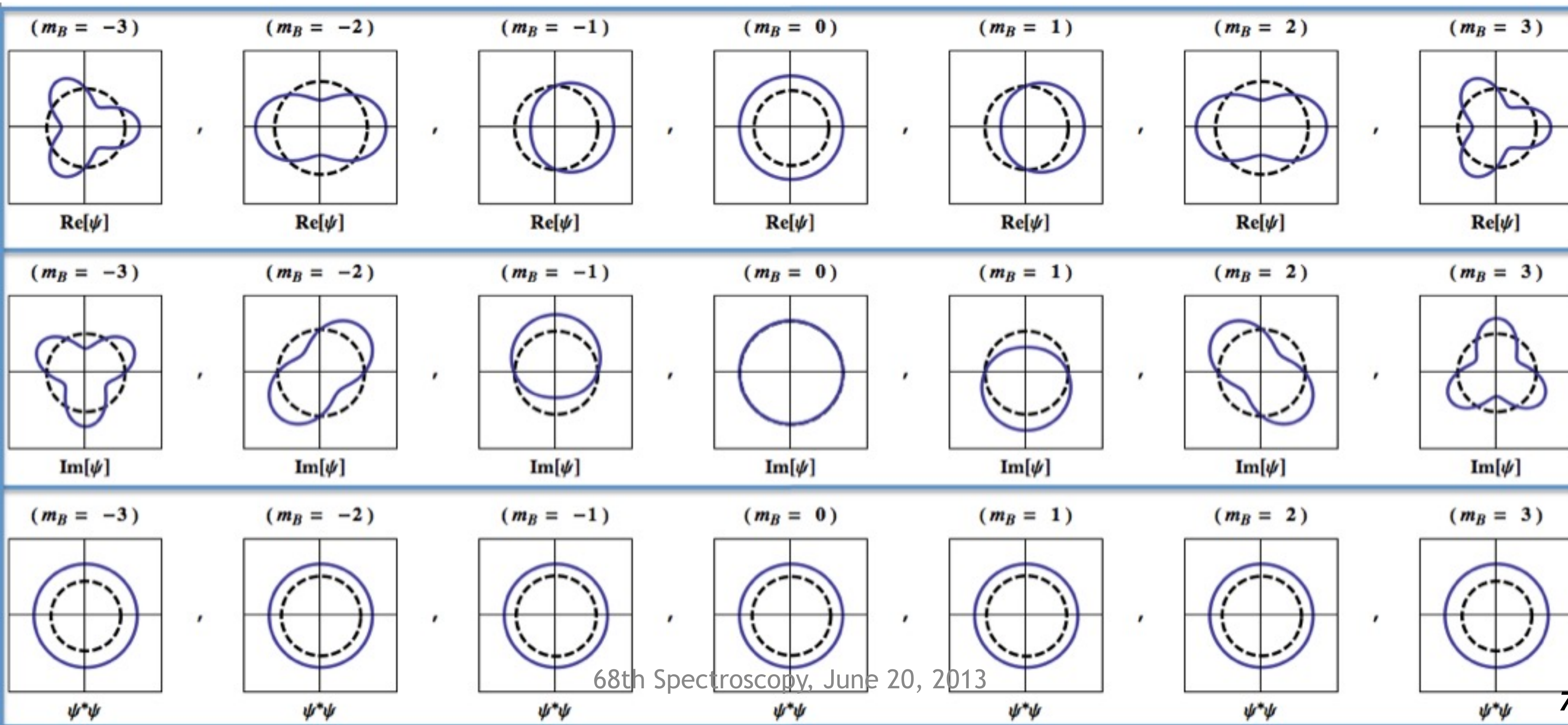


J=3

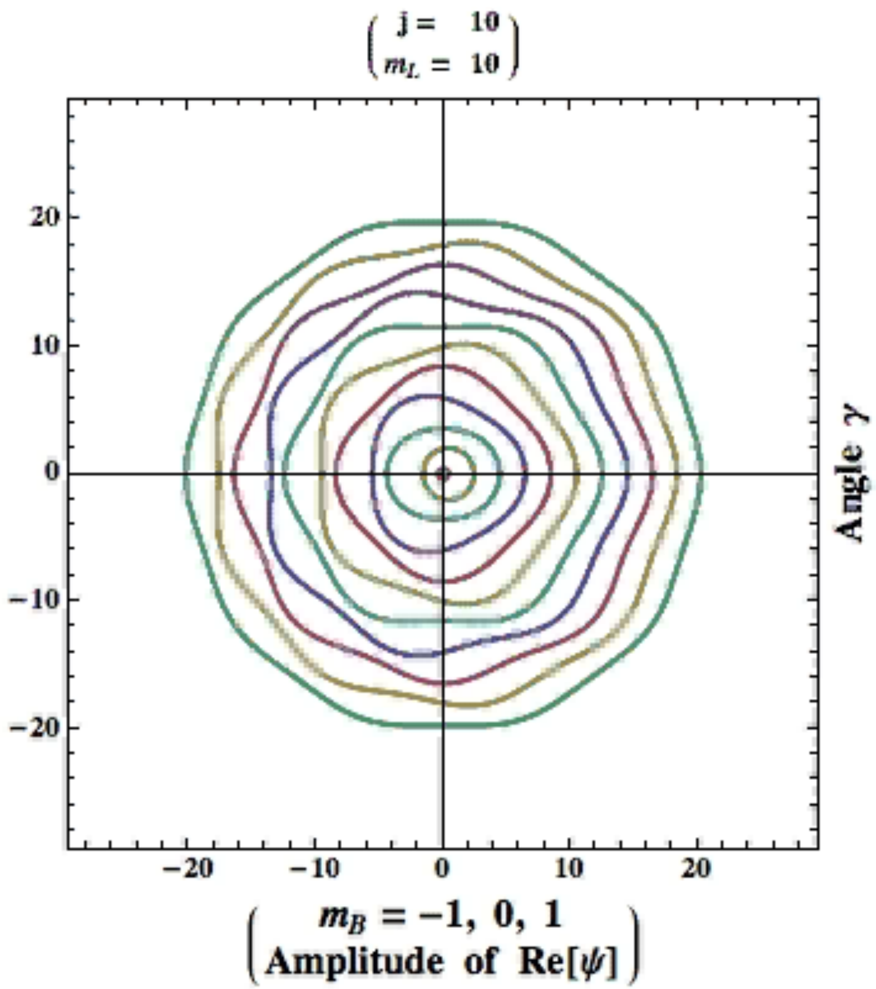
Rotor Wave Functions of Integer Spin (Boson) System

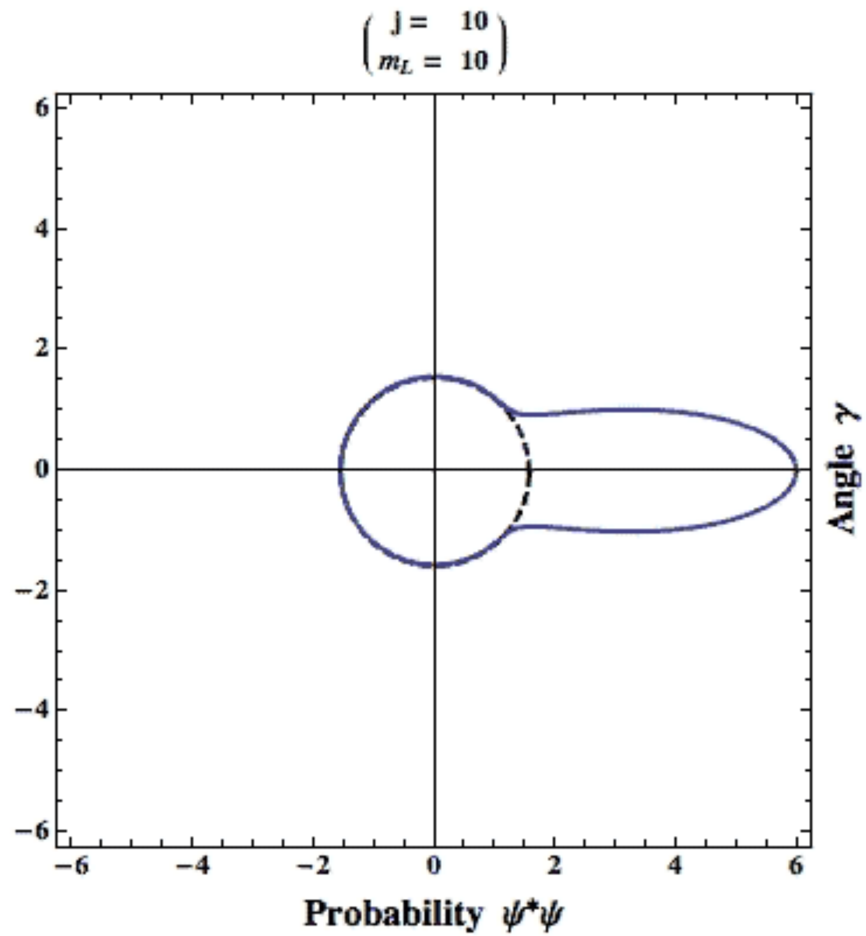
$$\left|{}^{\mathbf{j}}_{\mathbf{m_L,m_B}}\right\rangle = \frac{\mathbf{P^j_{m_L,m_B}}\,|0,0,0\rangle}{\sqrt{2\mathbf{j}+1}} = \frac{1}{\mathbf{N}}\int \mathbf{d}(\alpha,\beta,\gamma)\,\mathbf{D^j_{m_L,m_B}}^{*}(\alpha,\beta,\gamma)\,\mathbf{R}(\alpha,\beta,\gamma)\,|\mathbf{0,0,0}\rangle$$

$$= \frac{\sqrt{2\mathbf{j}+1}}{8\pi^2}\int_0^{2\pi}\mathbf{d}\alpha\int_0^{\pi}\sin\beta\,\mathbf{d}\beta\int_0^{2\pi}\mathbf{d}\gamma\,\mathbf{D^j_{m_L,m_B}}^{*}(\alpha,\beta,\gamma)\;|\alpha,\beta,\gamma\rangle \qquad (4.8)$$



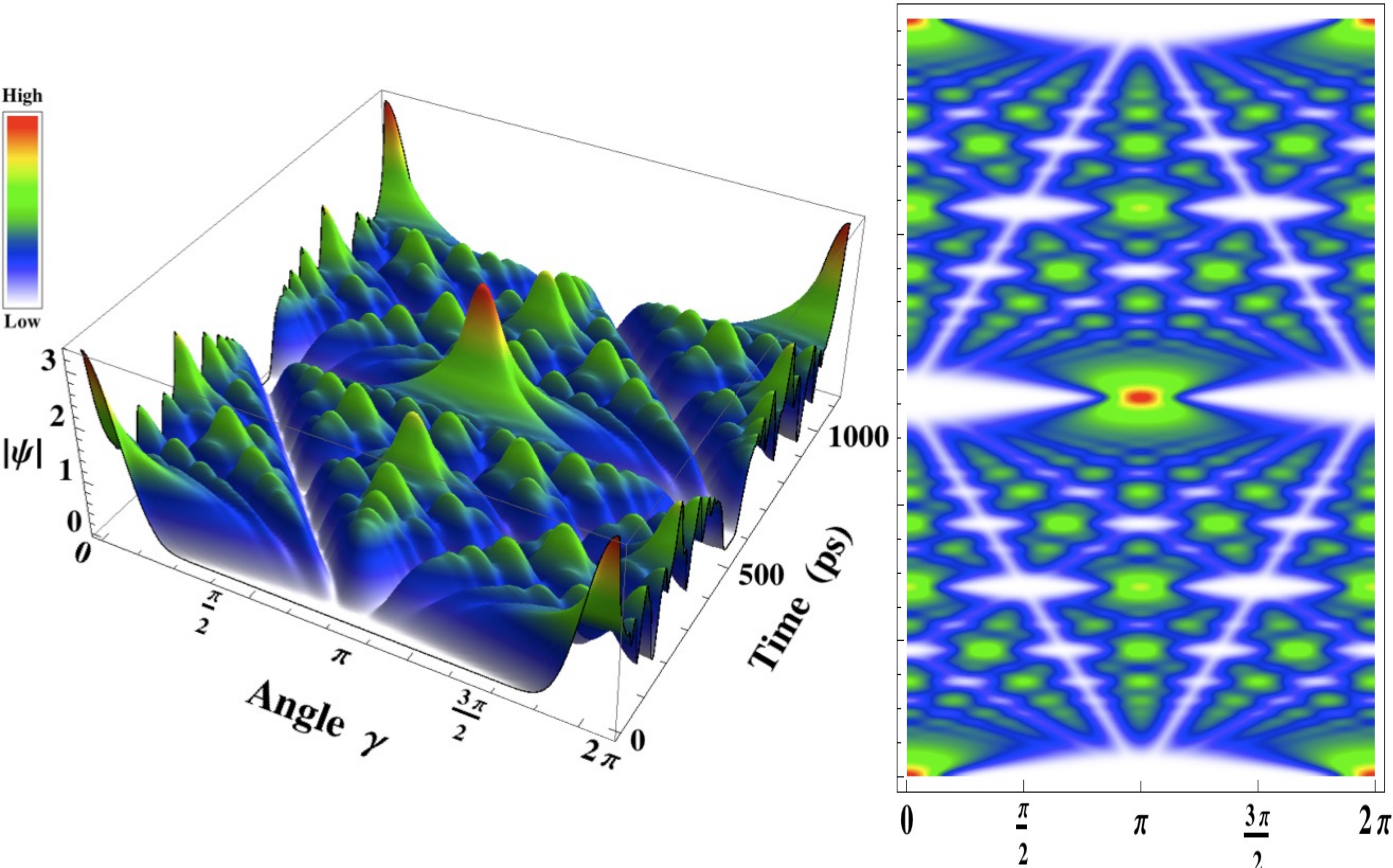
Dynamics of Integer Spin (Boson) System





Space-Time-plot

Rotor Wave Packet Propagation of Integer Spin (Boson) System



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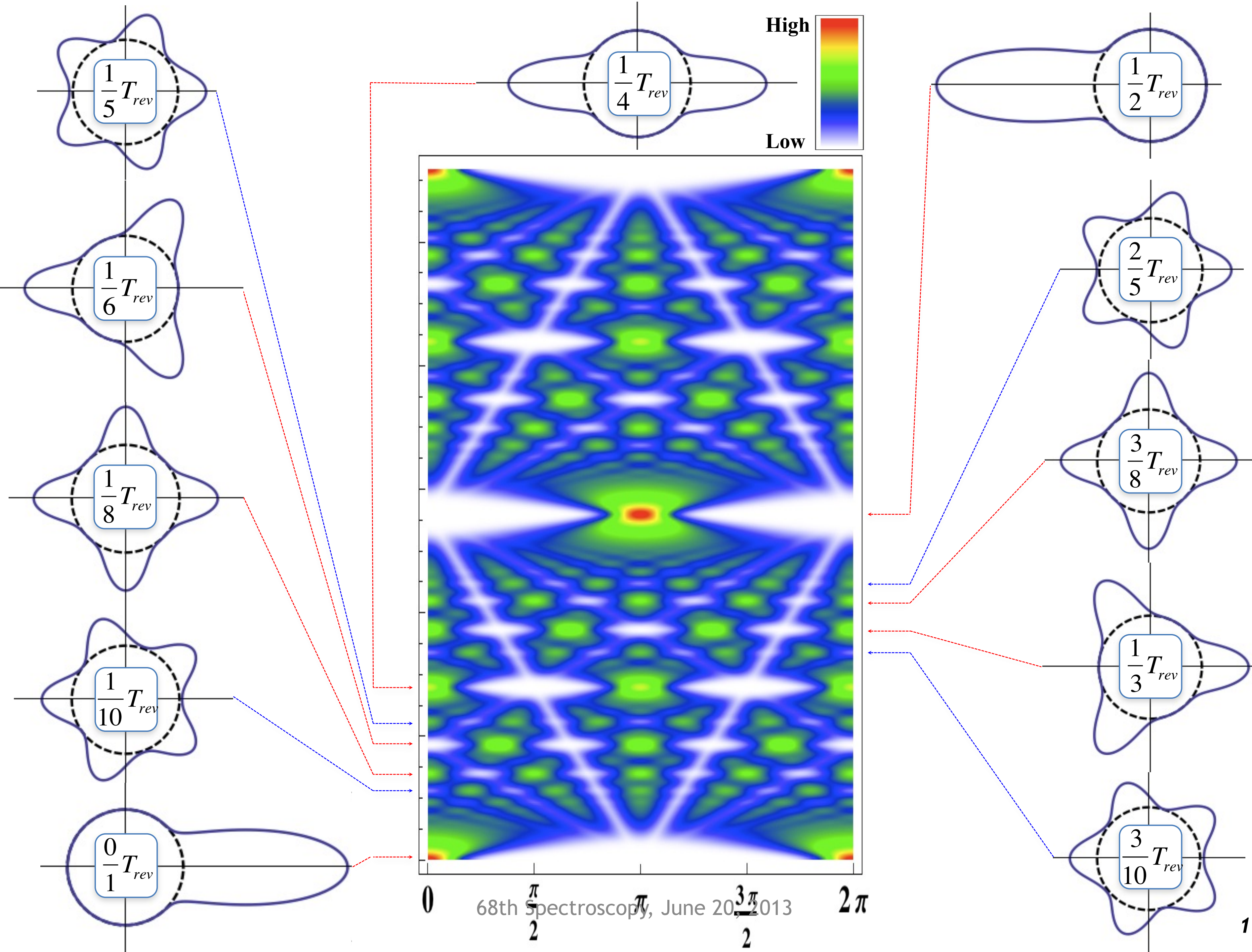
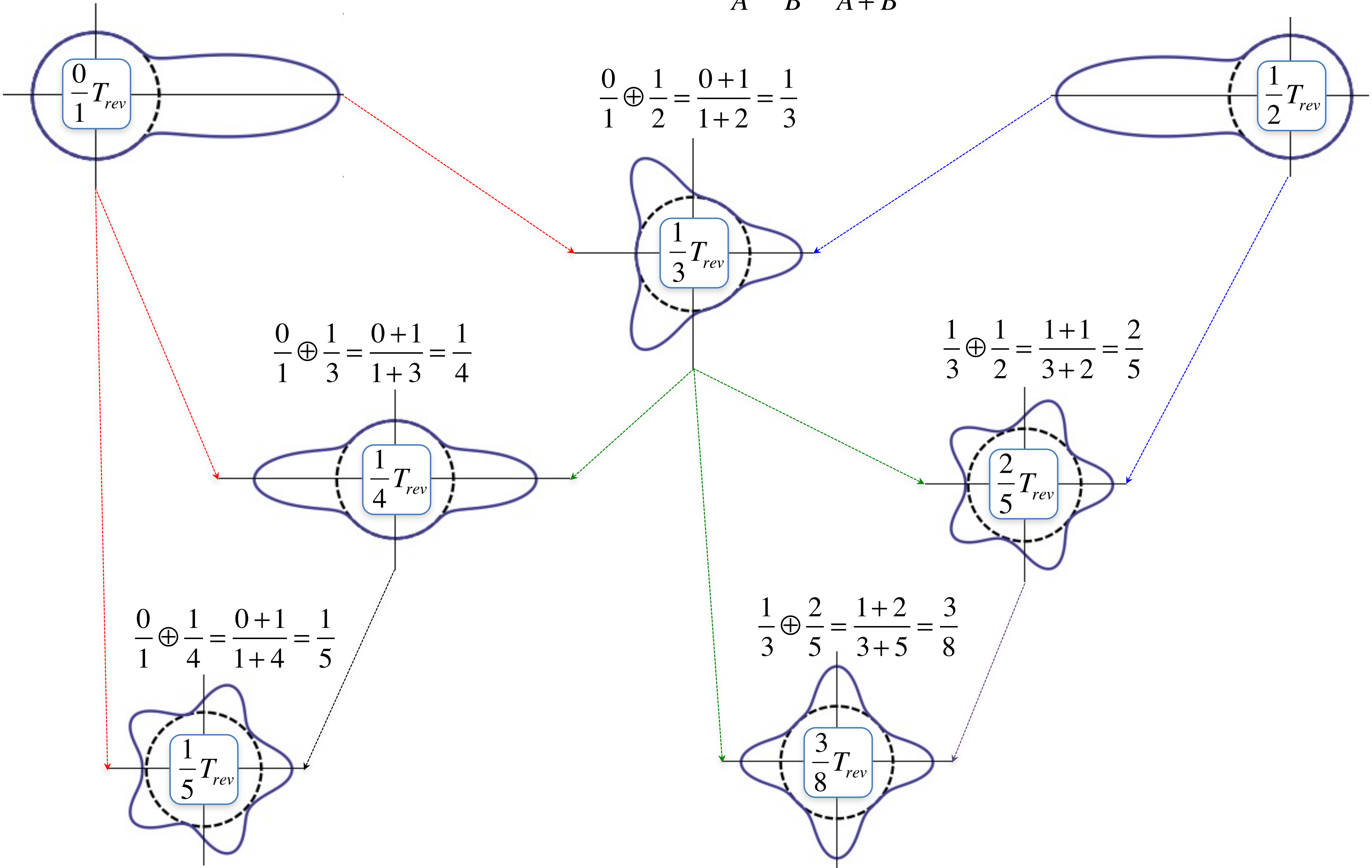

High
Low
$\frac{1}{5}T_{rev}$
$\frac{1}{4}T_{rev}$
$\frac{1}{2}T_{rev}$
$\frac{1}{6}T_{rev}$
$\frac{2}{5}T_{rev}$
$\frac{1}{8}T_{rev}$
$\frac{3}{8}T_{rev}$
$\frac{1}{10}T_{rev}$
$\frac{1}{3}T_{rev}$
$\frac{0}{1}T_{rev}$
$\frac{3}{10}T_{rev}$
0
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π

Half-integer Spinning Rotors exhibit Farey-sum Revivals

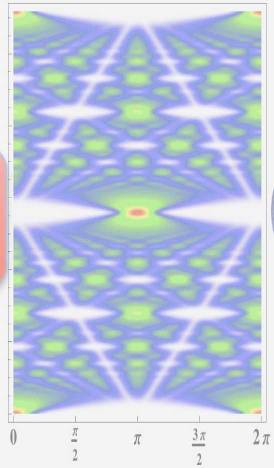
$$Farey-sum-Rule: \frac{a}{A} \oplus \frac{b}{B} = \frac{a+b}{A+B}$$

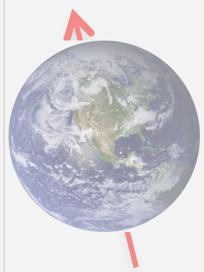


Outline

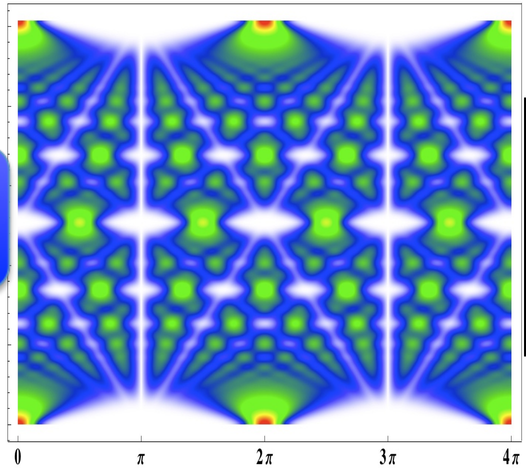
Motivation

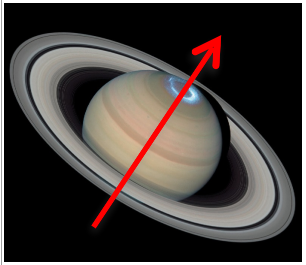
Integer Spin (Symmetric Rotor)





Half-integer Spin (Symmetric Rotor)

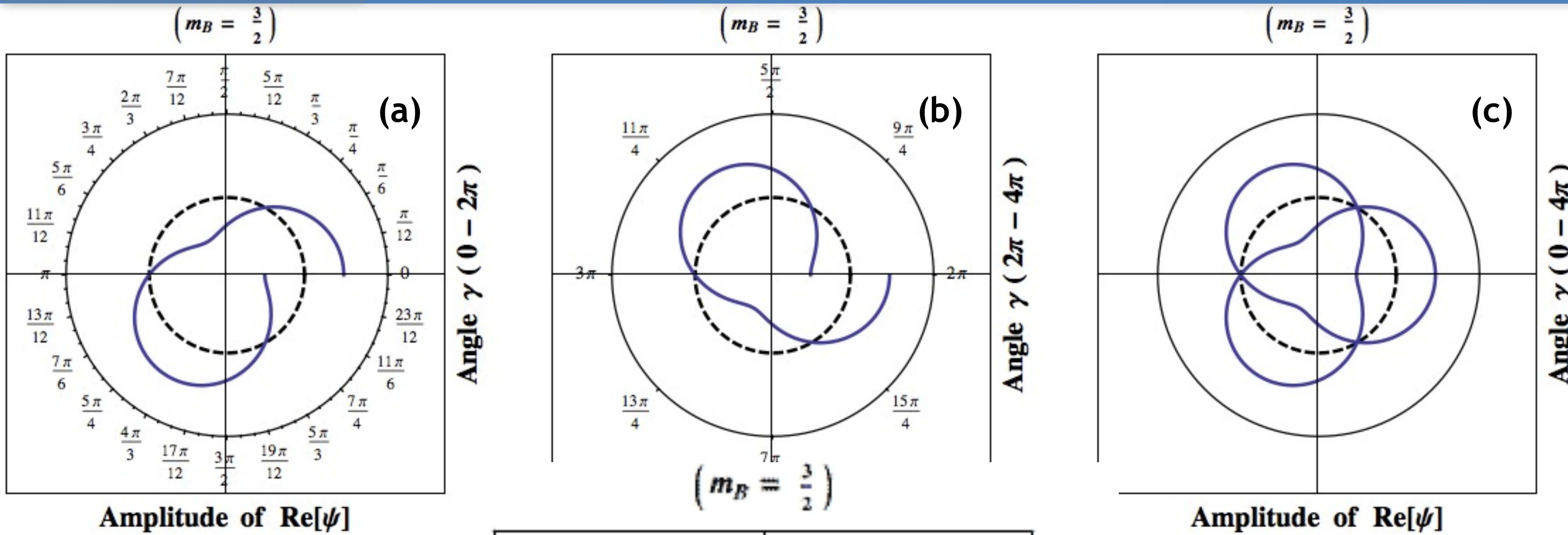


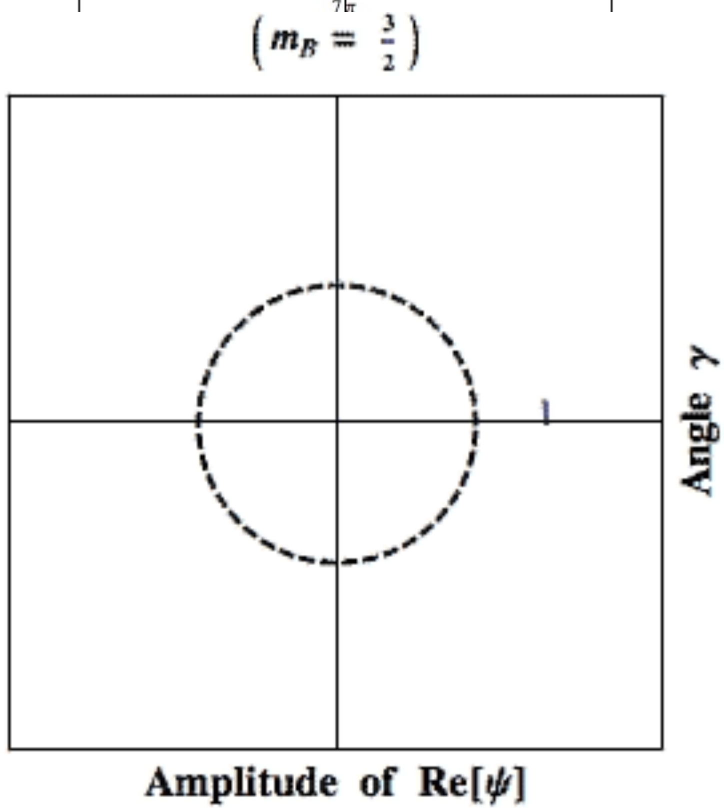


Summary

Unique 4π Rotation

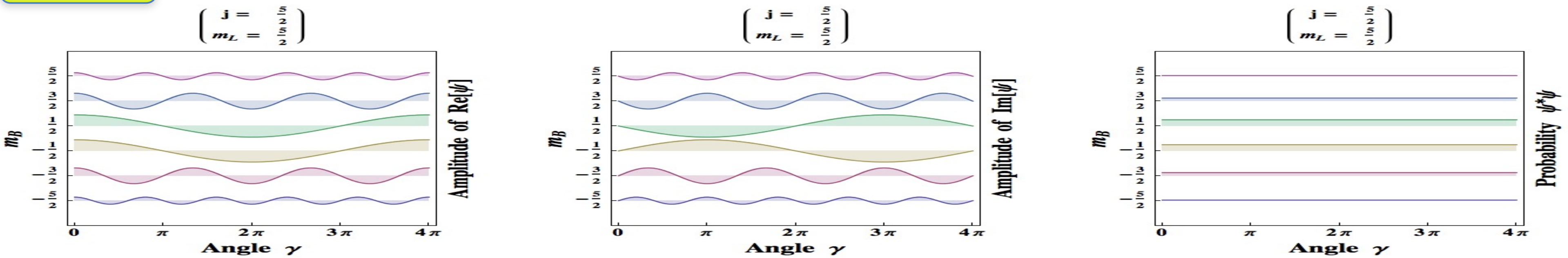
Rotor Wave Functions of Half-Integer Spin (Fermion) System

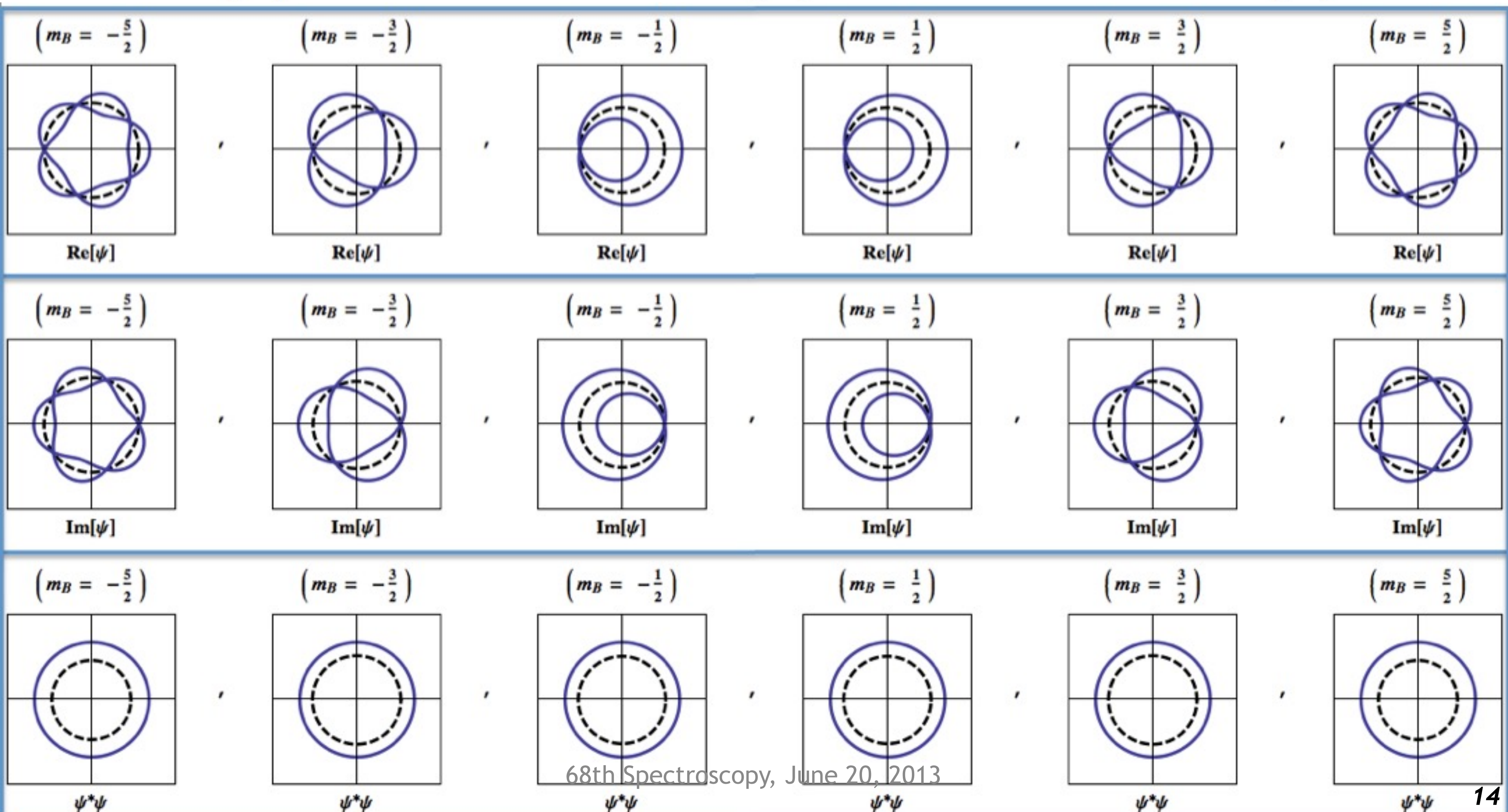




J=5/2

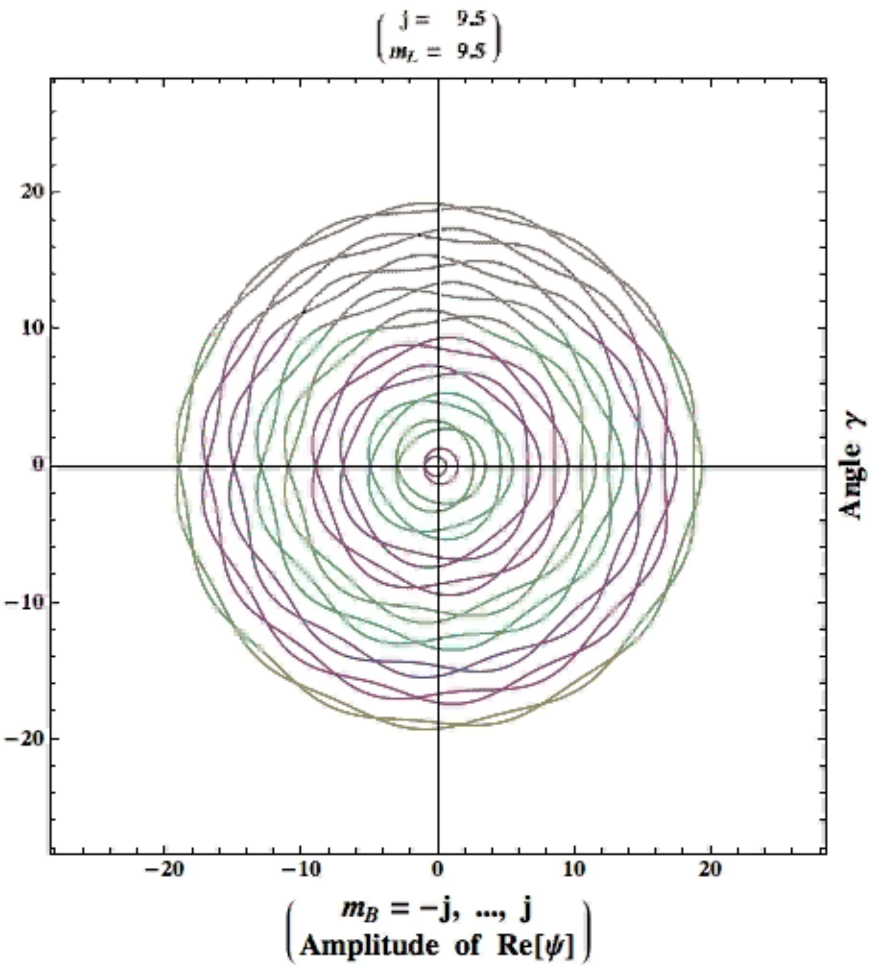
Rotor Wave Functions of Half-Integer Spin (Fermion) System

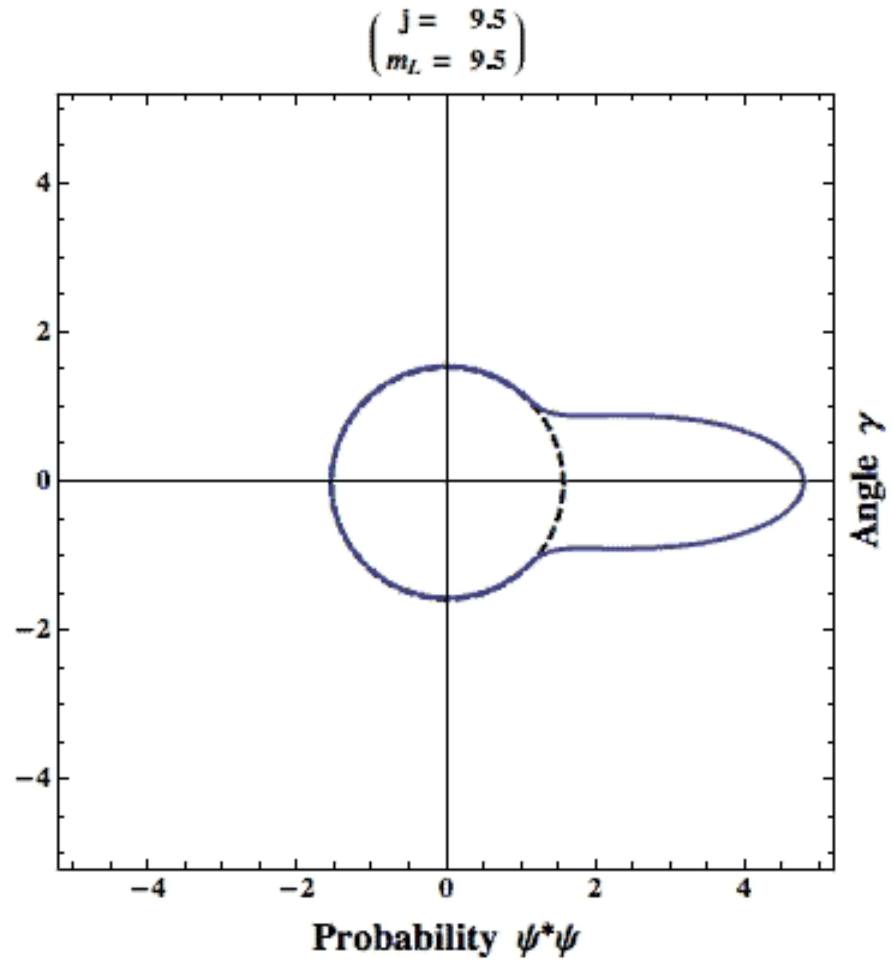




J=9.5

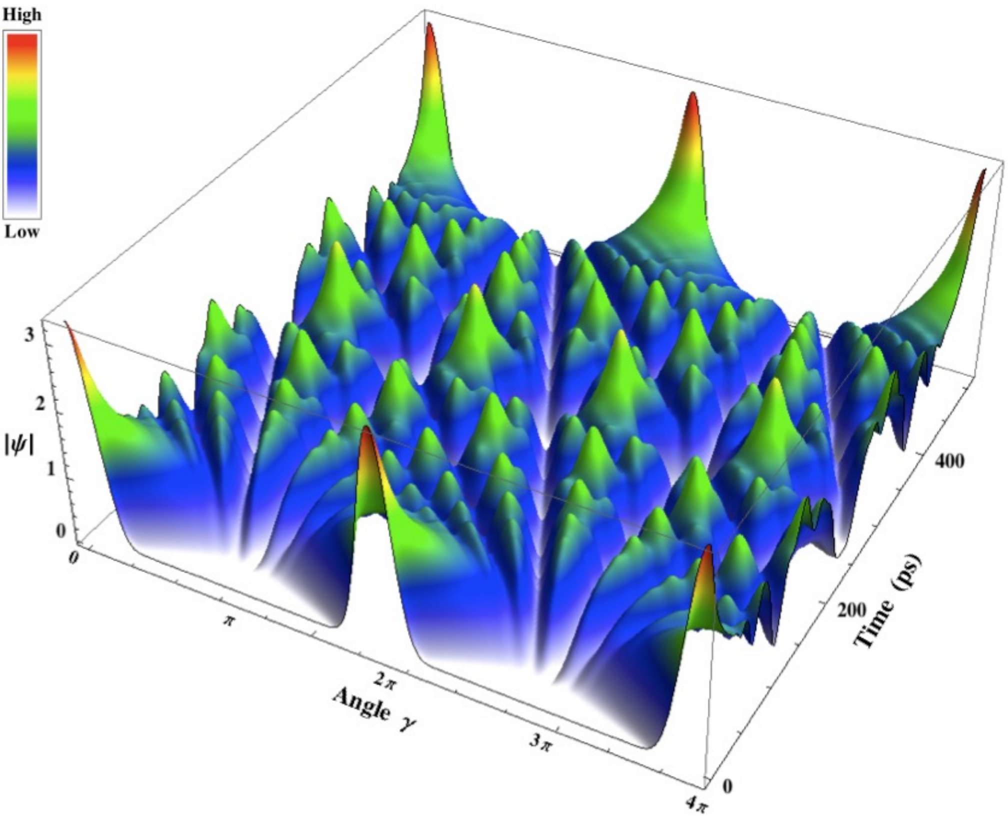
Dynamics of Integer Spin (Fermion) System

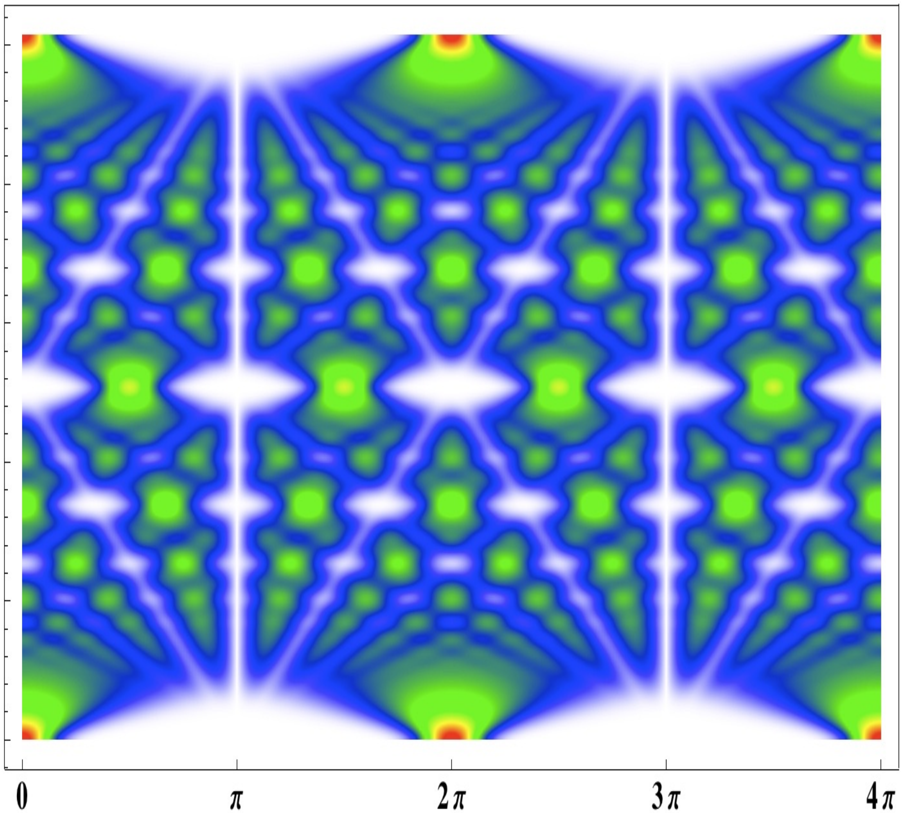




Space-Time-plot

Rotor Wave Packet Propagation of Half-integer Spin (Fermion) System





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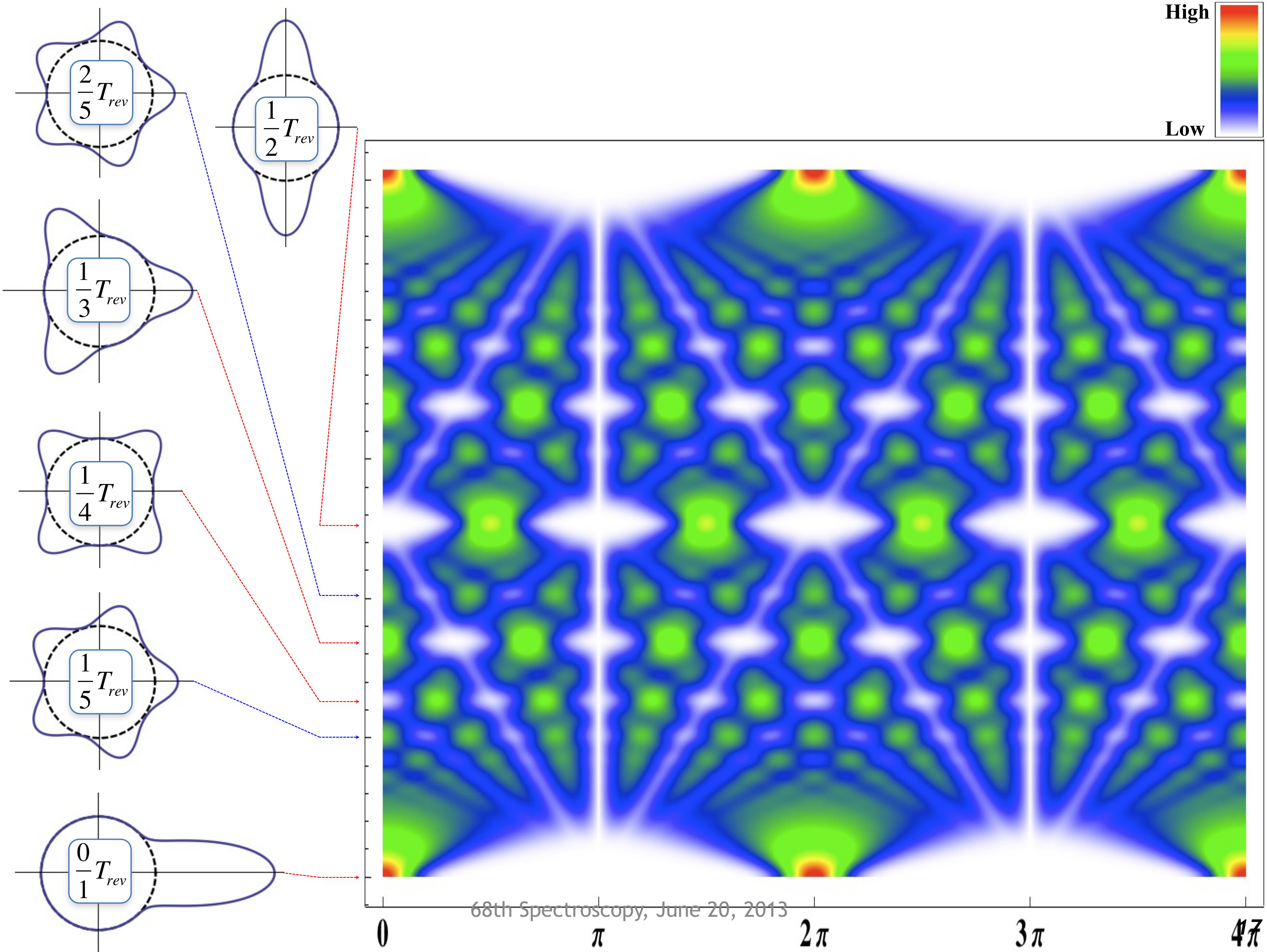

High
Low
$\frac{2}{5}T_{rev}$
$\frac{1}{2}T_{rev}$
$\frac{1}{3}T_{rev}$
$\frac{1}{4}T_{rev}$
$\frac{1}{5}T_{rev}$
$\frac{0}{1}T_{rev}$
0
π
2π
3π
4π

Half-integer Spinning Rotors exhibit Farey-sum Revivals

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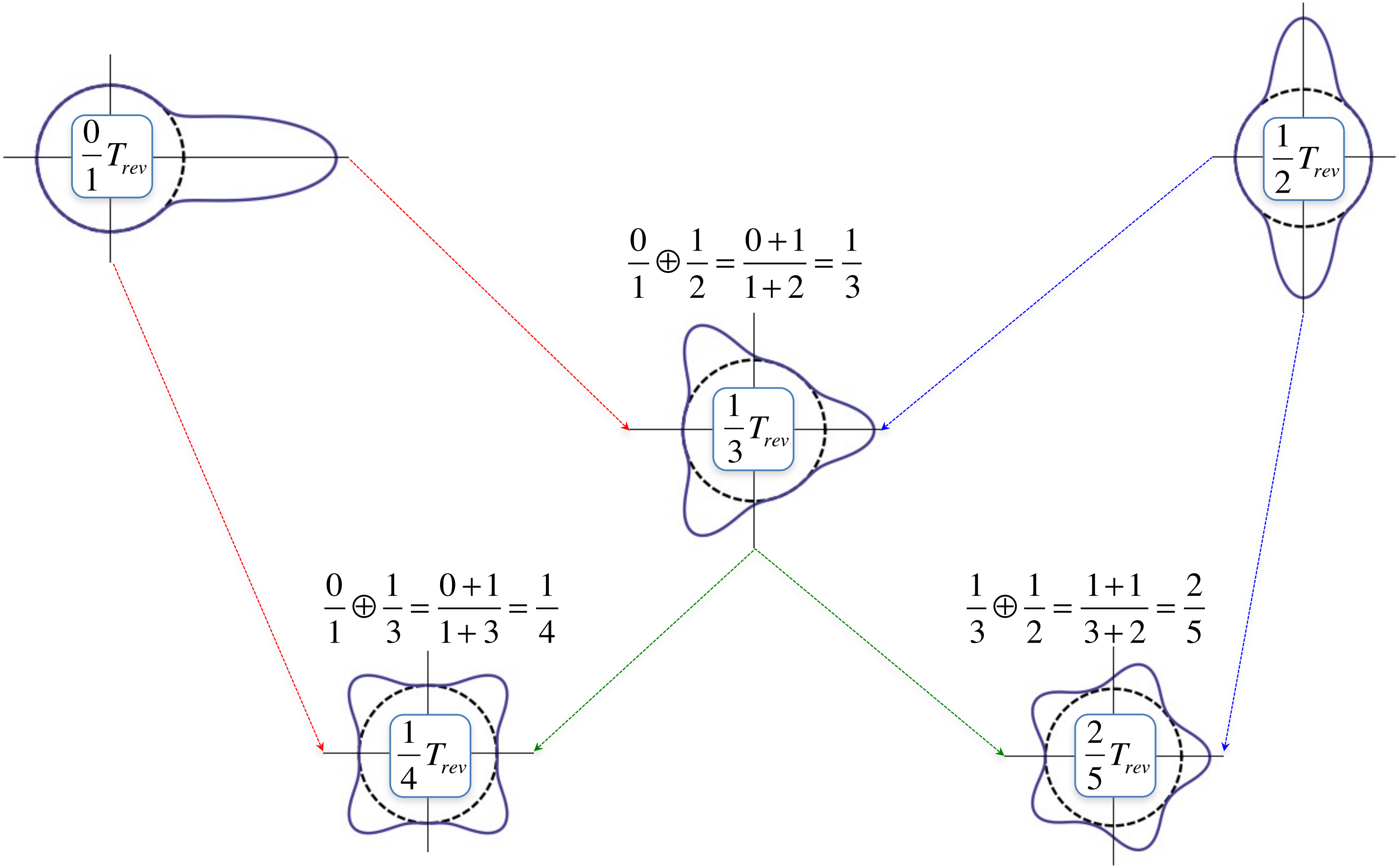
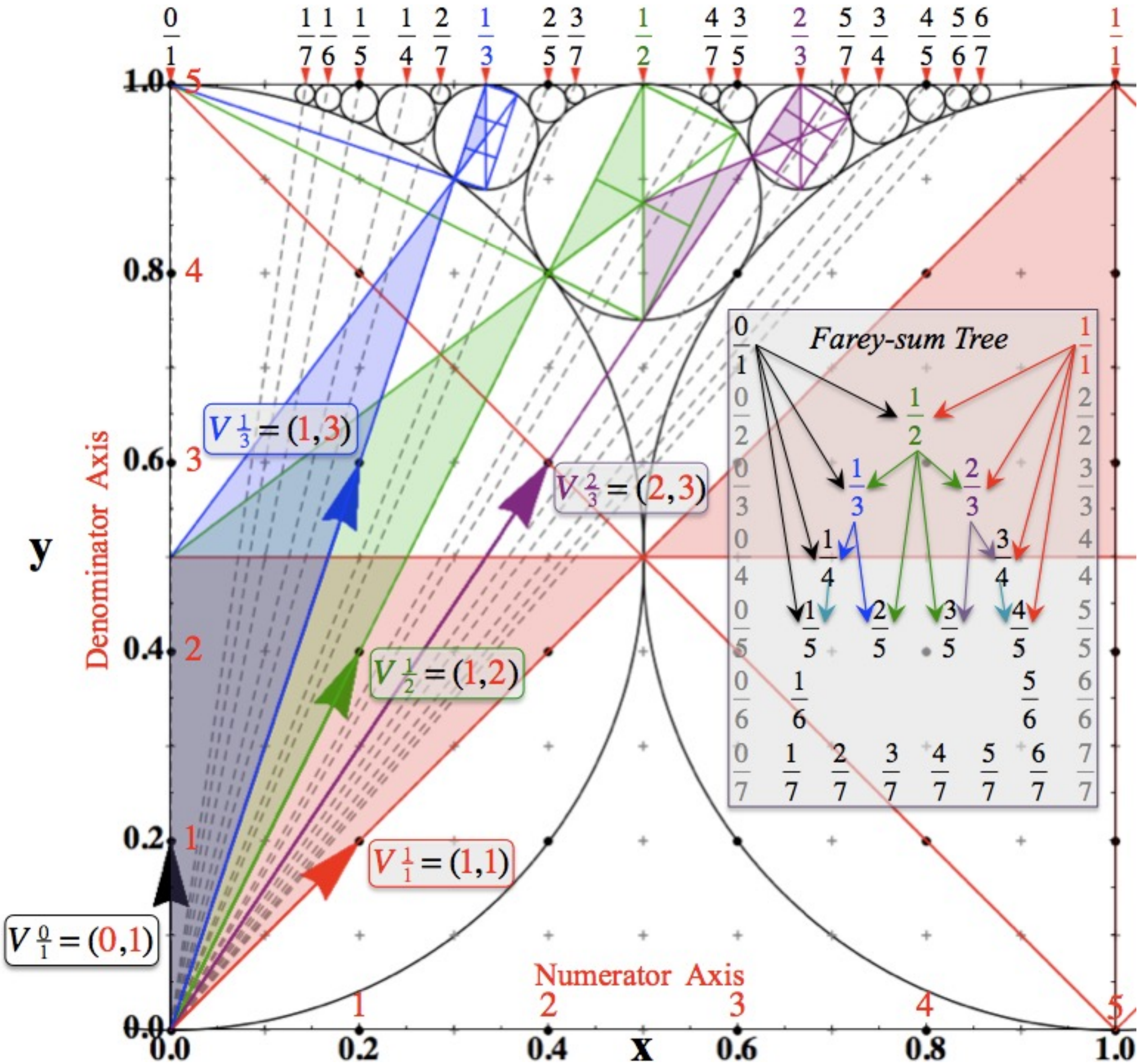


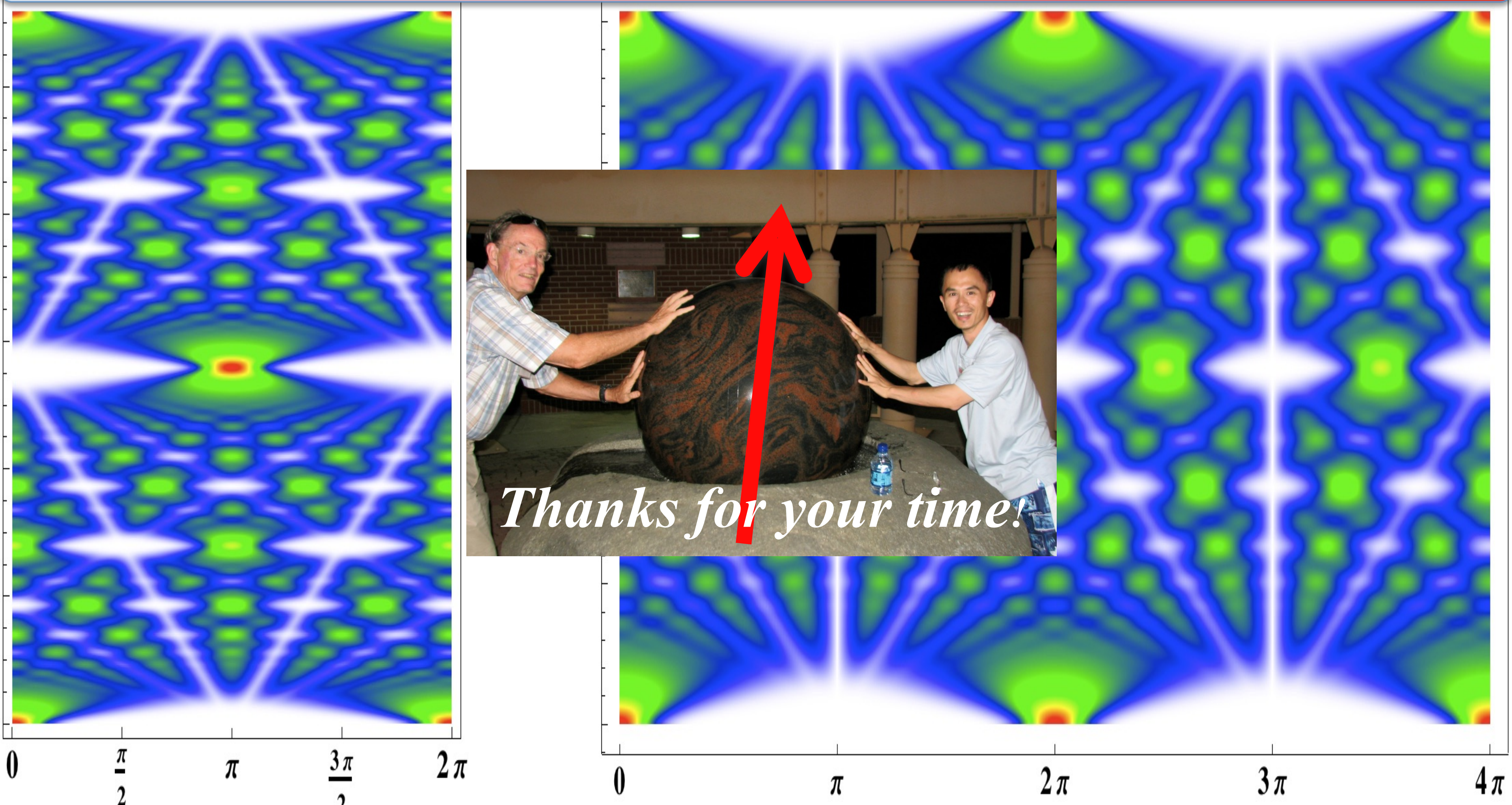
The coming next talk will address the curious connection of Farey-sum and Ford-circles



A. Z. Li and W. G. Harter, Physical Review Letters (submitted 2013).

Summary

Both Integer and Half-integer Spinning Rotors exhibit Farey-sum Revivals



Pave the way for more rich dynamic behaviors in Asymmetric Rotors