RESONANCE AND REVIVALS

I. QUANTUM ROTOR AND INFINITE-WELL DYNAMICS

William G. Harter and Alvason Zhenhua Li
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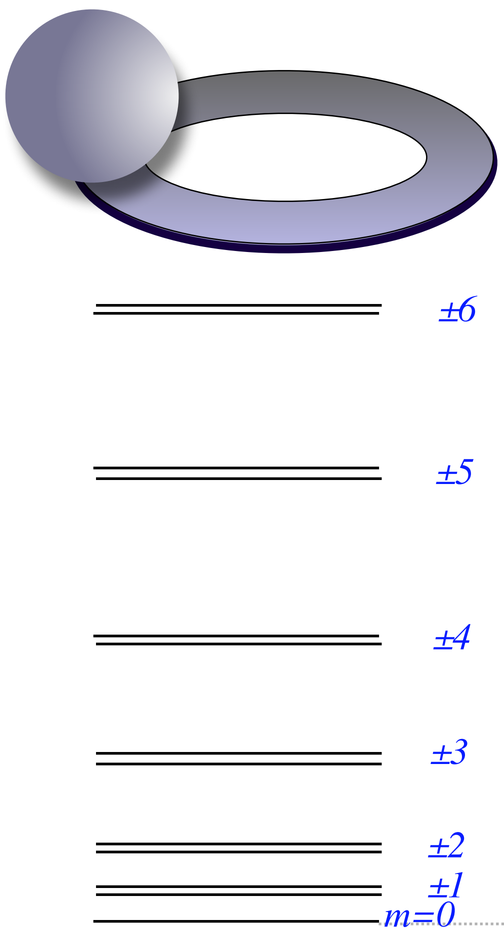

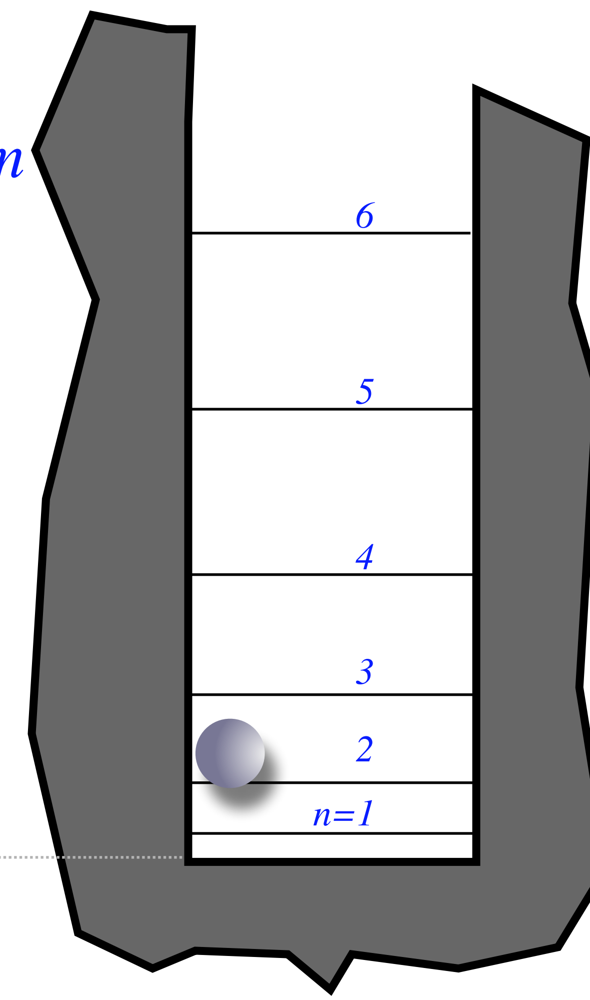

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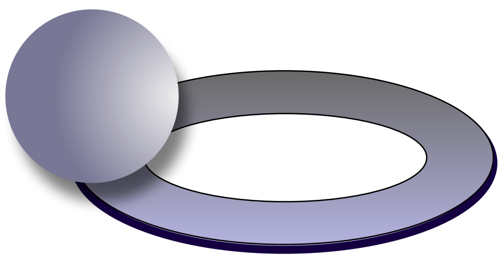

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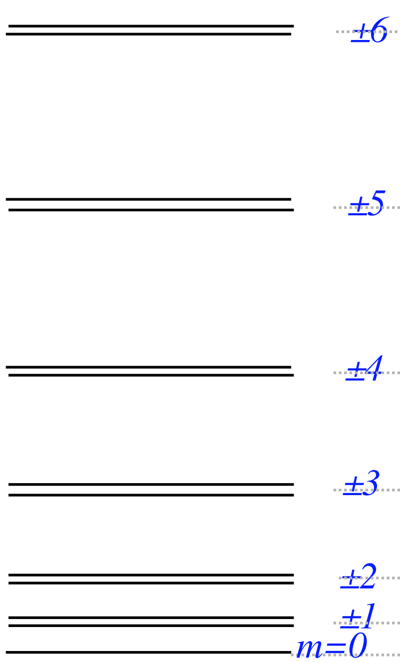
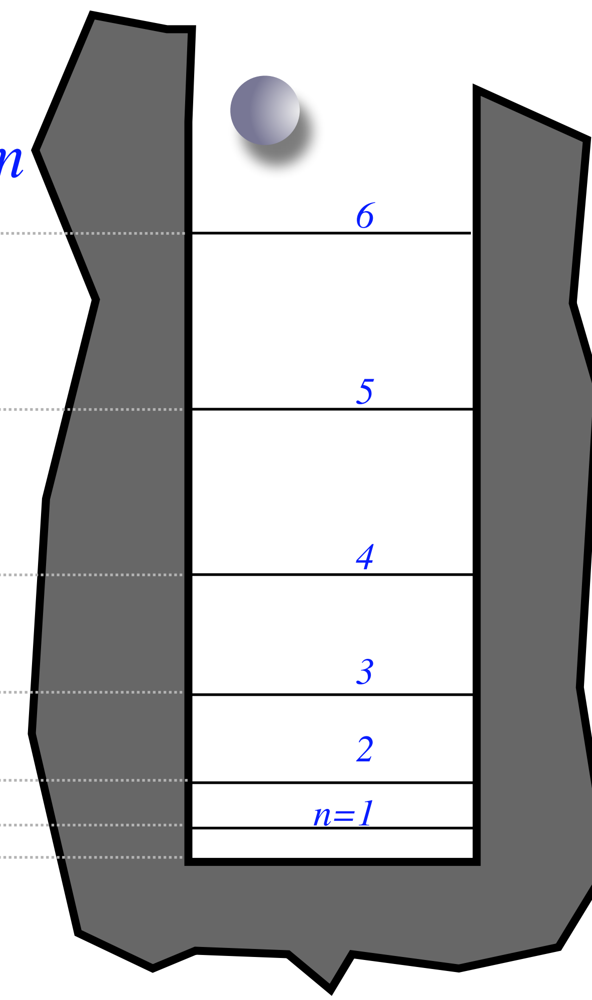


Won't talk about ∞-well

Rotor revival structure includes anything ∞-well can do….
…and is easier to explain.



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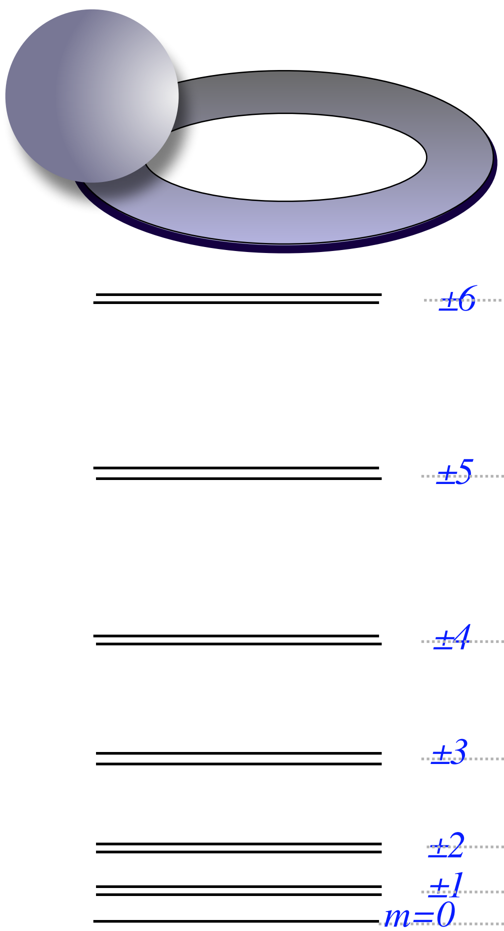

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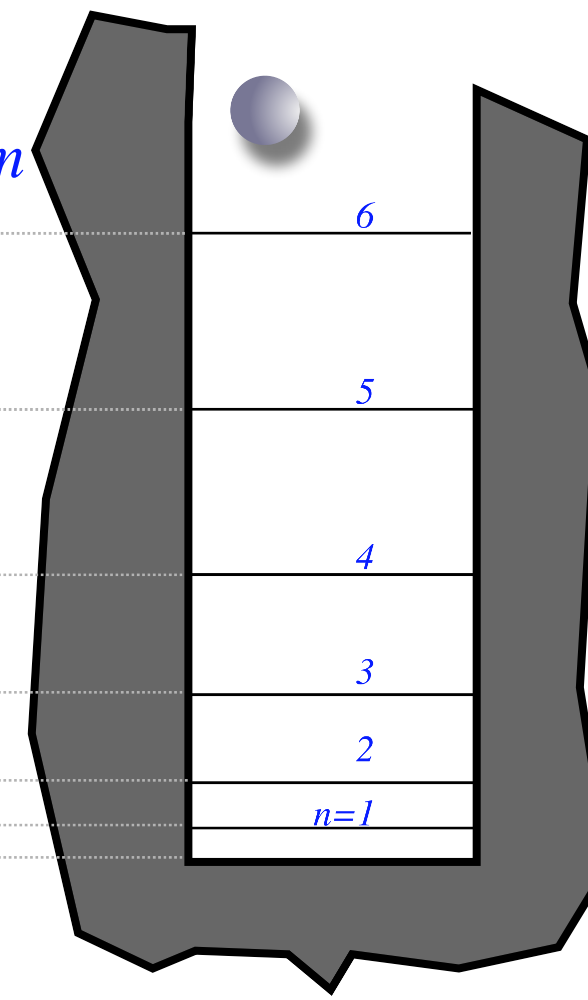

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Some Early History of Quantum Revivals

J.H. Eberly *et. al. Phys. Rev.* **A 23**,*236 (1981)*	Laser QuantumCavityDynamic revivals
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So we thought we'd put this revival business to bed! Then...

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So we thought we’d put this revival business to bed! Then this...

More recent story of Quantum Revivals

Anne B. McCoy *Chem. Phys. Lett.* **501**, *603(2011)*...reminds me that Morse potential is integer-analytic.

Leads to cool Morse revivals in: *Following Talk RJ05 by Li:*
Resonance&Revivals II. MORSE OSCILLATOR AND DOUBLE MORSE WELL DYNAMICS.

So now we’re having a revival-revival!

...and, in words by Joannie Mitchell, I find:
“I didn’t really know... revivals … at all.”

What do revivals look like?

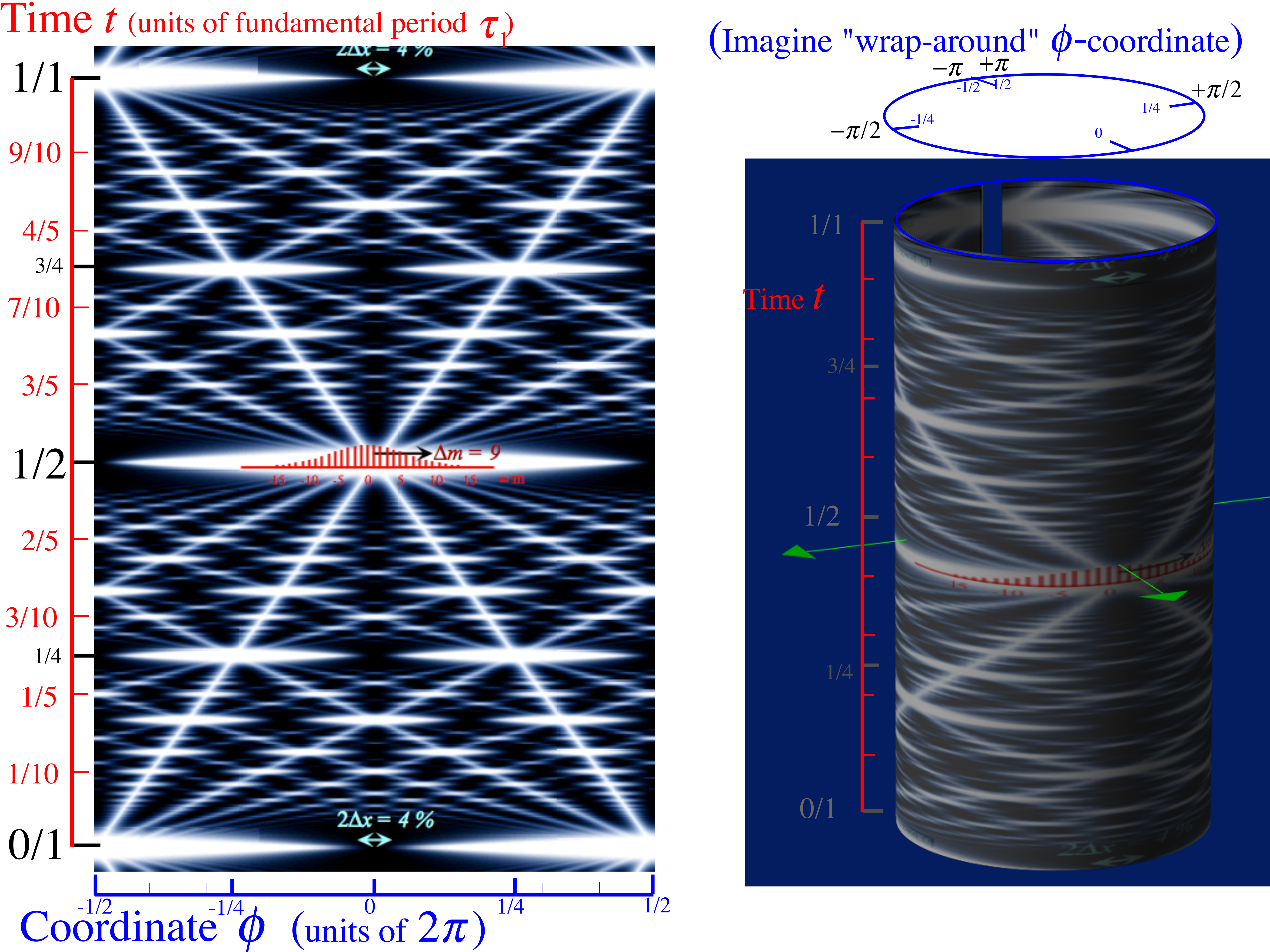
(...in space-time…)


BROOKLYN TABERNACLE DELIVERANCE CENTER
Apostle WILLIAM J. DENSON, Pastor, presents
HOLY GHOST MIRACLE TENT
CRUSADE
NIGHTLY SERVICES SUNDAY Through FRIDAY with PRAYER AT 6:30 P.M. DAY SERVICES MONDAY Through FRIDAY From 10:00 A.M. to 12:30 P.M.
SUNDAY MORNING SERVICE BEGIN AT 9:30 A.M.
COME And EXPECT
A MIRACLE
SUPERNATURAL
EXPLOSION


SALVATION - DIVINE HEALING
TENT REVIVAL
BEGINS
June 24 - July 4
7:45 Nightly
Except Sunday
LOCATION
Junction 319 & 98
Medart, Florida
Rev. Jimmie Dobbs
EVANGELIST
of Jacksonville, Fla.
FOR PEOPLE OF ALL FAITHS

What do revivals look like?
(...in space-time…)

OK,
let’s try that again...
with
quantum
revivals...



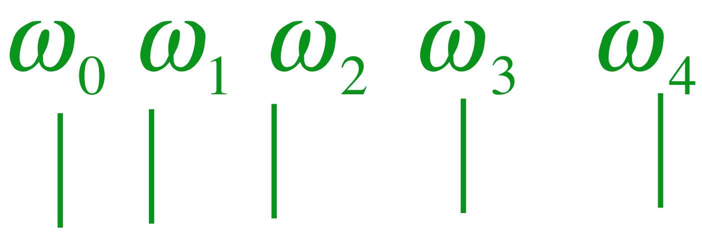
Time t (units of fundamental period τ1)
1/1
9/10
4/5
3/4
7/10
3/5
1/2
2/5
3/10
1/4
1/5
1/10
0/1
2Δx = 4 %
Δm = 9
-15 -10 -5 0 5 10 15 = m
-1/2
-1/4
0
1/4
1/2
Coordinate φ (units of 2π)
(Imagine "wrap-around" φ-coordinate)
−π +π
-1/2 1/2
+π/2
1/4
−π/2
-1/4
0
1/1
Time t
3/4
1/2
1/4
0/1

Observable dynamics of N-level-system state $|\Psi\rangle$

Depends on Fourier spectrum of probability distribution $\langle\Psi|\Psi\rangle$

$$|\Psi\rangle = \sum_{n=0}^{N} e^{-i\omega_n t}\psi_n$$

...But individual eigenfrequencies ω_n are not directly observable...



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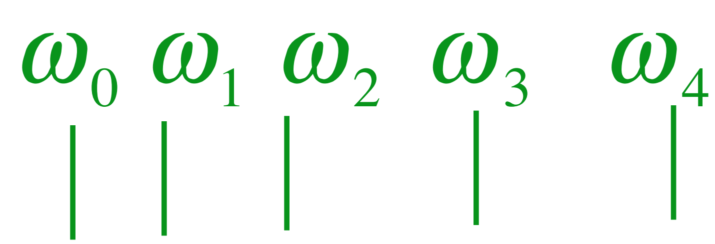
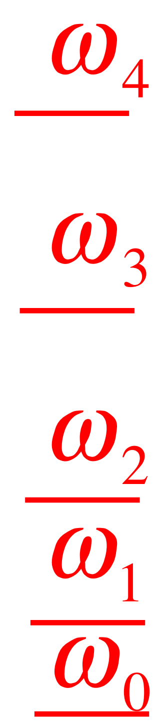


$$\langle\Psi| = \sum_{m=0}^{N} e^{+i\omega_m t}\psi_m^*$$



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$$|\Psi\rangle = \sum_{n=0}^{N} e^{-i\omega_n t}\psi_n$$

$$\langle\Psi|\Psi\rangle = \sum_{n=0}^{N} e^{i(\omega_m-\omega_n)t}\psi_m^*\psi_n$$

$$= \sum_{m,n=0}^{N} e^{i\Delta_{mn}t}\ \rho_{mn}$$

ω_0 ω_1 ω_2 ω_3 ω_4

$$\langle\Psi| = \sum_{m=0}^{N} e^{+i\omega_m t}\psi_m^*$$

ω_4

ω_3

ω_2

ω_1

ω_0

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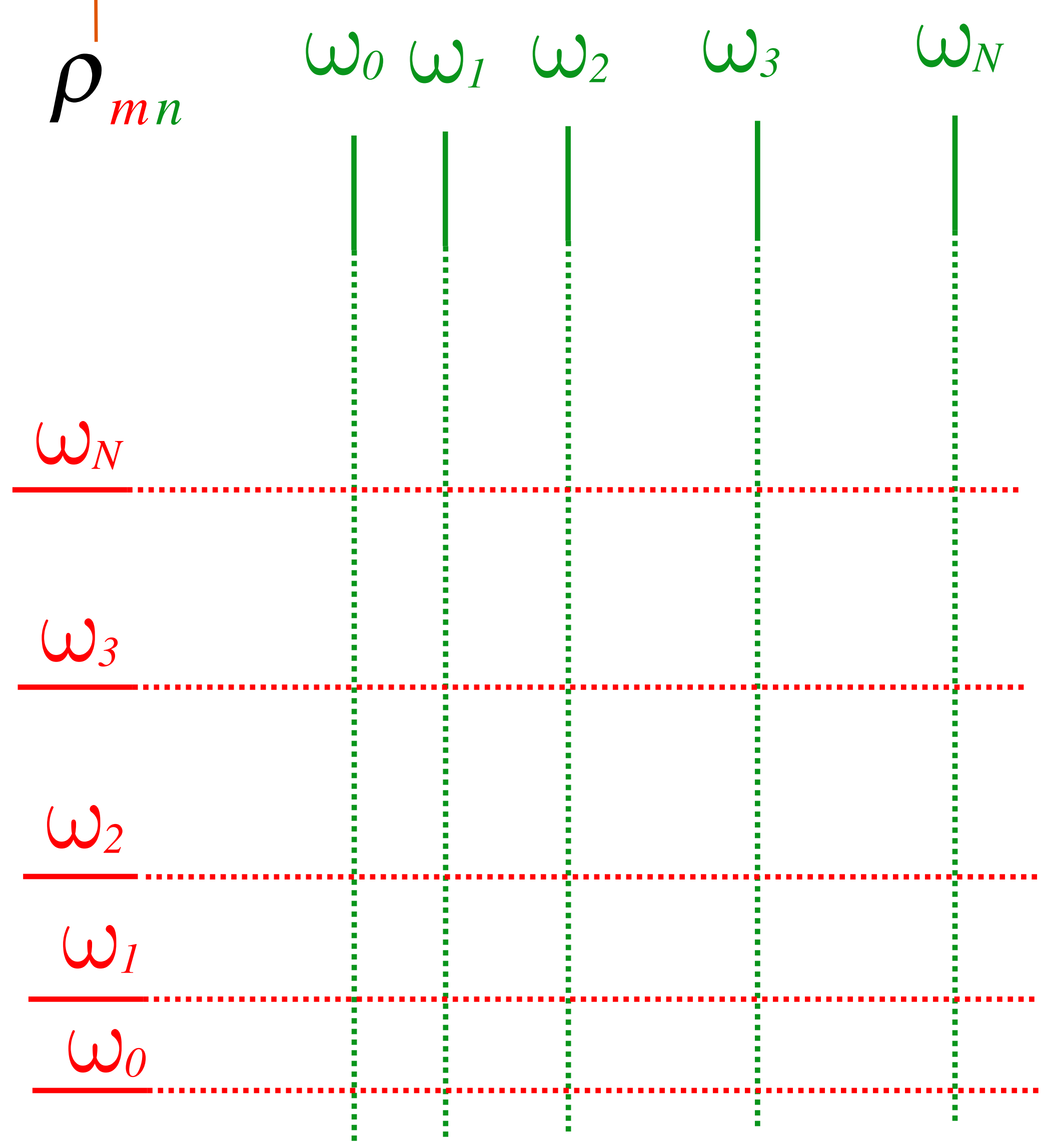

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...only *differences* $\Delta_{mn} = (\omega_m - \omega_n)$

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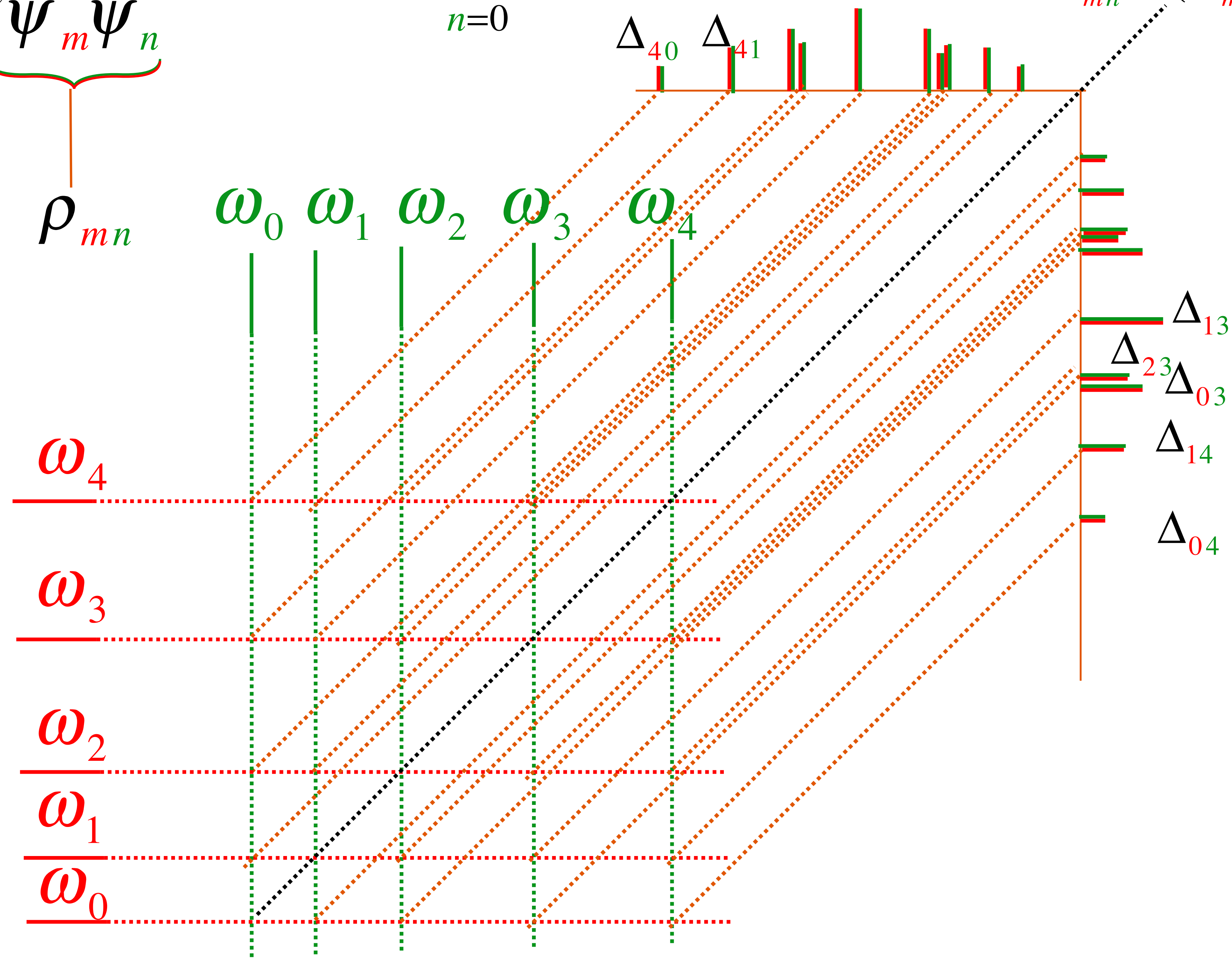

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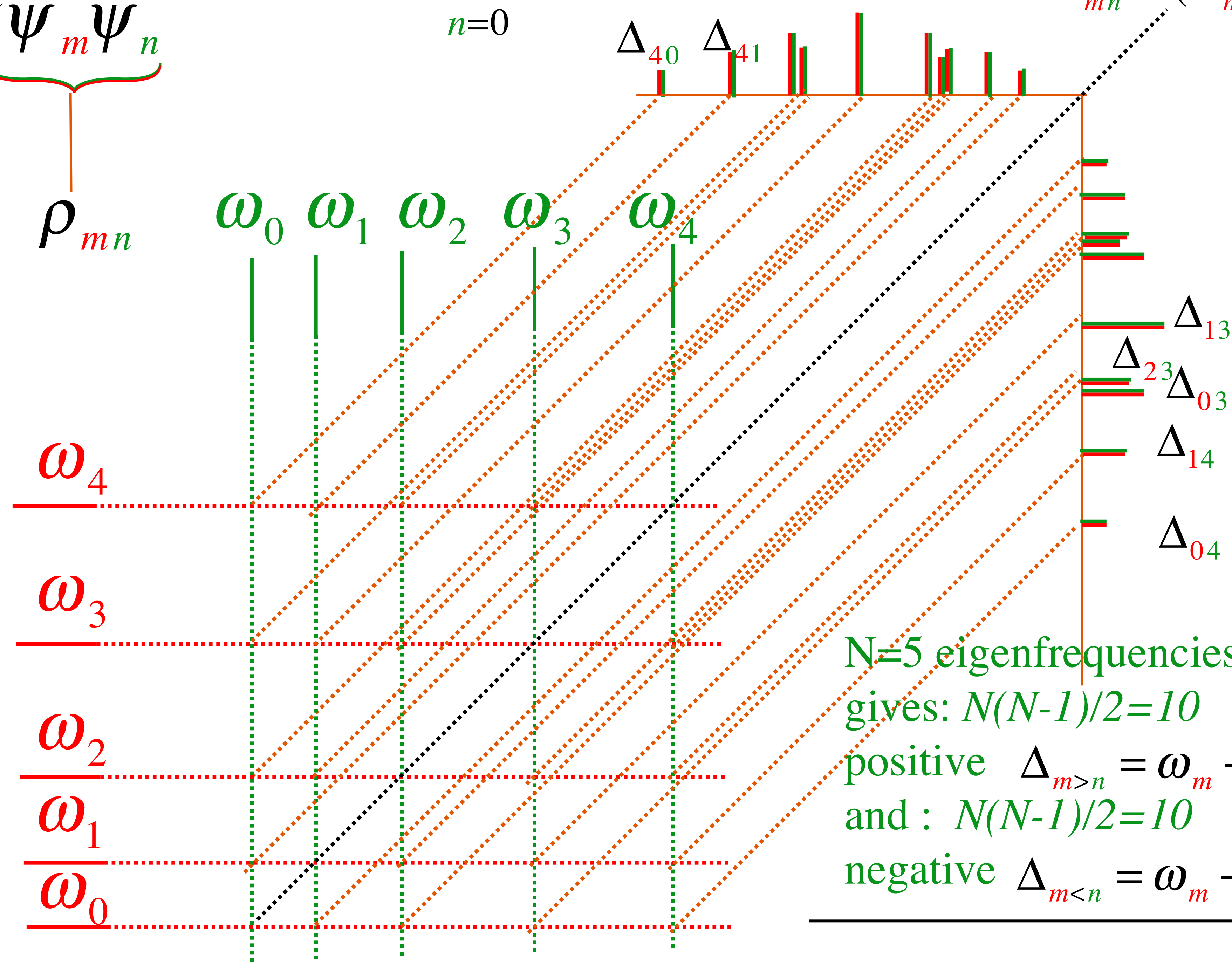

N=5 eigenfrequencies gives: *N(N-1)/2=10* positive $\Delta_{m>n} = \omega_m - \omega_n$

and : *N(N-1)/2=10* negative $\Delta_{m<n} = \omega_m - \omega_n$

Observable dynamics of *N*-level-system state $|\Psi\rangle$

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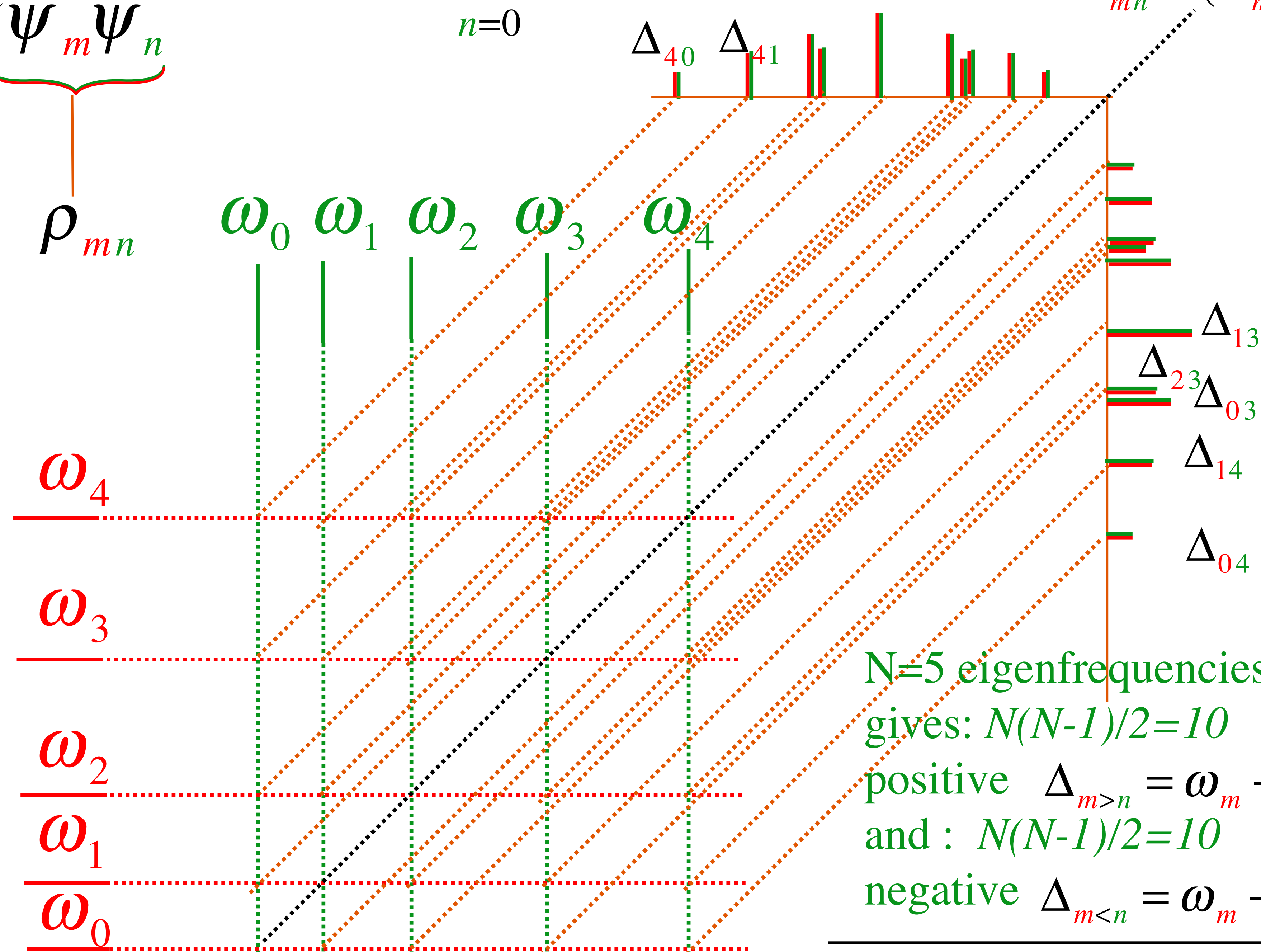

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and : *N(N-1)/2=10* negative $\Delta_{m<n} = \omega_m - \omega_n$

...that is, *N(N-1)/2=10* observable beats $|\Delta_{m<n}| = |\omega_m - \omega_n|$

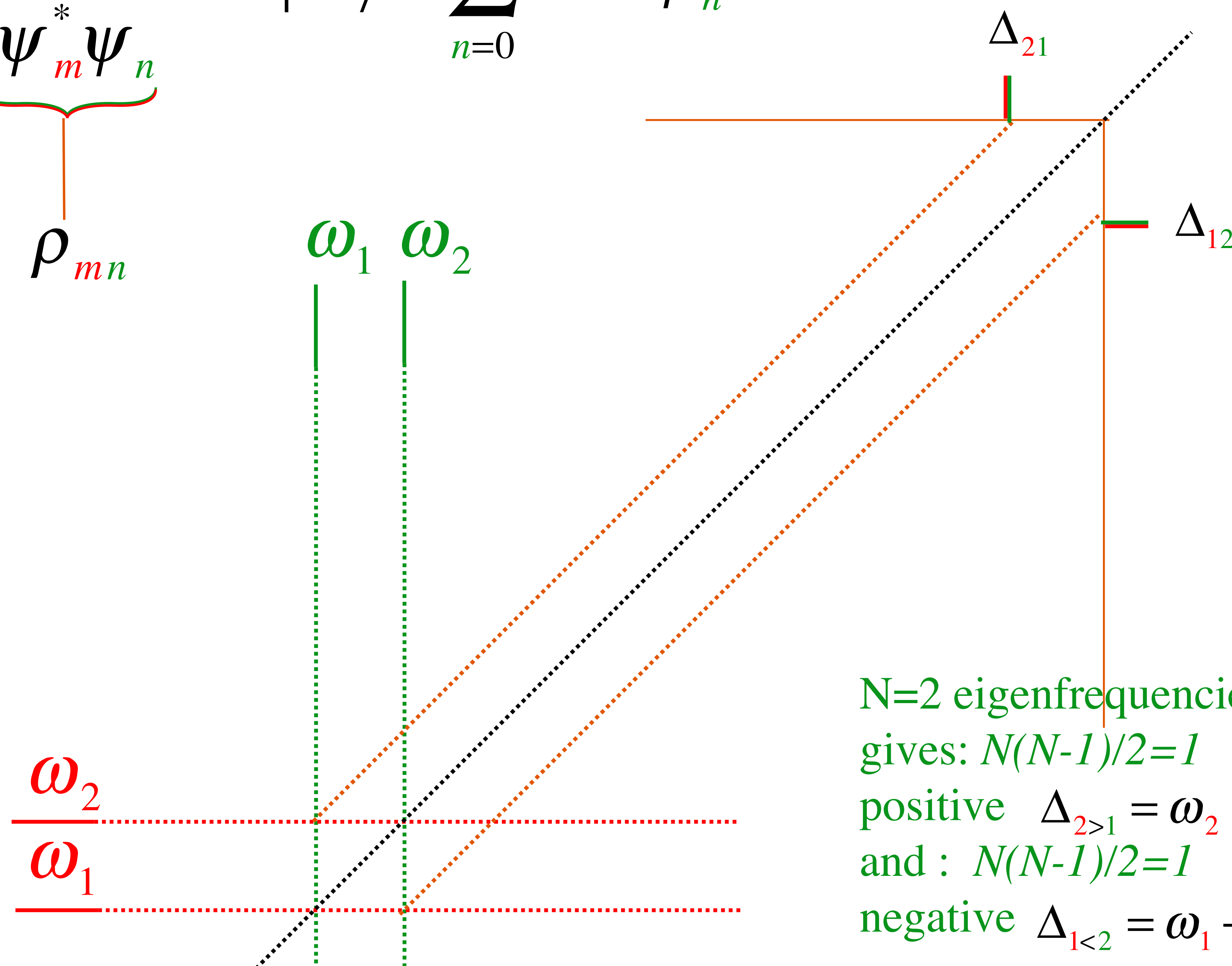
Observable dynamics of 2-level-system state $|\Psi\rangle$

Fourier spectrum of $\langle\Psi|\Psi\rangle$ has *ONE* beat frequency $\Delta_{21} = -\Delta_{12}$

$$|\Psi\rangle = \sum_{n=0}^{N} e^{-i\omega_n t}\psi_n$$

$$\langle\Psi|\Psi\rangle = \sum_{n=0}^{N} e^{i(\omega_m - \omega_n)t}\psi_m^*\psi_n = \sum_{m,n=0}^{N} e^{i\Delta_{mn}t}\rho_{mn}$$

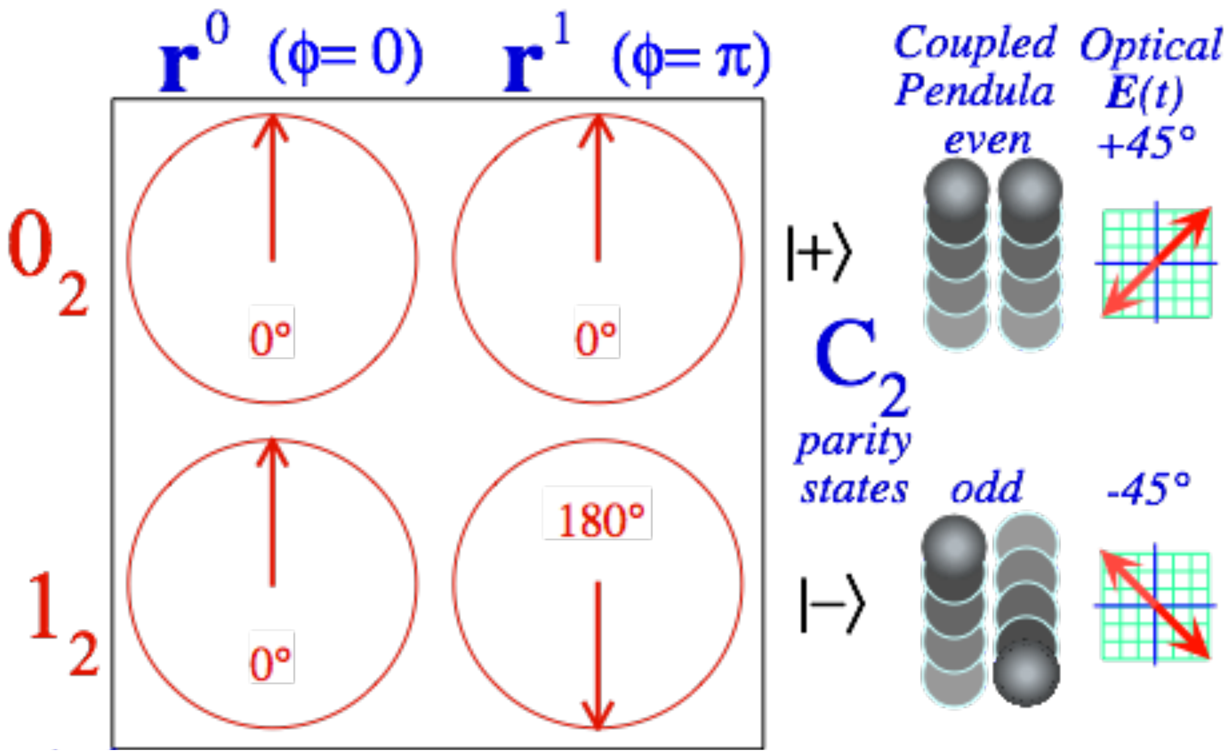
$$\langle\Psi| = \sum_{m=0}^{N} e^{+i\omega_m t}\psi_m^*$$



N=2 eigenfrequencies gives: *N(N-1)/2=1* positive $\Delta_{2>1} = \omega_2 - \omega_1$ and : *N(N-1)/2=1* negative $\Delta_{1<2} = \omega_1 - \omega_2$

...that is, *N(N-1)/2=1* observable beat $|\Delta_{m<n}| = |\omega_m - \omega_n|$

2-level-system and C_2 symmetry beat dynamics



C₂ Character Table describes eigenstates

symmetric A_1

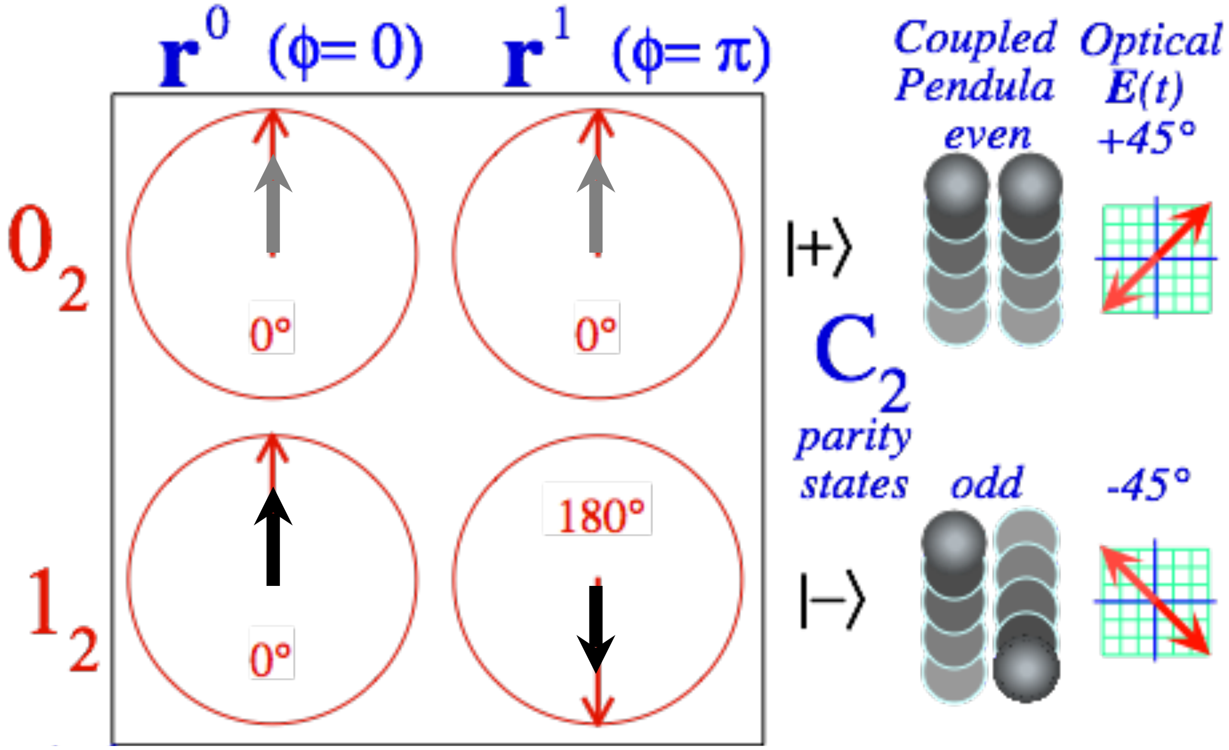
vs.

antisymmetric A_2

	$1 = r^0$	$r = r^1$
$0 \bmod 2$	1	1
$\pm 1 \bmod 2$	1	-1

2-level-system and C_2 symmetry beat dynamics

C_2 *Phasor*-Character Table



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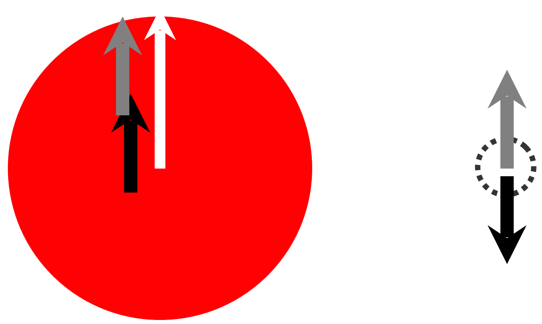

Phasor C_2 Characters describe local state beats

Initial sum

2-level-system and C_2 symmetry beat dynamics

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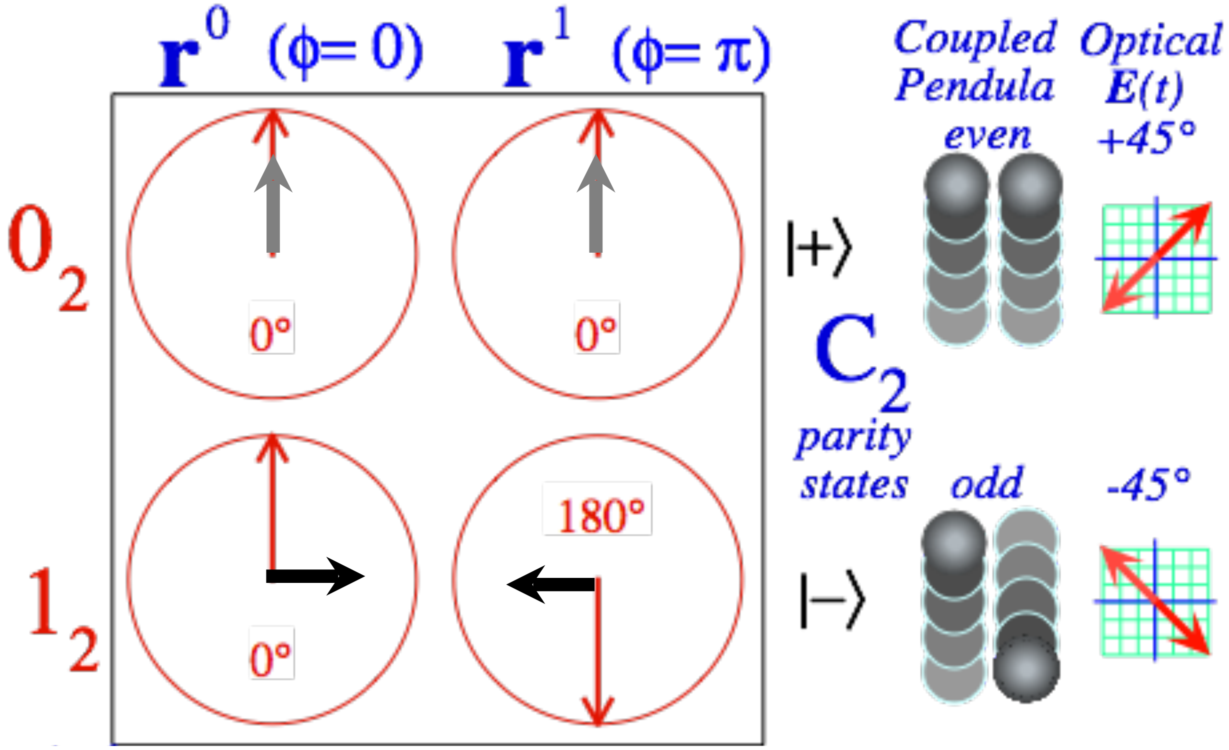

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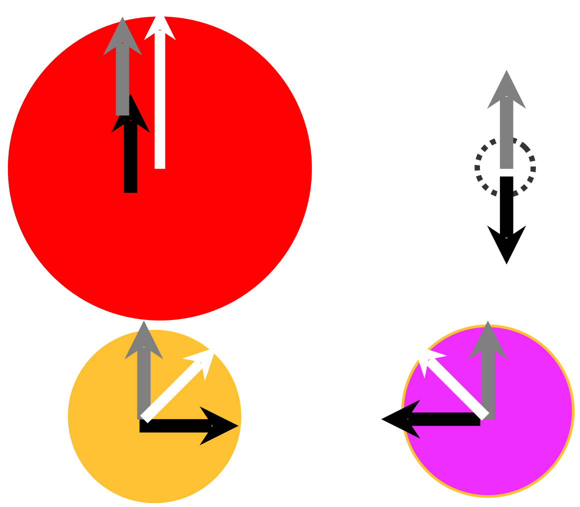

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1/4-beat

2-level-system and C_2 symmetry beat dynamics

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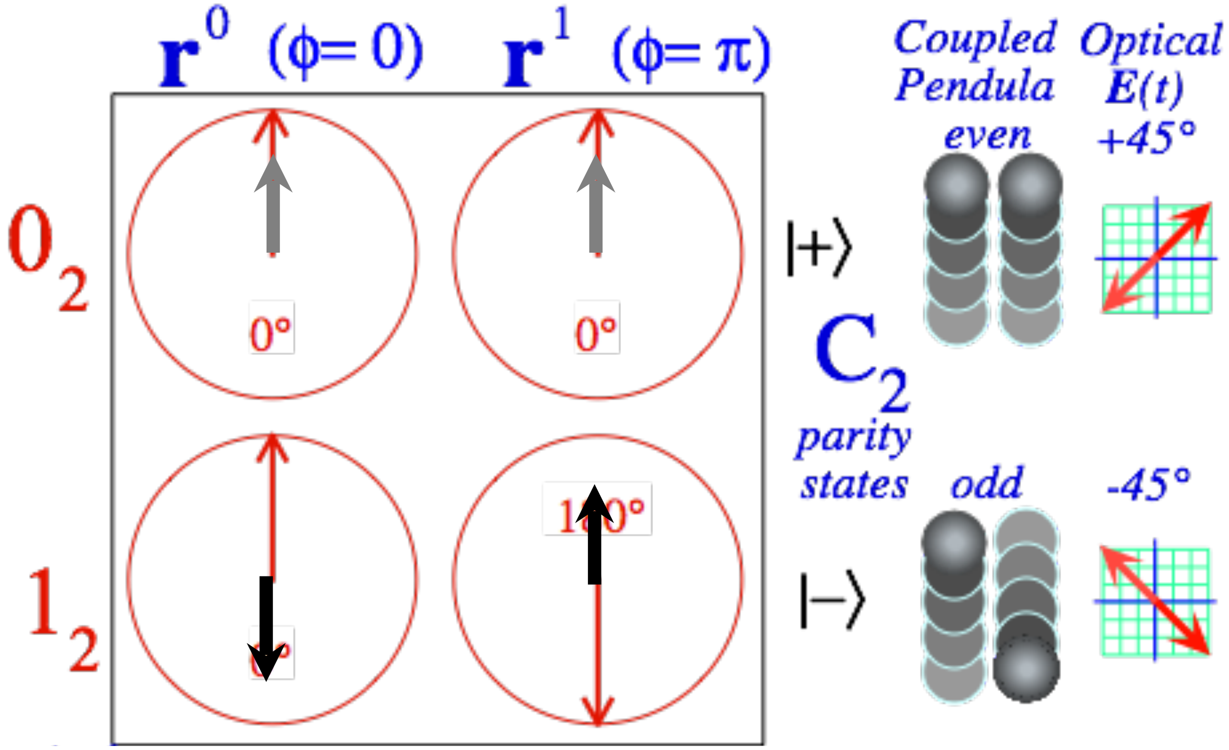

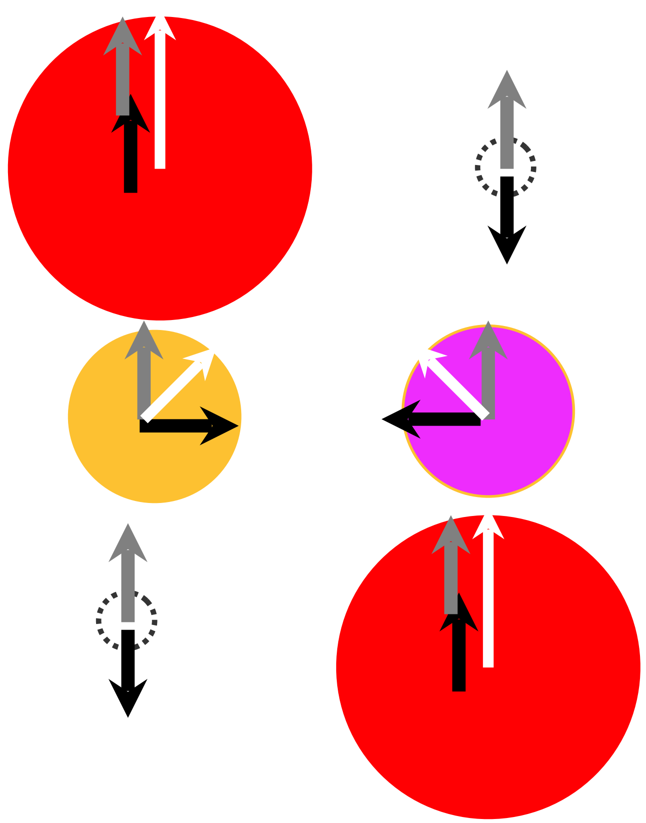

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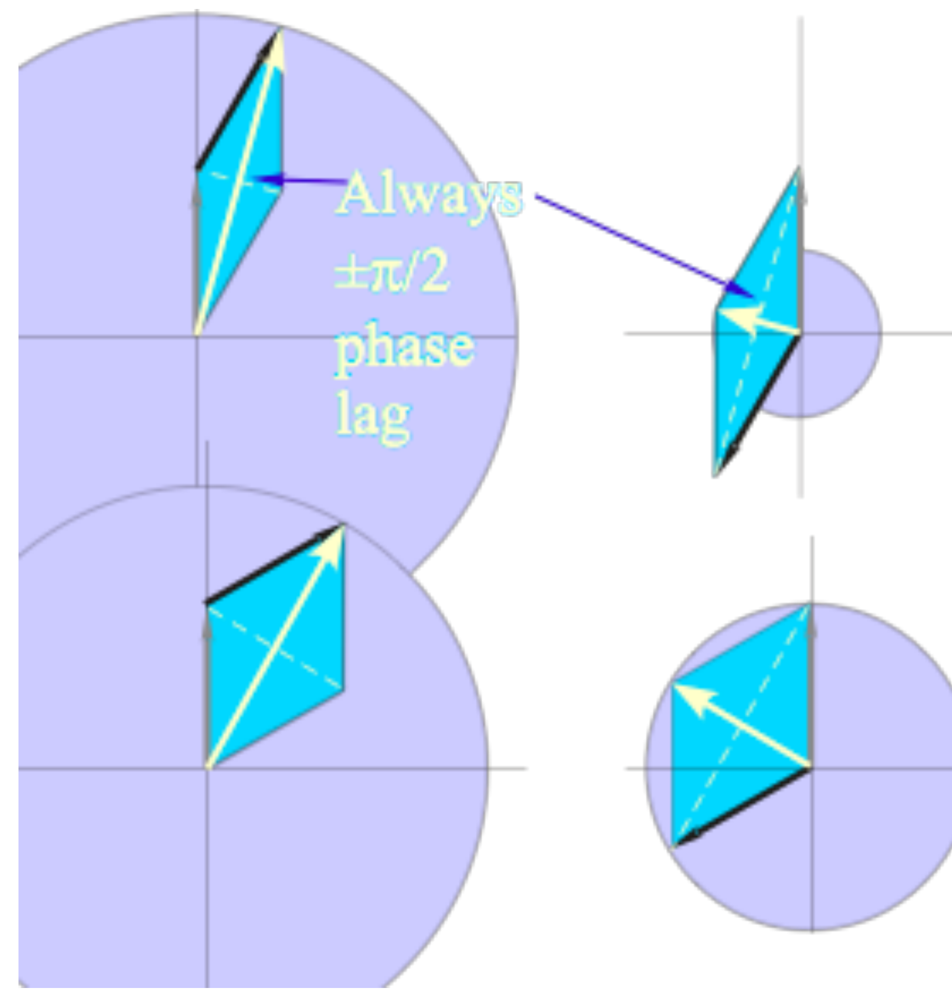
1/2-beat



2-level-system and C_2 symmetry beat dynamics

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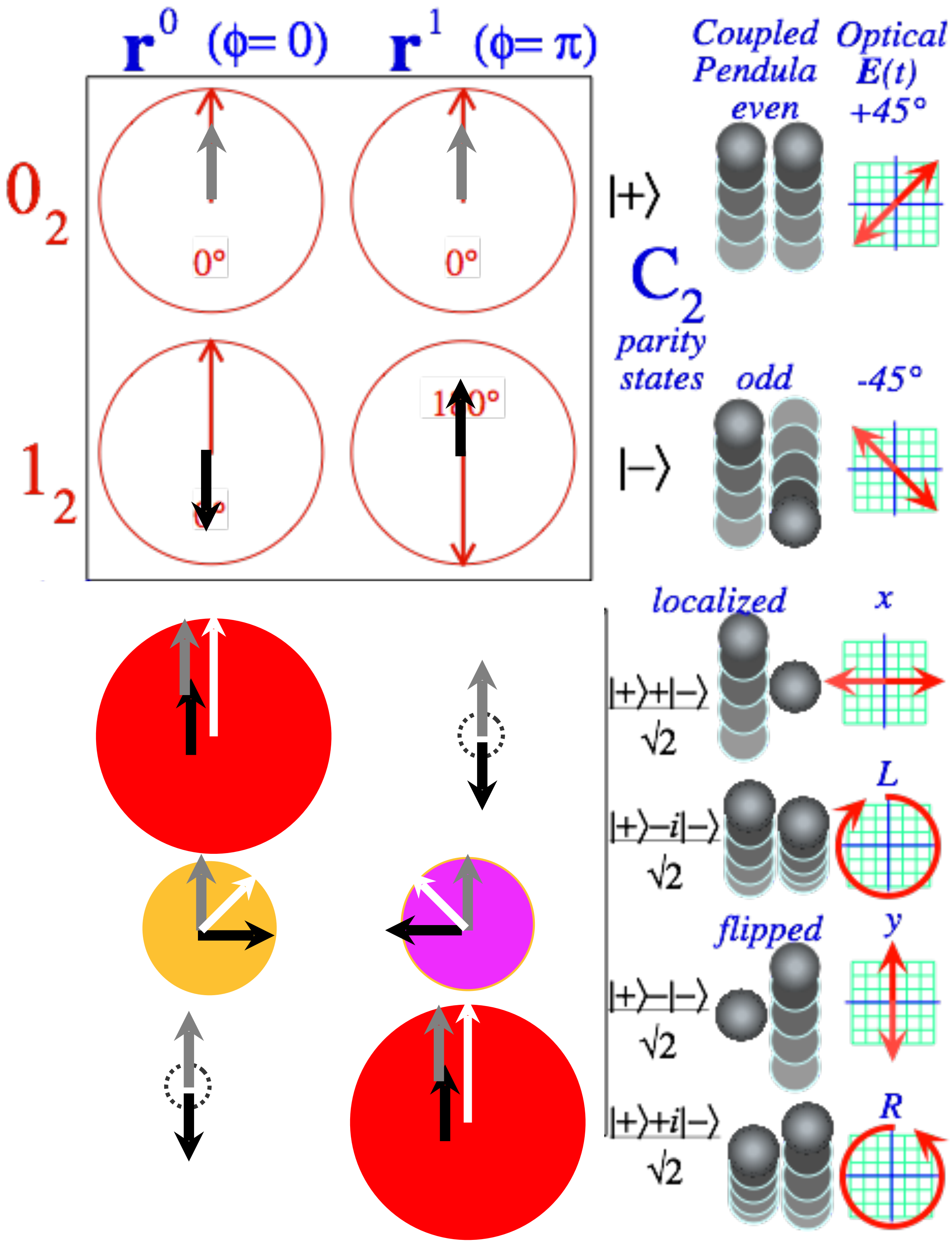

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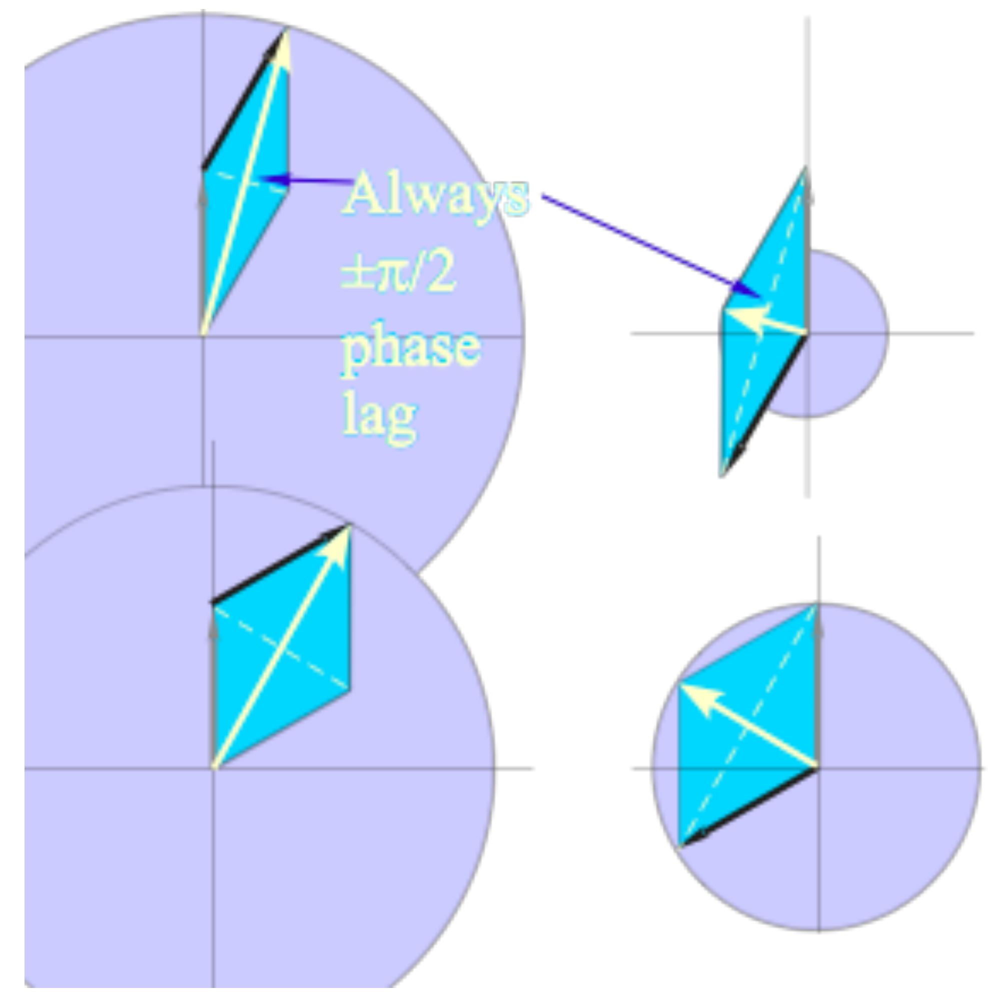
Phasor C_2 Characters describe local state beats

Initial sum

1/4-beat

1/2-beat

3/4-beat



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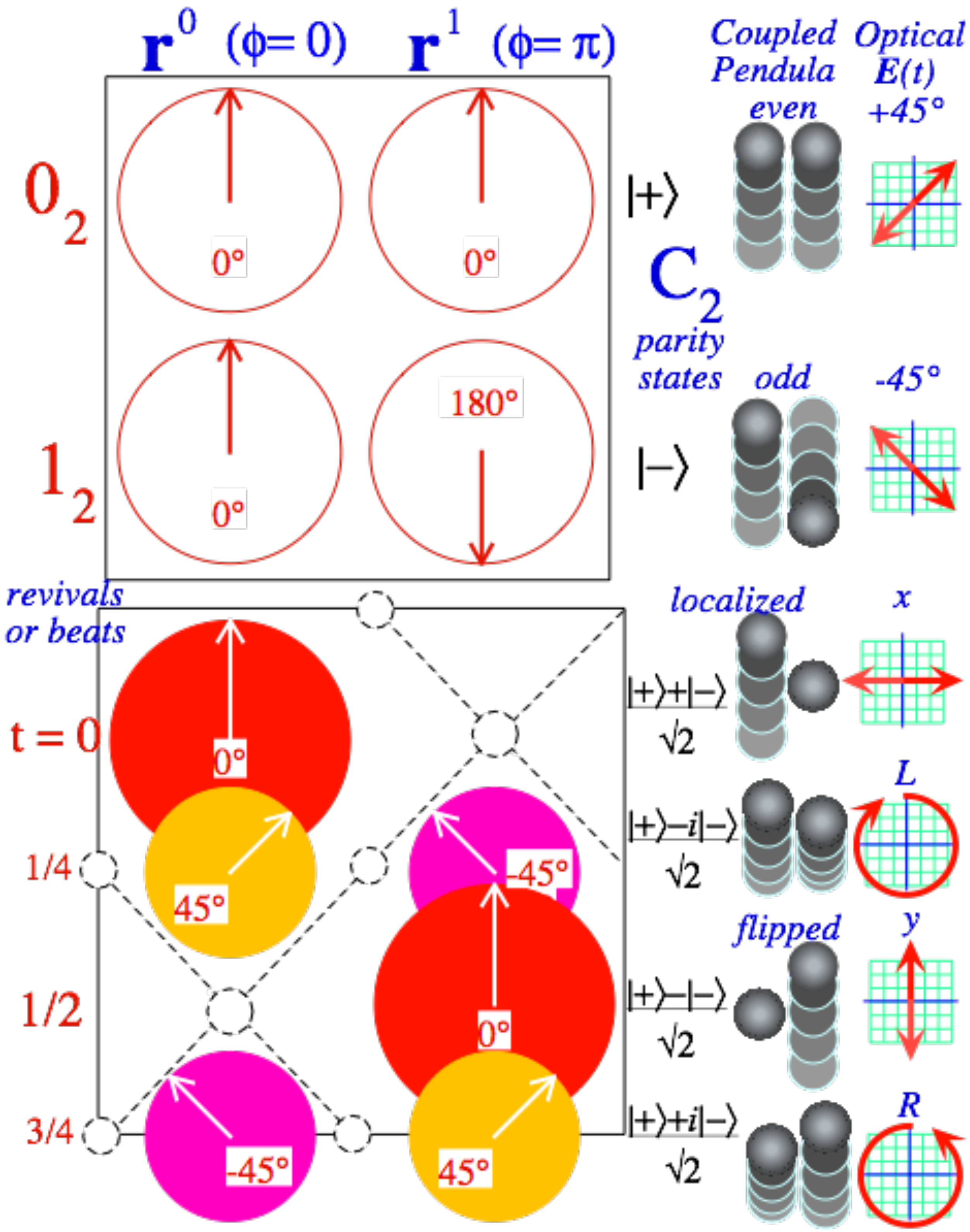
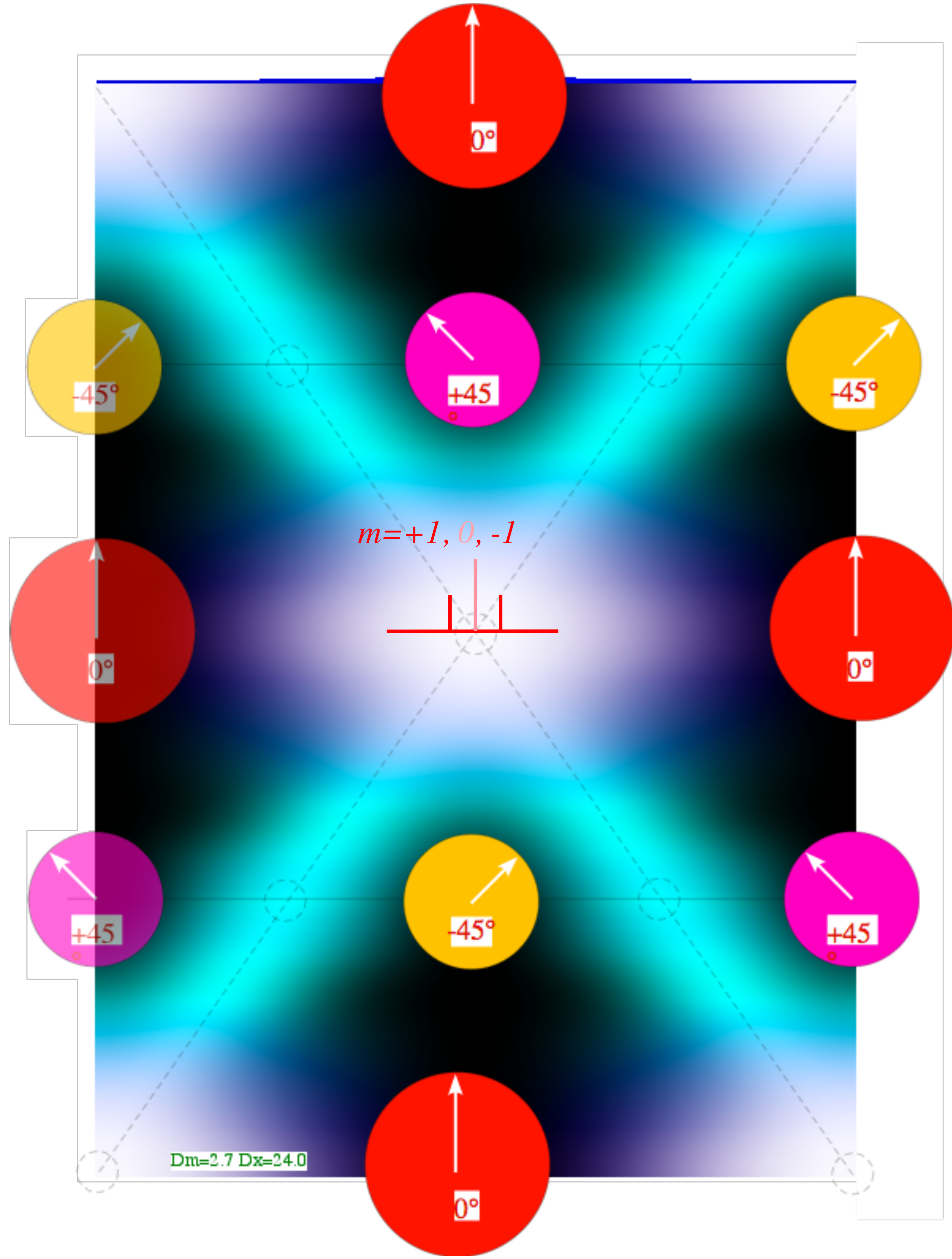




What do revivals look like?

...in *per*-space-time…

(… that is:

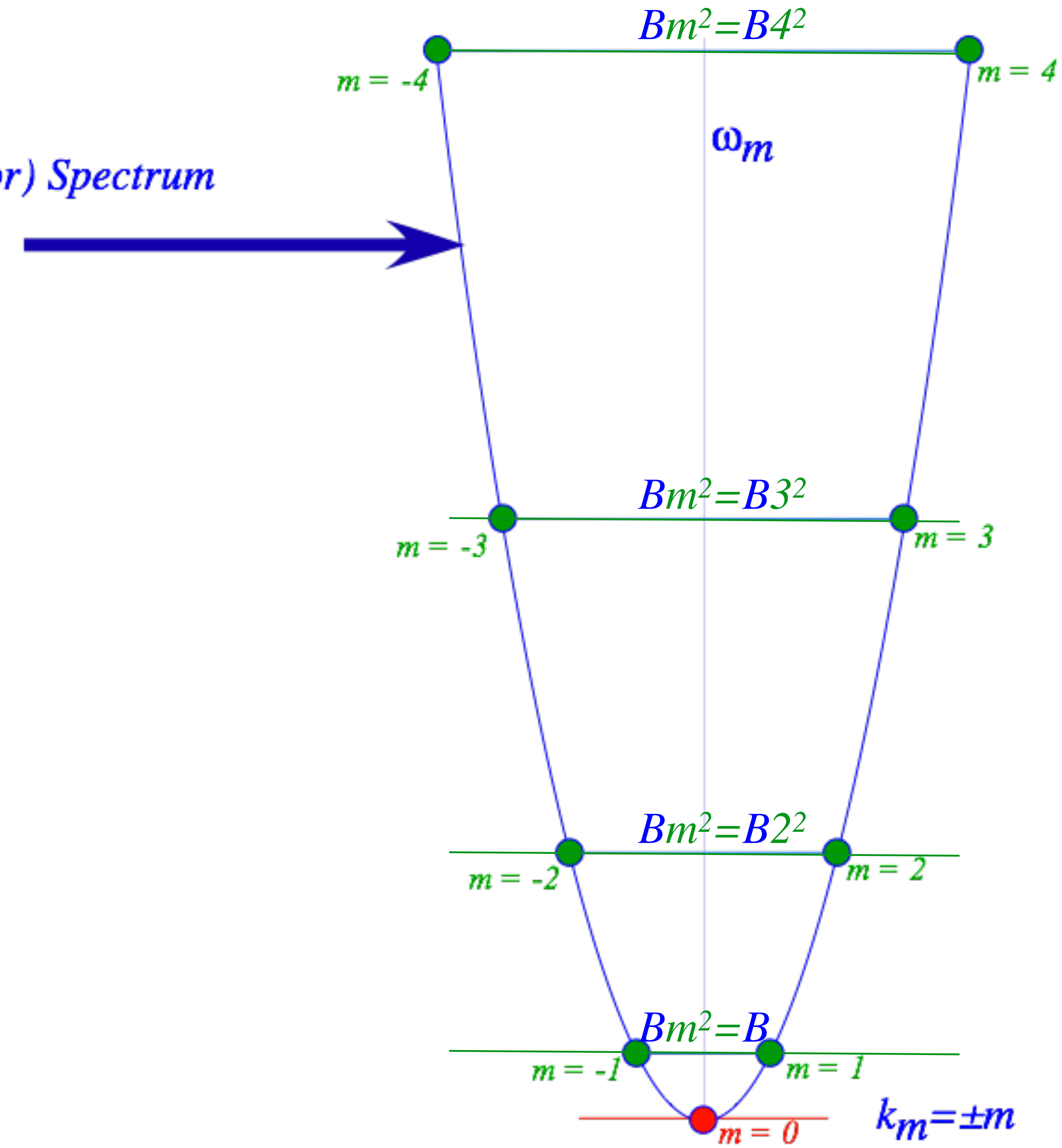
frequency ω_m radian/sec.

vs

k-vector k_m radian/cm)

N-level-system and revival-beat wave dynamics

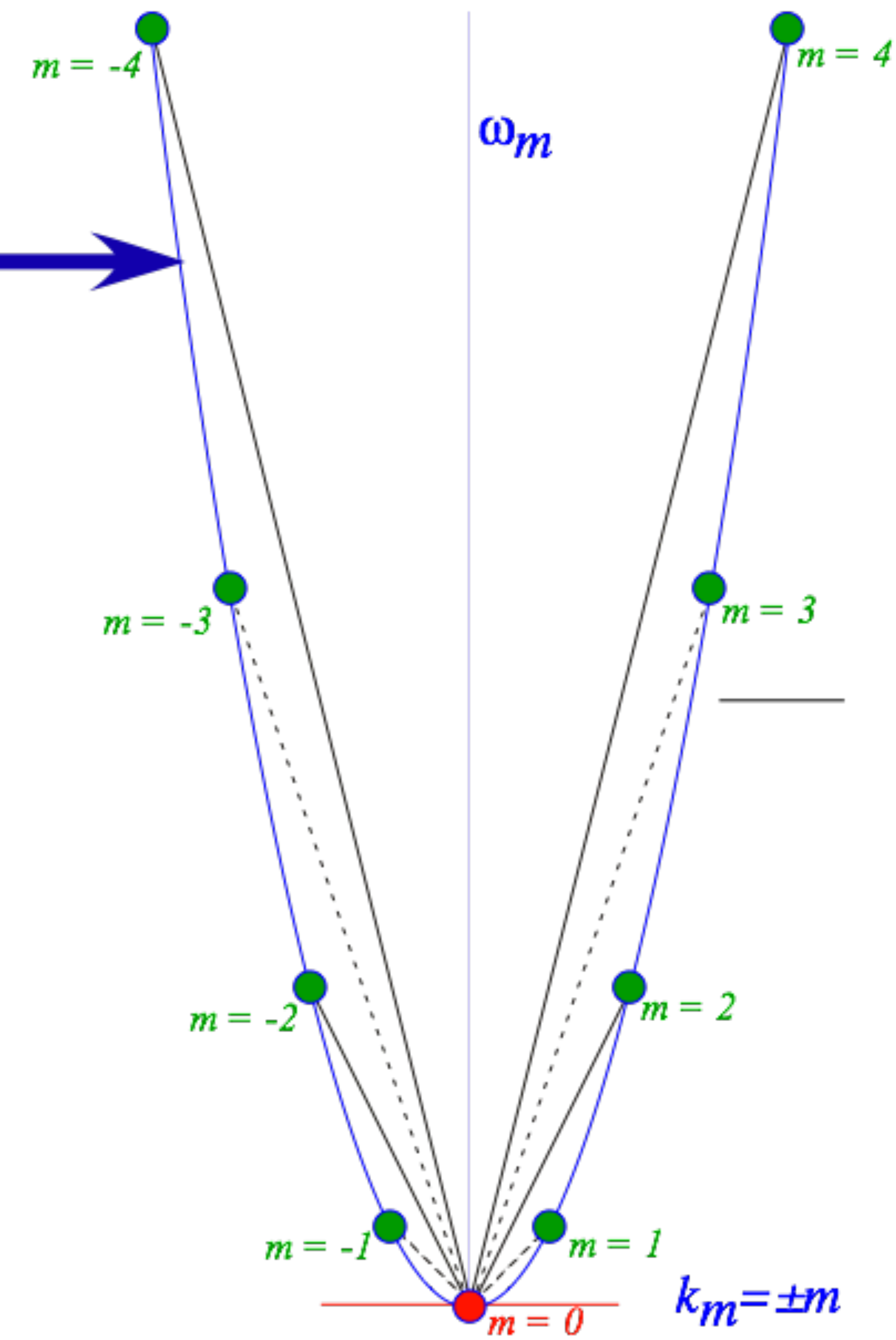
Levels
for
Quadratic (Bohr-Rotor) Spectrum
$\omega_m = Bm^2$
$k_m = \pm m$



N-level-system and revival-beat wave dynamics

Possible wave velocities
for
Quadratic (Bohr-Rotor) Spectrum
$\omega_m = Bm^2$
$k_m = \pm m$

$$V_{phase} = \frac{\omega_m}{k_m} = \frac{Bm^2}{m} = mB$$



N-level-system and revival-beat wave dynamics

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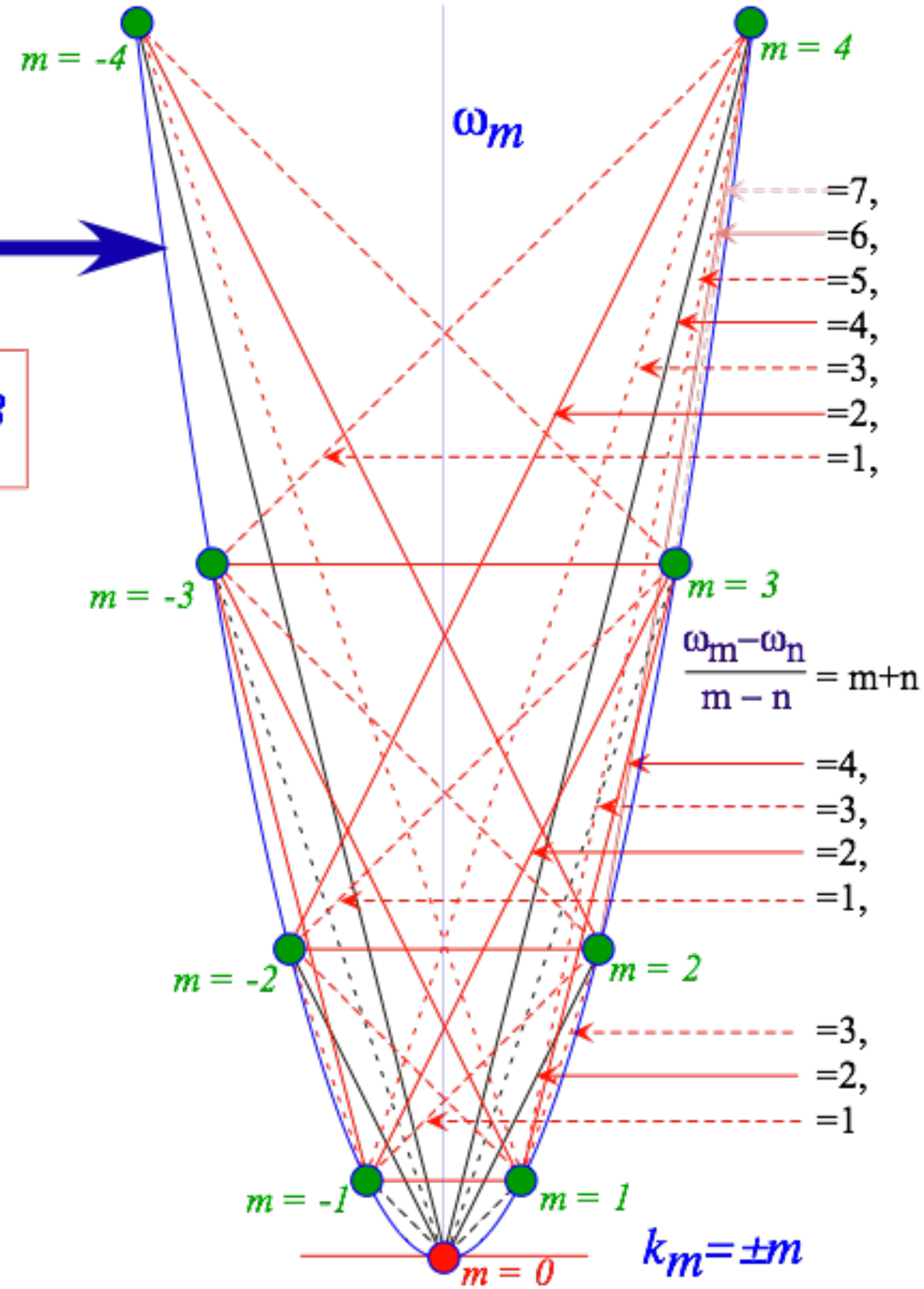
$$V_{group} = \frac{\omega_m - \omega_n}{k_m - k_n} = \frac{m^2 - n^2}{m \pm n} B = (m \pm n)B$$



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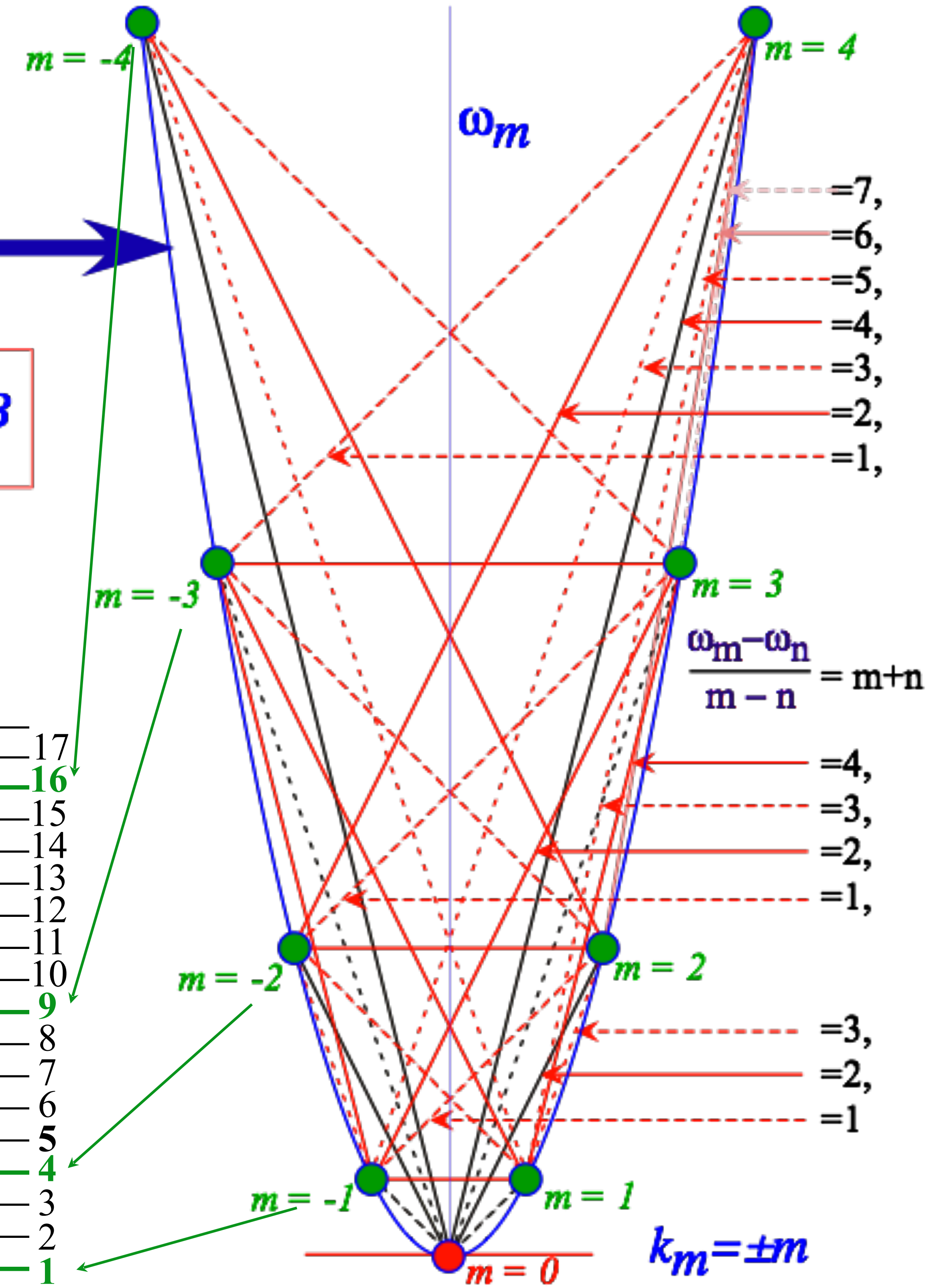
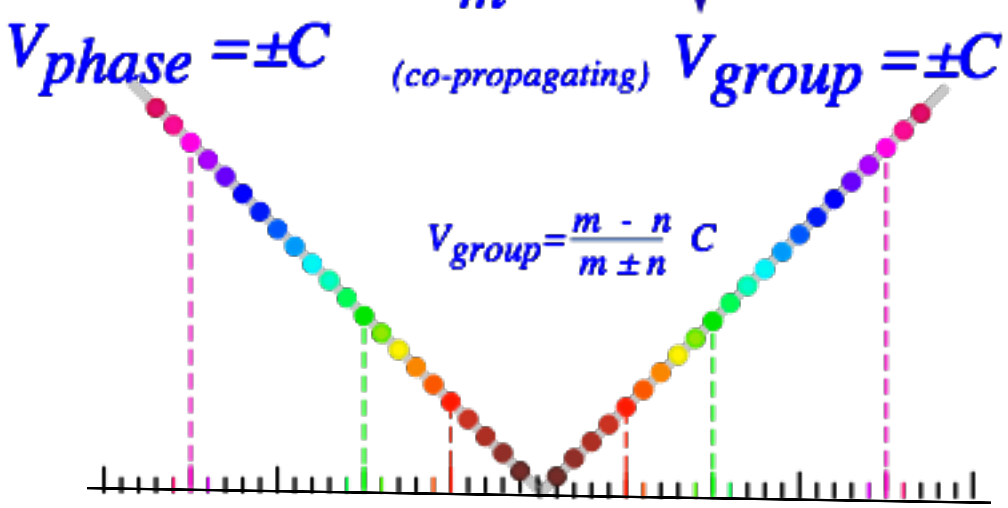
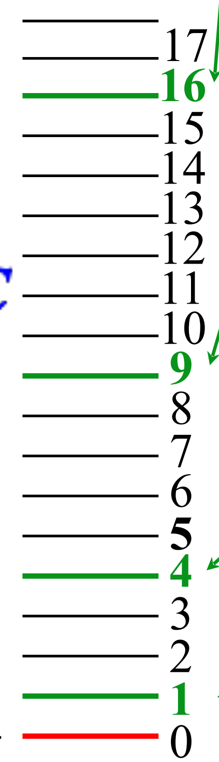


Possible wave velocities
for
Linear (Optical) Spectrum

$\omega_m = C|m|^1$

$k_m = m$

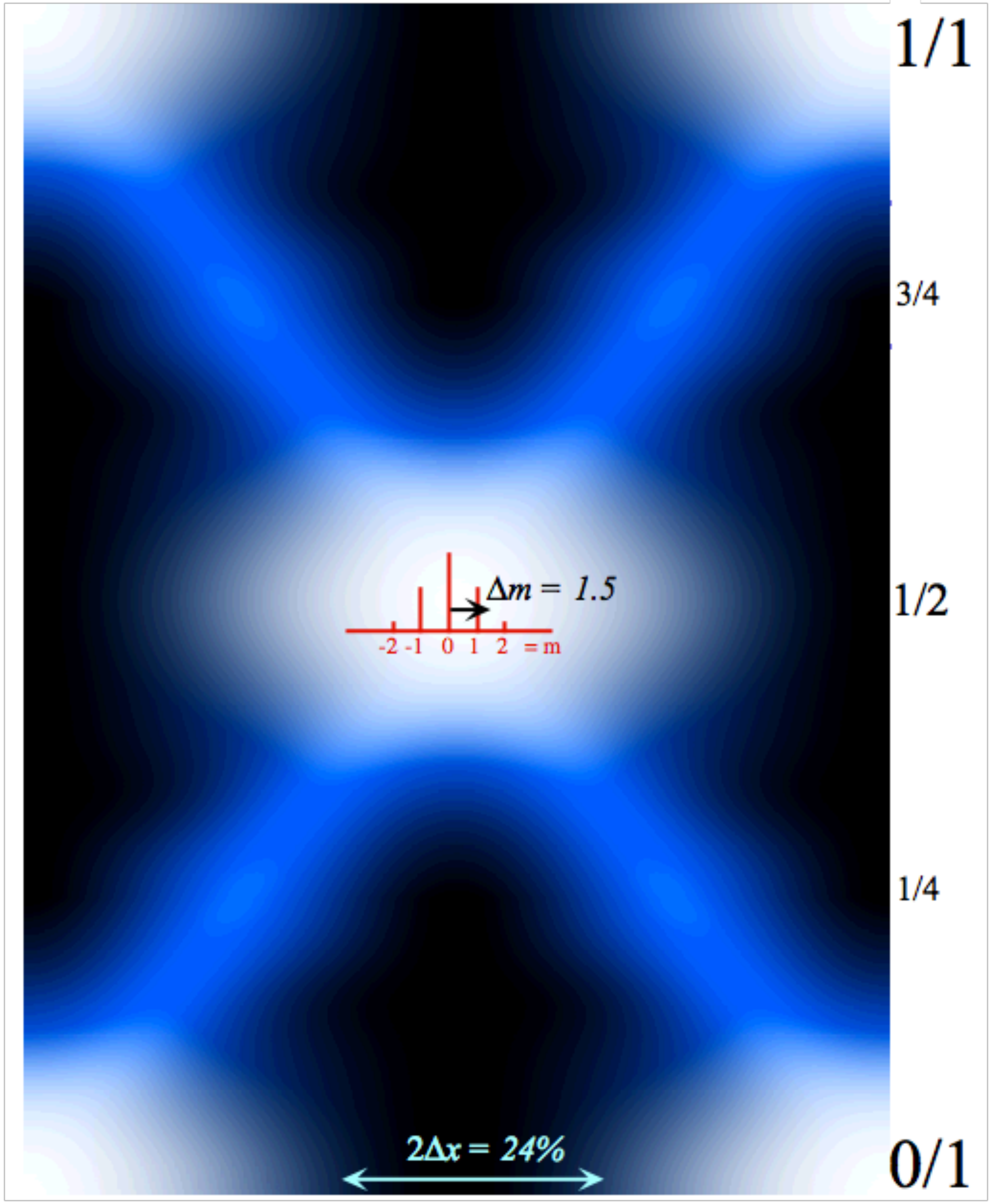




Harmonic Oscillator level spectrum contains the **Rotor Levels** as a <u>subset</u>

N-level-system and revival-beat wave dynamics

(Just 2-levels (0, ±1) (and some ±2) excited)



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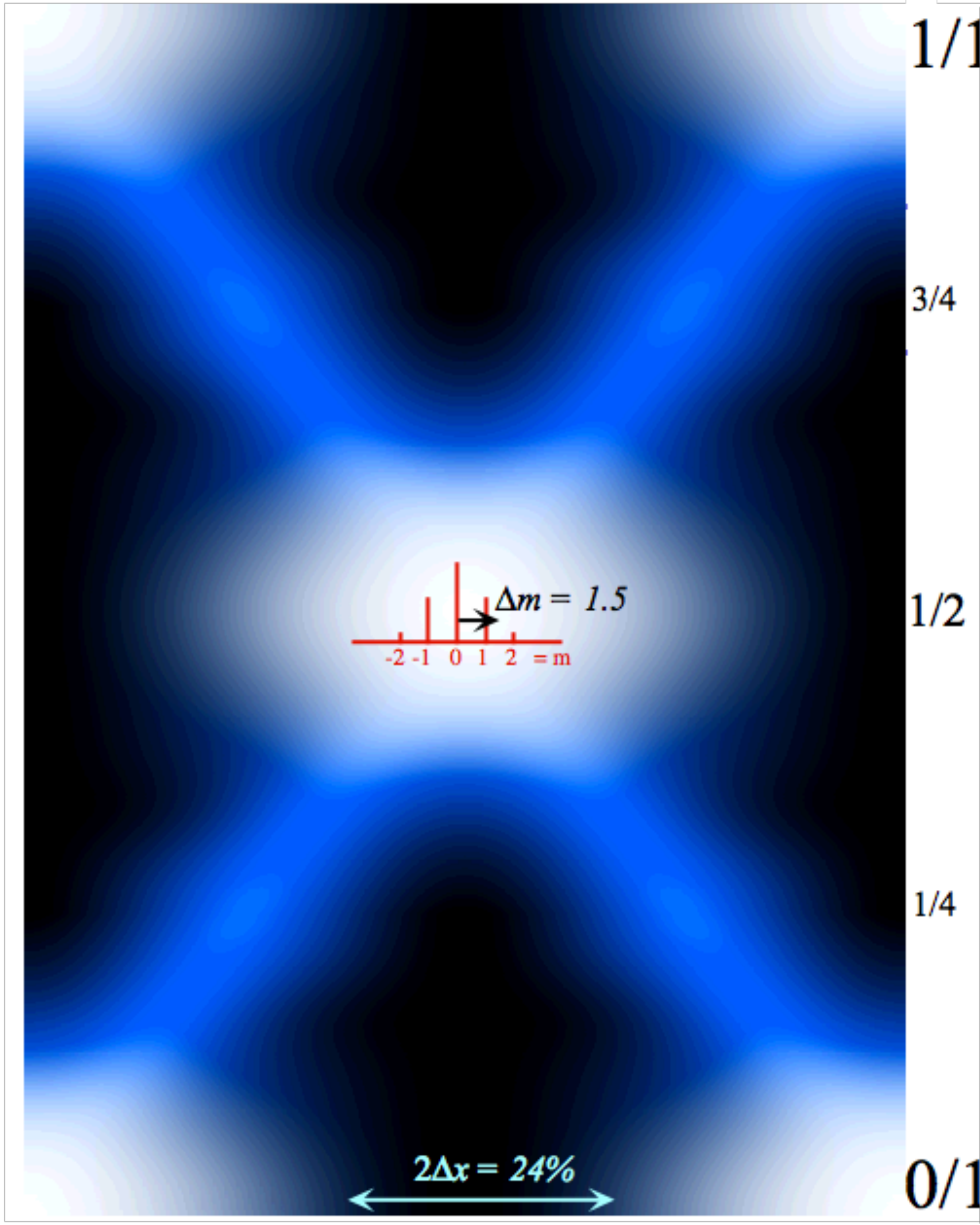
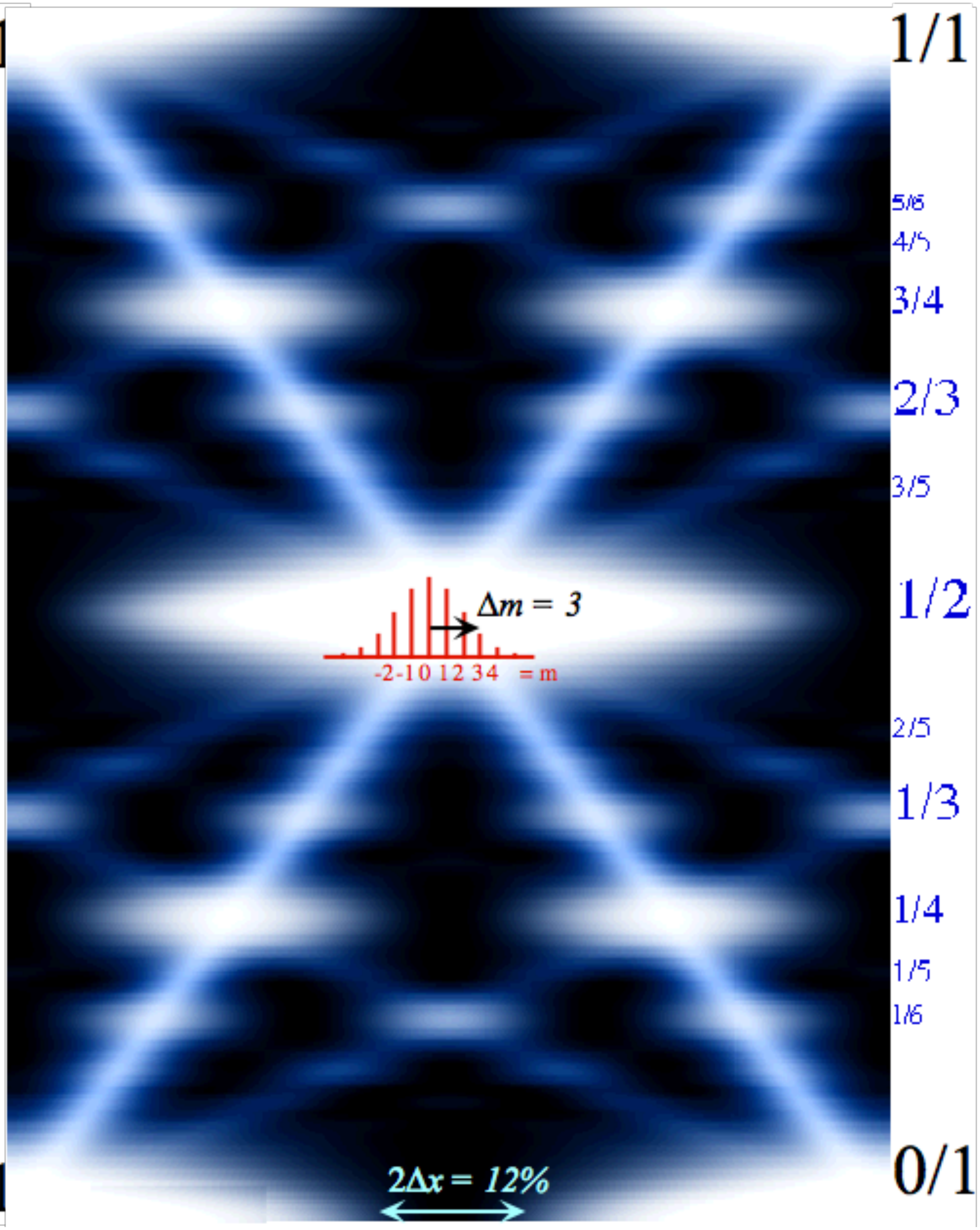


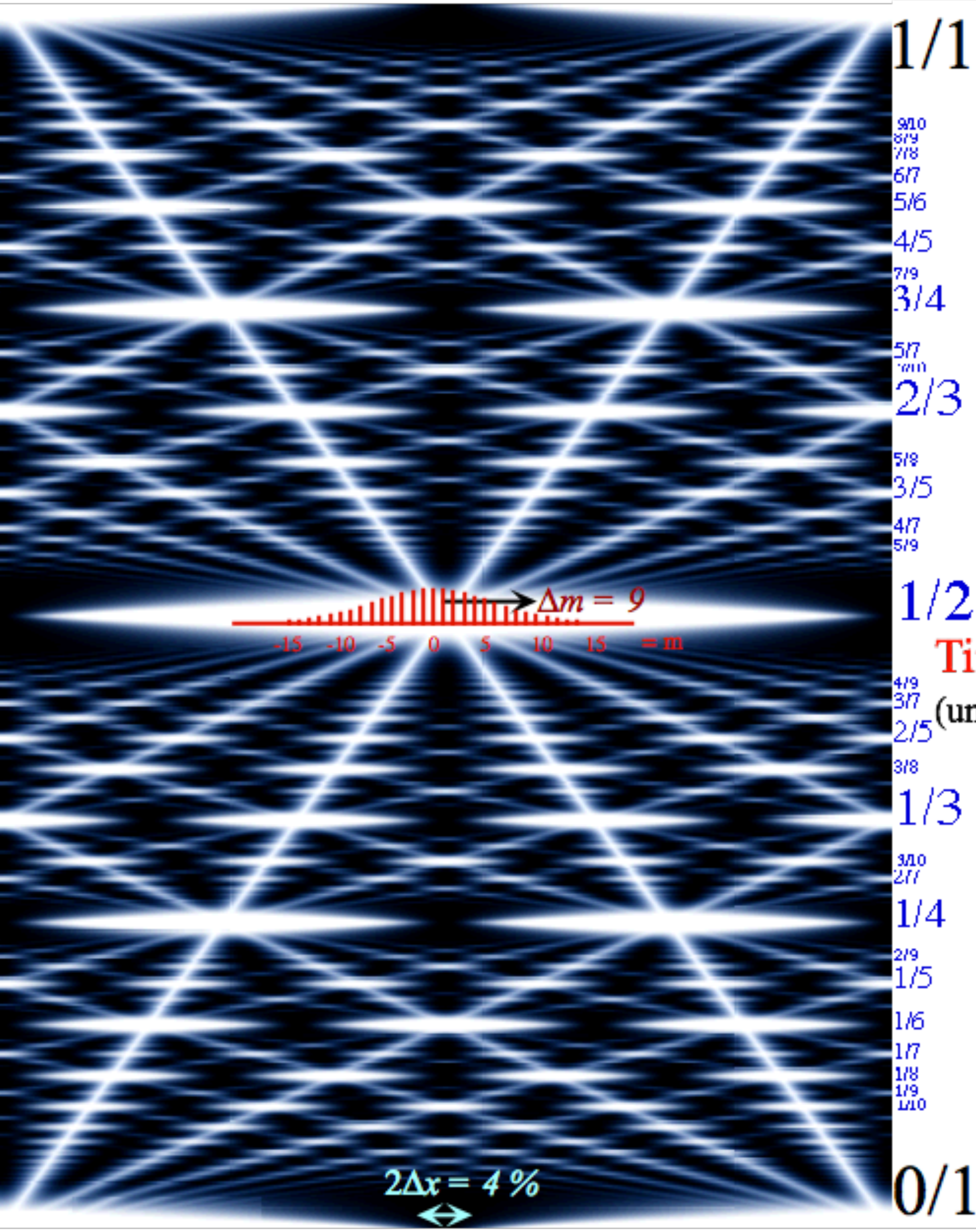
(4-levels (0, ±1,±2,±3) (and some ±4) excited)

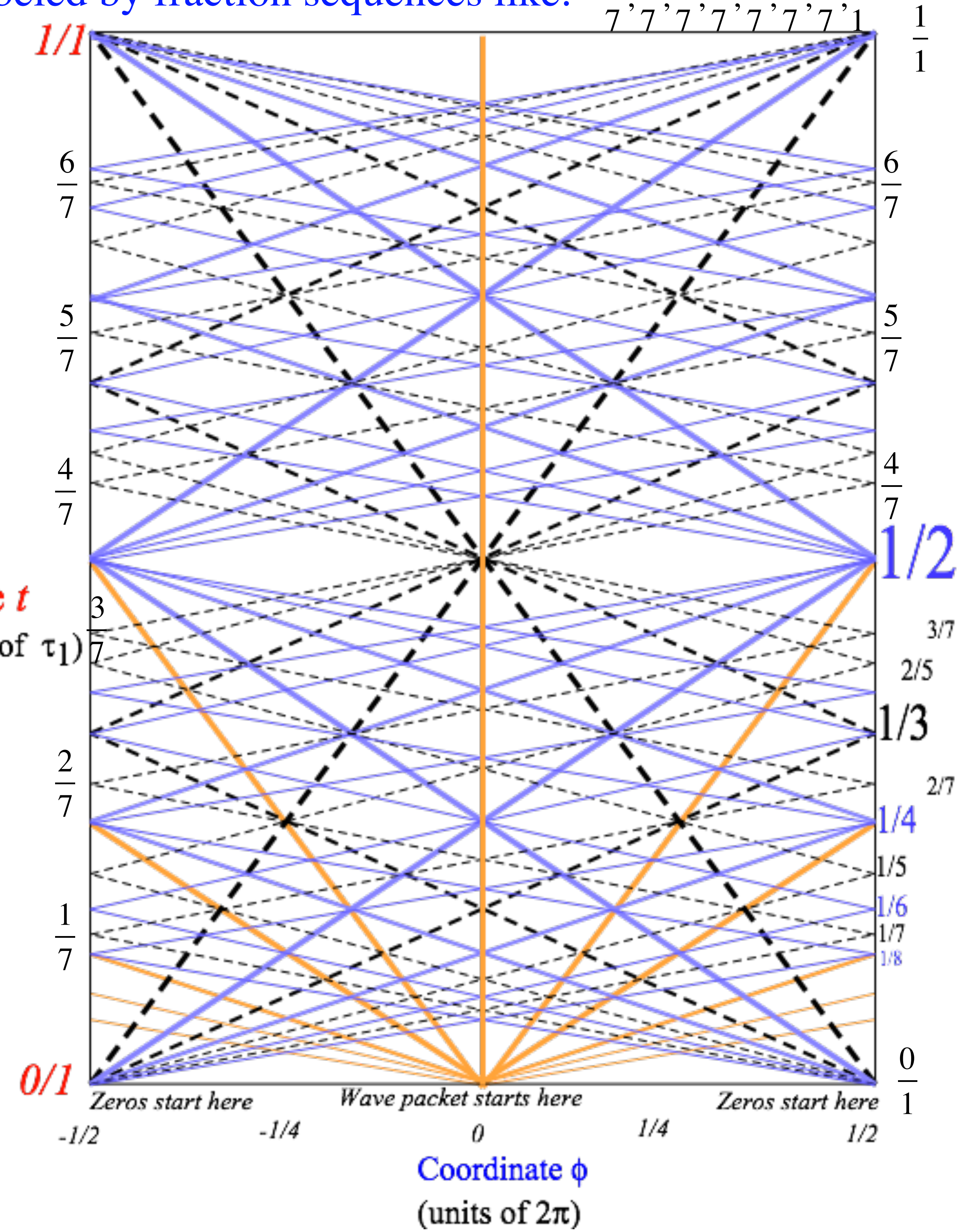


N-level-system and revival-beat wave dynamics

(9 or10-levels (0, ±1, ±2, ±3, ±4,..., ±9, ±10, ±11...) excited)

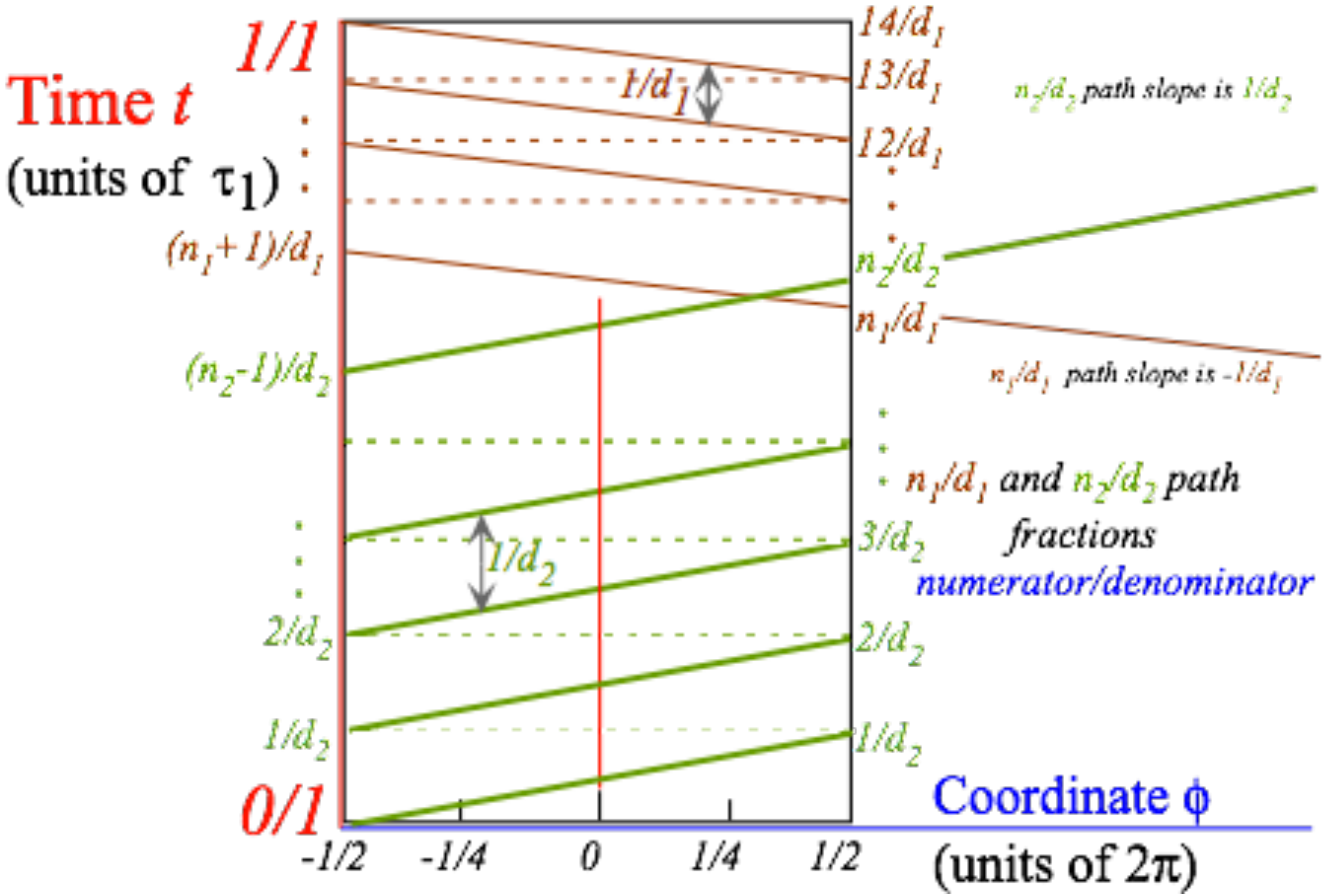
Zeros (clearly) and “particle-packets” (faintly) have paths labeled by fraction sequences like: $\frac{0}{7}, \frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}, \frac{1}{1}$





Farey Sum algebra of revival-beat wave dynamics

Label by *numerators N* and *denominators D* of rational fractions *N/D*



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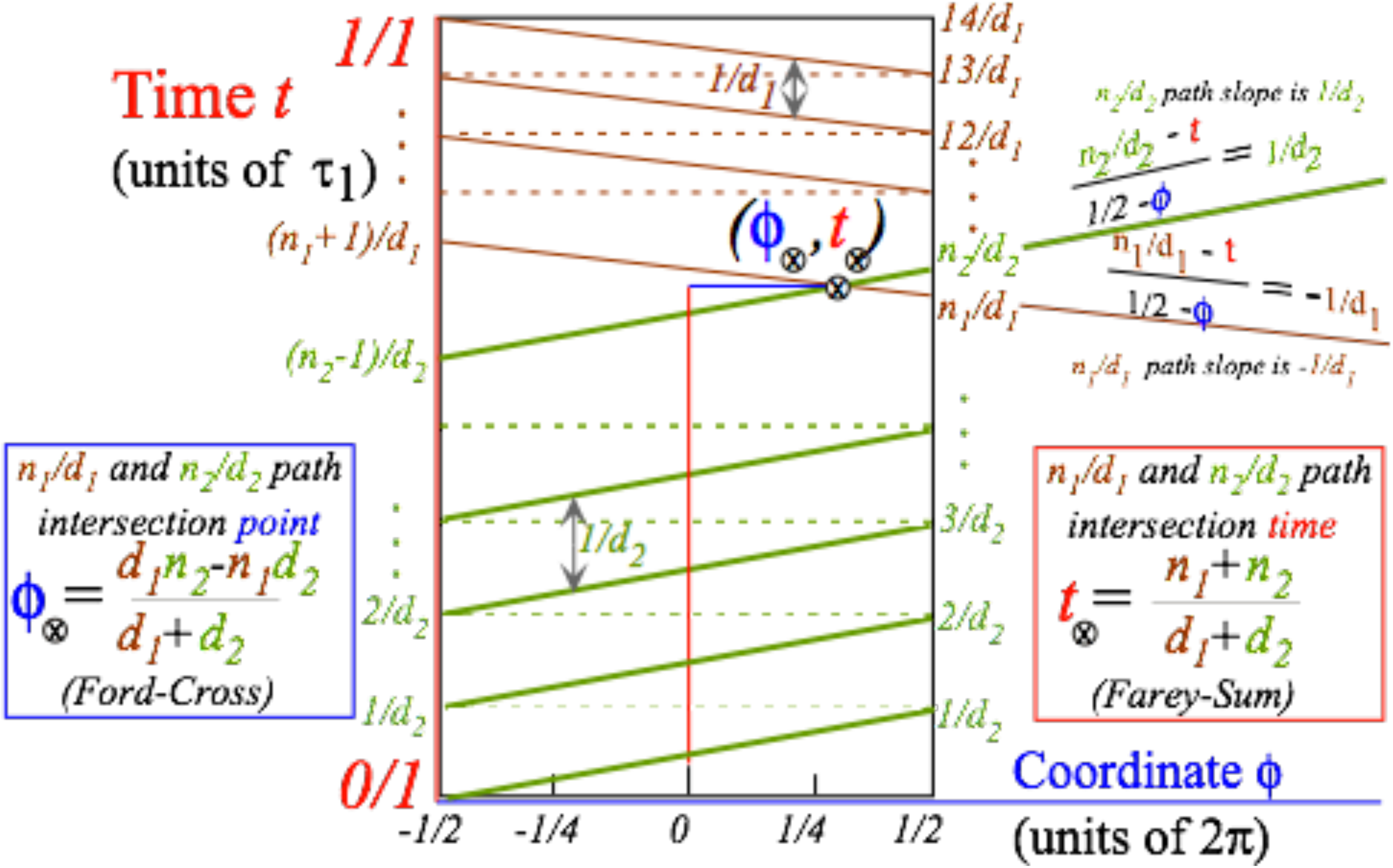
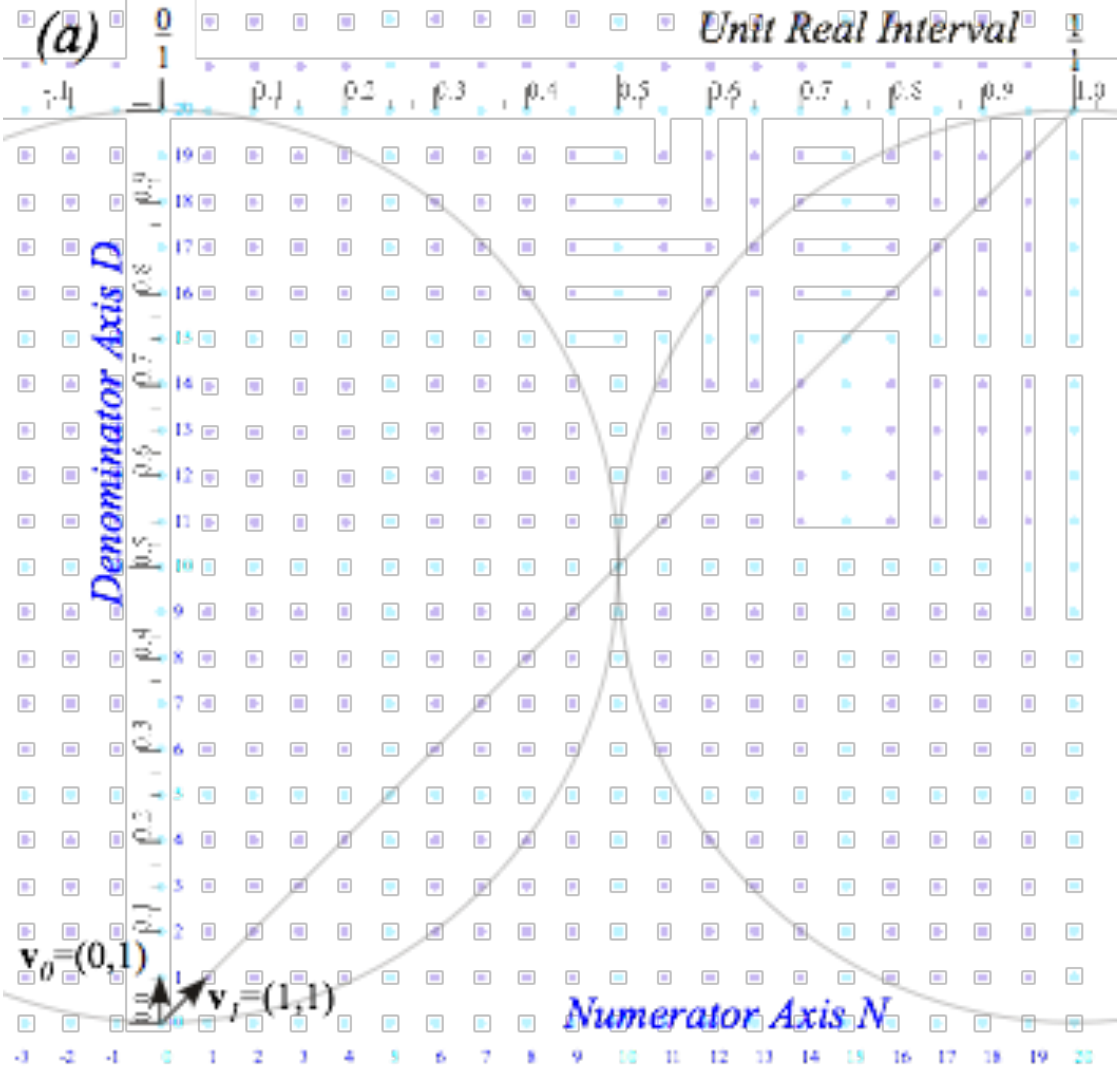


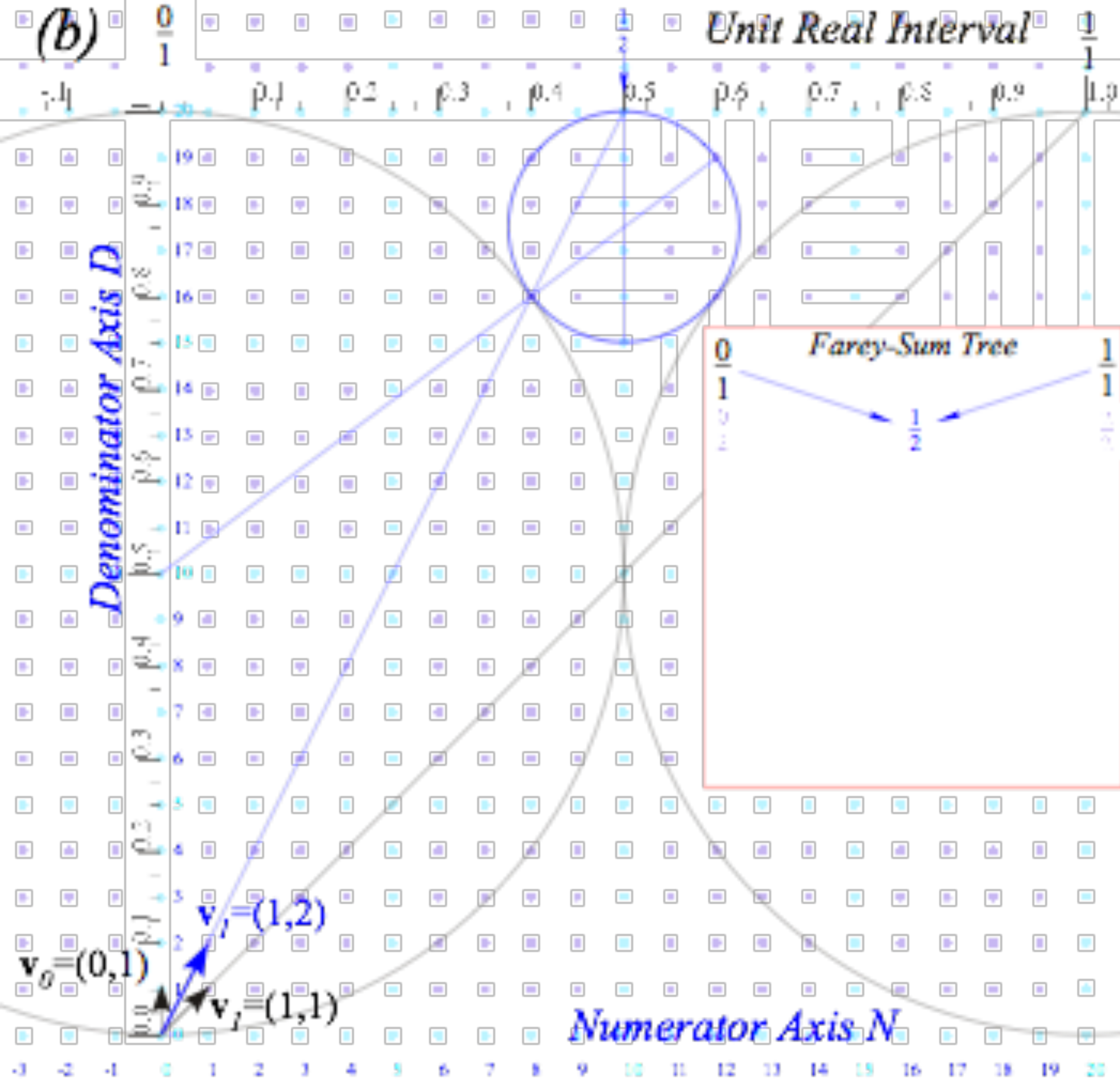
A Lesson in *Rational Fractions N/D*

(...that you can take home for your kids!)



Farey Sum related to vector sum and *Ford Circles*

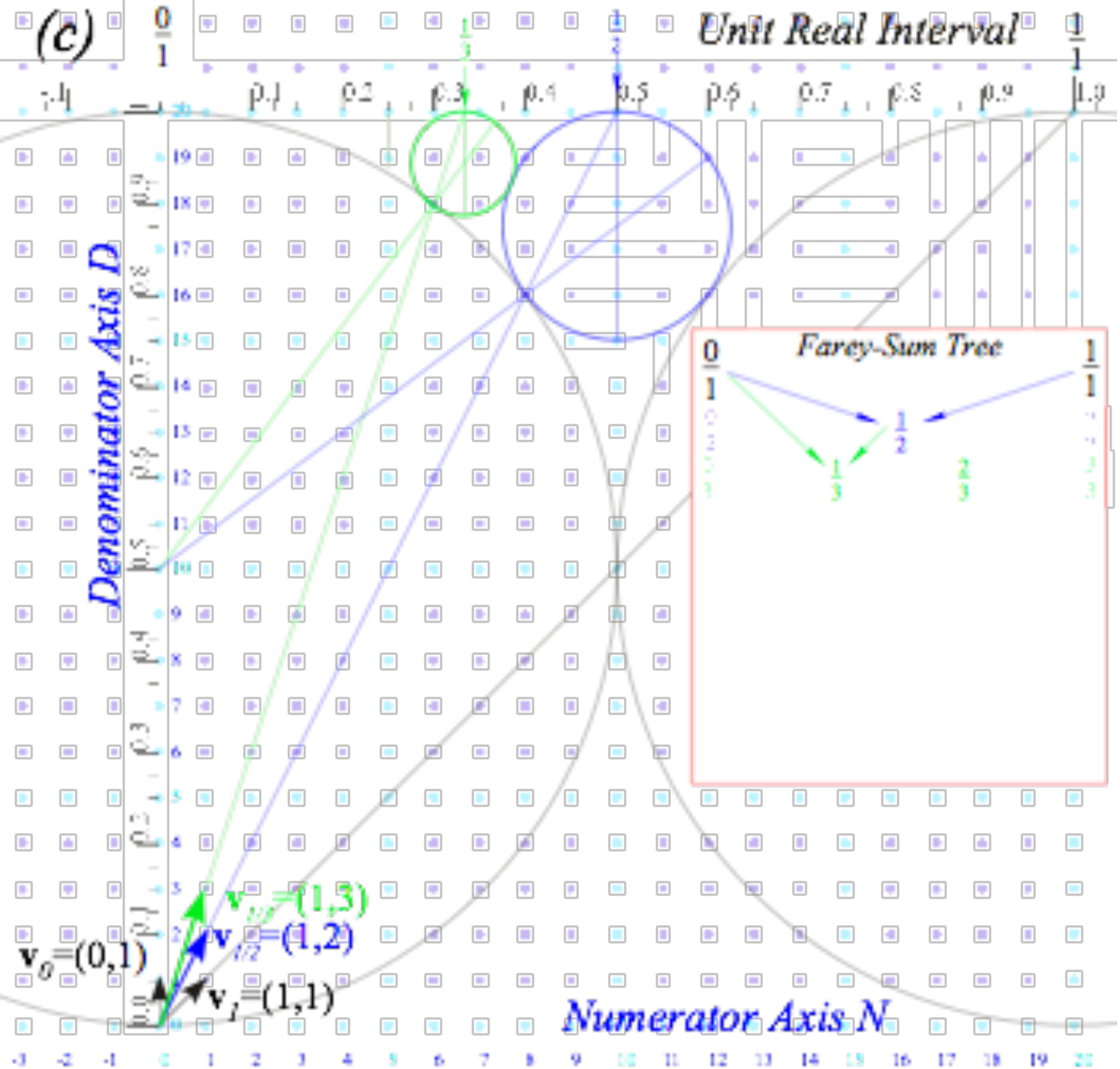
1/1-circle has diameter *1*



Farey Sum related to vector sum and *Ford Circles*

1/1-circle has diameter *1*

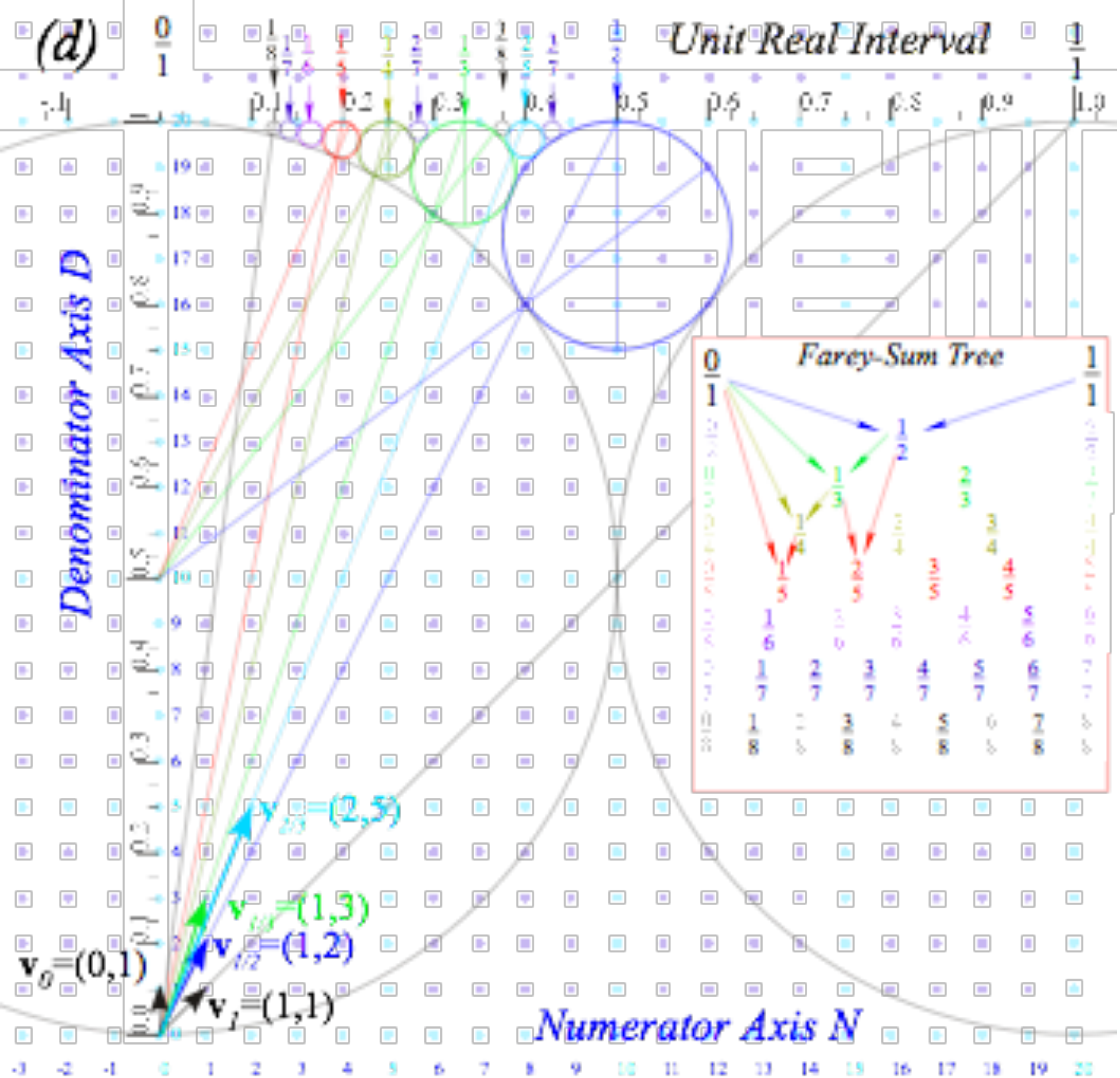
1/2-circle has diameter $1/2^2=1/4$



Farey Sum related to vector sum and *Ford Circles*

1/2-circle has diameter $1/2^2=1/4$

1/3-circles have diameter $1/3^2=1/9$

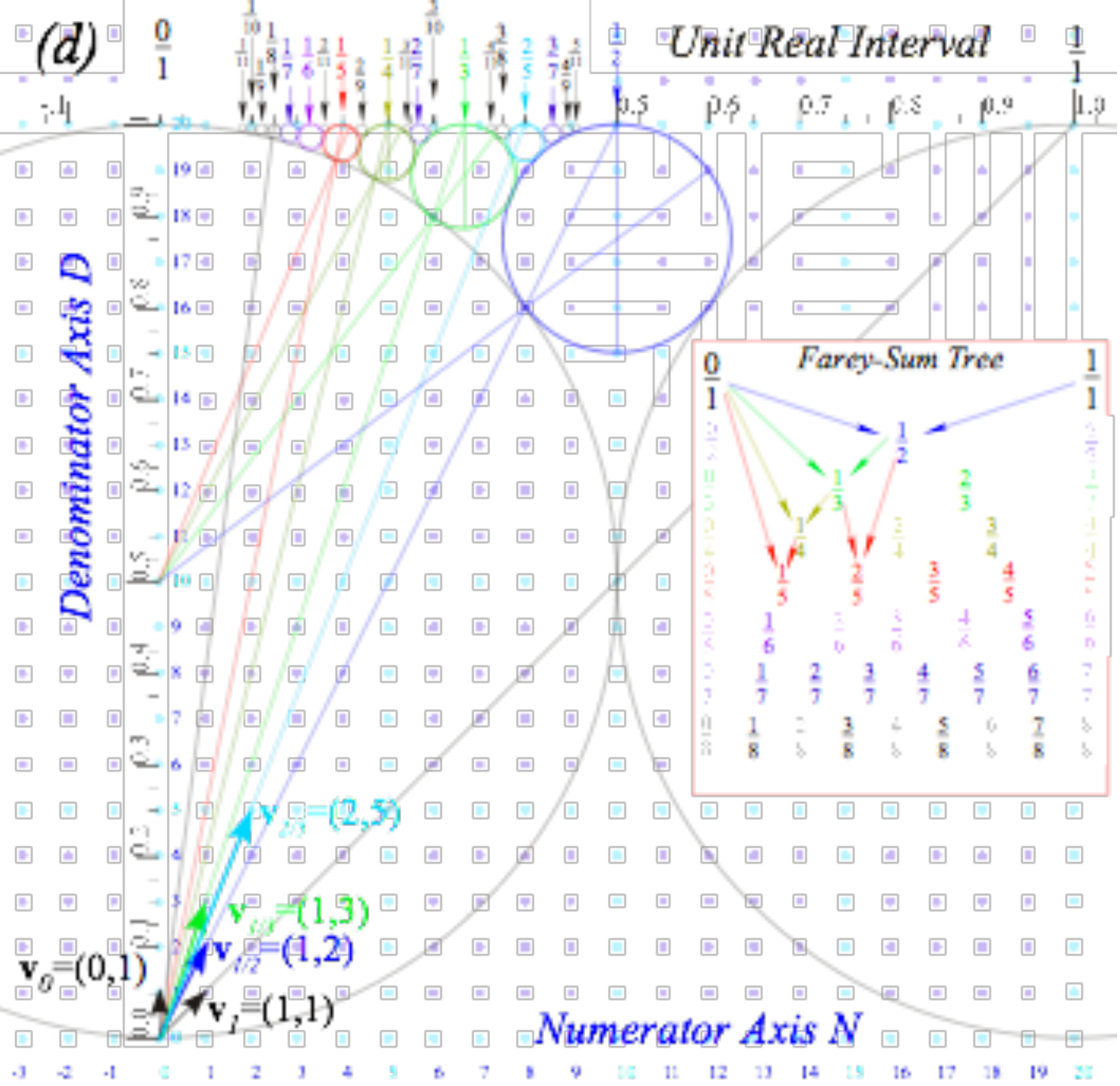


Farey Sum related to vector sum and *Ford Circles*

1/2-circle has diameter $1/2^2=1/4$

1/3-circles have diameter $1/3^2=1/9$

n/d-circles have diameter $1/d^2$



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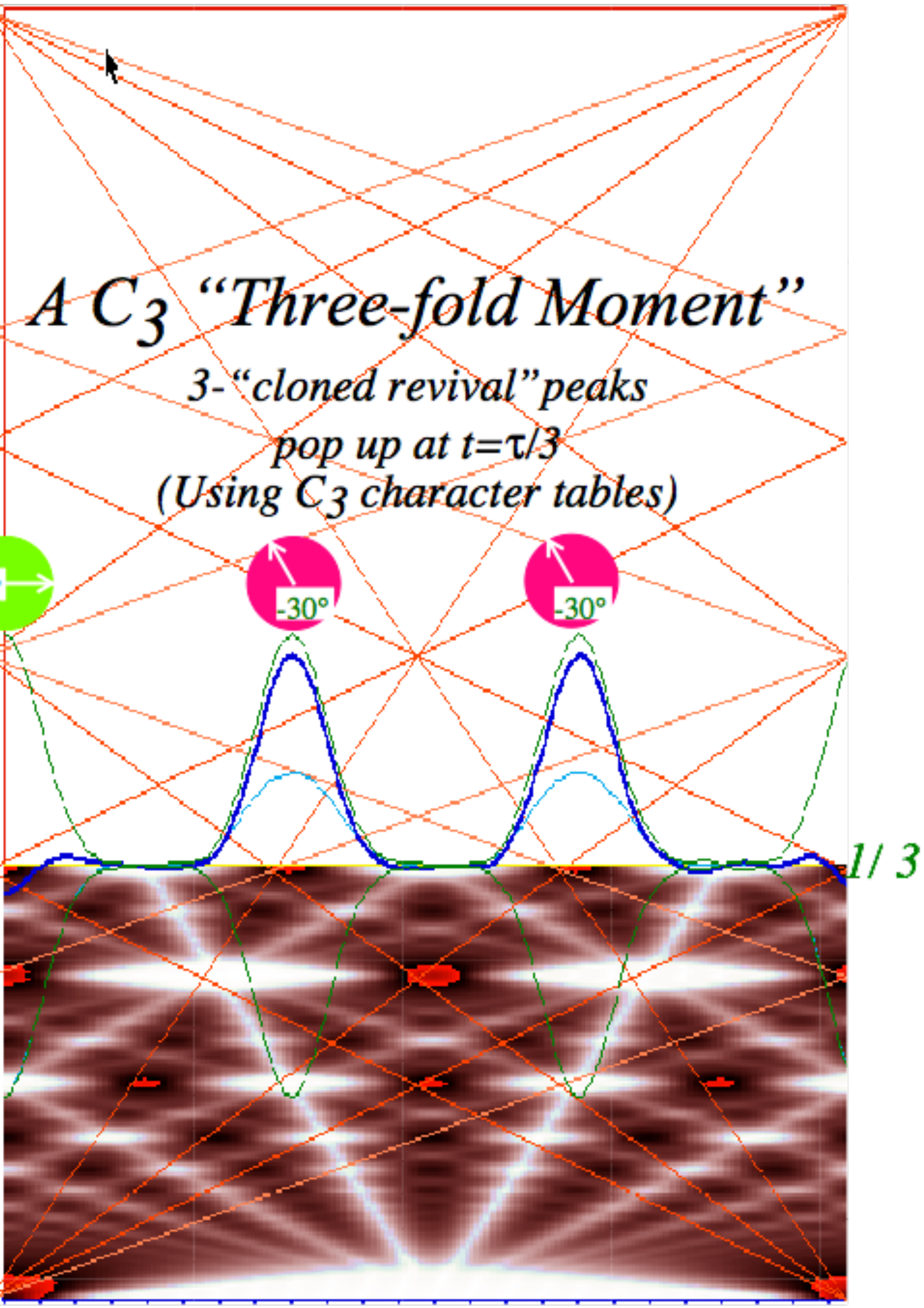
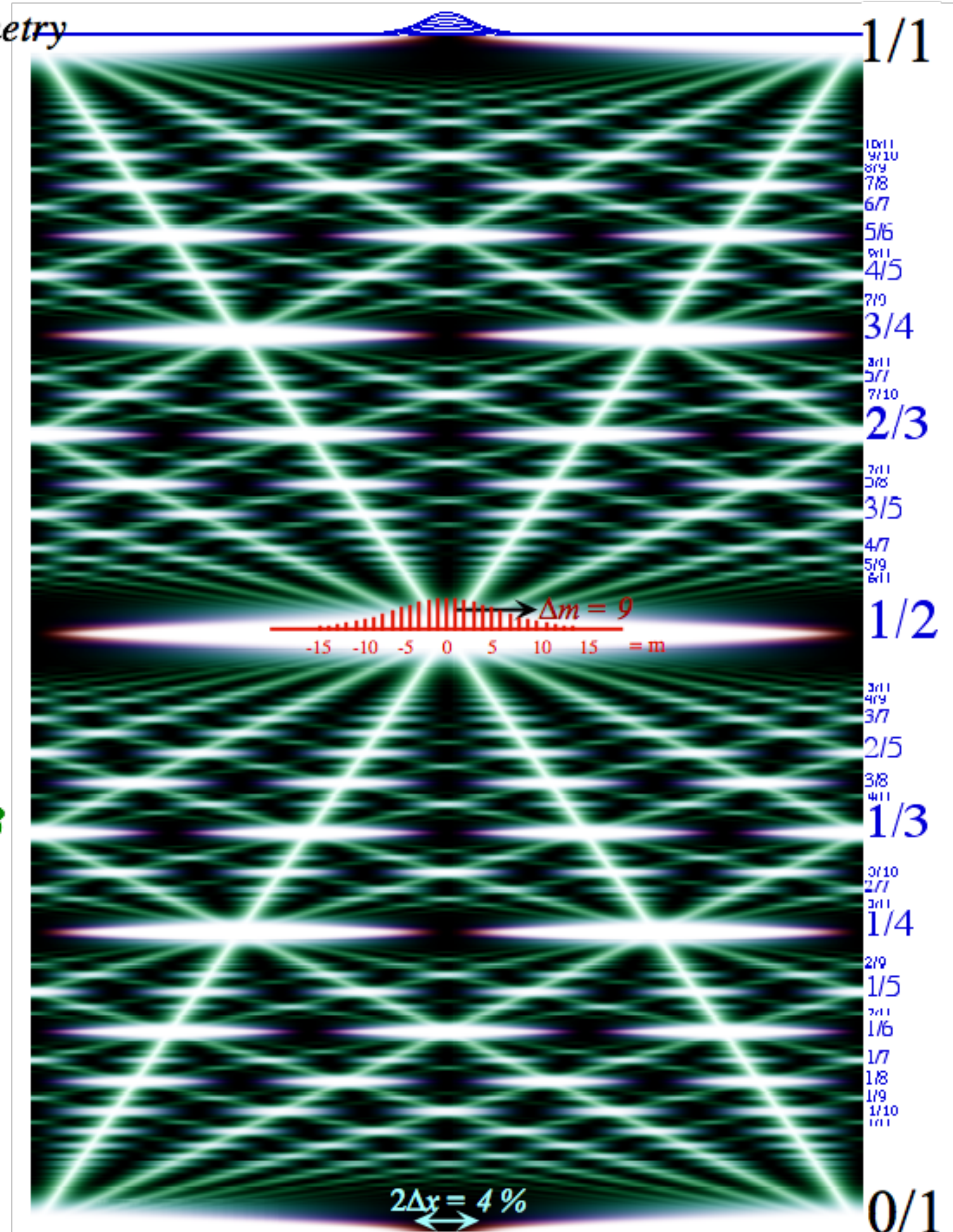
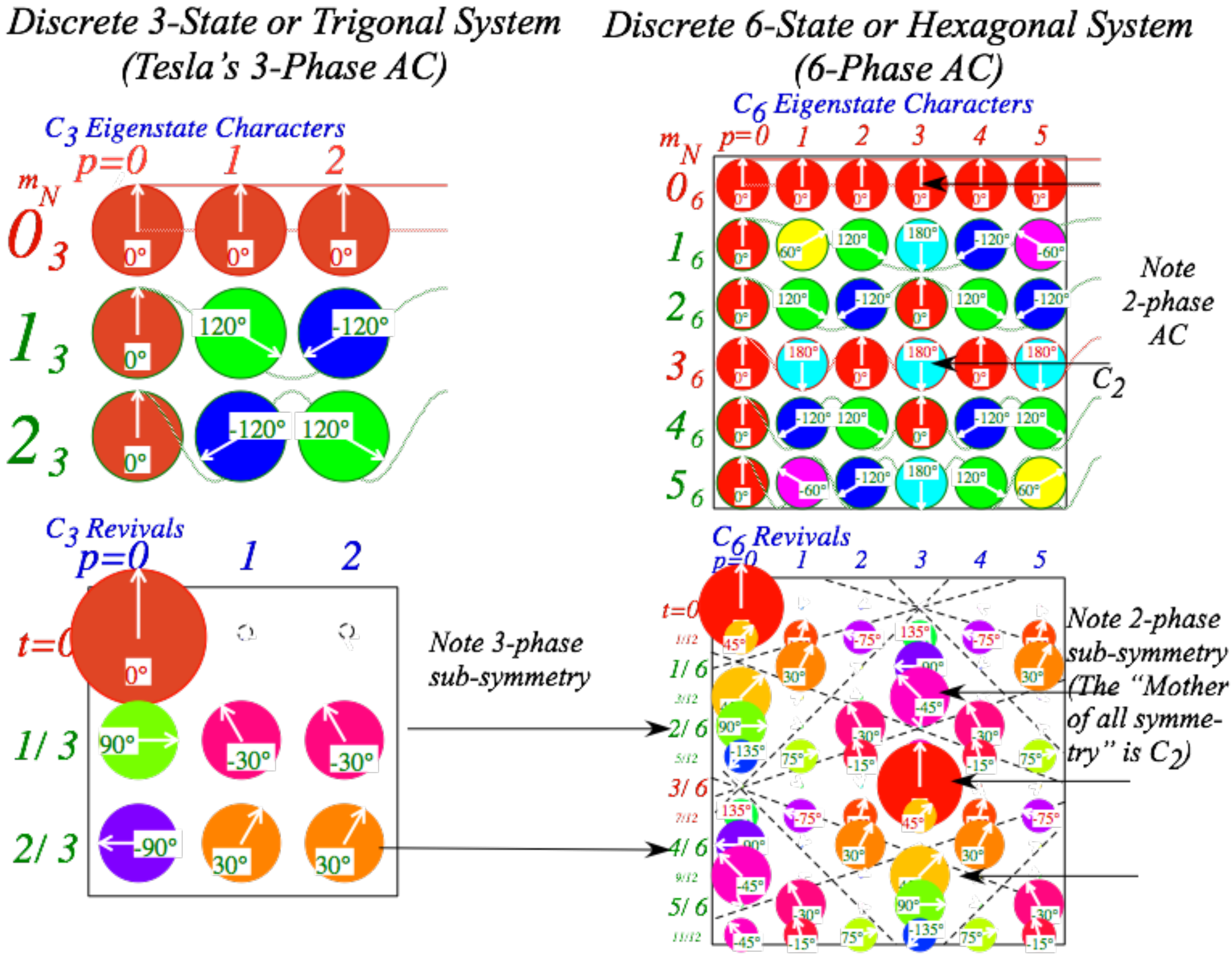
C_m algebra of revival-phase dynamics

Quantum rotor fractional take turns at Cn symmetry





C_m algebra of revival-phase dynamics



Summary

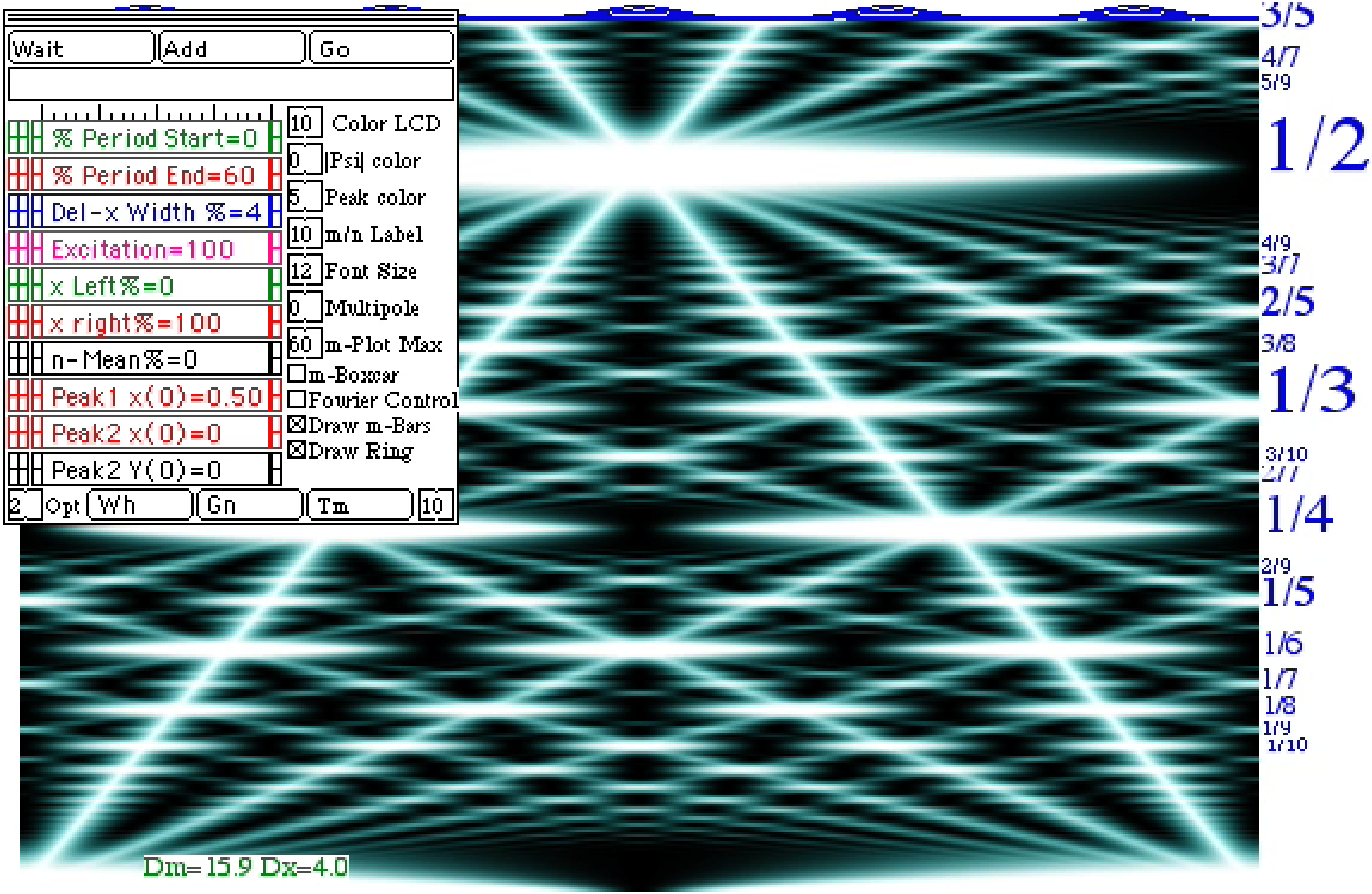
Quantum rotor revivals obey wonderfully simple geometry, number, and group theoretical analysis
and
as the next talk will show...

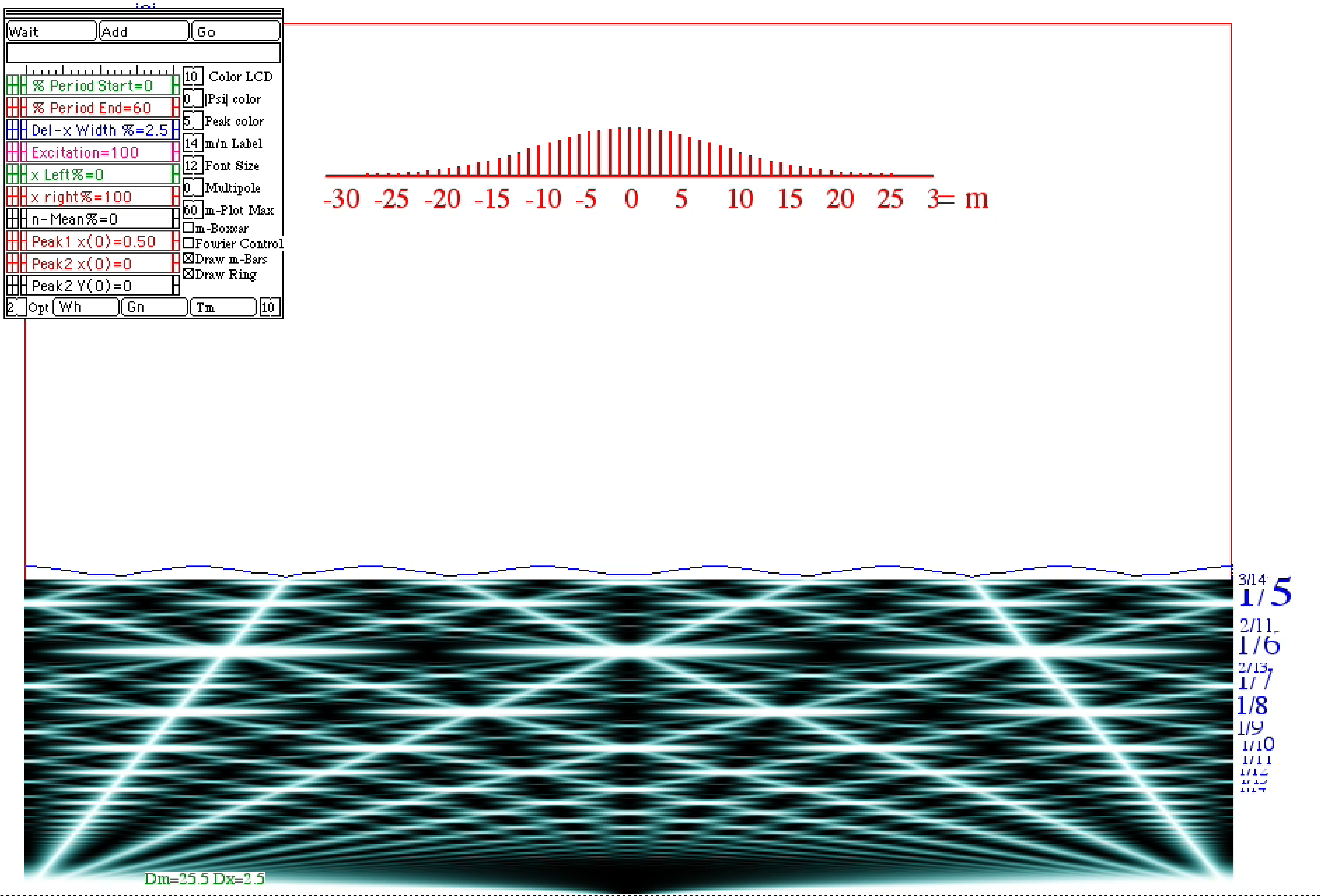
Summary

Quantum rotor revivals obey wonderfully simple geometry, number, and group theoretical analysis
and
as the next talk will show…

"I still don't really know... revivals … at all."

Simulation of revival-intensity dynamics

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Add
Go
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% Period End=60
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x Left%=0
x right%=100
n-Mean%=0
Peak1 x(0)=0.50
Peak2 x(0)=0
Peak2 Y(0)=0
10 Color LCD
0 |Psi| color
5 Peak color
14 m/n Label
12 Font Size
0 Multipole
60 m-Plot Max
m-Boxcar
Fourier Control
Draw m-Bars
Draw Ring
2 Opt
Wh
Gn
Tm
10
-30 -25 -20 -15 -10 -5 0 5 10 15 20 25 3= m
3/14
1/5
2/11
1/6
2/13
1/7
1/8
1/9
1/10
1/11
Dm=25.5 Dx=2.5