Improving STEM Teaching and Learning
William G. Harter and Tyle Reimer
Sponsored by Al Calabrese

Finding spectacular and interesting examples
…and simpler models to explain how they work

Deriving the simplest collision theory (By ruler&compass geometry)
Galileo’s relativity (an approximation)
*Project Ball* and problems with selling superball missiles to Whammo Co.

Having fun with examples by stretching theory to its limits
The *Astroblaster* (Superball towers and connection to Supernovae and toothpaste)
The *Monster Mash* (Crushing a bouncing particle)
The *Tiny-Big-Bang* (Watching Bohr wave-packet blow up…and down)
…and how not to add fractions on the *Titanic*
…and how not to add fractions on the *Titanic* 
(Farey’s sum & Ford’s circles)

Showing cultural and historical connection to examples
The *Trebuchet* (and how we owe almost everything to its mechanics)

Unifying Relativity with Quantum Theory (Why a *Men In Black* candidate shot little Suzy)
If 2-ball-collisions give *Classical Mechanics*, what do 2-laser-beam collisions give?
Lasers make relativistic (Minkowski) space-time (x,ct)-coordinates and (ω,ck), too
Find spectacular and interesting examples

**Multistage Throwing Devices**

**Superball Missiles**
(1965-2004)

**The Trebuchet**
(~$10^3$ BC-1520?)

*What Galileo might have tried to solve*

*What Galileo did solve*

(simple harmonic pendulum)

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*Am. J. Phys. 39, 656 (1971)*
(A class project)

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**Multistage Throwing Devices**

**Superball Missiles**
(1965-2004)

*X2 from Adv. Mechanics 2016*
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**Multistage Throwing Devices**

**Superball Missiles**
*(1965-2004)*

A problem in **space-time**: *(60mph cell-faxing 4ton SUV rear-ends 10mph 1ton VW)*

Before collision.....

After collision...what velocities?
Find spectacular and interesting examples
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A problem in space-time: (60mph cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Before collision....

After collision...what velocities?

Simulator
Elastic Collision
Dual Panel Space vs Space and Time vs. Space(Minkowski)

Simulator
Inelastic Collision
Dual Panel Space vs Space and Time vs. Space(Minkowski)

*Launch Car Collision Web Simulator
http://www.uark.edu/ua/modphys/markup/CMMotionWeb.html
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A problem in *space-time*: (60mph cell-faxing 4ton SUV
rear-ends 10mph 1ton VW)

Before collision.....

After collision...what velocities?

Let’s see if we can solve this *easily* with just **one** (or one-and-a-half) axiom(s)

Axiom-1: All mass or masses keep their total momentum until it is changed by some outsider.
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A problem in space-time: (60mph cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Before collision.....

After collision...what velocities?

Let’s see if we can solve this easily with just one (or one-and-a-half) axiom(s)

**Axiom-1: All mass or masses keep their total momentum until it is changed by some outsider.**

Conventional solution:
Look up the usual momentum and energy formulas/axioms and solve...

\[ \sum mV_i(\text{initial}) = \sum mV_i(\text{final}) \]
\[ \sum mV^2_i(\text{initial}) = \sum mV^2_i(\text{final}) \]

...But an UNconventional way is quicker and more revealing.....
..... (Just have to draw 2 lines!)

V_{SUV} and V_{VW} change violently but **total momentum** is constant

\[ \mathbf{P}_{\text{Total}} = M_{SUV} V_{SUV} + M_{VW} V_{VW} \]
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A problem in **space-time** : (60mph cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

**Before collision.....**

**After collision...what velocities?**

...But an UNconventional way is quicker and more revealing.....

(Just have to draw 2 lines!)

**Inventor of “Occam’s Razor”**

**Conventional solution:**
Look up the usual momentum and energy formulas/axioms and solve:

\[
\sum mV_i(\text{initial}) = \sum mV_i(\text{final})
\]

\[
\sum mV_i^2(\text{initial}) = \sum mV_i^2(\text{final})
\]

\[V_{\text{SUV}}\text{ and } V_{\text{VW}}\text{ change violently but } P_{\text{Total}}\text{ is constant}
\]

\[P_{\text{Total}} = M_{\text{SUV}} V_{\text{SUV}} + M_{\text{VW}} V_{\text{VW}}\]
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A problem in *space-time*:

**Before collision.....**

**1 mile**


**1 minute (60 sec.)**

-1  -0.8  -0.6  -0.4  -0.2  0

**Velocity-velocity Plot**

**Conventional solution:**
Look up the usual *momentum* and *energy* formulas/axioms and solve:

\[
\Sigma m_i V_i(\text{initial}) = \Sigma m_i V_i(\text{final})
\]

\[
\Sigma m_i V_i^2(\text{initial}) = \Sigma m_i V_i^2(\text{final})
\]

...But an UNconventional way is quicker and more revealing.....

...... (Just have to draw 2 lines!)

But, where-oh-where does it end?

Inventor of “Occam’s Razor”

**V**_{SU} and **V**_{VW} change violently but **total momentum** is constant

\[
P_{\text{Total}} = M_{SU} V_{SU} + M_{VW} V_{VW}
\]
Find spectacular and interesting examples … and simpler models to explain how they work

A problem in space-time:

Before collision.....

1 mile
-1  -0.8  -0.6  -0.4  -0.2


1 minute (60 sec.)

After collision...what velocities?

Perfectly elastic case

Kabong!

Totally inelastic case

Ka-runch!

But, where-oh-where does it end?

"Ka-runch!" ends on 45° line

slope = \frac{\Delta V}{\Delta t} = \frac{M_{SUV}}{-m_{VW}} = \frac{\Delta V}{\Delta t}\text{ SUV}

(V_{SUV} = V_{VW})

...But an UNconventional way is quicker and more revealing.....

..... (Just have to draw 2 lines!)

Conventional solution:

Look up the usual momentum and energy formulas/axioms and solve...

\[ \sum m_i v_i (\text{initial}) = \sum m_i v_i (\text{final}) \]

\[ \sum m_i v_i^2 (\text{initial}) = \sum m_i v_i^2 (\text{final}) \]

Inventor of “Occam’s Razor”

V_{SUV} and V_{VW} change violently but total momentum is constant

\[ P_{Total} = M_{SUV} V_{SUV} + M_{VW} V_{VW} \]
(a) During collision......

During collision......

Before collision: INITIAL

After Ka-runch ! collision......

Velocity-velocity plot

of Axiom-1:

\[
M_{SUV} V_{SUV} + M_{VW} V_{VW} = \text{constant} = P_{\text{Total}} = 250
\]

\[
V_{VW} = -\frac{M_{SUV}}{M_{VW}} V_{SUV} + \frac{P_{\text{Total}}}{M_{VW}} = -4 V_{SUV} + 250
\]

...with a simple Cartesian line-plot.

It's a simple Cartesian equation...

\[
4 \cdot x + 1 \cdot y = 250
\]

\[
y = 250 - 4 \cdot x
\]

\[
5 \cdot x = 250 \quad \text{...with } y = x = 50
\]

...with a simple Cartesian line-plot.

(b) After Ka-runch ! collision......

After Ka-runch ! collision......

It’s a simple Cartesian equation...

\[
4 \cdot x + 1 \cdot y = 250
\]

\[
y = 250 - 4 \cdot x
\]

\[
5 \cdot x = 250 \quad \text{...with } y = x = 50
\]
(a) During collision...

Velocity-velocity plot of Axiom-1:

\[ M_{SUV} \vec{V}_{SUV} + M_{VW} \vec{V}_{VW} = \text{constant} = P_{Total} = 250 \]

Before collision: INITIAL

\[ \vec{V}_{VW} = -\frac{M_{SUV}}{M_{VW}} \vec{V}_{SUV} + \frac{P_{Total}}{M_{VW}} \]

= \(-4 \vec{V}_{SUV} + 250 \)

After Ka-runch collision...

\[ \text{FINAL} \]

\[ \vec{V}_{VW} \]

10 mph

100 mph

50 mph

0

(b) After Ka-runch collision...

\[ \text{FINAL} \]

\[ \vec{V}_{VW} \]

10 mph

100 mph

50 mph

0

After Ka-Bong collision...

\[ \text{FINAL} \text{ Ka-Bong} \]

\[ \vec{V}_{VW} \]

10 mph

100 mph

50 mph

0
Auto-paused at time = 1.83
BounceIt Simulation: frictionless 1D-track with elastic bumper cars bouncing between walls

(a) Wall-2

Car-2
(VW)
$M_2 = 1$

Car-1
(SUV)
$M_2 = 4$

Wall-1

(b) $v^{FIN-2}$ $V_2$

$v^{FIN-3}$

$V^{FIN} = (40, 90)$

$v^{FIN} = (60, 10)$

$v^{FIN-2} = (40, -90)$

$v^{IN} = (50, 50)$

$v^{IN-3}$

From Fig. 2.4
Developing Conservation-of-Momentum
The key axiom of mechanics leading to Conservation-of-Energy Theorem

If and only if... there is T-Symmetry
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Unifying Relativity with Quantum Theory (Why a *Men In Black* candidate shot little Suzy)
  If 2-ball-collisions give *Classical Mechanics*, what do 2-laser-beam collisions give?
    Lasers make relativistic (Minkowski) space-time (x,ct)-coordinates and (ω,ck), too
A problem in **space-time** : (60 mph Cell-faxing 4ton SUV rear-ends 10 mph 1ton VW)

Geometry of Galilean translation (A **symmetry transformation**)

If you increase your velocity by 50 mph,...

...the rest of the world appears to be 50 mph **slower**

(In some direction x,y, or z...)

(In that direction...)

(a) Galileo transforms to **COM** frame

Galileo Galilei
1564-1642
A problem in space-time: (60 mph Cell-faxing 4ton SUV rear-ends 10 mph 1ton VW)

Geometry of Galilean translation (A symmetry transformation)
If you increase your velocity by 50 mph,... (In some direction x,y, or z...)
...the rest of the world appears to be 50 mph slower (In that direction...)

(a) Galileo transforms to COM frame
(b) ... and to five or six other reference frames

Fig. 2.5a in Unit 1

Fig. 2.5b in Unit 1
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*Multistage Throwing Devices*

*SUPERBALL MISSLES (1965-2004)*

http://www.uark.edu/ua/modphys/markup/BounceItWeb.html?scenario=6300

_Am. J. Phys. 39, 656 (1971)
(A class project)_
The X-2 Pen launcher and Superball Collision Simulator*

The X-2 pen-launcher

Superball penetration depth

\[ d \approx \frac{r^2}{2R} \]

ballpoint pen
\( M_2 = 10 \text{gm} \)

Superball
\( M_1 = 70 \text{gm} \)

bounce plate
\( M_0 = 10 \text{kg} \)

Fig. 3.1
(Unit 1)

\( m_1 = 70.000 \text{ g} \)
\( m_2 = 10.000 \text{ g} \)

V1 = -1.000 cm/s
V2 = -1.000 cm/s

m1

m2

http://www.uark.edu/ua/modphys/markup/BounceltWeb.html?scenario=6300

*Launch Generic Superball Collision Web Simulator

http://www.uark.edu/ua/modphys/markup/BounceltWeb.html?scenario=1007

(With g=0 and 70:10 mass ratio)
Fig. 3.3 (Unit 1)

(a) Super-elastic 2nd-body bounce

(b) 2-Bang Model

(c) n-Body Supernova Superballs

(Still Bigger BANG!)

(Bigger BANG!)

Fig. 3.4 (Unit 1)

This 1st bang is a floor-bounce of $M_1$ off very massive plate/Earth $M_0$

1st bang: $M_1$ off floor

2nd bang: $m_2$ off $M_1$

3rd bang: $m_2$ off ceiling

Ballpoint pen

$M_2 = 10$ gm

Superball penetration depth

$d = \frac{v^2}{2R}$

Bounce plate $M_0 = 10$ kg

((With $g=0$ and 70:10 mass ratio))

http://www.uark.edu/ua/modphys/markup/BounceItWeb.html?scenario=1007

http://www.uark.edu/ua/modphys/testing/markup/BounceItWeb.html

(With $g$ and 70:35 mass ratio)
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Unit 1
Fig. 8.2a-b
4-Body IBM Geometry
Fig. 8.2c-d
4-Equal-Body Geometry

BounceIt Simulation: 4-Ball Tower w/ $m_k/m_{k+1} = 3$

Opposite of continuous wave dynamics introduced in Unit 2

BounceIt Simulation: 4-Ball Tower w/ $m_k/m_{k+1} = 1$
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A story of Stirling Colgate (Palmolive) and core-collapse supernovae

Source
http://hubblesite.org/newscenter/archive/releases/2007/10/image/a/

Author
NASA, ESA, P. Challis, and R. Kirshner (Harvard-Smithsonian Center for Astrophysics)

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**Core-burning nuclear fusion stages for a 25-solar mass star**

<table>
<thead>
<tr>
<th>Process</th>
<th>Main fuel</th>
<th>Main products</th>
<th>Temperature (Kelvin)</th>
<th>Density (g/cm³)</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>hydrogen burning</td>
<td>hydrogen</td>
<td>helium</td>
<td>7×10⁷</td>
<td>10</td>
<td>10⁷ years</td>
</tr>
<tr>
<td>triple-alpha process</td>
<td>helium</td>
<td>carbon, oxygen</td>
<td>2×10⁸</td>
<td>2000</td>
<td>10⁶ years</td>
</tr>
<tr>
<td>carbon burning process</td>
<td>carbon</td>
<td>Ne, Na, Mg, Al</td>
<td>8×10⁸</td>
<td>10⁶</td>
<td>10³ years</td>
</tr>
<tr>
<td>neon burning process</td>
<td>neon</td>
<td>O, Mg</td>
<td>1.6×10⁹</td>
<td>10⁷</td>
<td>3 years</td>
</tr>
<tr>
<td>oxygen burning process</td>
<td>oxygen</td>
<td>Si, S, Ar, Ca</td>
<td>1.8×10⁹</td>
<td>10⁷</td>
<td>0.3 years</td>
</tr>
<tr>
<td>silicon burning process</td>
<td>silicon</td>
<td>nickel (decays into iron)</td>
<td>2.5×10⁹</td>
<td>10⁸</td>
<td>5 days</td>
</tr>
</tbody>
</table>

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Within a massive, evolved star (a) the onion-layered shells of elements undergo fusion, forming a nickel-iron core (b) that reaches Chandrasekhar mass and starts to collapse. The inner part of the core is compressed into neutrons (c), causing infalling material to bounce (d) and form an outward-propagating shock front (red). The shock starts to stall (e), but it is re-invigorated by neutrino interaction. The surrounding material is blasted away (f), leaving only a degenerate remnant.
Stirling Colgate

From Wikipedia, the free encyclopedia

Stirling Auchtincross Colgate (November 14, 1925 – December 1, 2013) was an American physicist at Los Alamos National Laboratory and a professor emeritus of physics, past president at the New Mexico Institute of Mining and Technology (New Mexico Tech), and an heir to the Colgate toothpaste family fortune. He was America's premier diagnostician of thermonuclear weapons during the early years at the Lawrence Livermore National Laboratory in California. While much of his involvement with physics is still highly classified, he made many contributions in the open literature including physics education and astrophysics. He was born in New York City in 1925, to Henry Auchtincross and Jeanette Thurber (née Pruyne) Colgate.

..an amusing off-color aside story of Stirling Colgate’s NMIMT resignation...

(Not told in Wikipedia!)

Quote

- "I was always enamored with explosives, and eventually I graduated to dynamite and then nuclear bombs."
Multiple-collision accelerator assembly
US 5256071 A

ABSTRACT
A device comprising several highly elastic objects is presented whose purpose is to demonstrate an unobvious consequence of fundamental laws of physics—the acceleration of an object to high speed by multiple collisions among a series of heavier objects moving at slower speed. The objects, each of different mass, are arrayed in close proximity in order of decreasing mass with their centers lying along a straight line. This arrangement of the assembly of objects is maintained by a constraining element which permits the assembly axis to be oriented in any desired direction and permits the assembly to be moved or manipulated as a unit in any desired way without destroying the arrangement of objects. In the preferred embodiment the elastic objects are polybutadiene balls (12), the constraining element is an interior guide-pin (10) fastened in the largest ball and extending radially therefrom, on which the remaining balls can slide freely because of diametrical holes formed in them. In use this multiple-collision accelerator assembly is suspended in vertical orientation, with the largest ball downward, by holding the tip-end of the guide-pin which extends beyond the littlest ball. The assembly is then dropped onto a solid surface (14), the striking of which produces a sharp impulse that is transmitted from the largest ball, through the assembly, causing the littlest ball to be projected to a height many times that from which the assembly was dropped.

1st publication describing theory and experiment of this device 20 years before.

Velocity Amplification in Collision Experiments Involving Superballs
William G. Harter¹ (class of WGH)

¹ University of Southern California, Los Angeles, California 90007

Am. J. Phys. 39, 656 (1971); http://dx.doi.org/10.1119/1.1986253

(Point allowing patent over previous 1973 proposal (4))

Now I have to pay APS for my own paper.)
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The Classical “Monster Mash”

Classical introduction to Heisenberg “Uncertainty” Relations

\[ v_2 = \frac{\text{const.}}{Y} \quad \text{or:} \quad Y \cdot v_2 = \text{const.} \]

is analogous to: \[ \Delta x \cdot \Delta p = N \cdot \hbar \]

Unit 1
Fig. 6.4

* Link to BounceIt “Monster Mash” \( x_2(t) \) animation
(Note: Time sense is inverted)
\[ V_2 = +0.064 \hat{i} + 0 \hat{j} \text{ cm/s} \]
\[ V_1 = -9.98 \times 10^{-4} \hat{i} + 0 \hat{j} \text{ cm/s} \]

Time = 34.276 s
\[ \Delta T = +0.002 \text{ s} \]
\[ E = +2.54 \times 10^{-4} \text{ erg} \]
Force constant = +5000000.000
Force power = +6.000
Drag (Collision) = +0

* Link to BounceIt “Monster Mash” \( V_2 \) vs \( x_2 \) animation
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$$\Delta m = 9$$

$$\Delta x = 4\%$$

(Time $t$ (units of fundamental period $\tau_1$)

(Imagine "wrap-around" $\phi$-coordinate)
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Farey Sum related to vector sum and **Ford Circles**

1/1-circle has diameter 1

**Farey-Sum** of fractions 0/1 and 1/1 is 1/2

That is vector sum $\mathbf{v}_0 + \mathbf{v}_1 = (1,2) = \mathbf{v}_2$

**Numerator Axis N**

**Denominator Axis D**

**Unit Real Interval**
Farey Sum related to vector sum and Ford Circles

1/1-circle has diameter $l$

$v_0$ circle radius intersecting $v_2$ line...points to center of $v_2$ Ford $1/2$-Circle

This vector $v_2$ points to real value $1/2 = 0.5$

Farey-Sum of fractions $0/1$ and $1/1$ is $1/2$

That is vector sum $v_0 + v_1 = (1, 2) = v_2$

$Farey-Sum$ of fractions $0/1$ and $1/1$ is $1/2$

That is vector sum $v_0 + v_1 = (1, 2) = v_2$

$Farey-Sum$ of fractions $0/1$ and $1/1$ is $1/2$

That is vector sum $v_0 + v_1 = (1, 2) = v_2$
Farey Sum related to vector sum and Ford Circles

1/2-circle has diameter $1/2^2 = 1/4$

1/3-circles have diameter $1/3^2 = 1/9$
Thales Rectangles provide analytic geometry of fractal structure

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Chapter 1. The Trebuchet: A dream problem for Galileo?

**Fig. 2.1.1** An elementary ground-fixed trebuchet

(a) What Galileo Might Have Tried to Solve

(b) What Galileo Did Solve

-Feb 2.1.2 Galileo's (supposed) problem

[Simpel pendulum dynamics](http://www.uark.edu/ua/modphys/markup/TrebuchetWeb.html)

Trebuchet simulator

http://www.uark.edu/ua/modphys/markup/TrebuchetWeb.html
What Trebuchet mechanics is **really** good for...
**What Trebuchet mechanics is really good for...**

- **Early Human Agriculture and Infrastructure Building Activity**
  - Throwing
  - Slinging
  - Chopping
  - Splitting
  - Cultivating and Digging
  - Reaping
  - Hammering
  - Bull whip cracking
  - Fly-fishing

- **Later Human Recreational Activity**
  - Lacrosse
  - Hammer throwing
  - Baseball & Football
  - Batting
  - Golfing
  - Cultivating and Digging

- **“Ring-The-Bell” (at the Fair)**
  - Tennis serving

- **Space Probe “Planetary Slingshot”**

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**The Atlatl**
(Cahokia, IL 12th Century)
**Trebuchet analogy with racquet swing** - What we learn

**Early on**
(Gain the energy/momentum)

- Large force $F$ nearly parallel to velocity $v$ so $v$ increases rapidly.
- Force $F$ nearly perpendicular to velocity $v$ so $v$ increases very little.

**Later on**
(Steer or guide)

- Rotation of body $r_b$ provides most of energy of arm-racquet lever $L$.
- Energy Input
  Most of speed gained early by arm-racquet system $L$.

**Follow-Through**
Arm-racquet system $L$ flies nearly freely.

Small applied forces mostly for steering.

Ball hit occurs.

**Preparation**
Center-of-mass for semi-rigid arm-racquet system $L$ is "cocked."
An Opposite to Trebuchet Mechanics- The “Flinger”

Web Simulation: Trebuchet - "Flinger"

Early on
(Not much happening)

Not much increase in velocity \( v \)

Later on
(Last-minute “cram” for energy)

Maximum increase in velocity \( v \) just before \( m \) slides off end

Anti-analogy can be useful pedagogy

Trebuchet-like experiment

Flinger experiment

skateboard wheel swings

skateboard wheel slides on pool cue-stick

**Trebuchet model in lab frame**

Assume: Constant beam $\omega$

\[\omega (r_b + \ell) = v_{\text{rotation}}\]

\[v_{\text{beam frame}}(\text{trebuchet}) = \begin{cases} \omega^2 \left(2r_b \ell\right) & \text{half-cocked 6 o'clock} \\ \omega^2 \left(4r_b \ell\right) & \text{fully-cocked 9 o'clock} \end{cases}\]

\[v_{\text{lab frame}}(\text{trebuchet}) = \begin{cases} \omega (r_b + \ell + \sqrt{2\ell r_b}) & \text{half-cocked 6 o'clock} \\ \omega (r_b + \ell + 2\sqrt{\ell r_b}) & \text{fully-cocked 9 o'clock} \end{cases}\]

\[= \begin{cases} 5.00\omega \\ 5.82\omega \end{cases} = \begin{cases} 5.16\omega \\ 6.00\omega \end{cases} = \begin{cases} 5.00\omega \\ 5.82\omega \end{cases}\]

\[\omega = \begin{cases} 5.00\omega \\ 5.82\omega \end{cases} \quad \ell = \begin{cases} 1 \quad \ell = 1.5 \quad \ell = 2 \end{cases}\]

**Flinger model in lab frame**

\[\omega (r_b + \ell) = v_{\text{rotation}}\]

\[v_{\text{beam frame}}(\text{flinger}) = \omega^2 \ell (2r_b + \ell)\]

\[v_{\text{lab frame}}(\text{flinger}) = \omega \sqrt{(r_b + \ell)^2 + \ell (2r_b + \ell)} = \omega \sqrt{2(r_b + \ell)^2 - r_b^2}\]

\[(\text{compare})\]

\[= \begin{cases} 3.74\omega \\ 3.96\omega \\ 4.12\omega \end{cases} \quad \omega = \begin{cases} 5.00\omega \\ 5.82\omega \end{cases} \quad \ell = \begin{cases} 1 \quad \ell = 1.5 \quad \ell = 2 \end{cases}\]
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Men In Black candidate shot little Suzy

*Bad Suzy!*
Relativity and Quantum Theory need to be unified in *one* book *half* the size of those old tomes!

It’s called *Relawavity*. 
Standing wave 
\( (x, ct) \) - grid

\[ \psi = A e^{i(kx - \omega t)} + A e^{i(-kx - \omega t)} \]

Introducing optical space-time grids and per-space-time “baseball-diamonds”

\[ c \kappa \text{ units of } 300 \text{ THz} \]

BohrIt Web Simulation
2CW \( ct \) vs \( x \) Plot
\( \langle ck = \pm 2 \rangle \)
Standing wave \( (x,ct) \) - grid

\[
\psi = A e^{i(kx - \omega t)} + A e^{i(-kx - \omega t)}
\]

\[
P = \frac{1}{2}(R+L)
\]

\[
G = \frac{1}{2}(R-L)
\]

Introducing optical space-time grids and per-space-time “baseball-diamonds”

\[
e^{iR} + e^{iL} = e^{\frac{R+L}{2}} \left( e^{\frac{iR-L}{2}} + e^{-\frac{iR-L}{2}} \right)
\]

\[= 2e^{\frac{iR+L}{2}} \cos \frac{R-L}{2}\]

\[= 2e^{-i\omega t} \cos kx\]

\[R = kx - \omega t \text{ and: } L = -(kx - \omega t)\]
Three scenarios that look the same to Bob

Alice’s laser moving right at $u = 3c/5$
tuned to $\nu_A = 600\,\text{THz}$

Bob stationary

Carla’s laser moving right at $u = 3c/5$
tuned to $\nu_A = 600\,\text{THz}$

Alice’s laser stationary tuned to $\nu_A = 600\,\text{THz}$

Bob moving left at $u = -3c/5$

Carla’s laser stationary tuned to $\nu_A = 600\,\text{THz}$

Alice’s laser stationary tuned up to $\nu_A = 1200\,\text{THz}$

Bob stationary

Carla’s laser stationary tuned down to $\nu_A = 300\,\text{THz}$

Much cheaper to do the 3rd scenario!$!$
\[ \psi = A e^{i(k_R x - \omega_R t)} + A e^{i(-k_L x - \omega_L t)} \]

\[ P = \frac{1}{2}(R+L) \]

\[ G = \frac{1}{2}(R-L) \]

\[ G' = \frac{1}{2}(R'-L') \]

\[ P' = \frac{1}{2}(R'+L') \]

\[ e^{iR'} + e^{iL'} = e^{i\frac{R'+L'}{2}} \left( e^{i\frac{R'-L'}{2}} + e^{-i\frac{R'-L'}{2}} \right) \]

\[ = e^{i\frac{R'+L'}{2}} 2 \cos \frac{R'-L'}{2} \]

\[ = \psi'_{phase} \psi'_{group} \]

\[ R' = k_R x - \omega_R t \quad \text{and:} \quad L' = -k_L x - \omega_L t \]

\[ cK \quad \text{units of THz} \]

\[ \text{BohrIt Web Simulation} \]

\[ 2 \text{ CW Minkowski Plot} \]

\( ck = -1, +4 \)
Lorentz transformations...

Write $G'$ and $P'$ in terms of $G$ and $P$ using $\cosh \rho$ and $\sinh \rho$

$$G' = \begin{pmatrix} c \kappa_{\text{group}}' & \nu_{\text{group}}' \\ \nu_{\text{group}}' & 1 \end{pmatrix} = v_A \begin{pmatrix} \cosh \rho & \sinh \rho \\ \sinh \rho & \cosh \rho \end{pmatrix} = v_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

$$P' = \begin{pmatrix} c \kappa_{\text{phase}}' & \nu_{\text{phase}}' \\ \nu_{\text{phase}}' & 1 \end{pmatrix} = v_A \begin{pmatrix} \sinh \rho & \cosh \rho \\ \cosh \rho & \sinh \rho \end{pmatrix} = v_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Lorentz transform matrix
Two Famous-Name Coefficients

Review of Lect. 30 p.106

This number is called an: **Einstein time-dilation**
(dilated by 25% here)

This number is called a: **Lorentz length-contraction**
(contracted by 20% here)

<table>
<thead>
<tr>
<th>phase</th>
<th>( b_{\text{RED}}^{\text{Doppler}} )</th>
<th>( \frac{c}{V_{\text{phase}}} )</th>
<th>( \frac{\kappa_{\text{phase}}}{\kappa_{A}} )</th>
<th>( \frac{\tau_{\text{phase}}}{\tau_{A}} )</th>
<th>( \frac{\nu_{\text{phase}}}{\nu_{A}} )</th>
<th>( \frac{\lambda_{\text{phase}}}{\lambda_{A}} )</th>
<th>( \frac{V_{\text{phase}}}{V_{\text{group}}} )</th>
<th>( b_{\text{BLUE}}^{\text{Doppler}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>group</td>
<td>( \frac{1}{b_{\text{Doppler}}^{\text{BLUE}}} )</td>
<td>( \frac{V_{\text{group}}}{c} )</td>
<td>( \frac{\nu_{\text{group}}}{\nu_{A}} )</td>
<td>( \frac{\lambda_{\text{group}}}{\lambda_{A}} )</td>
<td>( \frac{\kappa_{\text{group}}}{\kappa_{A}} )</td>
<td>( \frac{\tau_{\text{group}}}{\tau_{A}} )</td>
<td>( \frac{c}{V_{\text{group}}} )</td>
<td>( \frac{1}{b_{\text{Doppler}}^{\text{RED}}} )</td>
</tr>
<tr>
<td>rapidity ( \rho )</td>
<td>( e^{-\rho} )</td>
<td>( \tanh \rho )</td>
<td>( \sinh \rho )</td>
<td>( \frac{\text{sech} \rho}{\text{cosh} \rho} )</td>
<td>( \text{csch} \rho )</td>
<td>( \coth \rho )</td>
<td>( e^{\rho} )</td>
<td></td>
</tr>
<tr>
<td>( \beta = \frac{u}{c} )</td>
<td>( \sqrt{1-\beta^2} )</td>
<td>( \frac{\beta}{1} )</td>
<td>( \frac{1}{\sqrt{1-\beta^2}} )</td>
<td>( \sqrt{1-\beta^2} )</td>
<td>( \frac{1}{1} )</td>
<td>( \sqrt{1-\beta^2} )</td>
<td>( \frac{1}{\beta} )</td>
<td>( \sqrt{1+\beta^2} )</td>
</tr>
<tr>
<td>value for ( \beta = 3/5 )</td>
<td>( \frac{1}{2} = 0.5 )</td>
<td>( \frac{3}{5} = 0.6 )</td>
<td>( \frac{3}{4} = 0.75 )</td>
<td>( \frac{4}{5} = 0.80 )</td>
<td>( \frac{5}{4} = 1.25 )</td>
<td>( \frac{4}{3} = 1.33 )</td>
<td>( \frac{5}{3} = 1.67 )</td>
<td>( \frac{2}{1} = 2.0 )</td>
</tr>
</tbody>
</table>
RelativIt Web Simulation - Relativistic Events in Main Lighthouse’s Space-Time Frame

Lighthouse Graph
Ref time $t = 0.86$ sec.
$v/c = -0.60$ litesec/sec.

Ship Graph
Ref time $t = 0.03$ sec.
$v/c = -0.60$ litesec/sec.

Click & Drag at bottom to control animation speed
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*All this physics of relativity is mostly simple trigonometry of optical wave interference!*

*And, it derives fundamentals of quantum theory, too!*
Trigonometric road maps

(a) $\sin(\sigma) = 0.6000 = 3/5$
$\tan(\sigma) = 0.7500 = 3/4$
$\cos(\sigma) = 0.8000 = 4/5$

Circle sector area
$\sigma B^2 = 0.6435 B^2$ for angle $\angle \sigma = 36.87^\circ$

(b) $\sin(\sigma) = 0.6000 = 3/5$
$\tan(\sigma) = 0.7500 = 3/4$
$\sec(\sigma) = 1.2500 = 5/4$
$\cos(\sigma) = 0.8000 = 4/5$
$cot(\sigma) = 1.3333 = 4/3$
$csc(\sigma) = 1.6667 = 5/3$

All this physics of relativity is mostly simple trigonometry of optical wave interference!

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Trigonometric road maps

(a) \( \sin(\sigma) = 0.6000 = \frac{3}{5} \)
\( \cos(\sigma) = 0.8000 = \frac{4}{5} \)

- Circule sector area \( B^2 \sigma = 0.6435B^2 \)
- angle \( \angle \sigma = 36.87^\circ \)

(b)

\( \sin(\rho) = 0.6000 = \tanh(\rho) = \frac{3}{5} \)
\( \tan(\sigma) = 0.7500 = \sinh(\rho) = \frac{3}{4} \)
\( \sec(\sigma) = 1.2500 = \cosh(\rho) = \frac{5}{4} \)
\( \cos(\sigma) = 0.8000 = \text{sech}(\rho) = \frac{4}{5} \)
\( \cot(\sigma) = 1.3333 = \csc(\rho) = \frac{4}{3} \)
\( \csc(\sigma) = 1.6667 = \coth(\rho) = \frac{5}{3} \)

All this physics of relativity is mostly simple trigonometry of optical wave interference!

And, it derives fundamentals of quantum theory, too!

Hyperbola unit-B sector arc-area \( \rho = 0.6931 \)
angle \( \angle \rho = \nu = 30.96^\circ \)
Trigonometric road maps

Need to see how trig road maps match the physical maps on next 2 pages.

All this physics of relativity is mostly simple trigonometry of optical wave interference!

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