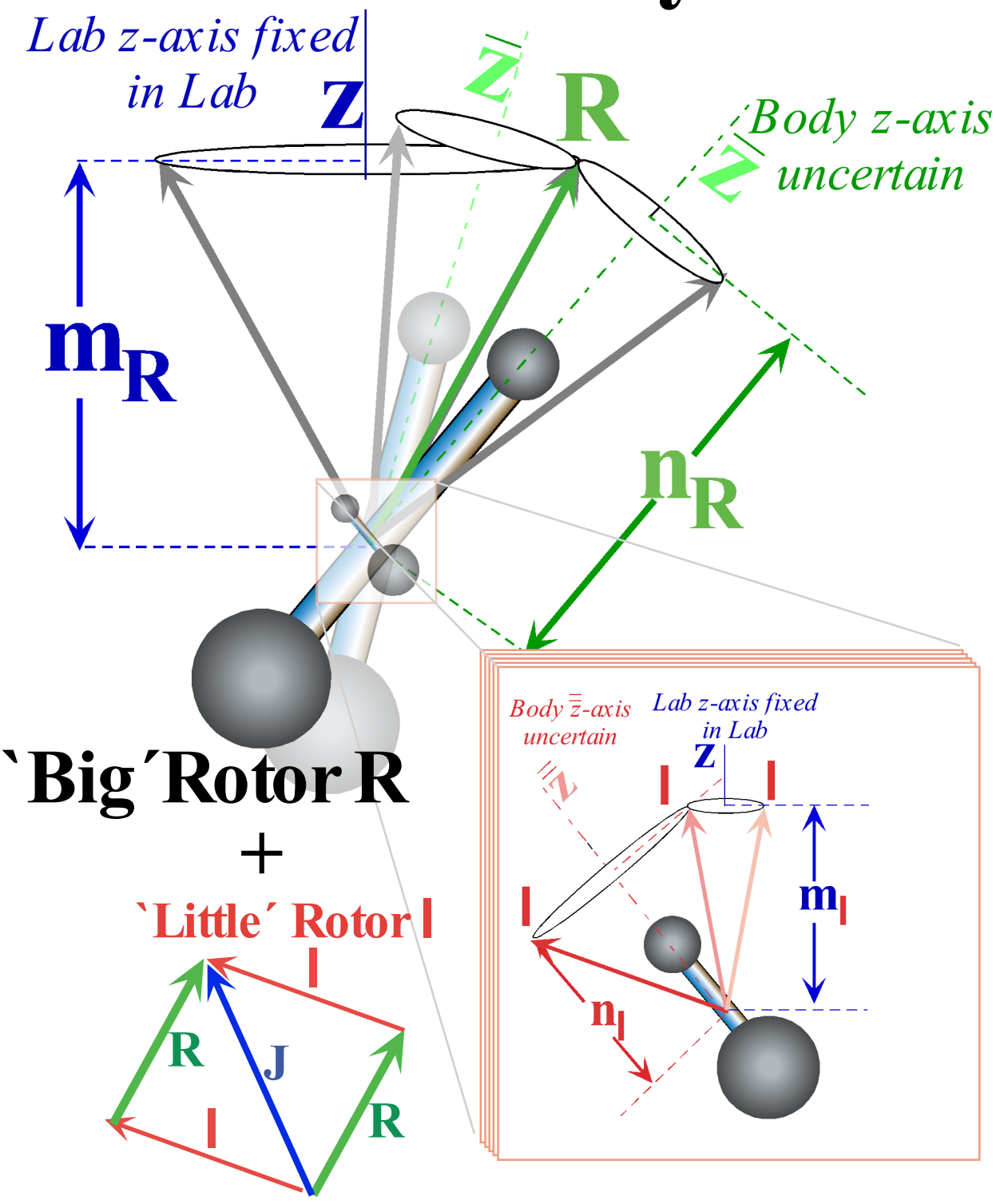
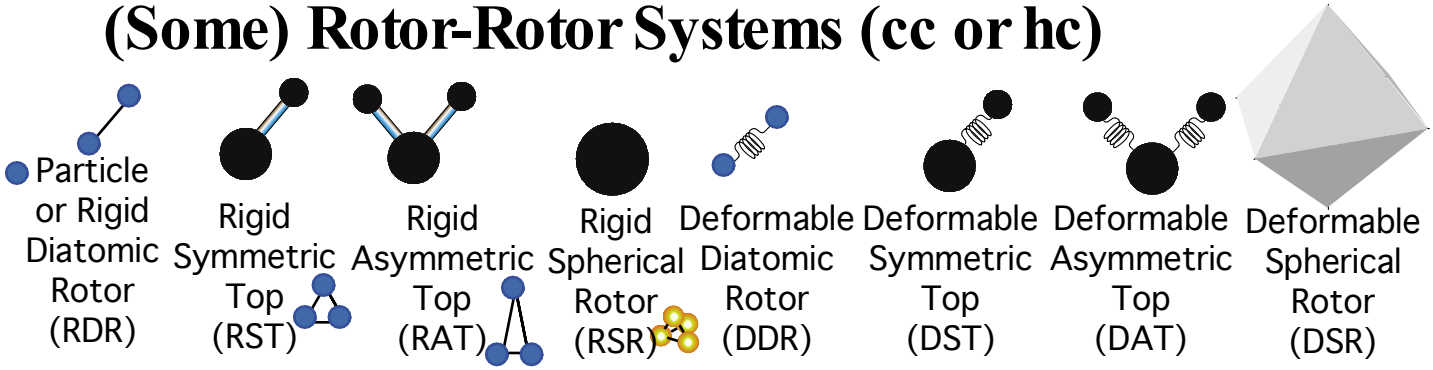
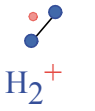
Concenetric RST-RST Rotor-Rotor System

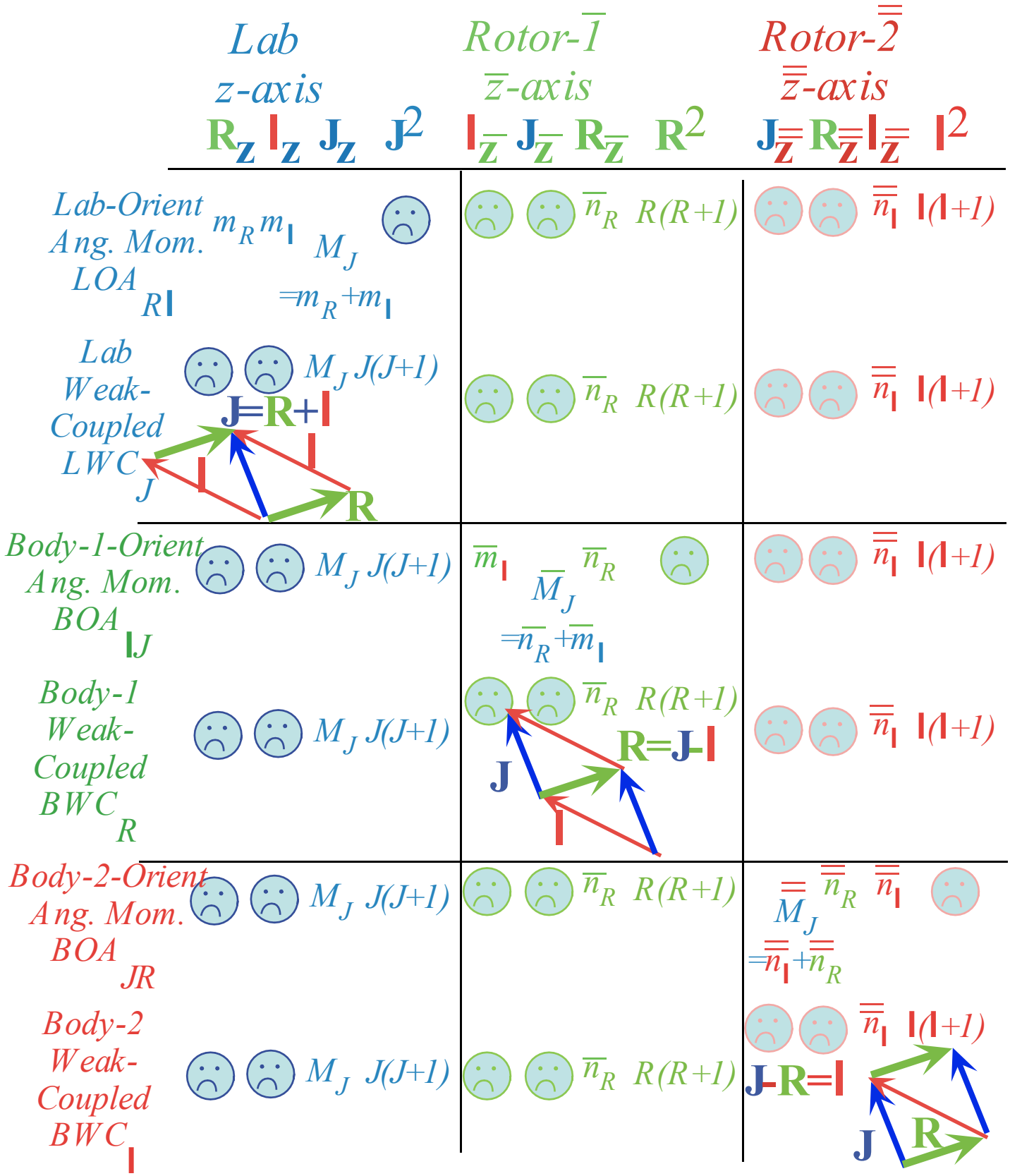


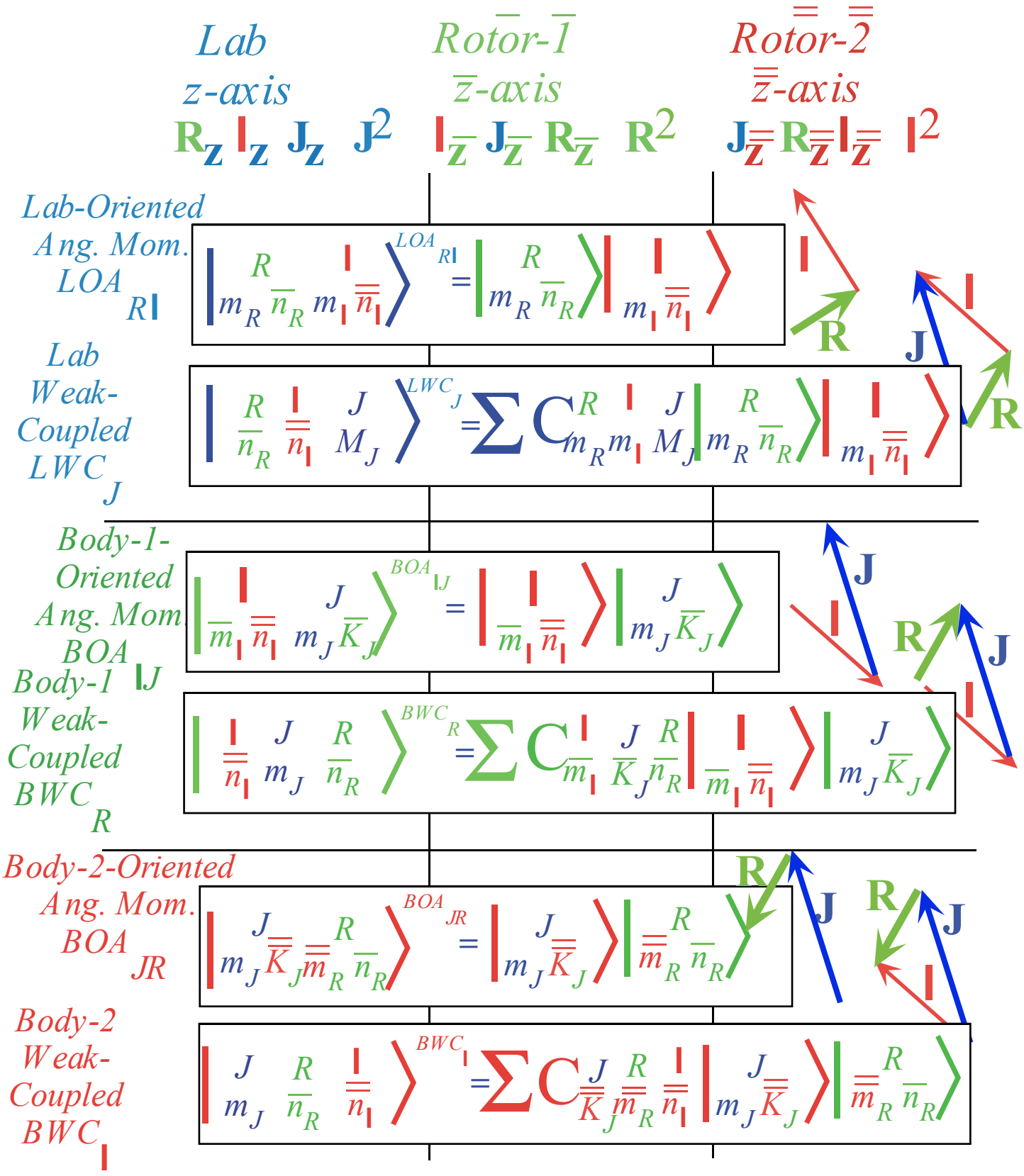
(Some) Rotor-Rotor Systems (cc or hc)



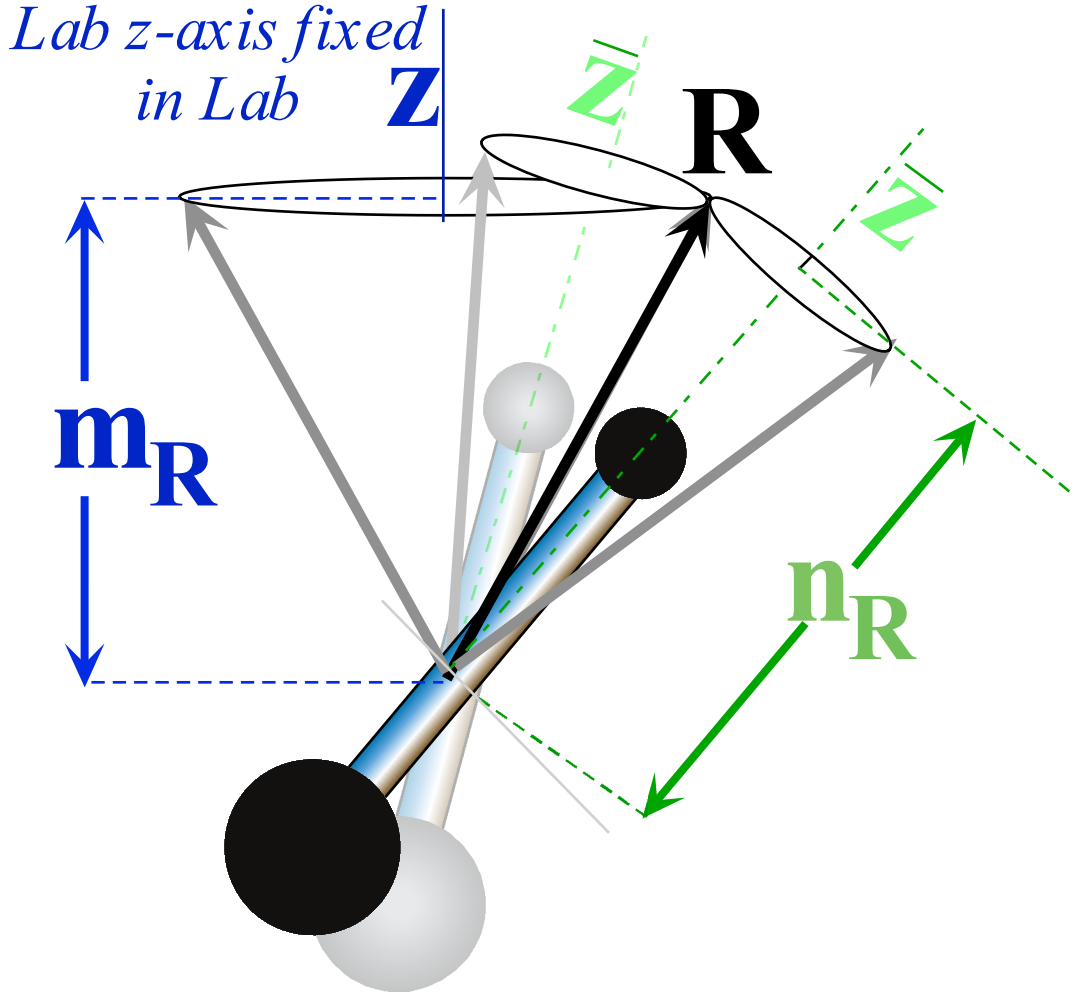
RDR	RDR-RDR	RDR-RST	RDR-RAT	RDR-RSR	RDR-DDR	RDR-DST	RDR-DAT	RDR-DSR
RST	RST-RDR	RST-RST	RST-RAT	RST-RSR	RST-DDR	RST-DST	RST-DAT	RST-DSR
RAT	RAT-RDR	RAT-RST	RAT-RAT	RAT-RSR	RAT-DDR	RAT-DST	RAT-DAT	RAT-DSR
RSR	RSR-RDR	RSR-RST	RSR-RAT	RSR-RSR	RSR-DDR	RSR-DST	RSR-DAT	RSR-DSR
DDR	DDR-RDR	DDR-RST	DDR-RAT	DDR-RSR	DDR-DDR	DDR-DST	DDR-DAT	DDR-DSR
DST	DST-RDR	DST-RST	DST-RAT	DST-RSR	DST-DDR	DST-DST	DST-DAT	DST-DSR
DAT	DAT-RDR	DAT-RST	DAT-RAT	DAT-RSR	DAT-DDR	DAT-DST	DAT-DAT	DAT-DSR
DSR	DSR-RDR	DSR-RST	DSR-RAT	DSR-RSR	DSR-DDR	DSR-DST	DSR-DAT	DSR-DSR




Lab
z-axis
R_z l_z J_z J^2
Rotor-$\bar{1}$
$\bar{z}$-axis
$l_{\bar{z}}$ $J_{\bar{z}}$ $R_{\bar{z}}$ R^2
Rotor-$\bar{\bar{2}}$
$\bar{\bar{z}}$-axis
$J_{\bar{\bar{z}}}$ $R_{\bar{\bar{z}}}$ $l_{\bar{\bar{z}}}$ l^2
Lab-Orient
Ang. Mom.
LOA
$R l$
$m_R m_l$ M_J
$=m_R+m_l$
$\bar{n}_R$ $R(R+1)$
$\bar{\bar{n}}_l$ $l(l+1)$
Lab
Weak-
Coupled
LWC
J
M_J $J(J+1)$
J=R+l
R
$\bar{n}_R$ $R(R+1)$
$\bar{\bar{n}}_l$ $l(l+1)$
Body-1-Orient
Ang. Mom.
BOA
$l J$
M_J $J(J+1)$
$\bar{m}_l$ $\bar{n}_R$ $\bar{M}_J$
$=\bar{n}_R+\bar{m}_l$
$\bar{\bar{n}}_l$ $l(l+1)$
Body-1
Weak-
Coupled
BWC
R
M_J $J(J+1)$
$\bar{n}_R$ $R(R+1)$
R=J-l
J
l
$\bar{\bar{n}}_l$ $l(l+1)$
Body-2-Orient
Ang. Mom.
BOA
JR
M_J $J(J+1)$
$\bar{n}_R$ $R(R+1)$
$\bar{\bar{M}}_J$ $\bar{\bar{n}}_R$ $\bar{\bar{n}}_l$
$=\bar{\bar{n}}_l+\bar{\bar{n}}_R$
Body-2
Weak-
Coupled
BWC
l
M_J $J(J+1)$
$\bar{n}_R$ $R(R+1)$
$\bar{\bar{n}}_l$ $l(l+1)$
J-R=l
J
R

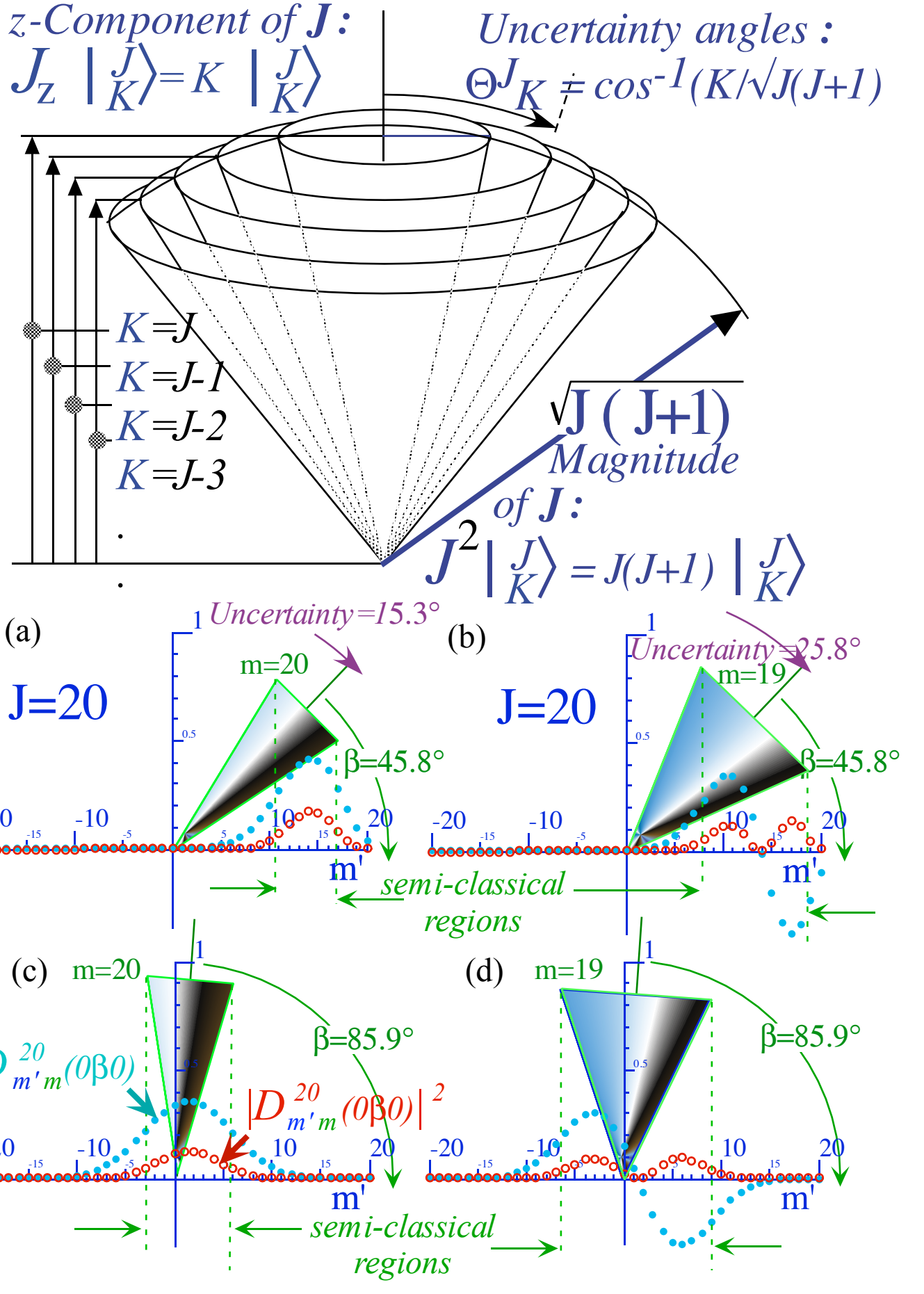

Lab
z-axis
Rotor-1
z̄-axis
Rotor-2
z̿-axis
R_z l_z J_z J^2
l_z̄ J_z̄ R_z̄ R^2
J_z̿ R_z̿ l_z̿ l^2
Lab-Oriented
Ang. Mom.
LOA
R l
Lab
Weak-
Coupled
LWC
J
Body-1-
Oriented
Ang. Mom.
BOA
Body-1 l J
Weak-
Coupled
BWC
R
Body-2-Oriented
Ang. Mom.
BOA
JR
Body-2
Weak-
Coupled
BWC
l
R
J
l
J
R
J
l
R
J
R
J
l

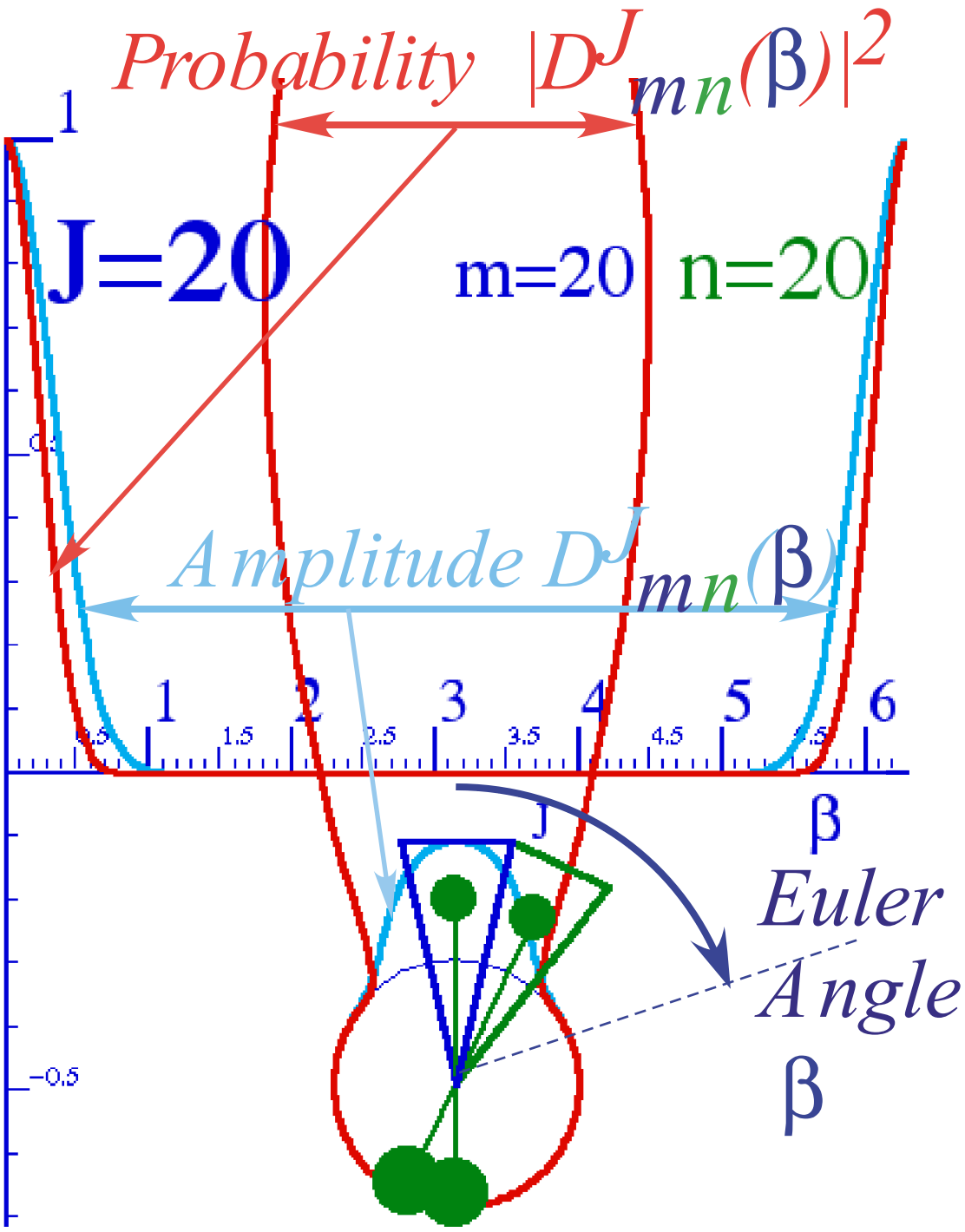
Single Rotor System



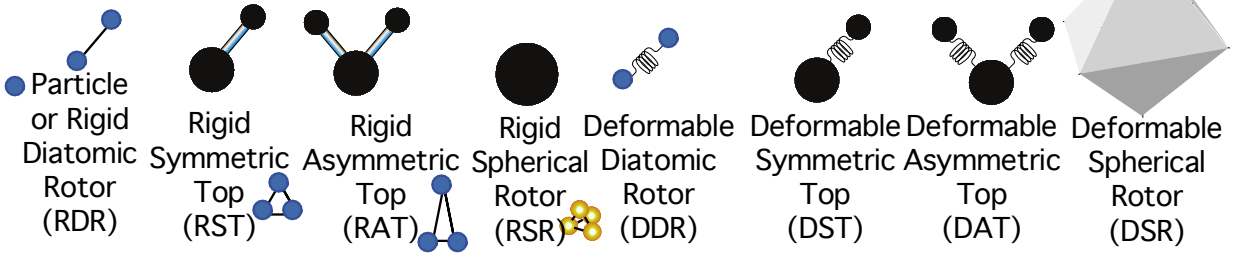
Rotor wavefunction is R(3) group irrep

$$\langle \alpha\beta\gamma |^{R}_{m_R n_R} \rangle = D^{R*}_{m_R n_R}(\alpha\beta\gamma)$$


z-Component of J :
$J_z \left| {}^{J}_{K} \right\rangle = K \left| {}^{J}_{K} \right\rangle$
Uncertainty angles :
$\Theta^{J}_{K} = \cos^{-1}(K/\sqrt{J(J+1)})$
K=J
K=J-1
K=J-2
K=J-3
$\sqrt{J(J+1)}$
Magnitude of J :
$J^2 \left| {}^{J}_{K} \right\rangle = J(J+1) \left| {}^{J}_{K} \right\rangle$
(a)
J=20
Uncertainty=15.3°
m=20
β=45.8°
m'
semi-classical regions
(b)
J=20
Uncertainty=25.8°
m=19
β=45.8°
m'
(c)
m=20
β=85.9°
$D^{20}_{m'm}(0\beta 0)$
$|D^{20}_{m'm}(0\beta 0)|^2$
m'
semi-classical regions
(d)
m=19
β=85.9°
m'


Probability $|D^J_{mn}(\beta)|^2$
1
J=20
m=20
n=20
Amplitude $D^J_{mn}(\beta)$
1
2
3
4
5
6
1.5
2.5
3.5
4.5
5.5
β
J
Euler
Angle
β
−0.5

Rotor-Rotor Systems (concentric or hetrocentric)

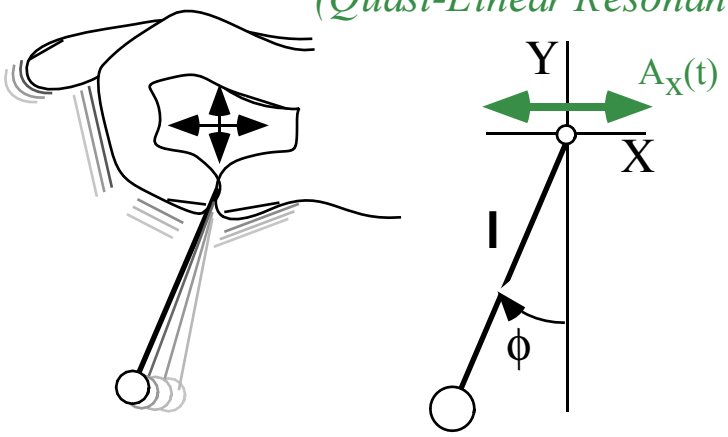


RDR	RDR-RDR 	RDR-RST	RDR-RAT	RDR-RSR	RDR-DDR	RDR-DST	RDR-DAT	RDR-DSR C_{60}-X_2
RST	RST-RDR	RST-RST	RST-RAT	RST-RSR	RST-DDR	RST-DST	RST-DAT	RST-DSR
RAT	RAT-RDR	RAT-RST	RAT-RAT	RAT-RSR	RAT-DDR	RAT-DST	RAT-DAT	RAT-DSR
RSR	RSR-RDR	RSR-RST	RSR-RAT	RSR-RSR	RSR-DDR	RSR-DST	RSR-DAT	RSR-DSR
DDR	DDR-RDR	DDR-RST	DDR-RAT	DDR-RSR	DDR-DDR	DDR-DST	DDR-DAT	DDR-DSR
DST	DST-RDR	DST-RST	DST-RAT	DST-RSR	DST-DDR	DST-DST	DST-DAT	DST-DSR
DAT	DAT-RDR	DAT-RST	DAT-RAT	DAT-RSR	DAT-DDR	DAT-DST	DAT-DAT	DAT-DSR
DSR	DSR-RDR	DSR-RST	DSR-RAT	DSR-RSR	DSR-DDR	DSR-DST	DSR-DAT	DSR-DSR

Hetrocentic Rotor-Rotors
Coupled rotation and translation (Throwing)

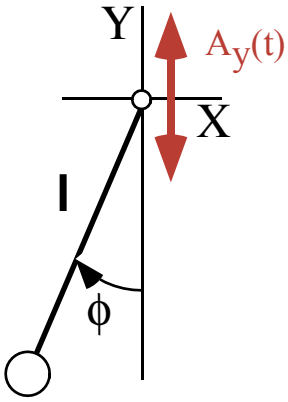
First non-human (or in-human) machines to do this:
trebuchets, whips.. (3000 BC-1542 AD)

X-stimulated pendulum:
(Quasi-Linear Resonance)



Forced Harmonic Resonance

Y-stimulated pendulum:
(Non-Linear Resonance)



Parametric Resonance

For small ϕ
($\sin\phi \sim \phi$) :

$$\frac{d^2\phi}{dt^2} + \frac{g}{l}\phi = \frac{A_X(t)}{l}$$

A Newtonian F=Ma equation

$$\frac{d^2\phi}{dt^2} + \left(\frac{g}{l} + \frac{A_y(t)}{l}\right)\phi = 0$$

A Schrodinger-like equation
(Time *t* replaces coord. *x*)

General ϕ:

$$\frac{d^2\phi}{dt^2} + \frac{g+A_y(t)}{l}\sin\phi + \frac{A_X(t)}{l}\cos\phi = 0$$

A Nasty equation!

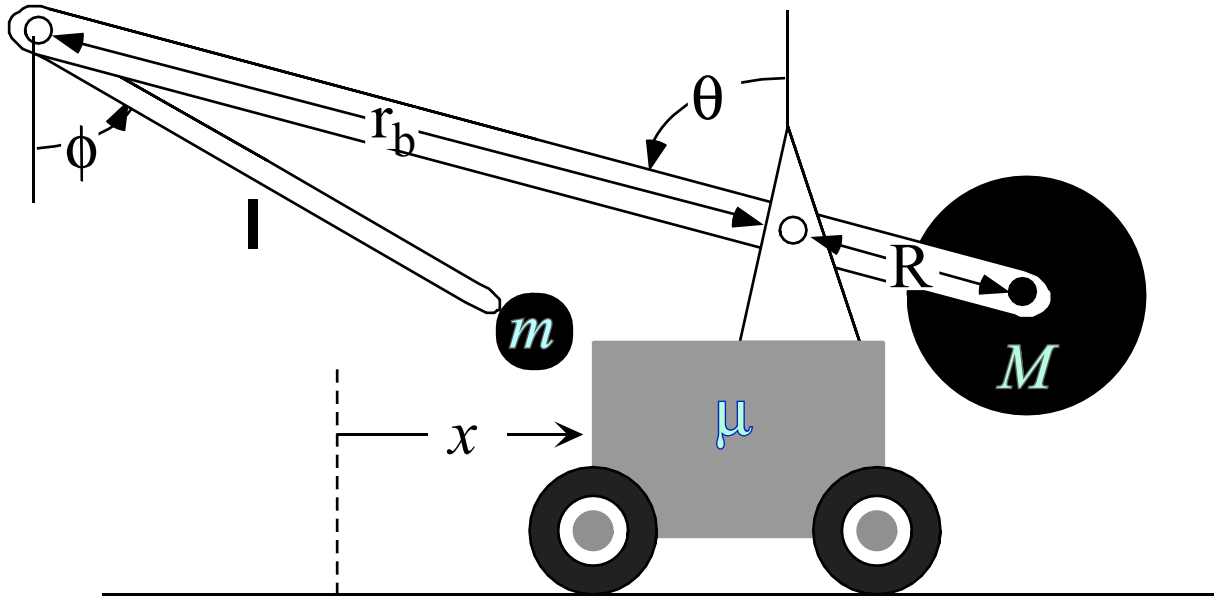
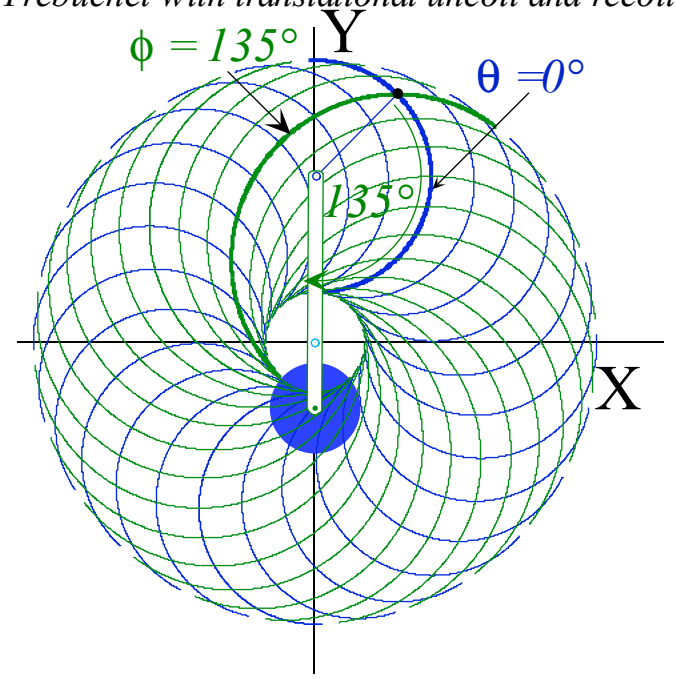
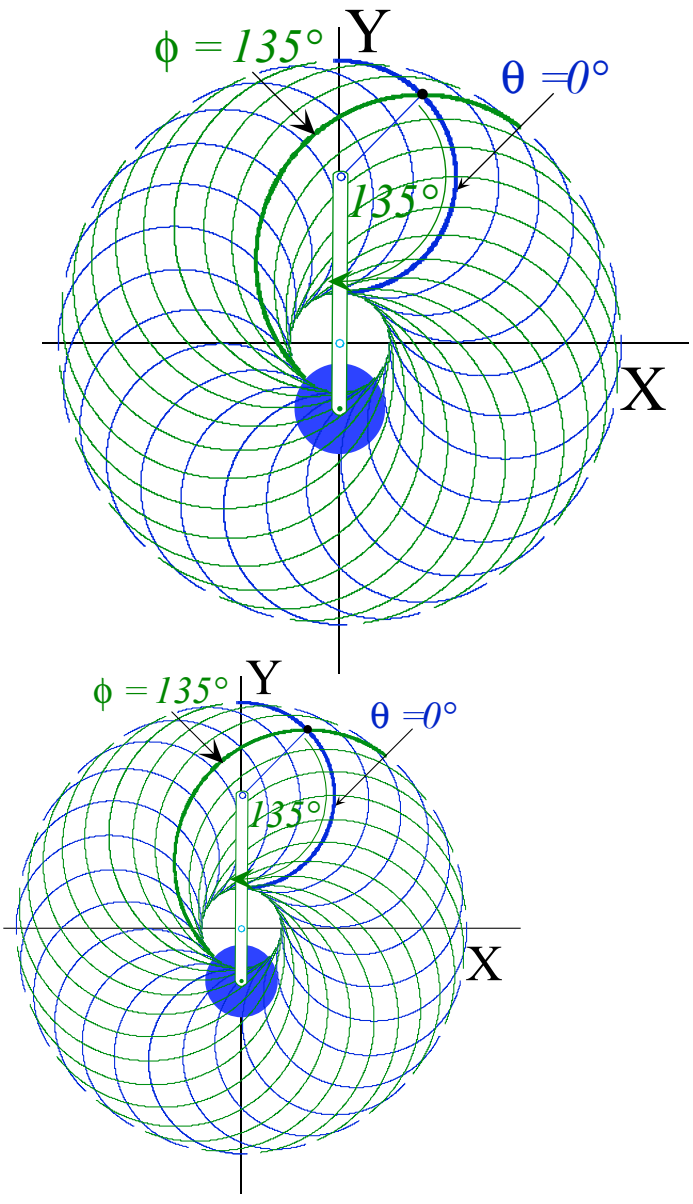


Fig. 10 Trebuchet with translational uncoil and recoil allowed.




ϕ = 135°
Y
θ =0°
135°
X
ϕ = 135°
Y
θ =0°
135°
X