Review Topics & Formulas for Unit 5

<table>
<thead>
<tr>
<th>$R''e^{ikx}$</th>
<th>$L''e^{-ikx}$</th>
<th>$R_1'e^{i\ell x}$</th>
<th>$L_1'e^{-i\ell x}$</th>
<th>$Re^{ikx}+Le^{-ikx}$</th>
</tr>
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<tbody>
<tr>
<td>$x = b'$</td>
<td>$x = a'$</td>
<td>$x = b$</td>
<td>$x = a$</td>
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Fig. 14.1.5 $C_2$-symmetric double barrier.

Figure showing $C_2$-symmetric double barrier $\chi = \cosh \kappa L - i \sinh 2\beta \sinh \kappa L$, and: $\xi = \cosh 2\beta \sinh \kappa L$.

\[ \chi = \cosh \kappa L - i \sinh 2\beta \sinh \kappa L, \quad (14.1.7) \]

\[ \cosh 2\beta = \frac{1}{2} \left( \frac{\kappa + k}{k} \right) = \frac{\kappa^2 + k^2}{2k\kappa}, \quad \sinh 2\beta = \frac{1}{2} \left( \frac{\kappa - k}{k} \right) = \frac{\kappa^2 - k^2}{2k\kappa} \quad (14.1.8) \]

Model Lorentz resonance function

\[ \left| \frac{1}{C_{11}(\omega)} \right|^2 = \left| \frac{c_n}{\omega - \omega_n + i\Gamma_n} \right|^2 = \left| \frac{c_n^2}{(\omega - \omega_n)^2 + \Gamma_n^2} \right| \quad (14.1.10) \]

resonance frequency $\omega_n$, resonance width $\Gamma_n$, resonance peak strength $|c_n/\Gamma_n|^2$

$\Gamma_n$ is the Lorenzian Half-Width at Half-Maximum (HWHM).

Fig. 14.1.18 $(N+1)$-barrier $(N)$-well potential

For $(E<V)$ are $k = \sqrt{(2E)}$, $\kappa = \sqrt{(2V-2E)}$, and $\sinh 2\beta = (\kappa^2-k^2)/(2k\kappa)$.

\[ \chi = \cosh \kappa L - i \sinh 2\beta \sinh \kappa L, \quad (14.1.17a) \]

For $(E>V)$ are $\ell = \sqrt{(2E-2V)}$, and $\cosh 2\alpha = (\ell^2+k^2)/(2k\ell)$.

\[ \chi = \cos \ell L + i \cosh 2\alpha \sin \ell L, \quad \text{and: } \xi = \sinh 2\alpha \sin \ell L. \quad (14.1.17b) \]
Pendulum model: 
\[ \mathbf{H}[\varepsilon_k] = \begin{pmatrix} H & -S & 0 \\ -S & H & -S \\ 0 & -S & H \end{pmatrix} \begin{pmatrix} 1 | \Psi \rangle \\ 2 | \Psi \rangle \\ 3 | \Psi \rangle \end{pmatrix} = \varepsilon_k \begin{pmatrix} 1 | \Psi \rangle \\ 2 | \Psi \rangle \\ 3 | \Psi \rangle \end{pmatrix} = \varepsilon_\lambda | \Psi \rangle \] (14.1.18)

\[ \varepsilon_m = H - 2S \cos \left( \frac{\pi m}{4} \right). \] (14.1.21b)

\[ \varepsilon_1 = \left( 1 + \sqrt{2} \right)/2 \]
\[ \varepsilon_2 = \left( 1 + 1 \right)/\sqrt{2} \]
\[ \varepsilon_3 = \left( 1 - \sqrt{2} \right)/2 \]

Kronig-Penney band conditions.

\[
\begin{align*}
(\text{for } E > V) & : \cos kW \cos \ell L - \frac{2E - V}{2\sqrt{E(E-V)}} \sin kW \sin \ell L \\
(\text{for } E < V) & : \cos kW \cosh \kappa L + \frac{V - 2E}{2\sqrt{V-E}} \sin kW \sinh \kappa L
\end{align*}
\]

\[ = \cos \phi \] (14.2.5b)

where rational units are used for energy.

\[ \phi = m \frac{2\pi}{N}, \quad k = \sqrt{2E}, \quad \ell = \sqrt{2(E-V)}, \quad \kappa = \sqrt{2(V-E)} \] (14.2.5c)

Bohr units

\[ \varepsilon_1^{\text{Bohr}}(A) = \frac{\hbar^2 \pi^2}{2M \mathcal{A}^2} = \left( \frac{1.05 \cdot 10^{-34} \text{J} \cdot \text{s}}{2 \cdot 9.109 \cdot 10^{-31} \text{kg} \cdot 1.602 \cdot 10^{-19} \text{J} \cdot \left( 1 \cdot 10^{-18} \text{m} \right)^2} \right) = \frac{3.76 \text{meV}}{A^2} \] (A in units of 100Å)

Our rational units: \[ \varepsilon_1^{\text{Bohr}}(A) = \frac{\pi^2}{2} \frac{1}{A^2} = 4.93 \] (for: \( A = 2 \) in 100Å units) (14.2.11)
Duality-relativity principle

\( D \text{ characters} \quad g = \begin{bmatrix} 1, r, r^2 \end{bmatrix} \{ i_1, i_2, i_3 \} \)

(15.1.4)

\[
\begin{array}{c|cccc}
D_2 & 1 & R_x & R_y & R_z \\
\hline
A_1 & 1 & 1 & 1 & 1 \\
B_1 & 1 & -1 & 1 & -1 \\
A_2 & 1 & 1 & -1 & -1 \\
B_2 & 1 & -1 & -1 & 1 \\
\end{array}
\]

(15.1.13)

\[
\begin{align*}
\text{Trace} D_A^A (g) & = \chi^A_A (g) \\
\text{Trace} D_A^B (g) & = \chi^A_B (g) \\
\text{Trace} D_A^{E_2} (g) & = \chi^{E_1}_A (g) \\
\end{align*}
\]

(15.1.20)

\[
\begin{align*}
D_{E_1}^{E_1} (g) & = \begin{pmatrix} 1 & 0 \\
0 & 1 \end{pmatrix} \\
D_{E_2^{E_2}} (g) & = \begin{pmatrix} 1 & 0 \\
0 & 1 \end{pmatrix}
\end{align*}
\]

(15.1.18)

Wigner-Weyl projection formula

\[
g = \sum_{\mu} \sum_{m} \sum_{n} D^{\mu}_{mn} (g) P^{\mu}_{mn} = D_A^A (g) P^{A}_{A} + D_A^B (g) P^{A}_{B} + D_{E_1}^{E_1} (g) P^{E_1}_{E_1} + D_{E_2}^{E_2} (g) P^{E_2}_{E_2}
\]

(15.1.20a)

\[
P^{\mu}_{mn} = \frac{\ell^{\mu}}{G} \sum_{g} \sum_{m} D^{\mu}_{mn} (g) g
\]

(15.1.20d)

\[
P^{\mu}_{mn} = \sum_{n'} \sum_{m'} D^{\mu}_{mn'} (g) P^{\mu}_{mn'}
\]

(15.1.21a)

\[
g P^{\mu}_{mn} = \sum_{n'} \sum_{m'} D^{\mu}_{n'n} (g) P^{\mu}_{m'n'}
\]

(15.1.21b)

grand D-orthonormality relations.

\[
D^{\mu}_{mn} (g) = \delta^{\mu \nu} \delta_{mn} \delta_{n'n'}
\]

(15.1.30)

\[
\mathbb{P}^{\mu} = \sum_{m} \sum_{m'} \mathbb{P}^{\mu}_{mn} (g) = \frac{\ell^{\mu}}{G} \sum_{g} \sum_{m} D^{\mu}_{mn} (g) g = \ell^{\mu} \sum_{g} \sum_{m} \chi^{\mu}_{mn} (g) g
\]

(15.2.5)

\[
\mathbb{P}^{\mu} = \sum_{m} \sum_{m'} \mathbb{P}^{\mu}_{mn} (g) = \ell^{\mu} \sum_{g} \sum_{m} D^{\mu}_{mn} (g) g = \ell^{\mu} \sum_{g} \sum_{m} \chi^{\mu}_{mn} (g) g
\]

(15.2.10)

\[
\chi^{\mu}_{mn} = \ell^{\mu} = \sqrt{\ell^{\mu}} C^G \chi^{\mu}_{mn}
\]

(15.2.10g)

Duality principle

\[
g | \mu \rangle = | g \rangle \cdot | \mu \rangle = | g^\dagger \rangle \cdot | \mu \rangle \quad \text{or:} \quad | g^\dagger \rangle | \mu \rangle = | g^\dagger \rangle = | g \rangle | \mu \rangle
\]

(15.3.8)

Duality-relativity principle

\[
\mu | t \rangle = t \cdot \mu | t \rangle = t \cdot | \mu \rangle = t \cdot | \mu \rangle = | \mu \rangle
\]

(15.3.9)

\[
\mu | \mu \rangle = \sum_{m} \sum_{m'} D^{\mu}_{mn} (g) | \mu \rangle | \mu \rangle = \sum_{m} \sum_{m'} D^{\mu}_{mn} (g) | \mu \rangle | \mu \rangle
\]

(15.3.10)
Regular representation of operators $g$ and dual operators $\overline{g}$.

\[
R^G_{h,f}(g) = \langle h | g | f \rangle = \delta_{h=gf} = \begin{cases} 
1 & \text{if } h = g \cdot f \\
0 & \text{if } h \neq g \cdot f 
\end{cases} = \delta_{f = h^\dagger g} \quad R^G_{h,f}(\overline{g}) = \langle h | \overline{g} | f \rangle = \langle \overline{1} | h^\dagger f \cdot g^\dagger | \overline{1} \rangle = \delta_{f = hg} \quad (15.3.11)
\]

Symmetry: $g \quad \mathbf{H} = \mathbf{H} \quad g$ of Hamiltonian $\mathbf{H} = H\overline{\mathbf{T}} + R\overline{\mathbf{r}} + R^*\mathbf{r}^2 + L\overline{\mathbf{I}}_j + M\overline{\mathbf{I}}_j + S\overline{\mathbf{I}}_j \quad (15.4.2a)$

Solution:

\[
H_{ab}^\mu = \sum_{g=1}^{G} \langle 1 | \mathbf{H} | g \rangle D^\mu_{ab}(g) \quad (15.4.5c)
\]