Review Topics & Formulas for Unit 5

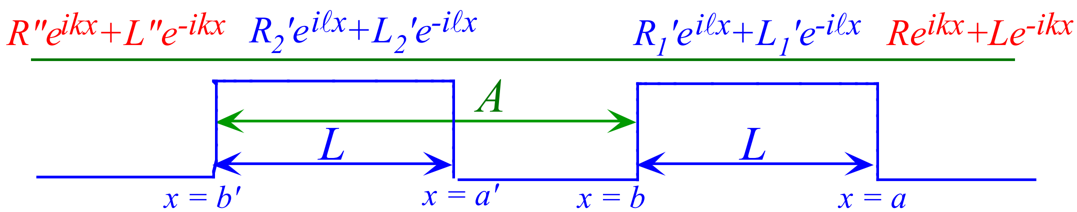

Fig. 14.1.5 C_2-symmetric double barrier .

$$\begin{pmatrix} R'' \\ L'' \end{pmatrix} = \begin{pmatrix} e^{i2kL}\chi^2 + e^{-i2kA}\xi^2 & -i\xi\left(e^{-i2kb}\chi^* + e^{-i2ka'}\chi\right) \\ i\xi\left(e^{i2kb}\chi + e^{i2ka'}\chi^*\right) & e^{-i2kL}\chi^2 + e^{i2kA}\xi^2 \end{pmatrix} \begin{pmatrix} R \\ L \end{pmatrix} \tag{14.1.6}$$

$$\chi = \cosh \kappa L - i \sinh 2\beta \ \sinh \kappa L, \text{ and: } \xi = \cosh 2\beta \ \sinh \kappa L, \tag{14.1.7}$$

$$\cosh 2\beta = \frac{1}{2}\left(\frac{\kappa}{k} + \frac{k}{\kappa}\right) = \frac{\kappa^2 + k^2}{2k\kappa}, \qquad \sinh 2\beta = \frac{1}{2}\left(\frac{\kappa}{k} - \frac{k}{\kappa}\right) = \frac{\kappa^2 - k^2}{2k\kappa} \tag{14.1.8}$$

Model Lorentz resonance function $\left|\frac{1}{C_{11}(\omega)}\right|^2 = \left|\frac{c_n}{\omega - \omega_n + i\Gamma_n}\right|^2 = \frac{|c_n|^2}{(\omega - \omega_n)^2 + \Gamma_n^2}$ (14.1.10)

resonance frequency ω_n , *resonance decay rate* Γ_n, , *resonance peak strength* $|c_n / \Gamma_n|^2$

Γ_n is the *Lorenztian Half-Width at Half-Maximum (HWHM)*.

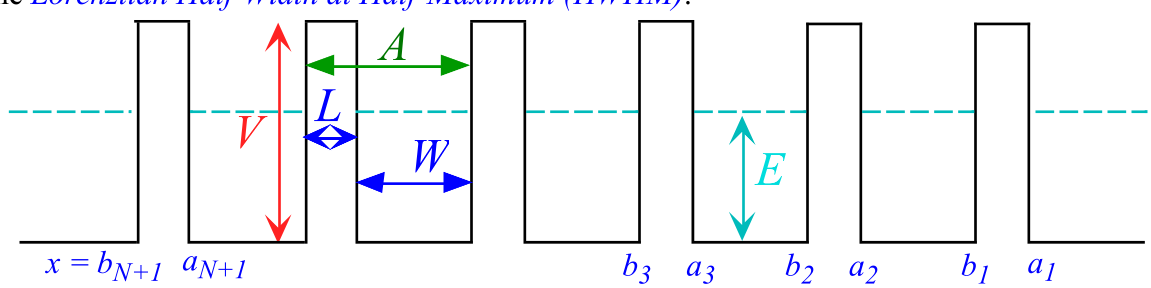

Fig. 14.1.18 (N+1)-barrier (N)-well potential

$$C^{N+1\,barrier} = C^{[N+1]} \cdots C' \cdot C =$$

$$\begin{pmatrix} e^{ikL}\chi^* & -ie^{-ik(a_{N+1}+b_{N+1})}\xi \\ ie^{ik(a_{N+1}+b_{N+1})}\xi & e^{-ikL}\chi \end{pmatrix} \cdots \begin{pmatrix} e^{ikL}\chi^* & -ie^{-ik(a_2+b_2)}\xi \\ ie^{ik(a_2+b_2)}\xi & e^{-ikL}\chi \end{pmatrix} \cdot \begin{pmatrix} e^{ikL}\chi^* & -ie^{-ik(a_1+b_1)}\xi \\ ie^{ik(a_1+b_1)}\xi & e^{-ikL}\chi \end{pmatrix} \tag{14.1.17a}$$

For ($E<V$) are $k = \sqrt{(2E)}$, $\kappa = \sqrt{(2V-2E)}$, and $\sinh 2\beta = (\kappa^2 - k^2)/(2k\kappa)$,

$$\chi = \cosh \kappa L - i \sinh 2\beta \ \sinh \kappa L, \text{ and: } \xi = \cosh 2\beta \ \sinh \kappa L, \tag{14.1.17a}$$

For ($E>V$) they are $\ell = \sqrt{(2E-2V)}$, and $\cosh 2\alpha = (\ell^2 + k^2)/(2k\ell)$.

$$\chi = \cos \ell L + i \cosh 2\alpha \ \sin \ell L, \text{ and: } \xi = \sinh 2\alpha \ \sin \ell L. \tag{14.1.17b}$$

Pendulum model: $$\mathbf{H}\left|\varepsilon_k\right\rangle = \begin{pmatrix} H & -S & 0 \\ -S & H & -S \\ 0 & -S & H \end{pmatrix}\begin{pmatrix} \langle 1|\Psi\rangle \\ \langle 2|\Psi\rangle \\ \langle 3|\Psi\rangle \end{pmatrix} = \varepsilon_k \begin{pmatrix} \langle 1|\Psi\rangle \\ \langle 2|\Psi\rangle \\ \langle 3|\Psi\rangle \end{pmatrix} = \varepsilon_k\left|\Psi\right\rangle \qquad (14.1.18)$$

$$\varepsilon_m = H - 2\,S\cos(\pi\ m/4)\,. \qquad (14.1.21b)$$

$$\begin{aligned} \langle\varepsilon_1| &= \begin{pmatrix} 1 & \sqrt{2} & 1 \end{pmatrix}/2 & \varepsilon_1 &= H-\sqrt{2}S \\ \langle\varepsilon_2| &= \begin{pmatrix} 1 & 0 & -1 \end{pmatrix}/\sqrt{2} & \varepsilon_2 &= H \\ \langle\varepsilon_3| &= \begin{pmatrix} 1 & -\sqrt{2} & 1 \end{pmatrix}/2 & \varepsilon_3 &= H+\sqrt{2}S \end{aligned} \qquad (14.1.21c)$$

Kronig-Penney band conditions.

$$\left.\begin{aligned} &(for\ E>V):\quad \cos kW\ \cos \ell L - \frac{2E-V}{2\sqrt{E(E-V)}}\sin kW\ \sin \ell L \\ &(for\ E<V):\quad \cos kW \cosh \kappa L + \frac{V-2E}{2\sqrt{E(V-E)}}\sin kW \sinh \kappa L \end{aligned}\right\} = \cos\phi \qquad (14.2.5b)$$

where rational units are used for energy.

$$\phi = m\frac{2\pi}{N}\,,\qquad k=\sqrt{2E}\,,\qquad \ell=\sqrt{2(E-V)}\,,\qquad \kappa=\sqrt{2(V-E)}\,. \qquad (14.2.5c)$$

Bohr units $$\varepsilon_1^{Bohr}(A) = \frac{\hbar^2}{2M}\frac{\pi^2}{A^2} = \frac{\left(1.05\cdot 10^{-34}\pi \mathrm{J\cdot s}\right)^2}{\left(2\cdot 9.109\cdot 10^{-31}\mathrm{kg}\right)}\frac{10^3\mathrm{meV}}{1.602\cdot 10^{-19}\mathrm{J}}\frac{1}{\left(A\cdot 10^{-8}\mathrm{m}\right)^2} \qquad (14.2.10a)$$

$$= \frac{3.76\mathrm{meV}}{A^2}\quad (A \text{ in units of } 100\text{Ä})$$

Our rational units: $$\varepsilon_1^{Bohr}(A) = \frac{\pi^2/2}{A^2} = \frac{4.93}{A^2} = 1.23\ \ (\text{for: } A=2 \text{ in } 100\text{Äunits}) \qquad (14.2.11)$$

D_2	$\mathbf{1}$	$\mathbf{R}_z$	$\mathbf{R}_y$	$\mathbf{R}_x$
A_1	1	1	1	1
B_1	1	-1	1	-1
A_2	1	1	−1	−1
B_2	1	-1	−1	1

(15.1.4)

D_3 *characters* $\mathbf{g}=$	$\mathbf{1}$	$\{\mathbf{r},\mathbf{r}^2\}$	$\{\mathbf{i}_1,\mathbf{i}_2,\mathbf{i}_3\}$
$TraceD^{A_1}(\mathbf{g})=\chi^{A_1}(\mathbf{g})$	1	1	1
$TraceD^{A_2}(\mathbf{g})=\chi^{A_2}(\mathbf{g})$	1	1	−1
$TraceD^{E_1}_{x_2y_2}(\mathbf{g})=\chi^{E_1}(\mathbf{g})$	2	−1	0

(15.1.13)

$\mathbf{g}=$	$\mathbf{1}$	$\mathbf{r}$	$\mathbf{r}^2$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$
$D^{E_1}_{c_3d_3}(\mathbf{g})=$	$\begin{pmatrix}1&0\\0&1\end{pmatrix}$	$\begin{pmatrix}\varepsilon_-&0\\0&\varepsilon_+\end{pmatrix}$	$\begin{pmatrix}\varepsilon_+&0\\0&\varepsilon_-\end{pmatrix}$	$\begin{pmatrix}0&\varepsilon_+\\\varepsilon_-&0\end{pmatrix}$	$\begin{pmatrix}0&\varepsilon_-\\\varepsilon_+&0\end{pmatrix}$	$\begin{pmatrix}0&1\\1&0\end{pmatrix}$

(15.1.8)

$\mathbf{g}=$	$\mathbf{1}$	$\mathbf{r}$	$\mathbf{r}^2$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$
$D^{E_1}_{x_2y_2}(\mathbf{g})=$	$\begin{pmatrix}1&0\\0&1\end{pmatrix}$	$\begin{pmatrix}-1/2&-\sqrt{3/2}\\\sqrt{3/2}&-1/2\end{pmatrix}$	$\begin{pmatrix}-1/2&\sqrt{3/2}\\-\sqrt{3/2}&-1/2\end{pmatrix}$	$\begin{pmatrix}-1/2&-\sqrt{3/2}\\-\sqrt{3/2}&1/2\end{pmatrix}$	$\begin{pmatrix}-1/2&\sqrt{3/2}\\\sqrt{3/2}&1/2\end{pmatrix}$	$\begin{pmatrix}1&0\\0&-1\end{pmatrix}$

Wigner-Weyl projection formula

$$\mathbf{g}=\sum_\mu\sum_m\sum_n D^\mu_{mn}(g)\,\mathbf{P}^\mu_{mn}=D^{A_1}(g)\mathbf{P}^{A_1}+D^{A_2}(g)\mathbf{P}^{A_2}+D^{E_1}_{11}(g)\mathbf{P}^{E_1}_{11}+D^{E_1}_{12}(g)\mathbf{P}^{E_1}_{12}$$
$$+D^{E_1}_{21}(g)\mathbf{P}^{E_1}_{21}+D^{E_1}_{22}(g)\mathbf{P}^{E_1}_{22} \qquad (15.1.20a)$$

$$\mathbf{P}^\mu_{mn}=\frac{\ell^\mu}{{}^oG}\sum_{\mathbf{g}}D^{\mu^*}_{mn}(g)\,\mathbf{g} \qquad (15.1.20d) \qquad\qquad \mathbf{P}^\mu_{jk}\mathbf{P}^\nu_{mn}=\delta^{\mu\nu}\delta_{km}\mathbf{P}^\mu_{jn} \qquad (15.1.20b)$$

$$\mathbf{g}\mathbf{P}^\mu_{mn}=\sum_{m'}D^\mu_{m'm}(g)\,\mathbf{P}^\mu_{m'n} \ (15.1.21a) \qquad \mathbf{P}^\mu_{mn}\mathbf{g}=\sum_{n'}D^\mu_{nn'}(g)\,\mathbf{P}^\mu_{mn'} \qquad (15.1.21b)$$

grand D-orthonormality relations.

$$D^\mu_{mn}\left(\mathbf{P}^{\mu'}_{m'n'}\right)=\delta^{\mu\mu'}\delta_{mm'}\delta_{nn'} \quad \text{or:} \quad \sum_{\mathbf{g}}D^\mu_{mn}(g)\,D^{\mu'^*}_{m'n'}(g)=\frac{{}^oG}{\ell^{\mu'}}\delta^{\mu\mu'}\delta_{mm'}\delta_{nn'} \qquad (15.1.30)$$

$$\mathbb{P}^\mu=\sum_{m=1}^{\ell^\mu}\mathbf{P}^\mu_{mm}=\frac{\ell^\mu}{{}^oG}\sum_{\mathbf{g}}\sum_{m=1}^{\ell^\mu}D^{\mu^*}_{mm}(\mathbf{g})\,\mathbf{g}=\frac{\ell^\mu}{{}^oG}\sum_{\mathbf{g}}\chi^{\mu^*}(\mathbf{g})\,\mathbf{g} \qquad \mathbf{c}_g=\sum_{ireps\,\mu}\frac{{}^oc_g\chi^\mu_g}{\ell^\mu}\,\mathbb{P}^\mu \qquad (15.2.5b)$$

$\mathbb{P}^\mu$ is the (μ)-th *all-commuting idempotent* $\mathbb{P}^\mu$ or *class projector*.

$$\chi^\mu{}_1=\ell^\mu=\sqrt{{}^\circ G\frac{\ell^\mu\chi_1^{\mu^*}}{{}^oG}}=\sqrt{{}^\circ G\left(\mathbf{c}_1\ \textit{coefficient in}\ \mathbf{P}^\mu\right)}=\sqrt{\left(\ell^\mu\right)^2} \qquad (15.2.10g)$$

Duality principle $\mathbf{g}|\mathbf{1}\rangle=|\mathbf{g}\rangle=\bar{\mathbf{g}}^\dagger|\mathbf{1}\rangle=\bar{\mathbf{g}}^{-1}|\mathbf{1}\rangle$, or: $\mathbf{g}^{-1}|\mathbf{1}\rangle=\mathbf{g}^\dagger|\mathbf{1}\rangle=|\mathbf{g}^{-1}\rangle=\bar{\mathbf{g}}|\mathbf{1}\rangle=\bar{\mathbf{g}}|\mathbf{1}\rangle$. (15.3.8)

Duality-relativity principle $\bar{\mathbf{g}}|\mathbf{t}\rangle=\mathbf{t}\cdot\mathbf{g}^\dagger\cdot\mathbf{t}^{-1}|\mathbf{t}\rangle=\mathbf{t}\cdot\mathbf{g}^\dagger\cdot\mathbf{t}^\dagger|\mathbf{t}\rangle$. (15.3.9)

$$\mathbf{g}\left|{}^{\mu}_{mn}\right\rangle=\sum_{m'=1}^{\ell^\mu}D^\mu_{m'm}(g)\left|{}^{\mu}_{m'n}\right\rangle \qquad \bar{\mathbf{g}}\left|{}^{\mu}_{mn}\right\rangle=\sum_{n'=1}^{\ell^\mu}D^{\mu^*}_{n'n}(g)\left|{}^{\mu}_{mn'}\right\rangle \qquad (15.3.10)$$

Regular representation of operators **g** and dual operators $\overline{\mathbf{g}}$.

$$R^G_{h,f}(\mathbf{g}) = \langle h|\mathbf{g}|f\rangle = \delta_{h=gf} = \left\{ \begin{array}{l} 1 \text{ if: } \mathbf{h}=\mathbf{g}\cdot\mathbf{f} \\ 0 \text{ if: } \mathbf{h}\neq\mathbf{g}\cdot\mathbf{f} \end{array} \right\} = \delta_{f^\dagger = h^\dagger g} \qquad R^G_{h,f}(\overline{\mathbf{g}}) = \langle h|\overline{\mathbf{g}}|f\rangle = \langle 1|\mathbf{h}^\dagger\mathbf{f}\cdot\mathbf{g}^\dagger|1\rangle = \delta_{f=hg} \quad (15.3.11)$$

Symmetry: $\mathbf{g\,H} = \mathbf{H\,g}$ of Hamiltonian $\mathbf{H} = H\overline{\mathbf{1}} + R\overline{\mathbf{r}} + R^*\overline{\mathbf{r}}^2 + L\overline{\mathbf{i}}_1 + M\overline{\mathbf{i}}_2 + S\overline{\mathbf{i}}_3$ (15.4.2a)

Solution: $$H^\mu_{ab} = \sum_{g=1}^{^\circ G} \langle\mathbf{1}|\mathbf{H}|\mathbf{g}\rangle D^{\mu *}_{ab}(g) \qquad (15.4.5c)$$