Review Topics & Formulas for Unit 2

Expo-Cosine Identity on 2-Component Counter-propagating Minkowski / Cartesian (standing) wave

Apply: $\frac{e^{ia}+e^{ib}}{2}=e^{i\frac{a+b}{2}}\frac{e^{i\frac{a-b}{2}}+e^{-i\frac{a-b}{2}}}{2}=e^{i\frac{a+b}{2}}\cos\frac{a-b}{2}$ to: $\Psi=\frac{e^{i(k_{\rightarrow}x-\omega_{\rightarrow}t)}+e^{i(k_{\leftarrow}x-\omega_{\leftarrow}t)}}{2}$

$$\Psi_{Minkowski}(x,t)=e^{i\frac{(k_{\rightarrow}+k_{\leftarrow})x-(\omega_{\rightarrow}+\omega_{\leftarrow})t}{2}}\cos\frac{(k_{\rightarrow}-k_{\leftarrow})x-(\omega_{\rightarrow}-\omega_{\leftarrow})t}{2}=e^{-i\omega_0 t'}\cos k_0 x'=\Psi_{Cartesian}(x',t')$$

Blue shifted: $\omega_{\rightarrow}=f\,\omega_0$, $k_{\rightarrow}=f\,k_0$, *Red shifted:* $\omega_{\leftarrow}=(1/f)\omega_0$, $k_{\leftarrow}=-(1/f)k_0$,

$$\Psi_{Minkowski}(x,t)=e^{i\frac{(f-1/f)k_0x-(f+1/f)\omega_0 t}{2}}\cos\frac{(f+1/f)k_0x-(f-1/f)\omega_0 t}{2}=e^{-i\omega_0 t'}\cos k_0 x'=\Psi_{Cartesian}(x',t')$$

Minkowski Wave velocities and Doppler f- factor using relativity postulate: $c=\omega_0/k_0=\omega_{\rightarrow}/k_{\rightarrow}=-\omega_{\leftarrow}/k_{\leftarrow}$

$$V_{phase}=\frac{\omega_{\rightarrow}+\omega_{\leftarrow}}{k_{\rightarrow}+k_{\leftarrow}}=\frac{f+1/f}{f-1/f}\frac{\omega_0}{k_0}\quad V_{group}=u=\frac{\omega_{\rightarrow}-\omega_{\leftarrow}}{k_{\rightarrow}-k_{\leftarrow}}=\frac{f-1/f}{f+1/f}\frac{\omega_0}{k_0}=\frac{f^2-1}{f^2+1}c$$

Let: $\beta=u/c$ solve for:

$$f=\sqrt{\frac{1+\beta}{1-\beta}},\quad 1/f=\sqrt{\frac{1-\beta}{1+\beta}},\quad f+1/f=\frac{1+\beta+1-\beta}{\sqrt{1-\beta^2}}=\frac{2}{\sqrt{1-\beta^2}},\ \text{and}\ f-1/f=\frac{1+\beta-1+\beta}{\sqrt{1-\beta^2}}=\frac{2\beta}{\sqrt{1-\beta^2}}.$$

$$\Psi_{Minkowski}(x,t)=e^{i\left\{\frac{\beta k_0 x}{\sqrt{1-\beta^2}}-\frac{\omega_0 t}{\sqrt{1-\beta^2}}\right\}}\cos\left[\frac{k_0x}{\sqrt{1-\beta^2}}-\frac{\beta\omega_0 t}{\sqrt{1-\beta^2}}\right]=e^{-i\omega_0 t'}\cos k_0x'=\Psi_{Cartesian}(x',t')$$

Equating Standing and Minkowskii wave phases gives Lorentz transformation:

$$\left[\frac{k_0x}{\sqrt{1-\beta^2}}-\frac{\beta\omega_0 t}{\sqrt{1-\beta^2}}\right]=k_0x' \qquad \textit{(cosines' phases)} \qquad x'=\frac{x-\beta ct}{\sqrt{1-\beta^2}}=x\cosh\theta-ct\sinh\theta$$

$$\left\{\frac{\beta k_0x}{\sqrt{1-\beta^2}}-\frac{\omega_0 t}{\sqrt{1-\beta^2}}\right\}=-\omega_0 t'=-k_0ct' \qquad (e^{i\{\ \}}\ \textit{phases}) \qquad ct'=\frac{-\beta x+ct}{\sqrt{1-\beta^2}}=-x\sinh\theta+ct\cosh\theta$$

Einstein dilation factor: $\Delta=1/\sqrt{1-\beta^2}=\cosh\theta=1/$*contraction factor:* $\sqrt{1-\beta^2}=\mathrm{sech}\,\theta$. *Doppler factor:* $f=e^{\theta}$.

Space-time (x,ct) invariants:
$(x+ct)(x-ct)=(x)^2-(ct)^2=(x'+ct')(x'-ct')=(x')^2-(ct')^2$
$(c^2\tau_A\tau_B)=(ct_A)(ct_B)-x_Ax_B=(ct'_A)(ct'_B)-x'_Ax'_B$
$(c\tau)^2=(ct)^2-x^2=(ct')^2-x'^2$

Wavevector-frequency (ck,ω) invariants:
$(ck+\omega)(ck-\omega)=(ck)^2-(\omega)^2=(ck'+\omega')(ck'-\omega')=(ck')^2-(\omega')^2$
$(\mu_A\mu_B)=(\omega_A)(\omega_B)-ck_Ack_B=(\omega'_A)(\omega'_B)-ck'_Ack'_B$
$(\mu)^2=(\omega)^2-k^2=(\omega')^2-k'^2$

Mixed (x,ct)-(ck,ω) invariant: Proper phase $=\Phi = kx-\omega t = k'x'-\omega' t'$ or: $c\Phi = ckx-\omega ct = ck'x'-\omega' ct'$

Requires Wavevector-frequency (ck,ω) Lorentz transformation:

$$ck' = \frac{ck-\beta\omega}{\sqrt{1-\beta^2}} = ck\cosh\theta - \omega\sinh\theta$$

$$\omega' = \frac{-\beta ck+\omega}{\sqrt{1-\beta^2}} = -ck\sinh\theta + \omega\cosh\theta$$

Planck Axiom

$E = \hbar\omega$

deBroglie Theorem

$\mathbf{p} = \hbar\mathbf{k}$

Bohr Quantum Radius r_{Bohr}

$$= a = \frac{4\pi\varepsilon_0}{me^2} = 0.528\mathring{A}$$

Dirac Diameter $2r_{Dirac} =$

$$ƛ_{Compton} = \frac{\hbar}{mc} = 3.68\cdot 10^{-13}$$

Relativisitic Dispersion

$$\hbar\omega = \sqrt{\left(mc^2\right)^2 + c^2(\hbar k)^2}$$

$$= mc^2 + \frac{1}{2m}(\hbar k)^2 + \ldots$$

Classical Radius $r_{Einstein} =$

$$\frac{e^2}{4\pi\varepsilon_0 mc^2} = 2.818\cdot 10^{-15}_{(m.)}$$

Electron Wavelength λ_e

$$\lambda_e(E\qquad) = h/\sqrt{2meE(in\ eV)}$$

$$= 1.23nm/\sqrt{E(in\ eV)}$$

Group Velocity: $V_{group} =$

$$\frac{d\omega}{dk} = \frac{c^2(\hbar k)}{\sqrt{\left(mc^2\right)^2 + c^2(\hbar k)^2}}$$

$$= \frac{c^2\hbar k}{\hbar\omega} = \frac{c^2 p}{E} = u = \frac{c^2}{V_{phase}}$$

Fine Structure Constant

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} = \frac{1}{137.036}$$

$$= \frac{r_{Einstein}}{ƛ_{Compton}} = \frac{ƛ_{Compton}}{r_{Bohr}}$$

Photon Wavelength λ_γ

$$\lambda_\gamma(E\qquad) = h/emcE(in\ eV)$$

$$= 1.24\mu m/E(in\ eV)$$