Relativistic Space-time Frames by Wave Interference

Review of basic light waves in space-time (x,t) or per-space-time (ω,k)
- Pulse Wave (PW) vs Continuous Wave (CW)
- Replacing Einstein-Wheeler “Clock-frames” with CW Laser-Phasors

Speed of light: Wrapping one’s mind around a universal c=299,792,458 m/s  (So Sorry, Galileo, this relativity is special!)

- Einstein PW Axioms versus Evenson CW Axioms (Occam’s Razor at Work)
- Only CW light clearly shows Doppler shift and T-symmetry

Geometric construction of wave-zero grids: Dueling lasers make lab frame space-time grid
Letting light make its Own(Eigen)-coordinates 
Continuous Wave (CW) grid based on $K_{\text{phase}}=(K_a+K_b)/2$ and $K_{\text{group}}=(K_a-K_b)/2$ vectors
Pulse Wave (PW) grid based on primitive $K_a=K_{\text{phase}}+K_{\text{group}}$ and $K_b=K_{\text{phase}}-K_{\text{group}}$ vectors

Thales geometry in phasor relativity (Galileo’s Revenge!)

How to use spacetime grids

How the same grids give relativistic classical and quantum mechanics (What’s the matter with MASS?)
It helps to introduce two *archetypes* of light waves and contrast them.

The first *(PW)* is a *Particle-like Wave* or part of a *Pulse-Wave* train. The second *(CW)* is a *Coherent Wave* or part of a *Continuous-Wave* train.

**(1) The PW archetype**

*PW* amplitude is **ZERO** everywhere except here...and here...and here...

*PW* amplitude...

...but has sharp **PEAKS**.

...is best defined by where it **IS**.

Ideal *PW* shape is a *Dirac Delta function*.

**(2) The CW archetype**

*CW* amplitude is **NON-zero** everywhere except here...and here...and here...and here...

...is mostly **NON-zero** with rounded crests and troughs.

...but has sharp **ZEROS**.

...is best defined by where it **IS NOT**.

Ideal *CW* shape is a *cosine wave* *(cos(ϕ))*. 

\[ c = 2.99792458 \times 10^8 \text{m/s} \approx 3 \times 10^8 \text{m/s} \approx 0.3 \mu \text{m/fs} \approx 1 \text{ft/ns} \]
**PW Pulse-Wave trains** versus **CW Continuous-Wave trains.**

(1) The PW archetype

PW amplitude is **ZERO** everywhere except here...and here...and here...

Real laser lab PW shape varies a lot...

...Gaussian? ...sawtooth?...square?...etc.

Quite a variety of shapes. (A jungle of possibilities!)

(2) The CW archetype

CW amplitude is **NON-zero** everywhere except here...and here...and here...and here...

Ideal CW **cosine wave** \((\cos(\phi))\) shape using **right triangle geometry**...

is found in student-calculators,

...infrared... red green blue ...ultra-violet... radio...microwave...

Real laser lab CW shape is very nearly a **cosine wave** and can be tuned precisely to any **frequency** (or **color** if it’s in visible spectrum)

All the same shape. (Differing only in **frequency**, **amplitude**, and **phase** )
PW forms are also called Wave Packets (WP) since they are interfering sums of many CW terms. CW terms are also called Color Waves or Fourier Spectral Components. (10-Cosine Waves make up this pulse.)

\[ \phi = kx - \omega t \]

CW terms interfering constructively (narrow regions of peaks) and destructively (wide regions of zeros). (this \( \phi \)-dimension is time and/or space)
PW widths reduce proportionally with more CW terms (greater Spectral width)

**Space-time width** (pulse width)

- $\Delta t = \tau$
- $\Delta t = \tau/2$
- $\Delta t = \tau/5$
- $\Delta t = \tau/10$
- $\Delta t = \tau/50$

**Spectral width** (harmonic frequency range)

- 1 CW term: $\Delta \nu = \nu = 1/\tau$
- 2 CW terms: $\Delta \nu = 2\nu$
- 5 CW terms: $\Delta \nu = 5\nu$
- 10 CW terms: $\Delta \nu = 10\nu$
- 50 CW terms: $\Delta \nu = 50\nu$

Fourier-Heisenberg product: $\Delta t \cdot \Delta \nu = 1$ (time-frequency uncertainty relation)

- frequency
- per-time
• Coordinate manifolds and frames

Old-fashioned meter-stick-clock frames

E. F. Taylor and J. A. Wheeler Spacetime Physics (Freeman San Francisco 1966)
• Coordinate manifolds and frames

Old-fashioned meter-stick-clock frames

E. F. Taylor and J. A. Wheeler Spacetime Physics (Freeman San Francisco 1966)
Optical wave coordinate manifolds and frames

Shining some light on light using complex phasor analysis

New-fashioned laser clocks & meter sticks
Complex Laser Phasor Clocks (Tesla’s AC “phasor”)

Old-fashioned meter-stick-clock frames

300 THz Laser plane wave \langle x, t | k, \omega \rangle = A e^{i(kx - \omega t)}

Amplitude A

Phase \phi = -\omega t

Re \Psi = \cos(kx - \omega t)

Im \Psi = \sin(kx - \omega t)
Complex Laser-Phasor Clocks  The physicist’s wave clocks consist of rows of *time phasors* turning *clockwise* with *negative* rotation and *negative* angular velocity $-\omega$.

Conventional Orientation (Im-up)

$$Ae^{-i\omega t} = A\cos \omega t - i A\sin \omega t$$

Amplitude $A$

Phase $\phi = -\omega t$

Real Part (position $x$)

Imaginary part

(velocity $v$ scaled by $\omega = 2\pi \nu$)
**Complex Laser-Phasor Clocks** The physicist’s wave clocks consist of rows of *time phasors* turning *clockwise* with *negative* rotation and *negative* angular velocity $-\omega$.

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- **Imaginary part** (velocity $v$ scaled by $\omega=2\pi\nu$)
  - $\phi = -\omega t$
  - $A\cos \omega t - iA\sin \omega t$

**Transverse Orientation (Re-up)**

- **Real Part** (position $x$)
  - $\phi = -\omega t$
  - $A\cos \omega t$

**Amplitude** $A$

**Phase** $\phi = -\omega t$

There is a mnemonic for this:

Imagination precedes reality by exactly one-quarter!

$Ae^{-i\omega t} = A\cos \omega t - iA\sin \omega t$
**Complex Laser-Phasor Clocks**  The physicist’s wave clocks consist of rows of *time phasors* turning *clockwise* with *negative* rotation and *negative* angular velocity $-\omega$.

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- **Imaginary part** (velocity $v$ scaled by $\omega=2\pi\nu$)
- **Real Part** (position $x$)

- **Amplitude $A$**
- **Phase $\phi=-\omega t$**

**Complex Laser-Phasor Clocks**  Transverse wave plots use the above (Real-up) orientation. We often plot the *Im-part* of wave, too. It “predicts” the wave $\tau/4$ later.

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Complex Laser-Phasor Clocks The physicist’s wave clocks consist of rows of time phasors turning clockwise with negative rotation and negative angular velocity $-\omega$.

Conventional Orientation (Im-up) Transverse Orientation (Re-up)

Real Part (position x) $A \cos \omega t - i A \sin \omega t$

Imaginary part (velocity v scaled by $\omega=2\pi\nu$)

Amplitude $A$

Phase $\phi=-\omega t$

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**Conventional Orientation (Im-up) vs. Transverse Orientation (Re-up)**

- **Imaginary part** (velocity $v$ scaled by $\omega=2\pi\nu$)
  - $Ae^{-i\omega t} = A\cos \omega t - iA\sin \omega t$
  - Amplitude $A$
  - Phase $\phi = -\omega t$
- **Real Part** (position $x$)
  - $A\cos \omega t$
  - $\phi = -\omega t$

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**There is a mnemonic for this:** *Imagination precedes reality by exactly one-quarter!*
New-fashioned laser clocks & meter sticks (contd.)

300THz Laser plane wave \(\langle x, t \mid k, \omega \rangle = A e^{i(kx - \omega t)}\)

(1.) Spacetime

\(x\) versus \(ct\)

\[\text{Re} \Psi = \cos(kx - \omega t)\]

\[\text{Im} \Psi = \sin(kx - \omega t)\]

“laser phasors”

Space \(x\)

Wavelength \(\lambda = 2\pi/k = 1/\kappa\)

Period \(\tau = 2\pi/\omega = 1/\nu\)

Speed of light

\[c = \lambda \cdot v = \omega / k = v / \kappa\]

\[c = 2.99792458 \times 10^8 \text{ m/s}\]

\[\approx 3 \times 10^8 \text{ m/s}\]

\[\approx 0.3 \mu \text{m/fs} \approx 1 \text{ ft/ns}\]
300THz Laser plane wave $\langle x, t | k, \omega \rangle = A e^{i(kx - \omega t)}$

### (1.) Spacetime

$x$ versus $ct$

- **Wavelength** $\lambda = 2\pi/k = 1/\kappa$
- **Wavevector** $k = 2\pi/\lambda = 2\pi\kappa$
- **Speed of light** $\nu \cdot \lambda = c$
- **Frequency** $\nu = \omega/2\pi$
- **Angular Frequency** $\omega = 2\pi/\tau$

### (2.) Per-Spacetime

$\omega$ versus $ck$

- **Period** $\tau = 2\pi/\omega = 1/\nu$

<table>
<thead>
<tr>
<th>SPACE</th>
<th>wavelength(meters)</th>
<th>wavenumber(per/meter)</th>
<th>wavevector(rdn.per/meter)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda = \frac{1}{\kappa} = \frac{2\pi}{k}$</td>
<td>$\kappa = \frac{1}{\lambda} = \frac{k}{2\pi}$</td>
<td>$k = \frac{2\pi}{\lambda} = 2\pi\kappa$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TIME</th>
<th>period(seconds)</th>
<th>Hz-frequency(per/sec.)</th>
<th>angular-frequency(rdn.per/sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\tau = \frac{1}{\nu} = \frac{2\pi}{\omega}$</td>
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Speed of light: Wrapping one’s mind around a universal $c=299,792,458\text{ m/s}$

(So Sorry, Galileo, this relativity is special!)

**Einstein Pulse Wave (PW) Axiom:** PW speed seen by all observers is $c$

- It’s going $-c$.
- It’s going $-c$. (Of course)
- It’s going $c$.
- It’s going $c$. (Of course)
- It’s going $c$.
- It’s going $c$.

A “road-runner” axiom is a “show-stopper”

**Pulse wave (PW) train**

$A_1\cos\omega t + A_2\cos 2\omega t + A_3\cos 3\omega t + A_4\cos 4\omega t + \ldots$

PW peaks precisely locate places where wave is.

Complicated
Speed of light: Wrapping one’s mind around a universal $c = 299,792,458 \text{ m/s}$

(So Sorry, Galileo, this relativity is special!)

**Einstein Pulse Wave (PW) Axiom:** PW speed seen by all observers is $c$

- It’s going -c.
- It’s going c. (Of course)
- It’s going c.

**Evenson Continuous Wave (CW) Axiom:** CW speed for all colors is $c$

- It’s going -c. It looks red!
- It’s going -c. It looks green. (Of course)
- It’s going c. It looks blue!
- It’s going c. It looks green. (Of course)
- It’s going c. It looks red!

PW peaks precisely locate places where wave is.

CW zeros precisely locate places where wave is not.

A “road-runner” axiom is a “show-stopper”

Using Occam’s Razor

Complicated

Simpler

More self-evident “must-be” axiom

**Using William of Ockham**

*1285-1349*

**Using Kenneth Evenson**

*1929-2002*

$c = 299,792,458 \text{ m/s}$

1024x788
How many kinds of 600THZ Green (or any other frequency) are there in space?
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**Linear dispersion:** \( \omega = ck \)

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**Linear dispersion:** $\omega = ck$

---

Linear dispersion means **NO** dispersion

Einstein PW is corollary of Evenson CW

- $\omega = ck$
- $\nu = c/\lambda$

---

How many kinds of 600THZ Green (or any other frequency) are there in space?

---

vacuum can’t support an $\infty$-number of "other speeds"

---

wavenumber $ck/2\pi$

(inverse wavelength $1/\lambda$)
Evenson CW Axiom ("All colors go c.") is only reasonable conclusion:

**Linear dispersion:** $\omega = ck$

Linear dispersion means **NO** dispersion

Einstein PW is corollary of Evenson CW

**frequency $\nu$**

$\omega = ck$

or:

$\nu = c/\lambda$

**wavenumber $ck/2\pi$**

(inverse wavelength $1/\lambda$)

vacuum can’t support an $\infty$-number of "other speeds"
Evenson CW Axiom ("All colors go c.") is only reasonable conclusion:

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Linear dispersion means **NO** dispersion

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What if **blue** were to travel 0.001% slower than **red** from a galaxy 9 billion light years away? (.and show up $10^5$ years late)

That would mean **Good-Bye Hubble Astronomy!**
If all colors always march in lock-step then any Doppler shift must be geometric factor, that is, the same multiplier for all colors.

If 300THz Doppler shifts to 600THz (1 octave-shift = 2.0)

Then 600THz Doppler shifts to 1200THz (1 octave-shift = 2.0)
If all colors always march in lock-step then any Doppler shift must be geometric factor, that is, the same multiplier for all colors.

If 300THz Doppler shifts to 600THz (1 octave-shift = 2.0)

Then 600THz Doppler shifts to 1200THz (1 octave-shift = 2.0)

Doppler shifts maintain frequency ratios (not differences)

1-D Doppler shifts \{red=e^{-\rho} ... blue=e^{+\rho}\} form a Lie Group

3-D Doppler shifts are hypercomplex elements of Lorentz Group
300THz Laser plane wave \( \langle x, t \mid k, \omega \rangle = A e^{i(kx - wt)} \)

**Spacetime**

- **x versus ct**
  - 300 THZ laser (Infrared)
  - Crest path (phase = 0)
  - Zero path (phase = \( \pi/2 \))
  - Wavelength \( \lambda = 2\pi/k = 1/\nu \)
  - Period \( \tau = 2\pi/\omega = 1/\nu \)

**Interfering wave pairs needed to make rest frame coordinates...**

**Per-Spacetime**

- \( \omega \) versus \( ck \)
  - Angular Frequency \( \omega \) per-time
  - Frequency \( \nu = \omega/2\pi \) per-sec.
  - Speed of light \( c = \omega/k = \nu/\kappa \)
  - 750THz or 400nm
  - 600THz or 500nm
  - 500THz or 600nm
  - 400THz or 750nm

**Single plane-wave meter-stick-clocks are too fast (can’t catch ‘em)**

(...But per-spacetime view is constant)
Analogy for Wave Interference (Beats) Look thru two different overlapping combs

Red comb: 57 teeth/in.

Green comb: 60 teeth/in.

Analogous wave sum:

- Where red crests line up with green crests, we find a Lump or Group.
- Where red crests line up with green troughs, we find a Zero or Node.

60-57 = 3 “lumps”/in.

60-57 = 3 $\left(\frac{1}{2}\text{-beats/unit } x \text{ or } t\right)$
Analogical for Wave Interference (Beats) Look thru two different overlapping combs.

Red comb: 57 teeth/in.
Green comb: 60 teeth/in.

60-57=3 “lumps”/in.

This bothersome 1/2 (as in spin-1/2) is a feature of relativity and quantum theory!
Collide PW for $\omega_{\rightarrow} = \omega_{\leftarrow}$
Collide PW for $\omega_\rightarrow=\omega_\leftarrow=\omega_o$

Collide CW for $\omega_\rightarrow=\omega_\leftarrow=\omega_0=ck_0$

$ck_\rightarrow=\omega_\rightarrow=2\pi600\text{THz}$

$ck_\leftarrow=-\omega_\leftarrow=-2\pi600\text{THz}$

Zero-lines make space-time “graphpaper”
Collide PW for $\omega_\rightarrow=\omega_\leftarrow=\omega_0$

Collide CW for $\omega_\rightarrow=\omega_\leftarrow=\omega_0=ck_0$

$ck_\rightarrow=\omega_\rightarrow=2\pi 600\text{THz}$

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To find zero lines of 600THZ Green CW sum we must factor that wave sum $e^{ia} + e^{ib}$
Collide PW for $\omega_{\rightarrow}=\omega_{\leftarrow}=$ $\omega_0$

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To find zero lines of 600THZ Green CW sum we must factor that wave sum $e^{ia} + e^{ib}$

Here’s how...

$$
\frac{e^{ia} + e^{ib}}{2} = e^{i\frac{a+b}{2}} \left( \frac{e^{i\frac{a-b}{2}} + e^{-i\frac{a-b}{2}}}{2} \right) \\
\text{Phase wave} \quad \text{Group wave}
$$
Collide PW for $\omega\rightarrow=\omega\leftarrow=\omega_0$

Collide CW for $\omega\rightarrow=\omega\leftarrow=\omega_0=ck_0$

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To find zero lines of 600THZ Green CW sum we must factor that wave sum $e^{ia} + e^{ib}$

Here’s how...

$$\frac{e^{ia} + e^{ib}}{2} = \frac{e^{ia+b/2} + e^{-ia-b/2}}{2}$$

Phase wave

$$\begin{align*}
\text{Real part} & \quad a+b \\
\text{Imaginary part} & \quad i\sin\frac{a+b}{2}
\end{align*}$$

$$\begin{align*}
\cos\frac{a+b}{2} + i\sin\frac{a+b}{2} & \quad \cos\frac{a-b}{2} \\
\cos\frac{a-b}{2} & \quad \cos\frac{a-b}{2}
\end{align*}$$
Collide PW for $\omega_{\rightarrow}=\omega_{\leftarrow}=\omega_0$

Collide CW for $\omega_{\rightarrow}=\omega_{\leftarrow}=\omega_0=ck_0$

to find zero lines of 600THZ Green CW sum we must factor that wave sum $e^{ia} + e^{ib}$

Here’s how...

$$e^{ia} + e^{ib} = \frac{e^{\frac{a+b}{2}}}{2} \left( e^{\frac{a-b}{2}} + e^{-\frac{a-b}{2}} \right)$$

$$= \left( \cos \frac{a+b}{2} + i \sin \frac{a+b}{2} \right) \left( \cos \frac{a-b}{2} \right)$$

Real part of: $\frac{e^{ia} + e^{ib}}{2} = \left( \cos \frac{a+b}{2} \right) \left( \cos \frac{a-b}{2} \right)$

Phase wave

Real part

Imaginary part

$ck_{\rightarrow}=\omega_{\leftarrow}=2\pi 600\text{THz}$

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Collide PW for $\omega_\rightarrow=\omega_\leftarrow=\omega_0$

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To find zero lines of 600THZ Green CW sum we must factor that wave sum $e^{ia} + e^{ib}$

Here’s how...

$$e^{ia} + e^{ib} = \frac{e^{\frac{a+b}{2}}}{2} e^{\frac{a-b}{2}} + e^{\frac{-i\frac{a-b}{2}}{2}}$$

Real part of:

$$\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right) =$$

$$= \left(\cos\left(\frac{k_\rightarrow+k_\leftarrow}{2}x-\omega_\rightarrow+\omega_\leftarrow\frac{t}{2}\right)\cos\left(\frac{k_\rightarrow-k_\leftarrow}{2}x-\omega_\rightarrow-\omega_\leftarrow\frac{t}{2}\right)\cos(0-\omega_0t)\cos(k_0x-0)\right)$$

Thursday, October 10, 2013
Collide PW for $\omega_-=\omega_-$

Collide CW for $\omega_+=\omega_-$

\[ e^{ia} + e^{ib} = \frac{e^{i\frac{a+b}{2}}}{2} + e^{i\frac{-a-b}{2}} \]

\[ \left( \cos \frac{a+b}{2} + i \sin \frac{a+b}{2} \right) \left( \cos \frac{a-b}{2} \right) \]

\[ \cos \left( \frac{k_x x - \omega t}{2} \right) \cos \left( \frac{k_x x - \omega t}{2} \right) \]

So zero lines of 600THZ Green CW sum make space-time “graphpaper”

Real part of:

\[ \frac{e^{ia} + e^{ib}}{2} = \left( \cos \frac{a+b}{2} \right)\left( \cos \frac{a-b}{2} \right) \]

$k_x x - \omega t$

PW peak paths

CW zero lines (white)
Zeros of head-on CW sum gives \((x, ct)\)-grid

**Spacetime**

\(x\) versus \(ct\)

\(CW\) square grid

**CW Standing Wave**

Rest Frame Coherent Wave paths (Cartesian grid)

Laser Time \(ct\)-axis

Laser Space \(x\)-axis

**Space \(x\)**

**Time \(ct\)**

Wavelength \(\lambda = \frac{2\pi}{k \pm v}\)

Period \(T = \frac{2\pi}{\omega = \frac{1}{v}}\)

1st Base” laser

Right-moving wave \(e^{i(kx - \omega t)}\)

600THz laser (green)

\(k = +2\)

2c

“3rd Base” laser

Left-moving wave \(e^{i(-kx - \omega t)}\)

600THz laser (green)

\(k = -2\)
Zeros of head-on CW sum gives \((x, ct)\)-grid

Find zeros by factoring sum:

\[ \Psi = e^{ia} + e^{ib} \]

\[ = e^{i(a+b)/2} \left( e^{i(a-b)/2} + e^{-i(a-b)/2} \right) \]

Phase factor:

\[ \exp(i(a+b)/2) = e^{i(\omega t)} \]

Group factor:

\[ 2\cos\left(\frac{a-b}{2}\right) = 2\cos(kx) \]
(a) CW squares
1 femtosecond
$1.0\, \text{fs} = 10^{-15}\, \text{s}$
1 micron
$1.0\, \mu \text{m} = 10^{-6}\, \text{m}$

(b) PW diamonds
PW laser
$\omega = 2c$
Need one more axiom (besides Evenson’s)...

**Detailed time reversal symmetry implies** $r=1/b$.

Approaching source *(600THz green)*

**Velocity Flip**

$+u \rightarrow -u$

**Time-Reversal Symmetry**

$d=1/r$

**Cause-Effect Flip**

Source $\rightarrow$ Receiver

Receding receiver sees Doppler **red-shift** of

1200THz source to 600THz *(600THz) $= r \cdot (1200THz)$ with $r=1/2$

Fixed receiver sees Doppler **blue-shift** of

600THz source to 1200THz

$b \cdot (600THz) = 1200THz$ with $b=2$

**Source Receiver**

Directed reversal symmetry demands

$r = 1/b$

Fixed source *(1200THz UV)*
Deriving Spacetime and per-spacetime coordinate geometry by:

1. Evenson CW axiom “All colors go c” keeps $K_A$ and $K_B$ on their baselines.
2. Time-Reversal axiom: $r = 1/b$
3. Half-Sum Phase $P = (R+L)/2$ and Half-Difference Group $G = (R-L)/2$
Deriving Spacetime and per-spacetime coordinate geometry by:

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---

**Laser Per-Spacetime**

- $\omega$ versus $ck$
- $\omega_1 = 1200\text{THz}$
- $\omega_2 = 600\text{THz}$
- $\omega_3 = 900\text{THz}$

**Atom Per-Spacetime**

- $\omega'$ versus $ck'$
- $\omega'_1 = 2.0 \cdot (2) = 4$
- $\omega'_3 = 1/2 \cdot (2) = 1$

**Lasers**

- Atom speed $u$
- Atom speed 0

**Groups**

- $K_A = (+4, 4)$
- $K_B = (-1, 1)$

**Baselines**

- New 1st baseline
- 1st base distance
- Halved 3rd base distance
- Doubled 1st base distance
Deriving Spacetime and per-spacetime coordinate geometry by:

(1) Evenson CW axiom “All colors go c” keeps $K_A$ and $K_B$ on their baselines.
(2) Time-Reversal axiom: $r = 1/b$
(3) Half-Sum Phase $P = (R + L)/2$ and Half-Difference Group $G = (R - L)/2$
Wavevector $\mathbf{k}$

Frequency $\omega$

<table>
<thead>
<tr>
<th>$\omega$</th>
<th>THz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>300</td>
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<tr>
<td>2</td>
<td>600</td>
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<tr>
<td>3</td>
<td>900</td>
</tr>
<tr>
<td>4</td>
<td>1200</td>
</tr>
</tbody>
</table>

Phase $\phi$

Group $\chi$

Doppler red-shift $\downarrow$ by factor $e^{-\frac{\phi}{2}}$

Doppler blue-shift $\uparrow$ by factor $e^{+\frac{\phi}{2}}$
Wavevector $ck$

Frequency $\omega$

Doppler blue-shift UP by factor $e^{\rho} = 2$

Doppler red-shift DN by factor $e^{-\rho} = 1/2$

Phase

Group

Wavevector $ck$

$\omega_0 e^{\rho} + \omega_0 e^{+\rho} = (5/2) \omega_0$

$\omega_0 e^{+\rho} = 2 \omega_0$

$(3/2) \omega_0$

$(1) \omega_0$

$(1/2) \omega_0$

$\omega_0 e^{-\rho} + \omega_0 e^{-\rho} = (5/2) \omega_0$

$\omega_0 e^{-\rho} = (3/2) \omega_0$

$\omega_0 e^{-\rho} = (1/2) \omega_0$
Wavevector $ck$

Frequency $\omega$

Doppler blue-shift UP by factor $e^\rho = 2$

Doppler red-shift DN by factor $e^-\rho = 1/2$

$\omega_0 e^\rho + \omega_0 e^{+\rho} = (5/2)\omega_0$

$\omega_0 e^{+\rho}$

$= 2\omega_0$

$\omega_0 e^{-\rho}$

$= \omega_0 \cosh \rho$

$= (3/2)\omega_0$

$= (1/2)\omega_0$

$\omega_0 e^{+\rho} + \omega_0 e^{-\rho} = (5/4)\omega_0$

$= \omega_0 \cos \rho$

Thursday, October 10, 2013
Wavevector \( ck \)

Frequency \( \omega \)

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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</thead>
<tbody>
<tr>
<td>1</td>
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</tr>
<tr>
<td>2</td>
<td>1500THz</td>
<td></td>
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</tr>
</tbody>
</table>

\[
\frac{\omega_{\text{e}^+} - \omega_{\text{e}^-}}{2} = \omega_0 \sinh \rho
\]

\[
= (3/4)\omega_0
\]

\[
\omega_{\text{e}^+} + \omega_{\text{e}^-} = (5/4)\omega_0
\]

\[
\omega_0 = \omega_0 \cosh \rho
\]

\[
(3/2)\omega_0 = \omega_0 \text{ e}^\rho \text{ DN by factor } e^{-\rho} = 1/2
\]

\[
(1)\omega_0 = \omega_0 \text{ e}^\rho \text{ UP by factor } e^{\rho} = 2
\]

\[
(1/2)\omega_0
\]

\[
\omega_{\text{e}^+} + \omega_{\text{e}^-} = (5/2)\omega_0
\]

\[
\omega_0 \text{ cosh } \rho
\]

\[
\omega_{\text{e}^+} - \omega_{\text{e}^-} = (3/2)\omega_0
\]
Wavevector $\mathbf{k}$

Frequency $\omega$

<table>
<thead>
<tr>
<th>Frequency (THz)</th>
<th>Phase</th>
</tr>
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<tbody>
<tr>
<td>300</td>
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<tr>
<td>1500</td>
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</tbody>
</table>

Doppler blue-shift

UP by factor $e^\rho = 2$

$\omega_0 e^\rho + \omega_0 e^{+\rho} = (5/2)\omega_0$

Doppler red-shift

DN by factor $e^{-\rho} = 1/2$

$\omega_0 e^{-\rho} + \omega_0 e^{+\rho} = (5/4)\omega_0$

Phase vector shifted from $\varpi(0,1)$ to $\varpi(\sinh \rho, \cosh \rho)$

$\omega_0 e^{\rho} - \omega_0 e^{-\rho} = \omega_0 \sinh \rho$

$\omega_0 e^{-\rho} + \omega_0 e^{+\rho} = (3/4)\omega_0$

$\omega_0 e^{-\rho} + \omega_0 e^{+\rho} = (3/2)\omega_0$

$\omega_0 e^{\rho} - \omega_0 e^{-\rho} = (1)\omega_0$

$\omega_0 e^{\rho} - \omega_0 e^{-\rho} = (1/2)\omega_0$

$\omega_0 e^{-\rho} + \omega_0 e^{+\rho} = (3/4)\omega_0$

$\omega_0 e^{-\rho} + \omega_0 e^{+\rho} = (5/4)\omega_0$

$\omega_0 e^{\rho} - \omega_0 e^{-\rho} = \omega_0 \cosh \rho$

$\omega_0 e^{\rho} - \omega_0 e^{-\rho} = \omega_0 \cosh \rho$

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$\omega_0 e^{\rho} - \omega_0 e^{-\rho} = \omega_0 \cosh \rho$
Wavevector $ck$

Frequency $\omega$

\[
\omega_0 e^{+\rho} - \omega_0 e^{-\rho} = \omega_0 \sinh \rho
\]

\[
= (3/4) \omega_0
\]

\[
= (450 \text{THz})
\]

\[
\frac{\omega_0 e^{+\rho} + \omega_0 e^{-\rho}}{2} = \omega_0 \cosh \rho
\]

Doppler red-shift
DN by factor $e^{-\rho} = 1/2$

Doppler blue-shift
UP by factor $e^{+\rho} = 2$

Phase vector shifted
from $\varpi(0,1)$ to $\varpi(\sinh \rho, \cosh \rho)$

Phase vector shifted
from $\varpi(0,1)$ to $\varpi(\sinh \rho, \cosh \rho)$

Phase vector shifted
from $\varpi(0,1)$ to $\varpi(\sinh \rho, \cosh \rho)$

Thursday, October 10, 2013
Wavevector $\mathbf{c}k$

Frequency $\omega$

Doppler red-shift DN by factor $e^{\rho} = 1/2$

Doppler blue-shift UP by factor $e^{-\rho} = 2$

Phase vector shifted from $\omega(0,1)$ to $\omega(sinh \rho, cosh \rho)$

Group vector shifted from $\omega(1,0)$ to $\omega(cos \rho, sinh \rho)$

\[
\frac{\omega_0 e^\rho - \omega_0 e^{-\rho}}{2} = \omega_0 sinh \rho = (3/4) \omega_0
\]

\[
\frac{\omega_0 e^\rho + \omega_0 e^{-\rho}}{2} = (5/4) \omega_0 = \omega_0 cosh \rho
\]
Wavevector $ck$

Frequency $\omega$

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Doppler red-shift DN by factor $e^{-\rho}=1/2$

Doppler blue-shift UP by factor $e^{+\rho}=2$

Phase vector shifted from $\varpi(0,1)$ to $\varpi(sinh \rho, cosh \rho)$

Group vector shifted from $\varpi(1,0)$ to $\varpi(sinh \rho, cosh \rho)$

$$\frac{\omega_0 e^{+\rho} - \omega_0 e^{-\rho}}{2} = \omega_0 sinh \rho$$

$= (3/4) \omega_0$

$(450THz)$

$$\omega_0 e^{-\rho} + \omega_0 e^{+\rho} = (5/4) \omega_0 = \omega_0 cosh \rho$$

Thursday, October 10, 2013
Wavevector $\mathbf{k}$

Frequency $\omega$

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<tr>
<th>Wavevector $\mathbf{k}$</th>
<th>Frequency $\omega$</th>
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Doppler red-shift
DN by factor $e^{-\rho}=1/2$

Doppler blue-shift
UP by factor $e^{\rho}=2$

Phase vector shifted from $\mathbf{\omega}(0,1)$ to $\mathbf{\omega}(\sinh \rho, \cosh \rho)$

Group vector shifted from $\mathbf{\omega}(1,0)$ to $\mathbf{\omega}(\cosh \rho, \sinh \rho)$

$$\frac{\omega e^{\rho} - \omega e^{-\rho}}{2} = \omega \sinh \rho$$

$$\frac{\omega e^{\rho} + \omega e^{-\rho}}{2} = \omega \cosh \rho$$

$$\omega_0 e^\rho - \omega_0 e^{-\rho} = (3/4) \omega_0$$

$$\omega_0 e^\rho + \omega_0 e^{-\rho} = (5/4) \omega_0$$

$$\omega_0 \cosh \rho = \omega_0$$
Wavevector $\mathbf{c}$

Frequency $\omega$

<table>
<thead>
<tr>
<th>Frequency (THz)</th>
<th>Group Vector Shifted</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>from $\omega (0,1)$ to $\omega (\sinh \rho, \cosh \rho)$</td>
</tr>
<tr>
<td>600</td>
<td>$\omega_0 e^\rho - \omega_0 e^{-\rho} = (3/4) \omega_0$</td>
</tr>
<tr>
<td>900</td>
<td>$\omega_0 \sinh \rho$</td>
</tr>
<tr>
<td>1200</td>
<td>$\omega_0 \cosh \rho$</td>
</tr>
<tr>
<td>1500</td>
<td>$\omega_0 e^\rho + \omega_0 e^{-\rho} = (5/4) \omega_0$</td>
</tr>
</tbody>
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Doppler red-shift DN by factor $e^{\rho} = 1/2$

Doppler blue-shift UP by factor $e^{\rho} = 2$
Wavevector $ck$

Frequency $\omega$

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- Doppler red-shift DN by factor $e^{-\rho} = 1/2$
- Doppler blue-shift UP by factor $e^{+\rho} = 2$

Phase vector shifted from $\omega(0,1)$ to $\omega(sinh \rho, cosh \rho)$

Group vector shifted from $\omega(1,0)$ to $\omega(sinh \rho, cosh \rho)$

$\omega_0 e^{+\rho} - \omega_0 e^{-\rho} = \omega_0 sinh \rho$

$\omega_0 e^{+\rho} + \omega_0 e^{-\rho} = (5/4) \omega_0 = \omega_0 cosh \rho$

$\omega_0 sinh \rho = (3/4) \omega_0$
PW for $\omega_+ = 2\omega_o, \omega_- = \omega_o/2$

$\frac{e^{ia} + e^{ib}}{2} = \frac{i^{a+b}}{e^{\frac{a+b}{2}}} \left( \frac{e^{\frac{a-b}{2}}}{2} + e^{-i\frac{a-b}{2}} \right)$

$= \begin{pmatrix}
    \cos \frac{a+b}{2} + i \sin \frac{a+b}{2} \\
    \cos \frac{a+b}{2} - i \sin \frac{a+b}{2}
\end{pmatrix}
\begin{pmatrix}
    \cos \frac{a-b}{2} \\
    \cos \frac{a-b}{2}
\end{pmatrix}$

Real part of:

$\frac{e^{ia} + e^{ib}}{2} = \left( \cos \frac{a+b}{2} \right) \left( \cos \frac{a-b}{2} \right)$

CW for $\omega_+ = 2\omega_o, \omega_- = \omega_o/2$

$c k_+ = \omega_+ = 2\pi 1200 \text{THz}$

$c k_- = -\omega_- = -2\pi 300 \text{THz}$
PW for $\omega_{\rightarrow} = 2\omega_0$, $\omega_{\leftarrow} = \omega_0/2$

$\cos \frac{a + b}{2} + i \sin \frac{a + b}{2} \left( \cos \frac{a - b}{2} \right)$

$e^{ia} + e^{ib} = e^{i \frac{a+b}{2}} \left( \cos \frac{a-b}{2} \right)$

$\frac{e^{ia} + e^{ib}}{2}$

$\frac{e^{i \frac{a+b}{2}}}{2} + e^{-i \frac{a-b}{2}}$

$\cos \left( \frac{k_{\rightarrow} + k_{\leftarrow}}{2} x - \frac{\omega_{\rightarrow} + \omega_{\leftarrow}}{2} t \right)$

$\cos \left( \frac{k_{\rightarrow} - k_{\leftarrow}}{2} x - \frac{\omega_{\rightarrow} - \omega_{\leftarrow}}{2} t \right)$

$\frac{1}{2} \cos \frac{a + b}{2} \left( \cos \frac{a - b}{2} \right)$

$\frac{1}{2} \cos \frac{a - b}{2} \left( \cos \frac{a + b}{2} \right)$

$\cos \left( \frac{k_{\rightarrow}}{2} x - \frac{\omega_{\rightarrow}}{2} t \right)$

$\cos \left( \frac{k_{\leftarrow}}{2} x - \frac{\omega_{\leftarrow}}{2} t \right)$

CW for $\omega_{\rightarrow} = 2\omega_0$, $\omega_{\leftarrow} = \omega_0/2$

$k_{\leftarrow} = -\omega_{\leftarrow} = -2\pi 300 \text{THz}$

$k_{\rightarrow} = \omega_{\rightarrow} = 2\pi 1200 \text{THz}$
PW for $\omega_\rightarrow = 2\omega_0, \omega_\leftarrow = \omega_0/2$

$cw$ for $\omega_\rightarrow = 2\omega_0, \omega_\leftarrow = \omega_0/2$

$c k_\rightarrow = \omega_\rightarrow = 2\pi 1200 \text{THz}$

$c k_\leftarrow = -\omega_\leftarrow = -2\pi 300 \text{THz}$

$$e^{ia} + e^{ib} = \frac{e^{i\frac{a+b}{2}}}{e^{i\frac{a-b}{2}}} \left( \frac{e^{i\frac{a-b}{2}}}{2} + e^{-i\frac{a-b}{2}} \right)$$

Real part of:

$$\frac{e^{ia} + e^{ib}}{2} = \left( \cos \frac{a+b}{2} \right) \left( \cos \frac{a-b}{2} \right)$$

Phase wave

Real part

Imaginary part

$$= \left( \cos \frac{a+b}{2} + i \sin \frac{a+b}{2} \right) \left( \cos \frac{a-b}{2} \right)$$

$$= \left( \cos \frac{2-1/2}{2c} \omega_0 x - \frac{2+1/2}{2c} \omega_0 t \right) \left( \cos \omega_0 x - \frac{2-1/2}{2c} \omega_0 t \right)$$
PW for $\omega_{\rightarrow}=2\omega_o, \omega_{\leftarrow}=\omega_o/2$

$\text{Real part of: } \frac{e^{ia}+e^{ib}}{2} = \left( \frac{\cos \frac{a+b}{2}}{2} \right) \left( \cos \frac{a-b}{2} \right) = \left( \cos \frac{\omega_o x-2\omega_o t}{2} \right) \left( \cos \frac{3\omega_o x-5\omega_o t}{4} \right)$

Real wave

Imaginary wave

$\text{Group wave}$

$\text{Phase wave}$

$\text{Real part of: } \frac{e^{i(a-b)/2}+e^{-i(a-b)/2}}{2} = \left( \cos \frac{a-b}{2} \right) \left( \cos \frac{a+b}{2} \right) = \left( \cos \frac{2\omega_o x-2\omega_o t}{2} \right) \left( \cos \frac{2\omega_o x-2\omega_o t}{2} \right)$

CW for $\omega_{\rightarrow}=2\omega_o, \omega_{\leftarrow}=\omega_o/2$

$ck_{\rightarrow}=\omega_{\rightarrow}=2\pi 1200\text{THz}$

$ck_{\leftarrow}=-\omega_{\leftarrow}=-2\pi 300\text{THz}$
PW for $\omega_\rightarrow= 2\omega_0, \omega_\leftarrow= \omega_0/2$

$$e^{ia} + e^{ib} = \frac{e^{i(a+b)/2}}{2} \left( e^{i(a-b)/2} + e^{-i(a-b)/2} \right)$$

Real part: $\cos \frac{a+b}{2} \left( \cos \frac{a-b}{2} \right)$

Imaginary part: $\frac{1}{2} \sin \frac{a+b}{2} \left( \cos \frac{a-b}{2} \right)$

$$V_{\text{Phase}} = \left( \frac{5}{4} \omega_0, \frac{3}{4} \omega_0 \right) = \frac{5}{3} c, \quad V_{\text{Group}} = \left( \frac{3}{4} \omega_0, \frac{5}{4} \omega_0 \right) = \frac{3}{3} c$$

CW for $\omega_\rightarrow= 2\omega_0, \omega_\leftarrow= \omega_0/2$

$$ck_\rightarrow=\omega_\rightarrow=2\pi 1200\text{THz} \quad ck_\leftarrow=-\omega_\leftarrow=-2\pi 300\text{THz}$$

$$\cos \left( \frac{k_\rightarrow + k_\leftarrow - \omega_\rightarrow + \omega_\leftarrow}{2} \right) \cos \left( \frac{2-1/2}{2c} \omega_0 x - \frac{2+1/2}{2} \omega_0 t \right)$$

$$\cos \left( \frac{3}{4c} \omega_0 x - \frac{5}{4} \omega_0 t \right)$$
PW for $\omega_\rightarrow=2\omega_0, \omega_\leftarrow=\omega_0/2$

$ck_\rightarrow=\omega_\rightarrow=2\pi 1200\text{THz}$

$ck_\leftarrow=-\omega_\leftarrow=-2\pi 300\text{THz}$

$V_{\text{Phase}} = \left( \frac{5}{4} \omega_0 \frac{3}{4} c \omega_0 \right) = \frac{5}{3} c$

$V_{\text{Group}} = \left( \frac{3}{4} \omega_0 \frac{5}{4} c \omega_0 \right) = \frac{3}{3} c$

$= \left( \cos \frac{a+b}{2} + i \sin \frac{a+b}{2} \right) \left( \cos \frac{a-b}{2} \right)$

$= \left( \frac{3}{4} \omega_0 \frac{5}{4} c \omega_0 \right) \cos \frac{5}{4} c \omega_0 x - \frac{3}{4} c \omega_0 t$
Laser lab views

Atom views (sees lab going $+u = \frac{3}{5}c$)

atom speed $-u = \frac{3}{5}c$
Euclidian Geometry for Per-spacetime Relativity

**Key Definition of Rapidity $\rho$**

- **Doppler blue shift:** $Bb = Be^{+\rho}$
- **Doppler red shift:** $Br = Be^{-\rho}$

**Relative Speed-Slope**

$\frac{u}{c} = \frac{\sinh \rho}{\cosh \rho} = \tanh \rho$

**Atom Per-time**

$\omega' = \frac{u}{c} = \tanh \rho$

**Key Results:**

- $\omega$ vs. $ck$
  - "winks" vs. "kinks"

  - $\omega = B \cosh \rho$
  - $ck = B \sinh \rho$

- **Group Velocity**
  - $\frac{\omega}{ck} = \frac{u}{c} = \tanh \rho$

- **Phase Velocity**
  - $\frac{ck}{\omega} = \frac{c}{u} = \coth \rho$

**Key Quantities**

- $\sinh \rho = \sqrt{1 - \frac{u^2}{c^2}}$
- $\cosh \rho = \sqrt{1 - \frac{u^2}{c^2}}$

**Lorentz-Einstein Factors**
Euclidian wave geometry with time-reversal symmetry imply dispersion hyperbolas: \( \omega = nB \cosh \rho \)

\[ B \sinh \rho = \frac{(B e^\rho - B e^{-\rho})}{2} \quad \text{and} \quad B \cosh \rho = \frac{(B e^\rho + B e^{-\rho})}{2} \]
Galileo’s Revenge!

In phasor (gauge) space
Galileo’s relativity is right!

Euclid’s 3-means (300 BC)
Geometric “heart” of wave mechanics

Thales (580BC) rectangle-in-circle
Relates to wave interference by (Galilean) phasor angular velocity addition

Linear velocity $V_{\text{group}}/c = u/c$

is \( \frac{\text{HALF-DIFF.}}{\text{HALF-SUM}} = \frac{3}{5} \)
New Relativity webapp  http://www.uark.edu/ua/modphys/testing/markup/RelativItWeb.html
$q = 0.693$

$B_{cosh}(q) = \frac{B_{e^q} + B_{e^{-q}}}{2}$

$B_{sinh}(q) = \frac{B_{e^q} - B_{e^{-q}}}{2}$

http://www.uark.edu/ua/modphys/testing/markup/RelaWavityWeb.html
Hyperbolic Functions

\[ \varphi = 0.6931 \]
\[ \text{Length}(\varphi) = 0.6931 \]
\[ \text{Area}(\varphi) = 0.6931 \]
\[ \sinh(\varphi) = 0.7500 \]
\[ \cosh(\varphi) = 1.2500 \]
\[ \tanh(\varphi) = 0.6000 \]
\[ \text{csch}(\varphi) = 1.3333 \]
\[ \text{sech}(\varphi) = 0.8000 \]
\[ \coth(\varphi) = 1.6667 \]

Circular Functions

\[ m_\varphi(\sigma) = 0.6435 \]
\[ \text{Length}(\sigma) = 0.6435 \]
\[ \text{Area}(\sigma) = 0.6435 \]
\[ \sin(\sigma) = 0.6000 \]
\[ \cos(\sigma) = 0.8000 \]
\[ \tan(\sigma) = 0.7500 \]
\[ \csc(\sigma) = 1.6667 \cdot \coth(\varphi) \]
\[ \sec(\sigma) = 1.2500 \]
\[ \cot(\sigma) = 1.3333 \]
The Circular Functions “Urban elite” versus the Hyperbolic Functions “Country-cousins”

They’re related by Legendre contact transformation: \[ L = p \cdot v - H \]

\[ \tan \sigma = \sinh \rho \]
Circular Geometry of Lagrangian Functions versus Hyperbolic Geometry of Hamiltonian Functions

The Circular Functions “Urban elite”
The Hyperbolic Functions “Country-cousins”

They’re related by Legendre contact transformation $L = p \cdot v - H$

\begin{align*}
\tan \sigma &= \sinh \rho \\
\sin \sigma &= \tanh \rho
\end{align*}
The Circular Functions “Urban elite” versus Hyperbolic Functions “Country-cousins”

They’re related by Legendre contact transformation $L = p\cdot v - H$

- $\tan \sigma = \sinh \rho$
- $\sin \sigma = \tanh \rho$
- $\cos \sigma = \sech \rho$
Circular Geometry of Lagrangian Functions versus Hyperbolic Geometry of Hamiltonian Functions

The Circular Functions “Urban elite”

They’re related by Legendre contact transformation \( L = p^*v - H \)

\[
\begin{align*}
\tan \sigma &= \sinh \rho \\
\sin \sigma &= \tanh \rho \\
\cos \sigma &= sech \rho \\
\sec \sigma &= cosh \rho
\end{align*}
\]
Circular Geometry of Lagrangian Functions versus Hyperbolic Geometry of Hamiltonian Functions

In spacetime: asimultaneity factor
velocity $u/c$
Lorentz contraction
Einstein time dilation

In per-spacetime: momentum
group velocity
-Lagrangian
Hamiltonian

- They’re related by Legendre contact transformation $L = p \cdot v - H$

- Old-fashioned notation:
  $\sin \sigma = \tanh \rho = u/c$
  $\sec \sigma = \cosh \rho = 1/\sqrt{1-u^2/c^2}$