Relativistic Space-time Frames by Wave Interference

Review of basic light waves in space-time (x,t) or per-space-time (ω,k)
  Pulse Wave (PW) vs Continuous Wave (CW)
  Replacing Einstein-Wheeler “Clock-frames” with CW Laser-Phasors

Speed of light: Wrapping one’s mind around a universal c=299,792,458 m/s
  (So Sorry, Galileo, this relativity is special!)
  Einstein PW Axioms versus Evenson CW Axioms (Occam’s Razor at Work)
  Only CW light clearly shows Doppler shift and T-symmetry

Geometric construction of wave-zero grids: Dueling lasers make lab frame space-time grid
  Letting light make its Own(Eigen)-coordinates
  Continuous Wave (CW) grid based on \( K_{\text{phase}} = (K_a + K_b)/2 \) and \( K_{\text{group}} = (K_a - K_b)/2 \) vectors
  Pulse Wave (PW) grid based on primitive \( K_a = K_{\text{phase}} + K_{\text{group}} \) and \( K_b = K_{\text{phase}} - K_{\text{group}} \) vectors

Galileo’s Revenge!

Spacetime frame simulations
  Relativity website - 2005 Pirelli Entrant ; URL is "http://www.uark.edu/ua/pirelli"
  or "http://www.uark.edu/ua/pirelli/html/default.html"

  New Relativity webapp http://www.uark.edu/ua/modphys/markup/RelativItWeb.html


Guest Lecture 1 by Prof. Bill Harter
Mon. 9.30.2013
Prof. Jiali’s Modern Physics Class

Tuesday, October 1, 2013
It helps to introduce two *archetypes* of light waves and contrast them.

The first (PW) is a *Particle-like Wave* or part of a *Pulse-Wave* train. The second (CW) is a *Coherent Wave* or part of a *Continuous-Wave* train.

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### (1) The PW archetype

**PW amplitude is ZERO** everywhere except here...and here...and here...

*PW amplitude...*  
*ZEROS.*  
*but has sharp PEAKS.*  
*is best defined by where it IS.*

Ideal *PW shape is a *Dirac Delta function.*

---

### (2) The CW archetype

**CW amplitude is NON-zero** everywhere except here...and here...and here...and here...

*CW amplitude...*  
*is mostly NON-zero* with rounded crests and troughs.  
*but has sharp ZEROS.*  
*is best defined by where it IS NOT.*

Ideal *CW shape is a cosine wave* \( \cos(\phi) \)
**PW Pulse-Wave trains** versus **CW Continuous-Wave trains.**

1. **The PW archetype**

   - PW amplitude is **ZERO** everywhere except here...and here...and here...

2. **The CW archetype**

   - CW amplitude is **NON-zero** everywhere except here...and here...and here...and here...

Ideal CW cosine wave \( \cos(\phi) \) shape using right triangle geometry...

is found in student-calculators,

\[
\cos(60^\circ) = 0.500
\]

... Ideal PW shape is the Dirac Delta function \( \delta(\phi) \)...

... is found in student-calculators,

\[
\int d\phi \delta(\phi) = 1
\]

This mathematical definition is not attainable in a laser lab.

(An infinite pulse uses all the energy in the universe!)

Real laser lab PW shape varies a lot...

...Gaussian?...sawtooth?...square?...etc.

Quite a variety of shapes. (A jungle of possibilities!)

Real laser lab CW shape is very nearly a cosine wave and can be tuned precisely to any frequency (or color if it’s in visible spectrum)

radio...microwave... ...infrared... (red green blue)...ultra-violet...

versus

All the same shape. (Differing only in frequency, amplitude, and phase)
PW forms are also called Wave Packets (WP) since they are interfering sums of many CW terms. CW terms are also called Color Waves or Fourier Spectral Components. 

\( \cos(\phi) + \cos(2\phi) + \cos(3\phi) + \cos(4\phi) + \cos(5\phi) + \cos(6\phi) + \cos(7\phi) + \cos(8\phi) + \cos(9\phi) + \cos(10\phi) \)

\( \phi = kx - \omega t \) (this \( \phi \)-dimension is time and/or space)

CW terms interfering constructively (narrow regions of peaks)

CW terms interfering destructively (wide regions of zeros)
$PW$ widths reduce proportionally with more $CW$ terms (greater *Spectral* width)

**Space-time width** (pulse width)

- $\Delta t = \tau$
  - fundamental period = $\tau$
  - More prone to interference
- $\Delta t = \tau/2$
  - More Wave-like
- $\Delta t = \tau/5$
  - More Particle-like
- $\Delta t = \tau/10$
  - Less prone to interference
- $\Delta t = \tau/50$

$\text{Fundamental frequency}$

1 *CW* term

$\Delta \nu = \nu = 1/\tau$

2 *CW* terms

$\Delta \nu = 2\nu$

5 *CW* terms

$\Delta \nu = 5\nu$

10 *CW* terms

$\Delta \nu = 10\nu$

50 *CW* terms

$\Delta \nu = 50\nu$

**Fourier-Heisenberg product:** $\Delta t \cdot \Delta \nu = 1$ (time-frequency uncertainty relation)

**Space-time width** (pulse width) vs. **Spectral width** (harmonic frequency range)

- Time dimension
- Frequency or per-time dimension

Less prone to interference more prone to interference

1 cosine wave (up to 2nd octave)

2 cosine waves (up to 5th)

5 cosine waves (up to 10th)

10 cosine waves (up to 20th)

50 cosine waves (up to 50th)

This dimension is time

This dimension is frequency or per-time

Tuesday, October 1, 2013
Coordinate manifolds and frames

Old-fashioned meter-stick-clock frames

E. F. Taylor and J. A. Wheeler Spacetime Physics (Freeman San Francisco 1966)
• *Coordinate manifolds and frames*

*Old-fashioned meter-stick-clock frames*

E. F. Taylor and J. A. Wheeler Spacetime Physics (Freeman San Francisco 1966)
• Optical wave coordinate manifolds and frames

Shining some light on light using complex phasor analysis

New-fashioned laser clocks & meter sticks

Complex Laser Phasor Clocks (Tesla’s AC “phasor”)

Amplitude $A = |\Psi|$  
Phase $\theta = (kx - \omega t)$

Real Part (position $x$) $A \cos \omega t$  
Imaginary part (velocity $v$ scaled by $\omega = 2\pi \nu$) $-i A \sin \omega t$

300THz Laser plane wave $\langle x, t | k, \omega \rangle = A e^{i(kx - \omega t)}$

$Re \Psi = \cos(kx - \omega t)$  
$Im \Psi = \sin(kx - \omega t)$
**Complex Laser-Phasor Clocks** The physicist’s wave clocks consist of rows of time phasors turning clockwise with negative rotation and negative angular velocity $-\omega$.

**Conventional Orientation (Im-up)**

- **Real Part** (position $x$)
  - $\cos \omega t$
  - $\sin \omega t$

- **Imaginary part** (velocity $v$ scaled by $\omega = 2\pi \nu$)
  - $-i A \sin \omega t$

**Amplitude** $A$

**Phase** $\phi = -\omega t$

There is a mnemonic for this: Imagination precedes reality by exactly one-quarter!
**Complex Laser-Phasor Clocks** The physicist's wave clocks consist of rows of *time phasors* turning *clockwise* with *negative* rotation and *negative* angular velocity $-\omega$.

Conventional Orientation (Im-up)  

Transverse Orientation (Re-up)

There is a mnemonic for this:  

**Imagination precedes reality by exactly one-quarter!**
**Complex Laser-Phasor Clocks**  The physicist’s wave clocks consist of rows of *time phasors* turning *clockwise* with *negative* rotation and *negative* angular velocity $-\omega$.

Conventional Orientation (Im-up)  

Transverse Orientation (Re-up)

**Complex Laser-Phasor Clocks**  Transverse wave plots use the above (Real-up) orientation. We often plot the *Im-part* of wave, too. It “predicts” the wave $\tau/4$ later.
**Complex Laser-Phasor Clocks** The physicist’s wave clocks consist of rows of *time phasors* turning *clockwise* with *negative* rotation and *negative* angular angular velocity $-\omega$.

**Conventional Orientation (Im-up)**

**Transverse Orientation (Re-up)**

The wave plots use the above (Real-up) orientation. We often plot the *Im-part* of wave, too. It “predicts” the wave $\tau/4$ later.

\[ A e^{-i\omega t} = A \cos \omega t - i A \sin \omega t \]

There is a mnemonic for this:

**Imagination precedes reality by exactly one-quarter!**

\[ A \cos \omega t - i A \sin \omega t \]
Complex Laser-Phasor Clocks  The physicist’s wave clocks consist of rows of time phasors turning clockwise with negative rotation and negative angular velocity $-\omega$.

Conventional Orientation (Im-up)  Transverse Orientation (Re-up)

Amplitude $A$

Phase $\phi = -\omega t$

Complex Laser-Phasor Clocks  Transverse wave plots use the above (Real-up) orientation. We often plot the Im-part of wave, too. It “predicts” the wave $\tau/4$ later.

There is a mnemonic for this: *Imagination precedes reality by exactly one-quarter!*
New-fashioned laser clocks & meter sticks (contd.)

300THz Laser plane wave \( \langle x, t \mid k, \omega \rangle = A e^{i(kx - \omega t)} \)

(1.) Spacetime

\( x \) versus \( ct \)

\[ \text{Re} \Psi = \cos(kx - \omega t) \]
\[ \text{Im} \Psi = \sin(kx - \omega t) \]

"laser phasors"

300 THZ laser (Infrared)

Wavelength \( \lambda = 2\pi / k = 1 / \kappa \)

Period \( \tau = 2\pi / \omega = 1 / \nu \)

Space \( x \)

Time \( ct \)

Speed of light
\[ c = \lambda \cdot \nu = \omega / k = \nu / \kappa \]
\[ c \approx 2.99792458 \times 10^8 \text{m/s} \]
\[ \approx 3 \times 10^8 \text{m/s} \]
\[ \approx 0.3 \mu \text{m/fs} \approx 1 \text{ft/ns} \]
**New-fashioned laser clocks & meter sticks (contd.)**

300THz Laser plane wave \( \langle x,t | k, \omega \rangle = A e^{i(kx - wt)} \)

(1.) **Spacetime**

\[ x \text{ versus } ct \]

Spacetime diagram with 300 THZ laser (Infrared)

- 300 THZ laser (Infrared)
- Crest path (phase = 0)
- Zero path (phase = -\( \pi / 2 \))
- Rough path (phase = +\( \pi \))

- Wavelength \( \lambda = 2\pi/k = 1/\kappa \)
- Speed of light \( c = \omega/k = \nu/\kappa \)

- Frequency \( \nu = \omega/2\pi \)
- Angular frequency \( \omega = 2\pi \nu \)

- Period \( \tau = 2\pi/\omega = 1/\nu \)

(2.) **Per-Spacetime**

\[ \omega \text{ versus } ck \]

- Frequency \( \nu = \omega/2\pi \)
- Angular frequency \( \omega = 2\pi \nu \)

- Speed of light \( c = \omega/k = \nu/\kappa \)

<table>
<thead>
<tr>
<th>SPACE</th>
<th>wavelength(meters)</th>
<th>wavenumber(per/meter)</th>
<th>wavevector(rdn.per/meter)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \lambda = \frac{1}{\kappa} = \frac{2\pi}{k} )</td>
<td>( \kappa = \frac{1}{\lambda} = \frac{k}{2\pi} )</td>
<td>( k = \frac{2\pi}{\lambda} = 2\pi\kappa )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TIME</th>
<th>period(seconds)</th>
<th>Hz-frequency(per/sec.)</th>
<th>angular-frequency(rdn.per/sec.)</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>( \tau = \frac{1}{\nu} = \frac{2\pi}{\omega} )</td>
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Speed of light: Wrapping one’s mind around a universal $c=299,792,458\text{m/s}$

(So Sorry, Galileo, this relativity is special!)

Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is $c$

A “road-runner” axiom is a “show-stopper”

Complicated

PW peaks precisely locate places where wave is.
Speed of light: Wrapping one’s mind around a universal \( c = 299,792,458 \text{ m/s} \)

(\textit{So Sorry, Galileo, this relativity is special!})

\textbf{Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is} \( c \)

\begin{itemize}
  \item It’s going \(-c\).
  \item It’s going \(-c\). (Of course)
  \item It’s going \(c\).
  \item It’s going \(c\). (Of course)
\end{itemize}

A “road-runner” axiom is a “show-stopper”

\textbf{PW peaks precisely locate places where wave is.}

\textbf{Continuous wave (CW) train}

\begin{itemize}
  \item \( A_1 \cos \omega t + A_2 \cos 2\omega t + A_3 \cos 3\omega t + A_4 \cos 4\omega t + \ldots \)
\end{itemize}

\textbf{Complicated}

\textbf{CW zeros precisely locate places where wave is not.}

\textbf{Evenson Continuous Wave (CW) axiom: CW speed for all colors is} \( c \)

\begin{itemize}
  \item It’s going \(-c\). It looks \textbf{red}!
  \item It’s going \(-c\). It looks \textbf{green}. (Of course)
  \item It’s going \(c\). It looks \textbf{blue}!
  \item It’s going \(c\). It looks \textbf{green}. (Of course)
  \item It’s going \(c\). It looks \textbf{red}!
\end{itemize}

600 THz (green)

\textbf{More self-evident “must-be” axiom}

\textbf{Sees Doppler blue shift}

\textbf{Laser source}

\textbf{Sees Doppler red shift}

\textbf{Simpler}
Evenson CW Axiom ("All colors go c.")

How many kinds of 600THZ Green (or any other frequency) are there in space?
Evenson CW Axiom ("All colors go c.")

How many kinds of 600THZ Green (or any other frequency) are there in space?
**Evenson CW Axiom** ("All colors go c.") is only reasonable conclusion:

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Evenson CW Axiom ("All colors go c.") is only reasonable conclusion:

**Linear dispersion**: \( \omega = ck \)

Linear dispersion means **NO dispersion**

Einstein PW is corollary of Evenson CW

How many kinds of 600THZ Green (or any other frequency) are there in space?
Evenson CW Axiom ("All colors go c.") is only reasonable conclusion:

**Linear dispersion:** $\omega = ck$

Linear dispersion means **NO** dispersion

Einstein PW is corollary of Evenson CW

what if blue were to travel 0.001% slower than red from a galaxy 9 billion light years away? (..and show up $10^5$ years late)

That would mean Good-Bye Hubble Astronomy!
If all colors always march in lock-step then any Doppler shift must be geometric factor, that is, the same multiplier for all colors.

If $300\text{THz}$ Doppler shifts to $600\text{THz}$ (1 octave-shift = 2.0)

Then $600\text{THz}$ Doppler shifts to $1200\text{THz}$ (1 octave-shift = 2.0)

Doppler shifts maintain frequency ratios (not differences)

1-D Doppler shifts $\{\text{red} = e^{-\rho} \ldots \text{blue} = e^{+\rho}\}$ form a Lie Group

3-D Doppler shifts are hypercomplex elements of Lorentz Group
New-fashioned laser clocks & meter sticks (contd.)

300THz Laser plane wave \( \langle x,t \mid k,\omega \rangle = Ae^{i(kx - wt)} \)

(1.) Spacetime

\( x \) versus \( ct \)

300 THZ laser (Infrared)

\( k = +1 \)

\( \omega = 1c \)

Wavelength \( \lambda = 2\pi/k = 1/\kappa \)

Period \( \tau = 2\pi/\omega = 1/\upsilon \)

Per-Spacetime

\( \omega \) versus \( ck \)

Angular Frequency \( \omega \) per-time

Frequency \( \nu = \omega/2\pi \) per-sec.

\( \nu \land \lambda = c \)

\( \nu \cdot \lambda = c = \omega/k = \upsilon/\kappa \)

Speed of light per-space \( \kappa \)

Interfering wave pairs needed to make rest frame coordinates...

Single plane-wave meter-stick-clocks are too fast (can’t catch ‘em)

(…But per-spacetime view is constant)

Spacetime vs. ct

Re\(\Psi\)

Im\(\Psi\)

Time

ct

Period

\( \tau = 2\pi/\omega = 1/\upsilon \)

Spacetime

x

Wavelength

\( \lambda = 2\pi/k = 1/\kappa \)

Interference wave pairs needed to make rest frame coordinates.

Tuesday, October 1, 2013
Analogy for Wave Interference (Beats) Look thru two different overlapping combs

Red comb: 57 teeth/in.
Green comb: 60 teeth/in.

60-57=3 “lumps”/in.

60-57=3 (1/2-beats/unit x or t)

Analogous wave sum

Light dark light dark light dark

Where red crests line up with green crests we find a Lump or Group

Where red crests line up with green troughs we find a Zero or Node

Group or Beat: where crests line up with crests

Node or Zero: where crests line up with troughs

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Analogogy for Wave Interference (Beats) Look thru two different overlapping combs

Red comb: 57 teeth/in.  
Green comb: 60 teeth/in.

Where red crests line up with green crests
we find a Lump or Group

Where red crests line up with green troughs
we find a Zero or Node

$60 - 57 = 3 \text{“lumps”}/\text{in.}$

$60 - 57 = 3 \left(1/2\text{-beats/unit x or t}\right)$

This bothersome $1/2$ (as in spin-$1/2$) is a feature of relativity and quantum theory!
Collide PW for $\omega \rightarrow = \omega \leftarrow$
Collide PW for $\omega_\rightarrow=\omega_\leftarrow=\omega_0$

Collide CW for $\omega_\rightarrow=\omega_\leftarrow=\omega_0=ck_0$

$ck_\rightarrow=\omega_\rightarrow=2\pi 600\text{THz}$

$ck_\leftarrow=-\omega_\leftarrow=-2\pi 600\text{THz}$

Zero-lines make space-time “graphpaper”
Collide PW for $\omega\rightarrow=\omega\leftarrow=\omega_0$

Collide CW for $\omega\rightarrow=\omega\leftarrow=\omega_0=ck_0$

$ck\rightarrow=\omega\leftarrow=2\pi 600\text{THz}$

$ck\leftarrow=-\omega\rightarrow=-2\pi 600\text{THz}$

To find zero lines of 600THZ Green CW sum we must factor that wave sum $e^{ia} + e^{ib}$
Collide PW for \( \omega_\rightarrow = \omega_\leftarrow = \omega_0 \)

\[
\text{To find zero lines of 600THZ Green CW sum we must factor that wave sum } \ e^{ia} + e^{ib} \\
\text{Here's how...}
\]

\[
\frac{e^{ia} + e^{ib}}{2} = \frac{i^{a+b}}{2e} \left( e^{i\frac{a-b}{2}} + e^{-i\frac{a-b}{2}} \right) \\
\text{Phase wave} = \left( \begin{array}{c}
\frac{2}{e} \\
\text{Group wave}
\end{array} \right)
\]
Collide PW for $\omega \rightarrow \omega \rightarrow \omega$  

Collide CW for $\omega \rightarrow \omega \leftarrow \omega = ck_0$  
$c k_\rightarrow=\omega_\rightarrow=2\pi 600THz$  
$c k_\leftarrow=\omega_\leftarrow=-2\pi 600THz$  

To find zero lines of 600THZ Green CW sum we must factor that wave sum $e^{ia} + e^{ib}$  

Here’s how...

$$e^{ia} + e^{ib} = \frac{e^{i(a+b)}}{2} + \left( e^{\frac{i(a-b)}{2}} + e^{-\frac{i(a-b)}{2}} \right)$$

$$= \left( \cos \frac{a+b}{2} + i \sin \frac{a+b}{2} \right) \left( \cos \frac{a-b}{2} \right)$$

Real part  
Imaginary part  
Phase wave  
Group wave
Collide PW for $\omega_\rightarrow = \omega_\leftarrow = \omega_0$

Collide CW for $\omega_\rightarrow = \omega_\leftarrow = \omega_0 = c k_0$

$ck_\rightarrow = \omega_\leftarrow = 2\pi 600\text{THz}$

$ck_\leftarrow = -\omega_\leftarrow = -2\pi 600\text{THz}$

To find zero lines of 600THZ Green CW sum we must factor that wave sum $e^{ia} + e^{ib}$

Here’s how...

$$\frac{e^{ia} + e^{ib}}{2} = \frac{e^{\frac{a+b}{2}}}{e^{\frac{a-b}{2}} + e^{-\frac{a-b}{2}}} = \frac{2}{\left(\cos \frac{a+b}{2}\right)\left(\cos \frac{a-b}{2}\right)}$$
Collide PW for $\omega_\rightarrow=\omega_\leftarrow=\omega_0$

Collide CW for $\omega_\rightarrow=\omega_\leftarrow=\omega_0=ck_0$

$ck_\rightarrow=\omega_\rightarrow=2\pi 600\text{THz}$

$ck_\leftarrow=-\omega_\leftarrow=-2\pi 600\text{THz}$

To find zero lines of 600THZ Green CW sum we must factor that wave sum $e^{ia} + e^{ib}$

Here’s how...

$$
\frac{e^{ia} + e^{ib}}{2} = e^{\frac{i(a+b)}{2}} \left( \cos \frac{a-b}{2} + i \sin \frac{a+b}{2} \right)
$$

Real part $e^{ia} + e^{ib}$

$$
\frac{2}{2} = \left( \cos \frac{a+b}{2} \right) \left( \cos \frac{a-b}{2} \right)
$$

Group wave

$\cos \left( \frac{k_\rightarrow+k_\leftarrow}{2} x - \omega_\rightarrow+\omega_\leftarrow t \right) \cos \left( \frac{k_\rightarrow-k_\leftarrow}{2} x - \omega_\rightarrow-\omega_\leftarrow t \right)$

Phase wave
Collide PW for $\omega_\rightarrow=\omega_\leftarrow$

$$e^{ia} + e^{ib} = \frac{e^{i\frac{a+b}{2}}}{2} + e^{i\frac{-a-b}{2}}$$

Real part: $\cos\left(\frac{a+b}{2}\right)$

Imaginary part: $\frac{1}{2}i\sin\left(\frac{a+b}{2}\right)$

Collide CW for $\omega_\rightarrow=\omega_\leftarrow$

$$ck_\rightarrow=\omega_\rightarrow=2\pi 600\text{THz}$$

$$ck_\leftarrow=-\omega_\leftarrow=-2\pi 600\text{THz}$$

So zero lines of 600THZ Green CW sum $e^{ia} + e^{ib}$ make space-time “graphpaper”
Zeros of head-on CW sum gives \((x, ct)\)-grid

**Find zeros by factoring sum:**

\[
\Psi = e^{ia} + e^{ib} = e^{i(a+b)/2} \left( e^{i(a-b)/2} + e^{-i(a-b)/2} \right)
\]

- **Phase factor:** 
  \[\exp(i(a+b)/2) = e^{-i\omega t}\]
- **Group factor:** 
  \[2\cos(a-b)/2 = 2\cos(kx)\]

**Per-Spacetime**

- **\(\omega\) versus \(ck\)**
- **“Baseball” Diamond**
- **Phase vector**
  \[\mathbf{P} = (\mathbf{K} + \mathbf{K})/2\]
- **Group vector**
  \[\mathbf{G} = (\mathbf{K} - \mathbf{K})/2\]
(a) **CW squares**

- **1 femtosecond**
- 1.0 fs = 10^{-15}s

- **1 micron**
- 1.0 \mu m = 10^{-6} \text{ meter}

(b) **PW diamonds**

- **PW laser**
- \omega_0 = 2c

- "patokey!"
Detailed time reversal symmetry implies $r=1/b$.

**Approaching source**

(600THz green)

Fixed receiver sees Doppler blue-shift of 600THz source to 1200THz with $b=2$

$\text{Doppler blue-shift of } 600\text{THz source to } 1200\text{THz}$

$\text{with } b=2$

**Receding receiver sees**

Doppler red-shift of 1200THz source to 600THz (600THz) = $r \cdot (1200\text{THz})$ with $r=1/2$

Detailed time reversal symmetry implies $r=1/b$. 

Velocity Flip $+u \leftrightarrow -u$

Time Reversal Symmetry demands $b = 1/r$

Cause-Effect Flip Source $\leftrightarrow$ Receiver

**Symmetry**

$\text{Symmetry}$

$\text{demands}$

$r = 1/b$

**Time Reversal Symmetry**

$\text{Time- Reversal Symmetry}$

$\text{demands}$

$r = 1/b$

**Source**

(600THz)

**Receiver**

(1200THz UV)
Deriving Spacetime and per-spacetime coordinate geometry by:

1. **Evenson CW axiom** “All colors go c” keeps \( K_A \) and \( K_B \) on their baselines.
2. **Time-Reversal axiom**: \( r = 1/b \)
3. **Half-Sum Phase** \( P = (R+L)/2 \) and **Half-Difference Group** \( G = (R-L)/2 \)
Deriving Spacetime and per-spacetime coordinate geometry by:

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3. Half-Sum Phase $P = (R + L)/2$ and Half-Difference Group $G = (R - L)/2$

Laser Per-Spacetime

$\omega$ versus $ck$

$1200\text{THz}$

$900\text{THz}$

$600\text{THz}$

$300\text{THz}$

3rd baseline

1st baseline

$\omega_1$

$\omega_2$

$\omega_3$

ATOM FRAME view of LASER WAVES

ATOM FRAME view of LASER WAVES

atom speed $-u$

atom speed $0$
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**LaserPer-Spacetime**

- $\omega$ versus $ck$
- $1200$ THz
- $900$ THz
- $600$ THz
- $300$ THz

**AtomPer-Spacetime**

- $\omega'$ versus $ck'$
- $1200$ THz
- $750$ THz or $400$ nm
- $600$ THz or $500$ nm
- $500$ THz or $600$ nm
- $400$ THz or $750$ nm

- New 1st baseline
- $K_A = (+4, 4)$
- $K_B = (-1, 1)$
- Halved 3rd base distance
- Doubled 1st base distance

**Example Calculations**

- $\omega' = 2 \cdot (2) = 4$
- $\omega' = 1/2 \cdot (2) = 1$
PW for $\omega \rightarrow = 2\omega_0, \omega \leftarrow = \omega_0/2$

$$\frac{e^{ia} + e^{ib}}{2} = e^{i\frac{a+b}{2}} \left( \frac{e^{i\frac{a-b}{2}}}{2} + e^{-i\frac{a-b}{2}} \right)$$

Phase wave

$$\begin{align*}
\text{Real part} & : \cos \frac{a+b}{2} + i \sin \frac{a+b}{2} \\
\text{Imaginary part} & : \cos \frac{a-b}{2}
\end{align*}$$

$\text{Real part of:} \quad \frac{e^{ia} + e^{ib}}{2} = \left( \cos \frac{a+b}{2} \right) \left( \cos \frac{a-b}{2} \right)$

CW for $\omega \rightarrow = 2\omega_0, \omega \leftarrow = \omega_0/2$

$$ck \rightarrow = \omega \rightarrow = 2\pi 1200 \text{THz}$$
$$ck \leftarrow = -\omega \leftarrow = -2\pi 300 \text{THz}$$
PW for $\omega \rightarrow = 2\omega_0$, $\omega \leftarrow = \omega_0/2$

$\frac{e^{ia} + e^{ib}}{2} = \frac{e^{i\frac{a+b}{2}}}{e^{\frac{a-b}{2}} + e^{-\frac{a-b}{2}}}$

$= \left( \cos \frac{a+b}{2} + i \sin \frac{a+b}{2} \right) \left( \cos \frac{a-b}{2} \right)$

Phase wave
Real part
Imaginary part

$\frac{ck_\rightarrow = \omega_\rightarrow = 2\pi 1200 \text{THz}}{ck_\leftarrow = \omega_\leftarrow = -2\pi 300 \text{THz}}$

CW for $\omega \rightarrow = 2\omega_0$, $\omega \leftarrow = \omega_0/2$

$\frac{ck_\rightarrow = \omega_\rightarrow = 2\pi 1200 \text{THz}}{ck_\leftarrow = -\omega_\leftarrow = -2\pi 300 \text{THz}}$

$\frac{e^{ia} + e^{ib}}{2} = \left( \cos \frac{a+b}{2} \right) \left( \cos \frac{a-b}{2} \right)$

Real part of:

$\frac{k_\rightarrow x - \omega_\rightarrow t}{k_\leftarrow x - \omega_\leftarrow t}$

$\frac{\cos \left( \frac{k_\rightarrow + k_\leftarrow}{2} x - \frac{\omega_\rightarrow + \omega_\leftarrow}{2} t \right)}{\cos \left( \frac{k_\rightarrow - k_\leftarrow}{2} x - \frac{\omega_\rightarrow - \omega_\leftarrow}{2} t \right)}$
PW for $\omega_\rightarrow = 2\omega_0, \omega_\leftarrow = \omega_0/2$

$e^{ia} + e^{ib} = \frac{e^{i(a+b)/2} + e^{-i(a+b)/2}}{2}$

$\frac{a+b}{2} \cos \frac{a+b}{2} + i \sin \frac{a+b}{2}$

$= \cos \frac{a+b}{2} + i \sin \frac{a+b}{2}$

Real part of: $\frac{e^{ia} + e^{ib}}{2} = \left( \cos \frac{a+b}{2} \right) \left( \cos \frac{a-b}{2} \right)$

$ck_\leftarrow = \omega_\leftarrow = -2\pi 300$THz

$\omega_\rightarrow = 2\omega_0, \omega_\leftarrow = \omega_0/2$

$ck_\rightarrow = \omega_\rightarrow = 2\pi 1200$THz

$\omega_\rightarrow = 2\omega_0, \omega_\leftarrow = \omega_0/2$

$\cos \left( \frac{2\omega_0 x - \omega_0 t + 1/2}{2c} \right) \cos \left( \frac{2\omega_0 x - \omega_0 t - 1/2}{2c} \right)$
PW for \( \omega \rightarrow = 2\omega_0, \omega \leftarrow = \omega_0/2 \)

\[
\begin{align*}
\frac{e^{ia} + e^{ib}}{2} &= e^{i\frac{a+b}{2}} \left( \frac{e^{i\frac{a-b}{2}} + e^{-i\frac{a-b}{2}}}{2} \right) \\
&= \left( \cos \frac{a+b}{2} + i \sin \frac{a+b}{2} \right) \left( \cos \frac{a-b}{2} \right)
\end{align*}
\]

\[
\begin{align*}
\text{Phase wave} &\quad \text{Real part} \quad \text{Imaginary part} \\
\text{Real part} &\quad \text{Imaginary part}
\end{align*}
\]

\[
\begin{align*}
\cos \left( \frac{k_+ + k_-}{2}x - \frac{\omega_0 + \omega}{2}t \right) &\quad \cos \left( \frac{k_-}{2}x - \frac{\omega}{2}t \right) \\
\cos \left( \frac{2-1/2}{2c} \omega x - \frac{2+1/2}{2} \omega_0 t \right) &\quad \cos \left( \frac{2+1/2}{2c} \omega x - \frac{2-1/2}{2} \omega_0 t \right)
\end{align*}
\]

\[
\begin{align*}
\cos \left( \frac{3}{4c} \omega x - \frac{5}{4} \omega_0 t \right) &\quad \cos \left( \frac{5}{4c} \omega x - \frac{3}{4} \omega_0 t \right)
\end{align*}
\]

\[
\begin{align*}
\text{Real part of:} &\quad \frac{e^{ia} + e^{ib}}{2} = \left( \cos \frac{a+b}{2} \right) \left( \cos \frac{a-b}{2} \right) = \\
k_\rightarrow x - \omega_\rightarrow t &\quad k_\leftarrow x - \omega_\leftarrow t
\end{align*}
\]

\[
\begin{align*}
c\kappa_\rightarrow = \omega_\rightarrow = 2\pi 1200 \text{THz} &\quad c\kappa_\leftarrow = -\omega_\leftarrow = -2\pi 300 \text{THz}
\end{align*}
\]
PW for $\omega_\rightarrow=2\omega_0$, $\omega_\leftarrow=\omega_0/2$

$$\frac{e^{ia} + e^{ib}}{2} = \frac{e^{i\frac{a+b}{2}}}{2} e^{\frac{a-b}{2}} + e^{-i\frac{a-b}{2}}$$

Real part

$$\cos \left( \frac{k_\rightarrow x - \omega_\rightarrow t}{2} \right) \cos \left( \frac{k_\leftarrow x - \omega_\leftarrow t}{2} \right)$$

Group wave

$$\frac{1}{2} \left( \cos \left( \frac{2+1/2}{2c} \omega_0 x - \frac{2-1/2}{2c} \omega_0 t \right) \cos \left( \frac{3}{4c} \omega_0 x - \frac{5}{4} \omega_0 t \right) \right)$$

Real

$$V_{Phase} = \left( \frac{5}{4} \omega_0 / \frac{3}{4c} \omega_0 \right) = \frac{5}{3} c, \quad V_{Group} = \left( \frac{3}{4} \omega_0 / \frac{5}{4c} \omega_0 \right) = \frac{3}{5} c$$
PW for $\omega_\rightarrow=2\omega_0, \omega_\leftarrow=\omega_0/2$

$ck_\rightarrow=\omega_\rightarrow=2\pi 1200\text{THz}$  
$ck_\leftarrow=-\omega_\leftarrow=-2\pi 300\text{THz}$

$\cos \left( \frac{3}{4} \omega_0 x - \frac{5}{4} \omega_0 t \right) \cos \left( \frac{5}{4} \omega_0 x - \frac{3}{4} \omega_0 t \right)$

$\omega_0 \rightarrow 2\omega_0, \omega_0 \leftarrow = \omega_0 / 2$

$\omega_0 \rightarrow 2\omega_0, \omega_0 \leftarrow = \omega_0 / 2$

$\cos \frac{a+b}{2} + i \sin \frac{a+b}{2} \left( \cos \frac{a-b}{2} \right)$

$V_{\text{Phase}} = \left( \frac{5}{4} \omega_0 / \frac{3}{4} c, \omega_0 \right) = \frac{5}{3} c$,  
$V_{\text{Group}} = \left( \frac{3}{4} \omega_0 / \frac{5}{4} c, \omega_0 \right) = \frac{3}{3} c$
Laser lab views

atom speed \( u = -\frac{3}{5}c \)

Atom views (sees lab going \( +u = \frac{3}{5}c \))
**LaserPer-Spacetime**

\[ \omega \text{ versus } ck \]

- \( \omega_1 \) → 1000THz
- \( \omega_2 \) → 900THz
- \( \omega_3 \) → 300THz

**AtomPer-Spacetime**

\[ \omega' \text{ versus } ck' \]

- \( \omega'_1 \) → 1200THz
- \( \omega'_2 \) → 1000THz
- \( \omega'_3 \) → 800THz
- \( \omega'_4 \) → 600THz

*Atom speed 0***

*Atom speed -u***

**Laser per-space ck**

**Atom per-space ck'**
Euclidian Geometry for Per-spacetime Relativity

Relative speed-slope:
\( \frac{u}{c} = \frac{\sinh \rho}{\cosh \rho} = \tanh \rho \)

**Key Definition of Rapidity \( \rho \)**

- **Doppler blue shift:**
  \( Bb = Be^{+\rho} \)

- **Doppler red shift:**
  \( Br = Be^{-\rho} \)

**Key Results:**

\[ \begin{align*}
\omega & \text{ vs. } ck \\
\omega & = B \cosh \rho \\
ck & = B \sinh \rho \\
\text{group velocity: } & \frac{\omega}{ck} = \frac{u}{c} = \tanh \rho \\
\text{phase velocity: } & \frac{ck}{\omega} = \frac{c}{u} = \coth \rho
\end{align*} \]

**Key Quantities**

\[ \begin{align*}
\sinh \rho &= \sqrt{1 - \frac{u^2}{c^2}} \\
\cosh \rho &= \sqrt{1 - \frac{u^2}{c^2}}
\end{align*} \]
Euclidian wave geometry with time-reversal symmetry imply dispersion hyperbolas: $\omega = nB \cosh \rho$

\[ B \sinh \rho = \frac{(B e^\rho - B e^{-\rho})}{2} \quad \text{and} \quad B \cosh \rho = \frac{(B e^\rho + B e^{-\rho})}{2} \]

Lab frame area...
equals
Atom frame area...
by time-reversal axiom: $r = 1/b$

...that implies hyperbolic invariants $r \cdot b = 1$
Euclid’s 3-means (300 BC)
Geometric “heart” of wave mechanics

Galileo’s Revenge!

In phasor (gauge) space
Galileo’s relativity is right!

Thales (580BC) rectangle-in-circle
Relates to wave interference by (Galilean)
phasor angular velocity addition

geometric mean: $\sqrt{1 \cdot 4} = 2$

$\frac{1}{2} [4-1] = 3/2$

(HALF-DIFFERENCE)

$\frac{1}{2} [4+1] = 5/2 = 5/2$

(HALF-SUM)

Linear velocity $V_{\text{group}}/c = u/c$
is (HALF-DIFF./HALF-SUM) = 3/5

Galileo’s Revenge!
In phasor (gauge) space
Galileo’s relativity is right!
New Relativity webapp [Link](http://www.uark.edu/ua/modphys/testing/markup/RelativItWeb.html)
$\varphi = 0.916$

$B_{\cosh(\varphi)} = (Be^{+\varphi} + Be^{-\varphi})/2$

$B_{\sinh(\varphi)} = (Be^{+\varphi} - Be^{-\varphi})/2$

http://www.uark.edu/ua/modphys/testing/markup/RelaWavityWeb.html
\[ \sigma = 0.780 = 44.700^\circ \]
\[ q = 0.874 \]

\[ g\text{-circle} \]

\[ E\text{nergy (E)} \]

\[ \text{Velocity aberation angle } \sigma \]

\[ \text{Momentum } \cdot c \]
\[ cp = B \sinh(q) \]

\[ \text{Hamiltonian} \]
\[ H = B \cosh(q) \]

\[ \text{-Lagrangian} \]
\[ -L = B \sech(q) \]

\[ \text{Rest Energy} \]
\[ B = Mc^2 \]

\[ \text{Velocity } \cdot Mc \]
\[ Mcu = B \tanh(q) \]

\[ p\text{-circle} \]

\[ b\text{-circle} \]

\[ \text{Per-Space (cp)} \]

http://www.uark.edu/ua/modphys/testing/markup/RelaWavityWeb.html
Hyperbolic Functions

\( q = 1.0405 \)
\( \text{Length}(q) = 1.0405 \)
\( \text{Area}(q) = 1.0405 \)
\( \sinh(q) = 1.2387 \)
\( \cosh(q) = 1.5920 \)
\( \tanh(q) = 0.7781 \)
\( \text{csch}(q) = 0.8073 \)
\( \text{sech}(q) = 0.6282 \)
\( \coth(q) = 1.2852 \)

http://www.uark.edu/ua/modphys/testing/markup/RelaWavityWeb.html