Relativity of interfering and galloping waves: Amplitude and SWR.

Unmatched amplitudes giving galloping waves

Standing Wave Ratio (SWR) and Standing Wave Quotient (SWQ)
  Analogy with group and phase

Galloping waves
  Analogy between wave galloping, Keplarian IHO orbits, and optical polarization
  Galloping dynamics algebra
  Waves that go back in time - The Feynman-Wheeler Switchback
  The Ship-Barn-and-Butler saga of confused causality

1st Quantization: Quantizing phase variables $\omega$ and $k$
  Understanding how quantum transitions require “mixed-up” states
  Closed cavity vs ring cavity

Relativistic effects on charge, current, and Maxwell Fields
  Current density changes by Lorentz asynchrony
  Magnetic B-field is relativistic $\sinh \rho$ 1st order-effect
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2-CW dynamics has two 1-CW amplitudes $A_\rightarrow$ and $A_\leftarrow$ that we now allow to be unmatched. ($A_\rightarrow \neq A_\leftarrow$)

$$A_\rightarrow e^{i(k_\rightarrow x - \omega_\rightarrow t)} + A_\leftarrow e^{i(k_\leftarrow x - \omega_\leftarrow t)} = e^{i(k_\Sigma x - \omega_\Sigma t)} [A_\rightarrow e^{i(k_\Delta x - \omega_\Delta t)} + A_\leftarrow e^{-i(k_\Delta x - \omega_\Delta t)}]$$

Waves have half-sum mean-phase rates $(k_\Sigma, \omega_\Sigma)$ and half-difference group rates $(k_\Delta, \omega_\Delta)$.

$$k_\Sigma = (k_\rightarrow + k_\leftarrow) / 2$$
$$\omega_\Sigma = (\omega_\rightarrow + \omega_\leftarrow) / 2$$

$$k_\Delta = (k_\rightarrow - k_\leftarrow) / 2$$
$$\omega_\Delta = (\omega_\rightarrow - \omega_\leftarrow) / 2$$
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2-CW dynamics has two 1-CW amplitudes \( A_\rightarrow \) and \( A_\leftarrow \) that we now allow to be unmatched. \( (A_\rightarrow \neq A_\leftarrow) \)

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\]

Waves have half-sum mean-phase rates \( (k_\Sigma, \omega_\Sigma) \) and half-difference group rates \( (k_\Delta, \omega_\Delta) \).

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k_\Sigma = (k_\rightarrow + k_\leftarrow) / 2 \quad \quad \quad k_\Delta = (k_\rightarrow - k_\leftarrow) / 2
\]

\[
\omega_\Sigma = (\omega_\rightarrow + \omega_\leftarrow) / 2 \quad \quad \quad \omega_\Delta = (\omega_\rightarrow - \omega_\leftarrow) / 2
\]

Also important is amplitude mean \( A_\Sigma = (A_\rightarrow + A_\leftarrow) / 2 \) and amplitude half-difference \( A_\Delta = (A_\rightarrow - A_\leftarrow) / 2 \).
Galloping waves due to unmatched amplitudes

2-CW dynamics has two 1-CW amplitudes $A_{\rightarrow}$ and $A_{\leftarrow}$ that we now allow to be unmatched. ($A_{\rightarrow} \neq A_{\leftarrow}$)

$$A_{\rightarrow} e^{i(k_{\rightarrow} x - \omega_{\rightarrow} t)} + A_{\leftarrow} e^{i(k_{\leftarrow} x - \omega_{\leftarrow} t)} = e^{i(k_{\Sigma} x - \omega_{\Sigma} t)} [A_{\rightarrow} e^{i(k_{\Delta} x - \omega_{\Delta} t)} + A_{\leftarrow} e^{-i(k_{\Delta} x - \omega_{\Delta} t)}]$$

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Also important is amplitude mean $A_{\Sigma} = (A_{\rightarrow} + A_{\leftarrow}) / 2$ and amplitude half-difference $A_{\Delta} = (A_{\rightarrow} - A_{\leftarrow}) / 2$.

Detailed wave motion depends on standing-wave-ratio $SWR$ or the inverse standing-wave-quotient $SWQ$.

$$SWR = \frac{(A_{\rightarrow} - A_{\leftarrow})}{(A_{\rightarrow} + A_{\leftarrow})}$$

$$SWQ = \frac{(A_{\rightarrow} + A_{\leftarrow})}{(A_{\rightarrow} - A_{\leftarrow})} = \frac{1}{SWR}$$
**Galloping waves due to unmatched amplitudes**

2-CW dynamics has two 1-CW amplitudes $A_→$ and $A←$ that we now allow to be unmatched. ($A_→ \neq A←$)

$$A_→ e^{i(k→ x - ω→ t)} + A← e^{i(k← x - ω← t)} = e^{i(kΣ x - ωΣ t)} [A_→ e^{i(kΔ x - ωΔ t)} + A← e^{-i(kΔ x - ωΔ t)}]$$

Waves have half-sum mean-phase rates $(kΣ, ωΣ)$ and half-difference group rates $(kΔ, ωΔ)$.

$$kΣ = (k→ + k←)/2$$
$$ωΣ = (ω→ + ω←)/2$$

$$kΔ = (k→ - k←)/2$$
$$ωΔ = (ω→ - ω←)/2$$

Also important is amplitude mean $AΣ = (A→ + A←)/2$ and amplitude half-difference $AΔ = (A→ - A←)/2$.

Detailed wave motion depends on standing-wave-ratio $SWR$ or the inverse standing-wave-quotient $SWQ$.

**Envelope–Min.**

$$SWR = \frac{(A→ - A←)}{(A→ + A←)}$$

**Envelope–Max.**

$$2AΣ = (A→ + A←)$$

**Envelop Maximum**

**Envelope Minimum**

$$2AΔ = (A→ - A←)$$

$$SWQ = \frac{(A→ + A←)}{(A→ - A←)} = \frac{1}{SWR}$$

Thursday, March 6, 2014
Galloping waves due to unmatched amplitudes

2-CW dynamics has two 1-CW amplitudes $A_{\rightarrow}$ and $A_{\leftarrow}$ that we now allow to be unmatched. ($A_{\rightarrow} \neq A_{\leftarrow}$)

\[ A_{\rightarrow} e^{i(k_{\rightarrow} x - \omega_{\rightarrow} t)} + A_{\leftarrow} e^{i(k_{\leftarrow} x - \omega_{\leftarrow} t)} = e^{i(k_{\Sigma} x - \omega_{\Sigma} t)} [A_{\rightarrow} e^{i(k_{\Delta} x - \omega_{\Delta} t)} + A_{\leftarrow} e^{-i(k_{\Delta} x - \omega_{\Delta} t)}] \]

Waves have half-sum mean-phase rates $(k_{\Sigma}, \omega_{\Sigma})$ and half-difference group rates $(k_{\Delta}, \omega_{\Delta})$.

\[ k_{\Sigma} = (k_{\rightarrow} + k_{\leftarrow}) / 2 \quad \omega_{\Sigma} = (\omega_{\rightarrow} + \omega_{\leftarrow}) / 2 \]
\[ k_{\Delta} = (k_{\rightarrow} - k_{\leftarrow}) / 2 \quad \omega_{\Delta} = (\omega_{\rightarrow} - \omega_{\leftarrow}) / 2 \]

Also important is amplitude mean $A_{\Sigma} = (A_{\rightarrow} + A_{\leftarrow}) / 2$ and amplitude half-difference $A_{\Delta} = (A_{\rightarrow} - A_{\leftarrow}) / 2$.

Detailed wave motion depends on standing-wave-ratio $SWR$ or the inverse standing-wave-quotient $SWQ$.

\[ SWR = \frac{(A_{\rightarrow} - A_{\leftarrow})}{(A_{\rightarrow} + A_{\leftarrow})} \quad SWQ = \frac{(A_{\rightarrow} + A_{\leftarrow})}{(A_{\rightarrow} - A_{\leftarrow})} = \frac{1}{SWR} \]

These are analogous to frequency ratios for group velocity $V_{\text{group}} < c$ and its inverse that is phase velocity $V_{\text{phase}} > c$.

\[ V_{\text{group}} = \frac{\omega_{\Delta}}{k_{\Delta}} = \frac{(\omega_{\rightarrow} - \omega_{\leftarrow})}{(k_{\rightarrow} - k_{\leftarrow})} = c \frac{(\omega_{\rightarrow} - \omega_{\leftarrow})}{(\omega_{\rightarrow} + \omega_{\leftarrow})} \]
\[ V_{\text{phase}} = \frac{\omega_{\Sigma}}{k_{\Sigma}} = \frac{(\omega_{\rightarrow} + \omega_{\leftarrow})}{(k_{\rightarrow} + k_{\leftarrow})} = c \frac{(\omega_{\rightarrow} + \omega_{\leftarrow})}{(\omega_{\rightarrow} - \omega_{\leftarrow})} \]
Galloping waves due to unmatched amplitudes

2-CW dynamics has two 1-CW amplitudes \( A_\to \) and \( A_\leftarrow \) that we now allow to be unmatched. \((A_\to \neq A_\leftarrow)\)

\[
A_\to e^{i(k_\to x - \omega_\to t)} + A_\leftarrow e^{i(k_\leftarrow x - \omega_\leftarrow t)} = e^{i(k_\Sigma x - \Omega_\Sigma t)} [A_\to e^{i(k_\Delta x - \Omega_\Delta t)} + A_\leftarrow e^{-i(k_\Delta x - \Omega_\Delta t)}]
\]

Waves have half-sum mean-phase rates \((k_\Sigma, \Omega_\Sigma)\) and half-difference group rates \((k_\Delta, \Omega_\Delta)\).

\[
k_\Sigma = (k_\to + k_\leftarrow)/2 \quad \quad k_\Delta = (k_\to - k_\leftarrow)/2
\]

\[
\omega_\Sigma = (\omega_\to + \omega_\leftarrow)/2 \quad \quad \omega_\Delta = (\omega_\to - \omega_\leftarrow)/2
\]

Also important is amplitude mean \( A_\Sigma = (A_\to + A_\leftarrow)/2 \) and amplitude half-difference \( A_\Delta = (A_\to - A_\leftarrow)/2 \).

Detailed wave motion depends on standing-wave-ratio \( SWR \) or the inverse standing-wave-quotient \( SWQ \).

These are analogous to frequency ratios for group velocity \( V_{\text{group}} < c \) and its inverse that is phase velocity \( V_{\text{phase}} > c \).

\[
V_{\text{group}} = \frac{\omega_\Delta}{k_\Delta} = \frac{(\omega_\to - \omega_\leftarrow)}{(k_\to - k_\leftarrow)} = c \frac{(\omega_\to - \omega_\leftarrow)}{(\omega_\to + \omega_\leftarrow)}
\]

\[
V_{\text{phase}} = \frac{\omega_\Sigma}{k_\Sigma} = \frac{(\omega_\to + \omega_\leftarrow)}{(k_\to + k_\leftarrow)} = c \frac{(\omega_\to + \omega_\leftarrow)}{(\omega_\to - \omega_\leftarrow)}
\]

\[
V_{\text{group}} = \frac{\omega_\Delta}{c k_\Delta} = \frac{(\omega_\to - \omega_\leftarrow)}{c(k_\to - k_\leftarrow)} = \frac{(\omega_\to - \omega_\leftarrow)}{(\omega_\to + \omega_\leftarrow)}
\]

\[
V_{\text{phase}} = \frac{\omega_\Sigma}{c k_\Sigma} = \frac{(\omega_\to + \omega_\leftarrow)}{c(k_\to + k_\leftarrow)} = \frac{(\omega_\to + \omega_\leftarrow)}{(\omega_\to - \omega_\leftarrow)} = \frac{c}{V_{\text{group}}}
\]
Galloping waves due to unmatched amplitudes

2-CW dynamics has two 1-CW amplitudes $A_\rightarrow$ and $A_\leftarrow$ that we now allow to be unmatched. ($A_\rightarrow \neq A_\leftarrow$)

$$A_\rightarrow e^{i(k_\rightarrow x - \omega_\rightarrow t)} + A_\leftarrow e^{i(k_\leftarrow x - \omega_\leftarrow t)} = e^{i(k_\Sigma x - \omega_\Sigma t)} [A_\rightarrow e^{i(k_\Delta x - \omega_\Delta t)} + A_\leftarrow e^{i(k_\Delta x - \omega_\Delta t)}]$$

Waves have half-sum mean-phase rates $(k_\Sigma, \omega_\Sigma)$ and half-difference group rates $(k_\Delta, \omega_\Delta)$.

$$k_\Sigma = (k_\rightarrow + k_\leftarrow) / 2$$
$$\omega_\Sigma = (\omega_\rightarrow + \omega_\leftarrow) / 2$$

$$k_\Delta = (k_\rightarrow - k_\leftarrow) / 2$$
$$\omega_\Delta = (\omega_\rightarrow - \omega_\leftarrow) / 2$$

Also important is amplitude mean $A_\Sigma = (A_\rightarrow + A_\leftarrow) / 2$ and amplitude half-difference $A_\Delta = (A_\rightarrow - A_\leftarrow) / 2$.

Detailed wave motion depends on standing-wave-ratio $SWR$ or the inverse standing-wave-quotient $SWQ$.

$$SWR = \frac{(A_\rightarrow - A_\leftarrow)}{(A_\rightarrow + A_\leftarrow)}$$
$$SWQ = \frac{(A_\rightarrow + A_\leftarrow)}{(A_\rightarrow - A_\leftarrow)} = \frac{1}{SWR}$$

These are analogous to frequency ratios for group velocity $V_{\text{group}} < c$ and its inverse that is phase velocity $V_{\text{phase}} > c$.

$$V_{\text{group}} = \frac{\omega_\Delta}{k_\Delta} = \frac{(\omega_\rightarrow - \omega_\leftarrow)}{(k_\rightarrow - k_\leftarrow)} = \frac{c (\omega_\rightarrow - \omega_\leftarrow)}{(\omega_\rightarrow + \omega_\leftarrow)}$$

$$V_{\text{phase}} = \frac{\omega_\Sigma}{k_\Sigma} = \frac{(\omega_\rightarrow + \omega_\leftarrow)}{(k_\rightarrow + k_\leftarrow)} = \frac{c (\omega_\rightarrow + \omega_\leftarrow)}{(\omega_\rightarrow - \omega_\leftarrow)}$$

$$V_{\text{group}} = \frac{\omega_\Delta}{c k_\Delta} = \frac{c (\omega_\rightarrow - \omega_\leftarrow)}{c(k_\rightarrow - k_\leftarrow)} = \frac{c (\omega_\rightarrow - \omega_\leftarrow)}{(\omega_\rightarrow + \omega_\leftarrow)}$$

$$V_{\text{phase}} = \frac{\omega_\Sigma}{c k_\Sigma} = \frac{c (\omega_\rightarrow + \omega_\leftarrow)}{c(k_\rightarrow + k_\leftarrow)} = \frac{c (\omega_\rightarrow + \omega_\leftarrow)}{(\omega_\rightarrow - \omega_\leftarrow)}$$

$$\frac{V_{\text{group}}}{c} = \frac{V_{\text{phase}}}{c}$$

is analogous to:

$$SWR = \frac{1}{SWQ}$$
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Waves that go back in time - The Feynman-Wheeler Switchback
Two extremes for Standing Wave Ratio

- SWR = 1
- SWR = 0

SWR = \pm 3/5

SWR = \pm 1/5

SWR = 0

Fig. 6.1 Monochromatic (1-frequency) 2-CW wave space-time patterns.
Two extremes for Standing Wave Ratio

SWR = 1

SWR = 0

SWR = -1/5

SWR = -3/5

SWR = +1/5

SWR = +3/5

Fig. 6.1 Monochromatic (1-frequency) 2-CW wave space-time patterns.
SWR = 1
Two extremes for Standing Wave Ratio
SWR = 0

...and
SWR = -1

(not shown in (x,ct) plots)
Two extremes for Standing Wave Ratio

1-frequency cases

\( \omega = 2c, k = 2 \)
\( \omega_i = 2c, k_i = -2 \)
\( u_{\text{GROUP}} = 0 \)
\( u_{\text{PHASE}} = \infty \)

2-frequency cases

\( \omega = 4c, k = 4 \)
\( \omega_i = 1c, k_i = -1 \)
\( u_{\text{GROUP}}/c = 3/5 \)
\( u_{\text{PHASE}}/c = 5/3 \)

SWR = 1 (not shown)

SWR = +3/5

SWR = +1/5

SWR = 0

SWR = -1/5

SWR = -3/5

SWR = -1 (not shown)

Same SWR cases viewed at \( u/c = 3/5 \)

...and SWR = -1

from: Fig. 4.5.2

from: Fig. 8.6.3

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Fig. 6.1 Monochromatic (1-frequency) 2-CW wave space-time patterns.

(a) $E_x = 0.2$
    $E_y = 0.8$

(b) $E_x = 0.4$
    $E_y = 0.6$

(c) $E_x = 0.5$
    $E_y = 0.5$

$\omega_y = 2c$, $k_y = 2$
$\omega_x = 4c$, $k_x = 4$
$u_{GROUP}/c = 3/5$
$u_{PHASE}/c = 5/3$

(d) $E_x = 0.6$
    $E_y = 0.4$

(e) $E_x = 0.8$
    $E_y = 0.2$

SWR = +3/5

1-frequency cases

Fig. 6.2 Dichromatic (2-frequency) 2-CW wave space-time patterns.

(a) $E_x = 0.2$
    $E_y = 0.8$

(b) $E_x = 0.4$
    $E_y = 0.6$

(c) $E_x = 0.5$
    $E_y = 0.5$

$\omega_y = 2c$, $k_y = -2$
$\omega_x = 1c$, $k_x = -1$
$u_{GROUP}/c = 0$

(d) $E_x = 0.6$
    $E_y = 0.4$

(e) $E_x = 0.8$
    $E_y = 0.2$

SWR = -3/5

2-frequency cases

Fig. 6.3 (a-g) Elliptic polarization ellipses relate to galloping waves in Fig. 6.1. (h-i) Kepler anomalies.
Fig. 6.1 Monochromatic (1-frequency) 2-CW wave space-time patterns.

Fig. 6.2 Dichromatic (2-frequency) 2-CW wave space-time patterns.

(a) $E_\perp = 0.2$
    $E_\parallel = 0.8$

$b/a = +1/1$

SWR = +3/5

(b) $E_\perp = 0.4$
    $E_\parallel = 0.6$

$b/a = +3/5$

SWR = +1/5

(c) $E_\perp = 0.5$
    $E_\parallel = 0.5$

$\omega_\perp = 4\omega_c$, $k_\perp = 4$

$\omega_\parallel = 1\omega_c$, $k_\parallel = 1$

$u_{\text{GROUP}} = c/3$

$u_{\text{PHASE}} = c/5/3$

$d/\omega = 0$

x-plane polarization

SWR = 0

(e) $E_\perp = 0.6$
    $E_\parallel = 0.4$

$b/a = -3/5$

SWR = -3/5

(f) $E_\perp = 0.8$
    $E_\parallel = 0.2$

$b/a = -1/1$

SWR = -3/5

Hallowing analogy

2-frequency cases

1-frequency cases

(b) $E_\perp = 0.4$
    $E_\parallel = 0.6$

$SWR = +1/5$

(c) $E_\perp = 0.5$
    $E_\parallel = 0.5$

$\omega_\perp = 2\omega_c$, $k_\perp = 2$

$\omega_\parallel = 2\omega_c$, $k_\parallel = -2$

$u_{\text{GROUP}} = 0$

$u_{\text{PHASE}} = \infty$

Fig. 6.3 (a-g) Elliptic polarization ellipses relate to galloping waves in Fig. 6.1. (h-i) Kepler anomalies.

Elliptic oscillator orbit

$SWR = b/a = 1/5$

$polarization analogy$

$SWR = +1$

right circular polarization

right moving wave

$SWR = +3/5$

right elliptical polarization

$SWR = +1/5$

left elliptical polarization

left moving wave

$y = b \sin \omega t$

$x = a \cos \omega t$

$\tan \phi(t) = \frac{y}{x} = \frac{b \sin \omega t}{a \cos \omega t}$

(i) Kepler anomaly relations

$\tan \phi(t) = SWR \tan \omega t$

Highest speed $v = 5$ at perigee $r = b$

Lowest speed $x = 1$ at apogee $r = a$

mean anomaly

eccentric anomaly
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Analogies between wave galloping, Keplerian IHO orbits, and optical polarization

We'll show wave galloping is analogous to Keplerian orbital motion of angles $\omega \cdot t$ and $\phi$ of orbits.

\[
\tan \phi(t) = \frac{b}{a} \tan \omega \cdot t
\]

from: Fig. 4.5.2  from: Fig. 8.6.3

QTforCA  CMwBang!

Unit 2 Ch.4  Unit 8 Ch.6
We’ll show wave galloping is analogous to Keplarian orbital motion of angles \( \omega \cdot t \) and \( \phi \) of orbits.

\[
\tan \phi(t) = \frac{b}{a} \tan \omega \cdot t
\]

**Elliptic oscillator orbit**

\( SWR = b/a = 1/5 \)

**Kepler anomaly relations**

\[
\tan \phi(t) = SWR \cdot \tan \omega t
\]

\[
\frac{y}{x} = \frac{b \sin \omega t}{a \cos \omega t} = SWR \cdot \tan \omega t
\]

from: Fig. 4.5.2

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Unit 2 Ch.4
An analogy between wave galloping, Keplarian IHO orbits, and optical polarization

We’ll show wave galloping is analogous to Keplarian orbital motion of angles \( \omega \cdot t \) and \( \phi \) of orbits.

\[
\tan \phi(t) = \frac{b}{a} \tan \omega \cdot t
\]

The eccentric anomaly time derivative of \( \phi \) (angular velocity) gallops between \( \omega \cdot b/a \) and \( \omega \cdot a/b \).

\[
\dot{\phi} = \frac{d\phi}{dt} = \frac{b}{a} \sec^2 \omega t = \frac{b}{a} \sec^2 \phi = \frac{\omega \cdot b/a}{a + \tan^2 \phi} = \frac{\omega \cdot b/a}{\cos^2 \omega t + (b/a)^2 \sin^2 \omega t} = \begin{cases} \\
\omega \cdot b/a & \text{for } \omega t = 0, \pi, 2\pi \ldots \\
\omega \cdot a/b & \omega t = \pi/2, 3\pi/2, \ldots \\
\end{cases}
\]

**Elliptic oscillator orbit**

\( SWR = b/a = 1/5 \)

**Kepler anomaly relations**

\[
\tan \phi(t) = SWR \cdot \tan \omega t \\
\tan \phi(t) = \frac{y}{x} = \frac{b \sin \omega t}{a \cos \omega t}
\]
We’ll show wave galloping is analogous to Keplarian orbital motion of angles $\omega \cdot t$ and $\phi$ of orbits.

The eccentric anomaly time derivative of $\phi$ (angular velocity) gallops between $\omega \cdot b/a$ and $\omega \cdot a/b$.

$$\tan \phi(t) = \frac{b}{a} \tan \omega \cdot t$$

The eccentric anomaly time derivative of $\phi$ (angular velocity) gallops between $\omega \cdot b/a$ and $\omega \cdot a/b$.

$$\dot{\phi} = \frac{d\phi}{dt} = \omega \cdot \frac{b \sec^2 \omega t}{a \sec^2 \phi} = \omega \cdot \frac{b \sec^2 \omega t}{a \left( \tan^2 \phi + \left( \frac{b}{a} \right)^2 \cdot \sin^2 \omega t \right)} = \left\{ \begin{array}{ll} \omega \cdot \frac{b}{a} & \text{for } \omega t = 0, \pi, 2\pi ... \\ \omega \cdot \frac{a}{b} & \omega t = \pi/2, 3\pi/2, ... \end{array} \right\}$$

The product of angular moment $r^2$ and $\dot{\phi}$ is orbital momentum, a constant proportional to ellipse area.

$$r^2 \frac{d\phi}{dt} = \text{constant} = (a^2 \cos^2 \omega t + b^2 \cdot \sin^2 \omega t) \frac{d\phi}{dt} = \omega \cdot ab$$

Analogy between wave galloping, Keplarian IHO orbits, and optical polarization.
An analogy between wave galloping, Keplarian IHO orbits, and optical polarization.

We’ll show wave galloping is analogous to Keplarian orbital motion of angles \( \omega \cdot t \) and \( \phi \) of orbits.

\[ \tan \phi(t) = \frac{b}{a} \tan(\omega \cdot t) \]

The eccentric anomaly time derivative of \( \phi \) (angular velocity) gallops between \( \omega \cdot b/a \) and \( \omega \cdot a/b \).\n
\[ \dot{\phi} = \frac{d\phi}{dt} = \frac{b}{a} \frac{b \sec^2(\frac{\omega t}{a} + 1 + \tan^2 \phi)}{sec^2 \phi} = \frac{\omega \cdot b / a}{\omega \cdot a / b} \frac{\omega \cdot b / a}{\omega \cdot b / a} \]

The product of angular moment \( r^2 \) and \( \dot{\phi} \) is orbital momentum, a constant proportional to ellipse area.

\[ r^2 \frac{d\phi}{dt} = constant = (a^2 \cos^2(\omega t) + b^2 \cdot \sin^2(\omega t)) \frac{d\phi}{dt} = \omega \cdot ab \]

Consider galloping wave zeros of a monochromatic wave having \( SWR = 1/5 \).

\[ 0 = \text{Re} \Psi(x,t) = \text{Re} \left[ A_\rightarrow e^{i(k_0 x - \omega_0 t)} + A_\leftarrow e^{i(-k_0 x - \omega_0 t)} \right] \text{ where: } \omega_\rightarrow = \omega_0 = \omega_\leftarrow = c k_0 = -c k_\leftarrow \]

\[ 0 = A_\rightarrow \left[ \cos k_0 x \cos \omega_0 t + \sin k_0 x \sin \omega_0 t \right] + A_\leftarrow \left[ \cos k_0 x \cos \omega_0 t - \sin k_0 x \sin \omega_0 t \right] \]

\[ (A_\rightarrow + A_\leftarrow) \left[ \cos k_0 x \cos \omega_0 t \right] = - (A_\rightarrow - A_\leftarrow) \left[ \sin k_0 x \sin \omega_0 t \right] \]

\[ E_\leftarrow = 0.4, \quad E_\rightarrow = 0.6 \]
Analogy between wave galloping, Keplarian IHO orbits, and optical polarization.

We’ll show wave galloping is analogous to Keplarian orbital motion of angles \( \phi \) and \( \phi \cdot t \) of orbits.

The eccentric anomaly time derivative of \( \phi \) (angular velocity) gallops between \( \omega \cdot b/a \) and \( \omega \cdot a/b \).

\[
\dot{\phi} = \frac{d\phi}{dt} = \frac{\omega \cdot b \sec^2 \omega t}{a \sec^2 \phi} = \frac{\omega \cdot b \sec^2 \omega t}{a(1 + \tan^2 \phi)} = \frac{\omega \cdot b / a}{\cos^2 \omega t + (b / a)^2 \cdot \sin^2 \omega t}
\]

The product of angular moment \( r^2 \) and \( \dot{\phi} \) is orbital momentum, a constant proportional to ellipse area.

\[
r^2 \frac{d\phi}{dt} = \text{constant} = (a^2 \cos^2 \omega t + b^2 \cdot \sin^2 \omega t) \frac{d\phi}{dt} = \omega \cdot ab
\]

Consider galloping wave zeros of a monochromatic wave having \( SWR = 1/5 \).

\[
0 = \text{Re} \Psi(x,t) = \text{Re} \left[ A_+ e^{i(k_0 x - \omega_0 t)} + A_- e^{i(-k_0 x - \omega_0 t)} \right]
\]

where: \( \omega_+ = \omega_0 = \omega_- = ck_0 = -ck_-

\[
0 = A_+ \left[ \cos k_0 x \cos \omega_0 t + \sin k_0 x \sin \omega_0 t \right] + A_- \left[ \cos k_0 x \cos \omega_0 t - \sin k_0 x \sin \omega_0 t \right]
\]

\[
\left( A_+ + A_- \right) \left[ \cos k_0 x \cos \omega_0 t \right] = -\left( A_+ - A_- \right) \left[ \sin k_0 x \sin \omega_0 t \right]
\]

Space \( k_0 x \) varies with time \( \omega_0 t \) in the same way that eccentric anomaly \( \phi \) varies with \( \omega \cdot t \).

\[
\tan k_0 x = -SWR \cdot \cot \omega_0 t = SWR \cdot \tan \omega_0 \bar{t}
\]

where: \( \omega_0 \bar{t} = \omega_0 t - \pi / 2 \).
The product of angular moment

\[ \phi(t) = \frac{b}{a} \tan \omega \cdot t \]

The eccentric anomaly time derivative of \( \phi \) (angular velocity) gallops between \( \omega \cdot b/a \) and \( \omega \cdot a/b \).

\[ \dot{\phi} = \frac{d\phi}{dt} = \omega \cdot \frac{b \sec^2 \omega t}{a \sec^2 \phi} = \omega \cdot \frac{b \sec^2 \omega t}{1 + \tan^2 \phi} = \frac{\omega \cdot b / a}{\cos^2 \omega t + (b/a)^2 \cdot \sin^2 \omega t} = \begin{cases} \omega \cdot b / a & \text{for } \omega t = 0, \pi, 2\pi \ldots \\ \omega \cdot a / b & \omega t = \pi / 2, 3\pi / 2, \ldots \end{cases} \]

The product of angular moment \( r^2 \) and \( \dot{\phi} \) is orbital momentum, a constant proportional to ellipse area.

\[ r^2 \frac{d\phi}{dt} = \text{constant} = (a^2 \cos^2 \omega t + b^2 \cdot \sin^2 \omega t) \frac{d\phi}{dt} = \omega \cdot ab \]

Consider galloping wave zeros of a monochromatic wave having \( SWR = 1/5 \).

\[ 0 = \text{Re} \Psi(x, t) = \text{Re} \left[ A_\rightarrow e^{i(k_0 x - \omega_0 t)} + A_\leftarrow e^{i(-k_0 x + \omega_0 t)} \right] \quad \text{where: } \omega_\rightarrow = \omega_0 = \omega_\leftarrow = \pm c k_0 = -\omega_\leftarrow \]

\[ 0 = A_\rightarrow \left[ \cos k_0 x \cos \omega_0 t + \sin k_0 x \sin \omega_0 t \right] + A_\leftarrow \left[ \cos k_0 x \cos \omega_0 t - \sin k_0 x \sin \omega_0 t \right] \]

\[ (A_\rightarrow + A_\leftarrow) \left[ \cos k_0 x \cos \omega_0 t \right] = - (A_\rightarrow - A_\leftarrow) \left[ \sin k_0 x \sin \omega_0 t \right] \]

Space \( k_0 x \) varies with time \( \omega_0 t \) in the same way that eccentric anomaly \( \phi \) varies with \( \omega \cdot t \).

\[ \tan k_0 x = -SWR \cdot \cot \omega_0 t = SWR \cdot \tan \omega_0 \overline{t} \quad \text{where: } \omega_0 \overline{t} = \omega_0 t - \pi / 2 \]

Speed of galloping wave zeros is the time derivative of root location \( x \) in units of light velocity \( c \).

\[ \frac{dx}{dt} = c \cdot SWR \frac{\sec^2 \omega_0 \overline{t}}{\sec^2 k_0 x} = \frac{c \cdot SWR}{\cos^2 \omega_0 \overline{t} + SWR^2 \cdot \sin^2 \omega_0 \overline{t} = \begin{cases} c \cdot SWR & \text{for } \overline{t} = 0, \pi, 2\pi \ldots \\ c \cdot SWQ & \overline{t} = \pi / 2, 3\pi / 2, \ldots \end{cases} \]
Wave-Zero Speed-Limits

Standing Wave Ratio $SWR$ and Quotient $SWQ$

$SWR = \frac{(E_\rightarrow - E_\leftarrow)}{(E_\rightarrow + E_\leftarrow)} = 1/SWQ$

$SWR = 1/5$

$SWQ = 5$

$E_\leftarrow = 0.5, \ E_\rightarrow = 0.5$

Speed of galloping wave zeros is the time derivative of root location $x$ in units of light velocity $c$.

$$\frac{dx}{dt} = c \cdot SWR \frac{\sec^2 \omega_0 \bar{t}}{\sec^2 k_0 x} = \frac{c \cdot SWR}{\cos^2 \omega_0 \bar{t} + SWR^2 \cdot \sin^2 \omega_0 \bar{t}} = \begin{cases} c \cdot SWR & \text{for: } \bar{t} = 0, \pi, 2\pi... \\ c \cdot SWQ & \bar{t} = \pi / 2, 3\pi / 2,... \end{cases}$$
Wave-Zero Speed-Limits

Standing Wave Ratio SWR and Quotient SWQ

$$SWR = (E_\rightarrow - E_\leftarrow)/(E_\rightarrow + E_\leftarrow) = 1/SWQ$$

$$SWR = 1/5$$

SWR is “resting” at $$(1/5)c$$

$$SWQ = 5$$

Wave zeros “galloping” at $$5c$$

$$E_\leftarrow = 0.5, E_\rightarrow = 0.5$$

SWR = 1 is analogous to (1,i) Right Circular Polarization

SWR = 0 is analogous to (1,0) x-Plane Linear Polarization

SWR = -1 is analogous to (1,-i) Left Circular Polarization
Staircase Galloping

Speed of galloping

\[ SWR = \frac{1}{2} \]

cancelled by group velocity

\[ u_{GROUP}/c = -\frac{1}{2}. \]
Unmatched amplitudes giving galloping waves

*Standing Wave Ratio (SWR) and Standing Wave Quotient (SWQ)*

Analogy with group and phase

Galloping waves

Analogy between wave galloping, *Keplarian IHO orbits, and optical polarization*

Galloping dynamics algebra

Waves that go back in time - *The Feynman-Wheeler Switchback*

The Ship-Barn-and-Butler saga of confused causality
Fig. 2.B.10 Lighthouse plot of two Happenings

Happening 1
Before
Happening 2

Brief faster-than-light travel
Fig. 2.B.10 Lighthouse plot of two Happenings

Fig. 2.B.11 Ship plot of two Happenings

Happening 1
Before

Happening 2

Brief faster-than-light travel

Brief travel back-in-time

(creation)

(annihilation)
Waves that go back in time - The Feynman-Wheeler Switchback

Minkowski Zero-Grids are Spacetime Switchbacks for
$-u_{\text{GROUP}} < \text{SWR} < 0$

- $\omega_{\rightarrow} = 4c$, $\omega_{\leftarrow} = 1c$
- $k_{\rightarrow} = 4$, $k_{\leftarrow} = -1$
- $u_{\text{GROUP}} = c3/5$, $u_{\text{PHASE}} = c5/3$

**Group zero speed limit**

$$u_{\text{GROUP}} + \text{SWR}$$

$$\frac{1}{1 + u_{\text{GROUP}} \cdot \text{SWR}}$$

$$c^2$$

$$= 5c/11$$

**Phase “anti-zero” going “back-in-time”**

- $E_{\leftarrow} = 0.5$, $E_{\rightarrow} = 0.5$

Wave zero-anti-zero annihilation and creation occur together at the same spacetime point for $\text{SWR} = 0$

- $E_{\leftarrow} = 0.6$, $E_{\rightarrow} = 0.4$

Wave zero-anti-zero annihilation and creation occur separately at different spacetime points for $-u_{\text{GROUP}} < \text{SWR} < 0$
At High Speed 2-CW Modes Look More Like 1-CW Beams

Various combinations of opposite-k 1-CW beams occur with open boundaries.

\[ E_{\text{wave}}: E = E_\rightarrow e^{i(k_\rightarrow x - \omega_\rightarrow t)} + E_\leftarrow e^{i(k_\leftarrow x - \omega_\leftarrow t)} \]

is related to \( \Psi_{\text{wave}}: \Psi = \psi_\rightarrow e^{i(k_\rightarrow x - \omega_\rightarrow t)} + \psi_\leftarrow e^{i(k_\leftarrow x - \omega_\leftarrow t)} \)

**Standing Wave Ratio (or Quotient)**

\[ \text{SWR} = (E_\rightarrow - E_\leftarrow) / (E_\rightarrow + E_\leftarrow) = 1/\text{SWQ} \]

**Wave Group (or Phase) Velocity**

\[ u_{\text{GROUP}} / c = (\omega_\rightarrow - \omega_\leftarrow) / (\omega_\rightarrow + \omega_\leftarrow) = c / u_{\text{PHASE}} \]

**1-frequency case:** \( \omega_\rightarrow = 2c, \ k_\rightarrow = 2, \ \omega_\leftarrow = 2c, \ k_\leftarrow = -2 \)

gives: \( u_{\text{GROUP}} = 0 \) and \( u_{\text{PHASE}} = \infty \)

**2-frequency case:** \( \omega_\rightarrow = 4c, \ k_\rightarrow = 4, \ \omega_\leftarrow = 1c, \ k_\leftarrow = -1 \)

gives: \( u_{\text{GROUP}} / c = 3/5 \) and \( u_{\text{PHASE}} / c = 5/3 \)

---

**Staircase Galloping**
1st Quantization: Quantizing phase variables $\omega$ and $k$

Understanding how quantum transitions require "mixed-up" states

Closed cavity vs ring cavity
Quantized $\omega$ and $k$  Counting wave kink numbers

If everything is made of waves then we expect *quantization* of everything because waves only thrive if *integral* numbers $n$ of their "kinks" fit into whatever structure (box, ring, etc.) they’re supposed to live. The numbers $n$ are called *quantum numbers*.

**OK box quantum numbers:**  
$n=1$  
$n=2$  
$n=3$  
$n=4$

(+ integers only)

Some **NOT OK numbers:**  
$n=0.67$  
$n=1.7$  
$n=2.59$  
$n=4$

too fat!  
too thin!  
wrong color again!  
...not tolerated!

misfits...

NOTE: We’re using “false-color” here.

This doesn’t mean a system’s energy can’t vary continuously between “OK” values $E_1, E_2, E_3, E_4,...$
Quantized $\omega$ and $k$

**Counting wave kink numbers**

If everything is made of waves then we expect *quantization* of everything because waves only thrive if *integral* numbers $n$ of their “kinks” fit into whatever structure (box, ring, etc.) they’re supposed to live. The numbers $n$ are called *quantum numbers*.

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*…misfits…*  

**…not tolerated!**

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*This doesn’t mean a system’s energy can’t vary continuously between “OK” values $E_1, E_2, E_3, E_4,...$*  

*In fact its state can be a linear combination of any of the “OK” waves $|E_1>, |E_2>, |E_3>, |E_4>,...$*
1st Quantization: Quantizing phase variables $\omega$ and $k$

Understanding how quantum transitions require “mixed-up” states

Closed cavity vs ring cavity
Quantized $\omega$ and $k$  

Counting wave kink numbers

If everything is made of waves then we expect quantization of everything because waves only thrive if integral numbers $n$ of their “kinks” fit into whatever structure (box, ring, etc.) they’re supposed to live. The numbers $n$ are called quantum numbers.

OK box quantum numbers:  

$n=1$  
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(+ integers only)

Some NOT OK numbers: $n=0.67$

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wrong color again!  
$n=4$

misfits...

...not tolerated!

NOTE: We’re using “false-color” here.

This doesn’t mean a system’s energy can’t vary continuously between “OK” values $E_1$, $E_2$, $E_3$, $E_4$, ...

In fact its state can be a linear combination of any of the “OK” waves $|E_1>$, $|E_2>$, $|E_3>$, $|E_4>$, ...

That’s the only way you get any light in or out of the system to “see” it.

$$|E_4>$$

$$|E_3>$$

$$|E_2>$$

$$|E_1>$$

frequency $\hbar \omega_{32} = E_3 - E_2$

frequency $\hbar \omega_{21} = E_2 - E_1$
Quantized \( \omega \) and \( k \) Counting wave kink numbers

If everything is made of waves then we expect quantization of everything because waves only thrive if integral numbers \( n \) of their “kinks” fit into whatever structure (box, ring, etc.) they’re supposed to live. The numbers \( n \) are called quantum numbers.

- **OK box quantum numbers:** \( n=1 \) \( n=2 \) \( n=3 \) \( n=4 \)

(\(+\) integers only)

Some NOT OK numbers: \( n=0.67 \) (too fat!) \( n=1.7 \) (too thin!) \( n=2.59 \) (wrong color again!) \( n=4 \) (misfits...)

NOTE: We’re using “false-color” here.

This doesn’t mean a system’s energy can’t vary continuously between “OK” values \( E_1, E_2, E_3, E_4, \ldots \)

In fact its state can be a linear combination of any of the “OK” waves \( |E_1\rangle, |E_2\rangle, |E_3\rangle, |E_4\rangle, \ldots \)

That’s the only way you get any light in or out of the system to “see” it.

\[
|E_4\rangle
\]

\[
|E_3\rangle
\]

\[
|E_2\rangle
\]

\[
|E_1\rangle
\]

These eigenstates are the only ways the system can “play dead”...

... “sleep with the fishes”...
Consider two lowest $E$-states by themselves

$|E_2\rangle$

$|E_1\rangle$
Consider two lowest E-states by themselves in time

\[ e^{-i\omega_1 t} |E_1\rangle \]

\[ e^{-i\omega_2 t} |E_2\rangle \]
Consider two lowest E-states by themselves in time

\[ |E_1\rangle \]

\[ |E_2\rangle \]

Now combine (add) them

\[ (|E_1\rangle + |E_2\rangle) / \sqrt{2} \]
Consider two lowest $E$-states by themselves in time

$$e^{-i\omega_1 t} |E_1\rangle$$

$$e^{-i\omega_2 t} |E_2\rangle$$

Now combine (add) them and let time roll!

$$\frac{(e^{-i\omega_1 t} |E_1\rangle + e^{-i\omega_2 t} |E_2\rangle)}{\sqrt{2}}$$
Consider two lowest E-states by themselves in time

\[ e^{-i\omega_1 t} |E_1\rangle \]

\[ e^{-i\omega_2 t} |E_2\rangle \]

Now combine (add) them and let time roll!

\[ \left( e^{-i\omega_1 t} |E_1\rangle + e^{-i\omega_2 t} |E_2\rangle \right) / \sqrt{2} \]

\[ (|E_1\rangle + |E_2\rangle) / \sqrt{2} \]
Consider two lowest $E$-states by themselves in time $e^{-i\omega_1 t} |E_1\rangle$

Now combine (add) them and let time roll!

$\left( e^{-i\omega_1 t} |E_1\rangle + e^{-i\omega_2 t} |E_2\rangle \right) / \sqrt{2}$
1st Quantization: Quantizing phase variables $\omega$ and $k$

Understanding how quantum transitions require “mixed-up” states

Closed cavity vs ring cavity
Quantized $\omega$ and $k$  

*Counting wave kink numbers*

If everything is made of waves then we expect *quantization* of everything because waves only thrive if *integral* numbers $n$ of their “kinks” fit into whatever structure (box, ring, etc.) they’re supposed to live. The numbers $n$ are called *quantum numbers*.

**OK box quantum numbers:**

- $n=1$
- $n=2$
- $n=3$
- $n=4$

*(+ integers only)*

Some **NOT OK numbers:**

- $n=0.67$  
  too fat!
- $n=1.7$  
  too thin!
- $n=2.59$  
  wrong color again!
- $n=4$  
  misfits...

...not tolerated!

**NOTE:** We’re using “false-color” here.

Rings tolerate a **zero** (kinkless) quantum wave but require ±*integral* wave number.

**OK ring quantum numbers:**

- $m=0$
- $m=\pm 1$
- $m=\pm 2$
- $m=3$

*(± integral number of wavelengths)*

Bohr’s models of *atomic spectra* (1913-1923) are beginnings of *quantum wave mechanics* built on *Planck-Einstein* (1900-1905) relation $E=\hbar \nu$. *DeBroglie* relation $p=\hbar /\lambda$ comes around 1923.
2nd Quantization: Quantizing amplitudes ("photons", "vibrons", and "what-ever-ons")
Introducing coherent states (What lasers use)
Analogy with \((\omega,k)\) wave packets
Wave coordinates need coherence
Quantized Amplitude Counting “photon” number

Planck’s relation $E = N\hbar\nu$ began as a tenative axiom to explain low-T light. Then he tried to disavow it! Einstein picked it up in his 1905 paper. Since then its use has grown enormously and continues to amaze, amuse (or bewilder) all who study it. A current view is that it represents the quantization of optical field amplitude. We picture this below as $N$-photon wave states for each box-mode of $m$ wave kinks.

Quantum field definitions have been called “2nd quantization” or “wave-waves”

NOTE: We’re using “false-color” here.
Quantized **Amplitude** Counting “photon” number

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Quantum field definitions have been called “2nd quantization” or “wave-waves”

NOTE: We’re using “false-color” here.

These are the fundamental “zero-point” or “vacuum” levels

These are the 1st excited or fundamental transition levels
Quantized **Amplitude** Counting “photon” number

Planck’s relation $E = N\hbar\omega$ began as a tenative axiom to explain low-T light. Then he tried to disavow it! Einstein picked it up in his 1905 paper. Since then its use has grown enormously and continues to amaze, amuse (or bewilder) all who study it. A current view is that it represents the quantization of optical field amplitude. We picture this below as $N$-photon wave states for each box-mode of $m$ wave kinks.
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Quantum field definitions have been called “2nd quantization” or “wave-waves”

NOTE: We’re using “false-color” here.
**Quantized Amplitude Counting “photon” number**

Planck’s relation $E=nh\nu$ began as a tenative axiom to explain low-T light. Then he tried to disavow it! Einstein picked it up in his 1905 paper. Since then its use has grown enormously and continues to amaze, amuse (or bewilder) all who study it. A current view is that it represents the quantization of optical field amplitude. We picture this below as $N$-photon wave states for each box-mode of $m$ wave kinks.

*Quantum field definitions have been called “2nd quantization” or “wave-waves”*

*NOTE: We’re using “false-color” here.*

---

**Quantized Wavenumber (“kink” or momentum number)**

1. $N_i=4$: 2nd excited levels
   - 4 red photons
2. $N_i=3$: 2nd excited levels
   - 3 red photons
3. $N_i=2$: 2nd excited levels
   - 2 red photons
4. $N_i=1$: 1st excited or fundamental transition levels
   - 1 red photon
5. $N_i=0$: Fundamental “zero-point” or “vacuum” levels
   - 0 red photons

---

These are the 2nd excited levels

These are the 1st excited or fundamental transition levels

These are the fundamental “zero-point” or “vacuum” levels
Quantum numbers $N$ of field or $n$, $m$,.. of modes are invariants and not changed by boosting velocity. Each mode fundamental frequency $\nu = n \nu_1$ and its $N$-photon multiples belong to invariant hyperbolas.

Boosted observers see distorted frequencies and lengths, but will agree on the numbers $n$ and $N$ of mode nodes and photons.

This is how light waves can “fake” some of the properties of classical “things” such as invariance or object permanence.

It takes at least TWO CW’s to achieve such invariance. One CW is not enough and cannot have non-zero invariant $N$. Invariance is an interference effect that needs at least two-to-tango!
2nd Quantization: Quantizing amplitudes ("photons", "vibrons", and "what-ever-ons")
Introducing coherent states (What lasers use)
Analogy with \((\omega, k)\) wave packets
Wave coordinates need coherence
Coherent States: Oscillator Amplitude Packets analogous to Wave Packets

We saw how adding CW’s (Continuous Waves m=1,2,3...) can make PW (Pulse Wave) or WP (Wave Packet) that is more like a classical “thing” with more localization in space x and time t.

Analogy:
Adding photons (Quantized amplitude N=0,1,2...) can make a CS (Coherent State) or OAP (Oscillator Amplitude Packet) that is more like a classical wave oscillation with more localization in field amplitude.

Pure photon states have localized (certain) N but delocalized (uncertain) amplitude and phase. OAP states have delocalized (uncertain) N but more localized (certain) amplitude and phase.
**Coherent States (contd.) Spacetime wave grid is impossible without coherent states**

Pure photon number $N$-states would make useless spacetime coordinates.

Total uncertainty of amplitude and phase makes the count pattern a wash. To see grids *some N-uncertainty is necessary!*

Coherent-$\alpha$-states are defined by continuous amplitude-packet parameter $\alpha$ whose square is average photon number $\overline{N}=|\alpha|^2$. Coherent-states make better spacetime coordinates for larger $\overline{N}=|\alpha|^2$.

Coherent-state uncertainty in photon number (and mass) varies with amplitude parameter $\Delta N \sim \alpha \sim \sqrt{N}$ so a coherent state with $\overline{N}=|\alpha|^2 = 10^6$ only has a 1-in-1000 uncertainty $\Delta N \sim \alpha \sim \sqrt{N}=1000$. 

www.uark.edu/ua/pirelli/php/coherent_vs_photon_2.php
Relativistic effects on charge, current, and Maxwell Fields

*Current density changes by Lorentz asynchrony*

*Magnetic B-field is relativistic $\sinh \rho$ 1st order-effect*
Relativistic effects on charge, current, and Maxwell Fields

Observer velocity is zero relative to (+) line of charge
wire appears neutral

(+), Charge fixed
(-), Charge moving to right (Negative current density)
(+), Charge density is Equal to the (-) Charge density
Relativistic effects on charge, current, and Maxwell Fields

Observer velocity is zero relative to (+) line of charge

(+ ) Charge fixed (-) Charge moving to right (Negative current density \( \vec{j}(x,t) \))

(+ ) Charge density is Equal to the (-) Charge density \( \rho(x,t)=0 \)
Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz asynchrony

Asynchrony due to off-diagonal $\begin{pmatrix} \sinh \rho \\ \cosh \rho \end{pmatrix}$ (a 1st-order effect)

in Lorentz transform:

$\begin{pmatrix} \cosh \rho & \sinh \rho \\ \sinh \rho & \cosh \rho \end{pmatrix} \sim \begin{pmatrix} 1 & \frac{v}{c} \\ \frac{v}{c} & 1 \end{pmatrix}$

Observer has $q^{[\text{+}]}$ “test-charge”

Observer velocity is $+v$ relative to $(\text{+})$ line of charge

wire appears positive $(\text{+})$
(repulsive to observer $q^{[\text{+}]}$)

(+) Charge fixed  (-) Charge moving to right (Negative current density $\mathbf{j}(x,t)$)
(+) Charge density is Greater than (-) Charge density (Positive $\rho(x,t) > 0$)
Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz *asynchrony*

Asynchrony due to off-diagonal $\sinh \rho$ (a 1st-order effect)

In Lorentz transform:

$$\begin{pmatrix} \cosh \rho & \sinh \rho \\ \sinh \rho & \cosh \rho \end{pmatrix} \sim \begin{pmatrix} 1 & v/c \\ v/c & 1 \end{pmatrix}$$

Observer has $q[+]$

“test-charge”

Observer velocity is $+v$ relative to (+) line of charge

(+ Charge fixed (-) Charge moving to right (Negative current density $j(x,t)$) (Positive $\rho(x,t) > 0$)

(+ Charge density is Greater than (-) Charge density

wire appears positive (+) (repulsive to observer $q[+]$)

Thursday, March 6, 2014
Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz asynchrony

Asynchrony due to off-diagonal $\begin{pmatrix} \sinh \rho & \sinh \rho \\ \sinh \rho & \cosh \rho \end{pmatrix}$ (a 1st-order effect)

in Lorentz transform:

$\begin{pmatrix} \cosh \rho & \sinh \rho \\ \sinh \rho & \cosh \rho \end{pmatrix} \sim \begin{pmatrix} 1 & v/c \\ v/c & 1 \end{pmatrix}$

observer has $q^{[+]}$

“test-charge”

Observer velocity is $-v$ relative to (+) line of charge

wire appears negative (-) (attractive to observer $q^{[+]}$)

(+) Charge fixed (-) Charge moving to right (Negative current density $\vec{J}(x,t)$)

(+) Charge density is Less than (-) Charge density (Negative $\rho(x,t) < 0$)

(+) Charge fixed (-) Charge moving to right (Negative current density $\vec{J}(x,t)$)

(+) Charge density is Less than (-) Charge density (Negative $\rho(x,t) < 0$)
Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz asynchrony

Asynchrony due to off-diagonal \( \sinh \rho \) (a 1\(^{st}\)-order effect)

in Lorentz transform:

\[
\begin{pmatrix}
\cosh \rho & \sinh \rho \\
\sinh \rho & \cosh \rho \\
\end{pmatrix} \sim \begin{pmatrix} 1 & v/c \\
v/c & 1 \end{pmatrix}
\]

observer has 

\( q^{[+]} \)

“test-charge”

Observer velocity is \(-v\) relative to (\(+\) line of charge

wire appears negative (-) (attractive to observer \( q^{[+]} \))

\((+)\) Charge fixed \((-)\) Charge moving to right \((Negative \ current\ density \ \vec{j}(x,t))\)

\((+)\) Charge density is \textit{Less} than \((-)\) Charge density \((Negative \ \rho(x,t)<0)\)
Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz asynchrony

Magnetic B-field is relativistic $\sinh \rho$ 1st order-effect
Magnetic B-field is relativistic \( \sinh \rho \) 1st order-effect

\[
\frac{\rho(-)}{\rho(+) = \frac{(+) \text{ charge separation}}{(-) \text{ charge separation}} = \frac{x(+) + x(-)}{x(-)}
\]

\[
\frac{\rho(-)}{\rho(+) = \frac{x(+) + 1 = \frac{uv}{c^2} + 1}{x(-)}}
\]

\[
\rho(+) - \rho(-) = \rho(+) \left( 1 - \frac{\rho(-)}{\rho(+) = \frac{uv}{c^2}} \right) = -\frac{uv}{c^2} \rho(+)
\]

Unit square: \( \frac{u/c}{1} = x(+) / y \)
\( \frac{v/c}{1} = y / x(-) \)
The electric force field $E$ of a charged line varies inversely with radius. The Gauss formula for force in mks units:

$$F = qE = q \left[ \frac{1}{4\pi\varepsilon_0} \frac{2\rho}{r} \right], \text{ where: } \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \frac{N \cdot m^2}{Coul}.$$  

$$F = qE = q \left[ \frac{1}{4\pi\varepsilon_0} \frac{2}{r} \left( -\frac{uv}{c^2} \rho(+) \right) \right] = -2 \frac{qv \rho(+)u}{4\pi\varepsilon_0 c^2 r} = -2 \times 10^{-7} \frac{I_q I_P}{r}$$

$I_q > 0$ charge up there. Yuk!

$I_q < 0$ charge up there. Yum!

$\frac{1}{4\pi\varepsilon_0} = 9 \cdot 10^9$
$c^2 = 9 \cdot 10^{-16}$
$\frac{1}{(4\pi\varepsilon_0 c^2)} = 10^{-7}$
The electric force field $\mathbf{E}$ of a charged line varies inversely with radius. The Gauss formula for force in mks units:

$$F = qE = q\left[\frac{1}{4\pi\varepsilon_0} \frac{2\rho}{r}\right]$$

where:

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N} \cdot \text{m}^2 \text{ Coul.}$$

$$F = qE = q\left[\frac{1}{4\pi\varepsilon_0} \frac{2}{r} \left(-\frac{u\nu}{c^2} \rho(+)\right)\right] = -2q\frac{\rho(+)u}{4\pi\varepsilon_0 c^2 r} = -2 \times 10^{-7} \frac{I_q I_{\rho}}{r}$$

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The magnetic B-field is relativistic $\sinh \rho$ 1st order-effect.

$1/4\pi\varepsilon_0 = 9 \cdot 10^9$

$c^2 = 9 \cdot 10^{-16}$

$1/(4\pi\varepsilon_0 c^2) = 10^{-7}$

I see excess (+) charge up there. Yuk!

I see excess (-) charge up there. Yum!
Using 4-vectors to EL Transform (charge-current) = (cρ, j)

\[
\begin{pmatrix}
  c\rho' \\
  j_x' \\
  j_y' \\
  j_z'
\end{pmatrix} =
\begin{pmatrix}
  \cosh \rho & \sinh \rho & \cdot & \cdot \\
  \sinh \rho & \cosh \rho & \cdot & \cdot \\
  \cdot & \cdot & 1 & \cdot \\
  \cdot & \cdot & \cdot & 1
\end{pmatrix}
\begin{pmatrix}
  c\rho \\
  j_x \\
  j_y \\
  j_z
\end{pmatrix}
\]