AMOP Lectures 8.0
Tue 2.25 2014

Relativity of transverse waves and 4-vectors

(Ch. 2-5 of CMwBang-Unit 8 Ch. 6 of QTforCA Unit 2)

Reviewing "Relawavity" geometry

Reviewing the stellar aberration angle σ vs. rapidity ρ

Pattern recognition: "Occam's Sword"

Introducing per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z)=(\omega,ck_x,ck_y,ck_z)$ transformation

More details of Lorentz boost of North-South-East-West plane-wave 4-vectors $(\omega_0,\omega_x,\omega_y,\omega_z)$

Thales-like construction of Lorentz boost in 2D and 3D

The spectral ellipsoid

Combination and interference of 4-vector plane waves (Idealized polarization case)

Combination group and phase waves define 4D Minkowski coordinates

Combination group and phase waves define wave guide dynamics

Waveguide dispersion and geometry

1^{st}-quantized cavity modes

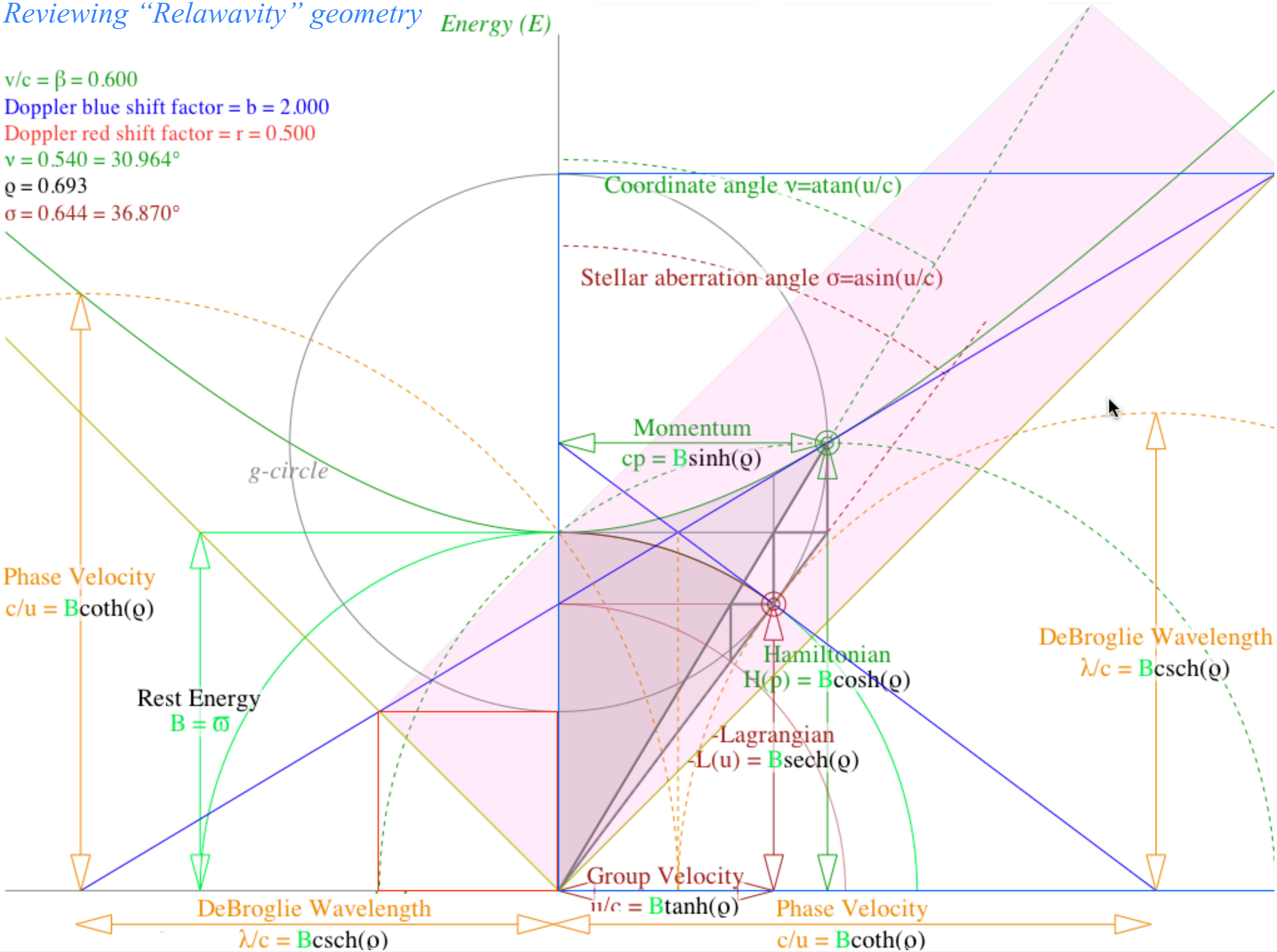
(And introducing 2^{nd}-quantized cavity modes)

Lorentz symmetry effects

How it makes momentum and energy be conserved

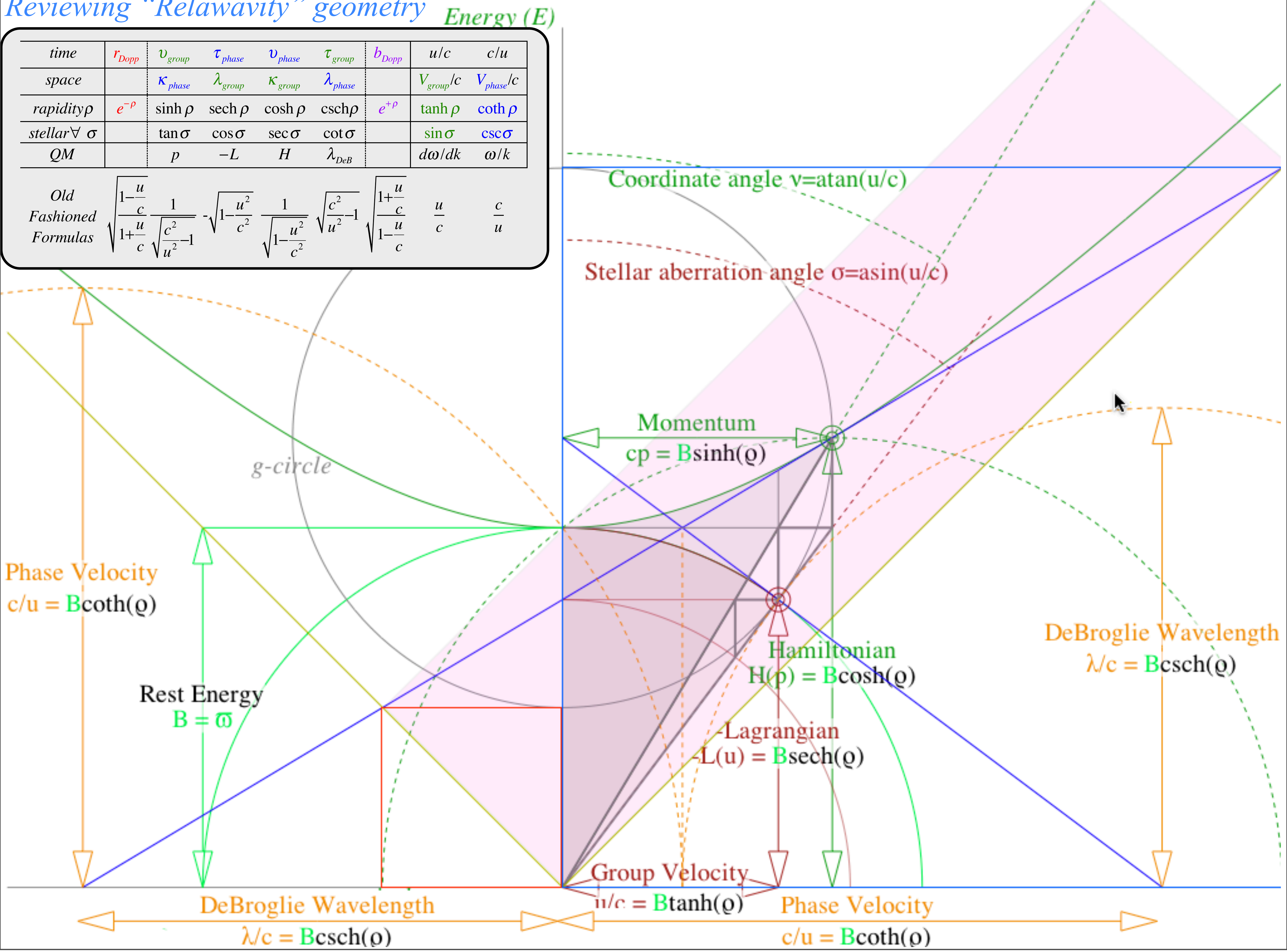
Reviewing "Relawavity" geometry
Reviewing the stellar aberration angle σ *vs. rapidity* ρ
Pattern recognition: "Occam's Sword"

Reviewing "Relawavity" geometry



Reviewing "Relawavity" geometry

time	r_{Dopp}	υ_{group}	τ_{phase}	υ_{phase}	τ_{group}	b_{Dopp}	u/c	c/u
space		κ_{phase}	λ_{group}	κ_{group}	λ_{phase}		V_{group}/c	V_{phase}/c
rapidity ρ	$e^{-\rho}$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$e^{+\rho}$	$\tanh\rho$	$\coth\rho$
stellar∀ σ		$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$		$\sin\sigma$	$\csc\sigma$
QM		p	$-L$	H	λ_{DeB}		$d\omega/dk$	ω/k
Old Fashioned Formulas	$\sqrt{\frac{1-\frac{u}{c}}{1+\frac{u}{c}}}$	$\frac{1}{\sqrt{\frac{c^2}{u^2}-1}}$	$\sqrt{1-\frac{u^2}{c^2}}$	$\frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$	$\sqrt{\frac{c^2}{u^2}-1}$	$\sqrt{\frac{1+\frac{u}{c}}{1-\frac{u}{c}}}$	$\frac{u}{c}$	$\frac{c}{u}$



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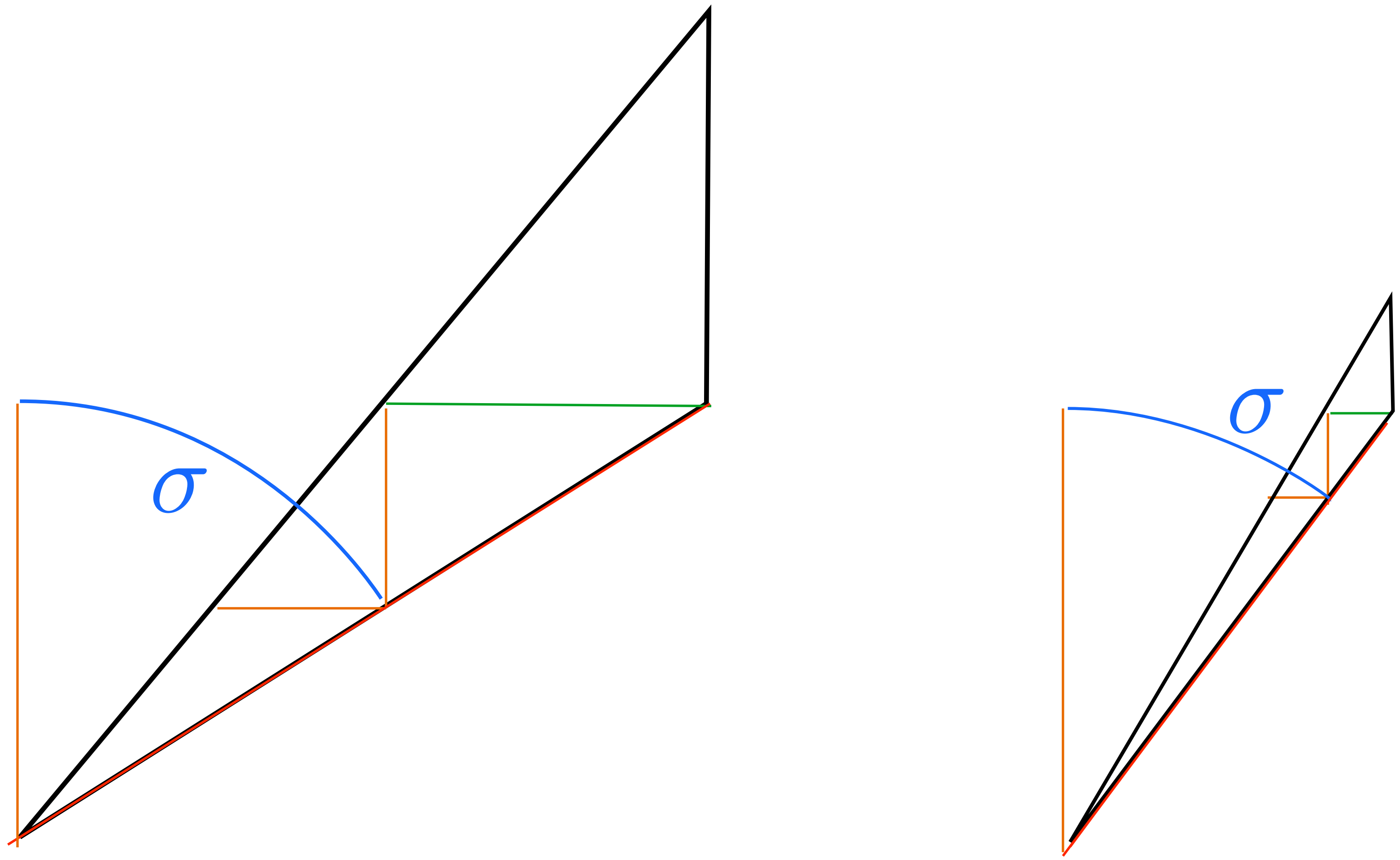


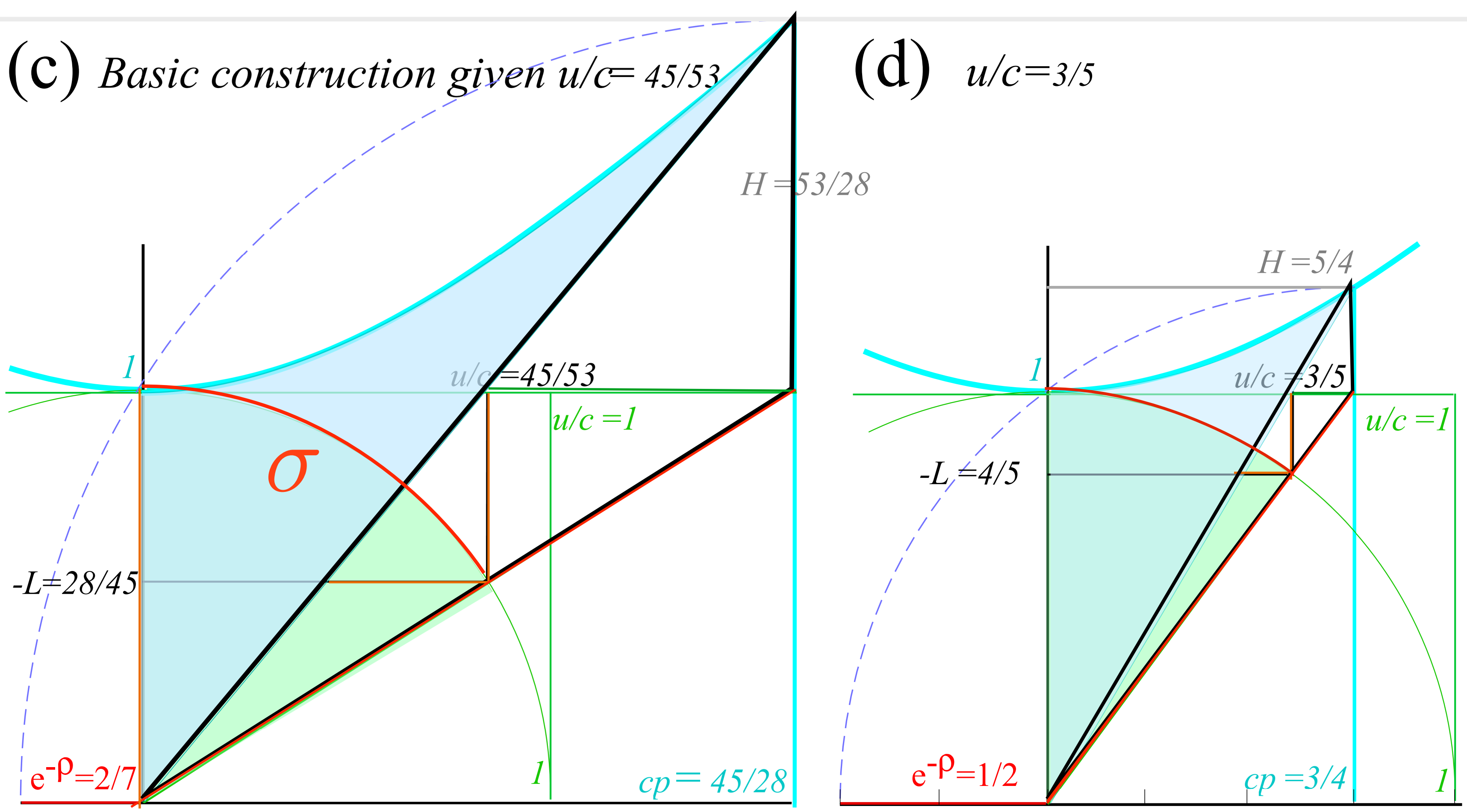
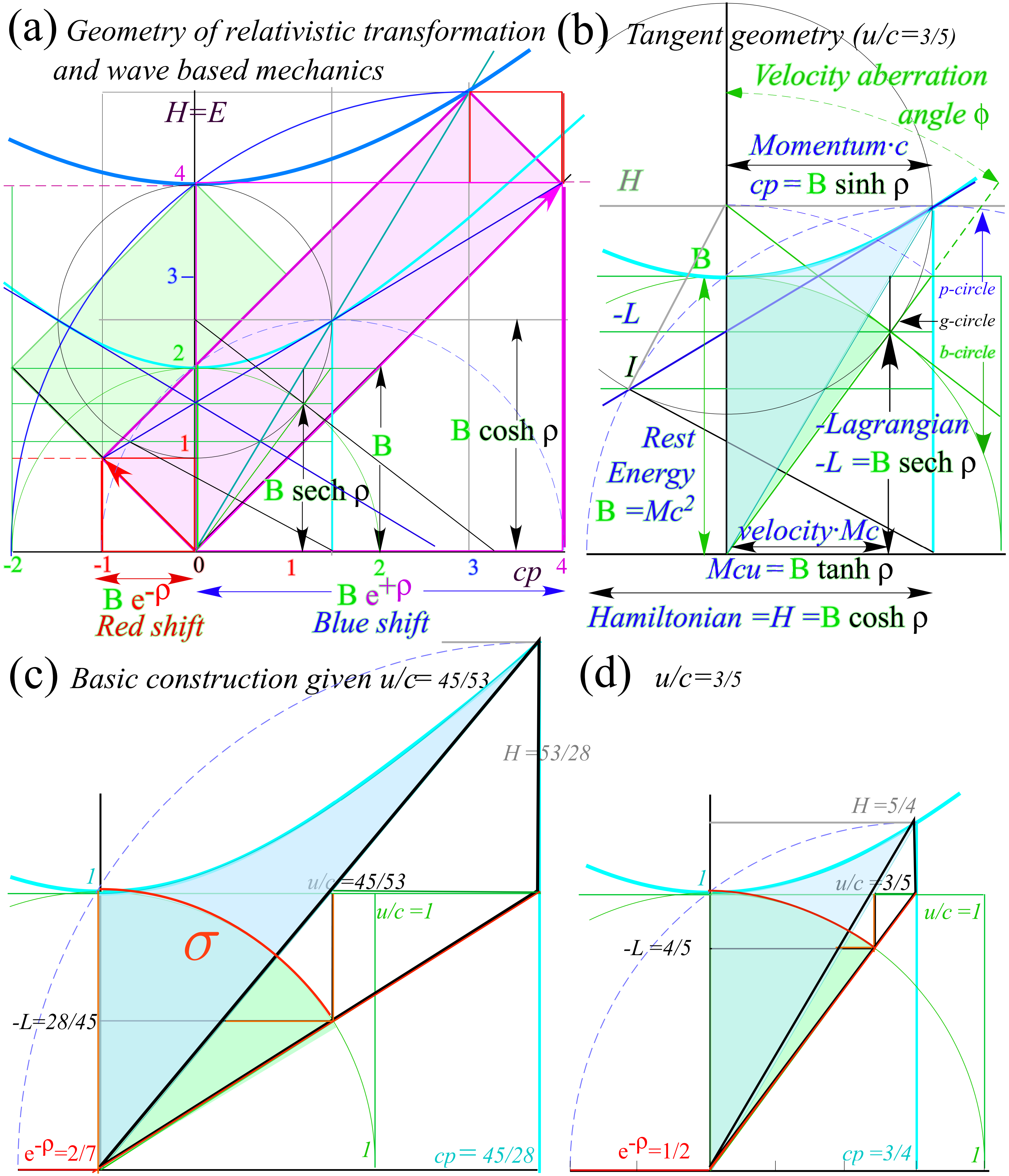


Fig. 5.5
Relativistic wave mechanics geometry.
(a) Overview.

(b-d) Details of contacting tangents.

from:Fig. 8.5.5
QTforCA
Unit 8 Ch.5



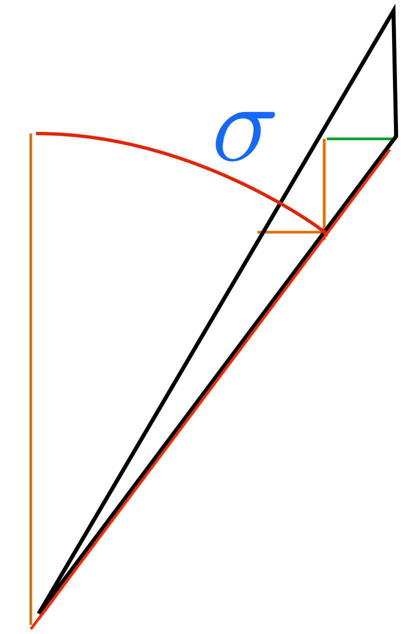


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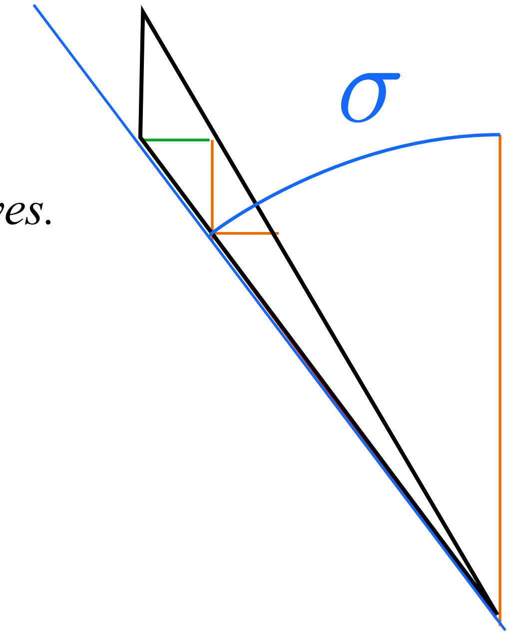
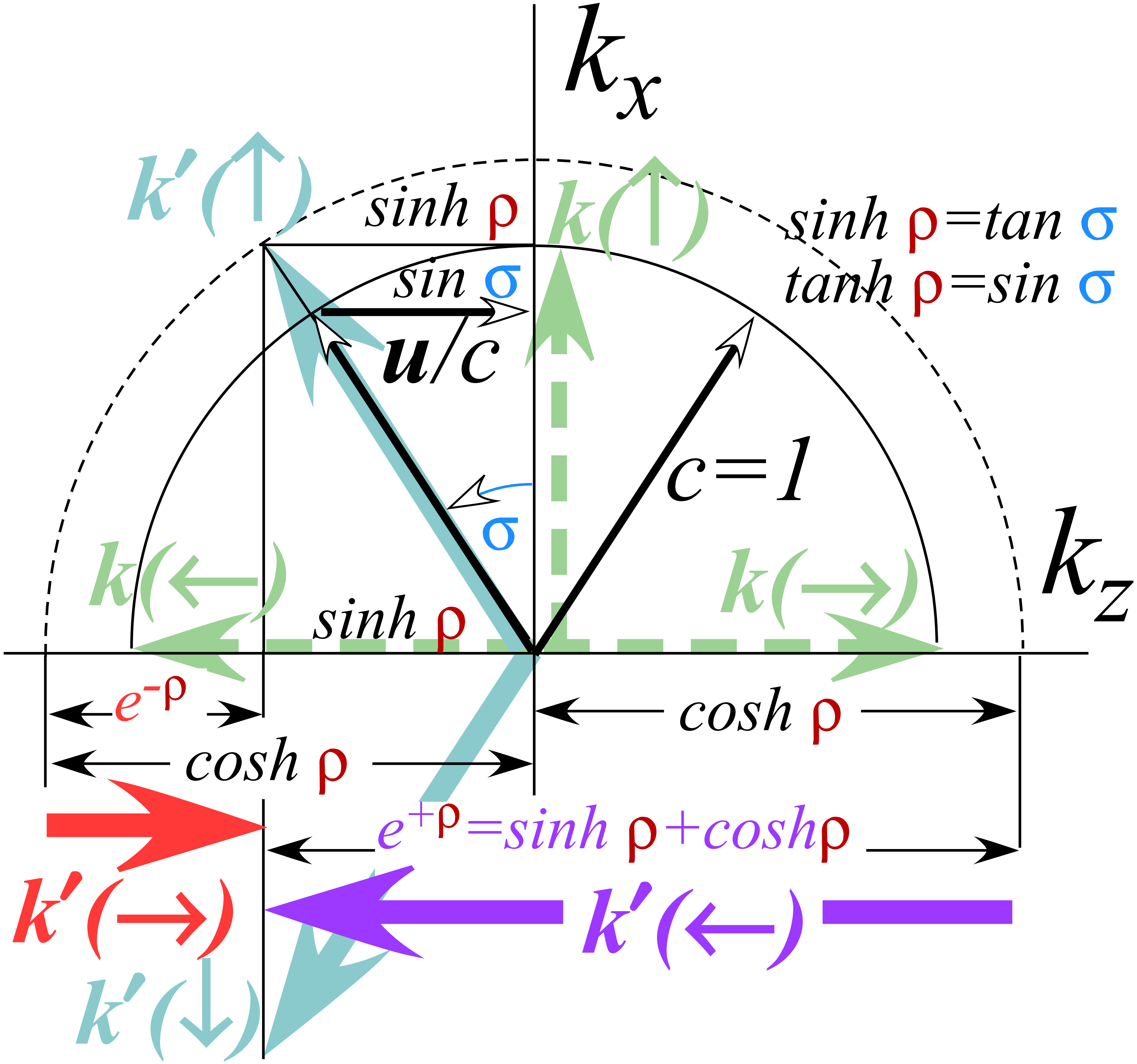
Fig. 5.10 CW cosmic speedometer.

Geometry of boosted counter-propagating waves.





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QTforCA
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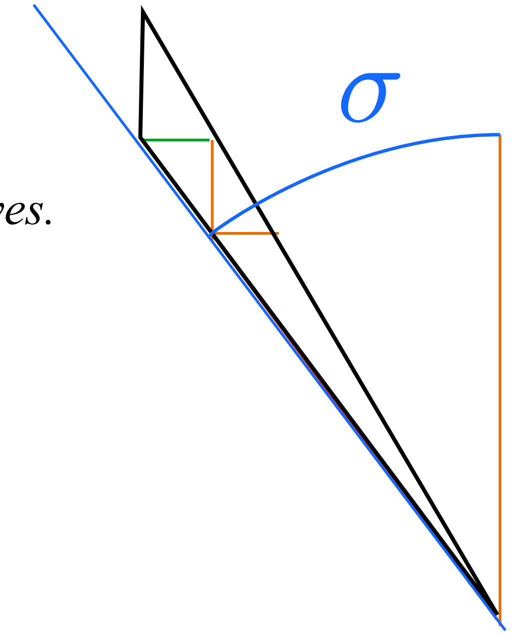

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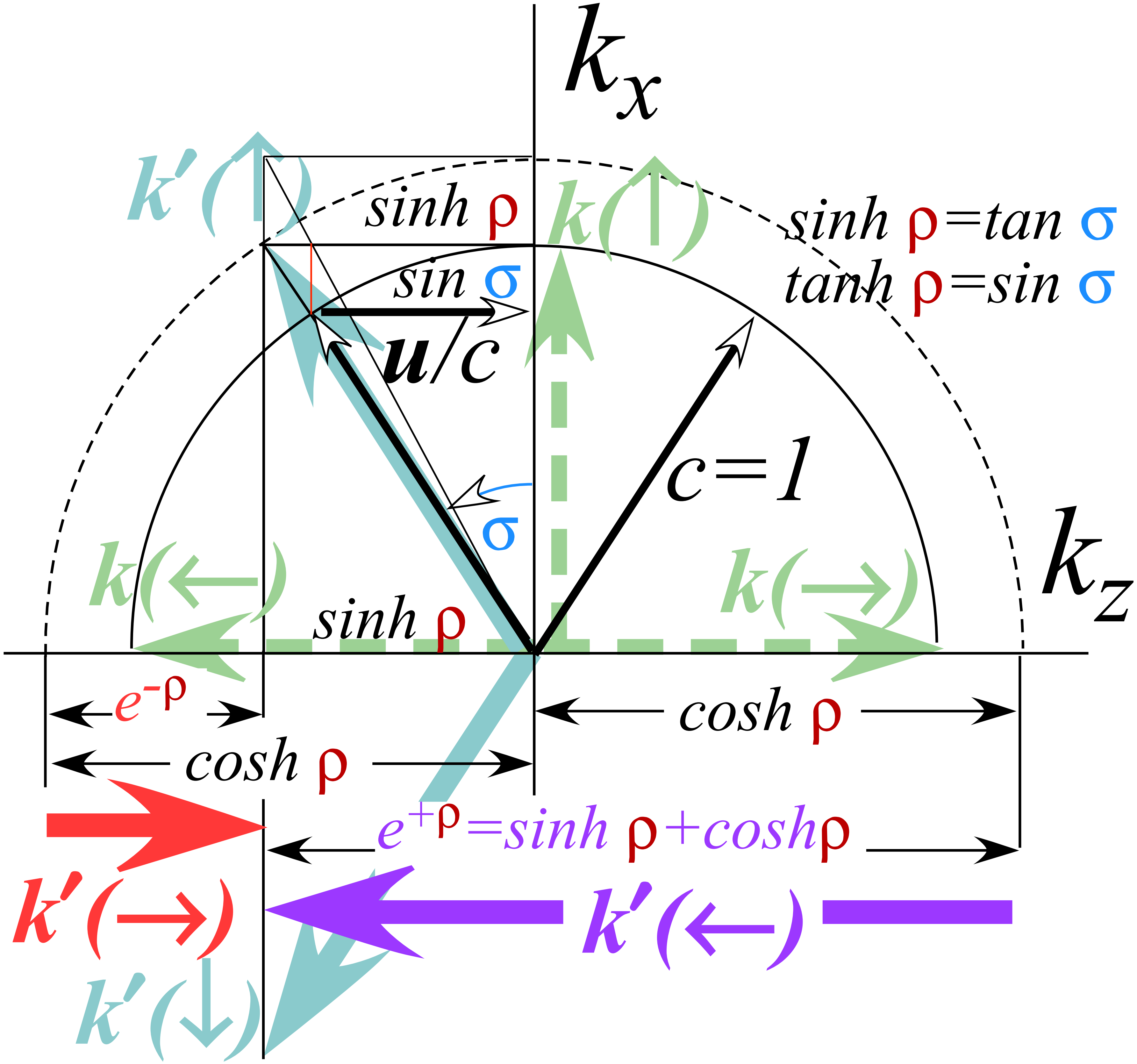

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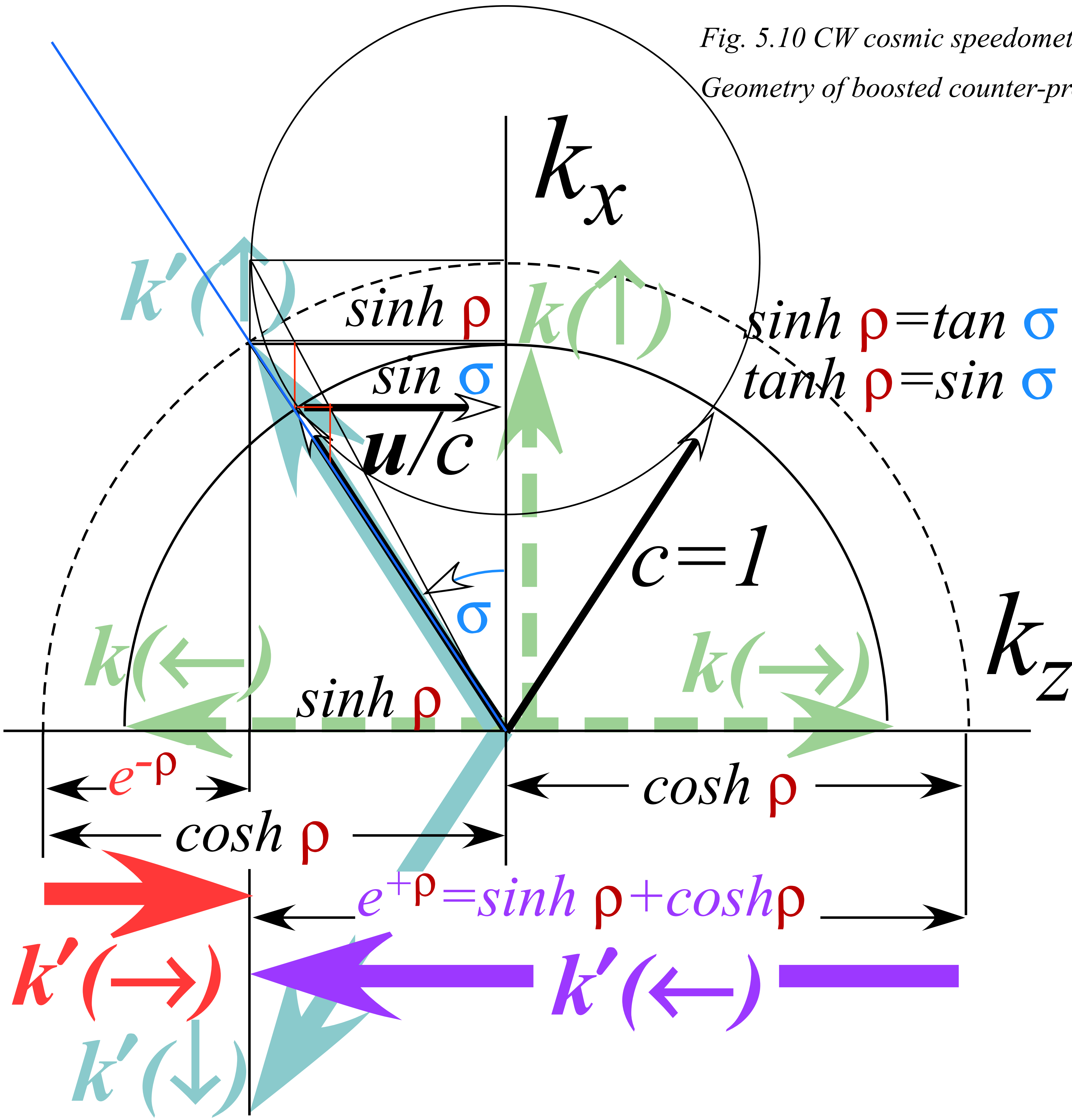

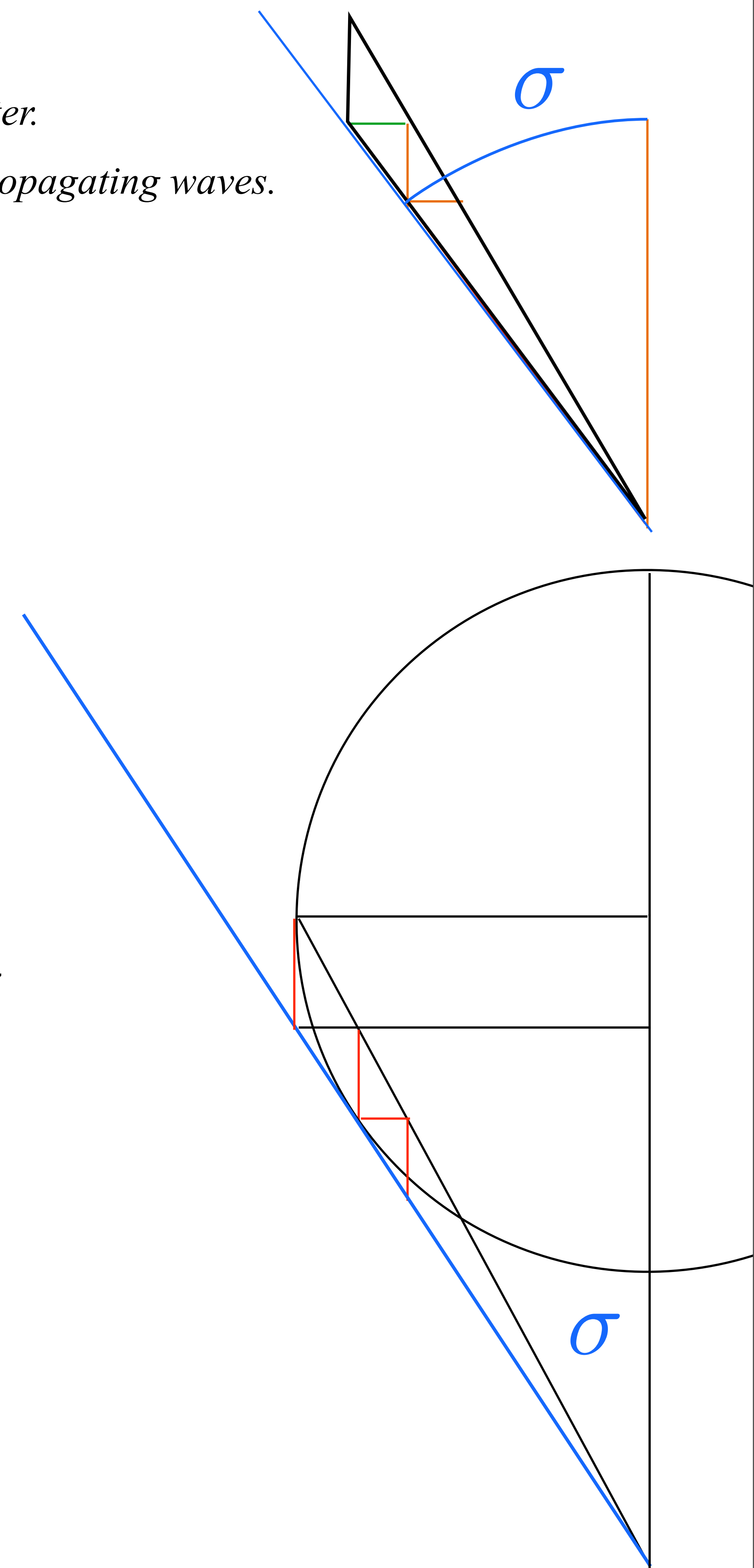

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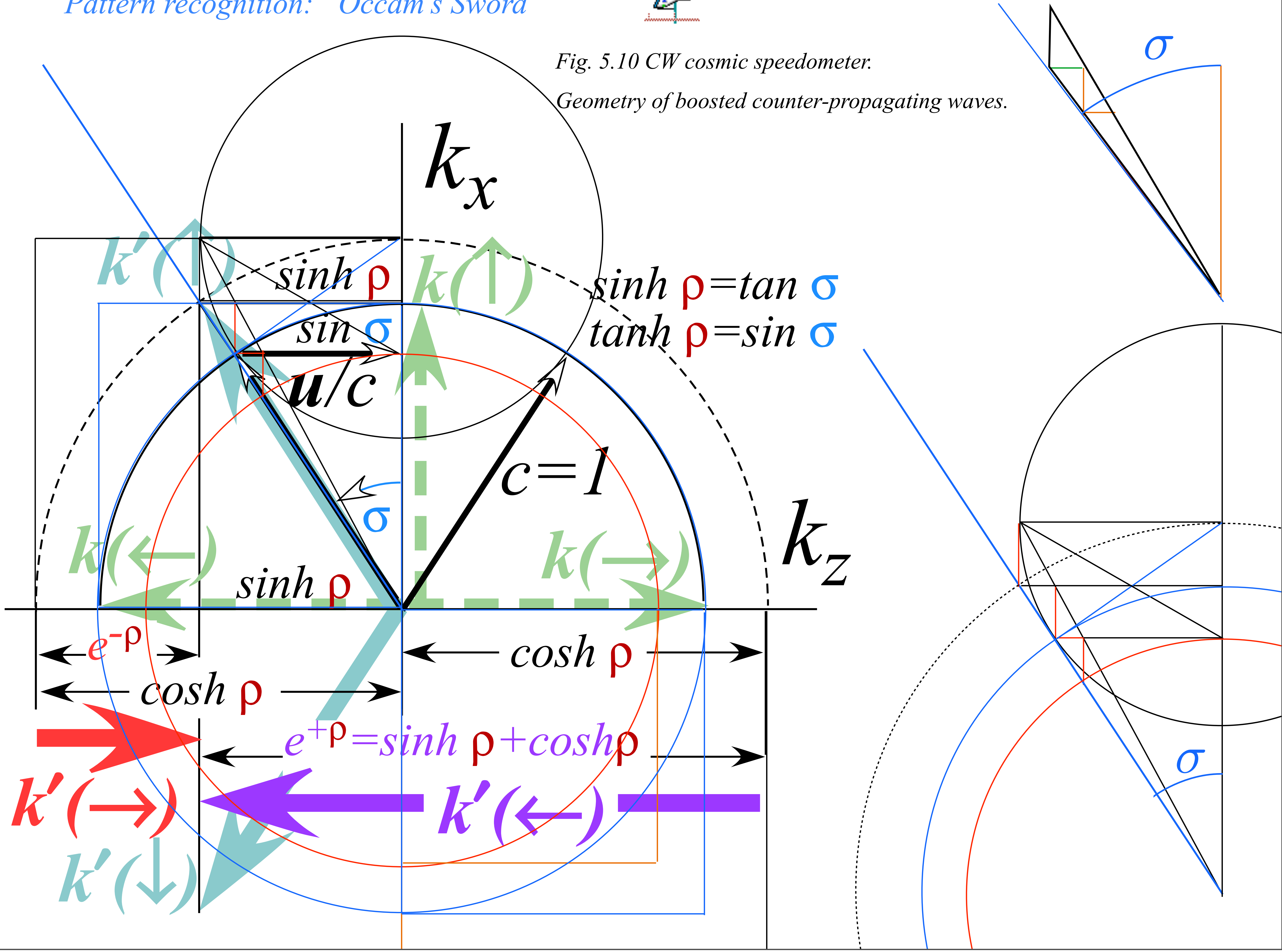

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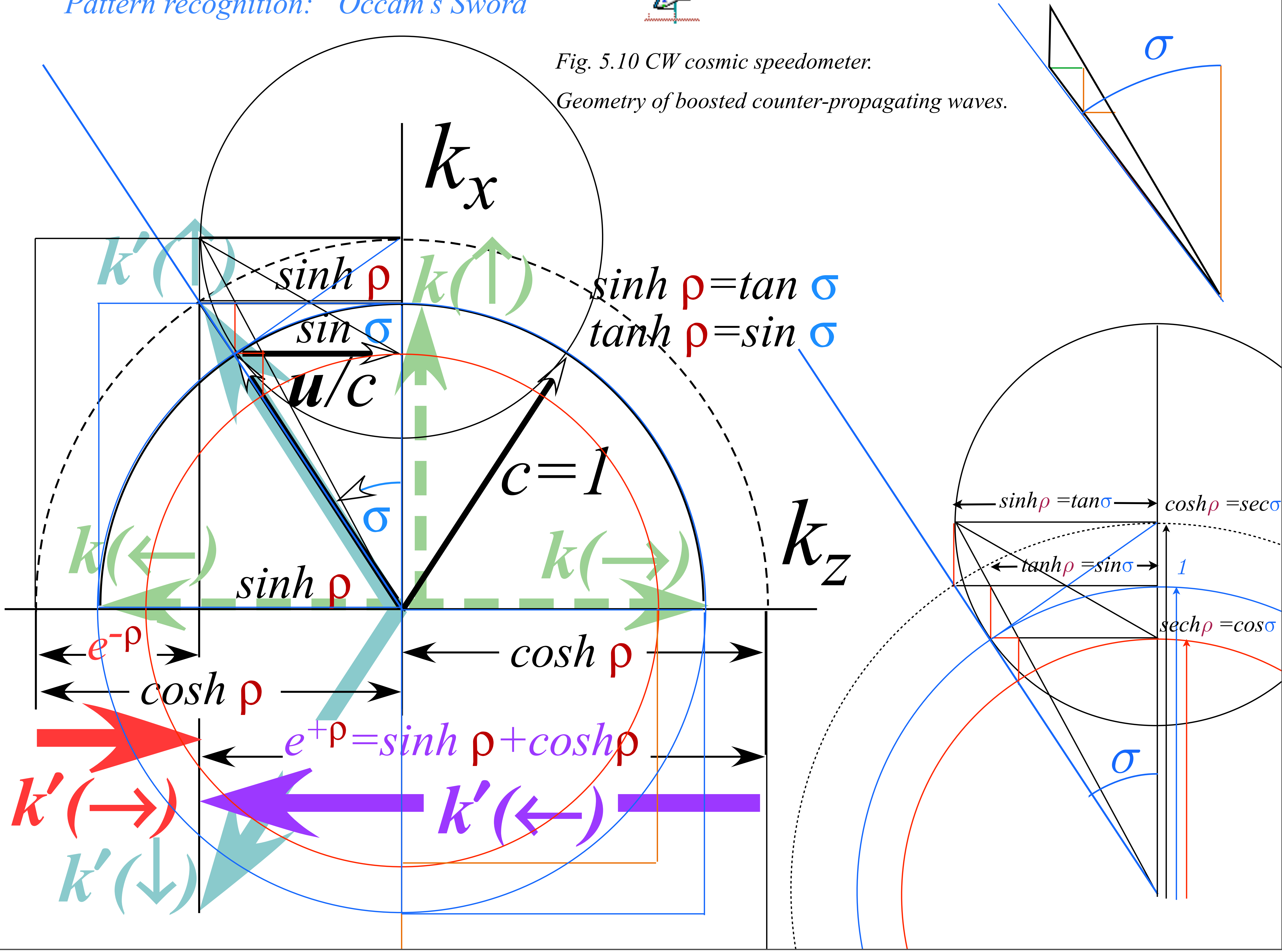

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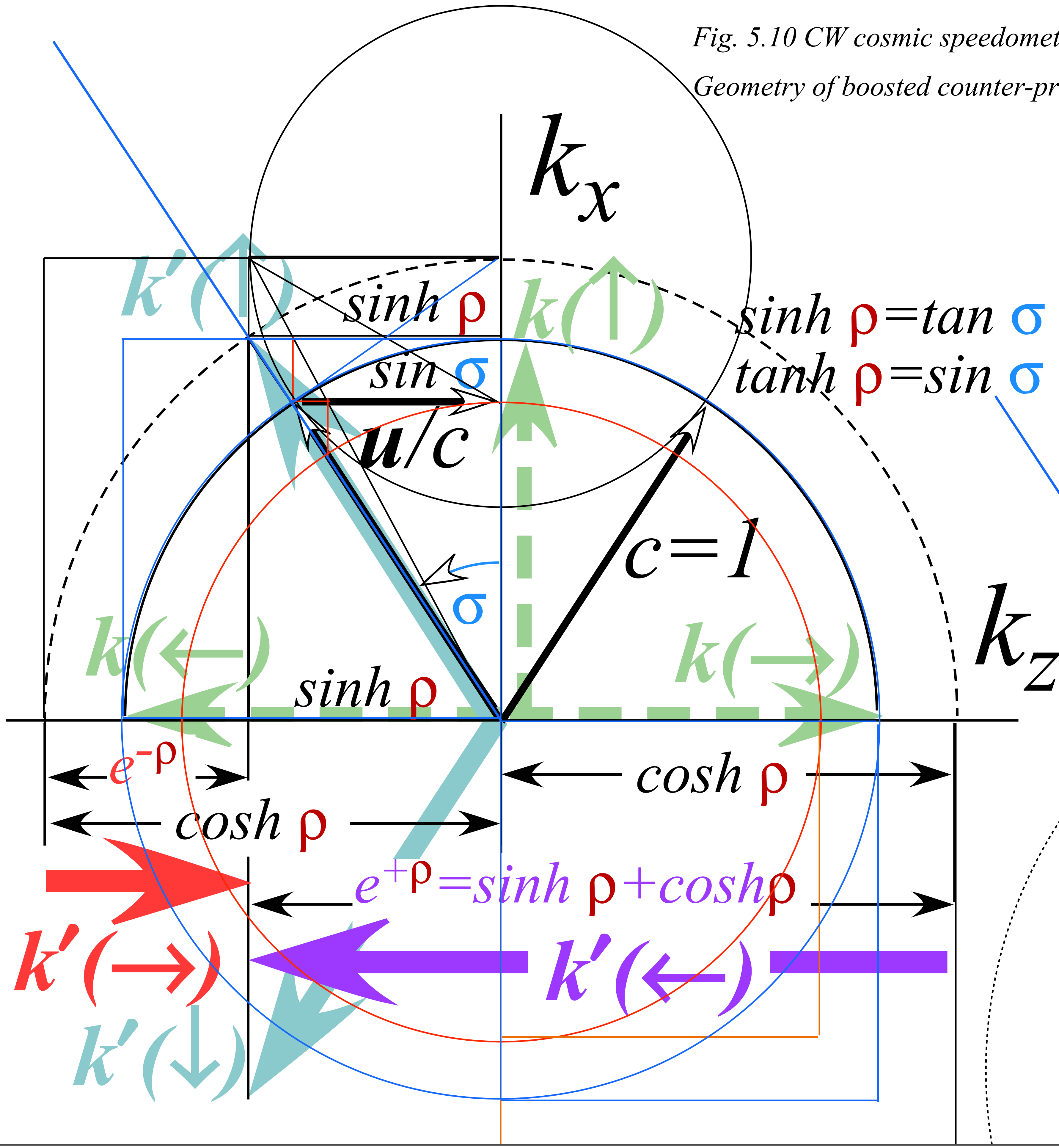
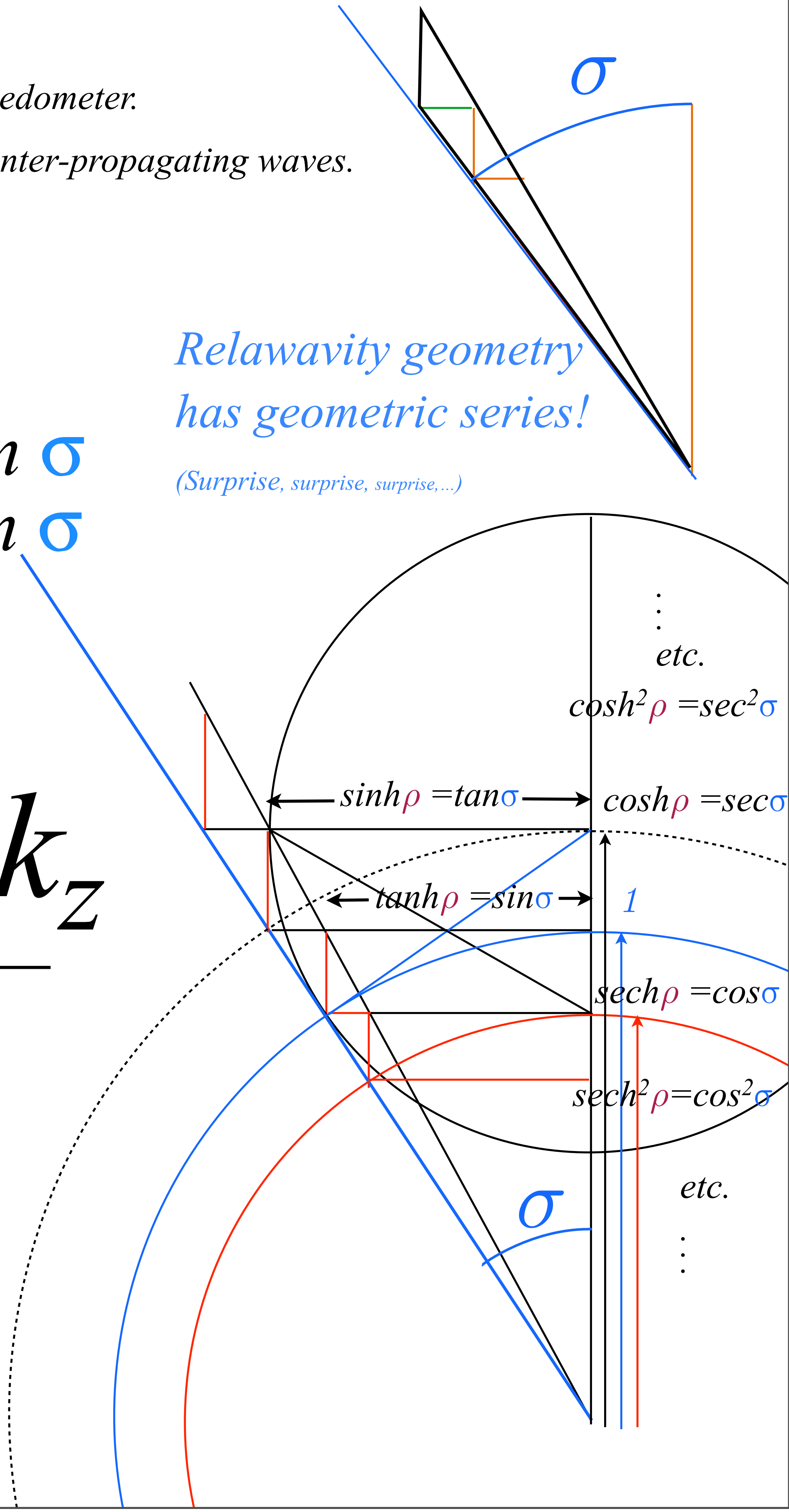


Relawavity geometry has geometric series!

(Surprise, surprise, surprise,...)



Introducing per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) = (\omega, ck_x, ck_y, ck_z)$ transformation

More details of Lorentz boost of North-South-East-West plane-wave 4-vectors $(\omega_0,\omega_x,\omega_y,\omega_z)$

Thales-like construction of Lorentz boost in 2D and 3D

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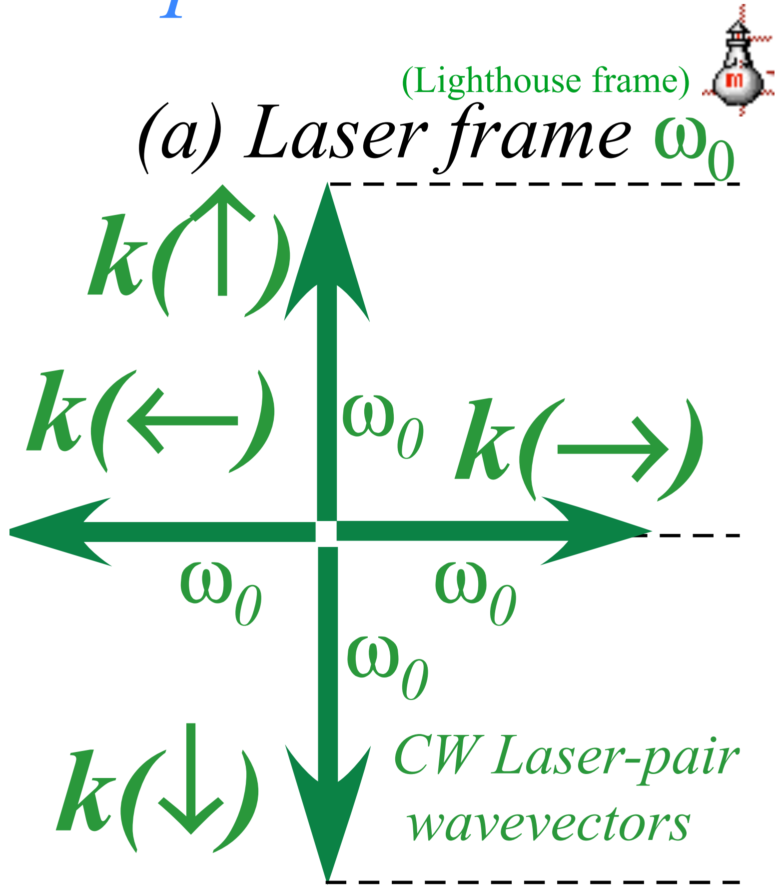

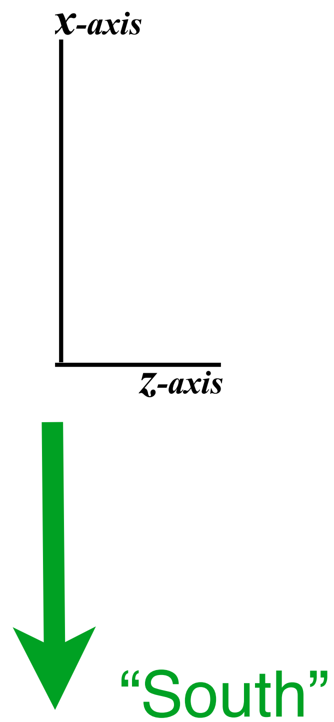

Suppose starlight in lighthouse frame is straight down x-axis : $\left(\omega_{\downarrow}, ck_{x\downarrow}, ck_{y\downarrow}, ck_{y\downarrow}\right) = \left(\omega_0, -\omega_0, 0, 0\right)$

from:Fig. 6.1.3
QTforCA
Unit 8 Ch.6

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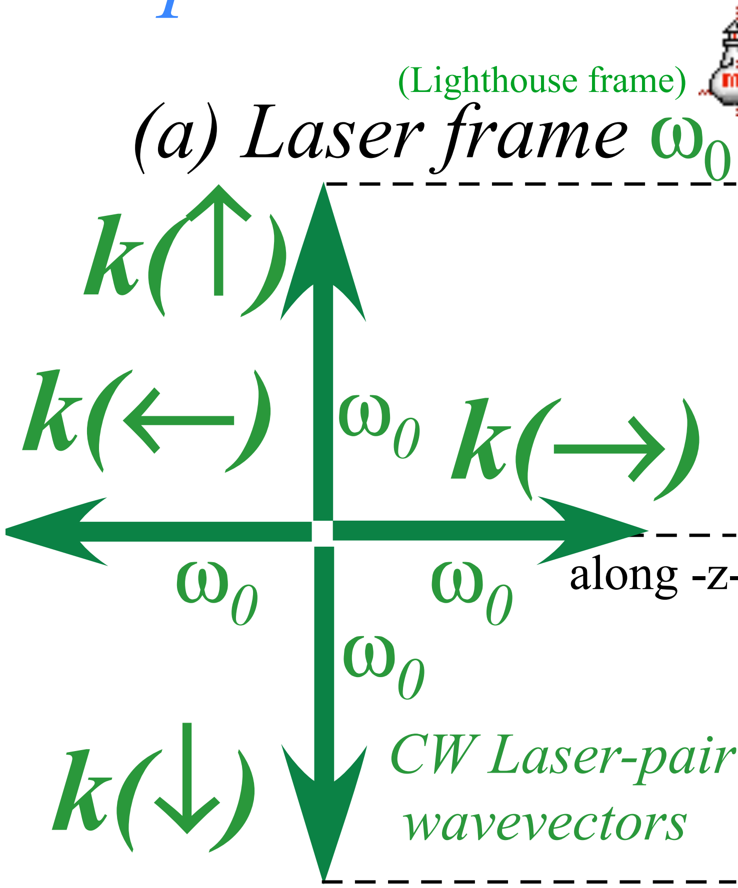

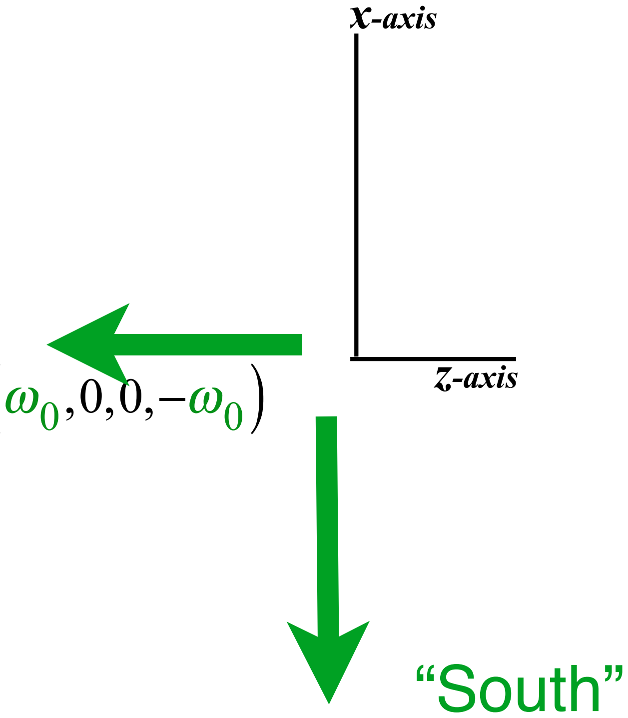

along -z-axis : $\left(\omega_{\leftarrow}, ck_{x\leftarrow}, ck_{y\leftarrow}, ck_{z\leftarrow}\right) = \left(\omega_0, 0, 0, -\omega_0\right)$

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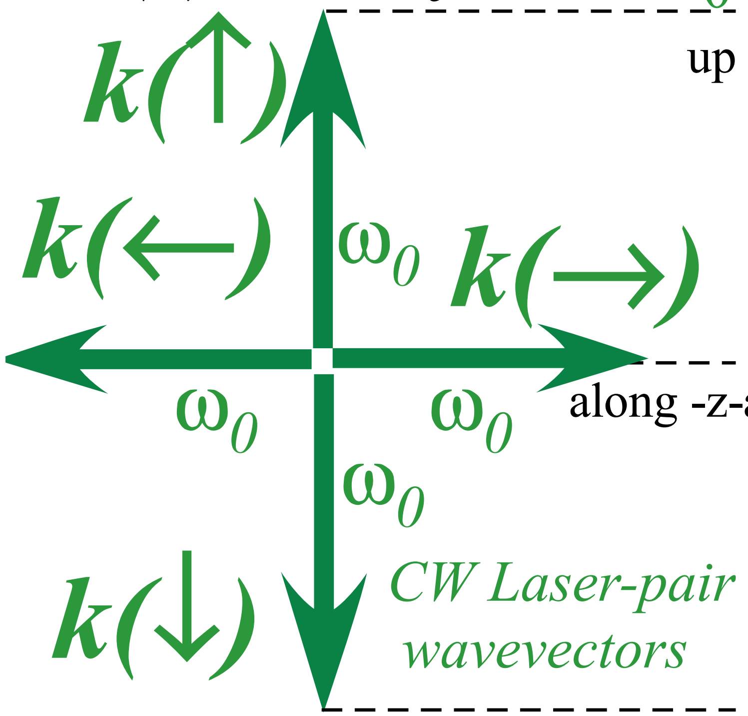
Per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z)=(\omega,ck_x,ck_y,ck_z)$ transformation

(Lighthouse frame)

(a) Laser frame ω_0



up +x-axis : $\left(\omega_{\uparrow},ck_{x\uparrow},ck_{y\uparrow},ck_{z\uparrow}\right)=\left(\omega_0,+\omega_0,0,0\right)$

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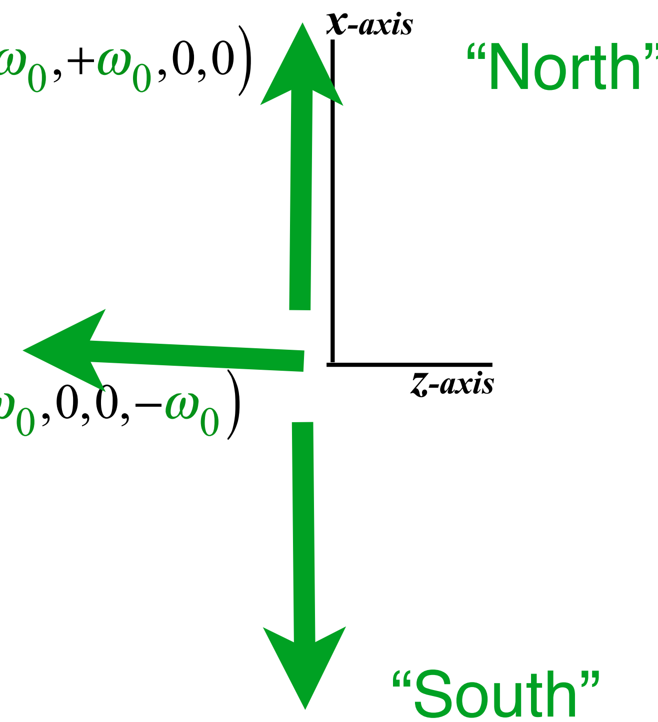

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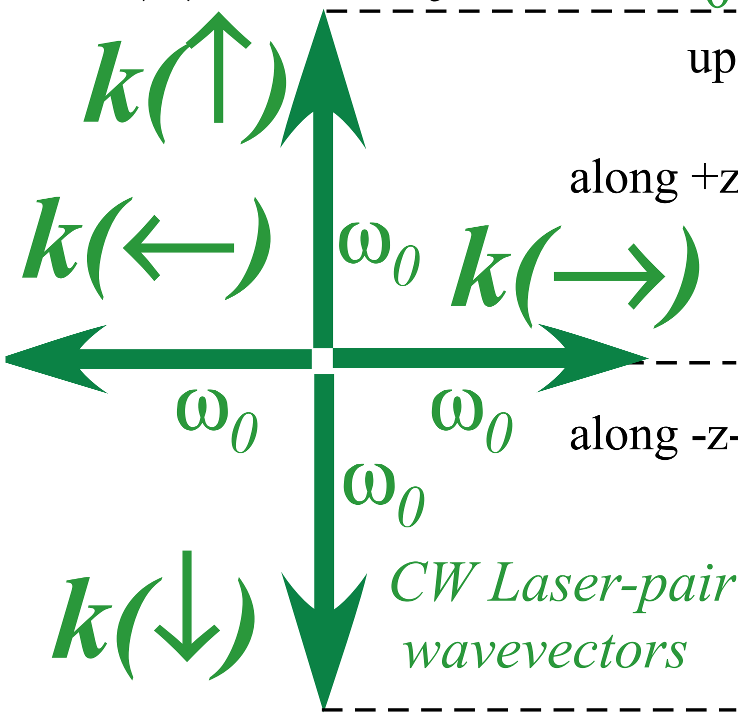

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along +z-axis : $\left(\omega_{\rightarrow}, ck_{x\rightarrow}, ck_{y\rightarrow}, ck_{z\rightarrow}\right) = \left(\omega_0, 0, 0, +\omega_0\right)$

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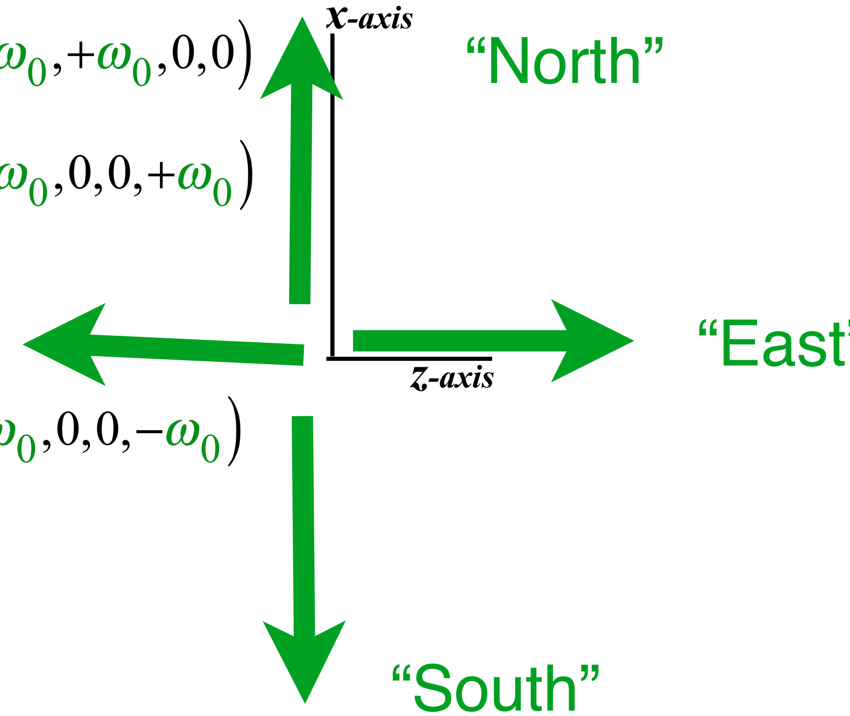

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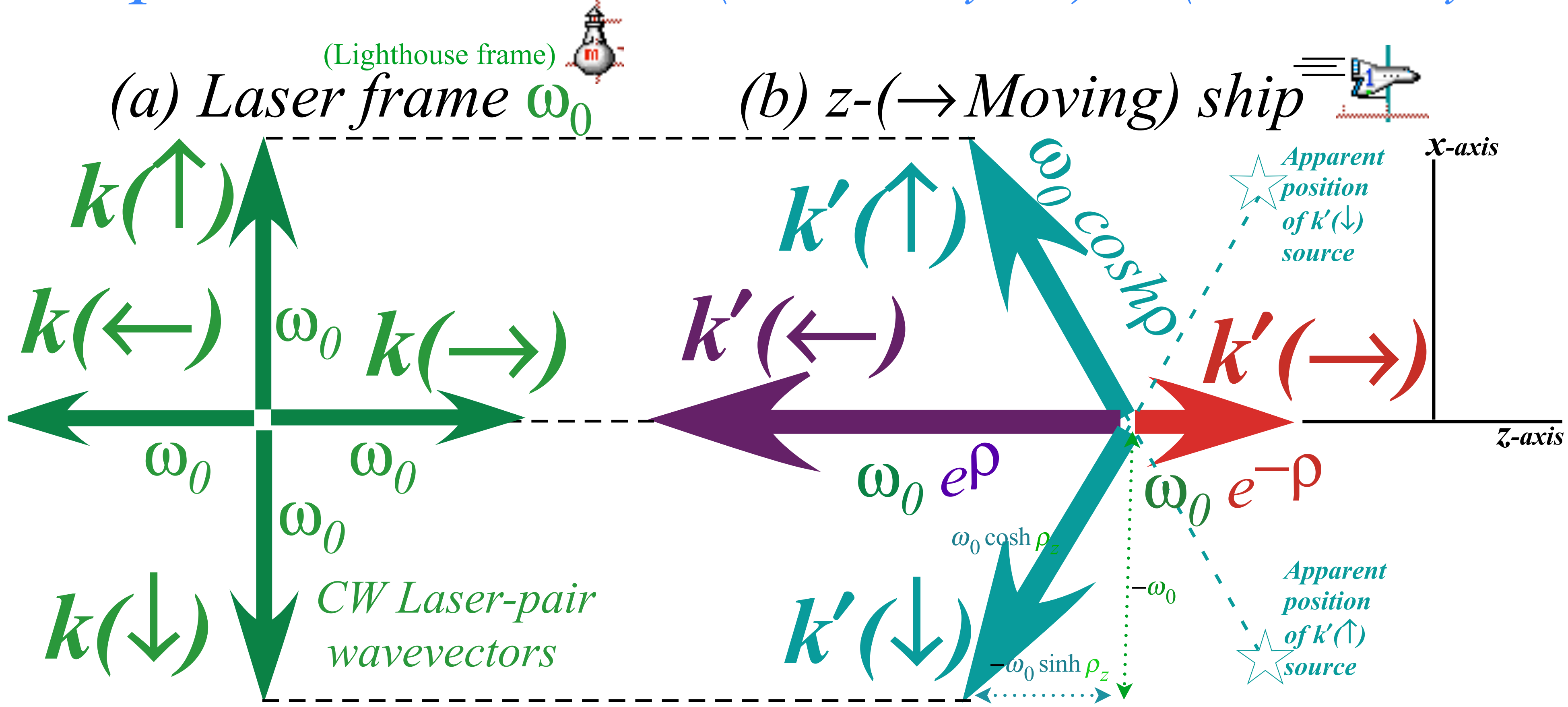

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from: Fig. 6.1.3 (modified) QTforCA Unit 8 Ch.6

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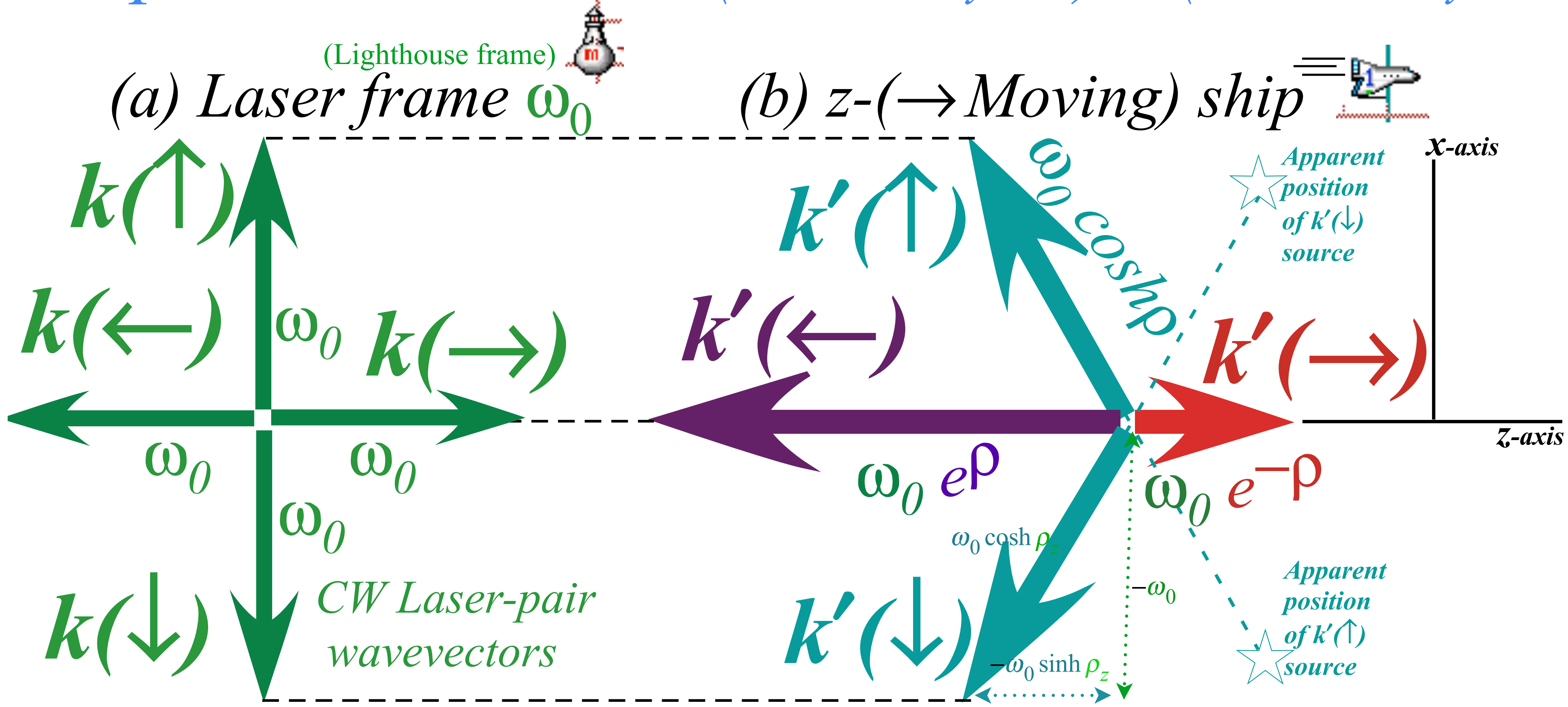

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$$\begin{pmatrix} \omega'_{\downarrow} \\ ck'_{x\downarrow} \\ ck'_{y\downarrow} \\ ck'_{z\downarrow} \end{pmatrix} = \begin{pmatrix} \cosh\rho_z & \cdot & \cdot & -\sinh\rho_z \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z \end{pmatrix} \begin{pmatrix} \omega_{\downarrow} \\ ck_{x\downarrow} \\ ck_{y\downarrow} \\ ck_{z\downarrow} \end{pmatrix} = \begin{pmatrix} \cosh\rho_z & \cdot & \cdot & -\sinh\rho_z \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z \end{pmatrix} \begin{pmatrix} \omega_0 \\ -\omega_0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \omega_0 \cosh\rho_z \\ -\omega_0 \\ 0 \\ -\omega_0 \sinh\rho_z \end{pmatrix}$$

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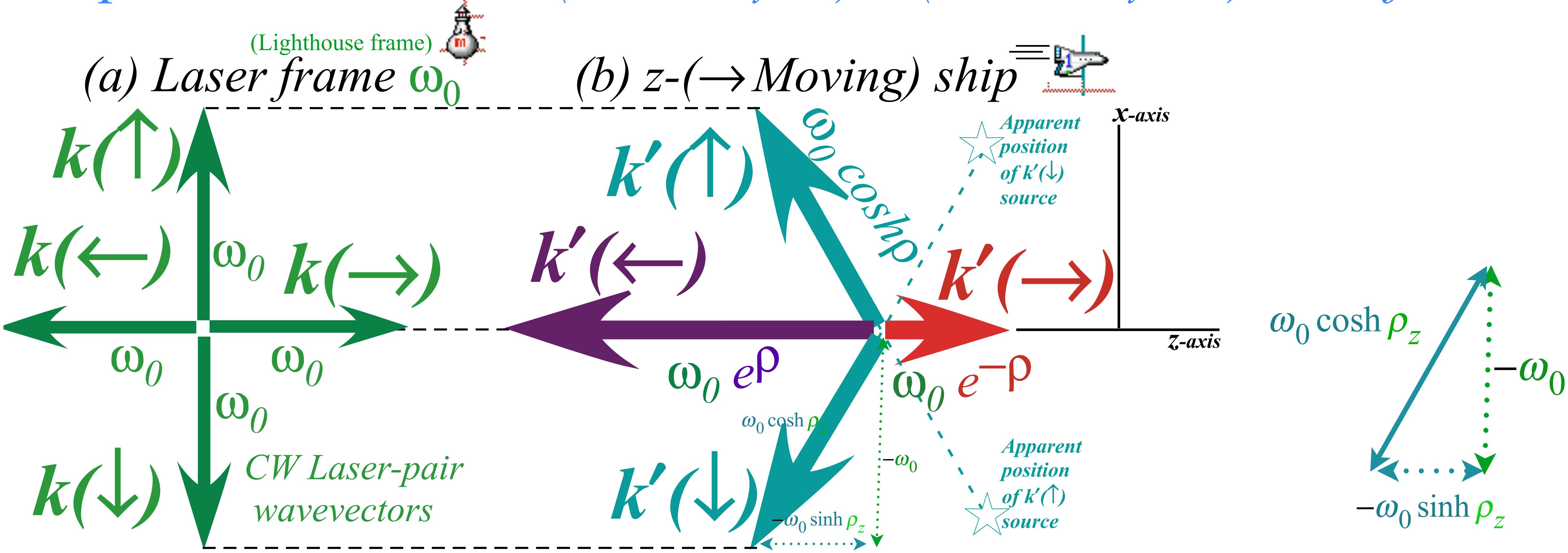

Suppose starlight in lighthouse frame is straight down x-axis : $\left(\omega_\downarrow, ck_{x\downarrow}, ck_{y\downarrow}, ck_{z\downarrow}\right) = \left(\omega_0, -\omega_0, 0, 0\right)$

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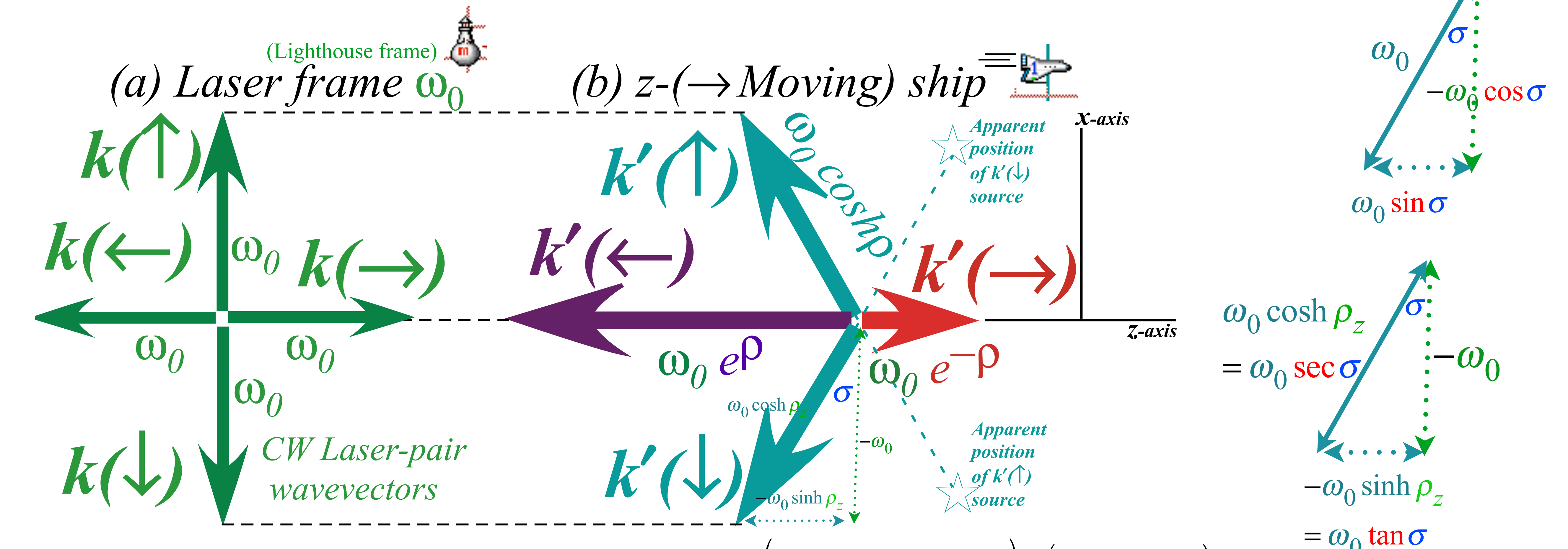
After the 4-vector transformation, $\omega_0=\omega_\downarrow$ is *transverse Doppler shifted* to $\omega_0\cosh\rho_z$, while $ck_z=0$ becomes $ck_z' = -\omega_0 \sinh\rho_z$.
(The x-component is unchanged: $ck_x' = -\omega_0 = ck_x$ and so is y-component: $ck_y' = -\omega_0 = ck_y$.)

from: Fig. 6.1.3 (modified) QTforCA Unit 8 Ch.6

Per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z)=(\omega,ck_x,ck_y,ck_z)$ transformation



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from:Fig. 6.1.3 (modified) QTforCA Unit 8 Ch.6

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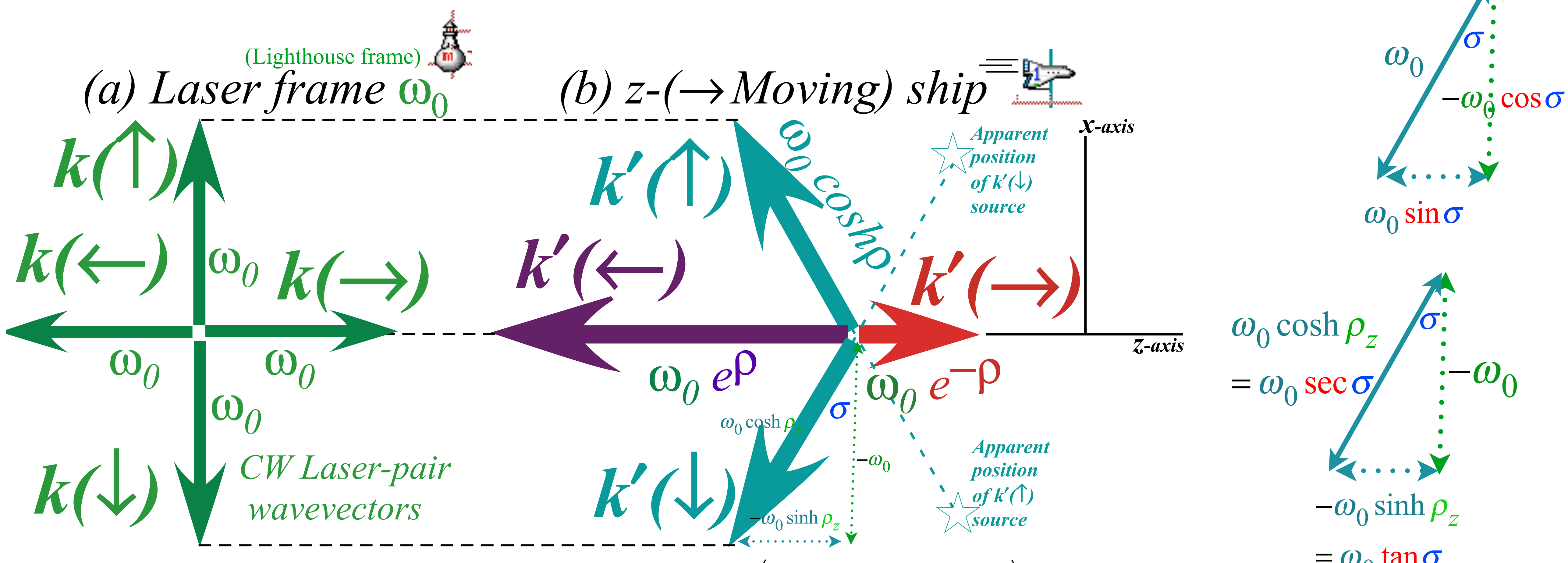

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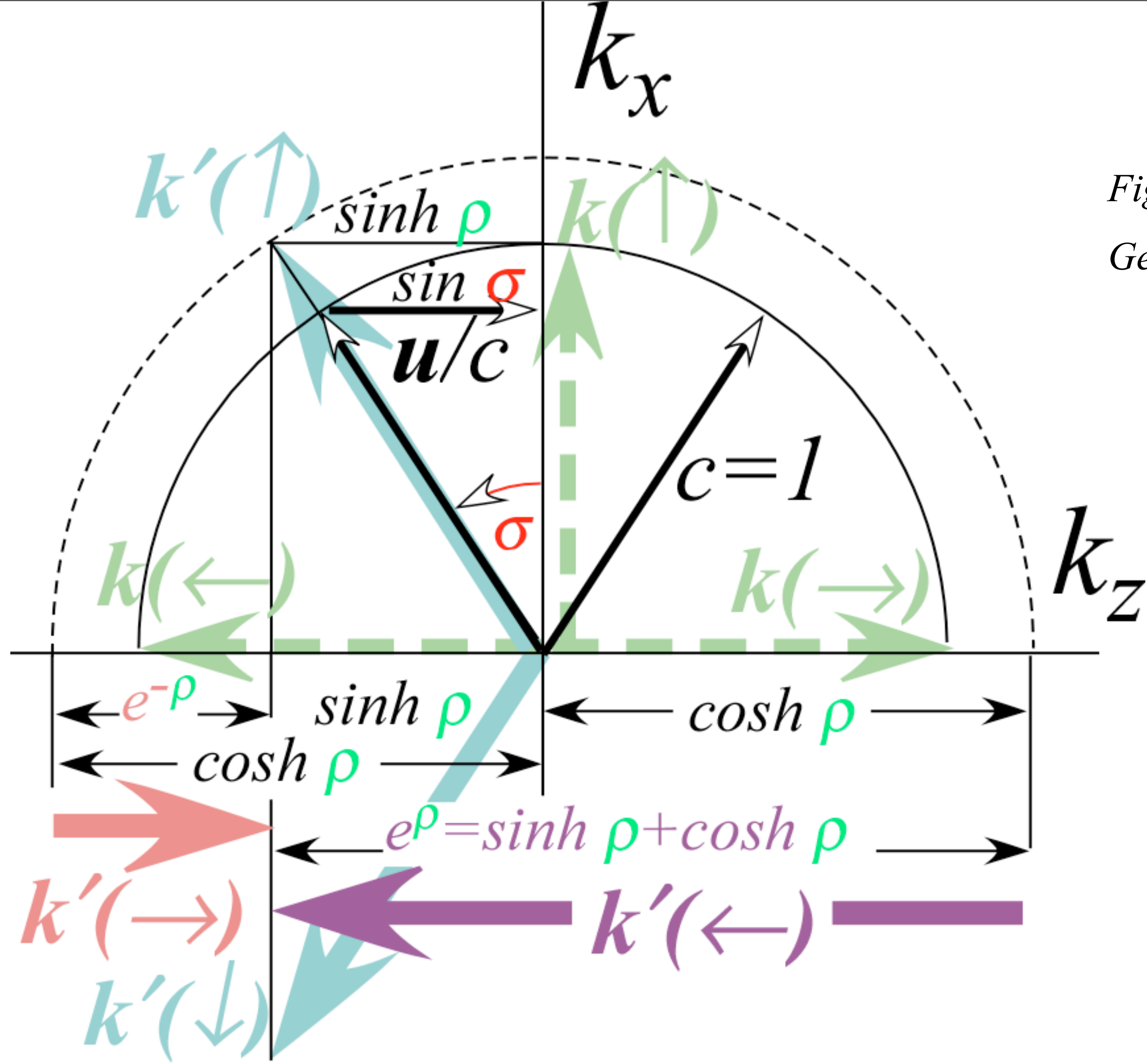
Recall hyperbolic invariant to Lorentz transform: $\omega^2 - c^2k^2 = \omega'^2 - c^2k'^2$ (=0 for 1-CW light)

The 4-vector form of this is: $\omega^2 - c^2\mathbf{k}\bullet\mathbf{k} = \omega'^2 - c^2\mathbf{k}'\bullet\mathbf{k}'$ (=0 " ")



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Geometry of Lorentz boost of counter-propagating waves.

from:Fig. 8.5.10
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QTforCA
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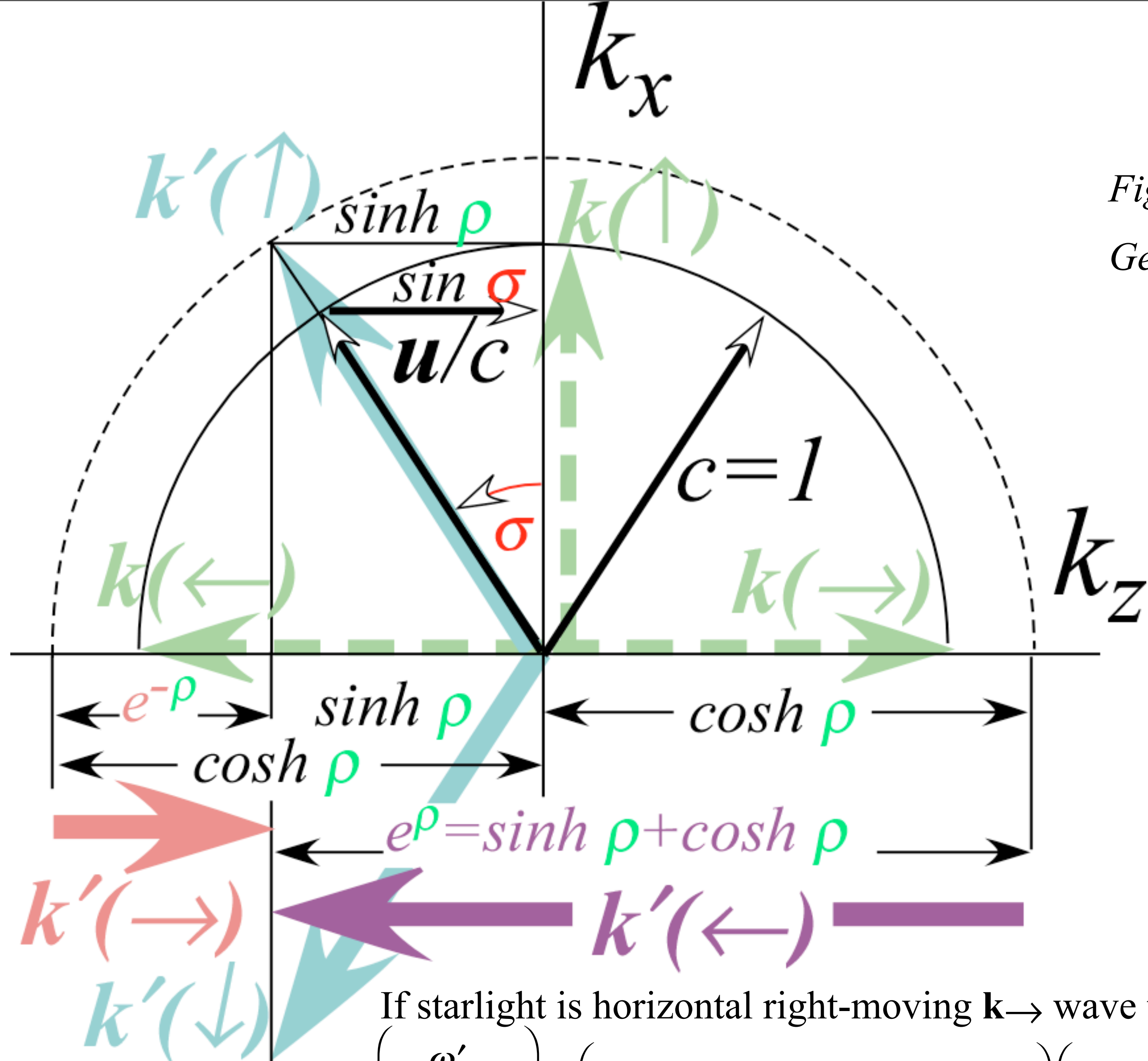


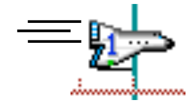


Fig. 5.10 CW cosmic speedometer.

Geometry of Lorentz boost of counter-propagating waves.

If starlight is horizontal right-moving $\mathbf{k}_{\rightarrow}$ wave then ship going $\mathbf{u}$ along z-axis sees :

$$\begin{pmatrix} \omega'_{\rightarrow} \\ ck'_{x\rightarrow} \\ ck'_{y\rightarrow} \\ ck'_{z\rightarrow} \end{pmatrix} = \begin{pmatrix} \cosh\rho_z & \cdot & \cdot & -\sinh\rho_z \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z \end{pmatrix} \begin{pmatrix} \omega_0 \\ 0 \\ 0 \\ +\omega_0 \end{pmatrix} = \omega_0 \begin{pmatrix} \cosh\rho_z - \sinh\rho_z \\ 0 \\ 0 \\ -\sinh\rho_z + \cosh\rho_z \end{pmatrix} = \begin{pmatrix} \omega_0 e^{-\rho_z} \\ 0 \\ 0 \\ -\omega_0 e^{-\rho_z} \end{pmatrix}$$

If starlight is horizontal left-moving $\mathbf{k}_{\leftarrow}$ wave then ship going $\mathbf{u}$ along z-axis sees :

$$\begin{pmatrix} \omega'_{\leftarrow} \\ ck'_{x\leftarrow} \\ ck'_{y\leftarrow} \\ ck'_{z\leftarrow} \end{pmatrix} = \begin{pmatrix} \cosh\rho_z & \cdot & \cdot & -\sinh\rho_z \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z \end{pmatrix} \begin{pmatrix} \omega_0 \\ 0 \\ 0 \\ -\omega_0 \end{pmatrix} = \omega_0 \begin{pmatrix} \cosh\rho_z + \sinh\rho_z \\ 0 \\ 0 \\ -\sinh\rho_z - \cosh\rho_z \end{pmatrix} = \begin{pmatrix} \omega_0 e^{+\rho_z} \\ 0 \\ 0 \\ -\omega_0 e^{+\rho_z} \end{pmatrix}$$

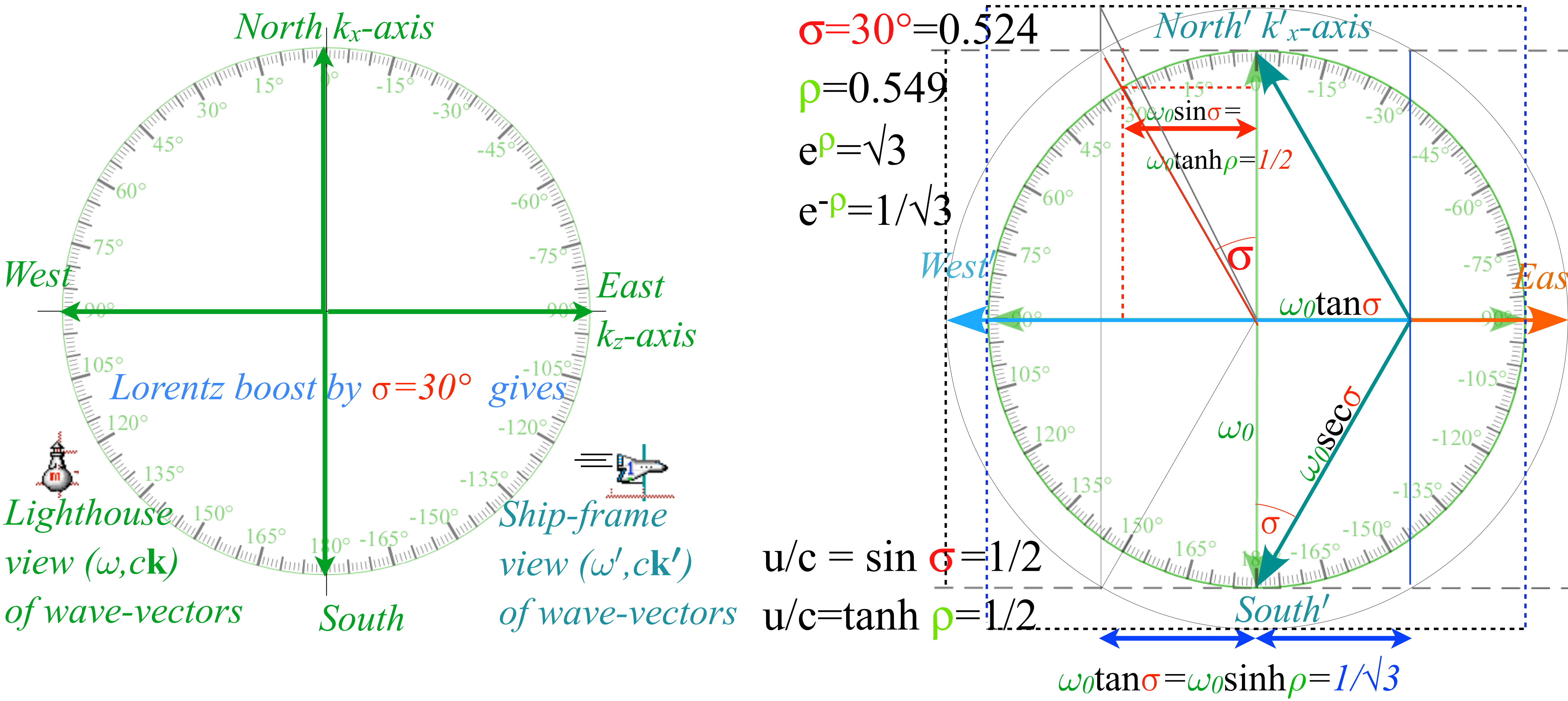
The usual longitudinal Doppler blue shifts $e^{+\rho_z}$ or Doppler red shifts $e^{-\rho_z}$ appear on both k-vector and frequency ω_0.

More details of Lorentz boost of North-South-East-West plane-wave 4-vectors ($\omega_0,\omega_x,\omega_y,\omega_z$)

Thales-like construction of Lorentz boost in 2D and 3D

The spectral ellipsoid

More details of Lorentz boost of North-South-East-West plane-wave 4-vectors $(\omega_0,\omega_x,\omega_y,\omega_z)$

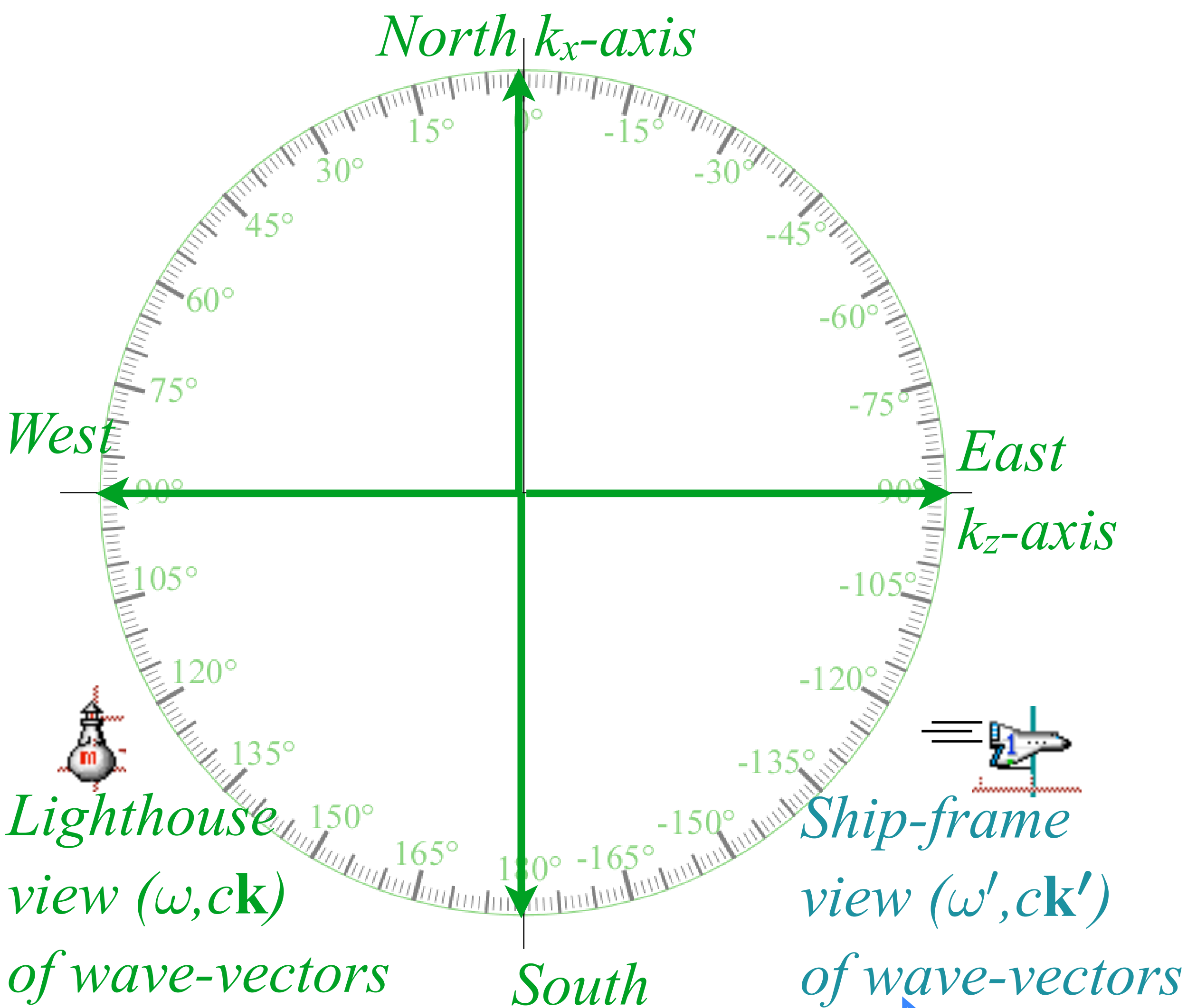


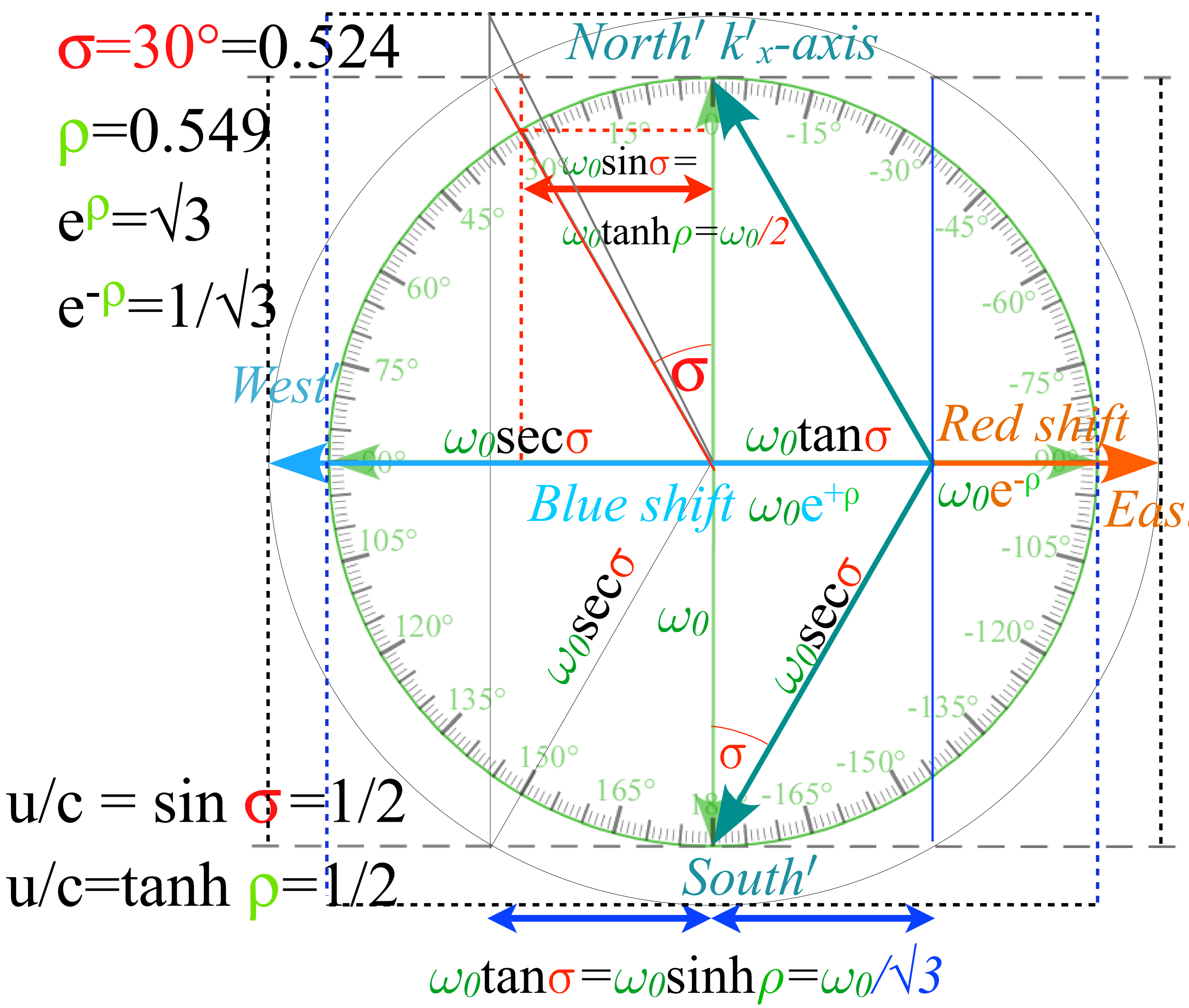
South starlight in lighthouse frame is straight down x-axis : $\left(\omega_{\downarrow},ck_{x\downarrow},ck_{y\downarrow},ck_{z\downarrow}\right)=\left(\omega_0,-\omega_0,0,0\right)$

$+\rho_z$-rapidity ship frame sees starlight Lorentz transformed to :$\left(\omega'_{\downarrow},ck'_{x\downarrow},ck'_{y\downarrow},ck'_{z\downarrow}\right)=\left(\omega_0\cosh\rho_z,-\omega_0,0,-\omega_0\sinh\rho_z\right)$

$$\begin{pmatrix}\omega'_{\downarrow}\\ ck'_{x\downarrow}\\ ck'_{y\downarrow}\\ ck'_{z\downarrow}\end{pmatrix}=\begin{pmatrix}\cosh\rho_z & \cdot & \cdot & -\sinh\rho_z\\ \cdot & 1 & \cdot & \cdot\\ \cdot & \cdot & 1 & \cdot\\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z\end{pmatrix}\begin{pmatrix}\omega_{\downarrow}\\ ck_{x\downarrow}\\ ck_{y\downarrow}\\ ck_{z\downarrow}\end{pmatrix}=\begin{pmatrix}\cosh\rho_z & \cdot & \cdot & -\sinh\rho_z\\ \cdot & 1 & \cdot & \cdot\\ \cdot & \cdot & 1 & \cdot\\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z\end{pmatrix}\begin{pmatrix}\omega_0\\ -\omega_0\\ 0\\ 0\end{pmatrix}=\begin{pmatrix}\omega_0\cosh\rho_z\\ -\omega_0\\ 0\\ -\omega_0\sinh\rho_z\end{pmatrix}=\begin{pmatrix}\omega_0\sec\sigma\\ -\omega_0\\ 0\\ -\omega_0\tan\sigma\end{pmatrix}$$

More details of Lorentz boost of North-South-East-West plane-wave 4-vectors $(\omega_0,\omega_x,\omega_y,\omega_z)$





Lorentz boost by $\sigma=30°$ or $e^{+\rho}=\sqrt{3}$

For ship going $u=c\tanh\rho$ along z-axis

West starlight $(\omega_0,0,0,-\omega_0)$ is *blue shifted by* $e^{+\rho}=\cosh\rho+\sinh\rho$

$$\begin{pmatrix}\omega'_{\leftarrow}\\ ck'_{x\leftarrow}\\ ck'_{y\leftarrow}\\ ck'_{z\leftarrow}\end{pmatrix}=\omega_0\begin{pmatrix}\cosh\rho_z+\sinh\rho_z\\ 0\\ 0\\ -\sinh\rho_z-\cosh\rho_z\end{pmatrix}=\begin{pmatrix}\omega_0 e^{+\rho_z}\\ 0\\ 0\\ -\omega_0 e^{+\rho_z}\end{pmatrix}$$

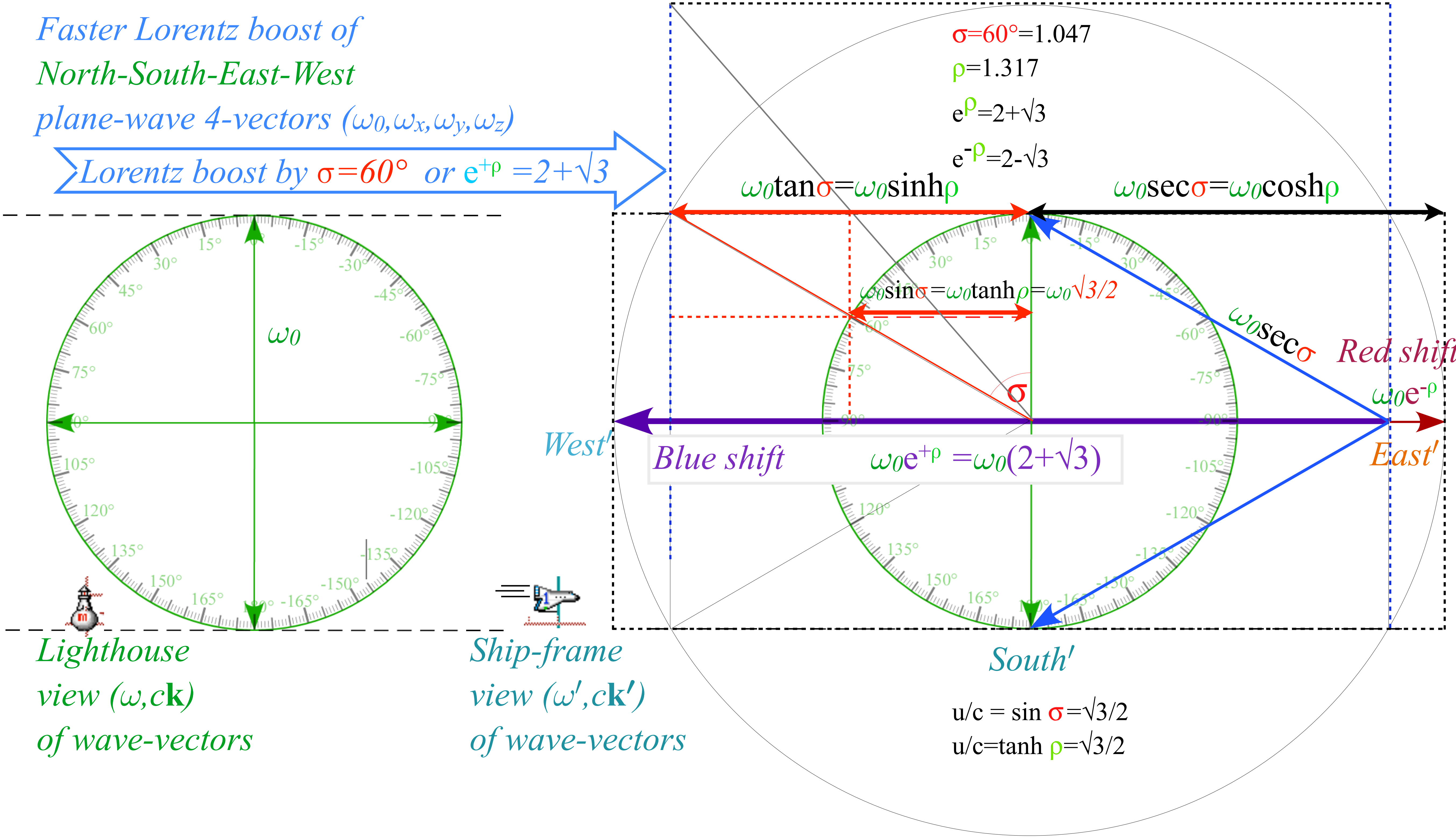
and *East* starlight $(\omega_0,0,0,+\omega_0)$ is *red shifted by* $e^{-\rho}=\cosh\rho-\sinh\rho$

$$\begin{pmatrix}\omega'_{\rightarrow}\\ ck'_{x\rightarrow}\\ ck'_{y\rightarrow}\\ ck'_{z\rightarrow}\end{pmatrix}=\omega_0\begin{pmatrix}\cosh\rho_z-\sinh\rho_z\\ 0\\ 0\\ -\sinh\rho_z+\cosh\rho_z\end{pmatrix}=\begin{pmatrix}\omega_0 e^{-\rho_z}\\ 0\\ 0\\ -\omega_0 e^{-\rho_z}\end{pmatrix}$$

Blue shift factor is $e^{+\rho}=\cosh\rho+\sinh\rho=\sec\sigma+\tan\sigma$

Red shift factor is $e^{-\rho}=\cosh\rho-\sinh\rho=\sec\sigma-\tan\sigma$

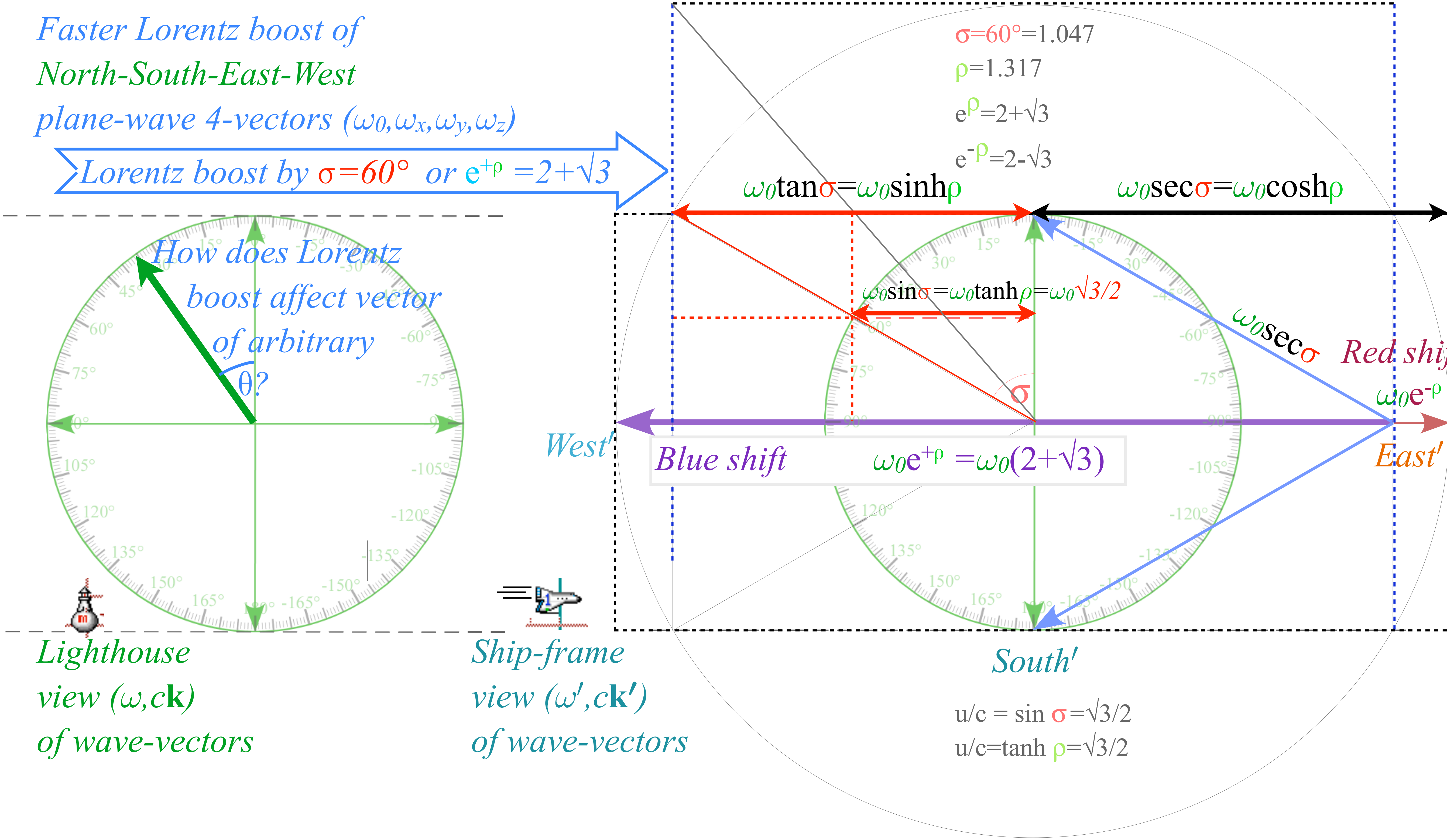
Faster Lorentz boost of
North-South-East-West
plane-wave 4-vectors $(\omega_0, \omega_x, \omega_y, \omega_z)$



More details of Lorentz boost of North-South-East-West plane-wave 4-vectors ($\omega_0,\omega_x,\omega_y,\omega_z$)

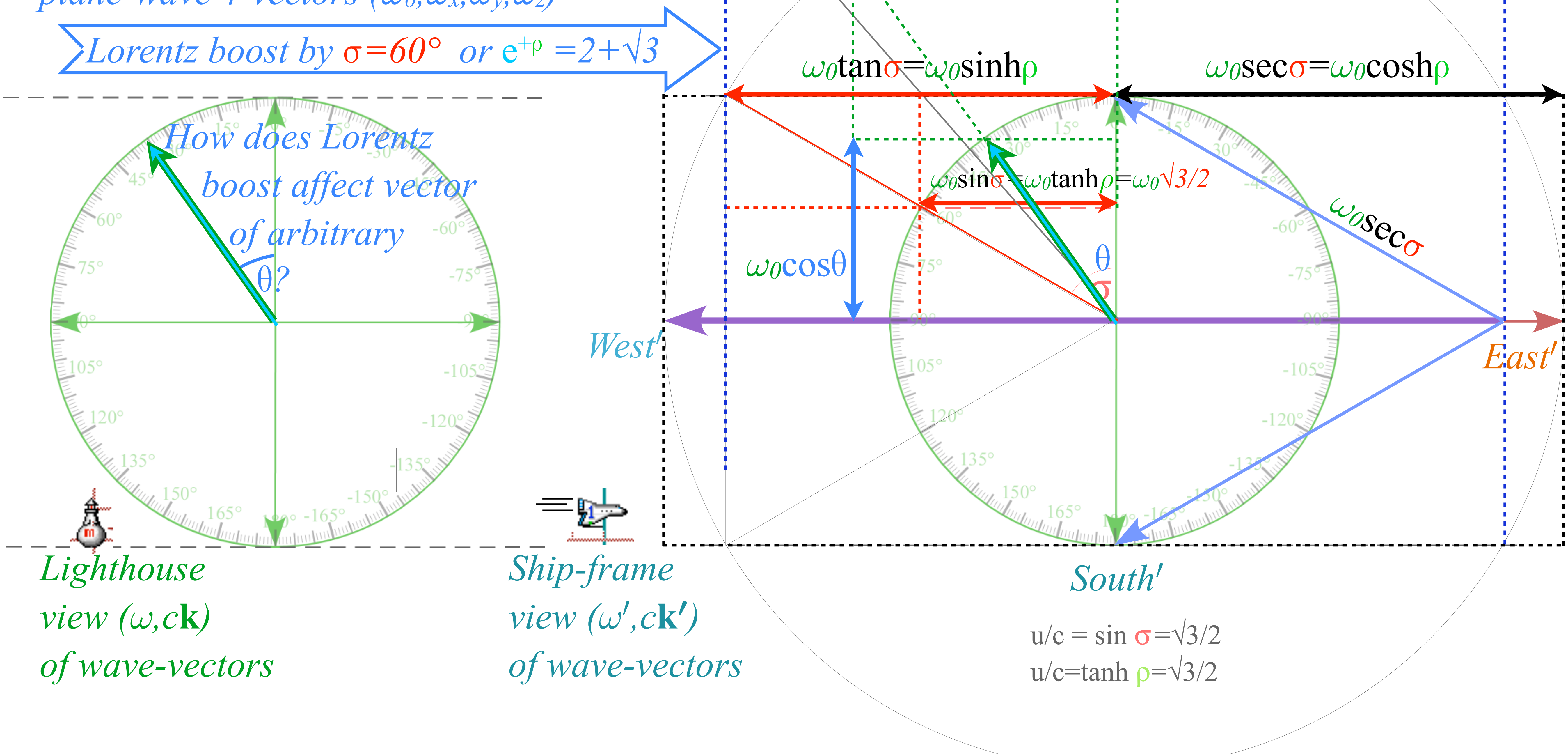
Thales-like construction of Lorentz boost in 2D and 3D

The spectral ellipsoid



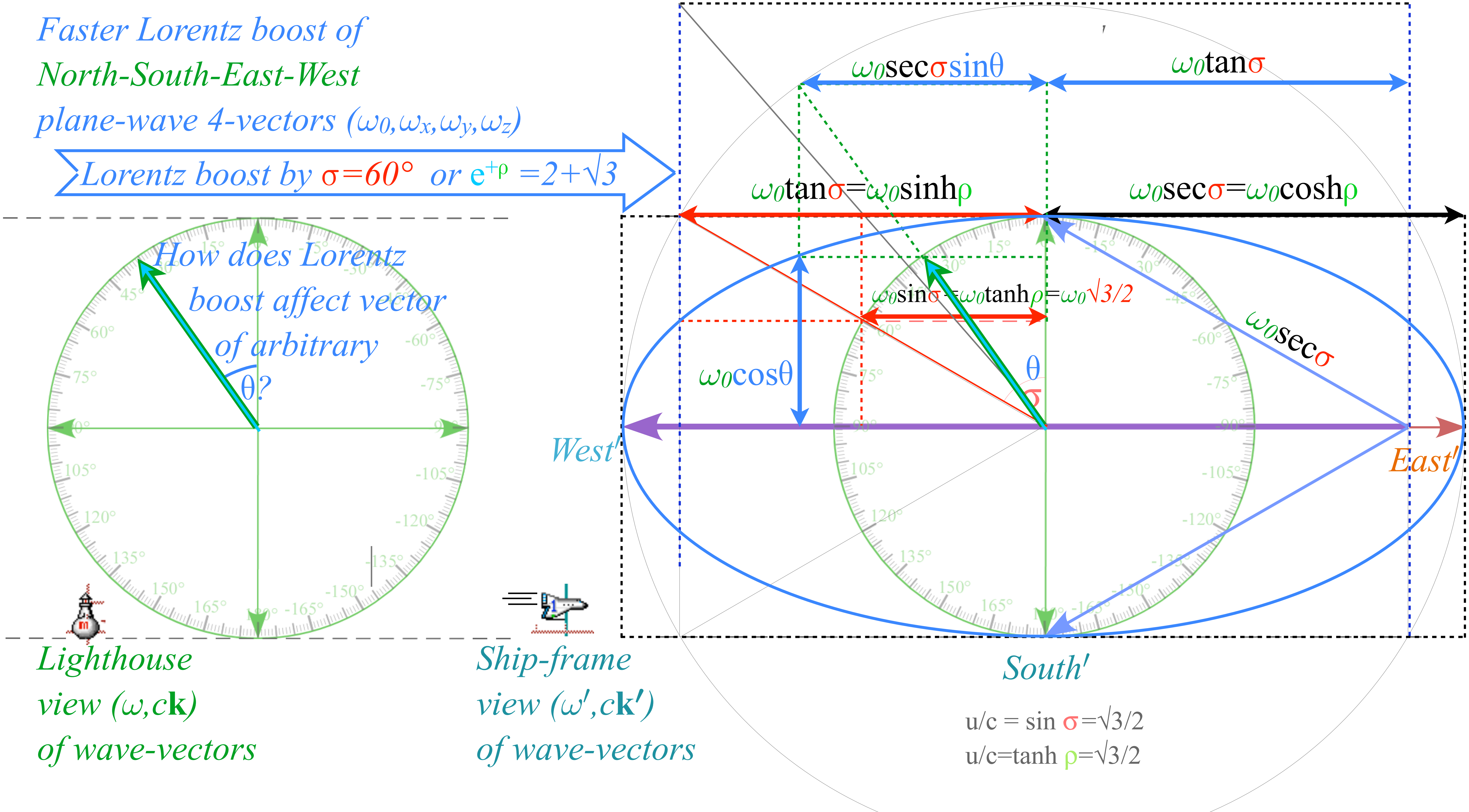
Faster Lorentz boost of
North-South-East-West
plane-wave 4-vectors $(\omega_0,\omega_x,\omega_y,\omega_z)$
Lorentz boost by $\sigma=60°$ or $e^{+\rho}=2+\sqrt{3}$
How does Lorentz boost affect vector of arbitrary θ?
Lighthouse view $(\omega,c\mathbf{k})$ of wave-vectors
Ship-frame view $(\omega',c\mathbf{k}')$ of wave-vectors
$\sigma=60°=1.047$
$\rho=1.317$
$e^{\rho}=2+\sqrt{3}$
$e^{-\rho}=2-\sqrt{3}$
$\omega_0\tan\sigma=\omega_0\sinh\rho$
$\omega_0\sec\sigma=\omega_0\cosh\rho$
$\omega_0\sin\sigma=\omega_0\tanh\rho=\omega_0\sqrt{3}/2$
$\omega_0\sec\sigma$
Red shift
$\omega_0 e^{-\rho}$
West′
East′
Blue shift
$\omega_0 e^{+\rho}=\omega_0(2+\sqrt{3})$
South′
$u/c=\sin\sigma=\sqrt{3}/2$
$u/c=\tanh\rho=\sqrt{3}/2$

Faster Lorentz boost of North-South-East-West plane-wave 4-vectors $(\omega_0,\omega_x,\omega_y,\omega_z)$



Let lab starlight ray at polar angle θ have $\mathbf{k}\uparrow\theta = \omega_0\,(1,\cos\theta,0,-\sin\theta)$. Then ship going $\mathbf{u}$ along z-axis sees :

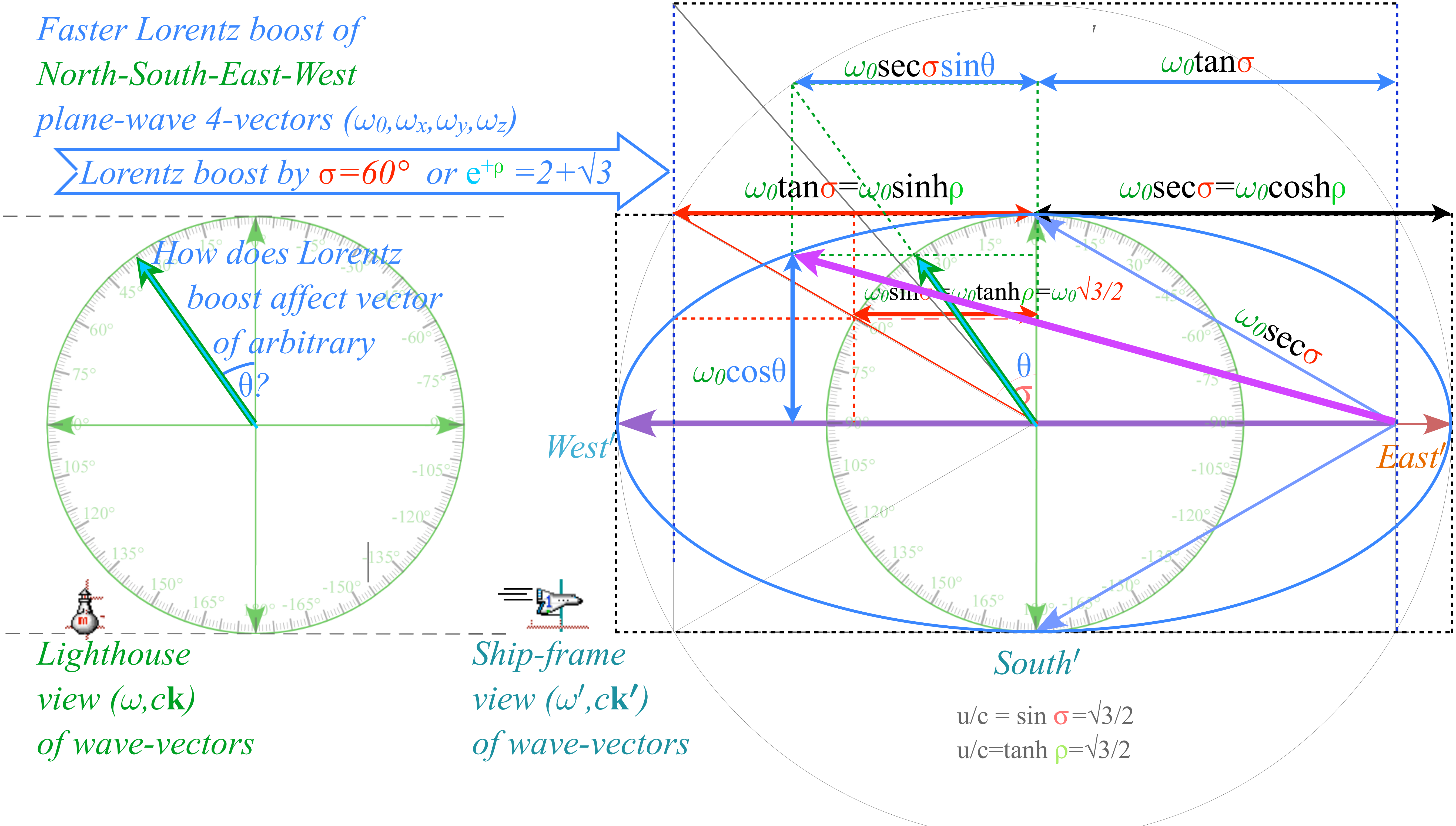
$$\begin{pmatrix}\omega'_{\uparrow\theta}\\ ck'_{x\uparrow\theta}\\ ck'_{y\uparrow\theta}\\ ck'_{z\uparrow\theta}\end{pmatrix}=\begin{pmatrix}\cosh\rho_z & \cdot & \cdot & -\sinh\rho_z\\ \cdot & 1 & \cdot & \cdot\\ \cdot & \cdot & 1 & \cdot\\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z\end{pmatrix}\begin{pmatrix}\omega_0\\ \omega_0\cos\theta\\ 0\\ -\omega_0\sin\theta\end{pmatrix}=\omega_0\begin{pmatrix}\cosh\rho_z+\sinh\rho_z\sin\theta\\ \cos\theta\\ 0\\ -\sinh\rho_z-\cosh\rho_z\sin\theta\end{pmatrix}=\omega_0\begin{pmatrix}\sec\sigma+\tan\sigma\sin\theta\\ \cos\theta\\ 0\\ -\tan\sigma-\sec\sigma\sin\theta\end{pmatrix}$$



Let lab starlight ray at polar angle θ have $\mathbf{k}\!\uparrow\theta = \omega_0\,(1,\cos\theta,0,-\sin\theta)$. Then ship going $\mathbf{u}$ along z-axis sees :

$$\begin{pmatrix} \omega'_{\uparrow\theta} \\ ck'_{x\uparrow\theta} \\ ck'_{y\uparrow\theta} \\ ck'_{z\uparrow\theta} \end{pmatrix} = \begin{pmatrix} \cosh\rho_z & \cdot & \cdot & -\sinh\rho_z \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z \end{pmatrix} \begin{pmatrix} \omega_0 \\ \omega_0\cos\theta \\ 0 \\ -\omega_0\sin\theta \end{pmatrix} = \omega_0 \begin{pmatrix} \cosh\rho_z + \sinh\rho_z\sin\theta \\ \cos\theta \\ 0 \\ -\sinh\rho_z - \cosh\rho_z\sin\theta \end{pmatrix} = \omega_0 \begin{pmatrix} \sec\sigma + \tan\sigma\sin\theta \\ \cos\theta \\ 0 \\ -\tan\sigma - \sec\sigma\sin\theta \end{pmatrix}$$

Faster Lorentz boost of North-South-East-West plane-wave 4-vectors $(\omega_0,\omega_x,\omega_y,\omega_z)$

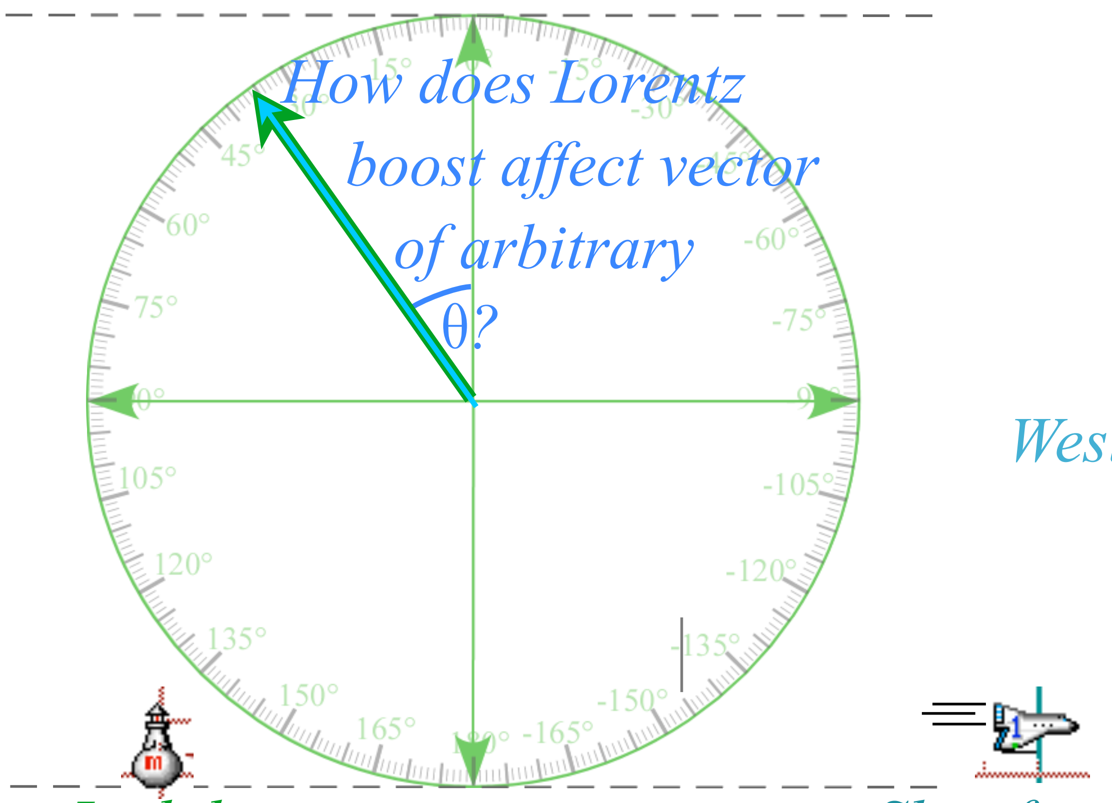


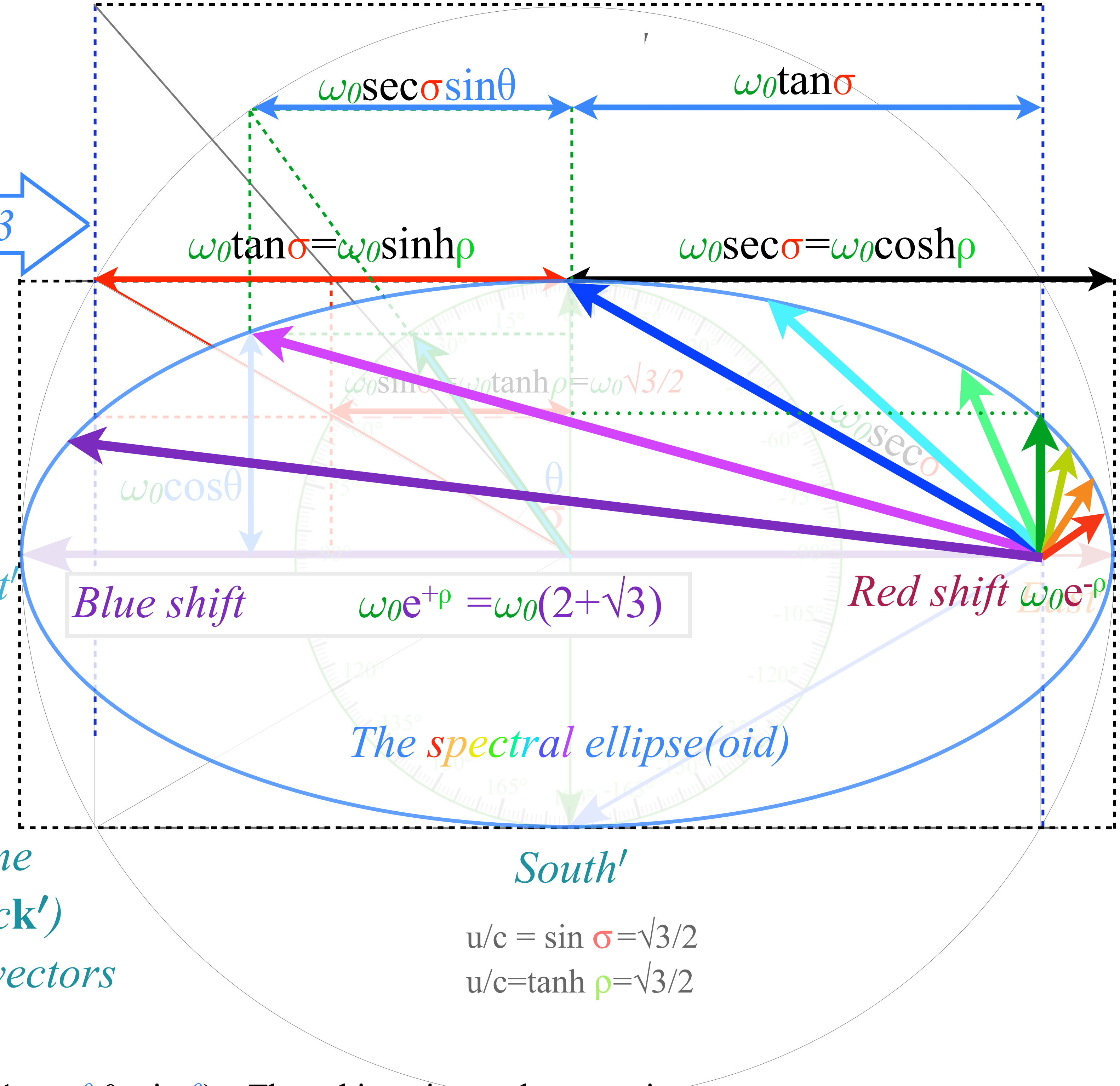
Let lab starlight ray at polar angle θ have $\mathbf{k}\uparrow\theta=\omega_0(1,\cos\theta,0,-\sin\theta)$. Then ship going $\mathbf{u}$ along z-axis sees :

$$\begin{pmatrix}\omega'_{\uparrow\theta}\\ ck'_{x\uparrow\theta}\\ ck'_{y\uparrow\theta}\\ ck'_{z\uparrow\theta}\end{pmatrix}=\begin{pmatrix}\cosh\rho_z & \cdot & \cdot & -\sinh\rho_z\\ \cdot & 1 & \cdot & \cdot\\ \cdot & \cdot & 1 & \cdot\\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z\end{pmatrix}\begin{pmatrix}\omega_0\\ \omega_0\cos\theta\\ 0\\ -\omega_0\sin\theta\end{pmatrix}=\omega_0\begin{pmatrix}\cosh\rho_z+\sinh\rho_z\sin\theta\\ \cos\theta\\ 0\\ -\sinh\rho_z-\cosh\rho_z\sin\theta\end{pmatrix}=\omega_0\begin{pmatrix}\sec\sigma+\tan\sigma\sin\theta\\ \cos\theta\\ 0\\ -\tan\sigma-\sec\sigma\sin\theta\end{pmatrix}$$

Faster Lorentz boost of
North-South-East-West
plane-wave 4-vectors $(\omega_0,\omega_x,\omega_y,\omega_z)$

Lorentz boost by $\sigma=60°$ *or* $e^{+\rho}=2+\sqrt{3}$



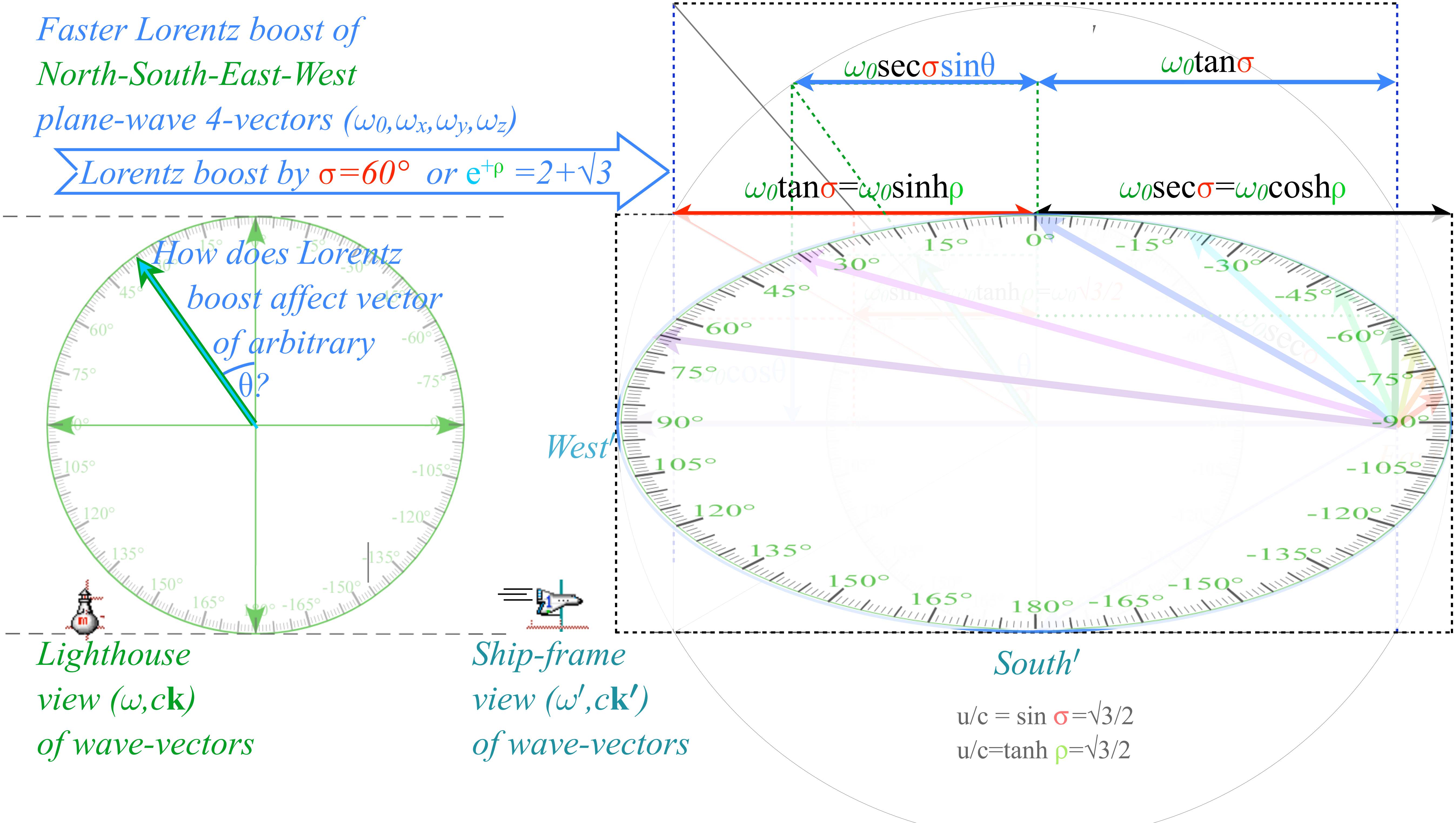


Lighthouse view $(\omega, c\mathbf{k})$ *of wave-vectors*

Ship-frame view $(\omega', c\mathbf{k}')$ *of wave-vectors*

Let lab starlight ray at polar angle θ have $\mathbf{k}\uparrow\theta = \omega_0\,(1,\cos\theta,0,-\sin\theta)$. Then ship going $\mathbf{u}$ along z-axis sees :

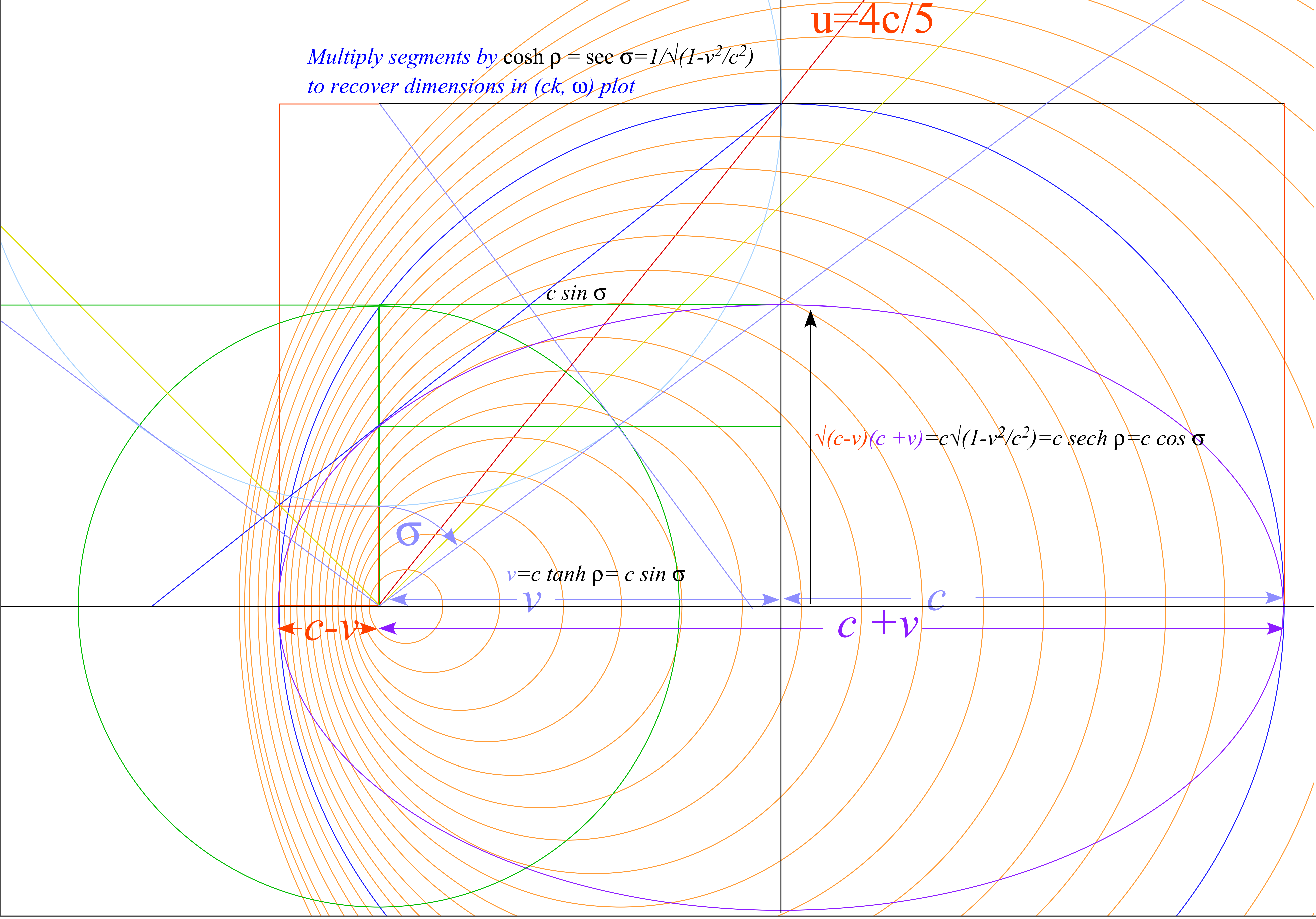
$$\begin{pmatrix}\omega'_{\uparrow\theta}\\ ck'_{x\uparrow\theta}\\ ck'_{y\uparrow\theta}\\ ck'_{z\uparrow\theta}\end{pmatrix}=\begin{pmatrix}\cosh\rho_z & \cdot & \cdot & -\sinh\rho_z\\ \cdot & 1 & \cdot & \cdot\\ \cdot & \cdot & 1 & \cdot\\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z\end{pmatrix}\begin{pmatrix}\omega_0\\ \omega_0\cos\theta\\ 0\\ -\omega_0\sin\theta\end{pmatrix}=\omega_0\begin{pmatrix}\cosh\rho_z+\sinh\rho_z\sin\theta\\ \cos\theta\\ 0\\ -\sinh\rho_z-\cosh\rho_z\sin\theta\end{pmatrix}=\omega_0\begin{pmatrix}\sec\sigma+\tan\sigma\sin\theta\\ \cos\theta\\ 0\\ -\tan\sigma-\sec\sigma\sin\theta\end{pmatrix}$$



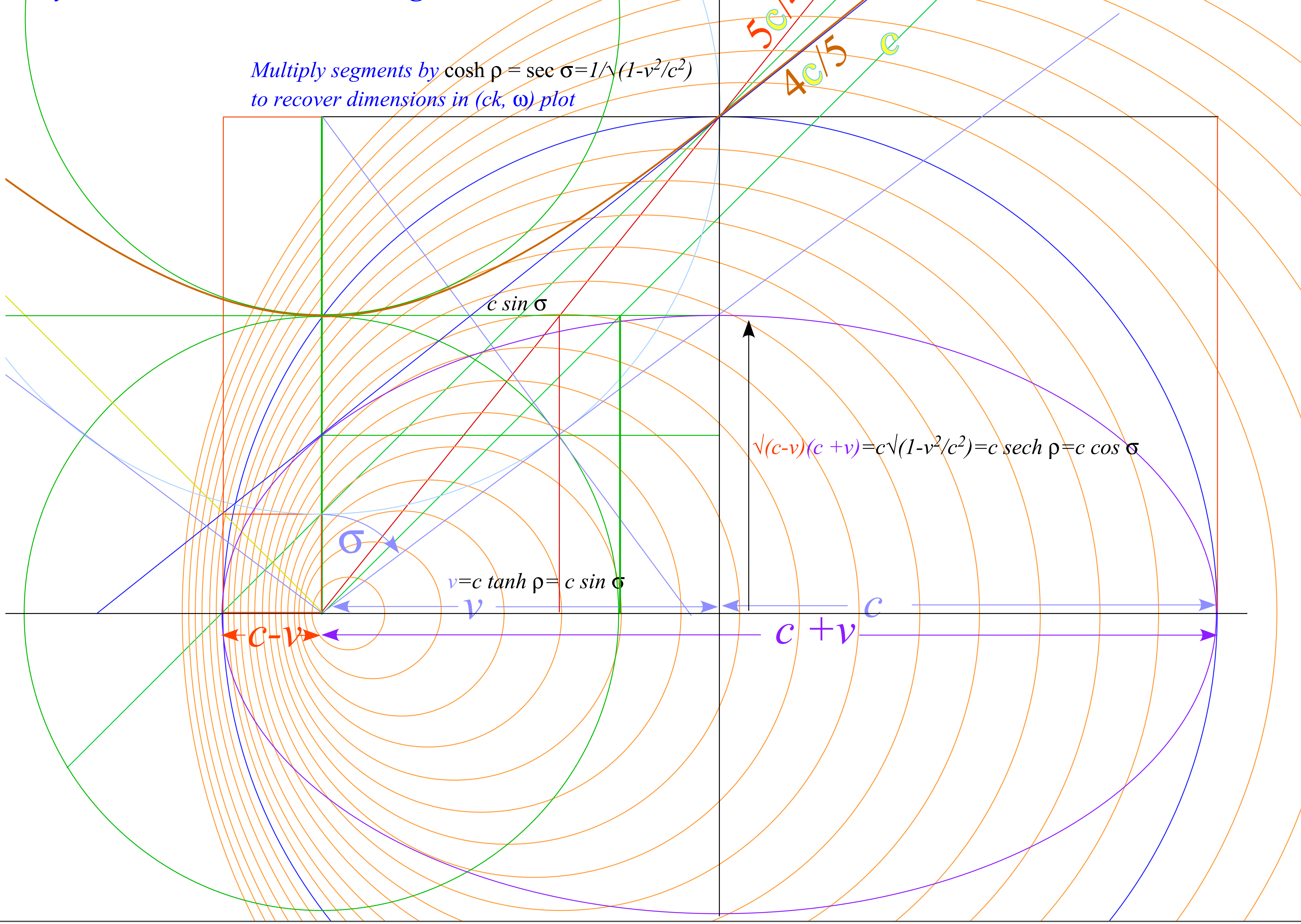
Let lab starlight ray at polar angle θ have $\mathbf{k}\uparrow\theta = \omega_0\,(1,\cos\theta,0,-\sin\theta)$. Then ship going **u** along z-axis sees :

$$\begin{pmatrix}\omega'_{\uparrow\theta}\\ ck'_{x\uparrow\theta}\\ ck'_{y\uparrow\theta}\\ ck'_{z\uparrow\theta}\end{pmatrix} = \begin{pmatrix}\cosh\rho_z & \cdot & \cdot & -\sinh\rho_z\\ \cdot & 1 & \cdot & \cdot\\ \cdot & \cdot & 1 & \cdot\\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z\end{pmatrix}\begin{pmatrix}\omega_0\\ \omega_0\cos\theta\\ 0\\ -\omega_0\sin\theta\end{pmatrix} = \omega_0\begin{pmatrix}\cosh\rho_z+\sinh\rho_z\sin\theta\\ \cos\theta\\ 0\\ -\sinh\rho_z-\cosh\rho_z\sin\theta\end{pmatrix} = \omega_0\begin{pmatrix}\sec\sigma+\tan\sigma\sin\theta\\ \cos\theta\\ 0\\ -\tan\sigma-\sec\sigma\sin\theta\end{pmatrix}$$

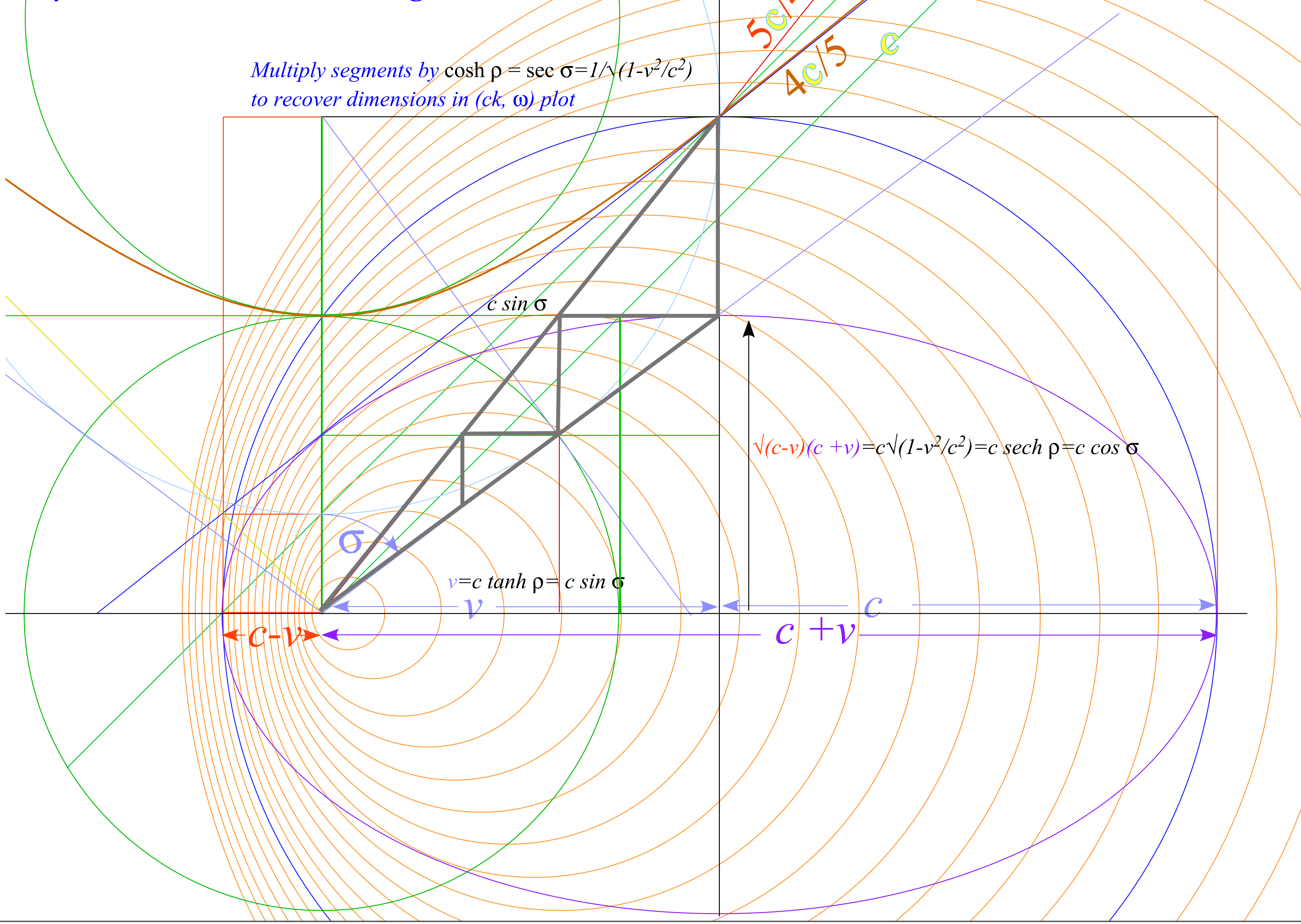
Space-Time Geometry



x-Space-y-Space Plot of wavefronts dropped by CW or PW source moving at u=4c/5

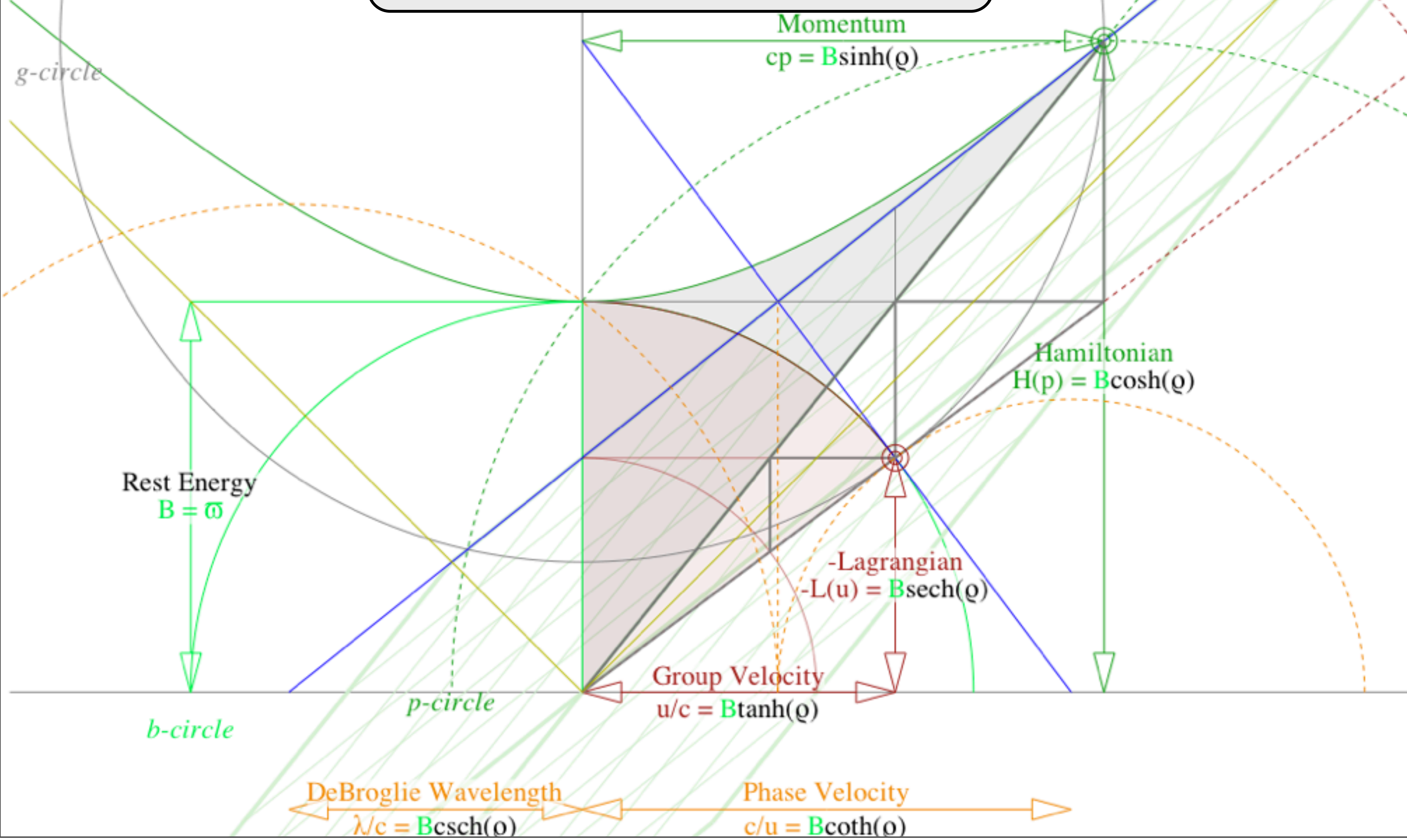


x-Space-y-Space Plot of wavefronts dropped by CW or PW source moving at $u=4c/5$



v/c = β = 0.800
Doppler blue shift factor = b = 3.001
Doppler red shift factor = r = 0.333
ν = 0.675 = 38.665°
ϱ = 1.099
σ = 0.928 = 53.143°

time	r_{Dopp}	υ_{group}	τ_{phase}	υ_{phase}	τ_{group}	b_{Dopp}	u/c	c/u
space		κ_{phase}	λ_{group}	κ_{group}	λ_{phase}		V_{group}/c	V_{phase}/c
rapidity ρ	$e^{-\rho}$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$e^{+\rho}$	$\tanh\rho$	$\coth\rho$
stellar∀ σ		$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$		$\sin\sigma$	$\csc\sigma$
QM		p	$-L$	H	λ_{DeB}		$d\omega/dk$	ω/k
Old Fashioned Formulas	$\sqrt{\frac{1-\frac{u}{c}}{1+\frac{u}{c}}}$	$\frac{1}{\sqrt{\frac{c^2}{u^2}-1}}$	$-\sqrt{1-\frac{u^2}{c^2}}$	$\frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$	$\sqrt{\frac{c^2}{u^2}-1}$	$\sqrt{\frac{1+\frac{u}{c}}{1-\frac{u}{c}}}$	$\frac{u}{c}$	$\frac{c}{u}$



Combination and interference of 4-vector plane waves (Idealized polarization case)

Combination group and phase waves define 4D Minkowski coordinates

Combination group and phase waves define wave guide dynamics

Waveguide dispersion and geometry

1st-quantized cavity modes

(And introducing 2nd-quantized cavity modes)

Combination and interference of 4-vector plane waves (Idealized amplitude case)

$$\Psi_{A_{\rightarrow},\omega_{\rightarrow},\mathbf{k}_{\rightarrow};A_{\leftarrow},\omega_{\leftarrow},\mathbf{k}_{\leftarrow}}(\mathbf{r},t) = A_{\rightarrow}e^{i(\mathbf{k}_{\rightarrow}\bullet\mathbf{r}\ -\ \omega_{\rightarrow}t)} + A_{\leftarrow}e^{i(\mathbf{k}_{\leftarrow}\bullet\mathbf{r}\ -\ \omega_{\leftarrow}t)}$$

2-CW-single-plane-polarized case: $\Psi_{\mathbf{k}}(\mathbf{r},t) = e^{i(\mathbf{k}_{\rightarrow}\bullet\mathbf{r}-\omega_{\rightarrow}t)} + e^{i(\mathbf{k}_{\leftarrow}\bullet\mathbf{r}-\omega_{\leftarrow}t)}$ *Idealized: Equal amplitudes and single plane polarization*

Factored into phase and group factors:

$$= e^{i\frac{(\mathbf{k}_{\rightarrow}+\mathbf{k}_{\leftarrow})\bullet\mathbf{r}-(\omega_{\rightarrow}+\omega_{\leftarrow})t}{2}}\, 2\cos\frac{(\mathbf{k}_{\rightarrow}-\mathbf{k}_{\leftarrow})\bullet\mathbf{r}-(\omega_{\rightarrow}-\omega_{\leftarrow})t}{2} = e^{i(\bar{\mathbf{K}}\bullet\mathbf{r}-\bar{\Omega}t)}\, 2\cos(\bar{\mathbf{k}}\bullet\mathbf{r}-\bar{\omega}t)$$

Phase (k,ω)

$$\frac{(\mathbf{k}_{\rightarrow}+\mathbf{k}_{\leftarrow})}{2} = \bar{\mathbf{K}}\ ,\qquad \frac{(\omega_{\rightarrow}+\omega_{\leftarrow})}{2} = \bar{\Omega}\ ,$$

Group (k,ω)

$$\bar{\mathbf{k}} = \frac{(\mathbf{k}_{\rightarrow}-\mathbf{k}_{\leftarrow})}{2},\qquad \bar{\omega} = \frac{(\omega_{\rightarrow}-\omega_{\leftarrow})}{2}.$$

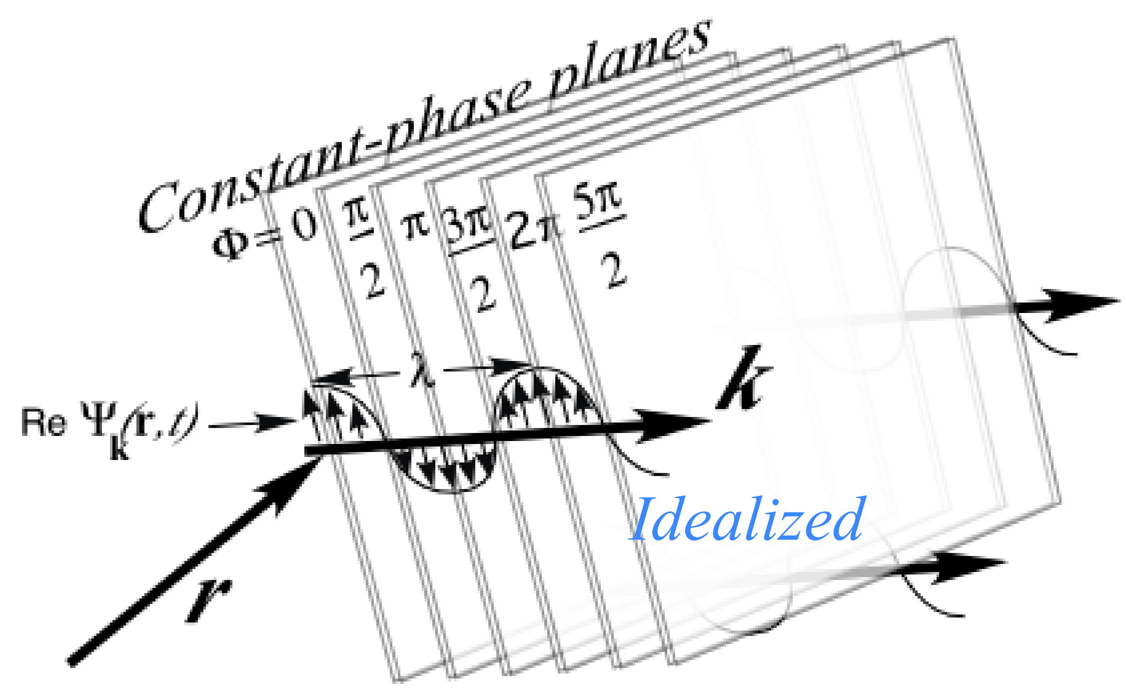


Fig. 6.1.1 Sketch of a 1-CW-single-plane-polarized plane wavefunction $\Psi_{\mathbf{k}}(\mathbf{r},t) = Ae^{i\Phi} = Ae^{i(\mathbf{k}\bullet\mathbf{r}-\omega t)}$ with wavevector **k**.

from:Fig. 6.1.1
QTforCA
Unit 2 Ch.6

Combination and interference of 4-vector plane waves (Idealized amplitude case)

$$\Psi_{A_\rightarrow,\omega_\rightarrow,\mathbf{k}_\rightarrow;A_\leftarrow,\omega_\leftarrow,\mathbf{k}_\leftarrow}(\mathbf{r},t) = A_\rightarrow e^{i(\mathbf{k}_\rightarrow\bullet\mathbf{r} - \omega_\rightarrow t)} + A_\leftarrow e^{i(\mathbf{k}_\leftarrow\bullet\mathbf{r} - \omega_\leftarrow t)}$$

2-CW-single-plane-polarized case: $\Psi_{\mathbf{k}}(\mathbf{r},t) = e^{i(\mathbf{k}_\rightarrow\bullet\mathbf{r}-\omega_\rightarrow t)} + e^{i(\mathbf{k}_\leftarrow\bullet\mathbf{r}-\omega_\leftarrow t)}$ *Idealized: Equal amplitudes and single plane polarization*

Factored into phase and group factors:

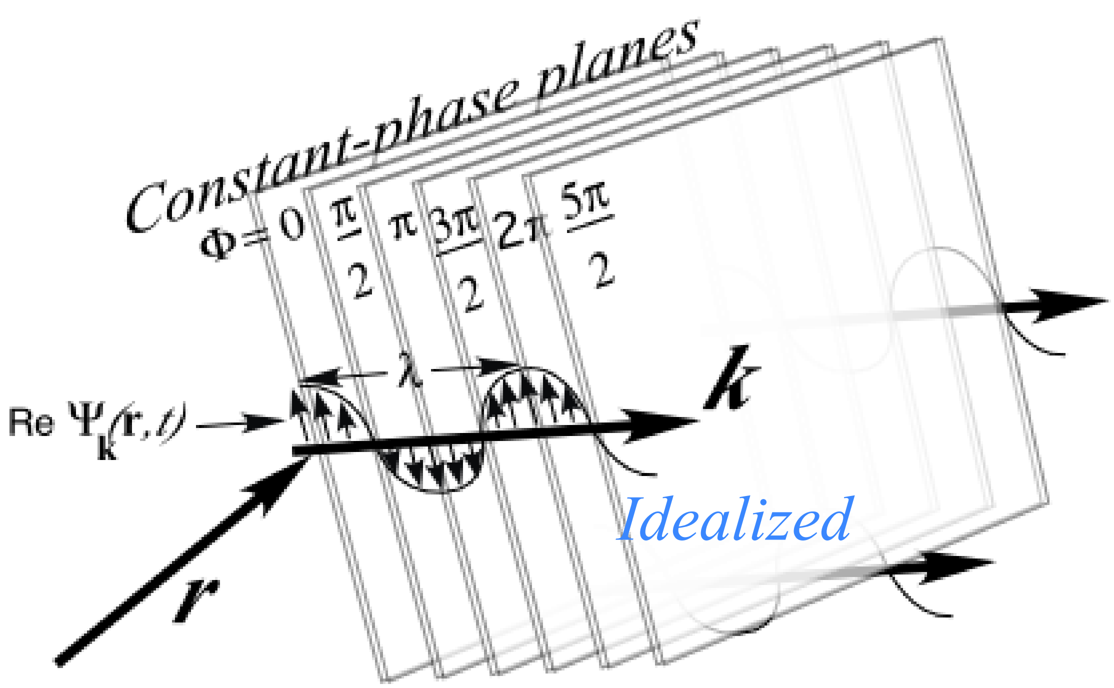
$$= e^{i\frac{(\mathbf{k}_\rightarrow+\mathbf{k}_\leftarrow)\bullet\mathbf{r}-(\omega_\rightarrow+\omega_\leftarrow)t}{2}}\, 2\cos\frac{(\mathbf{k}_\rightarrow-\mathbf{k}_\leftarrow)\bullet\mathbf{r}-(\omega_\rightarrow-\omega_\leftarrow)t}{2} = e^{i(\bar{\mathbf{K}}\bullet\mathbf{r}-\bar{\Omega}t)}\, 2\cos(\bar{\mathbf{k}}\bullet\mathbf{r}-\bar{\omega}t)$$

Phase (k,ω)

$$\frac{(\mathbf{k}_\rightarrow+\mathbf{k}_\leftarrow)}{2} = \bar{\mathbf{K}}, \qquad \frac{(\omega_\rightarrow+\omega_\leftarrow)}{2} = \bar{\Omega},$$

Group (k,ω)

$$\bar{\mathbf{k}} = \frac{(\mathbf{k}_\rightarrow-\mathbf{k}_\leftarrow)}{2}, \qquad \bar{\omega} = \frac{(\omega_\rightarrow-\omega_\leftarrow)}{2}.$$



from:Fig. 6.1.1
QTforCA
Unit 2 Ch.6

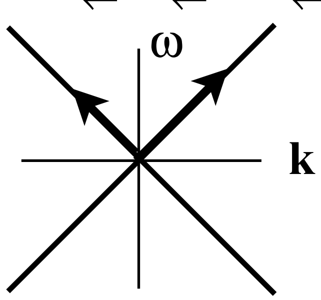
Fig. 6.1.1 Sketch of a 1-CW-single-plane-polarized plane wavefunction $\Psi_k(\mathbf{r},t) = Ae^{i\Phi} = Ae^{i(\mathbf{k}\bullet\mathbf{r}-\omega t)}$ *with wavevector* **k**.

Individual laser 4-vectors reside
on light cone or null-invariant.

Ship — Lighthouse — Laser lab

$$c^2\mathbf{k}'_\rightarrow\bullet\mathbf{k}'_\rightarrow - \omega'^2_\rightarrow = c^2\mathbf{k}_\rightarrow\bullet\mathbf{k}_\rightarrow - \omega^2_\rightarrow = c^2k_0^2 - \omega_0^2 = 0$$

$$c^2\mathbf{k}'_\leftarrow\bullet\mathbf{k}'_\leftarrow - \omega'^2_\leftarrow = c^2\mathbf{k}_\leftarrow\bullet\mathbf{k}_\leftarrow - \omega^2_\leftarrow = c^2k_0^2 - \omega_0^2 = 0$$



Combination and interference of 4-vector plane waves (Idealized amplitude case)

$$\Psi_{A_\rightarrow,\omega_\rightarrow,\mathbf{k}_\rightarrow;A_\leftarrow,\omega_\leftarrow,\mathbf{k}_\leftarrow}(\mathbf{r},t) = A_\rightarrow e^{i(\mathbf{k}_\rightarrow\bullet\mathbf{r}\,-\,\omega_\rightarrow t)} + A_\leftarrow e^{i(\mathbf{k}_\leftarrow\bullet\mathbf{r}\,-\,\omega_\leftarrow t)}$$

2-CW-single-plane-polarized case: $\Psi_{\mathbf{k}}(\mathbf{r},t) = e^{i(\mathbf{k}_\rightarrow\bullet\mathbf{r}-\omega_\rightarrow t)} + e^{i(\mathbf{k}_\leftarrow\bullet\mathbf{r}-\omega_\leftarrow t)}$ *Idealized: Equal amplitudes and single plane polarization*

Factored into phase and group factors:

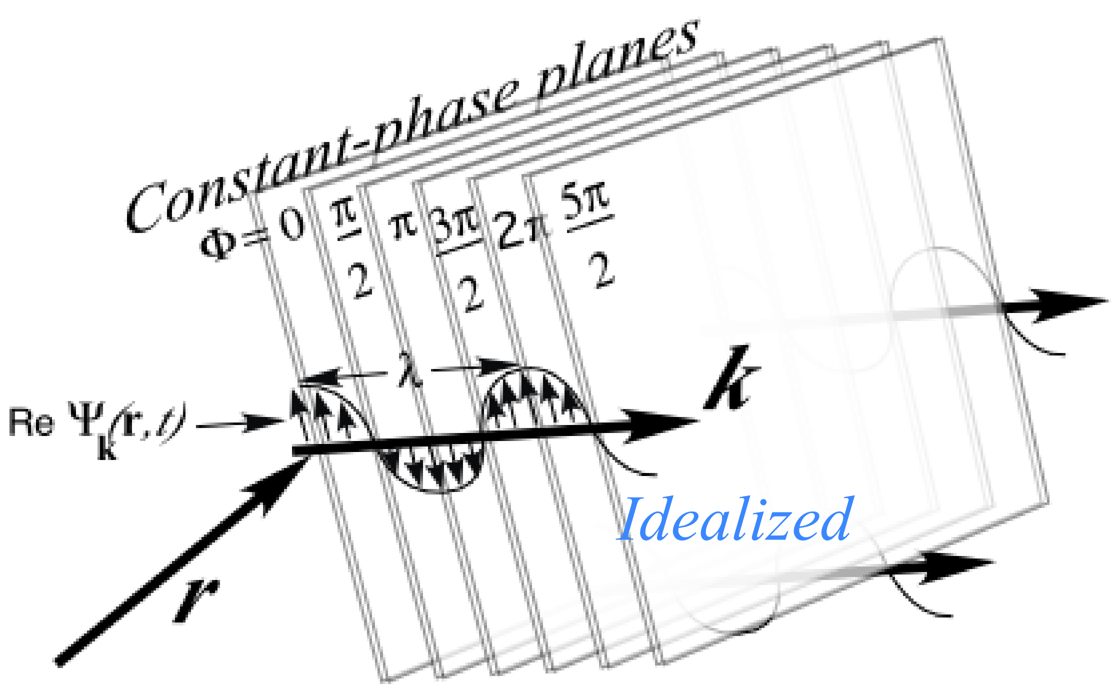
$$= e^{i\frac{(\mathbf{k}_\rightarrow+\mathbf{k}_\leftarrow)\bullet\mathbf{r}-(\omega_\rightarrow+\omega_\leftarrow)t}{2}}\, 2\cos\frac{(\mathbf{k}_\rightarrow-\mathbf{k}_\leftarrow)\bullet\mathbf{r}-(\omega_\rightarrow-\omega_\leftarrow)t}{2} = e^{i(\bar{\mathbf{K}}\bullet\mathbf{r}-\bar{\Omega}t)}\, 2\cos(\bar{\mathbf{k}}\bullet\mathbf{r}-\bar{\omega}t)$$

Phase (k,ω)

$$\frac{(\mathbf{k}_\rightarrow+\mathbf{k}_\leftarrow)}{2} = \bar{\mathbf{K}}\,, \qquad \frac{(\omega_\rightarrow+\omega_\leftarrow)}{2} = \bar{\Omega}\,,$$

Group (k,ω)

$$\bar{\mathbf{k}} = \frac{(\mathbf{k}_\rightarrow-\mathbf{k}_\leftarrow)}{2}\,, \qquad \bar{\omega} = \frac{(\omega_\rightarrow-\omega_\leftarrow)}{2}.$$



from:Fig. 6.1.1 QTforCA Unit 2 Ch.6

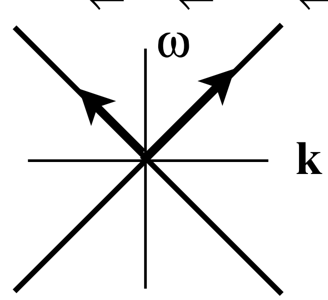
Fig. 6.1.1 Sketch of a 1-CW-single-plane-polarized plane wavefunction $\Psi_k(\mathbf{r},t) = Ae^{i\Phi} = Ae^{i(\mathbf{k}\bullet\mathbf{r}-\omega t)}$ *with wavevector* **k**.

Individual laser 4-vectors reside on light cone or null-invariant.

Ship | Lighthouse | Laser lab

$$c^2\mathbf{k}'_\rightarrow\bullet\mathbf{k}'_\rightarrow - \omega'^2_\rightarrow = c^2\mathbf{k}_\rightarrow\bullet\mathbf{k}_\rightarrow - \omega^2_\rightarrow = c^2k_0^2 - \omega_0^2 = 0$$

$$c^2\mathbf{k}'_\leftarrow\bullet\mathbf{k}'_\leftarrow - \omega'^2_\leftarrow = c^2\mathbf{k}_\leftarrow\bullet\mathbf{k}_\leftarrow - \omega^2_\leftarrow = c^2k_0^2 - \omega_0^2 = 0$$

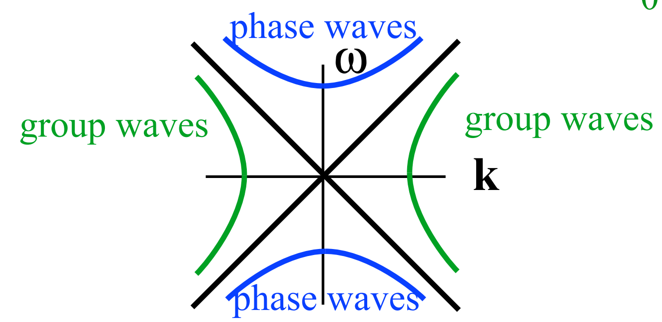


Sum and difference vectors are not on the light cone.

Ship | Lighthouse | Laser lab

$$\bar{\Omega}'^2 - c^2\bar{\mathbf{K}}'\bullet\bar{\mathbf{K}}' = \bar{\Omega}^2 - c^2\bar{\mathbf{K}}\bullet\bar{\mathbf{K}} = \omega_0^2 - 0 \quad = c^2k_0^2$$

$$\bar{\omega}'^2 - c^2\bar{\mathbf{k}}'\bullet\bar{\mathbf{k}}' = \bar{\omega}^2 - c^2\bar{\mathbf{k}}\bullet\bar{\mathbf{k}} = 0 - c^2\mathbf{k}_0\bullet\mathbf{k}_0 = -c^2k_0^2$$



Combination group and phase define 4D Minkowski coordinates (Idealized amplitude case)

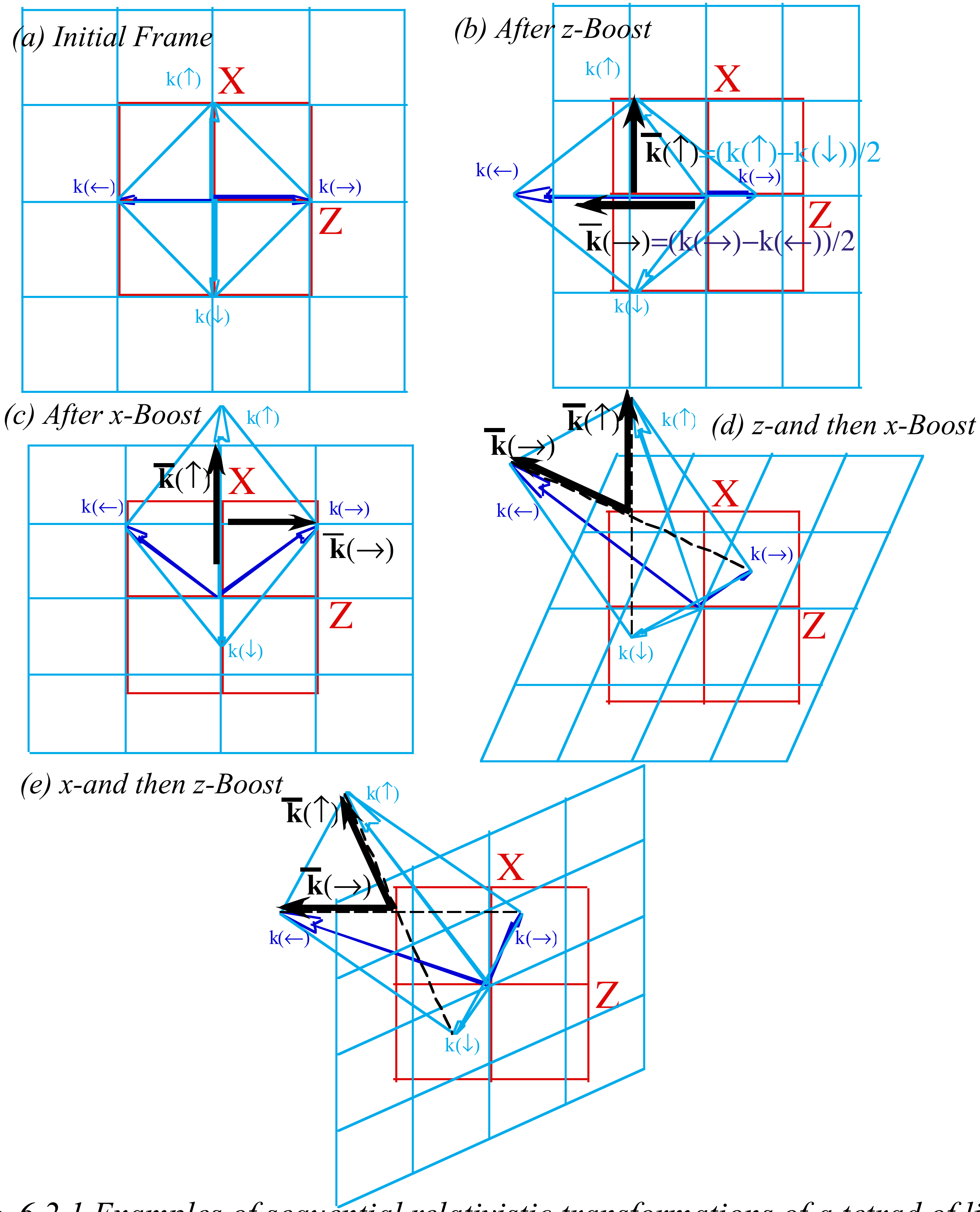


Fig. 6.2.1 Examples of sequential relativistic transformations of a tetrad of light wavevectors.

Future work: More efficient mapping Lorentz-Group operators and coordinate frames

from:Fig. 6.2.1 QTforCA Unit 2 Ch.6

Combination and interference of 4-vector plane waves (Idealized polarization case)

Combination group and phase waves define 4D Minkowski coordinates

Combination group and phase waves define wave guide dynamics

Waveguide dispersion and geometry

1^{st}-quantized cavity modes

(And introducing 2^{nd}-quantized cavity modes)

2-Dimensional wave mechanics: guided waves and dispersion in the "Hall of Mirrors"

Any two or three-dimensional wave will be seen to exceed the *c*-limit when it approaches an axis obliquely. It happens for plane waves. The phase velocities along coordinate axes are given by

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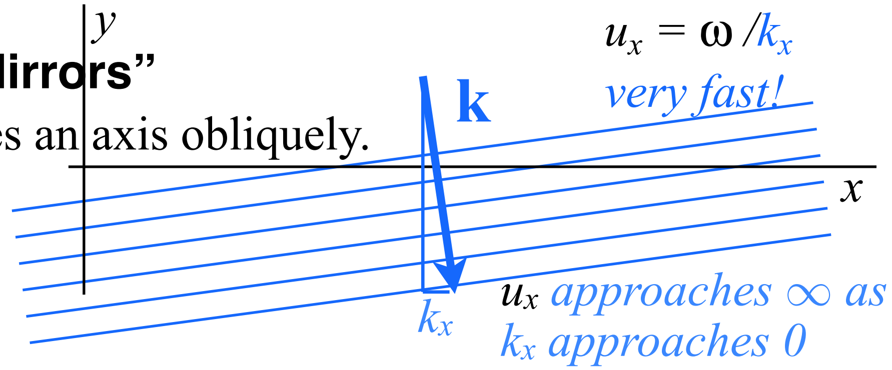
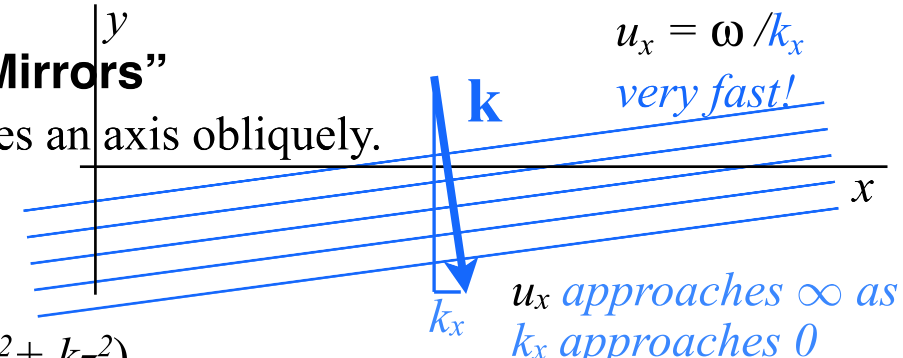


Waveguide dispersion and geometry

2-Dimensional wave mechanics: guided waves and dispersion in the “Hall of Mirrors”



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Thus, all the component phase velocities equal or exceed the phase velocity ω / k which is c for light!

A water waves exceeds c if it breaks parallel to shore so 'break-line" moves infinitely fast with $k_x = 0$.

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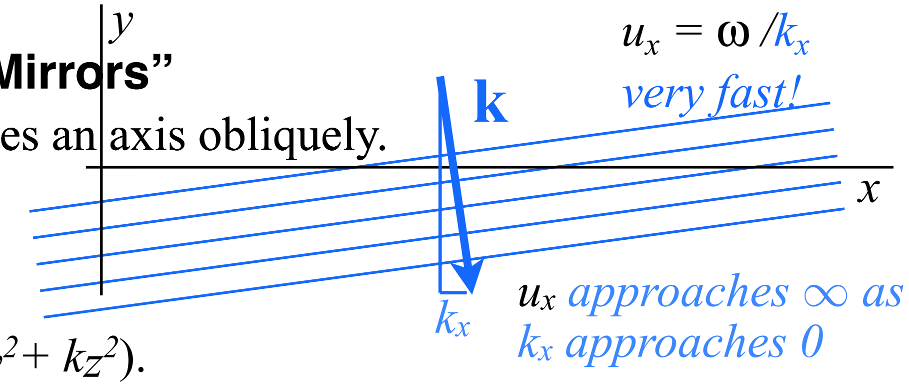

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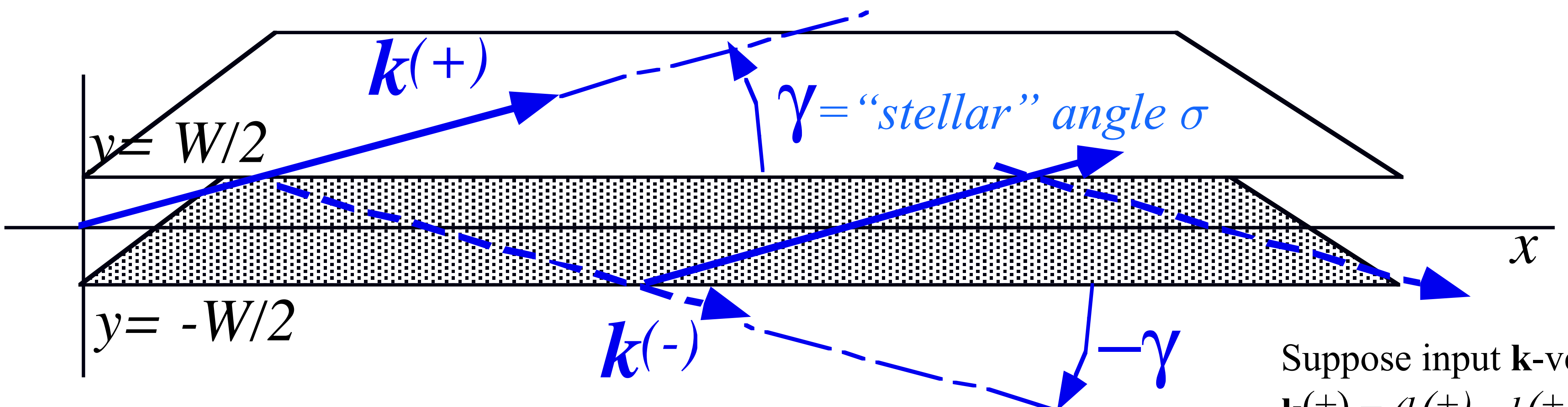

Fig. 6.3.1 "Hall of mirrors" model for an optical wave guide of width W.

Suppose input **k**-vector $\mathbf{k}^{(+)}$ enters at angle $+\gamma$.

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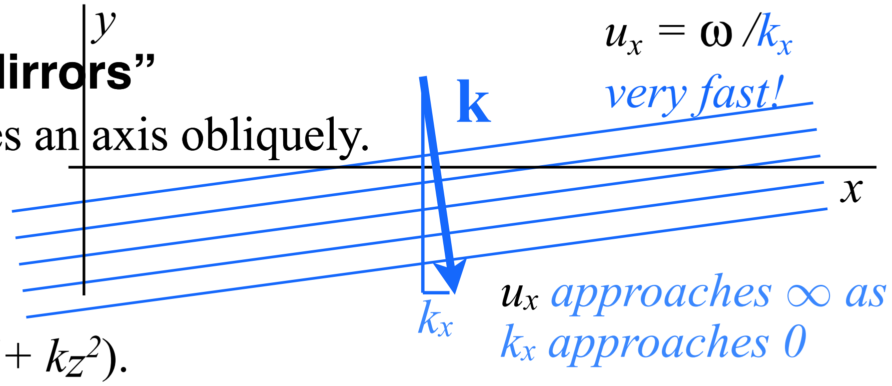
from:Fig. 6.3.1
QTforCA
Unit 2 Ch.6

2-Dimensional wave mechanics: guided waves and dispersion in the "Hall of Mirrors"



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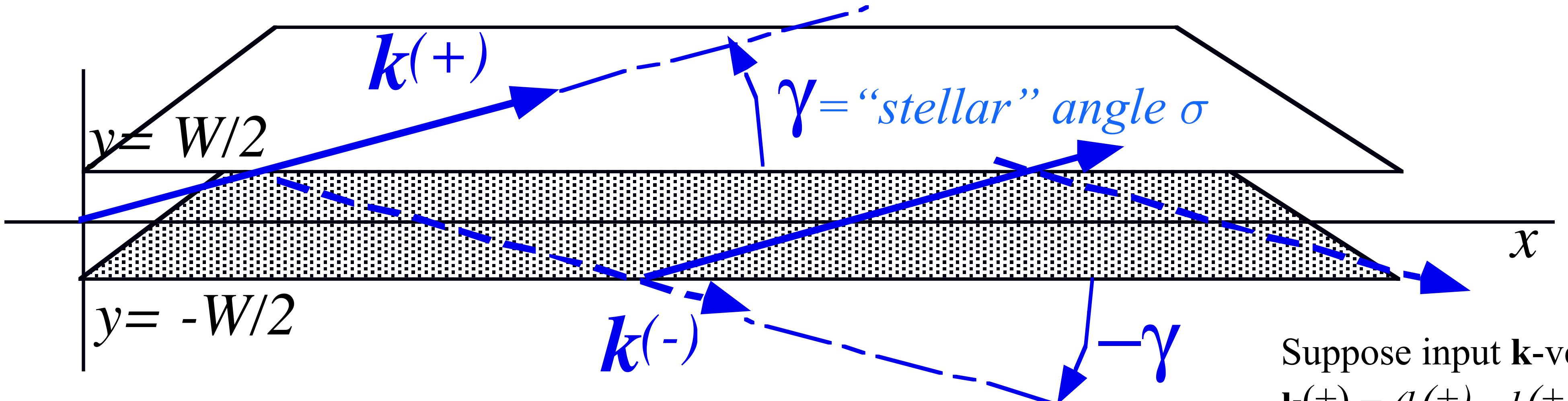

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from:Fig. 6.3.1
QTforCA
Unit 2 Ch.6

2-Dimensional wave mechanics: guided waves and dispersion in the "Hall of Mirrors"

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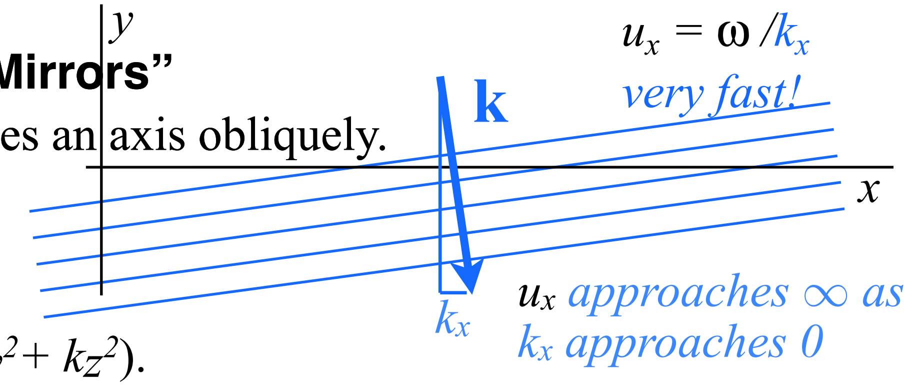

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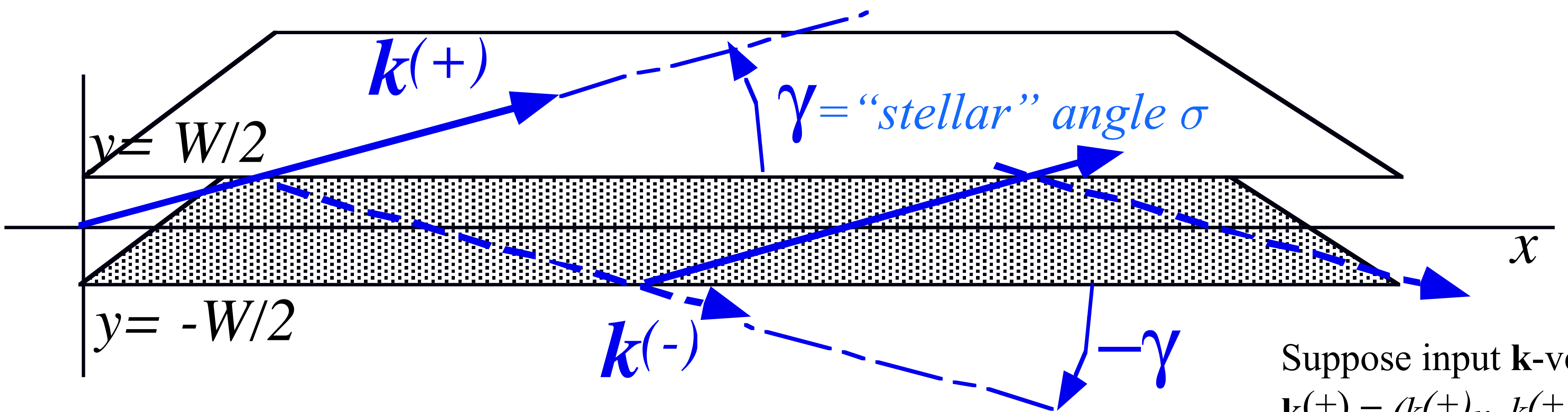

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$$= \exp i(kx \cos\gamma - \omega t) [\exp i(ky \sin\gamma) + \exp i(-ky \sin\gamma)]$$
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guide phase wave and group wave

from:Fig. 6.3.1
QTforCA
Unit 2 Ch.6

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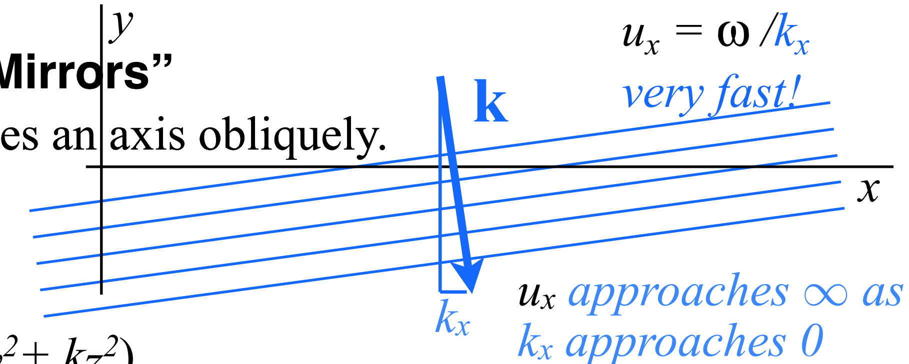

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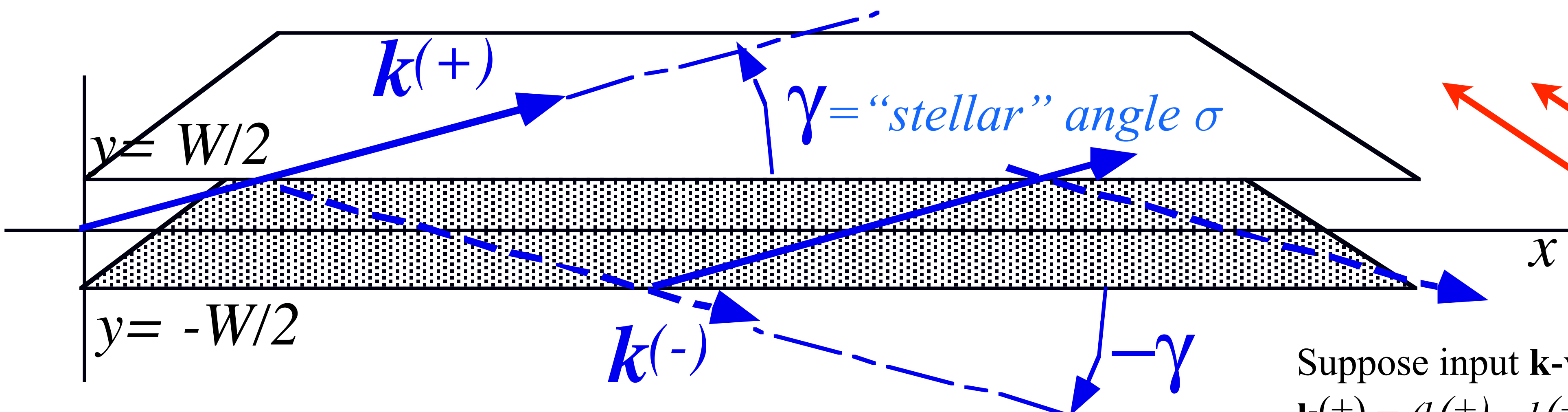
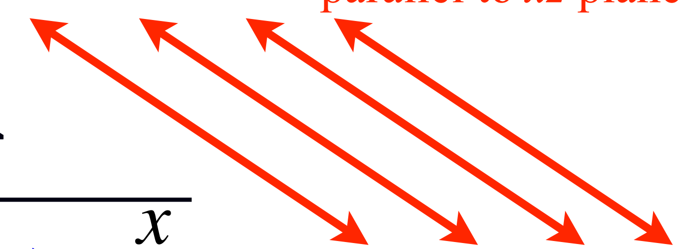


Fig. 6.3.1 "Hall of mirrors" model for an optical wave guide of width W.

Assume T_{ransverse}E_{lectric}-mode.
It always has **E** polarized parallel to xz plane



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guide phase wave and group wave

TE boundary conditions make group be zero on metal walls $y=\pm W/2$.

$$0=2 \cos(k(W/2) \sin\gamma) , \text{ or: } k(W/2) \sin\gamma = \pi/2 , \text{ or: } \sin\gamma = \pi/(kW)$$

from:Fig. 6.3.1
QTforCA
Unit 2 Ch.6

Combination and interference of 4-vector plane waves (Idealized polarization case)

- *Combination group and phase waves define 4D Minkowski coordinates*
- *Combination group and phase waves define wave guide dynamics*
  - *Waveguide dispersion and geometry*
    - *1st-quantized cavity modes*
    - *(And introducing 2nd-quantized cavity modes)*

Waveguide dispersion and geometry

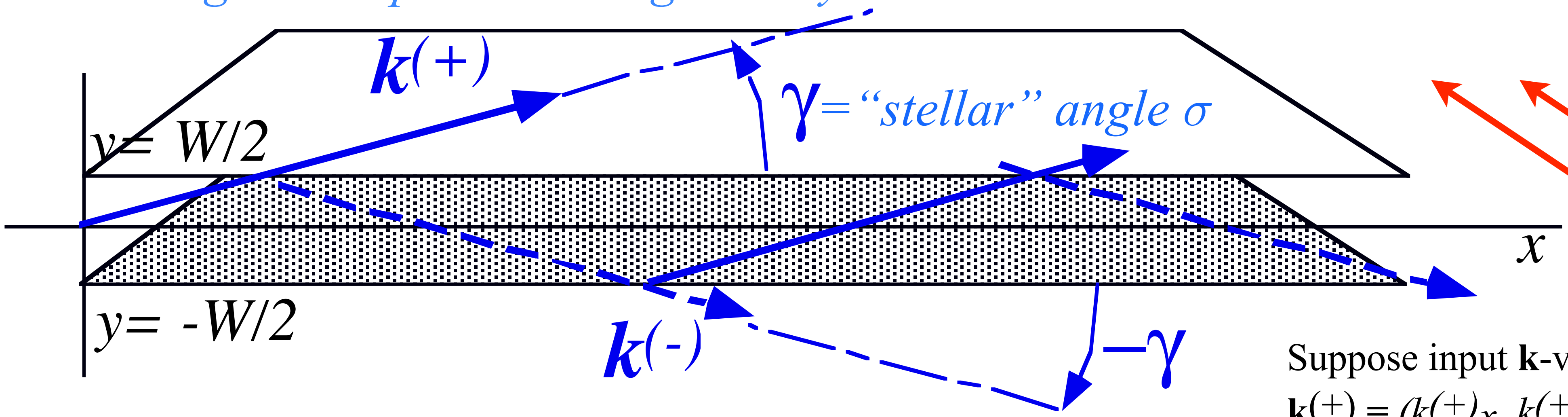
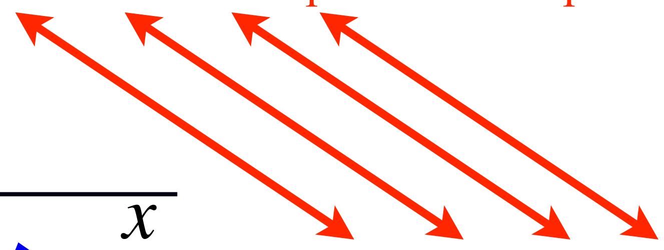


Fig. 6.3.1 "Hall of mirrors" model for an optical wave guide of width W.

Assume $T_{ransverse}E_{lectric}$-mode. It always has **E** polarized parallel to *xz* plane



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from:Fig. 6.3.1
QTforCA
Unit 2 Ch.6

Waveguide dispersion and geometry

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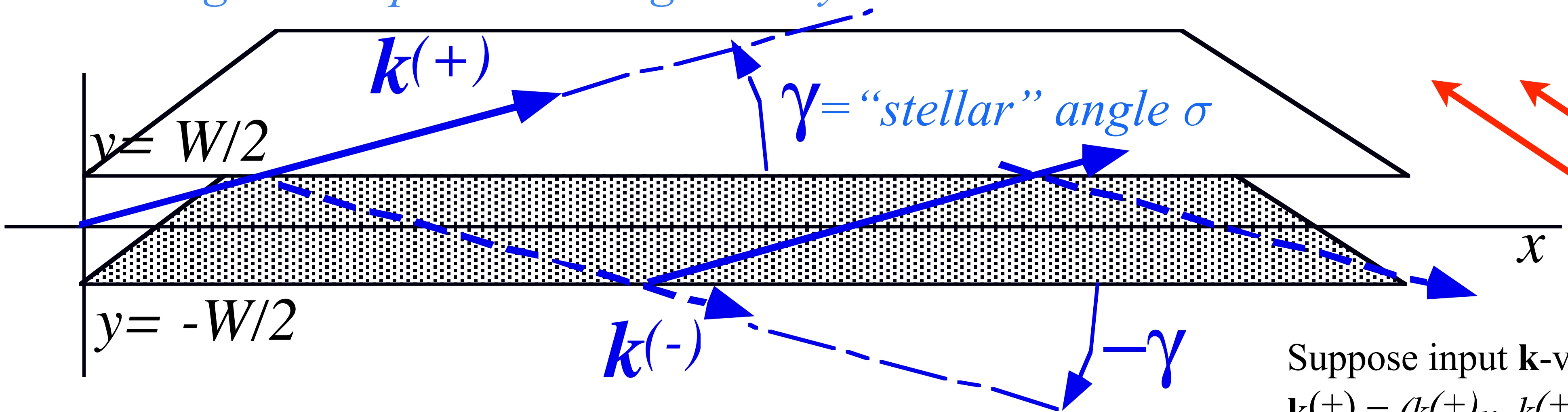

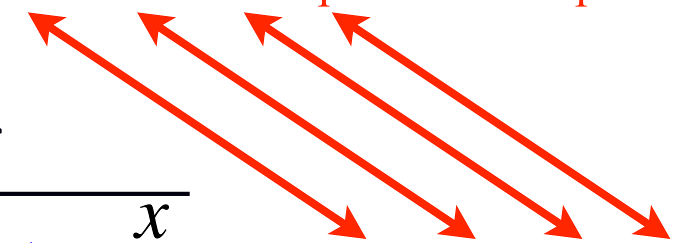

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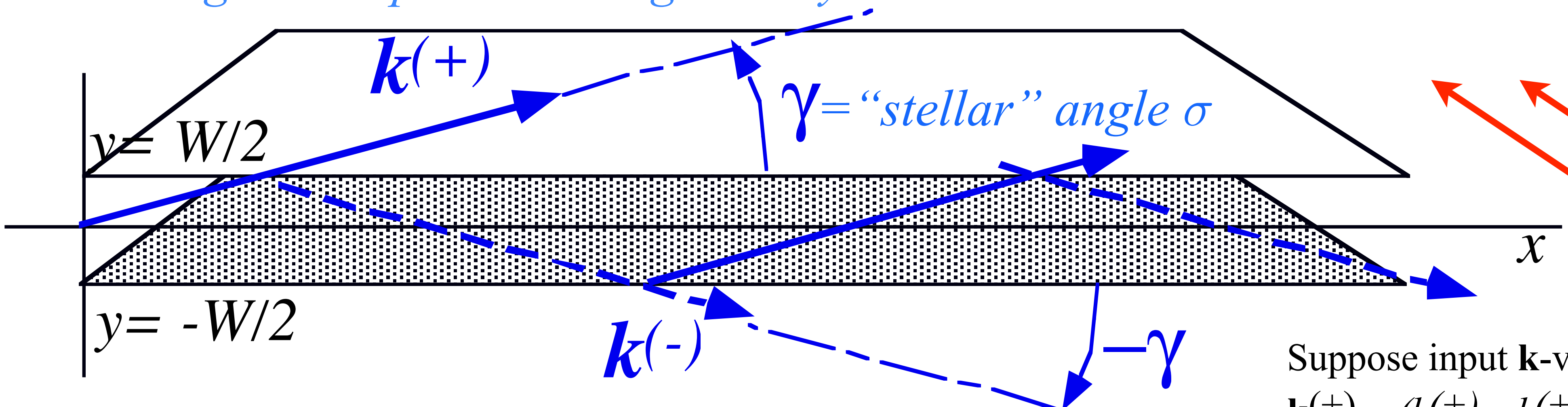
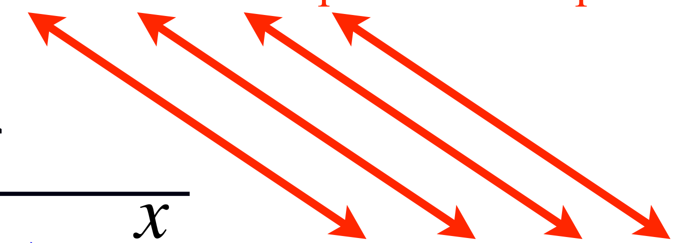
from:Fig. 6.3.1
QTforCA
Unit 2 Ch.6

Waveguide dispersion and geometry

Assume $T_{ransverse}E_{lectric}$-mode. It always has **E** polarized parallel to *xz* plane





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guide *phase wave* and *group wave*

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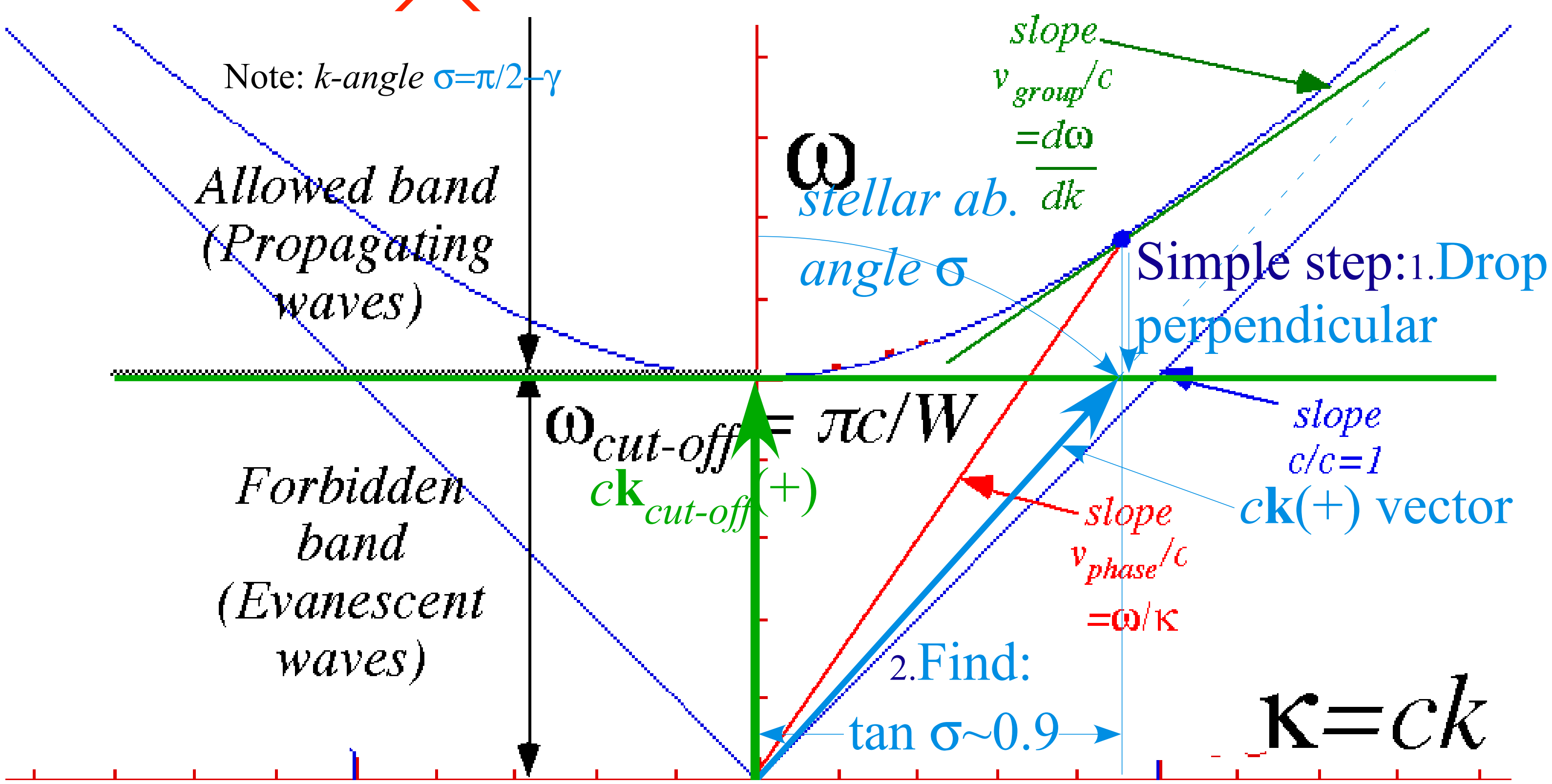


from:Fig. 6.3.2 (modified) QTforCA Unit 2 Ch.6

Fig. 6.3.2 Dispersion function for a fundamental TE wave guide mode

Waveguide dispersion and geometry

$$\omega = kc = c\sqrt{(k_x^2 + k_y^2 + k_z^2)} = c\sqrt{(k_x^2 + \pi^2/W^2)} = \sqrt{(c^2 k_x^2 + \omega_{cut}^2)}$$

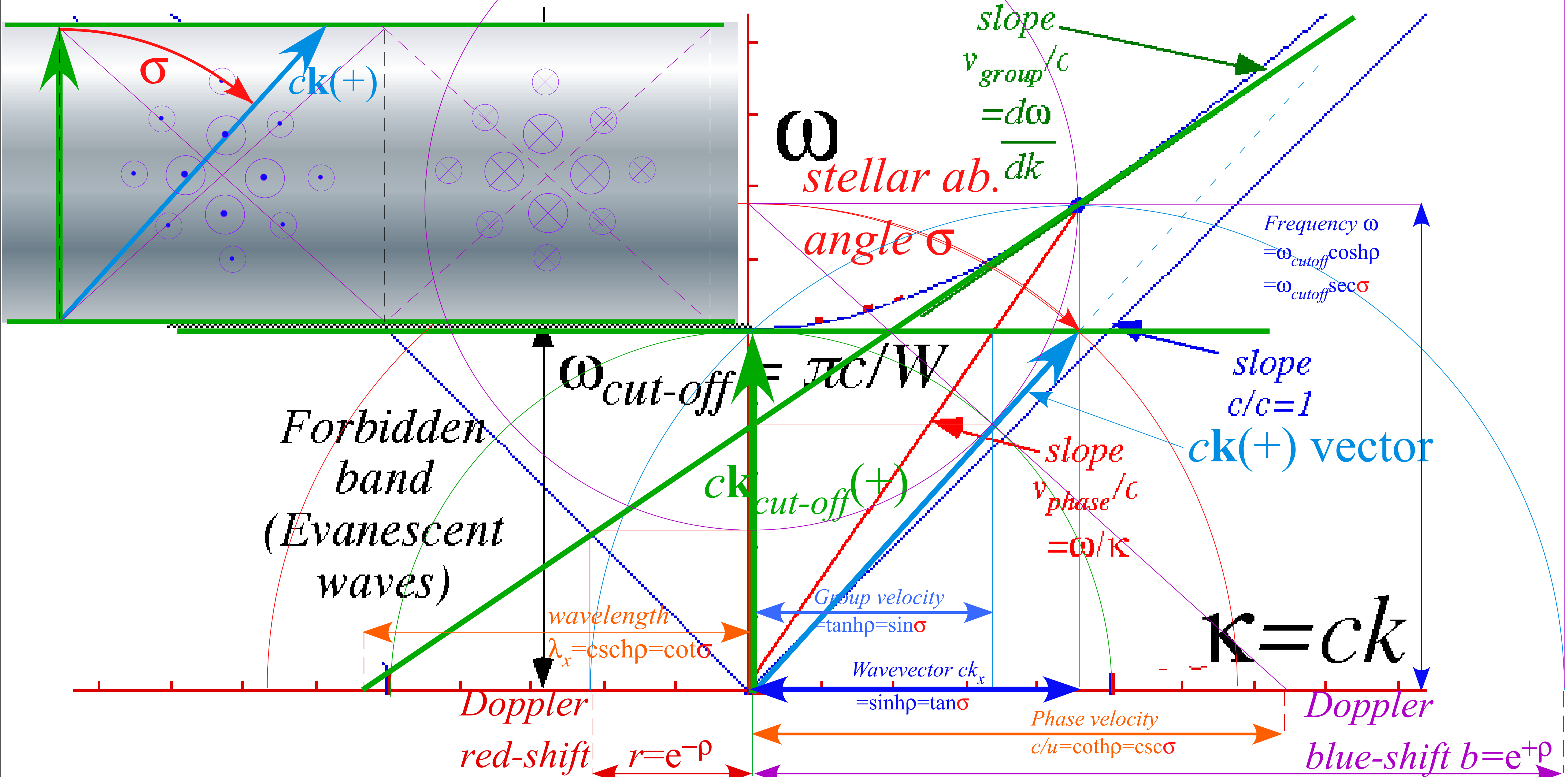


Fig. 6.3.2 Thales geometry of cavity or waveguide mode

from:Fig. 6.3.2 (modified) QTforCA Unit 2 Ch.6

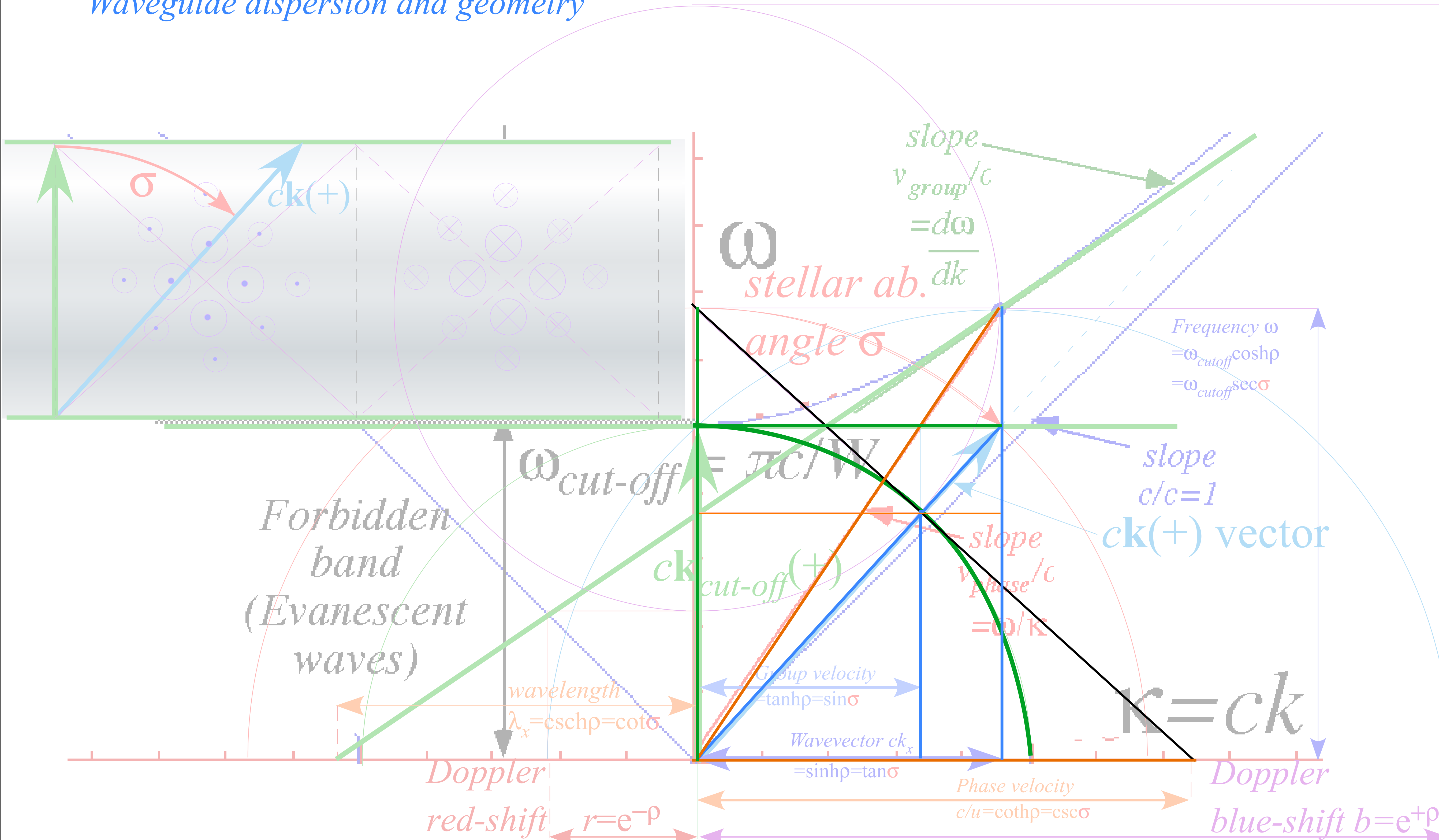
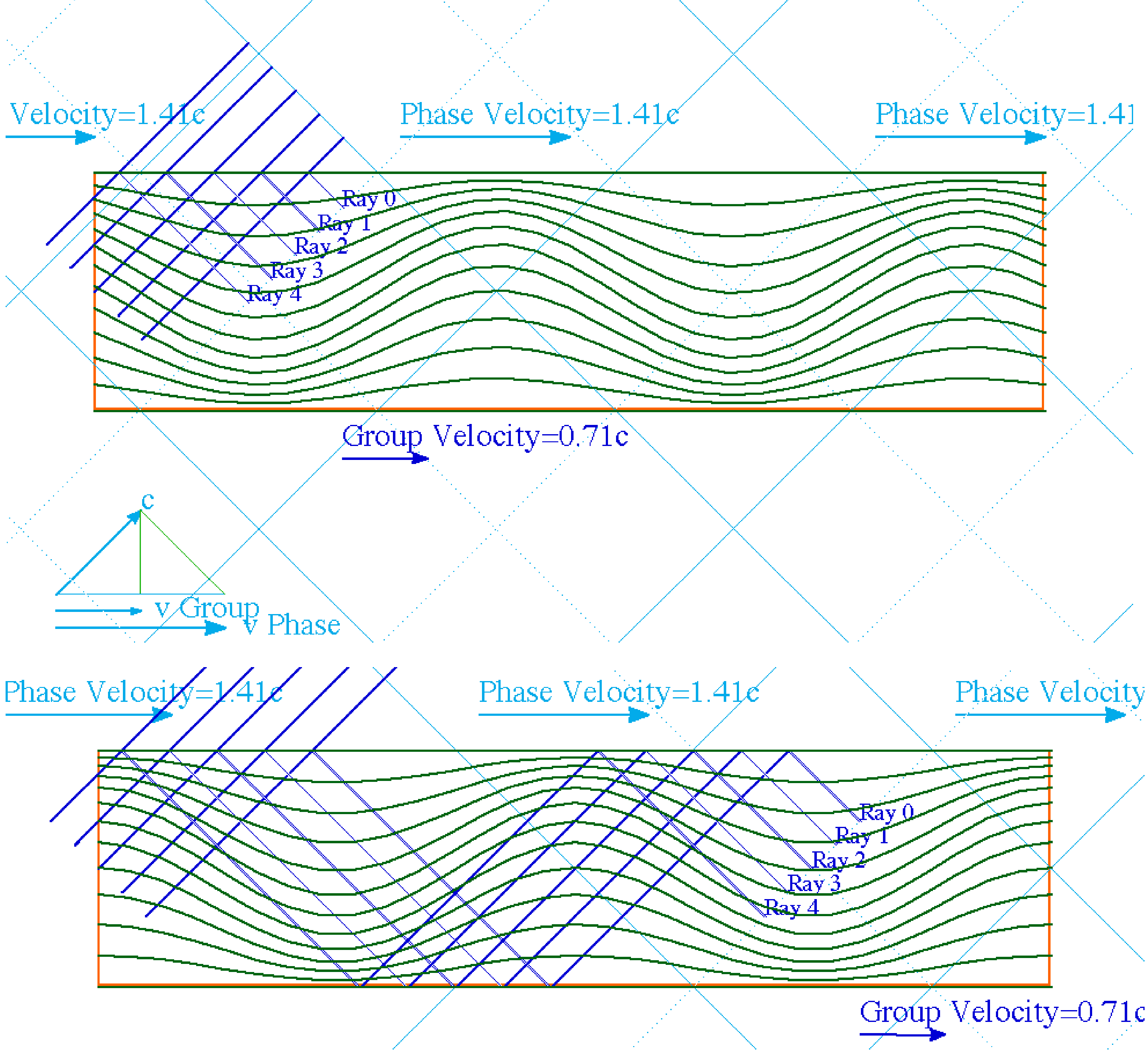


Fig. 6B.8 Thales geometry of cavity or waveguide mode

(Lecture 28 ends here)

Waveguide dispersion and geometry



Rare case!
Aberration angle
is σ=45°

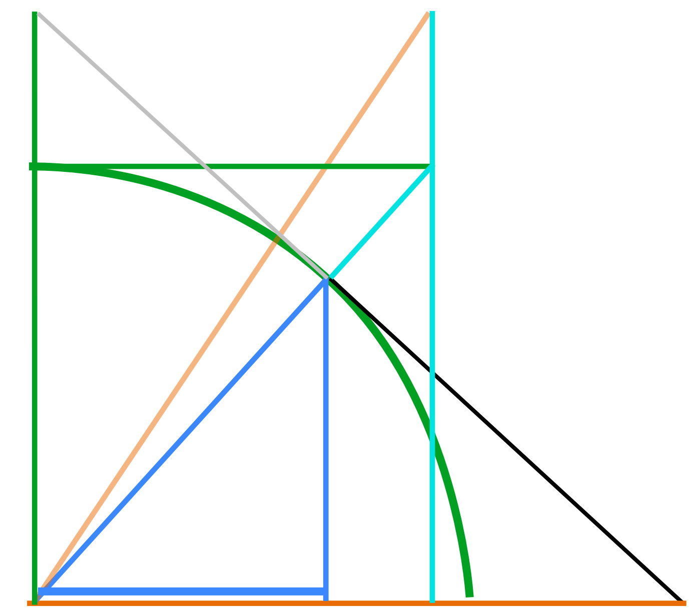


Fig. 6.3.4 Right moving guide wave with σ = 45°, $V_{phase}=\sqrt{2}c$, $V_{group}=c/\sqrt{2}$.

from:Fig. 6.3.4
QTforCA
Unit 2 Ch.6

Waveguide dispersion and geometry

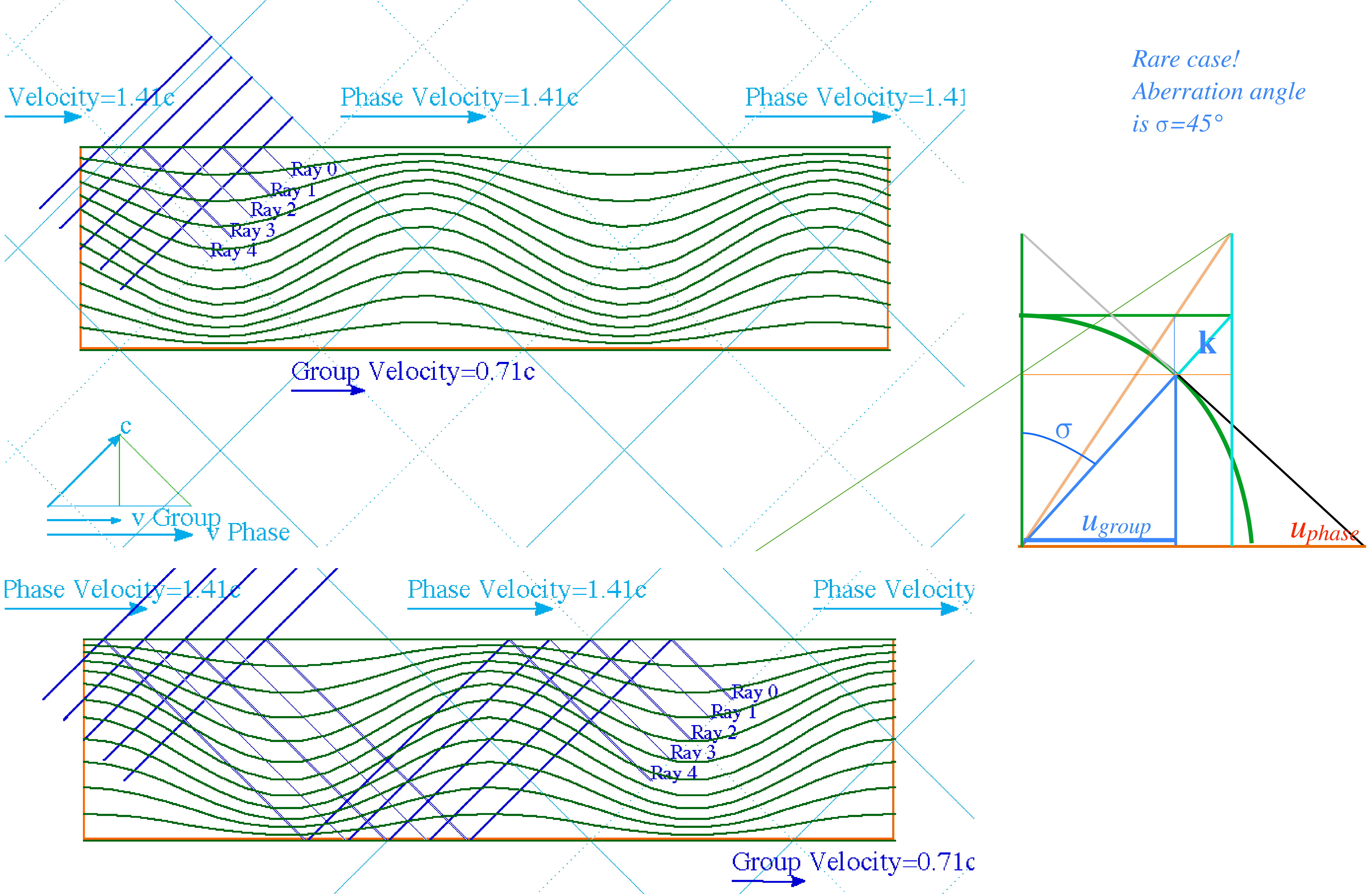


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Fig. 6.3.4 Right moving guide wave with $\sigma = 45°$, $V_{phase}=\sqrt{2}c$, $V_{group}=c/\sqrt{2}$.

from:Fig. 6.3.4
QTforCA
Unit 2 Ch.6

Waveguide dispersion and geometry

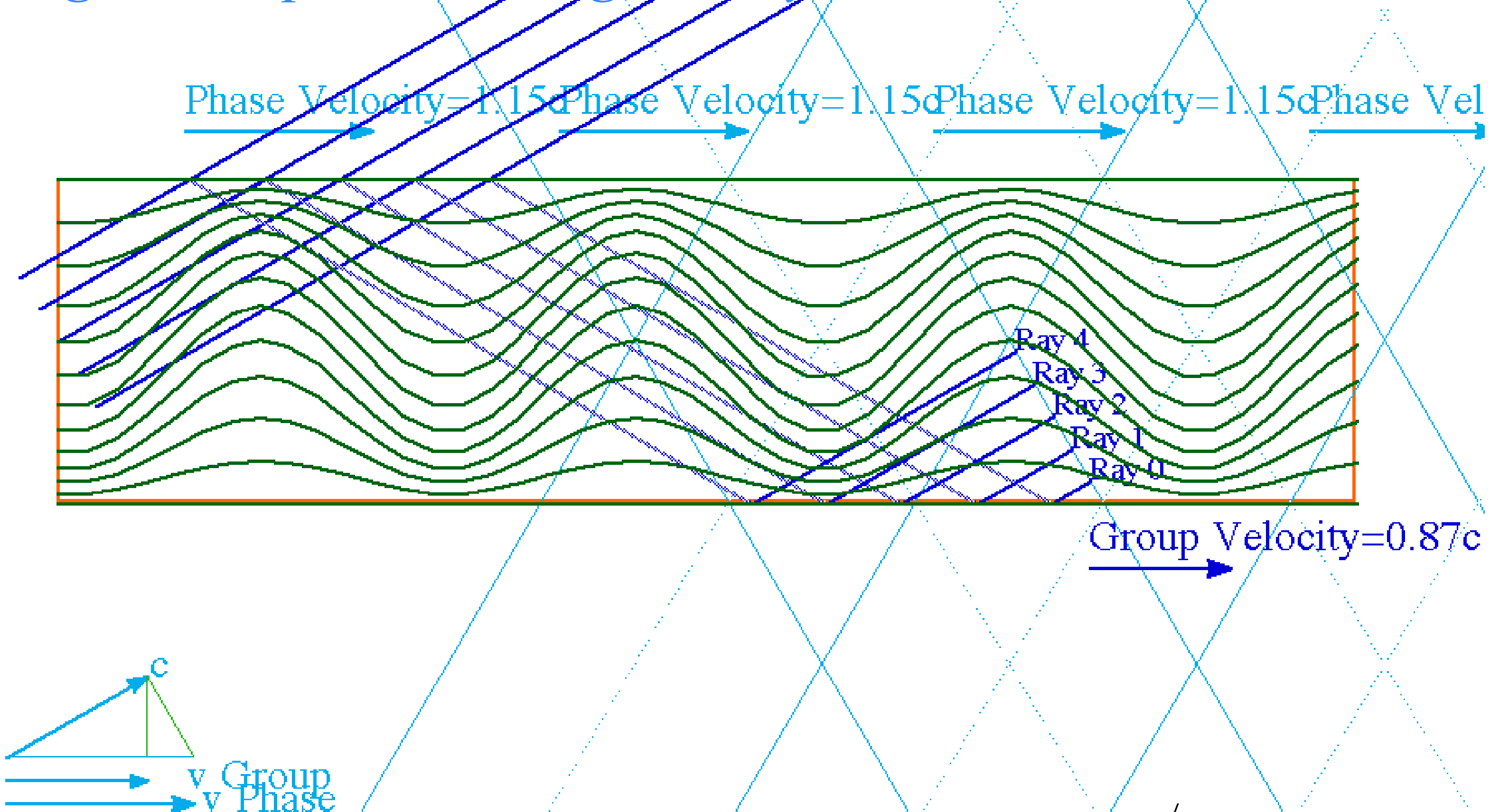
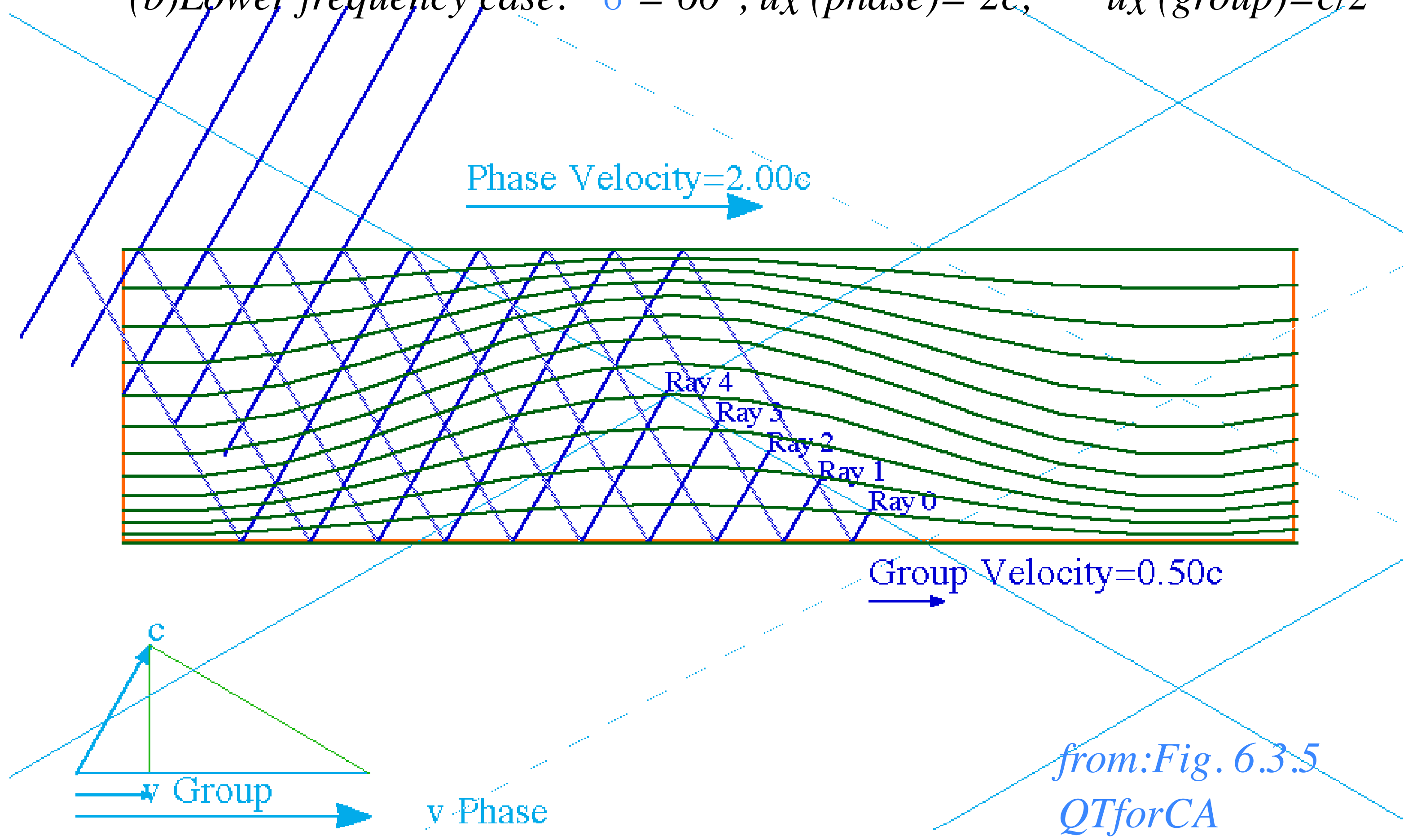


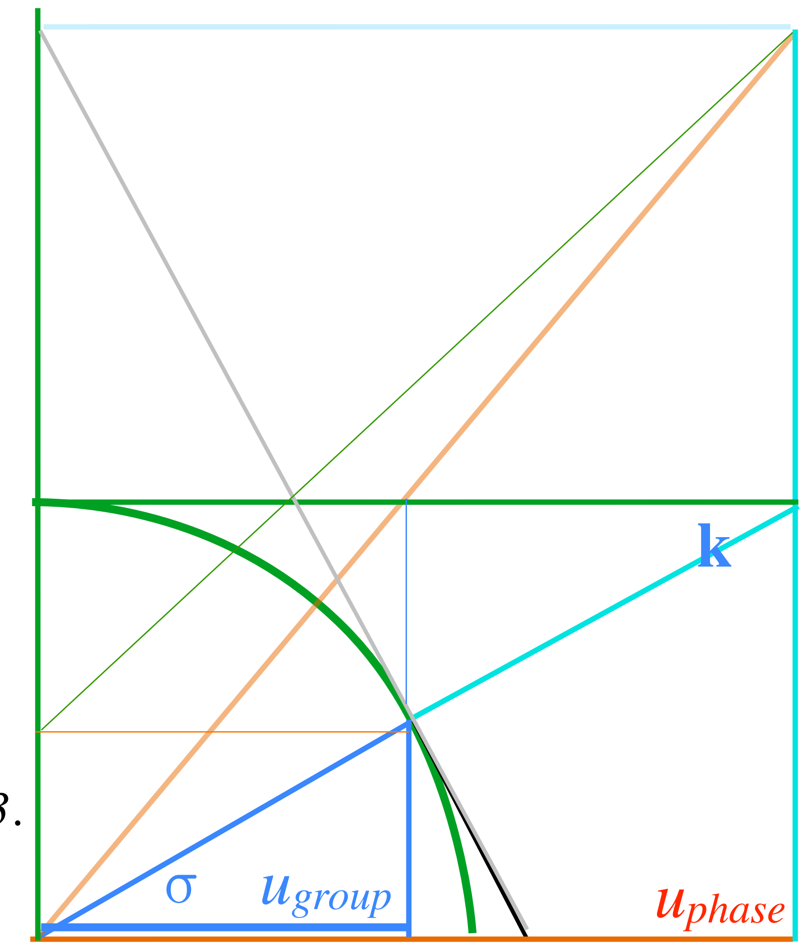
Fig. 6.3.5 Guide waves. (a)Higher frequency case: $\sigma = 30°$, u_x *(phase)*$=c\sqrt{3}/2c$, u_x *(group)*$=c2/\sqrt{3}$.

(b)Lower frequency case: $\sigma = 60°$, u_x *(phase)*$= 2c$, u_x *(group)*$=c/2$



$k_x = \sqrt{(\omega^2/c^2 - \pi^2/W^2)}$

$\omega = kc = \sqrt{(c^2 k_x^2 + \omega_{cut}^2)}$

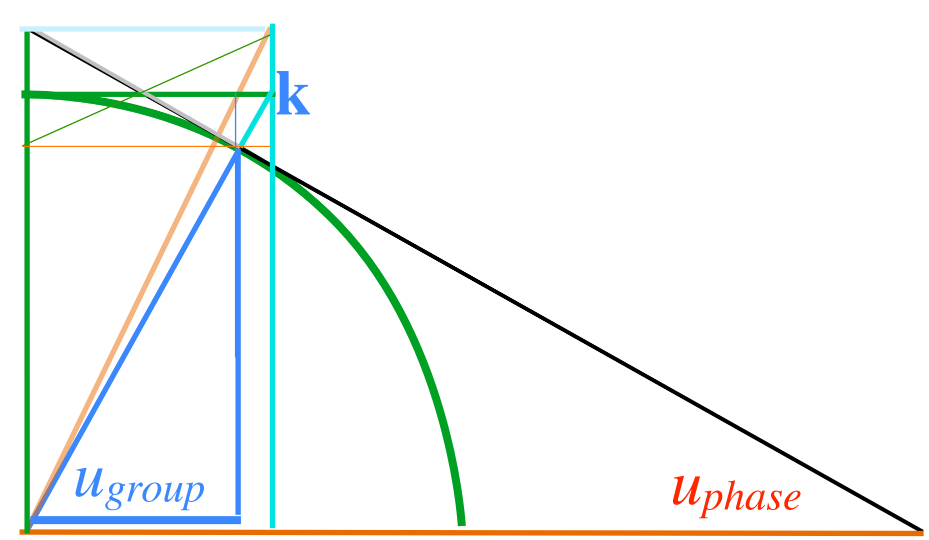


$u_x\ (phase) = \omega/k_x = c\omega/\sqrt{(\omega^2 - \pi^2c^2/W^2)}$

$= c/\cos\gamma = c/\sin\sigma = c \csc\sigma$

$u_x\ (group) = d\omega/dk_x = ck_x/\sqrt{(k_x^2 + \pi^2/W^2)}$

$= c\,(\omega^2 - \pi^2c^2/W^2)^{1/2}/\omega = c\cos\gamma = c\sin\sigma$



from:Fig. 6.3.5
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Unit 2 Ch.6

Combination and interference of 4-vector plane waves (Idealized polarization case)

Combination group and phase waves define 4D Minkowski coordinates

Combination group and phase waves define wave guide dynamics

Waveguide dispersion and geometry

➡ *1st-quantized cavity modes*

(And introducing 2nd-quantized cavity modes)

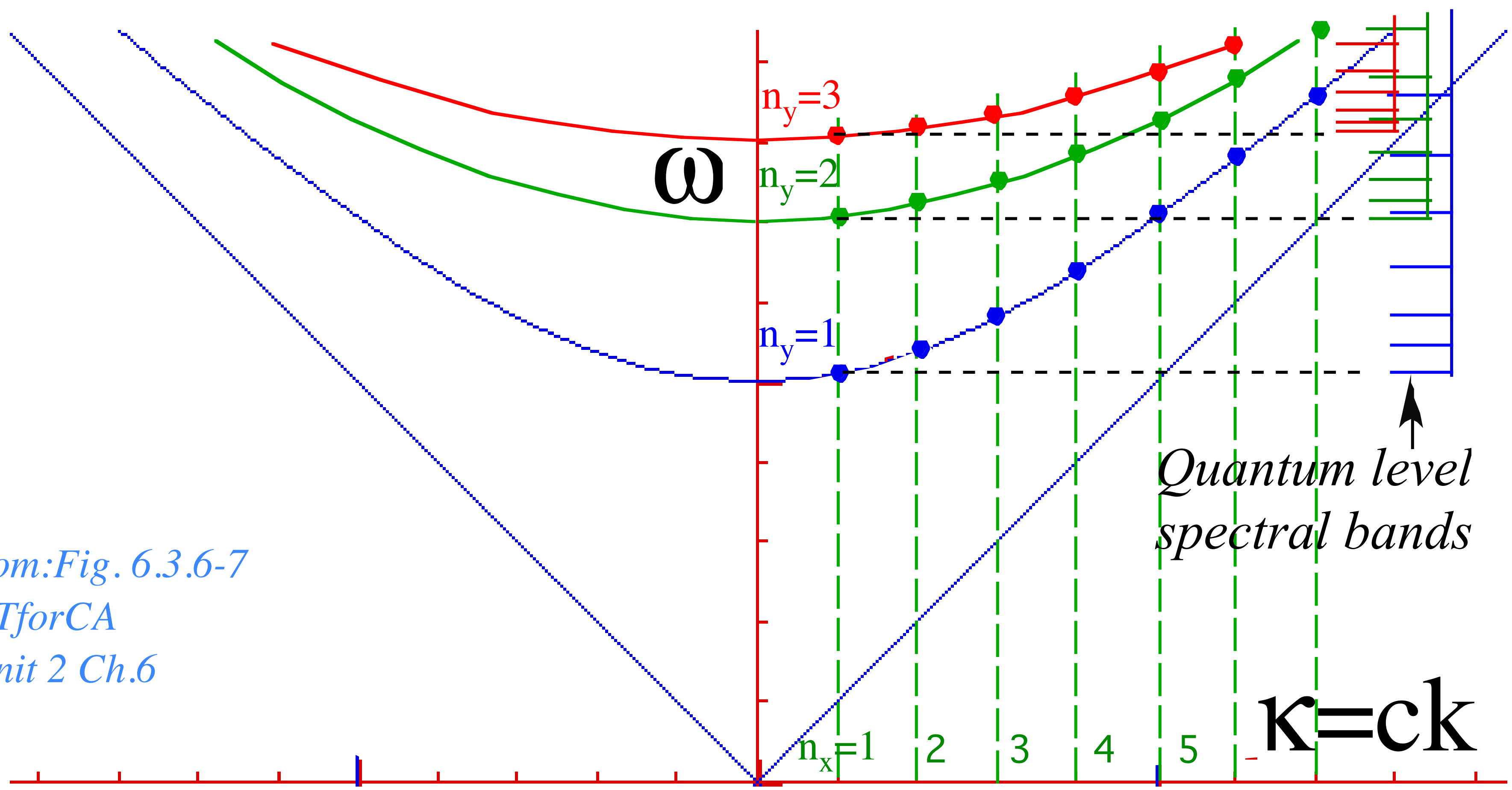
Cavity eigenfunctions and eigenvalues

Hall of Mirrors capped by a pair of doors at $x=0$ and $x=L$ becomes a *wave cavity* of length L.
The doors demand the wave electric field be zero at x-boundaries as well as along the walls. New boundary conditions:

$$k_x = k \cos\gamma = n_x \pi / L \qquad (n_x = 1, 2, \ldots)$$

Frequency bands are broken into discrete "quantized" values $\omega_{nx\, ny}$, one for each pair of integers or "quantum numbers" n_x and n_y .

$$\omega_{nx\, ny} = kc = c\sqrt{(k_x^2 + k_y^2 + k_z^2)} = c\sqrt{(n_x^2 \pi^2/L^2 + n_y^2 \pi^2/W^2)}$$



from:Fig. 6.3.6-7
QTforCA
Unit 2 Ch.6

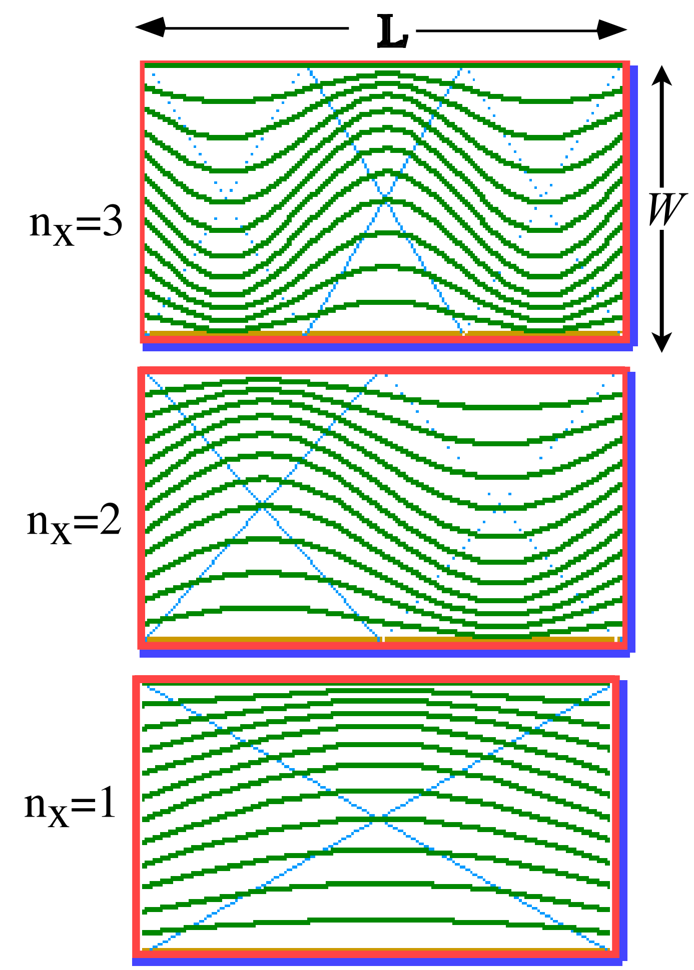


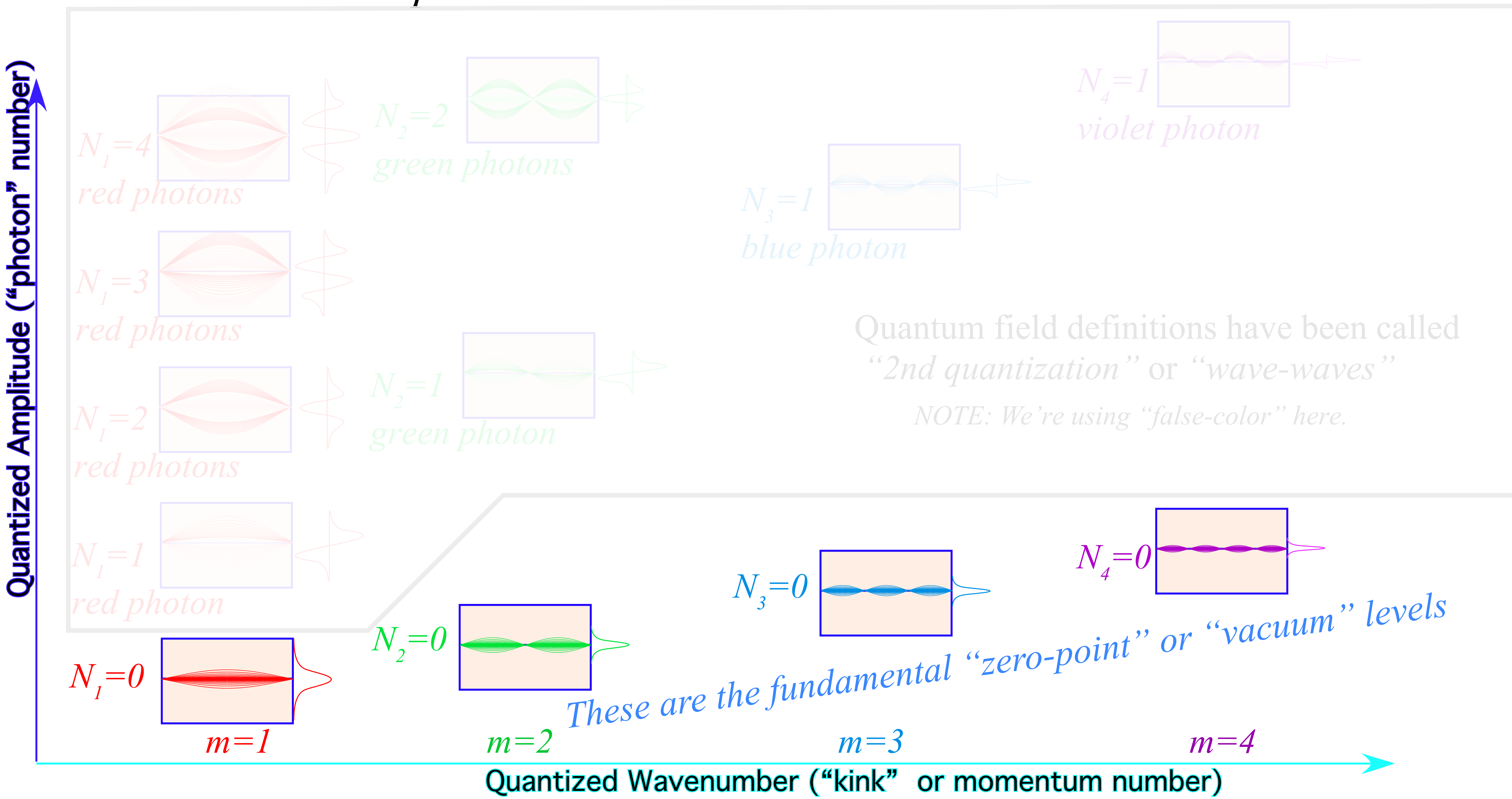
Fig. 6.3.7 Cavity modes for three lowest quantum numbers

Fig. 6.3.6 Cavity mode dispersion diagram showing overlapping and discrete ω and k values.

Quantized *Amplitude* *Counting "photon" number*

Planck's relation $E=Nh\upsilon$ began as a tenative axiom to explain low-T light. Then he tried to disavow it! Einstein picked it up in his 1905 paper. Since then its use has grown enormously and continues to amaze, amuse (or bewilder) all who study it.

A current view is that it represents the *quantization* of optical field *amplitude*. We picture this below as *N-photon* wave states for each box-mode of *m* wave kinks.



Quantized *Amplitude* *Counting "photon" number*

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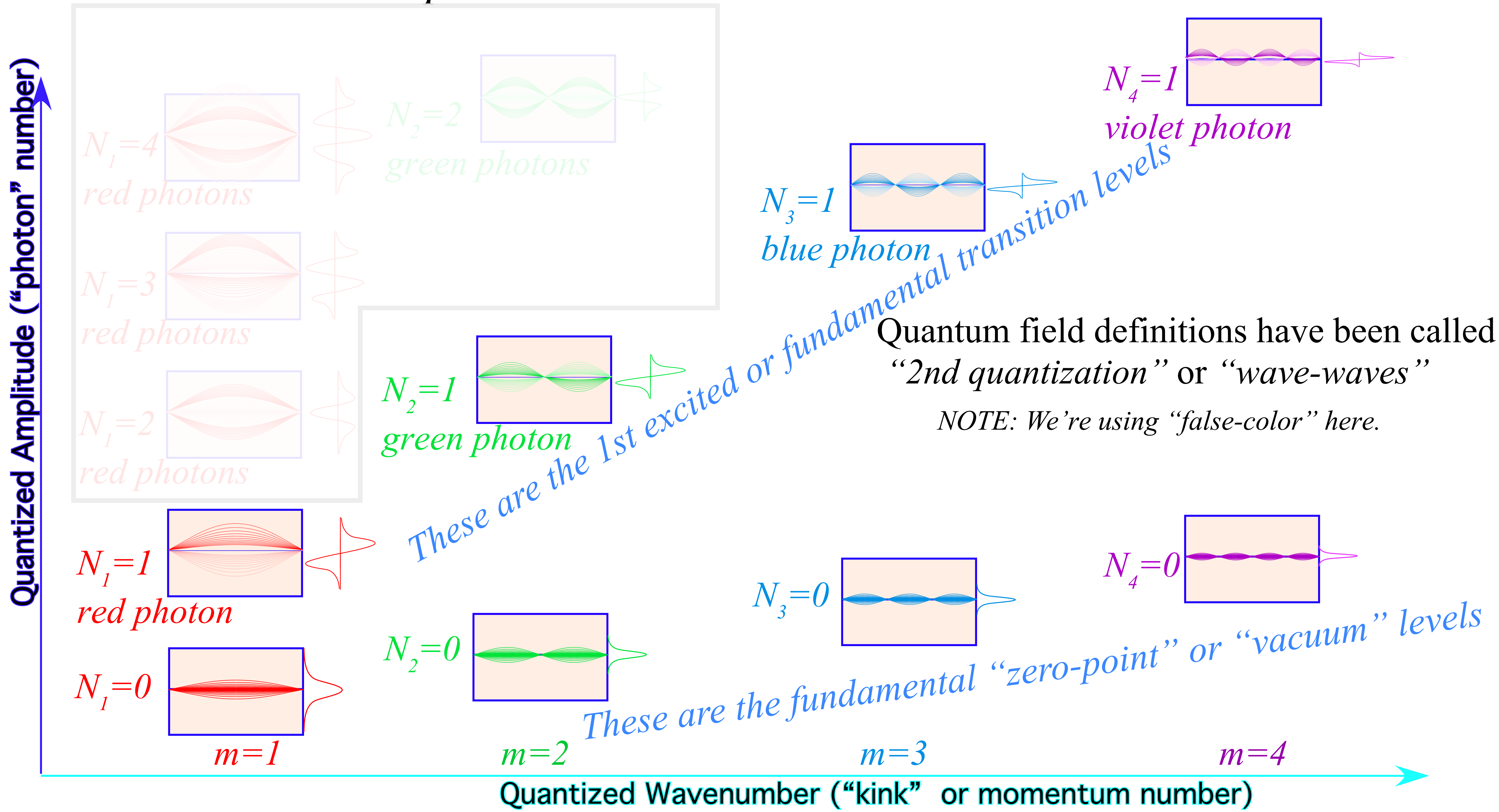

Lorentz symmetry effects

How it makes momentum and energy be conserved

A strength (and also, weakness) of CW axioms (1.1-2) is that they are *symmetry* principles

due to the Lorentz-Poincare isotropy of space-time (invariance to space-time translation $\mathbf{T}(\delta,\tau)$ in the vacuum).

Operator $\mathbf{T}$ has plane wave eigenfunctions $\psi_{k,\omega} = Ae^{i(kx-\omega t)}$ with roots-of-unity eigenvalues $e^{i(k\cdot\delta-\omega\cdot\tau)}$.

$$\left\langle \psi_{k,\omega}\right|\mathbf{T}^{\dagger} = \left\langle \psi_{k,\omega}\right| e^{-i(k\cdot\delta-\omega\cdot\tau)} \qquad (5.18a) \qquad\qquad \mathbf{T}\left|\psi_{k,\omega}\right\rangle = e^{i(k\cdot\delta-\omega\cdot\tau)}\left|\psi_{k,\omega}\right\rangle \qquad (5.18b)$$

This also applies to 2-part or "2-particle" product states $\Psi_{K,\Omega} = \psi_{k_1,\omega_1}\psi_{k_2,\omega_2}$ where exponents add *(k,*ω*)*-values of

each constituent to $K=k_1+k_2$ and $\Omega=\omega_1+\omega_2$, and $\mathbf{T}(\delta,\tau)$-eigenvalues also have that form $e^{i(K\cdot\delta-\Omega\cdot\tau)}$.

Matrix $\left\langle \Psi'_{K',\Omega'}\right|\mathbf{U}\left|\Psi_{K,\Omega}\right\rangle$ of **T**-symmetric evolution **U** is zero unless $K' = k'_1 + k'_2 = K$ and $\Omega' = \omega'_1 + \omega'_2 = \Omega$.

$$\left\langle \Psi'_{K',\Omega'}\right|\mathbf{U}\left|\Psi_{K,\Omega}\right\rangle = \left\langle \Psi'_{K',\Omega'}\right|\mathbf{T}^{\dagger}(\delta,\tau)\mathbf{U}\mathbf{T}(\delta,\tau)\left|\Psi_{K,\Omega}\right\rangle \qquad (\text{if } \mathbf{UT}=\mathbf{TU} \text{ for all } \delta \text{ and } \tau)$$

$$= e^{-i(K'\cdot\delta-\Omega'\cdot\tau)}e^{i(K\cdot\delta-\Omega\cdot\tau)}\left\langle \Psi'_{K',\Omega'}\right|\mathbf{U}\left|\Psi_{K,\Omega}\right\rangle = 0 \;\; \text{unless: } K' = K \text{ and: } \Omega' = \Omega$$

That's momentum (P=hK) and energy (E=hW) conservation!