Relativity of lightwaves and Lorentz-Minkowski coordinates IV.
(Ch. 0-3 of Unit 8)

More connections to conventional approach to relativity and old-fashioned formulas
Catching up to light (Coyote finally triumphs! Rest-frame at last.)
The most old-fashioned form(ula) of all: Thales & Euclid means
Galileo wins one! (...in gauge space) That “old-time” relativity (Circa 600BCE- 1905CE)

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Suppose you see two counter-propagating laser beams $\omega_R \rightarrow$ going right and $\omega_L \leftarrow$ going left.

Catching up to light (Coyote finally triumphs! Rest-frame at last.)

$$(\omega_R \rightarrow, c k_R \rightarrow) \text{ meets } (\omega_L \leftarrow, -c k_L \leftarrow)$$

$$= (4, +4c) \quad \quad = (1, -1c)$$
Catching up to light (Coyote finally triumphs! Rest-frame at last.)

Suppose you see two counter-propagating laser beams $\omega_{R\to}$ going right and $\omega_{L\leftarrow}$ going left.

Q1: How fast do you go to “catch up” to see both as the same color (frequency $\omega$)?

\[
(\omega_{R\to}, \ c k_{R\to}) \text{ meets } (\omega_{L\leftarrow}, -c k_{L\leftarrow}) = (4, +4c) = (1, -1c)
\]
Catching up to light (Coyote finally triumphs! Rest-frame at last.)

Suppose you see two counter-propagating laser beams $\omega \rightarrow$ going right and $\omega \leftarrow$ going left.

Q1: How fast do you go to “catch up” to see both as the same color (frequency $\omega_A$)?

Q2: What is that color (frequency $\omega_A$)?

$$(\omega_{R\rightarrow}, c k_{R\rightarrow}) \text{ meets } (\omega_{L\leftarrow}, -c k_{L\leftarrow})$$

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“Jeopardy” answers:

**A1:** How fast is the group velocity?
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Suppose you see two counter-propagating laser beams \( \omega_R \rightarrow \) going right and \( \omega_L \leftarrow \) going left.

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“Jeopardy” answers:

**A1:** How fast is the group velocity? 

\[
\frac{V_{\text{group}}}{c} = \frac{\omega_{R \rightarrow} - \omega_{L \leftarrow}}{ck_{R \rightarrow} - ck_{L \leftarrow}} = \frac{\omega_{R \rightarrow} - \omega_{L \leftarrow}}{\omega_{R \rightarrow} + \omega_{L \leftarrow}} = \frac{4 - 1}{4 + 1} = \frac{3}{5}
\]
Catching up to light (Coyote finally triumphs! Rest-frame at last.)

Suppose you see two counter-propagating laser beams \( \omega_{R\rightarrow} \) going right and \( \omega_{L\leftarrow} \) going left.

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A2: What is the geometric mean of \( \omega_{R\rightarrow} \) and \( \omega_{L\leftarrow} \)?
Catching up to light (Coyote finally triumphs! Rest-frame at last.)

Suppose you see two counter-propagating laser beams $\omega_R \rightarrow$ going right and $\omega_L \leftarrow$ going left.

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**A2:** What is the geometric mean of $\omega_R \rightarrow$ and $\omega_L \leftarrow$?

$$\omega_A = \sqrt{\omega_R \rightarrow \cdot \omega_L \leftarrow} = \sqrt{4 \cdot 1} = 2$$
Catching up to light (Coyote finally triumphs! Rest-frame at last.)

Suppose you see two counter-propagating laser beams $\omega_R \rightarrow$ going right and $\omega_L \leftarrow$ going left.

Q1: How fast do you go to “catch up” to see both as the same color (frequency $\omega_A$)?

Q2: What is that color (frequency $\omega_A$)?

“Jeopardy” answers:

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$$V_{\text{group}} = \frac{\omega_{R\rightarrow} - \omega_{L\leftarrow}}{c} = \frac{\omega_{R\rightarrow} - \omega_{L\leftarrow}}{c k_{R\rightarrow} - c k_{L\leftarrow}} = \frac{\omega_{R\rightarrow} - \omega_{L\leftarrow}}{\omega_{R\rightarrow} + \omega_{L\leftarrow}} = \frac{4 - 1}{4 + 1} = \frac{3}{5}$$

$A_2$: What is the geometric mean of $\omega_{R\rightarrow}$ and $\omega_{L\leftarrow}$?

$$\omega_A = \sqrt{\omega_{R\rightarrow} \cdot \omega_{L\leftarrow}} = \sqrt{4 \cdot 1} = 2$$

If you accelerate to $V_{\text{group}} = \frac{3}{5} c$ then you see...

...a standing wave... (assuming equal amplitudes, coherence, etc.)
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If you accelerate to \(V_{\text{group}} = \frac{3}{5} c\) then you see...

...a standing wave...(assuming equal amplitudes, coherence, etc.)

\[(\omega_{A \rightarrow}, ck_{A \rightarrow})\] meets \[(\omega_{A \leftarrow}, -ck_{A \leftarrow})\]
\[
= (2, +2c) \quad \rightarrow \quad (2, -2c)
\]

\[
(\omega_{\text{phase}}, ck_{\text{phase}}) \quad \text{and} \quad (\omega_{\text{group}}, ck_{\text{group}})
\]
\[
= (2, 0c) \quad \text{and} \quad (0, 2c)
\]
Catching up to light (Coyote finally triumphs! Rest-frame at last.)
The most old-fashioned form ula of all: Thales & Euclid means
Galileo wins one! (...in gauge space) That “old-time” relativity (Circa 600 BCE- 1905 CE)
Euclid’s 3-means (300 BC)
Geometric “heart” of wave mechanics

Thales (580BC) rectangle-in-circle
Relates to wave interference by (Galilean) phasor angular velocity addition

geometric mean: $\frac{1/2}{1 \cdot 4} = 2$
difference mean: $\frac{1/2}{[4-1]} = \frac{3}{2}$
(HALF-DIFFERENCE)
arithmetic mean: $\frac{1/2}{[4+1]} = \frac{5}{2} = \frac{5}{2}$
(HALF-SUM)
frequency

Linear velocity $V_{\text{group}}/c = u/c$
is (HALF-DIFF./HALF-SUM) = $\frac{3}{5}$

Fig. 3.3a Euclidian mean geometry for counter-moving waves of frequency 1 and 4. (300THz units).

Sites for animation:
http://www.uark.edu/ua/pirelli/php/means_1.php
http://www.uark.edu/ua/pirelli/php/half_sum_2.php
Fig. 3.1 Wave phasor addition. (a) Each phasor in a wave array is a sum (b) of two component phasors. (c) Phasor-relative views

Galileo’s revenge!
Galileo wins one (in gauge space)
Now we use Galilean relativity to add angular velocity, that is frequency \( \omega_a \) and \( \omega_b \), in phasor or "gauge" space. No "c-limit" evident. (So far at 18-fig. precision.)

\[ \sum \Psi_{A+B} = \Psi_A + \Psi_B \]

\[ \sum \Psi_{A-B} = \Psi_A - \Psi_B \]
That “old-time” relativity  *(Circa 600BCE- 1905CE)*

(“Bouncing-photons” in smoke & mirrors and Thales, again)

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Comparing Ship and Lighthouse views: Happening tables

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<tr>
<td>(Lighthouse space) $x = 0$</td>
<td>$x = -1.00 , c$</td>
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<td>(Lighthouse time) $t = 0$</td>
<td>$t = 2.00$</td>
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<td>$x' = 0$</td>
<td>$x' = c \Delta$</td>
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<td>(Ship time) $t' = 0$</td>
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Fig. 2.A.3 Happening 1 (1st blink hits ship) and 2 (2nd blink at Main) both happen at $t=2$. 
The ship and lighthouse saga

Happening 0.5: Main Lite blinks first time.

| Lighthouse | x = 0 | t = 1.00 |
| Ship | x' = 0 | t' = ?? |

Ship v/c (rel. to lighthouse) = -0.50

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**The ship and lighthouse saga**

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<tr>
<td>Ship:</td>
<td>$x' = 0$</td>
<td>$t' = \Delta = ????$</td>
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**Ship Time $t' = \Delta = ???$**

**Lighthouse $t = 1.00$**

**Ship $v/c$ (rel. to lighthouse) = -0.50**

**Ship Time $t' = \Delta = ???$**

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**Fig. 2.A.3** Happening 1 (1st blink hits ship) and 2 (2nd blink at Main) both happen at $t=2$. 

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Thursday, January 30, 2014
The ship and lighthouse saga

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Fig. 2.A.3 Happening 1 (1st blink hits ship) and 2 (2nd blink at Main) both happen at t=2.
The ship and lighthouse saga

Happening 0.5:
Main Lighthouse blinks first time.

| Lighthouse: x = 0 |
| Lighthouse: t = 1.00 |
| Ship: x' = 0 |
| Ship: t' = Δ = ?? |

Ship Time $t' = Δ = 1/\sqrt{1-v^2/c^2} = \cosh ρ$

$c^2Δ^2 = c^2 + v^2Δ^2$

$(c^2 - v^2)Δ^2 = c^2$

$Δ^2 = \frac{c^2}{(c^2 - v^2)} = \frac{1}{1 - v^2/c^2}$

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*Fig. 2.4.3 Happening 1 (1st blink hits ship) and 2 (2nd blink at Main) both happen at $t=2$.*
Happening 0:
Ship passes Main Lighthouse.
(Lighthouse space) \( x = 0 \)
(Lighthouse time) \( t = 0 \)
(Ship space) \( x' = 0 \)
(Ship time) \( t' = 0 \)

Happening 1: Ship gets hit by first blink from Main Lighthouse.

\[ \frac{c}{\Delta} = \sqrt{\frac{c^2 + v^2 \Delta^2}{c^2}} \]

\[ \Delta = \sqrt{\frac{c^2}{1-v^2/c^2}} = \cosh \rho = 1.15 \]

For \( u/c = 1/2 \)
\[ \Delta = \sqrt{\frac{1}{1-1/4}} = \sqrt{3} = 1.15.. \]

Happening 2: Main Lighthouse blinks second time.

\[ x = 0 \]
\[ t = 2.00 \]
\[ x' = 0 \]
\[ t' = 1.75 \]
\[ x' = c \Delta \]
\[ t' = 2\Delta = 2.30 \]

*Fig. 2.A.3 Happening 1 (1st blink hits ship) and 2 (2nd blink at Main) both happen at \( t=2 \).*
\[ \text{Ship Time } t' = \Delta = 1/\sqrt{(1-v^2/c^2)} = \cosh \rho = 1.15 \]

For \( u/c = 1/2 \)
\[ \Delta = 1/\sqrt{(1-1/4)} = 2/\sqrt{3} = 1.15 \ldots \]

Comparing Ship and Lighthouse views:

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<td>( x = 0 )</td>
</tr>
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<td>( x' = 2\Delta )</td>
</tr>
<tr>
<td>(Ship time) ( t' = 0 )</td>
<td>(Ship time) ( t' = (v+c)\Delta/c )</td>
<td>( t' = 2\Delta )</td>
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Fig. 2.4.3 Happening 1 (1st blink hits ship) and 2 (2nd blink at Main) both happen at \( t = 2 \).  Lecture 24 ended here
That “old-time” relativity (Circa 600BCE- 1905CE)

("Bouncing-photons" in smoke & mirrors and Thales, again)

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Fig. 2.B.5 Space-Space-Time plot of world lines for Lighthouses. North Lighthouse blink waves trace light cones.
That “old-time” relativity (Circa 600BCE-1905CE)

(“Bouncing-photons” in smoke & mirrors and Thales, again)

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Fig. 2.B.1 Town map according to a "tipsy" surveyor.

<table>
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<tr>
<td>(US surveyor)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x = 0$</td>
<td>$x = 0.5$</td>
<td>$x = 0$</td>
</tr>
<tr>
<td>$y = 0$</td>
<td>$y = 1.0$</td>
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</tr>
<tr>
<td>(French surveyor)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x' = 0$</td>
<td>$x' = 0$</td>
<td>$x' = -0.45$</td>
</tr>
<tr>
<td>$y' = 0$</td>
<td>$y' = 1.1$</td>
<td>$y' = 0.89$</td>
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Fig. 2.B.1 Town map according to a "tipsy" surveyor.  
Fig. 2.B.2 Diagram and formulas for reconciliation of the two surveyor's data.

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</table>

$x = x' \cos \theta + y' \sin \theta$

$y = -x' \sin \theta + y' \cos \theta$

$\cos \theta = \frac{1}{\sqrt{1 + \frac{b^2}{c^2}}}$

$\sin \theta = \frac{b / c}{\sqrt{1 + \frac{b^2}{c^2}}}$

$x' = x \cos \theta - y \sin \theta = \frac{x}{\sqrt{1 + \frac{b^2}{c^2}}} + \frac{-(b / c)y}{\sqrt{1 + \frac{b^2}{c^2}}}$

$y' = x \sin \theta + y \cos \theta = \frac{(b / c)x}{\sqrt{1 + \frac{b^2}{c^2}}} + \frac{y}{\sqrt{1 + \frac{b^2}{c^2}}}$
A politically incorrect analogy of rotational transformation and Lorentz transformation

Fig. 2.B.1 Town map according to a "tipsy" surveyor.

Fig. 2.B.2 Diagram and formulas for reconciliation of the two surveyor's data.

Object 0:
Town Square.

Object 1:
Saloon.

Object 2:
Gun Shoppe.

(US surveyor)

\[
\begin{align*}
  x &= 0 \\
  y &= 0 \\
\end{align*}
\]

(2nd surveyor)

\[
\begin{align*}
  x' &= 0 \\
  y' &= 0 \\
\end{align*}
\]

\[
\begin{align*}
  x &= x' \cos \theta + y' \sin \theta \\
  y &= -x' \sin \theta + y' \cos \theta \\
\end{align*}
\]

\[
\begin{align*}
  \cos \theta &= \frac{1}{\sqrt{1 + \frac{b^2}{c^2}}} \\
  \sin \theta &= \frac{b / c}{\sqrt{1 + \frac{b^2}{c^2}}} \\
\end{align*}
\]

Reminder: Component-based derivation is clumsy!
A politically incorrect analogy of rotational transformation and Lorentz transformation

Fig. 2.B.1 Town map according to a "tipsy" surveyor.

Fig. 2.B.2 Diagram and formulas for reconciliation of the two surveyor's data.

Reminder: Component-based derivation is clumsy!

Forget this!! It's too clumsy to generalize to 3D, 4D,...

Instead, use Dirac unit vectors $|x\rangle$, $|y\rangle$ and $|x'\rangle$, $|y'\rangle$.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td><em>(US surveyor)</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x = 0$</td>
<td>$x = 0.5$</td>
<td>$x = 0$</td>
</tr>
<tr>
<td>$y = 0$</td>
<td>$y = 1.0$</td>
<td>$y = 1.0$</td>
</tr>
<tr>
<td><em>(2nd surveyor)</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x' = 0$</td>
<td>$x' = 0$</td>
<td>$x' = -0.45$</td>
</tr>
<tr>
<td>$y' = 0$</td>
<td>$y' = 1.1$</td>
<td>$y' = 0.89$</td>
</tr>
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</table>

\[
\begin{align*}
\cos \theta &= \frac{1}{\sqrt{1 + \frac{b^2}{c^2}}} \\
\sin \theta &= \frac{b}{c} \frac{1}{\sqrt{1 + \frac{b^2}{c^2}}}
\end{align*}
\]
A politically incorrect analogy of rotational transformation and Lorentz transformation

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<td>y = 0</td>
</tr>
<tr>
<td>(2nd surveyor)</td>
<td>x' = 0</td>
<td>y' = 0</td>
</tr>
</tbody>
</table>

You may apply (Jacobian) transform matrix:

\[
\begin{pmatrix}
\langle x|x' \rangle & \langle x|y' \rangle \\
\langle y|x' \rangle & \langle y|y' \rangle
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\]

or the inverse (Kajobian) transformation:

\[
\begin{pmatrix}
\langle x'|x \rangle & \langle x'|y \rangle \\
\langle y'|x \rangle & \langle y'|y \rangle
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix}
\]

to any vector \( V = |V \rangle = |x \rangle \langle x|V \rangle + |y \rangle \langle y|V \rangle \)

\[
=|x \rangle \langle x'|V \rangle + |y \rangle \langle y'|V \rangle
\]
A politically incorrect analogy of rotational transformation and Lorentz transformation

Fig. 2.B.1 Town map according to a "tipsy" surveyor.

Fig. 2.B.2 Diagram and formulas for reconciliation of the two surveyor's data.

Object 0: Town Square.
Object 1: Saloon.
Object 2: Gun Shoppe.

(US surveyor) \( x = 0 \) \( y = 0 \)
(2nd surveyor) \( x' = 0 \) \( y' = 0 \)

\[ \begin{align*}
\text{(Jacobian) transformation \{V_x, V_y\} from \{V'_x, V'_y\}}: \\
V_x = \langle x | V \rangle = \langle x | 1 | V \rangle = \langle x | x' \rangle \langle x' | V \rangle + \langle x | y' \rangle \langle y' | V \rangle \\
V_y = \langle y | V \rangle = \langle y | 1 | V \rangle = \langle y | x' \rangle \langle x' | V \rangle + \langle y | y' \rangle \langle y' | V \rangle
\end{align*} \]

\[ \begin{align*}
\begin{array}{c|c|c}
\text{(US surveyor)} & \text{Saloon.} & \text{Gun Shoppe.} \\
\hline
x & 0.5 & 0 \\
y & 1.0 & 1.0
\end{array}
\]

\[ \begin{align*}
x' = x \cos \theta + y' \sin \theta \\
y = -x' \sin \theta + y' \cos \theta
\end{align*} \]

\[ \begin{align*}
\cos \theta &= \frac{1}{\sqrt{1 + \frac{b^2}{c^2}}} \\
\sin \theta &= \frac{b}{c \sqrt{1 + \frac{b^2}{c^2}}}
\end{align*} \]

Instead, use Dirac unit vectors \(|x\rangle, |y\rangle\) and \(|x'\rangle, |y'\rangle\)

You may apply (Jacobian) transform matrix:
\[
\begin{pmatrix}
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\langle y | x' \rangle & \langle y | y' \rangle
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\]
or the inverse (Kajobian) transformation:
\[
\begin{pmatrix}
\langle x | x' \rangle & \langle x | y' \rangle \\
\langle y | x' \rangle & \langle y | y' \rangle
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix}
\]

to any vector \( V = |V\rangle = |x\rangle \langle x | V \rangle + |y\rangle \langle y | V \rangle \)
\[
= |x\rangle \langle x | V \rangle + |y\rangle \langle y | V \rangle
\]
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Forget this!! It's too clumsy to generalize to 3D, 4D,...

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</tr>
<tr>
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<td></td>
</tr>
<tr>
<td>$x = 0.5$</td>
<td>$y = 1.0$</td>
<td></td>
</tr>
<tr>
<td>$x' = 0$</td>
<td>$y' = 1.1$</td>
<td></td>
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(Jacobian) transformation $\{V_xV_y\}$ from $\{V'_xV'_y\}$:

In matrix form:

$$
\begin{pmatrix}
V_x \\
V_y
\end{pmatrix}
= \begin{pmatrix}
\langle x|x'\rangle & \langle x|y'\rangle \\
\langle y|x'\rangle & \langle y|y'\rangle
\end{pmatrix}
\begin{pmatrix}
V'_x \\
V'_y
\end{pmatrix}
$$

You may apply (Jacobian) transform matrix:

$$
\begin{pmatrix}
\langle x|x'\rangle & \langle x|y'\rangle \\
\langle y|x'\rangle & \langle y|y'\rangle
\end{pmatrix}
= \begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
$$

or the inverse (Kajobian) transformation:

$$
\begin{pmatrix}
\langle x|x'\rangle & \langle x|y'\rangle \\
\langle y|x'\rangle & \langle y|y'\rangle
\end{pmatrix}
= \begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix}
$$

to any vector $V = |V\rangle = |x\rangle\langle x|V\rangle + |y\rangle\langle y|V\rangle$

$$
= |x\rangle\langle x'|V\rangle + |y\rangle\langle y'|V\rangle
$$
PLEASE!

Do NOT ever write

this:

\[
e_x' = |x'\rangle = \cos \theta |x\rangle - \sin \theta |y\rangle
\]

\[
e_y' = |y'\rangle = \sin \theta |x\rangle + \cos \theta |y\rangle
\]

like this:

\[
\begin{pmatrix}
  e_x' \\
  e_y'
\end{pmatrix}
= \begin{pmatrix}
  |x'\rangle \\
  |y'\rangle
\end{pmatrix}
= \begin{pmatrix}
  \cos \theta & -\sin \theta \\
  \sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
  |x\rangle \\
  |y\rangle
\end{pmatrix}
\]
PLEASE!

Do NOT ever write this:

\[ e_x = |x'\rangle = \cos\theta |x\rangle - \sin\theta |y\rangle \equiv R |x\rangle \]
\[ e_y = |y'\rangle = \sin\theta |x\rangle + \cos\theta |y\rangle \equiv R |y\rangle \]

(This is a useful abstract definition.)

like this:

\[ \begin{pmatrix} e_x \\ e_y \end{pmatrix} = \begin{pmatrix} |x'\rangle \\ |y'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |x\rangle \\ |y\rangle \end{pmatrix} \]

Here is a matrix representation of abstract definitions: \[ |x'\rangle \equiv R |x\rangle, \quad |y'\rangle \equiv R |y\rangle \]

\[
\begin{pmatrix}
V_x \\
V_y
\end{pmatrix} =
\begin{pmatrix}
|\langle x' | x\rangle \rangle & |\langle x' | y\rangle \rangle \\
|\langle y' | x\rangle \rangle & |\langle y' | y\rangle \rangle
\end{pmatrix}
\begin{pmatrix}
V'_x \\
V'_y
\end{pmatrix} =
\begin{pmatrix}
|\langle x | R x\rangle \rangle & |\langle x | R y\rangle \rangle \\
|\langle y | R x\rangle \rangle & |\langle y | R y\rangle \rangle
\end{pmatrix}
\begin{pmatrix}
V_x \\
V_y
\end{pmatrix} =
\begin{pmatrix}
|\langle x' | R x\rangle \rangle & |\langle x' | R y\rangle \rangle \\
|\langle y' | R x\rangle \rangle & |\langle y' | R y\rangle \rangle
\end{pmatrix}
\begin{pmatrix}
V'_x \\
V'_y
\end{pmatrix}
\]
(a) Rotation Transformation and Invariants

\[ x' = x \cos \theta - y \sin \theta = \frac{x}{\sqrt{1 + \frac{b^2}{c^2}}} - \frac{(b/c)y}{\sqrt{1 + \frac{b^2}{c^2}}} \]

\[ y' = x \sin \theta + y \cos \theta = \frac{(b/c)x}{\sqrt{1 + \frac{b^2}{c^2}}} + \frac{y}{\sqrt{1 + \frac{b^2}{c^2}}} \]

(b) Lorentz Transformation and Invariants

\[ x' = \frac{x}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{v}{c} \frac{ct}{\sqrt{1 - \frac{v^2}{c^2}}} = x \cosh \rho + y \sinh \rho \]

\[ ct' = \frac{ct}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{v}{c} \frac{x}{\sqrt{1 - \frac{v^2}{c^2}}} = x \sinh \rho + y \cosh \rho \]
That “old-time” relativity (Circa 600BCE- 1905CE)

(“Bouncing-photons” in smoke & mirrors and Thales, again)

The Ship and Lighthouse saga
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The straight scoop on “angle” and “rapidity” (They’re area!)

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Introducing the stellar aberration angle $\sigma$ vs. rapidity $\rho$

How Minkowski’s space-time graphs help visualize relativity
Group vs. phase velocity and tangent contacts
The straight scoop on “angle” and “rapidity” (They both are area!)

\[
x = \cosh \theta \\
y = \sinh \theta \\
y/x = \tanh \theta = v/c
\]

The “Area” being calculated is the total Gray Area between hyperbola pairs, X axis, and sloping u-line.

\[\text{Area} = \frac{1}{2} \text{base} \times \text{altitude} - \text{area under curve} = \frac{1}{2} xy - \int y \, dx\]

2005 Web version:
www.uark.edu/ua/pirelli/php/complex_phasors_1.php
The straight scoop on “angle” and “rapidity” (They both are area!)

\[
x = \cosh \theta \\
y = \sinh \theta \\
y/x = \tanh \theta = v/c
\]

The “Area” being calculated is the total Gray Area between hyperbola pairs, X axis, and sloping u-line.

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2014...Web-app versions:
http://www.uark.edu/ua/modphys/markup/RelaWavityWeb.html
The straight scoop on "angle" and "rapidity" (They both are area!)

\[ x = \cosh \theta \]
\[ y = \sinh \theta \]
\[ \frac{y}{x} = \tanh \theta = \frac{v}{c} \]

\[ A_{\text{area}} = 1 \cdot 1 - A_{\text{area}} - \int x \, dy \]

The "Area" being calculated is the total Gray Area between hyperbola pairs, X axis, and sloping u-line.

Circular Functions
- \( m_2(\sigma) = 0.8582 \)
- \( \text{Length}(\sigma) = 0.8582 \)
- \( \text{Area}(\sigma) = 0.8582 \)
- \( \sin(\sigma) = 0.7567 \)
- \( \tan(\sigma) = 1.1574 \)
- \( \sec(\sigma) = 1.5295 \)

Hyperbolic Functions
- \( q = 0.9884 \)
- \( \text{Area}(q) = 0.9884 \)
- \( \tanh(q) = 0.7567 \)
- \( \sinh(q) = 1.1574 \)
- \( \cosh(q) = 1.5295 \)
The straight scoop on “angle” and “rapidity” (They’re area!)

\[ y/x = \tanh \theta = v/c \]

\[ y = \sinh \rho \]

\[ x = \cosh \rho \]

The “Area” being calculated is the total Gray Area between hyperbola pairs, X axis, and sloping u-line

Useful hyperbolic identities

\[ \text{Area}_2 = \frac{1}{2} \text{base} \cdot \text{altitude} - \text{area under curve} = \frac{1}{2} xy - \int y \, dx \]

\[ \frac{\text{Area}_2}{2} = \frac{1}{2} \sinh \rho \cosh \rho - \int \sinh \rho \, d(\cosh \rho) \]

\[ \sinh^2 \rho = \left( \frac{e^\rho - e^{-\rho}}{2} \right)^2 = \frac{1}{4} \left( e^{2\rho} + e^{-2\rho} - 2 \right) = \frac{\cosh 2\rho - 1}{2} \]

\[ \sinh \rho \cosh \rho = \left( \frac{e^\rho - e^{-\rho}}{2} \right) \left( \frac{e^\rho + e^{-\rho}}{2} \right) = \frac{1}{4} \left( e^{2\rho} - e^{-2\rho} \right) = \frac{1}{2} \sinh 2\rho \]
The straight scoop on “angle” and “rapidity” (They’re area!)

\[ y/x = \tanh \theta = \frac{v}{c} \]

\[ y = \sinh \rho \]
\[ x = \cosh \rho \]

The “Area” being calculated is the total Gray Area between hyperbola pairs, X axis, and sloping u-line

Useful hyperbolic identities

\[ \sinh^2 \rho = \left( \frac{e^\rho - e^{-\rho}}{2} \right)^2 = \frac{1}{4} (e^{2\rho} + e^{-2\rho} - 2) = \frac{\cosh 2\rho - 1}{2} \]

\[ \sinh \theta \cosh \theta = \left( \frac{e^\theta - e^{-\theta}}{2} \right) \left( \frac{e^\theta + e^{-\theta}}{2} \right) = \frac{1}{4} (e^{2\theta} - e^{-2\theta}) = \frac{1}{2} \sinh 2\theta \]

\[ \int \cosh a\rho \, d\rho = \frac{1}{a} \sinh a\rho \]
The straight scoop on “angle” and “rapidity” (They’re area!)

\[ y/x = \tanh \theta = \frac{v}{c} \]

\[ y = \sinh \rho \]
\[ x = \cosh \rho \]

The “Area” being calculated is the total Gray Area between hyperbola pairs, X axis, and sloping u-line.

Useful hyperbolic identities

\[ \sinh^2 \rho = \left( \frac{e^\rho - e^{-\rho}}{2} \right)^2 = \frac{1}{4} (e^{2\rho} + e^{-2\rho} - 2) = \cosh 2\rho - 1 \]

\[ \sinh \rho \cosh \rho = \left( \frac{e^\rho - e^{-\rho}}{2} \right) \left( \frac{e^\rho + e^{-\rho}}{2} \right) = \frac{1}{4} (e^{2\rho} - e^{-2\rho}) = \frac{1}{2} \sinh 2\rho \]

\[ \int \cosh a\theta \, d\theta = \frac{1}{a} \sinh a\theta \]

Amazing result: Area = \( \rho \) is rapidity
That “old-time” relativity (Circa 600BCE- 1905CE)

(“Bouncing-photons” in smoke & mirrors and Thales, again)

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How Minkowski’s space-time graphs help visualize relativity

Group vs. phase velocity and tangent contacts
Galilean velocity addition becomes rapidity addition

From Lect. 22 p. 27 or eq. (3.6) in Ch. 3 of Unit 2:

Evenson axiom requires geometric Doppler transform: $$e^{\rho_{AB}} \cdot e^{\rho_{BC}} = e^{\rho_{AC}} = e^{\rho_{AB} + \rho_{BC}}$$

Easy to combine frame velocities using rapidity addition: $$\rho_{u+v} = \rho_u + \rho_v$$
Galilean velocity addition becomes rapidity addition

From Lect. 22 p. 27 or eq. (3.6) in Ch. 3 of Unit 2:

Evenson axiom requires geometric Doppler transform: 
\[ e^{\rho_{AB}} \cdot e^{\rho_{BC}} = e^{\rho_{AC}} = e^{\rho_{AB} + \rho_{BC}} \]

Easy to combine frame velocities using rapidity addition:
\[ \rho_{u+v} = \rho_u + \rho_v \]

\[
\frac{u'}{c} = \tanh(\rho_u + \rho_v) = \frac{\tanh \rho_u + \tanh \rho_v}{1 + \tanh \rho_u \tanh \rho_v} = \frac{\frac{u}{c} + \frac{v}{c}}{1 + \frac{u}{c} \cdot \frac{v}{c}}
\]

or:
\[
\frac{u'}{c} = \frac{u + v}{1 + \frac{u \cdot v}{c^2}}
\]

\[ \tanh(x + y) = \frac{\tanh x + \tanh y}{1 + \tanh x \tanh y} \]
**Galilean velocity addition becomes rapidity addition**

*From Lect. 22 p. 27 or eq. (3.6) in Ch. 3 of Unit 2:*

Evenson axiom requires geometric Doppler transform: \( e^{\rho_{AB}} \cdot e^{\rho_{BC}} = e^{\rho_{AC}} = e^{\rho_{AB} + \rho_{BC}} \)

Easy to combine frame velocities using rapidity addition:

\[ \frac{u'}{c} = \tanh(\rho_u + \rho_v) = \frac{\tanh \rho_u + \tanh \rho_v}{1 + \tanh \rho_u \tanh \rho_v} = \frac{u + v}{c + v} \]

or:

\[ u' = \frac{u + v}{1 + \frac{u \cdot v}{c^2}} \]

No longer does \((1/2 + 1/2)c\) equal \((1)c\)...

Relativistic result is:

\[ \frac{1}{2} + \frac{1}{2} c = \frac{1}{2} \cdot \frac{1}{5} = \frac{1}{5} c = \frac{4}{5} c \]
Galilean velocity addition becomes rapidity addition

From Lect. 22 p. 27 or eq. (3.6) in Ch. 3 of Unit 2:

Evenson axiom requires geometric Doppler transform: $e^{\rho_{AB}} \cdot e^{\rho_{BC}} = e^{\rho_{AC}} = e^{\rho_{AB} + \rho_{BC}}$

Easy to combine frame velocities using rapidity addition: $\rho_{u+v} = \rho_u + \rho_v$

\[
\frac{u'}{c} = \tanh(\rho_u + \rho_v) = \frac{\tanh \rho_u + \tanh \rho_v}{1 + \tanh \rho_u \tanh \rho_v} = \frac{u + v}{c} \cdot \frac{c}{c} = \frac{u}{1 + \frac{u \cdot v}{c^2}}
\]

or: $u' = \frac{u + v}{1 + \frac{u \cdot v}{c^2}}$

No longer does $(1/2 + 1/2)c$ equal $(1)c$…

Relativistic result is: $\frac{1 + 1}{2 + 2} c = \frac{1}{1 + \frac{1}{4}} c = \frac{1}{5} c = \frac{4}{5} c$

…but, $(1/2 + 1)c$ does equal $(1)c$… $\frac{1 + 1}{2 + 1} c = c$
That “old-time” relativity (Circa 600BCE- 1905CE)

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Introducing the stellar aberration angle $\sigma$ vs. rapidity $\rho$

How Minkowski’s space-time graphs help visualize relativity

Group vs. phase velocity and tangent contacts
Introducing the “Sin-Tan Rosetta Stone”  

(a) Circular Functions  
(Plane geometry)  

NOTE: Angle $\phi$ is now called stellar aberration angle $\sigma$  

www.uark.edu/ua/pirelli/php/complex_phasors_1.php
Introducing the “Sin-Tan Rosetta Stone”  

NOTE: Angle $\phi$ is now called stellar aberration angle $\sigma$  

Fig. 5.4 in Unit 8

2005 Web version:  
www.uark.edu/ua/pirelli/php/complex_phasors_1.php

2014...Web-app versions:  
http://www.uark.edu/ua/modphys/markup/RelaWavityWeb.html

Circular Functions  
\[
\begin{align*}
\sin(\phi) &= 0.7567 \\
\cos(\phi) &= 0.6538 \\
\tan(\phi) &= 1.1574 \\
\cot(\phi) &= 0.8940 \\
\sec(\phi) &= 1.5955 \\
\csc(\phi) &= 1.2833 \\
\end{align*}
\]
That “old-time” relativity (Circa 600BCE- 1905CE)

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How Minkowski’s space-time graphs help visualize relativity
Group vs. phase velocity and tangent contacts
Introducing the **stellar aberration angle** $\sigma$ vs. **rapidity** $\rho$

Together, rapidity $\rho=\ln b$ and stellar aberration angle $\sigma$ are parameters of relative velocity.

The rapidity $\rho=\ln b$ is based on longitudinal wave Doppler shift $b=e^\rho$ defined by $u/c=\tanh(\rho)$.

At low speed: $u/c\sim \rho$.

The stellar aberration angle $\sigma$ is based on the transverse wave rotation $R=e^{i\sigma}$ defined by $u/c=\sin(\sigma)$.

At low speed: $u/c\sim \sigma$.

*Fig. 5.6* Epstein’s cosmic speedometer with aberration angle $\sigma$ and transverse Doppler shift $\cosh \upsilon$. 

That “old-time” relativity (Circa 600BCE- 1905CE)

(“Bouncing-photons” in smoke & mirrors and Thales, again)

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A politically incorrect analogy of rotational transformation and Lorentz transformation
The straight scoop on “angle” and “rapidity” (They’re area!)
Galilean velocity addition becomes rapidity addition
Introducing the “Sin-Tan Rosetta Stone” (Thanks, Thales!)
Introducing the stellar aberration angle $\sigma$ vs. rapidity $\rho$

How Minkowski’s space-time graphs help visualize relativity
Group vs. phase velocity and tangent contacts
How Minkowski’s space-time graphs help visualize relativity

Note that in Lighthouse frame Happening 1 is simultaneous with Happening 2 at \( t=2.00 \text{sec} \).

2005 Web versions:

www.uark.edu/ua/pirelli/php/lighthouse_scenarios.php
2014...Web-app versions:

http://www.uark.edu/ua/modphys/markup/RelativItWeb.html
How Minkowski’s space-time graphs help visualize relativity (Here: \( r = \text{atanh}(1/2) = 0.549 \),

Note that in Lighthouse frame Happening 1 is simultaneous with Happening 2 at \( t = 2.00 \) sec.

...but, in Ship frame Happening 1 is at \( t' = 1.74 \) and Happening 2 is at \( t' = 2.30 \) sec.

Happening 1: Ship 1 is hit by Blink 1
Happening 2: Lighthouse emits Blink 2

Happening 2 won’t happen ‘til \( t = 2.00 \)

(Here: \( \rho = \text{Atanh}(1/2) = 0.55 \),
and: \( \sigma = \text{Asin}(1/2) = 0.52 \) or 30°)
How Minkowski’s space-time graphs help visualize relativity

Note that in Lighthouse frame Happening 1 is simultaneous with Happening 2 at $t=2.00\text{sec}$.

...but, in Ship frame Happening 1 is at $t'=1.74$ and Happening 2 is at $t'=2.30\text{sec}$.

Happening 1: Ship 1 is hit by Blink 1
Happening 2: Lighthouse emits Blink 2

That is $t'=2.30\text{ ship time}$

(Here: $\rho=\text{Atanh}(1/2)=0.55$, and $\sigma=\text{Asin}(1/2)=0.52$ or $30^\circ$)
2014...Web-app versions:

http://www.uark.edu/ua/modphys/markup/RelativItWeb.html
Ship time $t' = 2.313$

Ship $v/c (\text{Rel. to Lighthouse}) = 0.500$
Ship $v/c (\text{Rel. to Observer}) = 0.000$
Lighthouse $v/c (\text{Rel. to Observer}) = 0.500$

Ship Graph
Ref time $t = 2.31$ sec.
$v/c = -0.50$ litesec/sec.

2014... Web-app versions:

http://www.uark.edu/ua/modphys/Markup/RelativItWeb.html
That “old-time” relativity (Circa 600BCE- 1905CE)

(“Bouncing-photons” in smoke & mirrors and Thales, again)
The Ship and Lighthouse saga
Light-conic-sections make invariants
A politically incorrect analogy of rotational transformation and Lorentz transformation
The straight scoop on “angle” and “rapidity” (They’re area!)
Galilean velocity addition becomes rapidity addition
Introducing the “Sin-Tan Rosetta Stone” (Thanks, Thales!)
Introducing the stellar aberration angle $\sigma$ vs. rapidity $\rho$
How Minkowski’s space-time graphs help visualize relativity
Group vs. phase velocity and tangent contacts
Introducing the ‘‘Sin-Tan Rosetta Stone’’

NOTE: Angle $\phi$ is now called stellar aberration angle $\sigma$

Fig. 5.4 in Unit 8

(a) Circular Functions (plane geometry)

(b) Hyperbolic Functions ( spacetime geometry)

Circular arc area
$\varphi = 0.8934 = \text{angle}$
$\sin \varphi = 0.7792$
$\cos \varphi = 0.6267$
$\tan \varphi = 1.2433$
$\csc \varphi = 1.2833$
$\sec \varphi = 1.5955$
$\cot \varphi = 0.8043$

Hyperbolic arc area
$\rho = 1.0434 = \text{rapidity}$
$\sinh \rho = 1.2433$
$\cosh \rho = 1.5955$
$\tanh \rho = 0.7792$
$\csch \rho = 0.8043$
$\sech \rho = 0.6267$
$\coth \rho = 1.2833$

2005 Web version:
https://www.uark.edu/ua/pirelli/php/hyper_constrct.php
Introducing the “Sin-Tan Rosetta Stone”

NOTE: Angle $\phi$ is now called stellar aberration angle $\sigma$.

(a) Circular Functions
(plane geometry)

$\phi$
$cot \phi$
$csc \phi$
$\sin \phi$
$\cos \phi$
$\tan \phi$

Circular arc area
$\phi = 0.8934$ = angle
$\sin \phi = 0.7792$
$\cos \phi = 0.6267$
$\tan \phi = 1.2433$
$csc \phi = 1.2833$
$sec \phi = 1.5955$
$cot \phi = 0.8043$

(b) Hyperbolic Functions
(spacetime geometry)

$\rho$
$csc \rho$
$coth \rho$
$\sinh \rho$
$\cosh \rho$
$\tanh \rho$
$\coth \rho$

Hyperbolic arc area
$\rho = 1.0434$ = rapidity
$\sinh \rho = 1.2433$
$\cosh \rho = 1.5955$
$\tanh \rho = 0.7792$
$csc \rho = 0.8043$
$sech \rho = 0.6267$
$coth \rho = 1.2833$

2005 Web version:
https://www.uark.edu/ua/pirelli/php/hyper_constrct.php
Hyperbolic Functions

\( q = 1.1714 \)

Area(\( q \)) = 1.1714
\( \sinh(q) = 1.4582 \)
\( \cosh(q) = 1.7682 \)
\( \tanh(q) = 0.8247 \)
\( \csc(q) = 0.6858 \)
\( \sech(q) = 0.5656 \)
\( \coth(q) = 1.2125 \)

Circular Functions

m(\( \sigma \)) = 0.9697
\( \text{Length}(\sigma) = 0.9697 \)
\( \text{Area}(\sigma) = 0.9697 \)
\( \sin(\sigma) = 0.8247 \)
\( \cos(\sigma) = 0.5656 \)
\( \tan(\sigma) = 1.4582 \)
\( \csc(\sigma) = 1.2125 \)
\( \sec(\sigma) = 1.7682 \)
\( \cot(\sigma) = 0.6858 \)

2014...Web-app versions:

http://www.uark.edu/ua/modphys/markup/RelaWavityWeb.html

Thursday, January 30, 2014
Laser frequency = $B = 2 = 600\text{THz}$
Doppler blue shift factor = $b = 2.005$
Doppler red shift factor = $r = 0.499$
$q = 0.696$

CW Light Axioms
All colors go $c$: $\omega/k = c$ or L& R on diagonals
Time Reversal $(r \leftrightarrow b)$: $r = 1/b$

2014...Web-app versions:
http://www.uark.edu/ua/modphys/markup/RelativityWeb.html
Light Axioms

All colors go: $\omega/k = c$ or L&R on diagonals

Time Reversal ($r \leftrightarrow b$): $r = 1/b$
Frequency = $B = 2 = 600 \text{ THz}$

Doppler blue shift factor = $b = 2.005$

Doppler red shift factor = $r = 0.499$

Light Axioms

All colors go $c: \omega/k = c$ or L&R on diagonals

Time Reversal ($r \leftrightarrow b$): $r = 1/b$
\[ \frac{v}{c} = \beta = 0.600 \]
\[ \text{Doppler blue shift factor} = b = 2.000 \]
\[ \text{Doppler red shift factor} = r = 0.500 \]
\[ \nu = 0.540 = 30.964^\circ \]
\[ \eta = 0.693 \]
\[ \sigma = 0.644 = 36.870^\circ \]
\[ v/c = \beta = 0.600 \]
Doppler blue shift factor = \( b = 2.000 \)
Doppler red shift factor = \( r = 0.500 \)
\[ \nu = 0.540 = 30.964^\circ \]
\[ \eta = 0.693 \]
\[ \sigma = 0.644 = 36.870^\circ \]
\[ v/c = \beta = 0.600 \]

Doppler blue shift factor = \( b = 2.000 \)

Doppler red shift factor = \( r = 0.500 \)

\[ \nu = 0.540 = 30.964^\circ \]

\[ q = 0.693 \]

\[ \sigma = 0.644 = 36.870^\circ \]
Energy ($E$)

- Coordinate angle $\theta = \arctan(u/c)$
- Stellar aberration angle $\sigma = \arcsin(u/c)$

Momentum $p = B \sinh(\varphi)$

Hamiltonian $H(p) = B \cosh(\varphi)$

Lagrangian $L(u) = B \text{sech}(\varphi)$

Group Velocity $u/c = B \tanh(\varphi)$

Phase Velocity $c/u = B \coth(\varphi)$

DeBroglie Wavelength $\lambda/c = B \text{csch}(\varphi)$

Shirt factor $b = 2.000$

Shirt factor $r = 0.500$

964°

870°