AMOP Lecture 4.5
Thur 1.30.2014 -2.6.2014

Relativity of lightwaves and Lorentz-Minkowski coordinates IV.

(Ch. 0-3 of Unit 8)

More connections to conventional approach to relativity and old-fashioned formulas
Catching up to light (Coyote finally triumphs! Rest-frame at last.)
The most old-fashioned form(ula) of all: Thales & Euclid means
Galileo wins one! (...in gauge space) That "old-time" relativity (Circa 600BCE- 1905CE)
"Bouncing-photons" in smoke & mirrors
The Ship and Lighthouse saga
Light-conic-sections make invariants
A politically incorrect analogy of rotational transformation and Lorentz transformation
The straight scoop on "angle" and "rapidity" (They both are area!)
Galilean velocity addition becomes rapidity addition
Introducing the "Sin-Tan Rosetta Stone" (Thanks, Thales!)
Introducing the stellar aberration angle σ vs. rapidity ρ
How Minkowski's space-time graphs help visualize relativity
Group vs. phase velocity and tangent contacts

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$$=(4\ ,+4c) \qquad\qquad =(1\ ,-1c)$$

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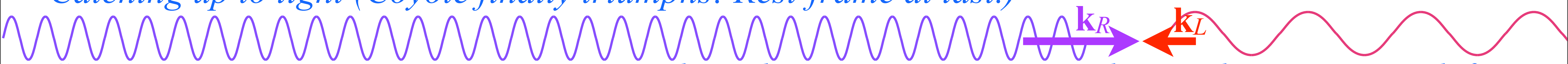

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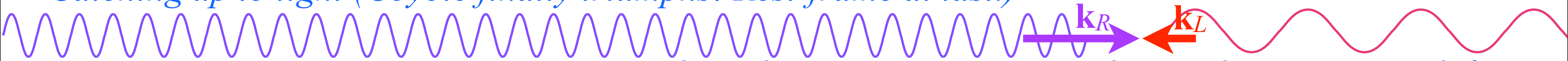

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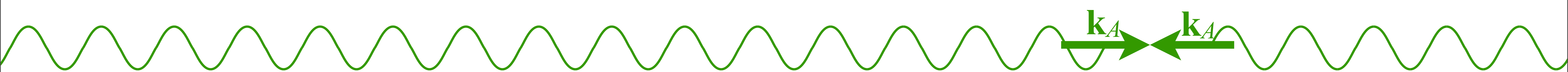

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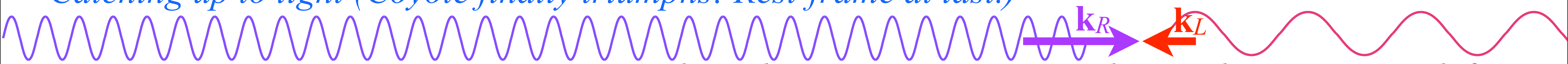

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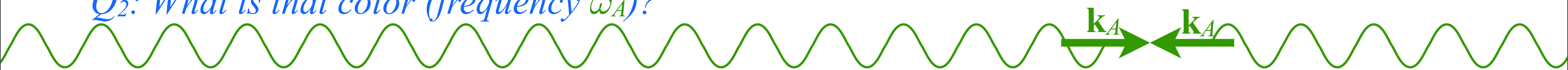
Q_2: What is that color (frequency ω_A)?



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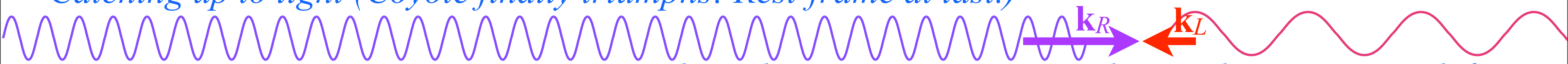

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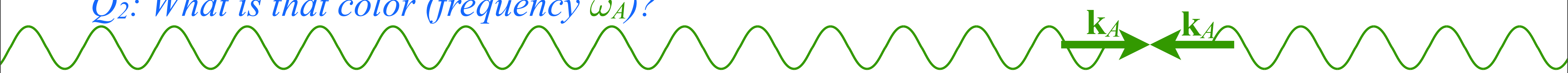

"Jeopardy" answers:

A_1:How fast is the group velocity?

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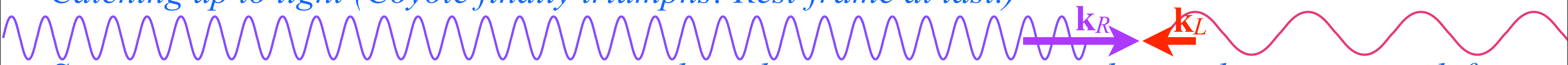

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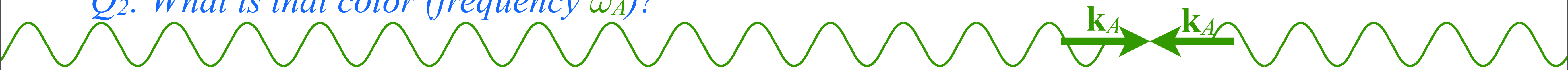

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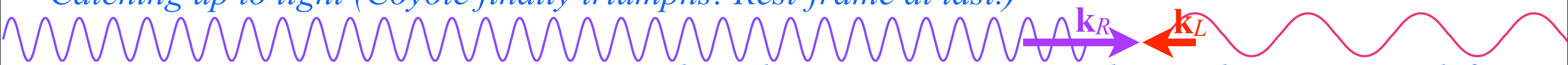

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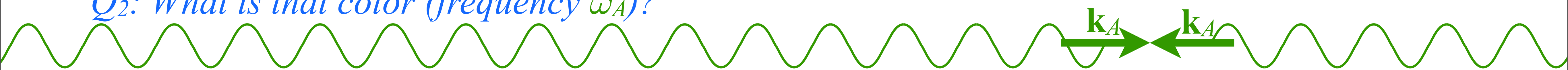

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A_2: What is the geometric mean of $\omega_{R\rightarrow}$ and $\omega_{L\leftarrow}$?

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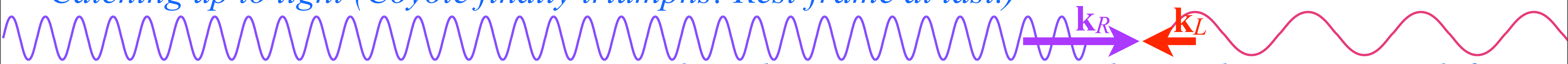

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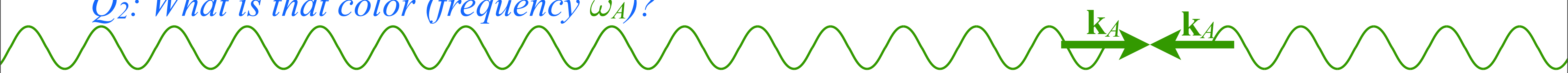

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A_2: What is the geometric mean of $\omega_{R\rightarrow}$ and $\omega_{L\leftarrow}$?

$$\omega_A=\sqrt{\omega_{R\rightarrow}\cdot\omega_{L\leftarrow}}=\sqrt{4\cdot 1}=2$$

$(\omega_{R\rightarrow}\ ,\ ck_{R\rightarrow})$ meets $(\omega_{L\leftarrow}\ ,\ -ck_{L\leftarrow})$

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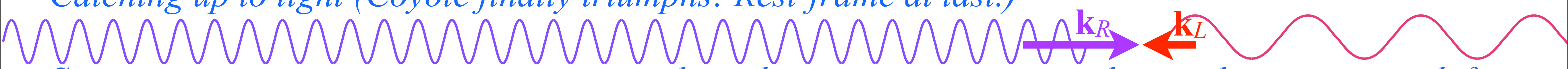

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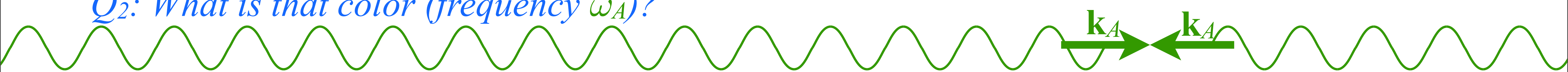

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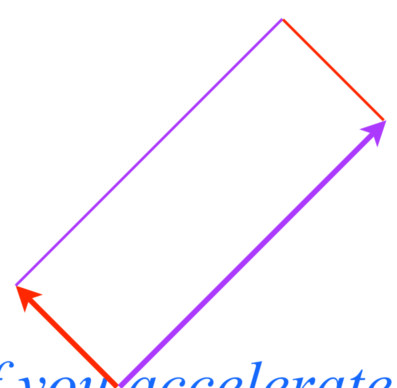
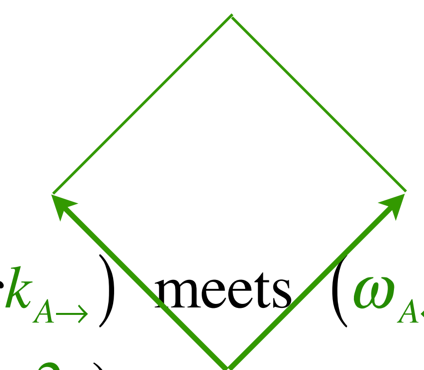


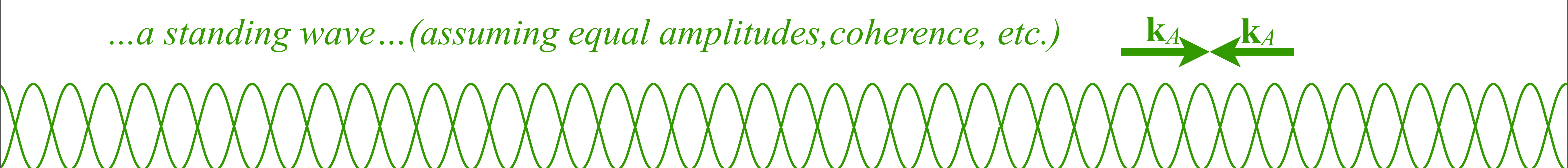
If you accelerate to $V_{group}=\frac{3}{5}c$ *then you see...*



$(\omega_{A\rightarrow}\ ,\ ck_{A\rightarrow})$ meets $(\omega_{A\leftarrow}\ ,\ -ck_{A\leftarrow})$

$=(2\ ,+2c)$ $\quad$ $=(2\ ,-2c)$

...a standing wave...(assuming equal amplitudes,coherence, etc.)



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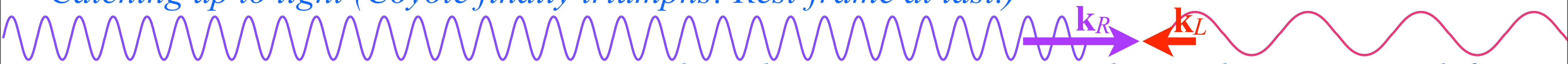

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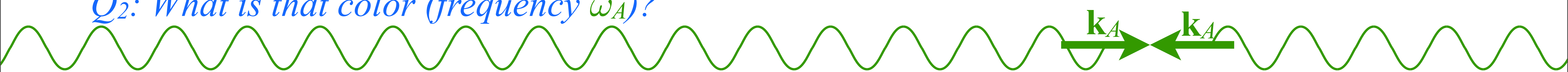

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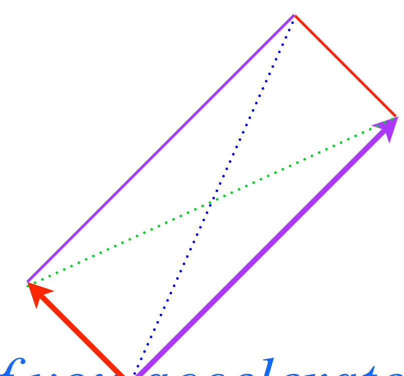

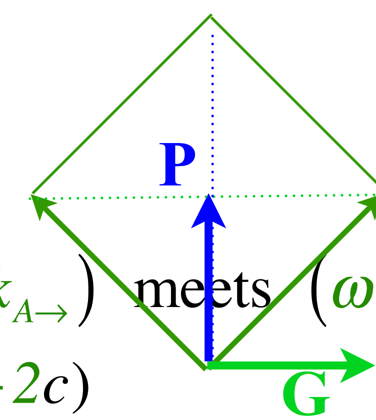

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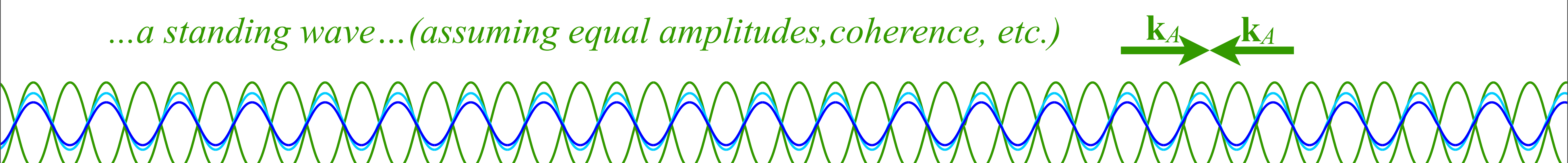

to become $(\omega_{phase}\ ,\ ck_{phase})$ and $(\omega_{group}\ ,\ ck_{group})$

$=(2\ ,\ 0c)$ $\qquad$ $=(0\ ,\ 2c)$

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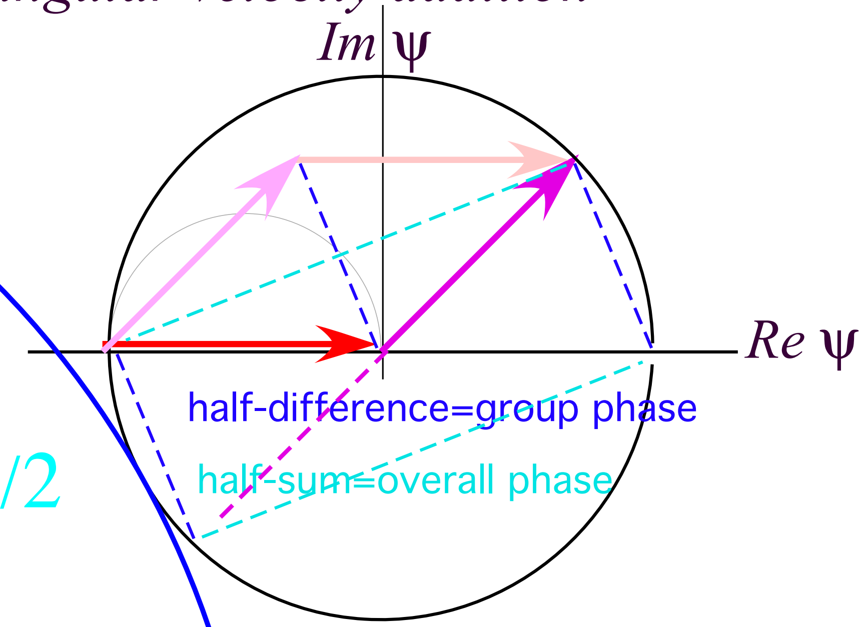
Galileo wins one! (...in gauge space) That "old-time" relativity (Circa 600BCE- 1905CE)

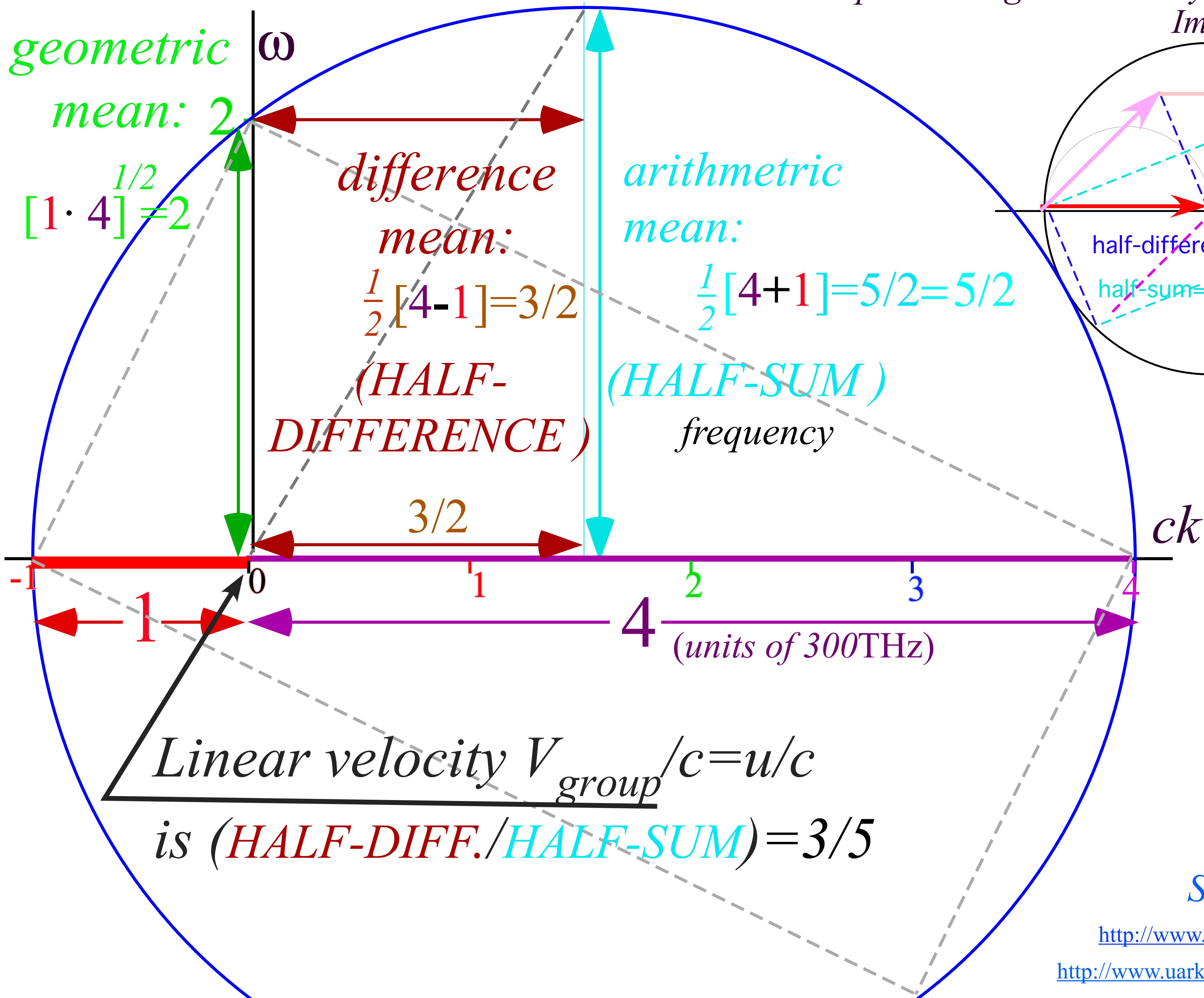
Euclid's 3-means (300 BC)

Geometric "heart" of wave mechanics

Thales (580BC) rectangle-in-circle

Relates to wave interference by (Galilean) phasor angular velocity addition





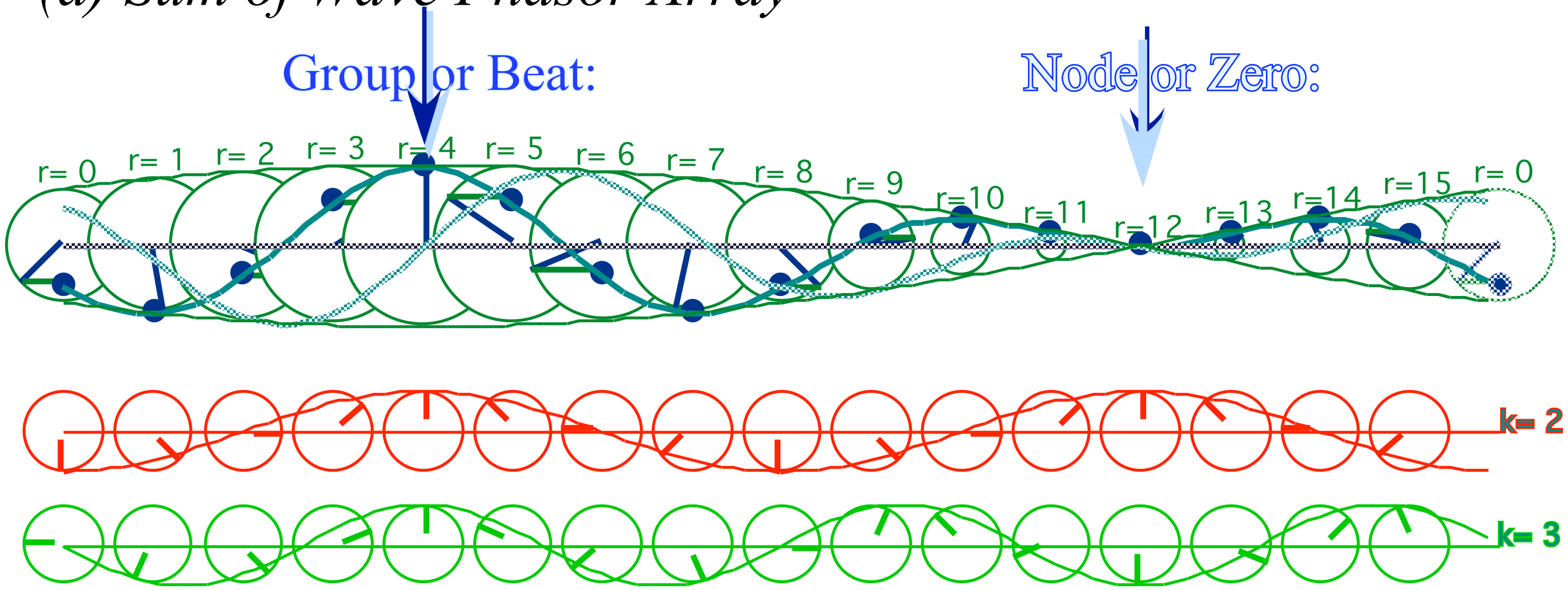
Sites for animation:

http://www.uark.edu/ua/pirelli/php/means_1.php

http://www.uark.edu/ua/pirelli/php/half_sum_2.php

Fig. 3.3a Euclidian mean geometry for counter-moving waves of frequency 1 and 4. (300THz units).

(a) Sum of Wave Phasor Array



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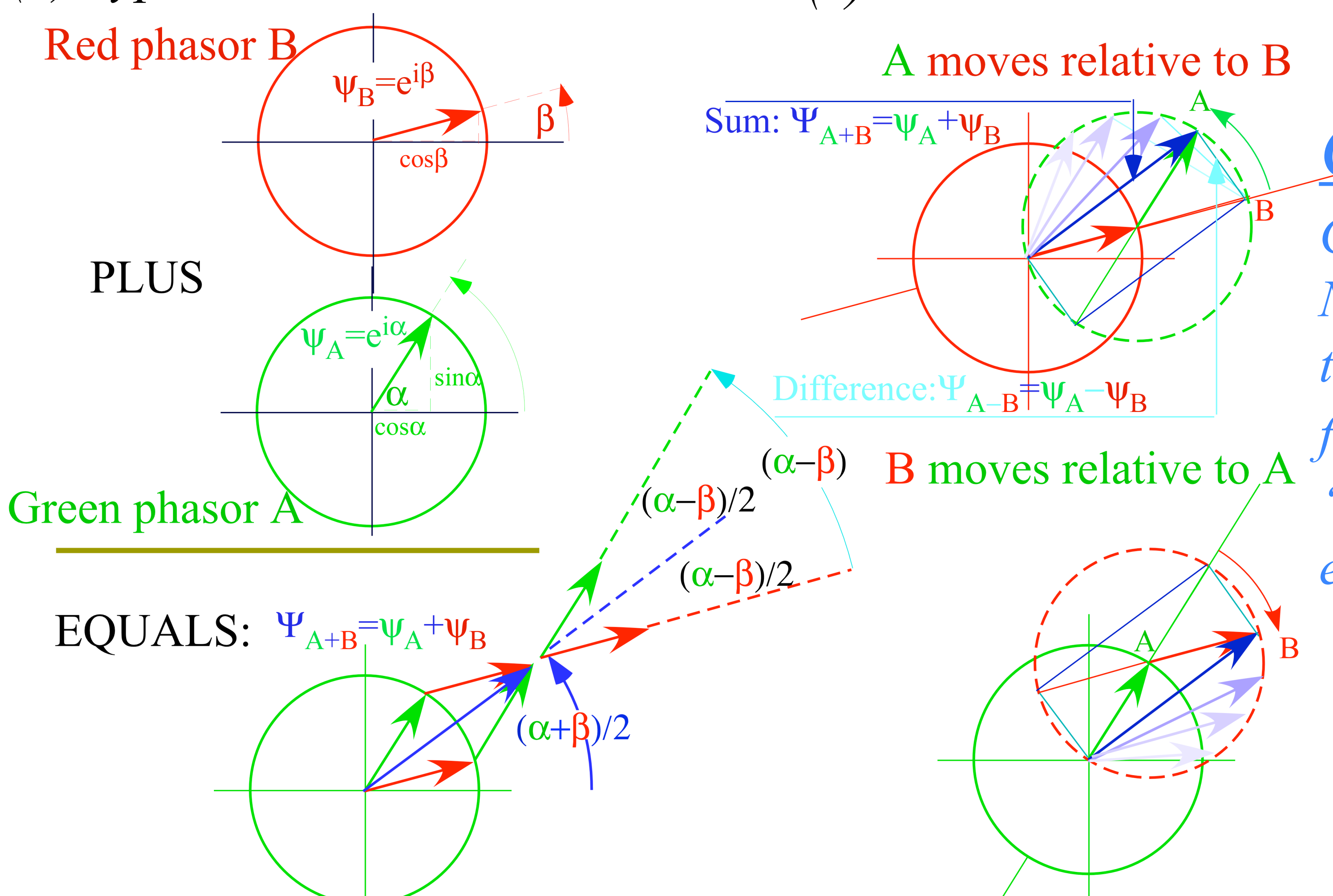
(b) Typical Phasor Sum:

(c) Phasor-relative views



Galileo's revenge!

Galileo wins one (in gauge space) Now we use Galilean relativity to add angular velocity, that is frequency ω_a and ω_b, in phasor or "gauge" space. No "c-limit" evident. (So far at 18-fig. precision.)

Fig. 3.1 Wave phasor addition. (a) Each phasor in a wave array is a sum (b) of two component phasors.

That "old-time" relativity *(Circa 600BCE- 1905CE)*

("Bouncing-photons" in smoke & mirrors and Thales, again)

The Ship and Lighthouse saga

Light-conic-sections make invariants

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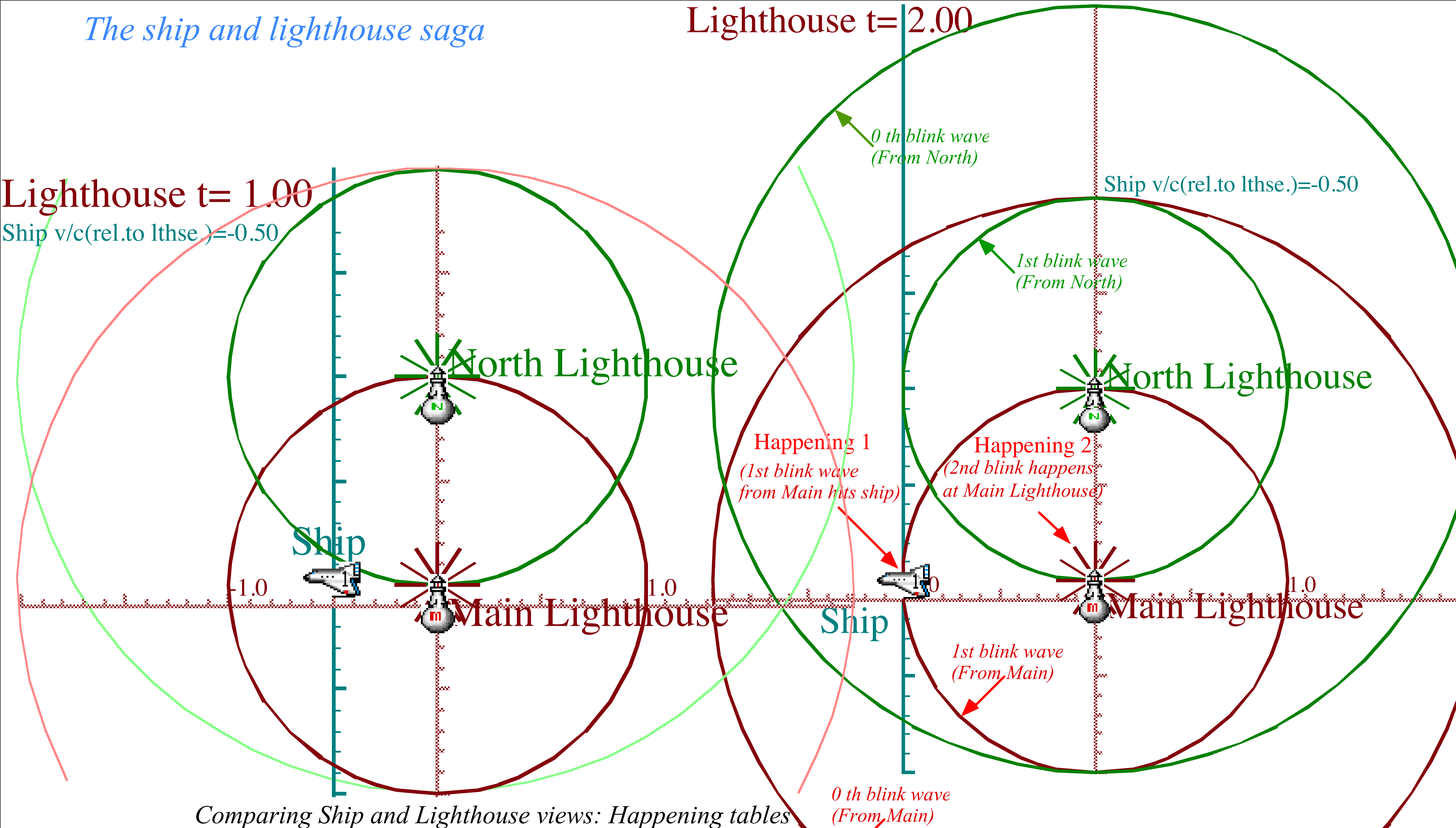


Comparing Ship and Lighthouse views: Happening tables

	Happening 0: Ship passes Main Lighthouse.	Happening 1: Ship gets hit by first blink from Main Lighthouse.	Happening 2: Main Lighthouse blinks second time.
(Lighthouse space)	$x = 0$	$x = -1.00\ c$	$x = 0$
(Lighthouse time)	$t = 0$	$t = 2.00$	$t = 2.00$
(Ship space)	$x' = 0$	$x' = 0$	$x' = c\,\Delta$
(Ship time)	$t' = 0$	$t' = 1.75$	$t' = 2\Delta = 2.30$

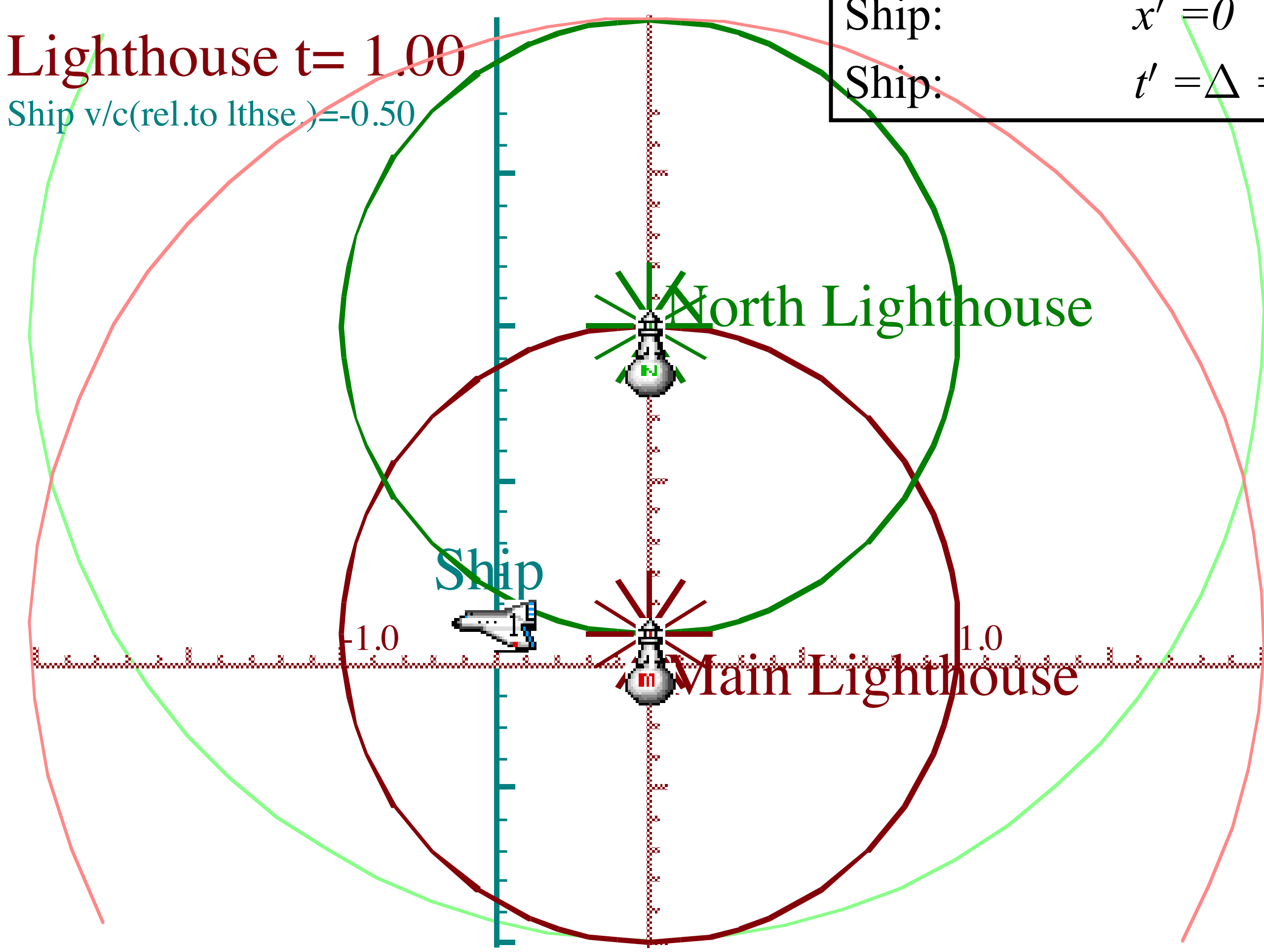
Fig. 2.A.3 Happening 1 (1st blink hits ship) and 2 (2nd blink at Main) both happen at t=2.

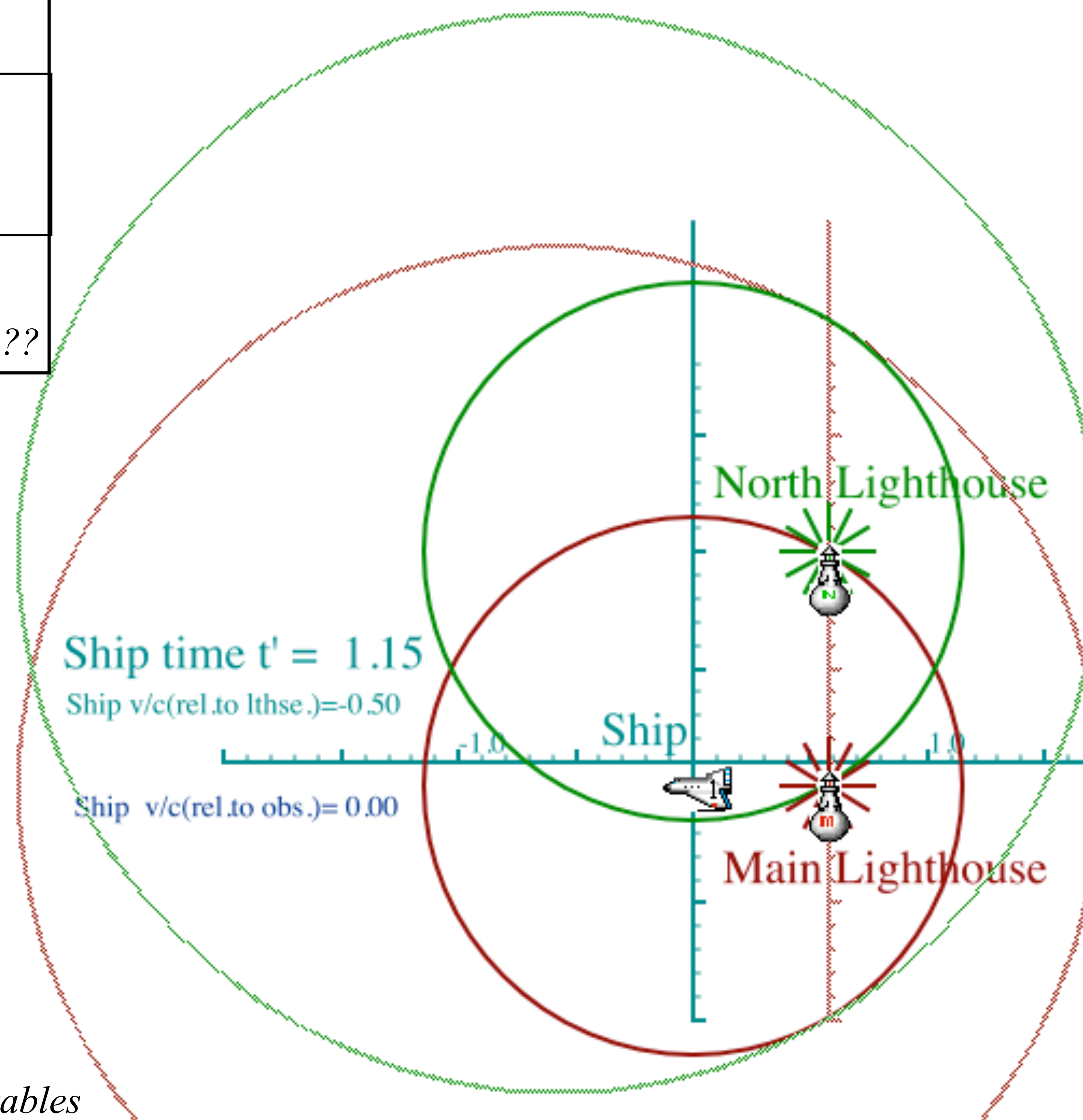
The ship and lighthouse saga

Happening 0.5:
Main Lite
blinks first time.

Lighthouse:	$x = 0$
Lighthouse:	$t = 1.00$
Ship:	$x' = 0$
Ship:	$t' = \Delta = ???$

Ship Time $t'=\Delta=??$





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The ship and lighthouse saga

Ship Time $t' = \Delta = ???$

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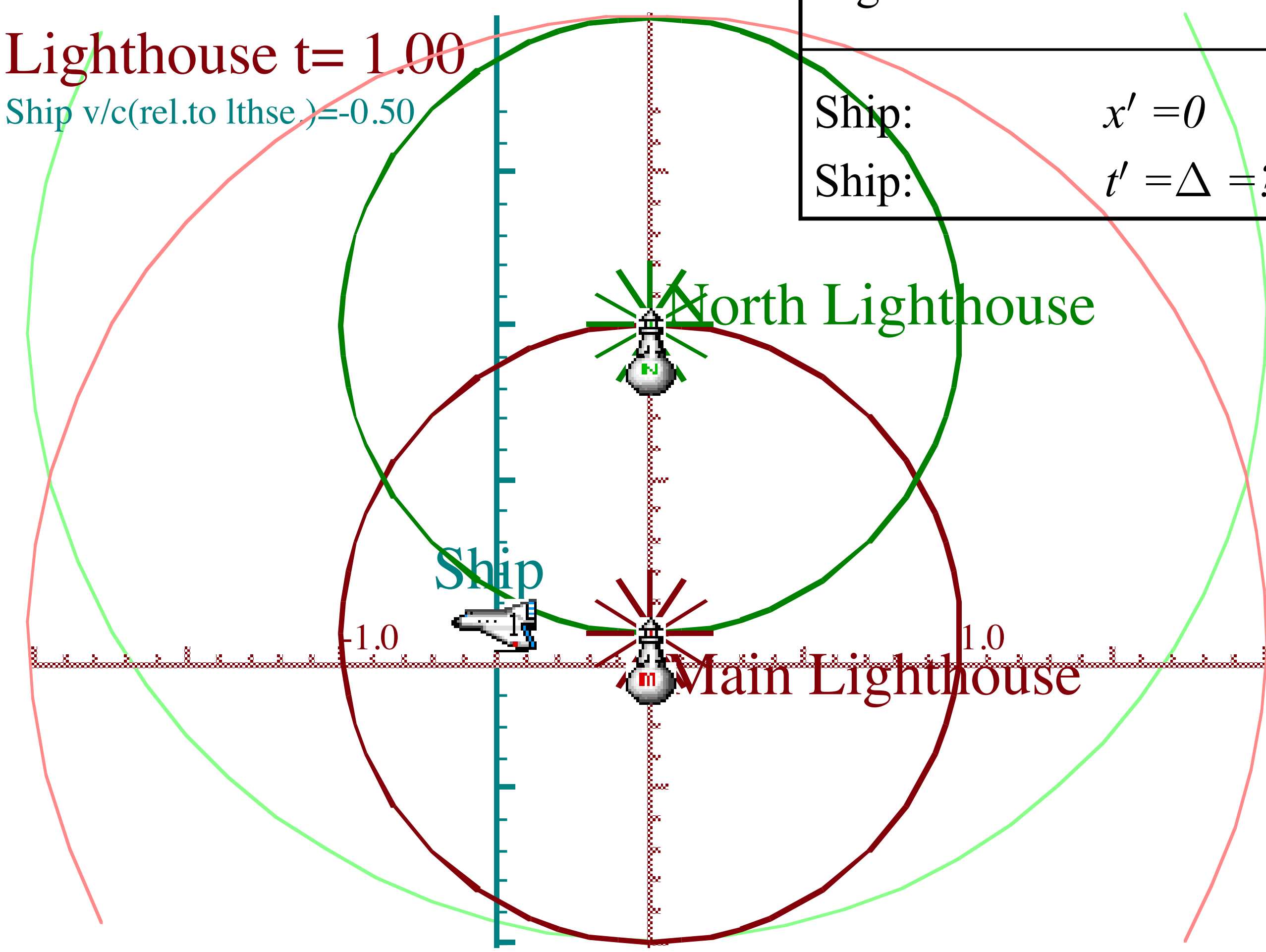

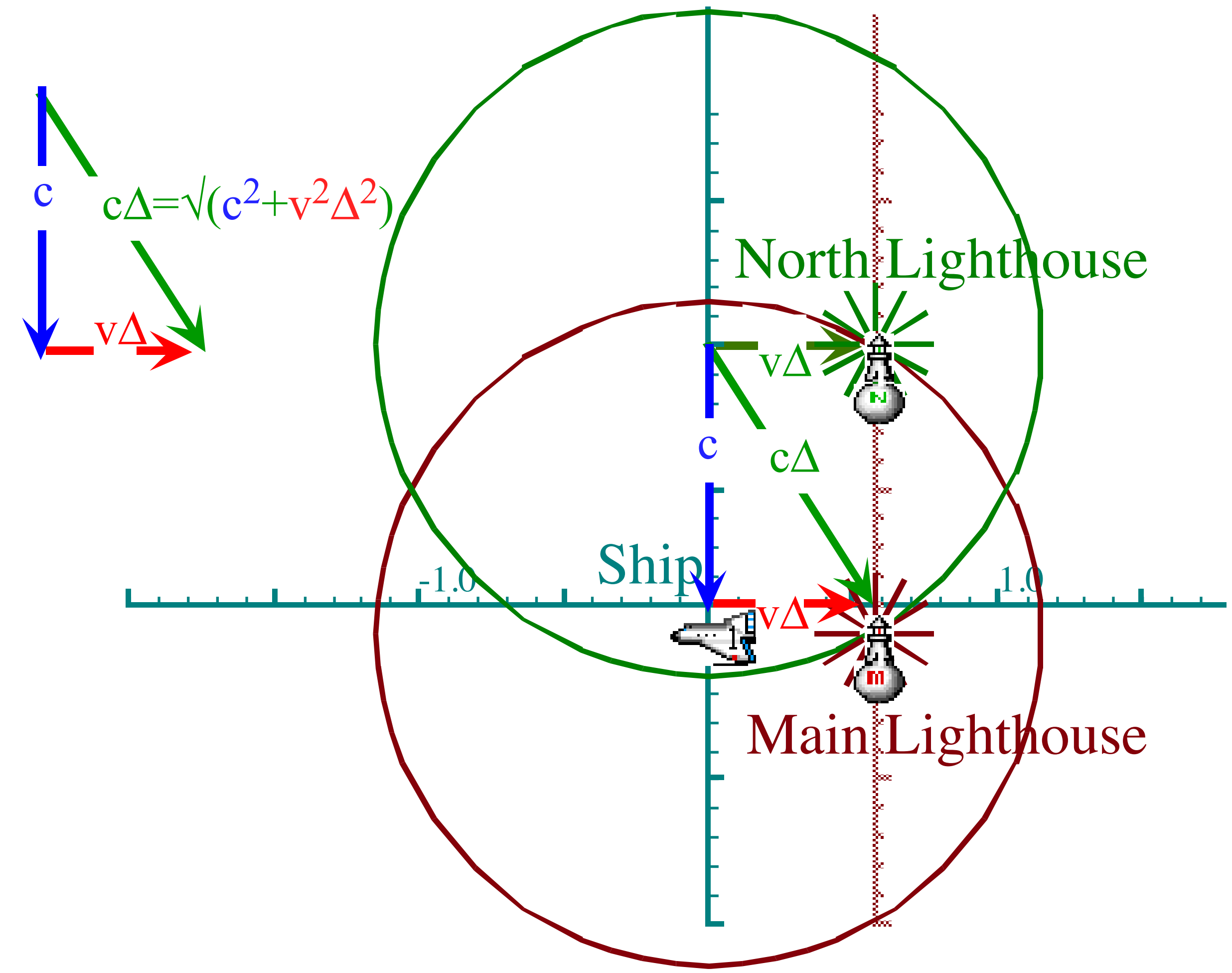

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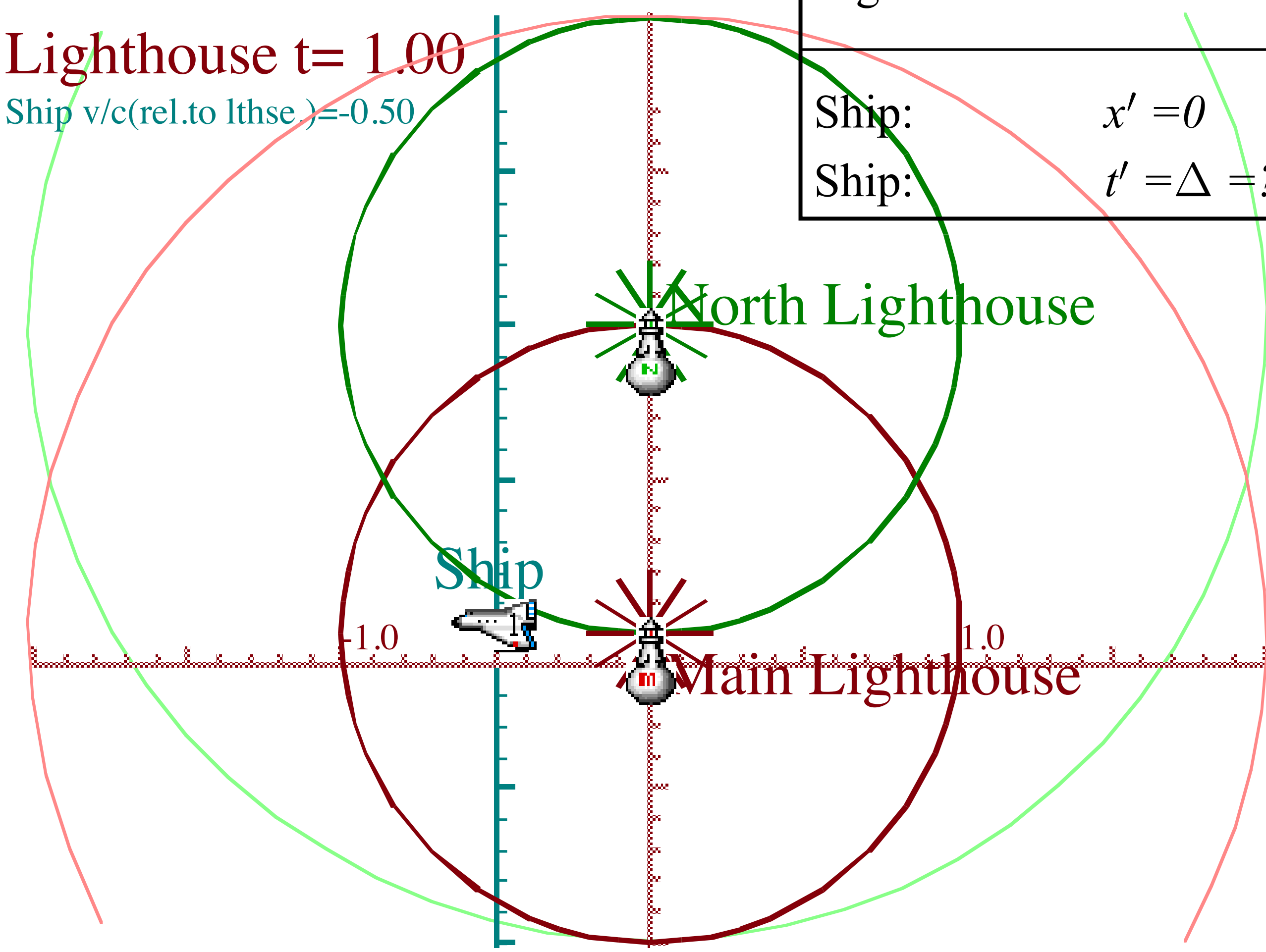
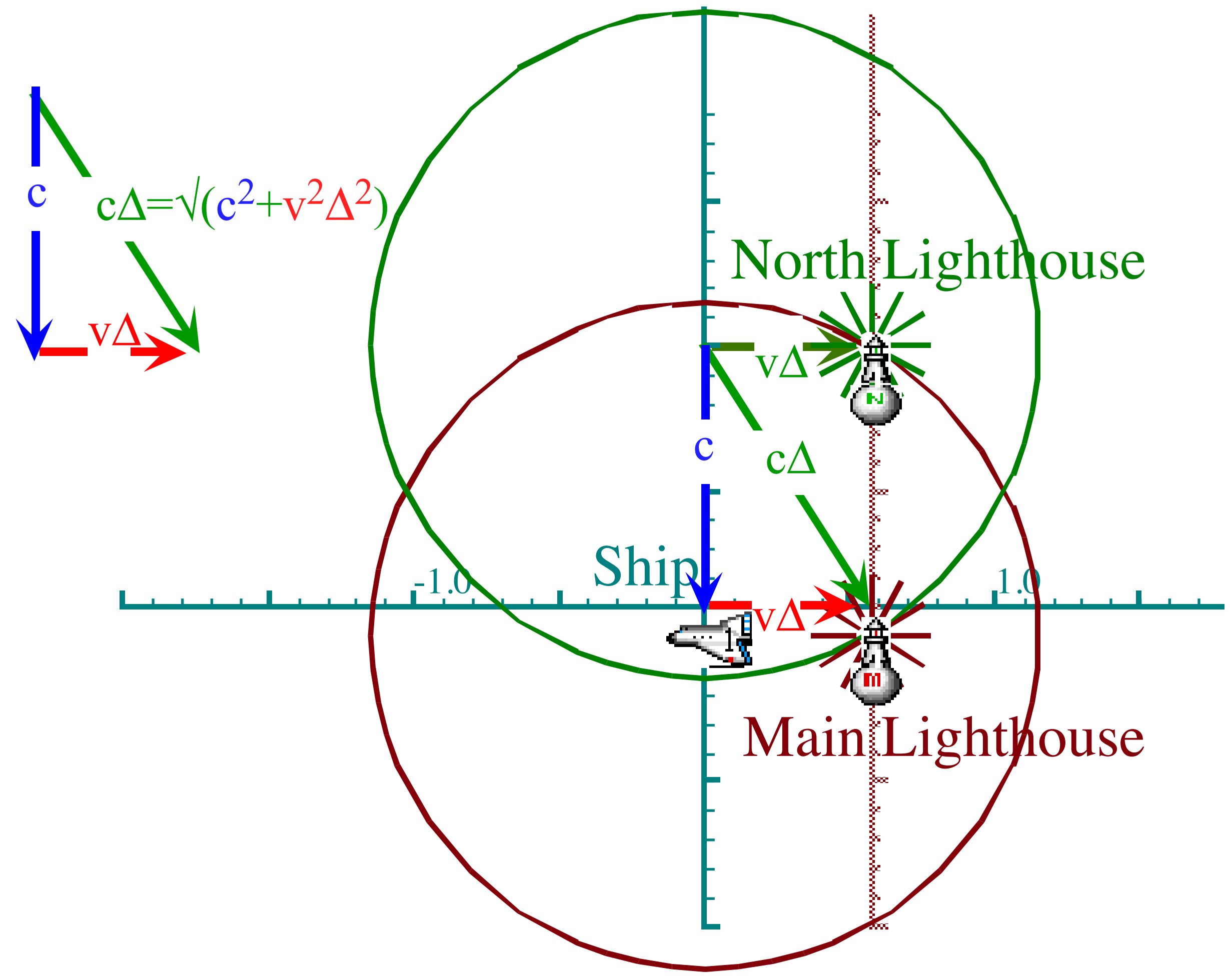
$$c^2\Delta^2 = c^2 + v^2\Delta^2$$

$$\left(c^2 - v^2\right)\Delta^2 = c^2$$





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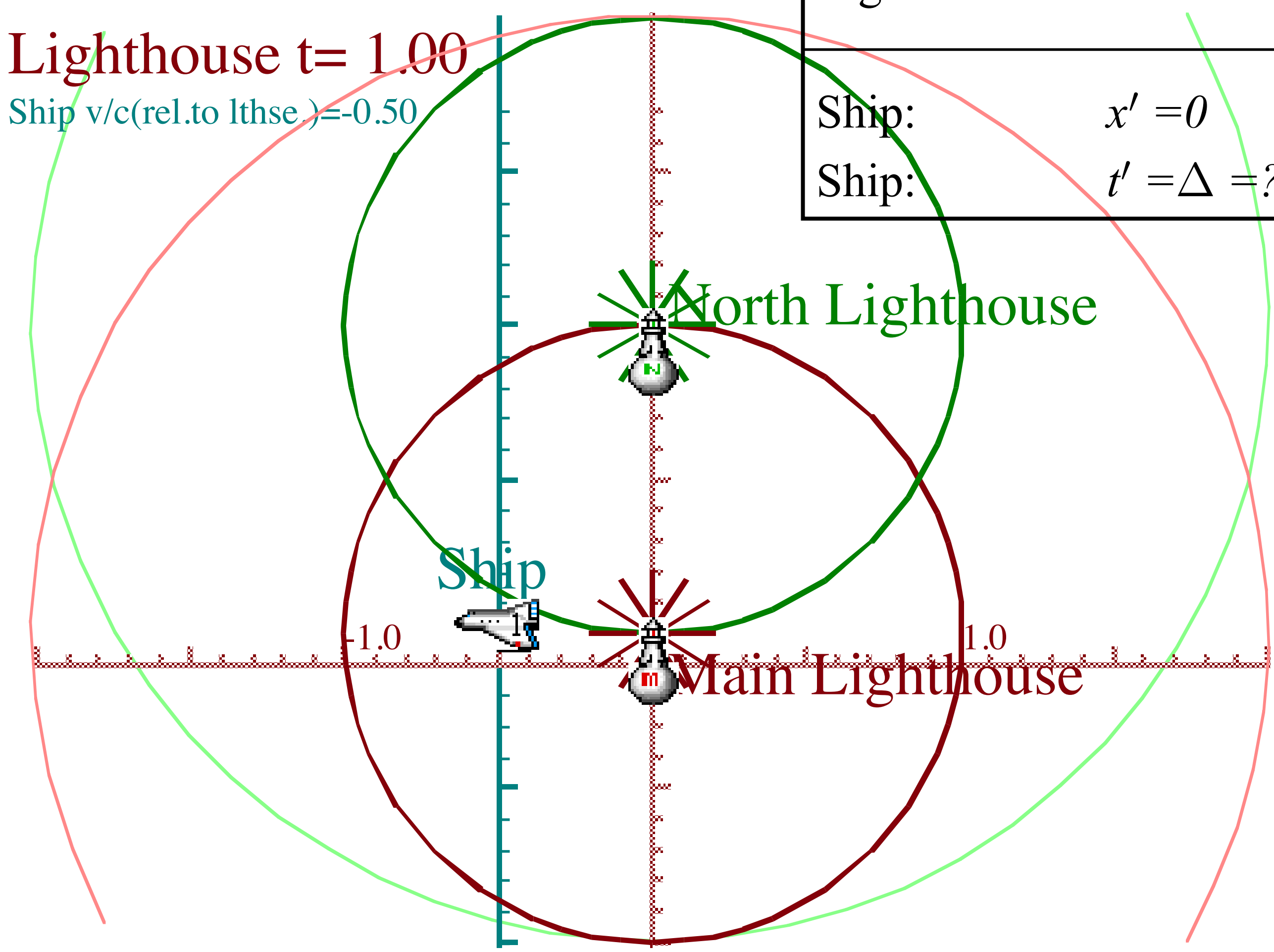
Lighthouse:	$t = 1.00$
Ship:	$x' = 0$
Ship:	$t' = \Delta = ???$

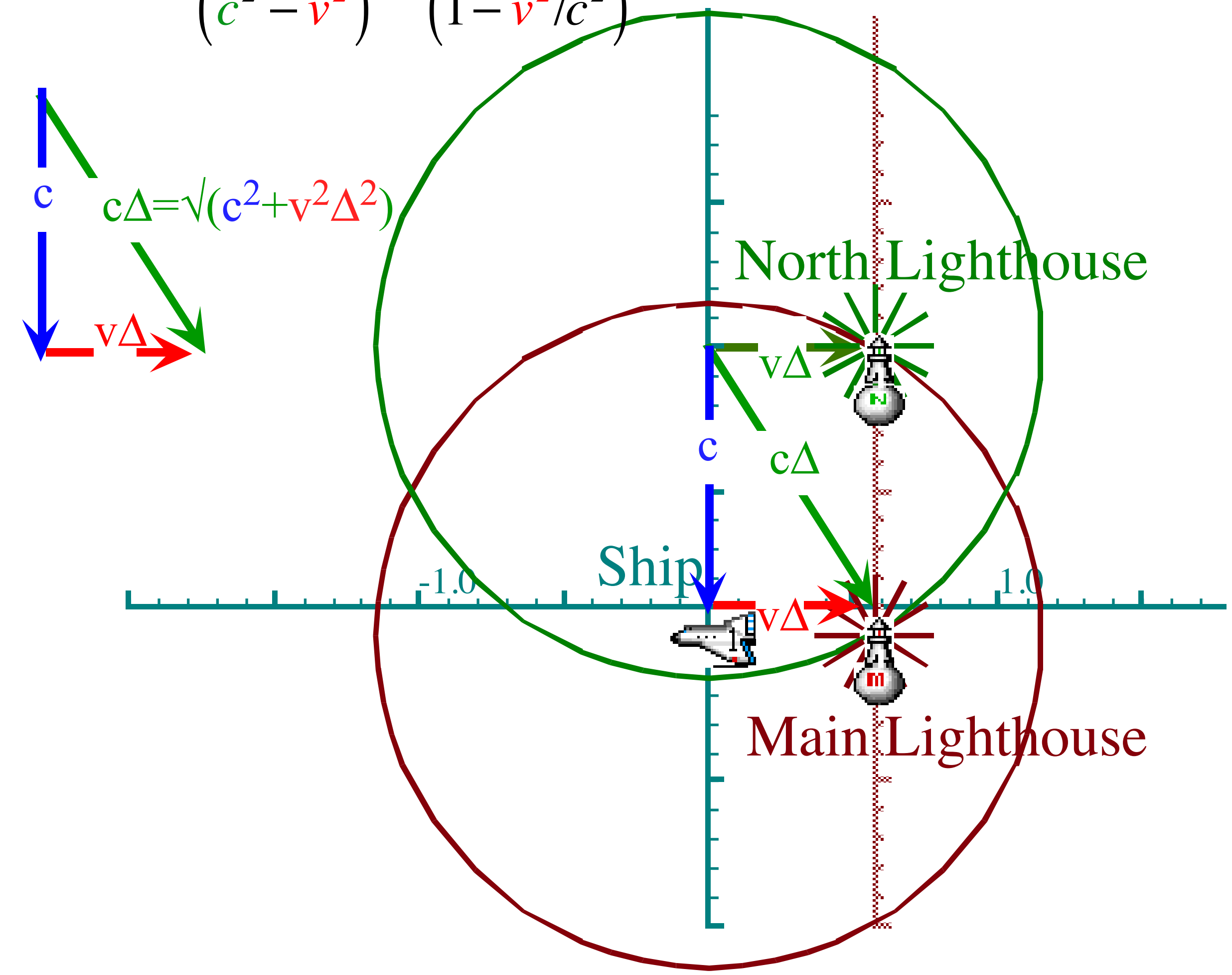


Ship Time $t' = \Delta = 1/\sqrt{(1-v^2/c^2)} = \cosh\rho$

$$c^2\Delta^2 = c^2 + v^2\Delta^2$$

$$\left(c^2 - v^2\right)\Delta^2 = c^2$$

$$\Delta^2 = \frac{c^2}{\left(c^2 - v^2\right)} = \frac{1}{\left(1 - v^2/c^2\right)}$$



Comparing Ship and Lighthouse views: Happening tables

Happening 0: Ship passes Main Lighthouse.	Happening 1: Ship gets hit by first blink from Main Lighthouse.	Happening 2: Main Lighthouse blinks second time.
(Lighthouse space) $x = 0$ (Lighthouse time) $t = 0$	$x = -1.00\,c$ $t = 2.00$	$x = 0$ $t = 2.00$
(Ship space) $x' = 0$ (Ship time) $t' = 0$	$x' = 0$ $t' = 1.75$	$x' = c\,\Delta$ $t' = 2\Delta = 2.30$

Fig. 2.A.3 Happening 1 (1st blink hits ship) and 2 (2nd blink at Main) both happen at t=2.

The ship and lighthouse saga

Happening 0.5: Main Lite blinks first time.	
Lighthouse:	$x = 0$
Lighthouse:	$t = 1.00$
Ship:	$x' = 0$
Ship:	$t' = \Delta = 1.15$

Ship Time $t' = \Delta = 1/\sqrt{(1-v^2/c^2)} = \cosh \rho = 1.15$

$$c^2\Delta^2 = c^2 + v^2\Delta^2$$

$$\left(c^2 - v^2\right)\Delta^2 = c^2$$

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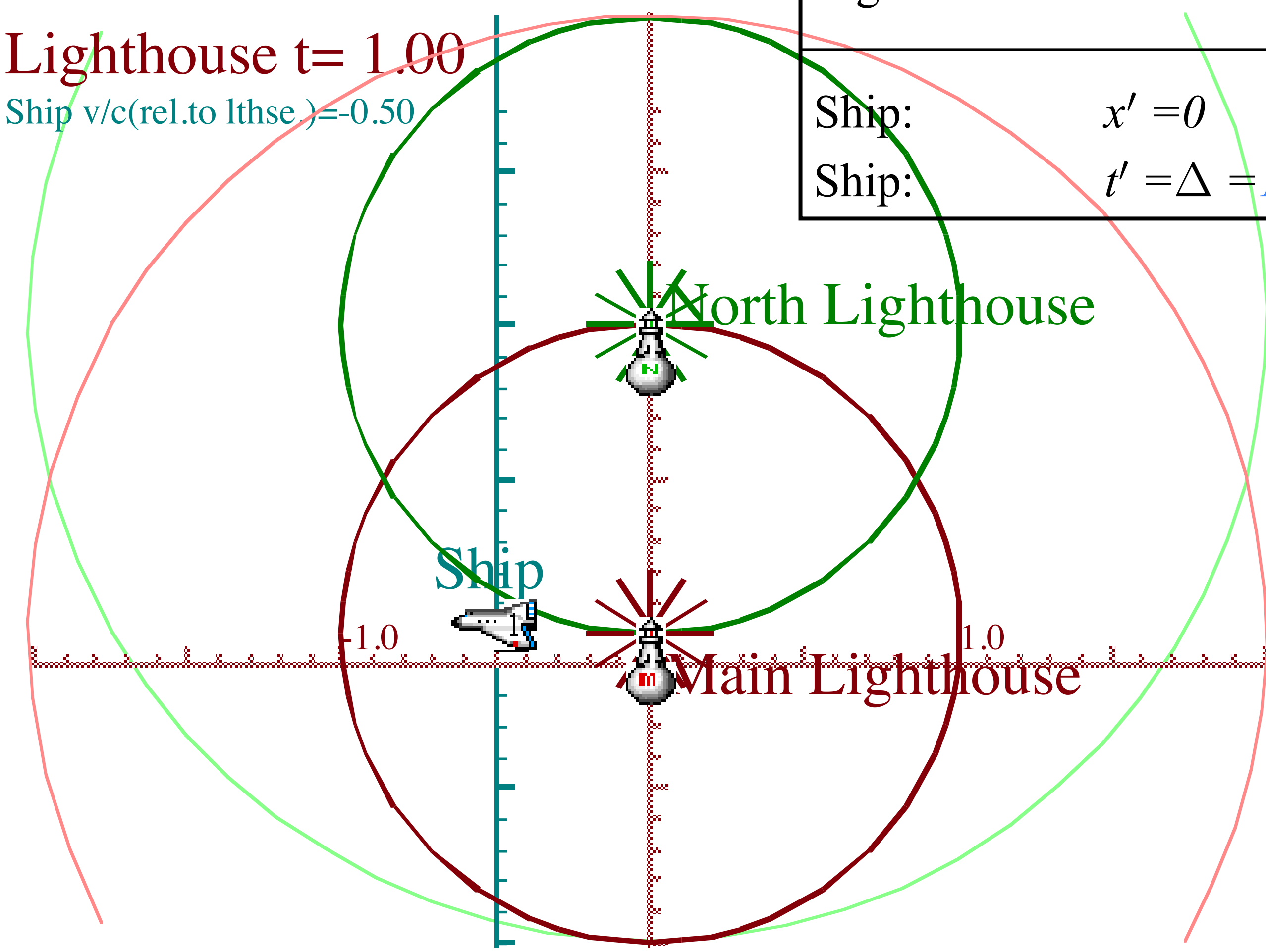

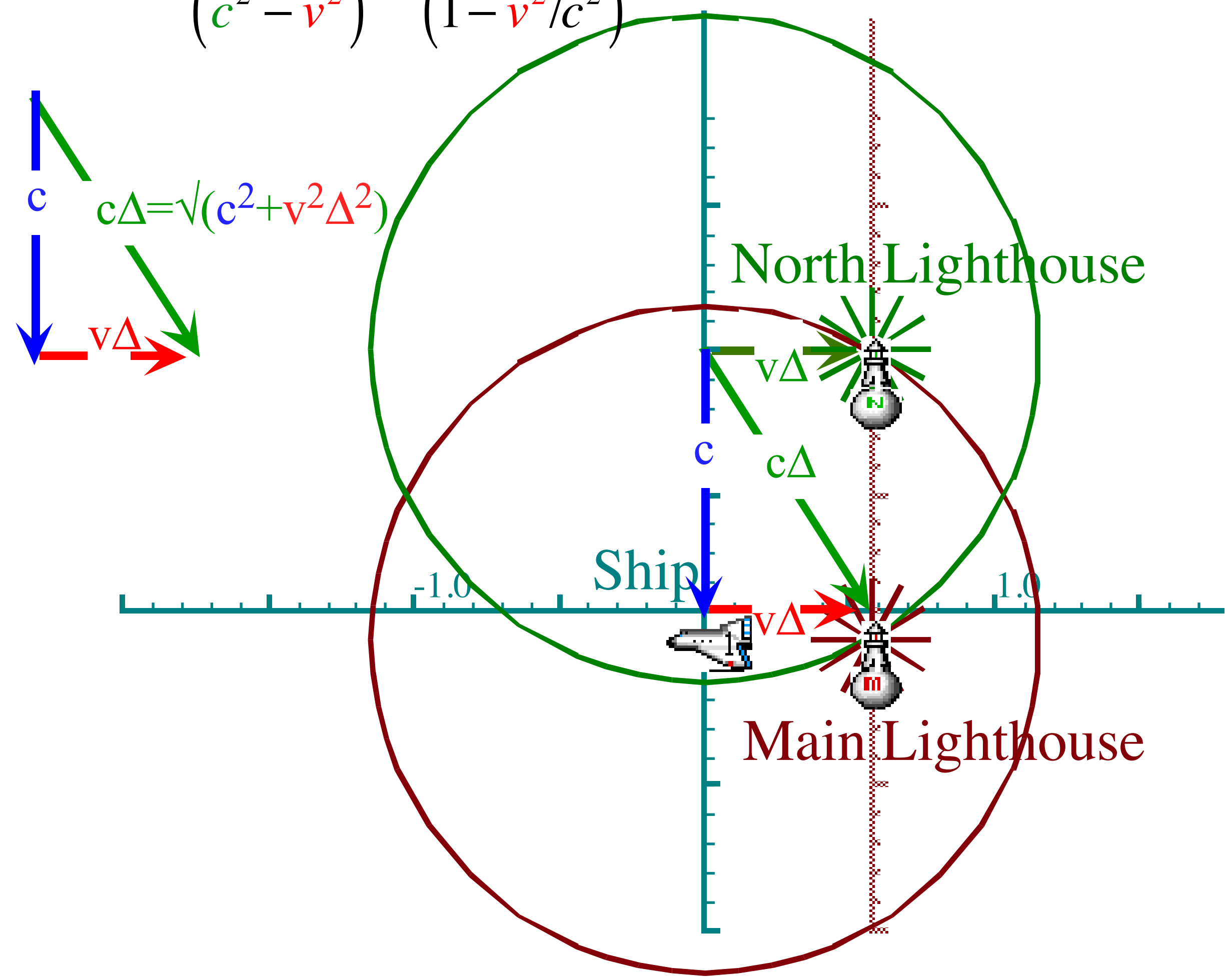

For u/c=1/2

$\Delta = 1/\sqrt{(1-1/4)} = 2/\sqrt{3} = 1.15..$

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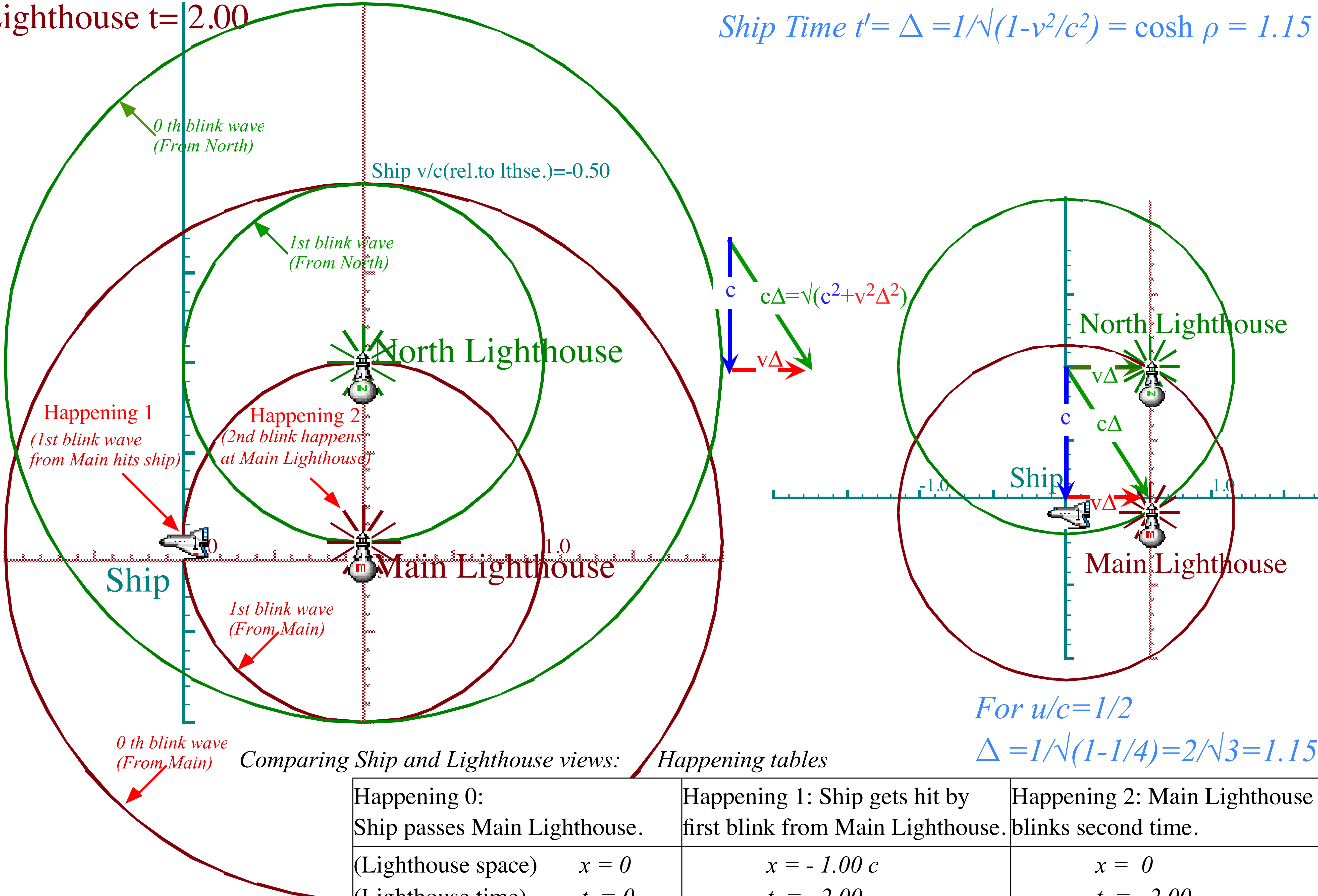
Fig. 2.A.3 Happening 1 (1st blink hits ship) and 2 (2nd blink at Main) both happen at t=2.

Lighthouse t= 2.00

Ship Time t′= Δ =1/√(1-v²/c²) = cosh *ρ = 1.15*



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Comparing Ship and Lighthouse views: *Happening tables*

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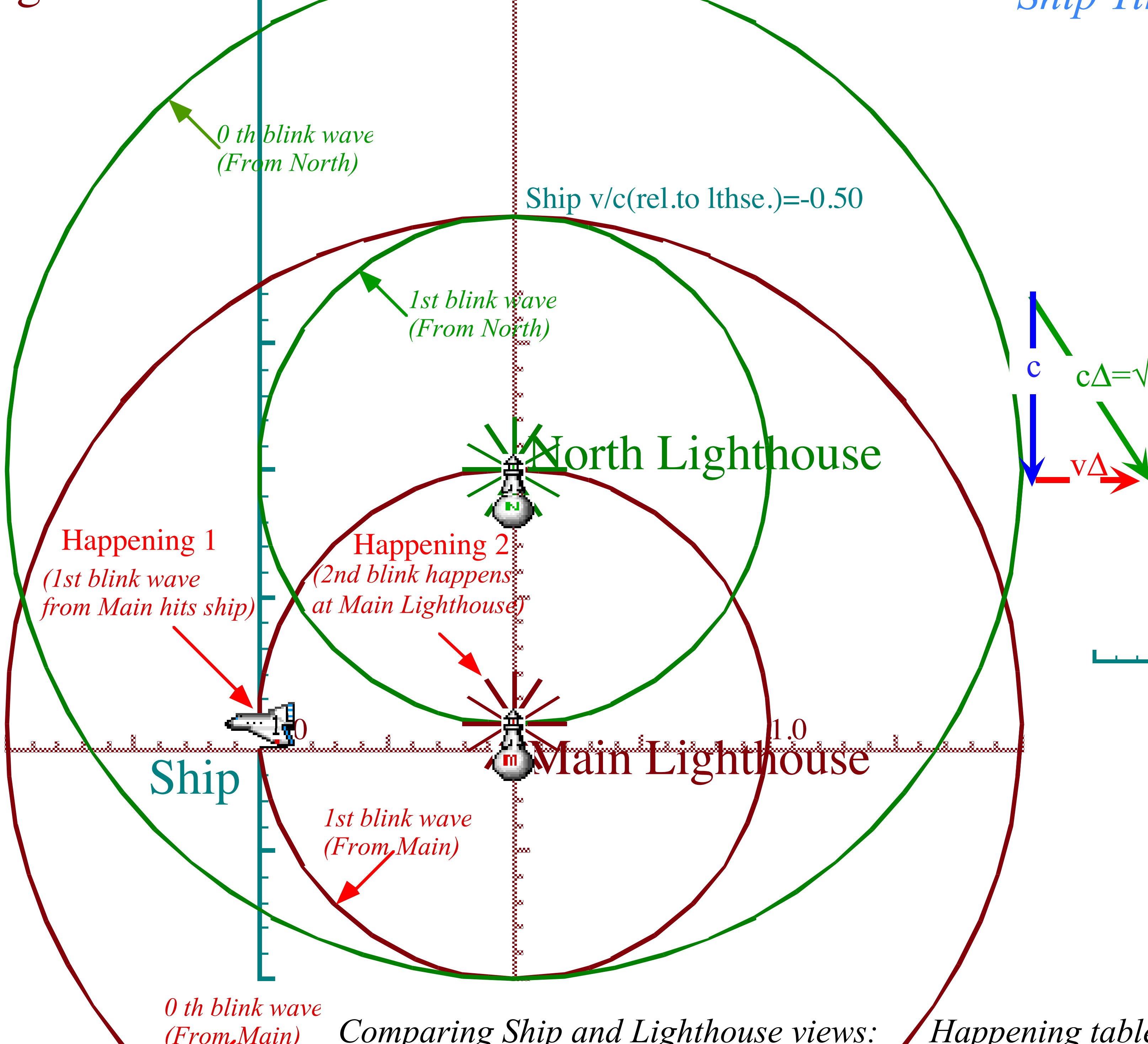

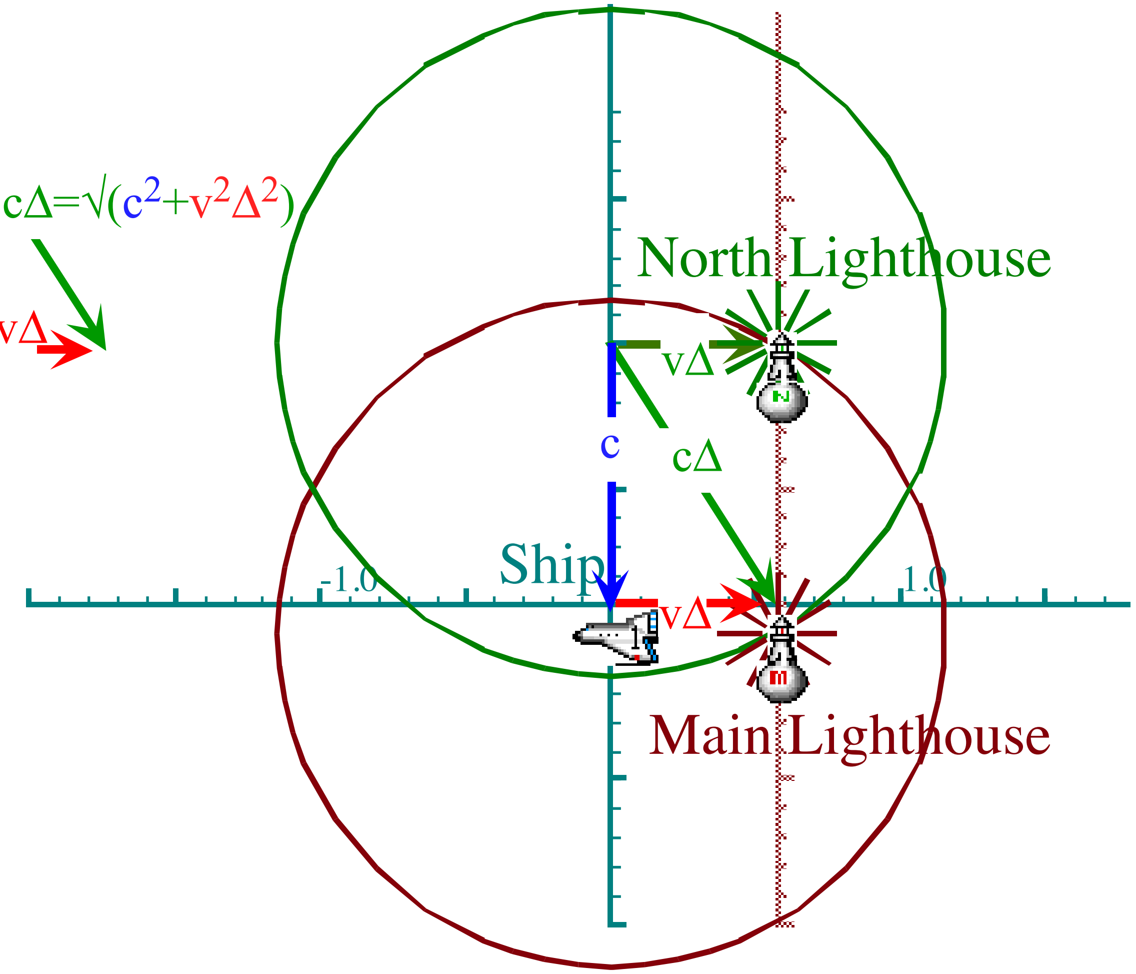

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Comparing Ship and Lighthouse views: Happening tables

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(Lighthouse space) $x = 0$ (Lighthouse time) $t = 0$	$x = -vc/(c-v)$ $t = c/(c-v)$	$x = 0$ $t = 2.00$
(Ship space) $x' = 0$ (Ship time) $t' = 0$	$x' = 0$ $t' = (v+c)\Delta/c$	$x' = 2v\Delta$ $t' = 2\Delta$

Fig. 2.A.3 Happening 1 (1st blink hits ship) and 2 (2nd blink at Main) both happen at t=2.

Lecture 24 ended here

That "old-time" relativity *(Circa 600BCE- 1905CE)*

("Bouncing-photons" in smoke & mirrors and Thales, again)

The Ship and Lighthouse saga

Light-conic-sections make invariants

A politically incorrect analogy of rotational transformation and Lorentz transformation

The straight scoop on "angle" and "rapidity" (They're area!)

Galilean velocity addition becomes rapidity addition

Introducing the "Sin-Tan Rosetta Stone" (Thanks, Thales!)

Introducing the stellar aberration angle σ vs. rapidity ρ

How Minkowski's space-time graphs help visualize relativity

Group vs. phase velocity and tangent contacts

Light-conic-sections make invariants

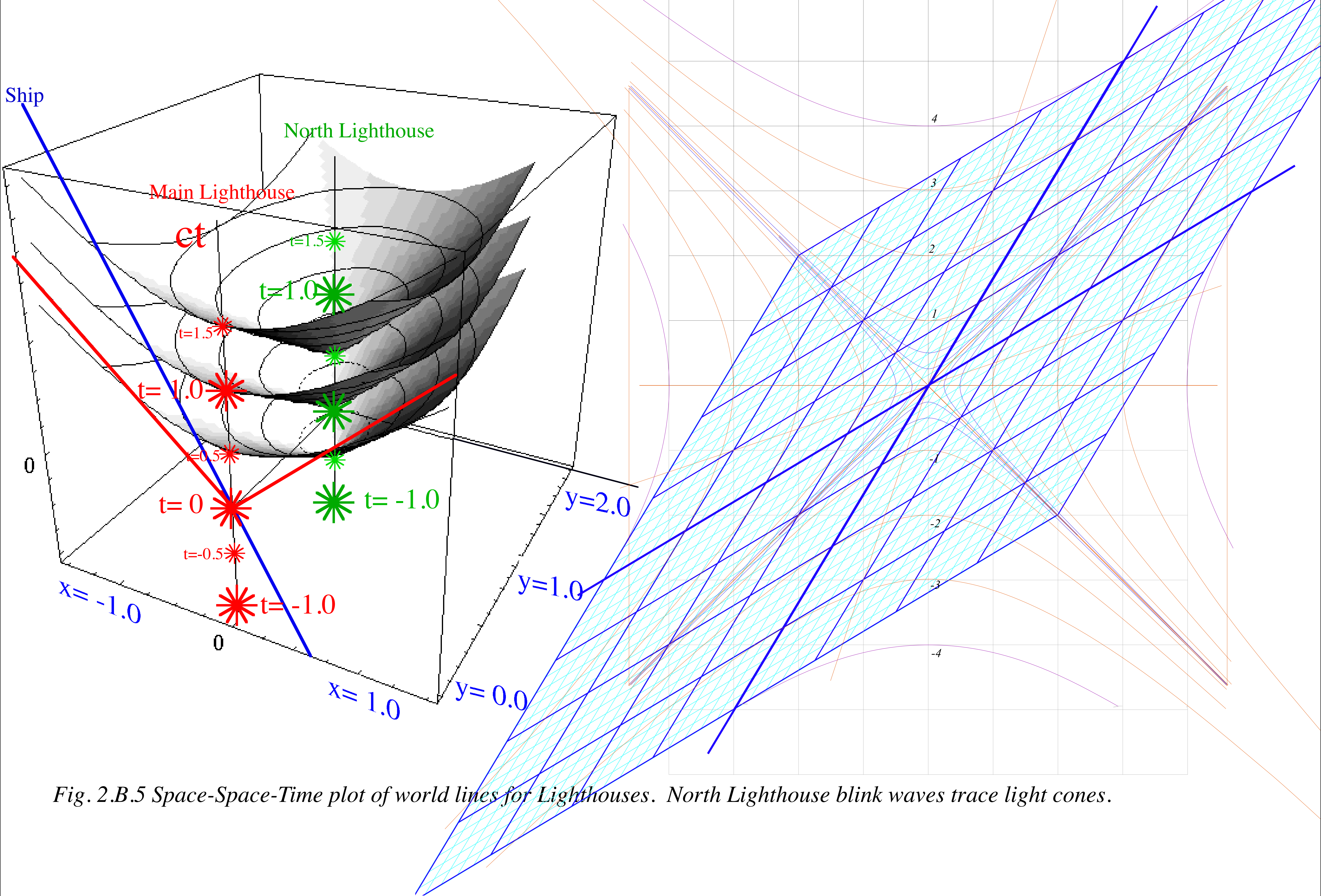

Fig. 2.B.5 Space-Space-Time plot of world lines for Lighthouses. North Lighthouse blink waves trace light cones.

That "old-time" relativity *(Circa 600BCE- 1905CE)*

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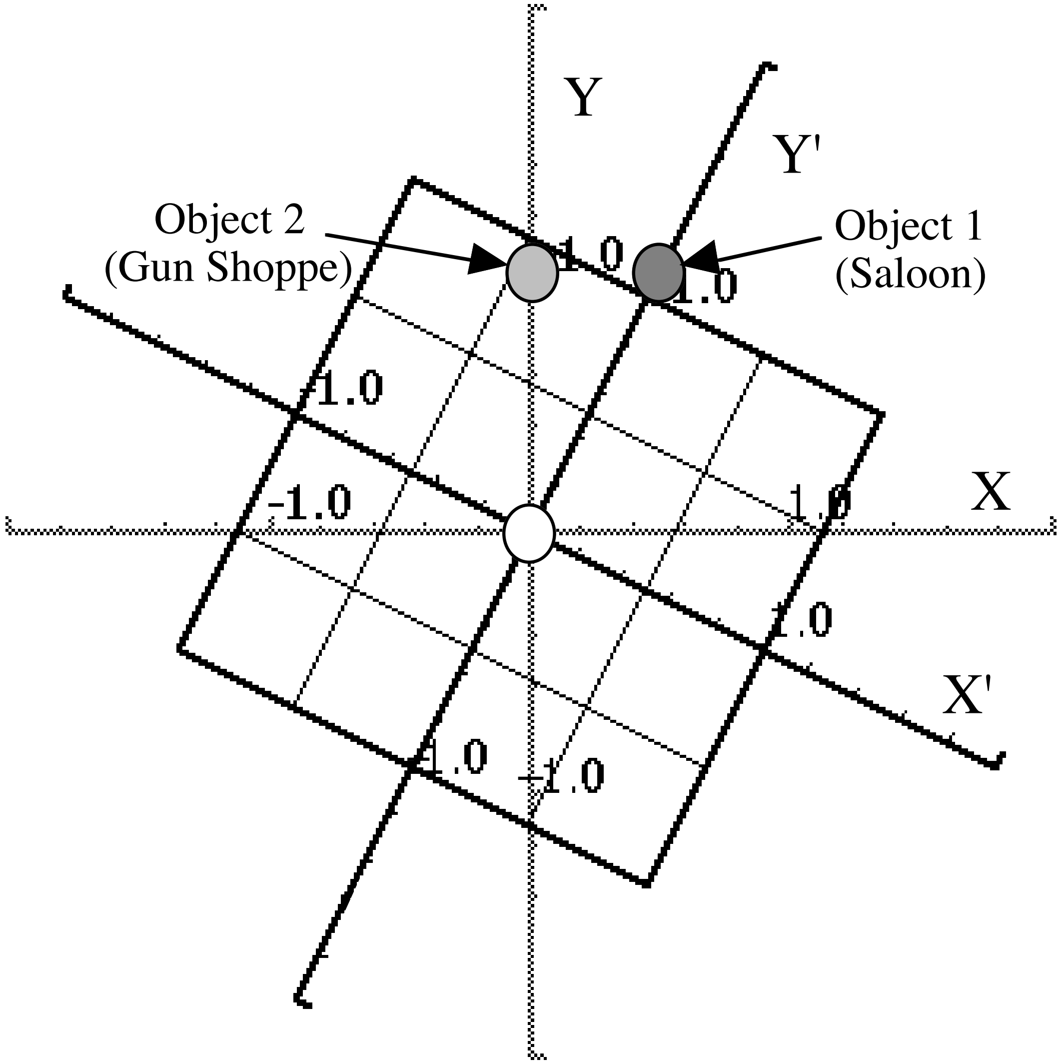
Fig. 2.B.1 Town map according to a "tipsy" surveyor.



Object 0: Town Square.	Object 1: Saloon.	Object 2: Gun Shoppe.
(US surveyor) $x = 0$, $y = 0$	$x = 0.5$, $y = 1.0$	$x = 0$, $y = 1.0$
(French surveyor) $x' = 0$, $y' = 0$	$x' = 0$, $y' = 1.1$	$x' = -0.45$, $y' = 0.89$

A politically incorrect analogy of rotational transformation and Lorentz transformation

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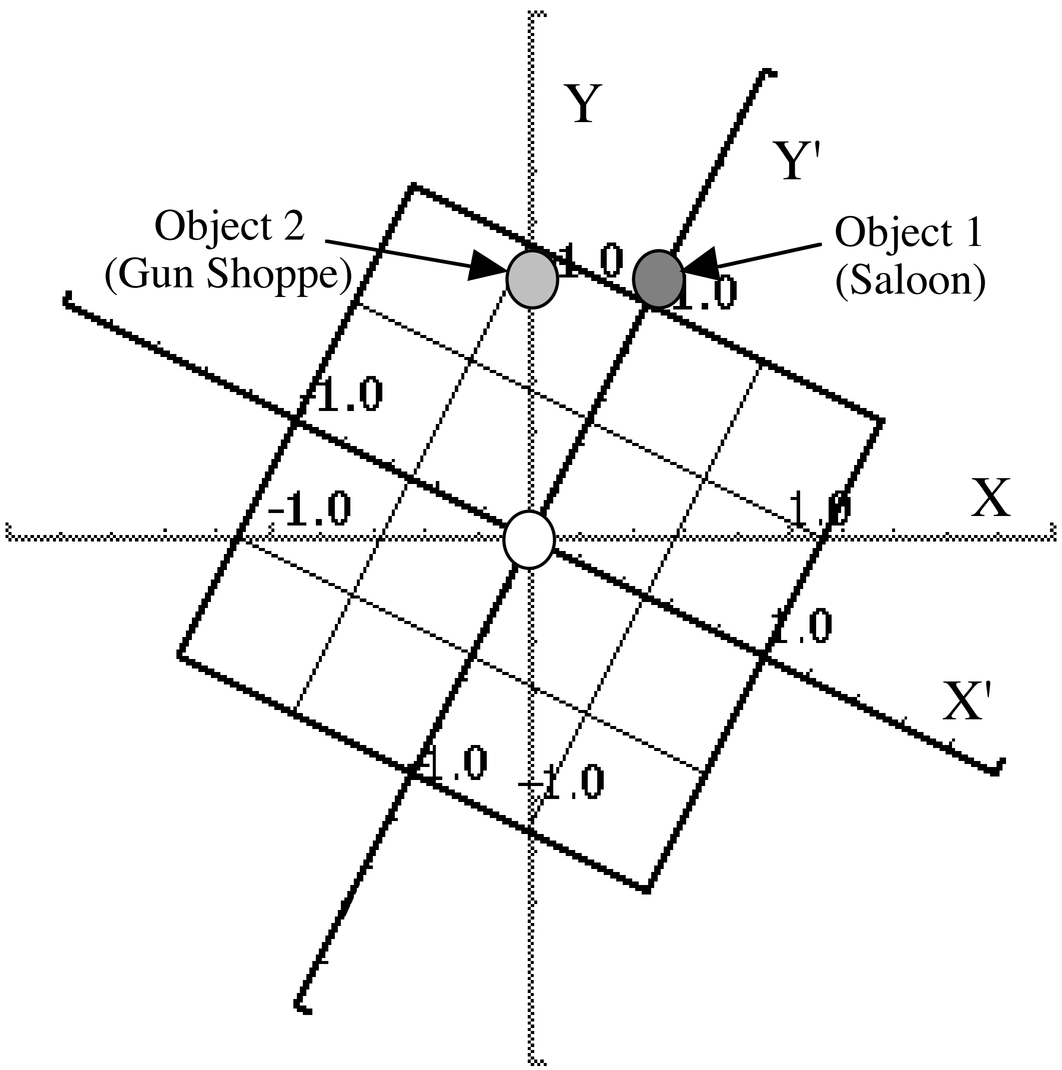
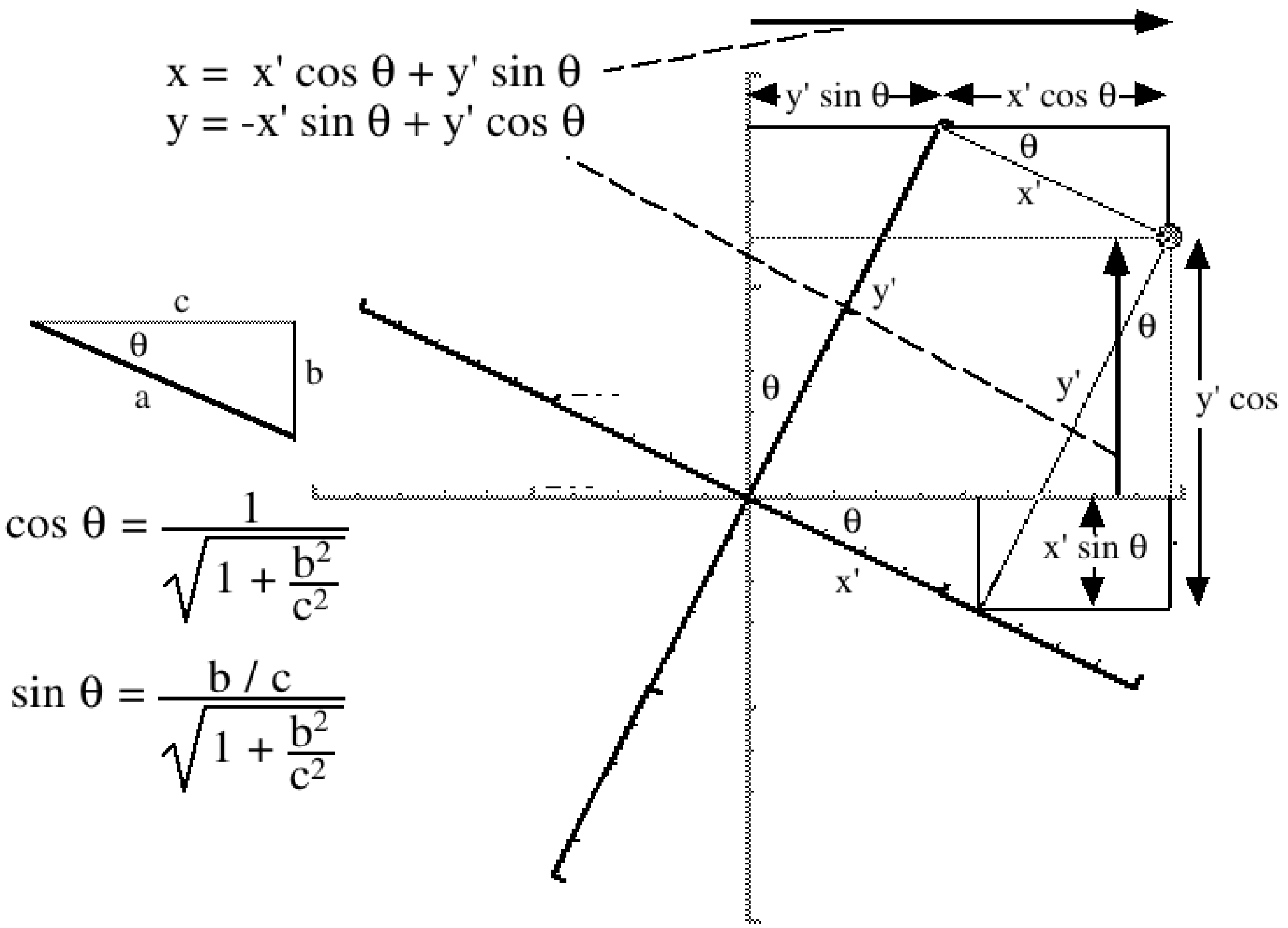


Fig. 2.B.2 Diagram and formulas for reconciliation of the two surveyor's data.



$$\cos\theta = \frac{1}{\sqrt{1+\frac{b^2}{c^2}}}$$

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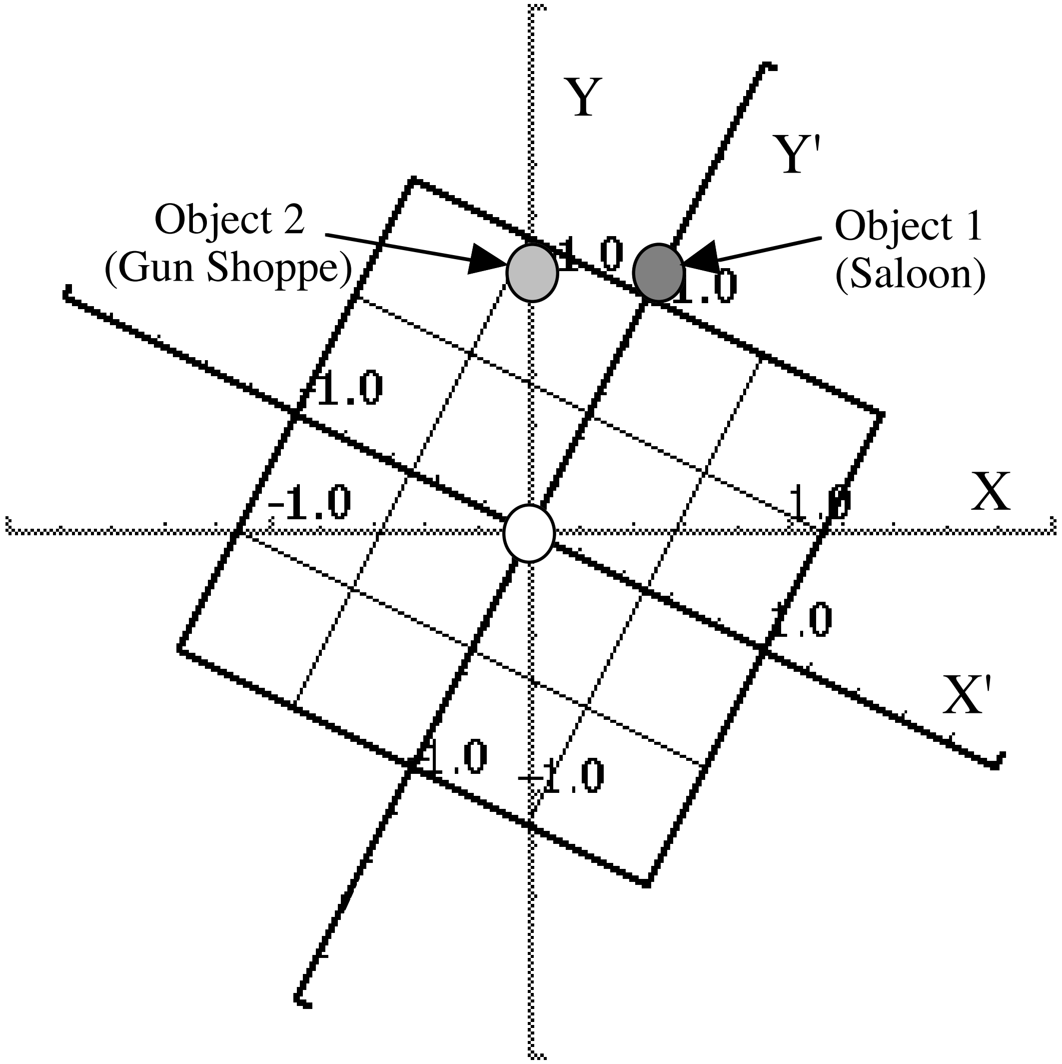
$$x' = x\cos\theta - y\sin\theta = \frac{x}{\sqrt{1+\frac{b^2}{c^2}}} + \frac{-(b/c)y}{\sqrt{1+\frac{b^2}{c^2}}}$$

$$y' = x\sin\theta + y\cos\theta = \frac{(b/c)x}{\sqrt{1+\frac{b^2}{c^2}}} + \frac{y}{\sqrt{1+\frac{b^2}{c^2}}}$$

A politically incorrect analogy of rotational transformation and Lorentz transformation



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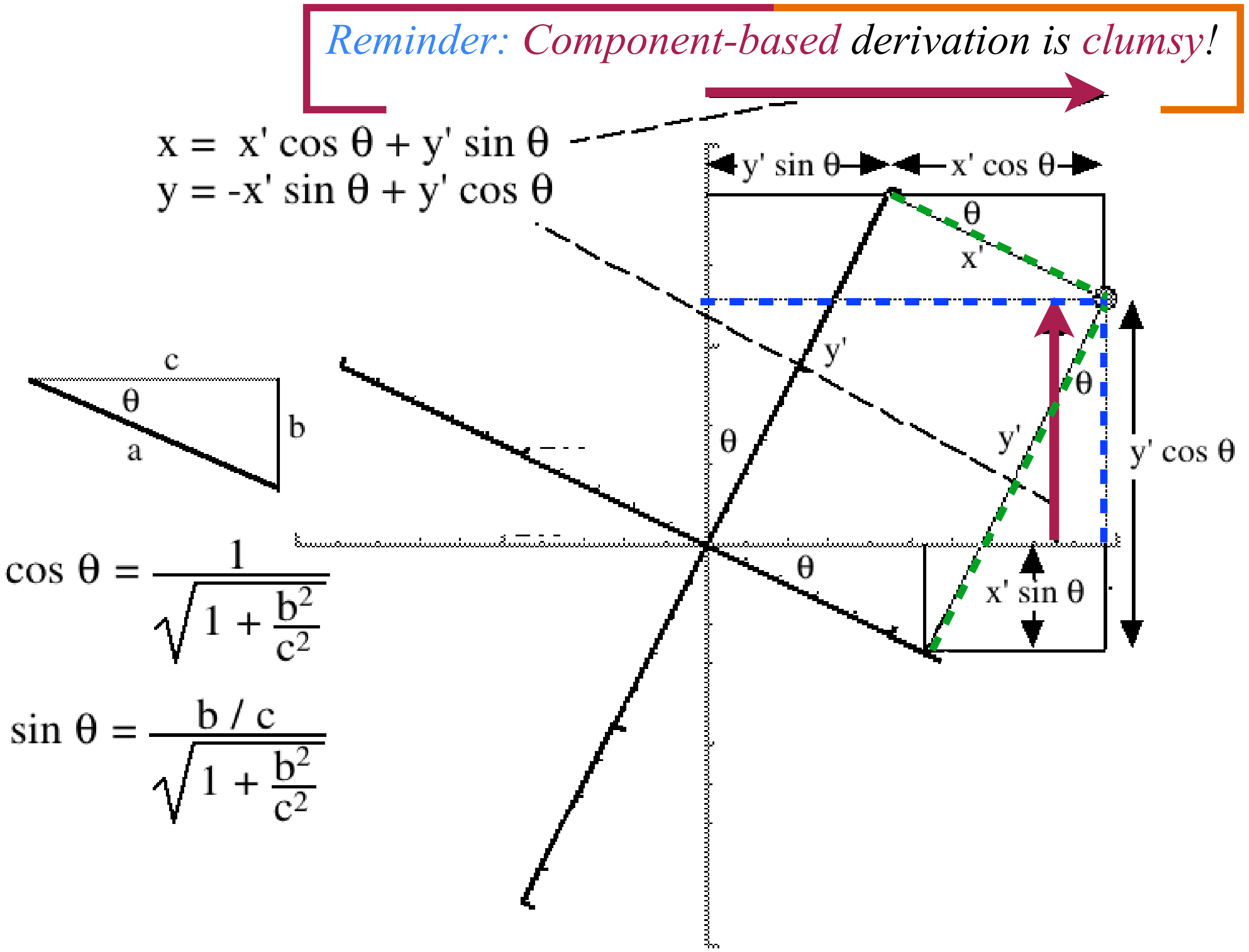

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Fig. 2.B.2 Diagram and formulas for reconciliation of the two surveyor's data.

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A politically incorrect analogy of rotational transformation and Lorentz transformation

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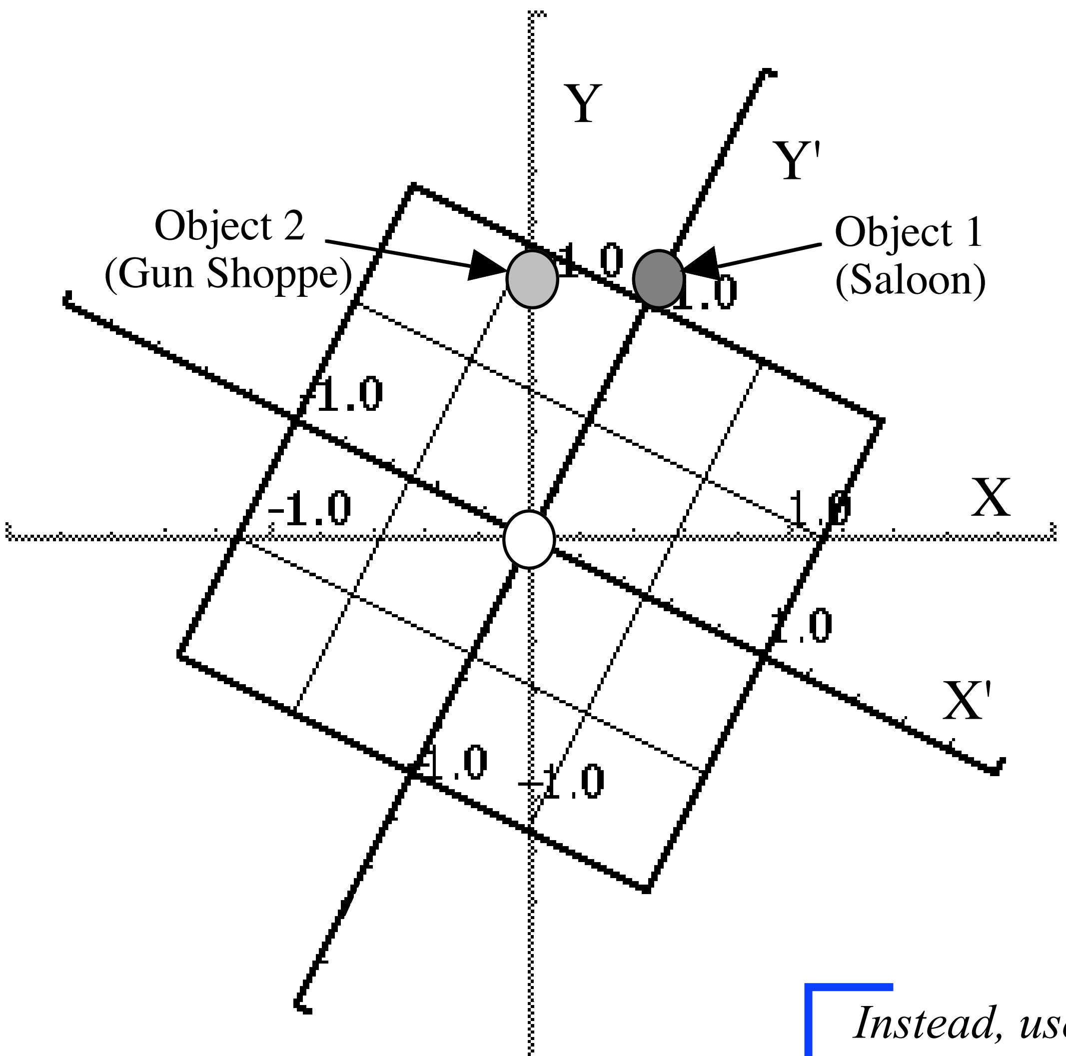

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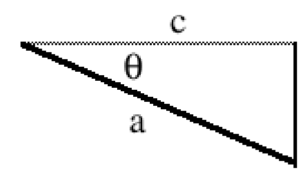
Reminder: Component-based derivation is clumsy!

x = x' cos θ + y' sin θ

y = -x' sin θ + y' cos θ

Forget this!! It's too clumsy to generalize to 3D, 4D,...



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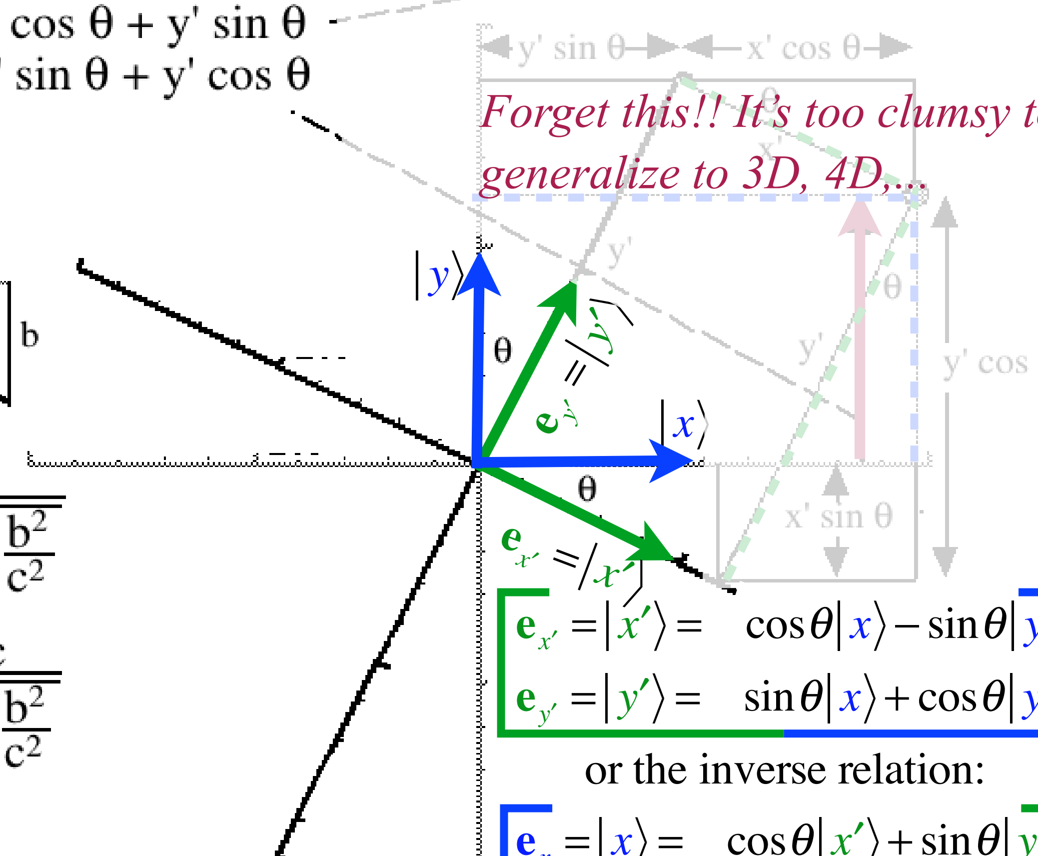

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A politically incorrect analogy of rotational transformation and Lorentz transformation

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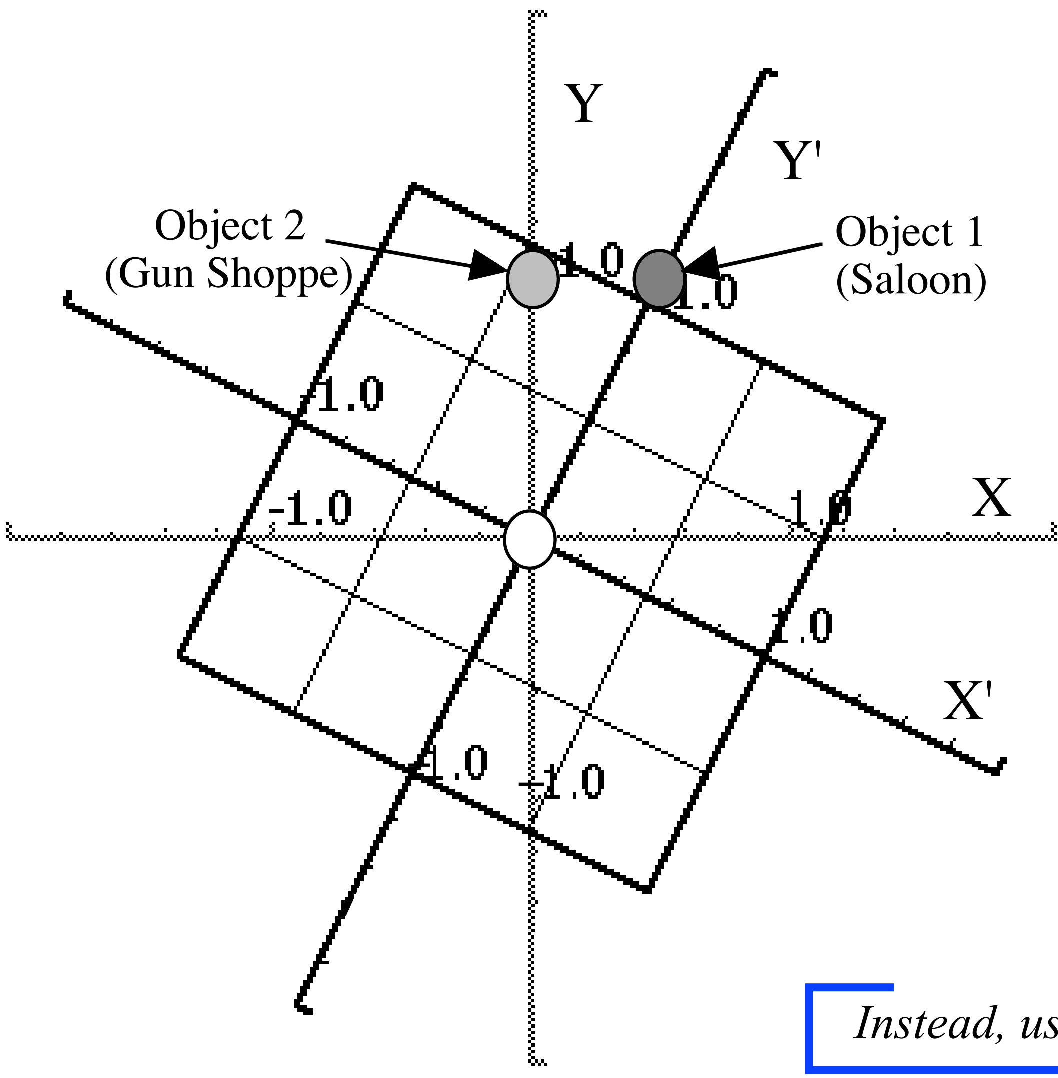

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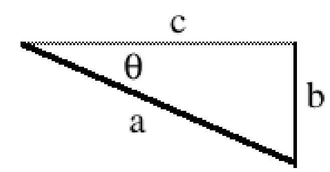

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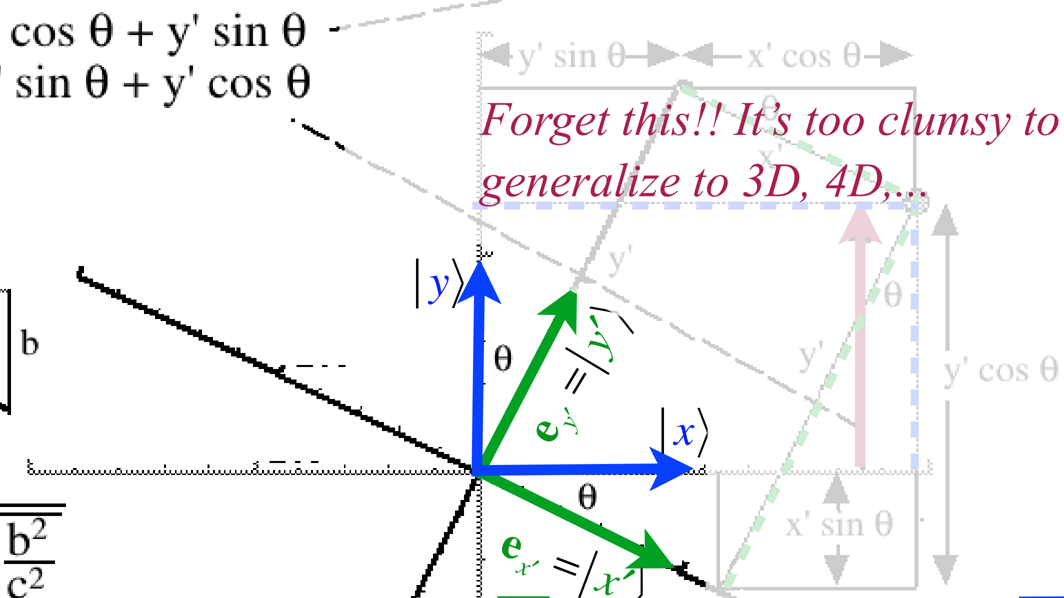

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You may apply (Jacobian) transform matrix:

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A politically incorrect analogy of rotational transformation and Lorentz transformation

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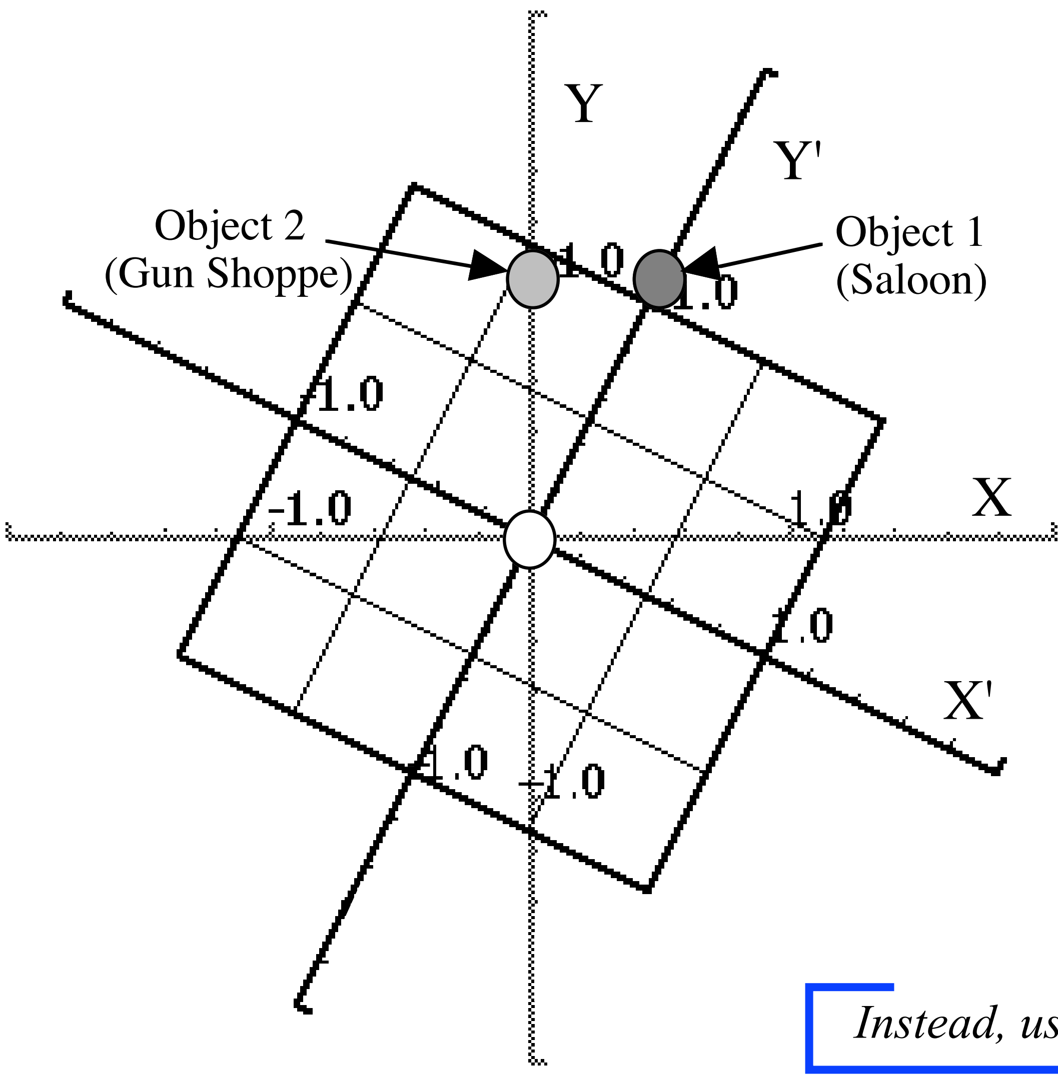

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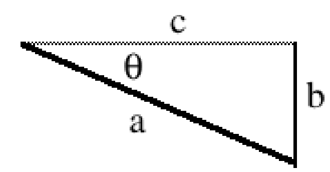

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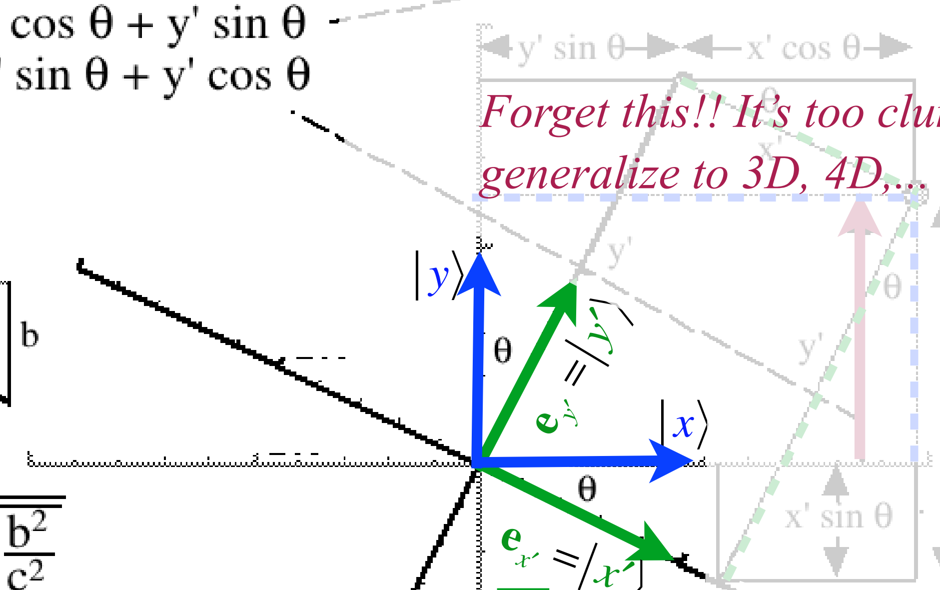

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(Jacobian) transformation $\{V_x V_y\}$ from $\{V_{x'} V_{y'}\}$:

$$V_x = \langle x|V\rangle = \langle x|1|V\rangle = \langle x|x'\rangle\langle x'|V\rangle + \langle x|y'\rangle\langle y'|V\rangle$$
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A politically incorrect analogy of rotational transformation and Lorentz transformation

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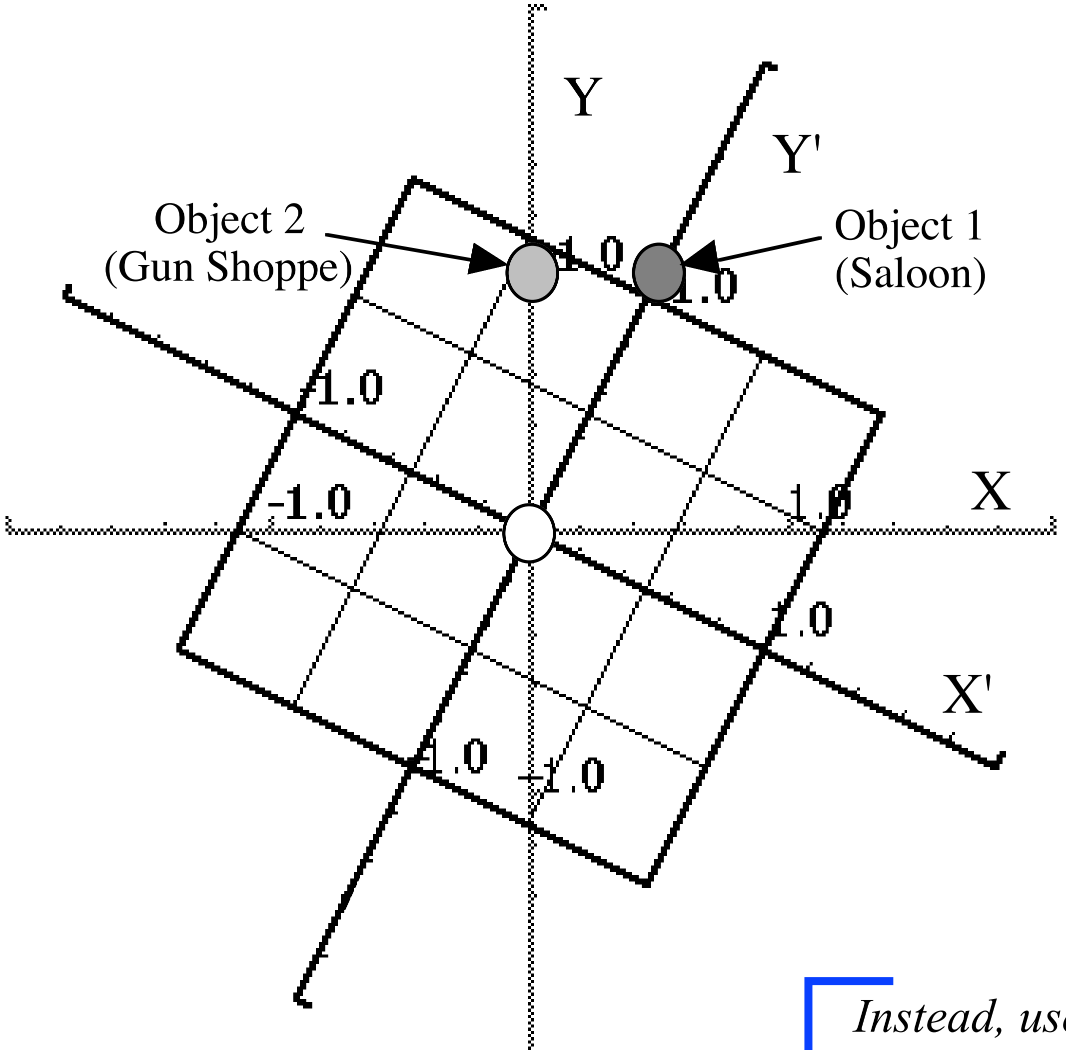

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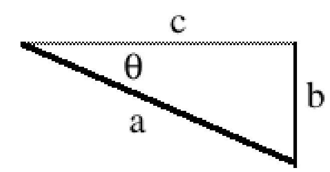

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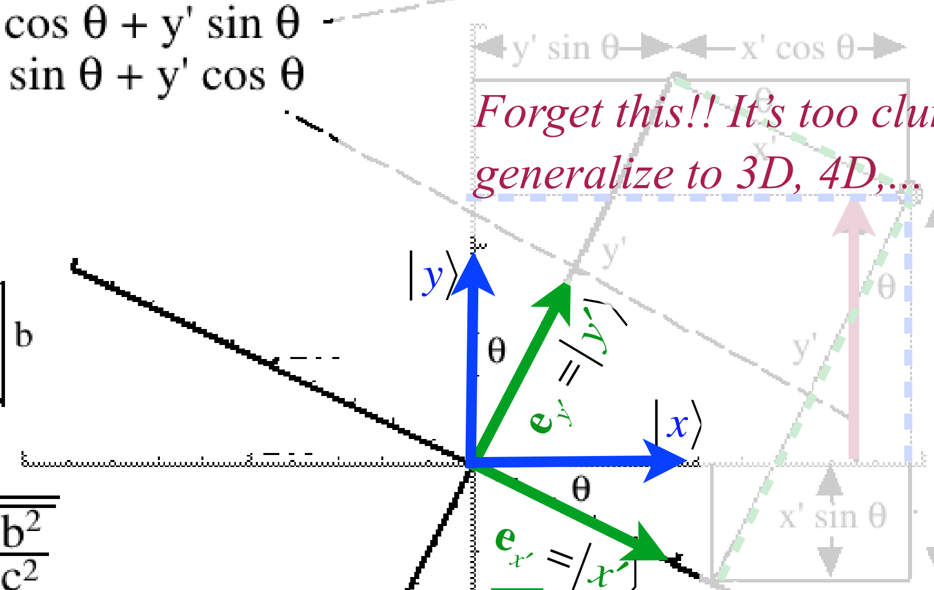

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in matrix form:

$$\begin{pmatrix} V_x \\ V_y \end{pmatrix} = \begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} \begin{pmatrix} V_{x'} \\ V_{y'} \end{pmatrix}$$

PLEASE!

Do NOT ever write

this:

$$\mathbf{e}_{x'} = |x'\rangle = \cos\theta|x\rangle - \sin\theta|y\rangle$$
$$\mathbf{e}_{y'} = |y'\rangle = \sin\theta|x\rangle + \cos\theta|y\rangle$$

like this:

$$\begin{pmatrix} \mathbf{e}_{x'} \\ \mathbf{e}_{y'} \end{pmatrix} = \begin{pmatrix} |x'\rangle \\ |y'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |x\rangle \\ |y\rangle \end{pmatrix}$$

PLEASE!

Do NOT ever write

this:

$$\mathbf{e}_{x'} = |x'\rangle = \cos\theta|x\rangle - \sin\theta|y\rangle \equiv \mathbf{R}|x\rangle$$

$$\mathbf{e}_{y'} = |y'\rangle = \sin\theta|x\rangle + \cos\theta|y\rangle \equiv \mathbf{R}|y\rangle$$

(This is a useful abstract definition.)

like this:

$$\begin{pmatrix} \mathbf{e}_{x'} \\ \mathbf{e}_{y'} \end{pmatrix} = \begin{pmatrix} |x'\rangle \\ |y'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |x\rangle \\ |y\rangle \end{pmatrix}$$

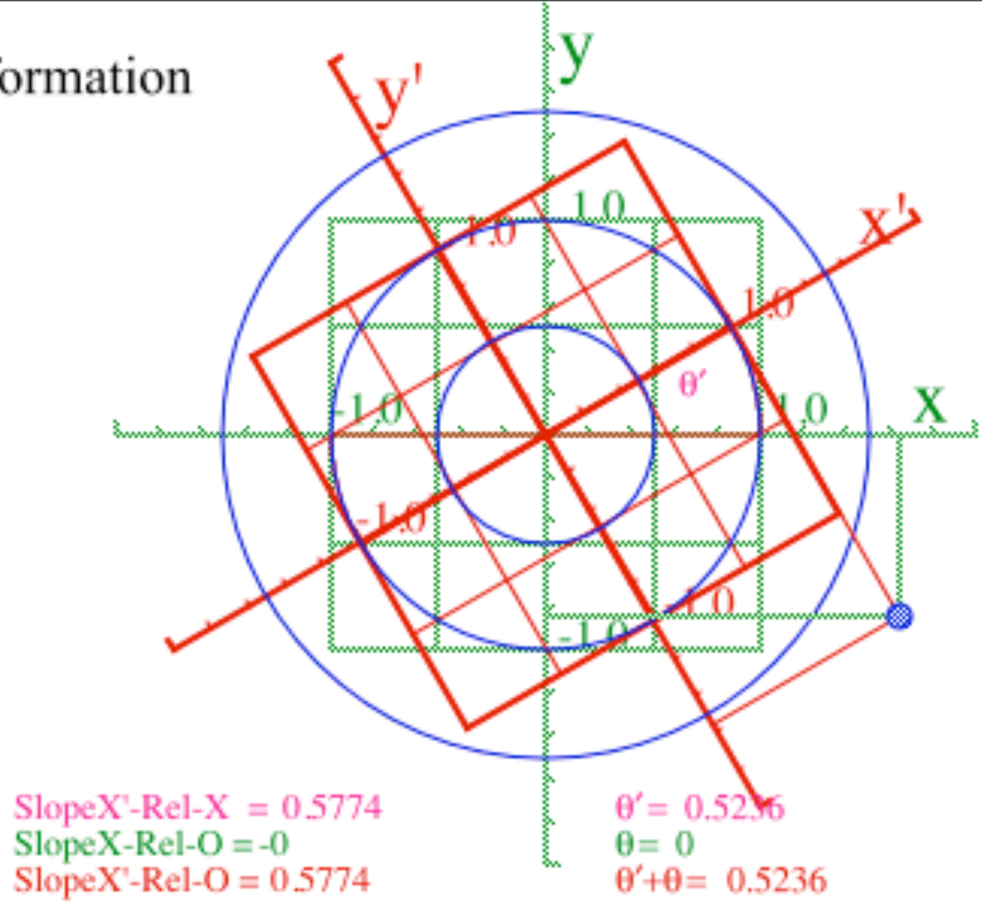
(Not helpful)

Here is a matrix representation of abstract definitions: $|x'\rangle \equiv \mathbf{R}|x\rangle$, $|y'\rangle \equiv \mathbf{R}|y\rangle$

$$\begin{pmatrix} V_x \\ V_y \end{pmatrix} = \begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} \begin{pmatrix} V_{x'} \\ V_{y'} \end{pmatrix} = \begin{pmatrix} \langle x|\mathbf{R}|x\rangle & \langle x|\mathbf{R}|y\rangle \\ \langle y|\mathbf{R}|x\rangle & \langle y|\mathbf{R}|y\rangle \end{pmatrix} \begin{pmatrix} V_{x'} \\ V_{y'} \end{pmatrix} = \begin{pmatrix} \langle x'|\mathbf{R}|x'\rangle & \langle x'|\mathbf{R}|y'\rangle \\ \langle y'|\mathbf{R}|x'\rangle & \langle y'|\mathbf{R}|y'\rangle \end{pmatrix} \begin{pmatrix} V_{x'} \\ V_{y'} \end{pmatrix}$$

(a) Rotation Transformation and Invariants

$x = 1.65$
$y = -0.85$
$x^2 + y^2 = 3.43$
$x' = 1.00$
$y' = -1.56$
$x'^2 + y'^2 = 3.43$

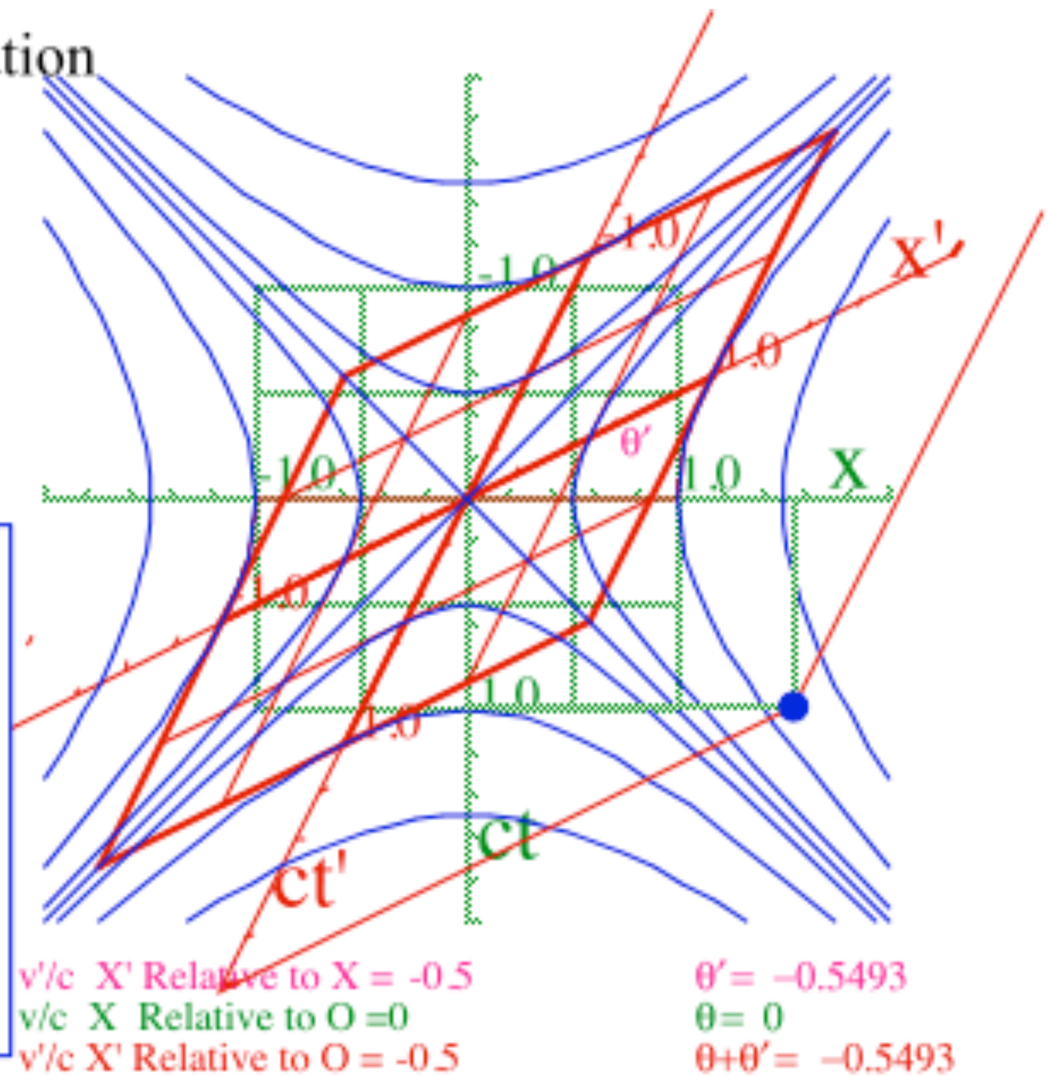


$$x' = x\cos\theta - y\sin\theta = \frac{x}{\sqrt{1+\frac{b^2}{c^2}}} + \frac{-(b/c)y}{\sqrt{1+\frac{b^2}{c^2}}}$$

$$y' = x\sin\theta + y\cos\theta = \frac{(b/c)x}{\sqrt{1+\frac{b^2}{c^2}}} + \frac{y}{\sqrt{1+\frac{b^2}{c^2}}}$$

(b) Lorentz Transformation and Invariants

$x = 1.5453$
$ct = 0.9819$
$x^2 - (ct)^2 = 1.42$
$x' = 2.3512$
$ct' = 2.0260$
$x'^2 - (ct')^2 = 1.42$



$$x' = \frac{x}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{\frac{v}{c}ct}{\sqrt{1-\frac{v^2}{c^2}}} = x\cosh\rho + y\sinh\rho$$

$$ct' = \frac{\frac{v}{c}x}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{ct}{\sqrt{1-\frac{v^2}{c^2}}} = x\sinh\rho + y\cosh\rho$$

That "old-time" relativity *(Circa 600BCE- 1905CE)*

("Bouncing-photons" in smoke & mirrors and Thales, again)

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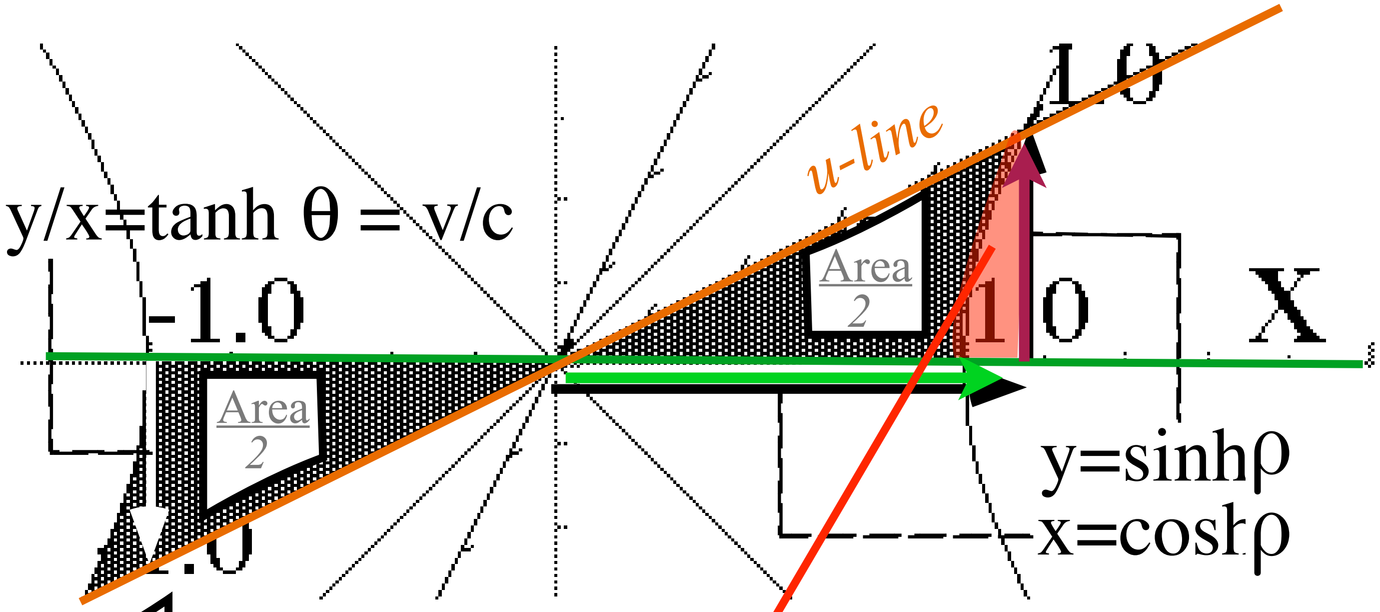


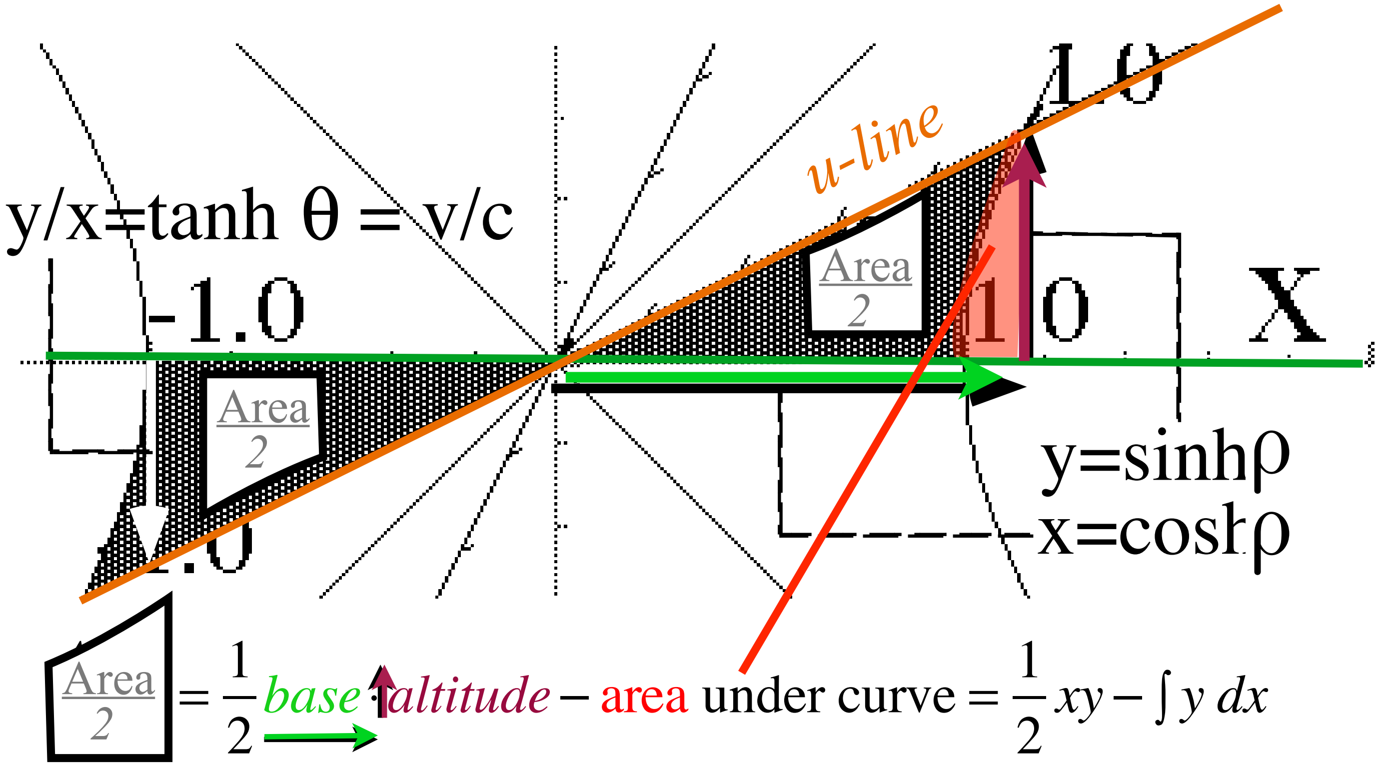
The "Area" being calculated is the <u>total</u> **Gray Area** *between hyperbola pairs, X axis, and sloping u-line*

$$\frac{Area}{2} = \frac{1}{2} \textit{base} \cdot \textit{altitude} - \text{area under curve} = \frac{1}{2} xy - \int y\, dx$$

2005 Web version:

www.uark.edu/ua/pirelli/php/complex_phasors_1.php

The straight scoop on "angle" and "rapidity" (They both are area!)



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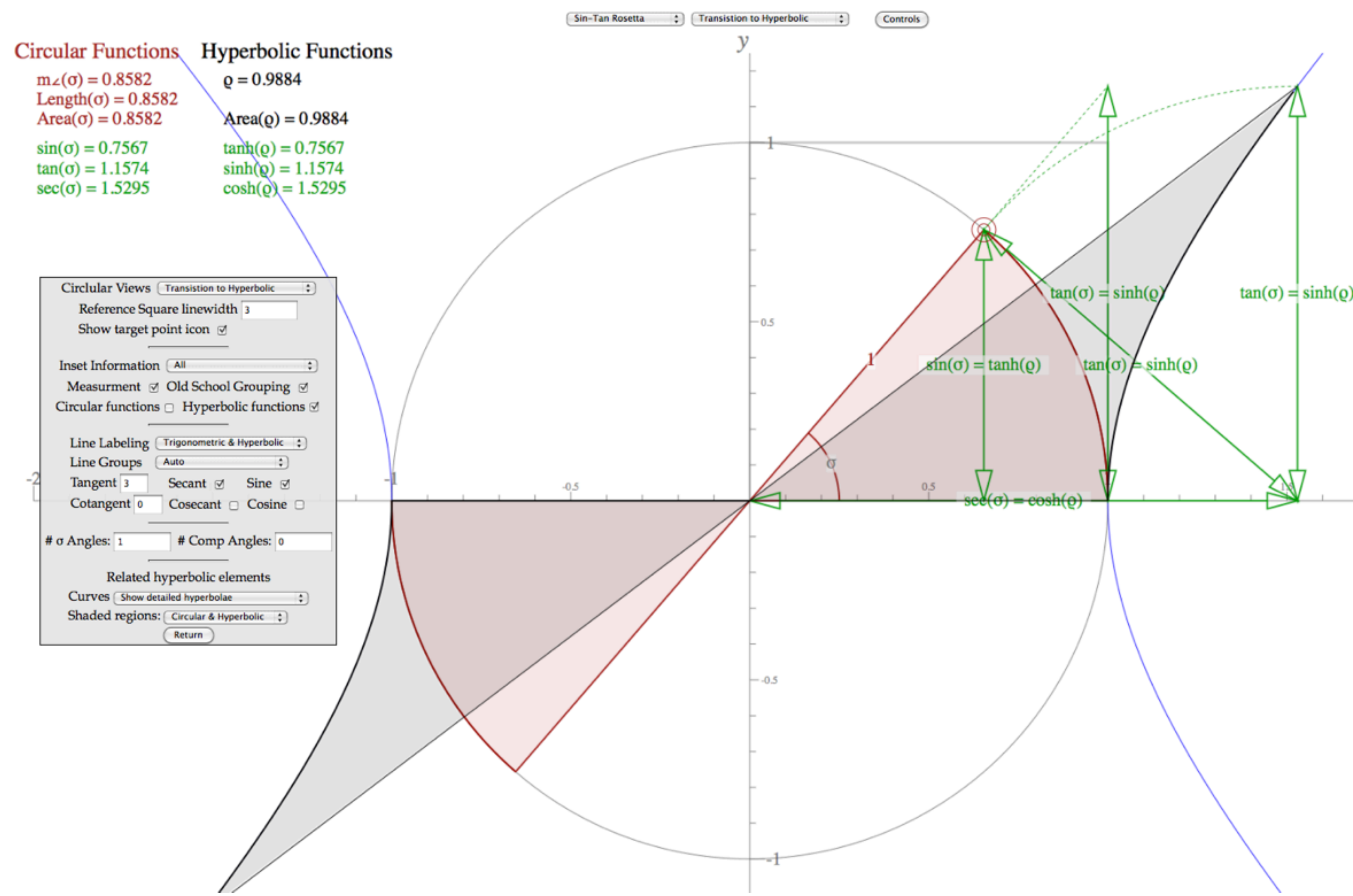
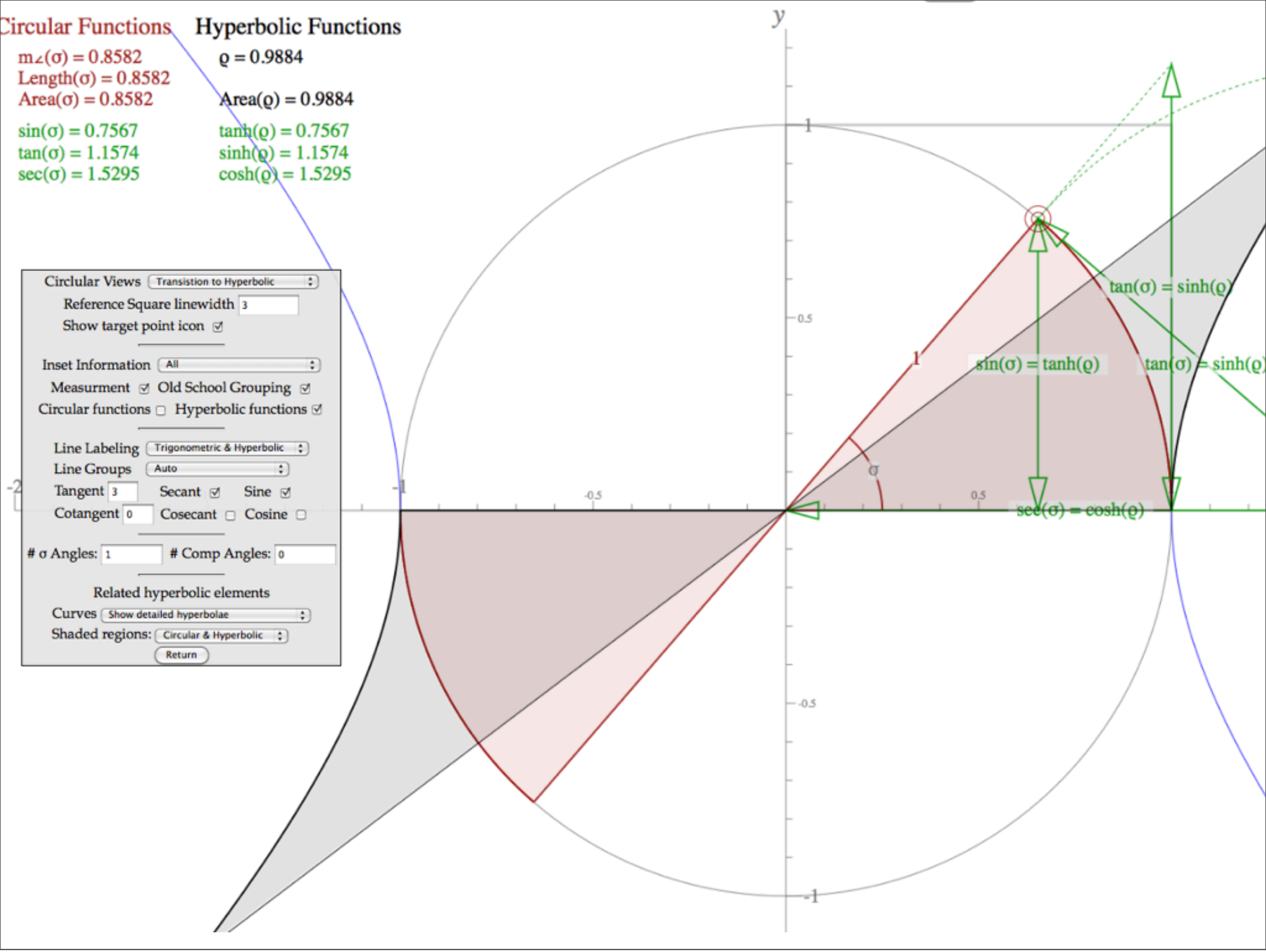
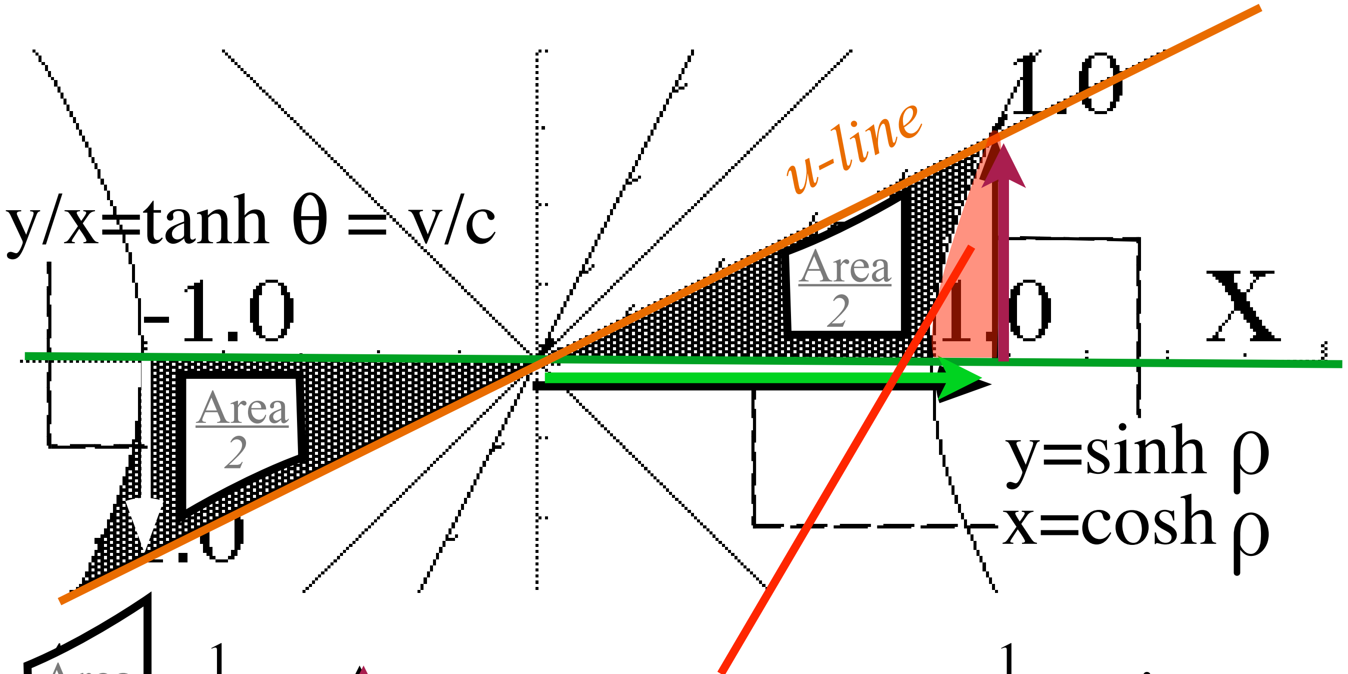
2014...Web-app versions:

http://www.uark.edu/ua/modphys/markup/RelaWavityWeb.html




Circular Functions
Hyperbolic Functions
m∠(σ) = 0.8582
Length(σ) = 0.8582
Area(σ) = 0.8582
ρ = 0.9884
Area(ρ) = 0.9884
sin(σ) = 0.7567
tan(σ) = 1.1574
sec(σ) = 1.5295
tanh(ρ) = 0.7567
sinh(ρ) = 1.1574
cosh(ρ) = 1.5295
Circlular Views
Transistion to Hyperbolic
Reference Square linewidth 3
Show target point icon
Inset Information
All
Measurment
Old School Grouping
Circular functions
Hyperbolic functions
Line Labeling
Trigonometric & Hyperbolic
Line Groups
Auto
Tangent 3
Secant
Sine
Cotangent 0
Cosecant
Cosine
σ Angles: 1
Comp Angles: 0
Related hyperbolic elements
Curves
Show detailed hyperbolae
Shaded regions:
Circular & Hyperbolic
Return
y
tan(σ) = sinh(ρ)
sin(σ) = tanh(ρ)
sec(σ) = cosh(ρ)
σ
1
0.5
-0.5
-1
-2

The straight scoop on "angle" and "rapidity" (They're area!)



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$$\frac{Area}{2} = \frac{1}{2} \sinh\rho \cosh\rho - \int \sinh\rho\, d(\cosh\rho)$$

Useful hyperbolic identities

$$\sinh^2\rho = \left(\frac{e^{\rho} - e^{-\rho}}{2}\right)^2 = \frac{1}{4}\left(e^{2\rho} + e^{-2\rho} - 2\right) = \frac{\cosh 2\rho - 1}{2}$$

$$\sinh\rho \cosh\rho = \left(\frac{e^{\rho} - e^{-\rho}}{2}\right)\left(\frac{e^{\rho} + e^{-\rho}}{2}\right) = \frac{1}{4}\left(e^{2\rho} - e^{-2\rho}\right) = \frac{1}{2}\sinh 2\rho$$

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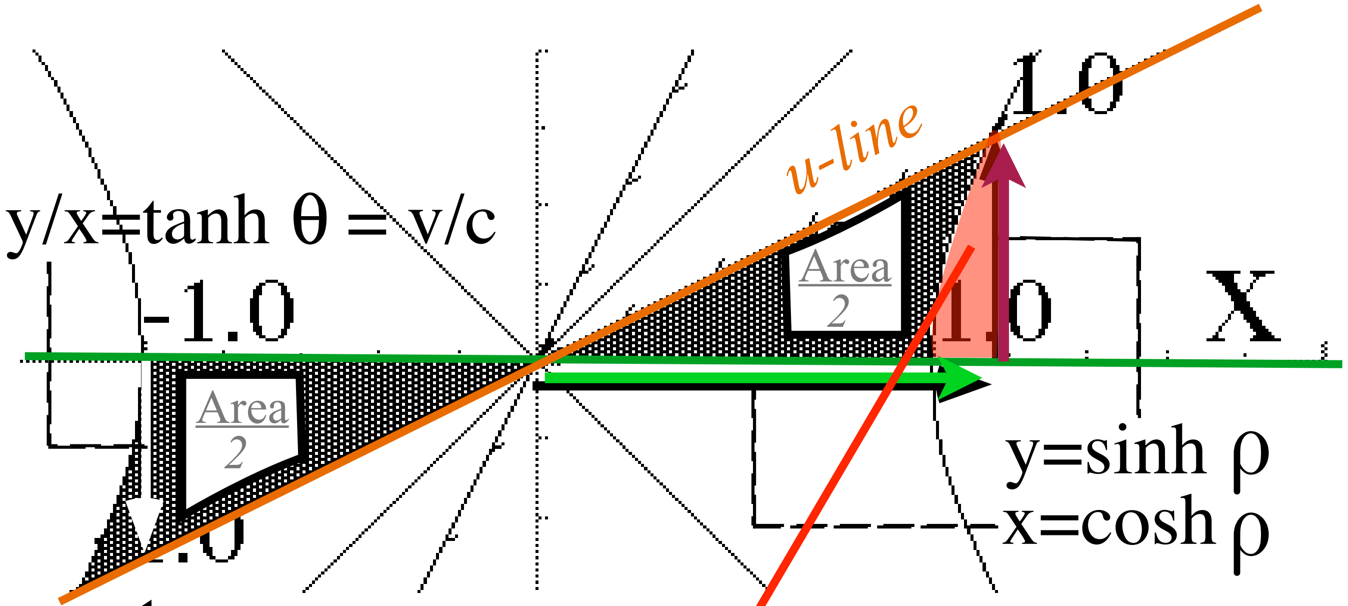

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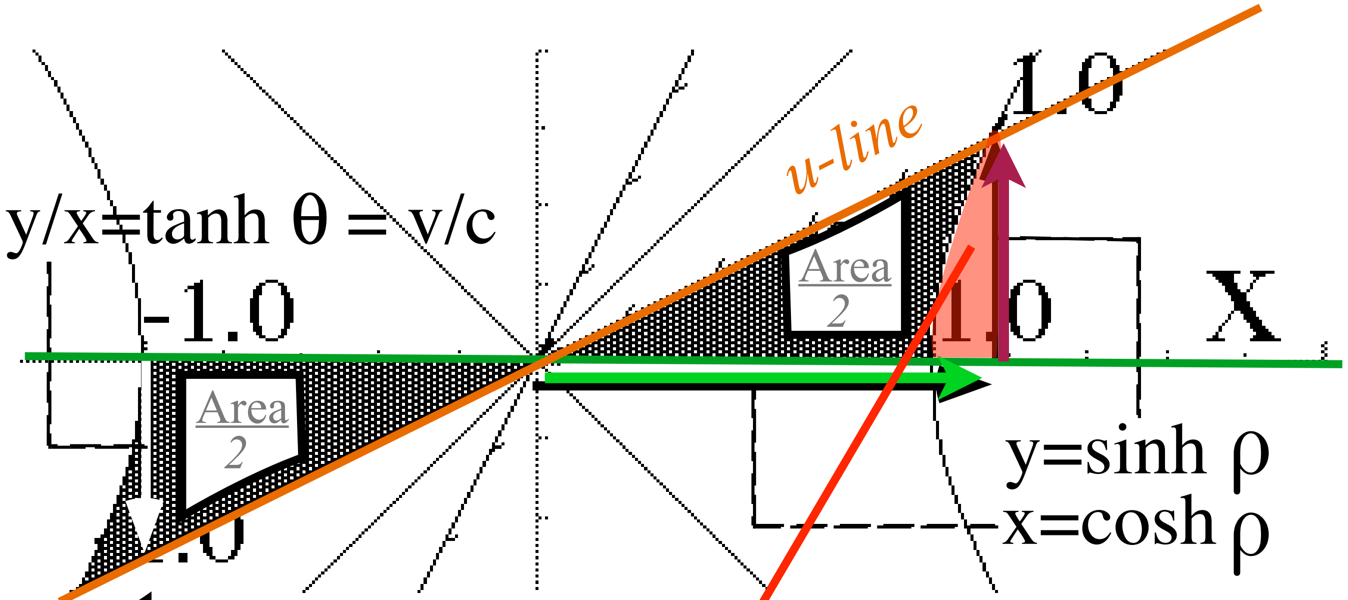
$$\sinh\theta \cosh\theta = \left(\frac{e^{\theta} - e^{-\theta}}{2}\right)\left(\frac{e^{\theta} + e^{-\theta}}{2}\right) = \frac{1}{4}\left(e^{2\theta} - e^{-2\theta}\right) = \frac{1}{2}\sinh 2\theta$$

$$\int \cosh a\rho \, d\rho = \frac{1}{a}\sinh a\rho$$

$$\frac{Area}{2} = \frac{1}{2}\sinh\rho\cosh\rho - \int \sinh^2\rho \, d\rho = \frac{1}{4}\sinh 2\rho - \int \frac{\cosh 2\rho - 1}{2} d\rho$$

The straight scoop on "angle" and "rapidity" (They're area!)



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$$= \frac{1}{4}\sinh 2\rho - \frac{1}{4}\sinh 2\rho + \int \frac{1}{2}\, d\rho$$

$$= \frac{\rho}{2}$$

Amazing result: ***Area*** $= \rho$ *is rapidity*

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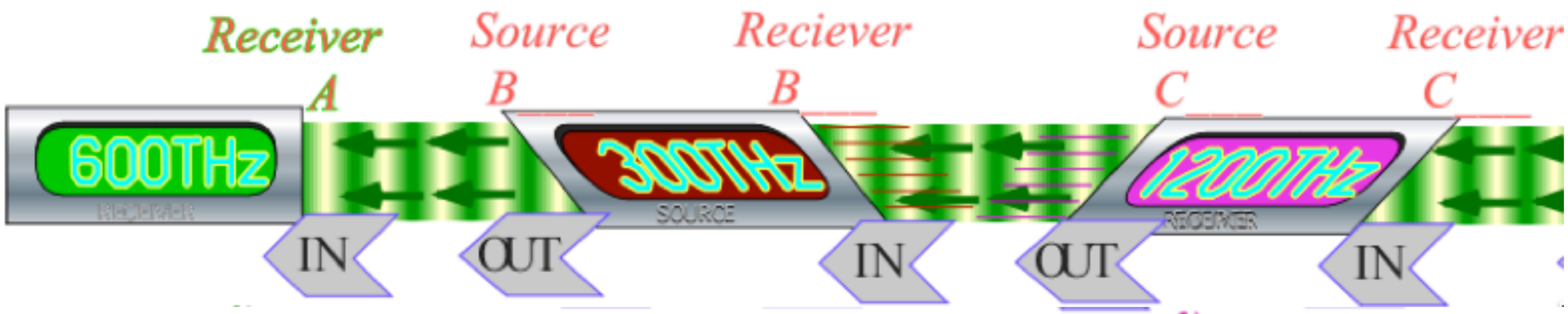
From Lect. 22 p. 27 or eq. (3.6) in Ch. 3 of Unit 2:

Evenson axiom requires *geometric* Doppler transform: $e^{\rho_{AB}} \cdot e^{\rho_{BC}} = e^{\rho_{AC}} = e^{\rho_{AB}+\rho_{BC}}$

Easy to combine frame velocities using *rapidity addition:* $\rho_{u+v} = \rho_u + \rho_v$



$\frac{\upsilon_A}{\upsilon_B} = b_{AB} = e^{\rho_{AB}} = 2$ $\qquad \rho_{AB} = ln(2) = 0.69$

$\frac{\upsilon_B}{\upsilon_C} = b_{BC} = e^{\rho_{BC}} = 1/4$ $\qquad \rho_{BC} = ln(1/4) = -1.38$

$\frac{\upsilon_C}{\upsilon_A} = b_{CA} = e^{\rho_{CA}} = 2$ $\qquad \rho_{CA} = ln(2) = 0.69$

$$\rho_{AB} + \rho_{BC} = \rho_{AC} = -\rho_{CA}$$

$$0.69 - 1.38 = -0.69$$

Galilean velocity addition becomes rapidity addition

From Lect. 22 p. 27 or eq. (3.6) in Ch. 3 of Unit 2:

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$$\frac{u'}{c} = \tanh(\rho_u + \rho_v) = \frac{\tanh\rho_u + \tanh\rho_v}{1+\tanh\rho_u \tanh\rho_v} = \frac{\frac{u}{c}+\frac{v}{c}}{1+\frac{u}{c}\frac{v}{c}}$$

$$\tanh(x+y) = \frac{\tanh x + \tanh y}{1+\tanh x \tanh y}$$

or: $$u' = \frac{u+v}{1+\frac{u\cdot v}{c^2}}$$

Galilean velocity addition becomes rapidity addition

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No longer does *(1/2+1/2)c* equal *(1)c*…

Relativistic result is: $$\frac{\frac{1}{2}+\frac{1}{2}}{1+\frac{1}{2}\frac{1}{2}}c = \frac{1}{1+\frac{1}{4}}c = \frac{1}{\frac{5}{4}}c = \frac{4}{5}c$$

Galilean velocity addition becomes rapidity addition

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...but, *(1/2+1)c* <u>does</u> equal *(1)c*…

$$\frac{\frac{1}{2} + 1}{1 + \frac{1}{2}1} c = c$$

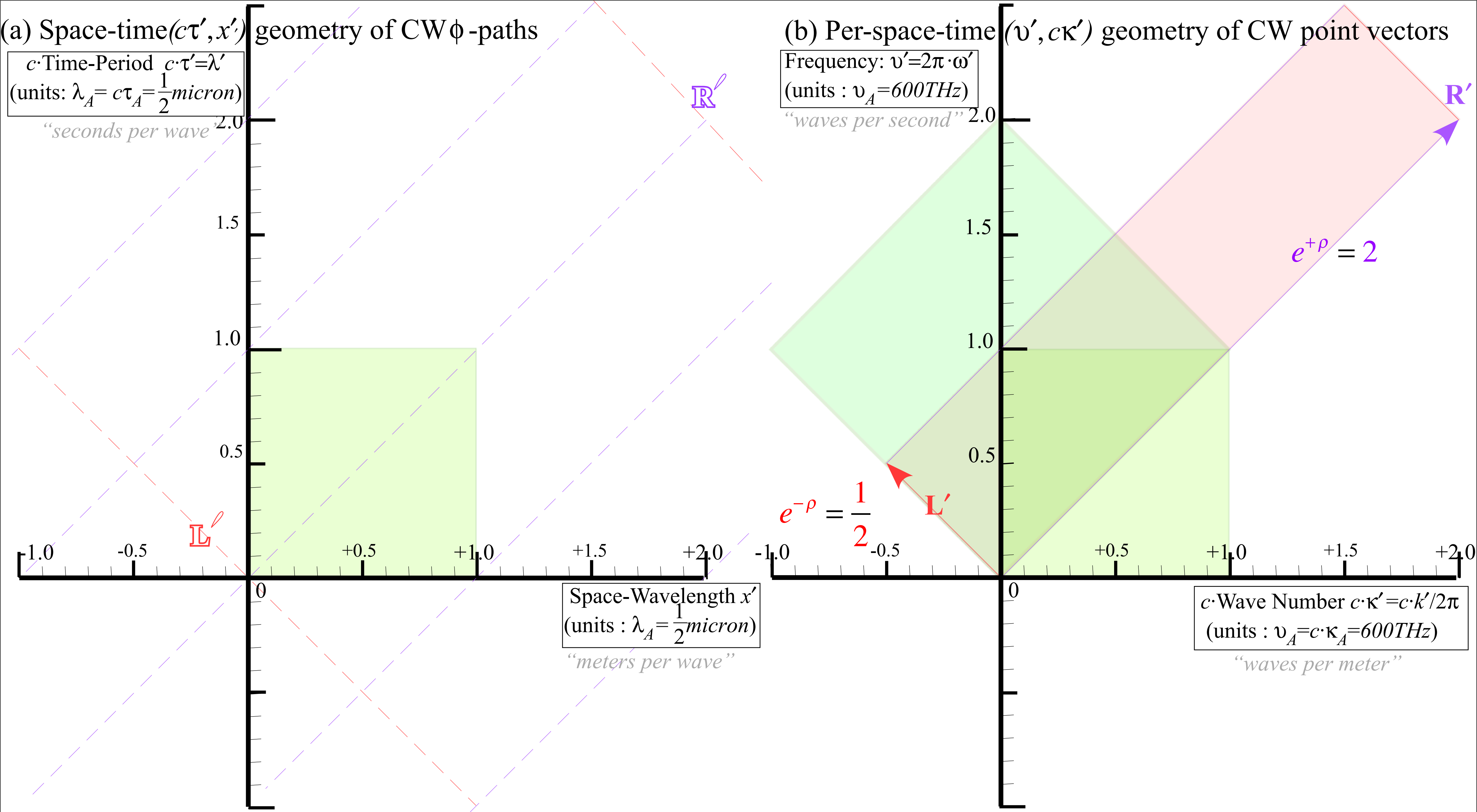
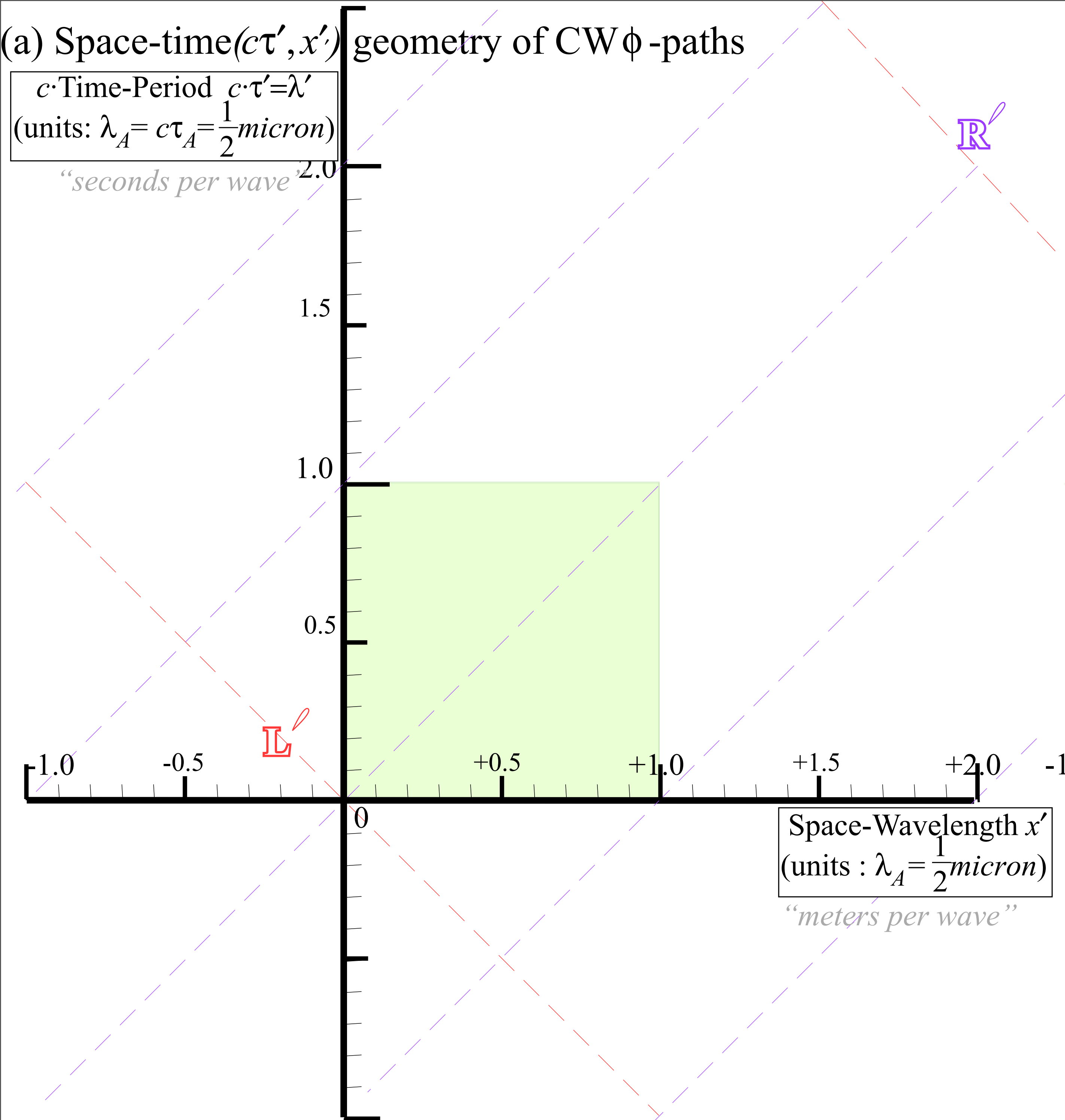
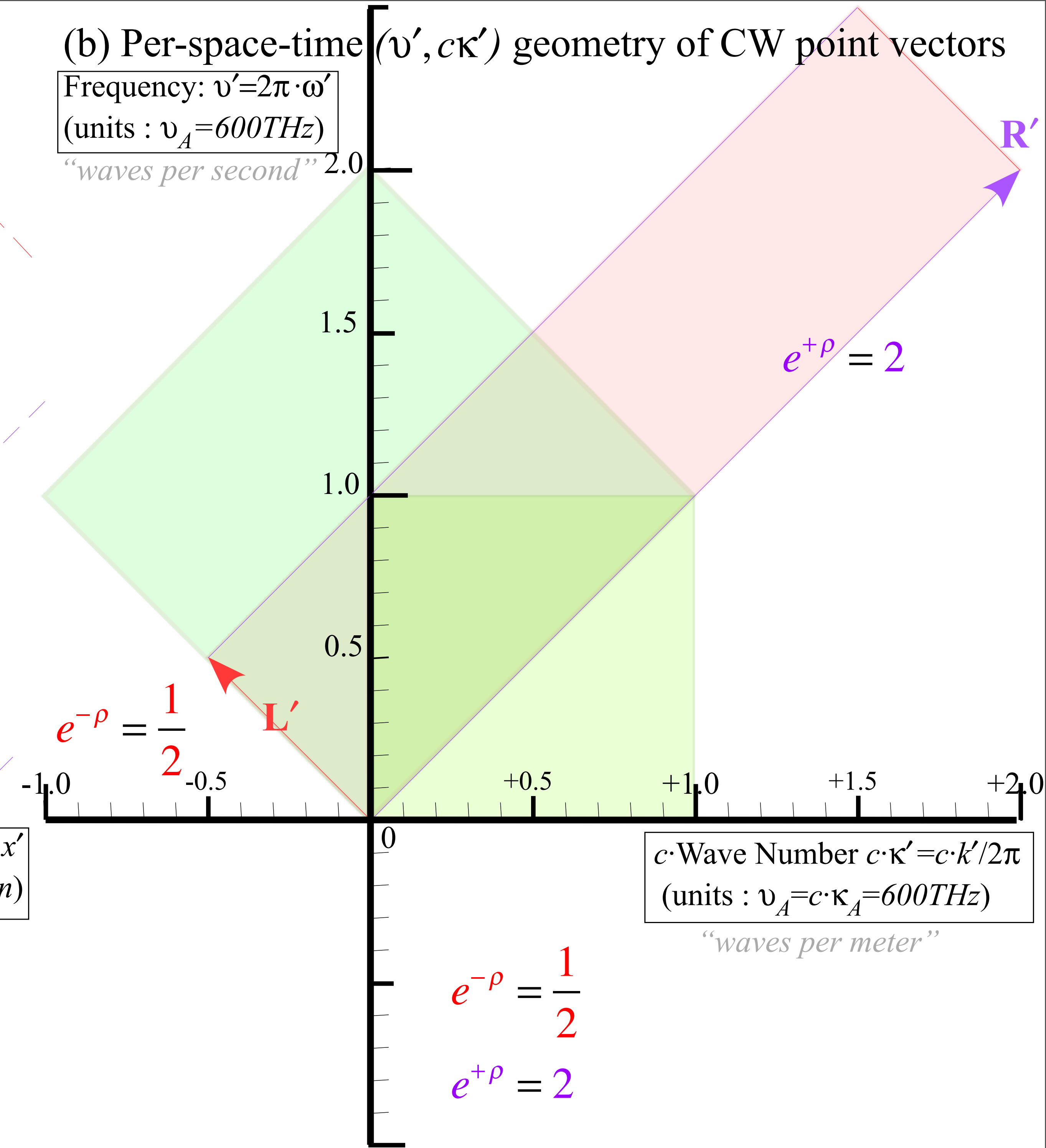


Fig. 7 SRQMbyR&C





$$\begin{pmatrix}\upsilon'_{phase}\\ c\kappa'_{phase}\end{pmatrix} = \mathbf{P}' = \frac{\mathbf{R}'+\mathbf{L}'}{2} = \upsilon_A \begin{pmatrix}\frac{e^{+\rho}+e^{-\rho}}{2}\\ \frac{e^{+\rho}-e^{-\rho}}{2}\end{pmatrix} = \upsilon_A\begin{pmatrix}\cosh\rho\\ \sinh\rho\end{pmatrix} = \upsilon_A\begin{pmatrix}\frac{5}{4}\\ \frac{3}{4}\end{pmatrix}$$

$$\begin{pmatrix}\upsilon'_{group}\\ c\kappa'_{group}\end{pmatrix} = \mathbf{G}' = \frac{\mathbf{R}'-\mathbf{L}'}{2} = \upsilon_A \begin{pmatrix}\frac{e^{+\rho}-e^{-\rho}}{2}\\ \frac{e^{+\rho}+e^{-\rho}}{2}\end{pmatrix} = \upsilon_A\begin{pmatrix}\sinh\rho\\ \cosh\rho\end{pmatrix} = \upsilon_A\begin{pmatrix}\frac{3}{4}\\ \frac{5}{4}\end{pmatrix}$$

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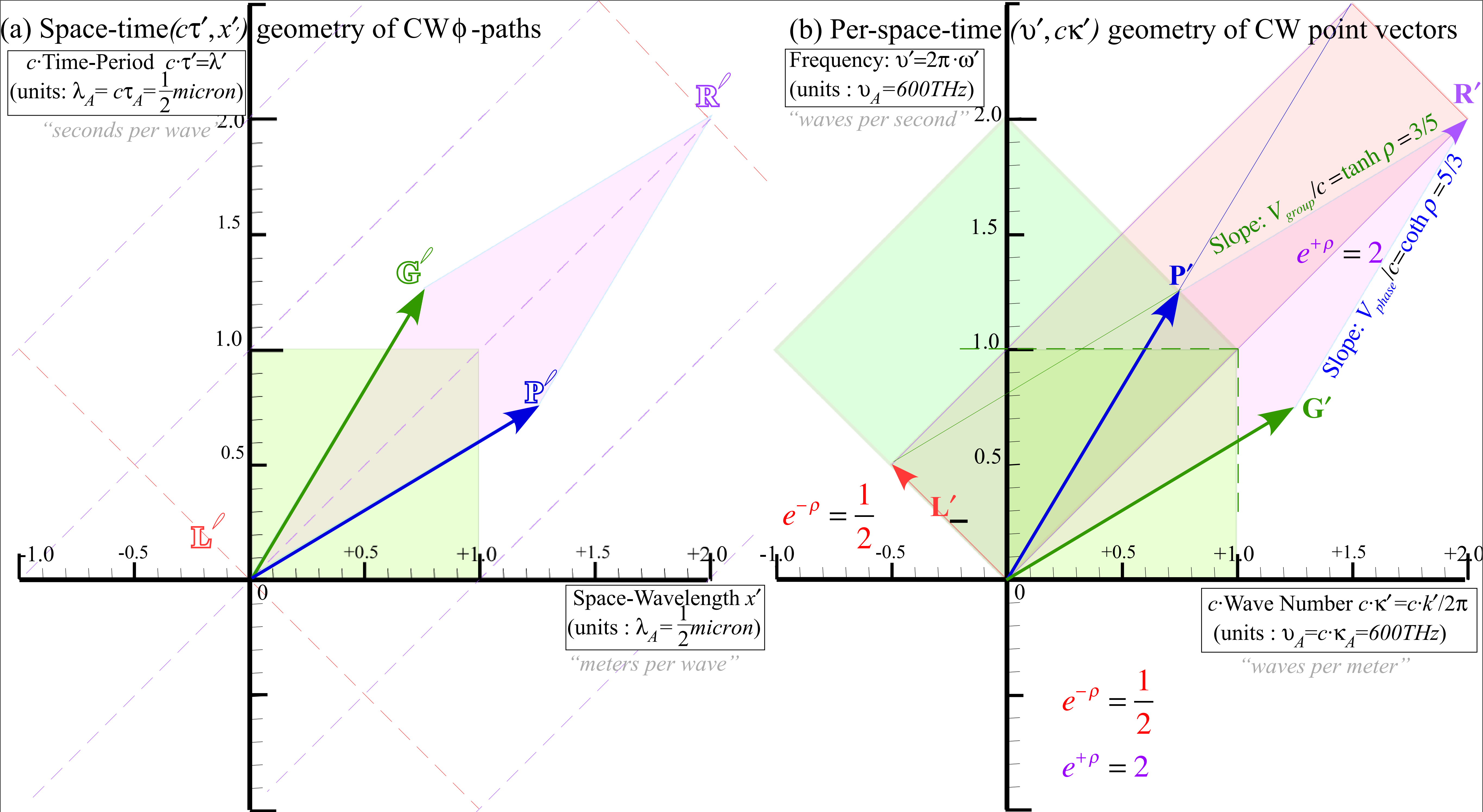

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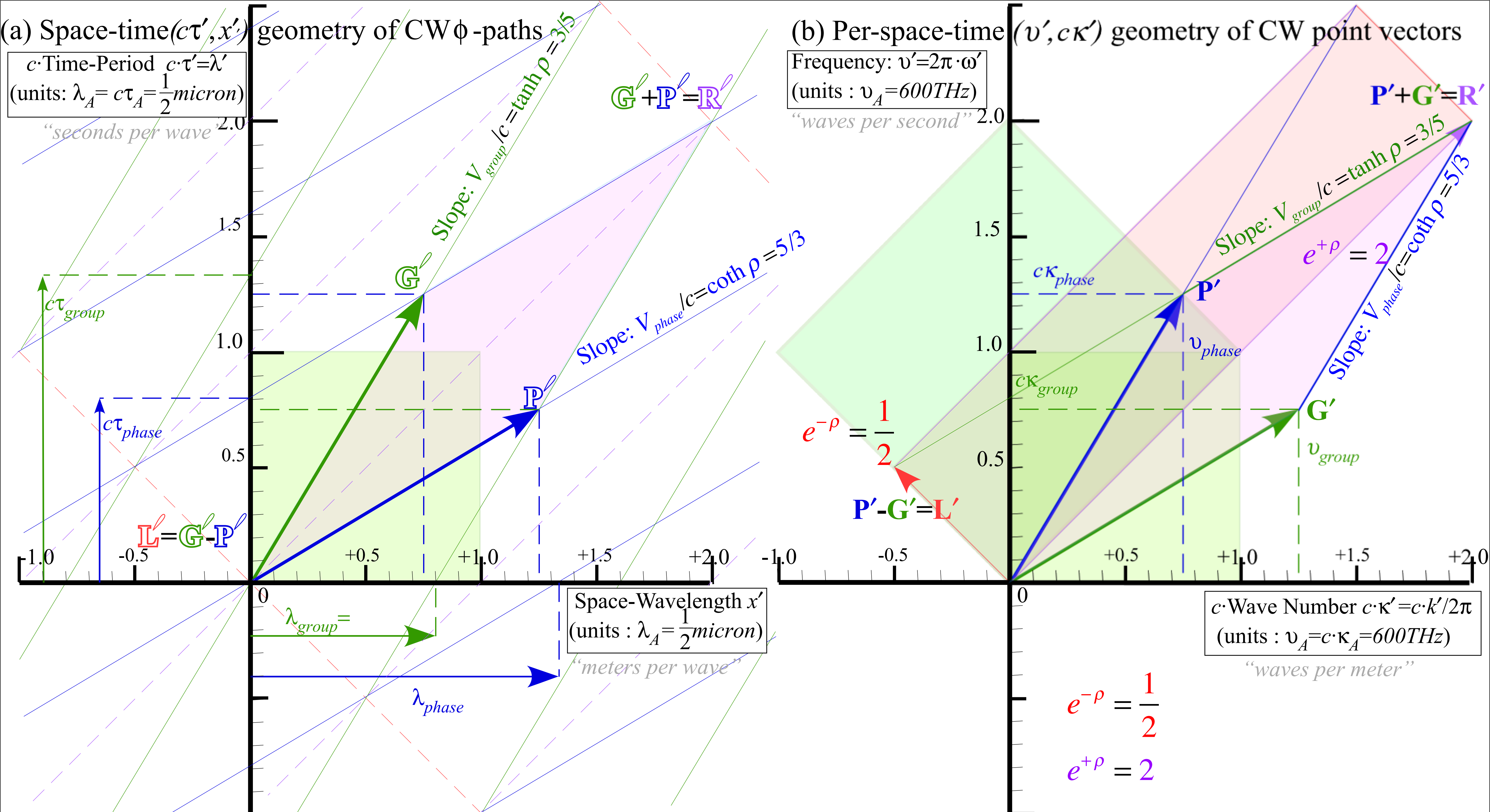

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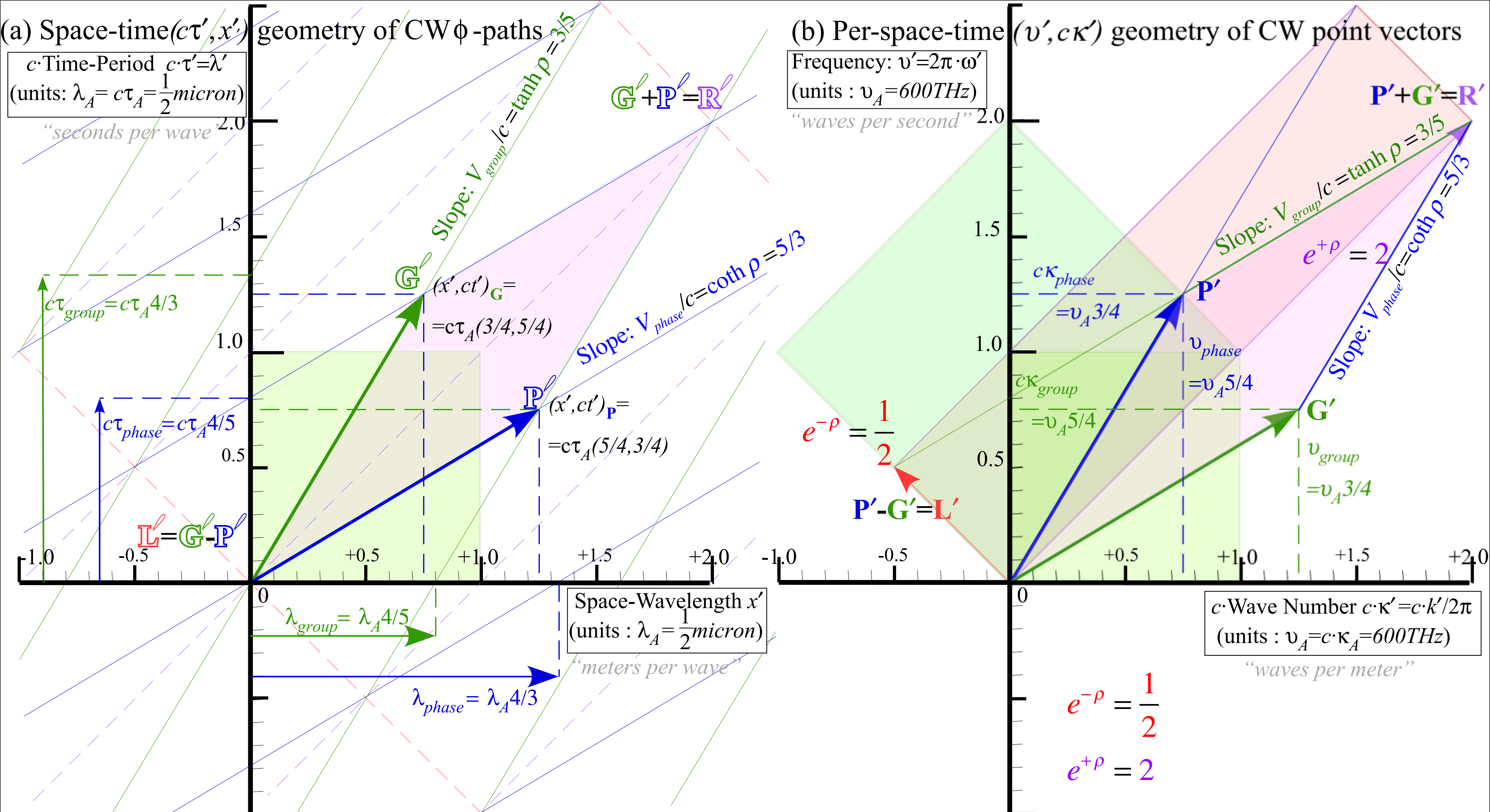

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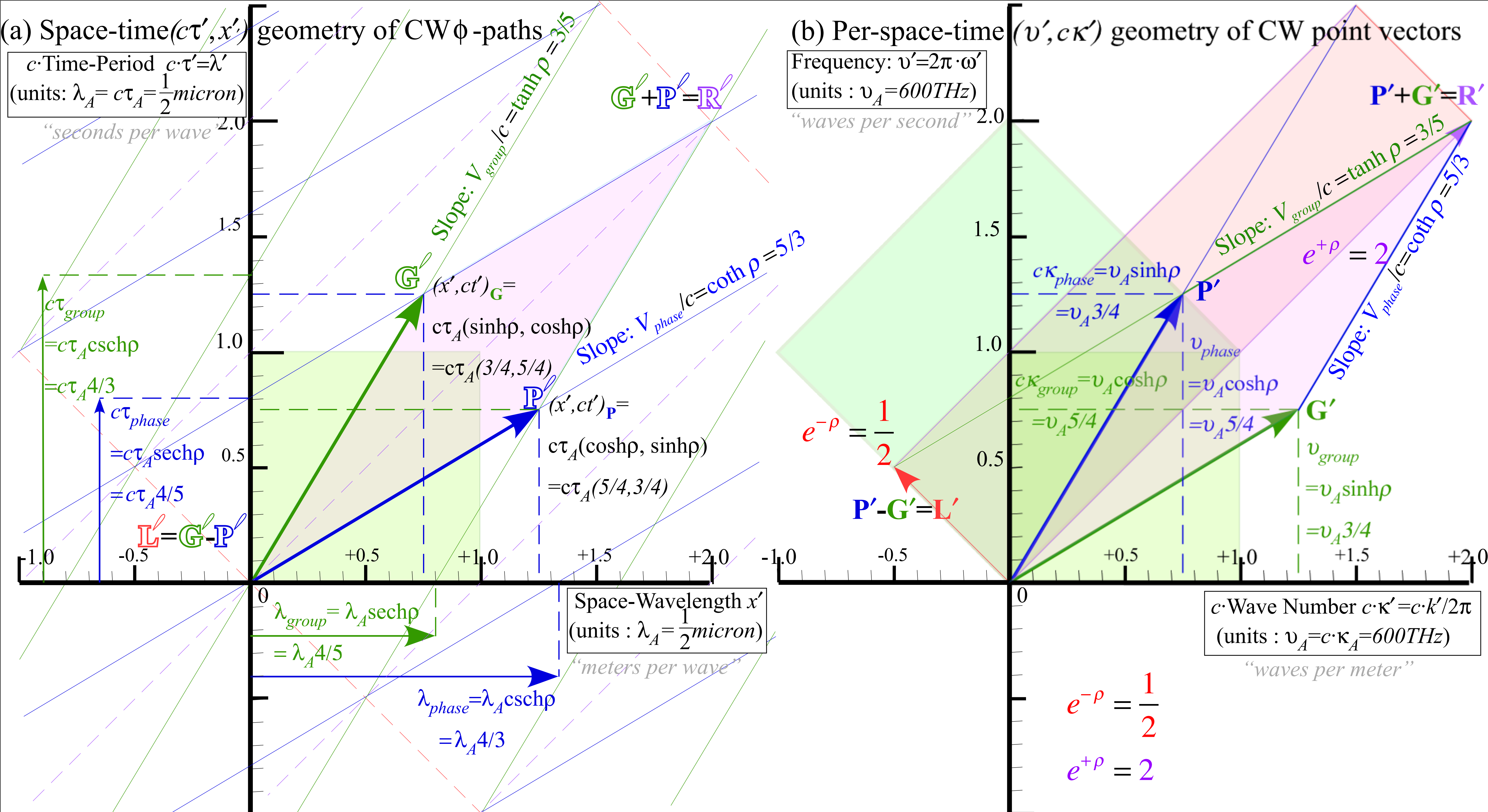

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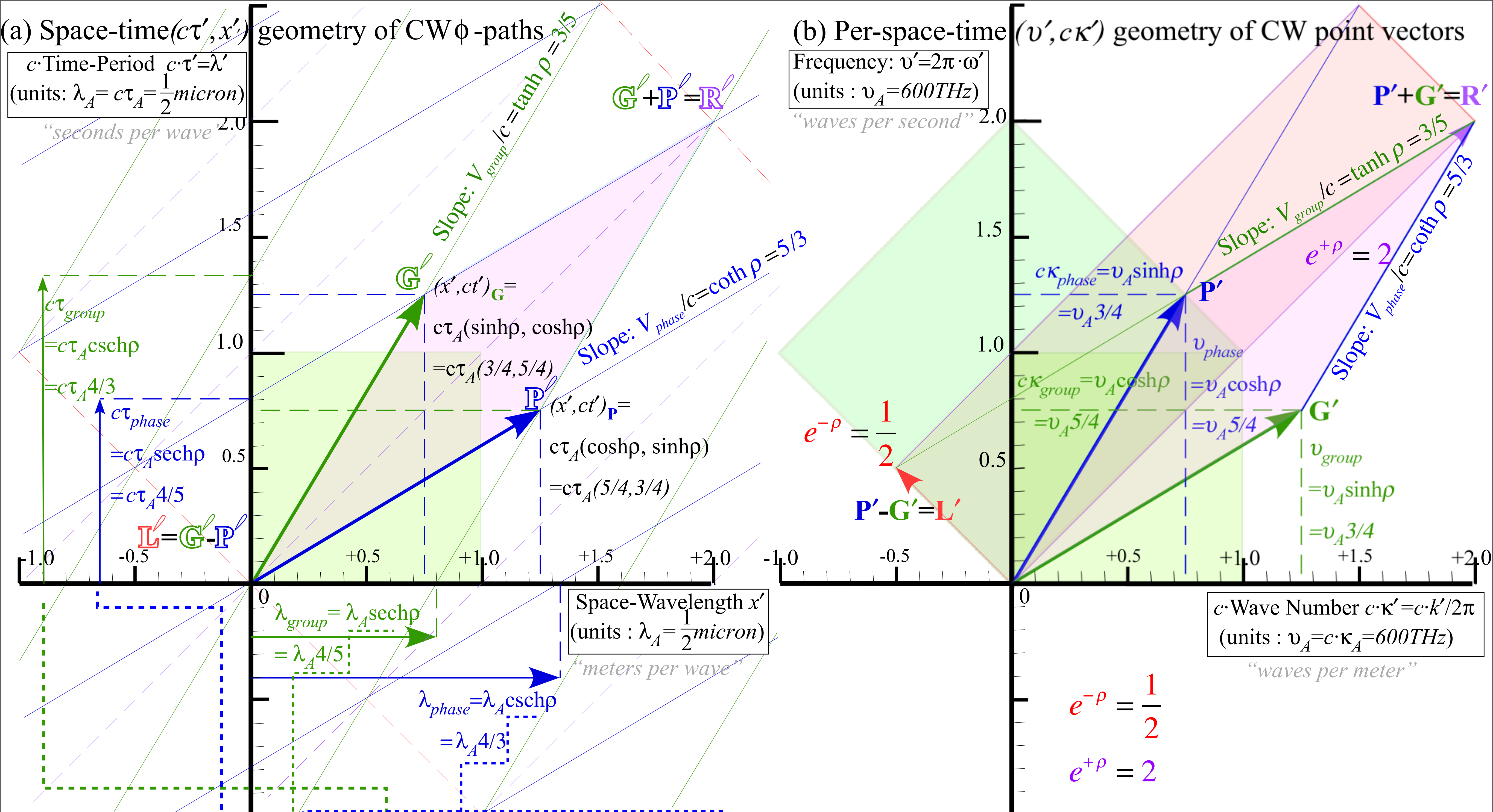
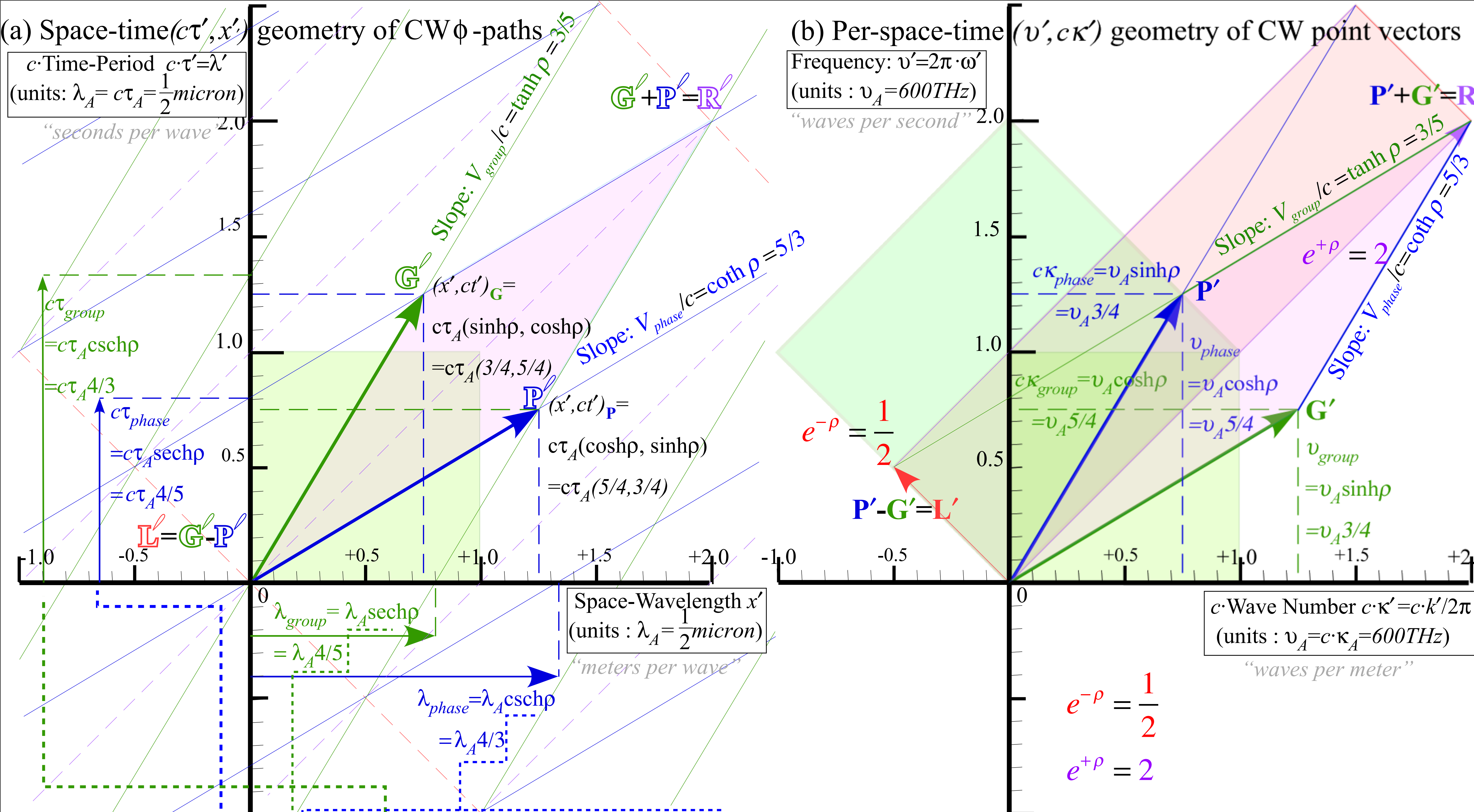


time	r_{Dopp}	υ_{group}	τ_{phase}	υ_{phase}	τ_{group}	b_{Dopp}	u/c	c/u
space		κ_{phase}	λ_{group}	κ_{group}	λ_{phase}		V_{group}/c	V_{phase}/c
	$e^{-\rho}$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$e^{+\rho}$	$\tanh\rho$	$\coth\rho$
	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{2}{1}$	$\frac{3}{5}$	$\frac{5}{3}$

$$\begin{pmatrix}\upsilon'_{phase}\\ c\kappa'_{phase}\end{pmatrix}=\mathbf{P}'=\frac{\mathbf{R}'+\mathbf{L}'}{2}=\upsilon_A\begin{pmatrix}\frac{e^{+\rho}+e^{-\rho}}{2}\\ \frac{e^{+\rho}-e^{-\rho}}{2}\end{pmatrix}=\upsilon_A\begin{pmatrix}\cosh\rho\\ \sinh\rho\end{pmatrix}=\upsilon_A\begin{pmatrix}\frac{5}{4}\\ \frac{3}{4}\end{pmatrix}$$

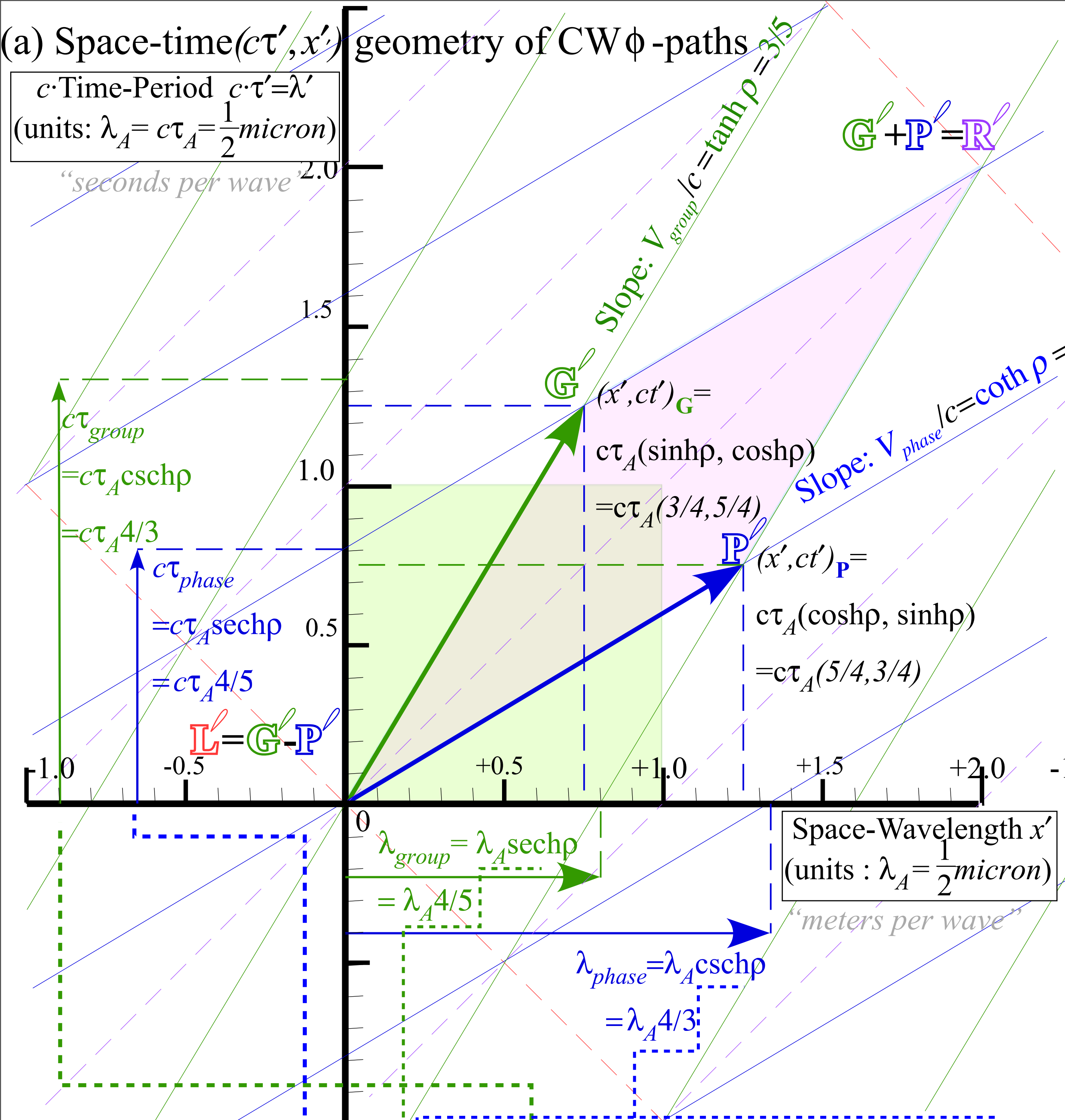
$$\begin{pmatrix}\upsilon'_{group}\\ c\kappa'_{group}\end{pmatrix}=\mathbf{G}'=\frac{\mathbf{R}'-\mathbf{L}'}{2}=\upsilon_A\begin{pmatrix}\frac{e^{+\rho}-e^{-\rho}}{2}\\ \frac{e^{+\rho}+e^{-\rho}}{2}\end{pmatrix}=\upsilon_A\begin{pmatrix}\sinh\rho\\ \cosh\rho\end{pmatrix}=\upsilon_A\begin{pmatrix}\frac{3}{4}\\ \frac{5}{4}\end{pmatrix}$$

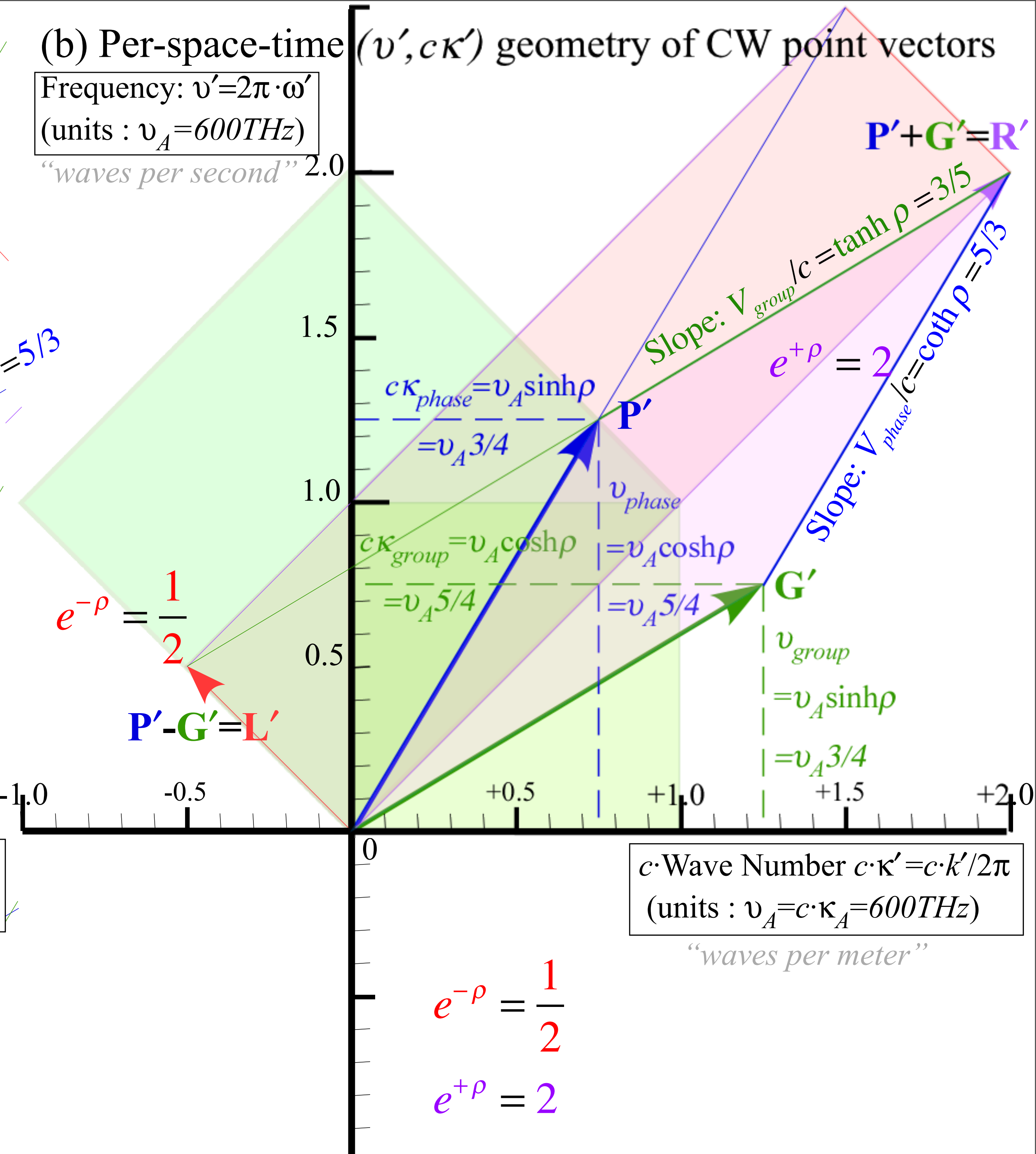


time	r_{Dopp}	υ_{group}	τ_{phase}	υ_{phase}	τ_{group}	b_{Dopp}	u/c	c/u
space		κ_{phase}	λ_{group}	κ_{group}	λ_{phase}		V_{group}/c	V_{phase}/c
rapidity ρ	$e^{-\rho}$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$e^{+\rho}$	$\tanh\rho$	$\coth\rho$
$\rho =$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{2}{1}$	$\frac{3}{5}$	$\frac{5}{3}$
stellar∀ σ		$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$		$\sin\sigma$	$\csc\sigma$

$$\begin{pmatrix}\upsilon'_{phase}\\ c\kappa'_{phase}\end{pmatrix} = \mathbf{P}' = \frac{\mathbf{R}'+\mathbf{L}'}{2} = \upsilon_A \begin{pmatrix}\frac{e^{+\rho}+e^{-\rho}}{2}\\ \frac{e^{+\rho}-e^{-\rho}}{2}\end{pmatrix} = \upsilon_A\begin{pmatrix}\cosh\rho\\ \sinh\rho\end{pmatrix} = \upsilon_A\begin{pmatrix}\frac{5}{4}\\ \frac{3}{4}\end{pmatrix}$$

$$\begin{pmatrix}\upsilon'_{group}\\ c\kappa'_{group}\end{pmatrix} = \mathbf{G}' = \frac{\mathbf{R}'-\mathbf{L}'}{2} = \upsilon_A \begin{pmatrix}\frac{e^{+\rho}-e^{-\rho}}{2}\\ \frac{e^{+\rho}+e^{-\rho}}{2}\end{pmatrix} = \upsilon_A\begin{pmatrix}\sinh\rho\\ \cosh\rho\end{pmatrix} = \upsilon_A\begin{pmatrix}\frac{3}{4}\\ \frac{5}{4}\end{pmatrix}$$





time	r_{Dopp}	υ_{group}	τ_{phase}	υ_{phase}	τ_{group}	b_{Dopp}	u/c	c/u
space		κ_{phase}	λ_{group}	κ_{group}	λ_{phase}		V_{group}/c	V_{phase}/c
rapidity ρ	$e^{-\rho}$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$e^{+\rho}$	$\tanh\rho$	$\coth\rho$
$\rho=$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{2}{1}$	$\frac{3}{5}$	$\frac{5}{3}$
stellar∀ σ		$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$		$\sin\sigma$	$\csc\sigma$
		p	L	H	λ_{DeB}			

$$\begin{pmatrix}\upsilon'_{phase}\\ c\kappa'_{phase}\end{pmatrix}=\mathbf{P'}=\frac{\mathbf{R'}+\mathbf{L'}}{2}=\upsilon_A\begin{pmatrix}\frac{e^{+\rho}+e^{-\rho}}{2}\\ \frac{e^{+\rho}-e^{-\rho}}{2}\end{pmatrix}=\upsilon_A\begin{pmatrix}\cosh\rho\\ \sinh\rho\end{pmatrix}=\upsilon_A\begin{pmatrix}\frac{5}{4}\\ \frac{3}{4}\end{pmatrix}$$

$$\begin{pmatrix}\upsilon'_{group}\\ c\kappa'_{group}\end{pmatrix}=\mathbf{G'}=\frac{\mathbf{R'}-\mathbf{L'}}{2}=\upsilon_A\begin{pmatrix}\frac{e^{+\rho}-e^{-\rho}}{2}\\ \frac{e^{+\rho}+e^{-\rho}}{2}\end{pmatrix}=\upsilon_A\begin{pmatrix}\sinh\rho\\ \cosh\rho\end{pmatrix}=\upsilon_A\begin{pmatrix}\frac{3}{4}\\ \frac{5}{4}\end{pmatrix}$$

A. Transformations and phase invariance

A laser phasor sketched in Fig.4 should be taken seriously as a gauge of time (clock) and of space (metric ruler) by giving time (wave period τ) and distance (wavelength λ) in Fig.7a. A time-stamp reading of phase ϕ at a particular space-time point should be equal for Alice and Bob in spite of having unequal readings (x, t) and (x', t') for that point and unequal frequency-wavevector readings (ω, k) and (ω', k') for a laser group-wave or its phase-wave.

$$\begin{aligned} \phi'_{phase} &\equiv k'_{phase}x' - \omega'_{phase}\, t' = k_{phase}x - \omega_{phase}\, t \equiv \phi_{phase} \\ \phi'_{group} &\equiv k'_{group}x' - \omega'_{group}\, t' = k_{group}x - \omega_{group}\, t \equiv \phi_{group} \end{aligned} \tag{20}$$

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$$\begin{aligned}\phi'_{phase} &\equiv k'_{phase}x' - \omega'_{phase}\,t' = k_{phase}x - \omega_{phase}\,t \equiv \phi_{phase}\\ \phi'_{group} &\equiv k'_{group}x' - \omega'_{group}\,t' = k_{group}x - \omega_{group}\,t \equiv \phi_{group}\end{aligned} \tag{20}$$

Bob's (ω',k') components are in (14) and (15). Alice's (ω,k) are the same with $\rho=0$. An Einstein-Lorentz Transformation (ELT) of Bob's (x',t') to Alice's (x,t) follows.

$$\begin{aligned}\phi_{phase} &\equiv x'\frac{\omega_A}{c}\cosh\rho - t'\omega_A\sinh\rho \;=0\cdot x - \omega_A\,t \quad\Rightarrow\quad ct = \;\; ct'\cosh\rho - x'\sinh\rho\\ \phi_{group} &\equiv x'\frac{\omega_A}{c}\sinh\rho - t'\omega_A\cosh\rho \;=\frac{\omega_A}{c}x - 0\cdot t \quad\Rightarrow\quad x = -ct'\sinh\rho + x'\cosh\rho\end{aligned} \tag{21}$$

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$$\begin{aligned} \phi_{phase} &\equiv x'\frac{\omega_A}{c}\cosh\rho - t'\omega_A\sinh\rho = 0\cdot x - \omega_A\,t \quad \Rightarrow \quad ct = ct'\cosh\rho - x'\sinh\rho \\ \phi_{group} &\equiv x'\frac{\omega_A}{c}\sinh\rho - t'\omega_A\cosh\rho = \frac{\omega_A}{c}x - 0\cdot t \quad \Rightarrow \quad x = -ct'\sinh\rho + x'\cosh\rho \end{aligned} \tag{21}$$

The ELT matrix form and its inverse complete the space-time side of Fig.7.

$$\begin{pmatrix} ct \\ x \end{pmatrix} = \begin{pmatrix} \cosh\rho & -\sinh\rho \\ -\sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} ct' \\ x' \end{pmatrix} \Rightarrow \begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \cosh\rho & +\sinh\rho \\ +\sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix} \tag{22}$$

A. Transformations and phase invariance

A laser phasor sketched in Fig.4 should be taken seriously as a gauge of time (clock) and of space (metric ruler) by giving time (wave period τ) and distance (wavelength λ) in Fig.7a. A time-stamp reading of phase ϕ at a particular space-time point should be equal for Alice and Bob in spite of having unequal readings (x,t) and (x',t') for that point and unequal frequency-wavevector readings (ω,k) and (ω',k') for a laser group-wave or its phase-wave.

$$\begin{aligned} \phi'_{phase} \equiv k'_{phase}x' - \omega'_{phase}\,t' = k_{phase}x - \omega_{phase}\,t \equiv \phi_{phase} \\ \phi'_{group} \equiv k'_{group}x' - \omega'_{group}\,t' = k_{group}x - \omega_{group}\,t \equiv \phi_{group} \end{aligned} \qquad (20)$$

Direct derivation of ELT uses base vectors $\mathbb{P}'$ and $\mathbb{G}'$ or $\mathbf{P}'$ and $\mathbf{G}'$ in (14) and (15).

$$\mathbf{P}' = \begin{pmatrix} \omega'_{phase} \\ ck'_{phase} \end{pmatrix} = \omega_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \begin{pmatrix} \omega_A \\ 0 \end{pmatrix}\cosh\rho + \begin{pmatrix} 0 \\ \omega_A \end{pmatrix}\sinh\rho = \mathbf{P}\cosh\rho + \mathbf{G}\sinh\rho \qquad (23)$$

$$\mathbf{G}' = \begin{pmatrix} \omega'_{group} \\ ck'_{group} \end{pmatrix} = \omega_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \begin{pmatrix} \omega_A \\ 0 \end{pmatrix}\sinh\rho + \begin{pmatrix} 0 \\ \omega_A \end{pmatrix}\cosh\rho = \mathbf{P}\sinh\rho + \mathbf{G}\cosh\rho \qquad (24)$$

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(a) Circlular Functions
(plane geometry)

www.uark.edu/ua/pirelli/php/complex_phasors_1.php

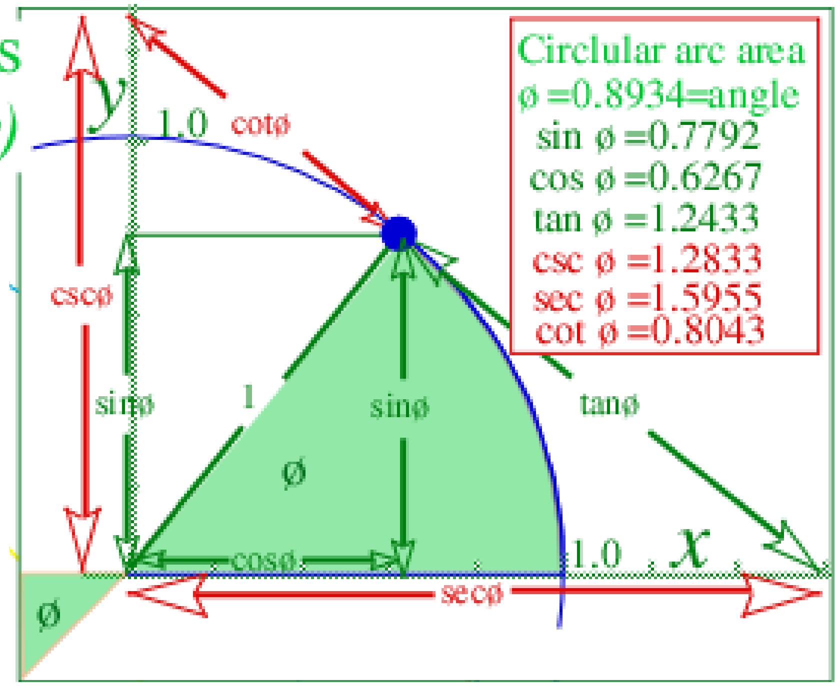

Fig. 5.4
in Unit 8

Introducing the "Sin-Tan Rosetta Stone"

NOTE: Angle ϕ is now called stellar aberration angle σ

(a) Circlular Functions
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2005 Web version:

www.uark.edu/ua/pirelli/php/complex_phasors_1.php

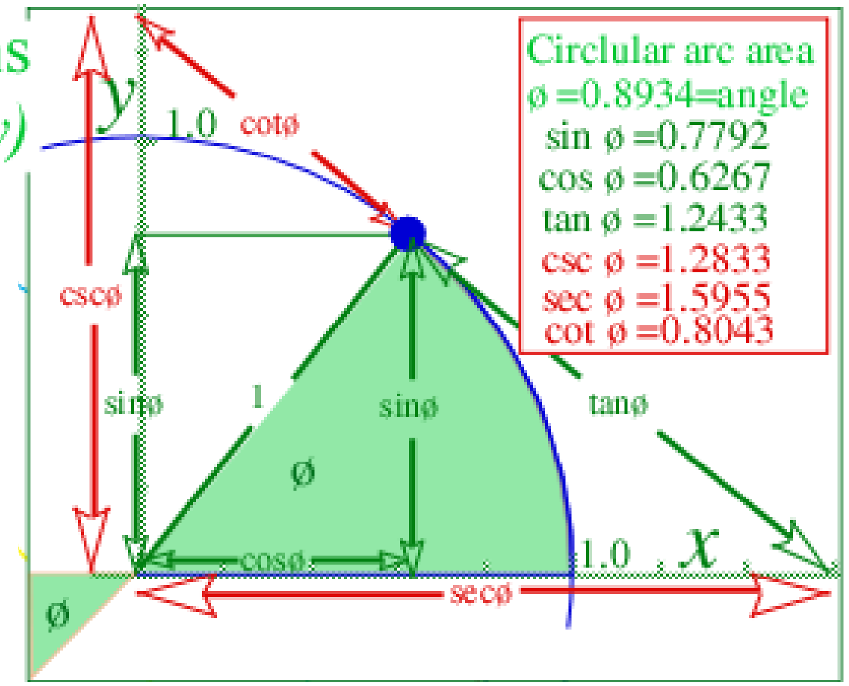
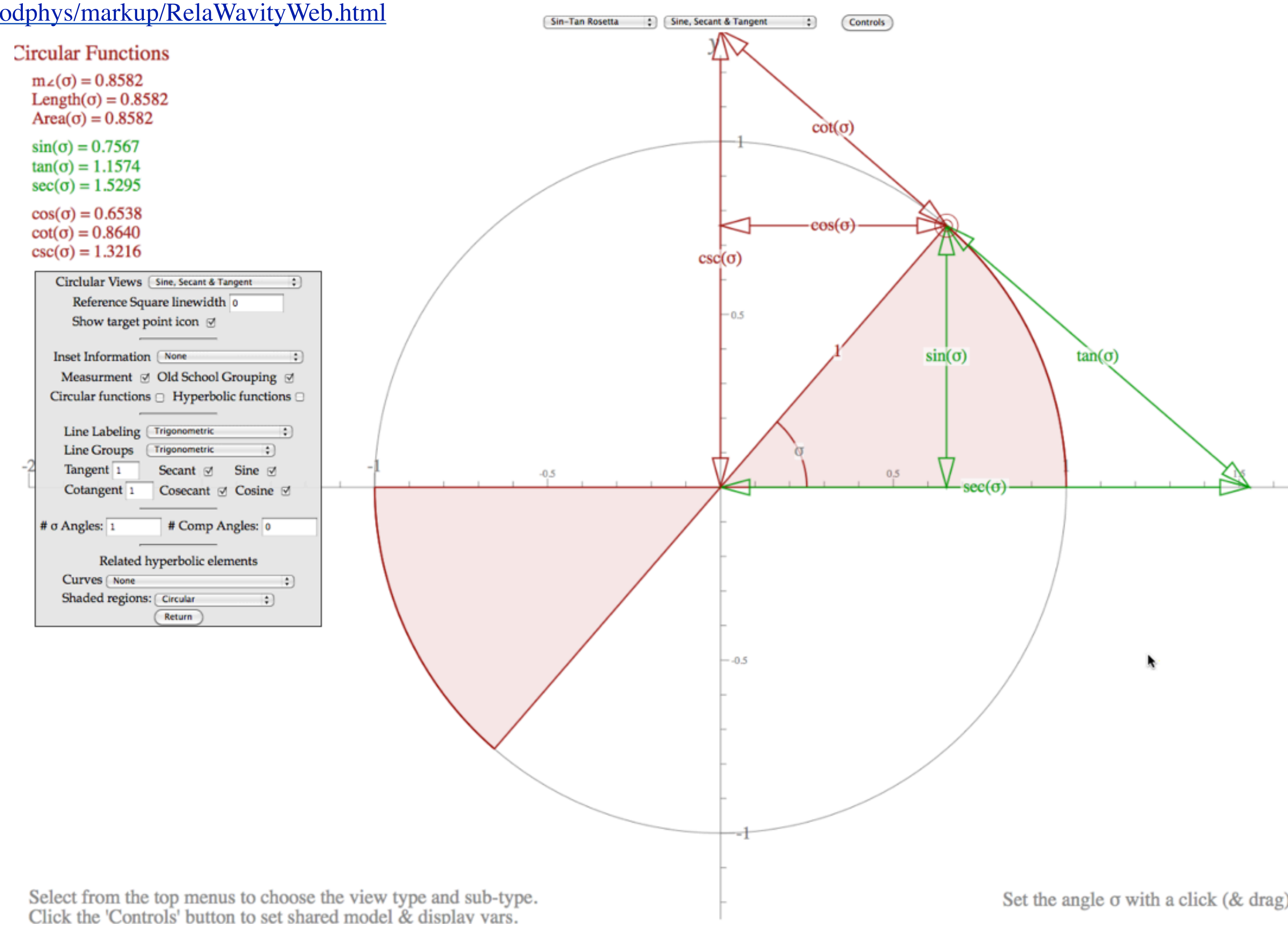
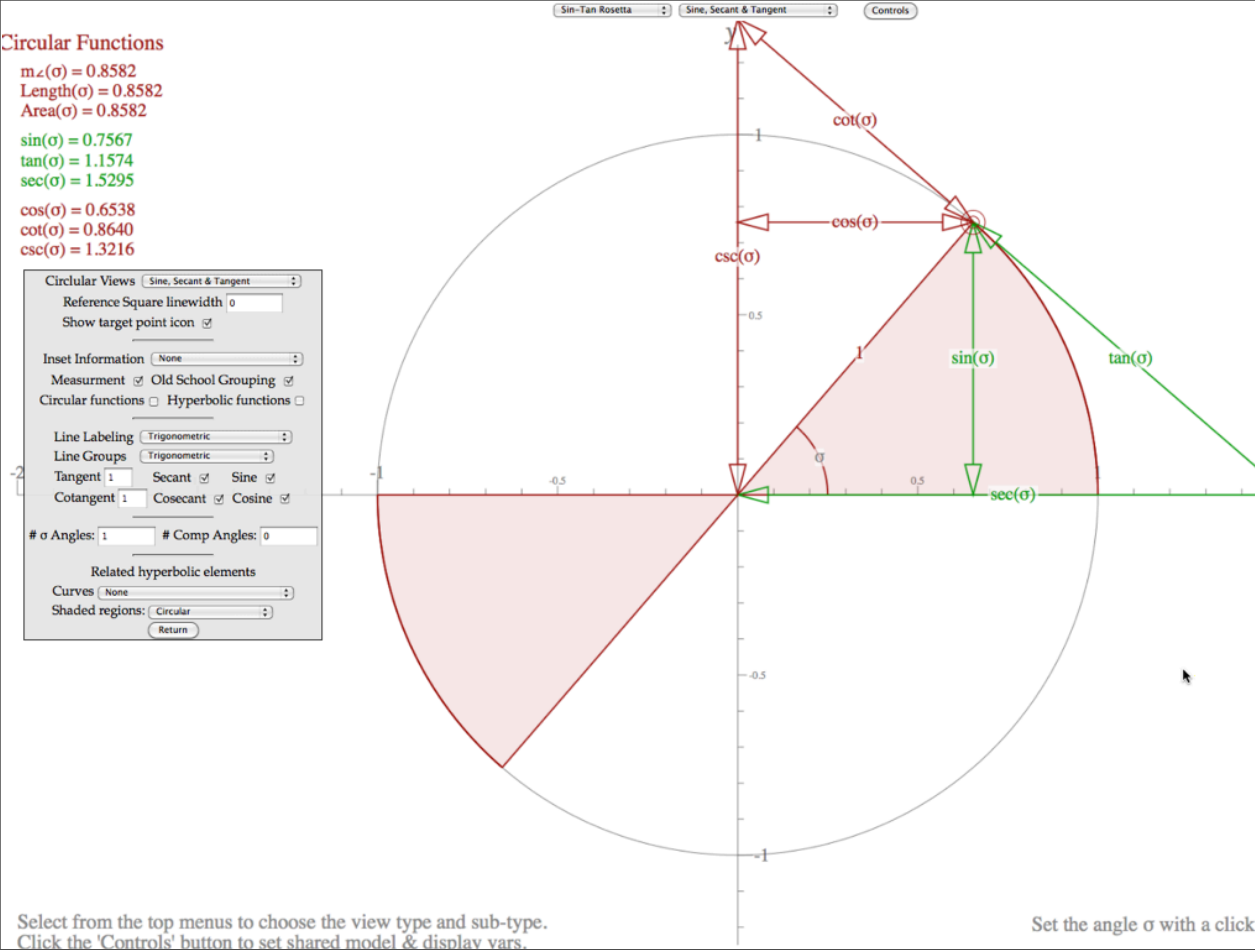


Fig. 5.4 in Unit 8

2014...Web-app versions:

http://www.uark.edu/ua/modphys/markup/RelaWavityWeb.html




Sin-Tan Rosetta
Sine, Secant & Tangent
Controls
Circular Functions
m∠(σ) = 0.8582
Length(σ) = 0.8582
Area(σ) = 0.8582
sin(σ) = 0.7567
tan(σ) = 1.1574
sec(σ) = 1.5295
cos(σ) = 0.6538
cot(σ) = 0.8640
csc(σ) = 1.3216
Circular Views
Sine, Secant & Tangent
Reference Square linewidth
0
Show target point icon
Inset Information
None
Measurment
Old School Grouping
Circular functions
Hyperbolic functions
Line Labeling
Trigonometric
Line Groups
Trigonometric
Tangent
1
Secant
Sine
Cotangent
1
Cosecant
Cosine
σ Angles:
1
Comp Angles:
0
Related hyperbolic elements
Curves
None
Shaded regions:
Circular
Return
cot(σ)
cos(σ)
csc(σ)
sin(σ)
tan(σ)
sec(σ)
σ
Select from the top menus to choose the view type and sub-type.
Click the 'Controls' button to set shared model & display vars.
Set the angle σ with a click

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Together, rapidity ρ=ln b and stellar aberration angle σ are parameters of relative velocity

The rapidity ρ=ln b is based on longitudinal wave Doppler shift $b=e^{\rho}$ defined by $u/c=\tanh(\rho)$. At low speed: $u/c \sim \rho$.

The stellar aberration angle σ is based on the transverse wave rotation $R=e^{i\sigma}$ defined by $u/c=\sin(\sigma)$. At low speed: $u/c \sim \sigma$.

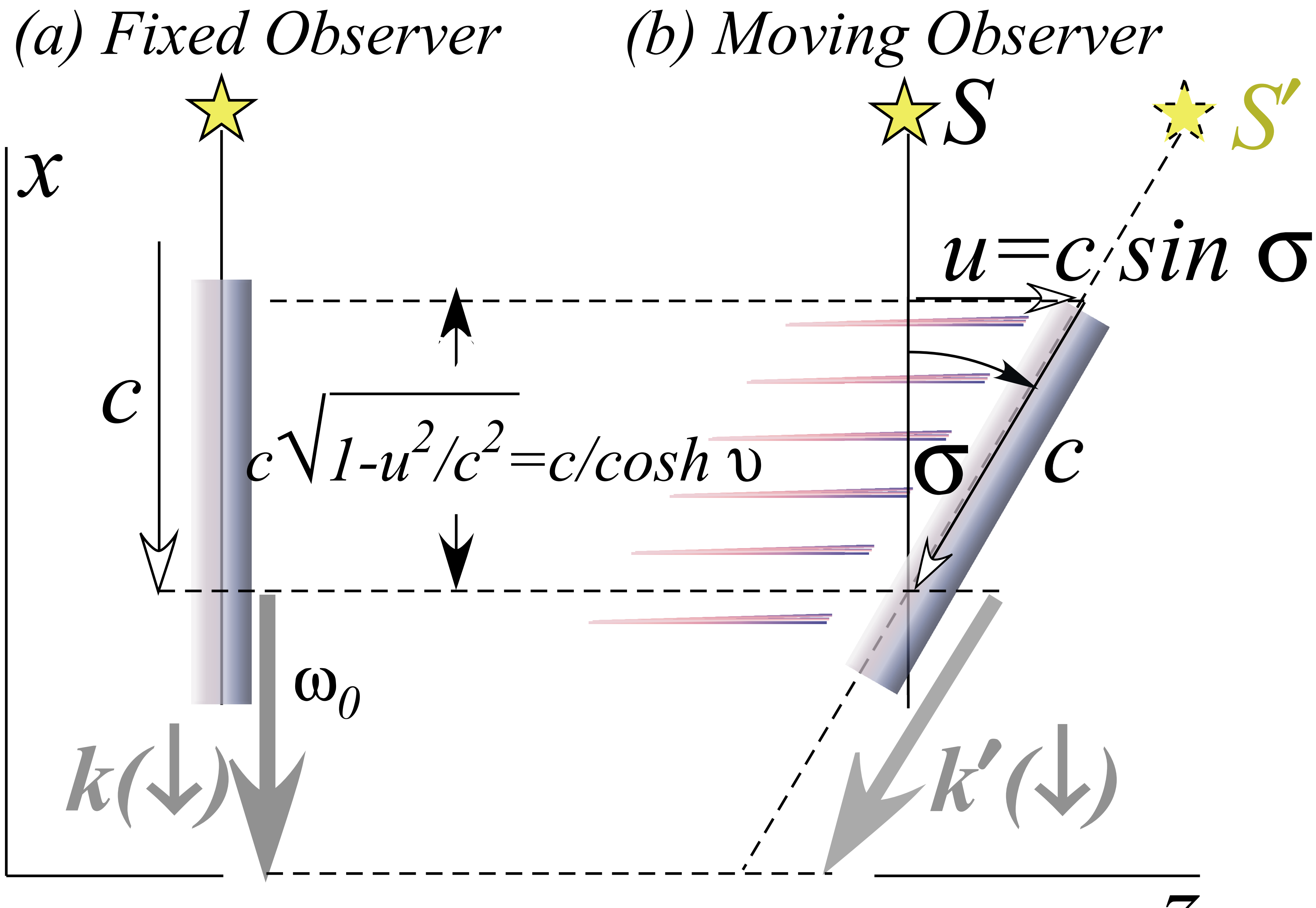

Fig. 5.6 Epstein's cosmic speedometer with aberration angle σ and transverse Doppler shift coshυ_Z.

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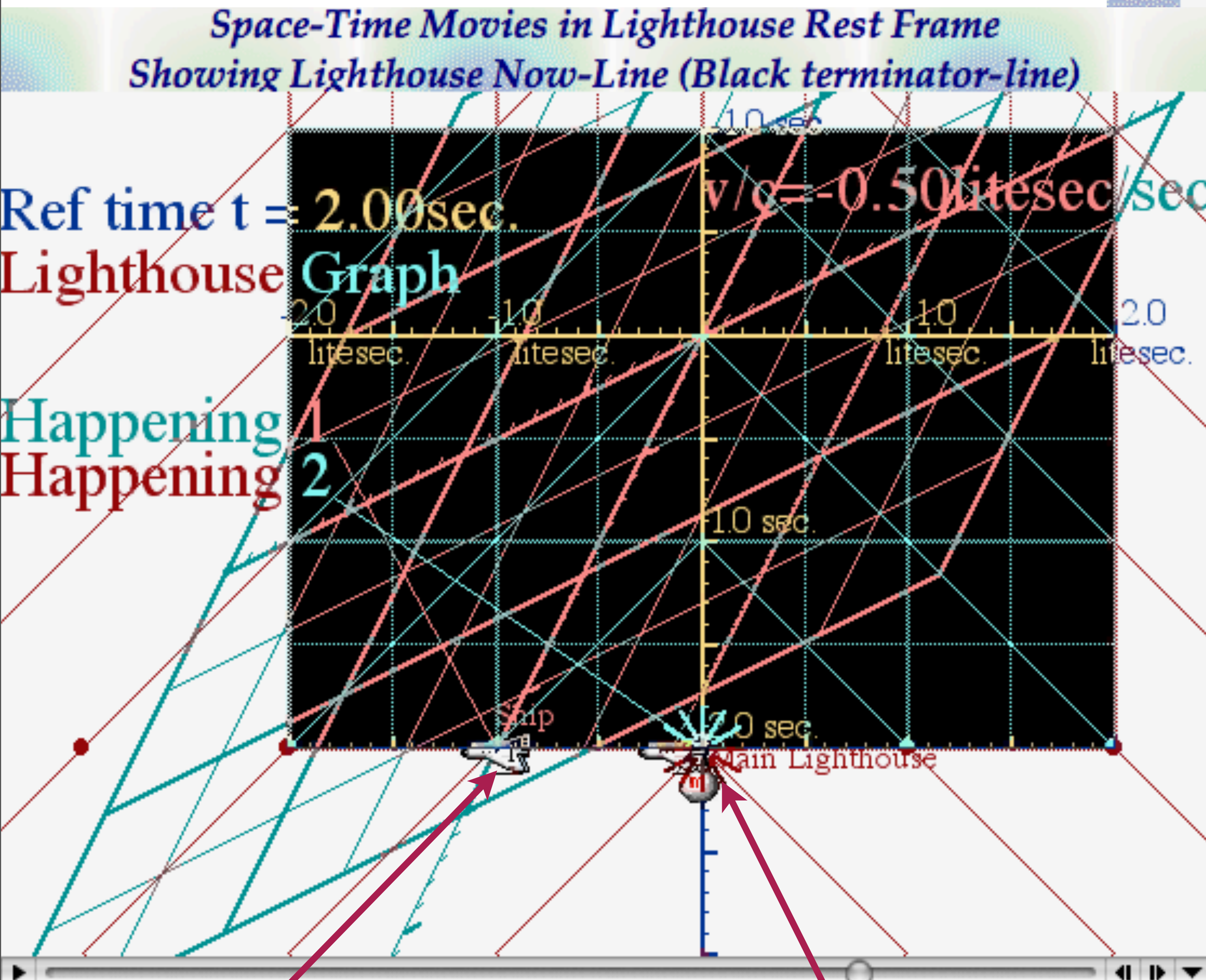
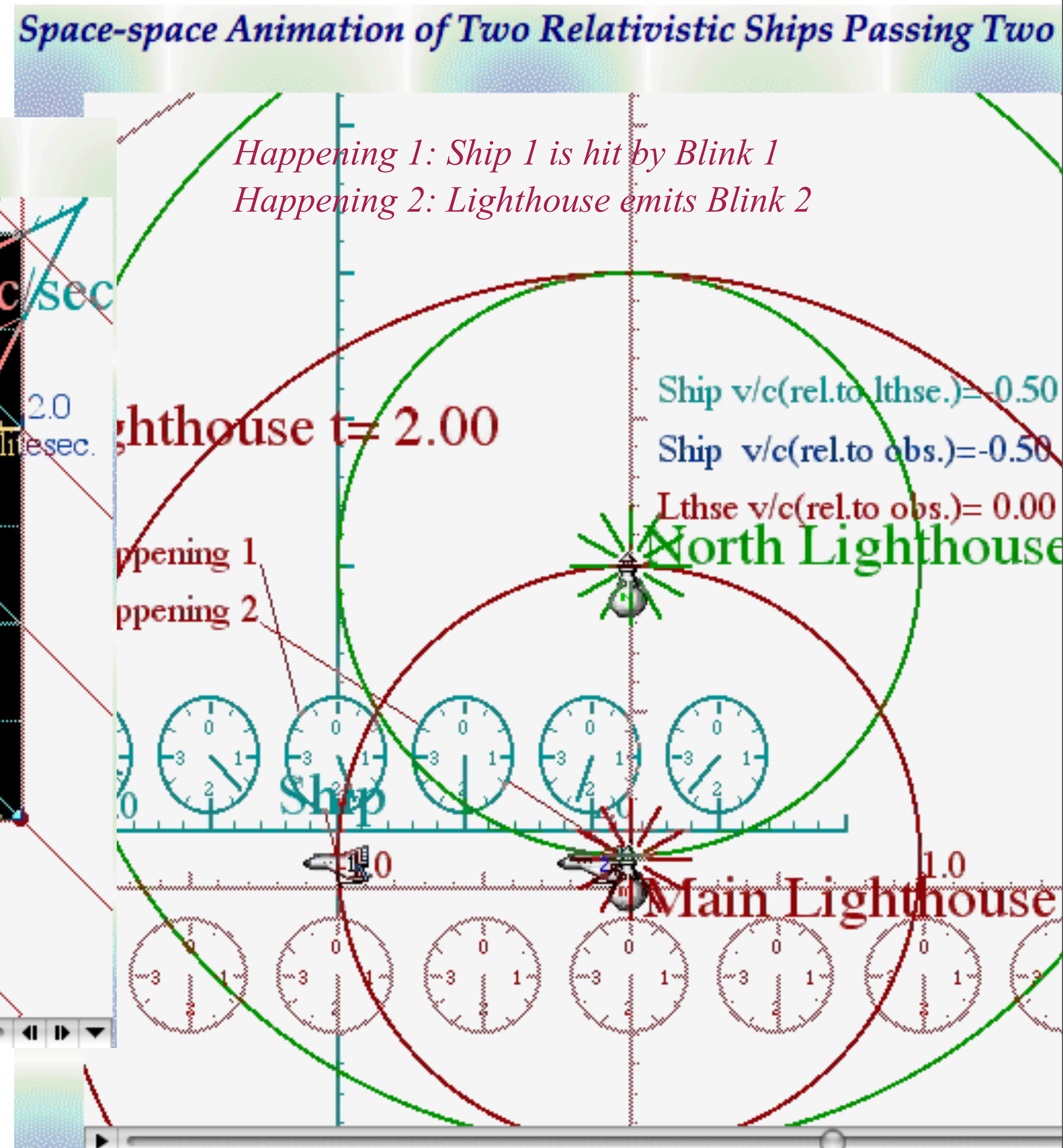
Note that in Lighthouse frame Happening 1 is simultaneous with Happening 2 at t=2.00sec.



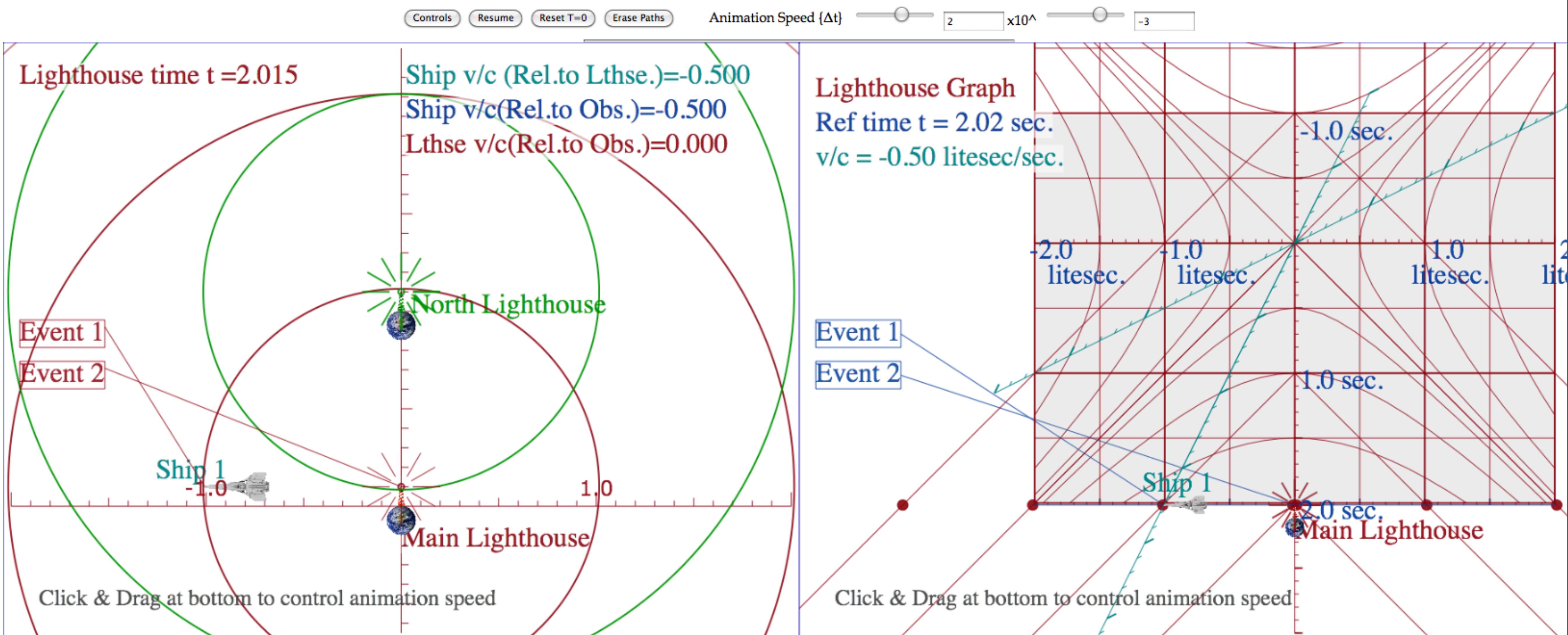


Happening 1

Happening 2

2005 Web versions:

www.uark.edu/ua/pirelli/php/lighthouse_scenarios.php



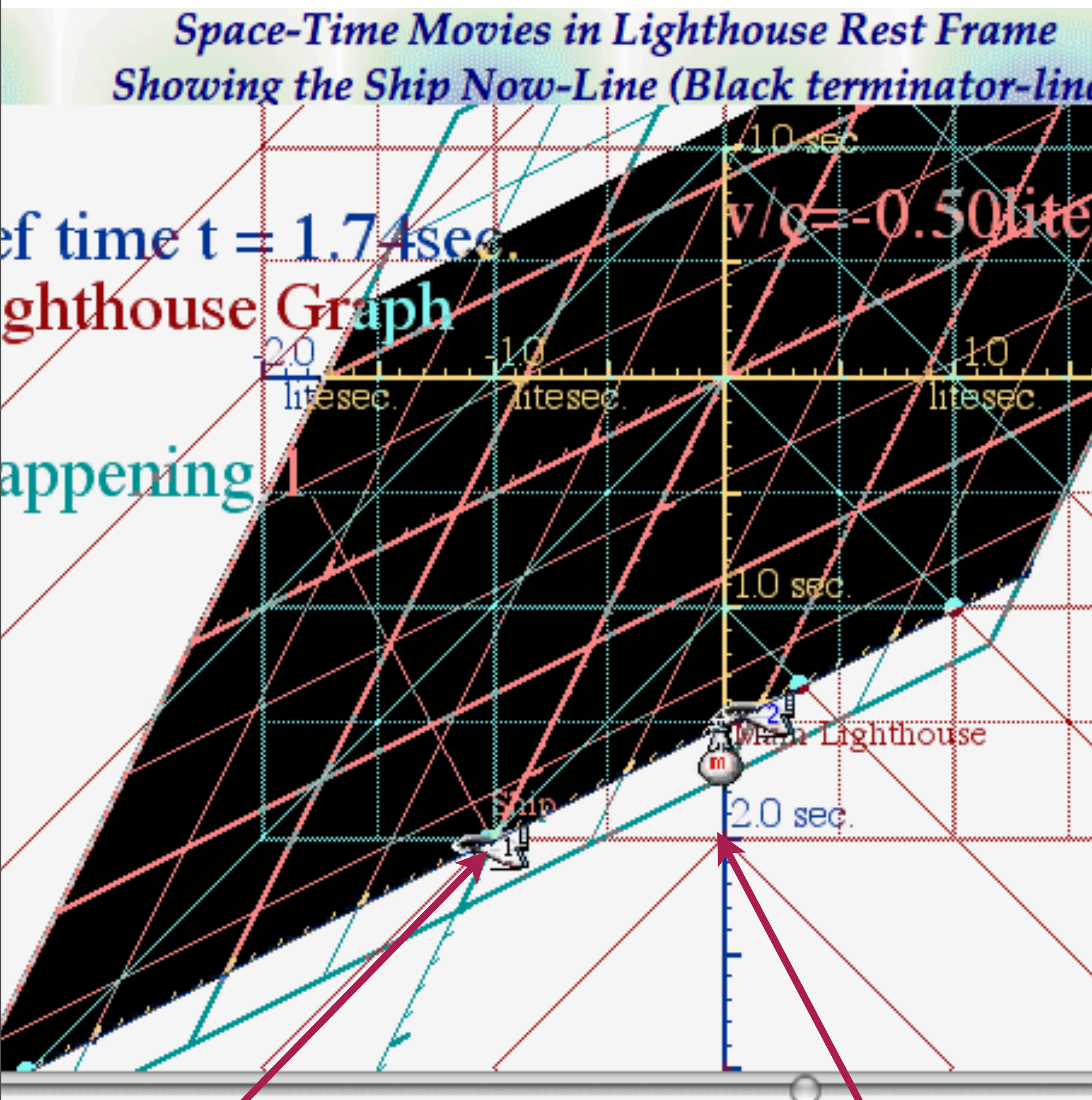
2014...Web-app versions:

http://www.uark.edu/ua/modphys/markup/RelativItWeb.html

How Minkowski's space-time graphs help visualize relativity (Here: r=atanh(1/2)=0.549,

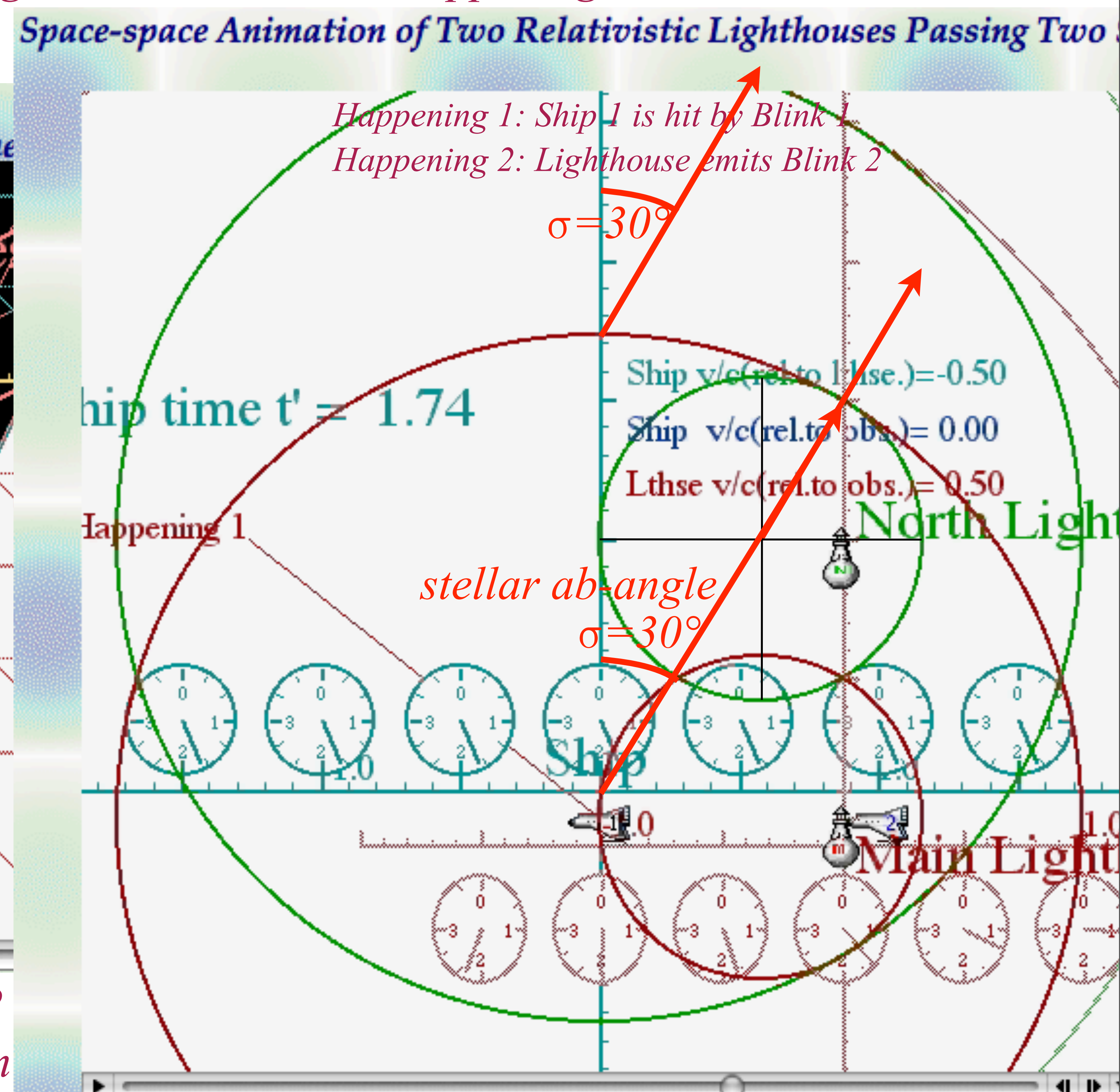
Note that in Lighthouse frame Happening 1 is simultaneous with Happening 2 at t=2.00sec.

...but, in Ship frame Happening 1 is at t'=1.74 and Happening 2 is at t'=2.30sec.



Happening 1

Happening 2 won't happen 'til t=2.00



(Here: ρ=Atanh(1/2)=0.55,

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www.uark.edu/ua/pirelli/php/lighthouse_scenarios.php

Note that in Lighthouse frame Happening 1 is simultaneous with Happening 2 at t=2.00sec.
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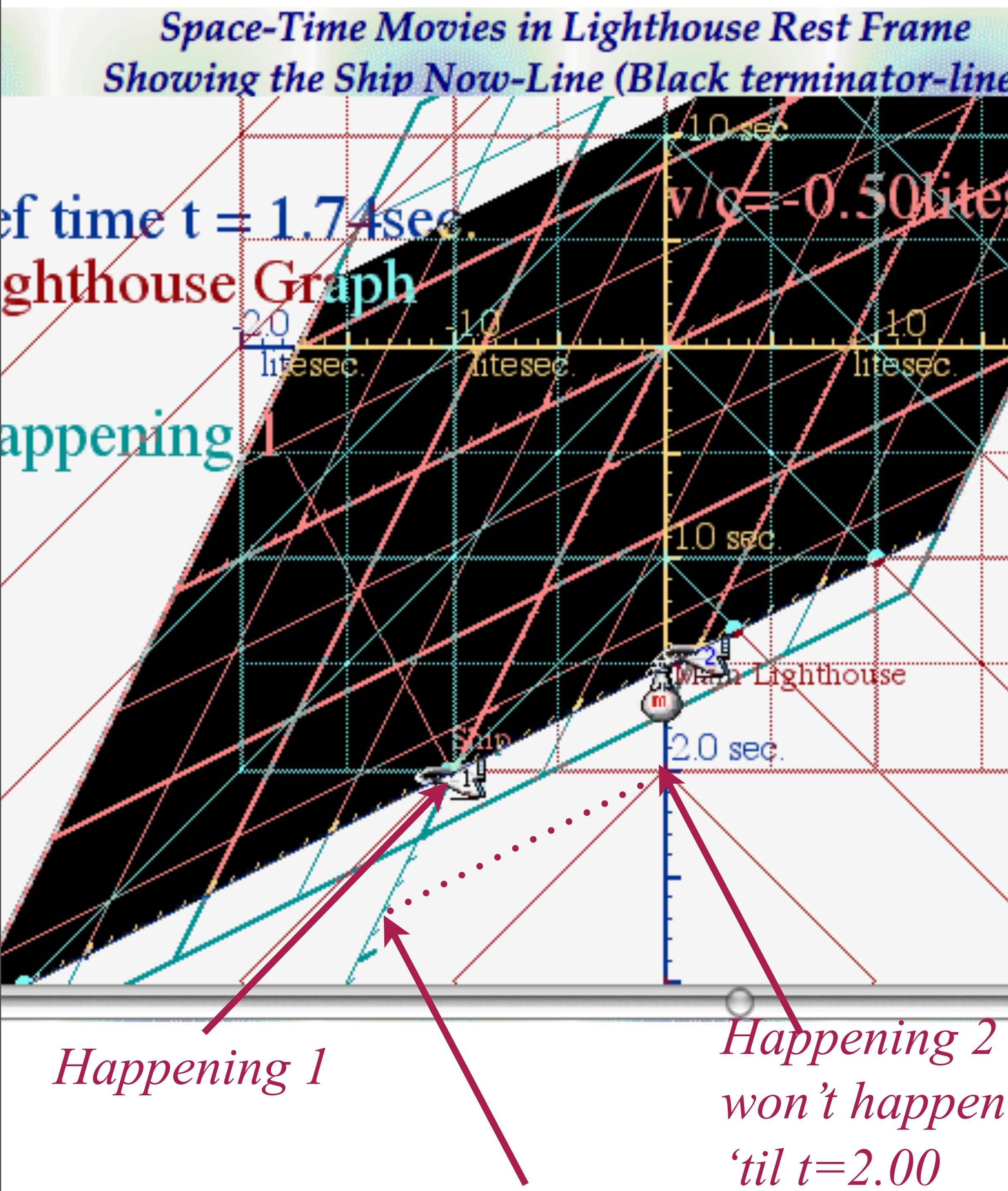

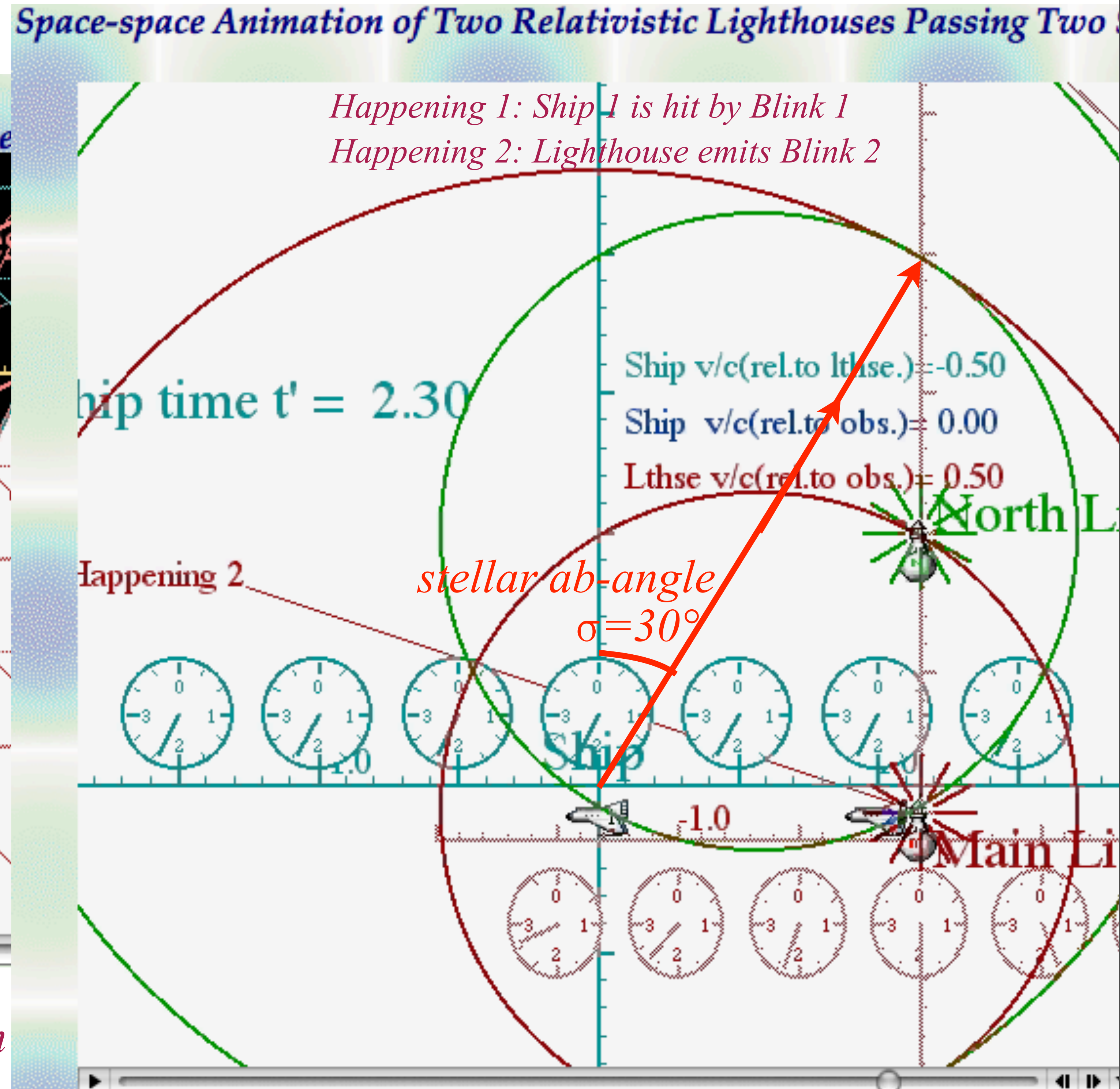

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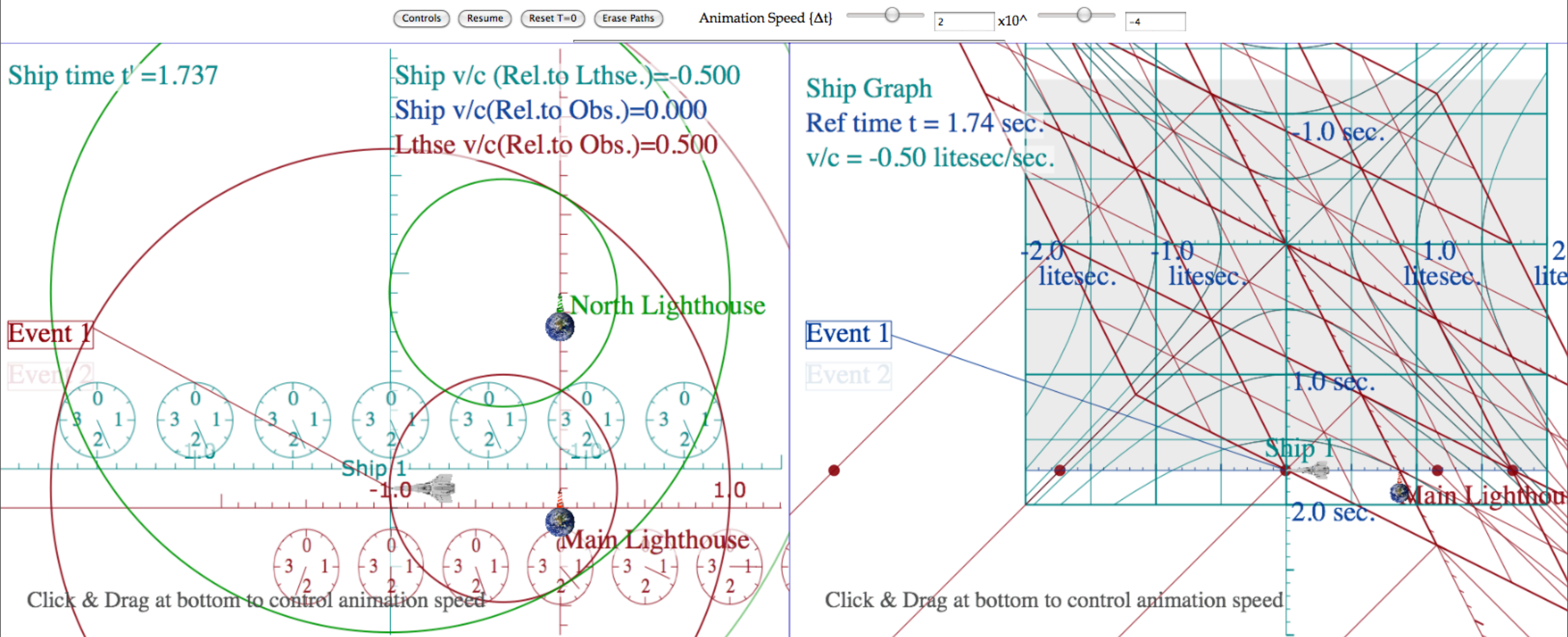

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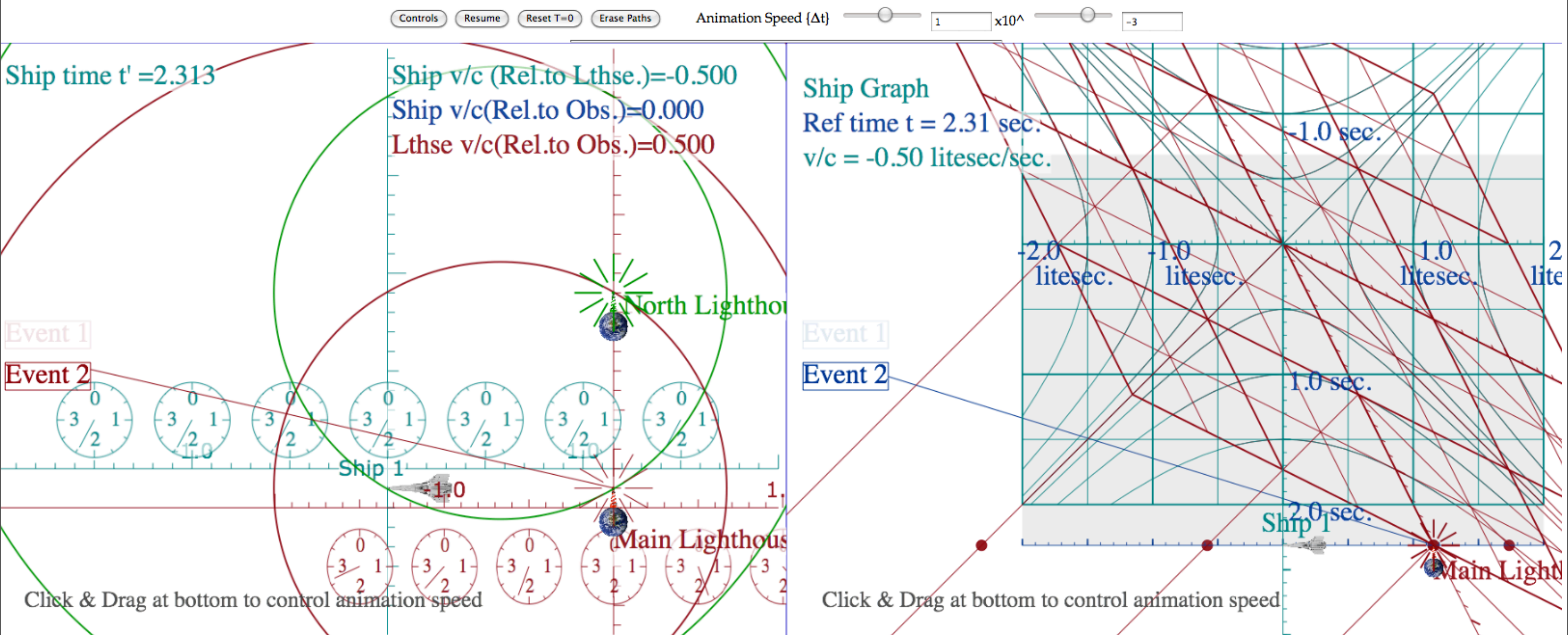

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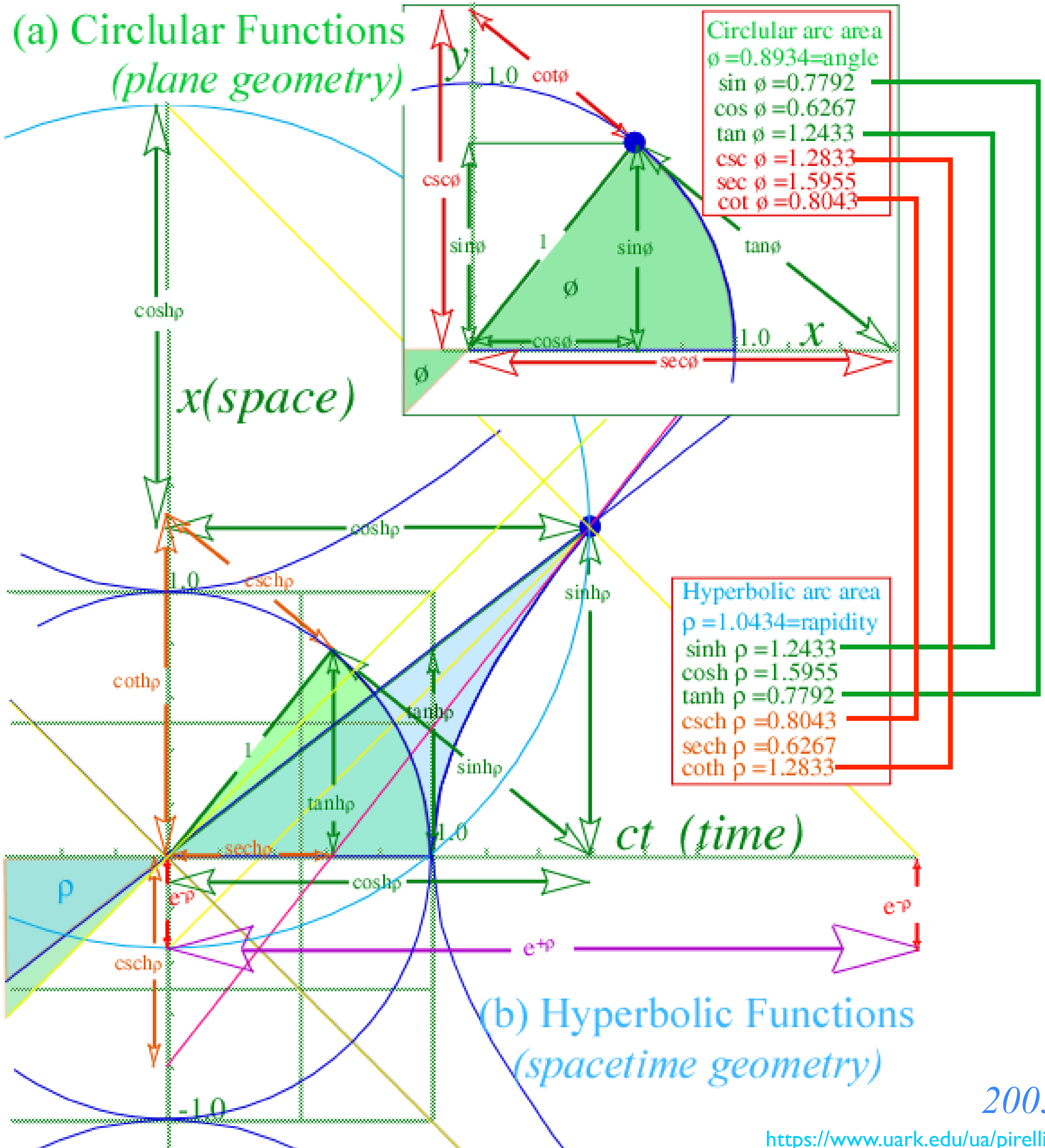

Fig. 5.4 in Unit 8

2005 Web version:
https://www.uark.edu/ua/pirelli/php/hyper_constrct.php

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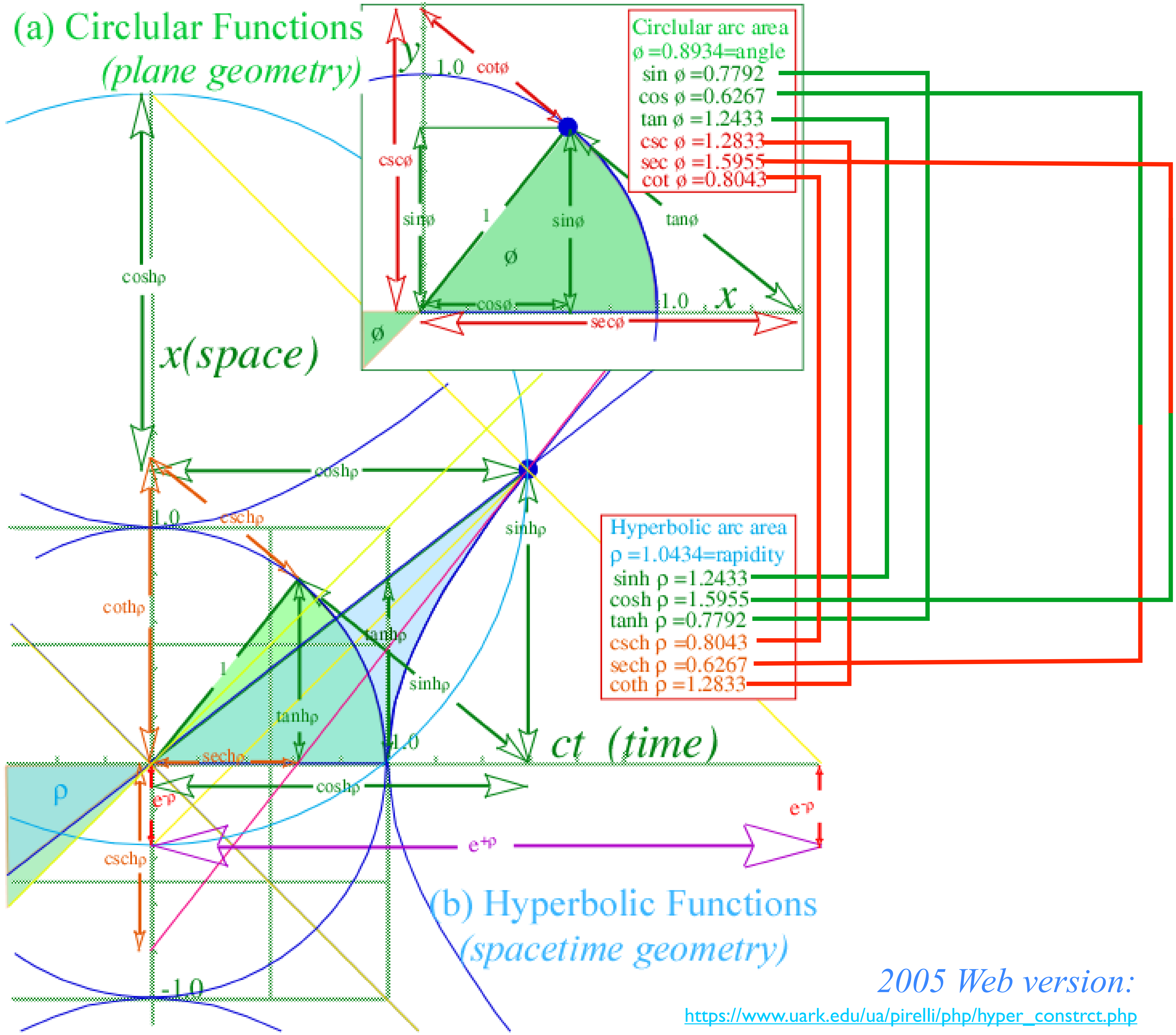

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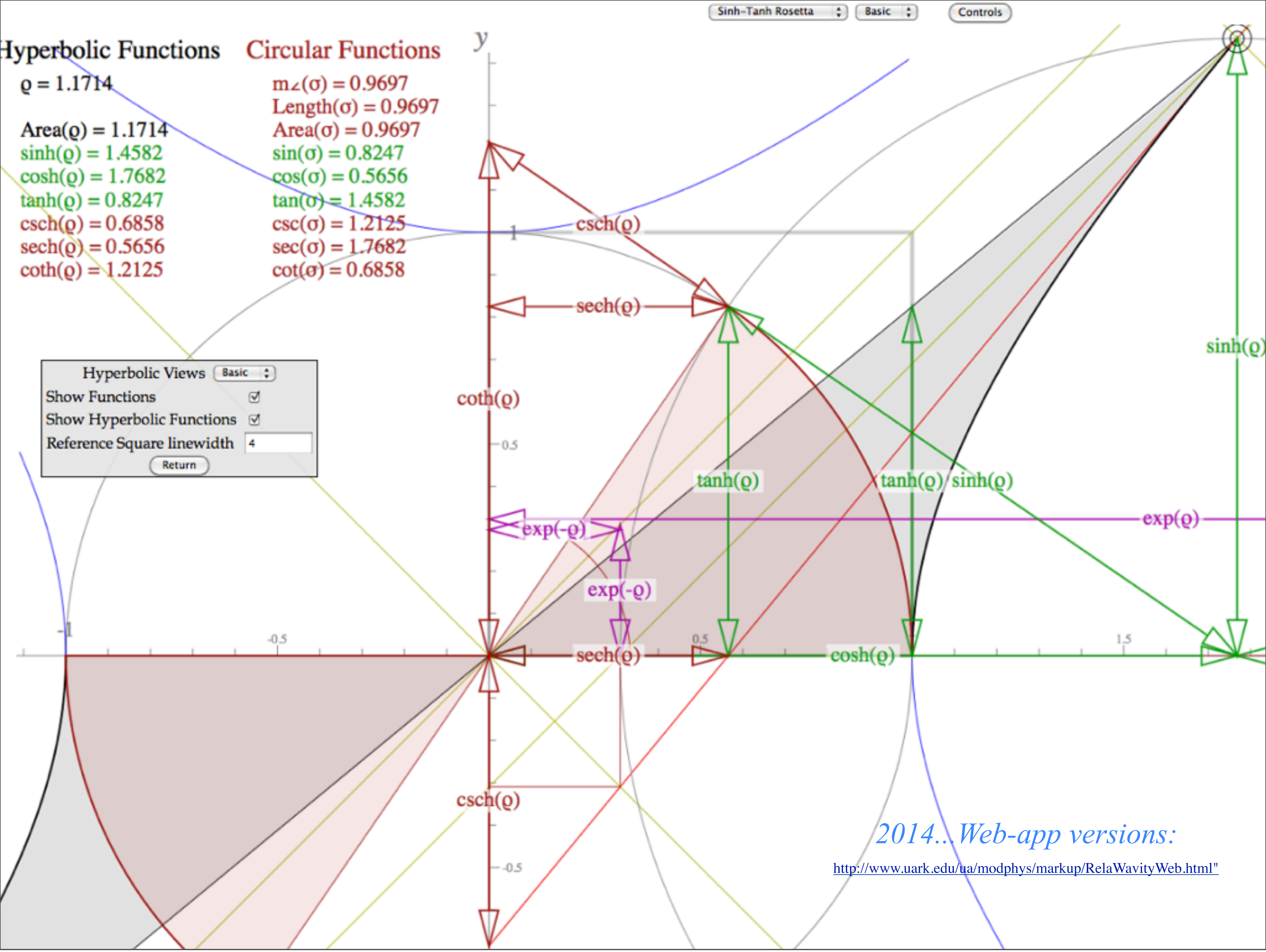

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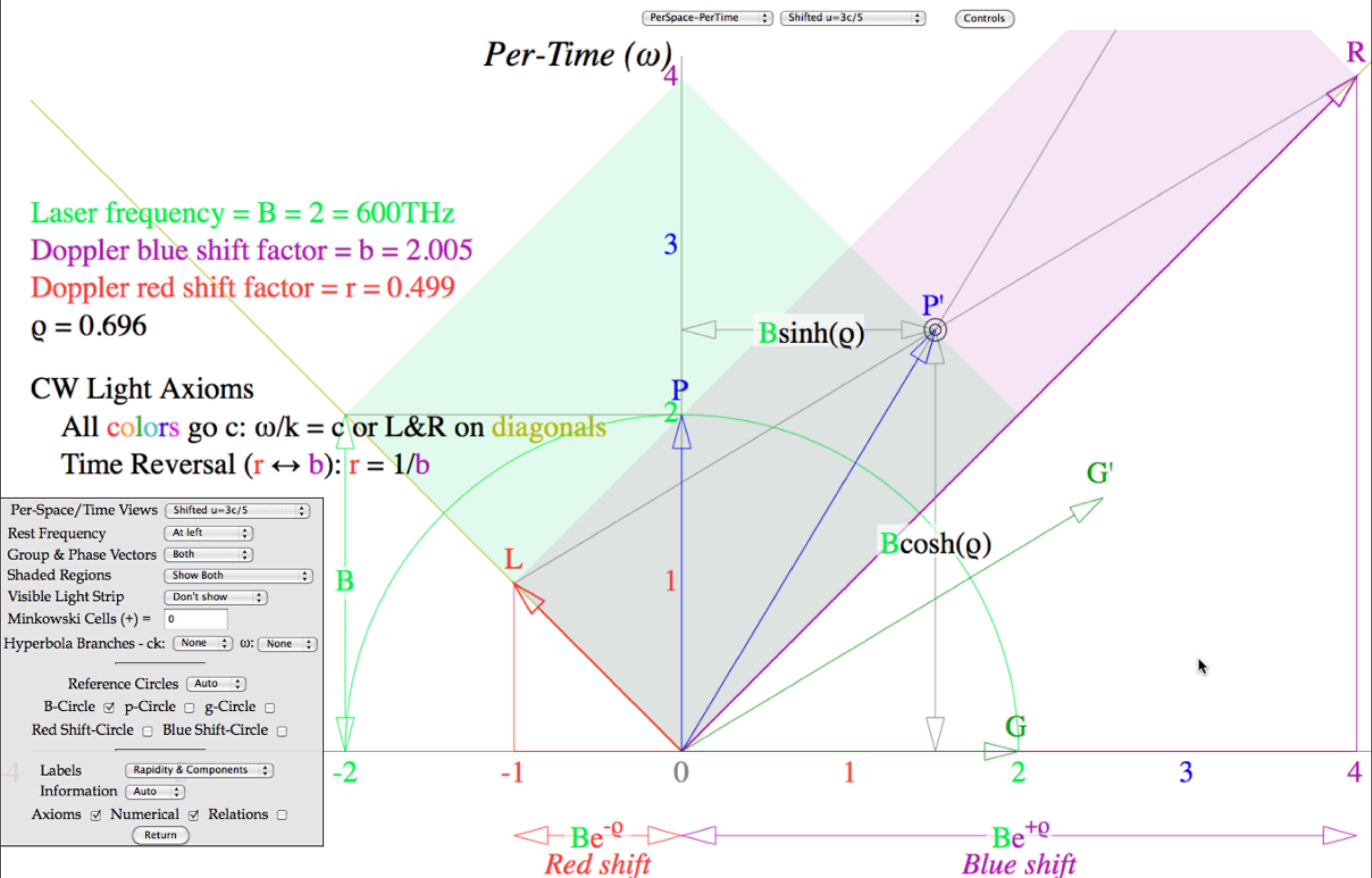
http://www.uark.edu/ua/modphys/markup/RelaWavityWeb.html"



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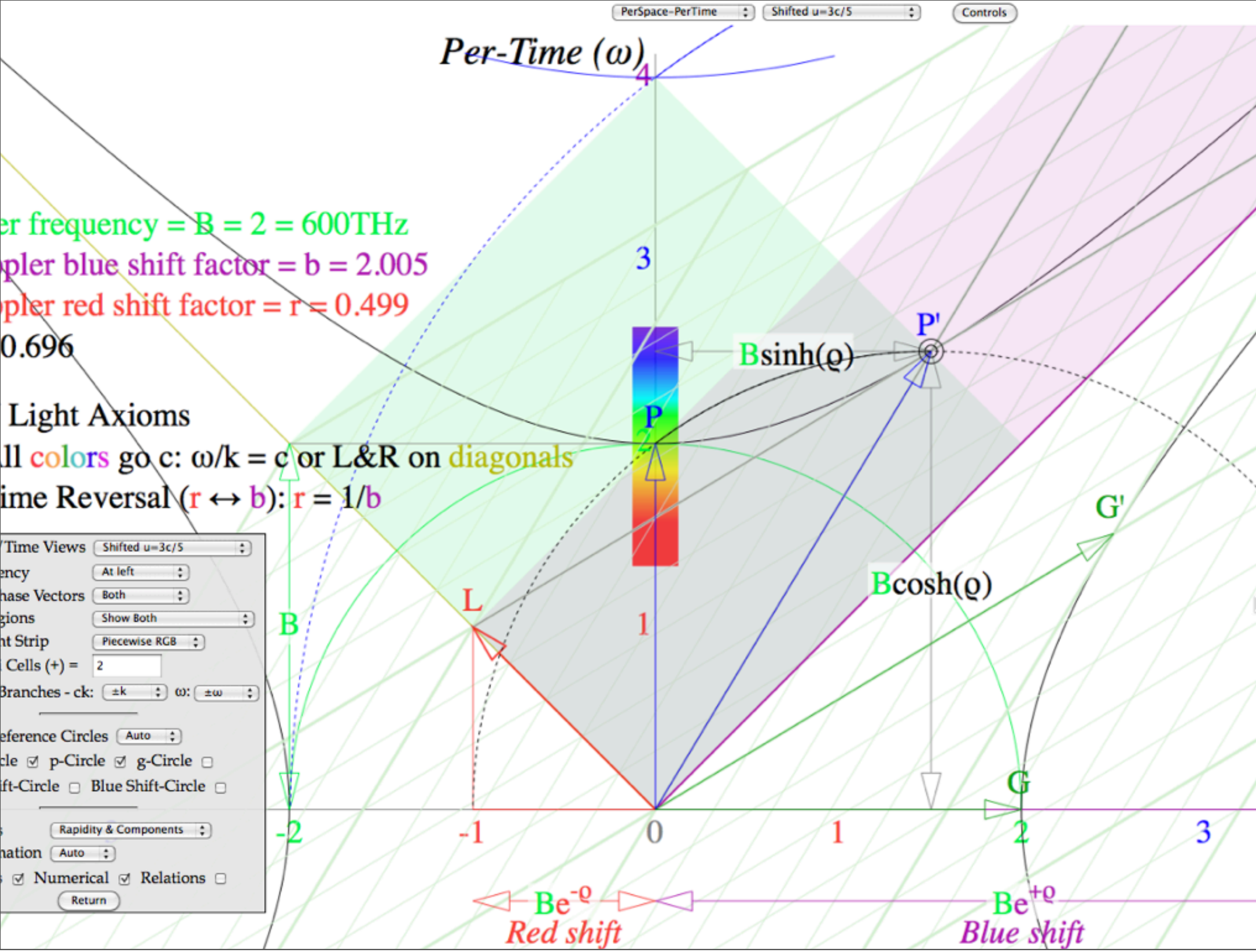
PerSpace-PerTime
Shifted u=3c/5
Controls
Per-Time (ω)
er frequency = B = 2 = 600THz
pler blue shift factor = b = 2.005
pler red shift factor = r = 0.499
0.696
Light Axioms
ll colors go c: ω/k = c or L&R on diagonals
ime Reversal (r ↔ b): r = 1/b
/Time Views
Shifted u=3c/5
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At left
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gions
Show Both
nt Strip
Piecewise RGB
Cells (+) =
2
Branches - ck:
±k
ω:
±ω
eference Circles
Auto
cle
p-Circle
g-Circle
ft-Circle
Blue Shift-Circle
Rapidity & Components
nation
Auto
Numerical
Relations
Return
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3
P'
Bsinh(ρ)
P
2
G'
Bcosh(ρ)
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-1
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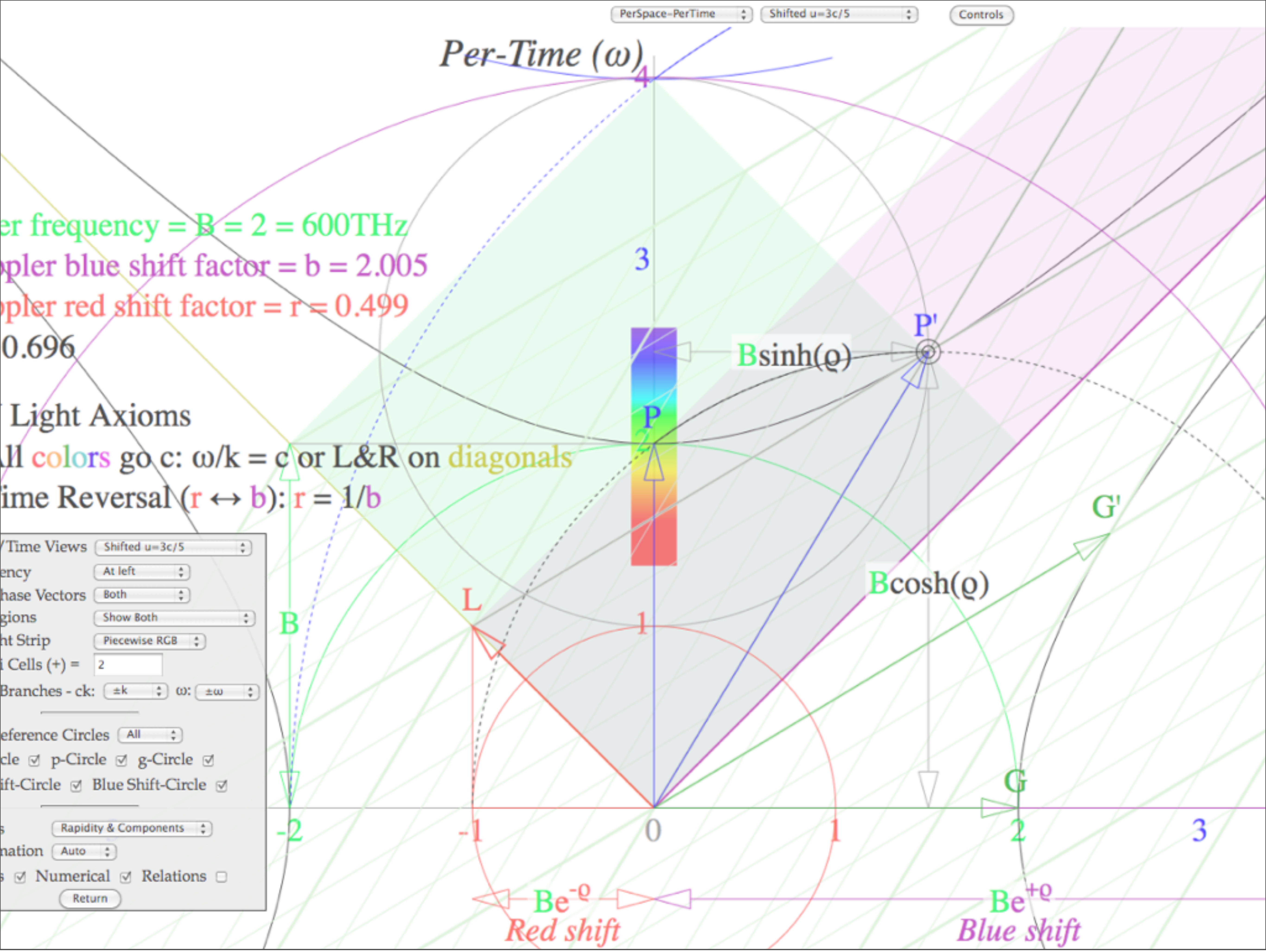
1
2
3
Be^-ρ
Red shift
Be^+ρ
Blue shift



PerSpace-PerTime
Shifted u=3c/5
Controls
Per-Time (ω)
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gions
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cle
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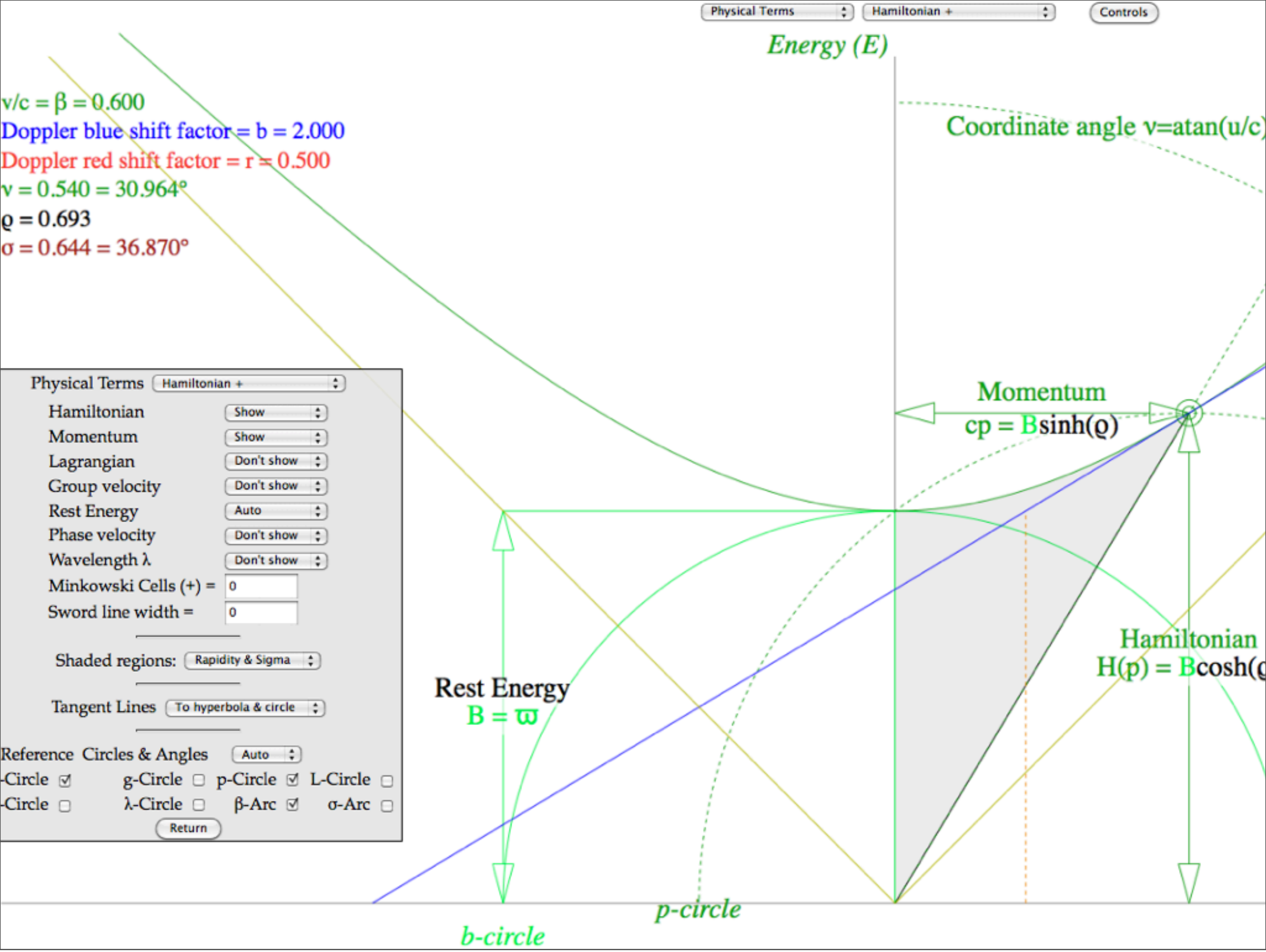
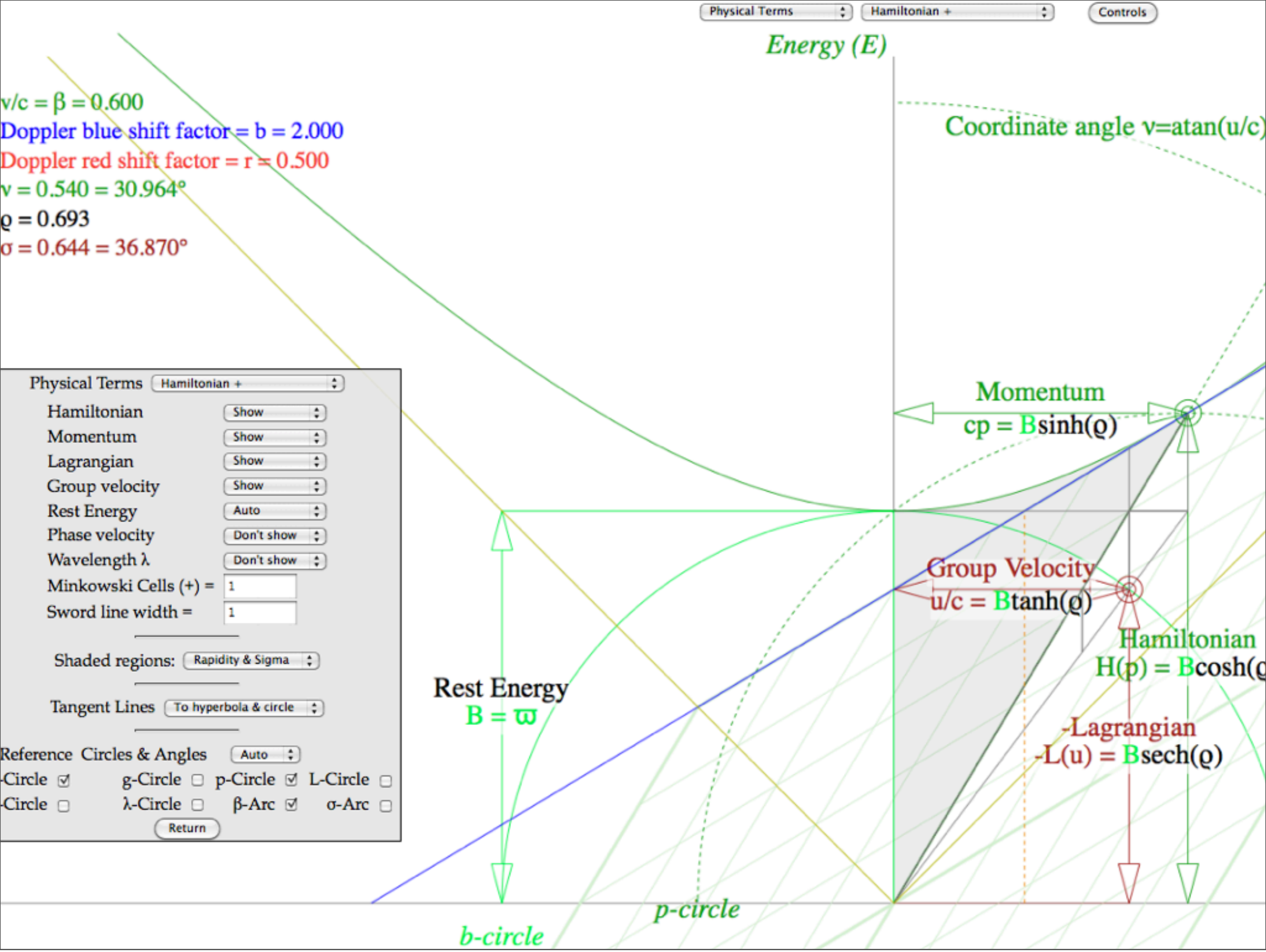
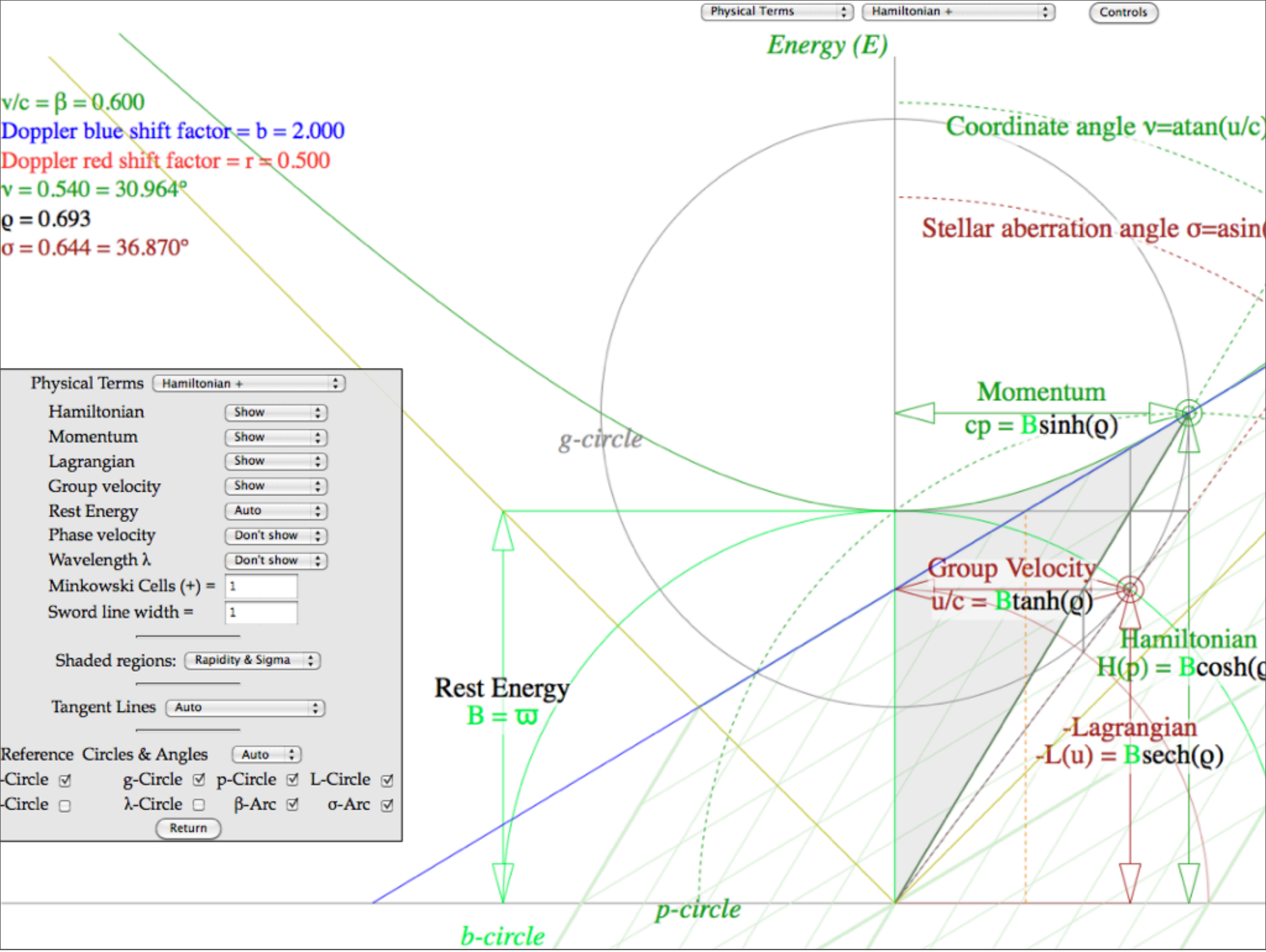
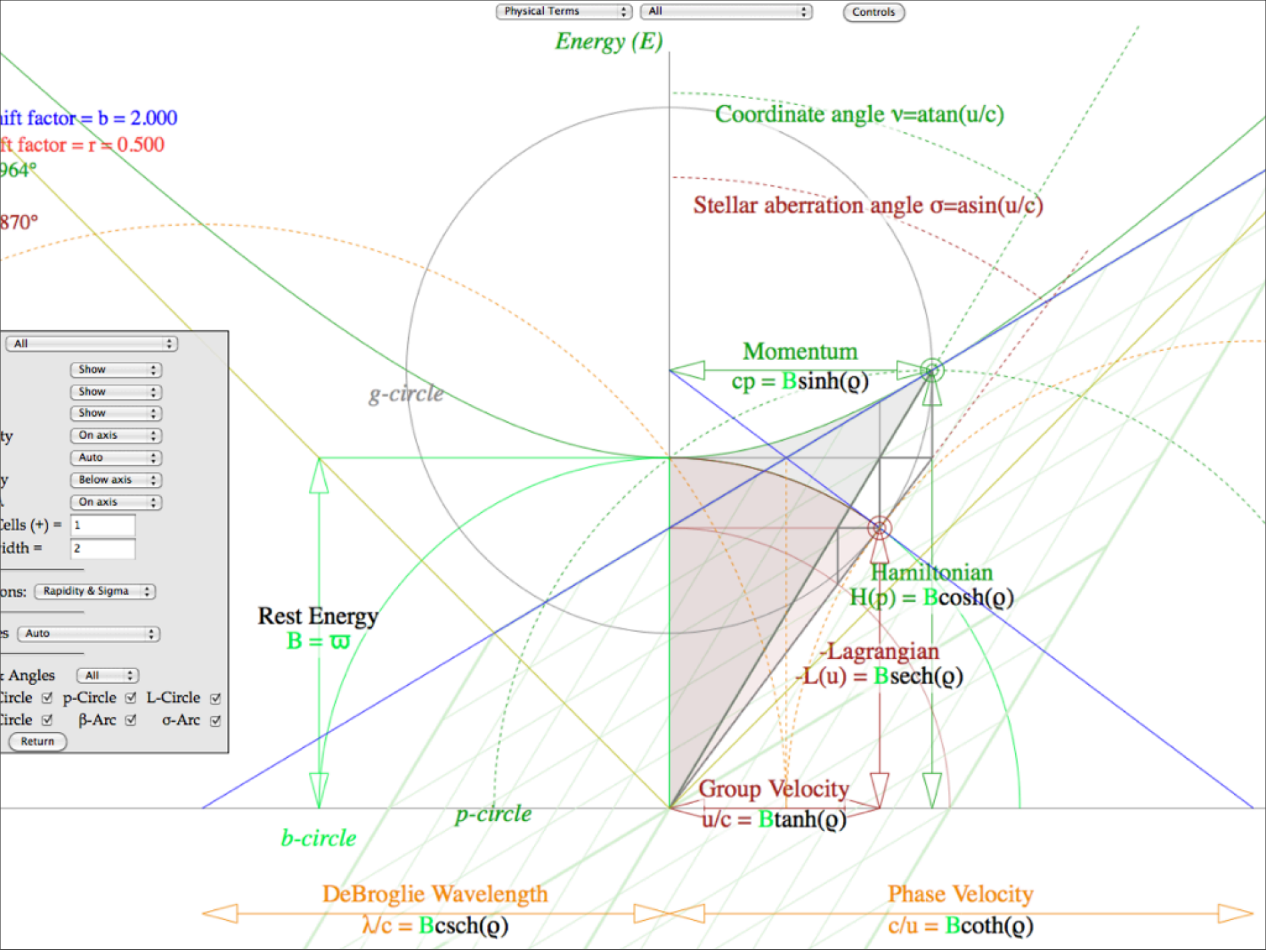
Relations
Return
4
3
P'
Bsinh(ρ)
P
2
G'
Bcosh(ρ)
L
B
1
G
-2
-1
0
1
2
3
Be^-ρ
Red shift
Be^+ρ
Blue shift



Physical Terms
Hamiltonian +
Controls
Energy (E)
v/c = β = 0.600
Doppler blue shift factor = b = 2.000
Doppler red shift factor = r = 0.500
ν = 0.540 = 30.964°
ϱ = 0.693
σ = 0.644 = 36.870°
Coordinate angle ν=atan(u/c)
Momentum
cp = Bsinh(ϱ)
Physical Terms
Hamiltonian +
Hamiltonian
Show
Momentum
Show
Lagrangian
Don't show
Group velocity
Don't show
Rest Energy
Auto
Phase velocity
Don't show
Wavelength λ
Don't show
Minkowski Cells (+) =
0
Sword line width =
0
Shaded regions:
Rapidity & Sigma
Tangent Lines
To hyperbola & circle
Reference Circles & Angles
Auto
-Circle
g-Circle
p-Circle
L-Circle
-Circle
λ-Circle
β-Arc
σ-Arc
Return
Rest Energy
B = ϖ
Hamiltonian
H(p) = Bcosh(ϱ
p-circle
b-circle


Physical Terms
Hamiltonian +
Controls
Energy (E)
v/c = β = 0.600
Doppler blue shift factor = b = 2.000
Doppler red shift factor = r = 0.500
ν = 0.540 = 30.964°
ϱ = 0.693
σ = 0.644 = 36.870°
Coordinate angle ν=atan(u/c)
Physical Terms Hamiltonian +
Hamiltonian Show
Momentum Show
Lagrangian Show
Group velocity Show
Rest Energy Auto
Phase velocity Don't show
Wavelength λ Don't show
Minkowski Cells (+) = 1
Sword line width = 1
Shaded regions: Rapidity & Sigma
Tangent Lines To hyperbola & circle
Reference Circles & Angles Auto
-Circle g-Circle p-Circle L-Circle
-Circle λ-Circle β-Arc σ-Arc
Return
Momentum
cp = Bsinh(ϱ)
Group Velocity
u/c = Btanh(ϱ)
Hamiltonian
H(p) = Bcosh(ϱ
Rest Energy
B = ϖ
-Lagrangian
-L(u) = Bsech(ϱ)
p-circle
b-circle


Physical Terms
Hamiltonian +
Controls
Energy (E)
v/c = β = 0.600
Doppler blue shift factor = b = 2.000
Doppler red shift factor = r = 0.500
ν = 0.540 = 30.964°
ρ = 0.693
σ = 0.644 = 36.870°
Coordinate angle ν=atan(u/c)
Stellar aberration angle σ=asin
Physical Terms Hamiltonian +
Hamiltonian Show
Momentum Show
Lagrangian Show
Group velocity Show
Rest Energy Auto
Phase velocity Don't show
Wavelength λ Don't show
Minkowski Cells (+) = 1
Sword line width = 1
Shaded regions: Rapidity & Sigma
Tangent Lines Auto
Reference Circles & Angles Auto
-Circle g-Circle p-Circle L-Circle
-Circle λ-Circle β-Arc σ-Arc
Return
Momentum
cp = Bsinh(ρ)
g-circle
Group Velocity
u/c = Btanh(ρ)
Hamiltonian
H(p) = Bcosh(
Rest Energy
B = ϖ
-Lagrangian
-L(u) = Bsech(ρ)
p-circle
b-circle


Physical Terms
All
Controls
Energy (E)
ift factor = b = 2.000
ft factor = r = 0.500
964°
870°
Coordinate angle ν=atan(u/c)
Stellar aberration angle σ=asin(u/c)
Momentum
cp = Bsinh(ϱ)
g-circle
Hamiltonian
H(p) = Bcosh(ϱ)
Rest Energy
B = ϖ
-Lagrangian
-L(u) = Bsech(ϱ)
Group Velocity
u/c = Btanh(ϱ)
p-circle
b-circle
DeBroglie Wavelength
λ/c = Bcsch(ϱ)
Phase Velocity
c/u = Bcoth(ϱ)
All
Show
Show
Show
ty
On axis
Auto
y
Below axis
On axis
Cells (+) =
1
idth =
2
ons:
Rapidity & Sigma
s
Auto
Angles
All
ircle
p-Circle
L-Circle
ircle
β-Arc
σ-Arc
Return