Relativity of wave-optics and Lorentz-Minkowski coordinates I.

Ch. 2 of Unit 8 CMwBang! and p.1-23 Relativity&QuantumTheory by Rule&Compass

1. Optical wave coordinates and frames
   Old-fashioned vs. New-fashioned spacetime frames
   Dueling lasers make lab frame space-time grid (CW or PW)
   Comparing Continuous-Wave (CW) vs. Pulse-Wave (PW) frames with Review of Light

2. Applying Occam’s razor to relativity axioms
   Einstein PW Axioms versus Evenson CW Axioms (Traditional: The "Roadrunner" Axiom)
   CW light clearly shows Doppler shifts
   Check that red is red is red,...green is green is green,...blue is blue is blue,... etc.
   Is dispersion linear? ... does astronomy work?... how about spectroscopy?
   Is Doppler a geometric factor or arithmetic sum?
   Introducing rapidity $\rho = \ln b$.
   That old Time-Reversal meta-Axiom (that is so-oo-o neglected!)

3. Spectral theory of Einstein-Lorentz relativity
   Applying Doppler Shifts to per-space-time ($ck, \omega$) graph
   CW Minkowski space-time coordinates ($x,ct$) and PW grids
   Relating Doppler Shifts $b$ or $r=1/b$ to velocity $u/c$ or rapidity $\rho$
   Connection: Conventional approach to relativity and old-fashioned formulas
1. Optical wave coordinates and frames

Old-fashioned vs. New-fashioned spacetime frames
Dueling lasers make lab frame space-time grid (CW or PW)
Comparing Continuous-Wave (CW) vs. Pulse-Wave (PW) frames
• **Optical wave coordinate manifolds and frames**

  *Shining some light on light using complex phasor analysis*

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**Old-fashioned meter-stick-clock frames**

E. F. Taylor and J. A. Wheeler Spacetime Physics (Freeman San Francisco 1966)

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**New-fashioned laser clocks & meter sticks**

Complex Phasor Clocks: Tesla’s AC “phasor”

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Quantum Phasor Clock

\[ \Psi = A e^{i(kx - \omega t)} = A \cos(kx - \omega t) + i A \sin(kx - \omega t) \]

Phase

\[ \theta = (kx - \omega t) \]

Amplitude or Magnitude

\[ A = |\Psi| \]

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**300THz Laser plane wave**

\[ \langle x, t | k, \omega \rangle = A e^{i(kx - wt)} \]

Re \( \Psi = \cos(kx - \omega t) \)

Im \( \Psi = \sin(kx - \omega t) \)
New-fashioned laser clocks & meter sticks (contd.)

Dual views:

1. Spacetime
   \( x \) versus \( ct \)
   - \( k = +1 \)
   - \( \omega = 1c \)
   - "laser phasors"
   - Period \( \tau = 2\pi/\omega = 1/\nu \)
   - Wavelength \( \lambda = 2\pi/k = 1/\kappa \)
   - \( \nu \lambda = c \)

2. Per-Spacetime
   \( \omega \) versus \( ck \)
   - \( \Delta \) Frequency
   - \( \nu = \omega/2\pi \)
   - Frequency
   - \( 1200\text{THz} \)
   - \( 900\text{THz} \)
   - \( 3\text{THz} \)
   - \( 2\text{THz} \)
   - \( 300\text{THz} \)

Single plane-wave meter-stick-clocks are too fast
(can't catch 'em)

Interfering wave pairs needed
to make rest frame coordinates...

(...But at least this view is constant)
1. Optical wave coordinates and frames

Old-fashioned vs. New-fashioned spacetime frames

Dueling lasers make lab frame space-time grid (CW or PW)

Comparing Continuous-Wave (CW) vs. Pulse-Wave (PW) frames
Zeros of head-on CW sum gives \((x, ct)-\text{grid}\)
Zeros of head-on CW sum gives \((x, ct)\)-grid

Find zeros by factoring sum:

\[
\Psi = e^{ia} + e^{ib} = e^{i(a+b)/2} \left(e^{i(a-b)/2} + e^{-i(a-b)/2}\right)
\]

Phase factor: \(\exp\left(\frac{a+b}{2}\right) = e^{i\omega t}\)

Group factor: \(2\cos\left(\frac{a-b}{2}\right) = 2\cos(kx)\)
1. Optical wave coordinates and frames

Old-fashioned vs. New-fashioned spacetime frames
Dueling lasers make lab frame space-time grid (CW or PW)

Comparing Continuous-Wave (CW) vs. Pulse-Wave (PW) frames
Newton’s “Fits” in Optical Interference
Newton complained that light waves have “fits” (what we now know as wave interference or resonance.) Examples of interference are head-on collision of two Continuous Waves (2-CW) or two Pulse Waves (PW)

Continuous Wave (CW) Addition

Pulse Wave (PW) Addition

http://www.uark.edu/ua/pirelli/php/waves_interfering_montage.php
Newton’s “Fits” in Optical Interference

Newton complained that light waves have “fits” (what we now know as wave interference or resonance.) Examples of interference are head-on collision of two Continuous Waves (2-CW) or two Pulse Waves (PW)

Continuous Wave (CW) Addition

Sharp zeros trace square grid (Peaks are diffuse)

Pulse Wave (PW) Addition

(Zeros are diffuse) Sharp peaks trace diamond grid

Pulse Wave (PW) sum compared with Continuous Wave (CW) sum

- **PW** waves are OFF (0) or ON (1)
- **PW** sum is Boolean \((0_L, 0_R, 0_L, 1_R, 1_L, 0_R, 1_L, 1_R)\).
- **PW** time peak-diamond paths are *wysiwyg*. (What you see is what you expect!)

\[
\begin{align*}
\text{PLUS} & \quad \text{EQUALS} \\
L+R & \quad \text{Left } L \quad \text{Right } R
\end{align*}
\]

- **CW** waves range continuously from -1 to +1
- **CW** sum is more subtle and nuanced interference.
- **CW** time zero-square paths are subtle results of the *half-sum* \(P\)-rule and the *half-difference* \(G\)-rule of phase \(P\) and group \(G\) zeros.

\[
\begin{align*}
P = R + L \\
G = R - L
\end{align*}
\]

http://www.uark.edu/ua/pirelli/php/waves_interfering_montage.php
Speed of light \( c \)
\[
c = \frac{\lambda}{\tau} = \frac{0.5 \cdot 10^{-6}}{5/3 \cdot 10^{-15}} = 3 \cdot 10^8
\]

(a) CW squares
1 femtosecond
1.0 fs = 10\(^{-15}\) s
1 micron
1.0 \( \mu m = 10^{-6} \) meter

(b) PW diamonds
PW laser
\( \omega_0 = 2c \)

"patooey!"
Light waves are the lead actors in our portrayal of relativity and quantum theory. This differs from standard treatments following Einstein’s original works that are based more on Newtonian notions of particles, bodies, and rigid frames.

Light, which Newton also regarded as fundamentally corpuscular (particle-like), had by the late 1800's been shown to have a fundamental wave nature due to work of (among others) Young, Huygens, and most notably, Maxwell.

Then one of Einstein’s 1905 works, following Planck’s 1900 hypothesis of light quanta, showed that light also had to have a particle-like nature.

Wave-particle duality may be understood by looking at interference properties of waves in general and how that applies in particular to light waves.

Relativity and quantum mechanics are practically the same subject when viewed in the light of wave interference/resonance, that is, light wave addition.
It helps to introduce two archetypes of light waves and contrast them.

The first \((PW)\) is a Particle-like Wave or part of a Pulse-Wave train.
The second \((CW)\) is a Coherent Wave or part of a Continuous-Wave train.

\textbf{(1) The PW archetype}

\(PW\) amplitude is \textbf{ZERO} everywhere except here...and here...and here...

\(PW\) amplitude...

...is mostly flat \textit{ZEROS}.
...but has sharp \textit{PEAKS}.
...is best defined by where it \textit{IS}.

Ideal \(PW\) shape is a \textit{Dirac Delta function}.

(Discussed on next page)

\textbf{(2) The CW archetype}

\(CW\) amplitude is \textbf{NON-zero} everywhere except here...and here...and here...and here...

\(CW\) amplitude...

...is mostly \textit{NON-zero} with rounded crests and troughs.
...but has sharp \textit{ZEROS}.
...is best defined by where it \textit{IS NOT}.

Ideal \(CW\) shape is a \textit{cosine wave} \((\cos(\phi))\)

(Discussed on next page)
...continuing to contrast two light wave archetypes:

**PW Pulse-Wave trains** versus **CW Continuous-Wave trains.**

1. **The PW archetype**

   *PW amplitude is **ZERO** everywhere except here...and here...and here...*

   Ideal *PW* shape is the **Dirac Delta function** $\delta(\phi)$...ininitely high at one point and zero elsewhere! (Definition: $\delta(\phi)=\infty$ if $\phi=0$ else $\delta(\phi)=0$ ) Also, its area is one! ( $\int d\phi \delta(\phi)=1$ ) This mathematical definition is not attainable in a laser lab.

   (An infinite pulse uses all the energy in the universe!)

   Real laser lab *PW* shape varies a lot...Gaussian?...sawtooth?...square?...etc.

2. **The CW archetype**

   *CW amplitude is **NON-zero** everywhere except here...and here...and here...and here...*

   Ideal *CW cosine wave* $(\cos(\phi))$ shape using right triangle geometry...is found in student-calculators,

   Real laser lab *CW* shape is very nearly a **cosine wave** and can be tuned precisely to any **frequency** (or **color** if it’s in **visible** spectrum)

   (Differing only in **frequency**, **amplitude**, and **phase**)

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**Diagram:**

- PW Pulse-Wave trains: A series of short, sharp peaks.
- CW Continuous-Wave trains: A smooth, continuous cosine wave.

**Additional Notes:**

- PW amplitude is zero everywhere except at specific points.
- CW amplitude is non-zero everywhere.
- Ideal PW shape is represented by the Dirac Delta function.
- Ideal CW shape is a cosine wave.
- Real PW and CW shapes can vary widely in the lab environment.
PW forms are also called Wave Packets (WP) since they are interfering sums of many CW terms (10-Cosine Waves make up this pulse).

CW terms are also called Color Waves or Fourier Spectral Components.

http://www.uark.edu/ua/pirelli/php/waves_pw_from_cw_anim.php
PW widths reduce proportionally with more CW terms (greater Spectral width)

**Space-time width** (pulse width)

\[ \Delta t = \tau \]

1 CW term
\[ \Delta \nu = \nu = 1/\tau \]

2 CW terms
\[ \Delta \nu = 2\nu \]

5 CW terms
\[ \Delta \nu = 5\nu \]

10 CW terms
\[ \Delta \nu = 10\nu \]

50 CW terms
\[ \Delta \nu = 50\nu \]

**Spectral width** (harmonic frequency range)

\[ \Delta \omega = 1\omega = \text{fundamental frequency} \]

\[ \Delta \omega = 2\omega \text{ (up to 2nd octave)} \]

\[ \Delta \omega = 5\omega \text{ (up to 5th)} \]

\[ \Delta \omega = 10\omega \text{ (up to 10th)} \]

\[ \Delta \omega = 50\omega \]

This dimension is **time**

This dimension is **frequency** or **per-time**

**Fourier-Heisenberg product:** \( \Delta t \cdot \Delta \nu = 1 \) (time-frequency uncertainty relation)

http://www.uark.edu/ua/pirelli/php/waves_pw_from_cw_width.php
How fast is light? Light goes one foot in a nano-second.

This may seem quite fast to us.

But, on a cosmic scale lightspeed is positively sub-glacial. In your lifetime light cannot cross one pixel (.) of the Hubble photo below.
Ways to recall (roughly) the speed \( c \) of light

**Light travels \(~\text{one foot in one nanosecond}\)...**

1 foot \(~1\) light-nanosecond

...or\(~\text{one billion feet in one second}\)...  

1 billion feet \(~1\) light-second

**Light travels \(~\frac{3}{10}\) of a meter in one nanosecond...**

\( \frac{3}{10} \) meter \(~1\) light-nanosecond

...or\(~3\) hundred million meters in one second...

300,000,000 meters \(~1\) light-second

Current Standard \( c \) Value  
K. M. Evenson - US NIST

\( c = 299,792,458 \) meters/second  
\( c = 186,282.397 \) miles/second

http://www.uark.edu/ua/pirelli/php/lightspeed_memnmonic.php
Think of light wave as pairs of steps:

Wave speed $c$ is \[
\text{Wave speed } c = \frac{\text{length per step}}{\text{time per step}}
\]

\[
c = \frac{\lambda}{\tau}
\]

length per step = wavelength $\lambda$

time per step = period $\tau$

length per step = wavelength $\lambda$

steps per time = Frequency $\nu = \frac{1}{\text{time per step}}$

Frequency $\nu = \frac{1}{\tau}$

http://www.uark.edu/ua/pirelli/php/wave_pair_steps.php

1 foot = 0.3048 meter

1/2 foot = 0.1524 meter

http://www.uark.edu/ua/pirelli/php/wave_pair_steps.php
Light speed using \textit{per-space} and \textit{per-time}

Wave speed $c$ is \[ c = \frac{\text{steps per second}}{\text{steps per meter}} \]

\[ c = \frac{\nu}{\kappa} \]

seconds per step = period $\tau$

\[ \nu = \frac{1}{\kappa} \text{ meters per step} \]

Wave-number $\kappa = \frac{1}{\lambda}$

\[ \nu = \frac{1}{\tau} \text{ seconds per step} \]

Frequency $\nu$

speed formulae:

\[ c = \frac{\nu}{\kappa} \quad c = \lambda \cdot \nu \quad c = \frac{\lambda}{\tau} \]

1 foot = 0.3048 meter

1/2 foot = 0.1524 meter
Continuous Wave in \textit{Spacetime} world defined by one vector in \textit{Per-Spacetime}

\[ c = \frac{x}{t} \]

\[ l = \frac{x}{ct} \]

\[ v = \frac{1}{cK} \]

\[ c = \frac{v}{K} \]
Color depends on light frequency $\nu$ ...

<table>
<thead>
<tr>
<th>Frequency $\nu = \omega / 2\pi$</th>
<th>1200 THz laser (vacuum UV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>750 THz (Waves per second)</td>
<td>blue</td>
</tr>
<tr>
<td>600 THz (Waves per second)</td>
<td>900 THz laser (near UV)</td>
</tr>
<tr>
<td>500 THz (Waves per second)</td>
<td>600 THz laser (green)</td>
</tr>
<tr>
<td>400 THz (Waves per second)</td>
<td>300 THz laser (near infrared)</td>
</tr>
</tbody>
</table>

http://www.uark.edu/ua/pirelli/php/color_freq.php
**Color depends on light frequency $\nu$ or wavelength $\lambda$...**

<table>
<thead>
<tr>
<th>Color</th>
<th>Frequency $\nu=\omega/2\pi$</th>
<th>Wavelength $\lambda=1/\kappa$</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>blue</strong></td>
<td>750 THz (Waves per second)</td>
<td>400 nm = 0.4 $\mu$m (meters per wave)</td>
<td>$\lambda=250$ nm $=250$ THz</td>
</tr>
<tr>
<td><strong>green</strong></td>
<td>600 THz (Waves per second)</td>
<td>500 nm = 0.5 $\mu$m (meters per wave)</td>
<td>$\lambda=200$ nm $=200$ THz</td>
</tr>
<tr>
<td><strong>orange</strong></td>
<td>500 THz (Waves per second)</td>
<td>600 nm = 0.6 $\mu$m (meters per wave)</td>
<td>$\lambda=100$ nm $=100$ THz</td>
</tr>
<tr>
<td><strong>red</strong></td>
<td>400 THz (Waves per second)</td>
<td>750 nm = 0.75 $\mu$m (meters per wave)</td>
<td>$\lambda=50$ nm $=50$ THz</td>
</tr>
</tbody>
</table>

**Very special mks frequency**

$547.72$ nm $=547.72$ THz
Color depends on light frequency $\nu$ or wavelength $\lambda$...

<table>
<thead>
<tr>
<th>Color</th>
<th>Wavespeed $c = \lambda \cdot \nu = \lambda / \tau = \nu / \kappa = \omega / k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue</td>
<td>Frequency $\nu = \omega / 2\pi$ 750 THz (Waves per meter)</td>
</tr>
<tr>
<td></td>
<td>Wavenumber $\kappa = k / 2\pi$ 2.5 Million/meter (Waves per meter)</td>
</tr>
<tr>
<td></td>
<td>Period $\tau = 1 / \nu$ 1/(750 T)sec. (Seconds per wave)</td>
</tr>
<tr>
<td></td>
<td>Wavelength $\lambda = 1 / \kappa$ 400 nm = 0.4 $\mu$m (meters per wave)</td>
</tr>
<tr>
<td>Green</td>
<td>Frequency $\nu = \omega / 2\pi$ 600 THz (Waves per second)</td>
</tr>
<tr>
<td></td>
<td>Wavenumber $\kappa = k / 2\pi$ 20,000/cm (Waves per centimeter)</td>
</tr>
<tr>
<td></td>
<td>Period $\tau = 1 / \nu$ 1/(600 T)sec. (Seconds per wave.)</td>
</tr>
<tr>
<td></td>
<td>Wavelength $\lambda = 1 / \kappa$ 500 nm = 0.5 $\mu$m (meters per wave)</td>
</tr>
<tr>
<td>Orange</td>
<td>Frequency $\nu = \omega / 2\pi$ 500 THz (Waves per second)</td>
</tr>
<tr>
<td></td>
<td>Wavenumber $\kappa = k / 2\pi$ 16,667/cm (Waves per centimeter)</td>
</tr>
<tr>
<td></td>
<td>Period $\tau = 1 / \nu$ 1/(500 T)sec. (Seconds per wave.)</td>
</tr>
<tr>
<td></td>
<td>Wavelength $\lambda = 1 / \kappa$ 600 nm = 0.6 $\mu$m (meters per wave)</td>
</tr>
<tr>
<td>Red</td>
<td>Frequency $\nu = \omega / 2\pi$ 400 THz (Waves per second)</td>
</tr>
<tr>
<td></td>
<td>Wavenumber $\kappa = k / 2\pi$ 13,333/cm (Waves per meter)</td>
</tr>
<tr>
<td></td>
<td>Period $\tau = 1 / \nu$ 1/(400 T)sec. (Seconds per wave.)</td>
</tr>
<tr>
<td></td>
<td>Wavelength $\lambda = 1 / \kappa$ 750 nm = 0.75 $\mu$m (meters per wave)</td>
</tr>
</tbody>
</table>

...but light speed $c = 299,792,458$ m/s is independent of Color

$547.72$ nm = 547.72 THz
1931 CIE “Slide Rule” for Human Perception

Frequency vs. Wavelength vs. perceived color
2. Applying Occam’s razor to relativity axioms

Einstein PW Axioms versus Evenson CW Axioms  (Traditional: The "Roadrunner" Axiom)

CW light clearly shows Doppler shifts
Check that red is red is red,...green is green is green,...blue is blue is blue,... etc.
Is dispersion linear? ... does astronomy work?... how about spectroscopy?
Is Doppler a geometric factor or arithmetic sum?

Introducing rapidity $\rho = \ln b$.

That old Time-Reversal meta-Axiom (that is so-oo-o neglected!)
Tradition: Start with the "Roadrunner" Axiom!

Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is $c$

A “road-runner” axiom is a “show-stopper”

PW peaks precisely locate places where wave is.
<table>
<thead>
<tr>
<th>#</th>
<th>Release date</th>
<th>Title</th>
<th>Duration</th>
<th>Credits</th>
<th>Pseudo-Latin names given</th>
<th>Acme Corporation devices used</th>
<th>Books Studied</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1949-9-17</td>
<td>Fast and Furry-ous</td>
<td>6:55</td>
<td>Michael Maltese</td>
<td>Acceleratii incredibus</td>
<td>ACME Super Outfit</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1952-5-24</td>
<td>Beep, Beep</td>
<td>6:45</td>
<td>Michael Maltese</td>
<td>Acceleratii incredibilis</td>
<td>Asprin, Matches, Rocket-Powered Roller Skates</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1952-8-23</td>
<td>Going! Going! Gosh!</td>
<td>6:25</td>
<td>Michael Maltese</td>
<td>Acceleratii incredibilis</td>
<td>an anvil, a weather balloon, a street cleaner's bin, and a fan</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1953-9-19</td>
<td>Zipping Along</td>
<td>6:55</td>
<td>Michael Maltese</td>
<td>Velocitius tremenjus</td>
<td>Giant Kite Kit, Bomb, Detonator, Nitroglycerin</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1954-8-14</td>
<td>Stop! Look! And Hasten!</td>
<td>7:00</td>
<td>Michael Maltese</td>
<td>Hot-roddicus supersonicus</td>
<td>Bird Seed, Triple Strength Fortified Leg Muscle Vitamins</td>
<td>ristol</td>
</tr>
<tr>
<td>6</td>
<td>1955-4-30</td>
<td>Ready, Set, Zoom!</td>
<td>6:55</td>
<td>Michael Maltese</td>
<td>Speedipus Rex</td>
<td>Glue</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1955-12-10</td>
<td>Guided Muscle</td>
<td>6:40</td>
<td>Michael Maltese</td>
<td>Velocitius detectibilus</td>
<td>Eabius almost anythings</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1956-5-5</td>
<td>Gee Whiz-z-z-z-z-z-</td>
<td>6:35</td>
<td>Michael Maltese</td>
<td>Delicius-delicius</td>
<td>Eabius birdius</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>1956-11-10</td>
<td>There They Go-Go-Go!</td>
<td>6:35</td>
<td>Michael Maltese</td>
<td>Dig-outius tid-bittius</td>
<td>ACME Triple Strength Battleship Steel Armor Plate, Rubber Band, Jet Bike</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1957-1-26</td>
<td>Scrambled Aches</td>
<td>6:50</td>
<td>Michael Maltese</td>
<td>Tastus supersonicus</td>
<td>ACME Dehydrated Boulders, Outboard Steam Roller</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>1957-9-14</td>
<td>Zoom and Bored</td>
<td>6:15</td>
<td>Michael Maltese</td>
<td>Birdibus zippibus</td>
<td>ACME Bumblebees</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>1958-4-12</td>
<td>Whoa, Be-Gone!</td>
<td>6:10</td>
<td>Michael Maltese</td>
<td>Birdius high-ballius</td>
<td>Tornado Seeds</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>1958-12-6</td>
<td>Hip Hip-Hurry!</td>
<td>6:13</td>
<td>Michael Maltese</td>
<td>digoutius-unbelievaelius</td>
<td>eabius-slobblius</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>1959-10-10</td>
<td>Wild About Hurry</td>
<td>6:45</td>
<td>Michael Maltese</td>
<td>Batoutahelius</td>
<td>Giant Elastic Rubber Band, 5 Miles of Railroad Track, Rocket Sled, Bird Seed, Iron Pellets, Indestructo Steel Ball</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>1960-1-9</td>
<td>Fastest with the Mostest</td>
<td>7:20</td>
<td>None</td>
<td>Velocitius incalcubili</td>
<td>Carnivorous slobblius</td>
<td></td>
</tr>
</tbody>
</table>
| No. | Year | Episode Title | Voice Actor(s) | Voice Actor(s) | Running Time | Villains
|-----|------|----------------|---------------|---------------|--------------|--------|
| 18  | 1960-10-8 | Hopalong Casualty | Chuck Jones | Chuck Jones | 6:05 | speedipus-ravens
| 19  | 1961-1-21 | Zip 'N Snort | Chuck Jones | Chuck Jones | 5:50 | digoutius-hot-rodis
| 20  | 1961-6-3 | Lickety-Splat | Chuck Jones, Abe Levitow | Chuck Jones | 6:20 | Fastius tasty-us
| 21  | 1961-11-11 | Beep Prepared | John Dunn, Chuck Jones | Chuck Jones, Maurice Noble | 6:00 | Tid-bittius velocitus
| 22  | 1962-6-2 | Adventures of the Road Runner | John Dunn, Chuck Jones, Michael Maltese | Chuck Jones | 26:00 | Super-Sonicus Idioticus
| 23  | 1963-12-28 | To Beep or Not to Beep | John Dunn, Chuck Jones | Chuck Jones, Maurice Noble | 6:35 | None
| 24  | 1964-6-6 | War and Pieces | John Dunn | Chuck Jones, Maurice Noble | 6:40 | Burn-em upus asphaltus
| 25  | 1965-1-1 | Zip Zip Hooray? | John Dunn | Chuck Jones | 6:15 | Super-Sonicus Idioticus
| 26  | 1965-2-1 | Road Runner a Go-Go | John Dunn | Chuck Jones | 6:05 | None
| 27  | 1965-2-27 | The Wild Chase | None | Friz Freleng, Hawley Pratt | 6:30 | None
| 28  | 1965-7-31 | Rushing Roulette | David Detliege | Robert McKimson | 6:20 | None
| 29  | 1965-8-21 | Run, Run, Sweet Road Runner | Rudy Larriva | Rudy Larriva | 6:00 | None
| 30  | 1965-9-18 | Tired and Feathered | Rudy Larriva | Rudy Larriva | 6:20 | None
| 31  | 1965-10-9 | Boulder Wham! | Len Janson | Rudy Larriva | 6:30 | None
| 32  | 1965-10-30 | Just Plane Beep | Don Jurwich | Rudy Larriva | 6:45 | None
| 33  | 1965-11-13 | Hairied and Hurried | Nick Bennion | Rudy Larriva | 6:45 | None
| 34  | 1965-12-11 | Highway Runnery | Al Bertino | Rudy Larriva | 6:45 | None

<table>
<thead>
<tr>
<th>#</th>
<th>Date</th>
<th>Title</th>
<th>Time</th>
<th>Director</th>
<th>Voice Actor</th>
<th>Writer</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>1965-12-25</td>
<td>Chaser on the Rocks</td>
<td>6:45</td>
<td>Tom Dagenais</td>
<td>Rudy Larriva</td>
<td>None</td>
<td>None</td>
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<tr>
<td>36</td>
<td>1966-1-8</td>
<td>Shot and Bothered</td>
<td>6:30</td>
<td>Nick Bennion</td>
<td>Rudy Larriva</td>
<td>None</td>
<td>Suction Cups</td>
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<tr>
<td>37</td>
<td>1966-1-29</td>
<td>Out and Out Rout</td>
<td>6:00</td>
<td>Dale Hale</td>
<td>Rudy Larriva</td>
<td>None</td>
<td>No ACME labeled devices used.</td>
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<tr>
<td>38</td>
<td>1966-2-19</td>
<td>The Solid Tin Coyote</td>
<td>6:15</td>
<td>Don Jurwich</td>
<td>Rudy Larriva</td>
<td>None</td>
<td>None</td>
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<tr>
<td>39</td>
<td>1966-3-12</td>
<td>Clippety Clobbered</td>
<td>6:15</td>
<td>Tom Dagenais</td>
<td>Rudy Larriva</td>
<td>None</td>
<td>None</td>
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<tr>
<td>40</td>
<td>1966-11-5</td>
<td>Sugar and Spies</td>
<td>6:20</td>
<td>Tom Dagenais</td>
<td>Robert McKimson</td>
<td>None</td>
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<tr>
<td>41</td>
<td>1979-11-27</td>
<td>Freeze Frame</td>
<td>6:05</td>
<td>John W. Dunn, Chuck Jones</td>
<td>Chuck Jones</td>
<td>Semper food-ellus</td>
<td>Grotesques appetitus</td>
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<td>42</td>
<td>1980-5-21</td>
<td>Soup or Sonic</td>
<td>9:10</td>
<td>Chuck Jones</td>
<td>Chuck Jones, Phil Monroe</td>
<td>Ultra-sonicus ad infinitum</td>
<td>Nemesis ridiculii</td>
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<td>43</td>
<td>1994-12-21</td>
<td>Chariots of Fur³</td>
<td>7:00</td>
<td>Chuck Jones</td>
<td>Chuck Jones</td>
<td>Boulevardius-burnupius</td>
<td>Dogius ignarami</td>
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<td>44</td>
<td>2000-12-30</td>
<td>Little Go Beep</td>
<td>7:55</td>
<td>Kathleen Helpie-Shipley, Earl Kress</td>
<td>Spike Brandt</td>
<td>Morselus babyfatius tastius</td>
<td>Poor schnookius</td>
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<td>45</td>
<td>2003-11-1</td>
<td>The Whizzard of Ow</td>
<td>7:00</td>
<td>Chris Kelly</td>
<td>Bret Haaland</td>
<td>Geococcyx californianus⁴</td>
<td>Canis latrans⁴</td>
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<td>Film</td>
<td>2003-11-14</td>
<td>Looney Tunes: Back in Action</td>
<td>91:00</td>
<td>Larry Doyle</td>
<td>Joe Dante</td>
<td>None</td>
<td>Desertus operatus idioticus</td>
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<td>46</td>
<td>2010-7-30</td>
<td>Coyote Falls³</td>
<td>2:59</td>
<td>Tom Sheppard</td>
<td>Matthew O'Callaghan</td>
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<td>47</td>
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<td>3:03</td>
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<td>Matthew O'Callaghan</td>
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<tr>
<td>48</td>
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<td>Rabid Rider³</td>
<td>3:07</td>
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<td>Matthew O'Callaghan</td>
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<td>49</td>
<td>TBA</td>
<td>Untitled Wile E. Coyote and Road Runner Short Film</td>
<td>5:38</td>
<td>Tom Sheppard</td>
<td>Matthew O'Callaghan</td>
<td>None</td>
<td>None</td>
</tr>
</tbody>
</table>

Thursday, January 23, 2014
2. Applying Occam’s razor to relativity axioms

Einstein PW Axioms versus Evenson CW Axioms  (Traditional: The "Roadrunner" Axiom)

CW light clearly shows Doppler shifts

Check that red is red is red,...green is green is green,...blue is blue is blue,... etc.

Is dispersion linear?... does astronomy work?... how about spectroscopy?

Is Doppler a geometric factor or arithmetic sum?

Introducing rapidity $\rho = \ln b$. 

That old Time-Reversal meta-Axiom (that is so-oo-o neglected!)
Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is $c$

A "road-runner" axiom is a "show-stopper"

Using Occam’s Razor
(and Evenson’s lasers)

PW peaks precisely locate places where wave is.
Continuous wave (CW) train
$A_1 \cos \omega t + A_2 \cos 2\omega t + A_3 \cos 3\omega t + A_4 \cos 4\omega t + ...$

PW zeros precisely locate places where wave is not.

Evenson Continuous Wave (CW) axiom: CW speed for all colors is $c$

More self-evident “must-be” axiom

It’s going $c$.
It looks blue!

It’s going -c.
It looks red!

It’s going $c$.
It looks green.
(OF course)

600 THz (green)
Laser source
Sees Doppler blue shift
It’s going -c.
It looks green.
(OF course)

Sees Doppler red shift
It’s going c.
It looks red!

Sees Doppler blue shift
It’s going c.
It looks green.
(OF course)

$\phi$

$c = 299,792,458 \text{ m/s}$

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- **CW light clearly shows Doppler shifts**
- Check that *red is red is red,...green is green is green,...blue is blue is blue,... etc.*
- Is dispersion linear? ... does astronomy work?... how about spectroscopy?
- Is Doppler a geometric factor or arithmetic sum?
  - Introducing **rapidity** \( \rho = \ln b \).

*That old Time-Reversal meta-Axiom (that is so-oo-o neglected!)*
Doppler Blueshift
More “hits” per sec. if moving toward laser source

Doppler Redshift
Fewer “hits” per sec. if moving away from laser source

Doppler Shifts in Spacetime
$x$ versus $ct$

Period $\tau = 2\pi/\omega = 1/\nu$
Wavelength $\lambda = 2\pi/k = 1/\nu$

600THz laser (green)

Time $ct$

Space $x$

Doppler’s picture needs revision for light whose period and wavelength both shift.

Why?

So that all colors go the same speed!

$\nu \cdot \lambda = \frac{\lambda}{\tau} = \frac{\omega}{k} = c$

$\nu \cdot \lambda = \frac{\lambda}{\tau} = \frac{\omega}{k} = c$

etc.

Related subject matter at:
http://www.uark.edu/ua/pirelli/php/doppler SEGUE.php
**CW Axiom ("All colors go c.") based on Doppler effects**

Showing that Green is Green is Green...(and all the same speed)...

Any color (like 600THz green) may be made by any other color source Doppler shifted by some speed u (less than c)

How many ways can you make 600THz green?

Frequency \( \nu = 600THz \) matches receiver.

Lower frequency \( \nu \) source approaches so its Doppler factor \((1 < b < \infty)\) blue-shifts \( \nu \) to match a 600THz-tuned receiver.

Higher frequency \( \nu \) source recedes so its Doppler factor \((0 < r < 1)\) red-shifts \( \nu \) to match a 600THz-tuned receiver.

How many kinds of green exist? (It’s either 1 or \( \infty \)).

Related subject matter at:

http://www.uark.edu/ua/pirelli/php/doppler_cw_logic_3.php
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_Introducing rapidity \( \rho = \ln b \)._

_That old Time-Reversal meta-Axiom (that is so-oo-o neglected!)_
Evenson CW Axiom ("All colors go c.") is only reasonable conclusion:

**Linear dispersion**: $\omega = ck$

Linear dispersion means *NO* dispersion

Einstein PW is corollary of Evenson CW

- $\omega = ck$
- $\nu = c/\lambda$

Vacuum can’t support an $\infty$-number of "other speeds"
Evenson CW Axiom ("All colors go c.") is only reasonable conclusion:

**Linear dispersion:** $\omega = ck$

Linear dispersion means **NO** dispersion

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![Graph showing frequency vs. wavenumber](image)

- $\omega = ck$
- $\nu = c/\lambda$

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Linear dispersion means **NO dispersion**

Einstein PW is corollary of Evenson CW

What if **blue** were to travel 0.001% slower than **red** from a galaxy 9 billion light years away? (..and show up $10^5$ years late)

That would mean Good-Bye Hubble Astronomy!
What if $\nu = 600\text{THz}$ green excited an Ar atom but NOT a $\lambda = 0.500\mu\text{m}$ optical cavity? (or vice-versa?)

That would mean Good-Bye Light Amplification by Stimulated Emission of Radiation.
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![Graph showing frequency vs. wavenumber](image)

What if blue were to travel 0.001% slower than red from a galaxy 9 billion light years away? (..and show up $10^5$ years late)

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What if $\nu=600$THz green excited an Ar atom but NOT a $\lambda=0.500\mu$m optical cavity? (or vice-versa?)

That would mean Good-Bye Light Amplification by Stimulated Emission of Radiation.
Linear Dispersion (means NO dispersion) has all colors (Fourier components) march in “lockstep”

\[ \omega = (c_{\text{const.}}) \cdot k \]

NON-linear Dispersion (has dispersion) so different colors (Fourier components) go different speeds

\[ \omega = \omega(k) \]

See animation: www.uark.edu/ua/pirelli/php/train_PW_Occum_Evenson.php
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Is Doppler a geometric factor or arithmetic sum?
Introducing rapidity $\rho = \ln b$.

That old Time-Reversal meta-Axiom (that is so-oo-o neglected!)
If all colors always march in lock-step then any Doppler shift must be geometric factor, that is, the same multiplier for all colors.

*If 300THz Doppler shifts to 600THz (1 octave-shift = 2.0)*

Then 600THz Doppler shifts to 1200THz (1 octave-shift = 2.0)
If all colors always march in lock-step then any Doppler shift must be geometric factor, that is, the same multiplier for all colors.

If 300THz Doppler shifts to 600THz (1 octave-shift = 2.0)

Then 600THz Doppler shifts to 1200THz (1 octave-shift = 2.0)
If all colors always march in lock-step then any Doppler shift must be geometric factor, that is, the same multiplier for all colors.

If $300\text{THz}$ Doppler shifts to $600\text{THz}$ (1 octave-shift = 2.0)

Then $600\text{THz}$ Doppler shifts to $1200\text{THz}$ (1 octave-shift = 2.0)

Doppler shifts maintain frequency ratios (not differences)

1-D Doppler shifts $\{\text{red}=e^{-\rho} \ldots \text{blue}=e^{+\rho}\}$ form a Lie Group

3-D Doppler shifts are hypercomplex elements of Lorentz Group
2. Applying Occam’s razor to relativity axioms

*Einstein PW Axioms versus Evenson CW Axioms* (Traditional: The "Roadrunner" Axiom)

*CW light clearly shows Doppler shifts*

*Check that red is red is red, green is green is green, blue is blue is blue, etc.*

*Is dispersion linear? ... does astronomy work? ... how about spectroscopy?*

*Is Doppler a geometric factor or arithmetic sum?*

*Introducing rapidity* $\rho = \ln b$.

*That old Time-Reversal meta-Axiom (that is so-oo-o neglected!)*
Frequency blue shift \( b \) when Source-Receiver interval is >>CLOSING<<

\[
\frac{v_{IN}}{v_{OUT}} = \frac{v_{Receiver}}{v_{Source}} = b = e^{+|\rho|} > 1
\]

Defining Rapidity \( \rho \) as logarithm of Doppler

\[\rho = \ln(b \text{ or } r)\]

Frequency red shift \( r \) when Source-Receiver interval is <<OPENING>>

\[
\frac{v_{Receiver}}{v_{Source}} = r = e^{-|\rho|} < 1
\]
Source $\nu_A = 300\text{THz}$

$\nu_C = b_{CA} \nu_A$

$\nu_B = b_{BA} \nu_A$

$b = 2$
**Frequency blue shift** $b$ when Source-Receiver interval is **CLOSING**:

$$\frac{v_{IN}}{v_{OUT}} = \frac{v_{\text{Receiver}}}{v_{\text{Source}}} = b = e^{+|\rho|} > 1$$

**Defining Rapidity** $\rho$ as logarithm of Doppler:

$$\rho = \ln(b \text{ or } r)$$

**Frequency red shift** $r$ when Source-Receiver interval is **OPENING**:

$$\frac{v_{\text{Receiver}}}{v_{\text{Source}}} = r = e^{-|\rho|} < 1$$

Source $v_A = 300 \text{THz}$
\[ \nu_C = b_{CA} \quad \nu_A = b_{CB} \quad \nu_B \]

Source \( \nu_A = 300 \text{THz} \)

\( b = 2 \)
**Frequency blue shift b when Source-Receiver interval is >>CLOSING<<**

\[ \frac{v_{IN}}{v_{OUT}} = \frac{v_{Receiver}}{v_{Source}} = b = e^{+|\rho|} > 1 \]

Defining **Rapidity** \( \rho \) as logarithm of Doppler

\( \rho = \ln(b \text{ or } r) \)

**Frequency red shift r when Source-Receiver interval is <<OPENING>>**

\[ \frac{v_{Receiver}}{v_{Source}} = r = e^{-|\rho|} < 1 \]

**Source** \( v_A = 300 \text{THz} \)

- **Carla**
  - \( v_C = b_{CA} \)
  - \( v_A = b_{CB} \)
  - \( v_B = b_{BA} \)

- **Bob**
  - \( v_B = b_{BA} \)
  - \( v_A \)

- **Alice**
  - \( v_A \)
  - \( b = 2 \)
Source $\nu_A = 300\text{THz}$

\[ \nu_C = b_{CA} \nu_A = b_{CB} \nu_B = b_{CB} b_{BA} \nu_A \]

Thursday, January 23, 2014
**Frequency blue shift** $b$ when Source-Receiver interval is >>CLOSING<<

\[ \frac{v_{IN}}{v_{OUT}} = \frac{v_{Receiver}}{v_{Source}} = b = e^{+|\rho|} \]

Defining **Rapidity** $\rho$ as logarithm of Doppler

\[ \rho = \ln(b \text{ or } r) \]

**Frequency red shift** $r$ when Source-Receiver interval is <<OPENING>>

\[ \frac{v_{Receiver}}{v_{Source}} = r = e^{-|\rho|} < 1 \]

Source $v_A = 300$THz

Carla

Receiver C

$\nu_C = b \, v_{CA} \quad v_A = b \, v_{CB} \quad v_B$

$= b \, v_{CB} \, b \, v_{BA} \, v_A$

Bob

Receiver B

$\nu_B = b \, v_{BA} \, v_A$

$= b \, v_{BA} \, v_A$

Alice

Source $v_A = 300$THz
2 times 2 = 4 Doppler arithmetic

Source $\nu_A = 300THz$

$\nu_C = b_{CA} \nu_A = b_{CB} \nu_B = b_{CB} b_{BA} \nu_A$

This implies:

$b_{CA} = b_{CB} b_{BA}$

$b$-Product rule
Frequency blue shift $b$ when Source-Receiver interval is >>CLOSING<<

$$\frac{v_{IN}}{v_{OUT}} = \frac{v_{\text{Receiver}}}{v_{\text{Source}}} = b = e^{+|\rho|} > 1$$

Defining Rapidity $\rho$ as logarithm of Doppler

$$\rho = \ln(b \text{ or } r)$$

Frequency red shift $r$ when Source-Receiver interval is <<OPENING>>

$$\frac{v_{\text{Receiver}}}{v_{\text{Source}}} = r = e^{-|\rho|} < 1$$

2 times 2 = 4 Doppler arithmetic

Source $v_A = 300THz$

Carla

$\nu_C = b_{CA}$ $\nu_A = b_{CB}$ $\nu_B$

$= b_{CB} b_{BA}$ $\nu_A$

Bob

$\nu_B = b_{BA}$ $\nu_A$

Alice

$\nu_C = b_{CA}$ $\nu_A = b_{CB}$ $\nu_B$

This implies:

$b_{CA} = b_{CB} b_{BA}$

$b$-Product rule
**Frequency blue shift b when Source-Receiver interval is **

\[
\frac{v_{IN}}{v_{OUT}} = \frac{v_{Receiver}}{v_{Source}} = b = e^{+|\rho|} > 1
\]

**Defining Rapidity \( \rho \) as logarithm of Doppler**

\[\rho = \ln(b \text{ or } r)\]

**Examples:**

**Source** A ---**Receiver** B

- Receiver A
  - **IN**
  - **OUT**
  - \( b = 2 \)
  - \( \rho = \ln(2) = 0.69 \)

**Source** A ---**Receiver** B

- Receiver A
  - **IN**
  - **OUT**
  - \( r = \frac{1}{2} \)
  - \( \rho = \ln(\frac{1}{2}) = -0.69 \)

---

**Frequency red shift r when Source-Receiver interval is**

\[
\frac{v_{Receiver}}{v_{Source}} = r = e^{-|\rho|} < 1
\]

**Examples:**

**Source** A ---**Receiver** B

- **Source** A
  - **OUT**
  - **IN**
  - \( r = \frac{1}{2} \)
  - \( \rho = \ln(\frac{1}{2}) = -0.69 \)

---

**Time Reversal**

**Source** A ---**Receiver** B

- **Source** C
  - **IN**
  - **OUT**
  - \( r = \frac{1}{2} \)
  - \( \rho = \ln(\frac{1}{2}) = -0.69 \)
Each Doppler shift $\frac{v_A}{v_B}$ maps to a Lorentz transformation $T_{AB}$.

$\frac{v_A}{v_B} = b_{AB} = e^{\rho_{AB}} = 2$

$\rho_{AB} = \ln(2) = 0.69$

$\frac{v_B}{v_C} = b_{BC} = e^{\rho_{BC}} = 1/4$

$\rho_{BC} = \ln(1/4) = -1.38$

$\frac{v_C}{v_A} = b_{CA} = e^{\rho_{CA}} = 2$

$\rho_{CA} = \ln(2) = 0.69$
Each Doppler shift \( \frac{v_A}{v_B} \) maps to a Lorentz transformation \( T_{AB} \)

receiver \( A \)  \hspace{1cm} source \( B \)  \hspace{1cm} receiver \( B \)  \hspace{1cm} source \( C \)  \hspace{1cm} receiver \( C \)  \hspace{1cm} source \( A \)

\[ \frac{v_A}{v_B} = b_{AB} = e^{\rho_{AB}} = 2 \]
\[ \rho_{AB} = \ln(2) = 0.69 \]

\[ \frac{v_B}{v_C} = b_{BC} = e^{\rho_{BC}} = 1/4 \]
\[ \rho_{BC} = \ln(1/4) = -1.38 \]

\[ \frac{v_C}{v_A} = b_{CA} = e^{\rho_{CA}} = 2 \]
\[ \rho_{CA} = \ln(2) = 0.69 \]

Group product is represented:

\[ T_{AB} \cdot T_{BC} = T_{CA} \]

\[ \frac{v_A}{v_B} \frac{v_B}{v_C} = \frac{v_A}{v_C} = e^{\rho_{AB} + \rho_{BC}} \]

...and rapidity \( \rho_{AB} \) is a Galilean (arithmetic) parameter

To be shown: \( \rho_{AB} = \text{atanh}(u_{AB}/c) \) approaches \( u_{AB}/c \) for: \( \rho_{AB} \ll 1 \)
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Is Doppler a geometric factor or arithmetic sum?
Introducing rapidity \( \rho = \ln b \).

That old Time-Reversal meta-Axiom (that is so-oo-o neglected!)
Inverse to Lorentz transformation \( T_{AB} \) is \( T_{BA} \)

..just as the arithmetic inverse of \( \frac{\nu_A}{\nu_B} \) is \( \frac{\nu_B}{\nu_A} \)

..just as the arithmetic inverse... of \( e^{\rho_{AB}} \) is \( e^{\rho_{BA}} = e^{-\rho_{AB}} \)

..just as the arithmetic inverse... of \( \rho_{AB} \) is \( \rho_{BA} = -\rho_{AB} \)

Inverse to Lorentz transformation is just as the arithmetic inverse of

\[
\frac{T_{AB}}{\nu_A} \quad \text{is} \quad \frac{T_{BA}}{\nu_B}
\]

.. just as the arithmetic inverse... of \( e^{\rho_{AB}} \) is \( e^{\rho_{BA}} = e^{-\rho_{AB}} \)

.. just as the arithmetic inverse... of \( \rho_{AB} \) is \( \rho_{BA} = -\rho_{AB} \)

Detailed time reversal symmetry implies \( r=1/b \).

Approaching source (600THz green)

Velocity Flip

Receding receiver sees Doppler red-shift of 1200THz source to 600THz (600THz) = \( r(1200\text{THz}) \) with \( r=1/2 \)

3. **Spectral theory of Einstein-Lorentz relativity**

Applying **Doppler Shifts** to per-space-time \((ck, \omega)\) graph

CW Minkowski space-time coordinates \((x, ct)\) and PW grids

Relating **Doppler Shifts** \(b\) or \(r = 1/b\) to velocity \(u/c\) or rapidity \(\rho\)

Lorentz transformation

*Connection: Conventional approach to relativity and old-fashioned formulas*
Deriving Spacetime and per-spacetime coordinate geometry by:

1. Evenson CW axiom “All colors go c” keeps $K_A$ and $K_B$ on their baselines.
2. Time-Reversal axiom: $r = 1/b$
3. Half-Sum Phase $P = (R + L)/2$ and Half-Difference Group $G = (R - L)/2$

Laser Per-Spacetime

$\omega$ versus $ck$

$\omega_3$

$\omega_1$

$\omega_2$

$\omega_4$

1st baseline

3rd baseline

1200THz

900THz

600THz

300THz

750THz

600THz or 500nm

500THz

400THz

ATOM FRAME view of LASER WAVES
Fig. 5 in SR&QM

recall also:

p. 3-11 of Lect.1
Deriving Spacetime and per-spacetime coordinate geometry by:

1. Evenson CW axiom “All colors go c” keeps $K_A$ and $K_B$ on their baselines.
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3. Half-Sum Phase $P = (R + L)/2$ and Half-Difference Group $G = (R - L)/2$

**Laser Per-Spacetime**

- $\omega$ versus $ck$
- $\omega_1 = 1200$ THz
- $\omega_2 = 600$ THz
- $\omega_3 = 900$ THz

**Atom Per-Spacetime**

- $\omega'$ versus $ck'$
- $\omega'_1 = 2.0 \cdot (2) = 4$
- $\omega'_3 = 1/2 \cdot (2) = 1$

**New 1st baseline**

- $K_A = (+4, 4)$
- $K_B = (-1, 1)$

**3rd base distance**

- Halved
- Doubled

**1st base distance**
3. *Spectral theory of Einstein-Lorentz relativity*

Applying *Doppler Shifts* to per-space-time \((ck, \omega)\) graph

CW Minkowski space-time coordinates \((x, ct)\) and PW grids

Relating *Doppler Shifts* \(b\) or \(r = 1/b\) to velocity \(u/c\) or rapidity \(\rho\)

Lorentz transformation

*Connection: Conventional approach to relativity and old-fashioned formulas*
Deriving Spacetime and per-spacetime coordinate geometry by:

1. Evenson CW axiom "All colors go c" keeps $K_A$ and $K_B$ on their baselines.
2. Time-Reversal axiom: $r = \frac{1}{b}$
3. Half-Sum Phase $P = \frac{(R + L)}{2}$ and Half-Difference Group $G = \frac{(R - L)}{2}$
Deriving Spacetime and per-spacetime coordinate geometry by:

1. Evenson CW axiom "All colors go c" keeps $K_A$ and $K_B$ on their baselines.
2. Time-Reversal axiom: $r = 1/b$
3. Half-Sum Phase $P = (R + L)/2$ and Half-Difference Group $G = (R - L)/2$
Deriving Spacetime and per-spacetime coordinate geometry by

(1) Evenson CW axiom “All colors go c”, keeps K4 and K3 on their baselines.

(2) Time-Reverse axiom: r = 1/β

(3) Half-Sum Phase $F = (R+L)/2$ and Half-Difference Group $G = (R-L)/2$
Deriving Spacetime and per-spacetime coordinate geometry by:

1. Evenson CW axiom “All colors go c” keeps \( K_A \) and \( K_B \) on their baselines.
2. Time-Reversal axiom: \( r = l/b \)
3. Half-Sum Phase \( P = (R+L)/2 \) and Half-Difference Group \( G = (R-L)/2 \)
Deriving Spacetime and per-spacetime coordinate geometry by:

1. Evenso CW axiom "All colors go c" keeps \( K_A \) and \( K_R \) on their baselines.
2. Time-Reversal axiom: \( r = 1/b \)
3. Half-Sum Phase \( P = (R + L)/2 \) and Half-Difference Group \( G = (R - L)/2 \)

Phase-vector: \( P' = \begin{pmatrix} \omega'_{\text{phase}} \\ \chi'_{\text{phase}} \end{pmatrix} \)

in per-space-time \( (\omega, \chi) \)

\[
P' = \frac{R' + L'}{2}

= B

\[
B = \begin{pmatrix} e^\rho + e^{-\rho} \\ e^\rho - e^{-\rho} \end{pmatrix}

\]

\[
K_2 = R'

\]

\[
K_2 = L'

\]

\[
K_2 = (-1, -1, 0)

\]

\[
G = (+4, 4)

\]

\[
K_A \text{ base}

\]

\[
K_B \text{ base}

\]

\[
G \text{ base}

\]

\[
H \text{ base}

\]

\[
\text{Doublet}

\]

\[
\text{1st base distance}

\]

\[
\text{3rd base distance}

\]

\[
\text{Laser per-space}

\]

\[
\text{Atom per-space}

\]

\[
\omega' = 2\cdot(2) = 1

\]

Thursday, January 23, 2014
Deriving Spacetime and per-spacetime coordinate geometry by:

(1) **Evenson CW axiom** "All colors go c" keeps $K_A$ and $K_R$ of their baselines.

(2) **Time-Reversal axiom**: $r = 1/b$

(3) **Half-Sum Phase** $P = (R + L)/2$ and **Half-Difference Group** $G = (R - L)/2$

Phase-vector: $P' = \begin{pmatrix} \omega_{\text{phase}}' \\ c k'_{\text{phase}} \end{pmatrix}$

in per-space-time $(\omega, ck)$

$$K_2 = R'$$

$$K_2 = L'$$

$$G' = \frac{R' - L'}{2}$$

$$G' = \frac{e^\rho - e^{-\rho}}{2}$$

$$P' = \frac{R' + L'}{2}$$

$$P' = \frac{e^\rho + e^{-\rho}}{2}$$

in per-space-time $(\omega, ck)$
Deriving Spacetime and per-spacetime coordinate geometry by

1. Evenson CW axiom “All colors go c” keeps $K_A$ and $K_R$ of the baselines.

2. Time-Reversal axiom: $r = 1/b$

3. Half-Sum Phase $P = (R + L)/2$ and Half-Difference Group $G = (R - L)/2$

Phase-vector: $P' = \begin{pmatrix} \omega'_{\text{phase}} \\ cK'_{\text{phase}} \end{pmatrix}$

in per-space-time $(\omega, ck)$

Group-vector: $G' = \begin{pmatrix} \omega'_{\text{group}} \\ cK'_{\text{group}} \end{pmatrix}$

in per-space-time $(\omega, ck)$
Deriving Spacetime and per-spacetime coordinate geometry by:

1. **Evenson CW axiom** “All colors go c” keeps $K_A$ and $K_R$ on their baselines.
2. Time-Reversal axiom: $r = 1/b$
3. Half-Sum Phase $P = (R + L)/2$ and Half-Difference Group $G = (R - L)/2$

**Phase-vector:** $P' = \begin{pmatrix} \omega'_{\text{phase}} \\ ck'_{\text{phase}} \end{pmatrix}$

**Group-vector:** $G' = \begin{pmatrix} \omega'_{\text{group}} \\ ck'_{\text{group}} \end{pmatrix}$

in per-space-time $(\omega, ck)$
Deriving Spacetime and per-spacetime coordinate geometry by:

(1) 
Evenson CW axiom “All colors go c” keeps $K_A$ and $K_B$ on their baselines.

(2) Time-Reversal axiom: $r = 1/b$

(3) Half-Sum Phase $P = (R+L)/2$ and Half-Difference Group $G = (R-L)/2$

Phase-vector: $P' = \begin{pmatrix} \omega'_{\text{phase}} \\ ck'_{\text{phase}} \end{pmatrix}$

in per-space-time $(\omega, ck)$

Group-vector: $G' = \begin{pmatrix} \omega'_{\text{group}} \\ ck'_{\text{group}} \end{pmatrix}$

in per-space-time $(\omega, ck)$

Thursday, January 23, 2014
Deriving Spacetime and per-spacetime coordinate geometry by 

(1) Evenson CW axiom "All colors go c” keeps $K_A$ and $K_B$ in their baselines. 
(2) Time-Reversal axiom: $r = 1/b$ 
(3) Half-Sum Phase $P = (R+L)/2$ and Half-Difference Group $G = (R-L)/2$ 

"lab"($\omega$) axis 
"lab"(ct) axis 
"lab"(ck) axis 
"lab"(x) axis

$G_{in}(\omega,ck)$ 
$P_{in}(\omega,ck)$

This is per-space ($ck'$) axis in per-space-time or
This is space ($x'$) axis in space-time

This is per-time ($ct'$) axis in per-space-time or
This is time ($ct'$) axis in space-time

 Atom Per-Spacetime 

<table>
<thead>
<tr>
<th>$\omega$ versus $ck$</th>
<th>$1200$THz</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3$ - $900$THz</td>
<td></td>
</tr>
<tr>
<td>$2$ - $600$THz</td>
<td></td>
</tr>
<tr>
<td>$1$ - $300$THz</td>
<td></td>
</tr>
</tbody>
</table>

Laser Per-Spacetime 

<table>
<thead>
<tr>
<th>$\omega$ versus $ck'$</th>
<th>$1200$THz</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4$ - $1200$THz</td>
<td></td>
</tr>
<tr>
<td>$3$ - $900$THz</td>
<td></td>
</tr>
<tr>
<td>$2$ - $600$THz</td>
<td></td>
</tr>
<tr>
<td>$1$ - $300$THz</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 9 in SR&QM
recall also:
p. 3-11 of Lect.1
(c) Minkowski CW-grid

(d) Dispersion plot

Fig. 9 in SR&QM

recall also:
p. 3-11 of Lect.1
Einstein-Lorentz Transformation (ELT) of spacetime \((x, ct)\) coordinates...

\[
\begin{pmatrix}
  x'_{(any)} \\
  ct'_{(any)}
\end{pmatrix}_B =
\begin{pmatrix}
  \cosh \rho & \sinh \rho \\
  \sinh \rho & \cosh \rho
\end{pmatrix}
\begin{pmatrix}
  x_{(any)} \\
  ct_{(any)}
\end{pmatrix}_A

\Leftrightarrow
\begin{pmatrix}
  x_{(any)} \\
  ct_{(any)}
\end{pmatrix}_A =
\begin{pmatrix}
  \cosh \rho & -\sinh \rho \\
  -\sinh \rho & \cosh \rho
\end{pmatrix}
\begin{pmatrix}
  x'_{(any)} \\
  ct'_{(any)}
\end{pmatrix}_B
\]

...is based upon the same ELT of per-spacetime \((ck, \omega)\) coordinates...

\[
\begin{pmatrix}
  \omega'_{(any)} \\
  ck'_{(any)}
\end{pmatrix}_B =
\begin{pmatrix}
  \cosh \rho & \sinh \rho \\
  \sinh \rho & \cosh \rho
\end{pmatrix}
\begin{pmatrix}
  \omega_{(any)} \\
  ck_{(any)}
\end{pmatrix}_A
\]

where:
\[
\begin{pmatrix}
  \cosh \rho & \sinh \rho \\
  \sinh \rho & \cosh \rho
\end{pmatrix} =
\begin{pmatrix}
  \gamma & \beta \cdot \gamma \\
  \beta \cdot \gamma & \gamma
\end{pmatrix}
\]
3. Spectral theory of Einstein-Lorentz relativity

Applying Doppler Shifts to per-space-time \((ck, \omega)\) graph

CW Minkowski space-time coordinates \((x, ct)\) and PW grids

Relating Doppler Shifts \(b\) or \(r = 1/b\) to velocity \(u/c\) or rapidity \(\rho\)

Lorentz transformation

Connection: Conventional approach to relativity and old-fashioned formulas
Connection to conventional approach to relativity and old-fashioned formulas

Given phase and group wave formulas:

\[
\begin{align*}
\begin{pmatrix}
\omega'_{\text{phase}} \\
ck'_{\text{phase}}
\end{pmatrix}
&= \mathbf{P}' \equiv \frac{R' + L'}{2} = \omega_A \begin{pmatrix}
e^\rho + e^{-\rho} \\
2
\end{pmatrix} = \omega_A \begin{pmatrix}
cosh \rho \\
\sinh \rho
\end{pmatrix} = \omega_A \begin{pmatrix}
\frac{5}{4} \\
\frac{3}{4}
\end{pmatrix} \\
\begin{pmatrix}
\omega'_{\text{group}} \\
ck'_{\text{group}}
\end{pmatrix}
&= \mathbf{G}' \equiv \frac{R' + L'}{2} = \omega_A \begin{pmatrix}
e^\rho - e^{-\rho} \\
2
\end{pmatrix} = \omega_A \begin{pmatrix}
\sinh \rho \\
cosh \rho
\end{pmatrix} = \omega_A \begin{pmatrix}
\frac{3}{4} \\
\frac{5}{4}
\end{pmatrix}
\end{align*}
\]
Connection to conventional approach to relativity and old-fashioned formulas

Given phase and group wave formulas:

\[
\begin{align*}
\begin{pmatrix}
\omega'_{\text{phase}} \\
k'_{\text{phase}}
\end{pmatrix} &= \mathbf{P}' = \frac{\mathbf{R}'+\mathbf{L}'}{2} = \omega_A \begin{pmatrix}
\frac{e^\rho + e^{-\rho}}{2} \\
\frac{e^\rho - e^{-\rho}}{2}
\end{pmatrix} = \omega_A \begin{pmatrix}
\cosh \rho \\
\sinh \rho
\end{pmatrix} = \omega_A \begin{pmatrix}
\frac{5}{4} \\
\frac{3}{4}
\end{pmatrix} \\
\begin{pmatrix}
\omega'_{\text{group}} \\
k'_{\text{group}}
\end{pmatrix} &= \mathbf{G}' = \frac{\mathbf{R}'+\mathbf{L}'}{2} = \omega_A \begin{pmatrix}
\frac{e^\rho - e^{-\rho}}{2} \\
\frac{e^\rho + e^{-\rho}}{2}
\end{pmatrix} = \omega_A \begin{pmatrix}
\sinh \rho \\
\cosh \rho
\end{pmatrix} = \omega_A \begin{pmatrix}
\frac{3}{4} \\
\frac{5}{4}
\end{pmatrix}
\end{align*}
\]

Calculate phase velocity and group velocity of coordinate waves:

\[
\begin{align*}
\frac{V_{\text{phase}}}{c} &= \frac{\omega_{\text{phase}}}{ck_{\text{phase}}} = \frac{\cosh \rho}{\sinh \rho} = \frac{5}{3} = \coth \rho \\
\frac{V_{\text{group}}}{c} &= \frac{\omega_{\text{group}}}{ck_{\text{group}}} = \frac{\sinh \rho}{\cosh \rho} = \frac{3}{5} = \tanh \rho \\
\frac{V'_{\text{group}}}{c} &= \frac{u}{c} = \tanh \rho = \frac{e^\rho - e^{-\rho}}{e^\rho + e^{-\rho}} = \frac{b-b^{-1}}{b+b^{-1}} = \frac{b^2-1}{b^2+1} = \beta
\end{align*}
\]

old-fashioned relativity parameter \( \beta = \frac{u}{c} \)
Connection to conventional approach to relativity and old-fashioned formulas

Given phase and group wave formulas:

\[
\begin{pmatrix}
\omega'_{\text{phase}} \\
ck'_{\text{phase}}
\end{pmatrix} = \mathbf{p'} = \frac{R' + L'}{2} = \omega_A \begin{pmatrix}
\frac{e^\rho + e^{-\rho}}{2} \\
\frac{e^\rho - e^{-\rho}}{2}
\end{pmatrix} = \omega_A \begin{pmatrix}
\cosh \rho \\
\sinh \rho
\end{pmatrix} = \omega_A \begin{pmatrix}
\frac{5}{4} \\
\frac{3}{4}
\end{pmatrix}
\]

\[
\begin{pmatrix}
\omega'_{\text{group}} \\
ck'_{\text{group}}
\end{pmatrix} = \mathbf{g'} = \frac{R' + L'}{2} = \omega_A \begin{pmatrix}
\frac{e^\rho - e^{-\rho}}{2} \\
\frac{e^\rho + e^{-\rho}}{2}
\end{pmatrix} = \omega_A \begin{pmatrix}
\sinh \rho \\
\cosh \rho
\end{pmatrix} = \omega_A \begin{pmatrix}
\frac{3}{4} \\
\frac{5}{4}
\end{pmatrix}
\]

Calculate phase velocity and group velocity of coordinate waves:

\[
\frac{V_{\text{phase}}}{c} = \frac{\omega'_{\text{phase}}}{ck'_{\text{phase}}} = \frac{\cosh \rho}{\sinh \rho} = \frac{5}{3} = \coth \rho
\]

\[
\frac{V_{\text{group}}}{c} = \frac{\omega'_{\text{group}}}{ck'_{\text{group}}} = \frac{\sinh \rho}{\cosh \rho} = \frac{3}{5} = \tanh \rho
\]

\[
\frac{V'_{\text{group}}}{c} = \frac{u}{c} = \tanh \rho = \frac{e^\rho - e^{-\rho}}{e^\rho + e^{-\rho}} = \frac{b - b^{-1}}{b + b^{-1}} = \frac{b^2 - 1}{b^2 + 1} \equiv \beta
\]

\[
\beta = \frac{b^2 - 1}{b^2 + 1}
\]

Solve \(\frac{b^2 - 1}{b^2 + 1} = \beta\) for Doppler-blue factor:
Connection to conventional approach to relativity and old-fashioned formulas

Given phase and group wave formulas:

\[
\begin{align*}
\left( \frac{\omega_{\text{phase}}'}{c k_{\text{phase}}'} \right) &= \mathbf{P}' = \frac{\mathbf{R}' + \mathbf{L}'}{2} = \omega_A \left( \begin{array}{c} e^\rho + e^{-\rho} \\ e^\rho - e^{-\rho} \end{array} \right) \quad = \omega_A \left( \begin{array}{c} \cosh \rho \\ \sinh \rho \end{array} \right) = \omega_A \left( \begin{array}{c} \frac{5}{4} \\ \frac{3}{4} \end{array} \right) \\
\left( \frac{\omega_{\text{group}}'}{c k_{\text{group}}'} \right) &= \mathbf{G}' = \frac{\mathbf{R}' + \mathbf{L}'}{2} = \omega_A \left( \begin{array}{c} e^\rho - e^{-\rho} \\ e^\rho + e^{-\rho} \end{array} \right) \quad = \omega_A \left( \begin{array}{c} \sinh \rho \\ \cosh \rho \end{array} \right) = \omega_A \left( \begin{array}{c} \frac{3}{4} \\ \frac{5}{4} \end{array} \right)
\end{align*}
\]

Calculate phase velocity and group velocity of coordinate waves:

\[
\begin{align*}
\frac{V_{\text{phase}}}{c} &= \frac{\omega_{\text{phase}}'}{c k_{\text{phase}}'} = \frac{\cosh \rho}{\sinh \rho} = \frac{5}{3} = \coth \rho \\
\frac{V_{\text{group}}}{c} &= \frac{\omega_{\text{group}}'}{c k_{\text{group}}'} = \frac{\sinh \rho}{\cosh \rho} = \frac{3}{5} = \tanh \rho \\
\frac{V'}{c} &= \frac{u}{c} = \tanh \rho = \frac{e^\rho - e^{-\rho}}{e^\rho + e^{-\rho}} = \frac{b - b^{-1}}{b + b^{-1}} = \frac{b^2 - 1}{b^2 + 1} = \beta
\end{align*}
\]

Solve \( \frac{b^2 - 1}{b^2 + 1} = \beta \) for Doppler-blue factor:
Connection to conventional approach to relativity and old-fashioned formulas

Given phase and group wave formulas:

\[
\begin{pmatrix}
\omega'_{\text{phase}} \\
ck'_{\text{phase}}
\end{pmatrix} =
\begin{pmatrix}
P' = \frac{R' + L'}{2} = \omega_A \\
\frac{e^\rho + e^{-\rho}}{2} = \omega_A \frac{\cosh \rho}{\sinh \rho} = \omega_A \begin{pmatrix} 5 \\ 4 \\ 3 \\ 4 \end{pmatrix}
\end{pmatrix}
\]

\[
\begin{pmatrix}
\omega'_{\text{group}} \\
ck'_{\text{group}}
\end{pmatrix} =
\begin{pmatrix}
G' = \frac{R' + L'}{2} = \omega_A \\
\frac{e^\rho - e^{-\rho}}{2} = \omega_A \frac{\sinh \rho}{\cosh \rho} = \omega_A \begin{pmatrix} 3 \\ 4 \\ 5 \\ 4 \end{pmatrix}
\end{pmatrix}
\]

Calculate phase velocity and group velocity of coordinate waves:

\[
\frac{V_{\text{phase}}}{c} = \frac{\omega_{\text{phase}}}{ck_{\text{phase}}} = \frac{\cosh \rho}{\sinh \rho} = \frac{5}{3} = \coth \rho
\]

\[
b^2 - 1 = \beta b^2 + \beta
\]

\[
b^2 = \frac{1 + \beta}{1 - \beta}
\]

\[
b^2 - \beta b^2 = 1 + \beta
\]

Solve \( \frac{b^2 - 1}{b^2 + \beta} = \beta \) for Doppler-blue factor:

\[
\frac{V'_{\text{group}}}{c} = \frac{u}{c} = \tanh \rho = \frac{e^\rho - e^{-\rho}}{e^\rho + e^{-\rho}} = \frac{b - b^{-1}}{b + b^{-1}} = \frac{b^2 - 1}{b^2 + 1} = \beta
\]

old-fashioned relativity parameter \( \beta = u/c \)
Connection to conventional approach to relativity and old-fashioned formulas

Given phase and group wave formulas:

\[
\begin{pmatrix}
\omega'_{\text{phase}} \\
ck'_{\text{phase}}
\end{pmatrix}
= P' = \frac{R' + L'}{2} = \omega_A \begin{pmatrix}
\frac{e^\rho + e^{-\rho}}{2} \\
\frac{e^\rho - e^{-\rho}}{2}
\end{pmatrix} = \omega_A \begin{pmatrix}
\cosh \rho \\
\sinh \rho
\end{pmatrix} = \omega_A \begin{pmatrix}
\frac{5}{4} \\
\frac{3}{4}
\end{pmatrix}
\]

\[
\begin{pmatrix}
\omega'_{\text{group}} \\
ck'_{\text{group}}
\end{pmatrix}
= G' = \frac{R' + L'}{2} = \omega_A \begin{pmatrix}
\frac{e^\rho - e^{-\rho}}{2} \\
\frac{e^\rho + e^{-\rho}}{2}
\end{pmatrix} = \omega_A \begin{pmatrix}
\sinh \rho \\
\cosh \rho
\end{pmatrix} = \omega_A \begin{pmatrix}
\frac{3}{4} \\
\frac{5}{4}
\end{pmatrix}
\]

Calculate phase velocity and group velocity of coordinate waves:

\[
\begin{align*}
\frac{V_{\text{phase}}}{c} &= \frac{\omega_{\text{phase}}}{ck_{\text{phase}}} = \frac{\cosh \rho}{\sinh \rho} = \frac{5}{3} = \coth \rho \\
\frac{V_{\text{group}}}{c} &= \frac{\omega_{\text{group}}}{ck_{\text{group}}} = \frac{\sinh \rho}{\cosh \rho} = \frac{3}{5} = \tanh \rho
\end{align*}
\]

\[b^2 - 1 = \beta b^2 + \beta \]

\[b^2 = \frac{1 + \beta}{1 - \beta}\]

Solve \(b^2 - 1 = \beta\) for Doppler-blue factor:

\[b = \sqrt{\frac{1 + \beta}{1 - \beta}} = \sqrt{\frac{1 + u/c}{1 - u/c}} = \frac{1 + u/c}{\sqrt{1 - u^2/c^2}} = \frac{1 + \beta}{\lambda}
\]

\[\beta = u/c \]

old-fashioned relativity parameter

\[\lambda = \sqrt{1 - u^2/c^2}\]

old-fashioned Lorentz x-contraction parameter
Connection to conventional approach to relativity and old-fashioned formulas

Given phase and group wave formulas:

\[
\begin{pmatrix}
\omega'_{\text{phase}} \\
ck'_{\text{phase}}
\end{pmatrix} = \begin{pmatrix}
P' \\
2
\end{pmatrix} = \begin{pmatrix}
\frac{e^\rho + e^{-\rho}}{2} \\
\frac{e^\rho - e^{-\rho}}{2}
\end{pmatrix} = \begin{pmatrix}
\omega_A \\
\omega_A
\end{pmatrix} \begin{pmatrix}
\cosh \rho \\
\sinh \rho
\end{pmatrix} = \begin{pmatrix}
5 \\
4
\end{pmatrix}
\]

\[
\begin{pmatrix}
\omega'_{\text{group}} \\
ck'_{\text{group}}
\end{pmatrix} = \begin{pmatrix}
G' \\
2
\end{pmatrix} = \begin{pmatrix}
\frac{e^\rho - e^{-\rho}}{2} \\
\frac{e^\rho + e^{-\rho}}{2}
\end{pmatrix} = \begin{pmatrix}
\omega_A \\
\omega_A
\end{pmatrix} \begin{pmatrix}
\sinh \rho \\
\cosh \rho
\end{pmatrix} = \begin{pmatrix}
3 \\
4
\end{pmatrix}
\]

Calculate phase velocity and group velocity of coordinate waves:

\[
\frac{V_{\text{phase}}}{c} = \frac{\omega_{\text{phase}}}{c k_{\text{phase}}} = \frac{\cosh \rho}{\sinh \rho} = \frac{5}{3} = \coth \rho
\]

\[
\frac{V_{\text{group}}}{c} = \frac{\omega_{\text{group}}}{c k_{\text{group}}} = \frac{\sinh \rho}{\cosh \rho} = \frac{3}{5} = \tanh \rho
\]

\[
b^2 = \frac{1+\beta}{1-\beta}
\]

Solve \( \frac{b^2-1}{b^2+1}=\beta \) for Doppler-blue factor: \( b = \sqrt{\frac{1+\beta}{1-\beta}} = \sqrt{\frac{1+u/c}{1-u/c}} = \frac{1+u/c}{\sqrt{1-u^2/c^2}} = \frac{1+\beta}{\lambda} \)

Convert Lorentz parameter to hyper-function: \( \lambda = \sqrt{1-u^2/c^2} \)

\[
= \sqrt{1-\tanh^2 \rho} = \text{sech} \rho = \frac{1}{\cosh \rho} \equiv \frac{1}{\gamma}
\]

old-fashioned relativity parameter \( \beta = u/c \)

old-fashioned Lorentz x-contraction parameter \( \lambda = \sqrt{1-u^2/c^2} \)
Connection to conventional approach to relativity and old-fashioned formulas

Given phase and group wave formulas:

\[
\begin{pmatrix}
\omega_{\text{phase}}' \\
k'_{\text{phase}}
\end{pmatrix} =
\begin{pmatrix}
\omega' \\
k'
\end{pmatrix} =
\begin{pmatrix}
\frac{e^\rho + e^{-\rho}}{2} \\
\frac{e^\rho - e^{-\rho}}{2}
\end{pmatrix} =
\begin{pmatrix}
\cosh \rho \\
\sinh \rho
\end{pmatrix} =
\begin{pmatrix}
\frac{5}{4} \\
\frac{3}{4}
\end{pmatrix}
\]

\[
\begin{pmatrix}
\omega_{\text{group}}' \\
k'_{\text{group}}
\end{pmatrix} =
\begin{pmatrix}
\omega' \\
k'
\end{pmatrix} =
\begin{pmatrix}
\frac{e^\rho - e^{-\rho}}{2} \\
\frac{e^\rho + e^{-\rho}}{2}
\end{pmatrix} =
\begin{pmatrix}
\sinh \rho \\
\cosh \rho
\end{pmatrix} =
\begin{pmatrix}
\frac{3}{4} \\
\frac{5}{4}
\end{pmatrix}
\]

Calculate phase velocity and group velocity of coordinate waves:

\[
\frac{V_{\text{phase}}}{c} = \frac{\omega_{\text{phase}}'}{k'_{\text{phase}}} = \frac{\cosh \rho}{\sinh \rho} = \frac{5}{3} = \coth \rho
\]

\[
\frac{V_{\text{group}}}{c} = \frac{\omega_{\text{group}}'}{k'_{\text{group}}} = \frac{\sinh \rho}{\cosh \rho} = \frac{3}{5} = \tanh \rho
\]

\[
b^2 = \frac{1 + \beta}{1 - \beta}
\]

Solve \(b^2 = \beta\) for Doppler-blue factor:

\[
b = \frac{1 + \beta}{\sqrt{1 - \beta}} = \frac{1 + u / c}{\sqrt{1 - u^2 / c^2}} = \frac{1 + u / c}{\sqrt{1 - u^2 / c^2}} = \frac{1 + \beta}{\lambda}
\]

Convert Lorentz parameter to hyper-function:

\[
\lambda = \sqrt{1 - u^2 / c^2}
\]

\[
= \sqrt{1 - \tanh^2 \rho} = \text{sech} \rho = \frac{1}{\cosh \rho} = \frac{1}{\gamma}
\]

old-fashioned relativity parameter \(\beta = u / c\)

old-fashioned Lorentz x-contraction parameter \(\lambda = \sqrt{1 - u^2 / c^2}\)

old-fashioned Einstein t-dilation parameter \(\gamma = \frac{1}{\sqrt{1 - u^2 / c^2}}\)
Connection to conventional approach to relativity and old-fashioned formulas

Given phase and group wave formulas:

\[
\begin{align*}
\begin{pmatrix}
\omega'_{\text{phase}} \\
ck'_{\text{phase}}
\end{pmatrix} &= \mathbf{P}' = \frac{\mathbf{R}' + \mathbf{L}'}{2} = \omega_A \begin{pmatrix}
e^\rho + e^{-\rho} \\
e^\rho - e^{-\rho}
\end{pmatrix} = \omega_A \begin{pmatrix}
\cosh \rho \\
\sinh \rho
\end{pmatrix} = \omega_A \begin{pmatrix}
\frac{5}{4} \\
\frac{3}{4}
\end{pmatrix} \\
\begin{pmatrix}
\omega'_{\text{group}} \\
ck'_{\text{group}}
\end{pmatrix} &= \mathbf{G}' = \frac{\mathbf{R}' + \mathbf{L}'}{2} = \omega_A \begin{pmatrix}
e^\rho - e^{-\rho} \\
e^\rho + e^{-\rho}
\end{pmatrix} = \omega_A \begin{pmatrix}
\sinh \rho \\
cosh \rho
\end{pmatrix} = \omega_A \begin{pmatrix}
\frac{3}{4} \\
\frac{5}{4}
\end{pmatrix}
\end{align*}
\]

Calculate phase velocity and group velocity of coordinate waves:

\[
\begin{align*}
\frac{V_{\text{phase}}}{c} &= \frac{\omega_{\text{phase}}}{ck_{\text{phase}}} = \frac{\cosh \rho}{\sinh \rho} = \frac{5}{3} = \coth \rho \\
\frac{V_{\text{group}}}{c} &= \frac{\omega_{\text{group}}}{ck_{\text{group}}} = \frac{\sinh \rho}{cosh \rho} = \frac{3}{5} = \tanh \rho \\
b^2 &= \frac{1 + \beta}{1 - \beta}
\end{align*}
\]

Solve \(\frac{b^2 - 1}{b^2 + 1} = \beta\) for Doppler-blue factor:

\[
b = \sqrt{\frac{1 + \beta}{1 - \beta}} = \sqrt{\frac{1 + u/c}{1 - u/c}} = \frac{1 + u/c}{\sqrt{1 - u^2/c^2}} = 1 + \beta
\]

Convert Lorentz parameter to hyper-function:

\[
\lambda = \sqrt{1 - u^2/c^2} \\
= \sqrt{1 - \tanh^2 \rho} = \text{sech} \rho
\]

\[
\frac{1}{\lambda} = \gamma = \frac{1}{\sqrt{1 - u^2/c^2}} = \cosh \rho
\]
Connection to conventional approach to relativity and old-fashioned formulas

Given phase and group wave formulas:

\[
\begin{bmatrix}
\omega'_{\text{phase}} \\
ck'_{\text{phase}}
\end{bmatrix} = \mathbf{P}' = \frac{\mathbf{R}' + \mathbf{L}'}{2} = \omega_A \begin{bmatrix}
e^\rho + e^{-\rho} \\
e^\rho - e^{-\rho}
\end{bmatrix} = \omega_A \begin{bmatrix}
\cosh \rho \\
\sinh \rho
\end{bmatrix} = \omega_A \begin{bmatrix}
\frac{5}{4} \\
\frac{3}{4}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\omega'_{\text{group}} \\
ck'_{\text{group}}
\end{bmatrix} = \mathbf{G}' = \frac{\mathbf{R}' + \mathbf{L}'}{2} = \omega_A \begin{bmatrix}
e^\rho - e^{-\rho} \\
e^\rho + e^{-\rho}
\end{bmatrix} = \omega_A \begin{bmatrix}
\sinh \rho \\
\cosh \rho
\end{bmatrix} = \omega_A \begin{bmatrix}
\frac{3}{4} \\
\frac{5}{4}
\end{bmatrix}
\]

Calculate phase velocity and group velocity of coordinate waves:

\[
\frac{V_{\text{phase}}}{c} = \frac{\omega'_{\text{phase}}}{ck'_{\text{phase}}} = \frac{\cosh \rho}{\sinh \rho} = \frac{5}{3} = \coth \rho
\]

\[
V_{\text{phase}} = \frac{1 + \beta}{1 - \beta}
\]

Solve \( \frac{b^2 - 1}{b^2 + 1} = \beta \) for Doppler-blue factor:

\[
b = \sqrt{\frac{1 + \beta}{1 - \beta}} = \sqrt{\frac{1 + u/c}{1 - u/c}} = \frac{1 + u/c}{\sqrt{1 - u^2/c^2}} = \frac{1 + \beta}{\lambda}
\]

Convert Lorentz parameter to hyper-function:

\[
\lambda = \sqrt{1 - u^2/c^2} = \sech \rho
\]

Doppler-blue (again)

\[
b = e^\rho = \cosh \rho + \sinh \rho = \frac{1 + u/c}{\sqrt{1 - u^2/c^2}} = \frac{1 + \beta}{\lambda}
\]
Connection to conventional approach to relativity and old-fashioned formulas

Given phase and group wave formulas:

\[
\begin{pmatrix}
    \omega'_{\text{phase}} \\
    c k'_{\text{phase}}
\end{pmatrix} = \mathbf{P}' = \frac{\mathbf{R}'+\mathbf{L}'}{2} = \omega_A \begin{pmatrix}
    \frac{e^\rho + e^{-\rho}}{2} \\
    e^\rho - e^{-\rho}
\end{pmatrix} = \omega_A \begin{pmatrix}
    \cosh \rho \\
    \sinh \rho
\end{pmatrix} = \omega_A \begin{pmatrix}
    \frac{5}{4} \\
    \frac{3}{4}
\end{pmatrix}
\]

\[
\begin{pmatrix}
    \omega'_{\text{group}} \\
    c k'_{\text{group}}
\end{pmatrix} = \mathbf{G}' = \frac{\mathbf{R}'+\mathbf{L}'}{2} = \omega_A \begin{pmatrix}
    \frac{e^\rho - e^{-\rho}}{2} \\
    e^\rho + e^{-\rho}
\end{pmatrix} = \omega_A \begin{pmatrix}
    \sinh \rho \\
    \cosh \rho
\end{pmatrix} = \omega_A \begin{pmatrix}
    \frac{3}{4} \\
    \frac{5}{4}
\end{pmatrix}
\]

Calculate phase velocity and group velocity of coordinate waves:

\[
\frac{V_{\text{phase}}}{c} = \frac{\omega_{\text{phase}}}{ck_{\text{phase}}} = \frac{\cosh \rho}{\sinh \rho} = \frac{5}{3} = \coth \rho
\]

\[
b^2 = \frac{1+\beta}{1-\beta}
\]

Solve \(b^2 = \frac{1+\beta}{1-\beta}\) for Doppler-blue factor: \(b = \sqrt{\frac{1+\beta}{1-\beta}} = \sqrt{\frac{1+u/c}{1-u/c}} = \frac{1+u/c}{\sqrt{1-u^2/c^2}} \equiv \frac{1+\beta}{\lambda}
\]

Convert Lorentz parameter to hyper-function: \(\lambda = \frac{1}{\sqrt{1-u^2/c^2}} \equiv \frac{1}{\tanh^2 \rho} = \sech^2 \rho = \frac{1}{\cosh^2 \rho} \equiv \frac{1}{\gamma}
\]

Doppler-blue (again)

\[
b = e^\rho = \cosh \rho + \sinh \rho = \frac{1+u/c}{\sqrt{1-u^2/c^2}} \equiv \frac{1+\beta}{\lambda} = \frac{1}{\lambda} + \frac{\beta}{\lambda} = \frac{1}{\sqrt{1-u^2/c^2}} + \frac{u/c}{\sqrt{1-u^2/c^2}}
\]
Connection to conventional approach to relativity and old-fashioned formulas

Given phase and group wave formulas:

\[
\begin{align*}
\left( \omega'_{\text{phase}} \atop c k'_{\text{phase}} \right) &= P' = \frac{R' + L'}{2} = \omega_A \left( \frac{e^\rho + e^{-\rho}}{2} \right) = \omega_A \left( \frac{\cosh \rho}{\sinh \rho} \right) = \omega_A \left( \frac{3}{4} \right) \\
\left( \omega'_{\text{group}} \atop c k'_{\text{group}} \right) &= G' = \frac{R' + L'}{2} = \omega_A \left( \frac{e^\rho - e^{-\rho}}{2} \right) = \omega_A \left( \frac{\sinh \rho}{\cosh \rho} \right) = \omega_A \left( \frac{5}{4} \right)
\end{align*}
\]

Calculate phase velocity and group velocity of coordinate waves:

\[
\begin{align*}
\frac{V_{\text{phase}}}{c} &= \frac{\omega_{\text{phase}}}{c k_{\text{phase}}} = \frac{\cosh \rho}{\sinh \rho} = \frac{5}{3} = \coth \rho \\
\frac{V_{\text{group}}}{c} &= \frac{\omega_{\text{group}}}{c k_{\text{group}}} = \frac{\cosh \rho}{\sinh \rho} = \frac{3}{5} = \tanh \rho
\end{align*}
\]

Solve \( b^2 = \frac{1 + \beta}{1 - \beta} \) for Doppler-blue factor:

\[
b = \sqrt{\frac{1 + \beta}{1 - \beta}} = \sqrt{\frac{1 + u/c}{1 - u/c}} = \sqrt{\frac{1 - u^2/c^2}{\lambda}} = \frac{1 + \beta}{\lambda}
\]

Convert Lorentz parameter to hyper-function:

\[
\lambda = \sqrt{1 - u^2/c^2} = \frac{1 + \beta}{\lambda}
\]

Doppler-blue (again)

\[
b = e^\rho = \cosh \rho + \sinh \rho = \frac{1 + u/c}{\sqrt{1 - u^2/c^2}} \equiv \frac{1 + \beta}{\lambda}
\]

and old-fashioned asimultaneity coeff.:

\[
\frac{u/c}{\sqrt{1 - u^2/c^2}} = \sinh \rho
\]

\[
\frac{1}{\lambda} = \gamma = \frac{1}{\sqrt{1 - u^2/c^2}} = \cosh \rho
\]

old-fashioned Lorentz x-contraction parameter

\[
\lambda = \sqrt{1 - u^2/c^2} = \text{sech} \rho
\]

old-fashioned Einstein t-dilation parameter

\[
\frac{1}{\lambda} = \gamma = \frac{1}{\sqrt{1 - u^2/c^2}} = \cosh \rho
\]
Euclidian Geometry for Per-spacetime Relativity

Key Definition of Rapidity $\rho$

Doppler blue shift:

$$Bb = Be^+\rho$$

Doppler red shift:

$$Br = Be^-\rho$$

Key Results:

$$\omega \quad vs. \quad ck$$

"winks" vs. "kinks"

$$\omega = B \cosh \rho$$

$$ck = B \sinh \rho$$

Group velocity:

$$\frac{\omega}{ck} = \frac{u}{c} = \tanh \rho$$

Phase velocity:

$$\frac{ck}{c} = \coth \rho$$

$$\omega \quad u$$

$$B\sinh \rho = \frac{(Be^+\rho - Be^-\rho)}{2}$$

$$B\cosh \rho = \frac{(Be^+\rho + Be^-\rho)}{2}$$

Related material at "per space-per-time" setting of:

http://www.uark.edu/ua/modphys/testing/markup/RelaWavityWeb.html

Connection to conventional approach to relativity and old-fashioned formulas

Lorentz-Einstein factors

$$\sinh \rho = \sqrt{1 - \frac{u^2}{c^2}}$$

$$\cosh \rho = \sqrt{1 - \frac{u^2}{c^2}}$$
Euclidian wave geometry with time-reversal symmetry imply

Lab frame area...

equals

Atom frame area...

by time-reversal axiom: \( r = \frac{1}{b} \)

...that implies hyperbolic invariants

\( r \cdot b = 1 \)
Euclidian wave geometry with time-reversal symmetry imply
dispersion hyperbolas: $\omega = nB \cosh \rho$

$B \sinh \rho = (Be^+ - Be^-)/2$

$B \cosh \rho = (Be^+ + Be^-)/2$
Group velocity $u$ and phase velocity $c^2/u$ are hyperbolic tangent slopes.
Group velocity \( u \) and phase velocity \( c^2/u \) are hyperbolic tangent slopes

Rare but important case where

\[
\frac{d\omega}{dk} = \frac{\Delta \omega}{\Delta k}
\]

with LARGE \( \Delta k \)

(not infinitesimal)

Relativistic
group wave

speed \( u = c \tanh \rho \)
Group velocity $u$ and phase velocity $c^2/u$ are hyperbolic tangent slopes

Rare but important case where

$$\frac{d\omega}{dk} = \frac{\Delta \omega}{\Delta k}$$

with LARGE $\Delta k$

(not infinitesimal)

Relativistic
group wave

speed $u = c \tanh \rho$
Group velocity \( u \) and phase velocity \( c^2/u \) are hyperbolic tangent slopes

Rare but important case where

\[
\frac{d\omega}{dk} = \frac{\Delta\omega}{\Delta k}
\]

with LARGE \( \Delta k \)
(not infinitesimal)

Relativistic group wave speed \( u = c \tanh \rho \)
Group velocity \( u \) and phase velocity \( c^2/u \) are hyperbolic tangent slopes

Rare but important case where

\[
\frac{d\omega}{dk} = \frac{\Delta\omega}{\Delta k}
\]

with LARGE \( \Delta k \)

(not infinitesimal)

Relativistic

group wave

speed \( u = c \tanh \rho \)
Group velocity $u$ and phase velocity $c^2/u$ are hyperbolic tangent slopes

Rare but important case where

$$\frac{d\omega}{dk} = \frac{\Delta\omega}{\Delta k}$$

with LARGE $\Delta k$

(not infinitesimal)

Relativistic group wave speed $u = c \tanh \rho$

Newtonian speed $u \sim c \rho$

Low speed approximation

Rapidity $\rho$ approaches $u/c$
Connection to conventional approach to relativity and old-fashioned formulas

The most old-fashioned form(ula) of all: Thales & Euclid means
Connection to conventional approach to relativity and old-fashioned formulas

The most old-fashioned formula of all: Thales & Euclid means

Euclid’s 3-means (300 BC)
Geometric “heart” of wave mechanics

Thales (580 BC) rectangle-in-circle
Relates to wave interference by (Galilean) phasor angular velocity addition

![Diagram showing geometric means and related formulas](http://www.uark.edu/ua/pirelli/html/default.html)

http://www.uark.edu/ua/pirelli/php/phasors_2_3_zoom_anim.php

http://www.uark.edu/ua/pirelli/html/default.html

Linear velocity $V_{group}/c=u/c$ is $(\text{HALF-DIFF.}/\text{HALF-SUM})=3/5$
The detailed trigonometry of half-sum & difference angles is shown below. The wave is factored into a product of group and phase waves.

**Main Result:** Factoring algebraic sums helps to locate wave zeros.

\[
\begin{align*}
\cos \alpha + \cos \beta &= 2 \cos \frac{\alpha - \beta}{2} \cdot [\cos(\alpha + \beta)/2] \\
\sin \alpha + \sin \beta &= 2 \cos \frac{\alpha - \beta}{2} \cdot [\sin (\alpha + \beta)/2]
\end{align*}
\]

Sum is zeroed by *either* factor. Each factor’s zero line is a spacetime coordinate line.
Connection to conventional approach to relativity and old-fashioned formulas
The most old-fashioned form(ula) of all: Thales & Euclid means

The detailed trigonometry of half-sum & difference angles is shown below. The wave is factored into a product of group and phase waves.

Main Result: Factoring algebraic sums helps to locate wave zeros.
\[
\cos \alpha + \cos \beta = 2 \cos (\alpha - \beta) / 2 \quad \cdot \quad [\cos(\alpha+\beta)/2]
\]
\[
\sin \alpha + \sin \beta = 2 \cos (\alpha - \beta) / 2 \quad \cdot \quad [\sin (\alpha+\beta)/2]
\]

Sum is zeroed by either factor. Each factor’s zero line is a spacetime coordinate line.
LaserPer-Spacetime
$\omega$ versus $ck$

AtomPer-Spacetime
$\omega'$ versus $ck'$

atom speed -u

atom speed 0

Laser per-space
ck

Atom per-space
ck'

3rd baseline

1rd baseline

1 - 300THz

2 - 600THz

3 - 900THz

1 - 1200THz

750THz or 400nm

600THz or 500nm

500THz or 600nm

400THz or 750nm