AMOP Lectures 10
Tue. 3.12 . 2014

Relativity of 1st Quantization and electromagnetic fields

(Ch. 2-5 of CMwBang-Unit 8   Ch. 6 of QTforCA Unit 2)

1st Quantization: Quantizing phase variables ω and k
- *Understanding how quantum transitions require "mixed-up" states*
  - *Closed cavity vs ring cavity*

2nd Quantization: Quantizing amplitudes ("photons","vibrons", and "what-ever-ons")
- *Analogy with molecular Born-Oppenheimer-Approximate energy levels*
- *Introducing coherent states (What lasers use)*
  - *Analogy with (ω,k) wave packets*
  - *Wave coordinates need coherence*

Relativistic effects on charge, current, and magnetic fields
- *Current density changes by Lorentz asynchrony*
  - *Magnetic B-field is relativistic* $\sinh\rho$ *1st order-effect*

1st Quantization: Quantizing phase variables ω and k

Understanding how quantum transitions require "mixed-up" states

Closed cavity vs ring cavity

Quantized ω and *k* *Counting wave kink numbers*

If everything is made of waves then we expect *quantization* of everything because waves only thrive if *integral* numbers *n* of their "kinks" fit into whatever structure (box, ring, etc.) they're supposed to live. The numbers *n* are called *quantum numbers*.

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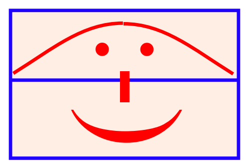
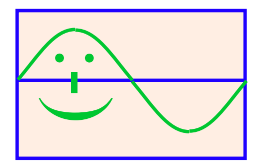
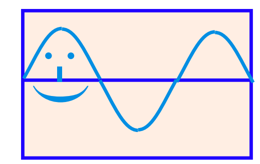
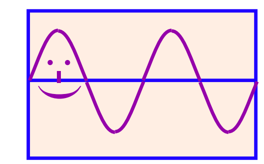
(+ integers only)

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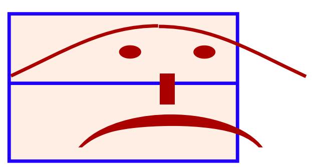

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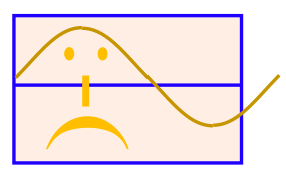

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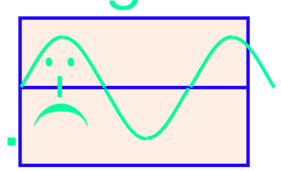
misfits...



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Quantized ω and *k* *Counting wave kink numbers*

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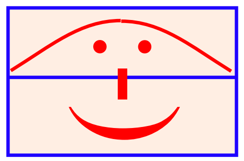

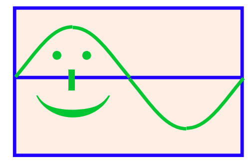

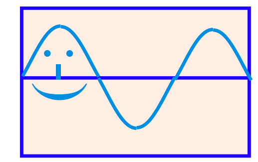

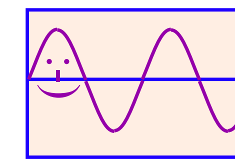

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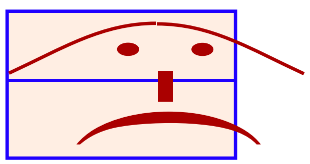

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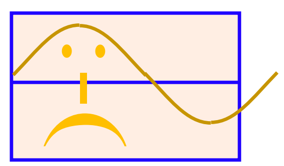

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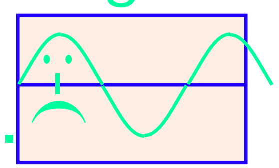

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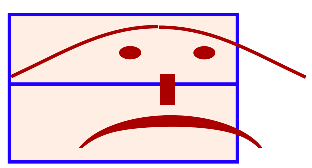

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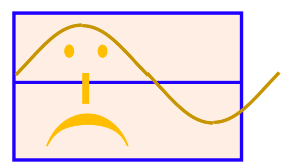

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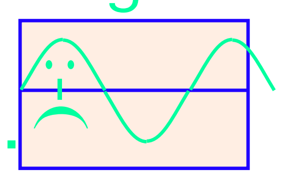

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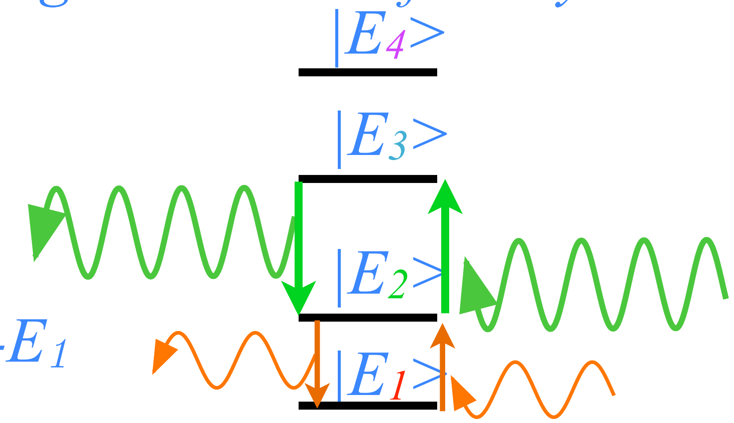
That's the only way you get any light in or out of the system to "see" it.



frequency $\hbar\omega_{32} = E_3 - E_2$

frequency $\hbar\omega_{21} = E_2 - E_1$

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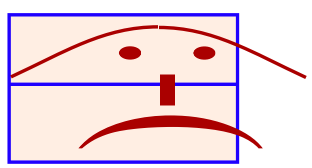

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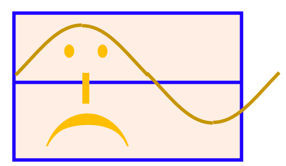

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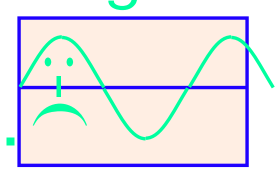

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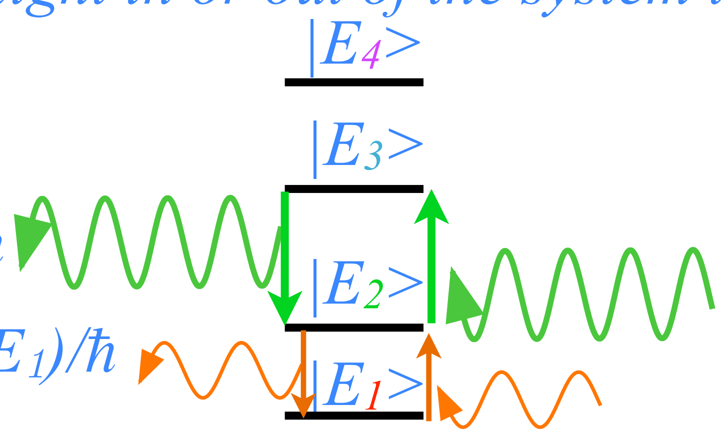
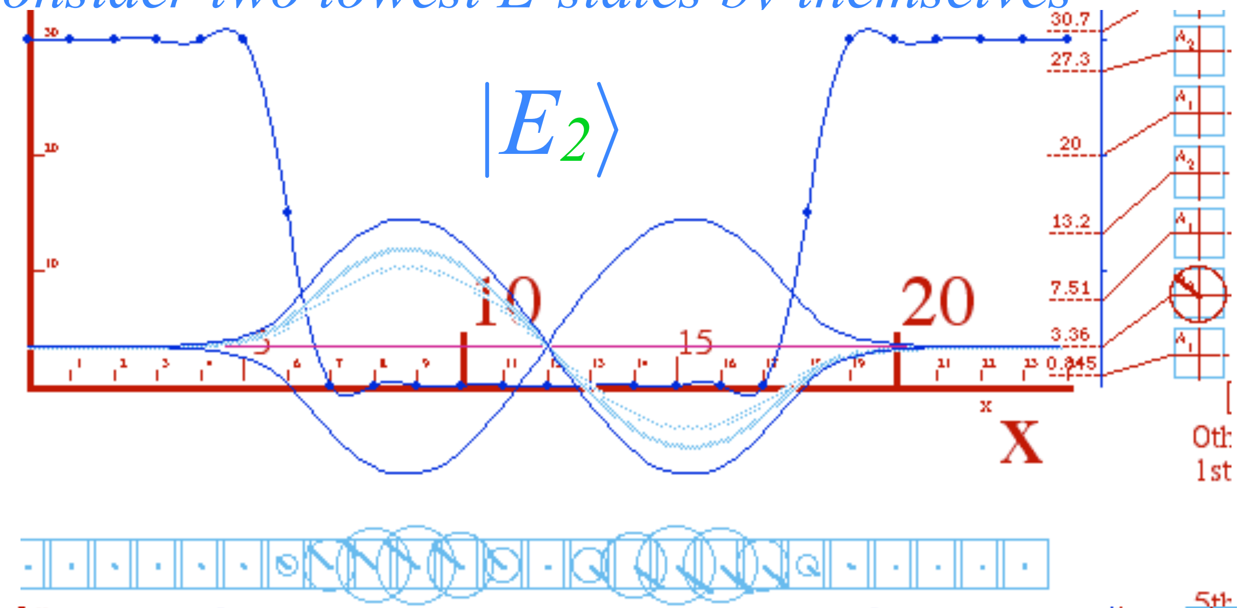
frequency $\omega_{32} = (E_3 - E_2)/\hbar$

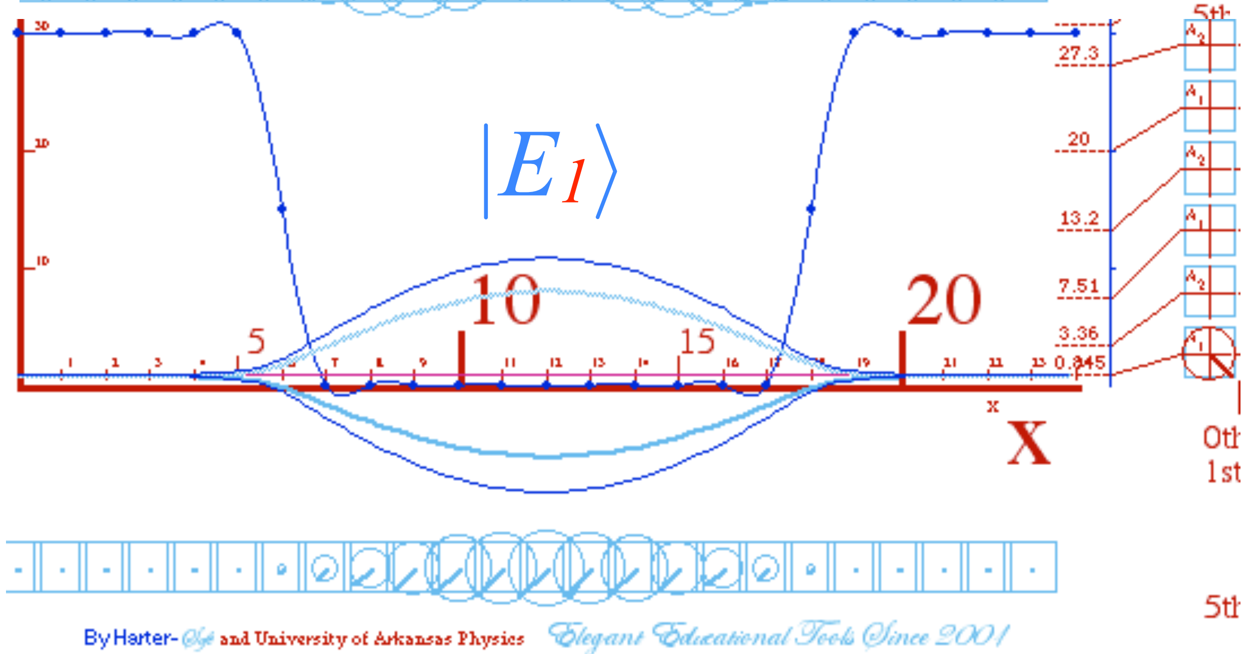
frequency $\omega_{21} = (E_2 - E_1)/\hbar$



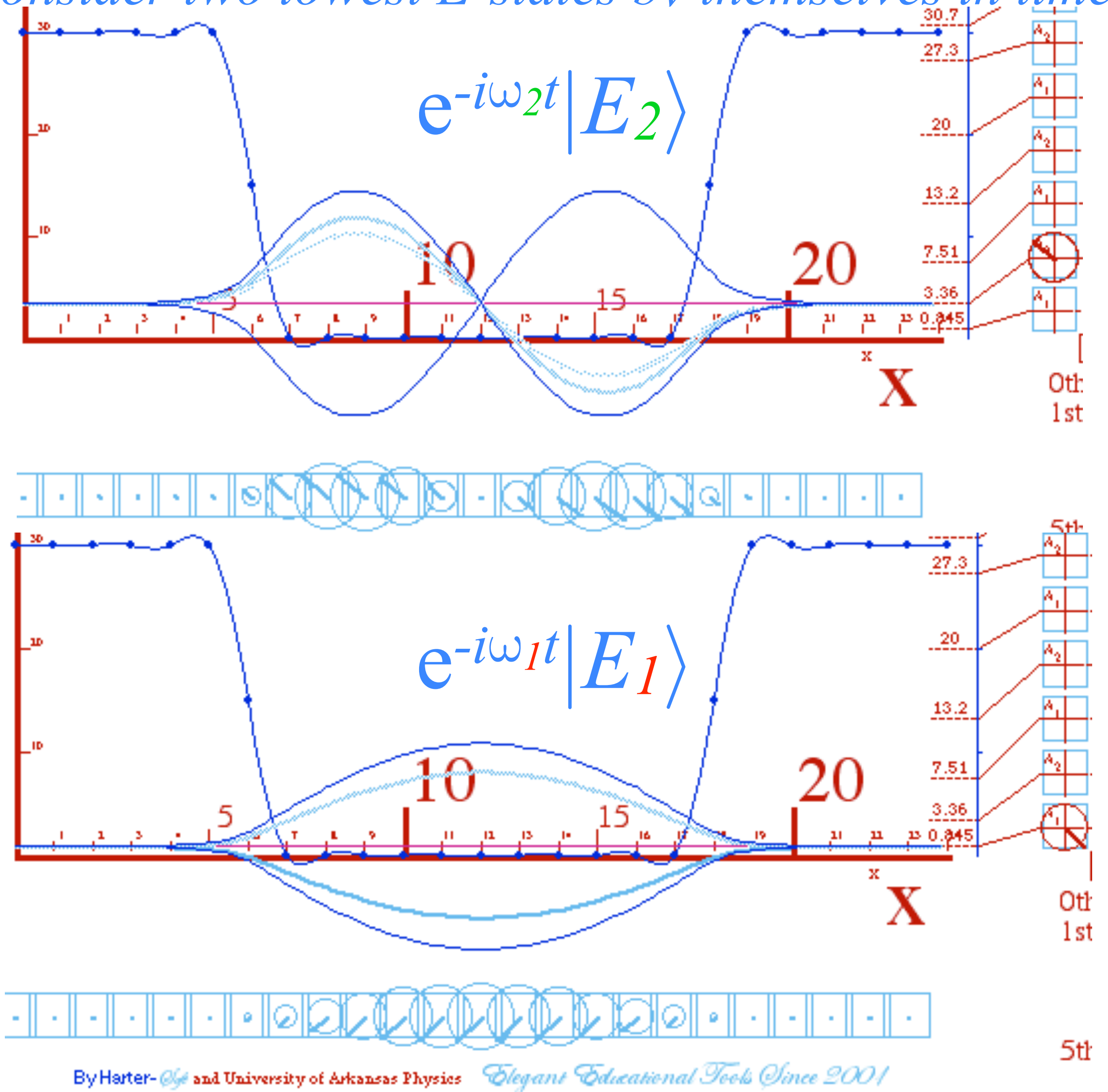
These eigenstates are the only ways the system can "play dead"... ... "sleep with the fishes"...

Consider two lowest E-states by themselves



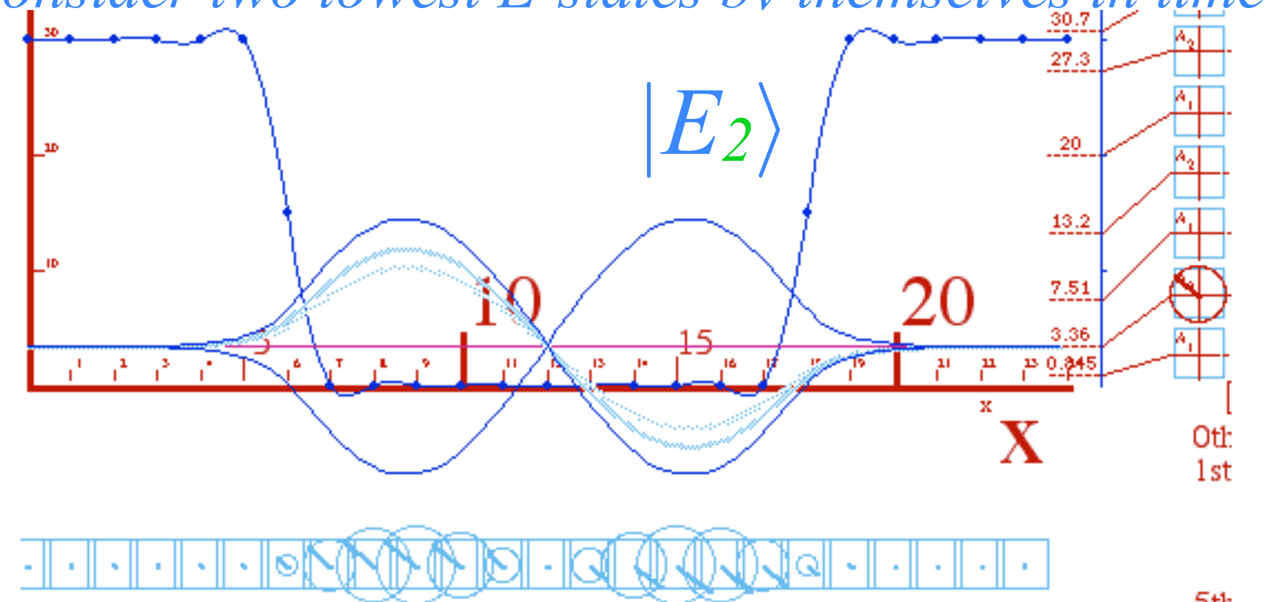


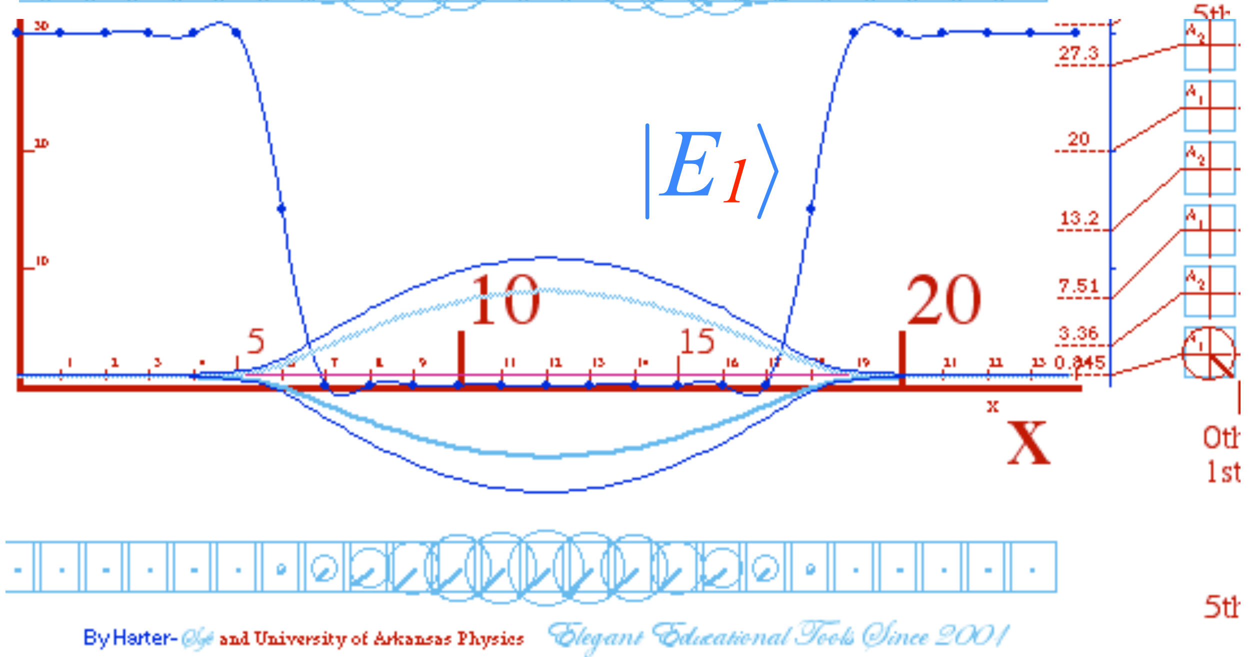
Consider two lowest E-states by themselves in time

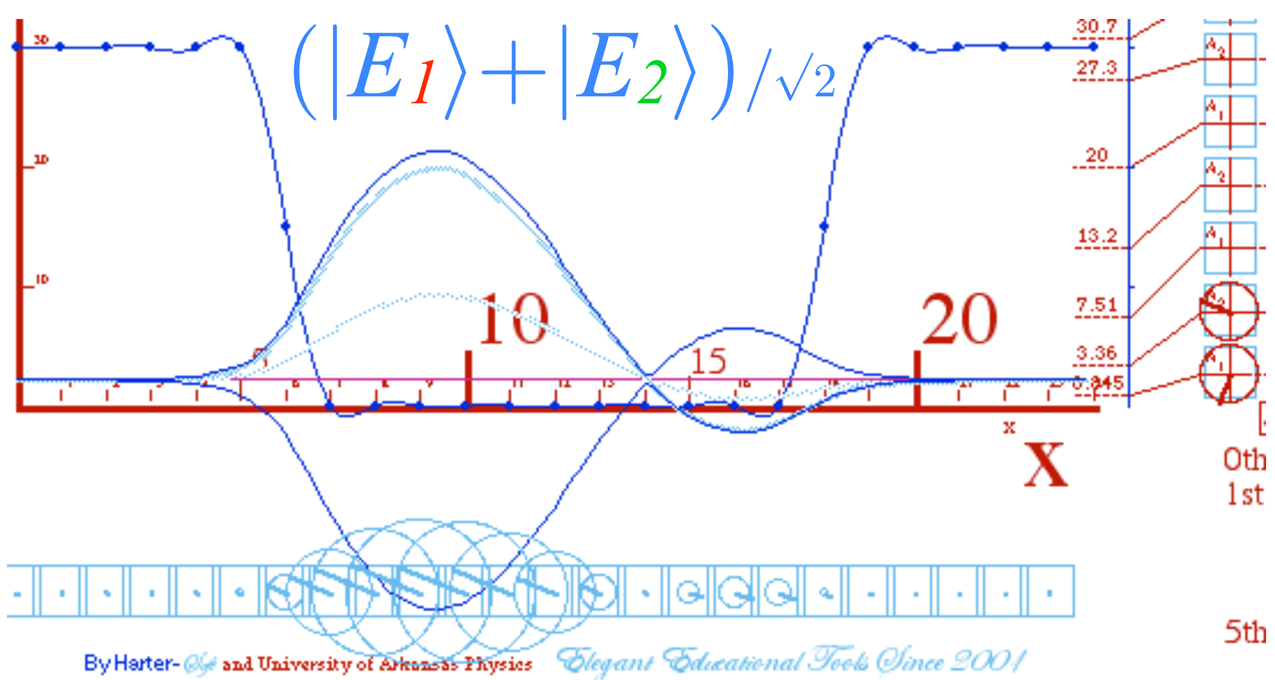


Consider two lowest E-states by themselves in time

Now combine (add) them







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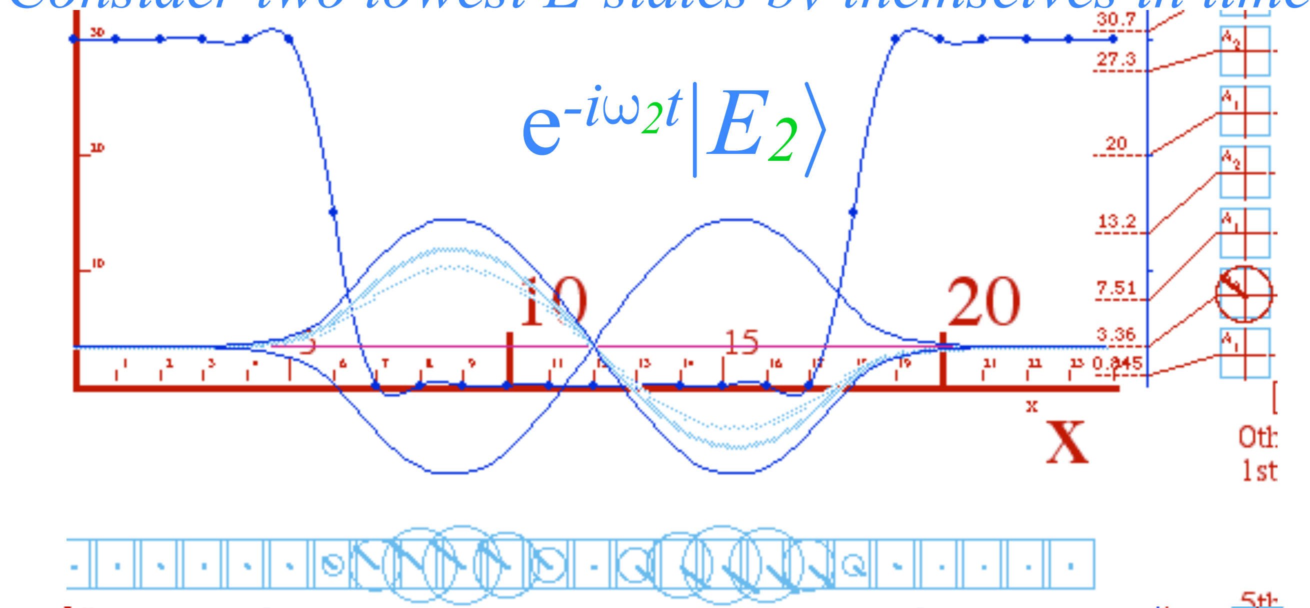
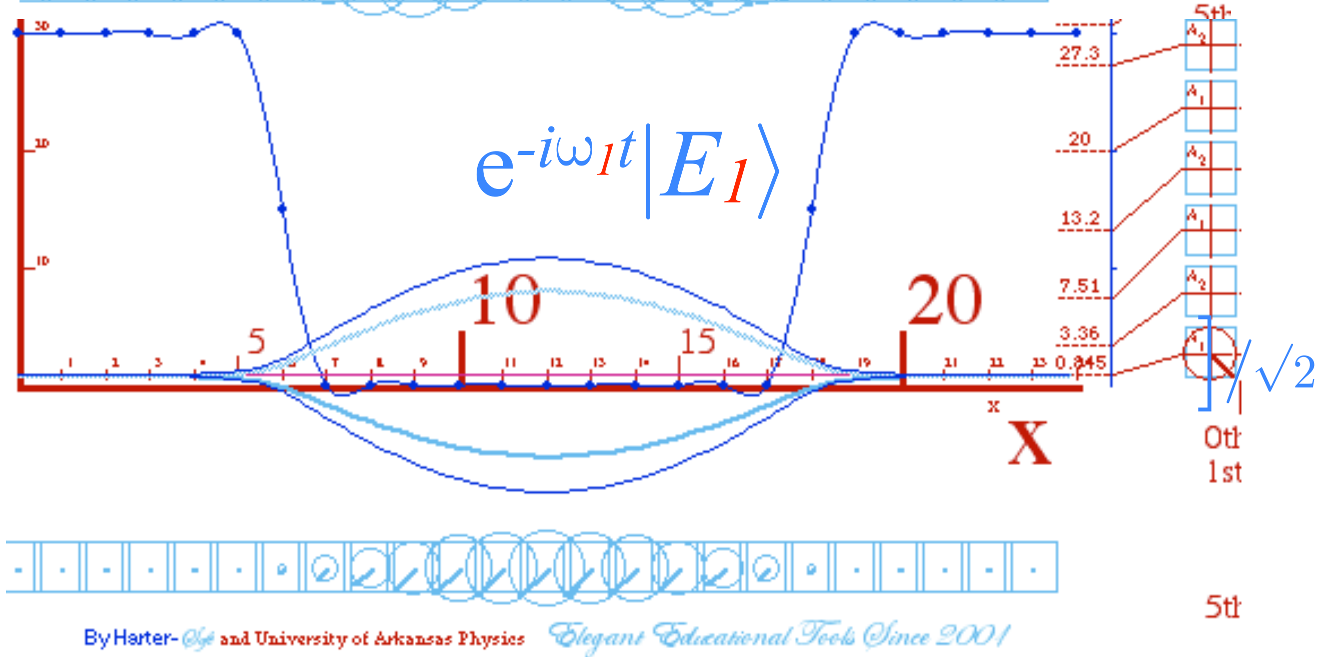
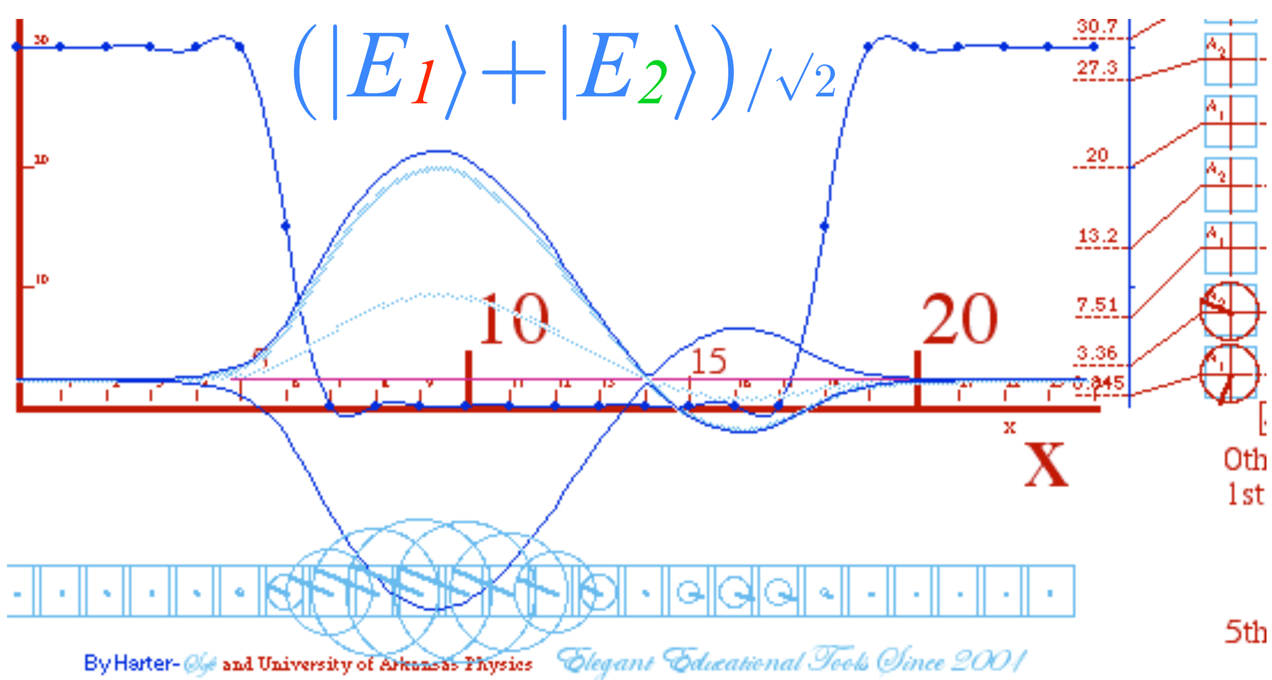
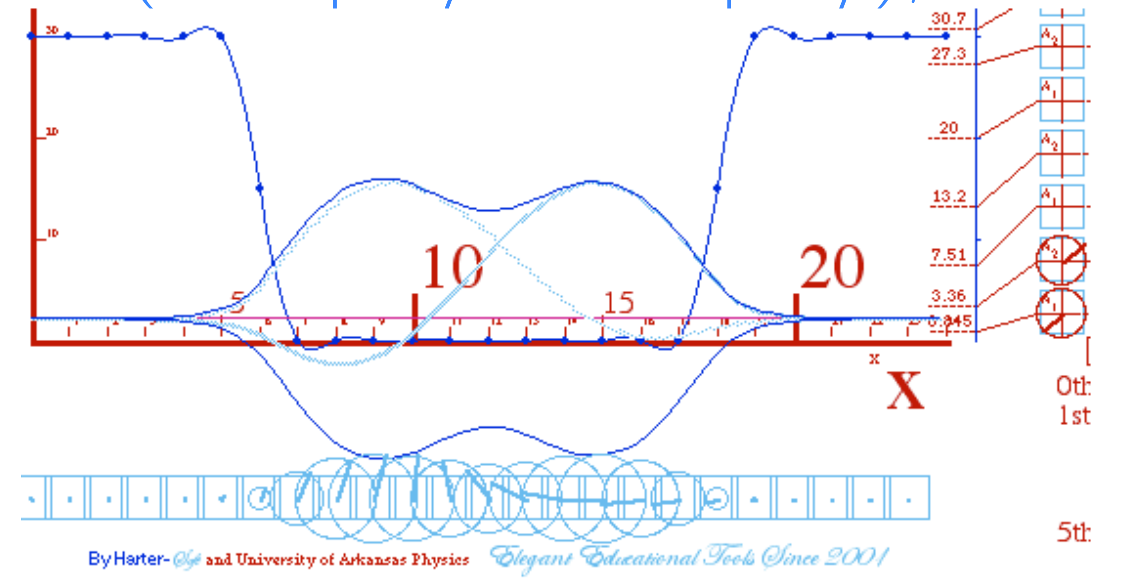






Now combine (add) them and let time roll!

$$(e^{-i\omega_1 t}|E_1\rangle + e^{-i\omega_2 t}|E_2\rangle)/\sqrt{2}$$



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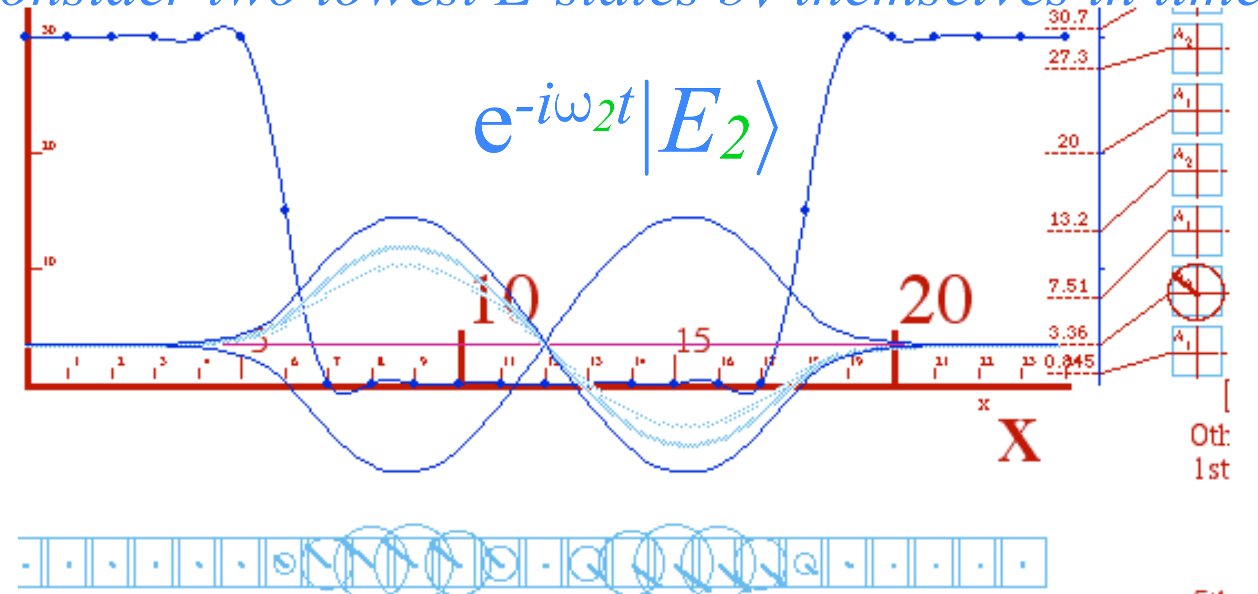

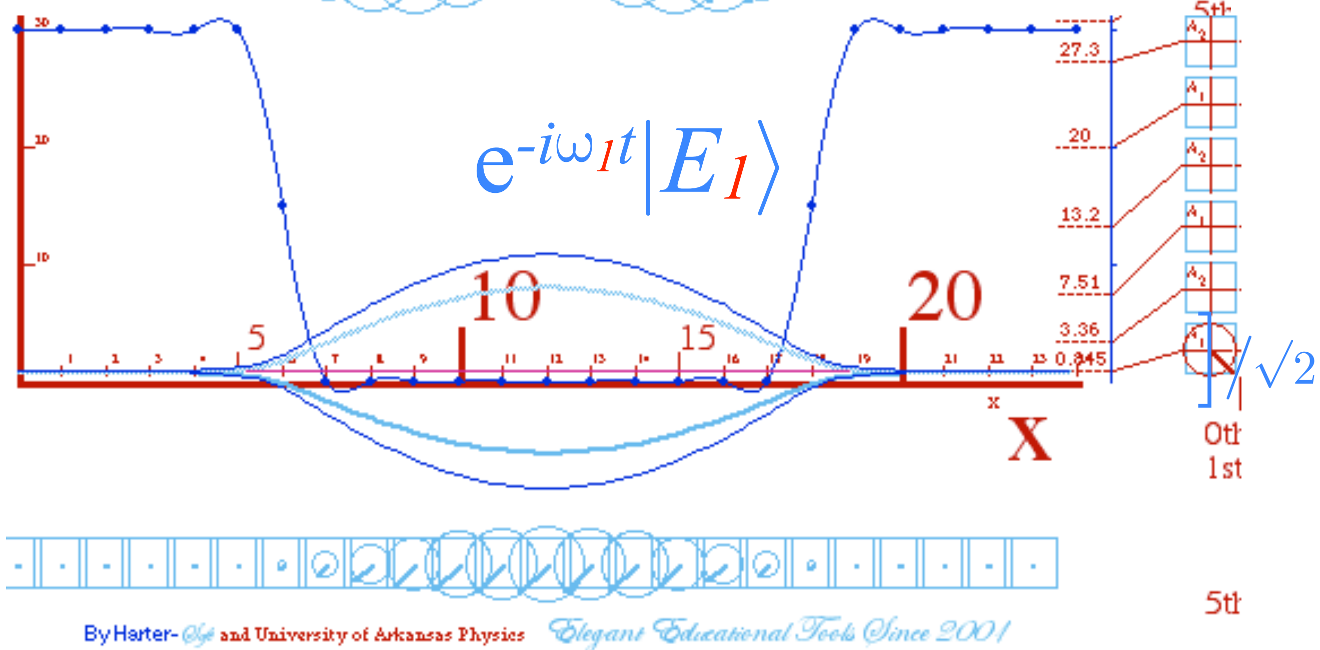

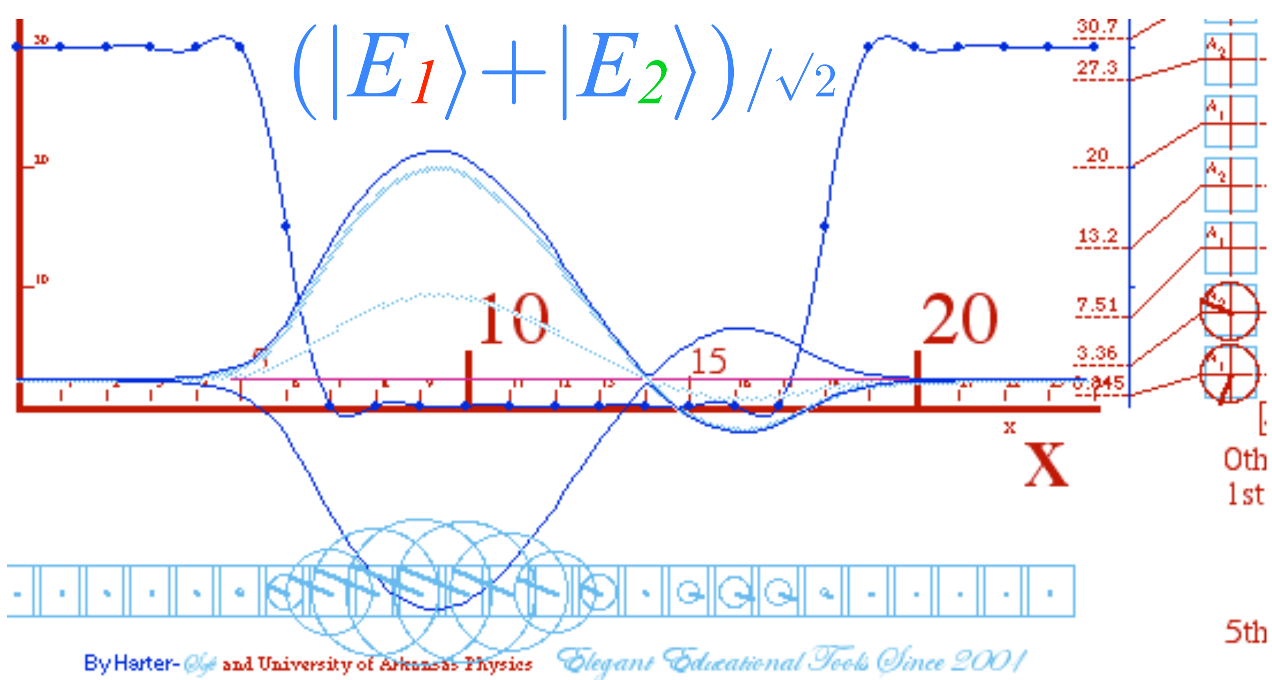

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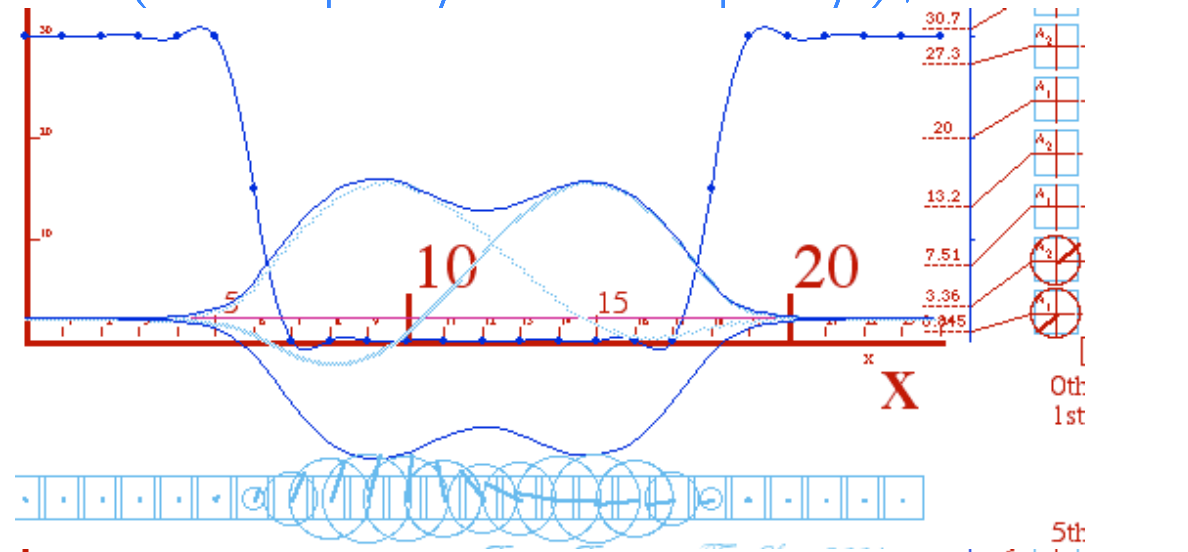
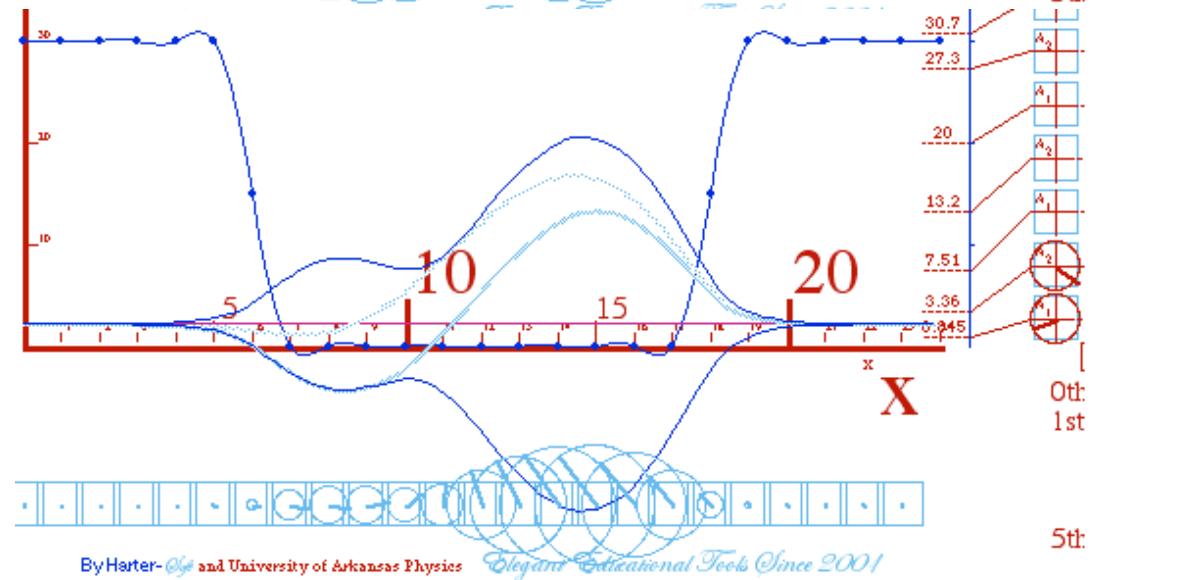
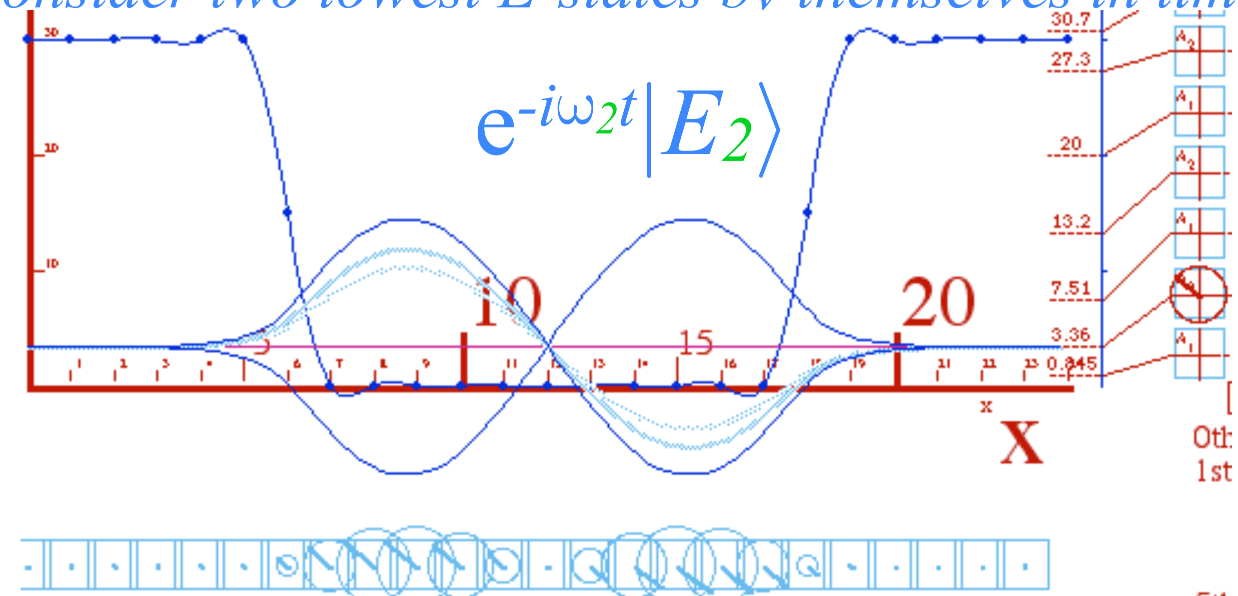


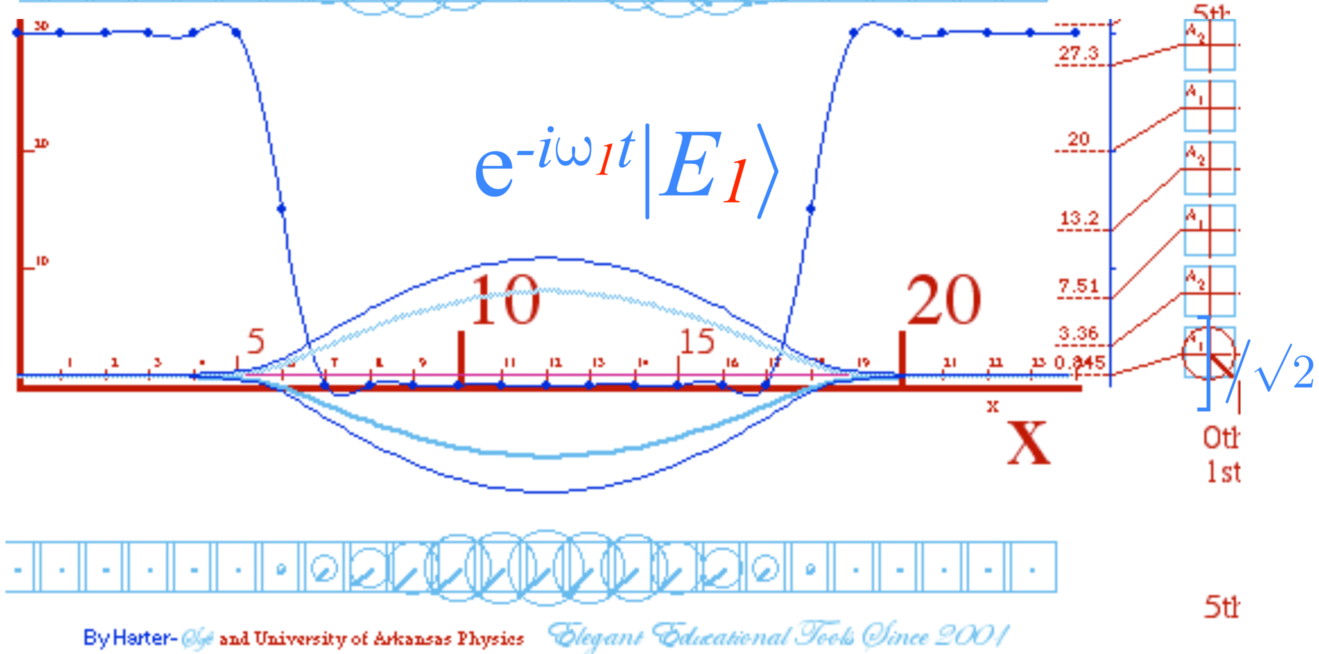


Consider two lowest E-states by themselves in time

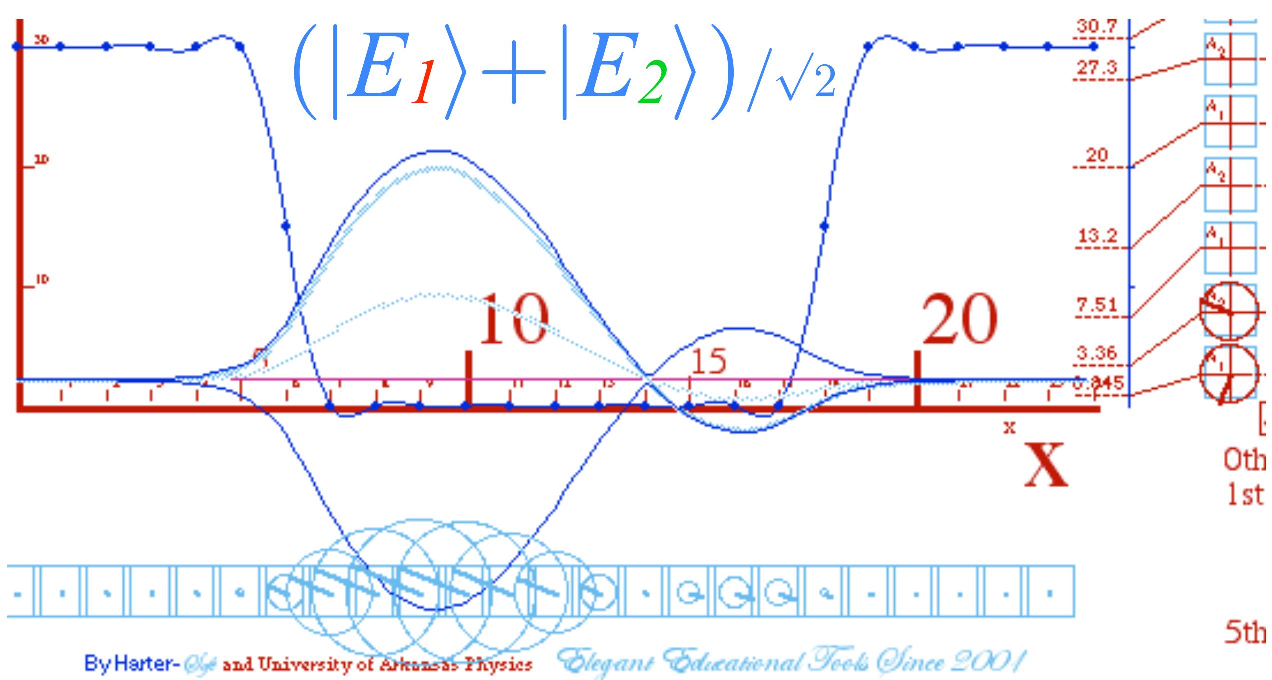
$e^{-i\omega_2 t}|E_2\rangle$



$e^{-i\omega_1 t}|E_1\rangle$



$(|E_1\rangle+|E_2\rangle)/\sqrt{2}$



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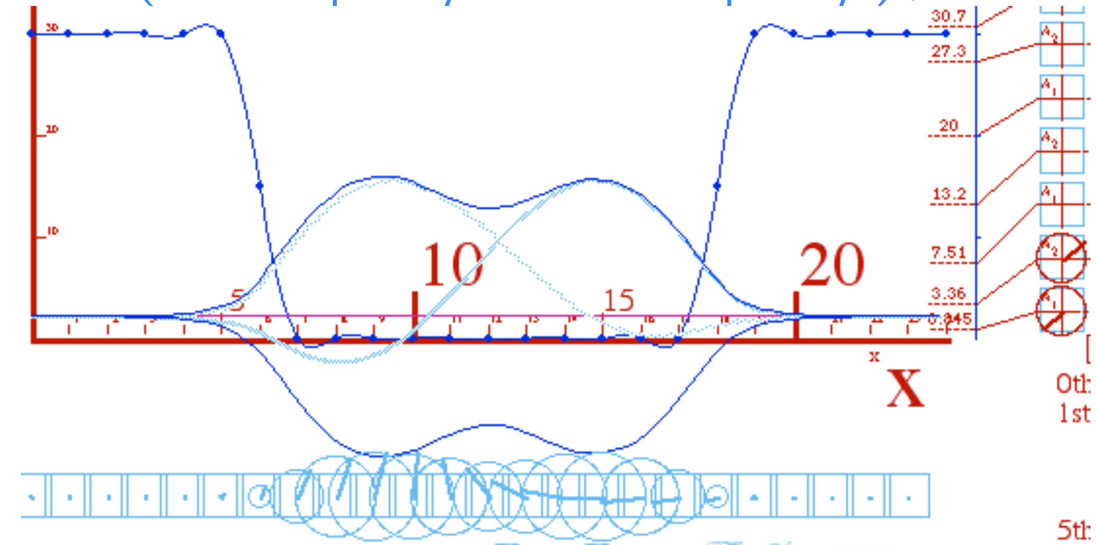

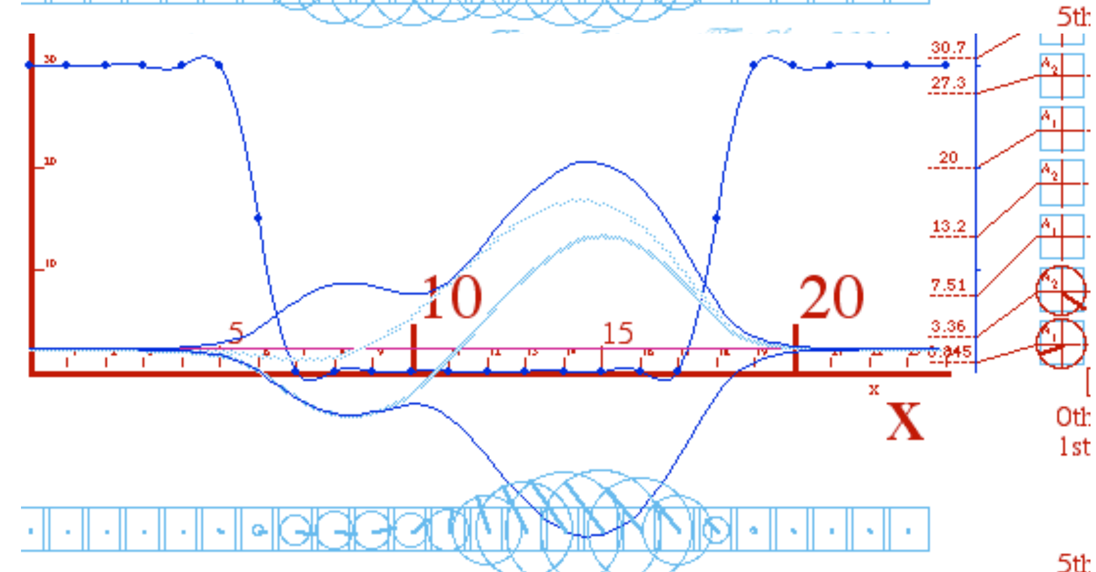

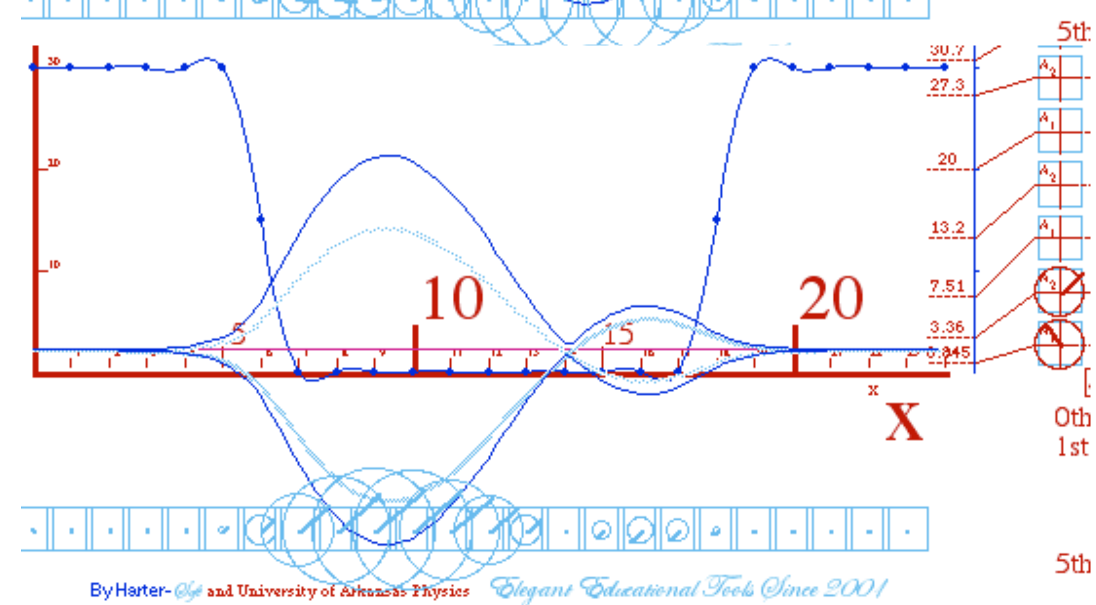

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too fat! too thin! wrong color again!

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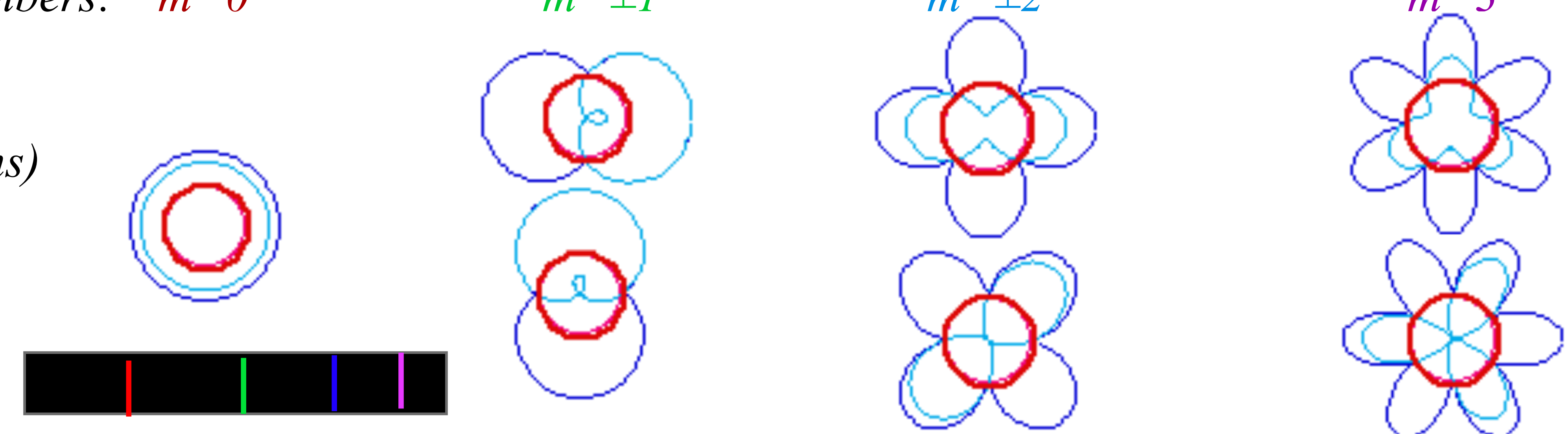
NOTE: We're using "false-color" here.

Rings tolerate a *zero* (kinkless) quantum wave but require ±*integral wave number.*

OK ring quantum numbers: $m=0$ $m=\pm1$ $m=\pm2$ $m=3$

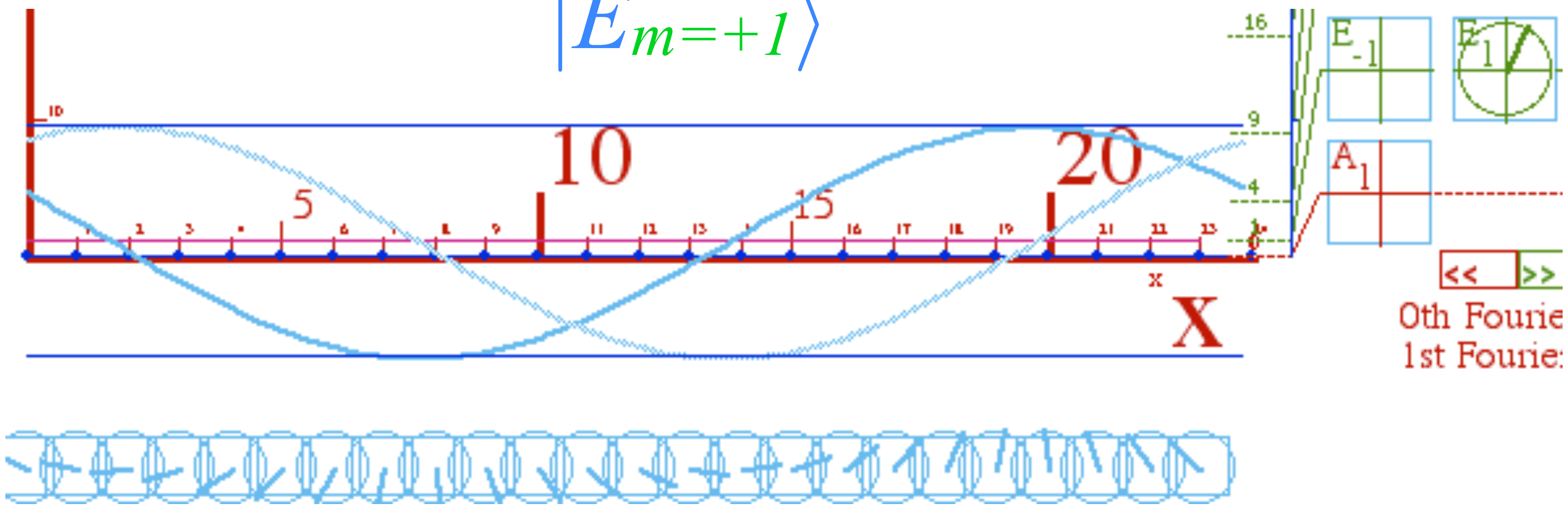
(± integral number of wavelengths)



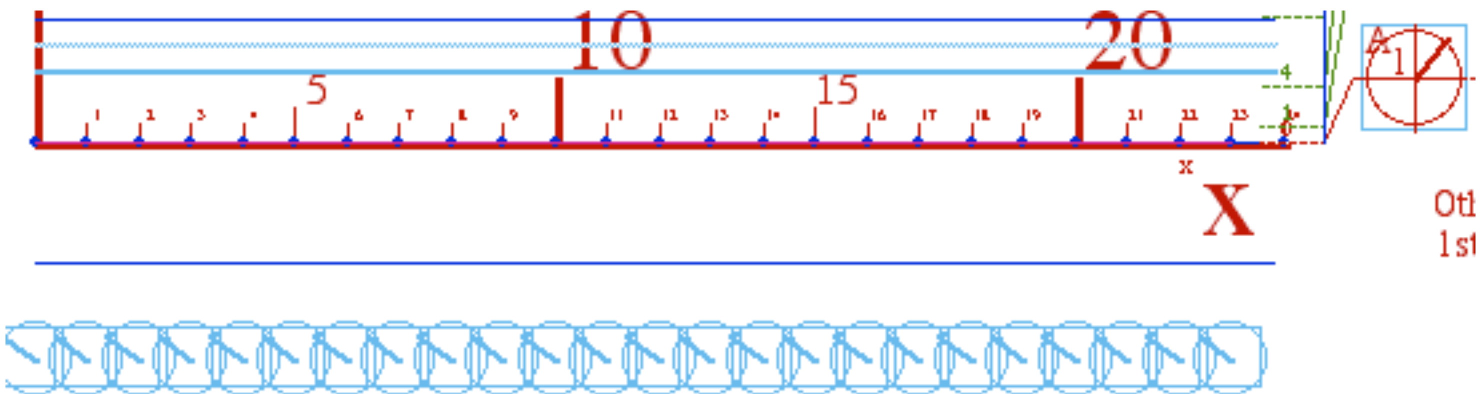
Bohr's models of *atomic spectra (1913-1923)* are beginnings of *quantum wave mechanics* built on *Planck-Einstein (1900-1905)* relation $E=h\upsilon$. *DeBroglie* relation $p=h/\lambda$ comes around *1923*.

Consider two lowest E-states by themselves

$\left|E_{m=+1}\right\rangle$



$\left|E_{m=0}\right\rangle$



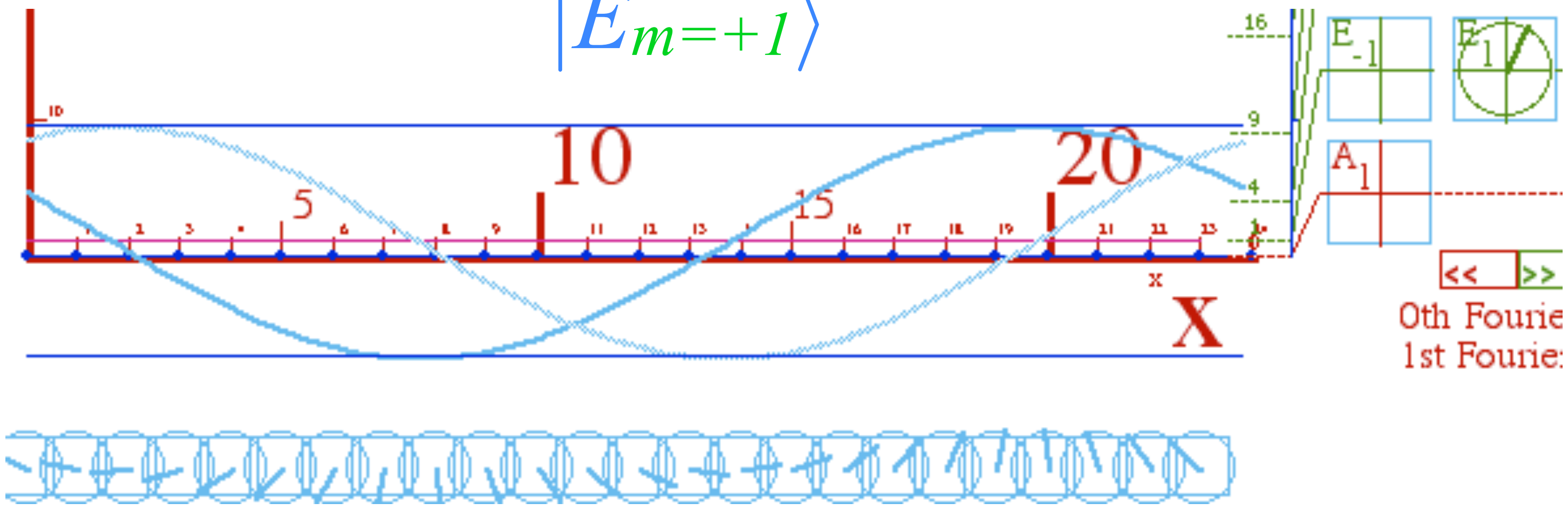
By Harter-Soft and University of Arkansas Physics *Elegant Educational Tools Since 2001*

Consider two lowest E-states by themselves

Now combine (add) them and let time roll!

$$\left(e^{-i\omega_0 t}\left|E_0\right\rangle + e^{-i\omega_{+1} t}\left|E_{+1}\right\rangle\right)/\sqrt{2}$$

$\left|E_{m=+1}\right\rangle$



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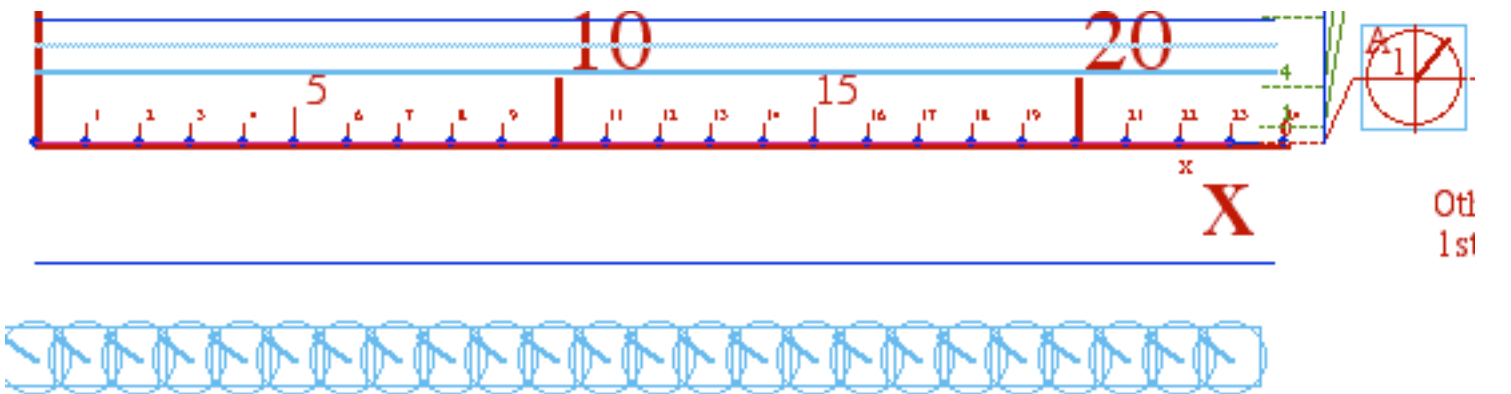

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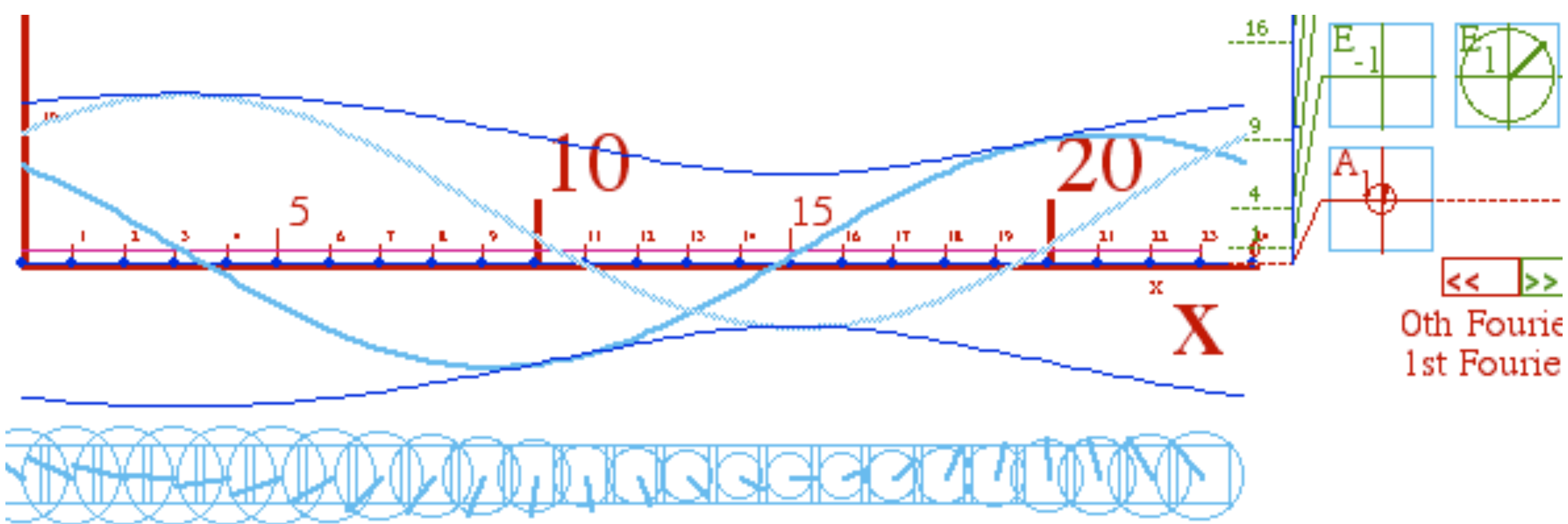

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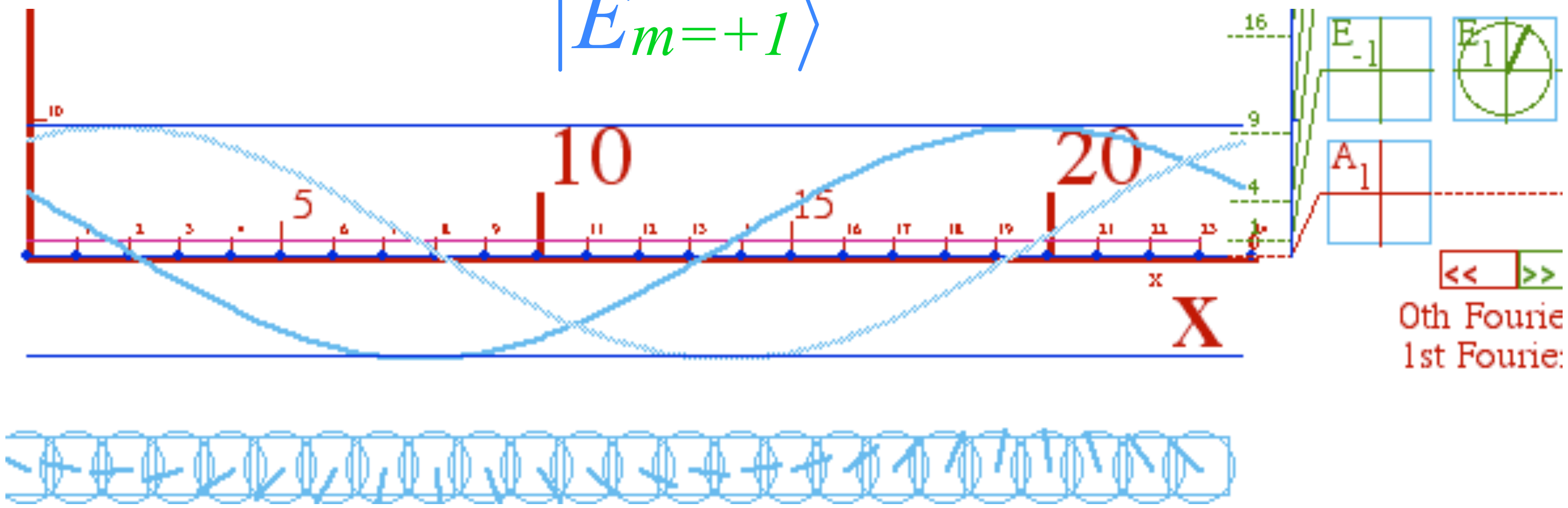
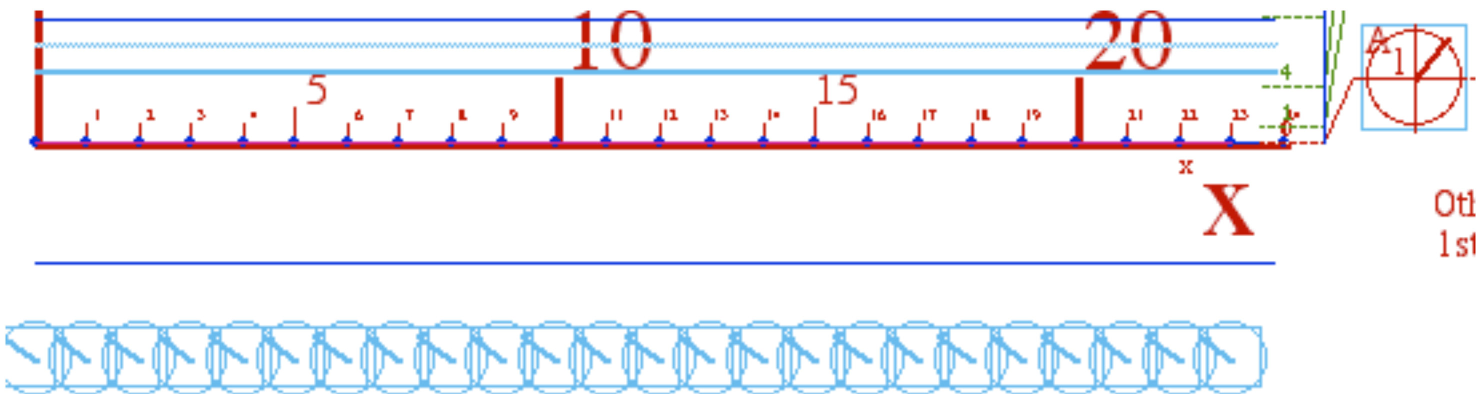
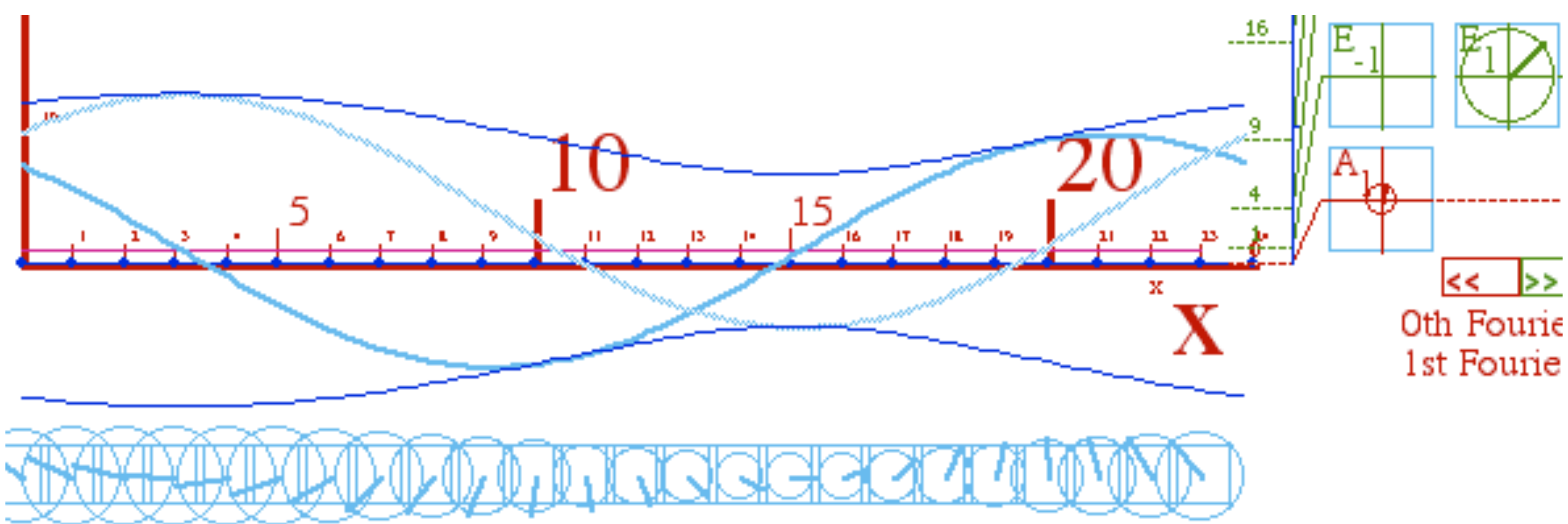
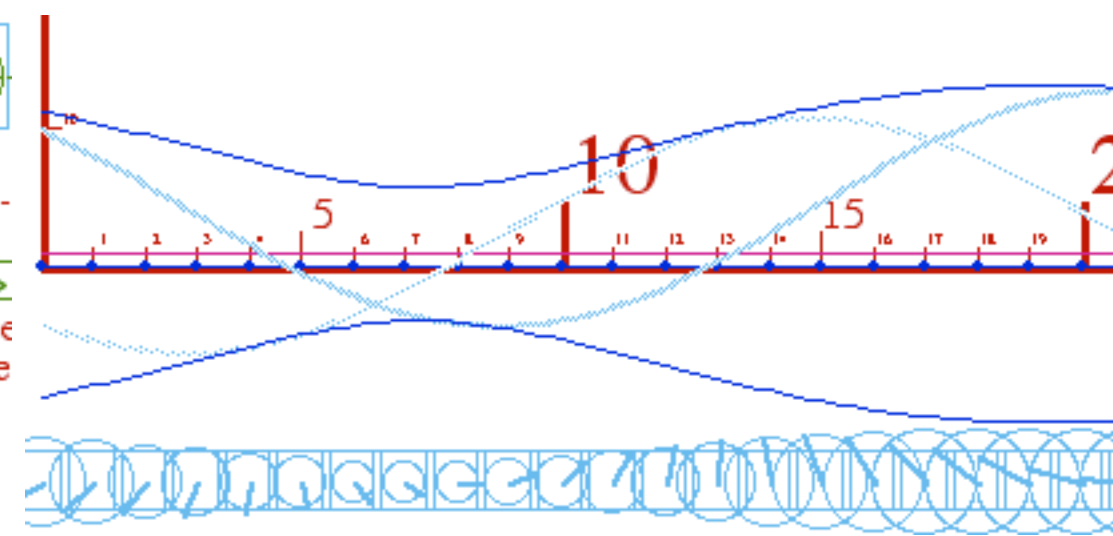


$|E_{m=0}\rangle$



(Just moves forward rigidly)





Consider two degenerate E-states by themselves

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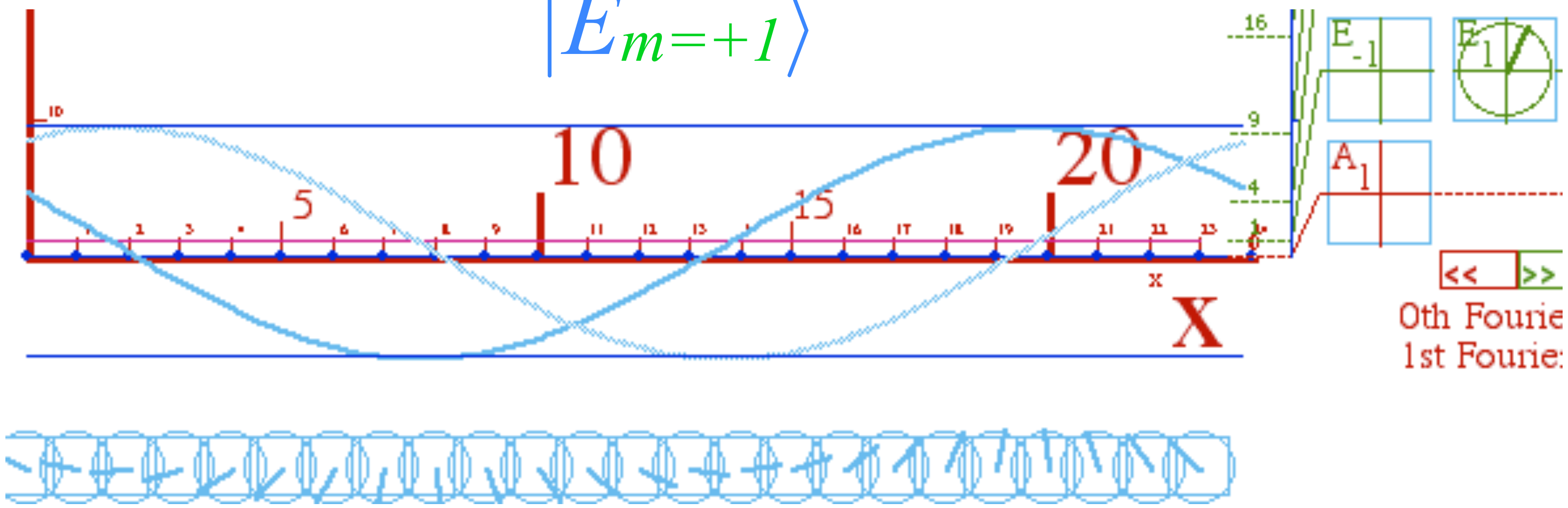
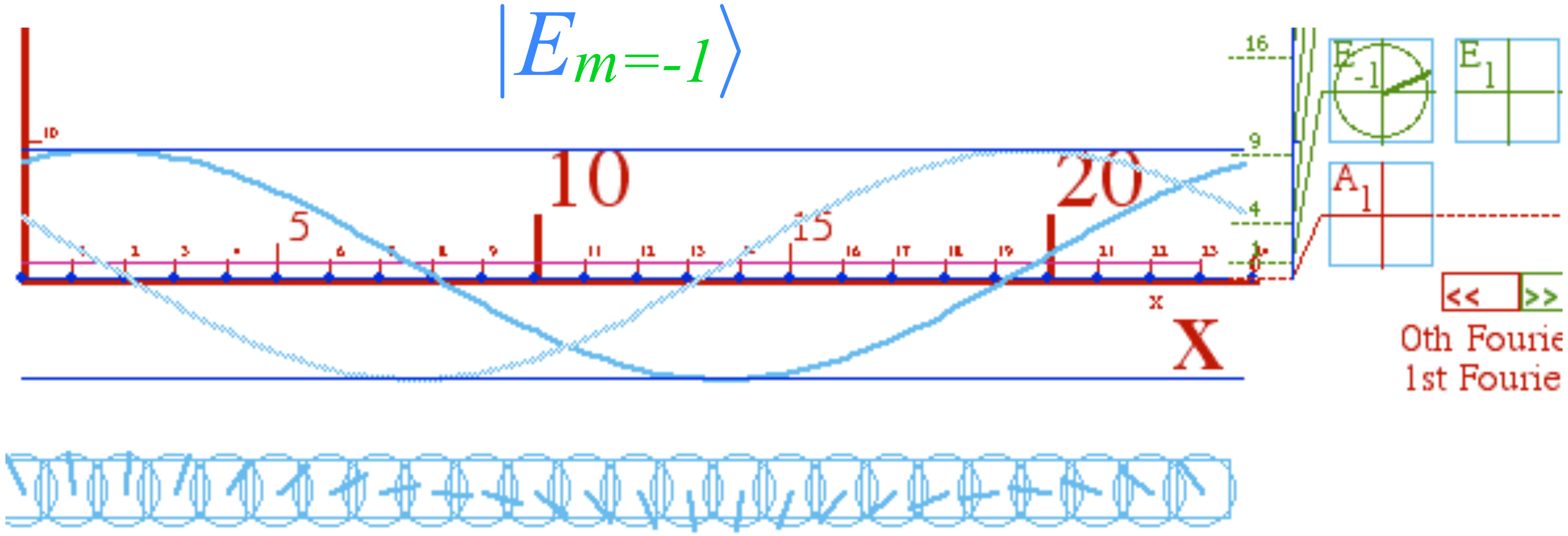


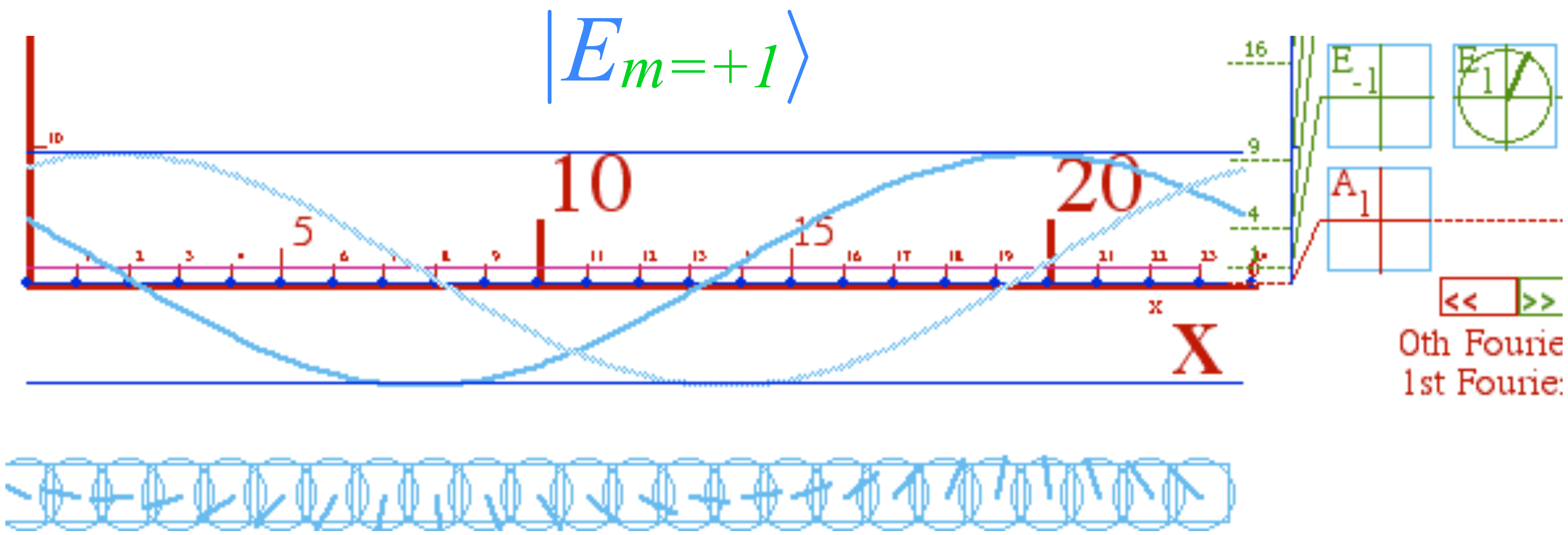
$\left|E_{m=-1}\right\rangle$

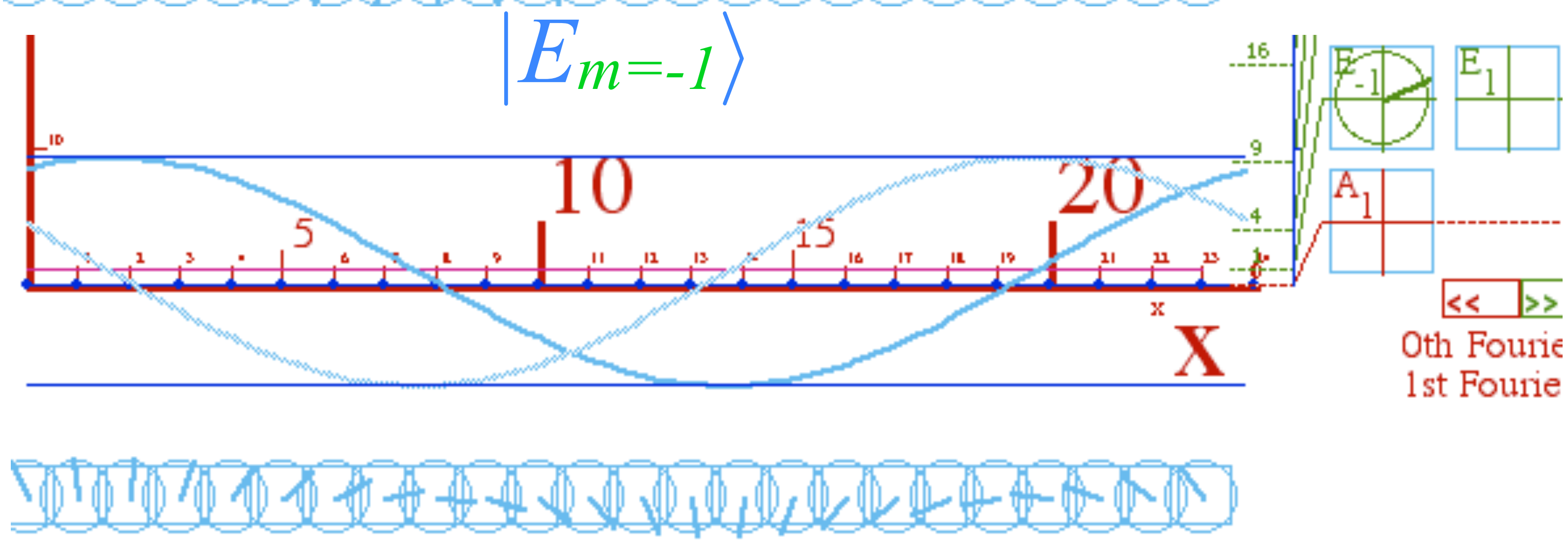


Consider two degenerate E-states by themselves

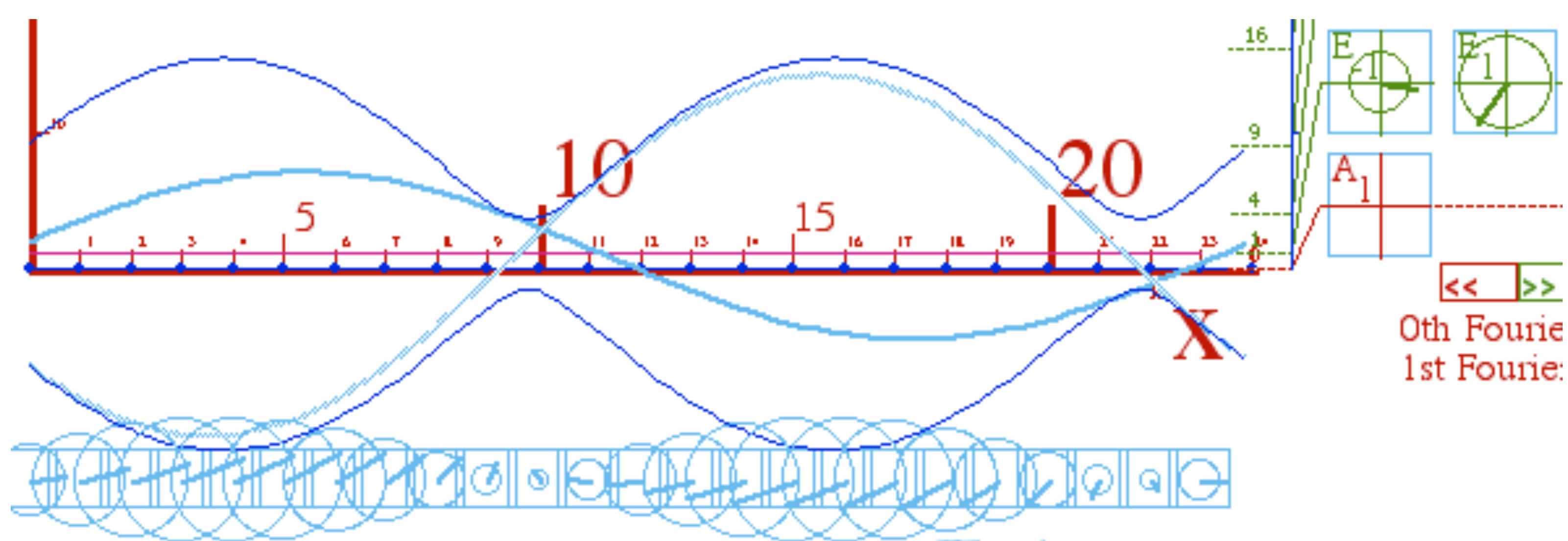
Now combine (add) them and let time roll!

$$\left(e^{-i\omega_{-1}t}|E_{-1}\rangle + e^{-i\omega_{+1}t}|E_{+1}\rangle\right)/\sqrt{2}$$

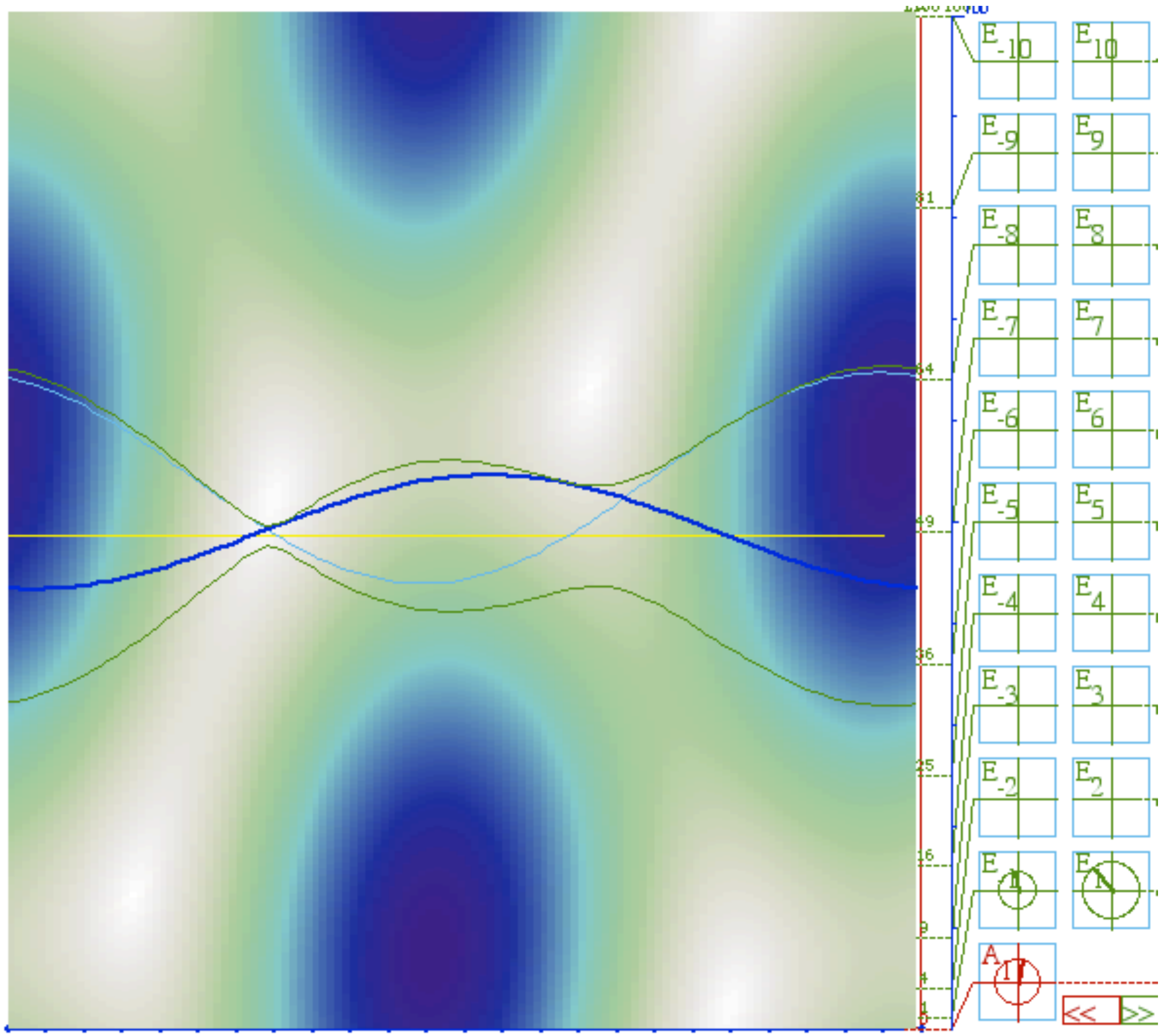




(Group wave is stationary
but phase can move or "gallop")



Consider more than two E-states combined...



2nd Quantization: Quantizing amplitudes ("photons","vibrons", and "what-ever-ons")

Analogy with molecular Born-Oppenheimer-Approximate energy levels

Introducing coherent states (What lasers use)

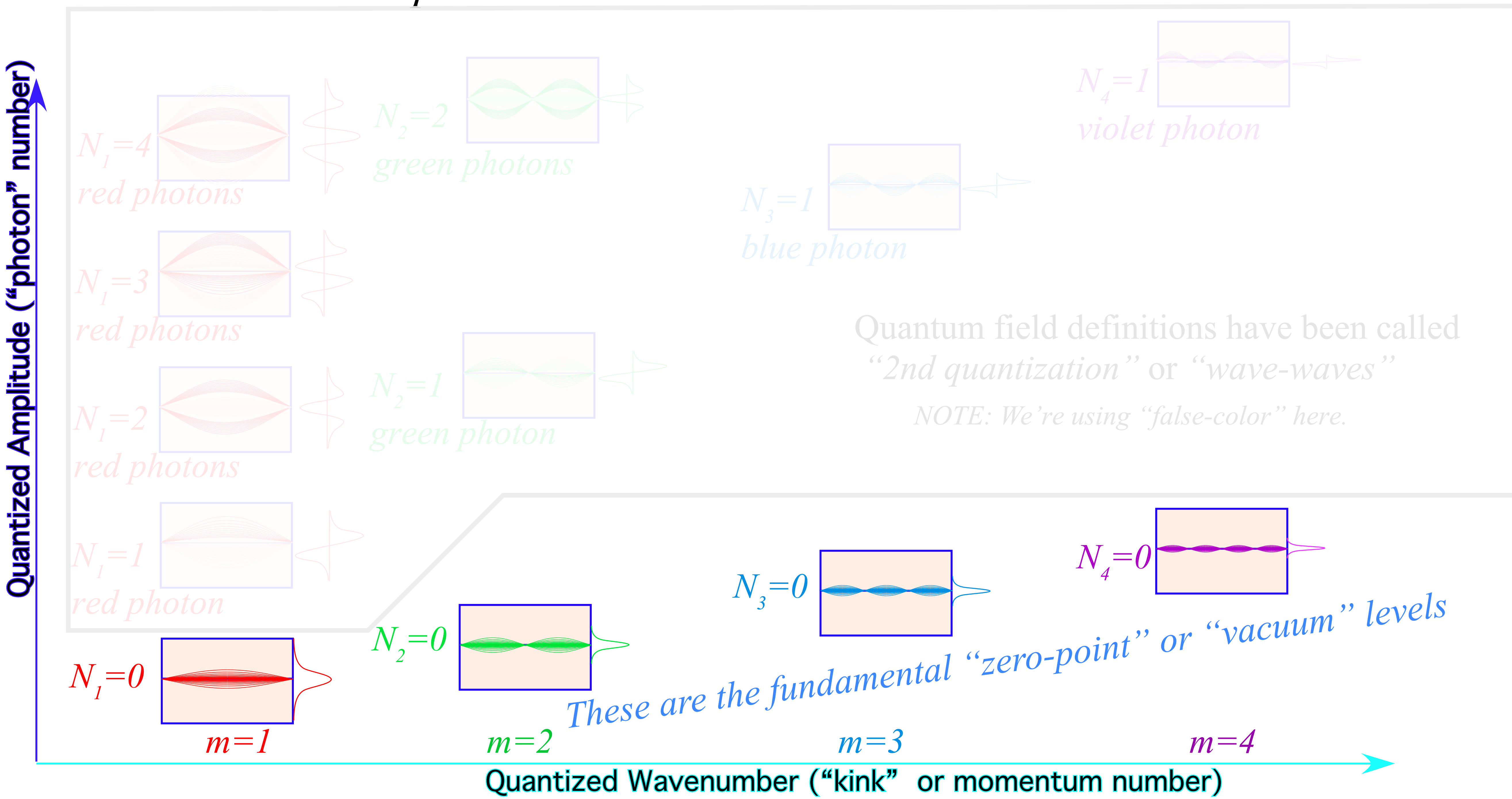
Analogy with (ω,k) wave packets

Wave coordinates need coherence

Quantized *Amplitude* *Counting "photon" number*

Planck's relation $E=Nh\upsilon$ began as a tenative axiom to explain low-T light. Then he tried to disavow it! Einstein picked it up in his 1905 paper. Since then its use has grown enormously and continues to amaze, amuse (or bewilder) all who study it.

A current view is that it represents the *quantization* of optical field *amplitude*. We picture this below as *N-photon* wave states for each box-mode of *m* wave kinks.



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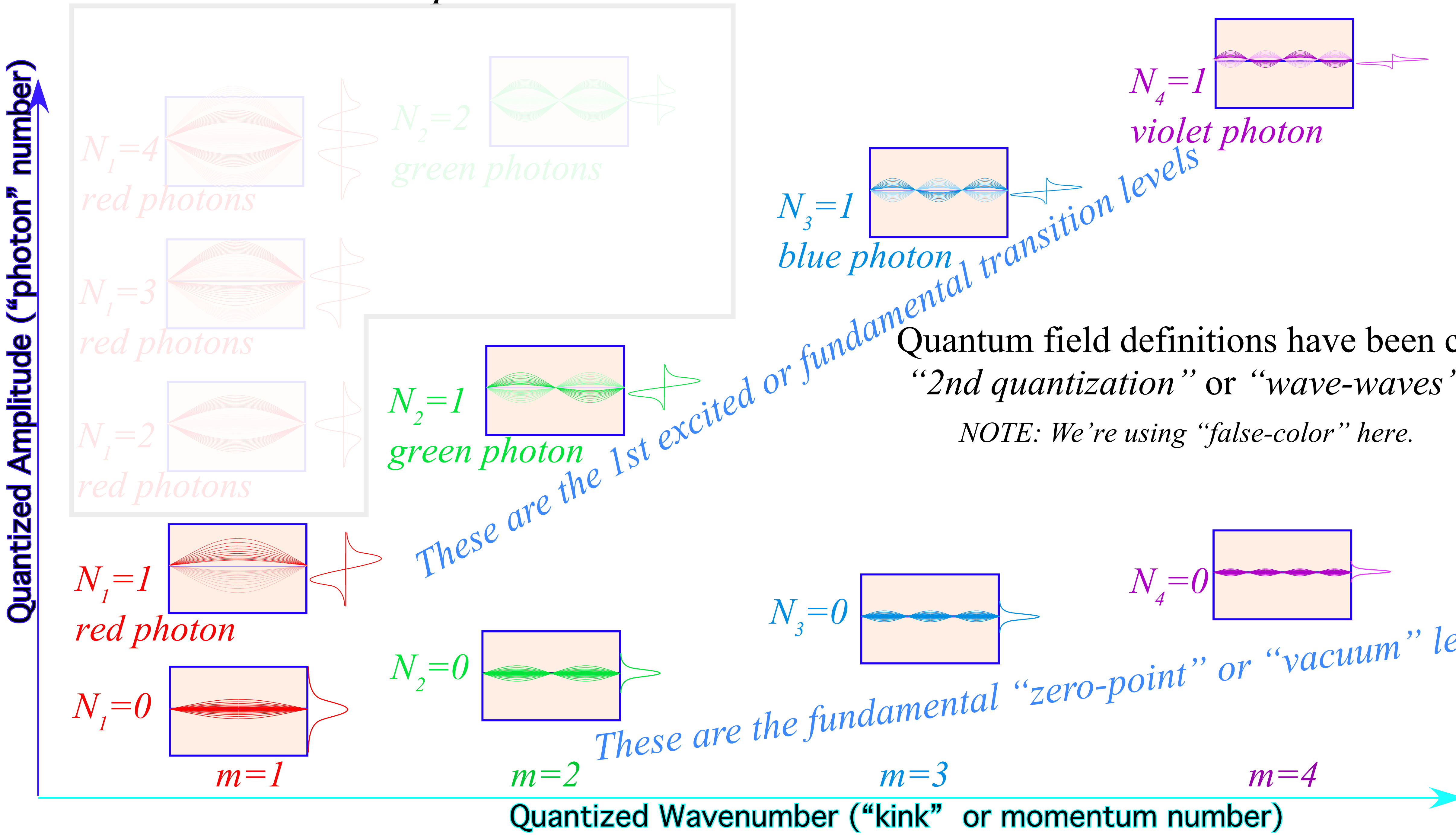

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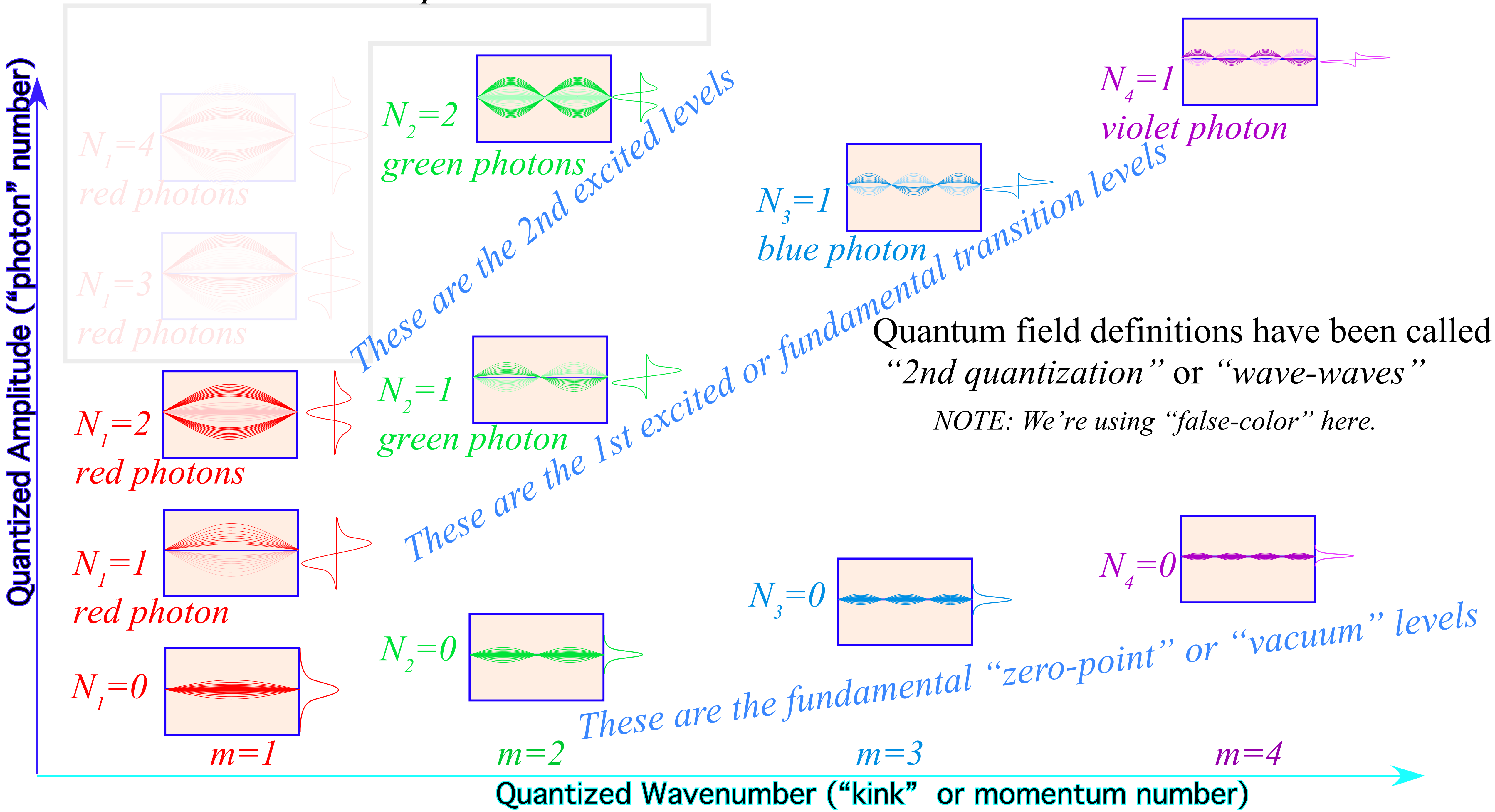

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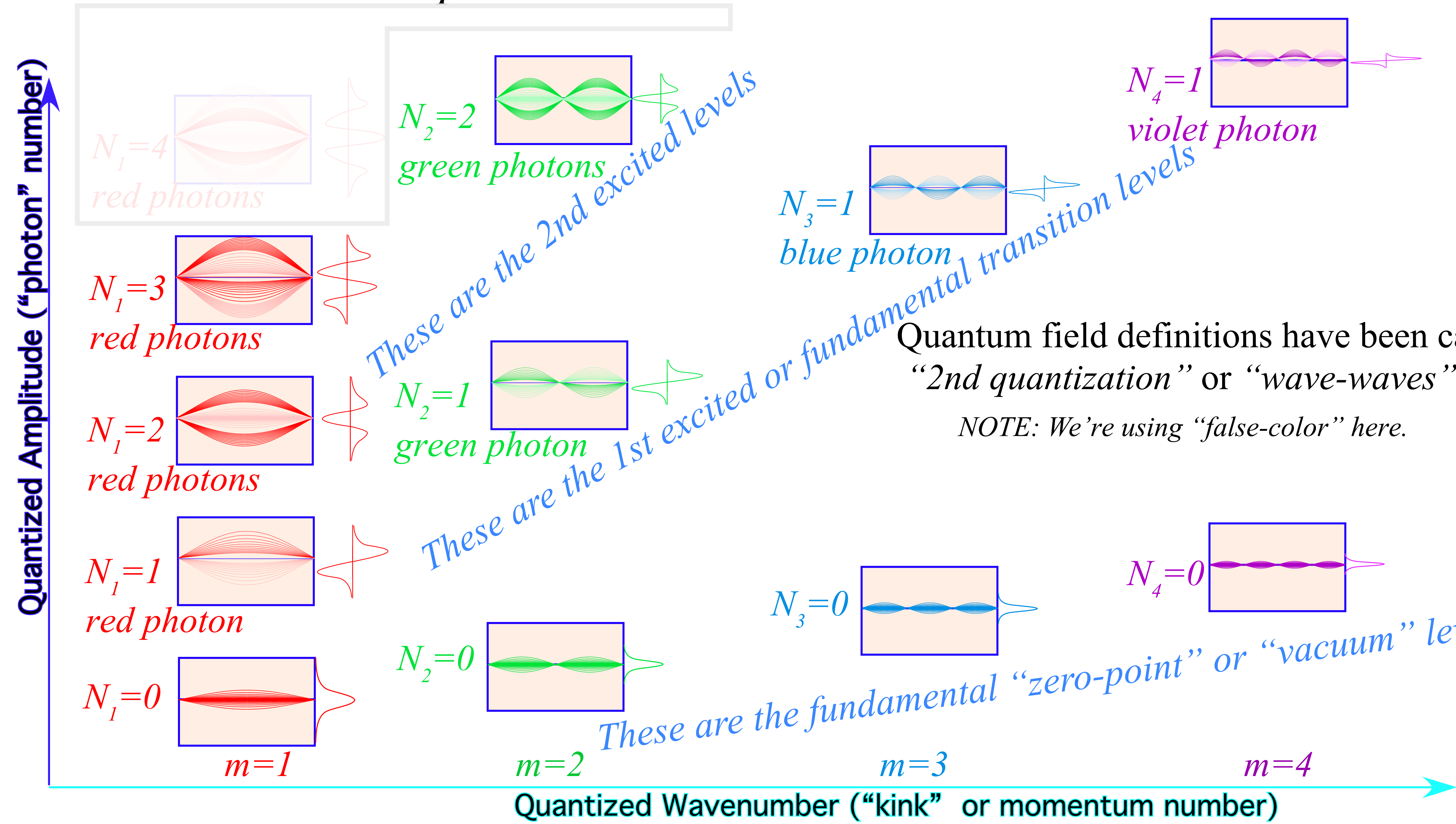

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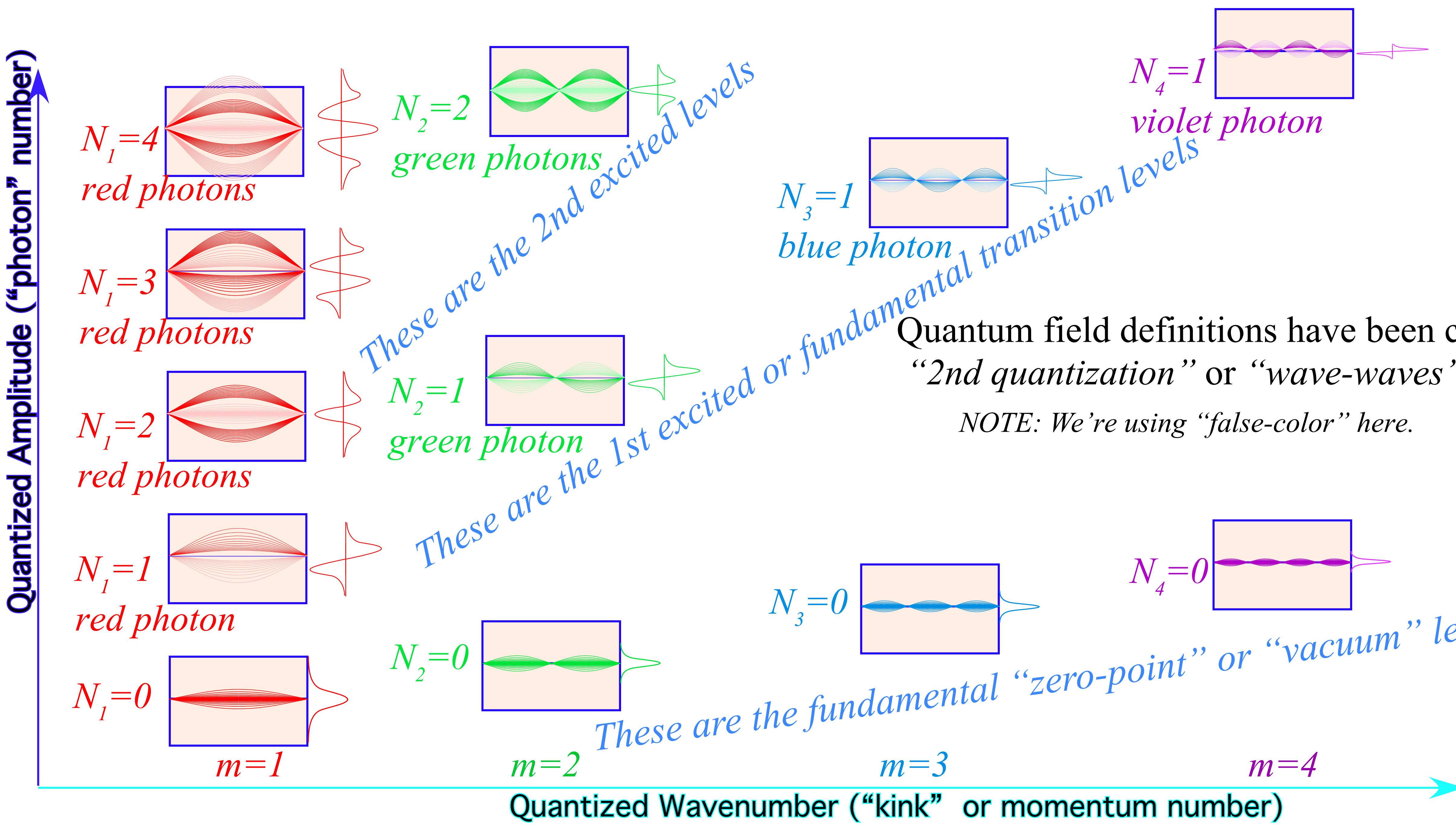
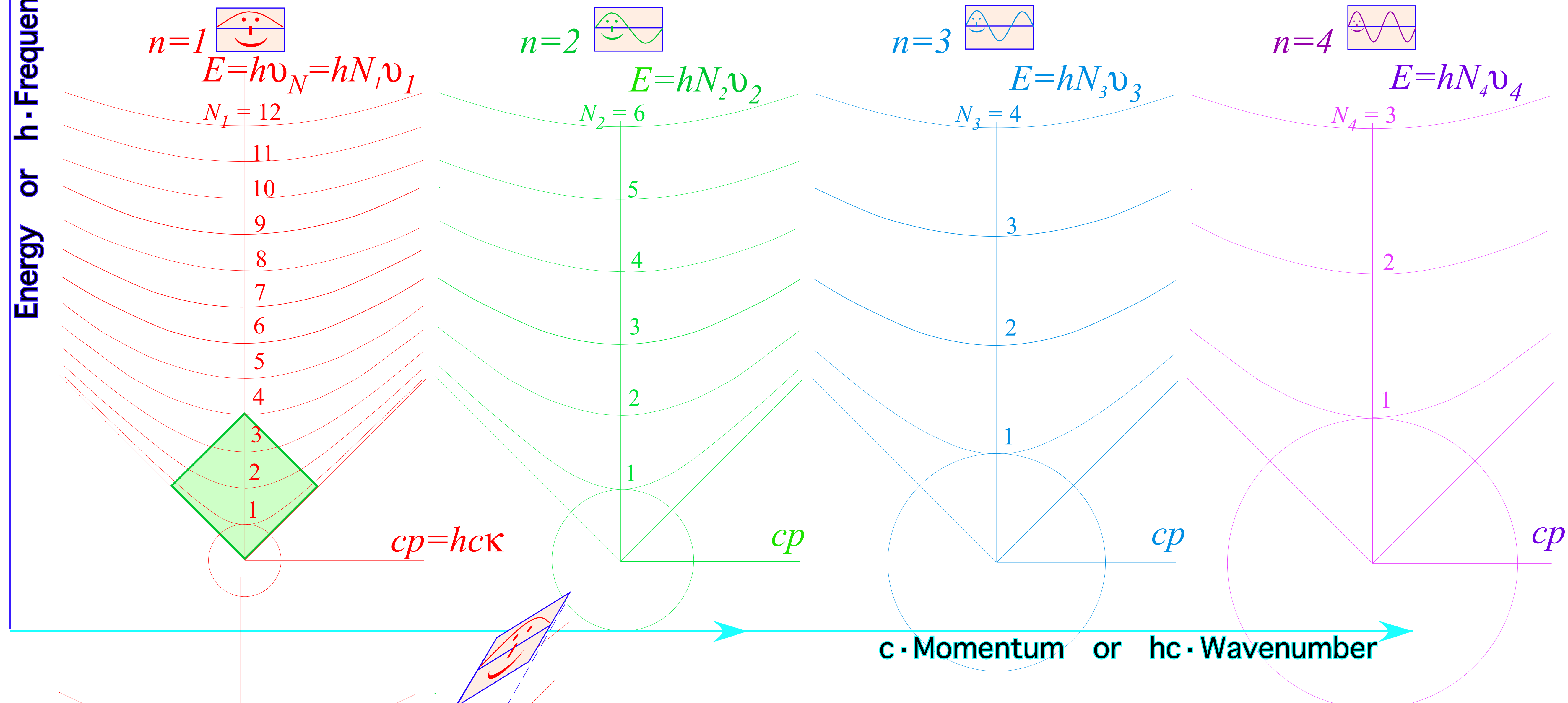
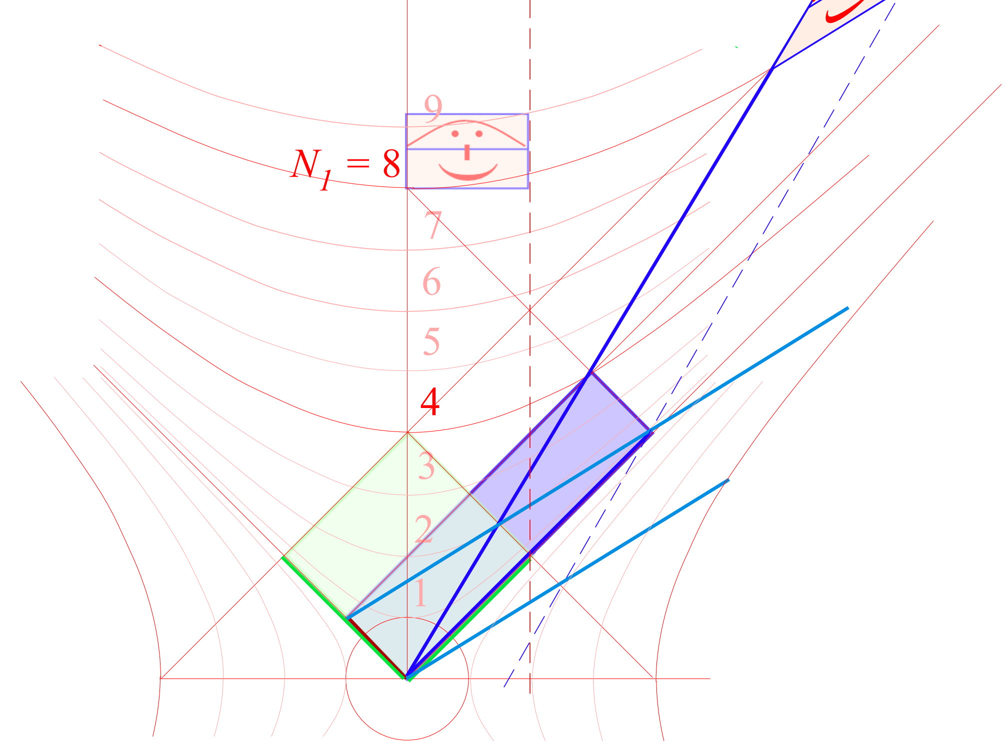


Quantum numbers *N* of field or *n, m,..* of modes are *invariants* and *not* changed by boosting velocity. Each mode fundamental frequency $\upsilon_n = n\upsilon_1$ and its *N*-photon multiples belong to invariant hyperbolas.





Boosted observers see distorted frequencies and lengths, but will agree on the *numbers n* and *N* of mode *nodes* and *photons*.

This is how light waves can “fake” some of the properties of classical “things” such as *invariance* or *object permanence*.

It takes at least *TWO CW*’s to achieve such invariance. One CW is not enough and cannot have non-zero invariant *N* . Invariance is an *interference* effect that needs at least *two-to-tango!*

www.uark.edu/ua/pirelli/php/quantized_2.php

2nd Quantization: Quantizing amplitudes ("photons","vibrons", and "what-ever-ons")

➡ *Analogy with molecular Born-Oppenheimer-Approximate energy levels*

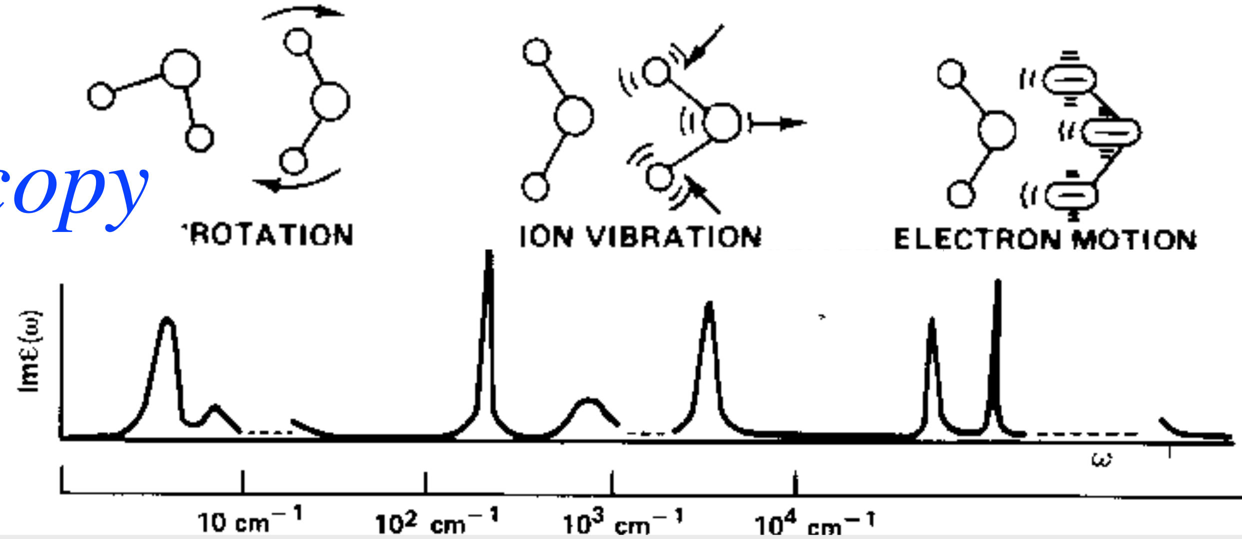
Introducing coherent states (What lasers use)

Analogy with (ω,k) wave packets

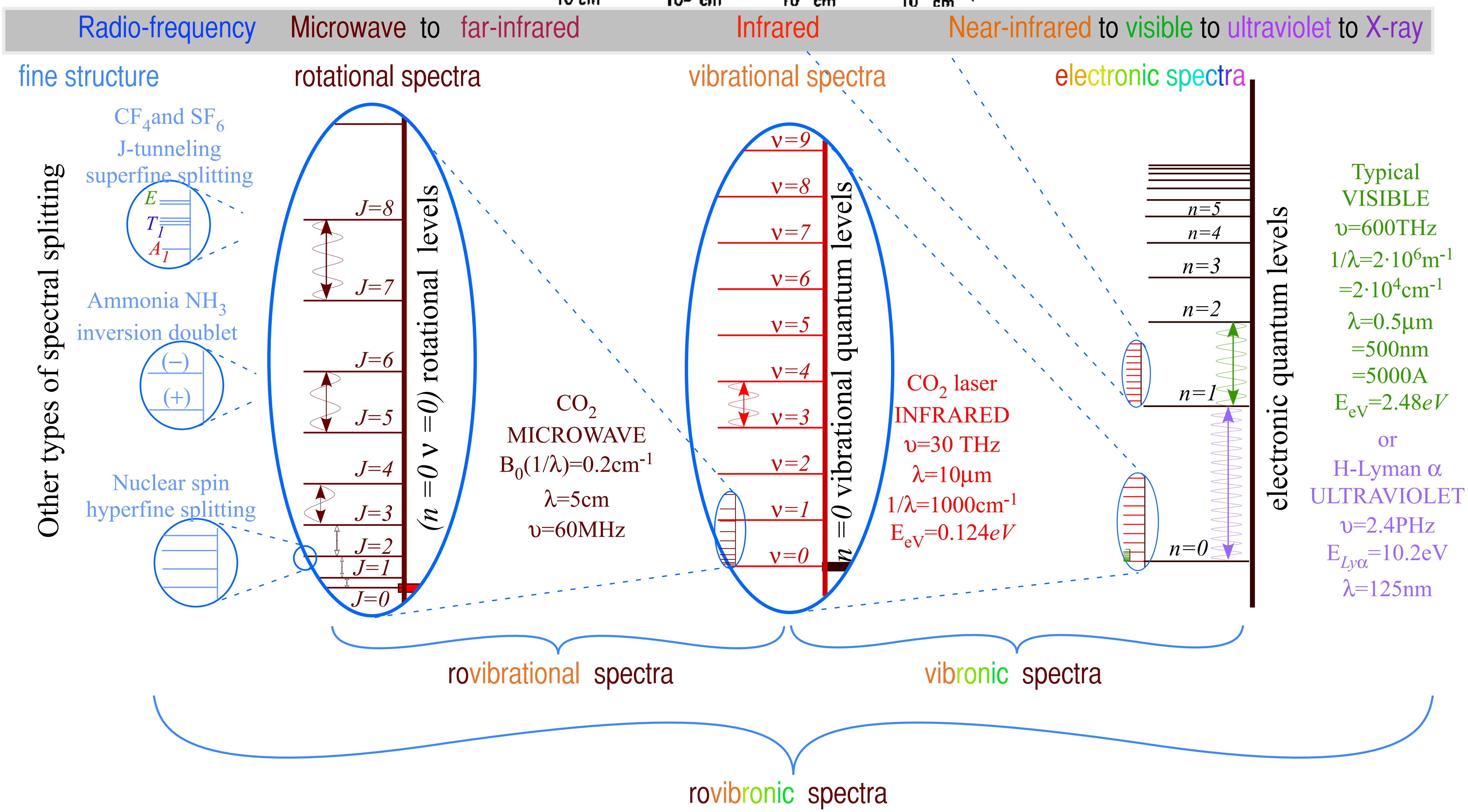
Wave coordinates need coherence

A sketch of modern molecular spectroscopy

From Fig. 6.5.5. Principles of Symmetry, Dynamics, and Spectroscopy W. G. Harter, Wiley Interscience, NY (1993)



The frequency hierarchy



Spectral Quantities

<u>Frequency υ</u>

<u>*Hertz*(sec^{-1})</u>

THz $10^{12}s^{-1}$

GHz $10^{9}s^{-1}$

MHz $10^{6}s^{-1}$

kHz $10^{3}s^{-1}$

<u>Wavelength λ</u>

<u>*meters*(m)</u>

fm $10^{-15}m$

pm $10^{-12}m$

nm $10^{-9}m$

μm $10^{-6}m$

mm $10^{-3}m$

cm $10^{-2}m$

km $10^{3}m$

<u>Wavenumber</u>

<u>*per meter*(m^{-1})</u>

cm^{-1} $10^{2}m^{-1}$

<u>Energy *ehυ*</u>

<u>*electonVolts*</u>

<u>(*eV*)</u>

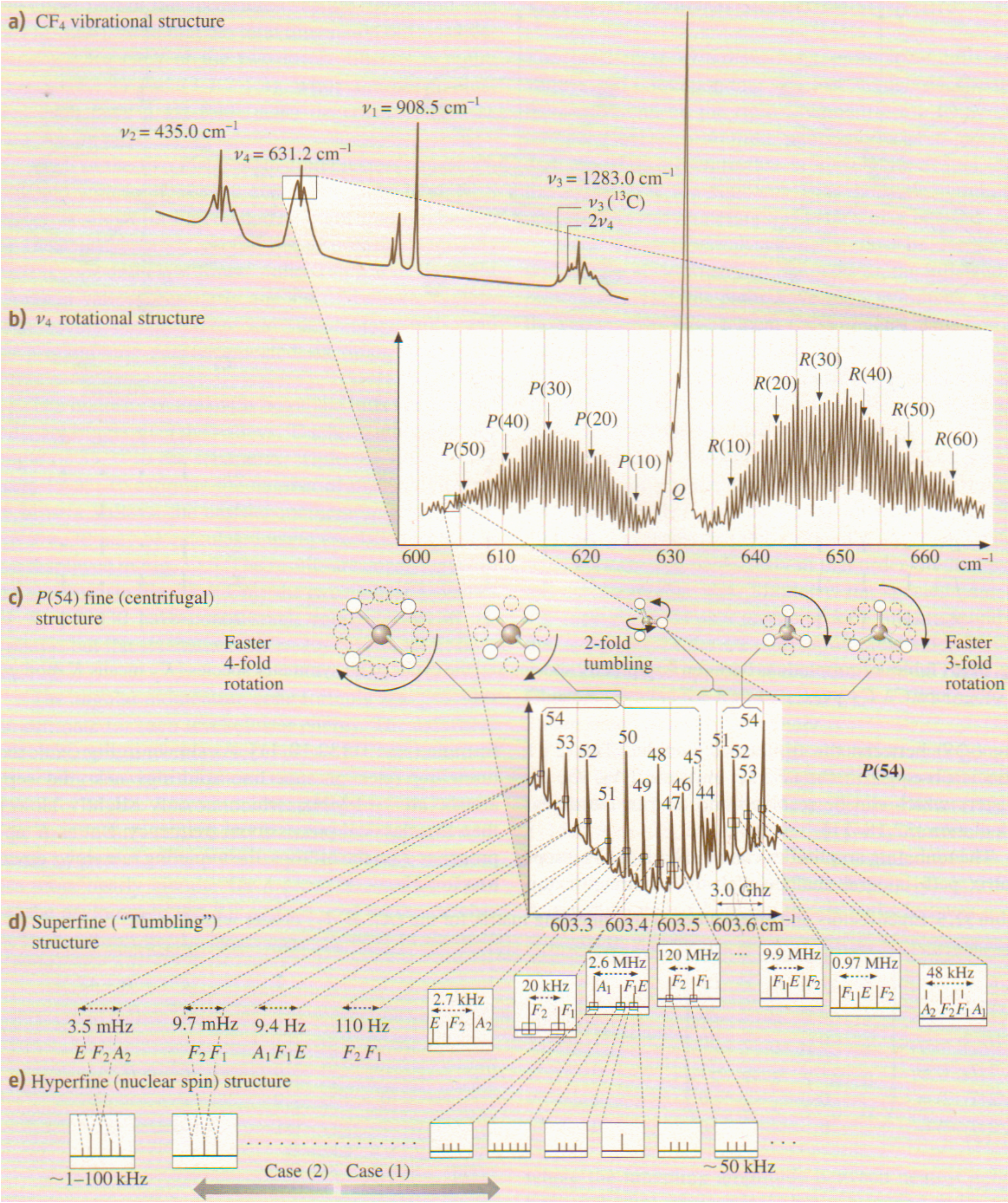
Example of frequency hierarchy for 16μm spectra of CF_4 (Freon-14)

W.G.Harter

Fig. 32.7

Springer Handbook of Atomic, Molecular, & Optical Physics

Gordon Drake Editor (2005)



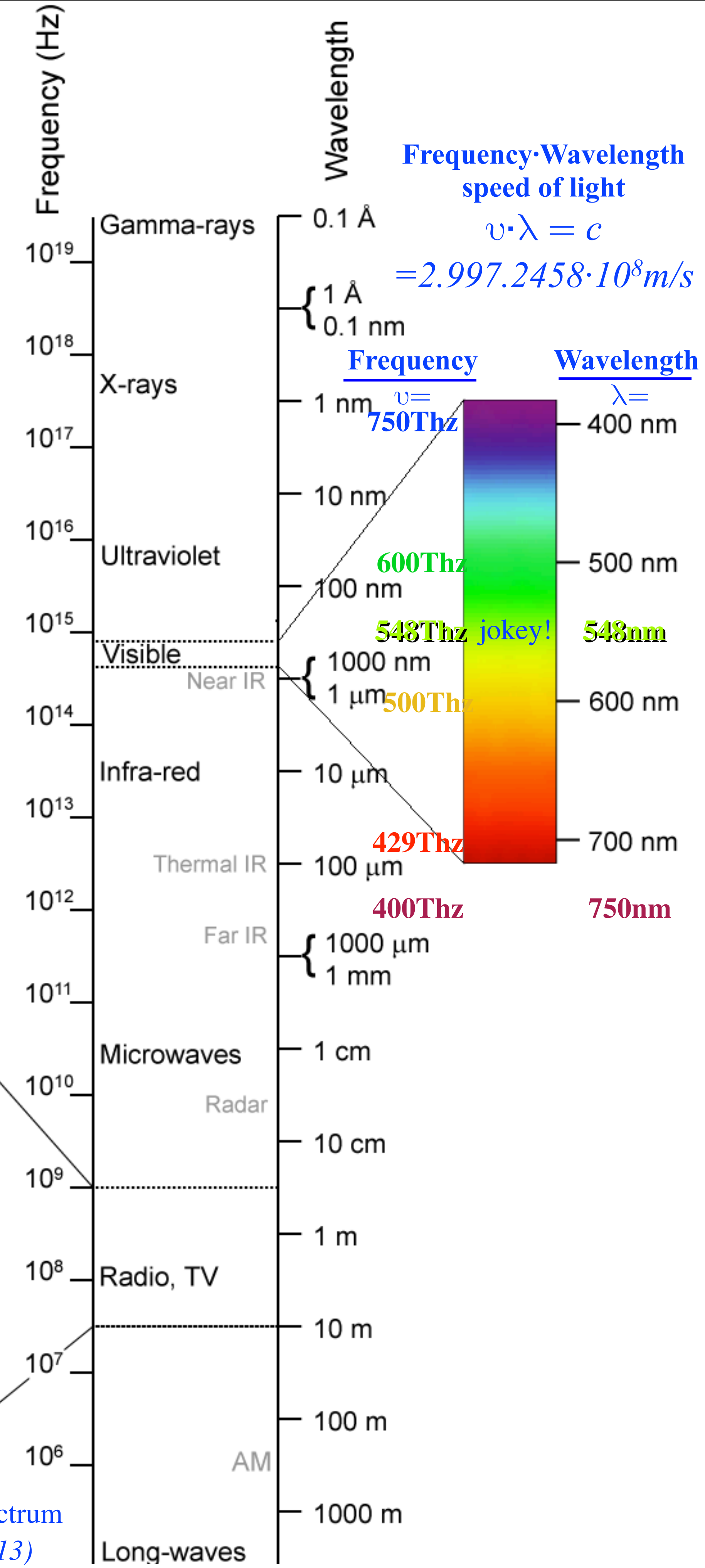
Units of frequency (Hz), wavelength (m), and energy (eV)

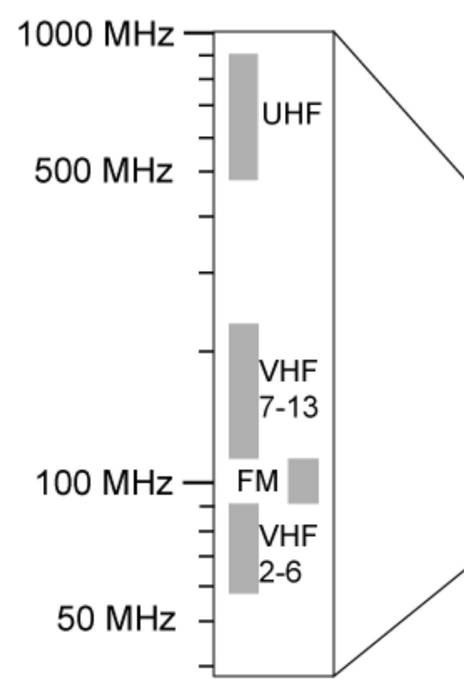
CLASS	FREQUENCY	WAVELENGTH	ENERGY
γ	300 EHz	1 pm	1.24 MeV
HX	30 EHz	10 pm	124 keV
	3 EHz	100 pm	12.4 keV
SX	300 PHz	1 nm	1.24 keV
EUV	30 PHz	10 nm	124 eV
NUV	3 PHz	100 nm	12.4 eV
NIR	300 THz	1 μm	1.24 eV
MIR	30 THz	10 μm	124 meV
FIR	3 THz	100 μm	12.4 meV
EHF	300 GHz	1 mm	1.24 meV
SHF	30 GHz	1 cm	124 μeV
UHF	3 GHz	1 dm	12.4 μeV
VHF	300 MHz	1 m	1.24 μeV
HF	30 MHz	10 m	124 neV
MF	3 MHz	100 m	12.4 neV
LF	300 kHz	1 km	1.24 neV
VLF	30 kHz	10 km	124 peV
VF/ULF	3 kHz	100 km	12.4 peV
SLF	300 Hz	1 Mm	1.24 peV
ELF	30 Hz	10 Mm	124 feV
	3 Hz	100 Mm	12.4 feV

From: Electromagnetic Spectrum
Wikepedia Commons (2013)

Exa: 10^{18}
Peta: 10^{15}
Tera: 10^{12}
Giga: 10^{9}
Mega: 10^{6}
kilo: 10^{3}

milli: 10^{-3}
micro: 10^{-6}
nano: 10^{-9}
pico: 10^{-12}
femto: 10^{-15}
atto: 10^{-18}





From: Electromagnetic Spectrum
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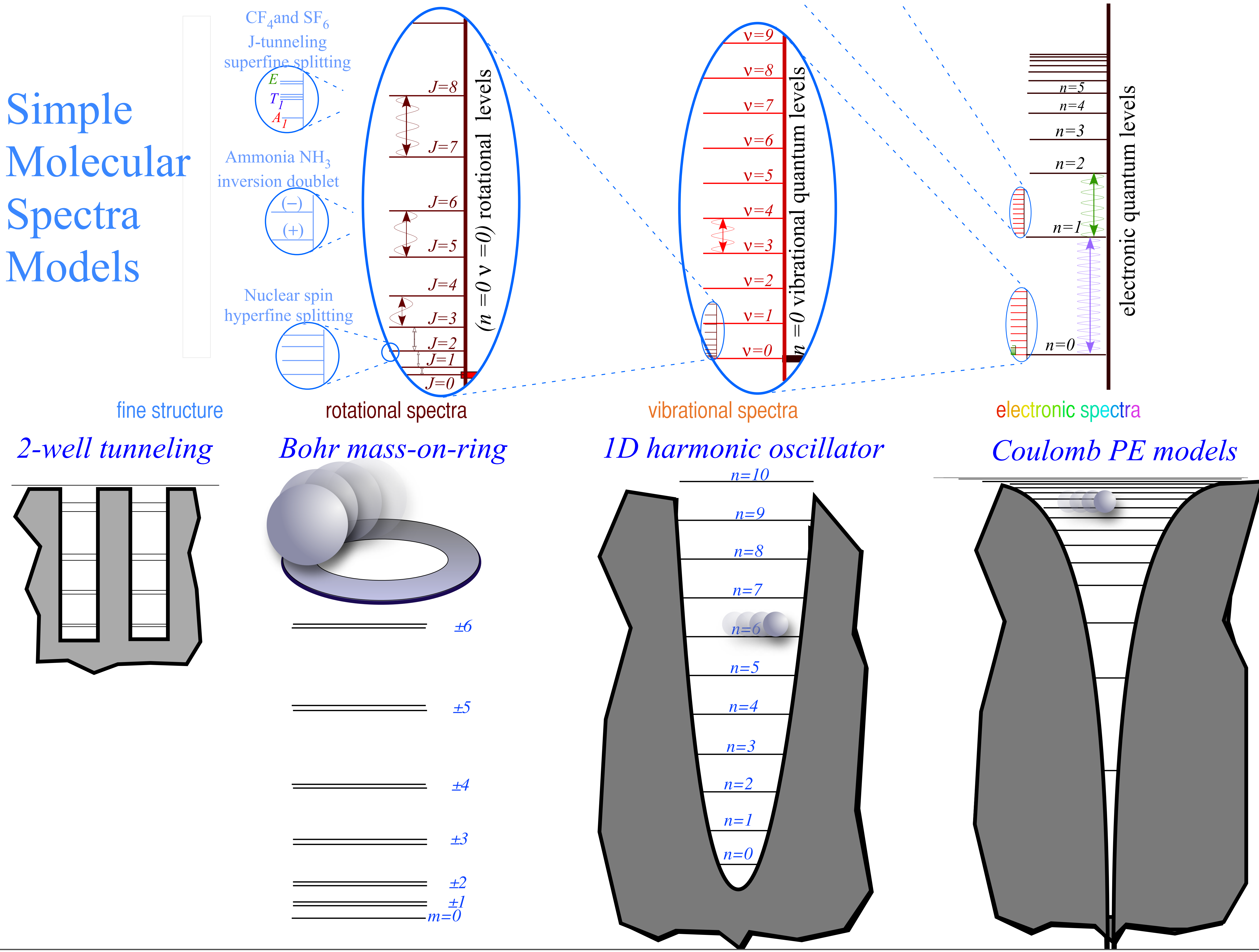
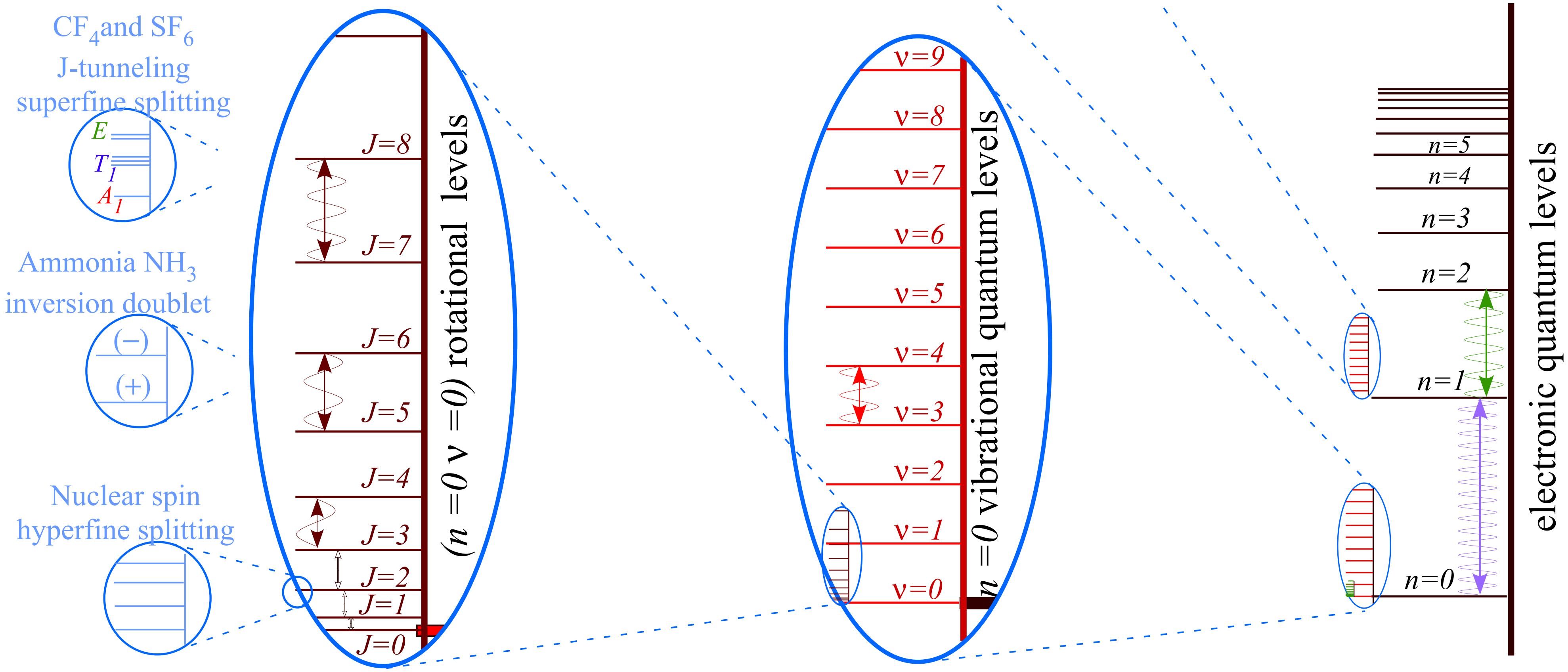
Simple Molecular Spectra Models



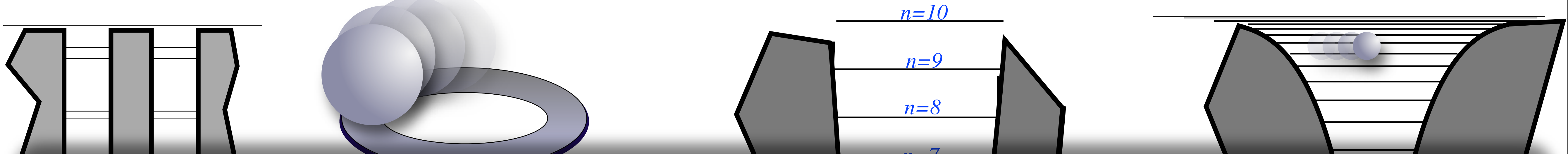
More Advanced Molecular Spectra Models

(Use symmetry group theory)



fine structure | rotational spectra | vibrational spectra | electronic spectra

2-well tunneling | *Bohr mass-on-a-ring* | *1D harmonic oscillator* | *Coulomb PE models*

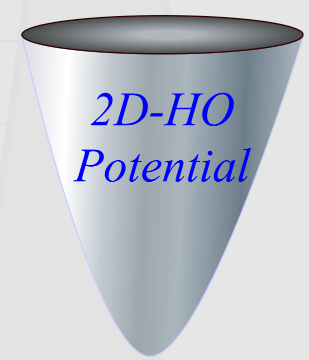


2-state U(2)-spin and quasi-spin tunneling models

3D R(3)-rotor and D-function lab-body wave models

2D harmonic oscillator and U(2) 2nd quantization

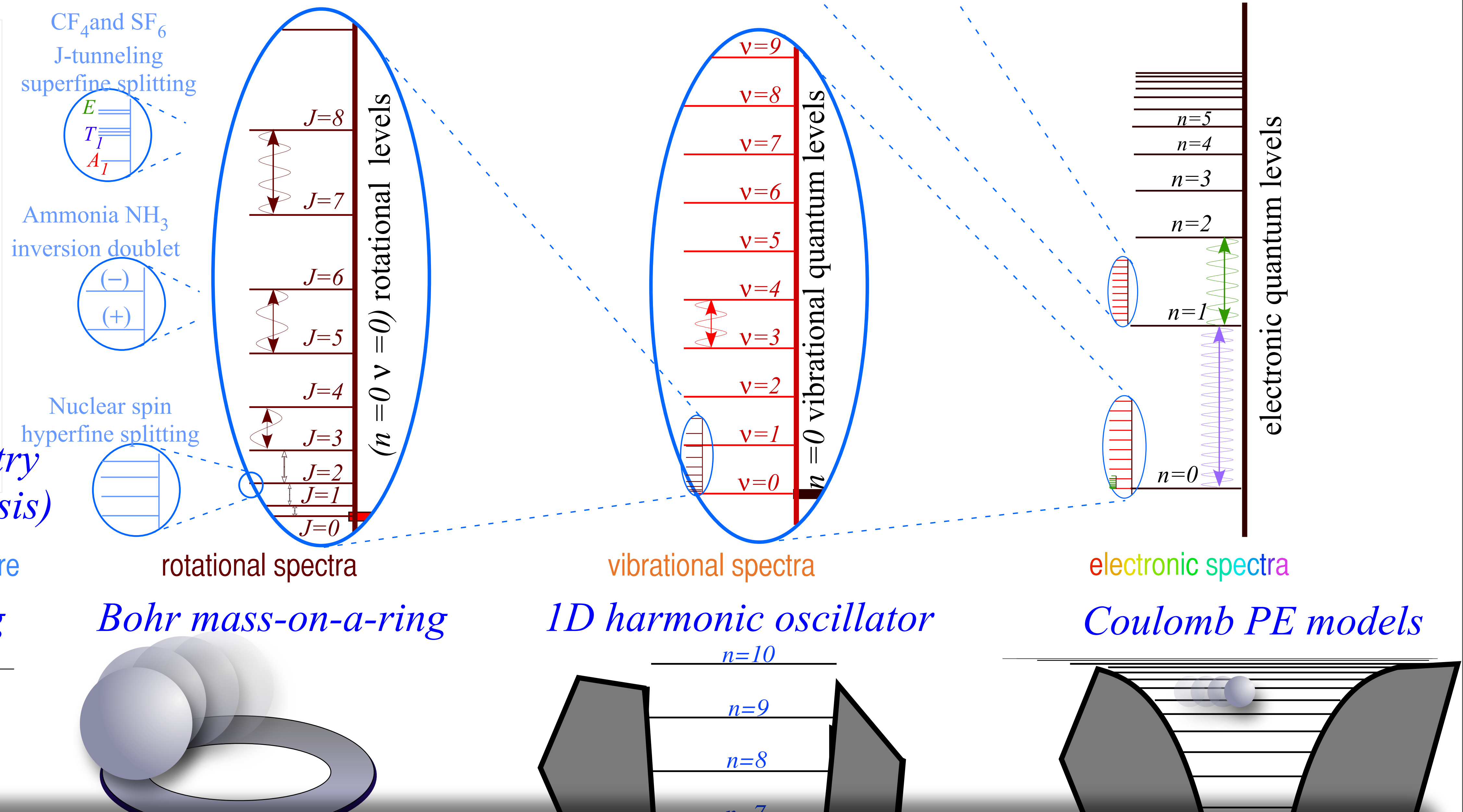
*U(m)*S$_n$ analysis of multi-electron states*



Rotational Energy Surface (RES) analysis of rovibronic tensor spectra

More Advanced Molecular Spectra Models

(Involve symmetry algebraic analysis)



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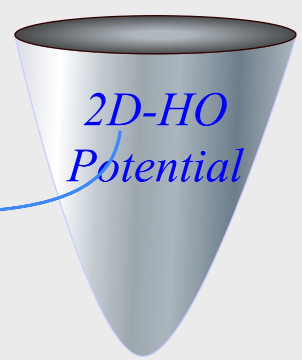


(closely connected)

Rotational Energy Surface (RES) analysis of rovibronic tensor spectra

Lecture 30 ended here

2nd Quantization: Quantizing amplitudes (“photons”,“vibrons”, and “what-ever-ons”)

Analogy with molecular Born-Oppenheimer-Approximate energy levels

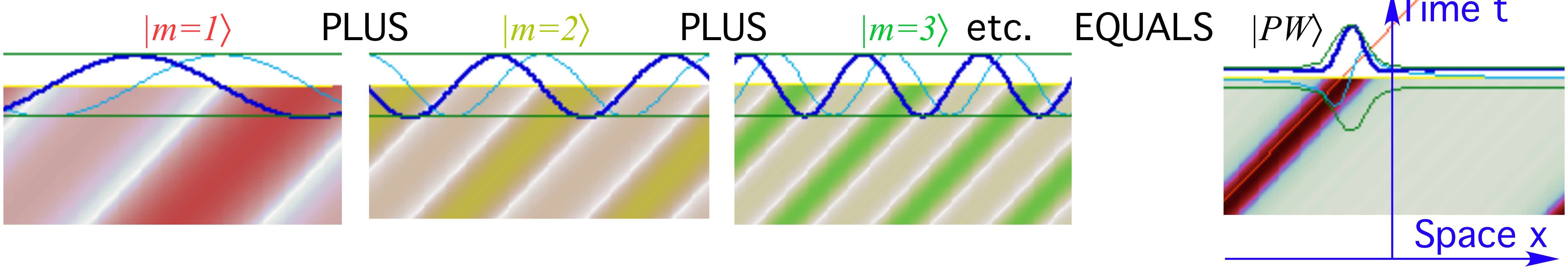
→ *Introducing coherent states (What lasers use)*

Analogy with (ω,k) wave packets

Wave coordinates need coherence

Coherent States: Oscillator Amplitude Packets analogous to Wave Packets

We saw how adding *CW*'s *(Continuous Waves m=1,2,3...)* can make *PW (Pulse Wave)* or *WP (Wave Packet)* that is more like a classical *"thing"* with more *localization* in space *x* and time *t*.



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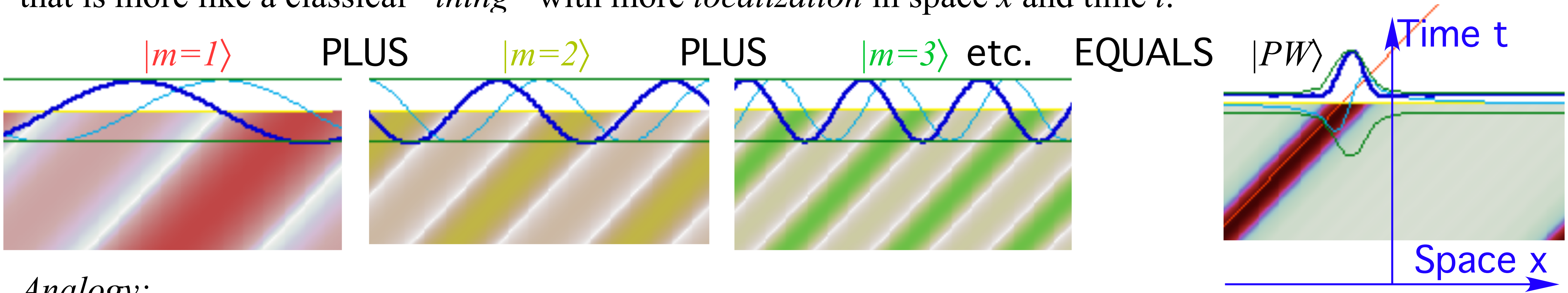
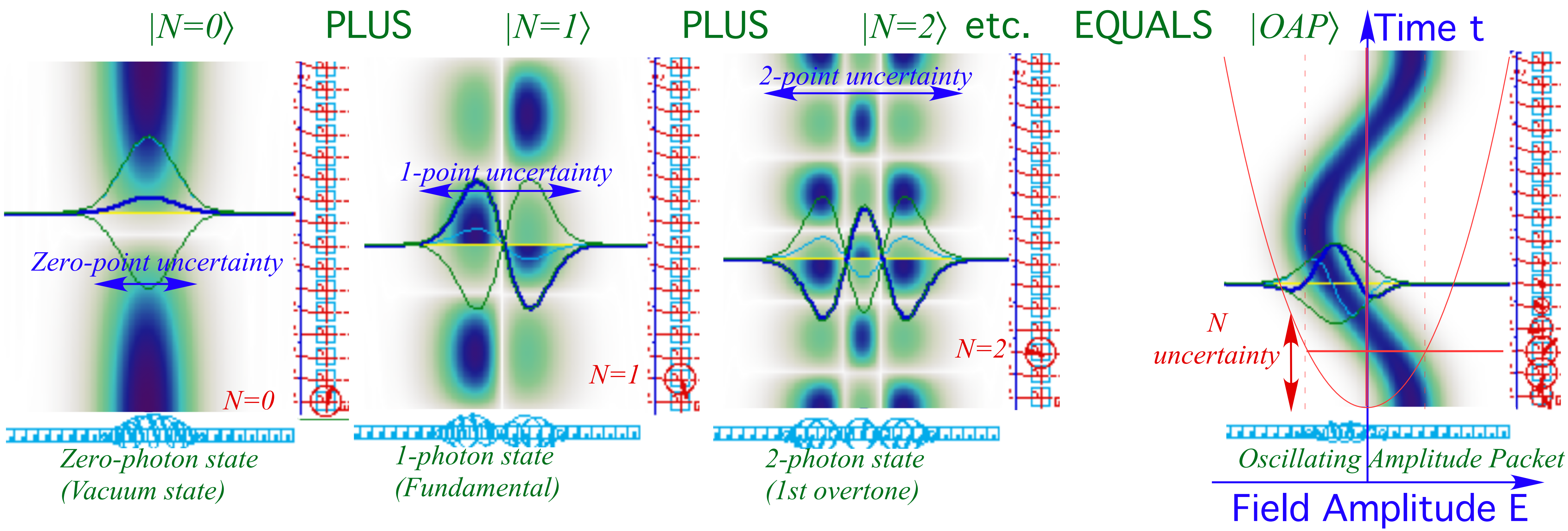


Analogy:

Adding *photons (Quantized amplitude N=0,1,2...)* can make a *CS (Coherent State)* or *OAP (Oscillator Amplitude Packet)* that is more like a *classical wave oscillation* with more *localization* in field amplitude.



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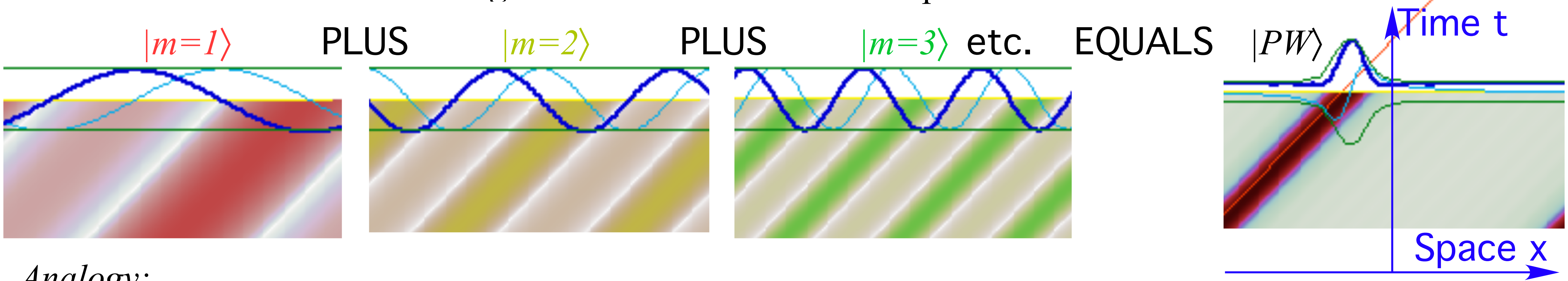

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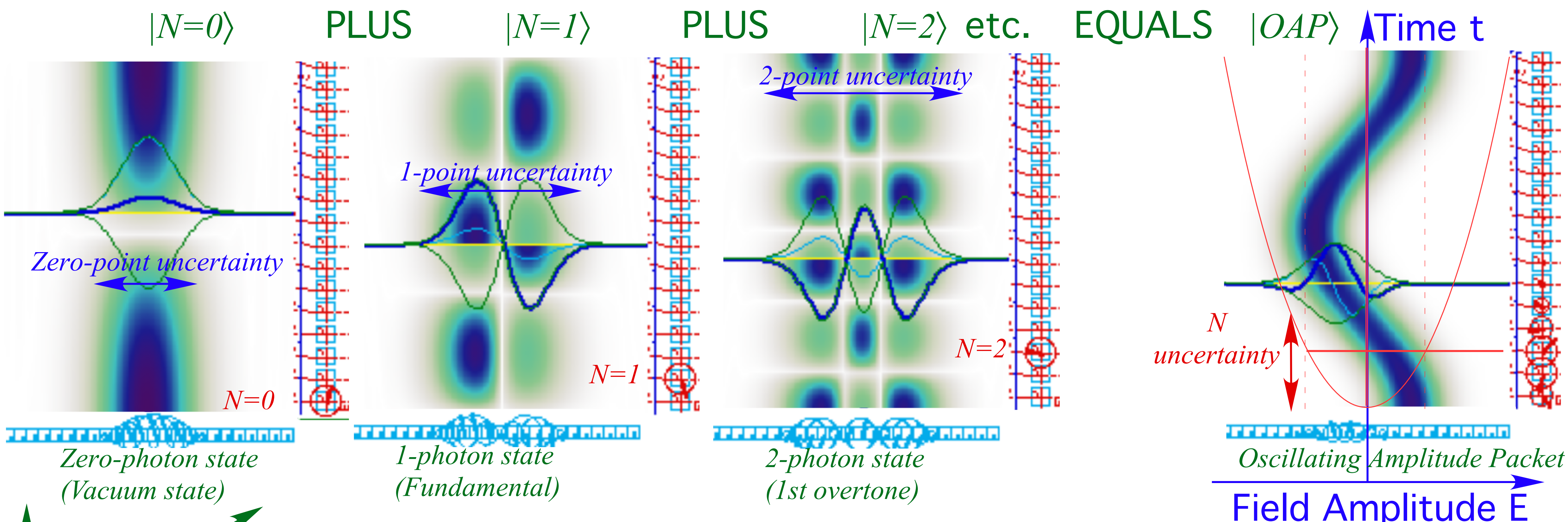

Pure photon states have localized (certain) *N* but *de*localized (*un*certain) amplitude and phase.

OAP states have *de*localized (*un*certain) *N* but more localized (certain) amplitude and phase.

www.uark.edu/ua/pirelli/php/coherent_vs_photon_I.php

Lecture 30 ended here

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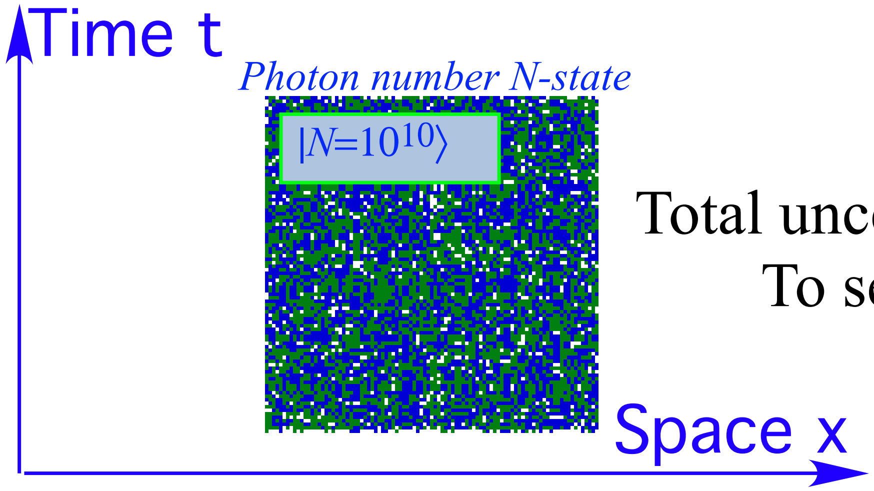
Coherent States(contd.) *Spacetime wave grid is impossible without coherent states*

Pure photon number *N*-states would make useless spacetime coordinates



Total uncertainty of amplitude and phase makes the count pattern a wash.
To see grids *some N-uncertainty is necessary!*

<u>*Coherent States(contd.)*</u> *Spacetime wave grid is impossible without coherent states*

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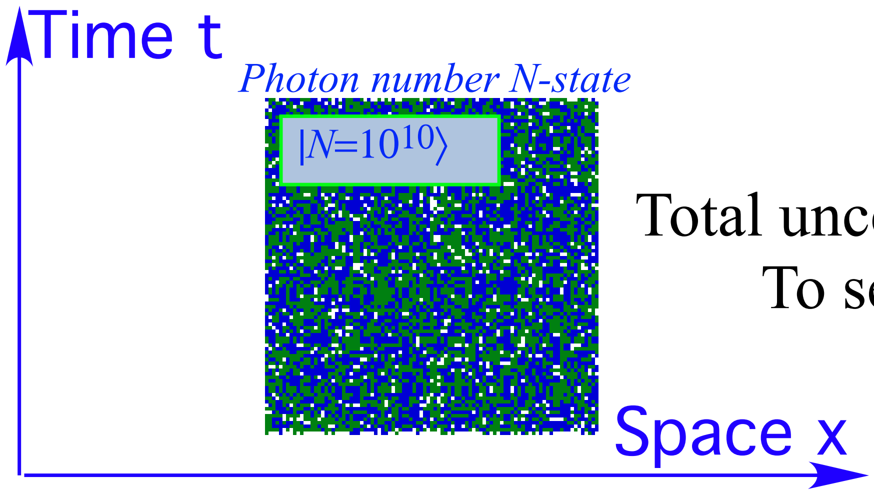
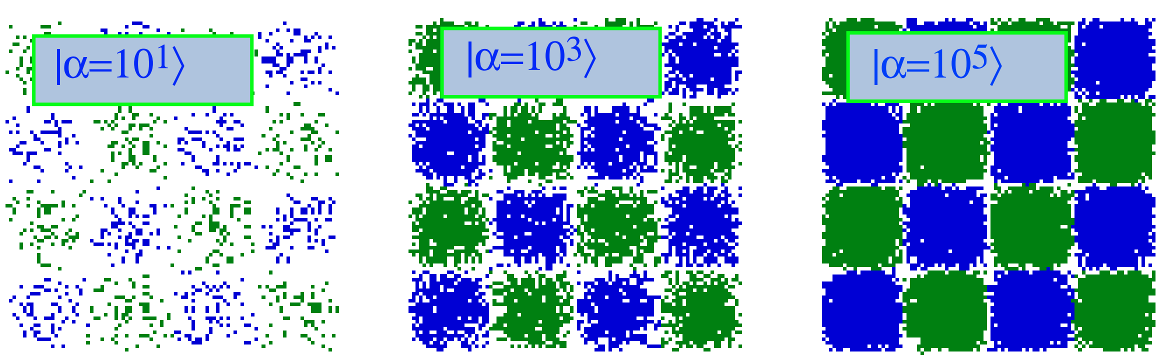


Total uncertainty of amplitude and phase makes the count pattern a wash.

To see grids *some N-uncertainty is necessary!*

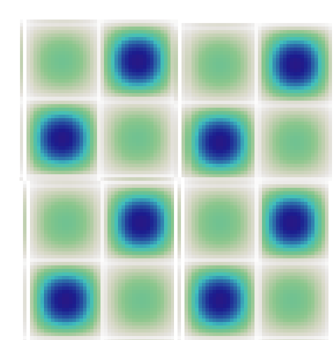
Coherent-α-states are defined by continuous amplitude-packet parameter α whose square is average photon number $\bar{N}=|\alpha|^2$. Coherent-states make better spacetime coordinates for larger $\bar{N}=|\alpha|^2$.

Quantum field coherent α-states



$\bar{N}=100$ $\Delta N=10$ | $\bar{N}=10^6$ $\Delta N=10^3$ | $\bar{N}=10^{10}$ $\Delta N=10^5$

Classical limit



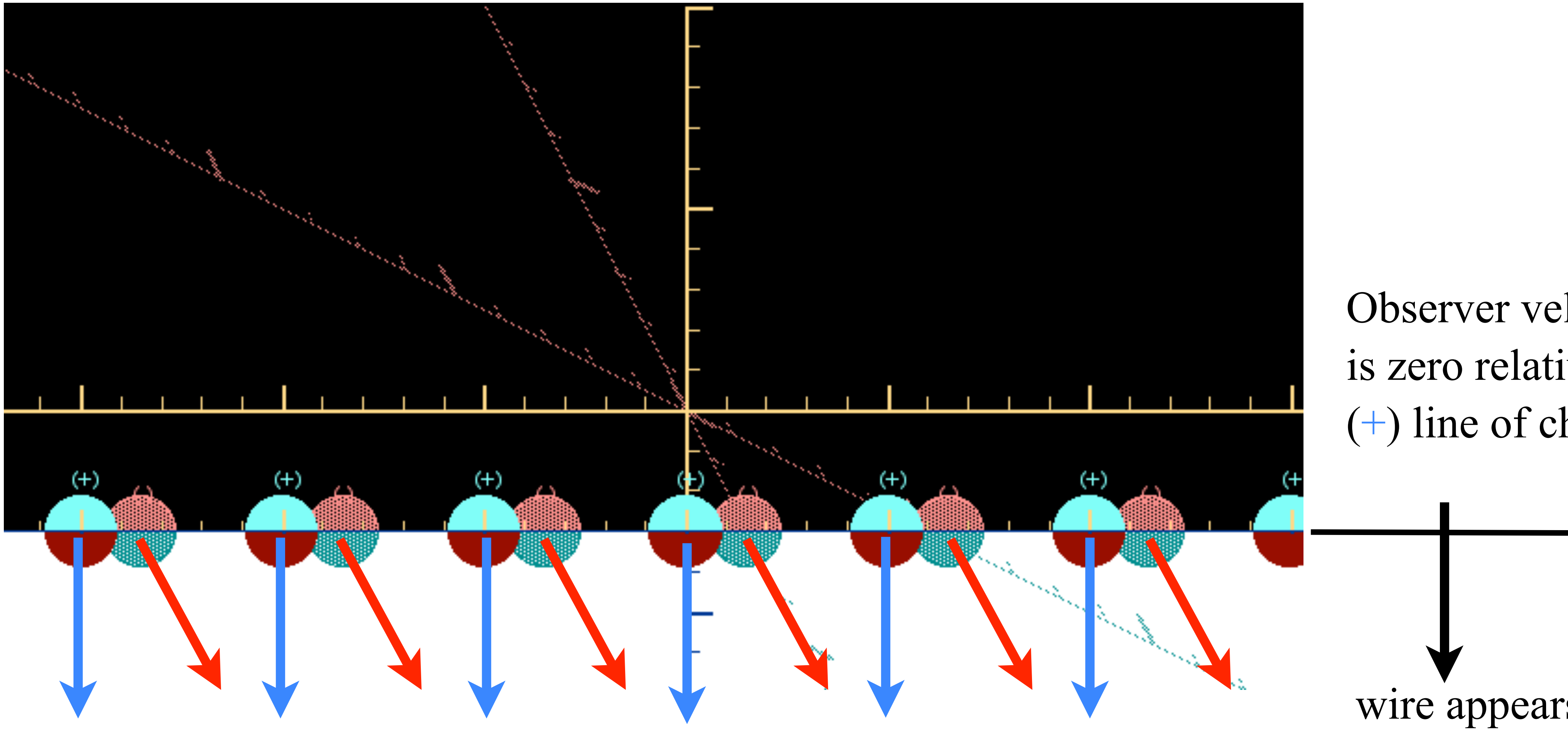
Coherent-state uncertainty in photon number (and mass) varies with amplitude parameter $\Delta N \sim \alpha \sim \sqrt{N}$ so a coherent state with $\bar{N}=|\alpha|^2=10^6$ only has a 1-in-1000 uncertainty $\Delta N \sim \alpha \sim \sqrt{N}=1000$.

Relativistic effects on charge, current, and Maxwell Fields

➔ *Current density changes by Lorentz asynchrony*

Magnetic B-field is relativistic $\sinh\rho$ *1st order-effect*

Relativistic effects on charge, current, and Maxwell Fields



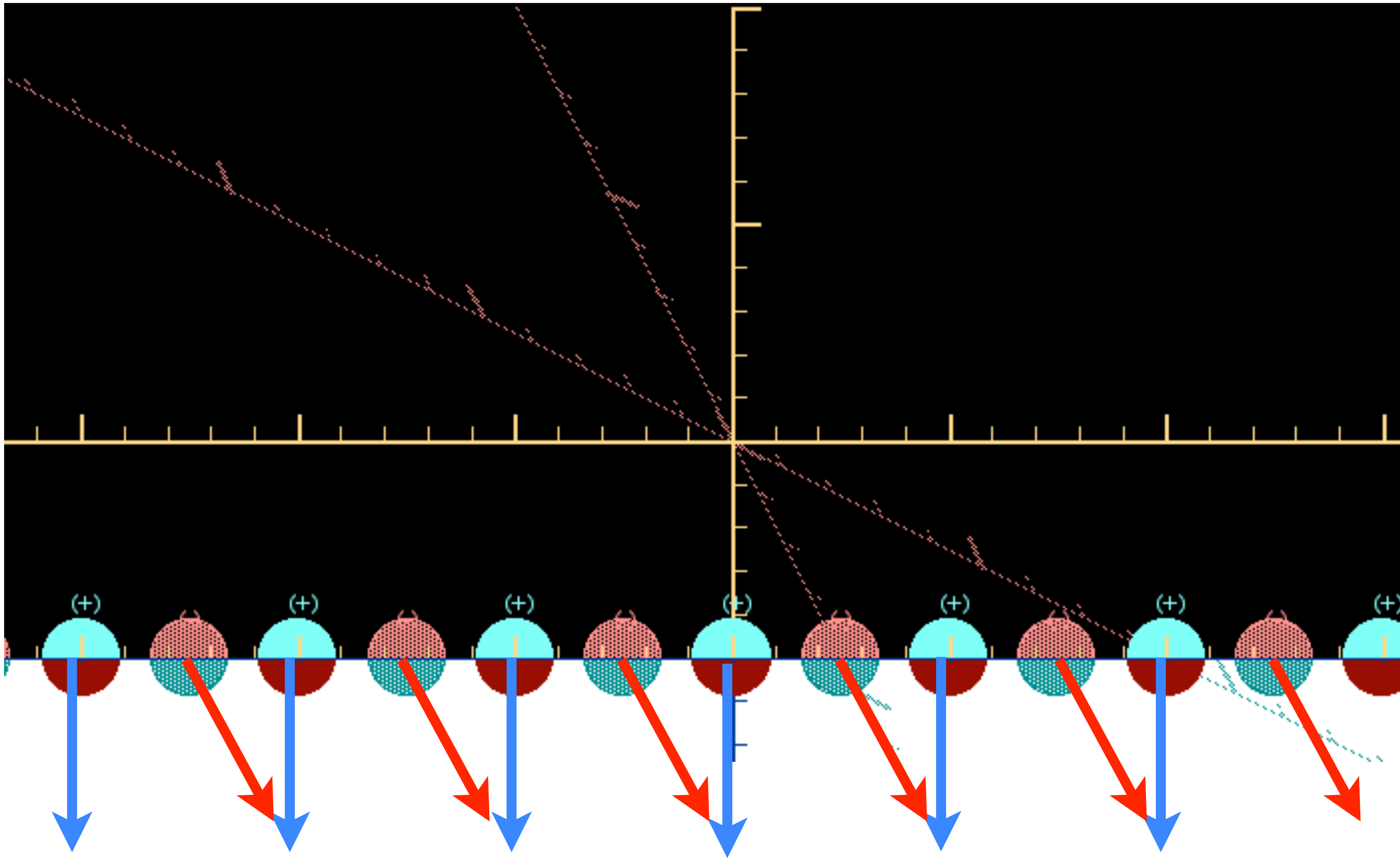
Observer velocity is zero relative to (+) line of charge

wire appears neutral

(+) Charge fixed (-) Charge moving to right (*Negative current density)*

(+) Charge density is Equal to the (-) Charge density

Relativistic effects on charge, current, and Maxwell Fields



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(+) Charge density is Equal to the (-) Charge density (*Zero* $\rho(x,t)=0$)

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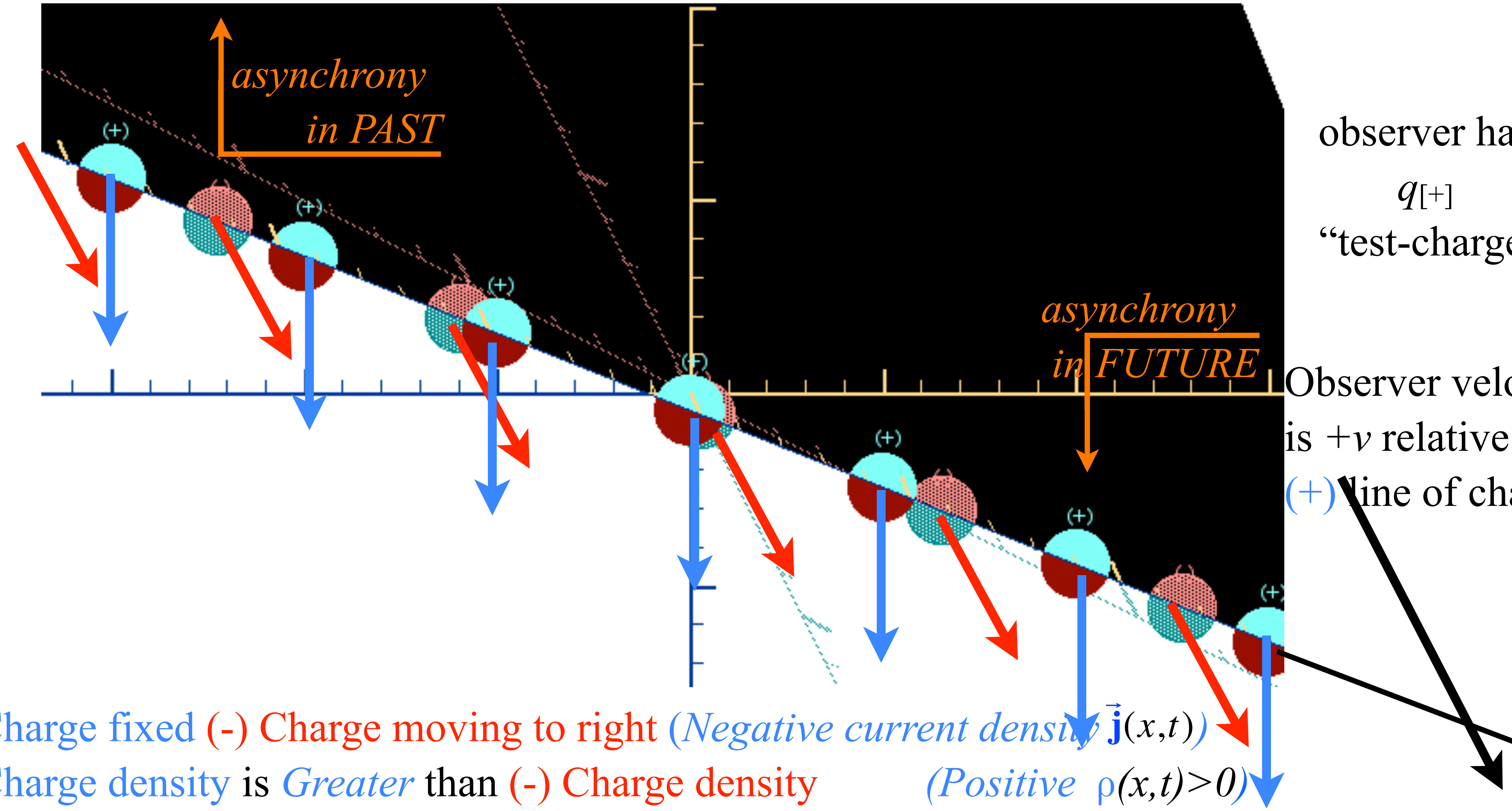
Current density changes by Lorentz asynchrony

Asynchrony due to off-diagonal $\sinh\rho$ (a 1st-order effect)

in Lorentz tranform : $\begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \sim \begin{pmatrix} 1 & v/c \\ v/c & 1 \end{pmatrix}$



observer has $q_{[+]}$ "test-charge"

Observer velocity is $+v$ relative to (+) line of charge

(+) Charge fixed (-) Charge moving to right (*Negative current density* $\vec{\mathbf{j}}(x,t)$)

(+) Charge density is *Greater* than (-) Charge density (*Positive* $\rho(x,t)>0$)

wire appears postive (+) (repulsive to observer $q_{[+]}$)

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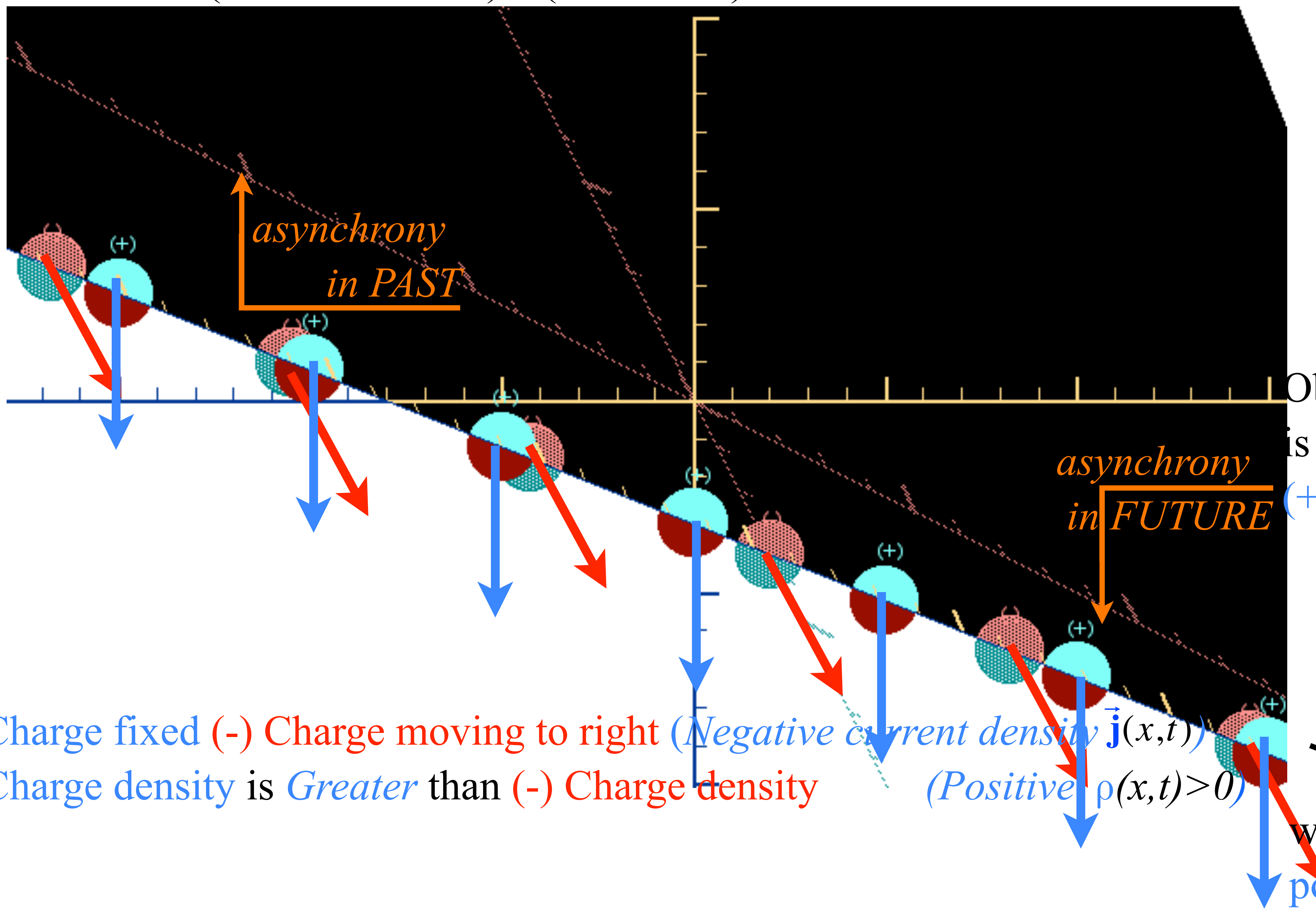

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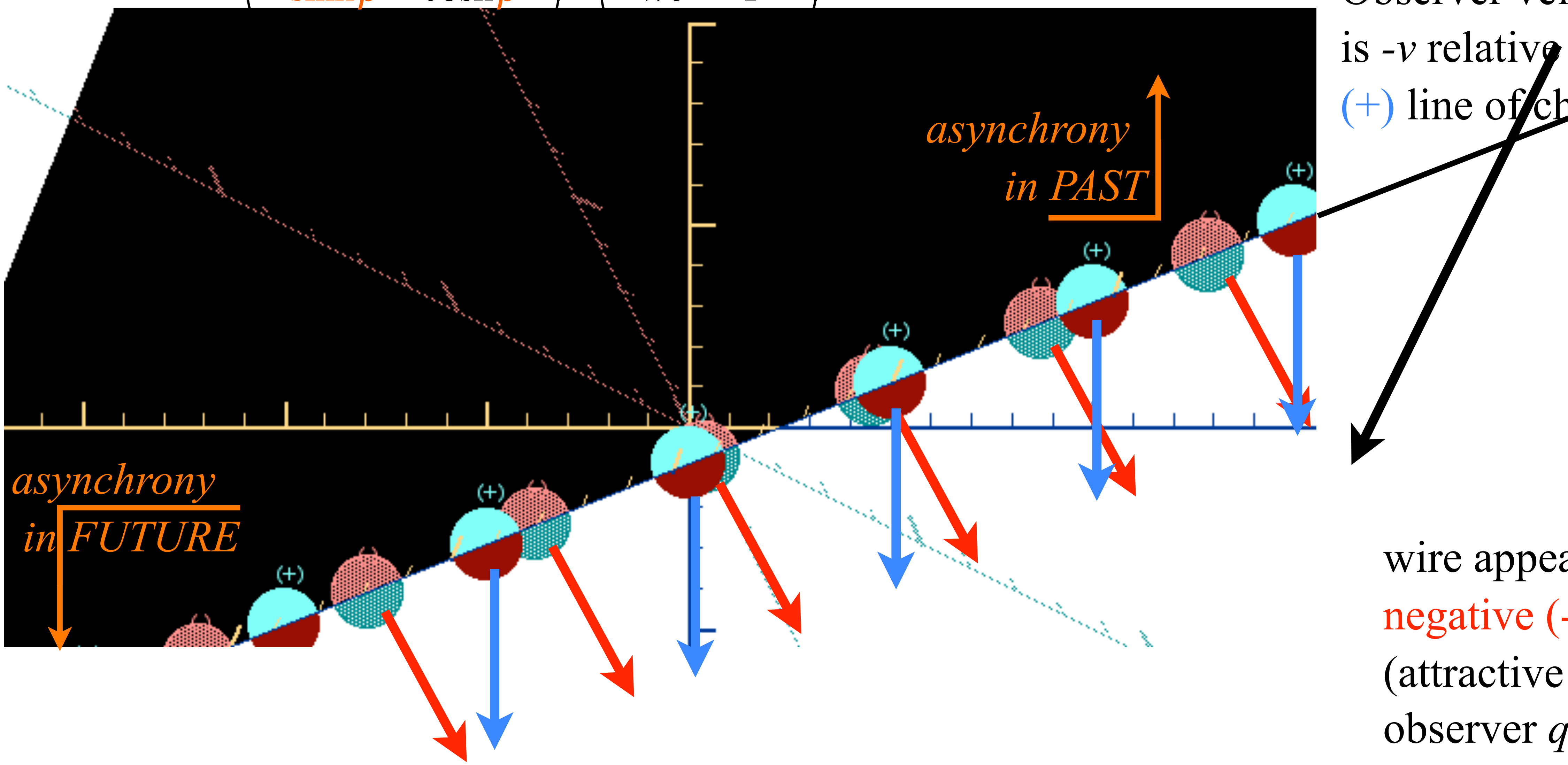
Observer velocity is -*v* relative to (+) line of charge



wire appears negative (-) (attractive to observer $q_{[+]}$)

(+) Charge fixed (-) Charge moving to right (*Negative current density* $\vec{\mathbf{j}}(x,t)$)

(+) Charge density is *Less* than (-) Charge density (*Negative* $\rho(x,t)<0$)

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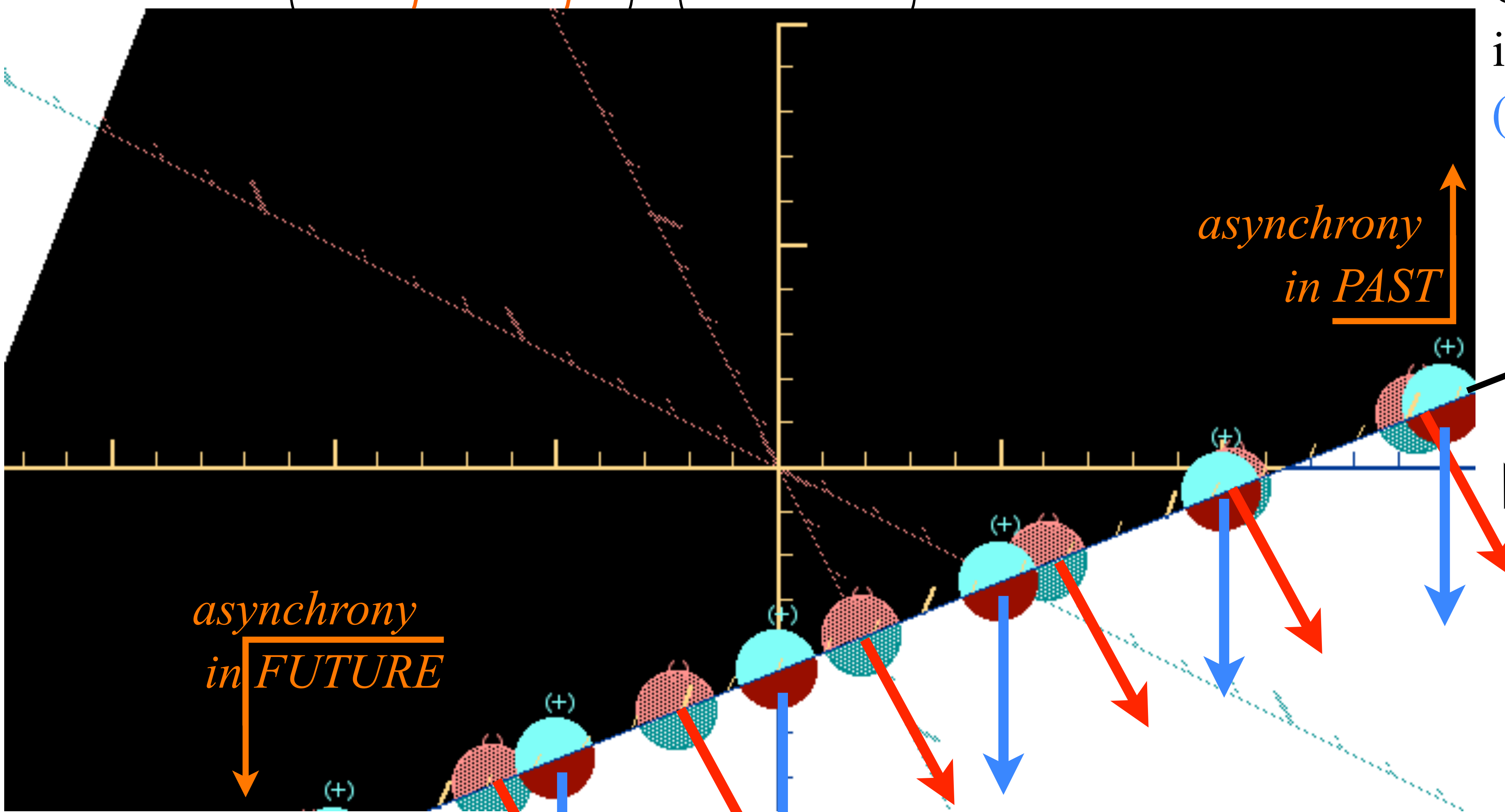
Observer velocity is -v relative to (+) line of charge



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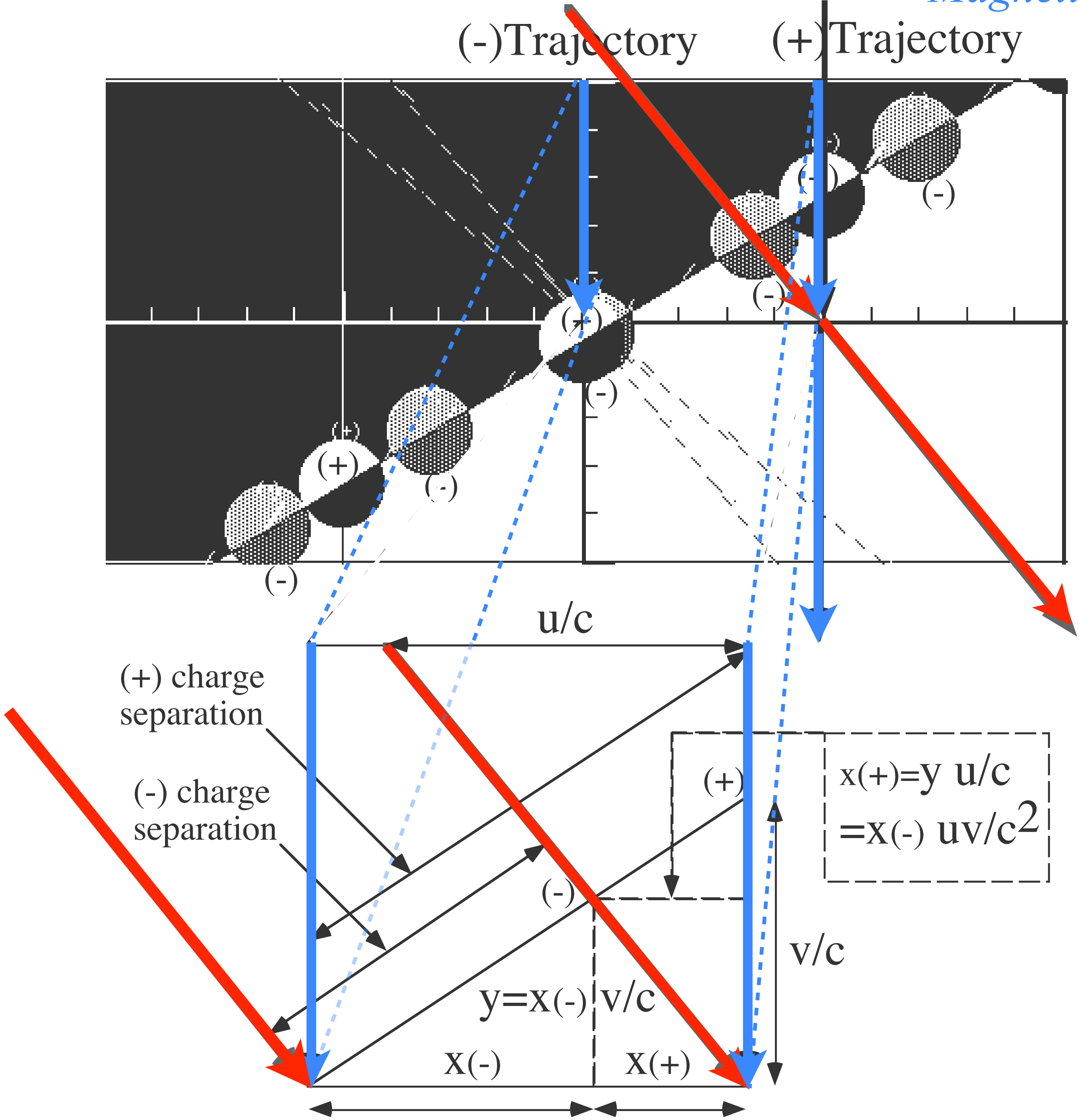
Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz asynchrony

➡ *Magnetic B-field is relativistic* $\sinh\rho$ *1st order-effect*

Magnetic B-field is relativistic sinhρ *1st order-effect*



$$\frac{\rho(-)}{\rho(+)} = \frac{(+)\text{ charge separation}}{(-)\text{ charge separation}} = \frac{x(+)+x(-)}{x(-)}$$

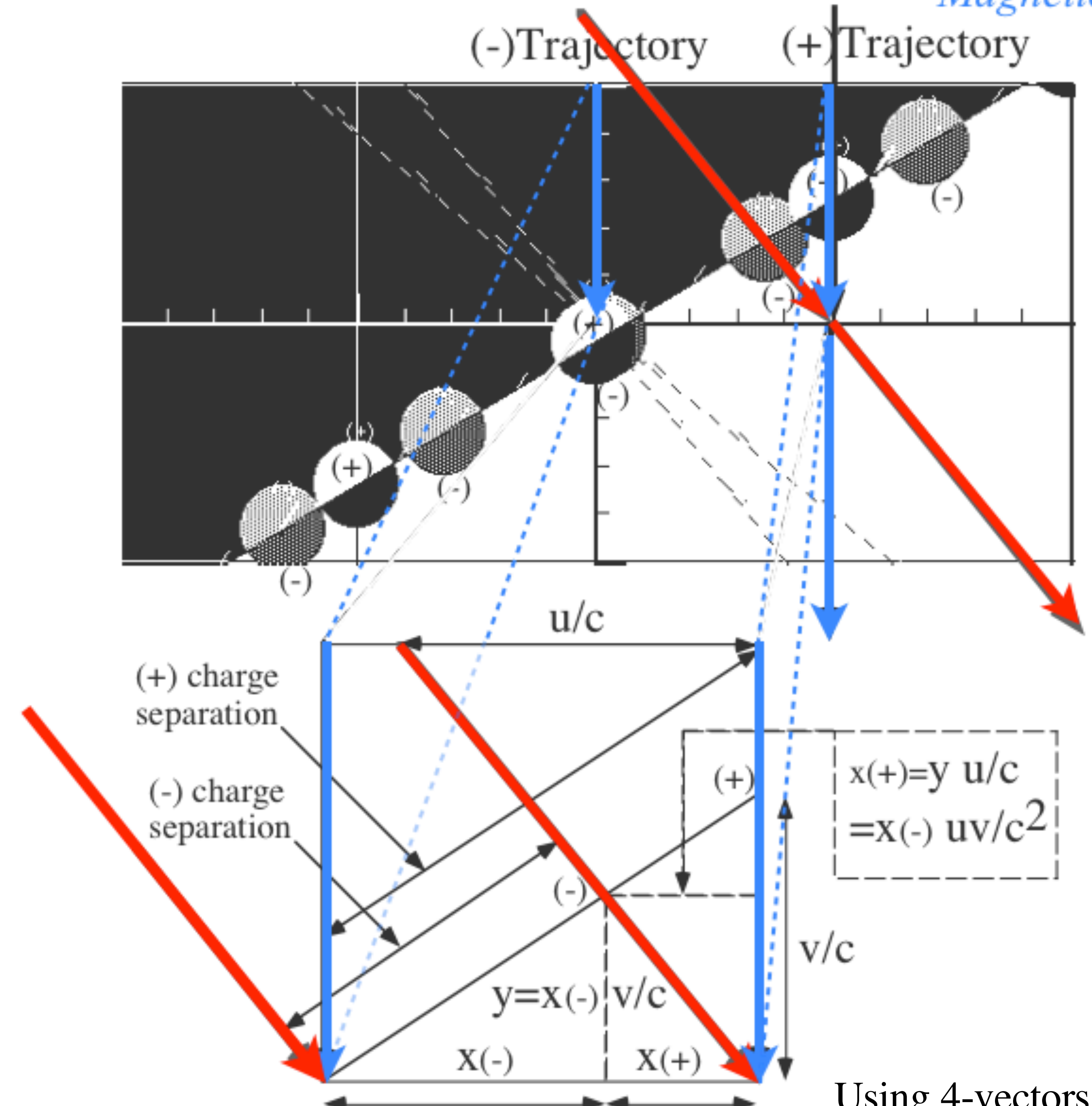
$$\frac{\rho(-)}{\rho(+)} = \frac{x(+)}{x(-)} + 1 = \frac{uv}{c^2} + 1$$

$$\rho(+) - \rho(-) = \rho(+)\left(1 - \frac{\rho(-)}{\rho(+)}\right) = -\frac{uv}{c^2}\rho(+)$$

Unit square: (u/c) /1 = x(+)/y

(v/c) /1 = y/x(-)

Magnetic B-field is relativistic sinhρ 1st order-effect



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Using 4-vectors to EL Transform (charge-current)=($c\rho$, **j**)

$$\begin{pmatrix} c\rho' \\ j_{x'} \\ j_{y'} \\ j_{z'} \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho & \cdot & \cdot \\ \sinh\rho & \cosh\rho & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & 1 \end{pmatrix} \begin{pmatrix} c\rho \\ j_x \\ j_y \\ j_z \end{pmatrix}$$

The electric force field **E** of a charged line varies inversely with radius. The Gauss formula for force in mks units :

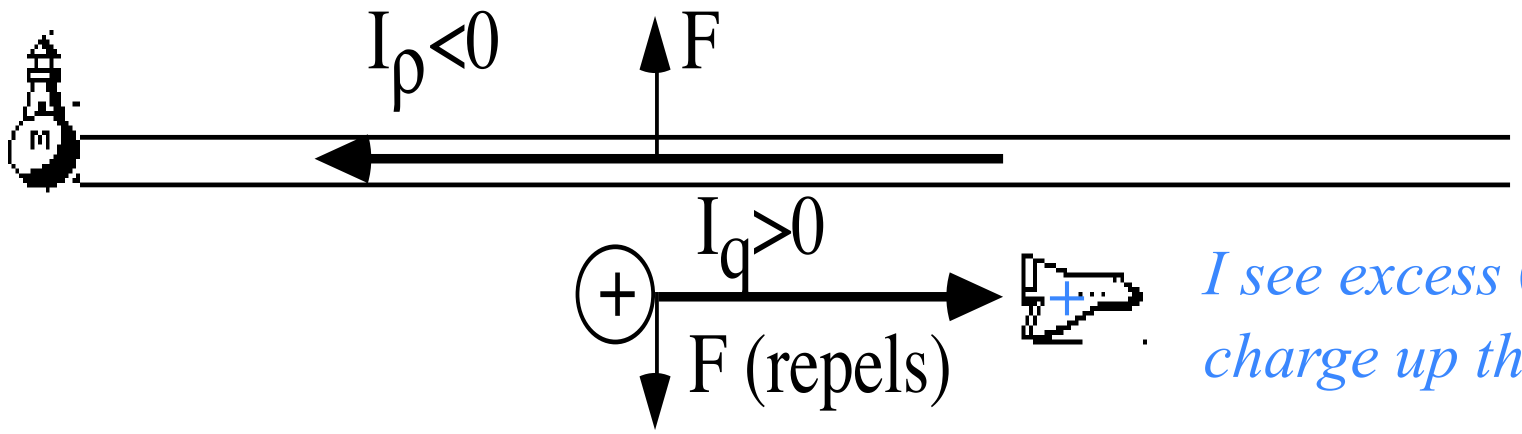
$$F = qE = q\left[\frac{1}{4\pi\varepsilon_0}\frac{2\rho}{r}\right] \quad , \quad \text{where:} \quad \frac{1}{4\pi\varepsilon_0} = 9\times 10^9 \frac{N\cdot m^2}{Coul.}$$

$$F = qE = q\left[\frac{1}{4\pi\varepsilon_0}\frac{2}{r}\left(-\frac{uv}{c^2}\rho(+)\right)\right] = -\frac{2\,qv\,\rho(+)u}{4\pi\varepsilon_0 c^2\, r} = -2\times 10^{-7}\frac{I_q\, I_\rho}{r}$$

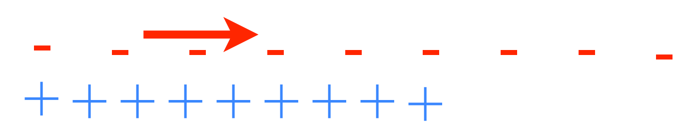
$1/4\pi\varepsilon_0 = 9\cdot 10^9$

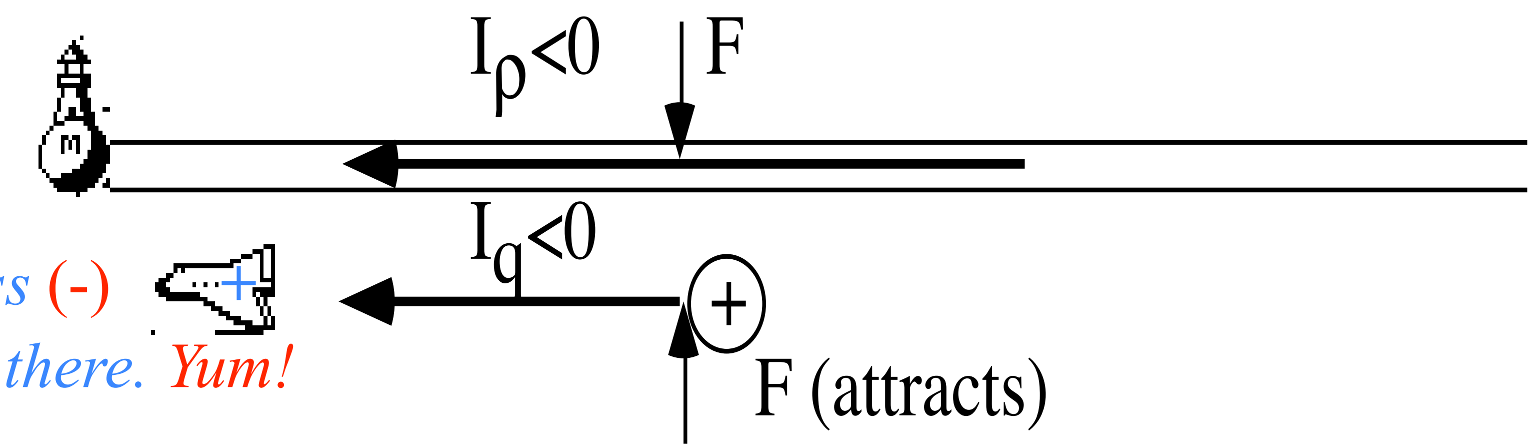
$c^2 = 9\cdot 10^{-16}$

$1/(4\pi\varepsilon_0 c^2) = 10^{-7}$

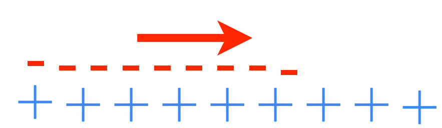


I see excess (+) charge up there. Yuk!





I see excess (-) charge up there. Yum!



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