Relativity of 1\textsuperscript{st} Quantization and electromagnetic fields

(Ch. 2-5 of CMwBang-Unit 8   Ch. 6 of QTforCA Unit 2 )

1\textsuperscript{st} Quantization: Quantizing phase variables $\omega$ and $k$

Understanding how quantum transitions require “mixed-up” states

Closed cavity vs ring cavity

2\textsuperscript{nd} Quantization: Quantizing amplitudes (“photons”, “vibrons”, and “what-ever-ons”)

Analogy with molecular Born-Oppenheimer-Approximate energy levels

Introducing coherent states (What lasers use)

Analogy with $(\omega,k)$ wave packets

Wave coordinates need coherence

Relativistic effects on charge, current, and magnetic fields

Current density changes by Lorentz \textit{asynchrony}

Magnetic B-field is relativistic $\sinh \rho$ 1\textsuperscript{st} order-effect
1st Quantization: Quantizing phase variables $\omega$ and $k$

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Closed cavity vs ring cavity
Quantized $\omega$ and $k$  

**Counting wave kink numbers**

If everything is made of waves then we expect quantization of everything because waves only thrive if integral numbers $n$ of their “kinks” fit into whatever structure (box, ring, etc.) they’re supposed to live. The numbers $n$ are called quantum numbers.

_OK box quantum numbers:_

- $n=1$
- $n=2$
- $n=3$
- $n=4$

(+ integers only)

_Some NOT OK numbers:_

- $n=0.67$
- $n=1.7$
- $n=2.59$
- $n=4$

too fat!  

too thin!  

wrong color again!  

misfits...

...not tolerated!

NOTE: We’re using “false-color” here.

This doesn’t mean a system’s energy can’t vary continuously between “OK” values $E_1$, $E_2$, $E_3$, $E_4$, ...
Quantized \( \omega \) and \( k \)  

*Counting wave kink numbers*

If everything is made of waves then we expect *quantization* of everything because waves only thrive if *integral* numbers \( n \) of their “kinks” fit into whatever structure (box, ring, etc.) they’re supposed to live. The numbers \( n \) are called *quantum numbers*.

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- \( n=4 \) misfits...

...not tolerated !

*NOTE: We’re using “false-color” here.*

This doesn’t mean a system’s energy can’t vary *continuously* between “OK” values \( E_1, E_2, E_3, E_4, \ldots \). In fact its state can be a linear combination of any of the “OK” waves \( |E_1>, |E_2>, |E_3>, |E_4>, \ldots \).
1st Quantization: Quantizing phase variables $\omega$ and $k$

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That’s the only way you get any light in or out of the system to “see” it.

\[ |E_4> \]

\[ frequency \ \hbar \omega_{32} = E_3 - E_2 \]

\[ frequency \ \hbar \omega_{21} = E_2 - E_1 \]
Quantized $\omega$ and $k$

Counting wave kink numbers

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In fact its state can be a linear combination of any of the “OK” waves $|E_1>$, $|E_2>$, $|E_3>$, $|E_4>$, ...

That’s the only way you get any light in or out of the system to “see” it.

$|E_4>$

$|E_3>$

$|E_2>$

$|E_1>$

These eigenstates are the only ways the system can “play dead”...

... “sleep with the fishes”...
Consider two lowest $E$-states by themselves.
Consider two lowest $E$-states by themselves in time

$$e^{-i\omega_2 t} |E_2\rangle$$

$$e^{-i\omega_1 t} |E_1\rangle$$
Consider two lowest $E$-states by themselves in time

Now combine (add) them
Consider two lowest $E$-states by themselves in time

\[ e^{-i\omega_2 t} |E_2\rangle \]

Now combine (add) them and let time roll!

\[ \frac{\left( e^{-i\omega_1 t} |E_1\rangle + e^{-i\omega_2 t} |E_2\rangle \right)}{\sqrt{2}} \]

\[ \frac{(|E_1\rangle + |E_2\rangle)}{\sqrt{2}} \]
Consider two lowest $E$-states by themselves in time
\[ e^{-i\omega_1 t} |E_1\rangle \]
\[ e^{-i\omega_2 t} |E_2\rangle \]
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Consider two lowest E-states by themselves in time
\[ e^{-i\omega_1 t} |E_1\rangle \]
\[ e^{-i\omega_2 t} |E_2\rangle \]
Now combine (add) them and let time roll!
\[ \left( e^{-i\omega_1 t} |E_1\rangle + e^{-i\omega_2 t} |E_2\rangle \right) / \sqrt{2} \]

\[ (|E_1\rangle + |E_2\rangle) / \sqrt{2} \]
1st Quantization: Quantizing phase variables $\omega$ and $k$

Understanding how quantum transitions require “mixed-up” states

Closed cavity vs ring cavity
Quantized $\omega$ and $k$  

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If everything is made of waves then we expect quantization of everything because waves only thrive if integral numbers $n$ of their “kinks” fit into whatever structure (box, ring, etc.) they’re supposed to live. The numbers $n$ are called quantum numbers.

**OK box quantum numbers:**

<table>
<thead>
<tr>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

(+ integers only)

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- $n=1.7$
  - too thin!
- $n=2.59$
  - wrong color again!
- $n=4$
  - misfits...
  - ...not tolerated!

**NOTE:** We’re using “false-color” here.

Rings tolerate a zero (kinkless) quantum wave but require $\pm$ integral wave number.

**OK ring quantum numbers:**

<table>
<thead>
<tr>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

($\pm$ integral number of wavelengths)

Bohr’s models of atomic spectra (1913-1923) are beginnings of quantum wave mechanics built on Planck-Einstein (1900-1905) relation $E=h\nu$. DeBroglie relation $p=\hbar/\lambda$ comes around 1923.
Consider two lowest $E$-states by themselves:

$$|E_{m=+1}\rangle$$

$$|E_{m=0}\rangle$$
Consider two lowest $E$-states by themselves

\[ |E_{m=0}\rangle \]

\[ |E_{m=+1}\rangle \]

Now combine (add) them and let time roll!

\[
\frac{e^{-i\omega_0 t} |E_0\rangle + e^{-i\omega_{+1} t} |E_{+1}\rangle}{\sqrt{2}}
\]
Consider two lowest E-states by themselves
\[ |E_{m=0}\rangle \]
\[ |E_{m=+1}\rangle \]

Now combine (add) them and let time roll!
\[
\left( e^{-i\omega t} |E_0\rangle + e^{-i\omega t} |E_1\rangle \right) / \sqrt{2}
\]

(Just moves forward rigidly)
Consider two degenerate $E$-states by themselves

$|E_{m=+1}\rangle$

$|E_{m=-1}\rangle$
Consider two degenerate $E$-states by themselves

$$|E_{m=+1}\rangle$$

Now combine (add) them and let time roll!

$$\left( e^{-i\omega t}|E_{-1}\rangle + e^{-i\omega t}|E_{+1}\rangle \right) / \sqrt{2}$$

(Group wave is stationary but phase can move or “gallop”)
Consider more than two E-states combined...
2nd Quantization: Quantizing amplitudes ("photons", "vibrons", and "what-ever-ons")
Analogy with molecular Born-Oppenheimer-Approximate energy levels
Introducing coherent states (What lasers use)
Analogy with $(\omega, k)$ wave packets
Wave coordinates need coherence
Quantized Amplitude Counting “photon” number

Planck’s relation $E = N\hbar\nu$ began as a tenative axiom to explain low-T light. Then he tried to disavow it! Einstein picked it up in his 1905 paper. Since then its use has grown enormously and continues to amaze, amuse (or bewilder) all who study it. A current view is that it represents the quantization of optical field amplitude. We picture this below as $N$-photon wave states for each box-mode of $m$ wave kinks.

Quantum field definitions have been called “2nd quantization” or “wave-waves”

NOTE: We’re using “false-color” here.

These are the fundamental “zero-point” or “vacuum” levels
Quantized Amplitude Counting “photon” number

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Quantum field definitions have been called “2nd quantization” or “wave-waves”

\[ N = 0 \] These are the fundamental “zero-point” or “vacuum” levels

\[ N = 1 \] These are the 1st excited or fundamental transition levels
Quantized Amplitude Counting “photon” number

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Quantum field definitions have been called “2nd quantization” or “wave-waves”

$N_1=4$ red photons
$N_1=3$ red photons
$N_1=2$ red photons
$N_1=1$ red photon
$N_1=0$ $m=1$

$N_2=2$ green photons
$N_2=1$ green photon
$N_2=0$

$N_3=1$ blue photon
$N_3=0$

$N_4=1$ violet photon
$N_4=0$

These are the 1st excited or fundamental transition levels

These are the 2nd excited levels

These are the fundamental “zero-point” or “vacuum” levels

Quantized Wavenumber (“kink” or momentum number)

NOTE: We’re using “false-color” here.
Quantized \textit{Amplitude} Counting “photon” number

Planck’s relation $E=N\hbar \nu$ began as a tenative axiom to explain low-T light. Then he tried to disavow it! Einstein picked it up in his 1905 paper. Since then its use has grown enormously and continues to amaze, amuse (or bewilder) all who study it. A current view is that it represents the \textit{quantization} of optical field \textit{amplitude}. We picture this below as $N$-\textit{photon} wave states for each box-mode of $m$ wave kinks.
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**Quantum field definitions have been called “2nd quantization” or “wave-waves”**

*NOTE: We’re using “false-color” here.*
Quantum numbers \( N \) of field or \( n, m, \ldots \) of modes are invariants and not changed by boosting velocity. Each mode fundamental frequency \( \nu_n = n\nu_1 \) and its \( N \)-photon multiples belong to invariant hyperbolas.

Boosted observers see distorted frequencies and lengths, but will agree on the numbers \( n \) and \( N \) of mode nodes and photons.

This is how light waves can “fake” some of the properties of classical “things” such as invariance or object permanence.

It takes at least TWO CW’s to achieve such invariance. One CW is not enough and cannot have non-zero invariant \( N \). Invariance is an interference effect that needs at least two-to-tango!
2nd Quantization: Quantizing amplitudes ("photons", "vibrons", and "what-ever-ons")

Analogy with molecular Born-Oppenheimer-Approximate energy levels

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Analogy with \((\omega,k)\) wave packets

Wave coordinates need coherence
A sketch of modern molecular spectroscopy

The frequency hierarchy

Radio-frequency | Microwave to far-infrared | Infrared | Near-infrared to visible to ultraviolet to X-ray

- **Fine structure**
  - CF₄ and SF₆ (J-tunneling superfine splitting)
  - Ammonia NH₃ inversion doublet
  - Nuclear spin hyperfine splitting

- **Rotational spectra**
  - CO₂ MICROWAVE
    - B₀(1/λ)=0.2 cm⁻¹
    - λ=5 cm
    - ν=60 MHz

- **Vibrational spectra**
  - CO₂ laser
    - INFRARED
      - ν=30 THz
      - λ=10 μm
      - 1/λ=1000 cm⁻¹
      - Eᵥ=0.124 eV

- **Electronic spectra**
  - Typical VISIBLE
    - ν=600 THz
    - λ=2×10⁻⁶ m
    - 1/λ=2×10⁶ cm⁻¹
    - λ=0.5 μm
    - 500 nm
    - 5000 Å
    - Eᵥ=2.48 eV

- **Wavenumber**
  - cm⁻¹
  - 10² m⁻¹

- **Energy**
  - eV
  - electron Volts

From Fig. 6.5.5.
Principles of Symmetry, Dynamics, and Spectroscopy
Example of frequency hierarchy for 16µm spectra of CF₄ (Freon-14)
W.G.Harter
Fig. 32.7
Springer Handbook of Atomic, Molecular, & Optical Physics
Gordon Drake Editor (2005)
### Units of frequency (Hz), wavelength (m), and energy (eV)

<table>
<thead>
<tr>
<th>CLASS</th>
<th>FREQUENCY</th>
<th>WAVELENGTH</th>
<th>ENERGY</th>
</tr>
</thead>
<tbody>
<tr>
<td>γ</td>
<td>300 EHz</td>
<td>1 pm</td>
<td>1.24 MeV</td>
</tr>
<tr>
<td>HX</td>
<td>30 EHz</td>
<td>10 pm</td>
<td>124 keV</td>
</tr>
<tr>
<td>SX</td>
<td>3 EHz</td>
<td>100 pm</td>
<td>12.4 keV</td>
</tr>
<tr>
<td>EUV</td>
<td>300 PHz</td>
<td>1 nm</td>
<td>1.24 keV</td>
</tr>
<tr>
<td>NUV</td>
<td>30 PHz</td>
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<td>124 eV</td>
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<td>300 THz</td>
<td>1 μm</td>
<td>1.24 eV</td>
</tr>
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<td>MIR</td>
<td>30 THz</td>
<td>10 μm</td>
<td>124 meV</td>
</tr>
<tr>
<td>FIR</td>
<td>3 THz</td>
<td>100 μm</td>
<td>12.4 meV</td>
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<tr>
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<td>300 GHz</td>
<td>1 mm</td>
<td>1.24 meV</td>
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<tr>
<td>SHF</td>
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<td>UHF</td>
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<td>1 dm</td>
<td>12.4 μeV</td>
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<td>VHF</td>
<td>30 MHz</td>
<td>1 m</td>
<td>1.24 μeV</td>
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<td>LF</td>
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<td>VLF</td>
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<td>100 km</td>
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<tr>
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<td>1.24 peV</td>
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<td>SLF</td>
<td>30 Hz</td>
<td>10 Mm</td>
<td>124 feV</td>
</tr>
<tr>
<td>ELF</td>
<td>3 Hz</td>
<td>100 Mm</td>
<td>12.4 feV</td>
</tr>
</tbody>
</table>

**Exa:** $10^{18}$  
**Peta:** $10^{15}$  
**Tera:** $10^{12}$  
**Giga:** $10^9$  
**Mega:** $10^6$  
**Kilo:** $10^3$  
**Milli:** $10^{-3}$  
**Micro:** $10^{-6}$  
**Nano:** $10^{-9}$  
**Pico:** $10^{-12}$  
**Femto:** $10^{-15}$  

**atto:** $10^{-18}$

---

**From:** Electromagnetic Spectrum  
**Wikipedia Commons** (2013)
Simple Molecular Spectra Models

- 2-well tunneling
- Bohr mass-on-ring
- 1D harmonic oscillator
- Coulomb PE models

Fine structure

Rotational spectra

Vibrational spectra

Electronic spectra

Ammonia NH$_3$

J-tunneling superfine splitting

Nuclear spin hyperfine splitting

CF$_4$ and SF$_6$
More Advanced Molecular Spectra Models
(Use symmetry group theory)

2-state U(2)-spin tunneling models
3D R(3)-rotor and D-function lab-body wave models
2D harmonic oscillator and U(2) 2nd quantization
U(m)\(*S_n\) analysis of multi-electron states

Rotational Energy Surface (RES) analysis of rovibronic tensor spectra
More Advanced Molecular Spectra Models

(Involve symmetry algebraic analysis)

2-well tunneling

2-state U(2)-spin and quasi-spin tunneling models

3D R(3)-rotor and D-function lab-body wave models

2D harmonic oscillator and U(2) 2nd quantization

U(m) * S_n analysis of multi-electron states

Rotational Energy Surface (RES) analysis of rovibronic tensor spectra

2D-HO Potential

CF_4 and SF_6 J-tunneling superfine splitting

Ammonia NH_3 inversion doublet

Nuclear spin hyperfine splitting

Bohr mass-on-a-ring

1D harmonic oscillator

Coulomb PE models

rotational spectra

vibrational spectra

electronic spectra

(\text{n=0, v=0)} rotational levels

(n=0, v=0) vibrational levels

(\text{n=0}) electronic quantum levels

\text{electronic quantum levels}

Wednesday, March 12, 2014
2nd Quantization: Quantizing amplitudes ("photons", "vibrons", and "what-ever-ons")
Analogy with molecular Born-Oppenheimer-Approximate energy levels
Introducing coherent states (What lasers use)
Analogy with (ω,k) wave packets
Wave coordinates need coherence
**Coherent States: Oscillator Amplitude Packets** analogous to **Wave Packets**

We saw how adding CW’s (Continuous Waves $m=1,2,3...$) can make PW (Pulse Wave) or WP (Wave Packet) that is more like a classical "thing" with more localization in space $x$ and time $t$. 

\[ |m=1\rangle \text{ PLUS } |m=2\rangle \text{ PLUS } |m=3\rangle \text{ etc. EQUALS } |PW\rangle \]

---

**Time $t$**

**Space $x$**
2nd Quantization: Quantizing amplitudes ("photons", "vibrons", and "what-ever-ons")
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\[
|m=1\rangle \quad \text{PLUS} \quad |m=2\rangle \quad \text{PLUS} \quad |m=3\rangle \quad \text{etc.} \quad \text{EQUALS} \quad |PW\rangle
\]

**Analogy:**
Adding photons (Quantized amplitude $N=0,1,2...$) can make a CS (Coherent State) or OAP (Oscillator Amplitude Packet) that is more like a classical wave oscillation with more localization in field amplitude.

\[
|N=0\rangle \quad \text{PLUS} \quad |N=1\rangle \quad \text{PLUS} \quad |N=2\rangle \quad \text{etc.} \quad \text{EQUALS} \quad |OAP\rangle
\]

- Zero-point uncertainty
- 1-point uncertainty
- 2-point uncertainty

Zero-photon state (Vacuum state)
1-photon state (Fundamental)
2-photon state (1st overtone)

\[N\text{-uncertainty}\]

Oscillating Amplitude Packet
Field Amplitude $E$
Coherent States: Oscillator Amplitude Packets analogous to Wave Packets
We saw how adding CW’s (Continuous Waves \(m=1,2,3\ldots\)) can make PW (Pulse Wave) or WP (Wave Packet) that is more like a classical “thing” with more localization in space \(x\) and time \(t\).

\[
|m=1\rangle \quad \text{PLUS} \quad |m=2\rangle \quad \text{PLUS} \quad |m=3\rangle \quad \text{etc.} \quad \text{EQUALS} \quad |PW\rangle
\]

Analogy:
Adding photons (Quantized amplitude \(N=0,1,2\ldots\)) can make a CS (Coherent State) or OAP (Oscillator Amplitude Packet) that is more like a classical wave oscillation with more localization in field amplitude.

\[
|N=0\rangle \quad \text{PLUS} \quad |N=1\rangle \quad \text{PLUS} \quad |N=2\rangle \quad \text{etc.} \quad \text{EQUALS} \quad |OAP\rangle
\]

Pure photon states have localized (certain) \(N\) but delocalized (uncertain) amplitude and phase.
OAP states have delocalized (uncertain) \(N\) but more localized (certain) amplitude and phase.
2nd Quantization: Quantizing amplitudes ("photons", "vibrons", and "what-ever-ons")
Analogy with molecular Born-Oppenheimer-Approximate energy levels
Introducing coherent states (What lasers use)
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Wave coordinates need coherence
Coherent States (contd.) Spacetime wave grid is impossible without coherent states

Pure photon number $N$-states would make useless spacetime coordinates

Total uncertainty of amplitude and phase makes the count pattern a wash. To see grids some $N$-uncertainty is necessary!
**Coherent States (contd.)** *Spacetime wave grid is impossible without coherent states*

Pure photon number $N$-states would make useless spacetime coordinates.

Total uncertainty of amplitude and phase makes the count pattern a wash. To see grids *some $N$-uncertainty is necessary!*

Coherent-$\alpha$-states are defined by continuous amplitude-packet parameter $\alpha$ whose square is average photon number $\bar{N}=|\alpha|^2$. Coherent-states make better spacetime coordinates for larger $\bar{N}=|\alpha|^2$.

Coherent-state uncertainty in photon number (and mass) varies with amplitude parameter $\Delta N \sim \alpha \sim \sqrt{\bar{N}}$ so a coherent state with $\bar{N}=|\alpha|^2=10^6$ only has a 1-in-1000 uncertainty $\Delta N \sim \alpha \sim \sqrt{\bar{N}}=1000$. 
Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz asynchrony

Magnetic B-field is relativistic \( \sinh \rho \) 1st order-effect
Relativistic effects on charge, current, and Maxwell Fields

Observer velocity is zero relative to (+) line of charge

(+) Charge fixed (-) Charge moving to right (Negative current density)
(+) Charge density is Equal to the (-) Charge density
Relativistic effects on charge, current, and Maxwell Fields

Observer velocity is zero relative to (+) line of charge

wire appears neutral

(+) Charge fixed (-) Charge moving to right (Negative current density $\vec{j}(x,t)$)

(+) Charge density is Equal to the (-) Charge density  \( \rho(x,t)=0 \)
Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz *asynchrony*

Asynchrony due to off-diagonal $\left( \begin{array}{cc} \sinh \rho & \cosh \rho \\ \cosh \rho & \sinh \rho \end{array} \right)$ (a 1\textsuperscript{st}-order effect)

in Lorentz tranform:

$\begin{pmatrix} \cosh \rho & \sinh \rho \\ \sinh \rho & \cosh \rho \end{pmatrix} \sim \begin{pmatrix} 1 & v/c \\ v/c & 1 \end{pmatrix}$

(+): Charge fixed  (-): Charge moving to right *(Negative current density $\vec{j}(x,t)$)*

(+): Charge density is *Greater* than (-): Charge density *(Positive $\rho(x,t)>0$)*

wire appears postive (+) (repulsive to observer $q[+]$)

Observer velocity is $+v$ relative to (+) line of charge
Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz asynchrony

Asynchrony due to off-diagonal $\text{Cosh} \rho \ 2 \times 2$ matrix (a 1st-order effect)

in Lorentz transform:

$$ \begin{pmatrix} \text{Cosh} \rho & \text{Sinh} \rho \\ \text{Sinh} \rho & \text{Cosh} \rho \end{pmatrix} \sim \begin{pmatrix} 1 & v/c \\ v/c & 1 \end{pmatrix}$$

Observer has $q^+[$ “test-charge”

Observer velocity $v$ is $+v$ relative to wire (line of charge)

(+)+ Charge fixed (-) Charge moving to right (Negative current density $j(x,t)$)

(+)+ Charge density is $>$ than (-) Charge density

(+) Charge density is $>$ Greater than (-) Charge density

$\rho(x,t)$ (Positive)

Wire appears positive (+) (repulsive to observer $q^+$)
Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz asynchrony

Asynchrony due to off-diagonal \( \begin{pmatrix} \sinh \rho & \cosh \rho \\ \sinh \rho & \cosh \rho \end{pmatrix} \) (a 1\(^{st}\)-order effect)

in Lorentz transform:

\[
\begin{pmatrix} \cosh \rho & \sinh \rho \\ \sinh \rho & \cosh \rho \end{pmatrix} \sim \begin{pmatrix} 1 & v/c \\ v/c & 1 \end{pmatrix}
\]

observer has \( q^+ \)

“test-charge”

Observer velocity is \(-v\) relative to \((+\) line of charge

wire appears negative (\(-\))
(attractive to observer \( q^+ \))

\((+\) Charge fixed \((-\) Charge moving to right (Negative current density \( \vec{\mathbf{j}}(x,t) \))

\((+\) Charge density is \textit{Less} than \((-\) Charge density (Negative \( \rho(x,t)<0 \))

\( \text{Wednesday, March 12, 2014} \)
Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz asynchrony

Asynchrony due to off-diagonal $\left( \begin{array}{cc} \sinh \rho & \cosh \rho \\ \sinh \rho & \cosh \rho \end{array} \right)$ (a 1$^{\text{st}}$-order effect)

in Lorentz transform:

\[
\begin{pmatrix}
\cosh \rho & \sinh \rho \\
\sinh \rho & \cosh \rho
\end{pmatrix} \sim \begin{pmatrix} 1 & \frac{v}{c} \\ \frac{v}{c} & 1 \end{pmatrix}
\]

observer has $q^{[+]}$

“test-charge”

Observer velocity is $-v$ relative to (+) line of charge

wire appears negative (-) (attractive to observer $q^{[+]}$)

(+) Charge fixed (-) Charge moving to right (Negative current density $\mathbf{j}(x,t)$)

(+) Charge density is Less than (-) Charge density (Negative $\rho(x,t)<0$)
Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz asynchrony

Magnetic B-field is relativistic $\sinh \rho$ 1st order-effect
Magnetic B-field is relativistic $\sinh \rho$ 1st order-effect

\[
\frac{\rho(-)}{\rho(+)} = \frac{(+) \text{ charge separation}}{(-) \text{ charge separation}} = \frac{x(+)+x(-)}{x(-)}
\]

\[
\rho(-) = \frac{x(+)}{x(-)} + 1 = \frac{uv}{c^2} + 1
\]

\[
\rho(+) - \rho(-) = \rho(+) \left( 1 - \frac{\rho(-)}{\rho(+)} \right) = -\frac{uv}{c^2} \rho(+)
\]

Unit square: \((u/c) / 1 = x(+)/y \)
\[(v/c) / 1 = y/x(-)\]
Using 4-vectors to EL Transform (charge-current) = \((c\rho, j)\)

\[
\begin{pmatrix}
  c\rho' \\
  j_x' \\
  j_y' \\
  j_z'
\end{pmatrix} = 
\begin{pmatrix}
  \cosh \rho & \sinh \rho & . & . \\
  \sinh \rho & \cosh \rho & . & . \\
  . & . & 1 & . \\
  . & . & . & 1
\end{pmatrix}
\begin{pmatrix}
  c\rho \\
  j_x \\
  j_y \\
  j_z
\end{pmatrix}
\]

\[
\rho(-) = \frac{\text{charge separation}}{\rho(+)} = \frac{x(+)+x(-)}{x(-)}
\]

\[
\frac{\rho(-)}{\rho(+)} = 1 + \frac{uv}{c^2}
\]

\[
\rho(+) - \rho(-) = \rho(+) \left(1 - \frac{\rho(-)}{\rho(+)}\right) = -\frac{uv}{c^2} \rho(+)
\]
The electric force field $\mathbf{E}$ of a charged line varies inversely with radius. The Gauss formula for force in mks units:

$$F = qE = q\left[\frac{1}{4\pi \varepsilon_0} \frac{2\rho}{r}\right], \quad \text{where:} \quad \frac{1}{4\pi \varepsilon_0} = 9 \times 10^9 \frac{N \cdot m^2}{\text{Coul}}.$$

$$F = qE = q\left[\frac{1}{4\pi \varepsilon_0} \frac{2}{r}\left(-\frac{uv}{c^2} \rho(+)\right)\right] = -\frac{2q v \rho(+) u}{4\pi \varepsilon_0 c^2 r} = -2 \times 10^{-7} \frac{I_q I_\rho}{r}$$

$I_\rho < 0$ \hspace{1cm} $I_q > 0$ \hspace{1cm} $F$ (repels)

$I_\rho < 0$ \hspace{1cm} $I_q < 0$ \hspace{1cm} $F$ (attracts)

I see excess (+) charge up there. Yuk!

I see excess (-) charge up there. Yum!
The electric force field $E$ of a charged line varies inversely with radius. The Gauss formula for force in mks units:

$$F = qE = q \left[ \frac{1}{4\pi \varepsilon_0} \frac{2\rho}{r} \right], \quad \text{where:} \quad \frac{1}{4\pi \varepsilon_0} = 9 \times 10^9 \frac{N \cdot m^2}{Coul}. $$

$$F = qE = q \left[ \frac{1}{4\pi \varepsilon_0} \frac{2}{r} \left( -\frac{uv}{c^2} \rho(+) \right) \right] = -\frac{2 q v \rho(+) u}{4\pi \varepsilon_0 c^2 r} = -2 \times 10^{-7} \frac{I_q I_\rho}{r}$$

$1/4\pi \varepsilon_0 = 9 \times 10^9$
$c^2 = 9 \times 10^{-16}$
$1/(4\pi \varepsilon_0 c^2) = 10^{-7}$

$\frac{1}{4\pi \varepsilon_0} \approx 9 \times 10^9$
$c^2 = 9 \times 10^{-16}$
$1/(4\pi \varepsilon_0 c^2) = 10^{-7}$

**Magnetic B-field is relativistic sinh $\rho$ 1st order-effect**

$\sinh \rho$ 1st order-effect

**Wednesday, March 12, 2014**