Lecture 35.

Serial Compton scattering and accelerating frames II

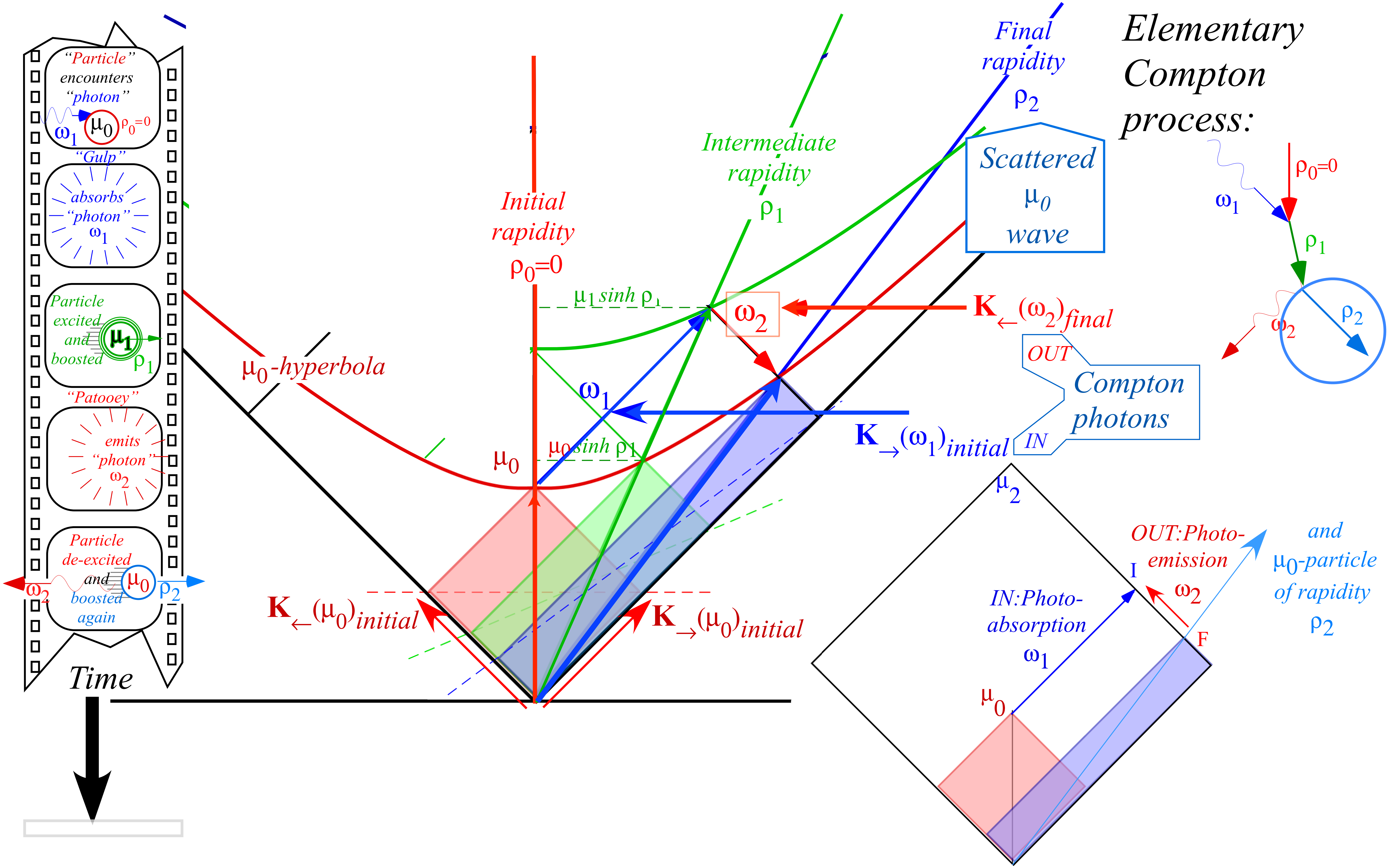
(Ch. 7-8 of Unit 2   4.24.12)

Serial Compton scattering and acceleration plot
- *Geometric construction*
- *Compton wavelength and formulae*
  - *Some numerology: Which is bigger...H-atom or an electron?*
- *Bouncing pulse wave (PW) vs (CW) shrinking laser*

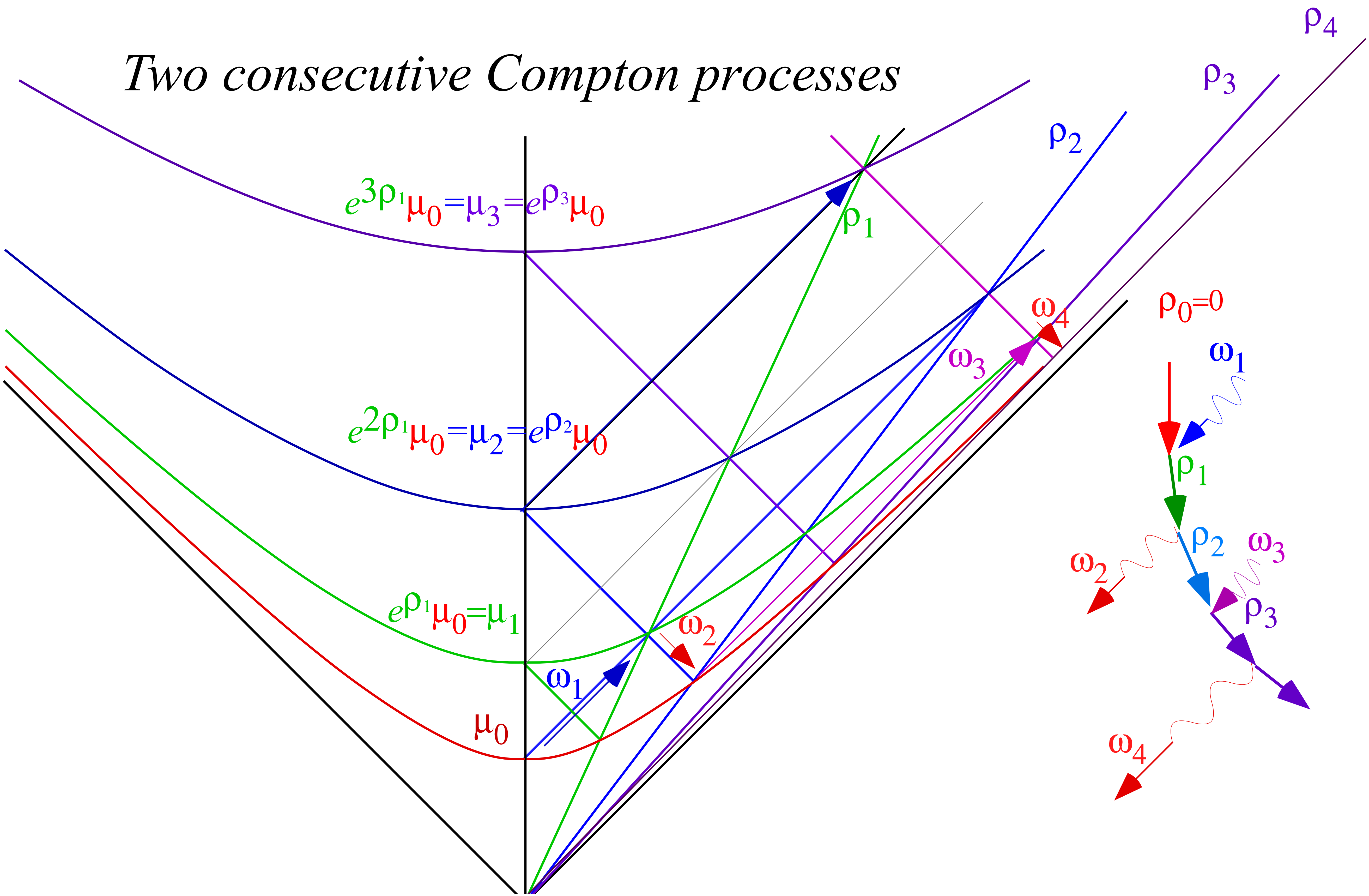
Lecture 34 review

Wave frames of varying acceleration
- *Relativistic acceleration*
  - *Optical "Einstein elevator" and flying-saucer-trailer*
- *Biggest mystery of all: Pair production*

Elementary Compton process:



Two consecutive Compton processes



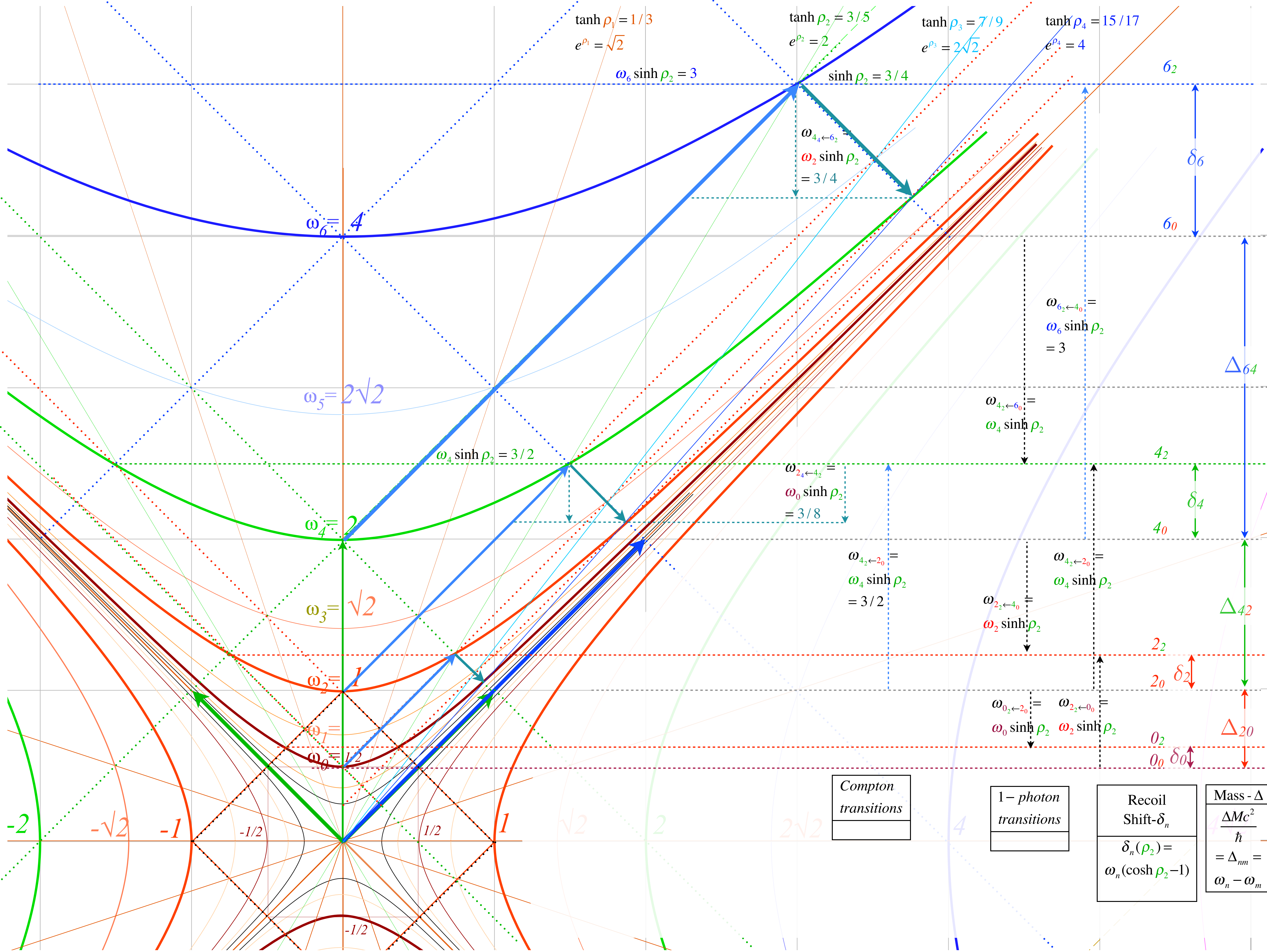
Serial Compton scattering and acceleration plot

→ *Geometric construction*

Compton wavelength and formulae

Some numerology: Which is bigger...H-atom or an electron?

Bouncing pulse wave (PW) vs (CW) shrinking laser



tanh ρ₁ = 1/3
e^ρ₁ = √2
tanh ρ₂ = 3/5
e^ρ₂ = 2
tanh ρ₃ = 7/9
e^ρ₃ = 2√2
tanh ρ₄ = 15/17
e^ρ₄ = 4
ω₆ sinh ρ₂ = 3
sinh ρ₂ = 3/4
ω₆ = 4
ω₄←₆ =
ω₂ sinh ρ₂
= 3/4
ω₅ = 2√2
ω₄ sinh ρ₂ = 3/2
ω₄ = 2
ω₃ = √2
ω₂ = 1
ω₁ =
ω₀ = 1/2
ω₆←₄ =
ω₆ sinh ρ₂
= 3
ω₄←₆ =
ω₄ sinh ρ₂
ω₂←₄ =
ω₀ sinh ρ₂
= 3/8
ω₄←₂ =
ω₄ sinh ρ₂
= 3/2
ω₄←₂ =
ω₄ sinh ρ₂
ω₂←₄ =
ω₂ sinh ρ₂
ω₀←₂ =
ω₀ sinh ρ₂
ω₂←₀ =
ω₂ sinh ρ₂
δ6
Δ64
δ4
Δ42
δ2
Δ20
δ0
-2
-√2
-1
-1/2
1/2
1
-1/2
Compton transitions
1 − photon transitions
Recoil Shift-δₙ
δₙ(ρ₂) = ωₙ(cosh ρ₂ − 1)
Mass - Δ
ΔMc²/ħ = Δₙₘ = ωₙ − ωₘ

Serial Compton scattering and acceleration plot

Geometric construction

→ *Compton wavelength and formulae*

Some numerology: Which is bigger...H-atom or an electron?

Bouncing pulse wave (PW) vs (CW) shrinking laser

$\tanh\rho_1 = 1/3$
$e^{\rho_1} = \sqrt{2}$

$\omega_6 \sinh\rho_2 = 3$

$\tanh\rho_2 = 3/5$
$e^{\rho_2} = 2$

$\sinh\rho_2 = 3/4$

$\tanh\rho_3 = 7/9$
$e^{\rho_3} = 2\sqrt{2}$

$\tanh\rho_4 = 15/17$
$e^{\rho_4} = 4$

Compton Wavelength formula

$$\lambda_{IN} - \lambda_{FIN} = \lambda_{2_4 \leftarrow 4_2} - \lambda_{4_2 \leftarrow 2_0} = 2\pi c\left(\frac{1}{\omega_{2_4 \leftarrow 4_2}} - \frac{1}{\omega_{4_2 \leftarrow 2_0}}\right)$$

$$= 2\pi c\left(\frac{1}{e^{-\rho_2}\omega_2 \sinh\rho_2} - \frac{1}{e^{+\rho_2}\omega_2 \sinh\rho_2}\right)$$

$$= 2\pi c\left(\frac{1}{e^{-\rho_2}} - \frac{1}{e^{+\rho_2}}\right)\frac{1}{\omega_2 \sinh\rho_2}$$

$$= \frac{2\pi c}{\omega_2}\left(\frac{e^{+\rho_2} - e^{-\rho_2}}{1}\right)\frac{1}{\sinh\rho_2}$$

$$= \frac{2\pi c}{\omega_2}(2) = \frac{2\pi c\hbar}{M_2 c^2}(2) = \frac{2\pi\hbar}{M_2 c}(2) = \frac{h}{M_2 c}(2)$$

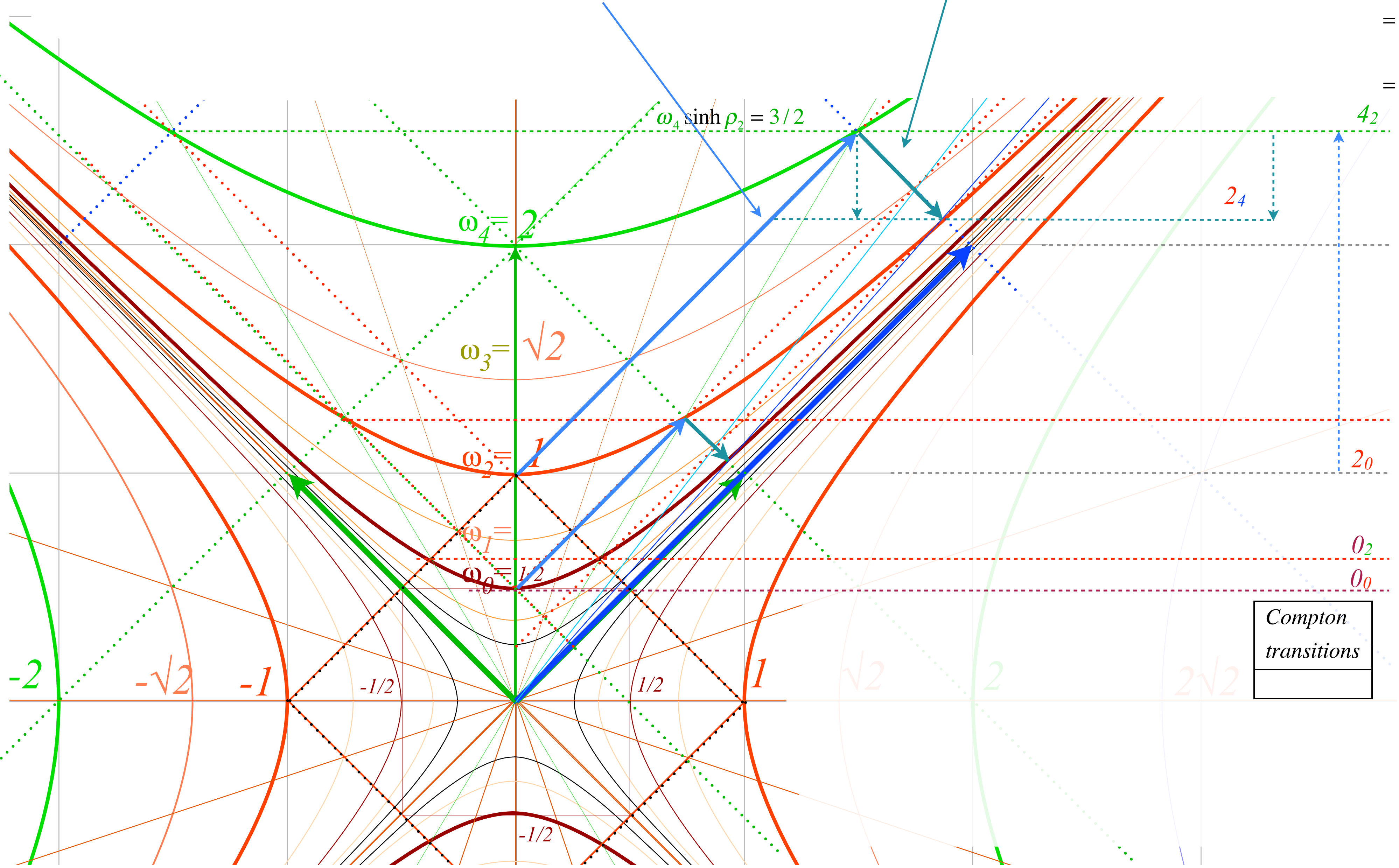
$$= 2 \cdot \text{Compton wavelength} = 2 \cdot \frac{h}{M_2 c}$$

$\omega_{4_2 \leftarrow 2_0} =$
$\omega_4 \sinh\rho_2$
$= e^{+\rho_2}\omega_2 \sinh\rho_2$
$= 3/2$

Compton IN

$\omega_{2_4 \leftarrow 4_2} =$
$\omega_0 \sinh\rho_2$
$= e^{-\rho_2}\omega_2 \sinh\rho_2$
$= 3/8$

Compton FIN



Serial Compton scattering and acceleration plot

Geometric construction

Compton wavelength and formulae

→ *Some numerology: Which is bigger...H-atom or an electron?*

Bouncing pulse wave (PW) vs (CW) shrinking laser

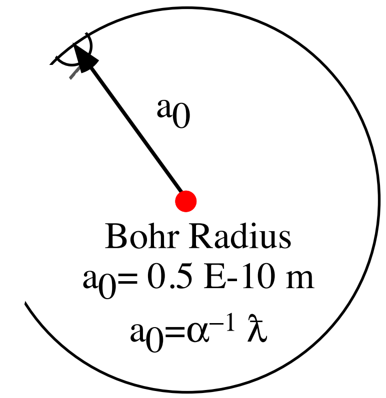


Fig.8A.2 Various electron radii and their relative sizes related by fine-structure constant α = 1/137.

Bohr model has electron orbiting at radius *r* so centrifugal force balances Coulomb attraction to the opposite charged proton.

$$\frac{m_e v^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2} \quad \text{or:} \quad \frac{m_e v^2 r^2}{r} = \frac{e^2}{4\pi\varepsilon_0} \quad \text{or:} \quad r = \frac{4\pi\varepsilon_0 m_e v^2 r^2}{e^2} = \frac{4\pi\varepsilon_0 (m_e v r)^2}{m_e e^2} = \frac{4\pi\varepsilon_0 \ell^2}{m_e e^2}$$

Bohr hypothesis: orbital momentum ℓ is a multiple N of $\hbar$ or

$$\ell = m_e v r = N\hbar \qquad (N = 1, 2, \ldots).$$

This gives the *atomic Bohr radius a_0 =0.05nm*

$$r = \frac{4\pi\varepsilon_0 \hbar^2}{m_e e^2} N^2 \left(= r_{Bohr} = 5.28 \cdot 10^{-11}\ m. = 0.528\ \ddot{A} \text{ for } N=1 \right)$$

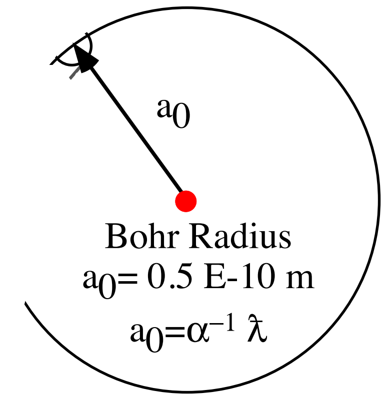


Fig.8A.2 Various electron radii and their relative sizes related by fine-structure constant α = 1/137.

Bohr model has electron orbiting at radius *r* so centrifugal force balances Coulomb attraction to the opposite charged proton.

$$\frac{m_e v^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2} \quad \text{or:} \quad \frac{m_e v^2 r^2}{r} = \frac{e^2}{4\pi\varepsilon_0} \quad \text{or:} \quad r = \frac{4\pi\varepsilon_0 m_e v^2 r^2}{e^2} = \frac{4\pi\varepsilon_0 (m_e v r)^2}{m_e e^2} = \frac{4\pi\varepsilon_0 \ell^2}{m_e e^2}$$

Bohr hypothesis: orbital momentum ℓ is a multiple N of $\hbar$ or

$$\ell = m_e\, v\, r = N\hbar \qquad (N = 1, 2, \ldots).$$

This gives the *atomic Bohr radius a_0 =0.05nm*

$$r = \frac{4\pi\varepsilon_0 \hbar^2}{m_e e^2} N^2 \left(= r_{Bohr} = 5.28 \cdot 10^{-11}\ m. = 0.528\ \ddot{A} \text{ for } N{=}1 \right)$$

It also implies rear-relativistic electron orbit speed *v* that is fraction *1/N* of *0.073c*.

$$\frac{v}{c} = \frac{\ell}{m_e r c} = \frac{N\hbar}{m_e r_{Bohr} c} = \frac{N\hbar}{m_e c} \frac{m_e e^2}{4\pi\varepsilon_0 \hbar^2 N^2} = \frac{1}{N} \frac{e^2}{4\pi\varepsilon_0 \hbar c} \quad \left(= 7.29 \cdot 10^{-3} = \frac{1}{137.} \text{ for } N{=}1 \right)$$

The dimensionless ratio $\alpha = e^2/(4\pi\varepsilon_0 \hbar c) = 1/137.036$ is called the *fine-structure constant* α.

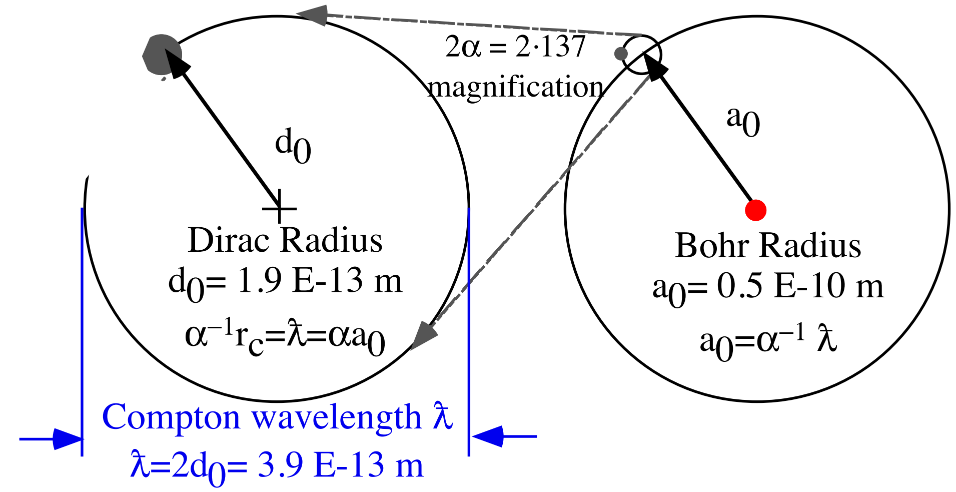


Fig.8A.2 Various electron radii and their relative sizes related by fine-structure constant $\alpha = 1/137$.

Bohr model has electron orbiting at radius r so centrifugal force balances Coulomb attraction to the opposite charged proton.

$$\frac{m_e v^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2} \qquad \text{or:} \quad \frac{m_e v^2 r^2}{r} = \frac{e^2}{4\pi\varepsilon_0} \qquad \text{or:} \quad r = \frac{4\pi\varepsilon_0 m_e v^2 r^2}{e^2} = \frac{4\pi\varepsilon_0 (m_e v r)^2}{m_e e^2} = \frac{4\pi\varepsilon_0 \ell^2}{m_e e^2}$$

Bohr hypothesis: orbital momentum ℓ is a multiple N of $\hbar$ or

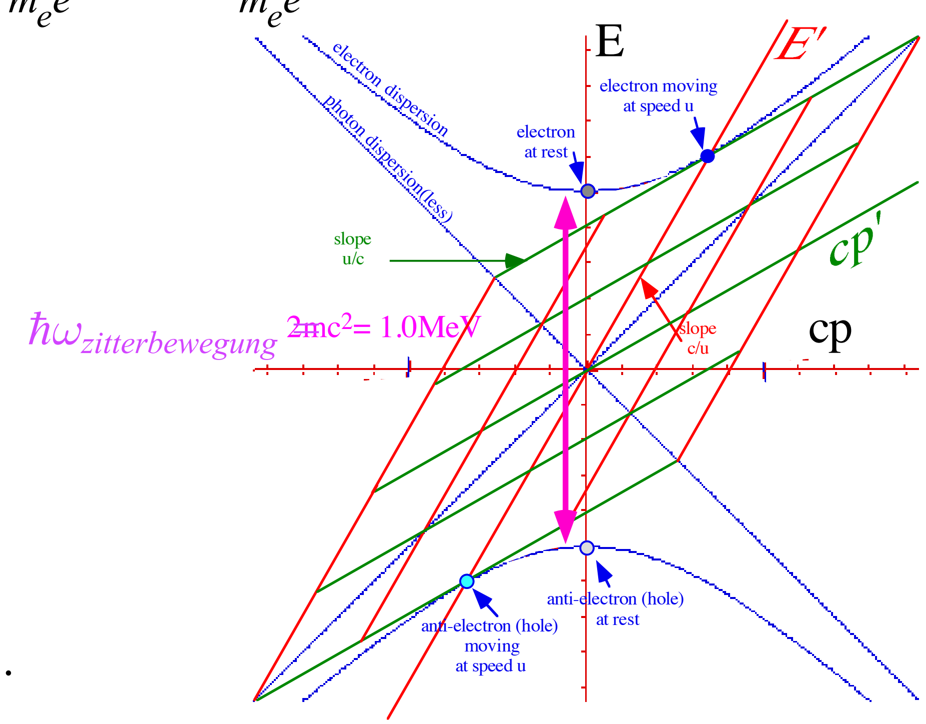
$$\ell = m_e\, v\, r = N\,\hbar \qquad (N = 1, 2, \ldots).$$

This gives the *atomic Bohr radius* $a_0 = 0.05nm$

$$r = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2} N^2 \left(= r_{Bohr} = 5.28 \cdot 10^{-11}\ m. = 0.528\ \ddot{A} \text{ for } N{=}1\right)$$

It also implies rear-relativistic electron orbit speed v that is fraction $1/N$ of $0.073c$.

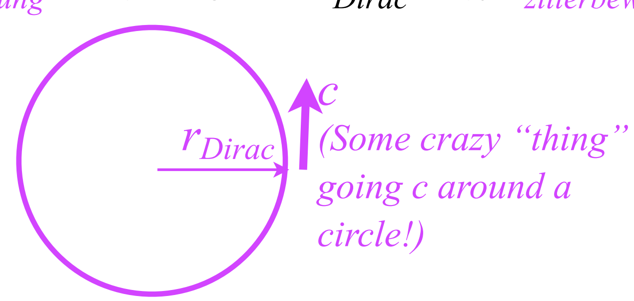
$$\frac{v}{c} = \frac{\ell}{m_e r c} = \frac{N\hbar}{m_e r_{Bohr} c} = \frac{N\hbar}{m_e c}\frac{m_e e^2}{4\pi\varepsilon_0\hbar^2 N^2} = \frac{1}{N}\frac{e^2}{4\pi\varepsilon_0\hbar c} \quad \left(= 7.29 \cdot 10^{-3} = \frac{1}{137.} \text{ for } N{=}1\right)$$



The *dimensionless* ratio $\alpha = e^2/(4\pi\varepsilon_0\hbar c) = 1/137.036$ is called the *fine-structure constant* α.

Now, some *numerology* of Dirac's electron radius involving *zwitterbewegung* where $\omega_{zitterbewegung} = 2mc^2/\hbar = 1.56 \cdot 10^{21}$*(radian)Hz*

$\omega_{zitterbewegung}\, r = c$ or $r_{Dirac} = c/\omega_{zitterbewegung} = \hbar/2mc = 1.93 \cdot 10^{-13}\ m$ relates to the *Compton wavelength* $\lambda\!\!\!^{-} = \hbar/mc = 3.8616 \cdot 10^{-13}\ m$



Reduced Compton wavelength: $2\pi\,\lambda\!\!\!^{-} = h/mc = 2.4263 \cdot 10^{-12}$
or Compton "circumference"

$2.4263102175 \pm 33 \times 10^{-12}$ m

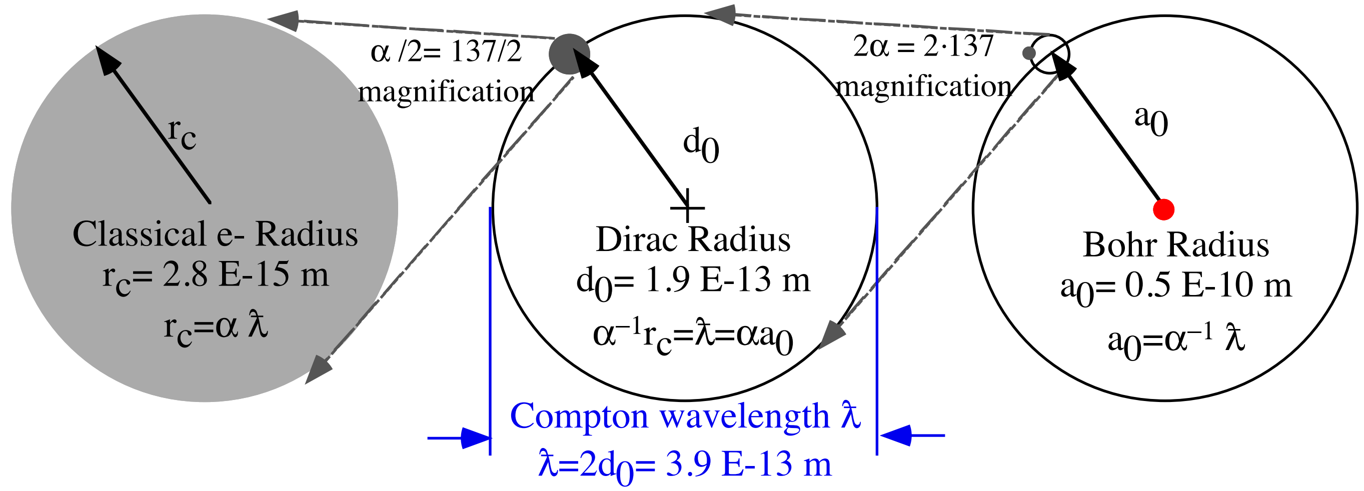


Fig.8A.2 Various electron radii and their relative sizes related by fine-structure constant α = 1/137.

The classical radius of the electron defined by setting its electrostatic PE to $m_e c^2$:

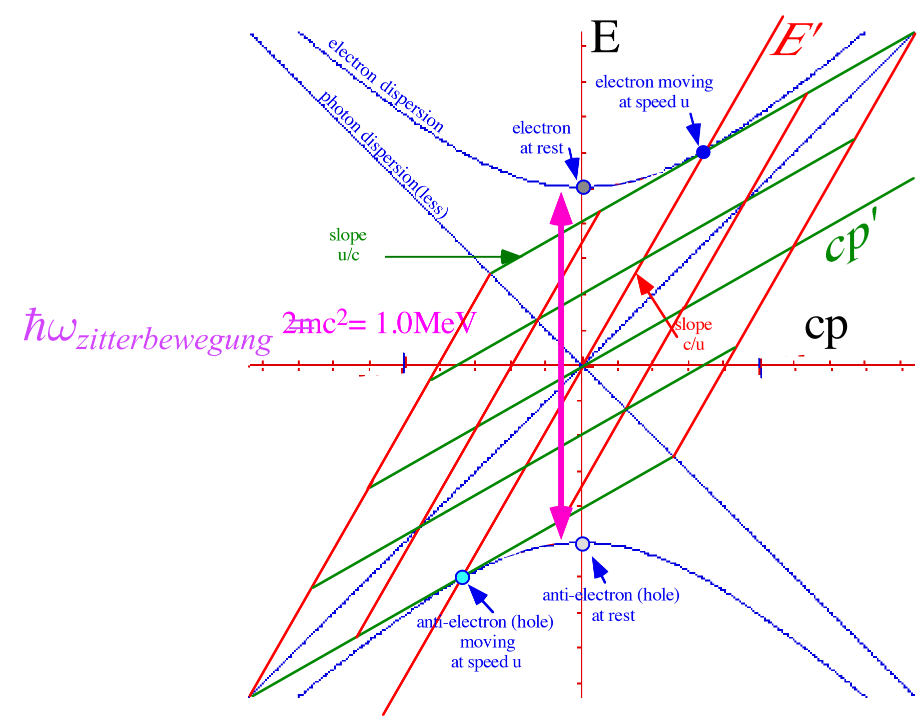
$$e^2/(4\pi e_0\, r_{classical}) = m_e c^2 \quad \text{or} \quad r_{classical} = e^2/(4\pi e_0\, m_e c^2) = 2.8\cdot 10^{-15}\ m.$$

Another fine-structure ratio to r_{Bohr}.

$$\frac{r_{Classical}}{r_{Bohr}} = \frac{e^2/4\pi\varepsilon_0 m_e c^2}{4\pi\varepsilon_0 \hbar^2/m_e e^2} = \left(\frac{e^2}{4\pi\varepsilon_0 \hbar c}\right)^2 = \left(\frac{1}{137.}\right)^2$$

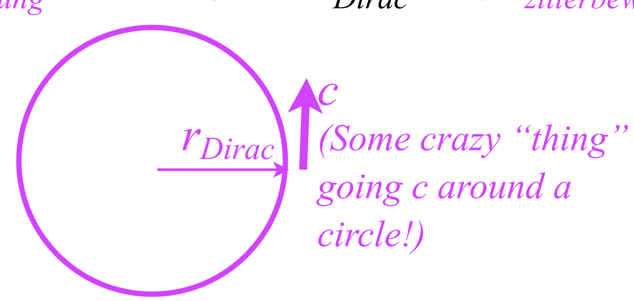
As a final numerological exercise, find angular momentum $\ell = m_e v r$ of fictitious *"zitterbewegung"* orbit inside the electron.
With $v=c$ and $r = r_{Dirac}$ the following is obtained.

$$\ell = m_e c\, r_{Dirac} = m_e c\hbar/(2m_e c)$$
$$= \hbar/2$$



Now, some *numerology* of Dirac's electron radius involving *zwitterbewegung* where $\omega_{zitterbewegung} = 2mc^2/\hbar = 1.56\cdot 10^{21}(radian)Hz$

$\omega_{zitterbewegung}\, r = c$ or $r_{Dirac} = c/\omega_{zitterbewegung} = \hbar/2mc = 1.93\cdot 10^{-13}\ m$ relates to the *Compton wavelength* $ƛ=\hbar/mc=3.8616\cdot 10^{-13}\ m$



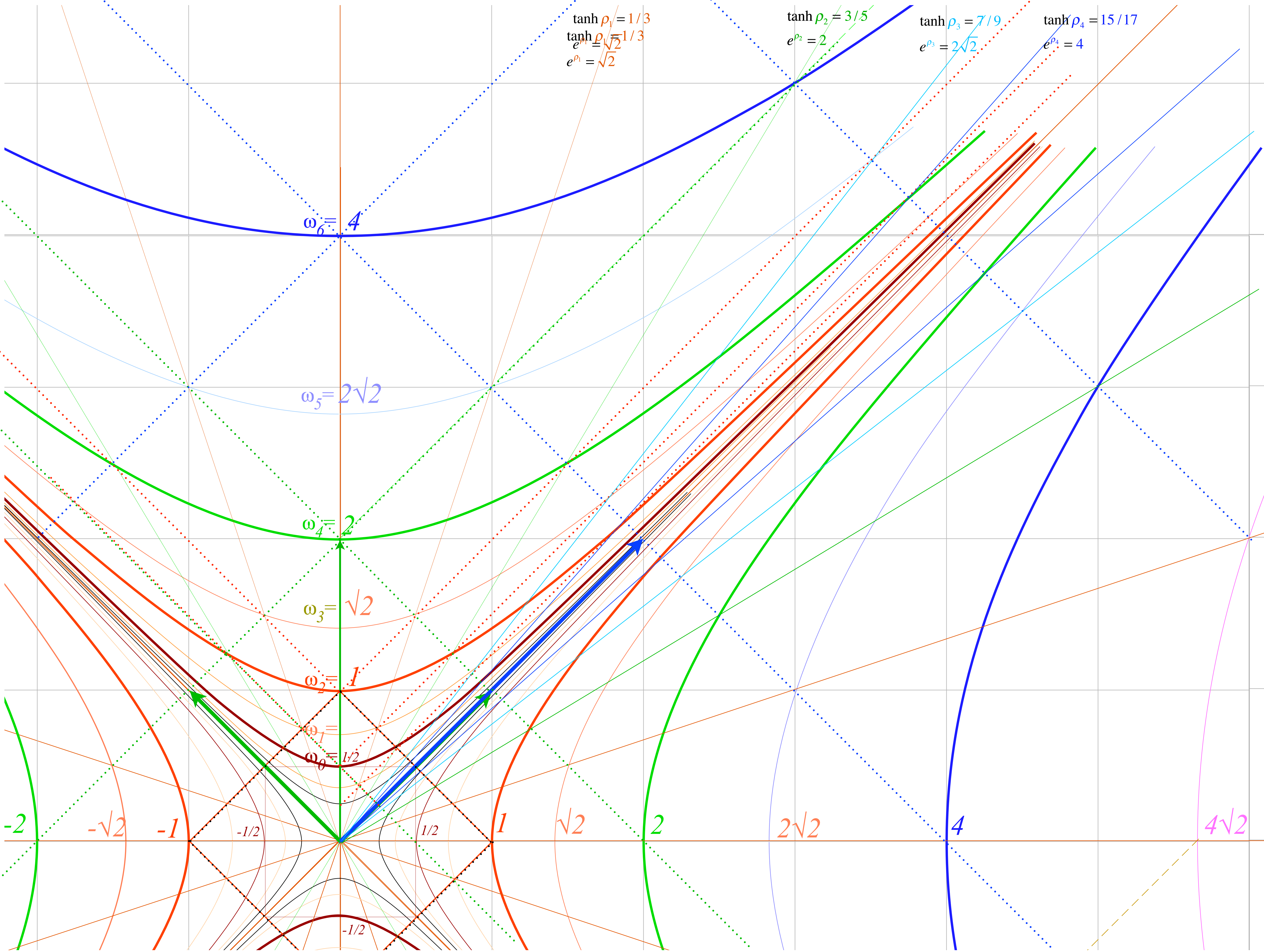
Serial Compton scattering and acceleration plot

Geometric construction

Compton wavelength and formulae

Some numerology: Which is bigger...H-atom or an electron?

→ *Bouncing pulse wave (PW) vs (CW) shrinking laser*



tanh ρ1 = 1/3
e^ρ1 = √2
tanh ρ2 = 3/5
e^ρ2 = 2
tanh ρ3 = 7/9
e^ρ3 = 2√2
tanh ρ4 = 15/17
e^ρ4 = 4
ω6 = 4
ω5 = 2√2
ω4 = 2
ω3 = √2
ω2 = 1
ω1 =
ω0 = 1/2
-2
-√2
-1
-1/2
1/2
1
√2
2
2√2
4
4√2
-1/2

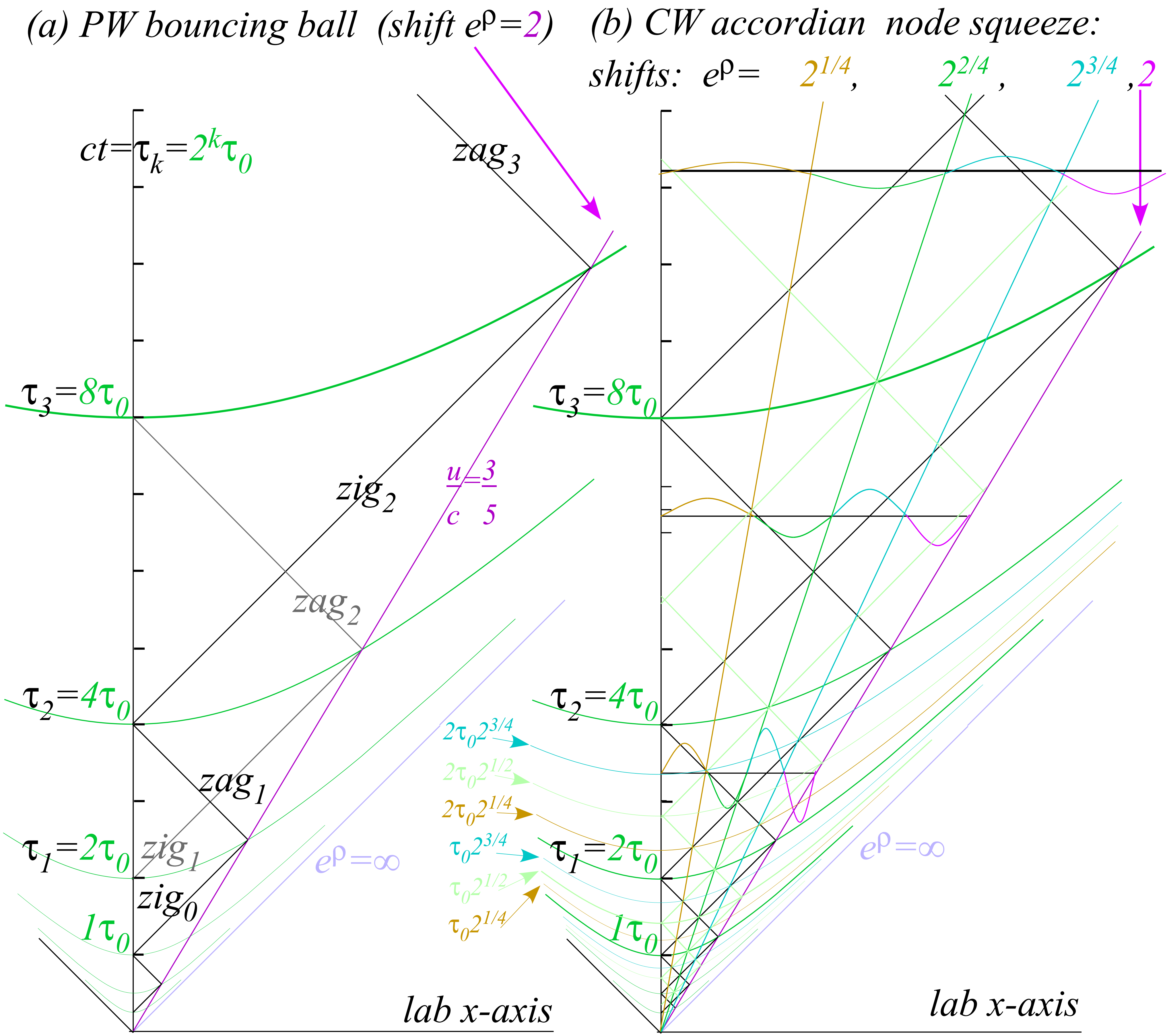
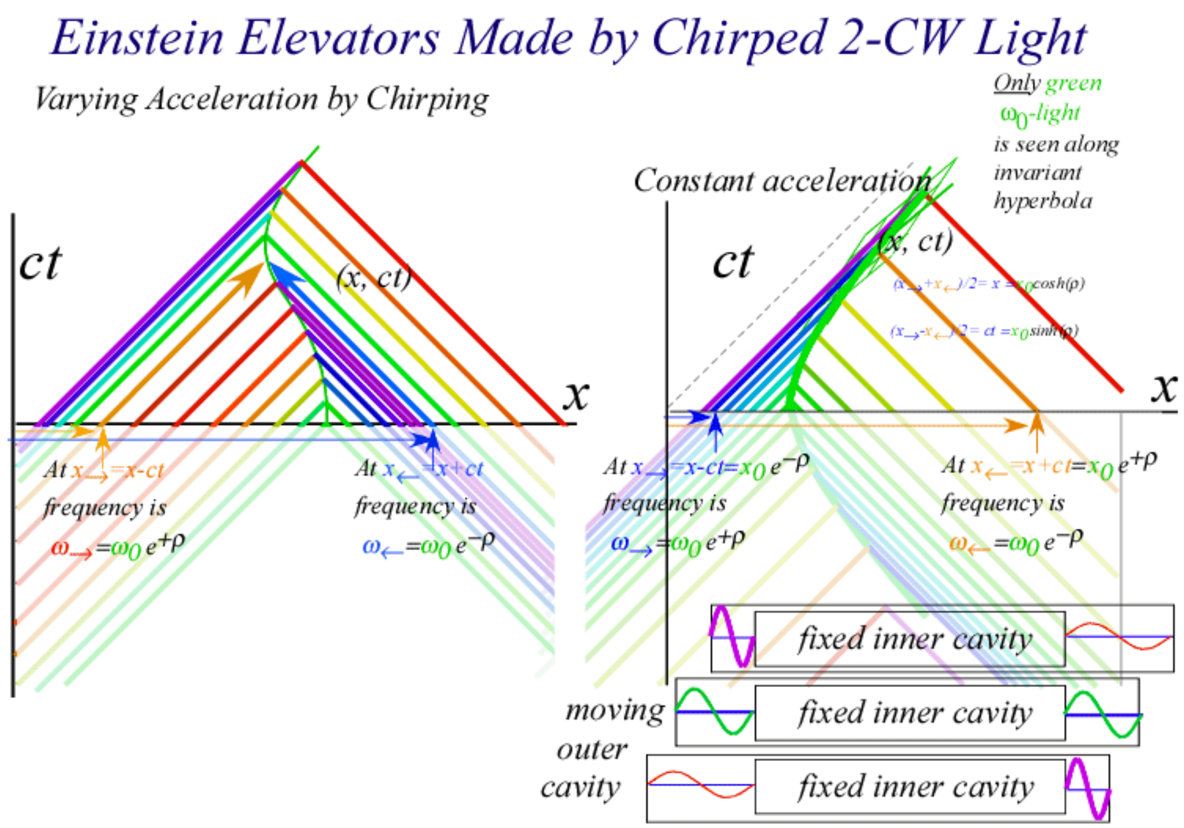


Fig. 7.8 Space-time nets (a) PW zigzag paths bounce. (b) CW nodes squeeze like an accordian.



Wave frames of varying acceleration

→ *Relativistic acceleration*

Optical "Einstein elevator" and flying-saucer-trailer

Biggest mystery of all: Pair production

Acceleration by chirping laser pairs

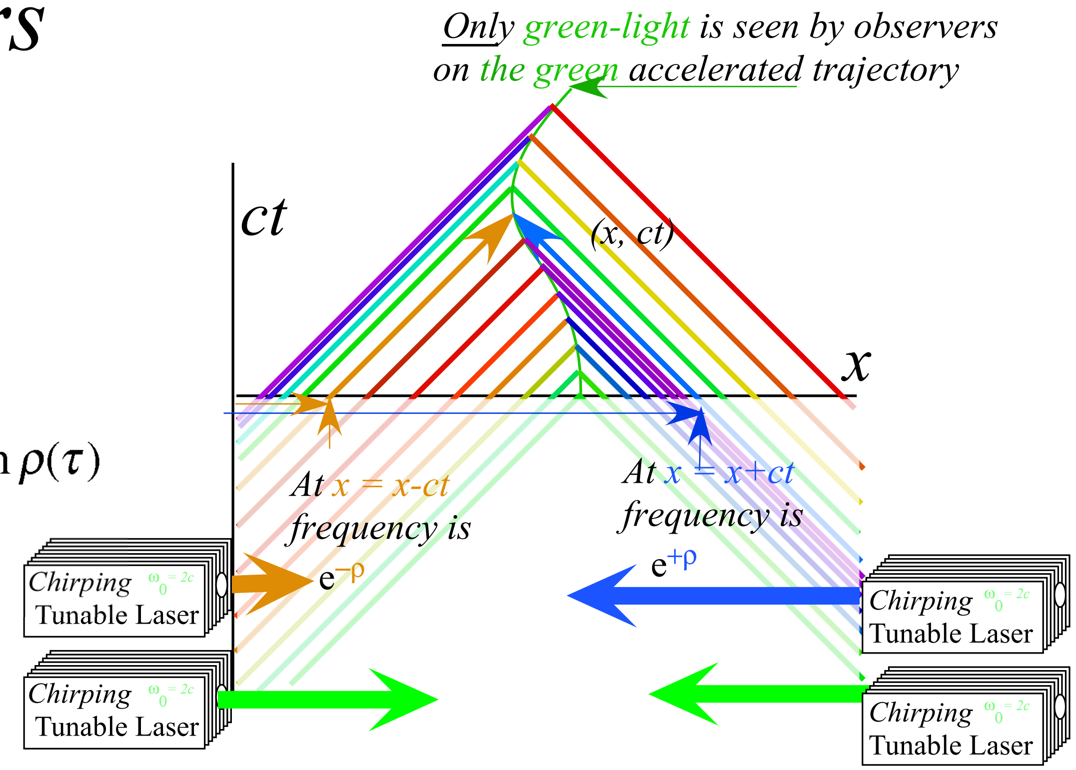
Varying acceleration (General case)

Varying local acceleration $\rho = \rho(\tau)$

$$u = \frac{dx}{dt} = c\tanh(\tau)$$

$$\frac{dt}{d\tau} = \cosh\rho(\tau) \qquad \frac{dx}{d\tau} = \frac{dx}{dt}\frac{dt}{d\tau} = c\tanh\rho(\tau)\cosh\rho(\tau) = c\sinh\rho(\tau)$$

$$ct = c\int\cosh\rho(\tau)\,d\tau \qquad x = c\int\sinh\rho(\tau)\,d\tau$$



Constant local acceleration $\rho = \frac{g\tau}{c}$ "*Einstein Elevator*"

$$ct = c\int\cosh\frac{g\tau}{c}\,d\tau \qquad x = c\int\sinh\frac{g\tau}{c}\,d\tau$$

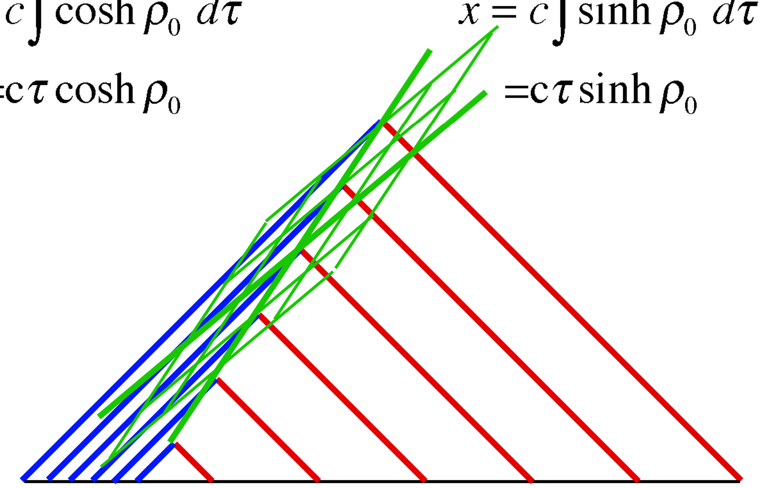
$$= \frac{c^2}{g}\sinh\frac{g\tau}{c} \qquad = \frac{c^2}{g}\cosh\frac{g\tau}{c}$$

Previous examples involved constant velocity

Constant velocity $\rho = \rho_0 = const.$ "*Lorentztransformation*"

$$ct = c\int\cosh\rho_0\,d\tau \qquad x = c\int\sinh\rho_0\,d\tau$$

$$= c\tau\cosh\rho_0 \qquad = c\tau\sinh\rho_0$$



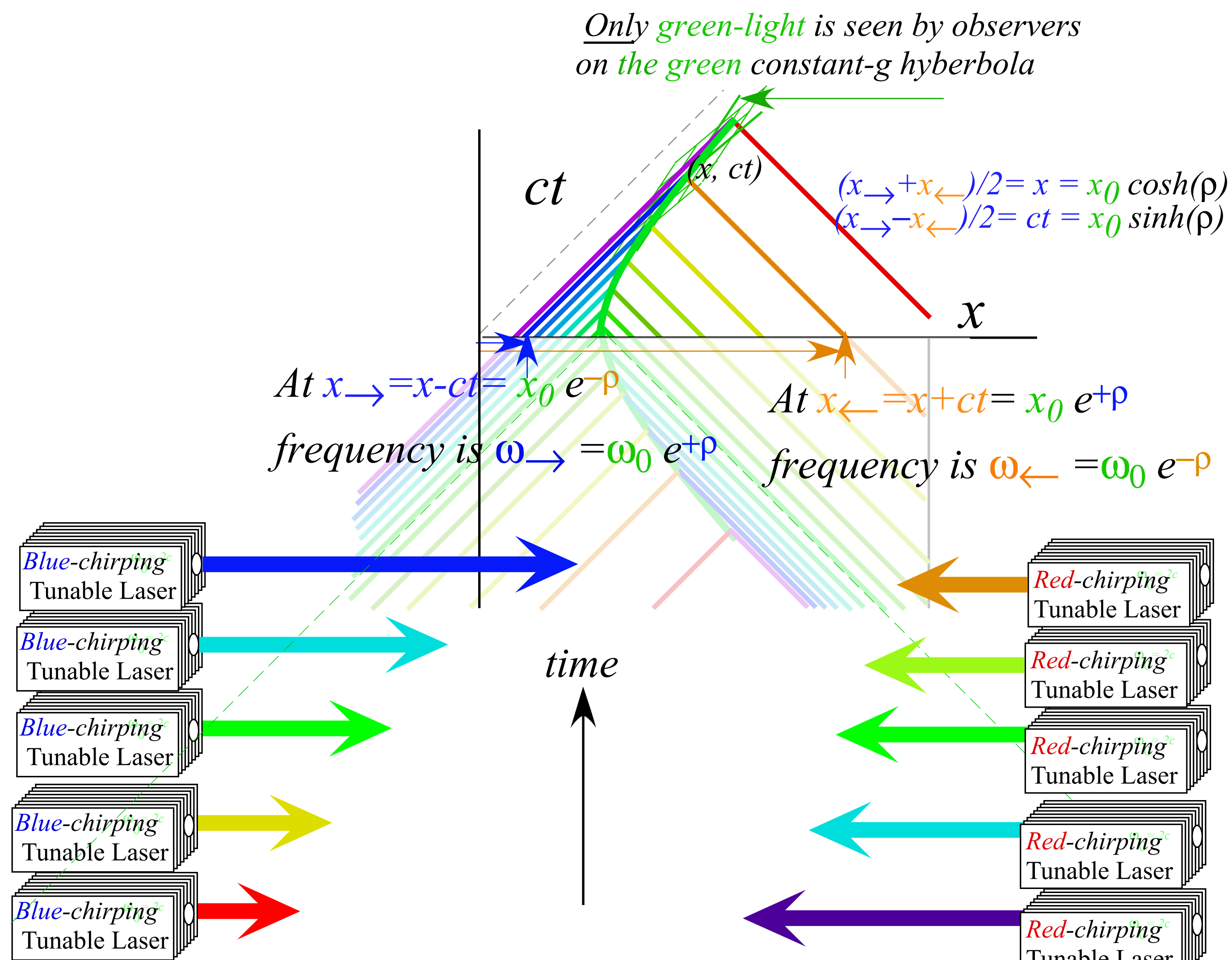


Fig. 8.1 Optical wave frames by red-and-blue-chirped lasers (a)Varying acceleration (b)Constant g

Geometrical Chirps Gives Accelerated Minkowski Grids

Coherent waves make space-time coordinates of accelerating ship & trailer(s) for region inside asymptotic "event horizon"

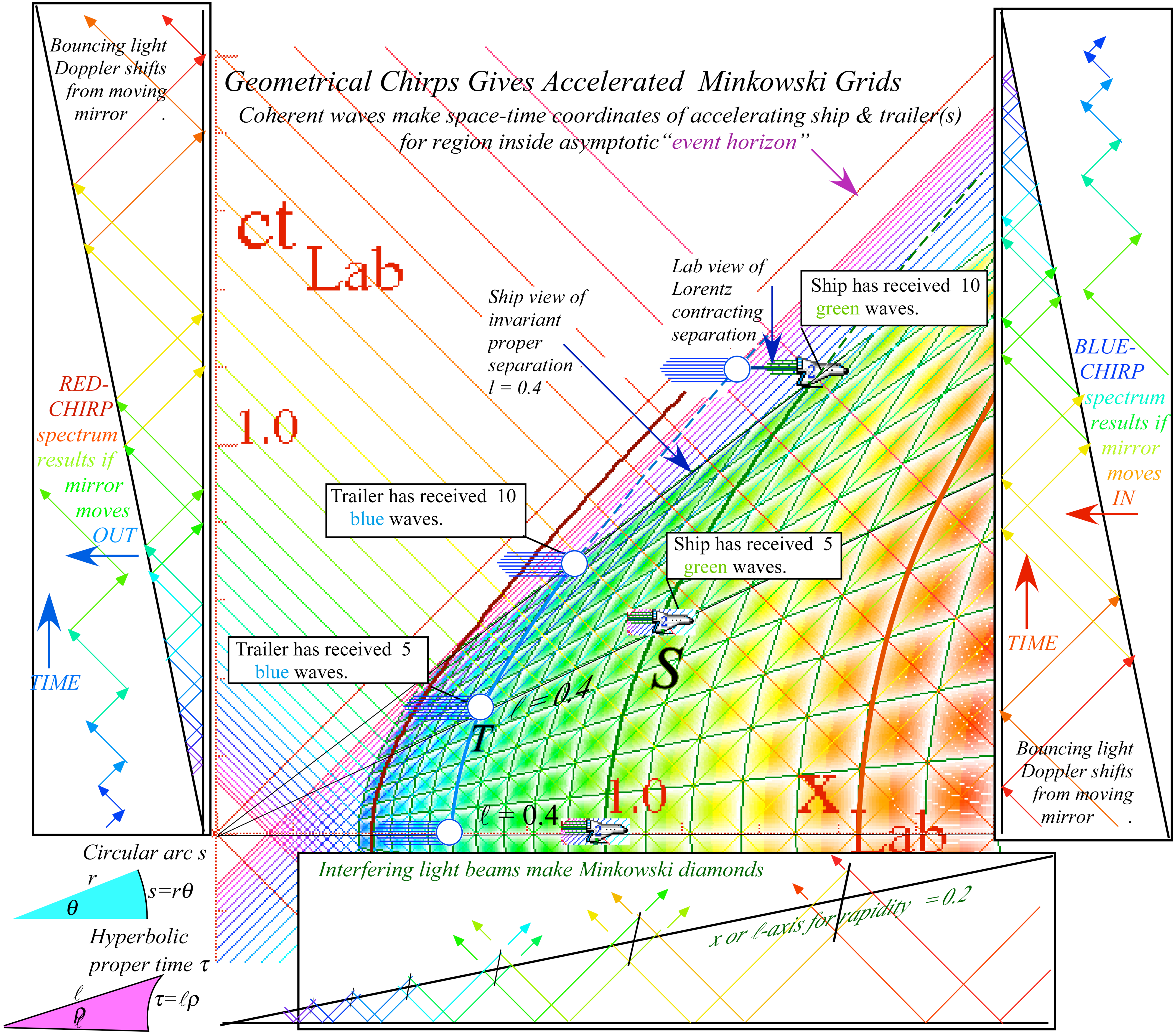
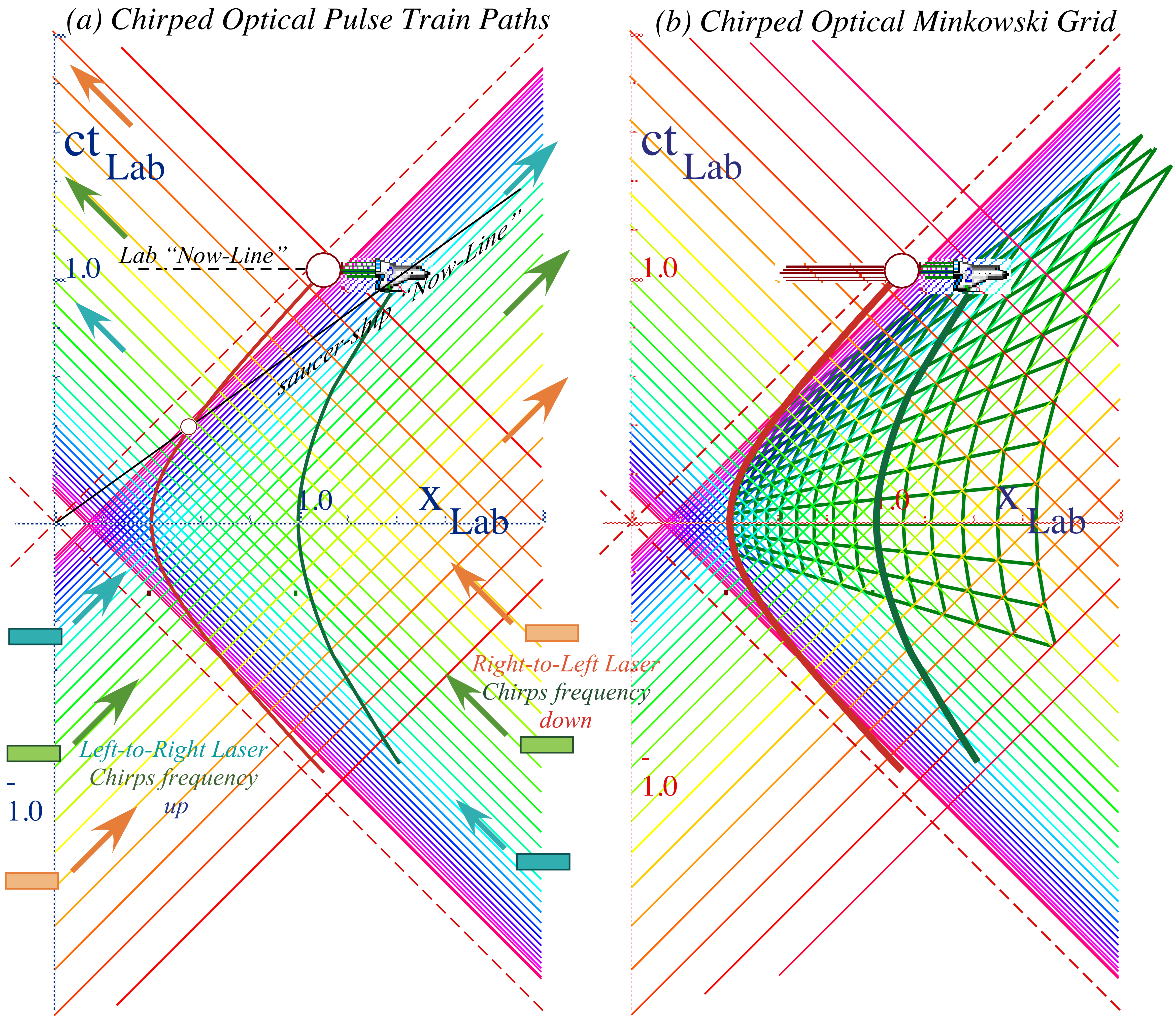
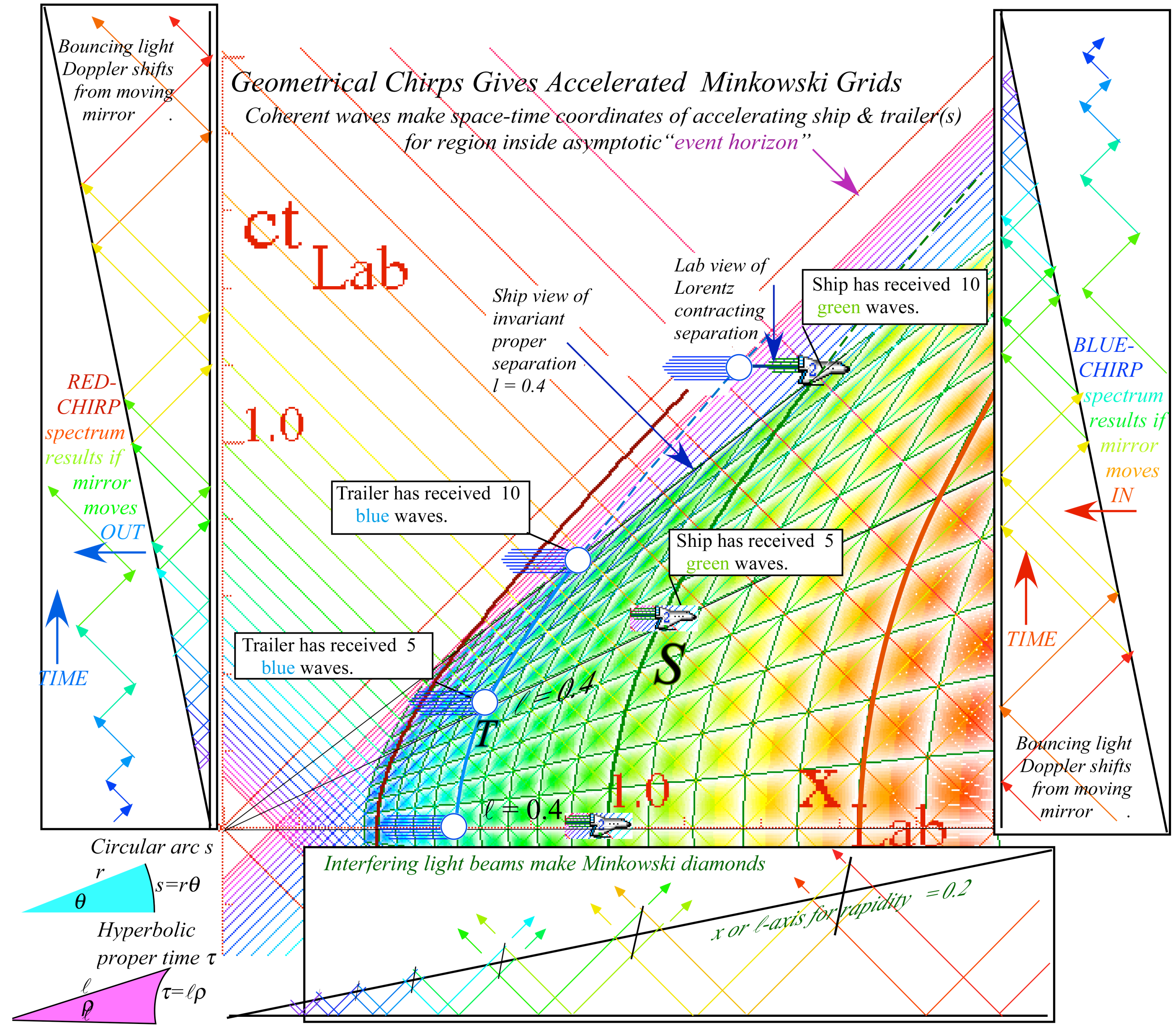


Fig. 8.2 Accelerated reference frames and their trajectories painted by chirped coherent light


(a) Chirped Optical Pulse Train Paths
(b) Chirped Optical Minkowski Grid
ct
Lab
Lab "Now-Line"
1.0
saucer-ship
"Now-Line"
1.0
x
Lab
-
1.0
Left-to-Right Laser
Chirps frequency
up
Right-to-Left Laser
Chirps frequency
down
ct
Lab
1.0
x
Lab
-
1.0



$$x = c\int \sinh\left(\frac{g\tau}{c}\right) d\tau = \frac{c^2}{g}\cosh\left(\frac{g\tau}{c}\right) \qquad (8.3b)$$

$$ct = c\int \cosh\left(\frac{g\tau}{c}\right) d\tau = \frac{c^2}{g}\sinh\left(\frac{g\tau}{c}\right) \qquad (8.3a)$$

Paths closer to the left hand blue-chirping laser have a higher g than flatter ones nearer the red-chirping one.

Each hyperbola has different but fixed location ℓ, color ω, and artificial gravity $g$that, by (8.3), are proper invariants of each path.

$$x^2-(ct)^2 = \ell^2 , \qquad \text{where:} \quad \ell=c^2/g \qquad (8.4)$$

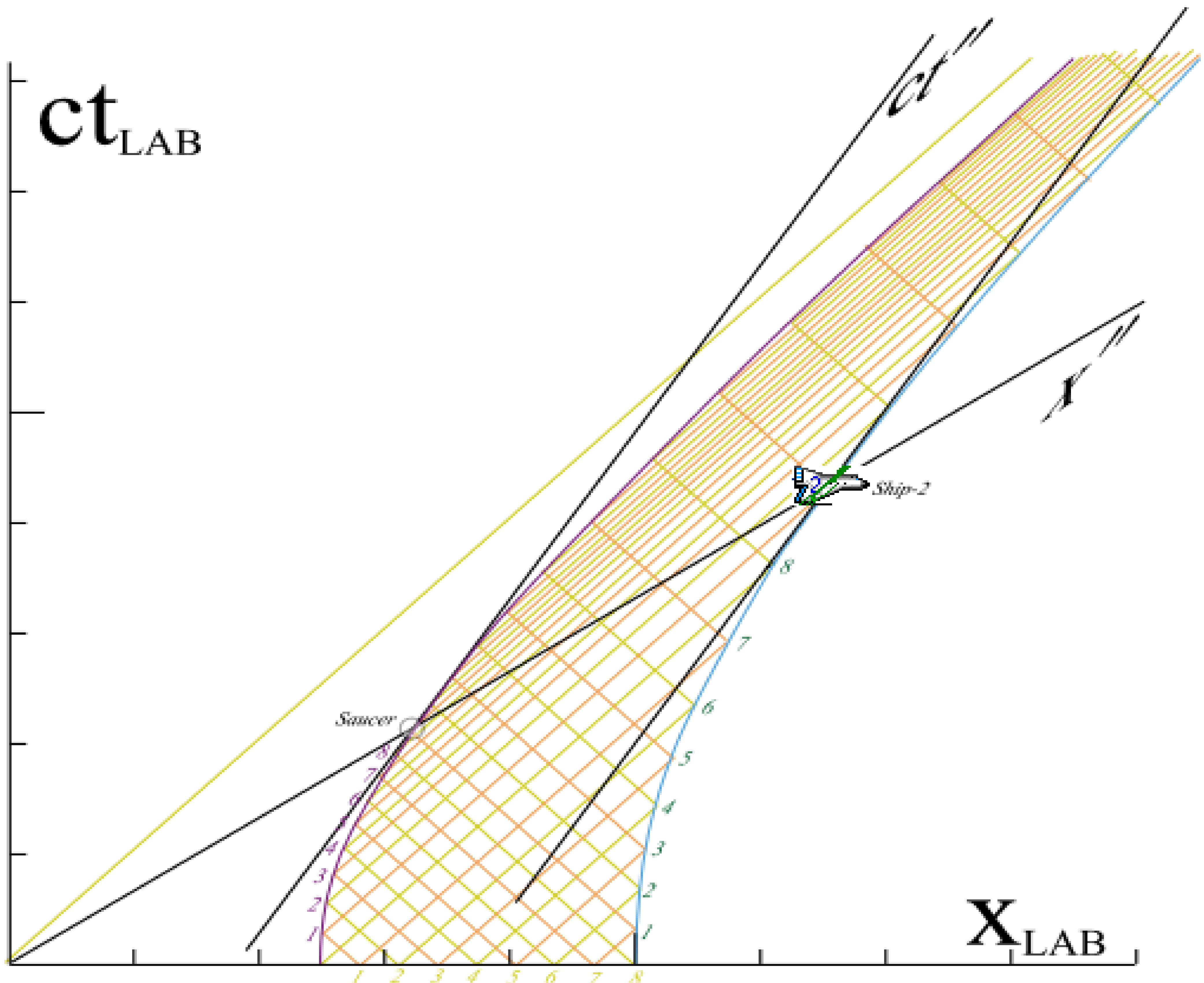
Frequency ω and acceleration g vary inversely with the path's proper location ℓ relative to origin.

$$\omega\ \ell = \omega\ c^2/g = \omega_0\ c^2/g_0 = const. \qquad (8.5)$$

Rapidity $\rho=g\tau/c$ in (8.3) has proper time be a product of hyperbolic radius ℓ in (8.4) and "angle" ρ.

$$c\tau = \rho\ c^2/g = \ell\,\rho \qquad (8.6)$$

This is analogous to a familiar circular arc length formula $s = r\ \phi$. Both have a singular center.



ct_LAB
ct''
x''
Ship-2
Saucer
X_LAB
1
2
3
4
5
6
7
8

Wave frames of varying acceleration

Relativistic acceleration

Optical "Einstein elevator" and flying-saucer-trailer

→ *Biggest mystery of all: Pair production*

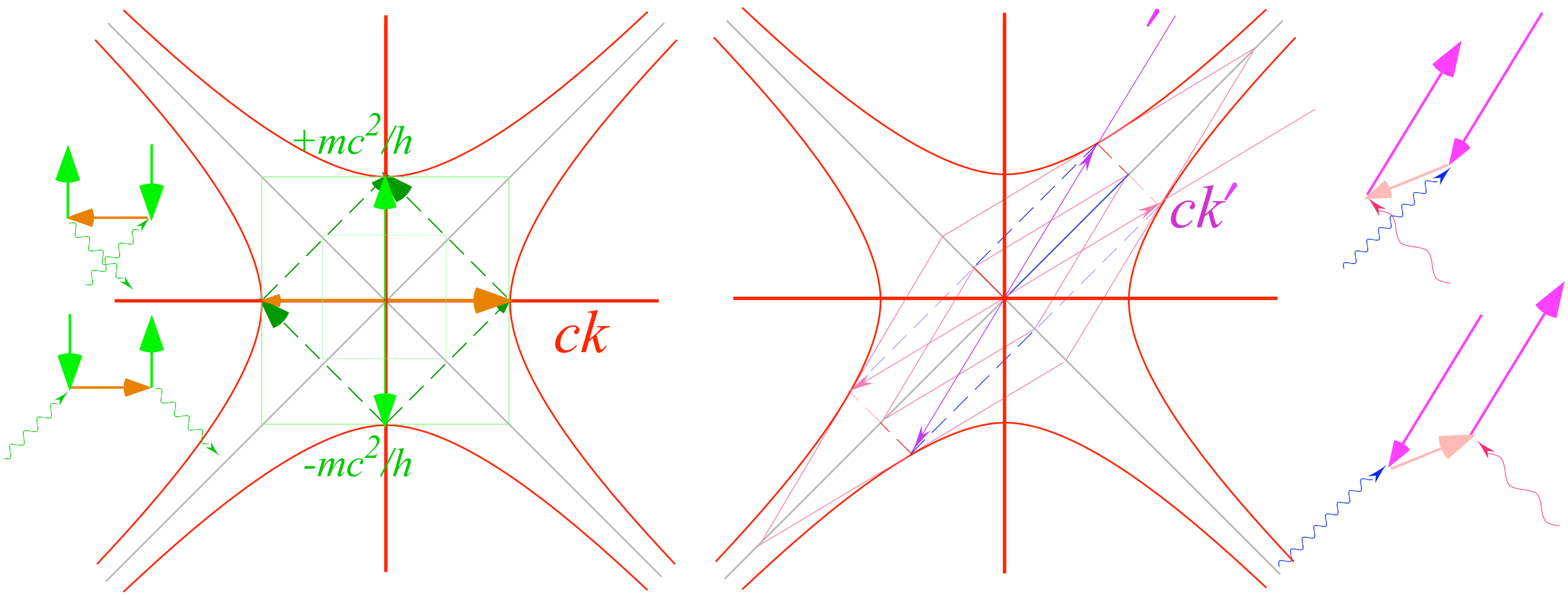
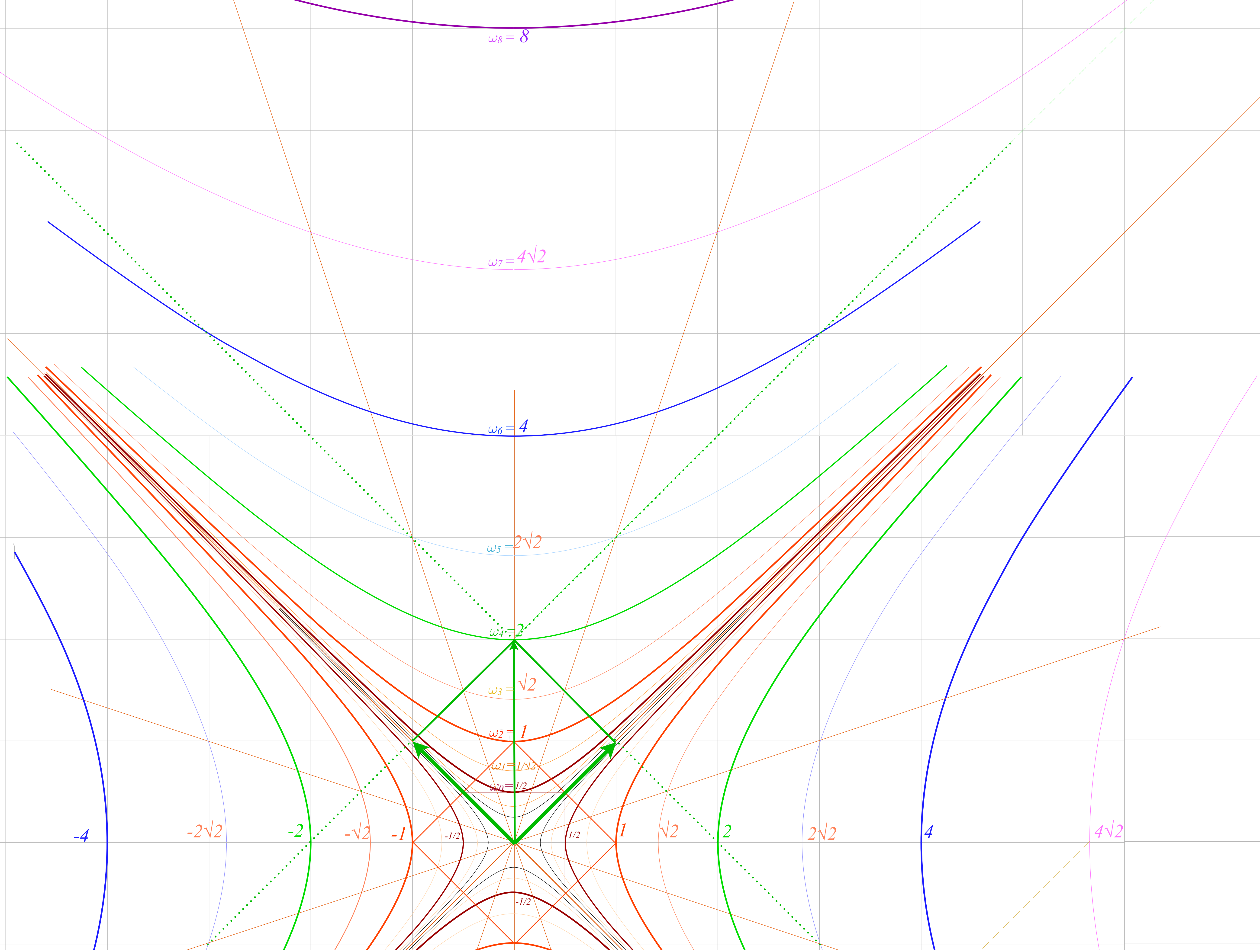
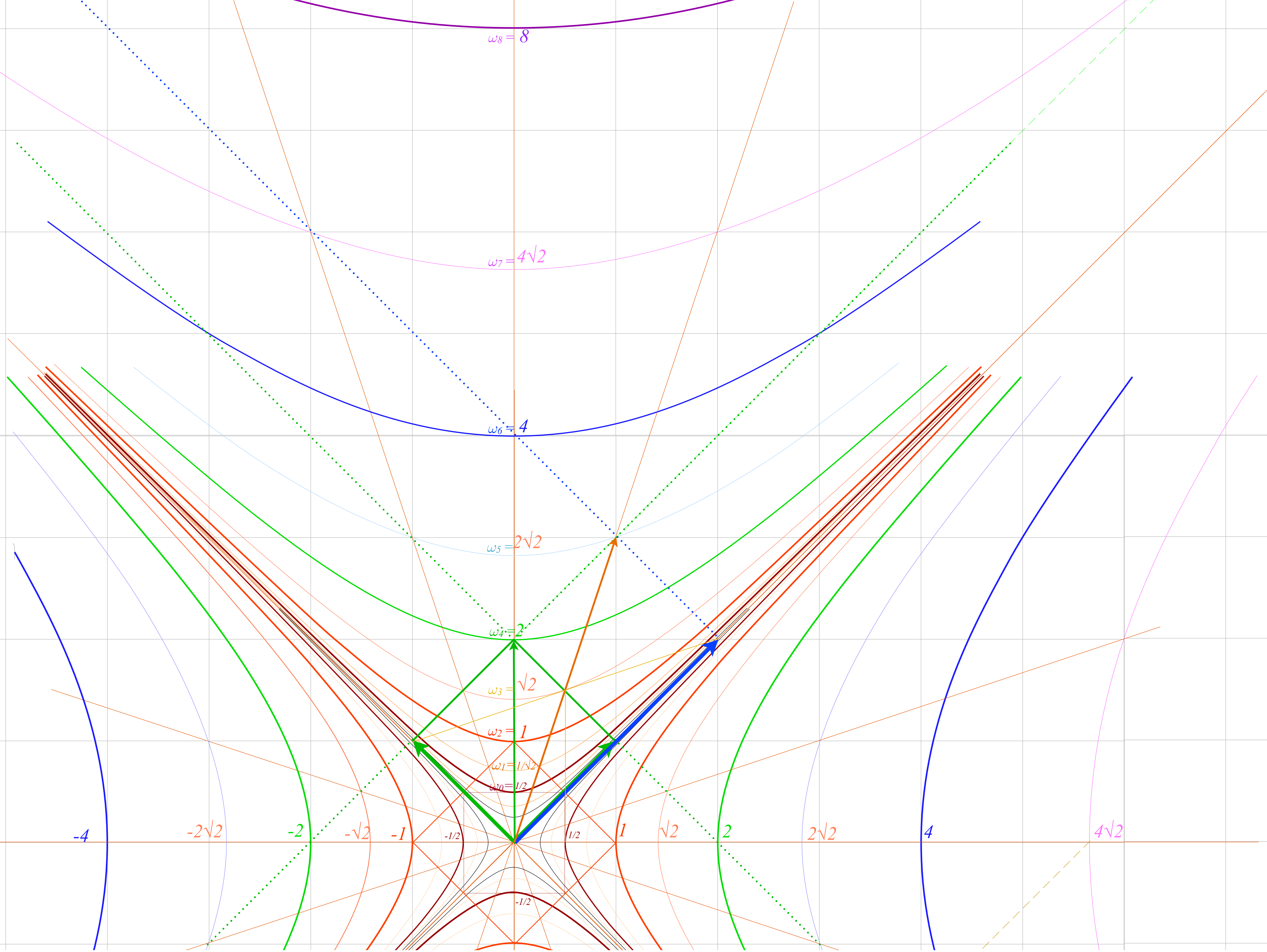
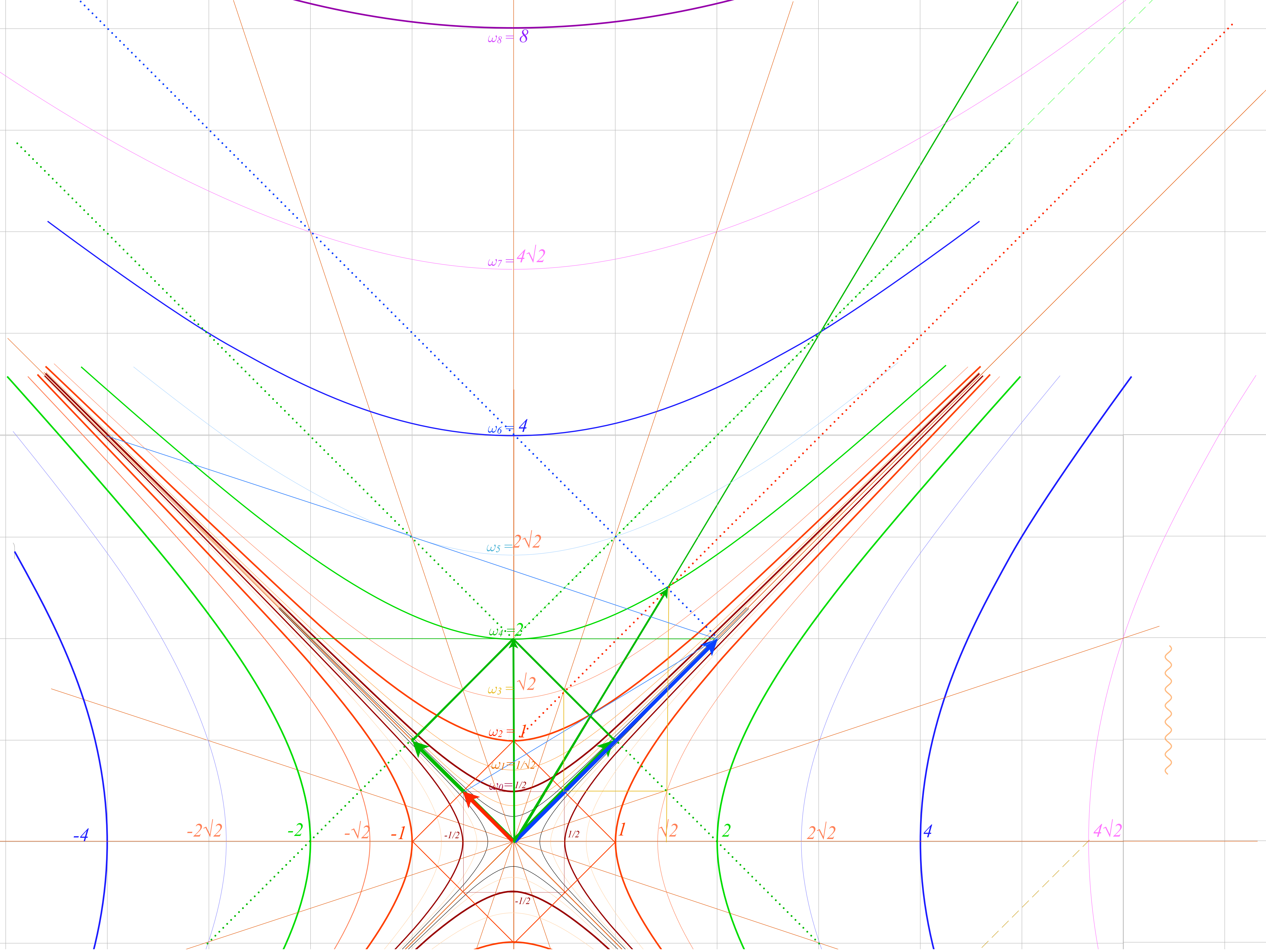


Fig. 8.3 Dirac matter-antimatter dispersion relations and pair-creation-destruction processes.


ω8 = 8
ω7 = 4√2
ω6 = 4
ω5 = 2√2
ω4 = 2
ω3 = √2
ω2 = 1
ω1 = 1/√2
ω0 = 1/2
-4
-2√2
-2
-√2
-1
-1/2
1/2
1
√2
2
2√2
4
4√2
-1/2



ω8 = 8
ω7 = 4√2
ω6 = 4
ω5 = 2√2
ω4 = 2
ω3 = √2
ω2 = 1
ω1 = 1/√2
ω0 = 1/2
-4
-2√2
-2
-√2
-1
-1/2
1/2
1
√2
2
2√2
4
4√2
-1/2


ω8 = 8
ω7 = 4√2
ω6 = 4
ω5 = 2√2
ω4 = 2
ω3 = √2
ω2 = 1
ω1 = 1/√2
ω0 = 1/2
-4
-2√2
-2
-√2
-1
-1/2
1/2
1
√2
2
2√2
4
4√2
-1/2