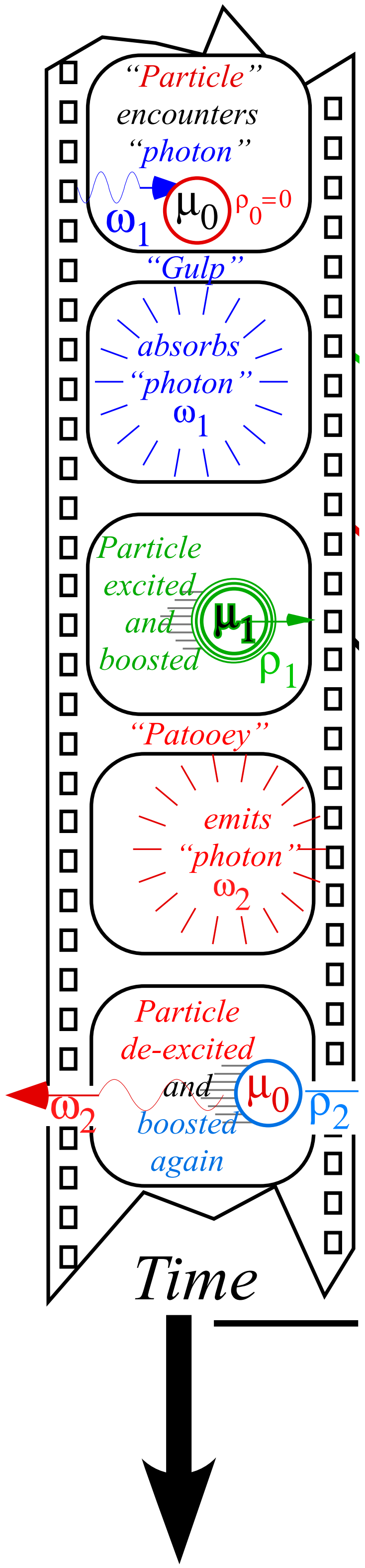
Lecture 34.

Serial Compton scattering and accelerating frames I.

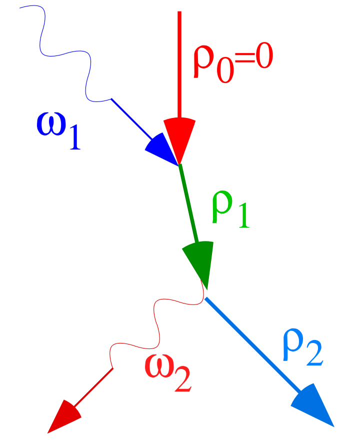
(Ch. 7-8 of Unit 2   4.23.12)

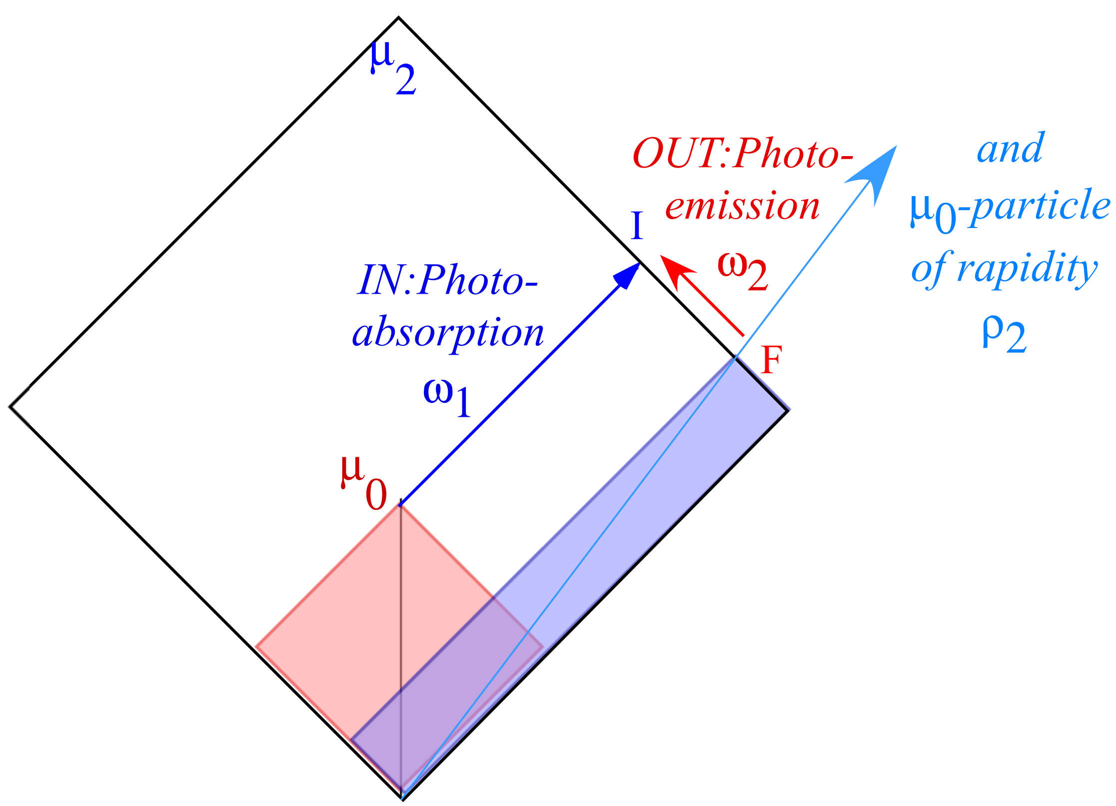
Review of fundamental 1-and2-photon processes and their Feynman diagrams
- *Space-time view of light scattering*

Serial Compton scattering and acceleration plot
- *Geometric construction*
- *Compton wavelength and formulae* — *Lecture 34 ended here*
  - *Some numerology: Which is bigger...H-atom or an electron?*
- *Bouncing pulse wave (PW) vs (CW) shrinking laser*

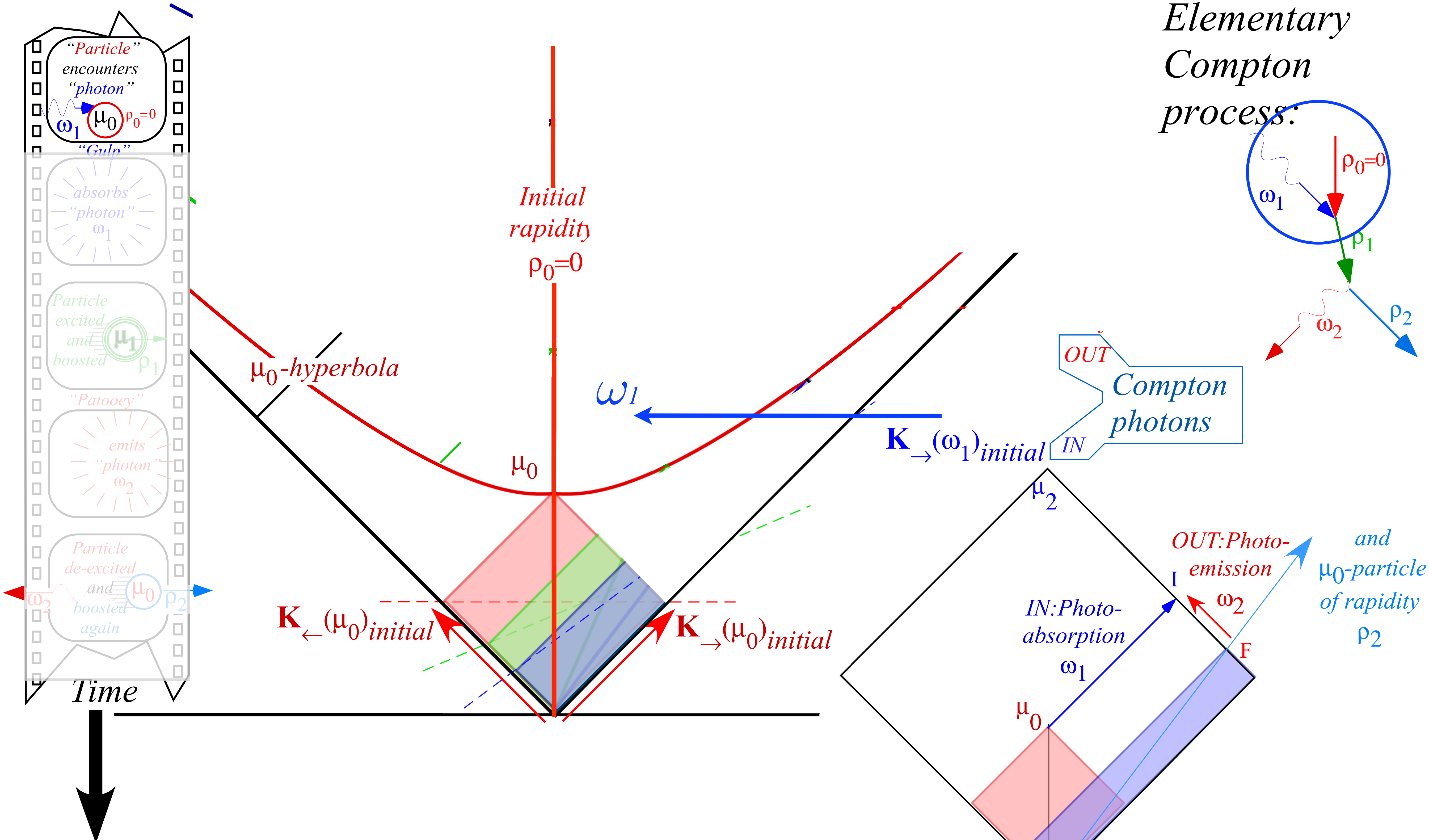


Elementary Compton process:

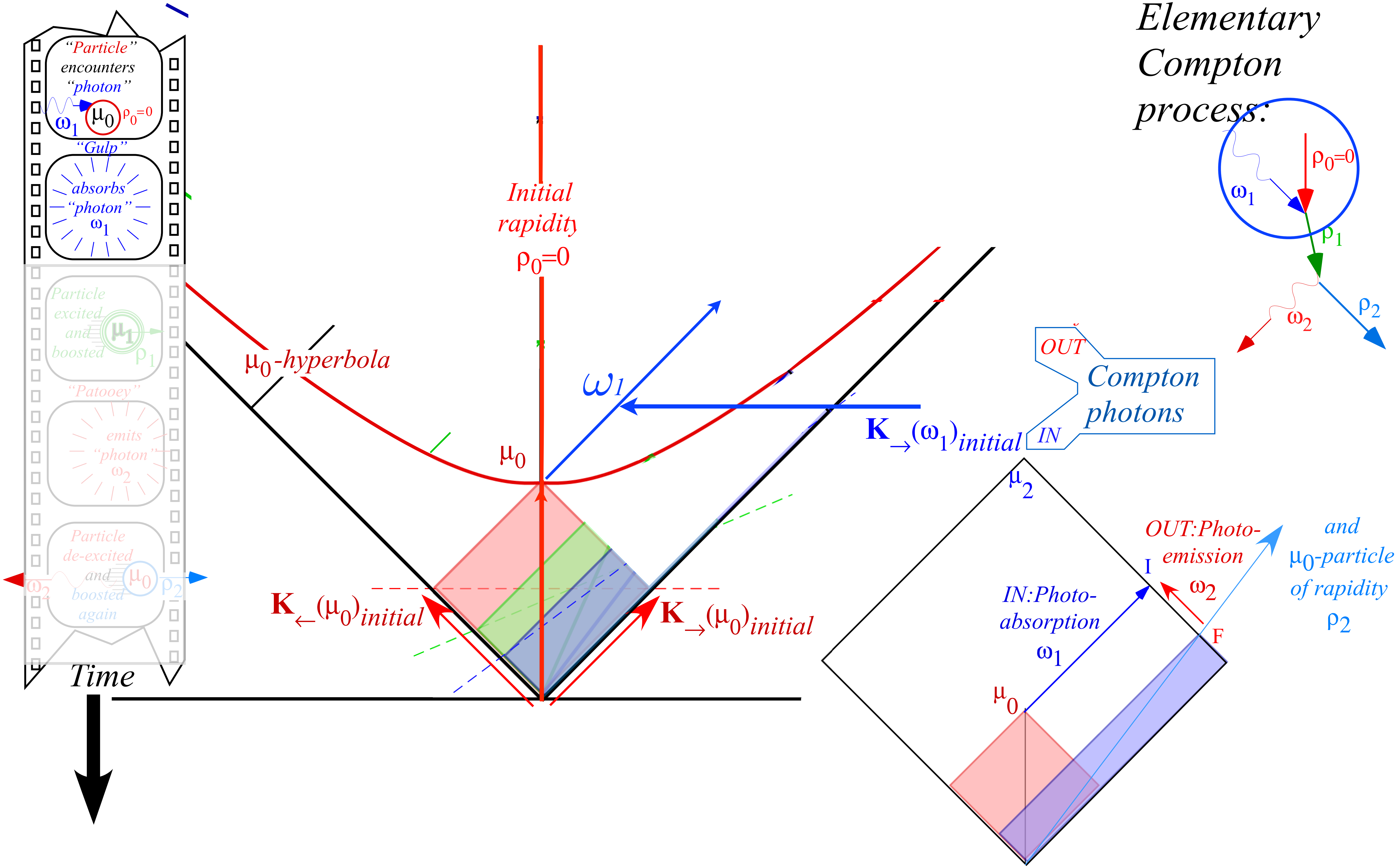




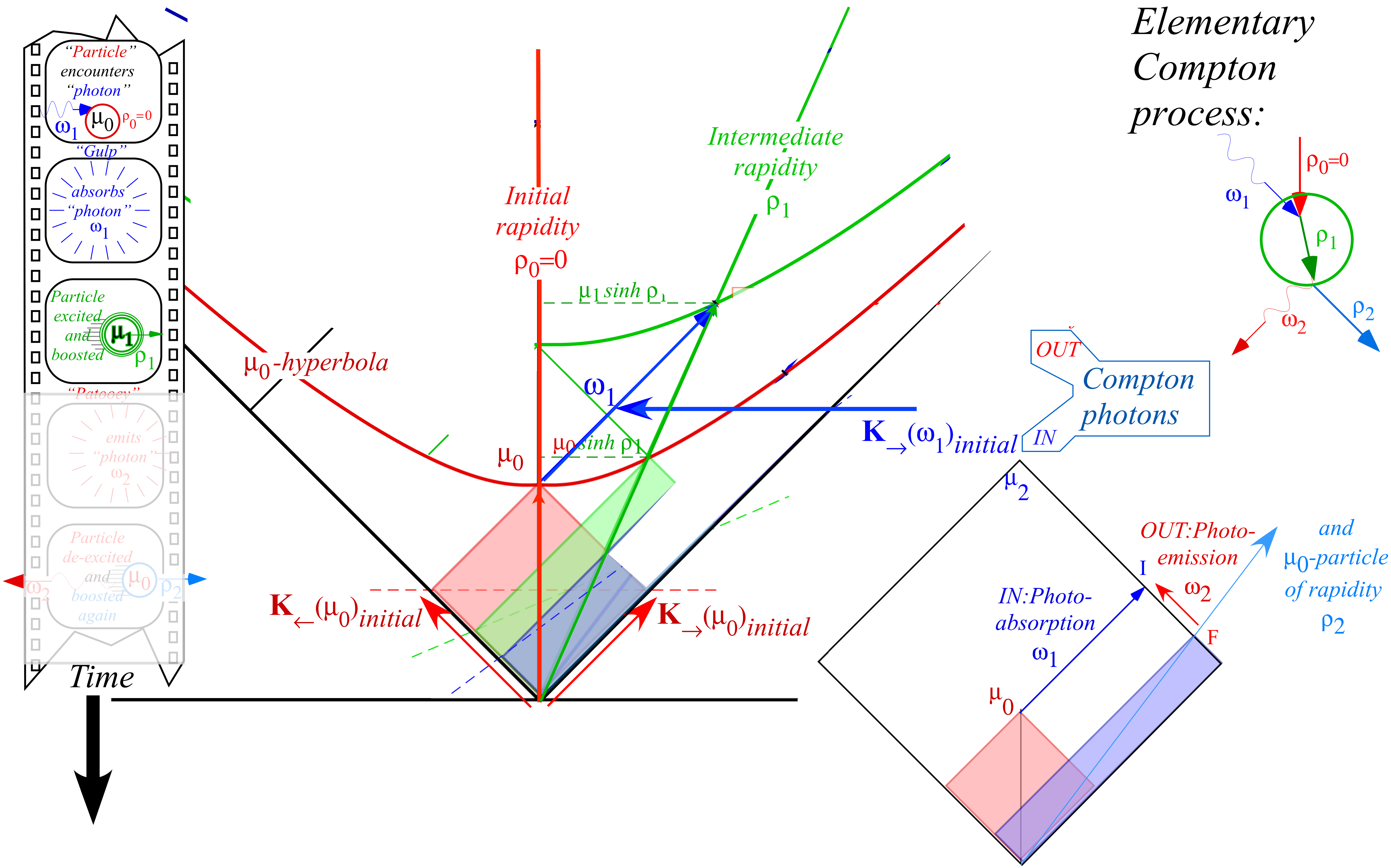
Elementary Compton process:



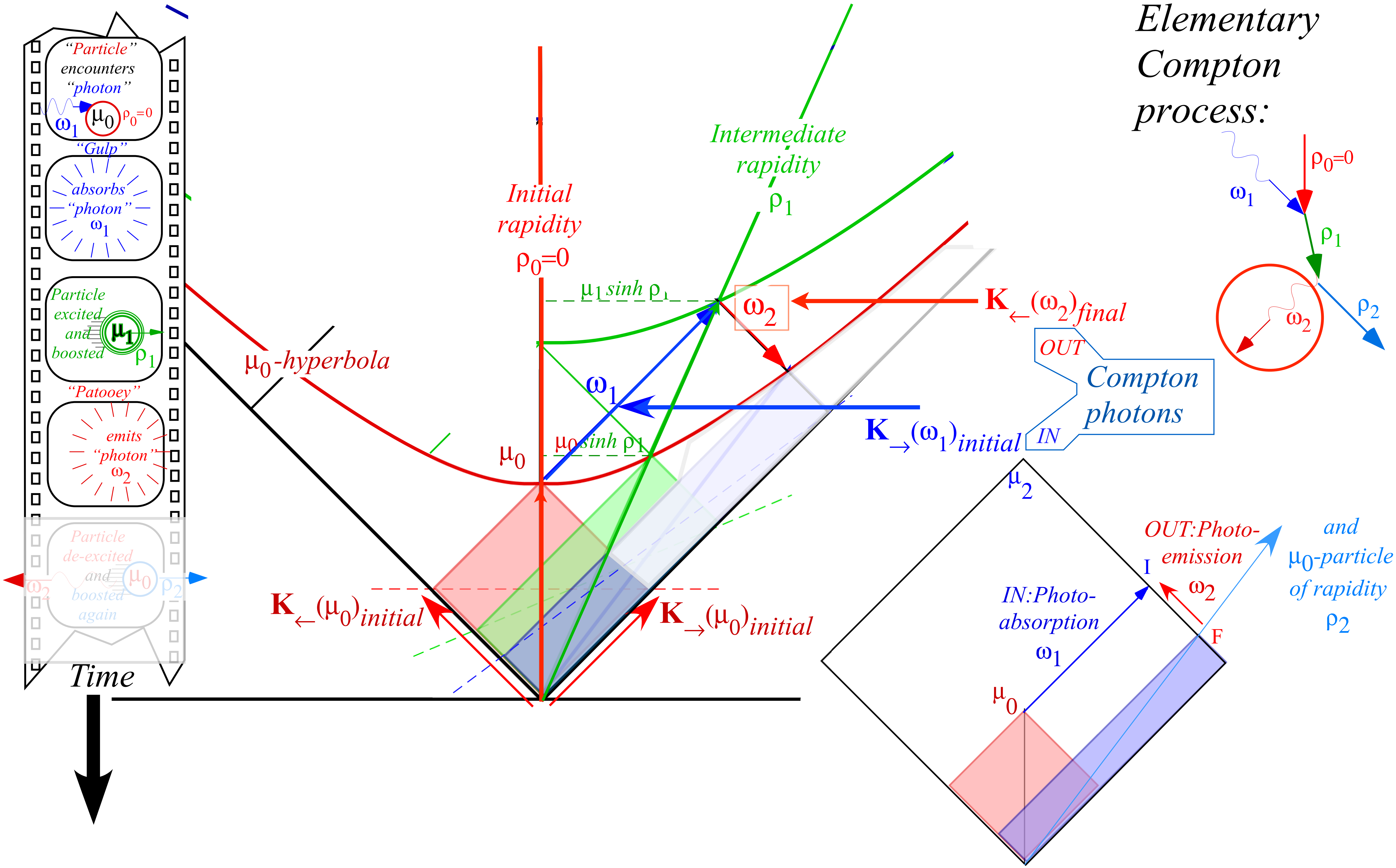
Elementary Compton process:

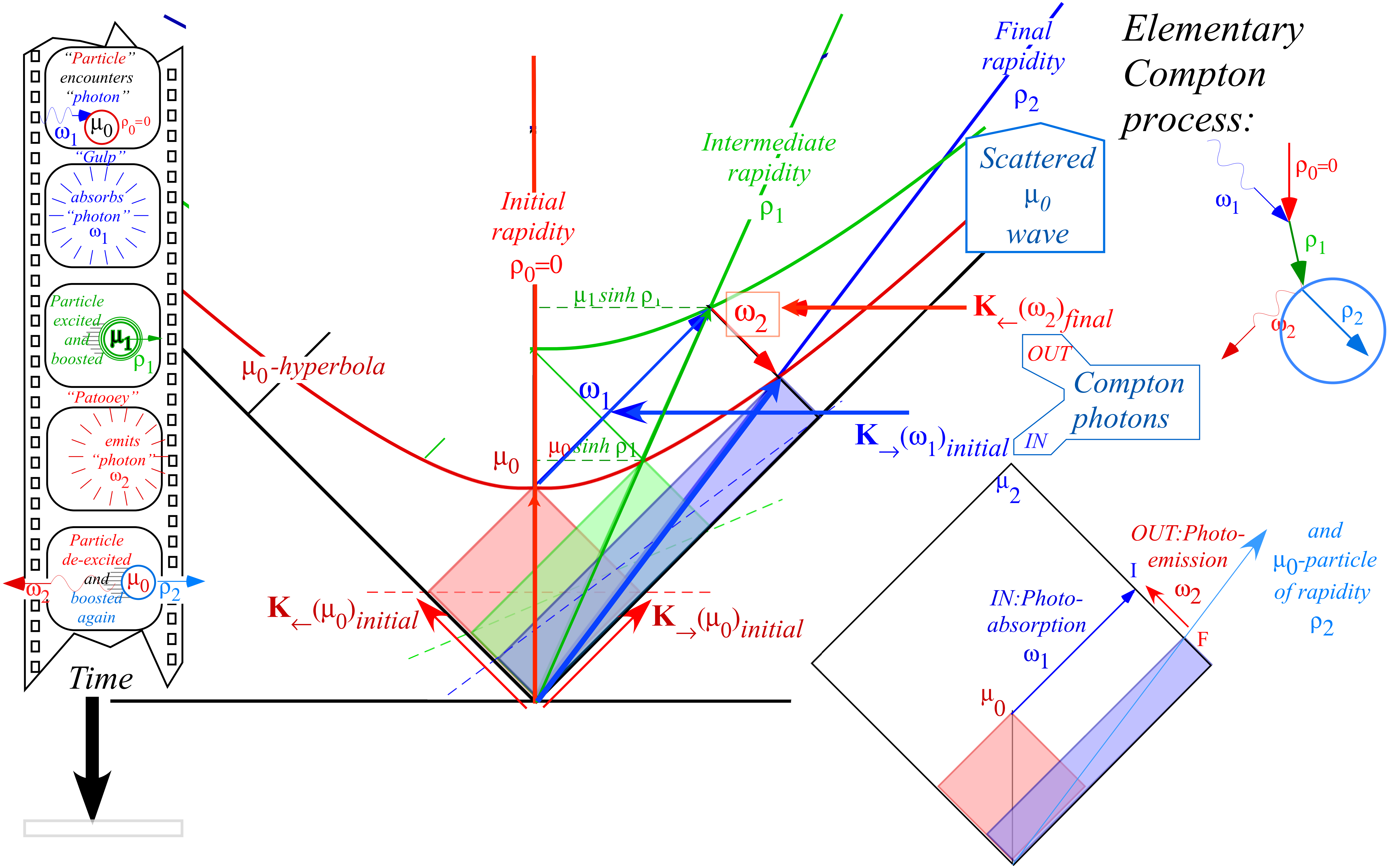


Elementary Compton process:



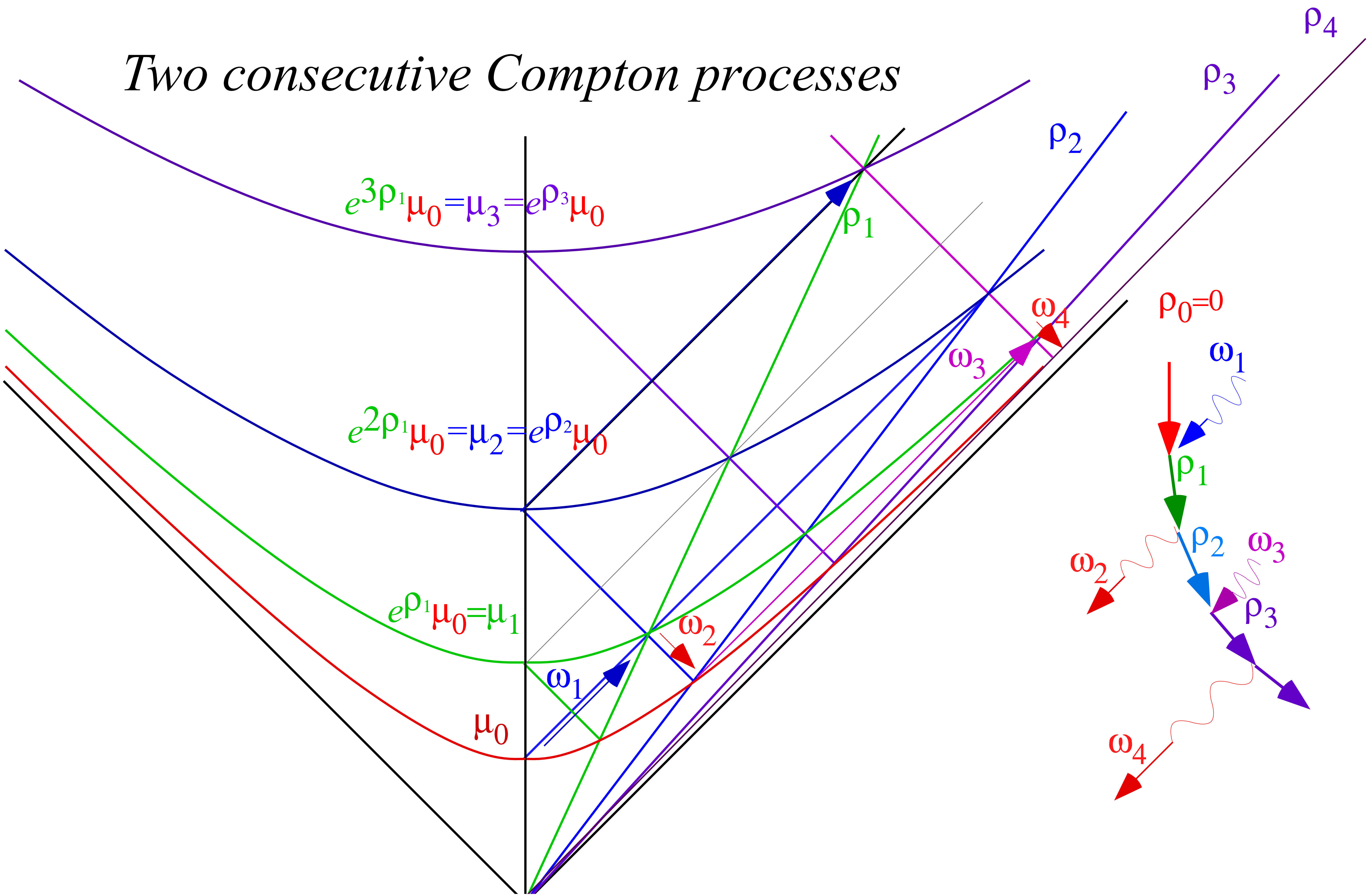
Elementary Compton process:





Elementary Compton process:
"Particle" encounters "photon"
ω_1 μ_0 ρ_0=0
"Gulp"
absorbs "photon" ω_1
Particle excited and boosted μ_1 ρ_1
"Patooey"
emits "photon" ω_2
Particle de-excited and boosted again μ_0 ω_2 ρ_2
Time
Initial rapidity ρ_0=0
Intermediate rapidity ρ_1
Final rapidity ρ_2
Scattered μ_0 wave
μ_1 sinh ρ_1
μ_0 sinh ρ_1
μ_0-hyperbola
μ_0
ω_1
ω_2
$K_{\leftarrow}(\omega_2)_{final}$
OUT
Compton photons
IN
$K_{\rightarrow}(\omega_1)_{initial}$
$K_{\leftarrow}(\mu_0)_{initial}$
$K_{\rightarrow}(\mu_0)_{initial}$
ρ_0=0
ω_1
ρ_1
ω_2
ρ_2
μ_2
OUT:Photo-emission
ω_2
I
F
IN:Photo-absorption
ω_1
μ_0
and μ_0-particle of rapidity ρ_2

Two consecutive Compton processes



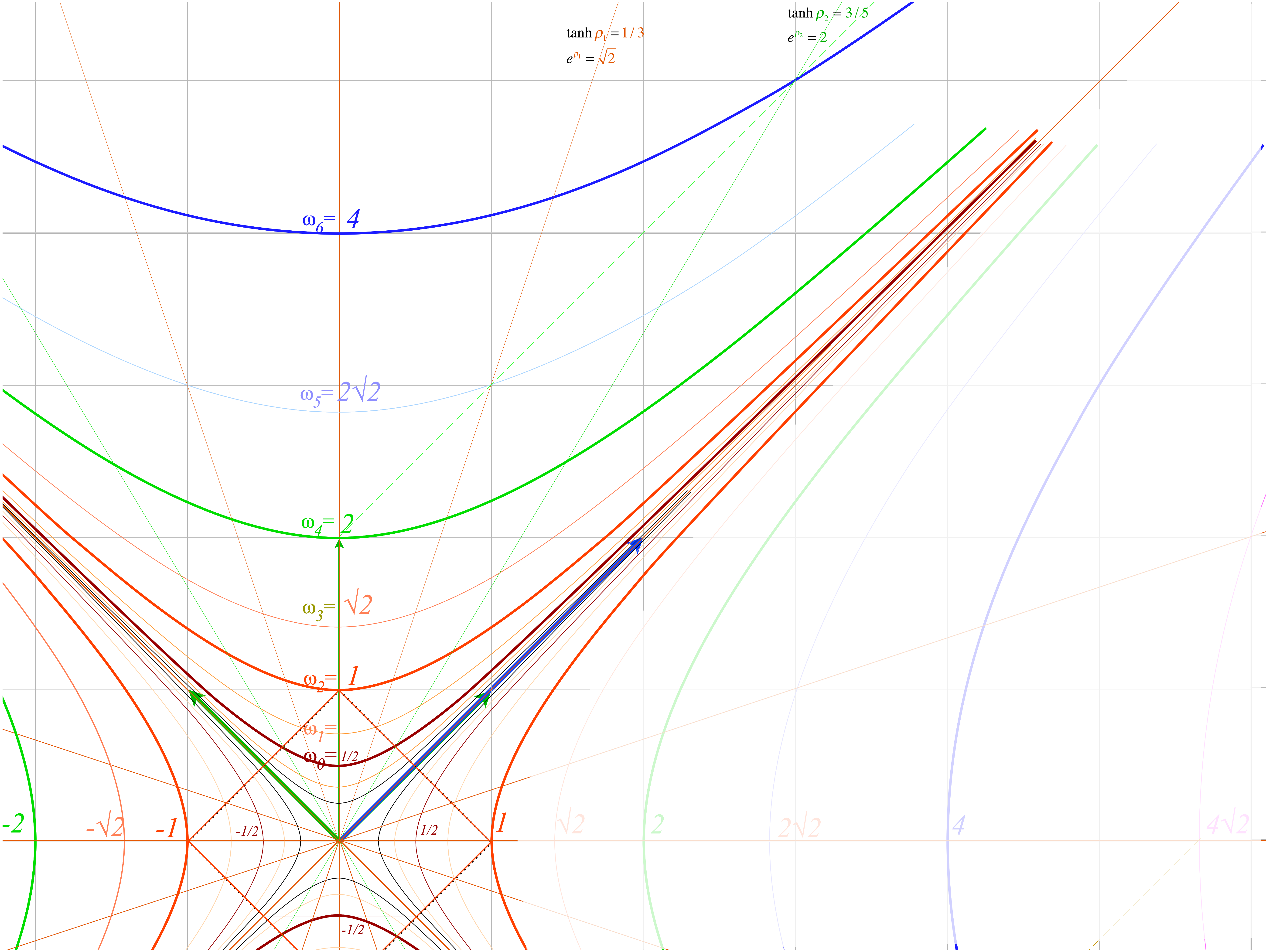
Serial Compton scattering and acceleration plot

→ *Geometric construction*

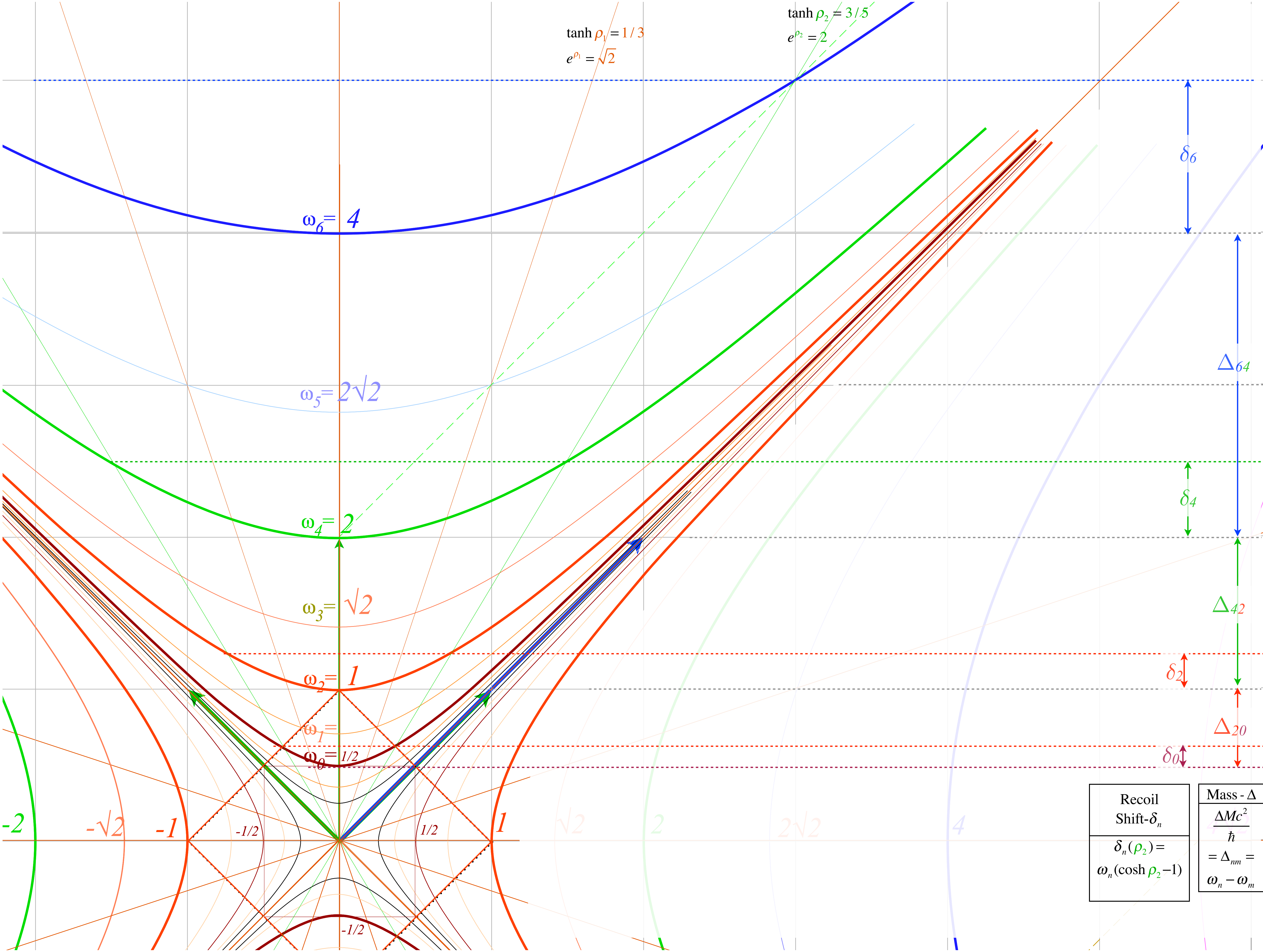
Compton wavelength and formulae

Some numerology: Which is bigger...H-atom or an electron?

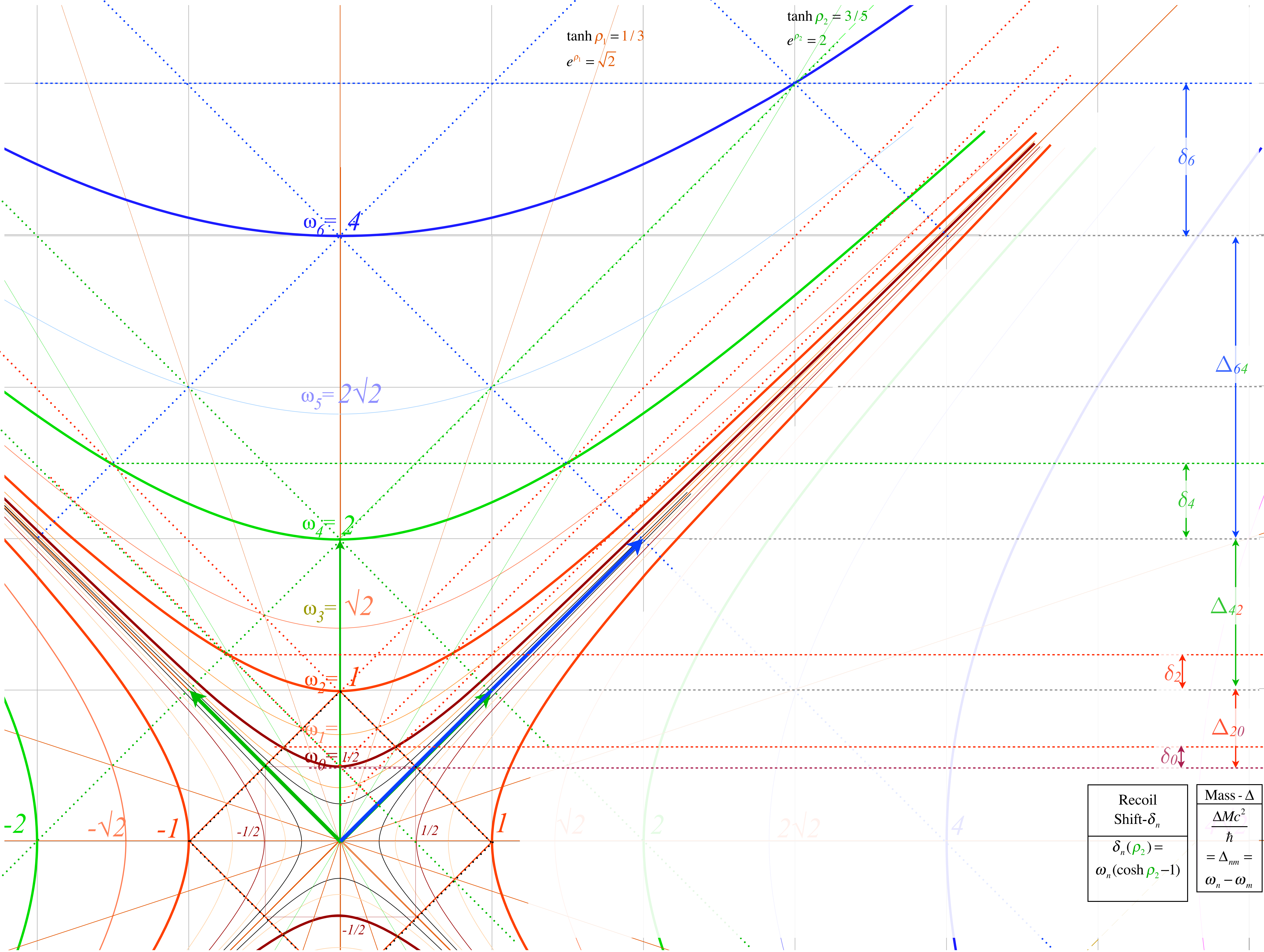
Bouncing pulse wave (PW) vs (CW) shrinking laser

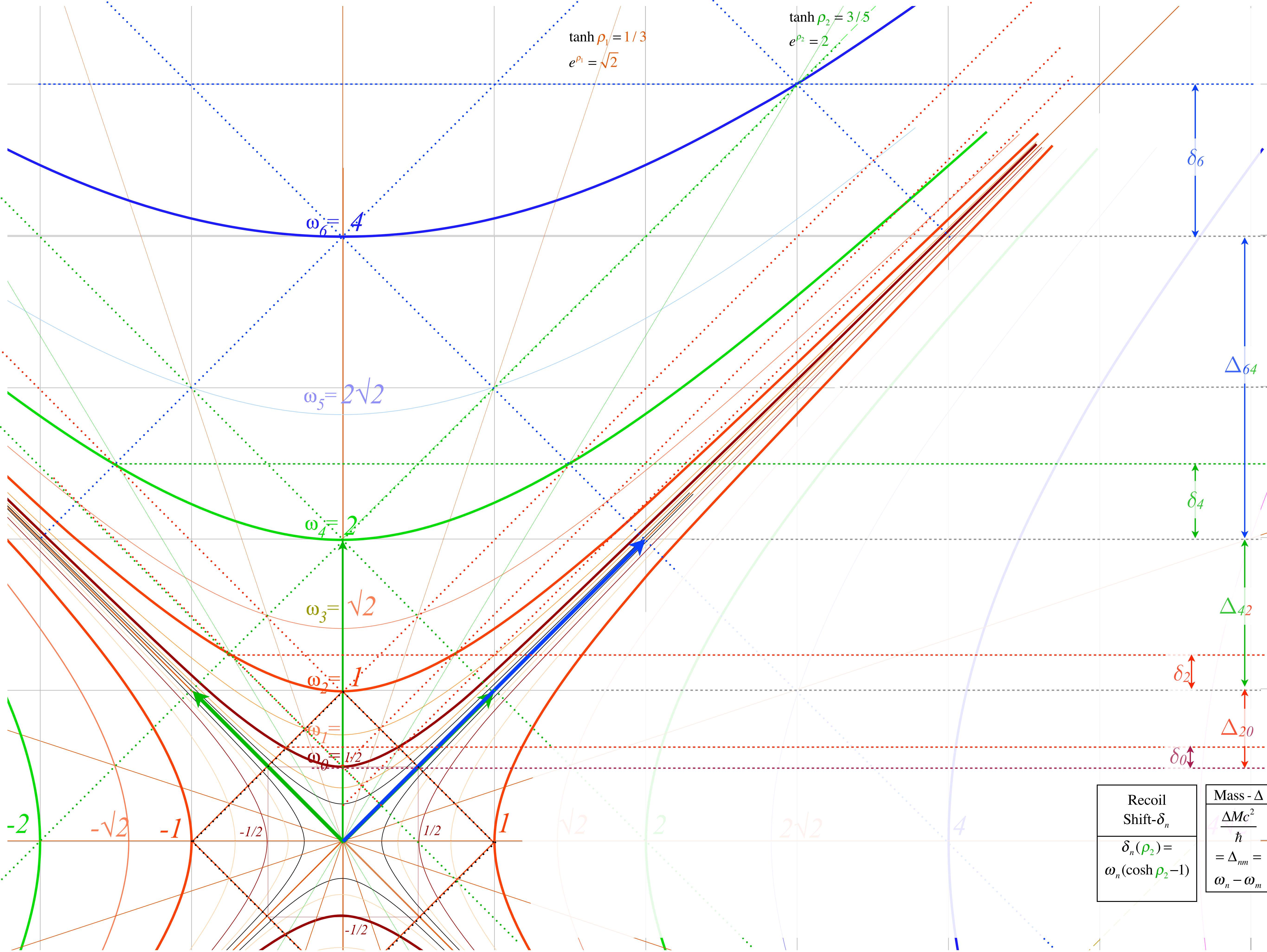


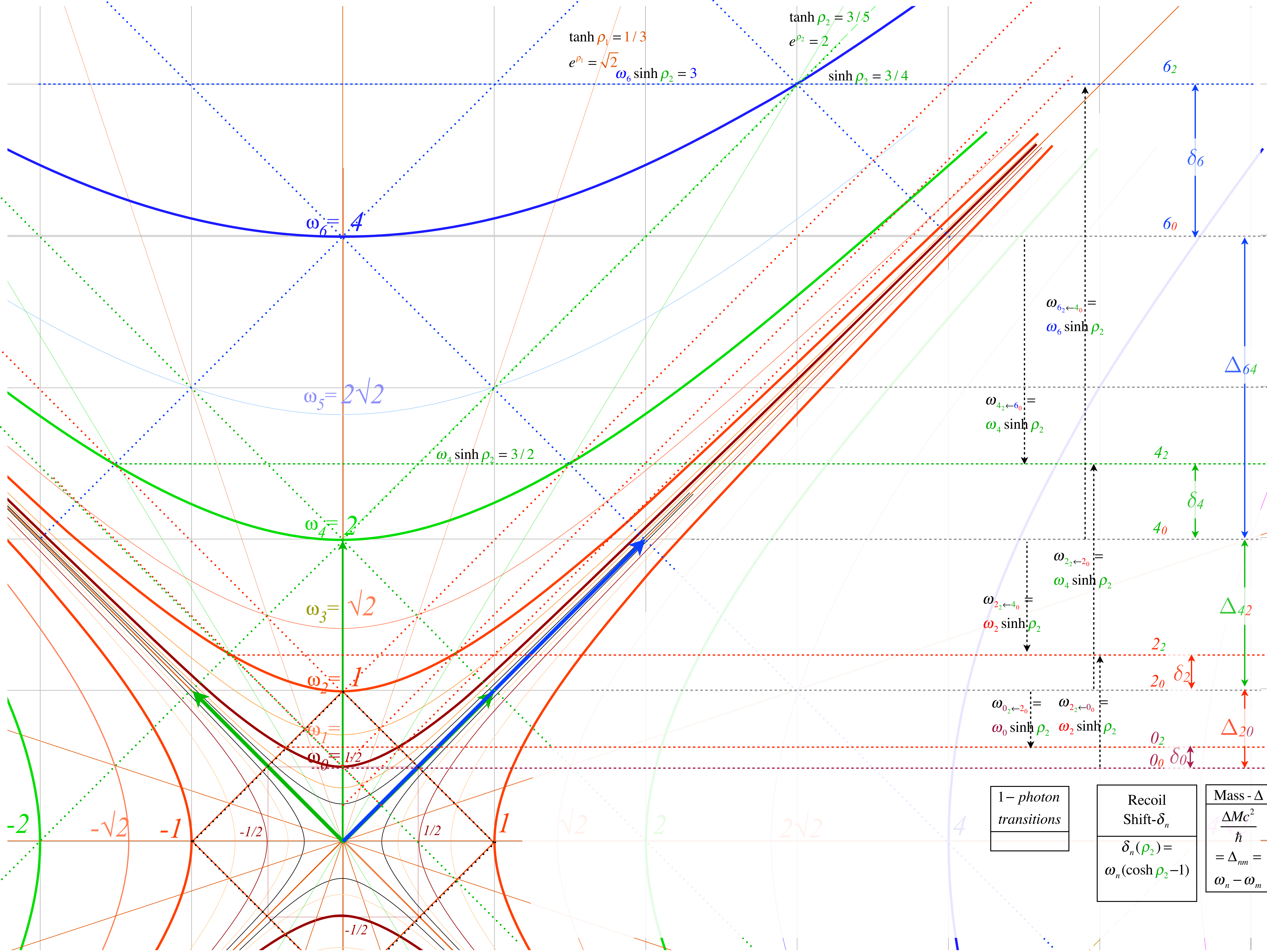
tanh ρ_1 = 1/3
$e^{\rho_1} = \sqrt{2}$
tanh ρ_2 = 3/5
e^{ρ_2} = 2
ω_6= 4
ω_5= 2√2
ω_4= 2
ω_3= √2
ω_2= 1
ω_1=
ω_0= 1/2
-2
-√2
-1
-1/2
1/2
1
√2
2
2√2
4
4√2
-1/2

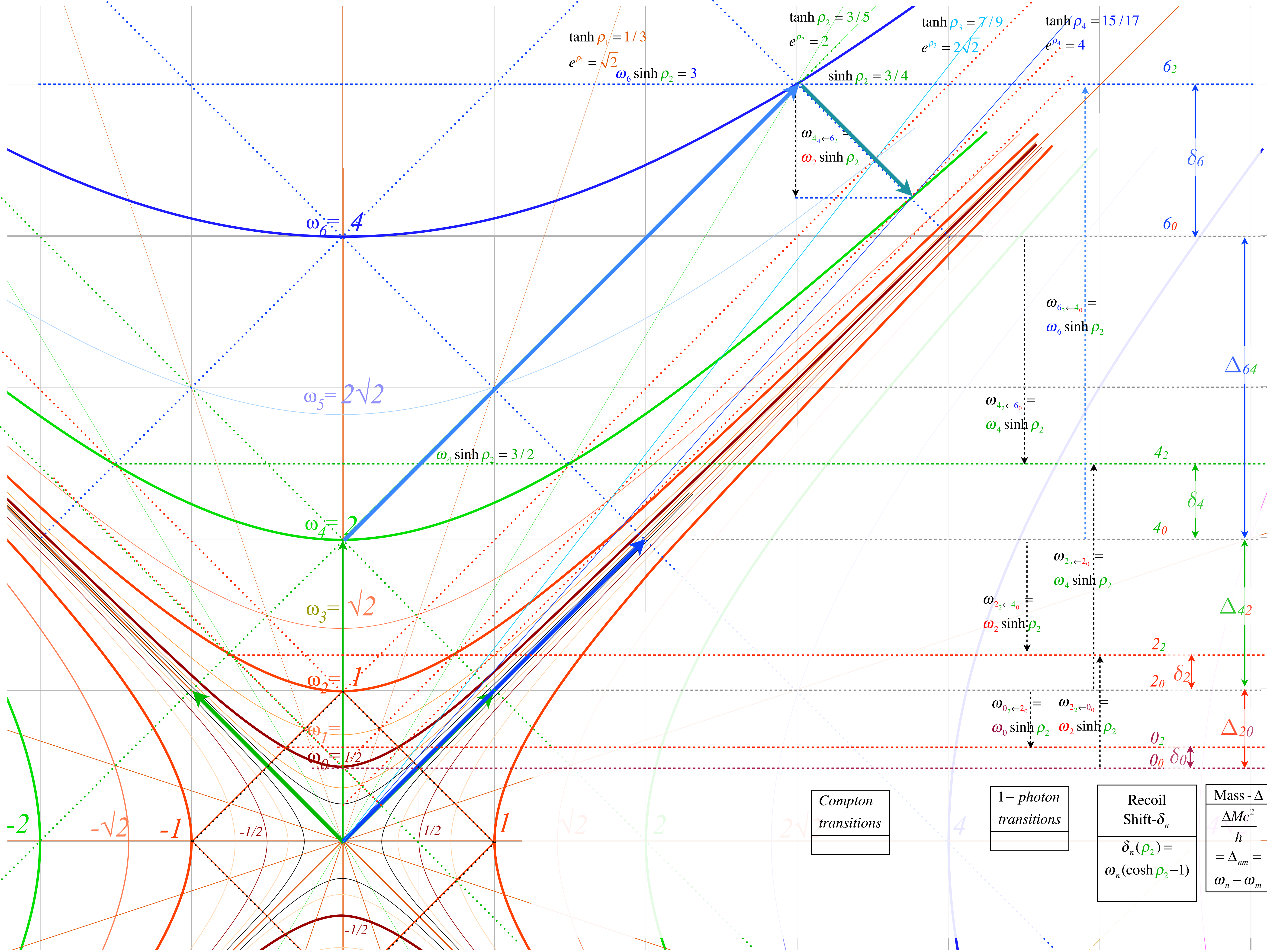


tanh $\rho_1 = 1/3$
$e^{\rho_1} = \sqrt{2}$
tanh $\rho_2 = 3/5$
$e^{\rho_2} = 2$
$\omega_6 = 4$
$\omega_5 = 2\sqrt{2}$
$\omega_4 = 2$
$\omega_3 = \sqrt{2}$
$\omega_2 = 1$
$\omega_1 =$
$\omega_0 = 1/2$
δ_6
Δ_{64}
δ_4
Δ_{42}
δ_2
Δ_{20}
δ_0
-2
$-\sqrt{2}$
-1
-1/2
1/2
1
-1/2
Recoil Shift-δ_n
$\delta_n(\rho_2) = \omega_n(\cosh\rho_2 - 1)$
Mass - Δ
$\frac{\Delta Mc^2}{\hbar} = \Delta_{nm} = \omega_n - \omega_m$

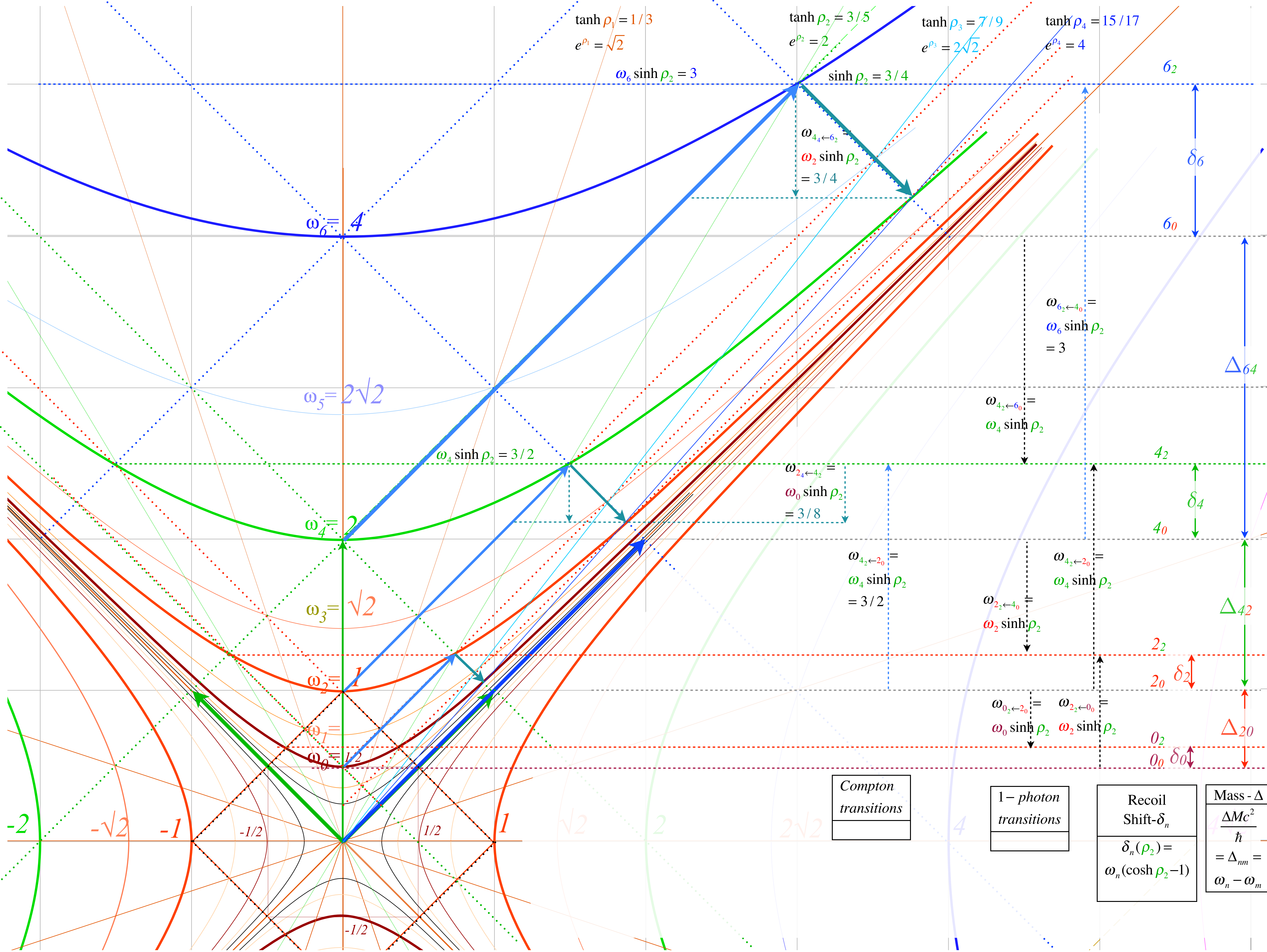

tanh ρ1 = 1/3
e^ρ1 = √2
tanh ρ2 = 3/5
e^ρ2 = 2
ω6= 4
ω5= 2√2
ω4= 2
ω3= √2
ω2= 1
ω1=
ω0= 1/2
δ6
Δ64
δ4
Δ42
δ2
Δ20
δ0
-2
-√2
-1
-1/2
1/2
1
√2
2
2√2
4
-1/2
Recoil Shift-δn
δn(ρ2) = ωn(cosh ρ2 −1)
Mass - Δ
ΔMc²/ħ
= Δnm =
ωn − ωm


tanh ρ1 = 1/3
e^ρ1 = √2
tanh ρ2 = 3/5
e^ρ2 = 2
ω6 = 4
ω5 = 2√2
ω4 = 2
ω3 = √2
ω2 = 1
ω1 =
ω0 = 1/2
δ6
Δ64
δ4
Δ42
δ2
Δ20
δ0
-2
-√2
-1
-1/2
1/2
1
√2
2
2√2
4
-1/2
Recoil Shift-δn
δn(ρ2) = ωn(cosh ρ2 −1)
Mass - Δ
ΔMc²/ħ = Δnm = ωn − ωm


tanh $\rho_1 = 1/3$
$e^{\rho_1} = \sqrt{2}$
$\omega_6 \sinh \rho_2 = 3$
tanh $\rho_2 = 3/5$
$e^{\rho_2} = 2$
sinh $\rho_2 = 3/4$
$\omega_6 = 4$
$\omega_5 = 2\sqrt{2}$
$\omega_4 \sinh \rho_2 = 3/2$
$\omega_4 = 2$
$\omega_3 = \sqrt{2}$
$\omega_2 = 1$
$\omega_1 =$
$\omega_0 = 1/2$
-2
$-\sqrt{2}$
-1
-1/2
1/2
1
-1/2
6_2
δ_6
6_0
Δ_{64}
4_2
δ_4
4_0
Δ_{42}
2_2
δ_2
2_0
Δ_{20}
0_2
0_0
δ_0
$\omega_{6_2 \leftarrow 4_0} =$
$\omega_6 \sinh \rho_2$
$\omega_{4_2 \leftarrow 6_0} =$
$\omega_4 \sinh \rho_2$
$\omega_{2_2 \leftarrow 2_0} =$
$\omega_4 \sinh \rho_2$
$\omega_{2_2 \leftarrow 4_0} =$
$\omega_2 \sinh \rho_2$
$\omega_{0_2 \leftarrow 2_0} =$
$\omega_0 \sinh \rho_2$
$\omega_{2_2 \leftarrow 0_0} =$
$\omega_2 \sinh \rho_2$
1 − photon transitions
Recoil Shift-δ_n
$\delta_n(\rho_2) = \omega_n(\cosh \rho_2 - 1)$
Mass - Δ
$\frac{\Delta M c^2}{\hbar} = \Delta_{nm} = \omega_n - \omega_m$



tanh $\rho_1 = 1/3$
$e^{\rho_1} = \sqrt{2}$
$\omega_6 \sinh \rho_2 = 3$
tanh $\rho_2 = 3/5$
$e^{\rho_2} = 2$
sinh $\rho_2 = 3/4$
tanh $\rho_3 = 7/9$
$e^{\rho_3} = 2\sqrt{2}$
tanh $\rho_4 = 15/17$
$e^{\rho_4} = 4$
6_2
$\omega_{4_4 \leftarrow 6_2} =$
$\omega_2 \sinh \rho_2$
δ_6
$\omega_6 = 4$
6_0
$\omega_{6_2 \leftarrow 4_0} =$
$\omega_6 \sinh \rho_2$
Δ_{64}
$\omega_5 = 2\sqrt{2}$
$\omega_{4_2 \leftarrow 6_0} =$
$\omega_4 \sinh \rho_2$
$\omega_4 \sinh \rho_2 = 3/2$
4_2
δ_4
$\omega_4 = 2$
4_0
$\omega_{2_2 \leftarrow 2_0} =$
$\omega_4 \sinh \rho_2$
$\omega_3 = \sqrt{2}$
$\omega_{2_2 \leftarrow 4_0} =$
$\omega_2 \sinh \rho_2$
Δ_{42}
2_2
δ_2
$\omega_2 = 1$
2_0
$\omega_1 =$
$\omega_{0_2 \leftarrow 2_0} =$
$\omega_0 \sinh \rho_2$
$\omega_{2_2 \leftarrow 0_0} =$
$\omega_2 \sinh \rho_2$
Δ_{20}
$\omega_0 = 1/2$
0_2
0_0
δ_0
Compton transitions
1 − photon transitions
Recoil Shift-δ_n
$\delta_n(\rho_2) =$
$\omega_n(\cosh \rho_2 - 1)$
Mass - Δ
$\frac{\Delta Mc^2}{\hbar}$
$= \Delta_{nm} =$
$\omega_n - \omega_m$
-2
$-\sqrt{2}$
-1
-1/2
1/2
1
$\sqrt{2}$
2
$2\sqrt{2}$
4
-1/2



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tanh ρ2 = 3/5
e^ρ2 = 2
tanh ρ3 = 7/9
e^ρ3 = 2√2
tanh ρ4 = 15/17
e^ρ4 = 4
ω6 sinh ρ2 = 3
sinh ρ2 = 3/4
62
ω4_4←6_2 =
ω2 sinh ρ2
= 3/4
δ6
ω6= 4
60
ω6_2←4_0 =
ω6 sinh ρ2
= 3
Δ64
ω5= 2√2
ω4_2←6_0 =
ω4 sinh ρ2
ω4 sinh ρ2 = 3/2
42
ω2_4←4_2 =
ω0 sinh ρ2
= 3/8
δ4
ω4= 2
40
ω4_2←2_0 =
ω4 sinh ρ2
= 3/2
ω4_2←2_0 =
ω4 sinh ρ2
ω3= √2
ω2_2←4_0 =
ω2 sinh ρ2
Δ42
22
δ2
ω2= 1
20
ω0_2←2_0 =
ω0 sinh ρ2
ω2_2←0_0 =
ω2 sinh ρ2
Δ20
ω1=
02
ω0= 1/2
00
δ0
Compton transitions
1 – photon transitions
Recoil Shift-δn
δn(ρ2) = ωn(cosh ρ2 – 1)
Mass - Δ
ΔMc²/ħ
= Δnm =
ωn – ωm
-2
-√2
-1
-1/2
1/2
1
√2
2
2√2
4
-1/2

Serial Compton scattering and acceleration plot

- *Geometric construction*
- → *Compton wavelength and formulae*
  - *Some numerology: Which is bigger...H-atom or an electron?*
- *Bouncing pulse wave (PW) vs (CW) shrinking laser*

$\tanh\rho_1 = 1/3$ $\quad e^{\rho_1} = \sqrt{2}$

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$\tanh\rho_3 = 7/9$ $\quad e^{\rho_3} = 2\sqrt{2}$

$\tanh\rho_4 = 15/17$ $\quad e^{\rho_4} = 4$

$\omega_6 \sinh\rho_2 = 3$

$\sinh\rho_2 = 3/4$

Compton Wavelength formula

$$\lambda_{IN} - \lambda_{FIN} = \lambda_{2_4\leftarrow 4_2} - \lambda_{4_2\leftarrow 2_0} = 2\pi c\left(\frac{1}{\omega_{2_4\leftarrow 4_2}} - \frac{1}{\omega_{4_2\leftarrow 2_0}}\right)$$

$$= 2\pi c\left(\frac{1}{e^{-\rho_2}\omega_2 \sinh\rho_2} - \frac{1}{e^{+\rho_2}\omega_2 \sinh\rho_2}\right)$$

$$= 2\pi c\left(\frac{1}{e^{-\rho_2}} - \frac{1}{e^{+\rho_2}}\right)\frac{1}{\omega_2 \sinh\rho_2}$$

$$= \frac{2\pi c}{\omega_2}\left(\frac{e^{+\rho_2} - e^{-\rho_2}}{1}\right)\frac{1}{\sinh\rho_2}$$

$$= \frac{2\pi c}{\omega_2}(2) = \frac{2\pi c\hbar}{M_2 c^2}(2) = \frac{2\pi\hbar}{M_2 c}(2) = \frac{h}{M_2 c}(2)$$

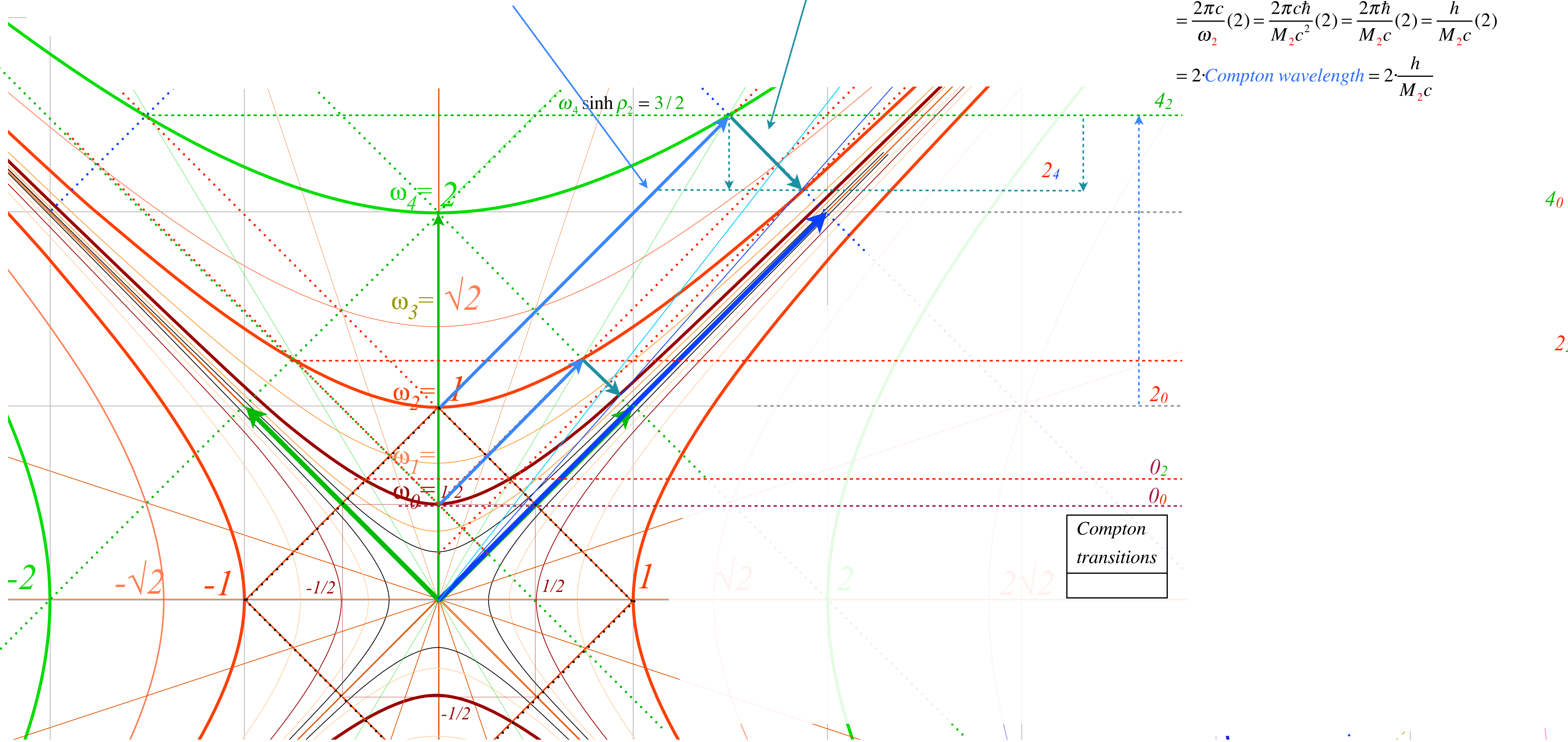
$$= 2\cdot \text{Compton wavelength} = 2\cdot\frac{h}{M_2 c}$$

$\omega_{4_2\leftarrow 2_0} =$
$\omega_4 \sinh\rho_2$
$= e^{+\rho_2}\omega_2 \sinh\rho_2$
$= 3/2$

Compton IN

$\omega_{2_4\leftarrow 4_2} =$
$\omega_0 \sinh\rho_2$
$= e^{-\rho_2}\omega_2 \sinh\rho_2$
$= 3/8$

Compton FIN



Serial Compton scattering and acceleration plot

Geometric construction

Compton wavelength and formulae

→ *Some numerology: Which is bigger...H-atom or an electron?*

Bouncing pulse wave (PW) vs (CW) shrinking laser

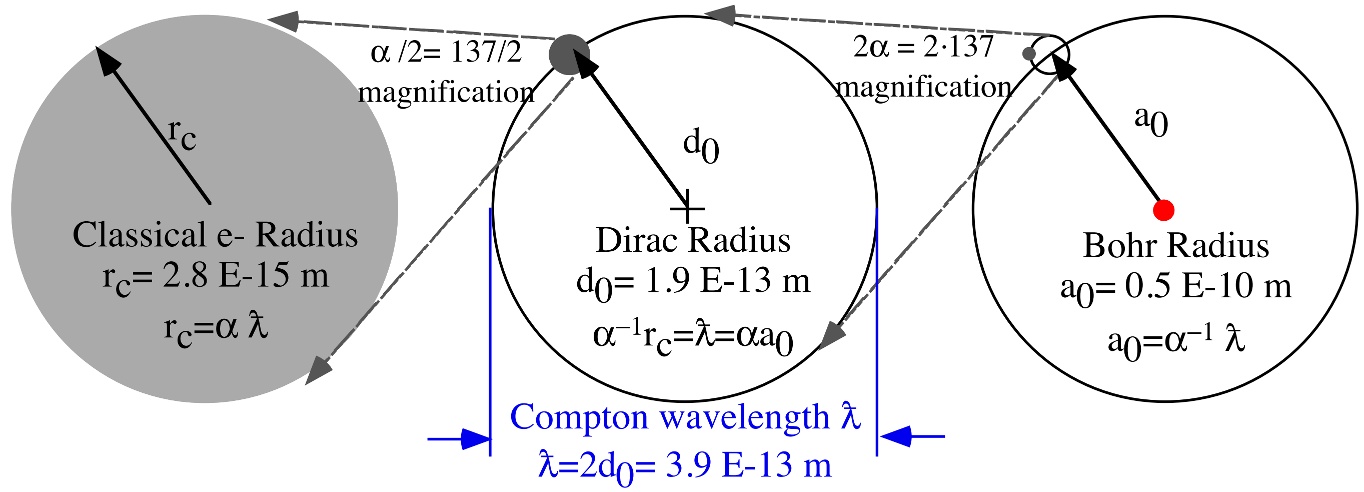


Fig.8A.2 Various electron radii and their relative sizes related by fine-structure constant α = 1/137.

Bohr model has electron orbiting at radius *r* so centrifugal force balances Coulomb attraction to the opposite charged proton.

$$\frac{mv^2}{r}=\frac{e^2}{4\pi\varepsilon_0 r^2}$$

Bohr hypothesis: orbital momentum ℓ is a multiple N of $\hbar$ or

$$\ell = m\,v\,r = N\,\hbar \qquad (N = 1,\, 2,\ldots).$$

This gives the *atomic Bohr radius a_0*

$$r=\frac{4\pi\varepsilon_0\hbar^2}{me^2}N^2\left(=r_{Bohr}=5.28E-11\ m. =0.528\ \ddot{A}\ \text{for}\ N{=}1\right)$$

It also implies rear-relativistic electron speed v given as follows.

$$\frac{v}{c}=\frac{\ell}{mrc}=\frac{1}{N}\frac{e^2}{4\pi\varepsilon_0\hbar c}\quad\left(=7.31\text{E-}3=\frac{1}{137.}\ \text{for}\ N{=}1\right)$$

The ratio $\alpha=e^2/(4\pi\varepsilon_0\hbar c)=1/137.036$ is called the *fine-structure constant* α.

Now, do some *numerology* and so-called Dirac's radius involving $\omega_{zwitterbegung}$ where $\omega_{zwitterbegung} = 2mc^2/\hbar = 1.56E21 (radian)Hz$

$\omega_{zwitterbegung}\, r = c$ or $r_{Dirac} = c\,/\,\omega_{zwitterbegung} = \hbar\,/2mc = 1.93\ E\ \text{-}13\ m$ relates to the *Compton wavelength*

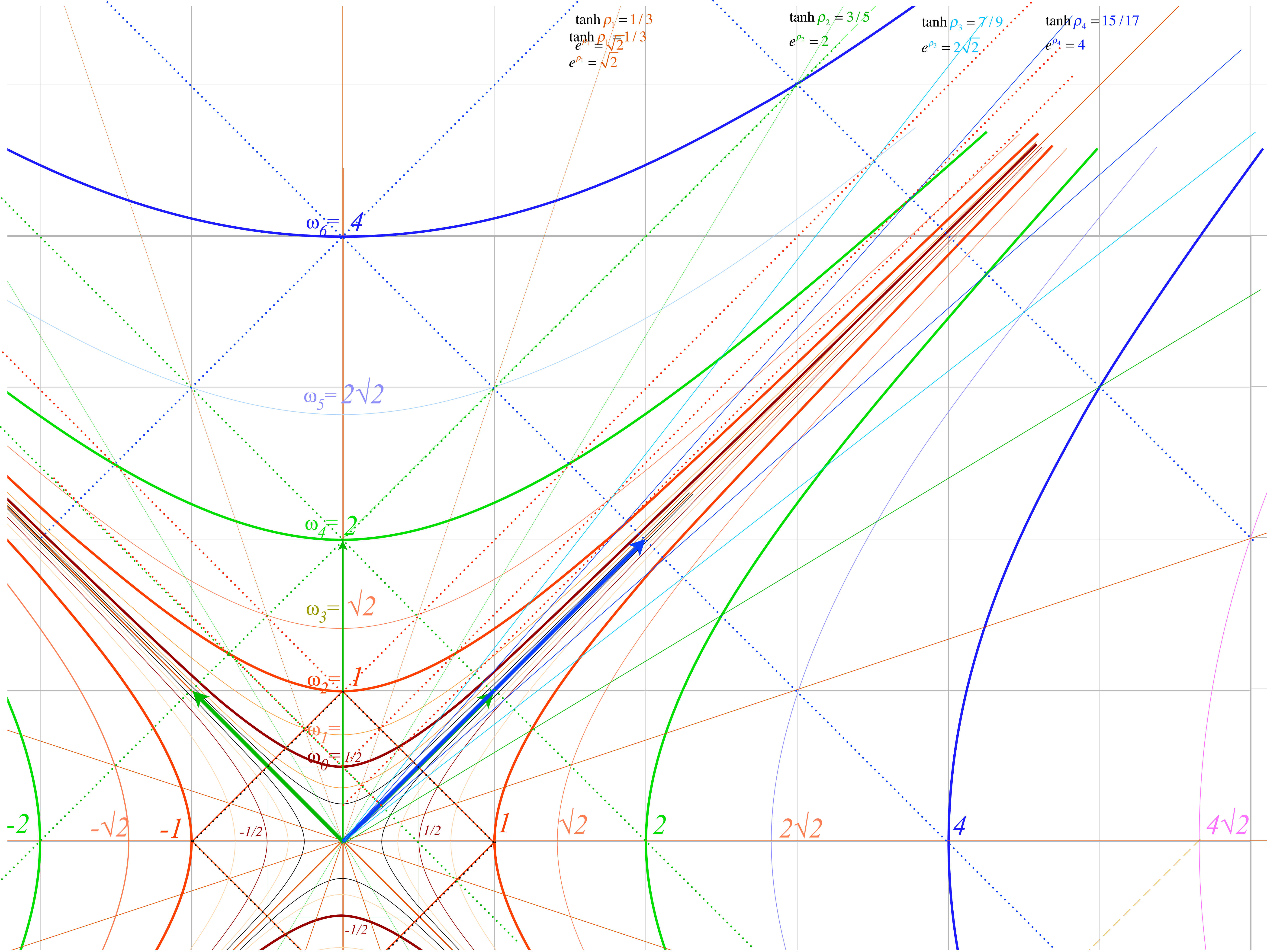
Serial Compton scattering and acceleration plot

Geometric construction

Compton wavelength and formulae

Some numerology: Which is bigger...H-atom or an electron?

→ *Bouncing pulse wave (PW) vs (CW) shrinking laser*



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e^ρ1 = √2
tanh ρ2 = 3 / 5
e^ρ2 = 2
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tanh ρ4 = 15 / 17
e^ρ4 = 4
ω6 = 4
ω5 = 2√2
ω4 = 2
ω3 = √2
ω2 = 1
ω1 =
ω0 = 1/2
-2
-√2
-1
-1/2
1/2
1
√2
2
2√2
4
4√2
-1/2

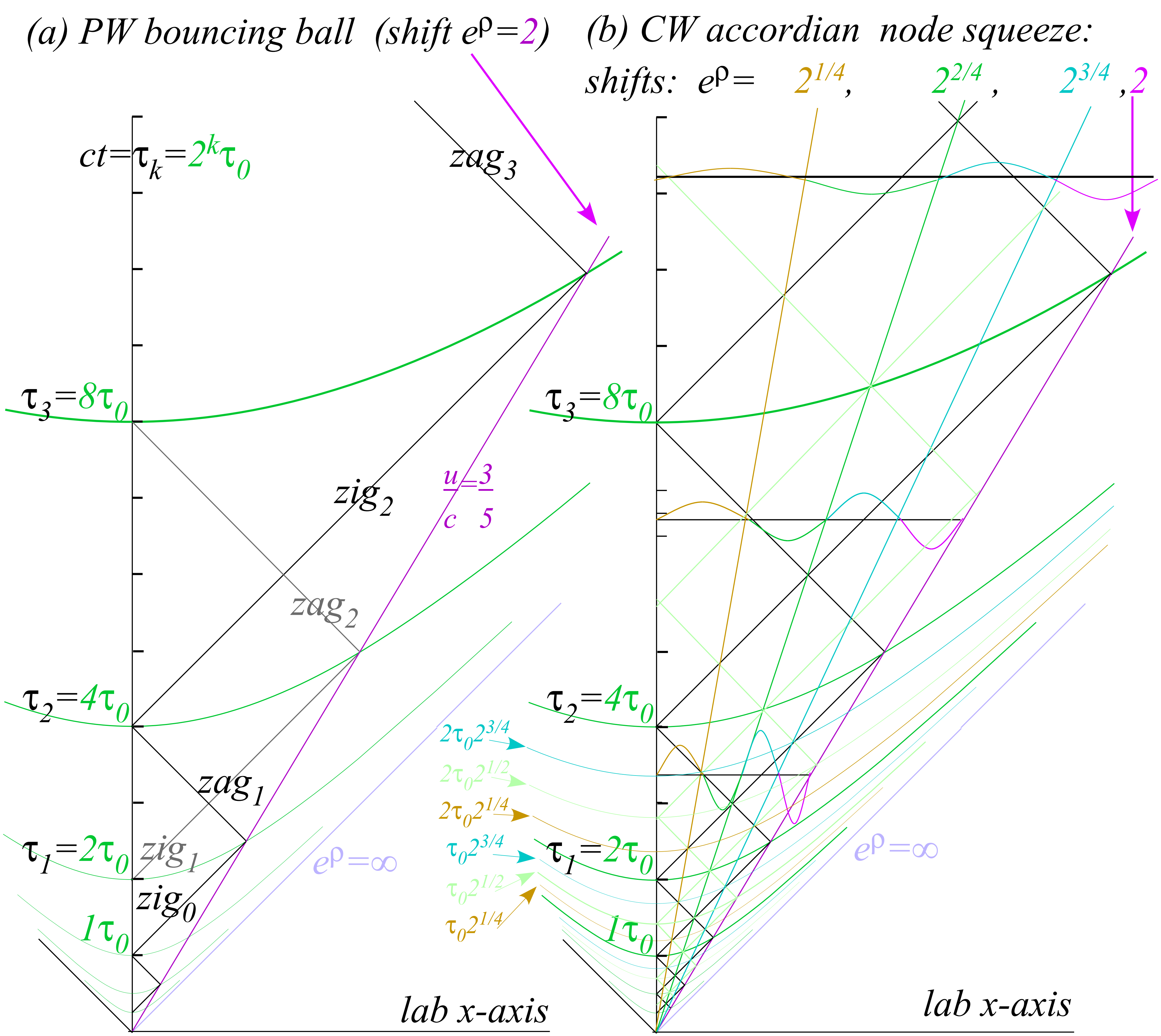

Fig. 7.8 Space-time nets (a) PW zigzag paths bounce. (b) CW nodes squeeze like an accordian.