Lecture 33.
Relativity of quantum collisions and energy momentum transfer
(Ch. 7 of Unit 2  4.19.12)

Fundamental 1-and 2-photon processes and their Feynman diagrams
(A) Absorption, (E) Emission, and (AE together) Compton scattering
“Exotic” processes (AA) pair-creation, (EE) Pair-annihilation

Wave geometry of 1-photon transitions and atomic recoil
1-Photon emission and recoil
Grotian 2-level diagrams vs. Feynman (ω,ck) diagrams
“Baseball diamond” geometric formulas (Rocket-science of Photon emission)
Feynman’s Father’s question: “Where is photon before it comes out?
An answer: that gives recoil frequency down-shift δ

1-Photon absorption and recoil
Similar diagrams and analysis gives recoil frequency up-shift δ

Wave geometry of 2-photon transitions and Compton scattering
2-Photon emission and recoil
Grotian 3-level diagrams vs. Feynman (ω,ck) diagrams
“Photon diamond” geometric formulas
Geometric frequency shifting
Fundamental light-matter processes:

Absorption $A$  

Emission $E$  

$AE$ Together  

(Compton Scattering)

1-photon processes

2-photon process
Fundamental processes and Feynman diagrams

**Fundamental light-matter processes:**

*Absorption A*

*Emission E*

*AE Together* (Compton Scattering)

"Exotic" processes: *AA Together* (Pair-Creation)

*EE Together* (Pair-Annihilation)

1-photon processes

2-photon processes

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Fundamental processes and Feynman diagrams

**Fundamental light-matter processes:**

**Absorption A**

**Emission E**

**AE Together**  
(Compton Scattering)

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1-photon processes

2-photon processes

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"Exotic" processes: **AA Together**  
(Pair-Creation)

**EE Together**  
(Pair-Annihilation)

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N-photon processes

2-CW approach to QED

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Hyper-complex geometry of optical interference...

...formulation of relativistic QM

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Fundamental processes and Feynman diagrams

Wave geometry of 1-photon transitions and Compton recoil

Grotian 2-level diagrams

Feynman ($\omega, ck$) diagrams
(1-photon)

$M$-to-$L'$ emission

$L'$-to-$M$ absorption

$L$-to-$M'$ absorption

$M'$-to-$L$ emission
Fundamental processes and Feynman diagrams

Wave geometry of 1-photon transitions and Compton recoil

Grotian 2-level diagrams

Feynman \((\omega,ck)\) diagrams (1-photon)

2-Level \((\omega,ck)\) “baseball” diamonds

\((\omega,ck)\) vector sum (energy-momentum conservation)
Wave geometry of 1-photon transitions and atomic recoil

1-Photon emission and recoil

Grotian 2-level diagrams vs. Feynman (ω,ck) diagrams

“Baseball diamond” geometric formulas (Rocket-science of Photon emission)

Feynman’s Father’s question: “Where is photon before it comes out?

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Similar diagrams and analysis gives recoil frequency up-shift δ
Fundamental processes and Feynman diagrams

Wave geometry of 1-photon transitions and Compton recoil

Grotian 2-level diagrams

Feynman (\(\omega,ck\)) diagrams
(1-photon)

2-Level (\(\omega,ck\)) “baseball” diamonds

Key recoil relations:
\[ \omega_m e^{-\rho} = \omega_{\ell} \]
\[ \rho = \ln \frac{M_{\ell}/M_m} \]

\(\omega,ck\) vector sum
(energy-momentum conservation)
Fundamental processes and Feynman diagrams

Wave geometry of 1-photon transitions and Compton recoil

Grotian 2-level diagrams

Feynman (ω,ck) diagrams
(1-photon)

2-Level (ω,ck) “baseball” diamonds

Key recoil relations:

\[ \omega_m e^{-\rho} = \omega_\ell \]

\[ \rho = \ln \frac{M_\ell}{M_m} \]

or:

\[ u \sim c \ln \frac{M_\ell}{M_m} \]

Photons are more like “rockets” than “bullets”
Wave geometry of 1-photon transitions and atomic recoil

1-Photon emission and recoil

Grotian 2-level diagrams vs. Feynman (ω,ck) diagrams

“Baseball diamond” geometric formulas (Rocket-science of Photon emission)

Feynman’s Father’s question: “Where is photon before it comes out?

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Similar diagrams and analysis gives recoil frequency up-shift δ
Photon Recoil Effects (Answers for Feynman’s father using geometric means)

\[ \text{\textit{R→Q'} \textit{Photo-Emission Process}} \]

Feynman tells of his father’s query, “Where’s the photon before an atom emits it?” and how he didn’t answer it (despite a high-$\$ MIT education). Here is a short answer that uses the geometric mean in a “baseball” diamond model of excited atom $R$. Back-emitted photon $\omega_{\text{RQ'}}$ is cut out (\( \mathcal{C} \)) of 3\textsuperscript{rd}-baseline but 1\textsuperscript{st} stays put.

\[ \text{Feynman diagram} \]

\[ \omega_{\text{RQ'}} \quad Q' \]

\[ 2 \omega' \]

\[ 2 \omega \]

\[ R \]
**Photon Recoil Effects** (Answers for Feynman’s father using geometric means)

Feynman tells of his father’s query, “Where’s the photon before an atom emits it?” and how he didn’t answer it (despite a high-$\$ MIT education). Here is a short answer that uses the *geometric mean* in a “baseball” diamond model of excited atom $R$. Back-emitted photon $\omega_{RQ'}$ is *cut out* ($\omega$) of 3$^{rd}$-baseline but 1$^{st}$ stays put.
Photon Recoil Effects (Answers for Feynman’s father using geometric means)

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Photon Recoil Effects (Answers for Feynman’s father using geometric means)

Feynman tells of his father’s query, “Where’s the photon before an atom emits it?” and how he didn’t answer it (despite a high-$\$ MIT education). Here is a short answer that uses the geometric mean in a “baseball” diamond model of excited atom $R$. Back-emitted photon $\omega_{RQ'}$ is cut out ($\phi'$) of 3$^{rd}$-baseline but 1$^{st}$ stays put.

Atom recoils from $R$ down to $Q'$ as it loses mass $\Delta M = M_R - M_Q$ or phase frequency $\Delta = 1$.

Atom 3$^{rd}$-baseline $\omega$ shrinks by Feynman’s father-factor $ff = \frac{\omega_3}{\omega}$. 

(Here we let: $ff = (2/3)/(3/2) = 4/9$)

\[ M_R c^2 / h = 2\omega = 3 \]
\[ M_Q c^2 / h = 2\omega' = 2 \]
\[ \omega = 3/2 \]
\[ \omega' = 1 \]
\[ \omega_3 = \omega \]
\[ \omega_1 = \omega \]
\[ \omega' = 1 \]
\[ \omega' = \sqrt{\omega_3 \cdot \omega_1} \]

Euclid mean
A short answer that uses the *geometric mean* in a “baseball” diamond model of excited atom $R$. Back-emitted photon $\omega_{RQ'}$ is *cut out* ($\delta$) of 3\textsuperscript{rd}-baseline but 1\textsuperscript{st} stays put.

Atom recoils from $R$ down to $Q'$ as it *loses mass* $\Delta M = M_R - M_Q$ or phase frequency $\Delta = 1$. 

$R \to Q'$ *Photo-Emission Process* (contd.)
A short answer that uses the **geometric mean** in a “baseball” diamond model of excited atom \( R \).

Back-emitted photon \( \omega_{RQ} \) is cut out \((\varphi)\) of 3\(^{rd}\)-baseline but 1\(^{st}\) stays put.

**Atom recoils from** \( R \) **down to** \( Q' \) **as it loses mass** \( \Delta M=M_R-M_Q \) **or phase frequency** \( \Delta=1 \).

**Atom 3\(^{rd}\)-baseline** \( \omega \) **shrinks by** Feynman’s-father-factor \( ff=\frac{\omega_3}{\omega} \).

\( ff \) **is square** \( f^2 \) **of Doppler red-shift factor** \( f=\frac{\omega}{\omega} = \frac{M_Q}{M_R} = e^{\rho_{\text{BOOST}}} \).

**Boosted atom**

\[ \rho_{\text{BOOST}}= \ln b = \ln \frac{M_R}{M_Q} \]
R→Q’ Photo-Emission Process (contd.)

A short answer that uses the geometric mean in a “baseball” diamond model of excited atom R.
Back-emitted photon $\omega_{RQ’}$ is cut out ($\phi$) of 3rd-baseline but 1st stays put.

Atom recoils from R down to Q’ as it loses mass $\Delta M = M_R - M_Q$ or phase frequency $\Delta = 1$.

Atom 3rd-baseline $\bar{\omega}$ shrinks by Feynman’s-father-factor $ff = \frac{\bar{\omega}^3}{\bar{\omega}} = \frac{M_Q}{M_R} = e^{\rho_{BOOST}}$.

Boosted atom $\rho_{BOOST} = \ln b = \ln \frac{M_R}{M_Q}$

Emitted photon $\omega_{RQ’} = (1 - ff) \bar{\omega}$

$= (1 - \frac{M_Q}{M_R}) \frac{M_R c^2}{2\hbar}$

Feynman’s father-factor $ff$

$ff \cdot \bar{\omega} = \omega’^3_3$

$ff \cdot \bar{\omega’} = \omega’^3_3$

Feynman diagram is a triangle in per-spacetime.
A short answer that uses the geometric mean in a “baseball” diamond model of excited atom \( R \).

Back-emitted photon \( \omega_{RQ} \) is cut out \((\varphi')\) of 3rd-baseline but 1st stays put.

Atom recoils from \( R \) down to \( Q' \) as it loses mass \( \Delta M = M_R - M_Q \) or phase frequency \( \Delta = 1 \).

Atom 3rd-baseline \( \varnothing \) shrinks by Feynman’s-father-factor \( ff = \frac{\omega_3}{\varnothing} \).

\( ff \) is square \( f^2 \) of Doppler red-shift factor \( f = \frac{\varnothing'}{\varnothing} = \frac{M_Q}{M_R} = e^{\rho_{BOOST}} \).

Boosted atom
\( \rho_{BOOST} = \ln b = \ln \frac{M_R}{M_Q} \)

Emitted photon
\( \omega_{RQ}' = (1 - ff) \varnothing \)
\( = (1 - \frac{M_Q}{M_R})M_Rc^2/2\hbar \)

\( \omega_1 = \frac{\omega}{\varnothing} \) Doppler blue shift-factor
\( b = 1/r \)

\( f = \frac{\varnothing}{\varnothing'} \) Doppler red shift-factor

\( \varnothing' = \sqrt{\varnothing_3' \cdot \varnothing_1} \)

Euclid mean
Wave geometry of 1-photon transitions and atomic recoil

1-Photon emission and recoil

Grotian 2-level diagrams vs. Feynman $(\omega,ck)$ diagrams

“Baseball diamond” geometric formulas (Rocket-science of Photon emission)

Feynman’s Father’s question: “Where is photon before it comes out?"

An answer: that gives recoil frequency down-shift $\delta$

1-Photon absorption and recoil

Similar diagrams and analysis gives recoil frequency up-shift $\delta$
Atoms lose only tiny fractions of mass $M_R$ to photons $\omega_{RQ'}$. (But, suppose a $\gamma$-emitter loses $\frac{1}{3}M_R$ as in the case here.) Here are the numbers for this case and approximations for atoms that only lose tiny mass. ($h\omega_{RQ'} \ll M_R c^2$)

- $M_R c^2 / \hbar = \omega = 3/2$ excited rest mass
- $M_Q c^2 / \hbar = \omega' = 1$ ground rest mass

$$f = \frac{M_Q}{M_R} = \frac{2}{3} \quad \text{Doppler red shift factor}$$

$$ff = \frac{f}{f} = \frac{4}{9} \quad \text{Feynman's-father factor}$$

Mass loss

$$\Delta M = \hbar \Delta / c^2 = (M_R - M_Q) \frac{c^2}{\hbar}$$

Recoil frequency

$$\Downarrow \text{Down-Shift } \delta$$

Feynman diagram

is a triangle in per-spacetime

\begin{align*}
\omega_R' &= \omega \\
\omega_Q' &= \omega \\
\omega_R &= \omega \\
\omega_Q &= \omega \\
\omega_R' &= 2\omega \\
\omega_Q' &= 2\omega \\
\omega_R &= \omega \\
\omega_Q &= \omega \\
\omega_R' &= 2\omega \\
\omega_Q' &= 2\omega \\
\end{align*}
R→Q’ Photo-Emission Process (contd.)
Atoms lose only tiny \textit{my} fractions of mass $M_R$ to photons $\omega_{RQ'}$. (But, suppose a $\gamma$-emitter loses $1/3M_R$ as in the case here.) Here are the numbers for this case and approximations for atoms that only lose tiny $\textit{my}$ mass. ($\hbar\omega_{RQ'}<<M_Rc^2$)

- $M_Rc^2/2\hbar=\bar{\omega}=3/2$ excited rest mass
- $f = \frac{M_Q}{M_R} = 2/3$ Doppler red shift factor
- $M_Qc^2/2\bar{\omega}'=1$ ground rest mass
- $ff = ff' = 4/9$ Feynman’s-father factor
- $b^2 = 9/4$

\[
\omega_{RQ'} = (1-ff)\bar{\omega} = (1- \frac{M_Q^2}{M_R^2})M_Rc^2/2\hbar
\]

\[
u_{\text{BOOST}} = c\frac{b^2-1}{b^2+1} = c\frac{5}{13}
\]

\[
\omega_{RQ'} = (1-4/9)3/2 = 5/6
\]

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\textbf{Feynman diagram}  
\textit{is a triangle in per-spacetime}
Atoms lose only tiny fractions of mass $M_R$ to photons $\omega_{RQ'}$. (But, suppose a $\gamma$-emitter loses $1/3M_R$ as in the case here.)

Here are the numbers for this case and approximations for atoms that only lose tiny mass. ($\hbar\omega_{RQ'} << M_Rc^2$)

$$M_Rc^2/2\hbar=\bar{\omega}=3/2\text{ excited rest mass}\quad f = \frac{M_Q}{M_R} = 2/3\quad \text{Doppler red shift factor}\quad b=3/2$$

$$M_Qc^2/2\hbar=\bar{\omega}'=1\quad \text{ground rest mass}\quad ff = ff' = 4/9\quad \text{Feynman’s-father factor}\quad b^2=9/4$$

![Feynman diagram](https://example.com/feynman_diagram.png)

**Boosted atom**

$$\rho_{\text{BOOST}} = \ln b = \ln \frac{M_R}{M_Q}$$

**Emitted photon**

$$\omega_{RQ'} = (1-ff)\bar{\omega} = (1- \frac{M_Q}{M_R})M_Rc^2/2\hbar$$

$$\omega_{RQ'} = (1-4/9)3/2 = 5/6$$

$$u_{\text{BOOST}} = c\frac{b^2-1}{b^2+1} = c\quad 5/13$$

$$\rho_{\text{BOOST}} = \ln b = \ln \frac{M_R}{M_Q}$$

**Recoil frequency**

Down-Shift $\delta$

$$\omega_{RQ'} = \frac{(M_R + M_Q)(M_R - M_Q)c^2}{2M_R} = \frac{(2M_Q + \hbar\Delta/c^2)\Delta}{2} = \frac{(M_Q\Delta + \hbar\Delta^2/2c^2)}{M_Q + \hbar\Delta/c^2}$$

**Feynman diagram**

is a triangle in per-spacetime

ck-axis

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Atoms lose only tiny fractions of mass $M_R$ to photons $\omega_{RQ'}$. (But, suppose a $\gamma$-emitter loses $1/3M_R$ as in the case here.) Here are the numbers for this case and approximations for atoms that only lose tiny mass. ($\hbar\omega_{RQ'} \ll M_Rc^2$)

$M_Rc^2/2\hbar=3/2$ excited rest mass \[ f = \frac{M_Q}{M_R} = 2/3 \quad \text{Doppler red shift factor} \quad b=3/2 \]

$M_Qc^2/2\hbar=1'$ ground rest mass \[ ff = f f = 4/9 \quad \text{Feynman’s-father factor} \quad b^2=9/4 \]

$\omega_{RQ'} = (1-ff)\omega = (1-\frac{M_Q^2}{M_R^2})M_Rc^2/2\hbar$

$\omega_{RQ'} = (1-4/9)3/2 = 5/6$

$\Delta = (M_R - M_Q)\frac{c^2}{\hbar}$

$\omega_{RQ'} = \frac{(M_R + M_Q)(M_R - M_Q)c^2}{2\hbar} = \frac{(2M_Q + \hbar\Delta/c^2)\Delta}{M_Q + \hbar\Delta/c^2}$

$\omega_{RQ'} = \frac{M_Qc^2\Delta/\hbar + \Delta^2/2}{M_Qc^2/\hbar + \Delta} \sim \Delta - \frac{\hbar\Delta}{M_Qc^2}$

Approximations for “soft-photon” atomic transitions

Usually, recoil shift $\delta$ and mass-loss $\Delta M=\hbar\Delta/c^2$ are negligible.
Wave geometry of 1-photon transitions and atomic recoil

1-Photon emission and recoil

Grotian 2-level diagrams vs. Feynman (ω,ck) diagrams

“Baseball diamond” geometric formulas (Rocket-science of Photon emission)

Feynman’s Father’s question: “Where is photon before it comes out?

An answer: that gives recoil frequency down-shift δ

1-Photon absorption and recoil

Similar diagrams and analysis gives recoil frequency up-shift δ
Photon Recoil Effects (contd.)

Q→R’ Photo-Absorption Process

Photon \( \omega_{QR'} \) is *pasted* onto the 1\textsuperscript{st}-baseline but 3\textsuperscript{rd}-base stays put.

Atom accelerates from Q up to R’ as it *gains mass* \( \Delta M = M_R - M_Q \) or phase frequency \( \Delta = 1 \).
Photon Recoil Effects (contd.)

Q→R’ Photo-Absorption Process

Photon $\omega_{QR'}$ is *pasted* onto the 1st-baseline but 3rd-base stays put. This boosts the atom by $\rho_{\text{BOOST}}=\ln f$.

Atom accelerates from Q up to $R'$ as it *gains mass* $\Delta M=M_R-M_Q$ or phase frequency $\Delta=1$. 

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Photon Recoil Effects (contd.)

Q→R’ Photo-Absorption Process

Photon $\omega_{QR'}$ is *pasted* onto the 1st-baseline but 3rd-base stays put. This boosts the atom by $\rho_{BOOST}=\ln f$.

Atom accelerates from Q up to R’ as it gains mass $\Delta M=M_R-M_Q$ or phase frequency $\Delta=1$. 

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Photon Recoil Effects (contd.)

Q→R’ Photo-Absorption Process

Photon \( \omega_{QR} \) is \textit{pasted} onto the 1\textsuperscript{st}-baseline but 3\textsuperscript{rd}-base stays put. This boosts the atom by \( \rho_{\text{BOOST}}=\ln f \).

Atom accelerates from Q up to R’ as it \textit{gains} mass \( \Delta M=M_{R}-M_{Q} \) or phase frequency \( \Delta=1 \).

Atom 1\textsuperscript{st}-baseline \( \omega \) grows by \textit{Feynman’s-father-factor} \( ff=\frac{\omega}{\omega'} \).

Doppler blue shift factor is \( b=f=\sqrt{ff} \) that relates \( \omega, \omega' \), and \( \omega_1' \).
Q→R’ Photo-Absorption Process (contd.)

\[ M_R c^2/\hbar = 2\omega = 3 \]
\[ M_Q c^2/\hbar = 2\omega = 2 \]

\[ \omega' = 3/2 \]
\[ \omega = 1 \]
\[ \omega_3 = \omega \]

Recoil frequency up-shift \( \delta \)

Mass gain \( \Delta M = \hbar \Delta / c^2 \)

Absorbed photon \( \omega_{QR'} = (ff-1)\omega \)
\[ = (\frac{M_R^2}{M_Q^2}-1) M_Q c^2 / 2\hbar \]

Absorbed atom
\[ \rho_{\text{BOOST}} = \ln f = \ln \frac{M_R}{M_Q} \]
\[ f = \frac{3}{2} \]
\[ \omega = 1 \]

\[ \omega_{QR'} = (9/4-1)1 = 5/4 \]

Approximations for “soft-photon” atomic transitions
Recoil shift \( \delta \) or mass-gain \( \Delta M = \hbar \Delta / c^2 \) assumed small.

\[ \omega_{QR'} = \frac{M_Q c^2 \Delta/\hbar + \Delta^2/2}{M_Q c^2/\hbar} \approx \Delta + \frac{\hbar \Delta^2}{2M_Q c^2} \]
Wave geometry of 2-photon transitions and Compton scattering
2-Photon emission and recoil
Grotian 3-level diagrams vs. Feynman (ω,ck) diagrams
“Photon diamond” geometric formulas
Geometric frequency shifting and “Compton stairs”
Wave geometry of 2-photon transitions and Compton scattering

Compton scattering
(Center of Momentum view)

Geometric 2-Level $(ck, \omega)$ diagram

\[ \omega_m \sinh \rho \]

\[ \hbar \omega_m = E_m \]

\[ \omega_m e^\rho = \frac{\omega_\ell}{2} \]

\[ \omega_m e^{-\rho} = \frac{\omega_h}{2} \]

\[ \omega_m \cosh \rho \]
Wave geometry of 2-photon transitions and Compton scattering
Fundamental processes and Feynman diagrams

Wave geometry of 2-photon transitions and Compton scattering

Compton scattering
(Center of Momentum view)

2-photon absorption
(Center of Momentum view)

Geometric 3-Level diamonds

$$\hbar \omega_h = E_h$$

$$\omega_m \sin \phi$$

$$\hbar \omega_m = E_m$$

$$\frac{\omega_m e^{-\phi}}{2} = \omega_l$$

$$\frac{\omega_m e^{+\phi}}{2} = \omega_h$$

$$\omega_m \cosh \phi$$
Wave geometry of 2-photon transitions and Compton scattering
2-Photon emission and recoil

Grotian 3-level diagrams vs. Feynman (ω,ck) diagrams
“Photon diamond” geometric formulas
Geometric frequency shifting and “Compton stairs”
To have \((\omega, k)\) conservation in transitions must have geometric series of \(\omega\)-levels: \(\omega_k, \omega_l, \omega_m, \omega_h, \ldots\)
Wave geometry of 2-photon transitions and Compton scattering
2-Photon emission and recoil
Grotian 3-level diagrams vs. Feynman ($\omega,ck$) diagrams
“Photon diamond” geometric formulas (Examples with geometric ratio of 2)
Geometric frequency shifting and “Compton stairs”
(a) $Q \rightarrow P'$

**Photo-Emission**

$\omega_3 = \frac{1}{4} \omega_3$

(b) $P \rightarrow Q'$

**Photo-Absorption**

$\omega_3' = \omega_3$

Feynman-father-factor of $ff = 4$

Doppler factor of $f = 2$ gives familiar recoil $u/c = 3/5$
Wave geometry of 2-photon transitions and Compton scattering

2-Photon emission and recoil

Grotian 3-level diagrams vs. Feynman ($\omega,ck$) diagrams

“Photon diamond” geometric formulas (Examples with geometric ratio of 2)

Geometric frequency shifting and “Compton stairs”
Fig. 7.7 Compton nets are congruent Compton staircases of transitions. (a) $f=2:1$ (b) $f=\sqrt{2}:1$
(a) PW bouncing ball (shift $e^\rho=2$)

(b) CW accordion node squeeze:
shifts: $e^\rho = 2^{1/4}, 2^{2/4}, 2^{3/4}, 2$

Fig. 7.8 Space-time nets (a) PW zigzag paths bounce. (b) CW nodes squeeze like an accordion.