Lecture 31.

Relativity of interfering and galloping waves: SWR and SWQ III.

(Ch. 4-6 of Unit 2  4.15.12)

1st Quantization: Quantizing phase variables $\omega$ and $k$

Understanding how quantum transitions require “mixed-up” states

Closed cavity vs Ring cavity

2nd Quantization: Quantizing amplitudes (“photons”, “vibrons”, and “what-ever-ons”)

Introducing coherent states (What lasers use)

Analogy with $(\omega,k)$ wave packets

Wave coordinates need coherence

Field Energy $= |E|^2 \varepsilon_0 \quad 1/4\pi\varepsilon_0 = 9 \cdot 10^9$

Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz asynchrony

Magnetic B-field is relativistic effect

Lecture 31 ended here
Review of Lecture 30

1st Quantization: Quantizing phase variables $\omega$ and $k$

Understanding how quantum transitions require “mixed-up” states

Closed cavity vs Ring cavity
Quantized ω and k

Counting wave kink numbers

If everything is made of waves then we expect quantization of everything because waves only thrive if integral numbers \( n \) of their “kinks” fit into whatever structure (box, ring, etc.) they’re supposed to live. The numbers \( n \) are called quantum numbers.

**OK box quantum numbers:**

\[
\begin{align*}
n &= 1 & n &= 2 & n &= 3 & n &= 4 \\
\text{(} & \text{+ integers only}\text{)}
\end{align*}
\]

**Some NOT OK numbers:**

\[
\begin{align*}
n &= 0.67 & n &= 1.7 & n &= 2.59 & n &= 4 \\
\text{too fat!} & \text{too thin!} & \text{wrong color again!} & \text{...not tolerated!}
\end{align*}
\]

**NOTE:** We’re using “false-color” here.

\[This\ doesn’t\ mean\ a\ system’s\ energy\ can’t\ vary\ continuously\ between\ “OK”\ values\ \( E_1, E_2, E_3, E_4, \ldots \)\]
Quantized $\infty$ and $k$

Counting wave kink numbers

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**OK box quantum numbers:**

$n=1$

$n=2$

$n=3$

$n=4$

(+ integers only)

Some **NOT OK numbers**: $n=0.67$

$n=1.7$

$n=2.59$

$n=4$

too fat!

too thin!

wrong color again!

misfits...

...not tolerated!

**NOTE:** We’re using “false-color” here.

This doesn’t mean a system’s energy can’t vary continuously between “OK” values $E_1, E_2, E_3, E_4, \ldots$

In fact its state can be a linear combination of any of the “OK” waves $|E_1\rangle, |E_2\rangle, |E_3\rangle, |E_4\rangle, \ldots$
Quantized $\omega$ and $k$  Counting wave kink numbers

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OK box quantum numbers: $n=1$ $n=2$ $n=3$ $n=4$

(+ integers only)

Some NOT OK numbers: $n=0.67$ $n=1.7$ $n=2.59$ $n=4$

too fat!  too thin!  wrong color again!  ...not tolerated!

NOTE: We’re using “false-color” here.

This doesn’t mean a system’s energy can’t vary continuously between “OK” values $E_1$, $E_2$, $E_3$, $E_4$, ...

In fact its state can be a linear combination of any of the “OK” waves $|E_1\rangle$, $|E_2\rangle$, $|E_3\rangle$, $|E_4\rangle$, ...

That’s the only way you get any light in or out of the system to “see” it.

$|E_4\rangle$

frequency $\hbar \omega_{32} = E_3 - E_2$

$|E_3\rangle$

frequency $\hbar \omega_{21} = E_2 - E_1$
Quantized $\omega$ and $k$  

**Counting wave kink numbers**  

If everything is made of waves then we expect **quantization** of everything because waves only thrive if **integral** numbers $n$ of their “kinks” fit into whatever structure (box, ring, etc.) they’re supposed to live. The numbers $n$ are called **quantum numbers**.  

**OK box quantum numbers:**  

$n=1$  

$n=2$  

$n=3$  

$n=4$  

(+) **integers only**  

Some **NOT OK numbers:**  

$n=0.67$ too fat!  

$n=1.7$ too thin!  

$n=2.59$ wrong color again!  

$n=4$ misfits...  

$\ddots$ ...not tolerated!  

**NOTE:** We’re using “false-color” here.  

This doesn’t mean a system’s energy can’t vary **continuously** between “OK” values $E_1, E_2, E_3, E_4,$...  

In fact its state can be a linear combination of any of the “OK” waves $|E_1>$, $|E_2>$, $|E_3>$, $|E_4>$,...  

That’s the only way you get any light in or out of the system to “see” it.  

These eigenstates are the only ways the system can “play dead”...  

... “sleep with the fishes”...
Quantized $\omega$ and $k$  

Counting wave kink numbers

If everything is made of waves then we expect *quantization* of everything because waves only thrive if *integral* numbers $n$ of their “kinks” fit into whatever structure (box, ring, etc.) they’re supposed to live. The numbers $n$ are called *quantum numbers*.

**OK box quantum numbers:**

- $n=1$
- $n=2$
- $n=3$
- $n=4$

(+ integers only)

Some **NOT OK numbers:**

- $n=0.67$
- $n=1.7$
- $n=2.59$
- $n=4$

too fat!  

too thin!  

wrong color again!  

misfits...  

...not tolerated !

**NOTE:** We’re using “false-color” here.

Rings tolerate a zero (kinkless) quantum wave but require ±*integral* wave number.

**OK ring quantum numbers:**

- $m=0$
- $m=±1$
- $m=±2$
- $m=3$

($±$ *integral number  

of wavelengths)

Bohr’s models of *atomic spectra* (1913-1923) are beginnings of *quantum wave mechanics* built on *Planck-Einstein* (1900-1905) relation $E=h\nu$. *DeBroglie* relation $p=h/\lambda$ comes around 1923.
2nd Quantization: Quantizing amplitudes ("photons", "vibrons", and "what-ever-ons")
Introducing coherent states (What lasers use)
Analogy with (ω,k) wave packets
Wave coordinates need coherence
Quantized Amplitude Counting “photon” number

Planck’s relation \(E = N\hbar\nu\) began as a tenative axiom to explain low-T light. Then he tried to disavow it! Einstein picked it up in his 1905 paper. Since then its use has grown enormously and continues to amaze, amuse (or bewilder) all who study it. A current view is that it represents the quantization of optical field amplitude. We picture this below as \(N\)-photon wave states for each box-mode of \(m\) wave kinks.
Quantized \textit{Amplitude} Counting “photon” number

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Quantum field definitions have been called “2nd quantization” or “wave-waves” \textit{NOTE: We’re using “false-color” here.}

These are the 1st excited or fundamental transition levels

These are the fundamental “zero-point” or “vacuum” levels

www.uark.edu/ua/pirelli/php/quantized_1.php
Quantized Amplitude Counting “photon” number

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Quantum field definitions have been called “2nd quantization” or “wave-waves”

NOTE: We’re using “false-color” here.

These are the fundamental “zero-point” or “vacuum” levels

$N_4 = 0$

These are the 1st excited or fundamental transition levels

$N_3 = 1$

$violet$ photon

$N_2 = 1$

$blue$ photon

These are the 2nd excited levels

$N_2 = 2$

$green$ photons

Quantized Wavenumber (“kink” or momentum number)

$N_1 = 4$

$red$ photons

$m = 1$

$N_1 = 3$

$red$ photons

$N_1 = 2$

$green$ photon

$N_1 = 1$

$red$ photon

$N_1 = 0$

$m = 2$

$N_2 = 0$

$m = 3$

$N_3 = 0$

$m = 4$

$N_4 = 1$
Quantized Amplitude Counting “photon” number

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Quantum numbers \( N \) of field or \( n, m \ldots \) of modes are *invariants* and *not* changed by boosting velocity. Each mode fundamental frequency \( \nu_n = n \nu_1 \) and its \( N \)-photon multiples belong to invariant hyperbolas.

Boosted observers see distorted frequencies and lengths, but will agree on the *numbers* \( n \) and \( N \) of mode *nodes* and *photons*.

This is how light waves can “fake” some of the properties of classical “things” such as *invariance* or *object permanence*.

It takes at least *TWO CW’s* to achieve such invariance. One CW is not enough and cannot have non-zero invariant \( N \). Invariance is an *interference* effect that needs at least *two-to-tango*!
Coherent States: Oscillator Amplitude Packets analogous to Wave Packets

We saw how adding CW’s (Continuous Waves \( m=1,2,3... \)) can make PW (Pulse Wave) or WP (Wave Packet) that is more like a classical “thing” with more localization in space \( x \) and time \( t \).

Analogy:
Adding photons (Quantized amplitude \( N=0,1,2... \)) can make a CS (Coherent State) or OAP (Oscillator Amplitude Packet) that is more like a classical wave oscillation with more localization in field amplitude.

Pure photon states have localized (certain) \( N \) but delocalized (uncertain) amplitude and phase. OAP states have delocalized (uncertain) \( N \) but more localized (certain) amplitude and phase.
**Coherent States (contd.)** *Spacetime wave grid is impossible without coherent states*

Pure photon number $N$-states would make useless spacetime coordinates

Total uncertainty of amplitude and phase makes the count pattern a wash. To see grids *some $N$-uncertainty is necessary!*

Coherent-$\alpha$-states are defined by continuous amplitude-packet parameter $\alpha$ whose square is average photon number $\overline{N}=|\alpha|^2$. Coherent-states make better spacetime coordinates for larger $\overline{N}=|\alpha|^2$.

Coherent-state uncertainty in photon number (and mass) varies with amplitude parameter $\Delta N \sim \alpha \sim \sqrt{N}$ so a coherent state with $\overline{N}=|\alpha|^2 = 10^6$ only has a 1-in-1000 uncertainty $\Delta N \sim \alpha \sim \sqrt{N}=1000$. 
Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz asynchrony

Magnetic B-field is relativistic effect
Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz asynchrony

(+): Charge fixed
(-): Charge moving to left (Negative current density)

(+): Charge density is Equal to the (-) Charge density

Observer velocity is zero relative to (+) line of charge

wire appears neutral
Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz asynchrony

Observer velocity is zero relative to (+) line of charge

(+): Charge fixed  (-): Charge moving to right (Negative current density)
(+): Charge density is Equal to the (-) Charge density
Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz asynchrony

(+): Charge fixed
(-): Charge moving to right (Negative current density)

(+): Charge density is Greater than (-): Charge density
Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz asynchrony

(+): Charge fixed
(-): Charge moving to left (Negative current density)
(+): Charge density is Greater than (-) Charge density

Observer velocity is (+) relative to (+) line of charge
Wire appears positive (+) (repulsive to +)
Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz asynchrony

(+): Charge fixed
(-): Charge moving to left (Negative current density)
(+): Charge density is Less than (-) Charge density

Observer velocity is (-) relative to (+) line of charge

Wire appears negative (-) (attractive to +)
Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz asynchrony

(+): Charge fixed
(-): Charge moving to left (Negative current density)

(+) Charge density is Less than (-) Charge density

Observer velocity is (-) relative to (+) line of charge

Wire appears negative (-) (attractive to +)
Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz asynchrony

Magnetic $B$-field is relativistic effect
\[
\frac{\rho(-)}{\rho(+) = \frac{(+)}{(-)} \text{ charge separation}} = \frac{x(+) + x(-)}{x(-)}
\]

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\frac{\rho(-)}{\rho(+) = \frac{(+)}{(-)} \text{ charge separation}} = \frac{x(+) + x(-)}{x(-)}
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\]

\[
\rho(-) = \frac{(+)}{(-)} = \frac{x(+) + x(-)}{x(-)}
\]

\[
\rho(-) = \frac{x(+) + x(-)}{x(-)}
\]

\[
\rho(+) - \rho(-) = \rho(+) \left( 1 - \frac{\rho(-)}{\rho(+)} \right) = -\frac{uv}{c^2} \rho(+)
\]

Unit square: \((u/c) / 1 = x(+) / y\)

\((v/c) / 1 = y / x(-)\)
Magnetic B-field is relativistic effect!

The electric force field $E$ of a charged line varies inversely with radius. The Gauss formula for force in mks units:

$$F = qE = q\left[\frac{1}{4\pi \varepsilon_0} \frac{2\rho}{r}\right], \text{ where: } \frac{1}{4\pi \varepsilon_0} = 9 \times 10^9 \frac{N \cdot m^2}{Coul}.$$

$$F = qE = q\left[\frac{1}{4\pi \varepsilon_0} \frac{2}{r}\left(-\frac{uv}{c^2} \rho(+)\right)\right] = -\frac{2}{4\pi \varepsilon_0} \frac{qv \rho(+)u}{c^2 r} = -2 \times 10^{-7} \frac{I_q I_\rho}{r}$$

$1/4\pi \varepsilon_0 = 9 \cdot 10^9$
$c^2 = 9 \cdot 10^{16}$
$1/(4\pi \varepsilon_0 c^2) = 10^{-7}$

I see excess (+) charge up there. Yuk!

I see excess (-) charge up there. Yum!
Magnetic B-field is relativistic effect!

The electric force field $E$ of a charged line varies inversely with radius. The Gauss formula for force in mks units:

$$F = qE = q\left[ \frac{1}{4\pi\varepsilon_0} \frac{2\rho}{r} \right], \quad \text{where:} \quad \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \frac{N\cdot m^2}{\text{Coul}}.$$

$$F = qE = q\left[ \frac{1}{4\pi\varepsilon_0} \frac{2\left( -\frac{u\nu v}{c^2} \rho(+) \right)}{r} \right] = -\frac{2q v \rho(+) u}{4\pi\varepsilon_0 c^2 r} = -2 \times 10^{-7} \frac{I_q I_\rho}{r}$$

$1/4\pi\varepsilon_0 = 9 \cdot 10^9$

$c^2 = 9 \cdot 10^{-16}$

$1/(4\pi\varepsilon_0 c^2) = 10^{-7}$

---

I see excess (+) charge up there. Yuk!

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Relating photons to Maxwell energy density and Poynting flux

Relativistic variation and invariance of frequency ($\omega,k$) and amplitudes

How probability $\psi$-waves and flux $\psi$-waves evolved

Properties of amplitude $\psi^*\psi$-squares

More on unmatched amplitudes AND unmatched frequencies AND unmatched quanta