Lecture 30.

Relativity of interfering and galloping waves: SWR and SWQ II.

(Ch. 4-6 of Unit 2   4.12.12)

Unmatched amplitudes giving galloping waves

- Standing Wave Ratio (SWR) and Standing Wave Quotient (SWQ)
  - Analogy with group and phase
- Analogy between wave galloping, Keplarian IHO orbits, and optical polarization
- Waves that go back in time - The Feynman-Wheeler Switchback

1st Quantization: Quantizing phase variables ω and k

- Understanding how quantum transitions require "mixed-up" states
  - Closed cavity vs Ring cavity

Lecture 30 ended here

Galloping waves due to unmatched amplitudes

2-CW dynamics has two 1-CW amplitudes $A_{\rightarrow}$ and $A_{\leftarrow}$ that we now allow to be *un*matched. $(A_{\rightarrow} \neq A_{\leftarrow})$

$$A_{\rightarrow} e^{i(k_{\rightarrow} x - \omega_{\rightarrow} t)} + A_{\leftarrow} e^{i(k_{\leftarrow} x - \omega_{\leftarrow} t)} = e^{i(k_{\Sigma} x - \omega_{\Sigma} t)} [A_{\rightarrow} e^{i(k_{\Delta} x - \omega_{\Delta} t)} + A_{\leftarrow} e^{-i(k_{\Delta} x - \omega_{\Delta} t)}]$$

Waves have half-sum mean-phase rates $(k_{\Sigma}, \omega_{\Sigma})$ and half-difference group rates $(k_{\Delta}, \omega_{\Delta})$.

$$k_{\Sigma} = (k_{\rightarrow} + k_{\leftarrow})/2 \qquad k_{\Delta} = (k_{\rightarrow} - k_{\leftarrow})/2$$

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Also important is amplitude mean $A_{\Sigma} = (A_{\rightarrow} + A_{\leftarrow})/2$ and half-difference $A_{\Delta} = (A_{\rightarrow} - A_{\leftarrow})/2$.

Detailed wave motion depends on standing-wave-ratio *SWR* or the inverse standing-wave-quotient *SWQ*.

$$SWR = \frac{(A_{\rightarrow} - A_{\leftarrow})}{(A_{\rightarrow} + A_{\leftarrow})} \qquad SWQ = \frac{(A_{\rightarrow} + A_{\leftarrow})}{(A_{\rightarrow} - A_{\leftarrow})}$$

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$$SWR = \frac{(A_{\rightarrow} - A_{\leftarrow})}{(A_{\rightarrow} + A_{\leftarrow})} \qquad SWQ = \frac{(A_{\rightarrow} + A_{\leftarrow})}{(A_{\rightarrow} - A_{\leftarrow})}$$

These are analogous to frequency ratios for group velocity $V_{group} < c$ and its inverse that is phase velocity $V_{phase} > c$.

$$V_{group} = \frac{\omega_{\Delta}}{k_{\Delta}} = \frac{(\omega_{\rightarrow} - \omega_{\leftarrow})}{(k_{\rightarrow} - k_{\leftarrow})} = c\frac{(\omega_{\rightarrow} - \omega_{\leftarrow})}{(\omega_{\rightarrow} + \omega_{\leftarrow})} \qquad V_{phase} = \frac{\omega_{\Sigma}}{k_{\Sigma}} = \frac{(\omega_{\rightarrow} + \omega_{\leftarrow})}{(k_{\rightarrow} + k_{\leftarrow})} = c\frac{(\omega_{\rightarrow} + \omega_{\leftarrow})}{(\omega_{\rightarrow} - \omega_{\leftarrow})}$$

$$\frac{V_{group}}{c} == \frac{c}{V_{phase}}$$

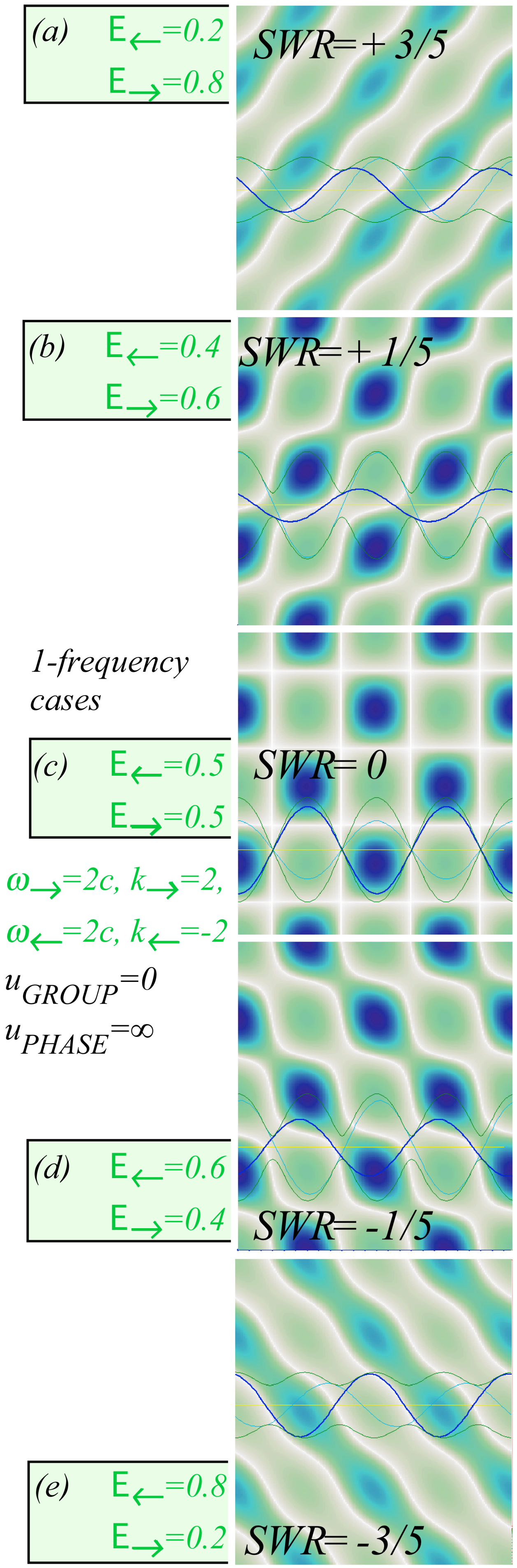
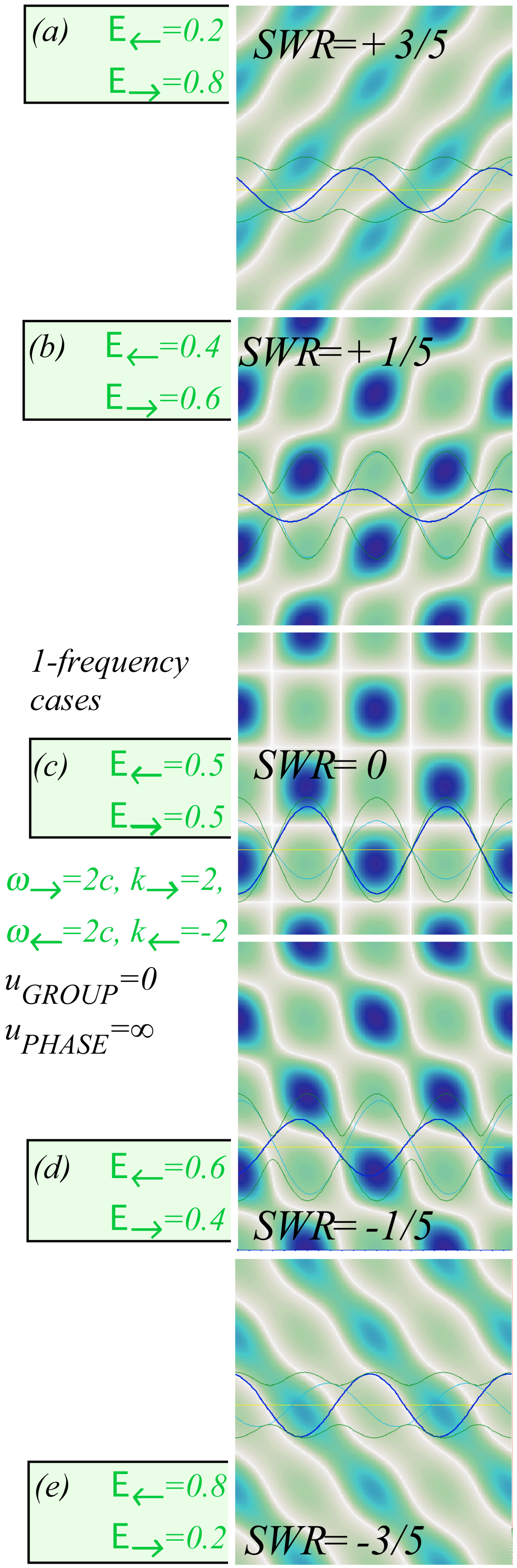


Fig. 6.1 Monochromatic (1-frequency) 2-CW wave space-time patterns.



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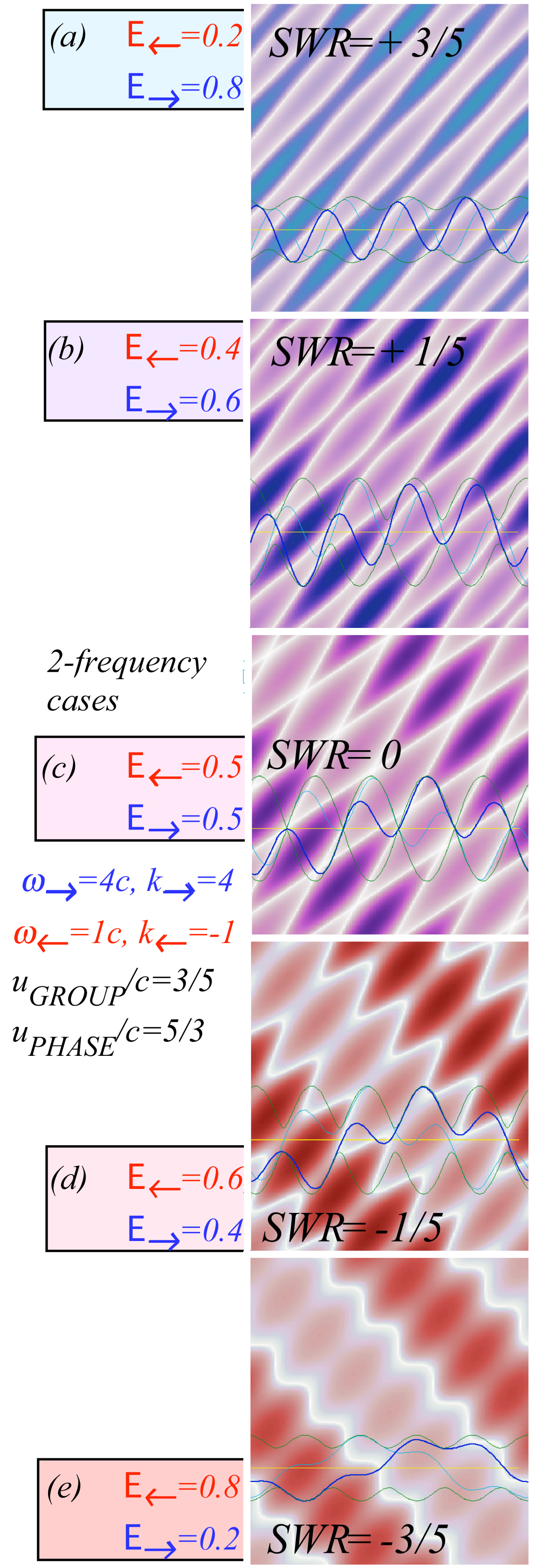
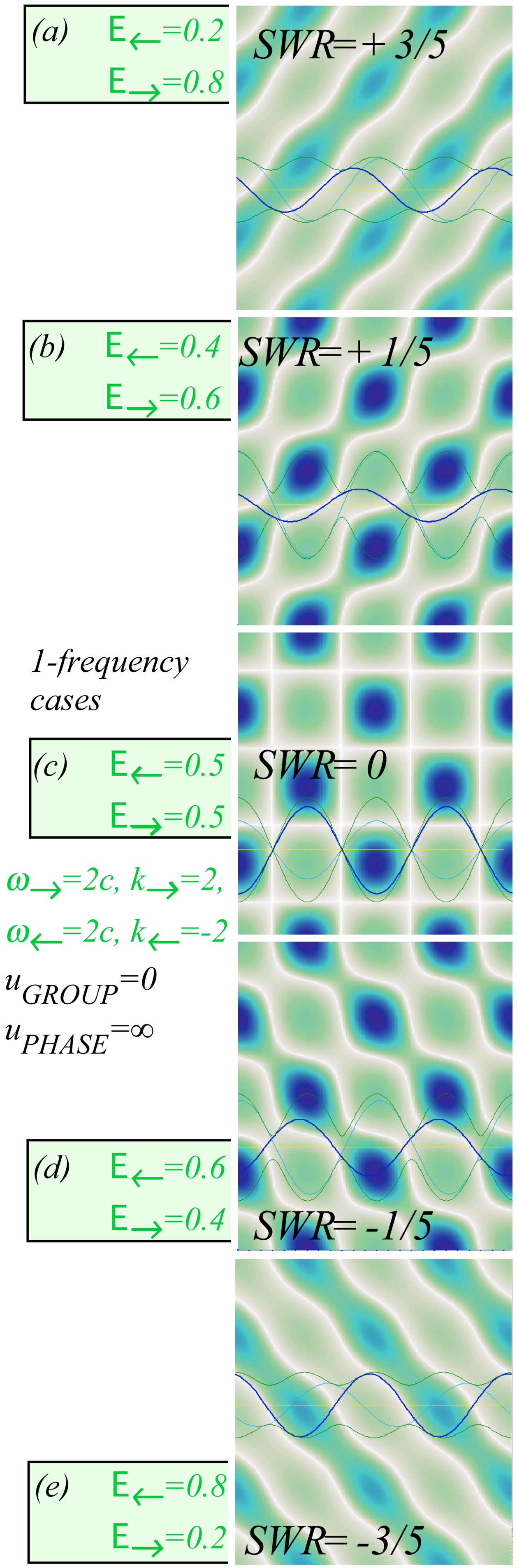


Fig. 6.2 Dichromatic (2-frequency) 2-CW wave space-time patterns.

www.uark.edu/ua/pirelli/php/amplitude_probability_2.php



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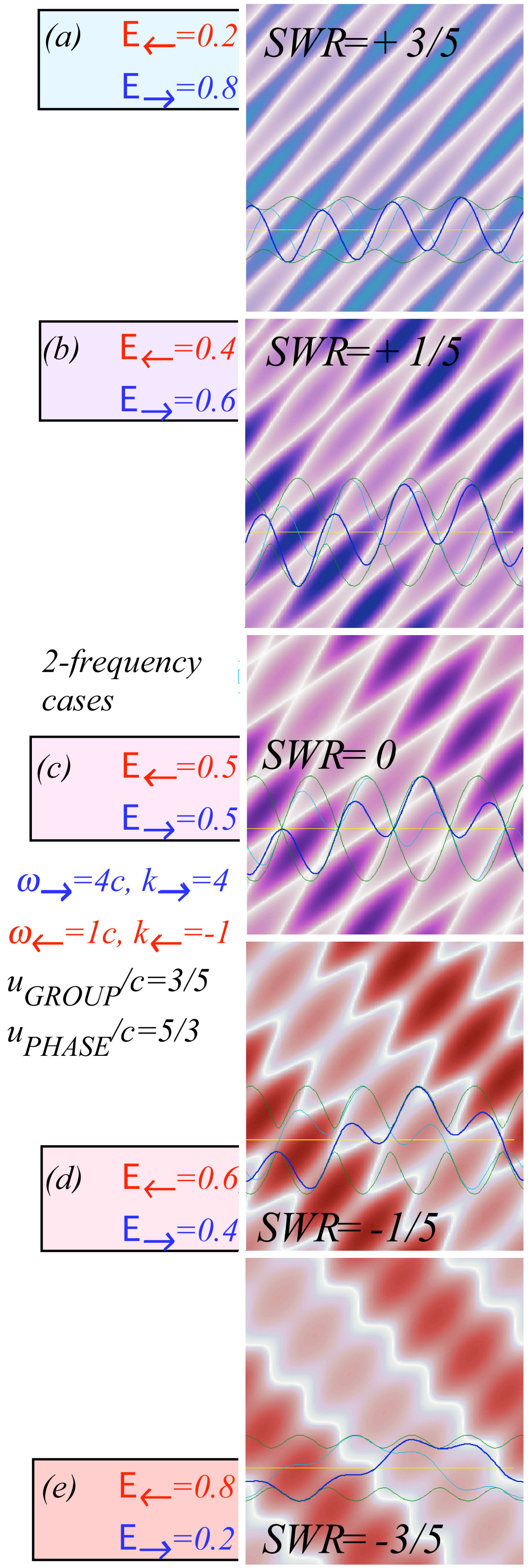

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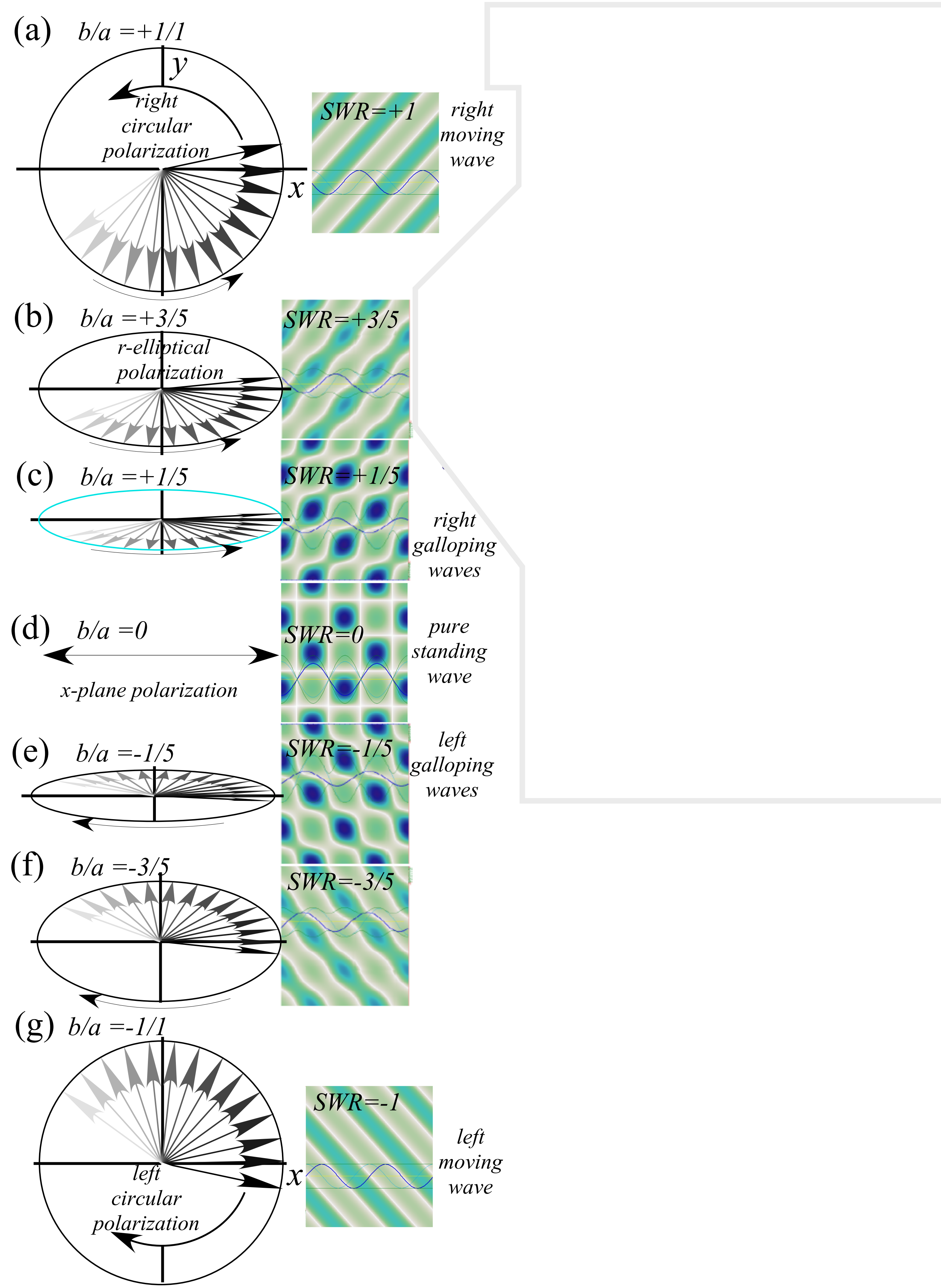
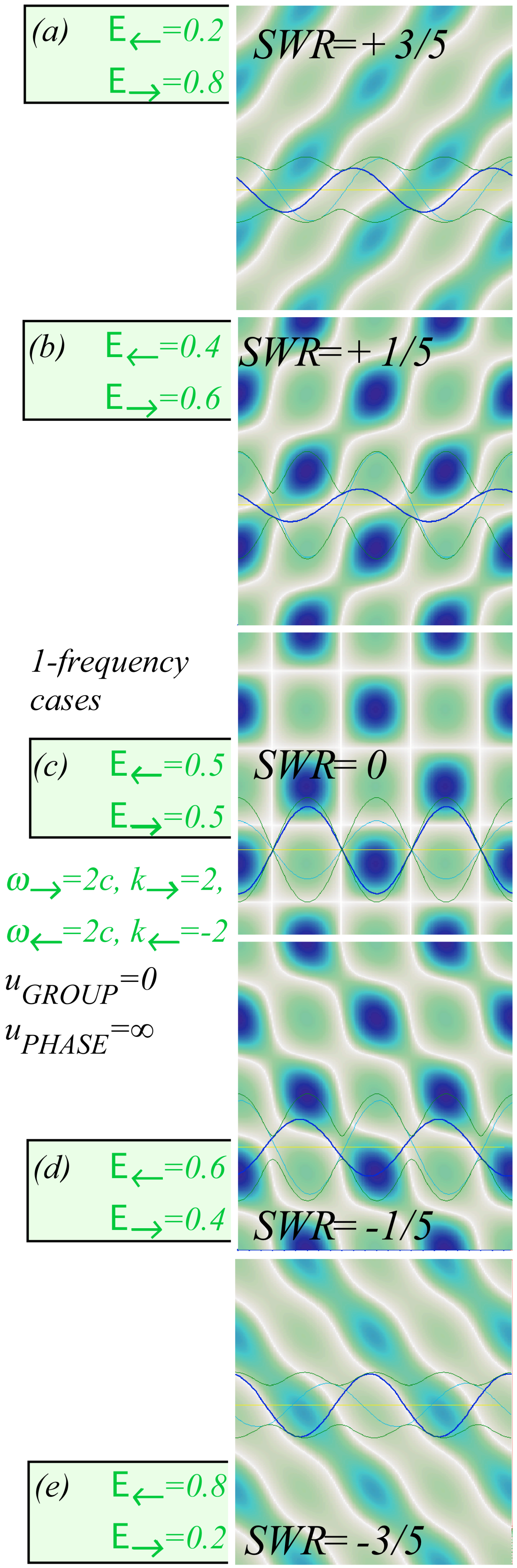


Fig. 6.3 (a-g) Elliptic polarization ellipses relate to galloping waves in Fig. 6.1. (h-i) Kepler anomalies.



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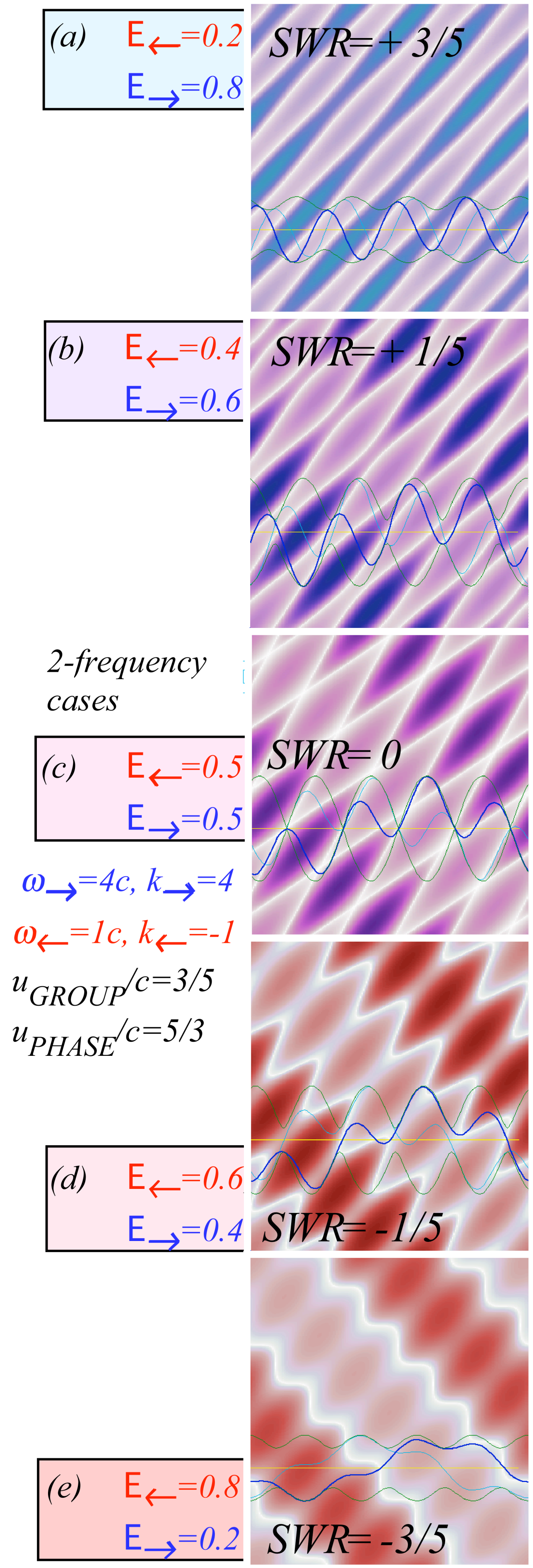

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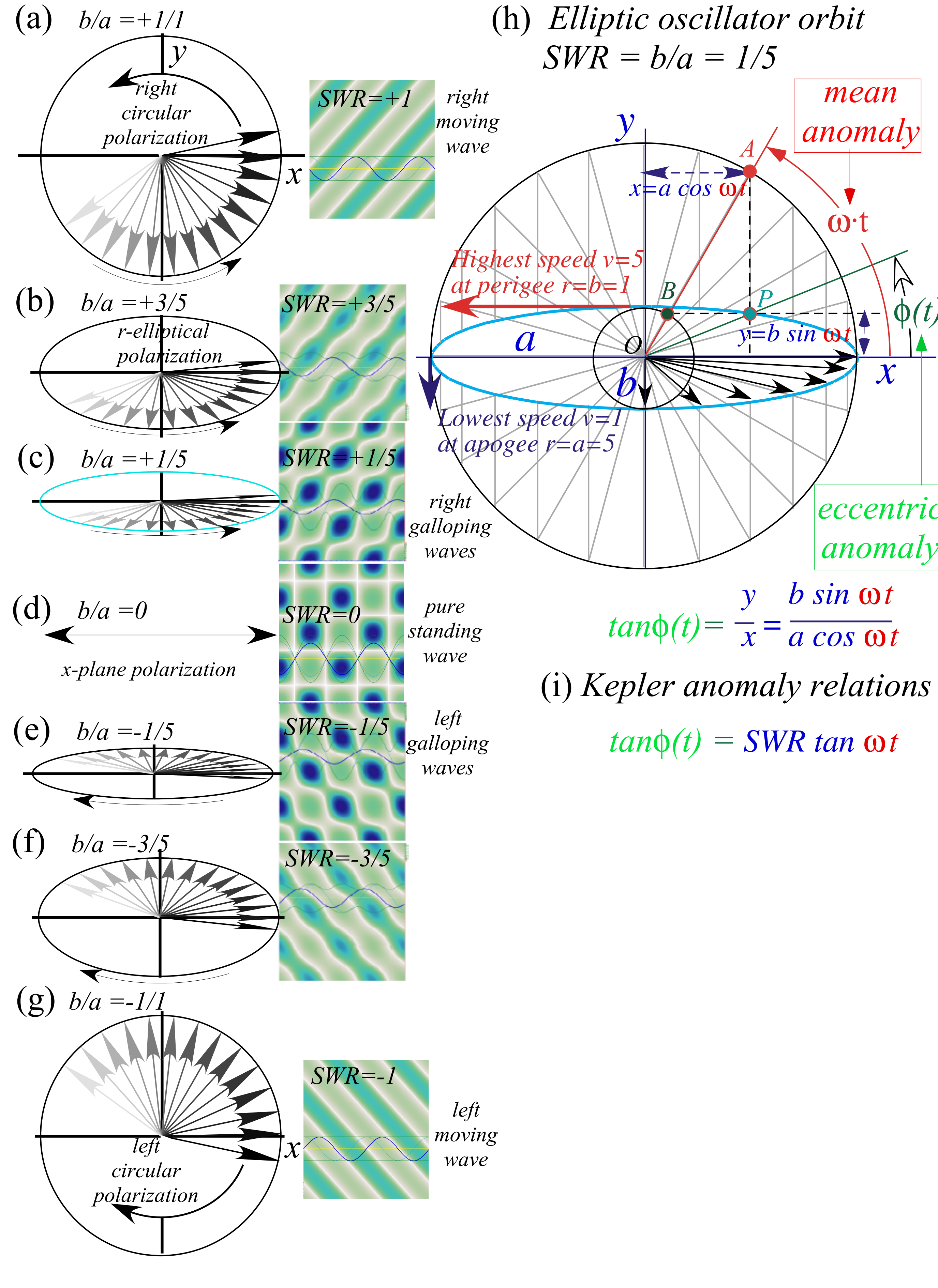

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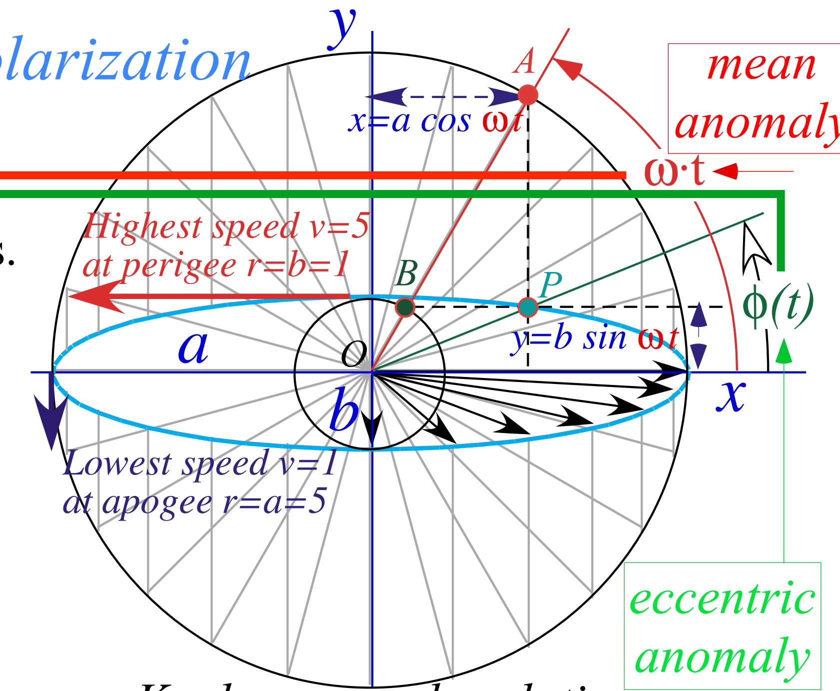
Analogy between wave galloping, Keplarian IHO orbits, and optical polarization

We'll show wave galloping is analogous to Keplarian orbital motion of angles $\omega\cdot t$ and ϕ of orbits.

$$\tan\phi(t) = \frac{b}{a}\tan\omega\cdot t$$



Kepler anomaly relations

$$\tan\phi(t) = \frac{y}{x} = \frac{b\sin\omega t}{a\cos\omega t} = SWR\cdot\tan\omega t$$

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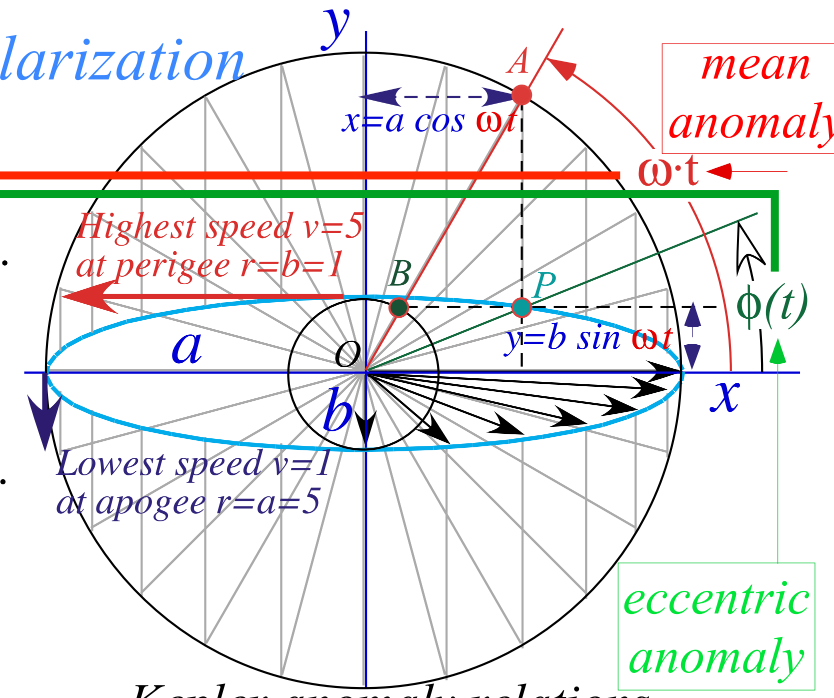

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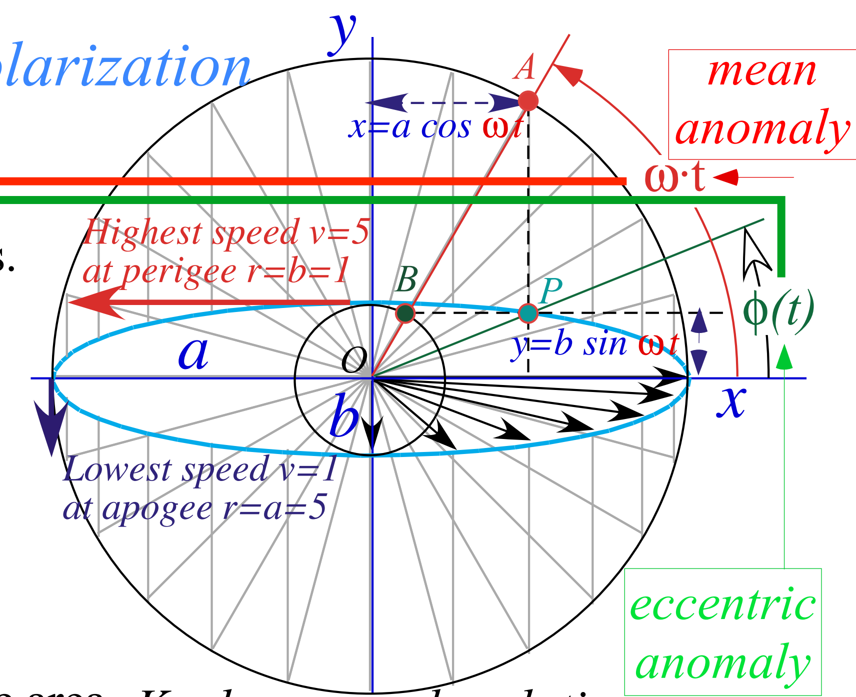

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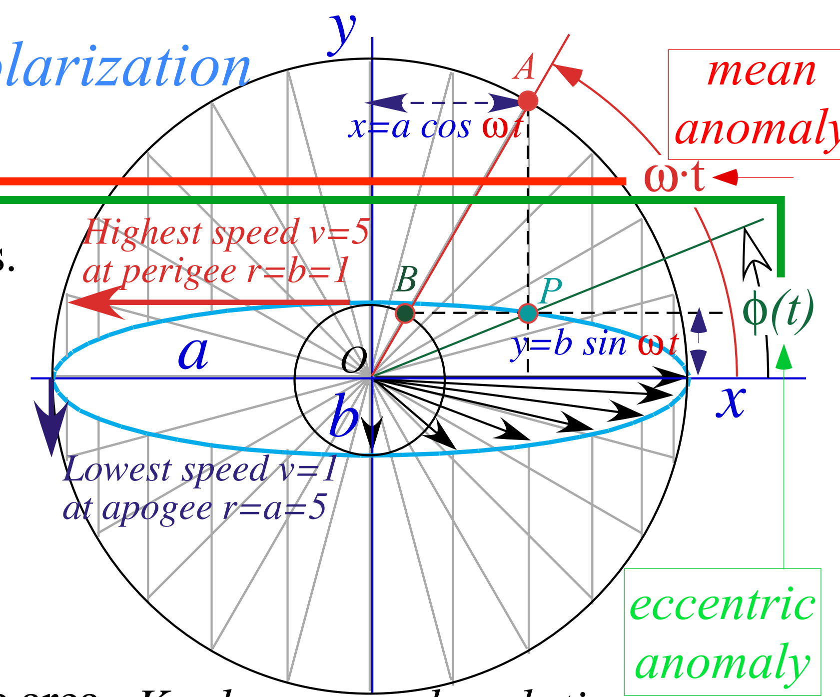

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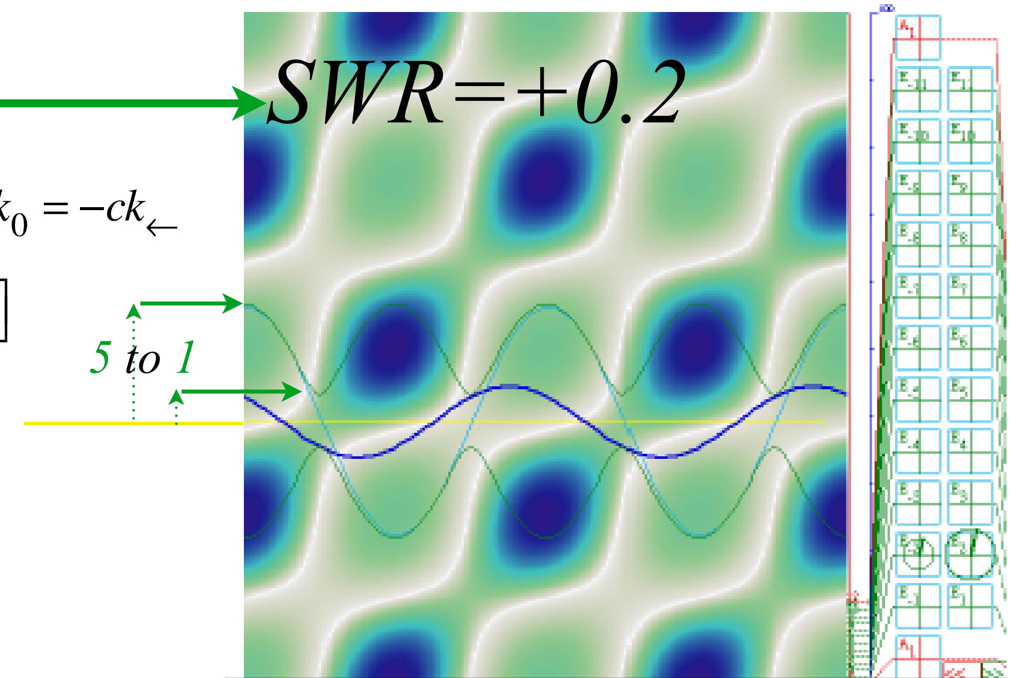

$$0 = \mathrm{Re}\,\Psi(x,t) = \mathrm{Re}\left[A_{\rightarrow}e^{i(k_0x-\omega_0 t)} + A_{\leftarrow}e^{i(-k_0x-\omega_0 t)}\right] \quad \text{where: } \omega_{\rightarrow} = \omega_0 = \omega_{\leftarrow} = ck_0 = -ck_{\leftarrow}$$

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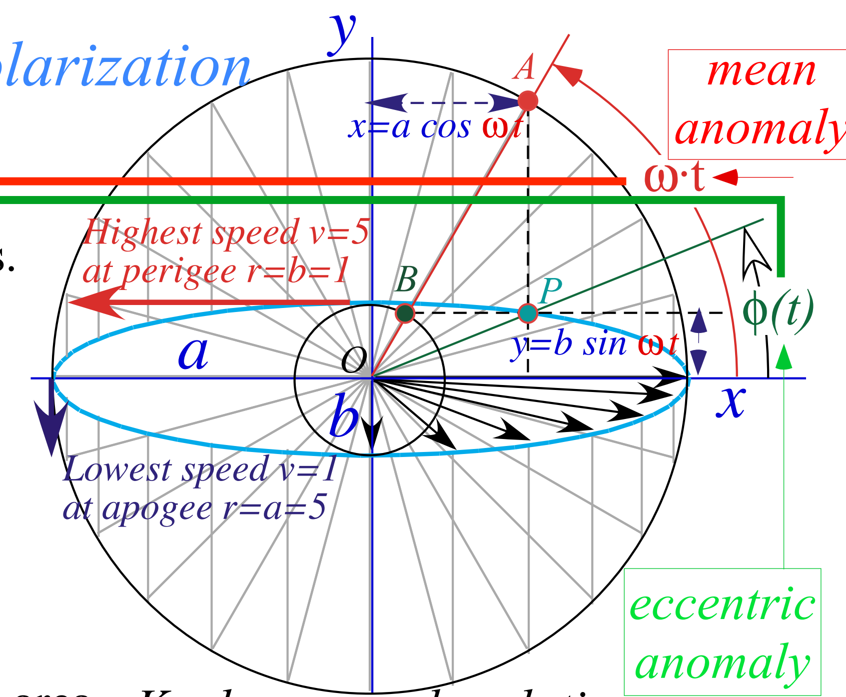

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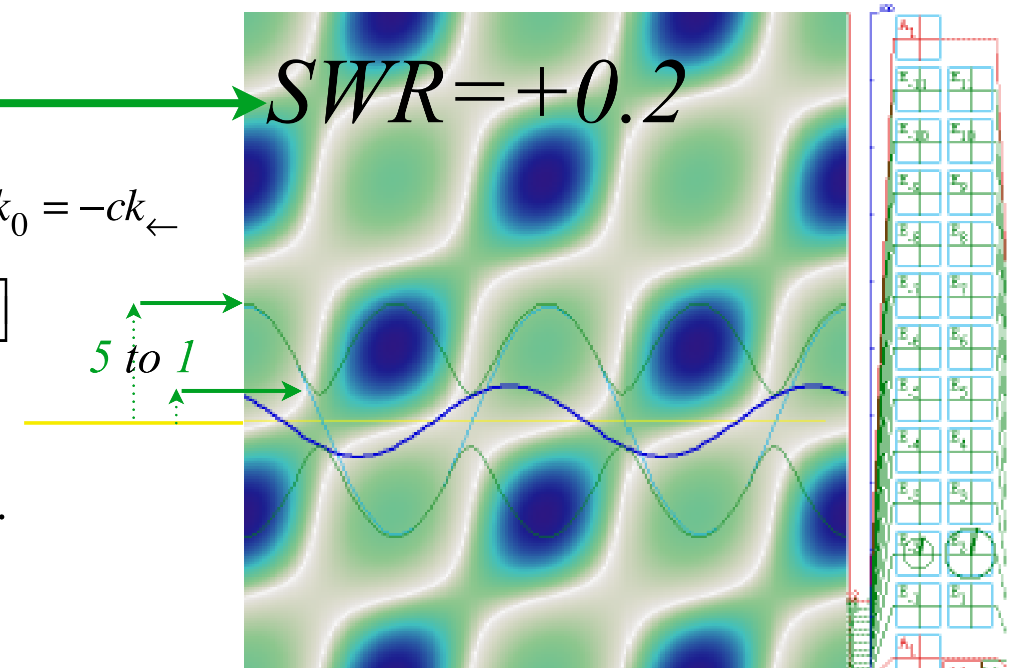

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Space k_0x varies with time ω_0t in the same way that eccentric anomaly ϕ varies with $\omega \cdot t$.

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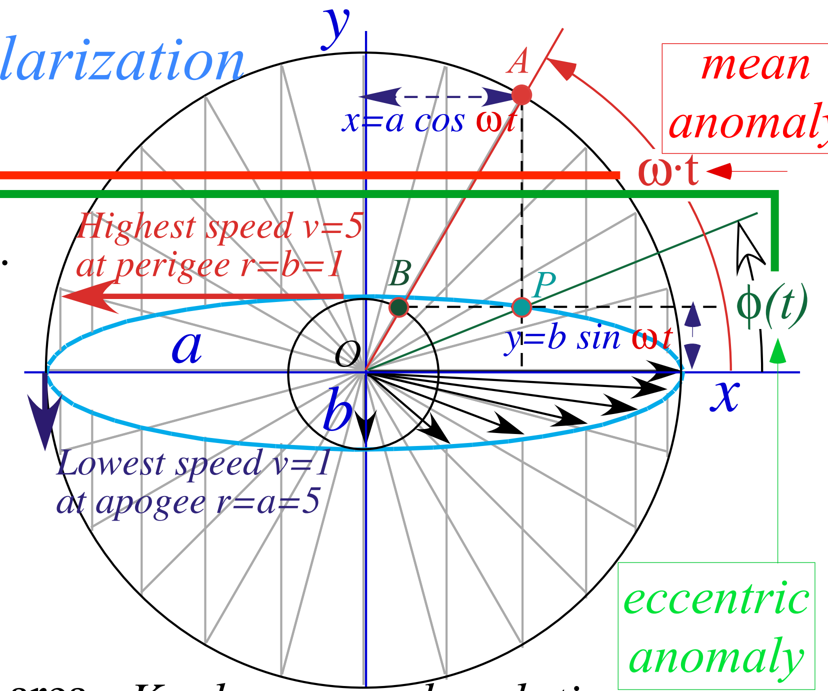
Analogy between wave galloping, Keplarian IHO orbits, and optical polarization



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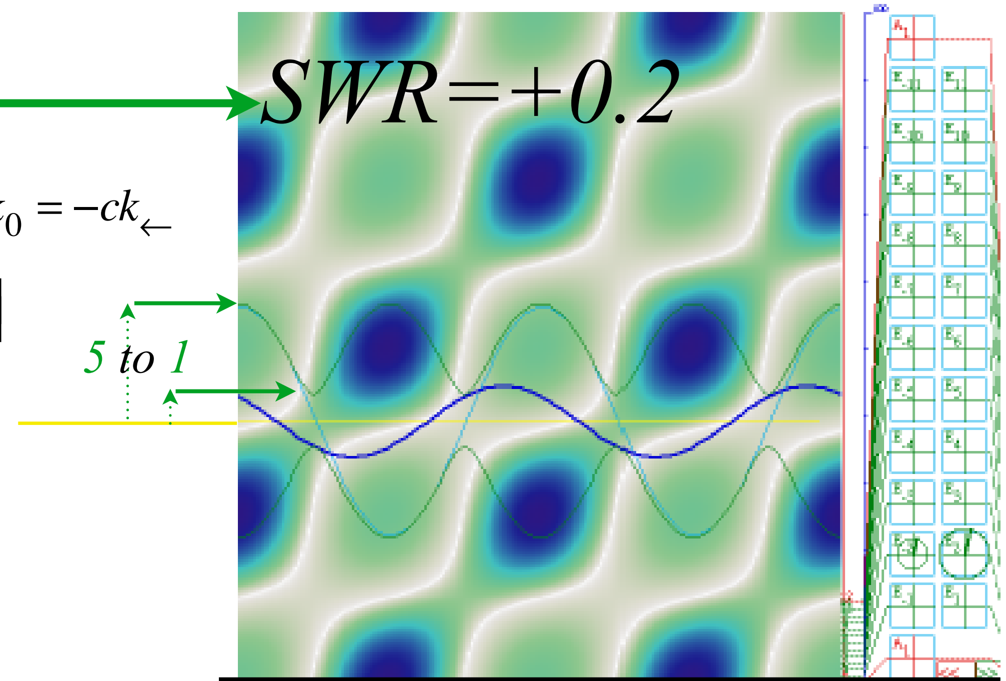

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$E_{\leftarrow}=0.4,\quad E_{\rightarrow}=0.6$

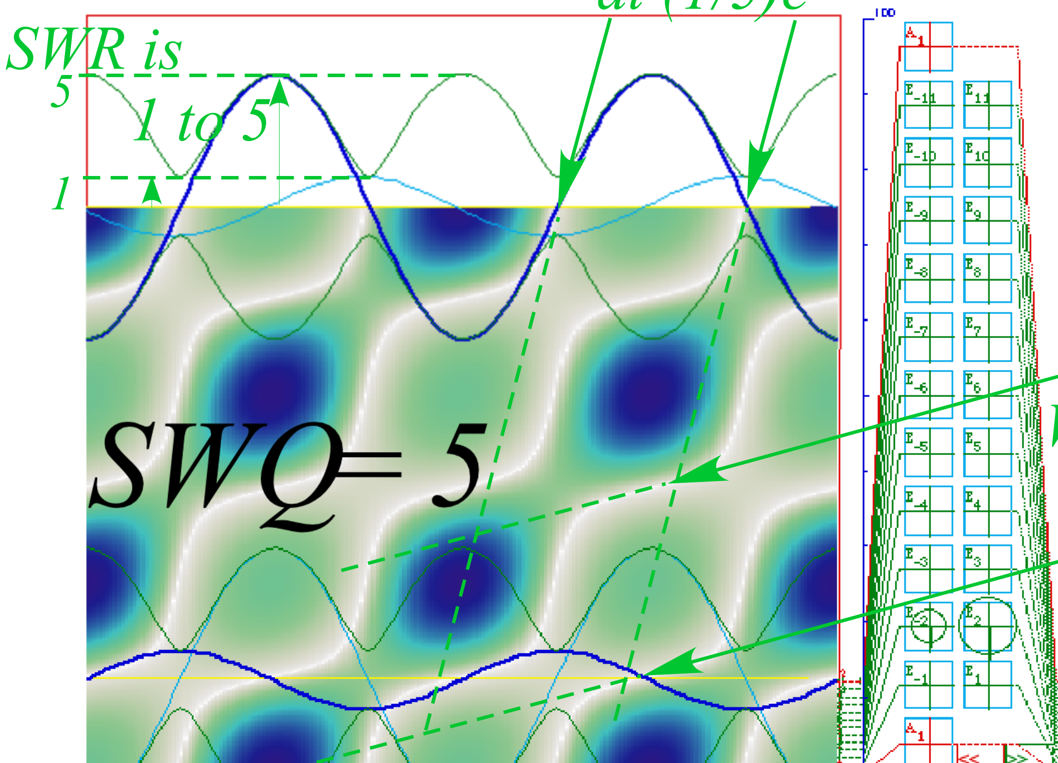
Speed of galloping wave zeros is the time derivative of root location x in units of light velocity c.

$$\frac{dx}{dt} = c\cdot SWR\frac{\sec^2\omega_0\bar{t}}{\sec^2 k_0x} = \frac{c\cdot SWR}{\cos^2\omega_0\bar{t} + SWR^2\cdot\sin^2\omega_0\bar{t}} = \begin{cases} c\cdot SWR & \text{for: } \bar{t} = 0,\ \pi,\ 2\pi\ldots \\ c\cdot SWQ & \bar{t} = \pi/2,\ 3\pi/2,\ldots \end{cases}$$

Wave-Zero Speed-Limits

Standing Wave Ratio SWR and Quotient SWQ

$$SWR=(E_{\rightarrow}-E_{\leftarrow})/(E_{\rightarrow}+E_{\leftarrow})=1/SWQ$$

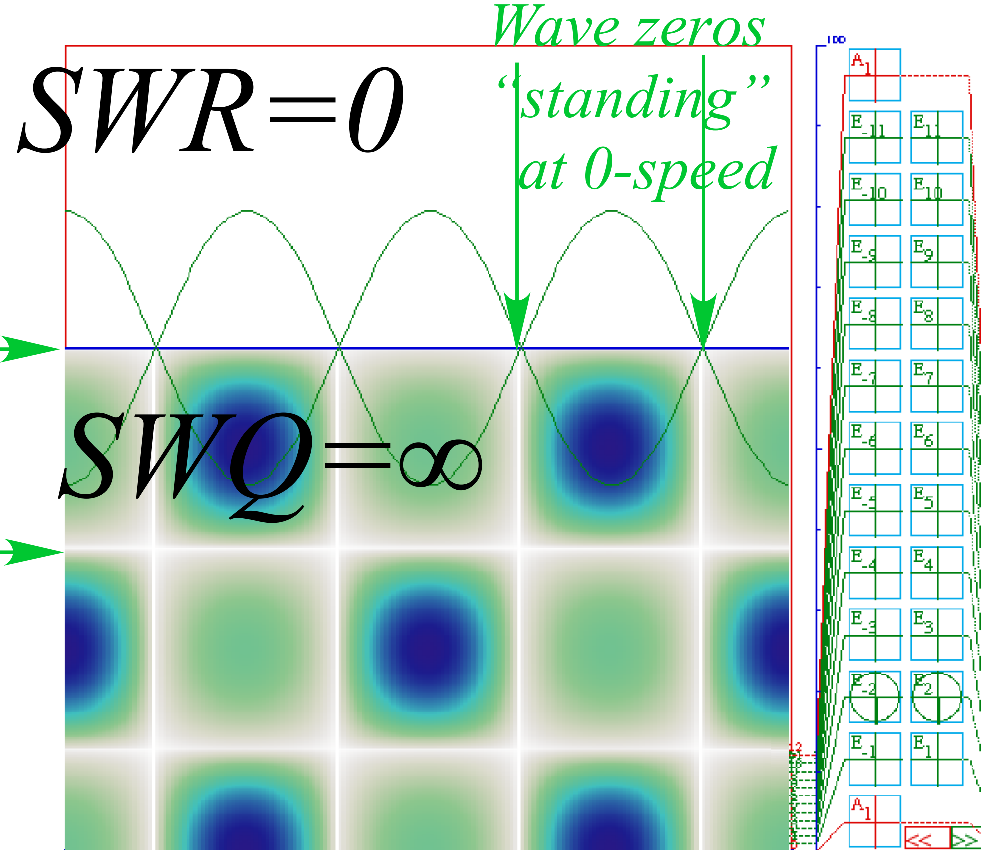


E =0.4, E =0.6

$\omega_{\rightarrow}=2c \quad \omega_{\leftarrow}=2c$

$k_{\rightarrow}=2 \quad k_{\leftarrow}=-2$

$u_{GROUP}=0 \ , \ u_{PHASE}=\infty$

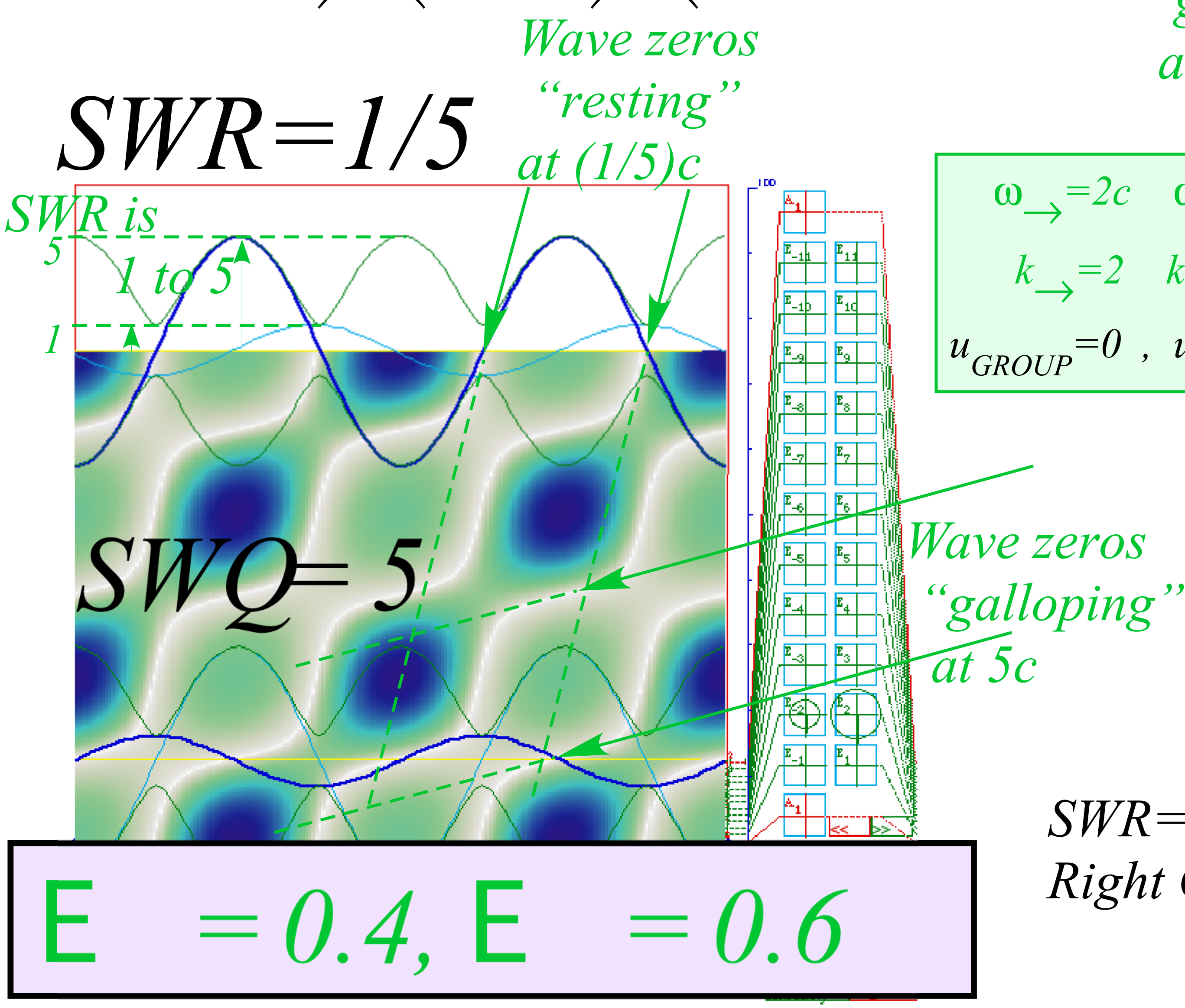


$E_{\leftarrow}=0.5, \ E_{\rightarrow}=0.5$

Wave-Zero Speed-Limits

Standing Wave Ratio SWR and Quotient SWQ

$$SWR=(E_{\rightarrow}-E_{\leftarrow})/(E_{\rightarrow}+E_{\leftarrow})=1/SWQ$$



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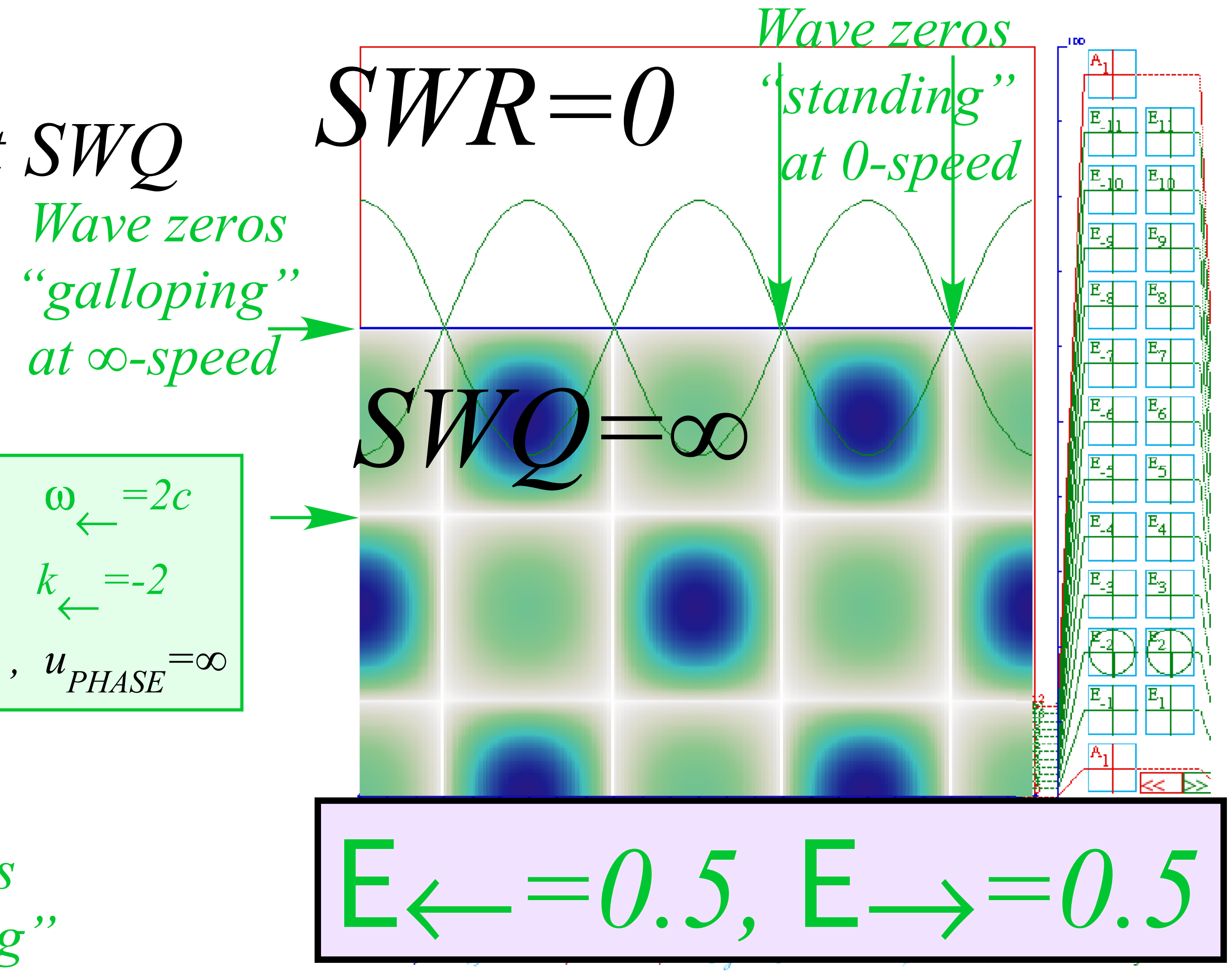
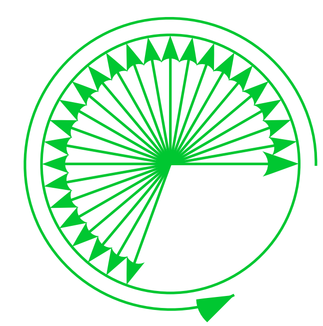
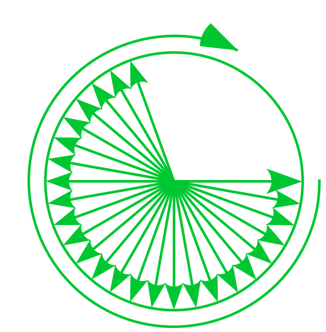


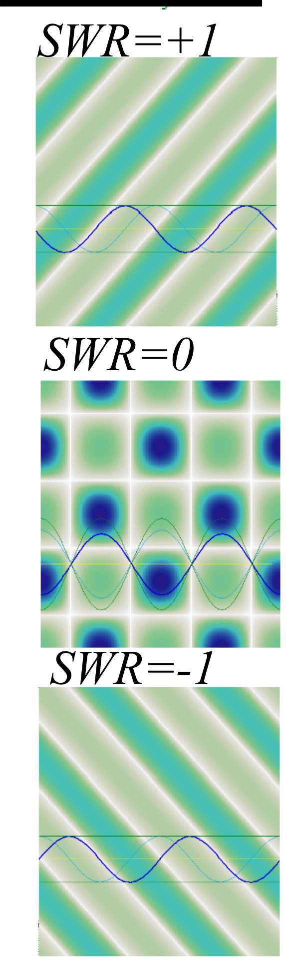
SWR=1 is analogous to (1,i)
Right Circular Polarization



SWR=0 is analogous to (1,0)
x-Plane Linear Polarization

SWR=-1 is analogous to (1,-i)
Left Circular Polarization





SWR=1/5 is analogous to (5-to-1)
Right Elliptic Polarization

Minkowski Zero-Grids are Spacetime *Switchbacks* for $-u_{GROUP}<SWR<0$

$\omega_{\rightarrow}=4c$	$\omega_{\leftarrow}=1c$
$k_{\rightarrow}=4,$	$k_{\leftarrow}=-1$
$u_{GROUP}=c3/5$	$u_{PHASE}=c5/3$

Group zero speed limit

$$\frac{u_{GROUP}+SWR}{1+u_{GROUP}\cdot\frac{SWR}{c^2}}=5c/11$$

$$\frac{\frac{3}{5}+\frac{-1}{5}}{1+\frac{3}{5}\frac{-1}{5}}=\frac{\frac{2}{5}}{\frac{22}{25}}=\frac{5}{11}$$

Group-zero speed $u_{GROUP}=c3/5$

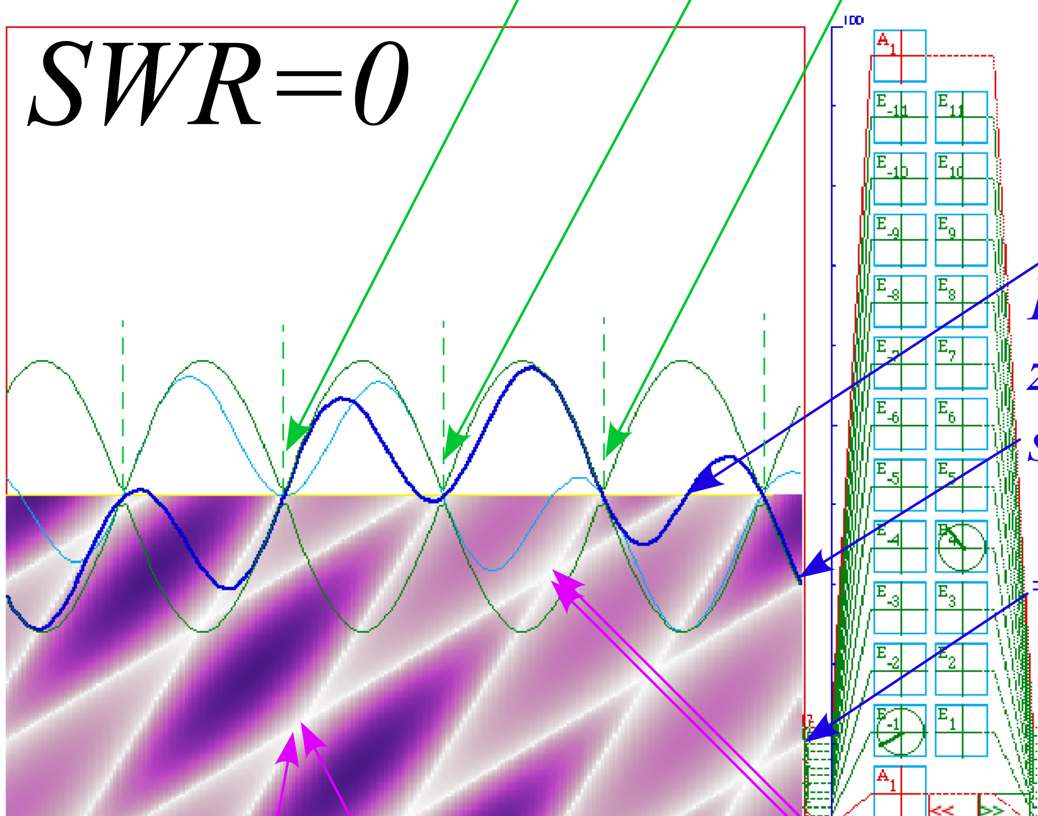
Phase zero speed $u_{PHASE}=c5/3$

Phase "anti-zero" going "back-in-time"

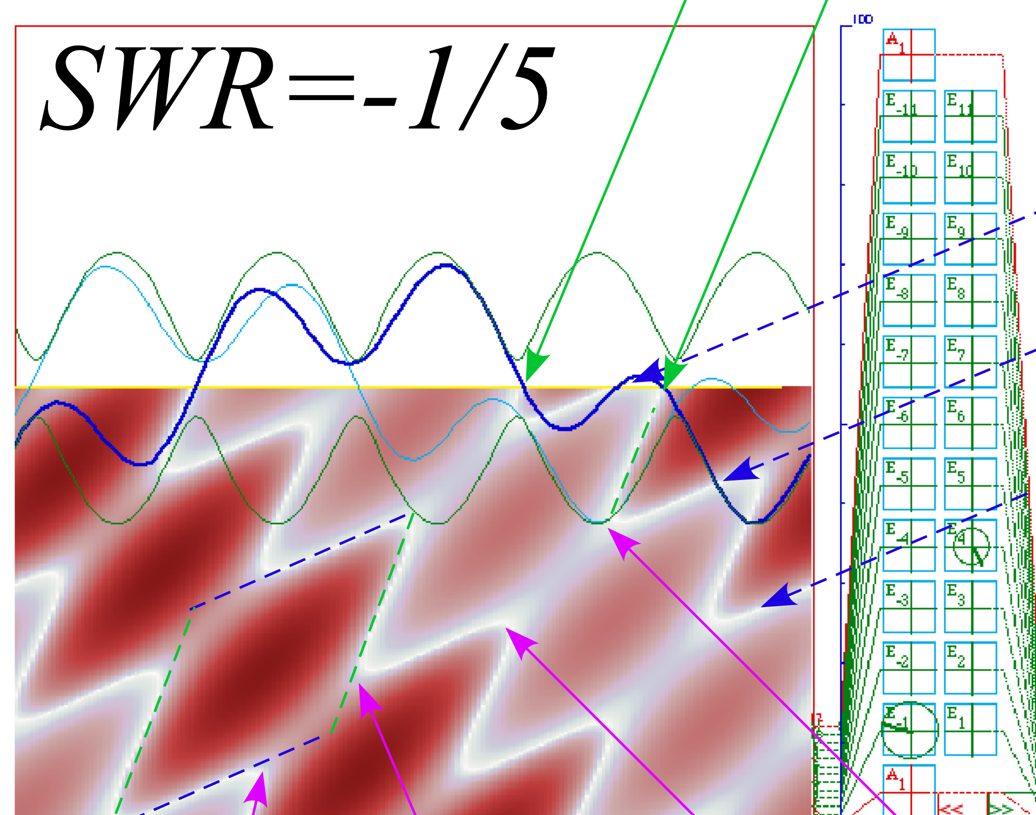
Phase zero speed limit

$$\frac{u_{PHASE}+SWR}{1+u_{PHASE}\cdot\frac{SWR}{c^2}}=11c/5$$

$$\frac{\frac{5}{3}+\frac{-1}{5}}{1+\frac{5}{3}\frac{-1}{5}}=\frac{\frac{22}{15}}{\frac{10}{25}}=\frac{11}{5}$$



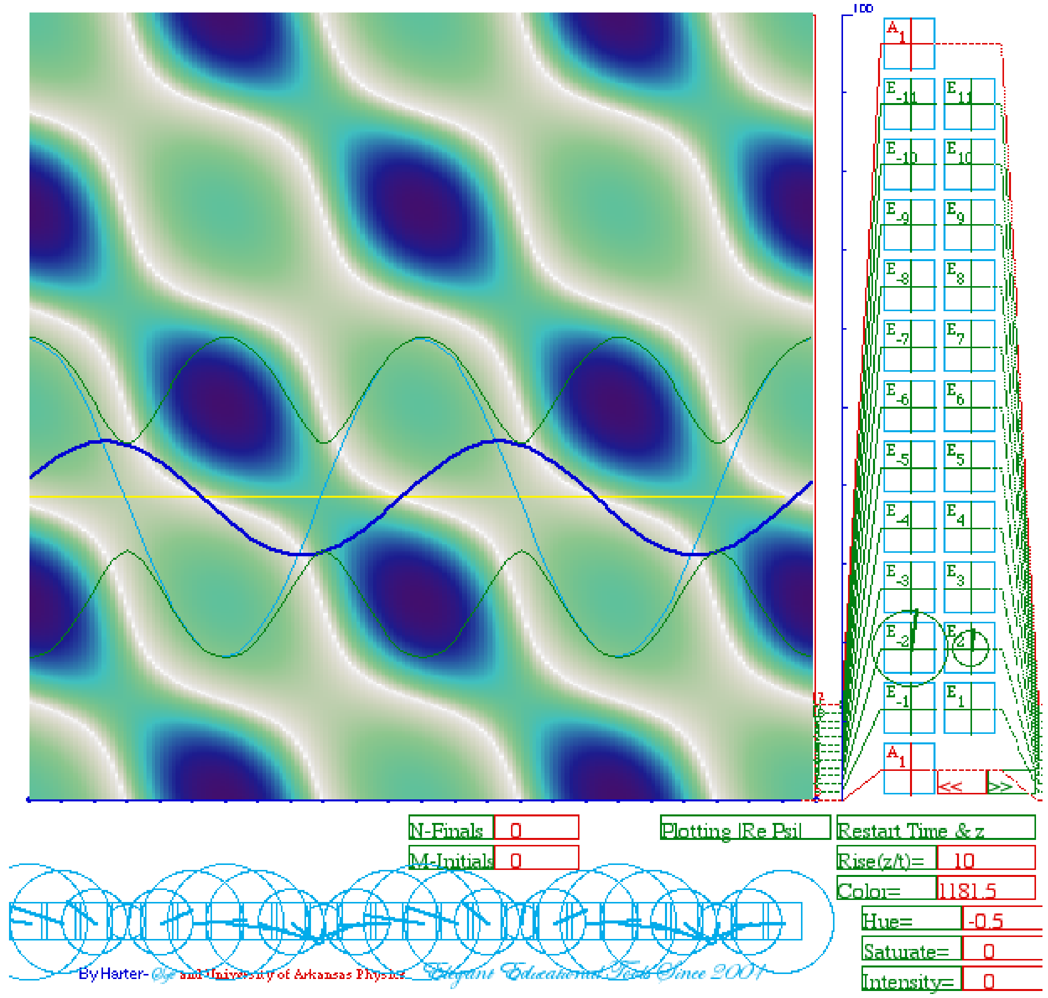
$E_{\leftarrow}=0.5$, $E_{\rightarrow}=0.5$



$E_{\leftarrow}=0.6$, $E_{\rightarrow}=0.4$

Wave zero-anti-zero annihilation *and* creation occur *together* at *the same* spacetime point for *SWR=0*

Wave zero-anti-zero annihilation and creation occur *separately* at *different* spacetime points for $-u_{GROUP}<SWR<0$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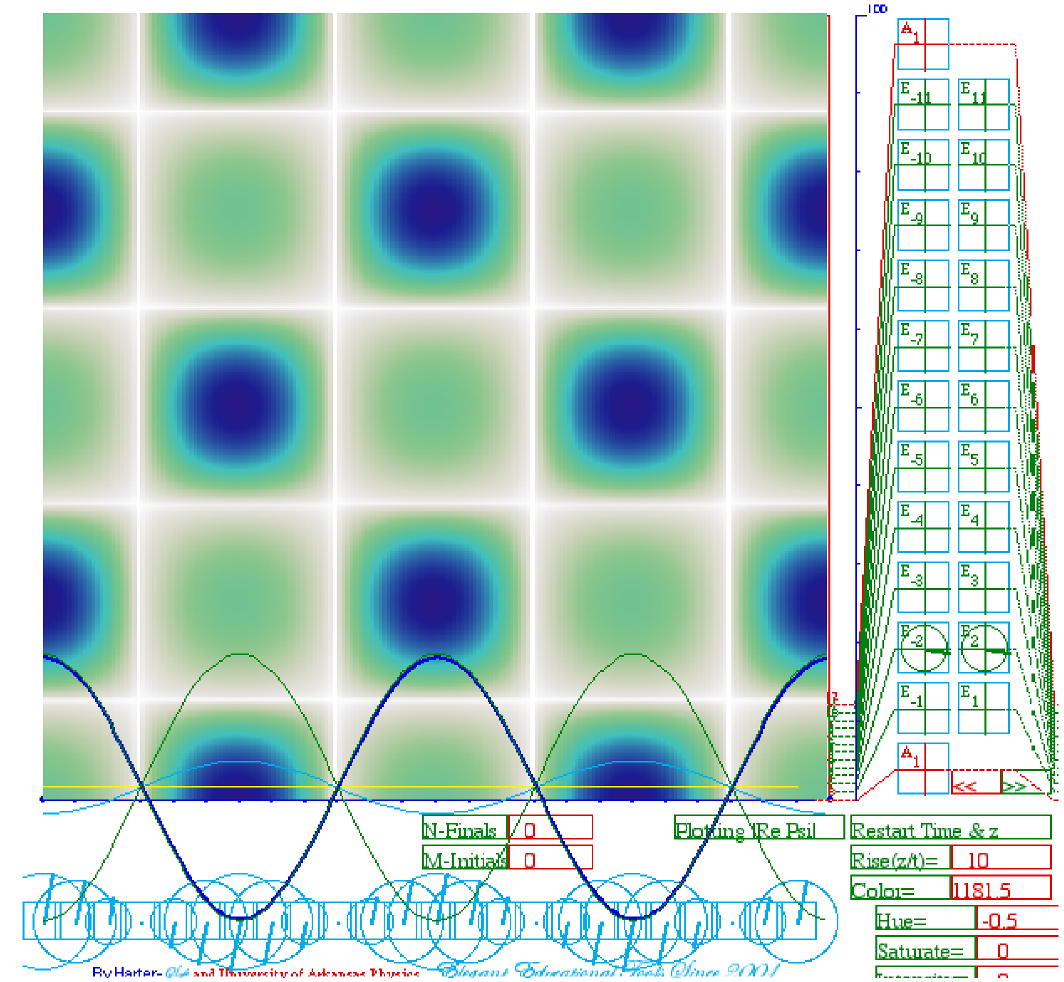
M-Initials 0
Plotting |Re Psi|
Restart Time & z
Rise(z/t)= 10
Color= 1181.5
Hue= -0.5
Saturate= 0
Intensity= 0
By Harter- and University of Arkansas Physics
Elegant Educational Tools Since 2001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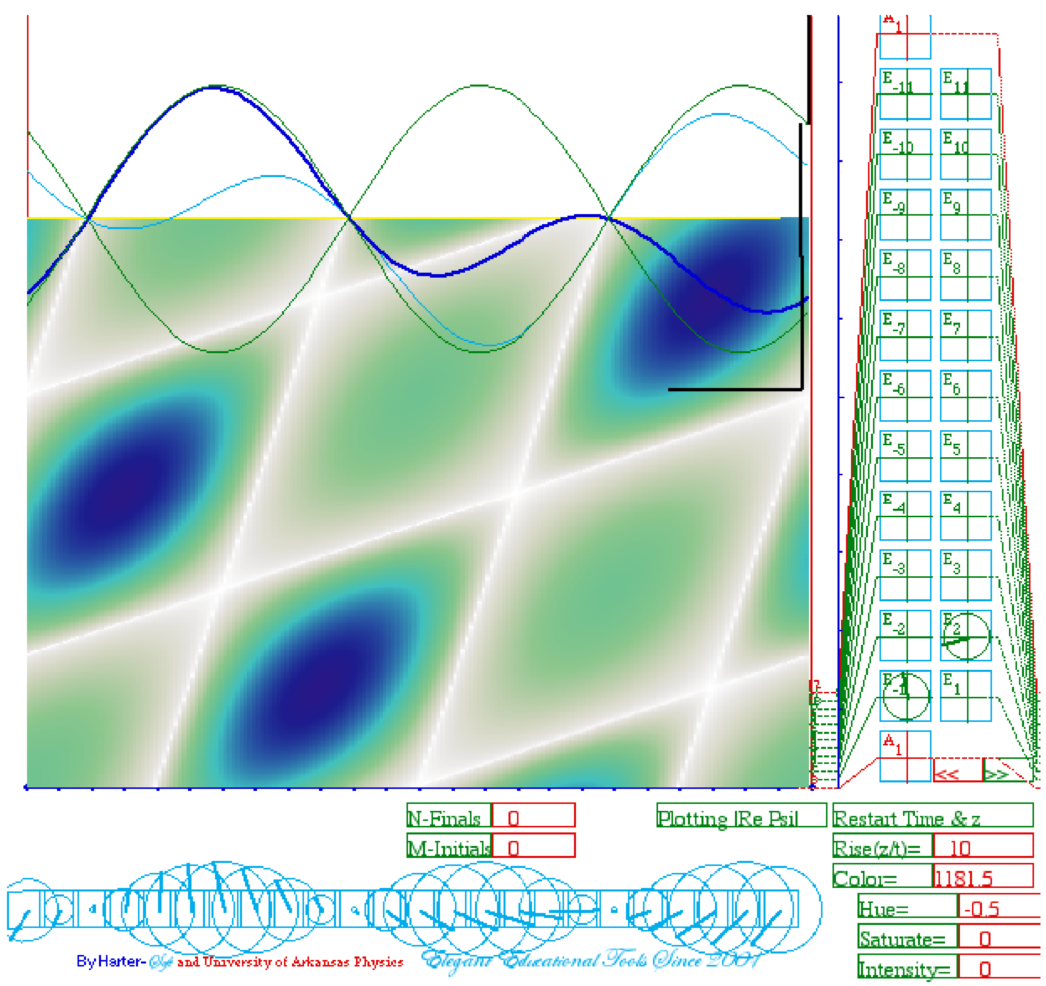
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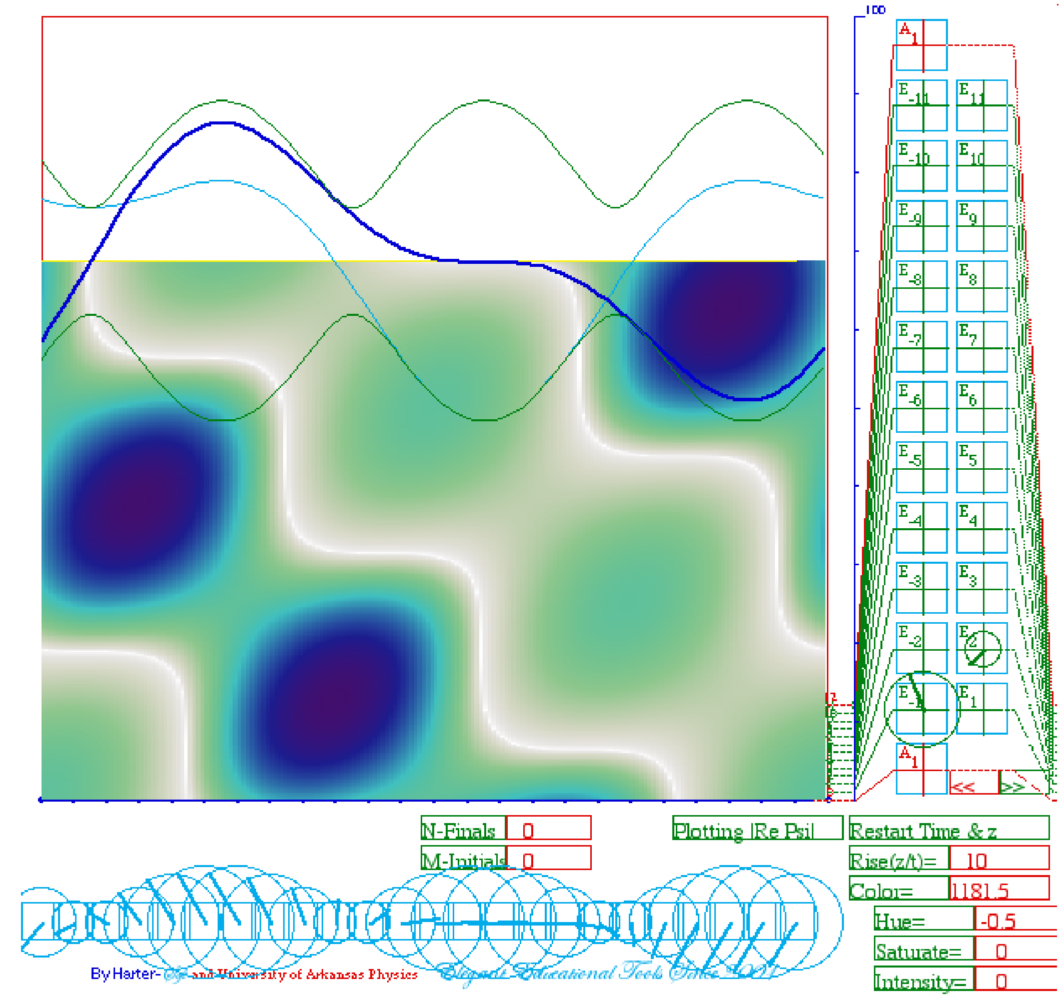
N-Finals 0
M-Initials 0
Plotting |Re Psi|
Restart Time & z
Rise(z/t)= 10
Color= 1181.5
Hue= -0.5
Saturate= 0
Intensity= 0
By Harter- and University of Arkansas Physics
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At High Speed 2-CW Modes Look More Like 1-CW Beams

$\psi = E\sqrt{\frac{\varepsilon_0}{h\upsilon}}$

Various combinations of *opposite-k 1-CW* beams occur with open boundaries.

E-*wave:* $E = E_{\rightarrow} e^{i(k_{\rightarrow}x - \omega_{\rightarrow}t)} + E_{\leftarrow} e^{i(k_{\leftarrow}x - \omega_{\leftarrow}t)}$ *is related to* Ψ-*wave:* $\Psi = \psi_{\rightarrow} e^{i(k_{\rightarrow}x - \omega_{\rightarrow}t)} + \psi_{\leftarrow} e^{i(k_{\leftarrow}x - \omega_{\leftarrow}t)}$

Standing Wave Ratio (or Quotient)

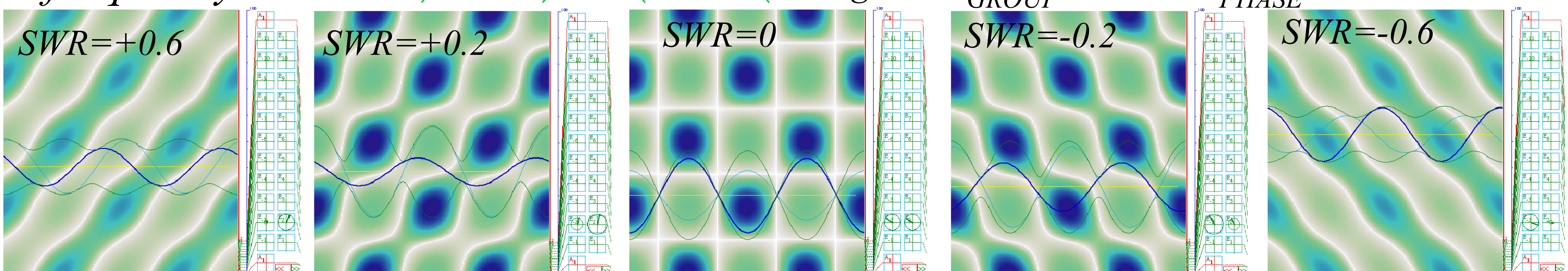
$SWR = (E_{\rightarrow} - E_{\leftarrow})/(E_{\rightarrow} + E_{\leftarrow}) = 1/SWQ$

key numbers

Wave Group (or Phase) Velocity

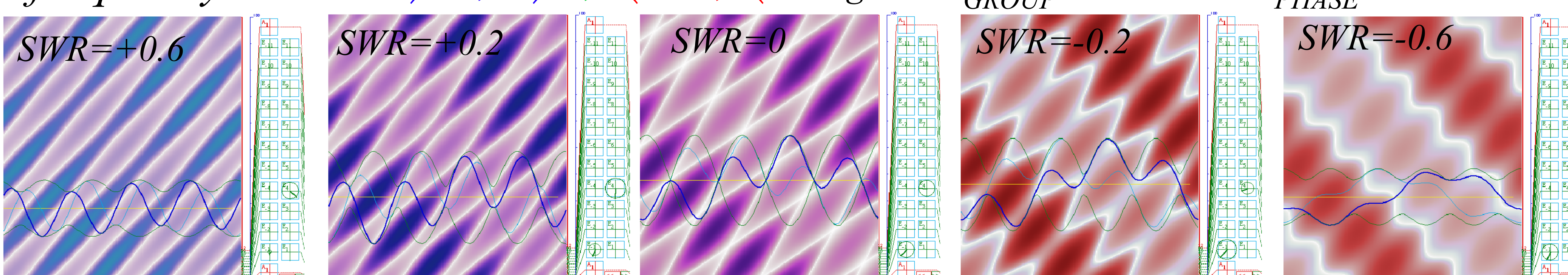
$u_{GROUP}/c = (\omega_{\rightarrow} - \omega_{\leftarrow})/(\omega_{\rightarrow} + \omega_{\leftarrow}) = c/u_{PHASE}$

1-frequency case : $\omega_{\rightarrow} = 2c,\ k_{\rightarrow} = 2,\ \omega_{\leftarrow} = 2c,\ k_{\leftarrow} = -2$ gives: $u_{GROUP} = 0$ and $u_{PHASE} = \infty$



$E_{\leftarrow} = 0.4,\ E_{\rightarrow} = 1.0$	$E_{\leftarrow} = 0.4,\ E_{\rightarrow} = 0.6$	$E_{\leftarrow} = 0.5,\ E_{\rightarrow} = 0.5$	$E_{\leftarrow} = 0.6,\ E_{\rightarrow} = 0.4$	$E_{\leftarrow} = 1.0,\ E_{\rightarrow} = 0.4$

2-frequency case : $\omega_{\rightarrow} = 4c,\ k_{\rightarrow} = 4,\ \omega_{\leftarrow} = 1c,\ k_{\leftarrow} = -1$ gives: $u_{GROUP}/c = 3/5$ and $u_{PHASE}/c = 5/3$



$E_{\leftarrow} = 0.4,\ E_{\rightarrow} = 1.0$	$E_{\leftarrow} = 0.4,\ E_{\rightarrow} = 0.6$	$E_{\leftarrow} = 0.5,\ E_{\rightarrow} = 0.5$	$E_{\leftarrow} = 0.6,\ E_{\rightarrow} = 0.4$	$E_{\leftarrow} = 1.0,\ E_{\rightarrow} = 0.4$

1st Quantization: Quantizing phase variables ω and k

Understanding how quantum transitions require "mixed-up" states

Closed cavity vs Ring cavity

<u>Quantized ω and *k*</u> *Counting wave kink numbers*

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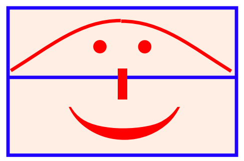
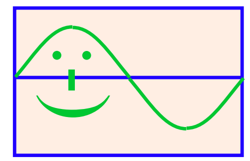
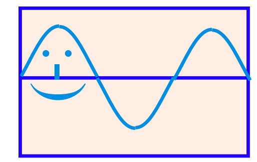
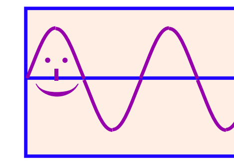
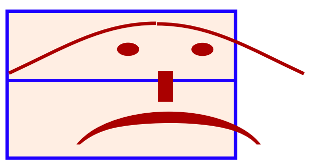
<u>OK</u> box quantum numbers: *n=1* *n=2* *n=3* *n=4*

(+ integers <u>only</u>)

Some
<u>NOT</u> OK numbers: *n=0.67* *n=1.7* *n=2.59* *n=4*

too fat!



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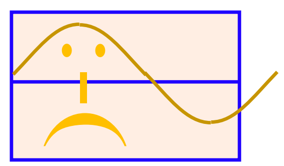
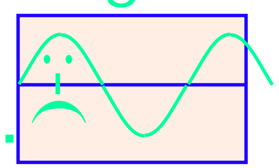


wrong color again!

misfits...



...not tolerated !

NOTE: We're using "false-color" here.

This doesn't mean a system's energy can't vary <u>continuously</u> between "OK" values E_1, E_2, E_3, E_4,...

Quantized ω and *k* *Counting wave kink numbers*

If everything is made of waves then we expect *quantization* of everything because waves only thrive if *integral* numbers *n* of their “kinks” fit into whatever structure (box, ring, etc.) they’re supposed to live. The numbers *n* are called *quantum numbers*.

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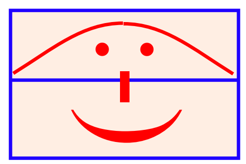

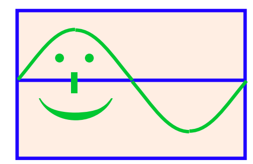

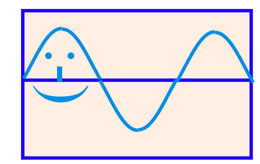

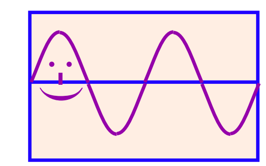

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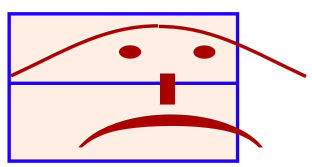

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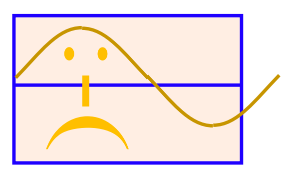
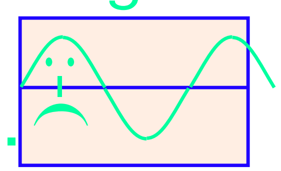


wrong color again!

misfits...



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In fact its state can be a linear combination of any of the “OK” waves $|E_1>$, $|E_2>$, $|E_3>$, $|E_4>$,...

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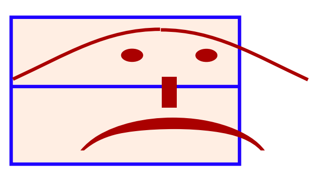

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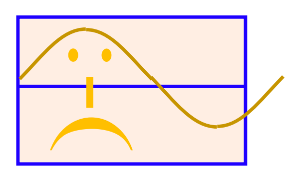

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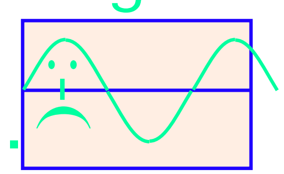

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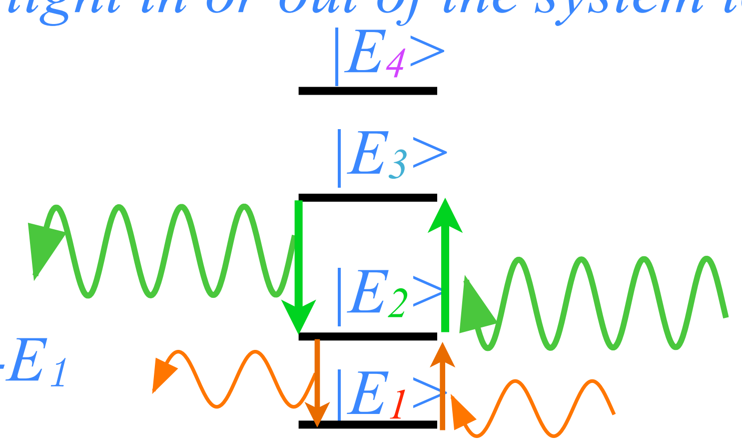
That's the only way you get any light in or out of the system to "see" it.

frequency $\hbar\omega_{32} = E_3 - E_2$

frequency $\hbar\omega_{21} = E_2 - E_1$



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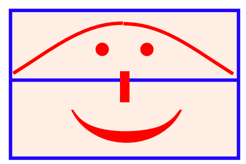
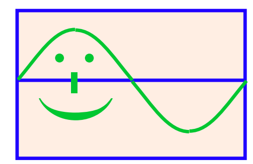
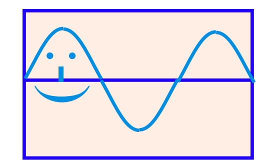
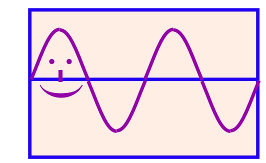

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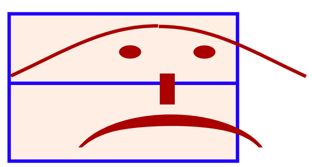

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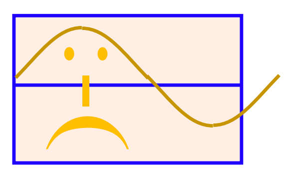
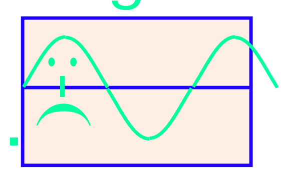


wrong color again!

misfits...



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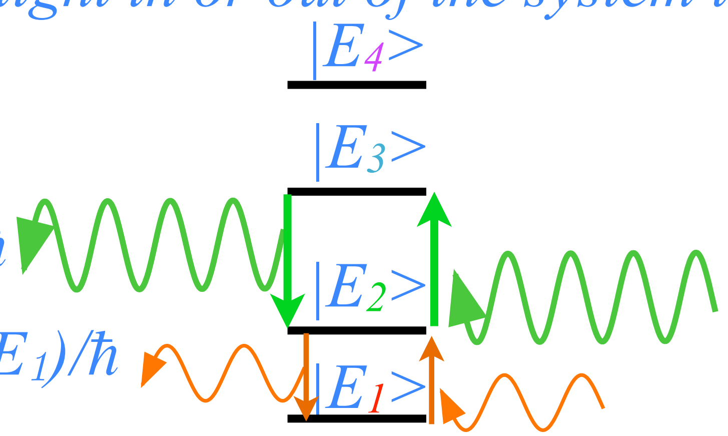
frequency $\omega_{32} = (E_3 - E_2)/\hbar$

frequency $\omega_{21} = (E_2 - E_1)/\hbar$



These eigenstates are the only ways the system can "play dead"... ... " sleep with the fishes"...

Quantized ω and *k* *Counting wave kink numbers*

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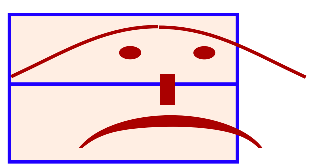

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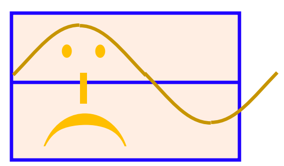

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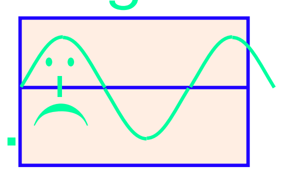


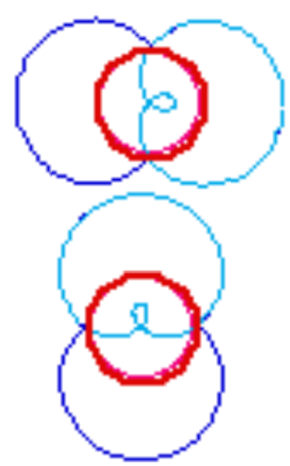
...not tolerated !

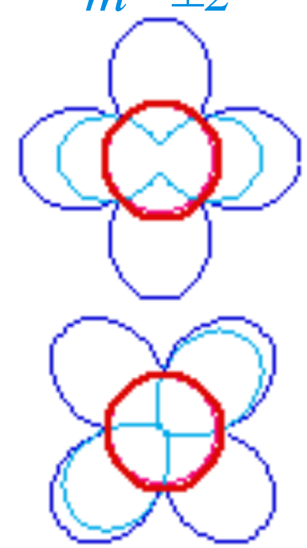
NOTE: We're using "false-color" here.

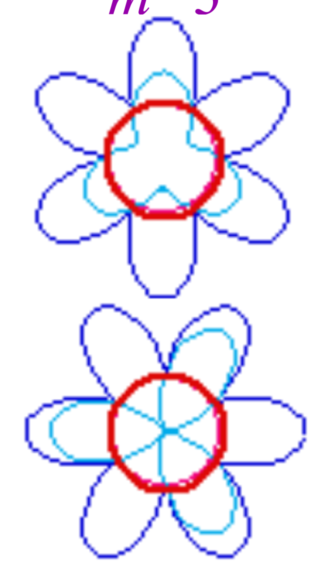
Rings tolerate a *zero* (kinkless) quantum wave but require ±*integral wave number.*

OK ring quantum numbers: *m=0* *m=±1* *m=±2* *m=3*

(± integral number of wavelengths)







Bohr's models of *atomic spectra (1913-1923)* are beginnings of *quantum wave mechanics* built on *Planck-Einstein (1900-1905)* relation $E=h\upsilon$. *DeBroglie* relation $p=h/\lambda$ comes around *1923*.

Lecture 30 ended here

2nd Quantization: Quantizing amplitudes ("photons","vibrons", and "what-ever-ons")

Introducing coherent states (What lasers use)

Analogy with (ω,k) wave packets

Wave coordinates need coherence

Quantized *Amplitude* *Counting "photon" number*

Planck's relation $E=Nh\upsilon$ began as a tenative axiom to explain low-T light. Then he tried to disavow it! Einstein picked it up in his 1905 paper. Since then its use has grown enormously and continues to amaze, amuse (or bewilder) all who study it.

A current view is that it represents the *quantization* of optical field *amplitude*. We picture this below as *N-photon* wave states for each box-mode of *m* wave kinks.