Lecture 30.

Relativity of interfering and galloping waves: SWR and SWQ II.

(Ch. 4-6 of Unit 2  4.12.12)

Unmatched amplitudes giving galloping waves

Standing Wave Ratio (SWR) and Standing Wave Quotient (SWQ)

Analogy with group and phase

Analogy between wave galloping, Keplarian IHO orbits, and optical polarization

Waves that go back in time - The Feynman-Wheeler Switchback

1st Quantization: Quantizing phase variables $\omega$ and $k$

Understanding how quantum transitions require “mixed-up” states

Closed cavity vs Ring cavity

Lecture 30 ended here
Galloping waves due to unmatched amplitudes

2-CW dynamics has two 1-CW amplitudes $A_{→}$ and $A_{←}$ that we now allow to be unmatched. ($A_{→} \neq A_{←}$)

$$A_{→} e^{i(k_{→} x - \omega_{→} t)} + A_{←} e^{i(k_{←} x - \omega_{←} t)} = e^{i(k_{Σ} x - \omega_{Σ} t)} [A_{→} e^{i(k_{Δ} x - \omega_{Δ} t)} + A_{←} e^{-i(k_{Δ} x - \omega_{Δ} t)}]$$

Waves have half-sum mean-phase rates $(k_{Σ}, \omega_{Σ})$ and half-difference group rates $(k_{Δ}, \omega_{Δ})$.

$$k_{Σ} = (k_{→} + k_{←}) / 2$$
$$\omega_{Σ} = (\omega_{→} + \omega_{←}) / 2$$

$$k_{Δ} = (k_{→} - k_{←}) / 2$$
$$\omega_{Δ} = (\omega_{→} - \omega_{←}) / 2$$
Galloping waves due to unmatched amplitudes

2-CW dynamics has two 1-CW amplitudes $A_\rightarrow$ and $A_\leftarrow$ that we now allow to be unmatched. $(A_\rightarrow \neq A_\leftarrow)$

$$A_\rightarrow e^{i(k_\rightarrow x - \omega_\rightarrow t)} + A_\leftarrow e^{i(k_\leftarrow x - \omega_\leftarrow t)} = e^{i(k_\Sigma x - \omega_\Sigma t)} [A_\rightarrow e^{i(k_\Delta x - \omega_\Delta t)} + A_\leftarrow e^{-i(k_\Delta x - \omega_\Delta t)}]$$

Waves have half-sum mean-phase rates $(k_\Sigma, \omega_\Sigma)$ and half-difference group rates $(k_\Delta, \omega_\Delta)$.

$$k_\Sigma = (k_\rightarrow + k_\leftarrow)/2$$
$$\omega_\Sigma = (\omega_\rightarrow + \omega_\leftarrow)/2$$

$$k_\Delta = (k_\rightarrow - k_\leftarrow)/2$$
$$\omega_\Delta = (\omega_\rightarrow - \omega_\leftarrow)/2$$

Also important is amplitude mean $A_\Sigma = (A_\rightarrow + A_\leftarrow)/2$ and half-difference $A_\Delta = (A_\rightarrow - A_\leftarrow)/2$.

Detailed wave motion depends on standing-wave-ratio $SWR$ or the inverse standing-wave-quotient $SWQ$.

$$SWR = \frac{(A_\rightarrow - A_\leftarrow)}{(A_\rightarrow + A_\leftarrow)}$$
$$SWQ = \frac{(A_\rightarrow + A_\leftarrow)}{(A_\rightarrow - A_\leftarrow)}$$
Galloping waves due to unmatched amplitudes

2-CW dynamics has two 1-CW amplitudes $A_\rightarrow$ and $A_\leftarrow$ that we now allow to be unmatched. ($A_\rightarrow \neq A_\leftarrow$)

$$A_\rightarrow e^{i(k_\rightarrow x - \omega_\rightarrow t)} + A_\leftarrow e^{i(k_\leftarrow x - \omega_\leftarrow t)} = e^{i(k_\Sigma x - \omega_\Sigma t)} [A_\rightarrow e^{i(k_\Delta x - \omega_\Delta t)} + A_\leftarrow e^{-i(k_\Delta x - \omega_\Delta t)}]$$

Waves have half-sum mean-phase rates $(k_\Sigma, \omega_\Sigma)$ and half-difference group rates $(k_\Delta, \omega_\Delta)$.

$$k_\Sigma = (k_\rightarrow + k_\leftarrow)/2 \quad k_\Delta = (k_\rightarrow - k_\leftarrow)/2$$
$$\omega_\Sigma = (\omega_\rightarrow + \omega_\leftarrow)/2 \quad \omega_\Delta = (\omega_\rightarrow - \omega_\leftarrow)/2$$

Also important is amplitude mean $A_\Sigma = (A_\rightarrow + A_\leftarrow)/2$ and half-difference $A_\Delta = (A_\rightarrow - A_\leftarrow)/2$.

Detailed wave motion depends on standing-wave-ratio $SWR$ or the inverse standing-wave-quotient $SWQ$.

$$SWR = \frac{(A_\rightarrow - A_\leftarrow)}{(A_\rightarrow + A_\leftarrow)} \quad SWQ = \frac{(A_\rightarrow + A_\leftarrow)}{(A_\rightarrow - A_\leftarrow)}$$

These are analogous to frequency ratios for group velocity $V_{\text{group}} < c$ and its inverse that is phase velocity $V_{\text{phase}} > c$.

$$V_{\text{group}} = \frac{\omega_\Delta}{k_\Delta} = \frac{(\omega_\rightarrow - \omega_\leftarrow)}{(k_\rightarrow - k_\leftarrow)} = c \frac{(\omega_\rightarrow - \omega_\leftarrow)}{(\omega_\rightarrow + \omega_\leftarrow)}$$
$$V_{\text{phase}} = \frac{\omega_\Sigma}{k_\Sigma} = \frac{(\omega_\rightarrow + \omega_\leftarrow)}{(k_\rightarrow + k_\leftarrow)} = c \frac{(\omega_\rightarrow + \omega_\leftarrow)}{(\omega_\rightarrow - \omega_\leftarrow)}$$

$$\frac{V_{\text{group}}}{c} = \frac{1}{V_{\text{phase}}}$$
Fig. 6.1 Monochromatic (1-frequency) 2-CW wave space-time patterns.
1-frequency cases

(c) $E_\rightarrow = 0.5$
$E_\leftarrow = 0.5$

$\omega_\rightarrow = 2c$, $k_\rightarrow = 2$
$\omega_\leftarrow = 2c$, $k_\leftarrow = -2$

$u_{GROUP} = 0$
$u_{PHASE} = \infty$

2-frequency cases

(c) $E_\rightarrow = 0.5$
$E_\leftarrow = 0.5$

$\omega_\rightarrow = 4c$, $k_\rightarrow = 4$
$\omega_\leftarrow = 1c$, $k_\leftarrow = -1$

$u_{GROUP} = 3/5$
$u_{PHASE} = 5/3$

Fig. 6.1 Monochromatic (1-frequency)
2-CW wave space-time patterns.

Fig. 6.2 Dichromatic (2-frequency)
2-CW wave space-time patterns.
Fig. 6.3 (a-g) Elliptic polarization ellipses relate to galloping waves in Fig. 6.1. (h-i) Kepler anomalies.
Fig. 6.3 (a-g) Elliptic polarization ellipses relate to galloping waves in Fig. 6.1. (h-i) Kepler anomalies.

\[ \tan \phi(t) = \frac{y}{x} = \frac{b \sin \omega t}{a \cos \omega t} \]

\[ \text{mean anomaly} \]

\[ \text{eccentric anomaly} \]

\[ \text{Highest speed } v = 5 \text{ at perigee } r = 1 \]

\[ \text{Lowest speed } x = 1 \text{ at apogee } r = 5 \]

\[ \text{SWR} = \frac{b}{a} = \frac{1}{5} \]

\[ \tan \phi(t) = \frac{y}{x} = \frac{b \sin \omega t}{a \cos \omega t} \]

\[ \text{KEPLER ANOMALY RELATIONS} \]

\[ \text{tan} \phi(t) = \text{SWR} \tan \omega t \]
Analogy between wave galloping, Keplarian IHO orbits, and optical polarization

We’ll show wave galloping is analogous to Keplarian orbital motion of angles $\omega \cdot t$ and $\phi$ of orbits.

$$\tan \phi(t) = \frac{b}{a} \tan \omega \cdot t$$
Analogical between wave galloping, Keplarian IHO orbits, and optical polarization.

We’ll show wave galloping is analogous to Keplarian orbital motion of angles \( \omega t \) and \( \phi \) of orbits.

The eccentric anomaly time derivative of \( \phi \) (angular velocity) gallops between \( \omega \cdot b/a \) and \( \omega \cdot a/b \).

\[
\dot{\phi} = \frac{d\phi}{dt} = \omega \cdot \frac{b \sec^2 \omega t}{a + \tan^2 \phi} = \frac{\omega \cdot b \cdot \sec^2 \omega t}{a + \tan^2 \phi} = \frac{\omega \cdot b / a}{\cos^2 \omega t + (b/a)^2 \cdot \sin^2 \omega t}
\]

\[
\dot{\phi} = \omega \cdot \frac{b}{a} \cdot \frac{\sec^2 \omega t}{1 + \tan^2 \phi} = \omega \cdot \frac{b}{a} \cdot \frac{\cos^2 \omega t}{1 + \sin^2 \omega t}
\]

\[
\begin{align*}
\frac{\omega \cdot b / a}{\cos^2 \omega t + (b/a)^2 \cdot \sin^2 \omega t} &= \begin{cases} 
\omega \cdot b / a & \text{for } \omega t = 0, \pi, 2\pi \ldots \\
\omega \cdot a / b & \text{for } \omega t = \pi / 2, 3\pi / 2, \ldots
\end{cases} \\
\end{align*}
\]

We see that:

\[
\frac{\tan \phi(t)}{\frac{\omega}{a} \cdot \cos \omega t} = \frac{\omega}{a} \cdot \tan \omega t
\]

Kepler anomaly relations:

\[
\begin{align*}
x &= a \cos \omega t \\
y &= b \sin \omega t
\end{align*}
\]
Analogies between wave galloping, Keplarian IHO orbits, and optical polarization

We’ll show wave galloping is analogous to Keplarian orbital motion of angles $\omega \cdot t$ and $\phi$ of orbits.

The eccentric anomaly time derivative of $\phi$ (angular velocity) gallops between $\omega \cdot b/a$ and $\omega \cdot a/b$.

\[
\frac{d\phi}{dt} = \omega \cdot \frac{b}{a} \sec^2 \phi = \omega \cdot \frac{b}{a} \frac{\sec^2 \phi}{1 + \tan^2 \phi} = \frac{\omega \cdot b}{a} \frac{\cos^2 \omega t + (b/a)^2 \cdot \sin^2 \omega t}{\cos^2 \omega t + (b/a)^2 \cdot \sin^2 \omega t} = \begin{cases} 
\omega \cdot b/a & \text{for } \omega t = 0, \pi, 2\pi, \ldots \\
\omega \cdot a/b & \omega t = \pi/2, 3\pi/2, \ldots 
\end{cases}
\]

The product of angular moment $r^2$ and $\dot{\phi}$ is orbital momentum, a constant proportional to ellipse area.

\[
r^2 \frac{d\phi}{dt} = \text{constant} = (a^2 \cos^2 \omega t + b^2 \cdot \sin^2 \omega t) \frac{d\phi}{dt} = \omega \cdot ab
\]
Analogy between wave galloping, Keplarian IHO orbits, and optical polarization.

We’ll show wave galloping is analogous to Keplarian orbital motion of angles \( \omega \cdot t\) and \( \phi \) of orbits.

\[
\tan \phi(t) = \frac{b}{a} \tan \omega \cdot t
\]

The eccentric anomaly time derivative of \( \phi \) (angular velocity) gallops between \( \omega \cdot b/a \) and \( \omega \cdot a/b \).

\[
\dot{\phi} = \frac{d\phi}{dt} = \omega \cdot \frac{b \sec^2 \omega t}{a \sec^2 \phi} = \omega \cdot \frac{b \sec^2 \omega t}{a + \tan^2 \phi} = \omega \cdot \frac{b/a}{\cos^2 \omega t + (b/a)^2 \cdot \sin^2 \omega t} = \left\{
\begin{array}{l}
\omega \cdot b/a \quad \text{for} \quad \omega t = 0, \pi, 2\pi, \\
\omega \cdot a/b \quad \omega t = \pi/2, 3\pi/2, 
\end{array}
\right.
\]

The product of angular moment \( r^2 \) and \( \dot{\phi} \) is orbital momentum, a constant proportional to ellipse area.

\[
r^2 \frac{d\phi}{dt} = \text{constant} = (a^2 \cos^2 \omega t + b^2 \cdot \sin^2 \omega t) \frac{d\phi}{dt} = \omega \cdot ab
\]

Consider galloping wave zeros of a monochromatic wave having \( SWR = 1/5 \).

\[
0 = \text{Re} \Psi(x,t) = \text{Re} \left[ A_\rightarrow e^{i(k_0 x \cdot \omega_0 t)} + A_\leftarrow e^{i(-k_0 x \cdot \omega_0 t)} \right] \quad \text{where:} \quad \omega_\rightarrow = \omega_0 = \omega_\leftarrow = ck_0 = -ck_0
\]

\[
0 = A_\rightarrow \left[ \cos k_0 x \cdot \cos \omega_0 t + \sin k_0 x \sin \omega_0 t \right] + A_\leftarrow \left[ \cos k_0 x \cdot \cos \omega_0 t - \sin k_0 x \sin \omega_0 t \right] \\
(A_\rightarrow + A_\leftarrow) \left[ \cos k_0 x \cdot \cos \omega_0 t \right] = - (A_\rightarrow - A_\leftarrow) \left[ \sin k_0 x \sin \omega_0 t \right]
\]

\[
\frac{E_\leftarrow}{E_\rightarrow} = 0.4, \quad \frac{E_\rightarrow}{E_\leftarrow} = 0.6
\]
The product of angular moment $r^2$ and $\dot{\phi}$ is orbital momentum, a constant proportional to ellipse area.

$$r^2 \frac{d\phi}{dt} = \text{constant} = (a^2 \cos^2 \omega t + b^2 \cdot \sin^2 \omega t) \frac{d\phi}{dt} = \omega \cdot ab$$

Consider galloping wave zeros of a monochromatic wave having $SWR = 1/5$.

$$0 = \Re \Psi(x,t) = \Re \left[ A_\rightarrow e^{i(k_0 x - \omega_0 t)} + A_\leftarrow e^{i(-k_0 x - \omega_0 t)} \right] \quad \text{where: } \omega_\rightarrow = \omega_0 = \omega_\leftarrow = c k_0 = -c k_\leftarrow$$

$$0 = A_\rightarrow \left[ \cos k_0 x \cos \omega_0 t + \sin k_0 x \sin \omega_0 t \right] + A_\leftarrow \left[ \cos k_0 x \cos \omega_0 t - \sin k_0 x \sin \omega_0 t \right]$$

$$= (A_\rightarrow + A_\leftarrow) [\cos k_0 x \cos \omega_0 t] - (A_\rightarrow - A_\leftarrow) [\sin k_0 x \sin \omega_0 t]$$

Space $k_0 x$ varies with time $\omega_0 t$ in the same way that eccentric anomaly $\phi$ varies with $\omega t$.

$$\tan k_0 x = -SWR \cdot \cot \omega_0 t = SWR \cdot \tan \omega_0 t$$

where: $\omega_0 = \omega_0 t - \pi / 2$

$$E_\Leftarrow = 0.4, \quad E_\rightarrow = 0.6$$

**Analogy between wave galloping, Keplarian IHO orbits, and optical polarization**

We’ll show wave galloping is analogous to Keplarian orbital motion of angles $\omega t$ and $\phi$ of orbits.

$$\tan \phi(t) = \frac{b}{a} \tan \omega t$$

The eccentric anomaly time derivative of $\dot{\phi}$ (angular velocity) gallops between $\omega \cdot b/a$ and $\omega \cdot a/b$.

$$\dot{\phi} = \frac{d\phi}{dt} = \omega \cdot \frac{b \sec^2 \omega t}{a \sec^2 \phi} = \omega \cdot \frac{b \sec^2 \omega t}{a + \tan^2 \phi} = \frac{\omega \cdot b / a}{\cos^2 \omega t + (b / a)^2 \cdot \sin^2 \omega t} = \begin{cases} \omega \cdot b / a & \text{for: } \omega t = 0, \pi, 2\pi \ldots \\ \omega \cdot a / b & \omega t = \pi / 2, 3\pi / 2, \ldots \end{cases}$$

**Kepler anomaly relations**

$$\tan \phi(t) = \frac{y}{x} = \frac{b \sin \omega t}{a \cos \omega t} = SWR \cdot \tan \omega t$$

**Highest speed $v=5$ at perigee $r=b=1$**

**Lowest speed $v=1$ at apogee $r=a=5$**

**Eccentric anomaly**

**Mean anomaly**

**True anomaly**

$SWR = +0.2$
We’ll show wave galloping is analogous to Keplarian orbital motion of angles $\omega \cdot t$ and $\phi$ of orbits.

The eccentric anomaly time derivative of $\phi$ (angular velocity) gallops between $\omega \cdot b/a$ and $\omega \cdot a/b$.

$$\dot{\phi} = \frac{d\phi}{dt} = \omega \cdot \frac{b}{a} \sec^{2} \frac{\omega t}{a} \sec^{2} \phi = \omega \cdot \frac{b}{a} \cdot \frac{\sec^{2} \phi}{1 + \tan^{2} \phi} = \frac{\omega \cdot b}{a} \cdot \frac{1}{\cos^{2} \frac{\omega t}{a} + \left(\frac{b}{a}\right)^{2} \cdot \sin^{2} \frac{\omega t}{b}}$$

The product of angular moment $r^{2}$ and $\dot{\phi}$ is orbital momentum, a constant proportional to ellipse area.

$$r^{2} \frac{d\phi}{dt} = \text{constant} \Rightarrow \left(\omega^{2} \cos^{2} \omega t + b^{2} \cdot \sin^{2} \omega t\right) \frac{d\phi}{dt} = \omega \cdot ab$$

Consider galloping wave zeros of a monochromatic wave having $SWR = 1/5$.

$$0 = \Re \Psi(x,t) = \Re \left[ A_{\rightarrow} e^{i(k_{0}x - \omega_{0}t)} + A_{\leftarrow} e^{i(-k_{0}x - \omega_{0}t)} \right]$$

where: $\omega_{\rightarrow} = \omega_{\leftarrow} = c k_{0} = -c k_{\leftarrow}$

$$0 = A_{\rightarrow} \left[ \cos k_{0}x \cos \omega_{0}t + \sin k_{0}x \sin \omega_{0}t \right] + A_{\leftarrow} \left[ \cos k_{0}x \cos \omega_{0}t - \sin k_{0}x \sin \omega_{0}t \right]$$

$$\left( A_{\rightarrow} + A_{\leftarrow} \right) \left[ \cos k_{0}x \cos \omega_{0}t \right] = -\left( A_{\rightarrow} - A_{\leftarrow} \right) \left[ \sin k_{0}x \sin \omega_{0}t \right]$$

Space $k_{0}x$ varies with time $\omega_{0}t$ in the same way that eccentric anomaly $\phi$ varies with $\omega \cdot t$.

$$\tan k_{0}x = -SWR \cdot \cot \omega_{0}t = SWR \cdot \tan \omega_{0}\bar{t}$$

where: $\omega_{0}\bar{t} = \omega_{0}t - \pi / 2$

Speed of galloping wave zeros is the time derivative of root location $x$ in units of light velocity $c$.

$$\frac{dx}{dt} = c \cdot SWR \cdot \frac{\sec^{2} \omega_{0}\bar{t}}{\sec^{2} k_{0}x} = \frac{c \cdot SWR}{\cos^{2} \omega_{0}\bar{t} + SWR^{2} \cdot \sin^{2} \omega_{0}\bar{t}}$$

$$\left( c \cdot SWR \right) \text{ for: } \bar{t} = 0, \pi, 2\pi$$

$$c \cdot SWQ \quad \bar{t} = \pi / 2, 3\pi / 2$$
Wave-Zero Speed-Limits

Standing Wave Ratio \( SWR \) and Quotient \( SWQ \)

\[
SWR = \frac{E_\rightarrow - E_\leftarrow}{E_\rightarrow + E_\leftarrow} = 1/SWQ
\]

\( SWR = 1/5 \)

\( SWQ = 5 \)

\( E_\leftarrow = 0.5, \quad E_\rightarrow = 0.6 \)

Wave zeros "resting" at \((1/5)c\)

Wave zeros "galloping" at \(5c\)

Wave zeros "standing" at \(0\)-speed
Wave-Zero Speed-Limits

Standing Wave Ratio SWR and Quotient SWQ

\[ SWR = \frac{(E_{-} - E_{\leftrightarrow})}{(E_{-} + E_{\leftrightarrow})} = 1/\text{SWQ} \]

**SWR = 1/5**

SWR is 1 to 5

SWQ = 5

Wave zeros "resting" at (1/5)c

Wave zeros "galloping" at 5c

\[ \omega_{-} = 2c, \quad \omega_{\leftrightarrow} = 2c \]

\[ k_{-} = 2, \quad k_{\leftrightarrow} = -2 \]

\[ u_{\text{GROUP}} = 0, \quad u_{\text{PHASE}} = \infty \]

**E \leftarrow = 0.5, E \rightarrow = 0.5**

**SWR = 1** is analogous to (1,i)

Right Circular Polarization

**SWR = 0** is analogous to (1,0)

x-Plane Linear Polarization

**SWR = -1** is analogous to (1,-i)

Left Circular Polarization
Waves that go back in time - The Feynman-Wheeler Switchback

Minkowski Zero-Grids are Spacetime Switchbacks for
\(-u_{\text{GROUP}} < S\text{WR} < 0\)

Group-zero speed
\(u_{\text{GROUP}} = \frac{3c}{5}\)

Phase zero speed
\(u_{\text{PHASE}} = \frac{3c}{5}\)

Phase “anti-zero” going “back-in-time”

Wave zero-anti-zero annihilation and creation occur together at the same spacetime point for \(S\text{WR}=0\)

Wave zero-anti-zero annihilation and creation occur separately at different spacetime points for \(-u_{\text{GROUP}} < S\text{WR} < 0\)
At High Speed 2-CW Modes Look More Like 1-CW Beams

Various combinations of opposite-\(k\) 1-CW beams occur with open boundaries. 

\[ E_{\text{-wave}} : E = E \rightarrow e^{i(k \cdot x - \omega t)} + E \leftarrow e^{i(k \cdot x - \omega t)} \] 

is related to \( \Psi \)-wave: 

\[ \Psi = \Psi \rightarrow e^{i(k \cdot x - \omega t)} + \Psi \leftarrow e^{i(k \cdot x - \omega t)} \]

Standing Wave Ratio (or Quotient) 

\[ SWR = \frac{(E \rightarrow - E \leftarrow)}{(E \rightarrow + E \leftarrow)} = 1/SWQ \]

Wave Group (or Phase) Velocity 

\[ u_{\text{GROUP}} / c = (\omega \rightarrow - \omega \leftarrow) / (\omega \rightarrow + \omega \leftarrow) = c / u_{\text{PHASE}} \]

1-frequency case: \( \omega \rightarrow = 2c, k \rightarrow = 2, \omega \leftarrow = 2c, k \leftarrow = -2 \) gives: \( u_{\text{GROUP}} = 0 \) and \( u_{\text{PHASE}} = \infty \)

2-frequency case: \( \omega \rightarrow = 4c, k \rightarrow = 4, \omega \leftarrow = 1c, k \leftarrow = -1 \) gives: \( u_{\text{GROUP}} / c = 3/5 \) and \( u_{\text{PHASE}} / c = 5/3 \)
1st Quantization: Quantizing phase variables $\omega$ and $k$

Understanding how quantum transitions require “mixed-up” states

Closed cavity vs Ring cavity
Quantized $\omega$ and $k$  

Counting wave kink numbers

If everything is made of waves then we expect quantization of everything because waves only thrive if integral numbers $n$ of their “kinks” fit into whatever structure (box, ring, etc.) they’re supposed to live. The numbers $n$ are called quantum numbers.

_OK_ box quantum numbers:  

$n=1$  

$n=2$  

$n=3$  

$n=4$

(+ integers only)

Some _NOT OK_ numbers:  

$n=0.67$  

too fat!

$n=1.7$  

too thin!

$n=2.59$  

wrong color again!

$n=4$  

misfits...  

...not tolerated!

NOTE: We’re using “false-color” here.

This doesn’t mean a system’s energy can’t vary _continuously_ between “OK” values $E_1, E_2, E_3, E_4,$...
**Quantized \( \omega \) and \( k \)**

Counting wave kink numbers

If everything is made of waves then we expect quantization of everything because waves only thrive if integral numbers \( n \) of their “kinks” fit into whatever structure (box, ring, etc.) they’re supposed to live. The numbers \( n \) are called quantum numbers.

**OK box quantum numbers:**

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Some **NOT OK numbers:**

- \( n=0.67 \) too fat!
- \( n=1.7 \) too thin!
- \( n=2.59 \) wrong color again!
- \( n=4 \) misfits...
- \( n=4 \) ...not tolerated!

**NOTE:** We’re using “false-color” here.

This doesn’t mean a system’s energy can’t vary continuously between “OK” values \( E_1, E_2, E_3, E_4, \ldots \). In fact its state can be a linear combination of any of the “OK” waves \( |E_1>, |E_2>, |E_3>, |E_4>, \ldots \).
Quantized $\omega$ and $k$ Counting wave kink numbers

If everything is made of waves then we expect quantization of everything because waves only thrive if integral numbers $n$ of their “kinks” fit into whatever structure (box, ring, etc.) they’re supposed to live. The numbers $n$ are called quantum numbers.

**OK box quantum numbers:**  
\begin{align*}
& n=1 \\
& n=2 \\
& n=3 \\
& n=4
\end{align*}

(+ integers only)

Some **NOT OK numbers:**  
\begin{align*}
& n=0.67 \\
& n=1.7 \\
& n=2.59 \\
& n=4
\end{align*}

---

This doesn’t mean a system’s energy can’t vary continuously between “OK” values $E_1$, $E_2$, $E_3$, $E_4$,.... In fact its state can be a linear combination of any of the “OK” waves $|E_1\rangle$, $|E_2\rangle$, $|E_3\rangle$, $|E_4\rangle$,.... That’s the only way you get any light in or out of the system to “see” it.

\[ |E_4\rangle \]

**frequency** $\hbar \omega_{32} = E_3 - E_2$

**frequency** $\hbar \omega_{21} = E_2 - E_1$

---

NOTE: We’re using “false-color” here.
Quantized $\omega$ and $k$  

_Counting wave kink numbers_

If everything is made of waves then we expect _quantization_ of everything because waves only thrive if _integral_ numbers $n$ of their “kinks” fit into whatever structure (box, ring, etc.) they’re supposed to live. The numbers $n$ are called _quantum numbers_.

_OK box quantum numbers:_  
$n=1$  $n=2$  $n=3$  $n=4$

(+ _integers only_)  

_Some NOT OK numbers:_  
$n=0.67$ too fat!  
$n=1.7$ too thin!  
$n=2.59$ wrong color again!  
$n=4$  

Misfits... ...not tolerated!

_NOTE: We’re using “false-color” here._

This doesn’t mean a system’s energy can’t vary _continuously_ between “OK” values $E_1$, $E_2$, $E_3$, $E_4$, …  

_In fact its state can be a linear combination of any of the “OK” waves $|E_1\rangle$, $|E_2\rangle$, $|E_3\rangle$, $|E_4\rangle$, …_  

_That’s the only way you get any light in or out of the system to “see” it._

$|E_4\rangle$  

These _eigenstates_ are the only ways the system can “play dead”…  
... “sleep with the fishes”…

_frequency $\omega_{32} = (E_3-E_2) / \hbar$  

_frequency $\omega_{21} = (E_2-E_1) / \hbar$
Quantized $\omega$ and $k$  

Counting wave kink numbers
If everything is made of waves then we expect quantization of everything because waves only thrive if integral numbers $n$ of their “kinks” fit into whatever structure (box, ring, etc.) they’re supposed to live. The numbers $n$ are called quantum numbers.

OK box quantum numbers: $n=1 \quad n=2 \quad n=3 \quad n=4$

$ (+ \text{ integers only})$

Some NOT OK numbers: $n=0.67$ too fat! $n=1.7$ too thin! $n=2.59$ wrong color again! $n=4$ misfits... $n=4$ ...not tolerated!

NOTE: We’re using “false-color” here.

Rings tolerate a zero (kinkless) quantum wave but require $\pm$integral wave number.

OK ring quantum numbers: $m=0 \quad m=\pm 1 \quad m=\pm 2 \quad m=3$

$(\pm \text{ integral number of wavelengths})$

Bohr’s models of atomic spectra (1913-1923) are beginnings of quantum wave mechanics built on Planck-Einstein (1900-1905) relation $E=h\nu$. DeBroglie relation $p=h/\lambda$ comes around 1923.
2nd Quantization: Quantizing amplitudes ("photons", "vibrons", and "what-ever-ons")

Introducing coherent states (What lasers use)

Analogy with (ω,k) wave packets

Wave coordinates need coherence
Quantized Amplitude Counting “photon” number

Planck’s relation $E=nh\nu$ began as a tenative axiom to explain low-T light. Then he tried to disavow it! Einstein picked it up in his 1905 paper. Since then its use has grown enormously and continues to amaze, amuse (or bewilder) all who study it. A current view is that it represents the quantization of optical field amplitude. We picture this below as $N$-photon wave states for each box-mode of $m$ wave kinks.

Quantum field definitions have been called “2nd quantization” or “wave-waves”

NOTE: We’re using “false-color” here.

These are the fundamental “zero-point” or “vacuum” levels