

Lecture 26.

Introduction to Relativistic Classical and Quantum Mechanics

(Ch. 2-5 of Unit 2 4.03.12)

Group vs. phase velocity and tangent contacts (Includes Lecture 25 review)

Reviewing “Sin-Tan Rosetta” geometry

How optical CW group and phase properties give relativistic mechanics

Three kinds of mass (Einstein rest mass, Galilean momentum mass, Newtonian inertial mass)

What’s the matter with light?

Bohr-Schrodinger (BS) approximation throws out Mc^2

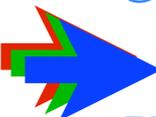
Deriving relativistic quantum Lagrangian-Hamiltonian relations

Feynman’s flying clock and phase minimization

Geometry of relativistic mechanics

Relativistic Classical and Quantum Mechanics

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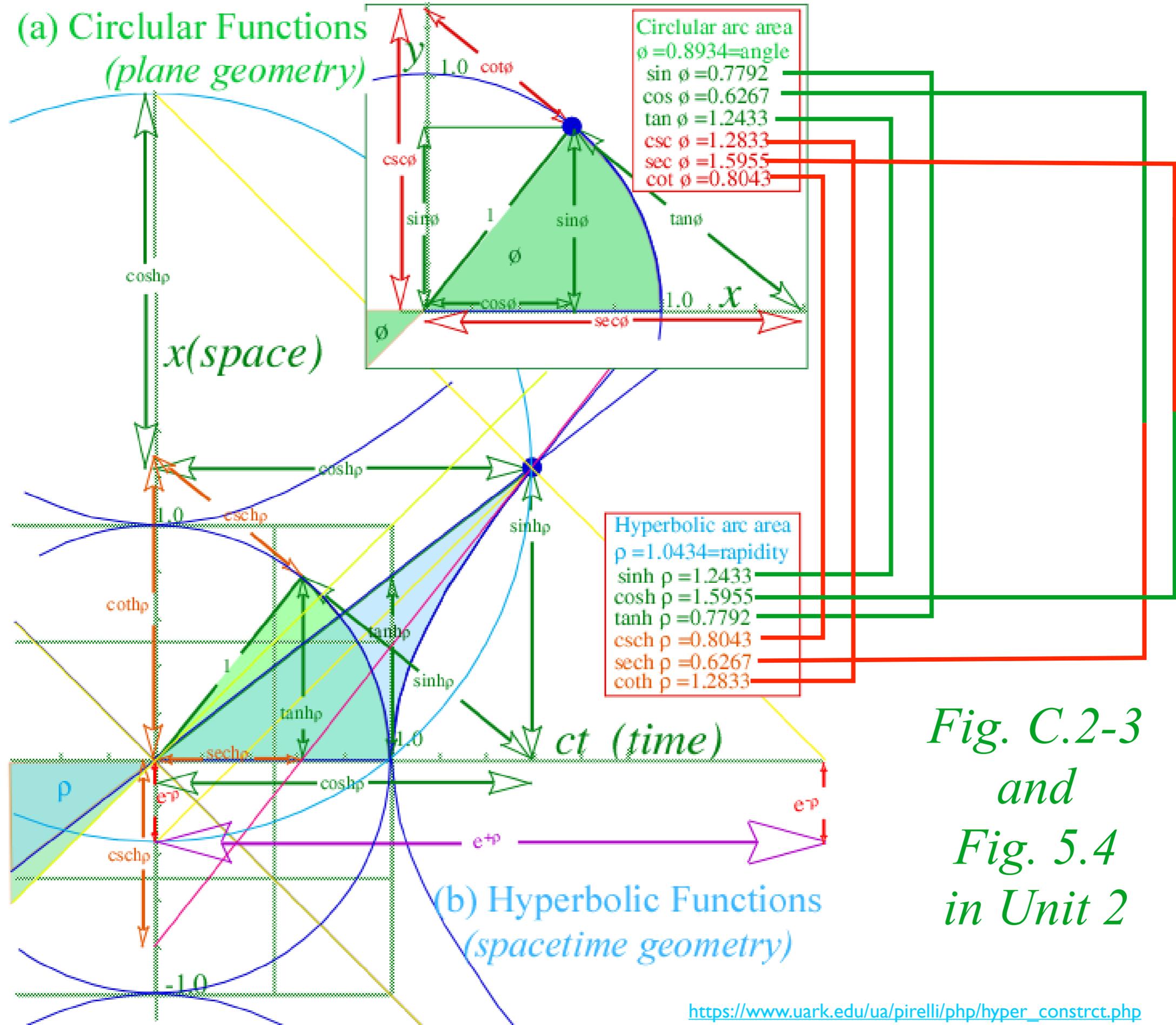
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Hyperbolic Function Values

Arc Area = $\rho = 1.1758$ {radii²}

$\sinh \rho = 1.4660$

$\cosh \rho = 1.7746$

$\tanh \rho = 0.8261$

$\operatorname{csch} \rho = 0.6821$

$\operatorname{sech} \rho = 0.5635$

$\operatorname{coth} \rho = 1.2105$

$\exp(\rho) = 3.2406$

$\exp(-\rho) = 0.3086$

Circular Function Values

$m\angle(\sigma) = 0.9722$ {radians}

Arc length(σ) = 0.9722 {radii}

Section Area(σ) = 0.9722 {radii²}

$\sin \sigma = 0.8261$

$\cos \sigma = 0.5635$

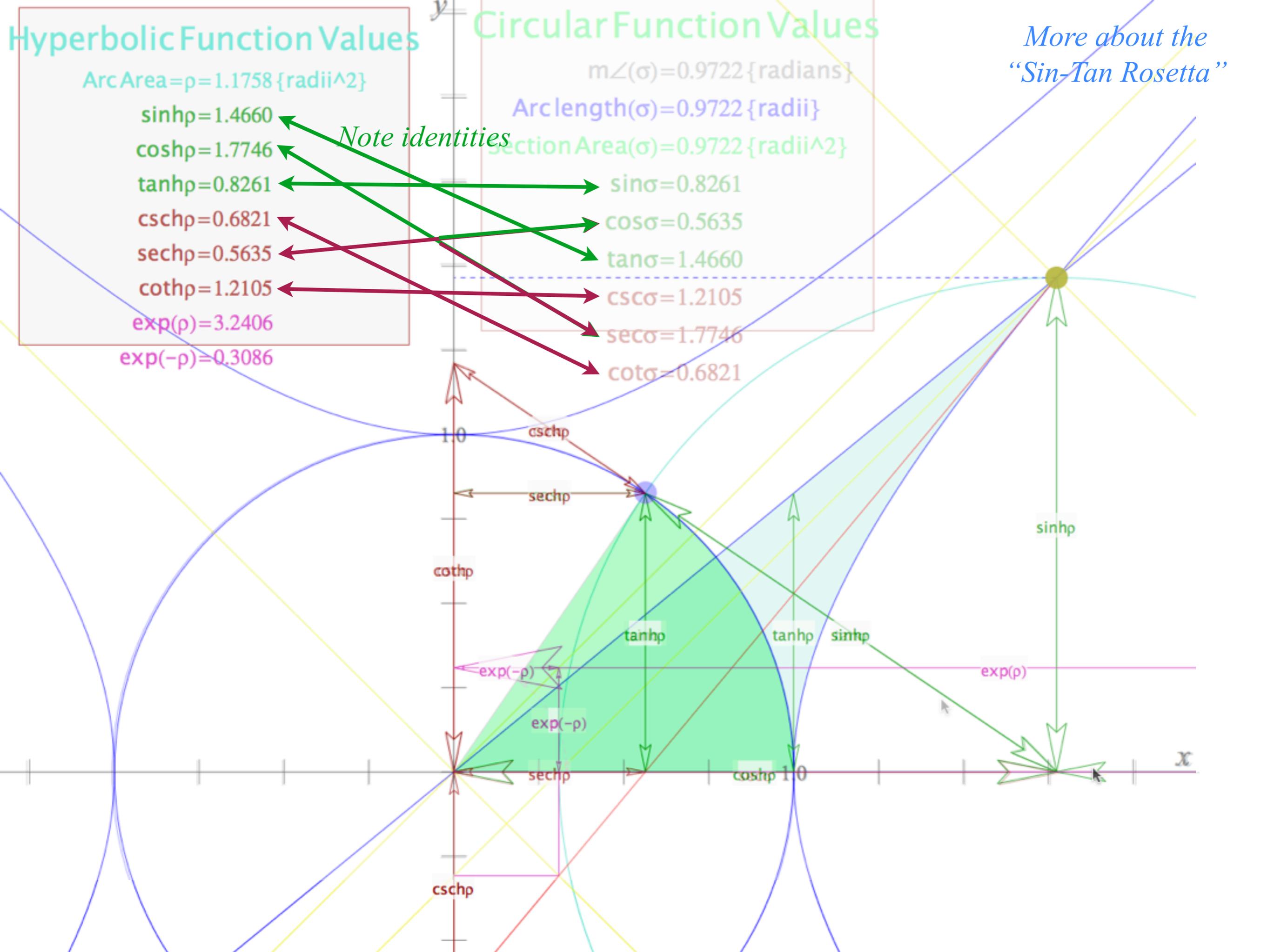
$\tan \sigma = 1.4660$

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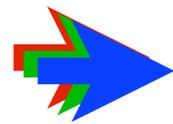
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More about the "Sin-Tan Rosetta"



Relativistic Classical and Quantum Mechanics



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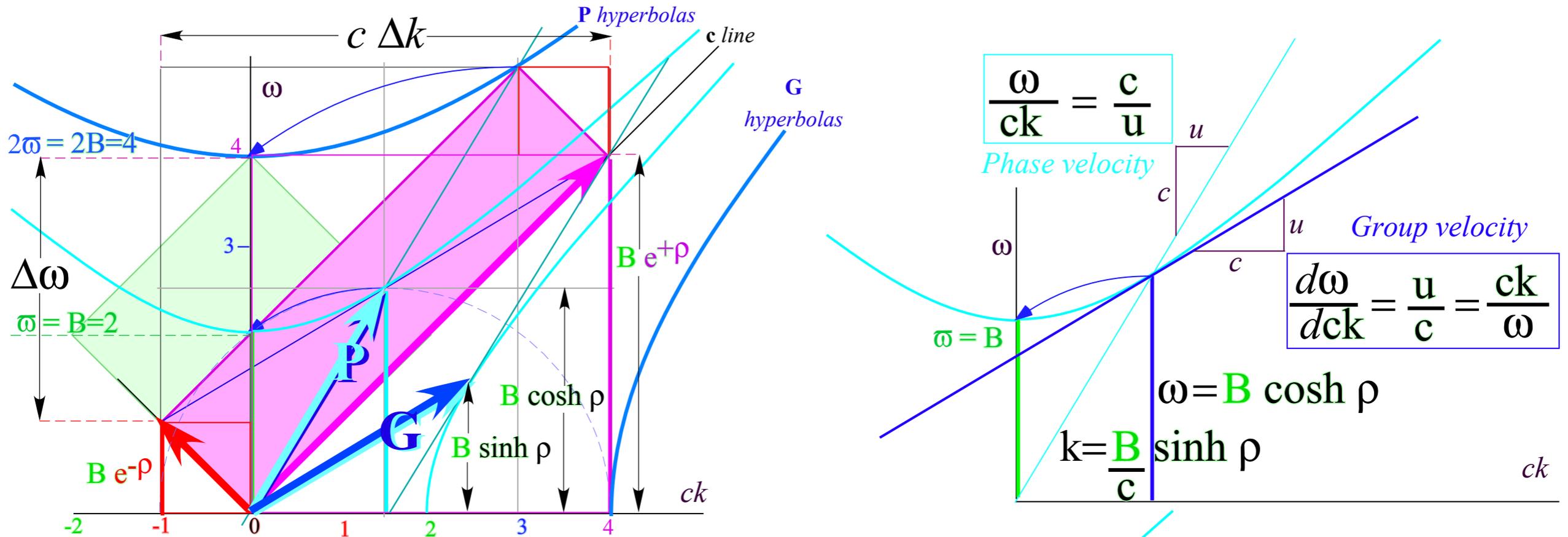
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Group vs. phase velocity and tangent contacts

Group velocity u and phase velocity c^2/u are hyperbolic tangent slopes

(From Fig. 2.3.4)

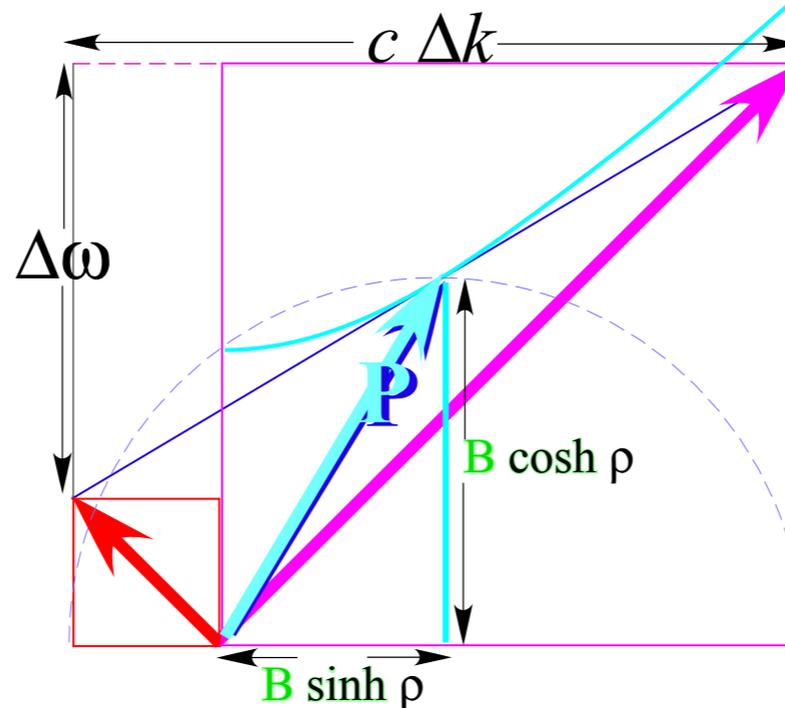


Rare but important case where

$$\frac{d\omega}{dk} = \frac{\Delta\omega}{\Delta k}$$

with **LARGE** Δk
(not infinitesimal)

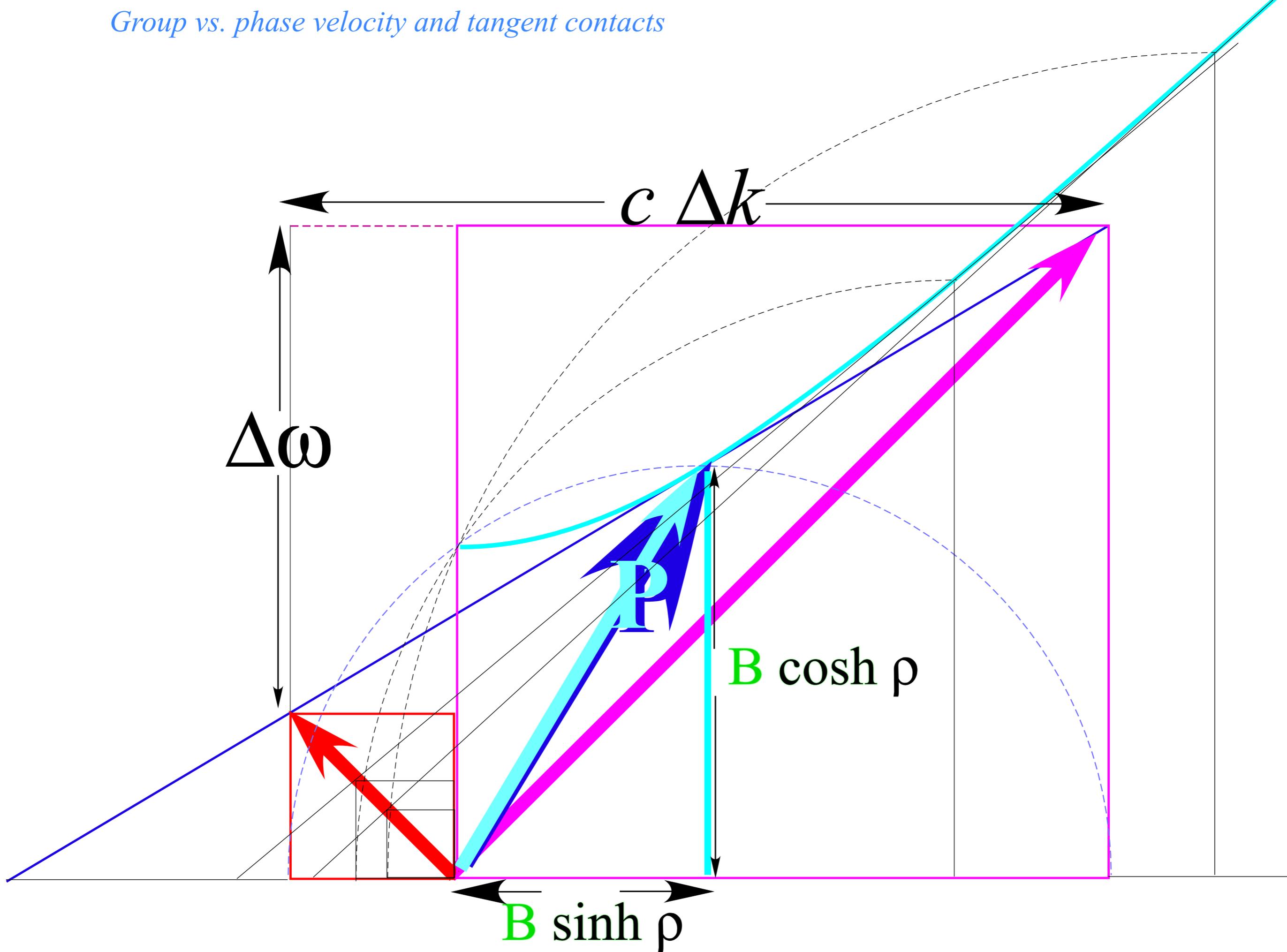
Relativistic
group wave
speed $u = c \tanh \rho$

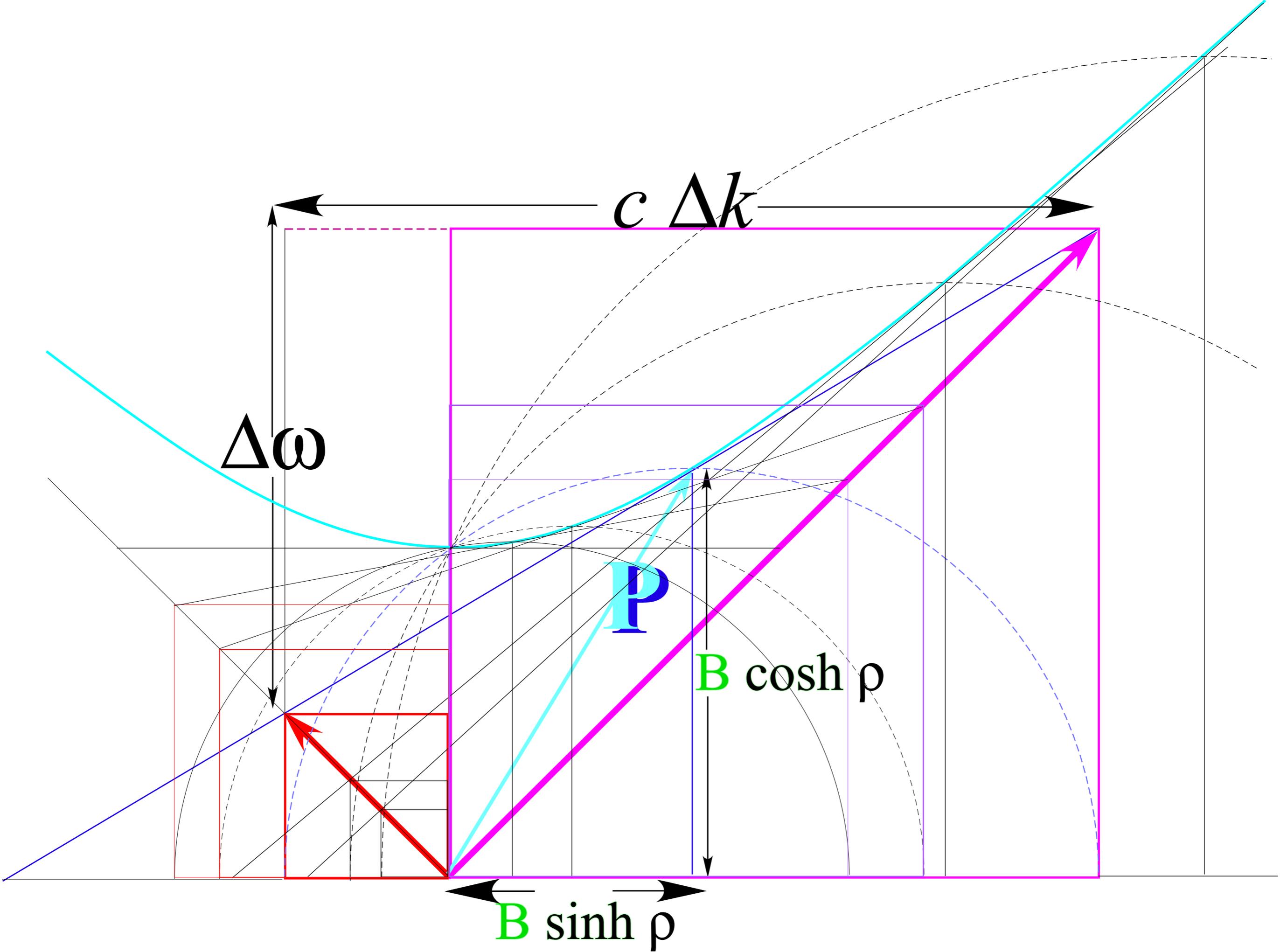


Newtonian speed $u \sim c\rho$
Low speed approximation
Rapidity ρ approaches u/c

Lecture 25 ended here

Group vs. phase velocity and tangent contacts

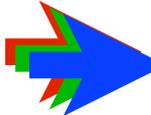




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Start with low speed approximations : $\omega = B \cosh \rho = B(1 + \frac{1}{2} \rho^2 + \dots)$ where: $\rho \approx \frac{u}{c}$

CW Axioms (“All colors go c.” and “ $r=1/b$ ”) imply hyperbolic dispersion then mechanics of matter

$$\omega = B \cosh \rho \cong B + \frac{1}{2} \frac{B}{c^2} u^2$$

These follow from
CW axioms

$$k = \frac{B}{c} \sinh \rho \cong \frac{B}{c^2} u$$

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$$k = \frac{B}{c} \sinh \rho \approx \frac{B}{c^2} u$$

$$E = \text{constant} + \frac{1}{2} M u^2$$

(Newton's energy)

$$p = M u$$

(Galileo's momentum)

So 2-CW-light frequency ω is like **energy** E while k -number is like **momentum** p ,

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So 2-CW-light frequency ω is like **energy** E while k -number is like momentum p , implies Planck's $E = s \cdot \omega$ scaling with factors: $s = \hbar = s$ equal to DeBroglie's $p = s \cdot k$.

$$E = s \omega = s B \cosh \rho \approx s B + \frac{1}{2} \frac{s B}{c^2} u^2$$

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$$p = s k = \frac{s B}{c} \sinh \rho \approx \frac{s B}{c^2} u$$

Both relations imply: $M = \frac{s B}{c^2}$ giving a (famous) *rest energy constant* : $s B = M c^2$

This then gives the famous *Einstein energy* E and also the *Einstein momentum* p

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$$E = s \omega = M c^2 \cosh \rho \cong M c^2 + \frac{1}{2} M u^2$$

$$p = s k = M c \sinh \rho \cong M u$$

$$= \frac{M c^2}{\sqrt{1 - u^2/c^2}}$$

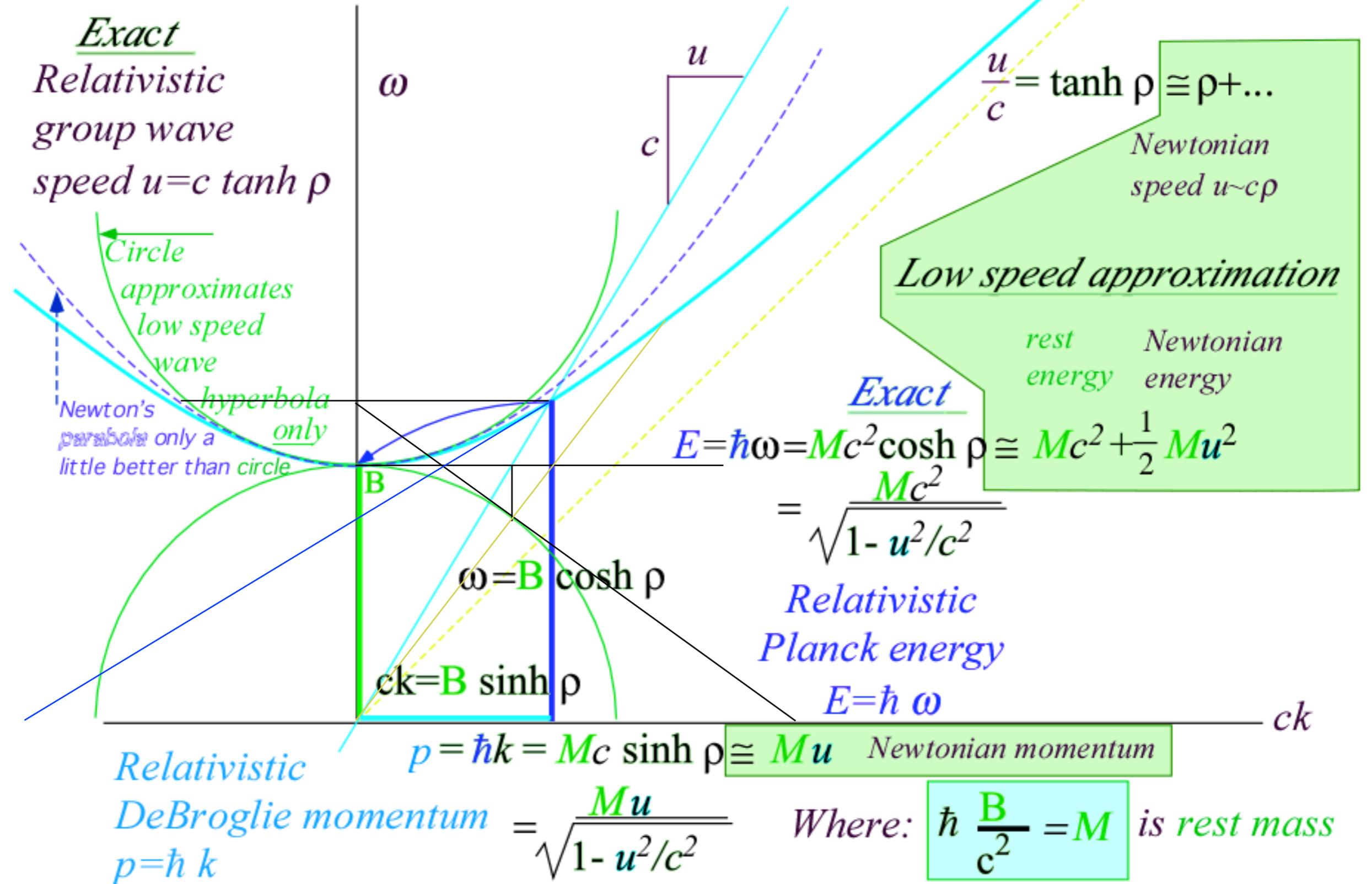
$$= \frac{M u}{\sqrt{1 - u^2/c^2}}$$

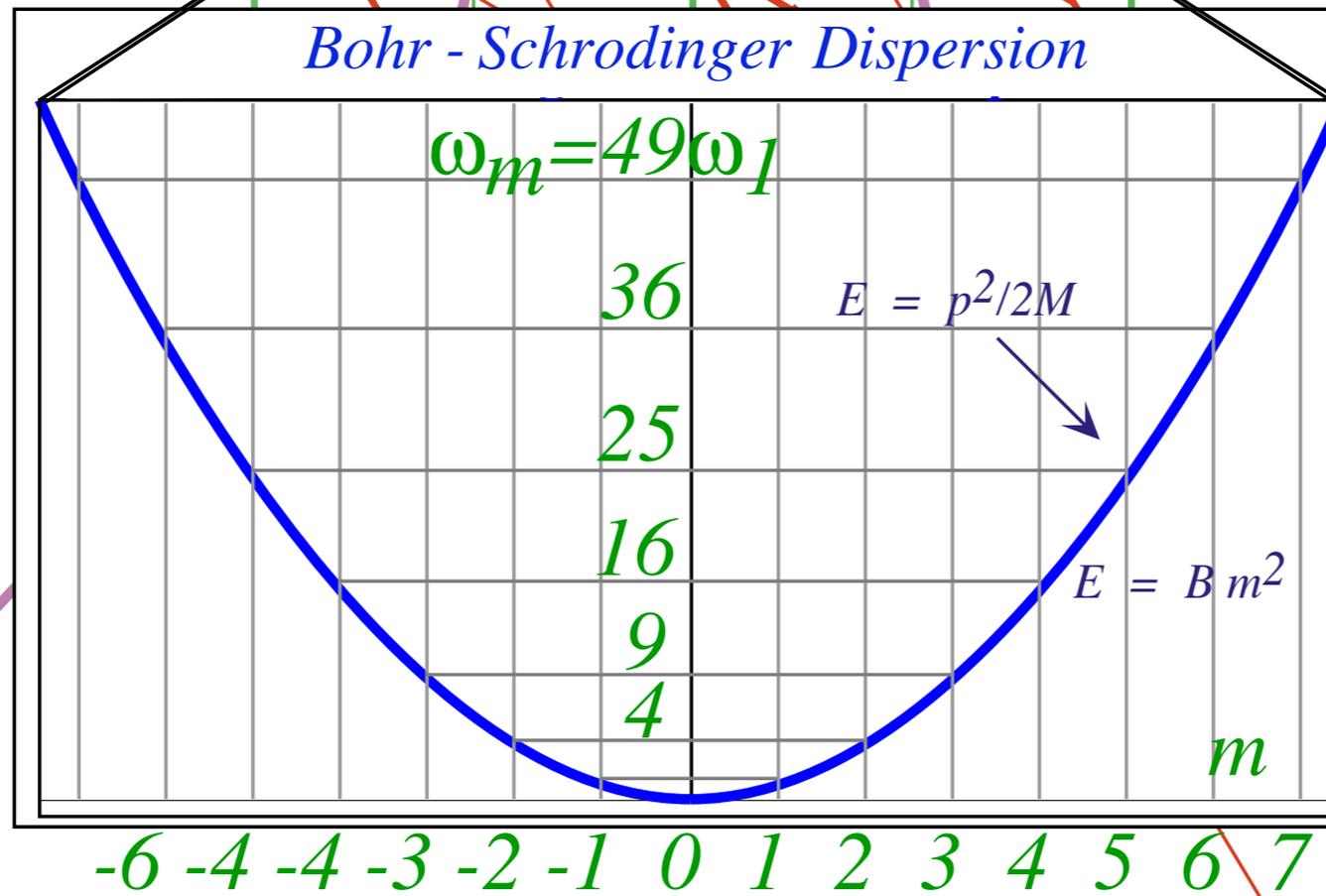
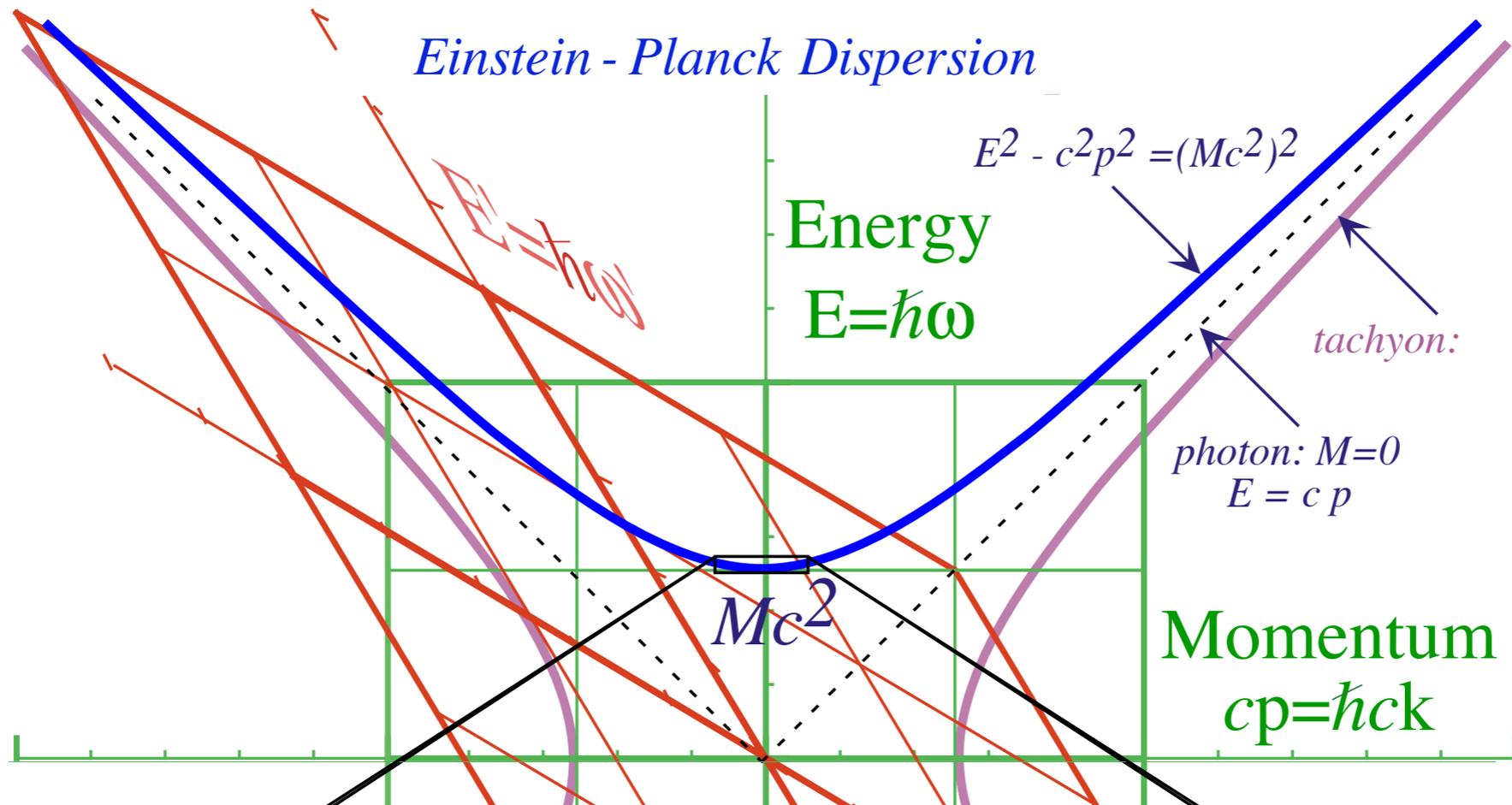
Rest energy ($u=0$): $\hbar \mathbf{B} = M c^2$

Rest momentum ($u=0$): $p=0$

Scale factors determined by experiment
Planck's constant
 $s = \hbar = 1.054572 \cdot 10^{-34} \text{ Joule} \cdot \text{s}$
 $h = 6.626069 \cdot 10^{-34} \text{ J} \cdot \text{s} = 2\pi \hbar$

Summary of geometry ω -vs- ck or E -vs- cp relations with velocity u or rapidity ρ





Relativistic Classical and Quantum Mechanics

Group vs. phase velocity and tangent contacts

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 *Three kinds of mass (Einstein rest mass, Galilean momentum mass, Newtonian inertial mass)*

What’s the matter with light?

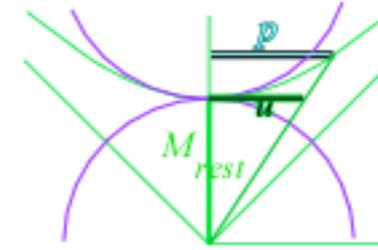
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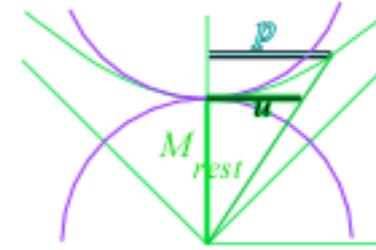
What's the Matter With Light? *Three definitions of optical mass*



1. Rest mass $M_N = h\nu_N/c^2$ based on Planck's law $E = h\nu_N = N h\nu_1$

Rest mass: $M_{rest} = E/c^2 = h\nu_N/c^2$ (Is invariant)

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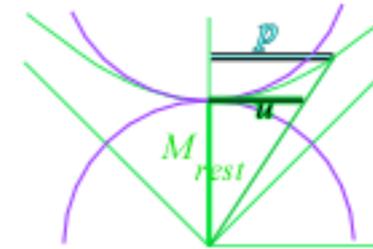
$$\text{Rest mass: } M_{rest} = E/c^2 = h\nu_N/c^2 \quad (\text{Is invariant})$$

2. Momentum mass is defined by Galileo's old formula $p = Mu$ with newer forms for momentum $p = M_{rest} u \cosh \rho$ $\rho = M_{rest} u / (1 - u^2/c^2)^{1/2}$ and group velocity $u = d\omega/dk$. It is the ratio p/u of momentum to velocity.

$$\text{Momentum mass: } M_{momentum} = p/u = M_{rest} \cosh \rho \quad (\text{Not invariant})$$

$$= M_{rest} / (1 - u^2/c^2)^{1/2}$$

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3. *Effective mass* is defined by Newton's old formula $F = Ma$ with newer forms for $F = dp/dt = \hbar dk/dt$ and $a = du/dt =$ to give $F/a = (\hbar dk/dt)(dt/du) = \hbar dk/du = \hbar/(du/dk)$. It is the ratio F/a of *change of momentum* to the *change of velocity*,

$$\text{Effective mass: } M_{effective} = \hbar/(du/dk) = \hbar/(d^2\omega/dk^2) \quad (\text{Not invariant})$$

$$= M_{rest} \cosh^3 \rho = M_{rest} / (1 - u^2/c^2)^{3/2}$$

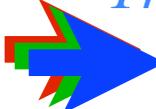
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Three kinds of mass for photon γ in CW relativistic theory

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$$M_{rest} = \frac{\hbar\omega_{proper}}{c^2}$$

(2) Galilean momentum mass

$$M_{mom} = p/u = \frac{\hbar k}{\frac{d\omega}{dk}}$$

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Equations (4.11) in Unit 2

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Equations (4.11) in Unit 2

A 2-CW 600THz cavity has zero total momentum p , but each photon adds a tiny mass M_γ to it.

$$M_\gamma = \hbar\omega/c^2 = \omega (1.2 \cdot 10^{-51}) kg \cdot s = 4.5 \cdot 10^{-36} kg \quad (\text{for: } \omega = 2\pi \cdot 600 \text{THz})$$

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A 1-CW state has no rest mass, but 1-photon momentum is a non-zero value $p_\gamma = M_\gamma c$. (*Galilean revenge II.*)

$$p_\gamma = \hbar k = \hbar\omega/c = \omega (4.5 \cdot 10^{-43}) \text{kg} \cdot \text{m} = 1.7 \cdot 10^{-27} \text{kg} \cdot \text{m} \cdot \text{s}^{-1} \quad (\text{for: } \omega = 2\pi \cdot 600 \text{THz})$$

Relativistic Classical and Quantum Mechanics

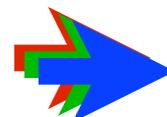
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$$E = \frac{Mc^2}{\sqrt{1 - u^2/c^2}} = Mc^2 \cosh \rho = Mc^2 \sqrt{1 + \sinh^2 \rho} = \sqrt{(Mc^2)^2 + (cp)^2}$$

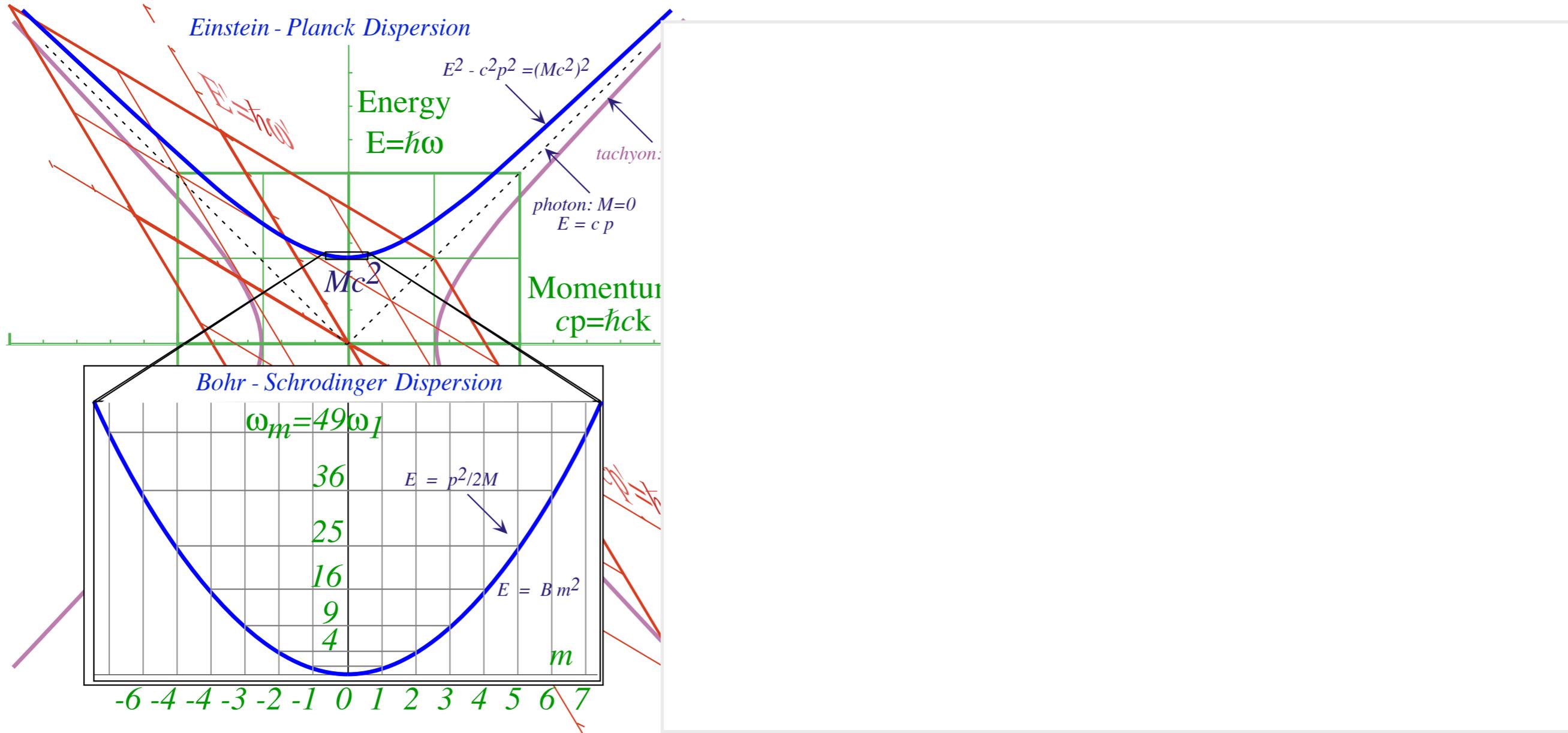
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The BS claim: may shift energy origin ($E=Mc^2, cp=0$) to ($E=0, cp=0$). (*Frequency is relative!*)

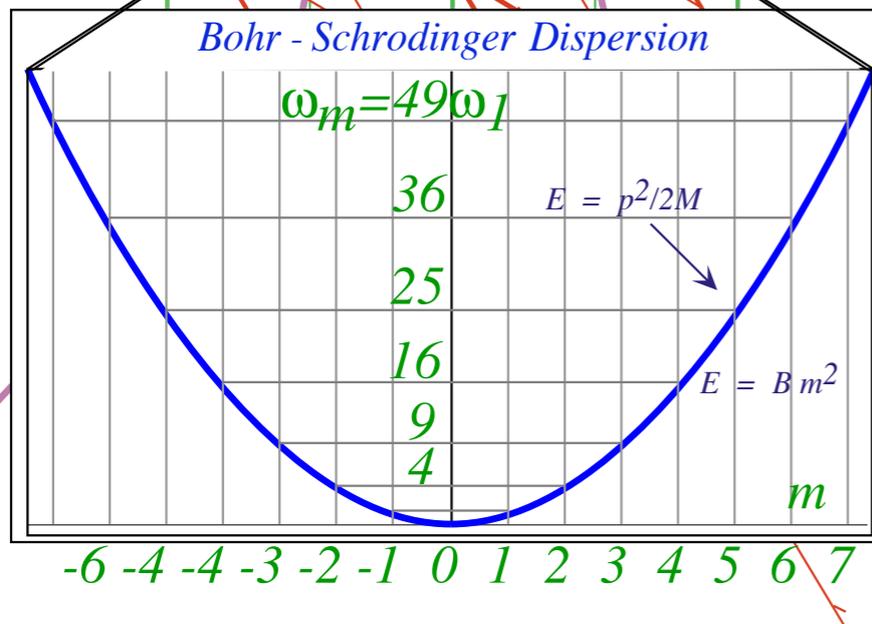
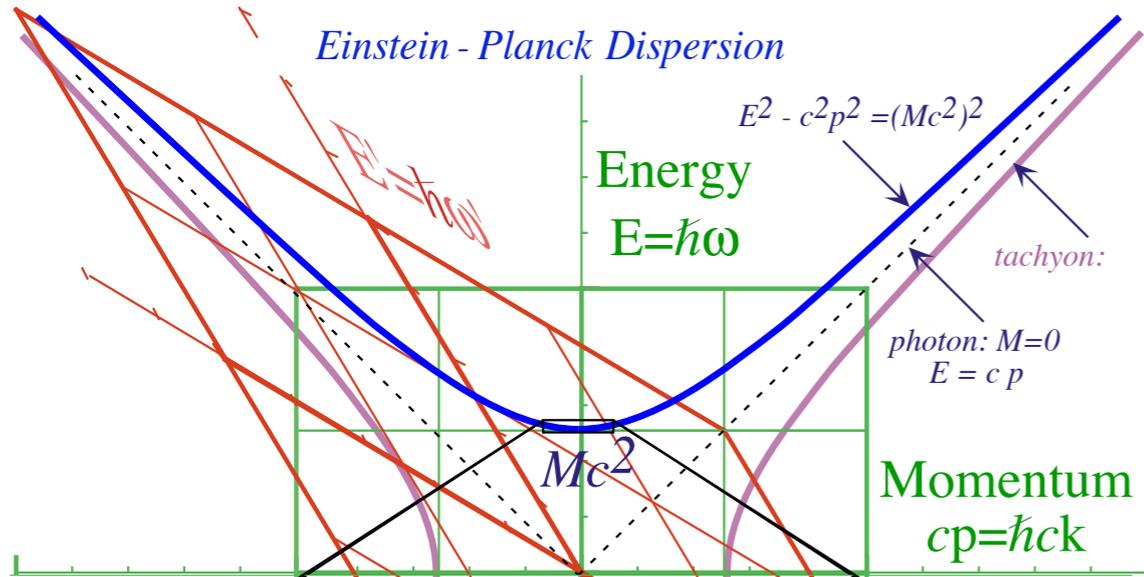


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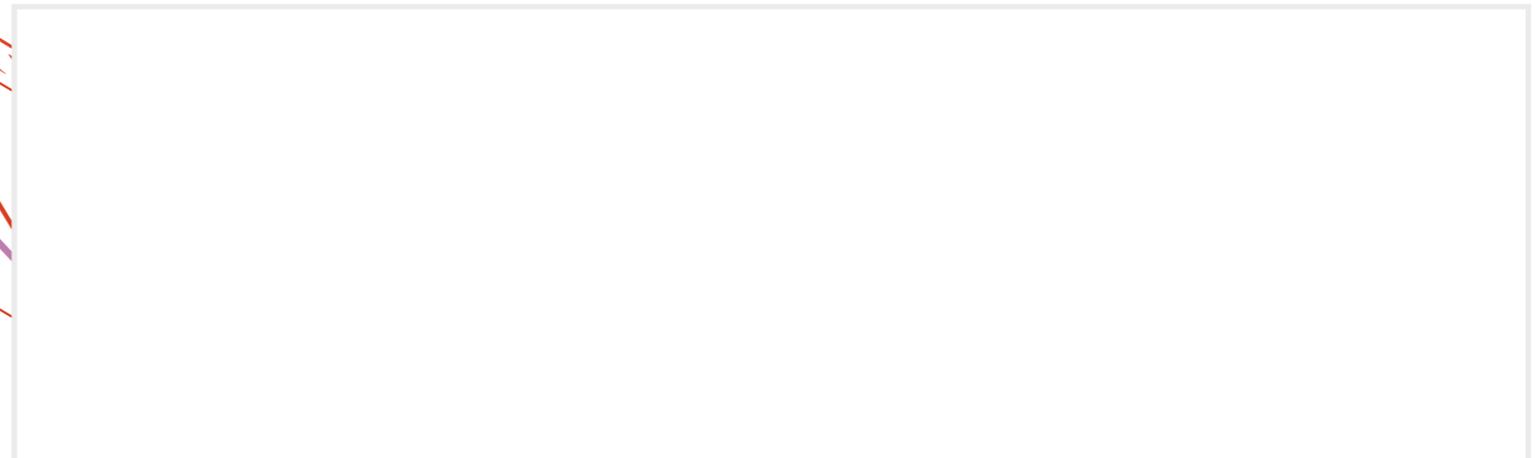
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Group velocity $u = V_{group} = \frac{d\omega}{dk}$ is a differential quantity unaffected by origin shift.

But, Phase velocity $\frac{\omega}{k} = V_{phase}$ is greatly reduced by deleting Mc^2 from $E = \hbar\omega$.

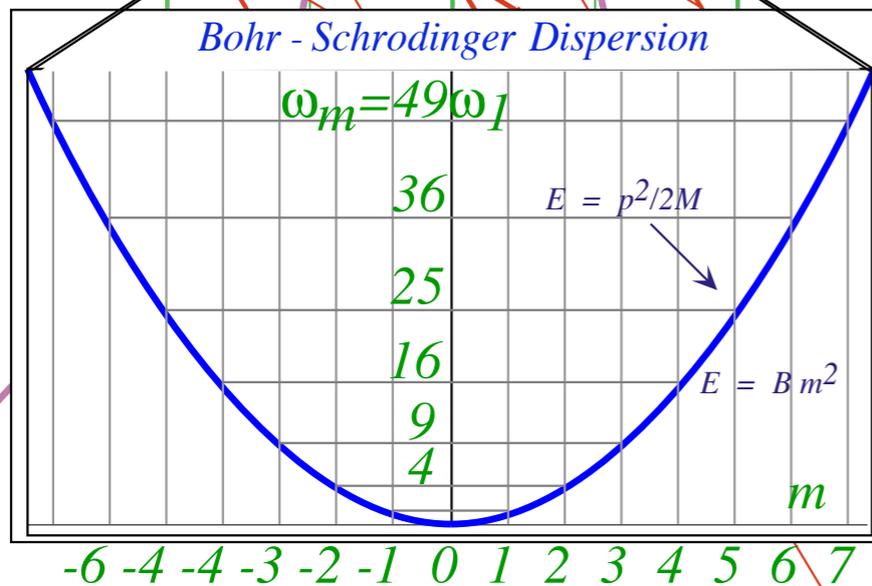
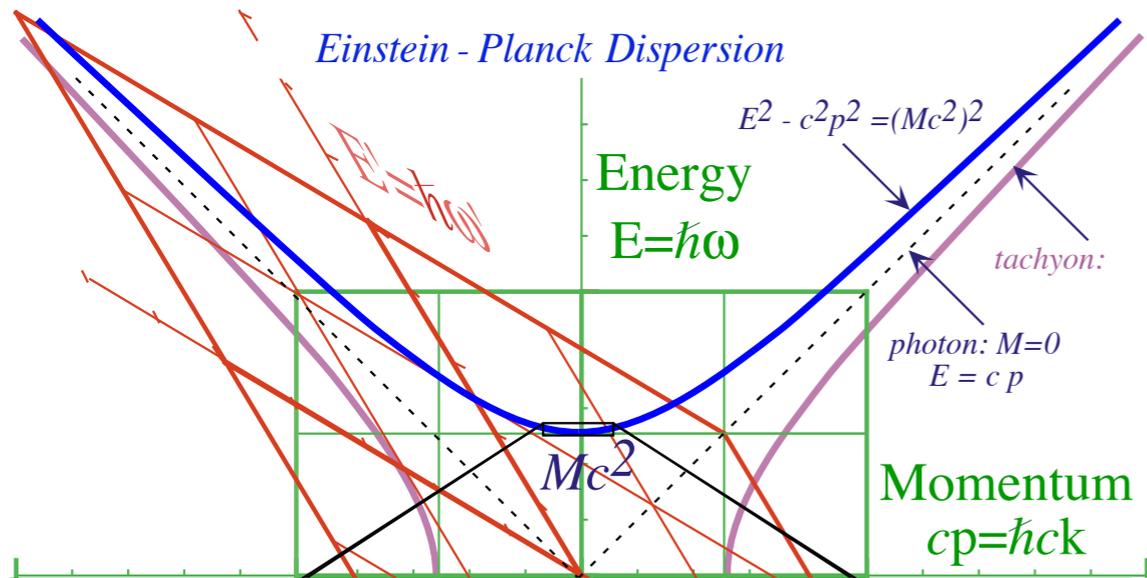


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Group velocity $u = V_{group} = \frac{d\omega}{dk}$ is a differential quantity unaffected by origin shift.

But, Phase velocity $\frac{\omega}{k} = V_{phase}$ is greatly reduced by deleting Mc^2 from $E = \hbar\omega$.

It slows from $V_{phase} = c^2/u$ to a sedate sub-luminal speed of $V_{group}/2$.

$$\omega_{BS}(k) = \frac{k^2}{2M} \quad \text{gives:} \quad V_{phase} = \frac{\omega_{BS}}{k} = \frac{k}{2M}$$

$$\text{and:} \quad V_{group} = \frac{d\omega_{BS}}{dk} = \frac{k}{M}$$

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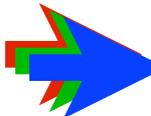
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Deriving relativistic quantum Lagrangian-Hamiltonian relations

Start with phase Φ and set $k=0$ to get product of proper frequency $\mu = Mc^2/\hbar$ and proper time τ

$$d\Phi = kdx - \omega dt = -\mu d\tau = -(Mc^2/\hbar) d\tau.$$

$$d\tau = dt \sqrt{1-u^2/c^2} = dt \operatorname{sech} \rho$$

Deriving relativistic quantum Lagrangian-Hamiltonian relations

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Differential action: $dS = L dt = p \cdot dx - H \cdot dt = \hbar k \cdot dx - \hbar \omega \cdot dt = \hbar d\Phi$

is Planck scale \hbar times differential phase: $dS = \hbar d\Phi$

Deriving relativistic quantum Lagrangian-Hamiltonian relations

Start with phase Φ and set $k=0$ to get product of proper frequency $\mu = Mc^2/\hbar$ and proper time τ

$$d\Phi = kdx - \omega dt = -\mu d\tau = -(Mc^2/\hbar) d\tau. \quad d\tau = dt \sqrt{(1-u^2/c^2)} = dt \operatorname{sech} \rho$$

Differential action: $dS = L dt = p \cdot dx - H \cdot dt = \hbar k \cdot dx - \hbar \omega \cdot dt = \hbar d\Phi$

is Planck scale \hbar times differential phase: $dS = \hbar d\Phi$

For constant u the Lagrangian is: $L = -\hbar\mu\tau = -Mc^2\sqrt{(1-u^2/c^2)} = -Mc^2\operatorname{sech} \rho = -Mc^2\cos \sigma$

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...with Poincare invariant: $L = p \cdot \dot{x} - H = p \cdot u - H$

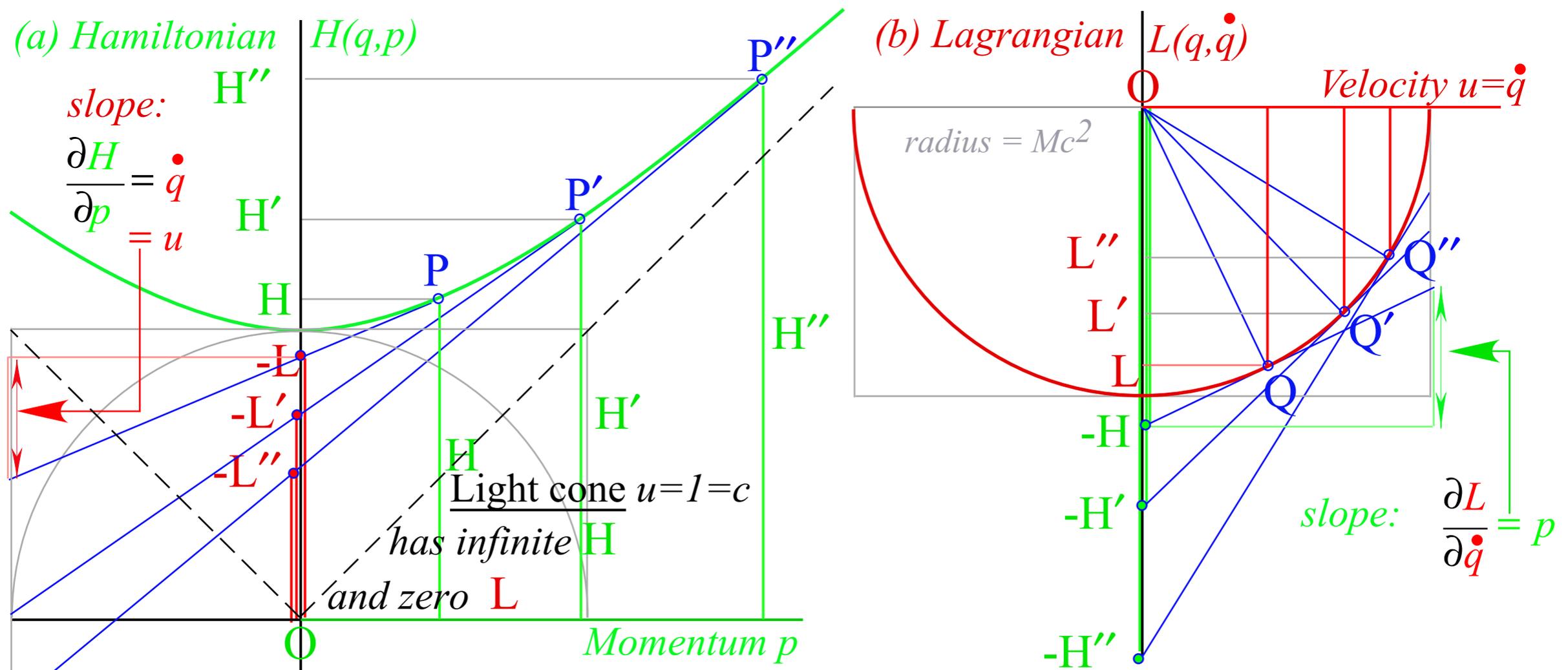


Fig. 5.1. Geometry of contact transformation between relativistic (a) Hamiltonian (b) Lagrangian

Relativistic Classical and Quantum Mechanics

Group vs. phase velocity and tangent contacts

Reviewing “Sin-Tan Rosetta” geometry

How optical CW group and phase properties give relativistic mechanics

Three kinds of mass (Einstein rest mass, Galilean momentum mass, Newtonian inertial mass)

What’s the matter with light?

Bohr-Schrodinger (BS) approximation throws out Mc^2

Deriving relativistic quantum Lagrangian-Hamiltonian relations

 *Feynman’s flying clock and phase minimization*

Geometry of relativistic mechanics

Feynman's flying clock and phase minimization

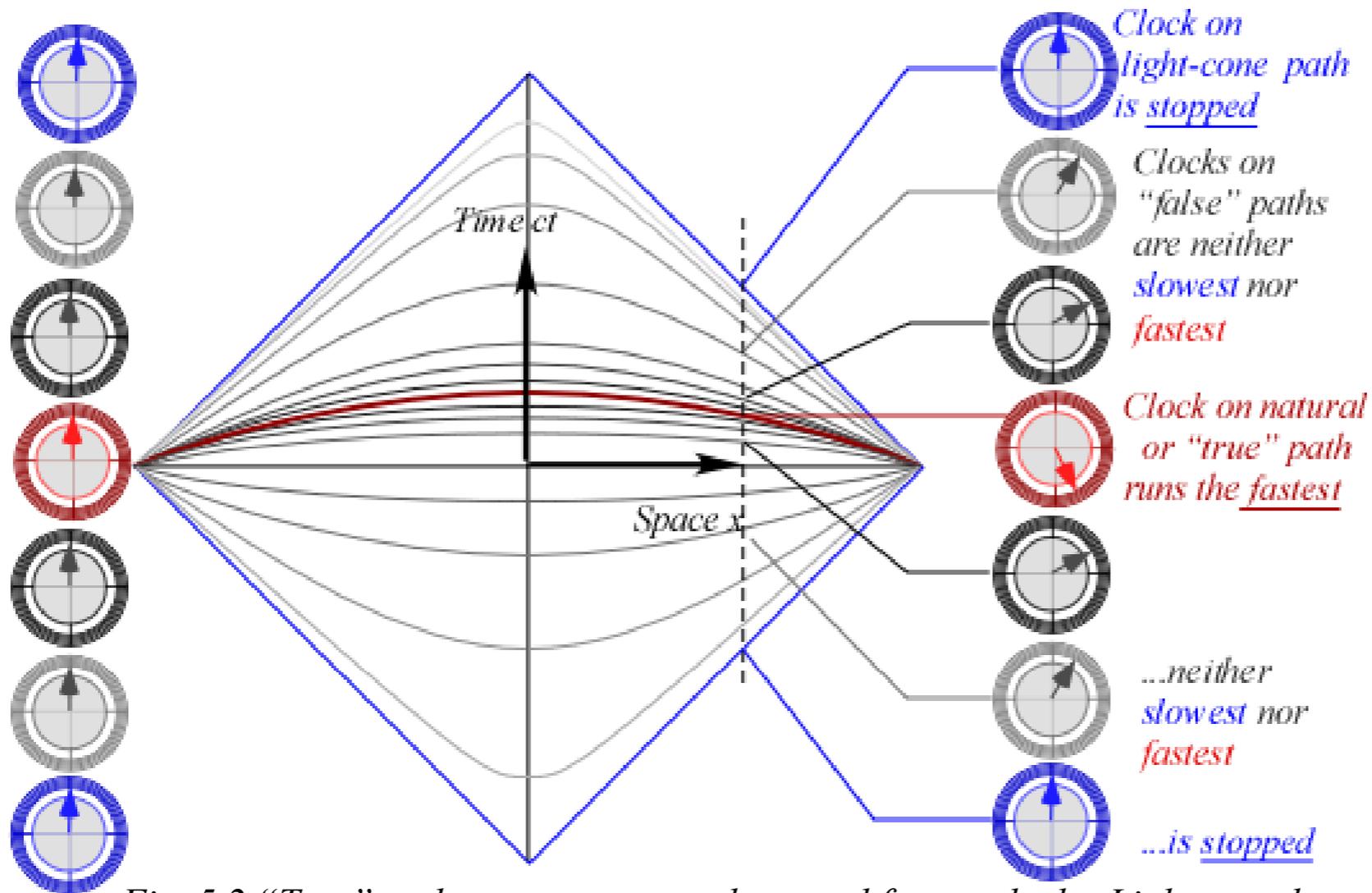


Fig. 5.2 "True" paths carry extreme phase and fastest clocks. Light-cone has only stopped clocks.

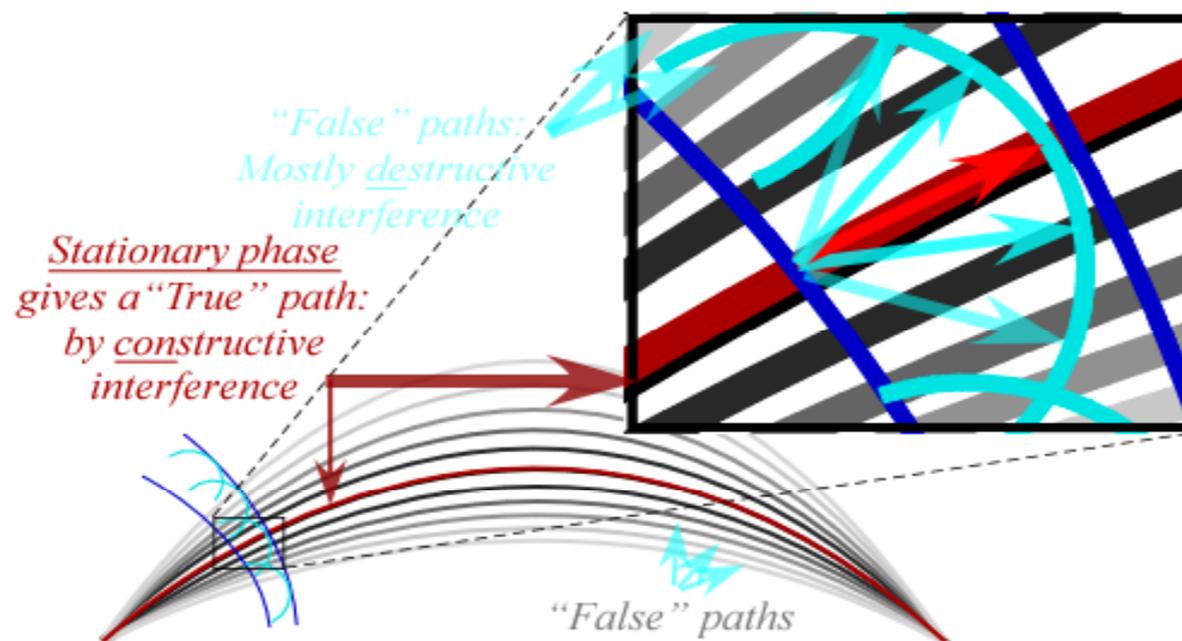


Fig. 5.3 Quantum waves interfere constructively on "True" path but mostly cancel elsewhere.

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