Lecture 23.

Relativity of lightwaves and Lorentz-Minkowski coordinates II.

(Ch. 0-3 of Unit 2   3.27.12)

3. Spectral theory of Einstein-Lorentz relativity   *(Includes Lecture 22 review)*

Applying Doppler Shifts to per-space-time (ck,ω) graph

CW Minkowski space-time coordinates (x,ct) and PW grids

Lecture 22 ended (about) here ←

Relating Doppler Shifts b or r=1/b to velocity u/c or rapidity ρ

Lorentz transformation

Lorentz length-contraction and Einstein time-dilation

4. Einstein-Lorentz symmetry

What happened to Galilean symmetry? (It moved to "gauge" space!)

Thale's construction and Euclid's means

Lecture 23 ended here ←

Time reversal symmetry gives hyperbolic invariants

per-space-time hyperbola

space-time hyperbola

Phase invariance

5. That "old-time" relativity ("Bouncing-photons" smoke & mirrors)

The ship and lighthouse saga

3. Spectral theory of Einstein-Lorentz relativity

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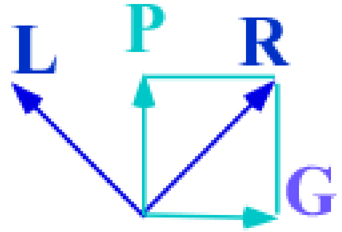
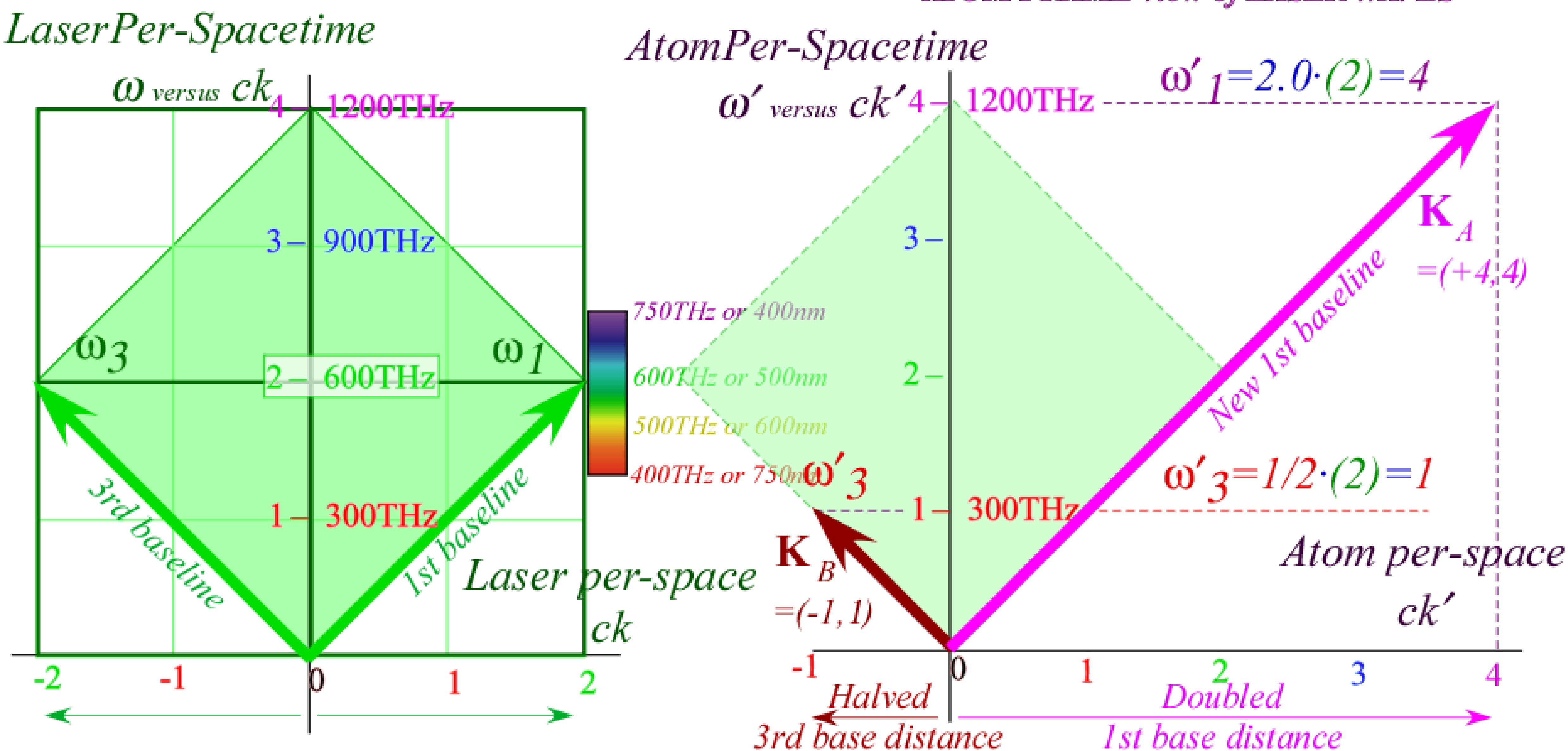
Deriving Spacetime and per-spacetime coordinate geometry by:

(1) *Evenson* CW axiom *"All colors go c"* keeps $\mathbf{K}_A$ and $\mathbf{K}_B$ *on their baselines.*

(2) Time-Reversal axiom: $r=1/b$

(3) Half-Sum Phase **P=(R+L)/2** and Half-Difference Group **G=(R−L)/2**





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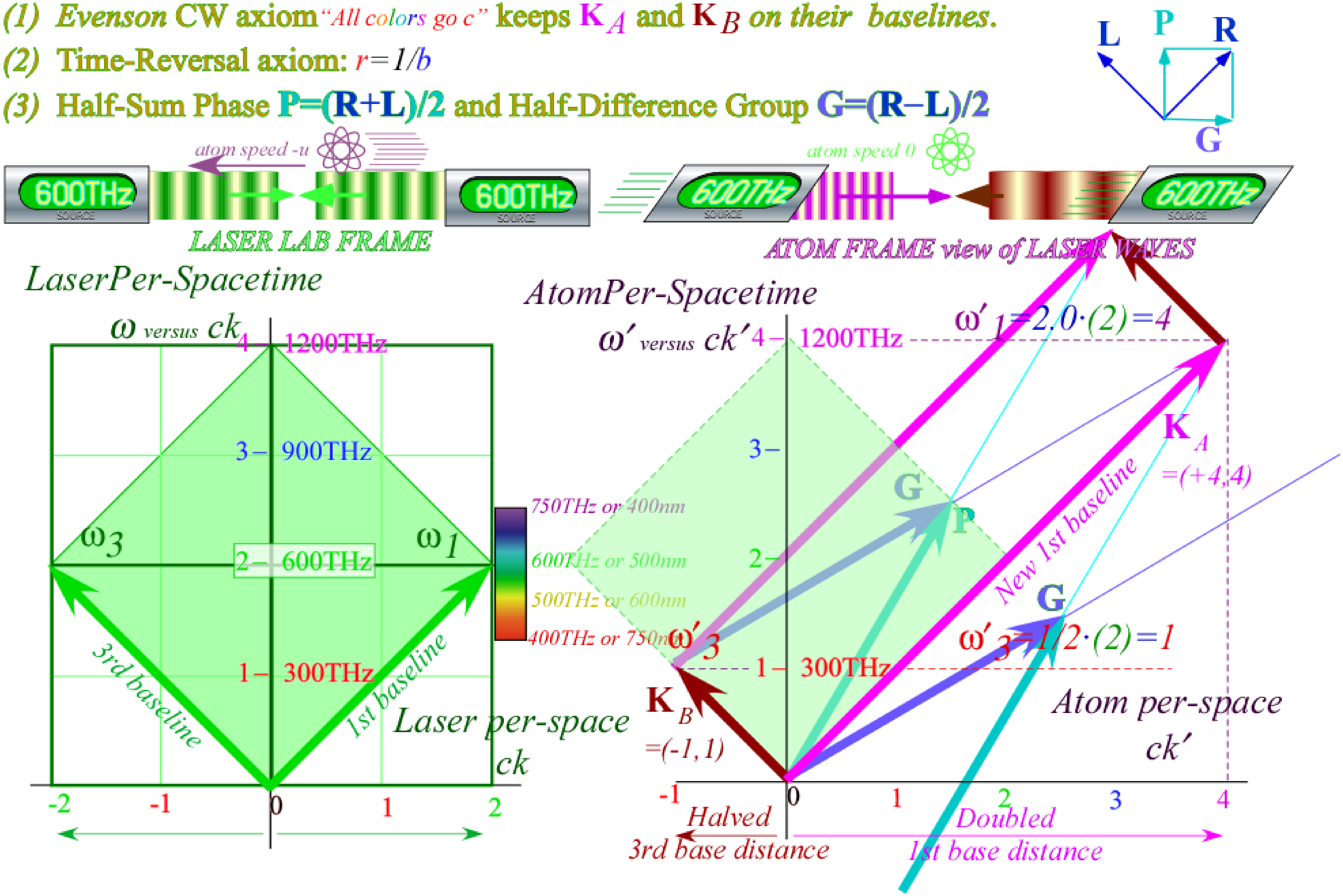

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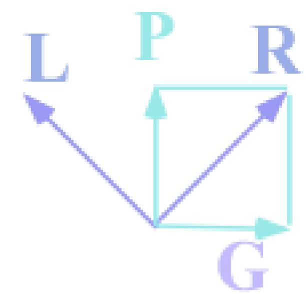

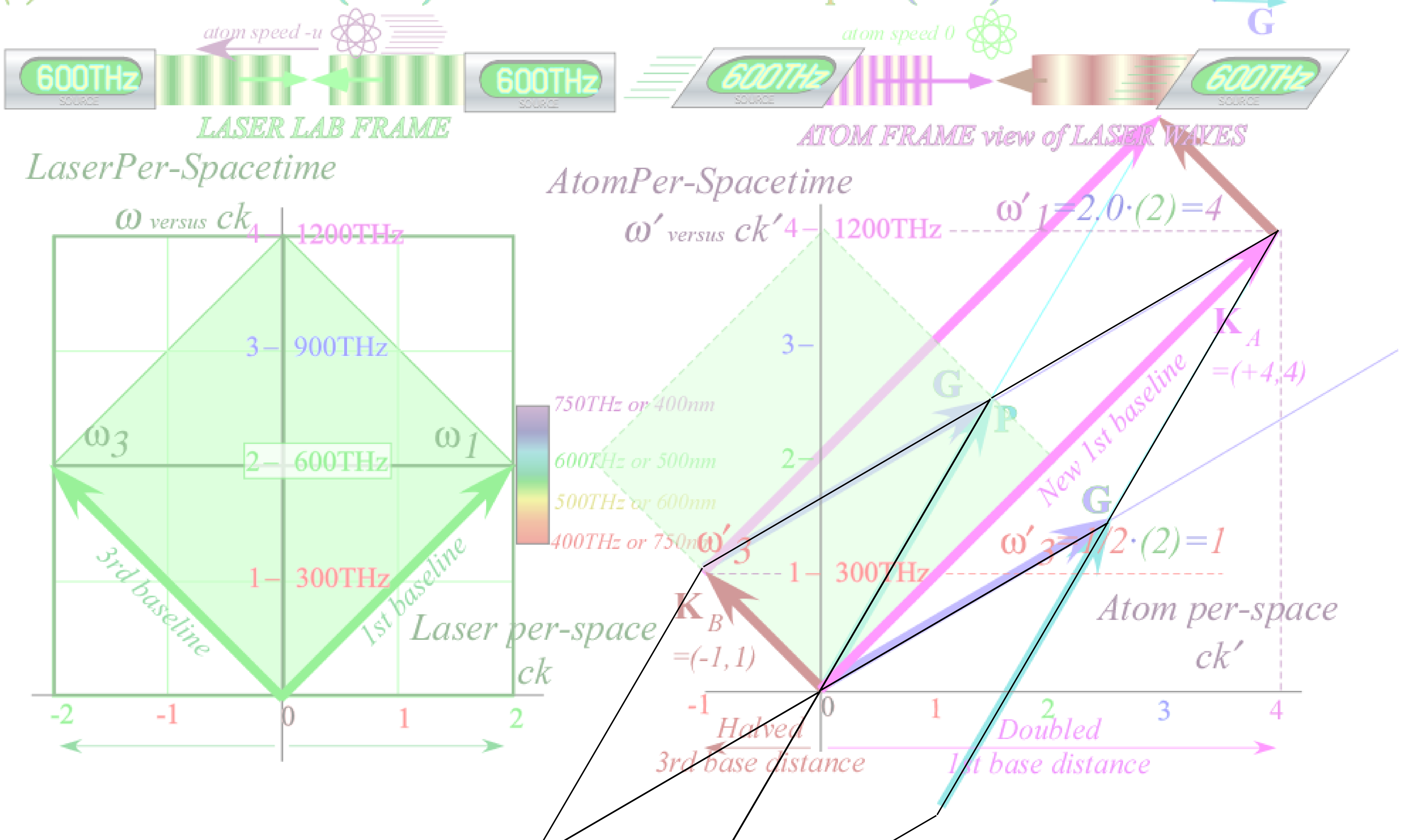

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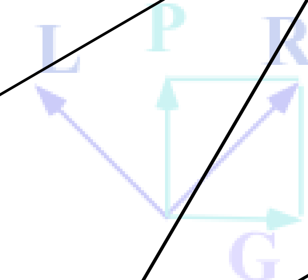

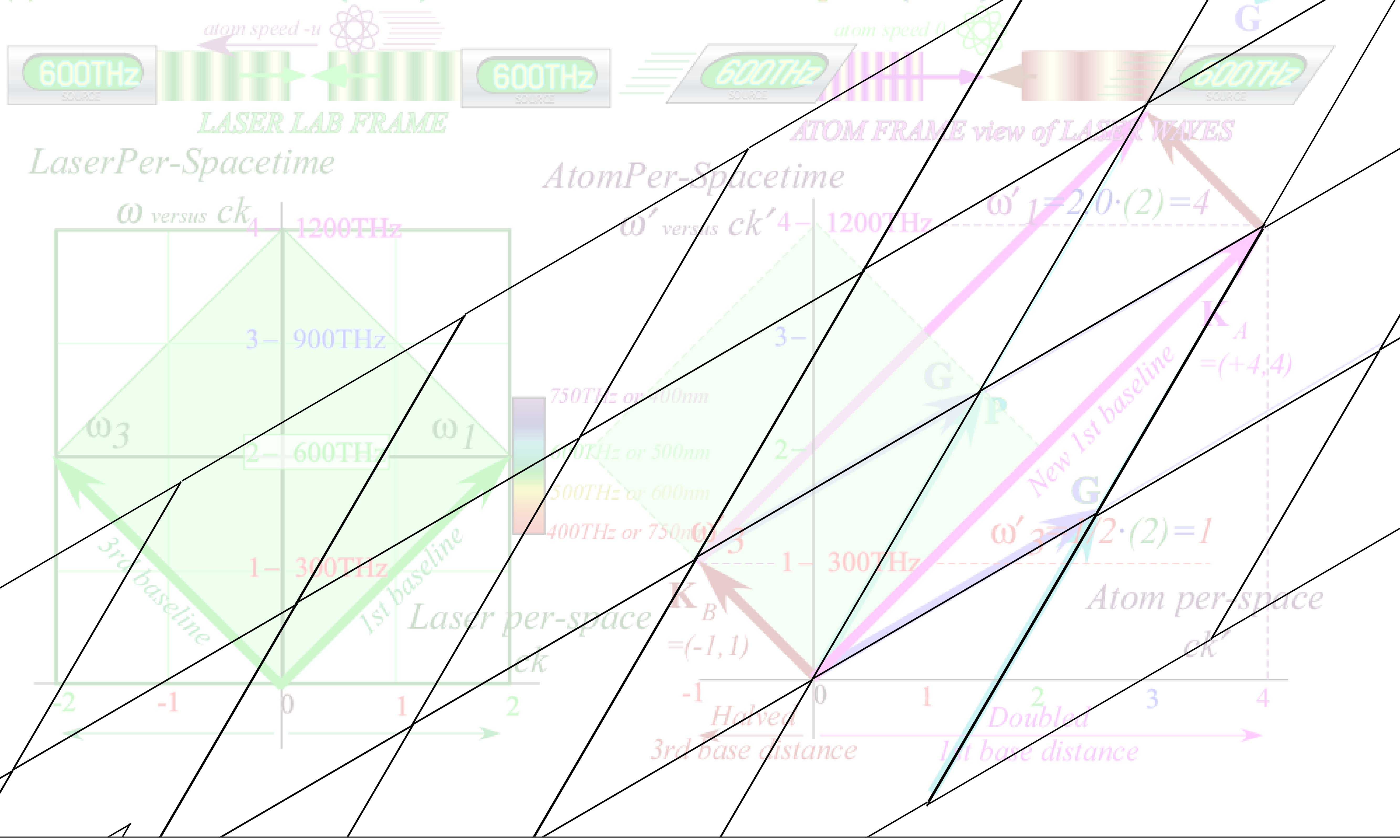

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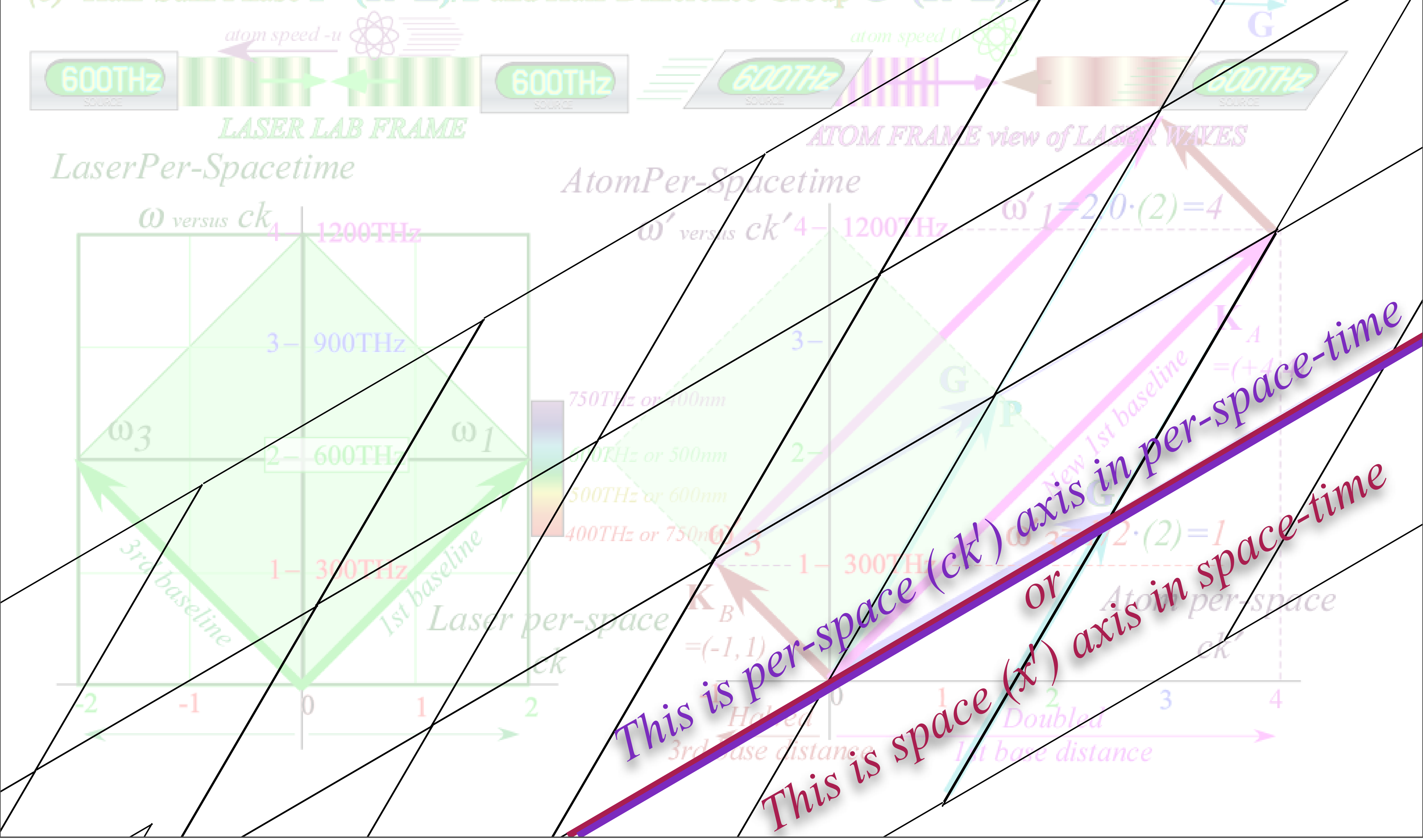

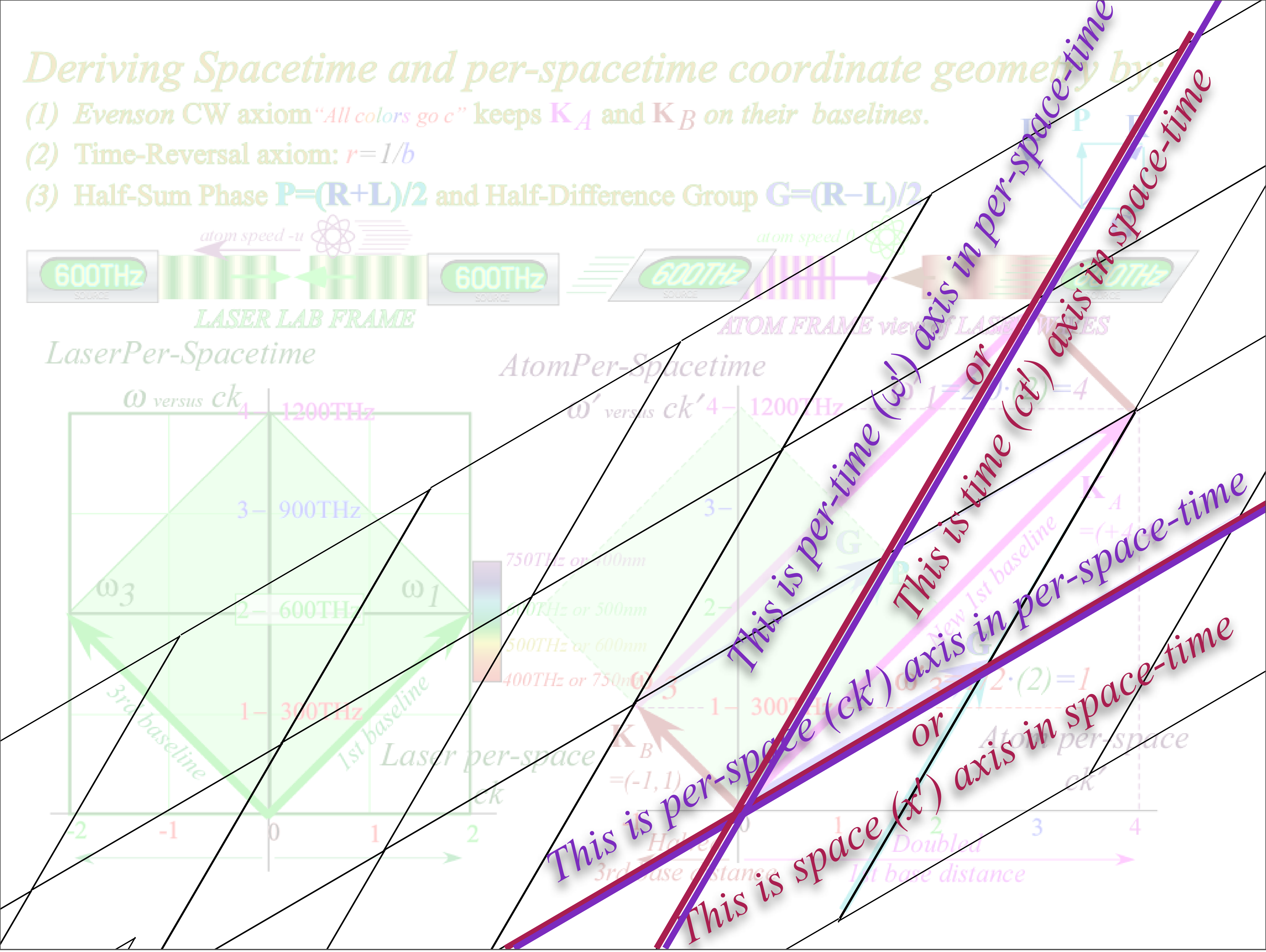

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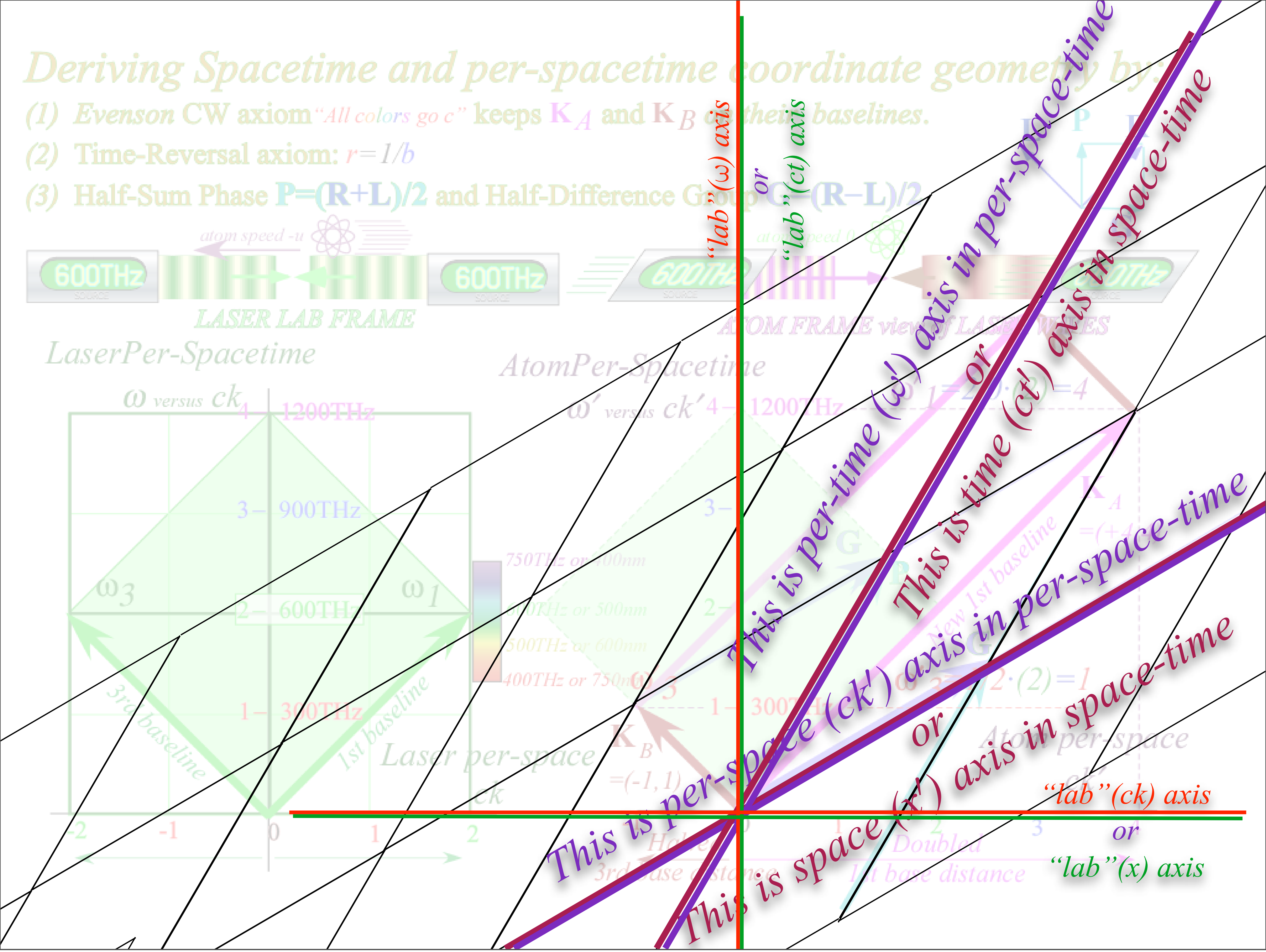
(3) Half-Sum Phase P=(R+L)/2 and Half-Difference Group G=(R−L)/2
atom speed -u
600THz
LASER LAB FRAME
atom speed 0
ATOM FRAME view of LASER WAVES
LaserPer-Spacetime
ω versus ck
1200THz
900THz
600THz
300THz
ω3
ω1
3rd baseline
1st baseline
Laser per-space
ck
750THz or 400nm
600THz or 500nm
500THz or 600nm
400THz or 750nm
AtomPer-Spacetime
ω′ versus ck′
K_B =(-1,1)
K_A
New 1st baseline
Atom per-space
ck′
Halved
3rd base distance
Doubled
1st base distance
This is per-time (ω′) axis in per-space-time
or
This is time (ct′) axis in space-time
This is per-space (ck′) axis in per-space-time
or
This is space (x′) axis in space-time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"lab"(ω) axis
or
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This is per-time (ω′) axis in per-space-time
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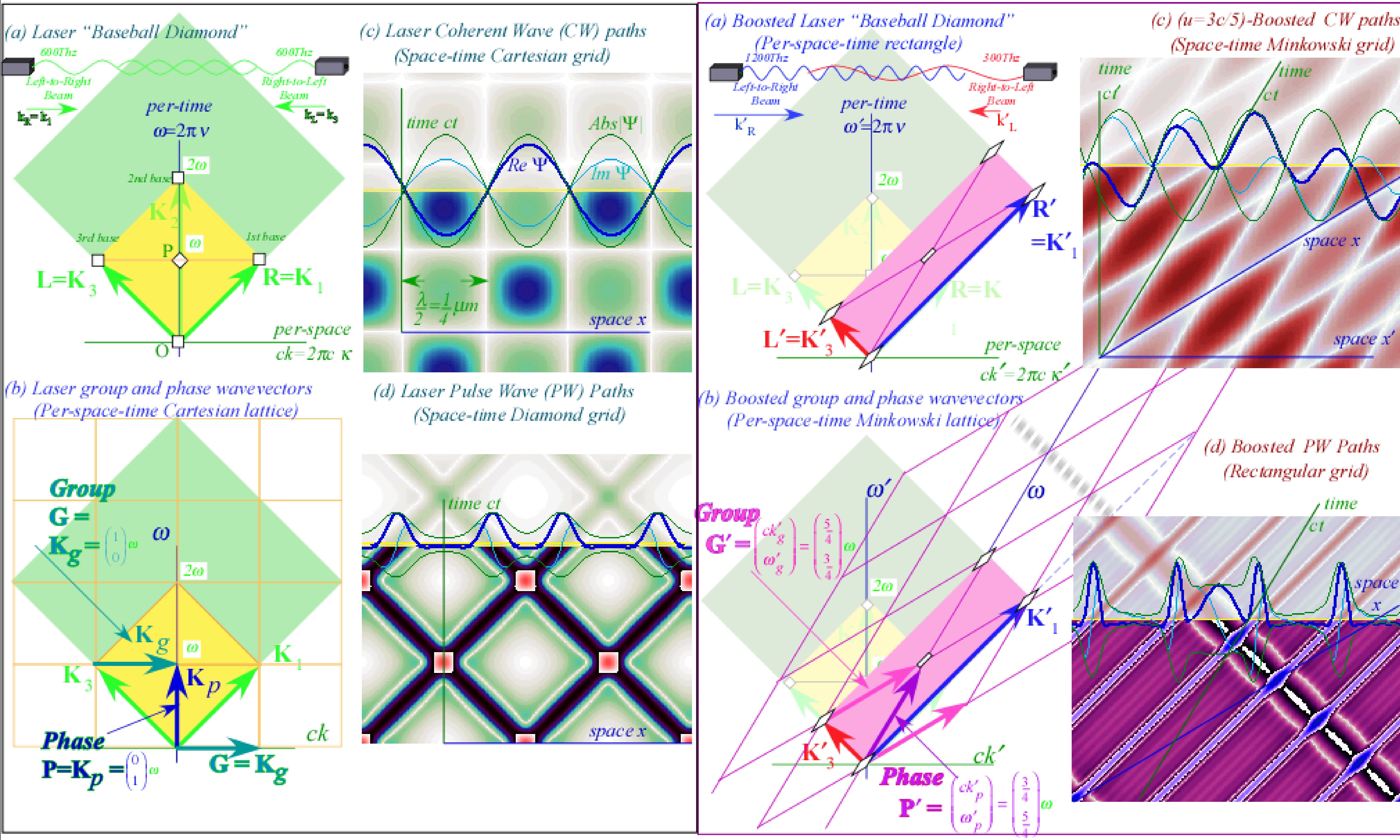

Laser lab views

atom speed $-u=-\frac{3}{5}c$

Atom views (sees lab going $+u=\frac{3}{5}c$)

Lecture 22 ended (approximately) here

3. Spectral theory of Einstein-Lorentz relativity

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Spectral development: Note B=2.0 here.

Euclidian Geometry for Per-spacetime Relativity

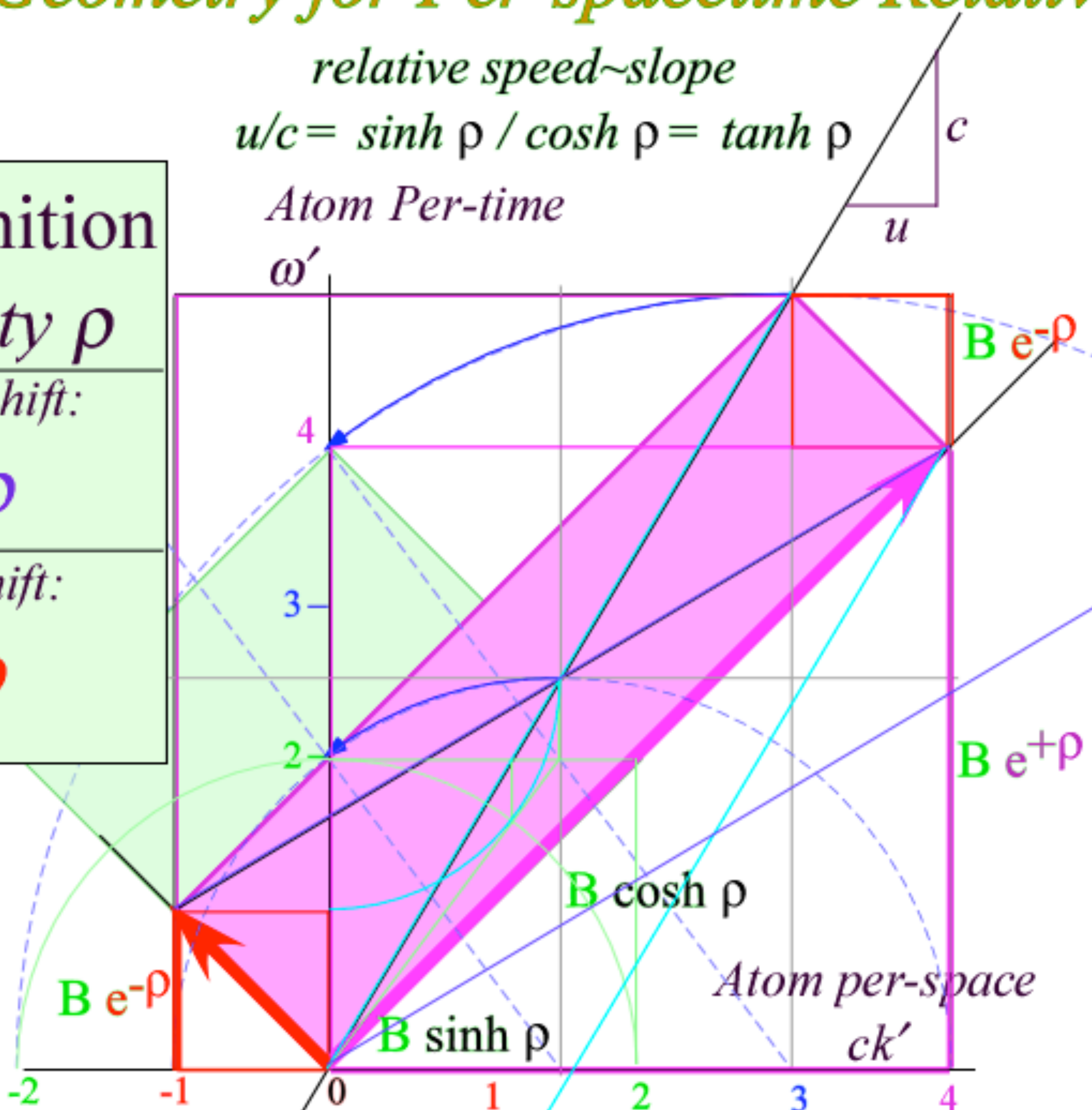
Key Definition of Rapidity ρ

Doppler blue shift:

$Bb = Be^{+\rho}$

Doppler red shift:

$Br = Be^{-\rho}$



$B \sinh\rho = (Be^{+\rho} - Be^{-\rho})/2$

$B \cosh\rho = (Be^{+\rho} + Be^{-\rho})/2$

Key Results:

ω vs. ck

"winks" vs. "kinks"

$\omega = B\cosh\rho$

$ck = B\sinh\rho$

group velocity:

$$\frac{ck}{\omega} = \frac{u}{c} = \tanh\rho$$

phase velocity:

$$\frac{\omega}{ck} = \frac{c}{u} = \coth\rho$$

Deriving relations between Doppler Shifts b or r=1/b and velocity u/c or rapidity ρ:

$$b = 1/r = e^{+\rho} = \cosh\rho + \sinh\rho$$

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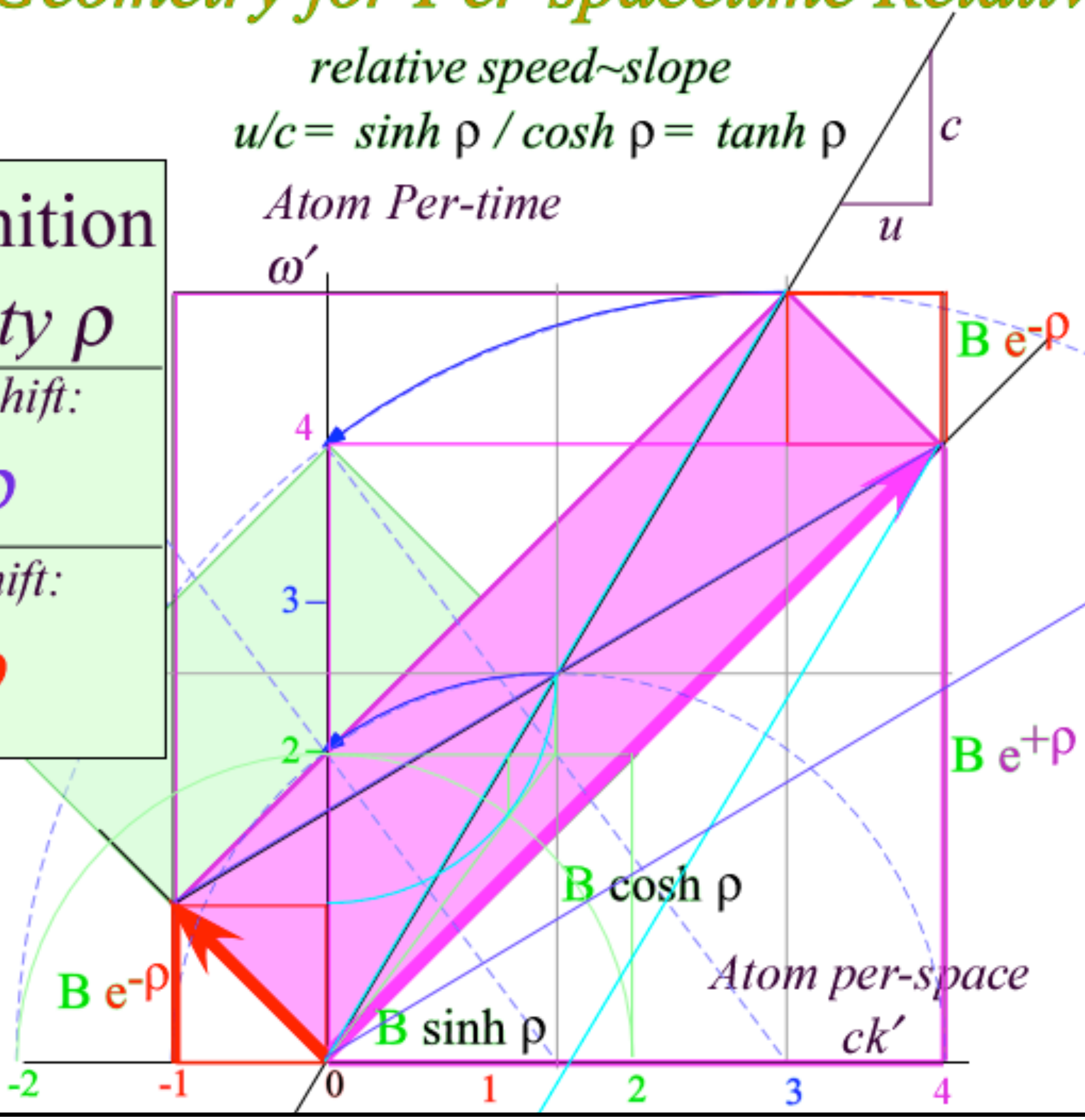
Euclidian Geometry for Per-spacetime Relativity



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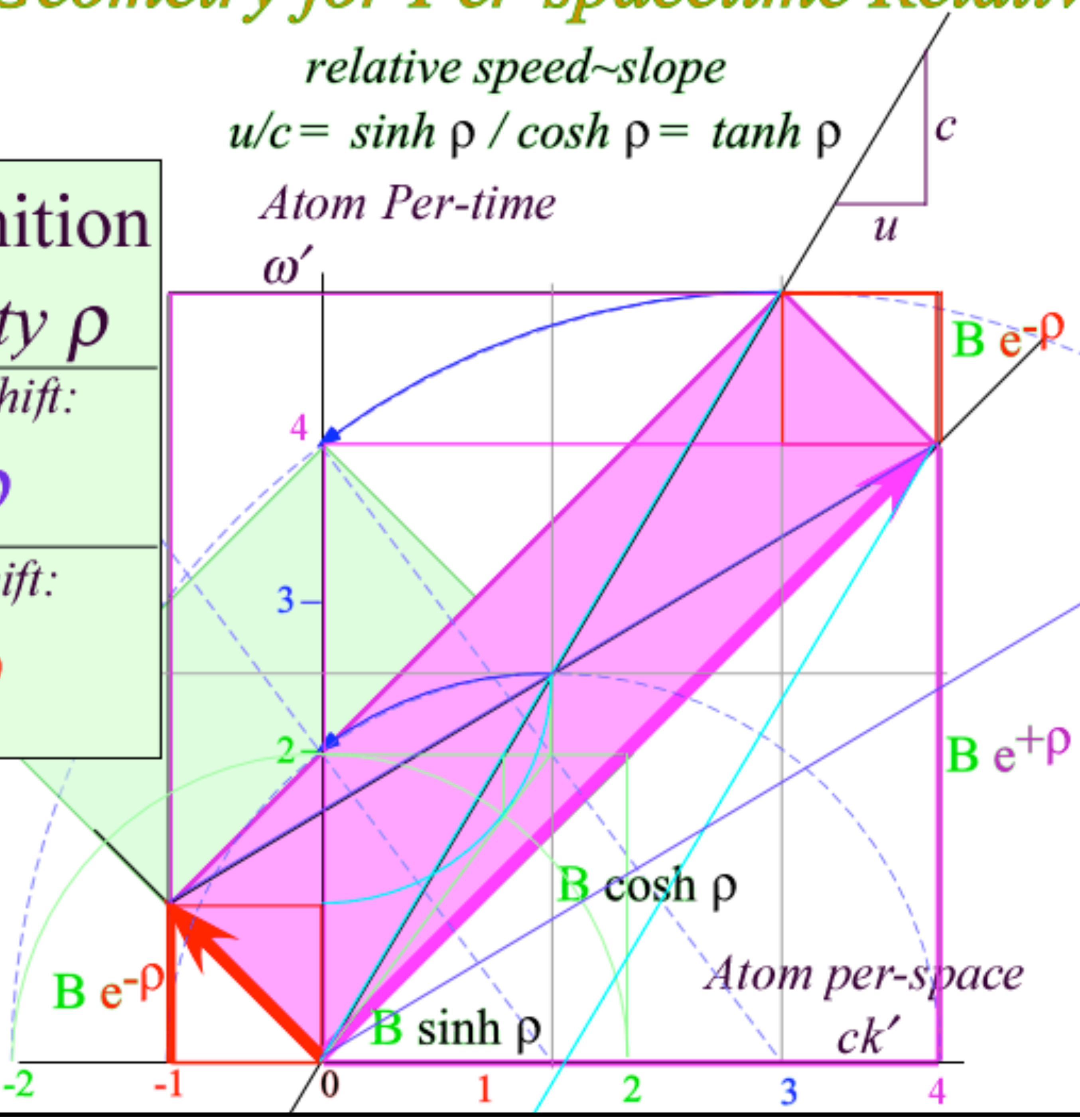

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$$\frac{b - 1/b}{b + 1/b} = \frac{\sinh\rho}{\cosh\rho} = \tanh\rho = \frac{u}{c} = \frac{b^2 - 1}{b^2 + 1} = \frac{1 - r^2}{1 + r^2}$$

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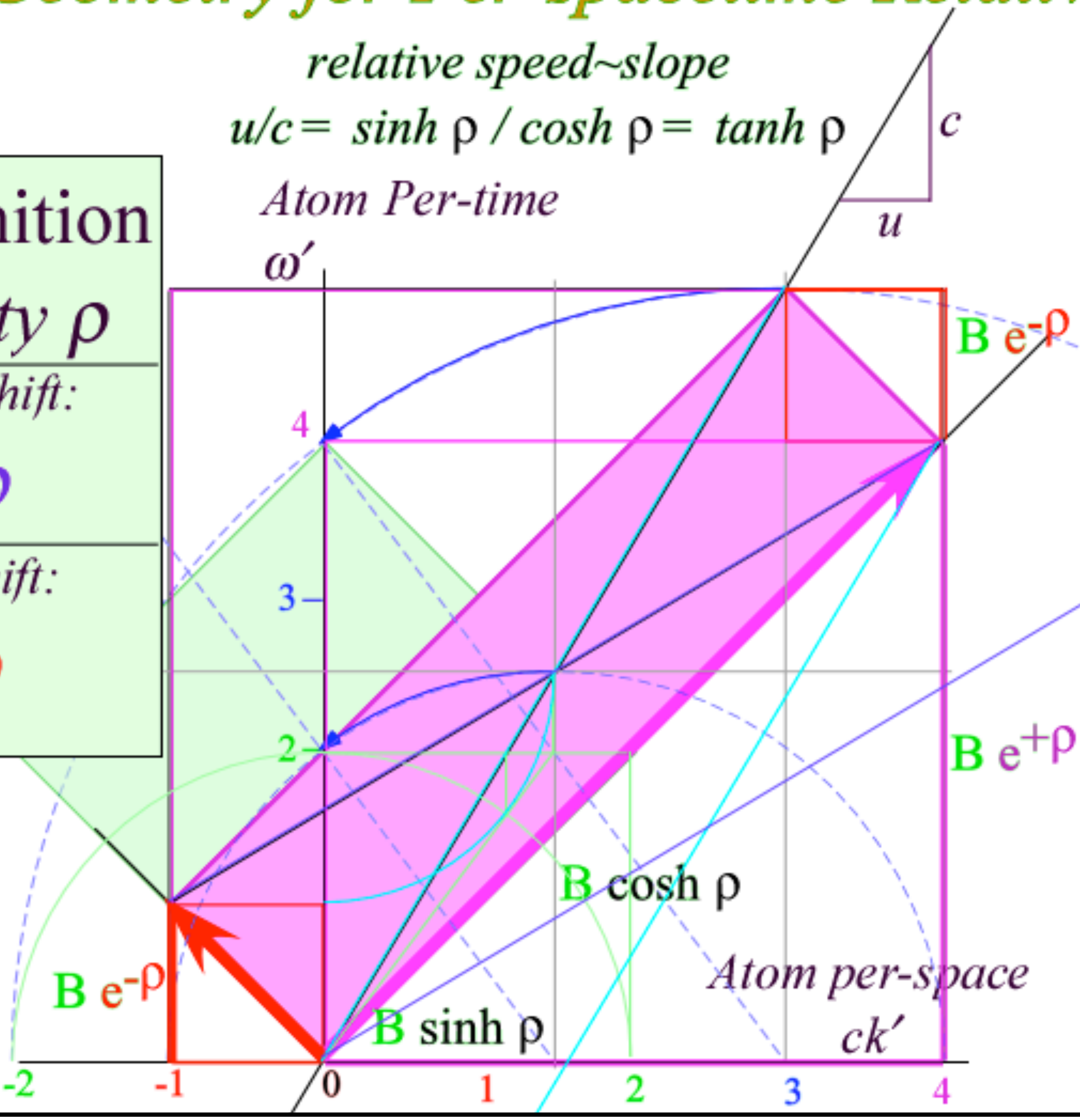

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$$r = 1/b = e^{-\rho} = \cosh\rho - \sinh\rho \qquad 1 = \cosh^2\rho - \sinh^2\rho$$

$$\frac{b-1/b}{b+1/b} = \frac{\sinh\rho}{\cosh\rho} = \tanh\rho = \frac{u}{c} = \frac{b^2-1}{b^2+1} = \frac{1-r^2}{1+r^2} \qquad \frac{1}{\cosh^2\rho} = 1 - \frac{\sinh^2\rho}{\cosh^2\rho} = 1 - \frac{u^2}{c^2}$$

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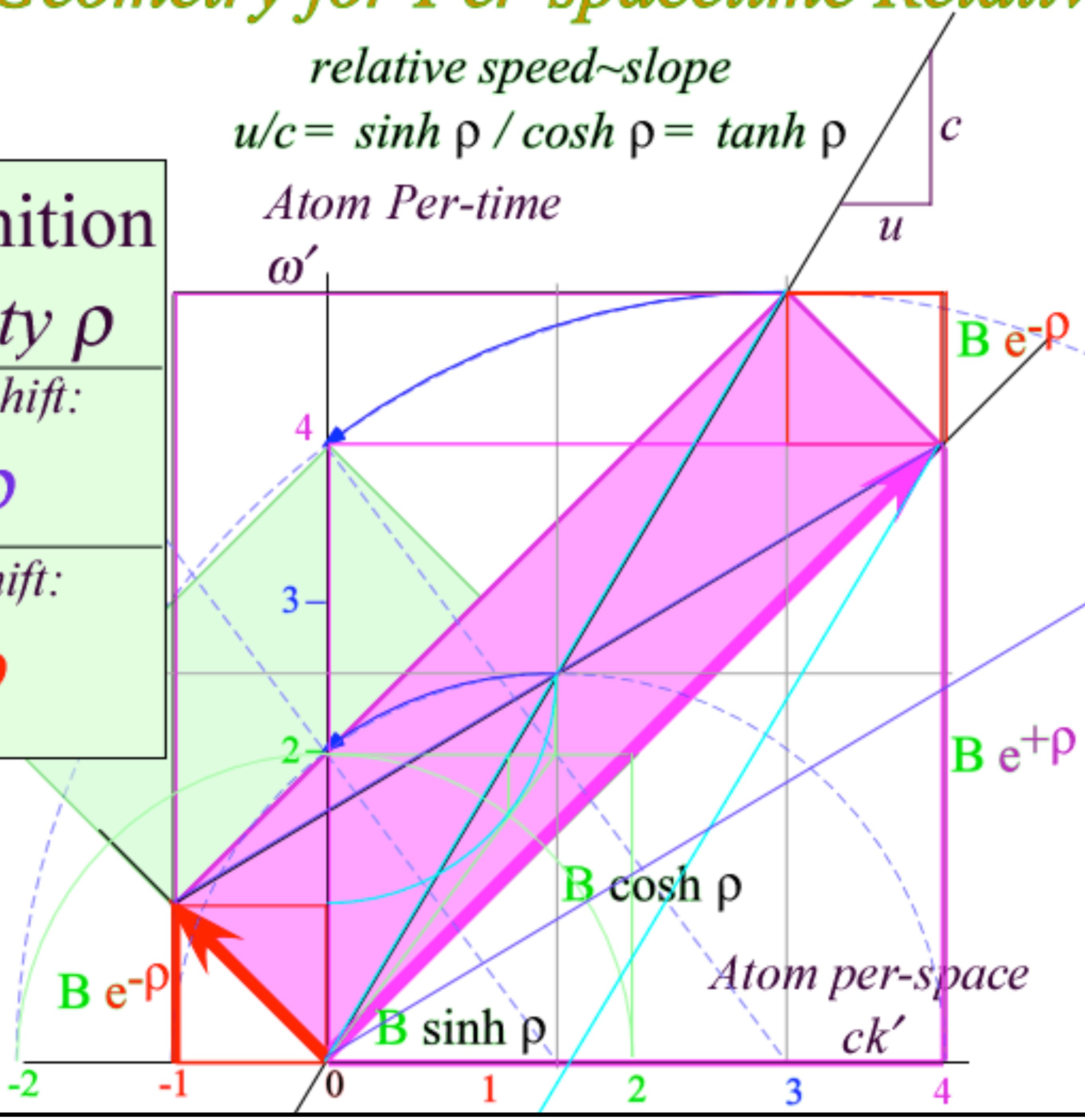

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$$br = 1 = (\cosh\rho + \sinh\rho)(\cosh\rho - \sinh\rho)$$

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$$\frac{1}{\cosh^2\rho} = 1 - \frac{\sinh^2\rho}{\cosh^2\rho} = 1 - \frac{u^2}{c^2}$$

$$\cosh\rho = \frac{1}{\sqrt{1-u^2/c^2}},$$

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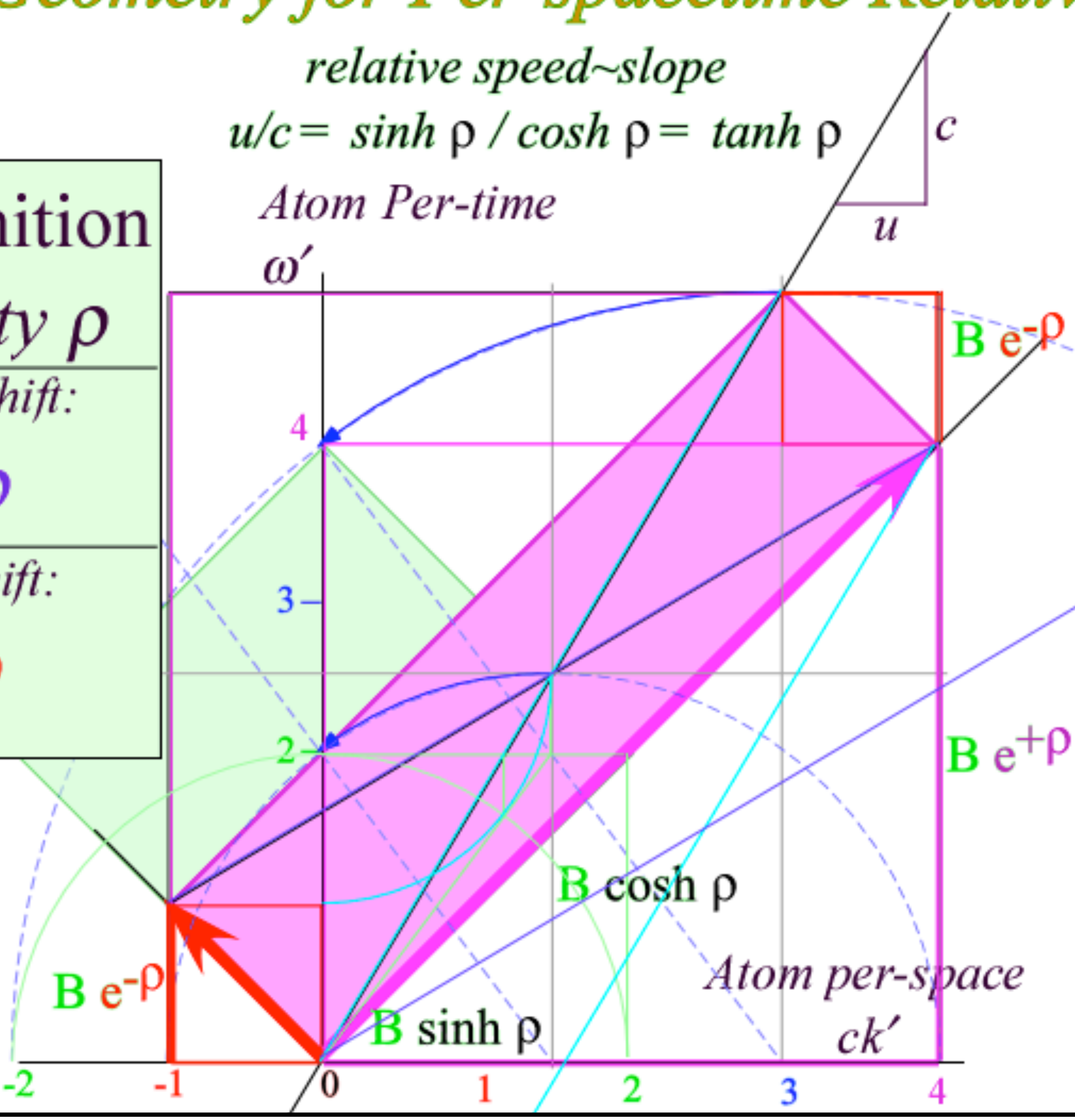
Euclidian Geometry for Per-spacetime Relativity



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$$\cosh\rho = \frac{1}{\sqrt{1-u^2/c^2}},$$

$$\sinh\rho = \frac{u/c}{\sqrt{1-u^2/c^2}}$$

$$b = \cosh\rho + \sinh\rho = \frac{1+u/c}{\sqrt{1-u^2/c^2}} = \sqrt{\frac{1+u/c}{1-u/c}}$$

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Euclidian Geometry for Per-spacetime Relativity

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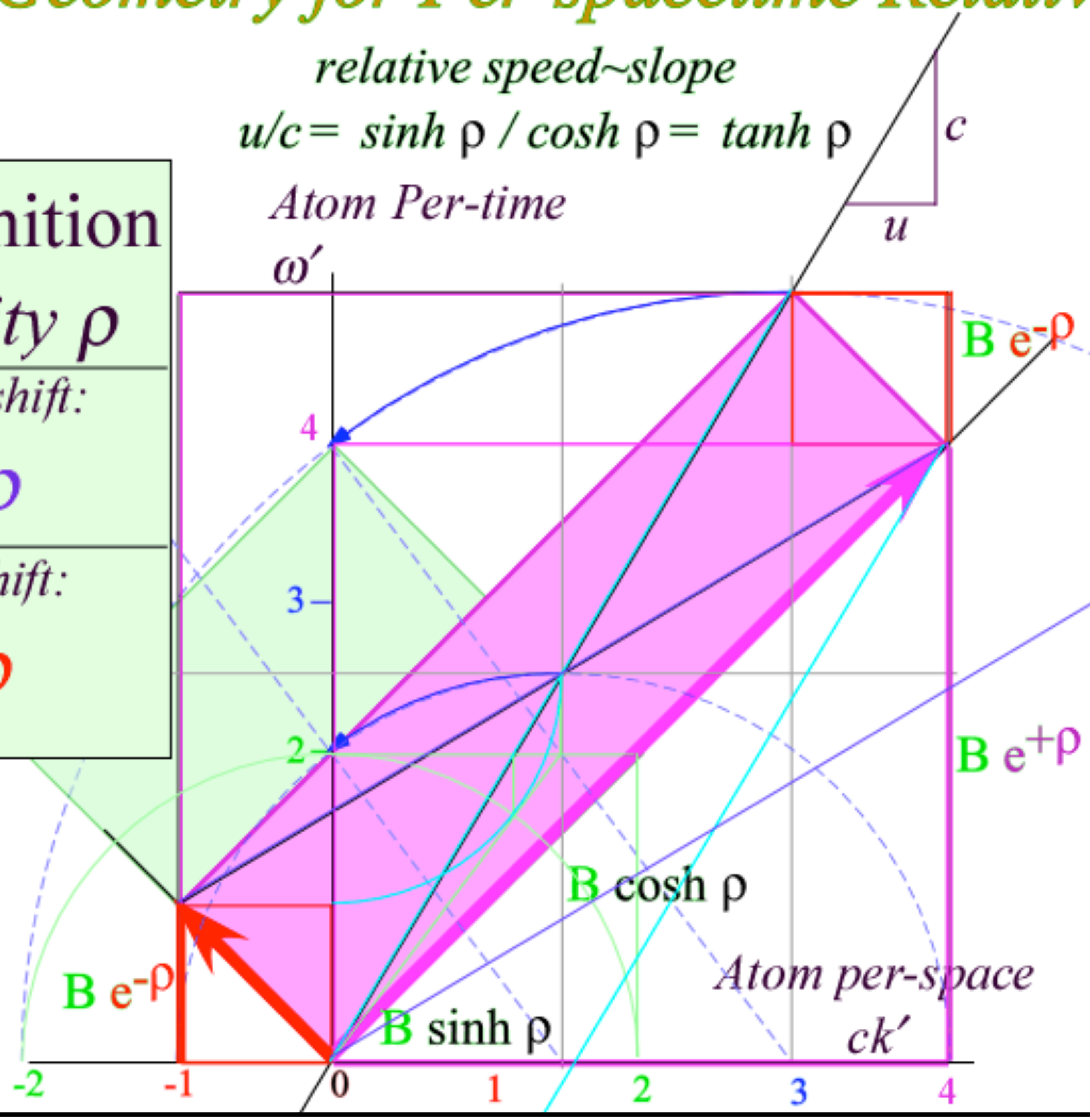

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$$b = \cosh\rho + \sinh\rho = \frac{1+u/c}{\sqrt{1-u^2/c^2}} = \sqrt{\frac{1+u/c}{1-u/c}}$$

$$r = \cosh\rho - \sinh\rho = \sqrt{\frac{1-u/c}{1+u/c}}$$

3. Spectral theory of Einstein-Lorentz relativity

Applying Doppler Shifts to per-space-time (ck,ω) graph

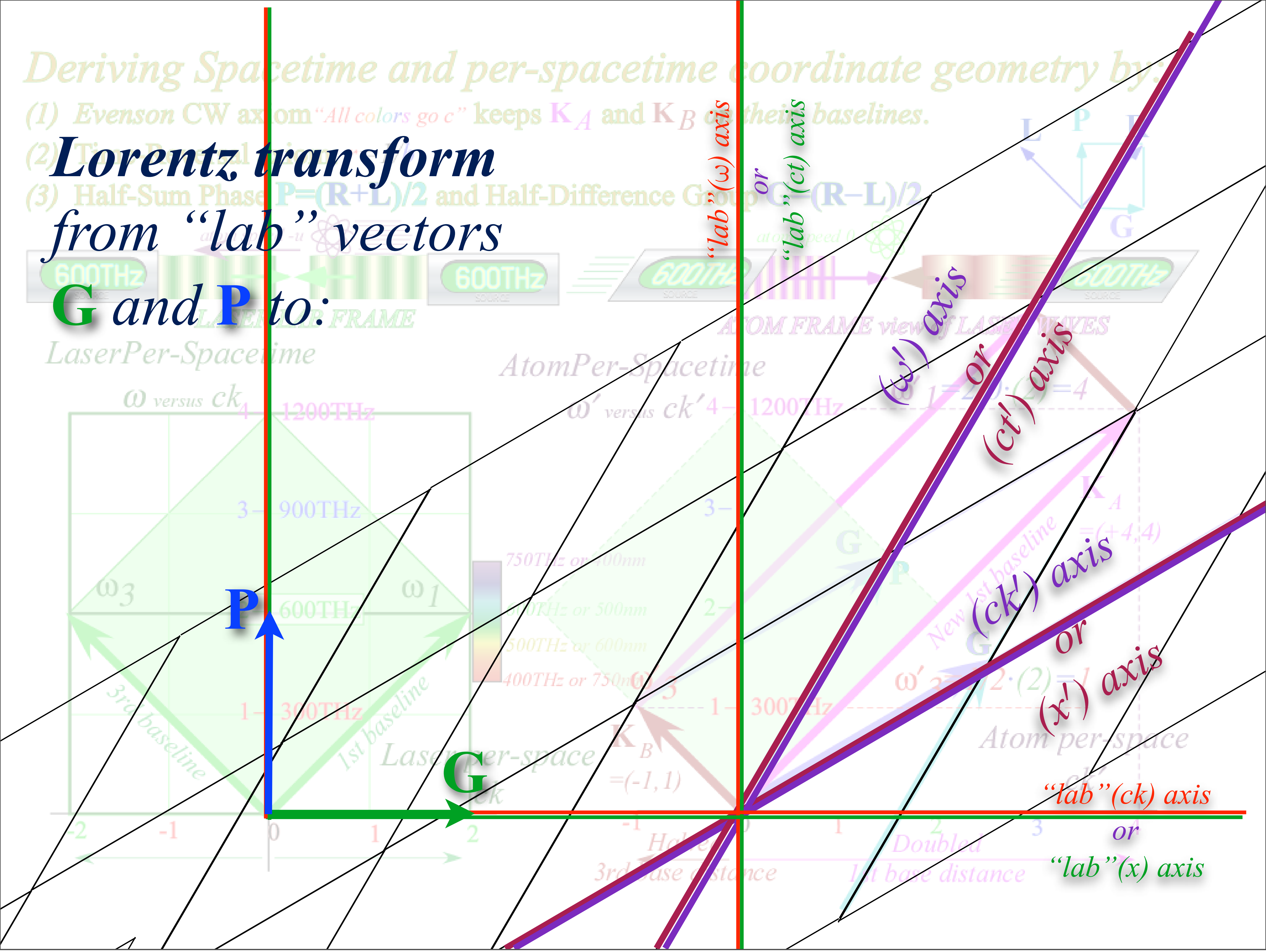
CW Minkowski space-time coordinates (x,ct) and PW grids

Relating Doppler Shifts b or $r=1/b$ to velocity u/c or rapidity ρ

Lorentz transformation

Lorentz length-contraction and Einstein time-dilation

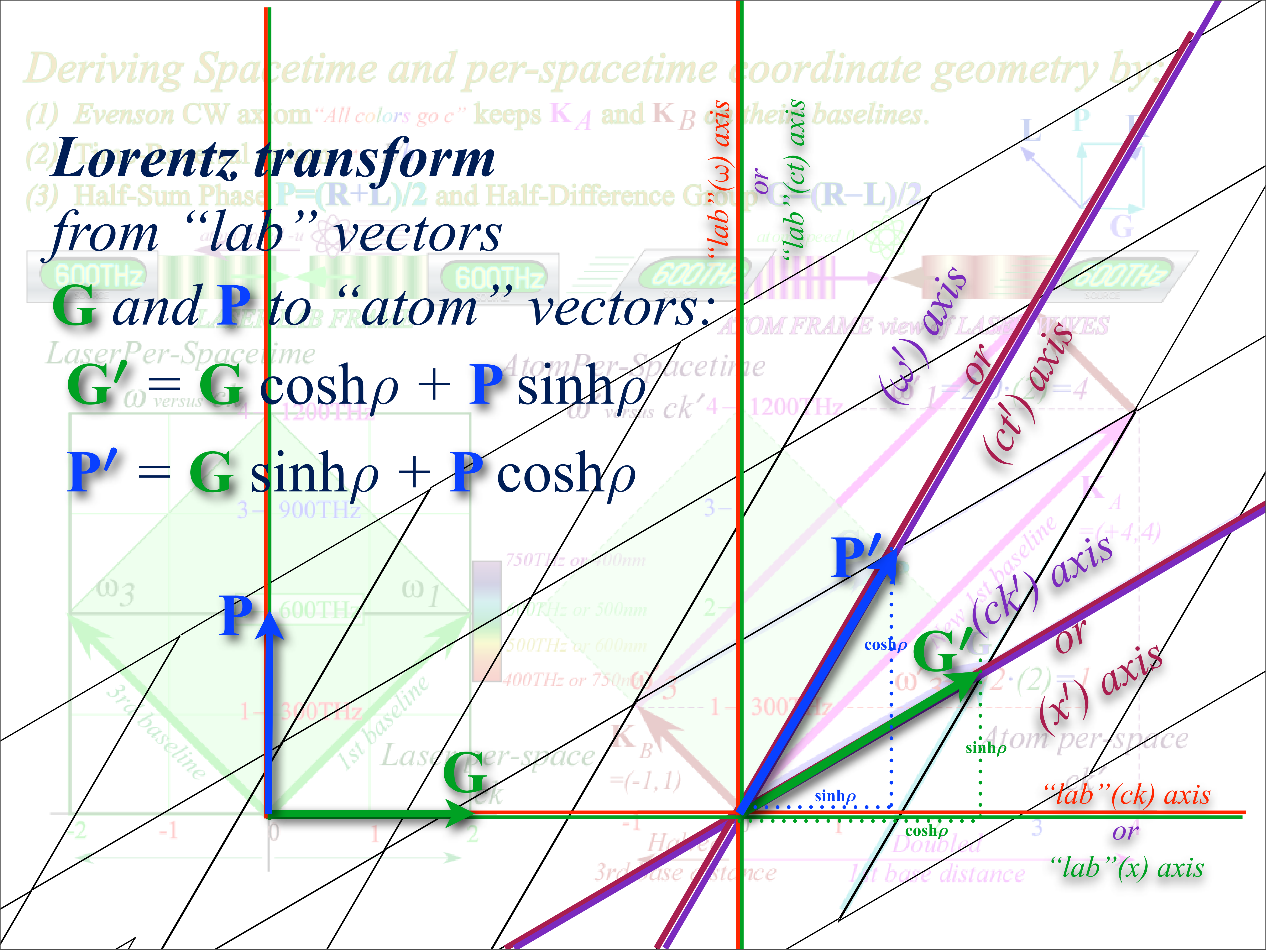
Lorentz transform from “lab” vectors **G** *and* **P** *to:*



Lorentz transform from "lab" vectors G and P to "atom" vectors:

$$\mathbf{G}' = \mathbf{G}\cosh\rho + \mathbf{P}\sinh\rho$$

$$\mathbf{P}' = \mathbf{G}\sinh\rho + \mathbf{P}\cosh\rho$$



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$$\mathbf{G}' = \mathbf{G}\cosh\rho + \mathbf{P}\sinh\rho \qquad \mathbf{G} = \mathbf{G}'\cosh\rho - \mathbf{P}'\sinh\rho$$

$$\mathbf{P}' = \mathbf{G}\sinh\rho + \mathbf{P}\cosh\rho \qquad \mathbf{P} = -\mathbf{G}'\sinh\rho + \mathbf{P}'\cosh\rho$$

Lorentz transform from "lab" vectors G and P to "atom" vectors:

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A "professional" notation: (Dirac's bra-kets $\langle\mathbf{A}|\mathbf{B}\rangle = \delta_{A,B}$ and: $|\mathbf{A}\rangle\langle\mathbf{A}| + |\mathbf{B}\rangle\langle\mathbf{B}| = \mathbf{1}$ *)*

Lorentz transformation operator L

$$L|\mathbf{G}\rangle = |\mathbf{G'}\rangle = |\mathbf{G}\rangle\langle\mathbf{G}|\mathbf{G'}\rangle + |\mathbf{P}\rangle\langle\mathbf{P}|\mathbf{G'}\rangle$$

$$L|\mathbf{P}\rangle = |\mathbf{P'}\rangle = |\mathbf{G}\rangle\langle\mathbf{G}|\mathbf{P'}\rangle + |\mathbf{P}\rangle\langle\mathbf{P}|\mathbf{P'}\rangle$$

$$\begin{pmatrix} \langle\mathbf{G}|\mathbf{G'}\rangle & \langle\mathbf{G}|\mathbf{P'}\rangle \\ \langle\mathbf{P}|\mathbf{G'}\rangle & \langle\mathbf{P}|\mathbf{P'}\rangle \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} = \begin{pmatrix} \langle\mathbf{G}|L|\mathbf{G}\rangle & \langle\mathbf{G}|L|\mathbf{P}\rangle \\ \langle\mathbf{P}|L|\mathbf{G}\rangle & \langle\mathbf{P}|L|\mathbf{P}\rangle \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1-u^2/c^2}} & \frac{u/c}{\sqrt{1-u^2/c^2}} \\ \frac{u/c}{\sqrt{1-u^2/c^2}} & \frac{1}{\sqrt{1-u^2/c^2}} \end{pmatrix}$$

Lorentz transform from "lab" vectors G and P to "atom" vectors:

$$\mathbf{G}' = \mathbf{G}\cosh\rho + \mathbf{P}\sinh\rho \qquad \mathbf{G} = \mathbf{G}'\cosh\rho - \mathbf{P}'\sinh\rho$$

$$\mathbf{P}' = \mathbf{G}\sinh\rho + \mathbf{P}\cosh\rho \qquad \mathbf{P} = -\mathbf{G}'\sinh\rho + \mathbf{P}'\cosh\rho$$

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Lorentz transformation operator L

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INVERSE Lorentz transformation L^{-1}

$$L^{-1}|\mathbf{G}'\rangle = |\mathbf{G}\rangle = |\mathbf{G}'\rangle\langle\mathbf{G}'|\mathbf{G}\rangle + |\mathbf{P}'\rangle\langle\mathbf{P}'|\mathbf{G}\rangle$$

$$L^{-1}|\mathbf{P}'\rangle = |\mathbf{P}\rangle = |\mathbf{G}'\rangle\langle\mathbf{G}'|\mathbf{P}\rangle + |\mathbf{P}'\rangle\langle\mathbf{P}'|\mathbf{P}\rangle$$

$$\begin{pmatrix} \langle\mathbf{G}'|\mathbf{G}\rangle & \langle\mathbf{G}'|\mathbf{P}\rangle \\ \langle\mathbf{P}'|\mathbf{G}\rangle & \langle\mathbf{P}'|\mathbf{P}\rangle \end{pmatrix} = \begin{pmatrix} \cosh\rho & -\sinh\rho \\ -\sinh\rho & \cosh\rho \end{pmatrix} = \begin{pmatrix} \langle\mathbf{G}'|L^{-1}|\mathbf{G}'\rangle & \langle\mathbf{G}'|L^{-1}|\mathbf{P}'\rangle \\ \langle\mathbf{P}'|L^{-1}|\mathbf{G}'\rangle & \langle\mathbf{P}'|L^{-1}|\mathbf{P}'\rangle \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1-u^2/c^2}} & \frac{-u/c}{\sqrt{1-u^2/c^2}} \\ \frac{-u/c}{\sqrt{1-u^2/c^2}} & \frac{1}{\sqrt{1-u^2/c^2}} \end{pmatrix}$$

Lorentz transform from "lab" vectors G and P to "atom" vectors:

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$$\begin{pmatrix} \langle\mathbf{G}|\mathbf{G}'\rangle & \langle\mathbf{G}|\mathbf{P}'\rangle \\ \langle\mathbf{P}|\mathbf{G}'\rangle & \langle\mathbf{P}|\mathbf{P}'\rangle \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} = \begin{pmatrix} \langle\mathbf{G}|L|\mathbf{G}\rangle & \langle\mathbf{G}|L|\mathbf{P}\rangle \\ \langle\mathbf{P}|L|\mathbf{G}\rangle & \langle\mathbf{P}|L|\mathbf{P}\rangle \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1-u^2/c^2}} & \frac{u/c}{\sqrt{1-u^2/c^2}} \\ \frac{u/c}{\sqrt{1-u^2/c^2}} & \frac{1}{\sqrt{1-u^2/c^2}} \end{pmatrix}$$

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$$\begin{pmatrix} \langle\mathbf{G}'|\mathbf{G}\rangle & \langle\mathbf{G}'|\mathbf{P}\rangle \\ \langle\mathbf{P}'|\mathbf{G}\rangle & \langle\mathbf{P}'|\mathbf{P}\rangle \end{pmatrix} = \begin{pmatrix} \cosh\rho & -\sinh\rho \\ -\sinh\rho & \cosh\rho \end{pmatrix} = \begin{pmatrix} \langle\mathbf{G}'|L^{-1}|\mathbf{G}'\rangle & \langle\mathbf{G}'|L^{-1}|\mathbf{P}'\rangle \\ \langle\mathbf{P}'|L^{-1}|\mathbf{G}'\rangle & \langle\mathbf{P}'|L^{-1}|\mathbf{P}'\rangle \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1-u^2/c^2}} & \frac{-u/c}{\sqrt{1-u^2/c^2}} \\ \frac{-u/c}{\sqrt{1-u^2/c^2}} & \frac{1}{\sqrt{1-u^2/c^2}} \end{pmatrix}$$

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$$\begin{pmatrix} \langle\mathbf{G}|\mathbf{G}'\rangle & \langle\mathbf{G}|\mathbf{P}'\rangle \\ \langle\mathbf{P}|\mathbf{G}'\rangle & \langle\mathbf{P}|\mathbf{P}'\rangle \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} = \begin{pmatrix} \langle\mathbf{G}|L|\mathbf{G}\rangle & \langle\mathbf{G}|L|\mathbf{P}\rangle \\ \langle\mathbf{P}|L|\mathbf{G}\rangle & \langle\mathbf{P}|L|\mathbf{P}\rangle \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1-u^2/c^2}} & \frac{u/c}{\sqrt{1-u^2/c^2}} \\ \frac{u/c}{\sqrt{1-u^2/c^2}} & \frac{1}{\sqrt{1-u^2/c^2}} \end{pmatrix}$$

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$p = \langle\mathbf{P}|\mathbf{V}\rangle = \langle\mathbf{P}|\mathbf{G}'\rangle\langle\mathbf{G}'|\mathbf{V}\rangle + \langle\mathbf{P}|\mathbf{P}'\rangle\langle\mathbf{P}'|\mathbf{V}\rangle = \langle\mathbf{P}|\mathbf{G}'\rangle g' + \langle\mathbf{P}|\mathbf{P}'\rangle p'$

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$$\begin{pmatrix}\langle\mathbf{G}'|\mathbf{G}\rangle & \langle\mathbf{G}'|\mathbf{P}\rangle\\ \langle\mathbf{P}'|\mathbf{G}\rangle & \langle\mathbf{P}'|\mathbf{P}\rangle\end{pmatrix}=\begin{pmatrix}\cosh\rho & -\sinh\rho\\ -\sinh\rho & \cosh\rho\end{pmatrix}=\begin{pmatrix}\langle\mathbf{G}'|L^{-1}|\mathbf{G}'\rangle & \langle\mathbf{G}'|L^{-1}|\mathbf{P}'\rangle\\ \langle\mathbf{P}'|L^{-1}|\mathbf{G}'\rangle & \langle\mathbf{P}'|L^{-1}|\mathbf{P}'\rangle\end{pmatrix}=\begin{pmatrix}\frac{1}{\sqrt{1-u^2/c^2}} & \frac{-u/c}{\sqrt{1-u^2/c^2}}\\ \frac{-u/c}{\sqrt{1-u^2/c^2}} & \frac{1}{\sqrt{1-u^2/c^2}}\end{pmatrix}$$

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in matrix notation: $\begin{pmatrix}g\\ p\end{pmatrix}=\begin{pmatrix}\langle\mathbf{G}|\mathbf{G}'\rangle & \langle\mathbf{G}|\mathbf{P}'\rangle\\ \langle\mathbf{P}|\mathbf{G}'\rangle & \langle\mathbf{P}|\mathbf{P}'\rangle\end{pmatrix}\begin{pmatrix}g'\\ p'\end{pmatrix}=\begin{pmatrix}\cosh\rho & \sinh\rho\\ \sinh\rho & \cosh\rho\end{pmatrix}\begin{pmatrix}g'\\ p'\end{pmatrix}$

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$$\begin{pmatrix} \langle\mathbf{G'}|\mathbf{G}\rangle & \langle\mathbf{G'}|\mathbf{P}\rangle \\ \langle\mathbf{P'}|\mathbf{G}\rangle & \langle\mathbf{P'}|\mathbf{P}\rangle \end{pmatrix} = \begin{pmatrix} \cosh\rho & -\sinh\rho \\ -\sinh\rho & \cosh\rho \end{pmatrix} = \begin{pmatrix} \langle\mathbf{G'}|L^{-1}|\mathbf{G'}\rangle & \langle\mathbf{G'}|L^{-1}|\mathbf{P'}\rangle \\ \langle\mathbf{P'}|L^{-1}|\mathbf{G'}\rangle & \langle\mathbf{P'}|L^{-1}|\mathbf{P'}\rangle \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1-u^2/c^2}} & \frac{-u/c}{\sqrt{1-u^2/c^2}} \\ \frac{-u/c}{\sqrt{1-u^2/c^2}} & \frac{1}{\sqrt{1-u^2/c^2}} \end{pmatrix}$$

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in matrix notation:
$$\begin{pmatrix} g \\ p \end{pmatrix} = \begin{pmatrix} \langle\mathbf{G}|\mathbf{G'}\rangle & \langle\mathbf{G}|\mathbf{P'}\rangle \\ \langle\mathbf{P}|\mathbf{G'}\rangle & \langle\mathbf{P}|\mathbf{P'}\rangle \end{pmatrix}\begin{pmatrix} g' \\ p' \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix}\begin{pmatrix} g' \\ p' \end{pmatrix}$$

Test it! In per-space-time space-time (g,p)=(ck,ω)… …In space-time (g,p)=(x, ct) it's the same!

$$\begin{pmatrix} ck \\ \omega \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix}\begin{pmatrix} ck' \\ \omega' \end{pmatrix} \qquad \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix}\begin{pmatrix} x' \\ ct' \end{pmatrix}$$

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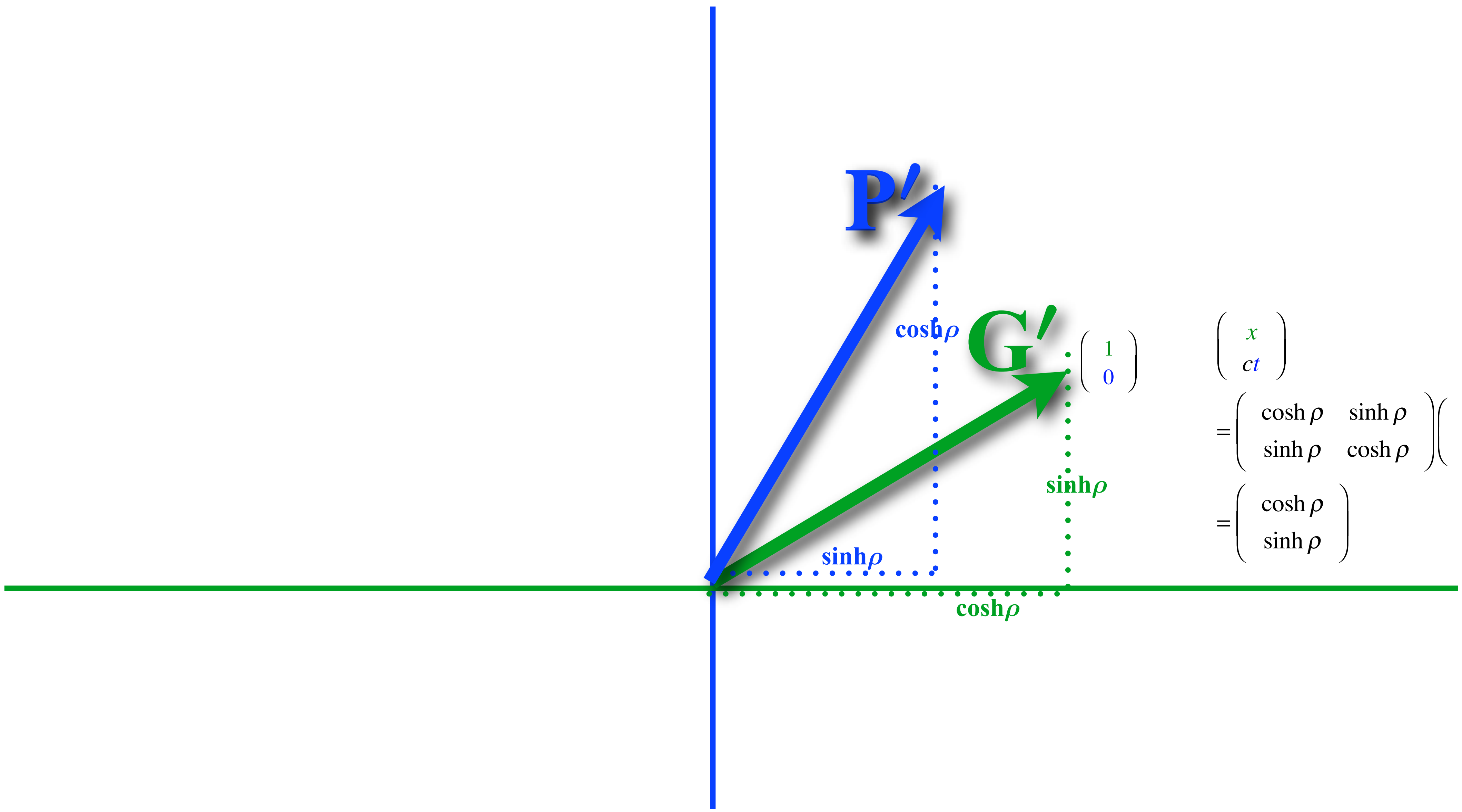


$$\begin{pmatrix}1\\0\end{pmatrix}$$

$$\begin{pmatrix}x\\ct\end{pmatrix} = \begin{pmatrix}\cosh\rho & \sinh\rho\\ \sinh\rho & \cosh\rho\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}\cosh\rho\\ \sinh\rho\end{pmatrix}$$

$$\begin{pmatrix}ck\\ \omega\end{pmatrix} = \begin{pmatrix}\cosh\rho & \sinh\rho\\ \sinh\rho & \cosh\rho\end{pmatrix}\begin{pmatrix}ck'\\ \omega'\end{pmatrix} \qquad \begin{pmatrix}x\\ ct\end{pmatrix} = \begin{pmatrix}\cosh\rho & \sinh\rho\\ \sinh\rho & \cosh\rho\end{pmatrix}\begin{pmatrix}x'\\ ct'\end{pmatrix}$$

3. Spectral theory of Einstein-Lorentz relativity

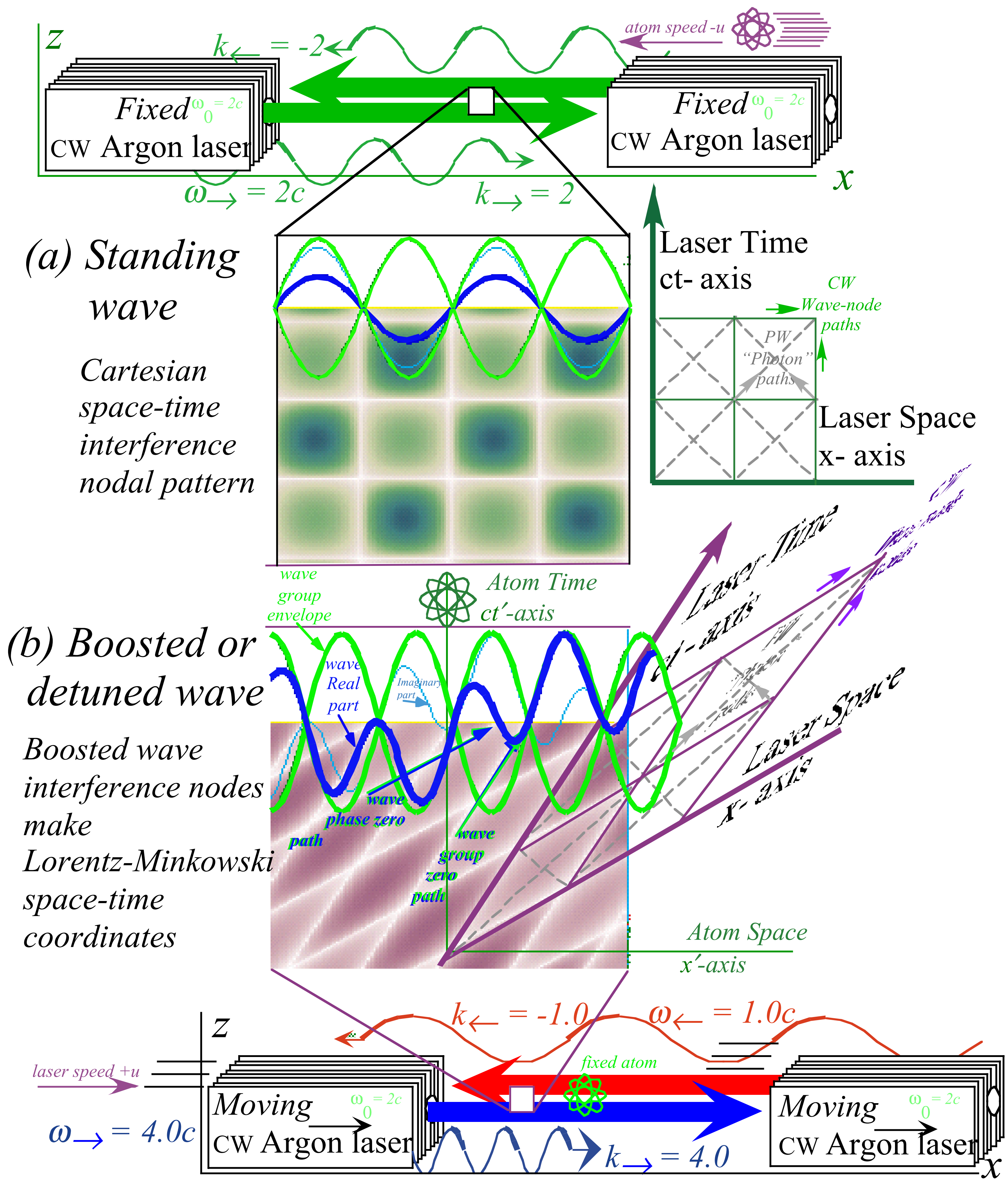
Applying Doppler Shifts to per-space-time (ck,ω) graph

CW Minkowski space-time coordinates (x,ct) and PW grids

Relating Doppler Shifts b or $r=1/b$ to velocity u/c or rapidity ρ

Lorentz transformation

Lorentz length-contraction and Einstein time-dilation


z
$k_{\leftarrow} = -2$
atom speed -u
Fixed $\omega_0 = 2c$
CW Argon laser
Fixed $\omega_0 = 2c$
CW Argon laser
x
$\omega_{\rightarrow} = 2c$
$k_{\rightarrow} = 2$
(a) Standing wave
Cartesian space-time interference nodal pattern
Laser Time ct- axis
CW Wave-node paths
PW "Photon" paths
Laser Space x- axis
wave group envelope
Atom Time ct′-axis
(b) Boosted or detuned wave
Boosted wave interference nodes make Lorentz-Minkowski space-time coordinates
wave Real part
Imaginary part
wave phase zero path
wave group zero path
Laser Time ct- axis
Laser Space x- axis
Atom Space x′-axis
z
$k_{\leftarrow} = -1.0$
$\omega_{\leftarrow} = 1.0c$
laser speed +u
fixed atom
Moving $\omega_0 = 2c$
CW Argon laser
Moving $\omega_0 = 2c$
CW Argon laser
$\omega_{\rightarrow} = 4.0c$
$k_{\rightarrow} = 4.0$
x

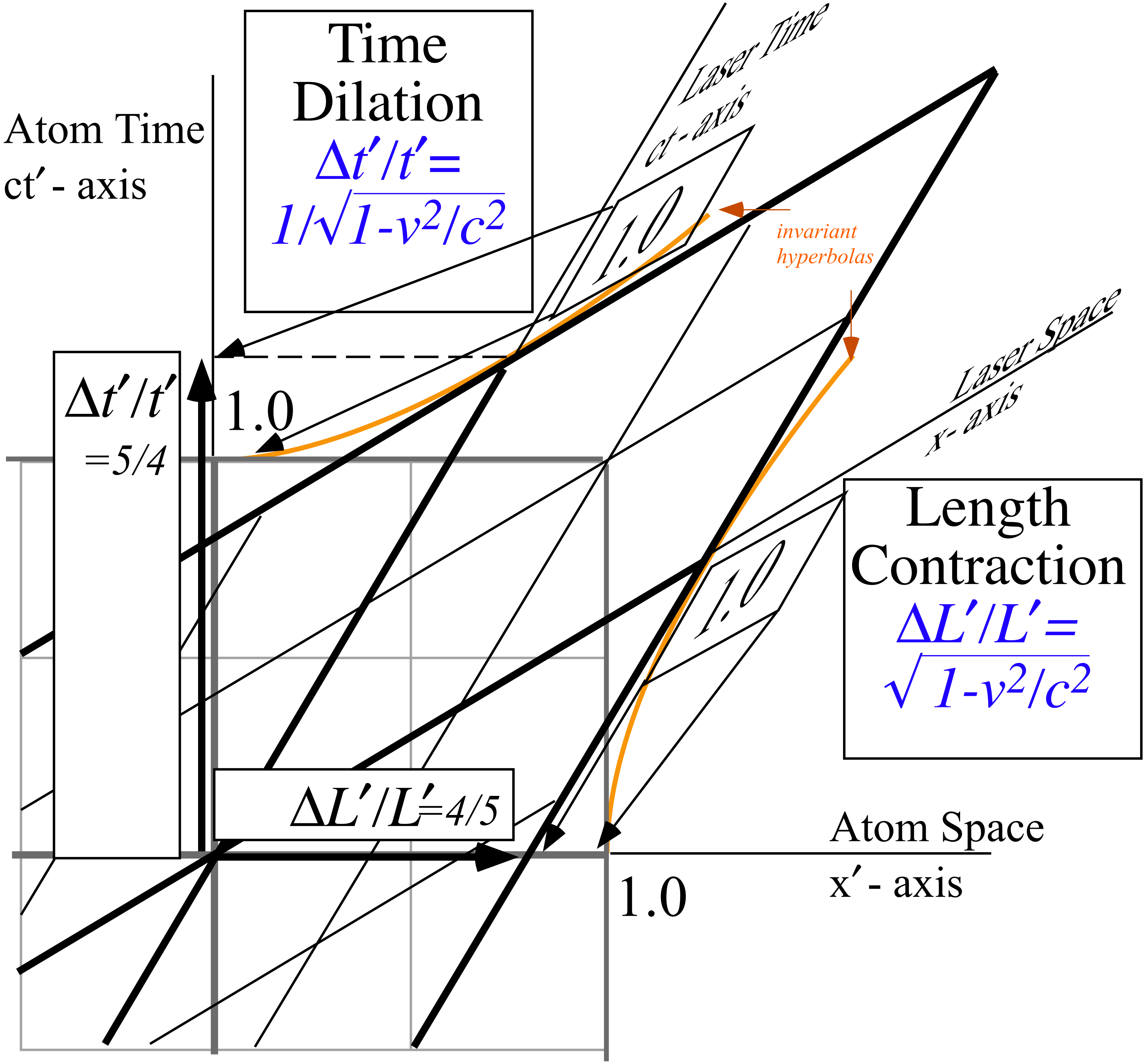


Fig. 2.4 Space-time grid intersections mark Lorentz contraction and Einstein time dilation.

4. Einstein-Lorentz symmetry

What happened to Galilean symmetry? *(It moved to "gauge" space!)*

Thale's construction and Euclid's means

Time reversal symmetry gives hyperbolic invariants

per-space-time hyperbola

space-time hyperbola

Phase invariance

Euclid's 3-means (300 BC)

Geometric "heart" of wave mechanics

Thales (580BC) rectangle-in-circle

Relates to wave interference by (Galilean) phasor angular velocity addition

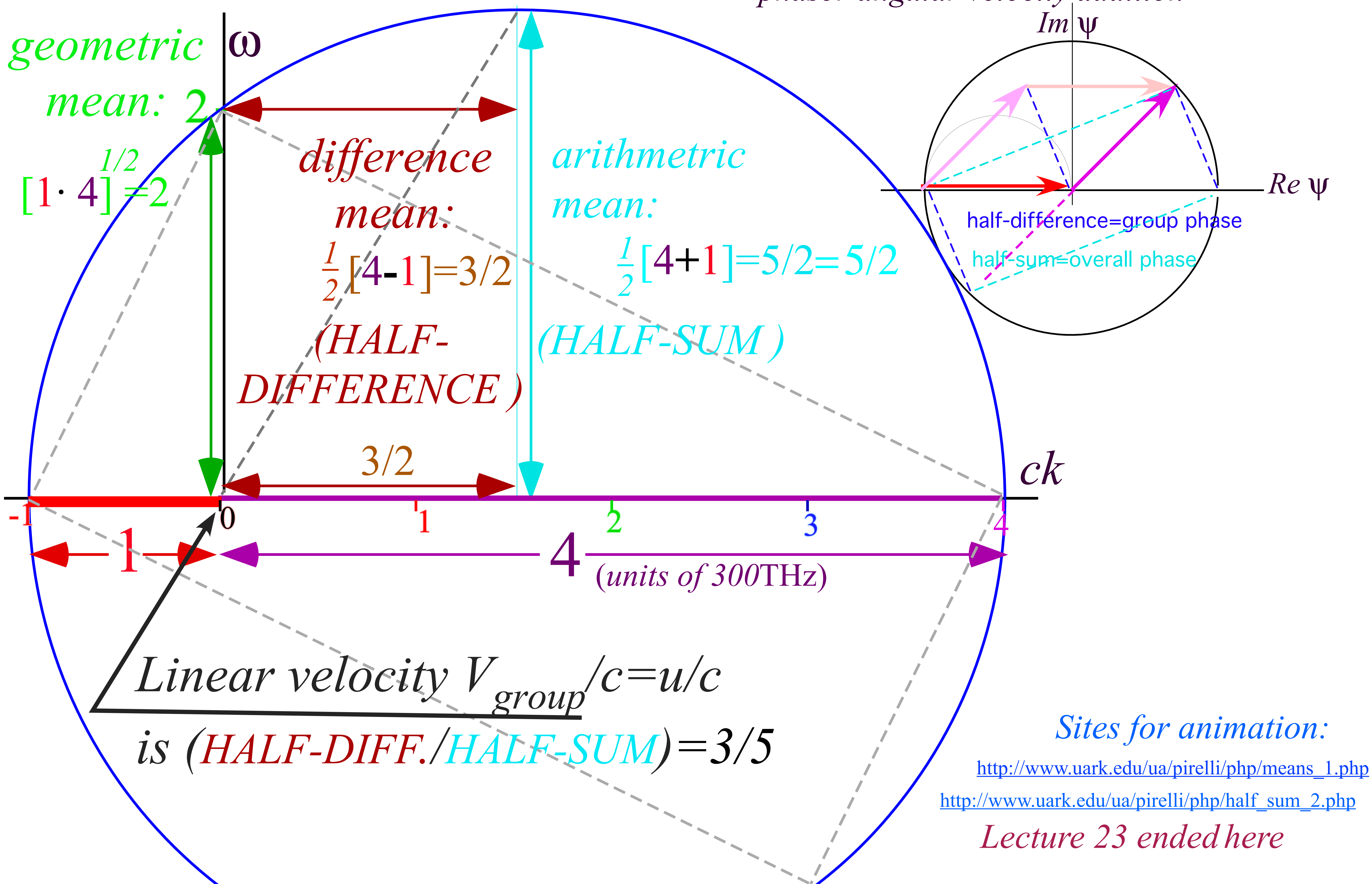


Fig. 3.3a Euclidian mean geometry for counter-moving waves of frequency 1 and 4. (300THz units).

http://www.uark.edu/ua/pirelli/php/means_1.php

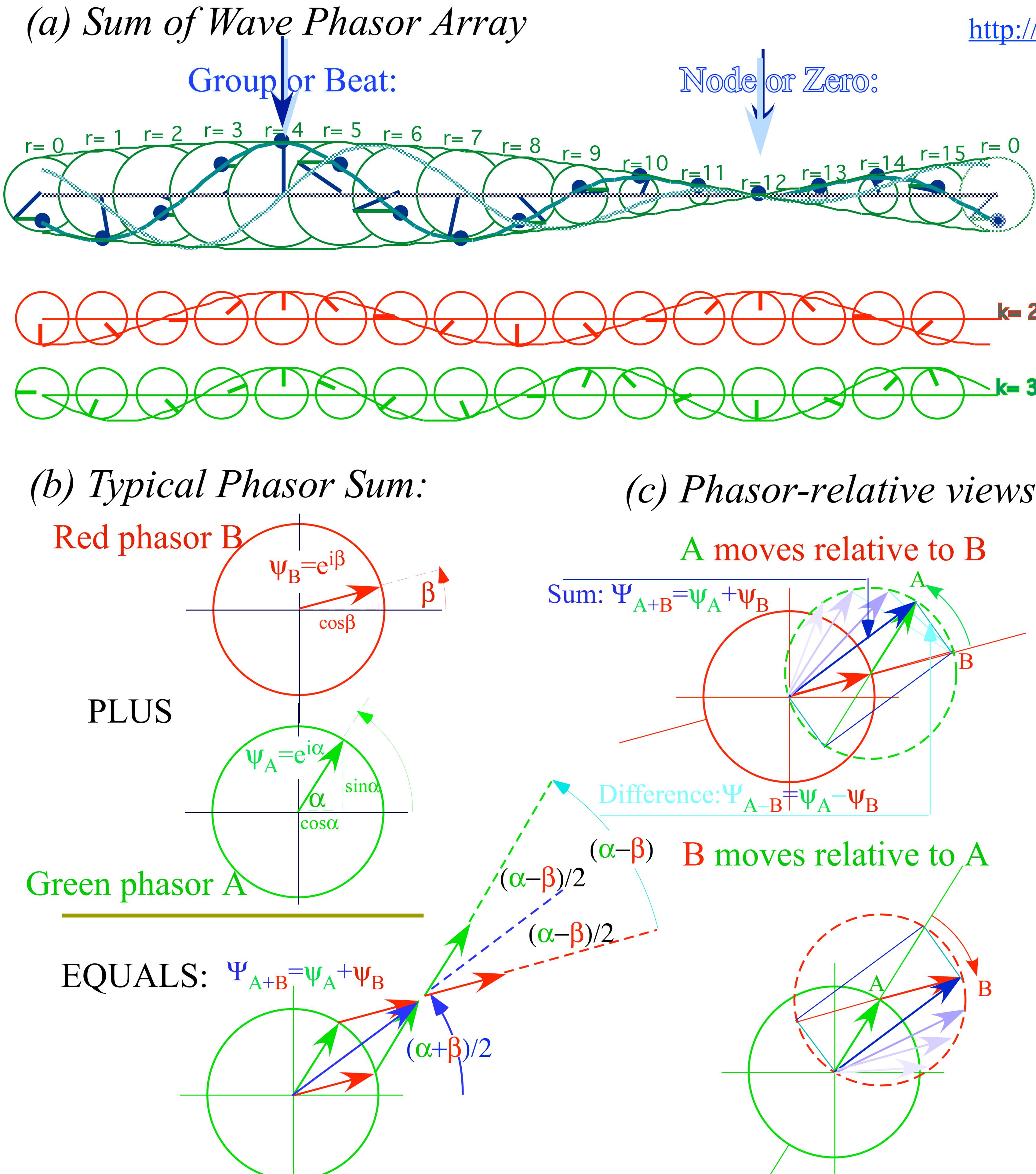


Fig. 3.1 Wave phasor addition. (a) Each phasor in a wave array is a sum (b) of two component phasors.