Lecture 22.

Relativity of wave-optics and Lorentz-Minkowski coordinates I.
(Ch. 2 of Unit 2)

1. Optical wave coordinates and frames
   Old-fashioned vs. New-fashioned spacetime frames
   Dueling lasers make lab frame space-time grid (CW or PW)
   Comparing Continuous-Wave (CW) vs. Pulse-Wave (PW) frames

2. Applying Occam’s razor to relativity axioms
   Einstein PW Axioms versus Evenson CW Axioms
   CW light clearly shows Doppler shifts
   Check that red is red is red,...green is green is green,...blue is blue is blue,... etc.
   Is dispersion linear? ... does astronomy work?... how about spectroscopy?
   Is Doppler a geometric factor or arithmetic sum?
   Introducing rapidity $\rho = \ln b$.
   That old Time-Reversal meta-Axiom (that is so-oo-o neglected!)

3. Spectral theory of Einstein-Lorentz relativity
   Applying Doppler Shifts to per-space-time $(ck, \omega)$ graph
   CW Minkowski space-time coordinates $(x, ct)$ and PW grids
   Relating Doppler Shifts $b$ or $r = 1/b$ to velocity $u/c$ or rapidity $\rho$
   Lorentz transformation

Lecture 22 ended (about) here
1. Optical wave coordinates and frames

Old-fashioned vs. New-fashioned spacetime frames
Dueling lasers make lab frame space-time grid (CW or PW)
Comparing Continuous-Wave (CW) vs. Pulse-Wave (PW) frames
• **Optical wave coordinate manifolds and frames**

*Shining some light on light using complex phasor analysis*

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**Old-fashioned meter-stick-clock frames**

E. F. Taylor and J. A. Wheeler Spacetime Physics (Freeman San Francisco 1966)

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**New-fashioned laser clocks & meter sticks**

Complex Phasor Clocks: Tesla's AC "phasor"

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Quantum Phasor Clock

\[ \Psi = Ae^{i(kx - \omega t)} \]

\[ = A \cos(kx - \omega t) + iA \sin(kx - \omega t) \]

Phasor clocks turn clockwise in time for positive \( \omega \)

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300THz Laser plane wave \( \langle x, t | k, \omega \rangle = Ae^{i(kx - wt)} \)

\[ Re \Psi = \cos(kx - \omega t) \]

\[ Im \Psi = \sin(kx - \omega t) \]
New-fashioned laser clocks & meter sticks (contd.)

Dual views:

1. Spacetime
   \(x\) versus \(ct\)

   \[k = +1\]
   \[\omega = \frac{1}{c}\]

   “laser phasors”

   Period \(\tau = \frac{2\pi}{\omega} = \frac{1}{\nu}\)

   Wavelength \(\lambda = \frac{2\pi}{k} = \frac{1}{\kappa}\)

   \(\nu \cdot \lambda = c\)

2. Per-Spacetime
   \(\omega\) versus \(ck\)

   \[\Delta \text{Frequency} \quad \omega \quad 3\]
   \[900\text{THz} \quad \nu = \frac{\omega}{2\pi} \quad \text{per-sec.}\]
   \[600\text{THz} \quad \nu - \lambda = c\]

   \[300\text{THz} \quad \nu + \lambda = c\]

   “1st Base”

Single plane-wave meter-stick-clocks are too fast
(can’t catch ’em)

Interfering wave pairs needed

to make rest frame coordinates...

(...But at least this view is constant)
1. Optical wave coordinates and frames

Old-fashioned vs. New-fashioned spacetime frames

Dueling lasers make lab frame space-time grid (CW or PW)

Comparing Continuous-Wave (CW) vs. Pulse-Wave (PW) frames
Zeros of head-on CW sum gives \((x, ct)\)-grid

Find zeros by factoring sum:

\[
\Psi = e^{ia} + e^{ib} = e^{i(a+b)/2} \left( e^{i(a-b)/2} + e^{-i(a-b)/2} \right)
\]

Phase factor:

\[\exp(i(a+b)/2) = e^{i(\omega t)}\]

Group factor:

\[2\cos\left(\frac{a-b}{2}\right) = 2\cos(kx)\]
1. Optical wave coordinates and frames
   Old-fashioned vs. New-fashioned spacetime frames
   Dueling lasers make lab frame space-time grid (CW or PW)
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Newton’s “Fits” in Optical Interference

Newton complained that light waves have “fits” (what we now know as wave interference or resonance.) Examples of interference are head-on collision of two Continuous Waves (2-CW) or two Pulse Waves (PW)

**Continuous Wave (CW) Addition**

![Continuous Wave (CW) Addition Diagram]

**Pulse Wave (PW) Addition**

![Pulse Wave (PW) Addition Diagram]

**Pulse Wave (PW) sum compared with Continuous Wave (CW) sum**

- **PW waves** are OFF (0) or ON (1)
- **PW sum** is Boolean (0_L,0_R),(0_L,1_R), (1_L,0_R),(1_L,1_R).
- **PW time peak-diamond** paths are 
  *wysiwyg*. (What you see is what you expect!)

- **Continuous Wave (CW) sum**
  - **CW waves** range continuously from -1 to +1
  - **CW sum** is more subtle and nuanced interference.
  - **CW time zero-square** paths are subtle results of the half-sum \( P \)-rule and the half-difference \( G \)-rule of phase \( P \) and group \( G \) zeros.
(a) CW squares

1 femtosecond
1.0 fs = 10^{-15} s

1 micron
1.0 μm = 10^{-6} meter

Speed of light $c$
$c = \frac{\lambda}{\tau} = \frac{0.5 \cdot 10^{-6}}{5/3 \cdot 10^{-15}} = 3 \cdot 10^8$

(b) PW diamonds

"pataooey!"
2. Applying Occam’s razor to relativity axioms

Einstein PW Axioms versus Evenson CW Axioms

CW light clearly shows Doppler shifts

Check that red is red is red, ... green is green is green, ... blue is blue is blue, ... etc.

Is dispersion linear? ... does astronomy work? ... how about spectroscopy?

Is Doppler a geometric factor or arithmetic sum?

Introducing rapidity $\rho = \ln b$.

That old Time-Reversal meta-Axiom (that is so-oo-o neglected!)
Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is c

A “road-runner” axiom is a “show-stopper”

Using Occam’s Razor
(and Evenson’s lasers)

Complicated

PW peaks precisely locate places where wave is.

Simpler

CW zeros precisely locate places where wave is not.

Evenson Continuous Wave (CW) axiom: CW speed for all colors is c

More self-evident “must-be” axiom
2. Applying Occam’s razor to relativity axioms

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That old Time-Reversal meta-Axiom (that is so-oo-o neglected!)
Doppler Blueshift
More “hits” per sec. if moving toward laser source

Doppler Redshift
Fewer “hits” per sec. if moving away from laser source

Doppler’s picture needs revision for light whose period and wavelength both shift.

Why?

...So that all colors go the same speed!

\[ \nu \cdot \lambda = \frac{\lambda}{\tau} = \frac{\omega}{k} = c \]

\[ \nu \cdot \lambda = \frac{\lambda}{\tau} = \frac{\omega}{k} = c \quad \text{etc.} \]
**CW Axiom ("All colors go c.") based on Doppler effects**

Showing that Green is Green is Green...(and all the same speed)... Any color (like 600THz green) may be made by any other color source Doppler shifted by some speed $u$ (less than $c$)

How many ways can you make 600THz green?

- Higher frequency $\nu$ source recedes so its Doppler factor ($0 < r < 1$) red-shifts $\nu$ to match a 600THz-tuned receiver.
- Frequency $\nu = 600$THz matches receiver.
- Lower frequency $\nu$ source approaches so its Doppler factor ($1 < b < \infty$) blue-shifts $\nu$ to match a 600THz-tuned receiver.

How many kinds of green exist? (It’s either 1 or $\infty$.)
2. Applying Occam’s razor to relativity axioms

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Introducing rapidity \( \rho = \ln b \).

That old Time-Reversal meta-Axiom (that is so-oo-o neglected!)
Evenson CW Axiom ("All colors go c.") is only reasonable conclusion:

**Linear dispersion:** $\omega = ck$

*Linear dispersion means NO dispersion*

*Einstein PW is corollary of Evenson CW*

- $\omega = ck$
- or:
- $\nu = c/\lambda$

**vacuum can’t support an \(\infty\)-number of “other speeds”**
Evenson CW Axiom ("All colors go c.") is only reasonable conclusion: **Linear dispersion**: $\omega = ck$

- Linear dispersion means **NO** dispersion
- Einstein PW is corollary of Evenson CW

What if **blue** were to travel 0.001% slower than **red** from a galaxy 9 billion light years away? (..and show up $10^5$ years late)

That would mean Good-Bye Hubble Astronomy!
**Evenson CW Axiom** ("All colors go c.") is only reasonable conclusion:

**Linear dispersion:** $\omega = ck$

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What if blue were to travel 0.001% slower than red from a galaxy 9 billion light years away? (..and show up $10^5$ years late)

That would mean Good-Bye Hubble Astronomy!

What if $\nu=600$THz green excited an Ar atom but NOT a $\lambda=0.500\mu m$ optical cavity? (or vice-versa?)

That would mean Good-Bye Light Amplification by Stimulated Emission of Radiation.
Linear Dispersion (means NO dispersion) has all colors (Fourier components) march in “lockstep”

\[ \omega = (\text{Const.}) \cdot k \]

NON-linear Dispersion (has dispersion) so different colors (Fourier components) go different speeds

\[ \omega = \omega(k) \]

See animation: www.uark.edu/ua/pirelli/php/train_PW_Occum_Evenson.php
2. Applying Occam’s razor to relativity axioms

Einstein PW Axioms versus Evenson CW Axioms

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Is dispersion linear? ... does astronomy work?... how about spectroscopy?

Is Doppler a geometric factor or arithmetic sum?

Introducing rapidity $\rho = \ln b$.

That old Time-Reversal meta-Axiom (that is so-oo-o neglected!)
If all colors always march in lock-step then any Doppler shift must be geometric factor, that is, the same multiplier for all colors.

If $300\text{THz}$ Doppler shifts to $600\text{THz}$ ($1 \text{octave-shift} = 2.0$)

Then $600\text{THz}$ Doppler shifts to $1200\text{THz}$ ($1 \text{octave-shift} = 2.0$)
If all colors always march in lock-step then any Doppler shift must be geometric factor, that is, the same multiplier for all colors.

If 300THz Doppler shifts to 600THz (1 octave-shift = 2.0)

Then 600THz Doppler shifts to 1200THz (1 octave-shift = 2.0)

Doppler shifts maintain frequency ratios (not differences)

1-D Doppler shifts \{\text{red}=e^{-\rho} \ldots \text{blue}=e^{+\rho}\} form a Lie Group

3-D Doppler shifts are hypercomplex elements of Lorentz Group
2. Applying Occam’s razor to relativity axioms

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Introducing rapidity $\rho = \ln b$.

That old Time-Reversal meta-Axiom (that is so-oo-o neglected!)
Frequency **blue shift $b$ when Source-Receiver interval is**

$$\frac{v_{\text{IN}}}{v_{\text{OUT}}} = \frac{v_{\text{Receiver}}}{v_{\text{Source}}} = b = e^{+|\rho|} > 1$$

Defining **Rapidity $\rho$ as logarithm of Doppler**

$$\rho = \ln(b \text{ or } r)$$

**CLOSING**

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Frequency **red shift $r$ when Source-Receiver interval is**

$$\frac{v_{\text{Receiver}}}{v_{\text{Source}}} = r = e^{-|\rho|} < 1$$

**OPENING**
**Frequency blue shift** $b$ when Source-Receiver interval is closing:

$$\frac{v_{IN}}{v_{OUT}} = \frac{v_{Receiver}}{v_{Source}} = b = e^{+|\rho|} > 1$$

**Examples:**

- **Receiver** $A$ \rightarrow **Source** $B$ with $v_{600THz} \rightarrow v_{300THz}$ and $v_{OUT} = b = 2$
  
  \[ \rho = \ln(2) = 0.69 \]

- **Source** $C$ \rightarrow **Receiver** $A$ with $v_{1200THz} \rightarrow v_{600THz}$ and $v_{IN} = c = 2$

**Frequency red shift** $r$ when Source-Receiver interval is opening:

$$\frac{v_{Receiver}}{v_{Source}} = r = e^{-|\rho|} < 1$$

**Examples:**

- **Source** $A$ \rightarrow **Receiver** $B$ with $v_{600THz} \rightarrow v_{300THz}$ and $v_{OUT} = r = 1/2$
  
  \[ \rho = \ln(1/2) = -0.69 \]

- **Receiver** $C$ \rightarrow **Source** $A$ with $v_{600THz} \rightarrow v_{1200THz}$ and $v_{IN} = c = 2$

**Defining Rapidity** $\rho$ as logarithm of Doppler:

\[ \rho = \ln(b \text{ or } r) \]

**Time Reversal**
Each Doppler shift $\frac{v_A}{v_B}$ maps to a Lorentz transformation $T_{AB}$.

\[
\begin{align*}
\frac{v_A}{v_B} &= b_{AB} = e^{\rho_{AB}} = 2 \\
\rho_{AB} &= \ln(2) = 0.69 \\
\frac{v_B}{v_C} &= b_{BC} = e^{\rho_{BC}} = 1/4 \\
\rho_{BC} &= \ln(1/4) = -1.38 \\
\frac{v_C}{v_A} &= b_{CA} = e^{\rho_{CA}} = 2 \\
\rho_{CA} &= \ln(2) = 0.69
\end{align*}
\]
Each Doppler shift $\frac{v_A}{v_B}$ maps to a Lorentz transformation $T_{AB}$

- Receiver $A$ Source $B$ Receiver $B$ Source $C$ Receiver $C$ Source $A$

$\frac{v_A}{v_B} = b_{AB} = e^{\rho_{AB}} = 2$
$\rho_{AB} = \ln(2) = 0.69$

$\frac{v_B}{v_C} = b_{BC} = e^{\rho_{BC}} = 1/4$
$\rho_{BC} = \ln(1/4) = -1.38$

$\frac{v_C}{v_A} = b_{CA} = e^{\rho_{CA}} = 2$
$\rho_{CA} = \ln(2) = 0.69$

Group product is represented:

$$T_{AB} \cdot T_{BC} = T_{CA}$$

$\frac{v_A}{v_B} \cdot \frac{v_B}{v_C} = \frac{v_A}{v_C}$
$e^{\rho_{AB}} e^{\rho_{BC}} = e^{\rho_{AC}} = e^{(\rho_{AB} + \rho_{BC})}$

...and rapidity $\rho_{AB}$ is a Galilean (arithmetic) parameter

To be shown: $\rho_{AB} = \text{atanh}(u_{AB}/c)$ approaches $(u_{AB}/c)$ for: $\rho_{AB} < < 1$
2. Applying Occam’s razor to relativity axioms

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That old Time-Reversal meta-Axiom (that is so-oo-o neglected!)
Inverse to Lorentz transformation \( T_{AB} \) is \( T_{BA} \)

.. just as the arithmetic inverse of \( \frac{v_A}{v_B} \) is \( \frac{v_B}{v_A} \)

.. just as the arithmetic inverse... of \( e^{\rho_{AB}} \) is \( e^{\rho_{BA}} = e^{-\rho_{AB}} \)

.. just as the arithmetic inverse... of \( \rho_{AB} \) is \( \rho_{BA} = -\rho_{AB} \)

**Detailed time reversal symmetry** implies \( r = 1/b \).

Approaching source (600THz green)

Receding receiver sees Doppler red-shift of 1200THz source to 600THz
(600THz)=\( r(1200\text{THz}) \) with \( r=1/2 \)

3. Spectral theory of Einstein-Lorentz relativity

Applying Doppler Shifts to per-space-time \((ck, \omega)\) graph

CW Minkowski space-time coordinates \((x, ct)\) and PW grids

Relating Doppler Shifts \(b\) or \(r = 1/b\) to velocity \(u/c\) or rapidity \(\rho\)

Lorentz transformation
Deriving Spacetime and per-spacetime coordinate geometry by:

1. Evenson CW axiom “All colors go c” keeps $K_A$ and $K_B$ on their baselines.
2. Time-Reversal axiom: $r = 1/b$
3. Half-Sum Phase $P = (R + L)/2$ and Half-Difference Group $G = (R - L)/2$

Laser Per-Spacetime

Atom Per-Spacetime

$\omega$ versus $ck$

$\omega'$ versus $ck'$

$K_A = (+4, 4)$

$K_B = (-1, 1)$

$\omega' = 2 \cdot (2) = 4$

$\omega' = 1/2 \cdot (2) = 1$

3rd baseline

1st baseline

300THz

600THz

750THz or 400nm

500THz or 600nm

400THz or 750nm

Laser per-space

ck

Atom per-space

ck'

Halved 3rd base distance

Doubled 1st base distance

Tuesday, March 27, 2012
3. **Spectral theory of Einstein-Lorentz relativity**

Applying **Doppler Shifts** to per-space-time \((ck, \omega)\) graph

CW Minkowski space-time coordinates \((x, ct)\) and PW grids

Relating **Doppler Shifts** \(b\) or \(r = 1/b\) to velocity \(u/c\) or rapidity \(\rho\)

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**LaserPer-Spacetime**

<table>
<thead>
<tr>
<th>$\omega$ versus $ck$</th>
<th>1200THz</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>900THz</td>
</tr>
<tr>
<td>2</td>
<td>600THz</td>
</tr>
<tr>
<td>1</td>
<td>300THz</td>
</tr>
</tbody>
</table>

**AtomPer-Spacetime**

<table>
<thead>
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<th>$\omega'$ versus $ck'$</th>
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<tr>
<td>4</td>
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<td>2</td>
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</tbody>
</table>

- $K_A = (+4,4)$
- $K_B = (-1,1)$
- $\omega'_1 = 2.0 \cdot (2) = 4$
- $\omega'_3 = 1.2 \cdot (2) = 1$

---

**Solvent Coordination**
Deriving Spacetime and per-spacetime coordinate geometry by:

1. **Evenson CW axiom** “All colors go c” keeps $K_A$ and $K_B$ on their baselines.
2. Time-Reversal axiom: $r = 1/b$
3. Half-Sum Phase $P = (R + L)/2$ and Half-Difference Group $G = (R - L)/2$
Deriving Spacetime and per-spacetime coordinate geometry by:

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Deriving Spacetime and per-spacetime coordinate geometry by:

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2. Time-Reverse axiom: $r = 1/b$
3. Half-Sum Phase $P = (R+L)/2$ and Half-Difference Group $G = (R-L)/2$
Deriving Spacetime and per-spacetime coordinate geometry by...

(1) Evenson CW axiom "All colors go c" keeps $K_A$ and $K_B$ on their baselines.

(2) Time-Reversal axiom: $r = 1/b$

(3) Half-Sum Phase $P = (R+L)/2$ and Half-Difference Group $G = (R-L)/2$

This is per-space ($ck'$) axis in per-space-time or
This is space ($x'$) axis in space-time

This is per-time ($\omega'$) axis in per-space-time or
This is time ($ct'$) axis in space-time

"lab" ($\omega$) axis or
"lab" ($ct$) axis or
"lab" ($ck$) axis or
"lab" ($x$) axis

LaserPer-Spacetime

AtomPer-Spacetime

3rd baseline 1st baseline

This is per-space ($ck'$) axis in per-space-time or
This is space ($x'$) axis in space-time

"lab" ($ck$) axis or
"lab" ($x$) axis

Tuesday, March 27, 2012