Geometry and Motion of Isotropic Harmonic Oscillators
(Ch. 9 and Ch. 11 of Unit 1)

Geometry of idealized “Sophomore-physics Earth”
Coulomb field outside Isotropic Harmonic Oscillator (IHO) field inside
Contact-geometry of potential curve(s)
“Crushed-Earth” models: 3 key energy “steps” and 4 key energy “levels”
Earth matter vs nuclear matter:
Introducing the “neutron starlet” and “Black-Hole-Earth”

Isotropic harmonic oscillator dynamics in 1D, 2D, and 3D
Sinusoidal space-time dynamics derived by geometry
Isotropic harmonic oscillator orbits in 1D and 2D (You get 3D for free!)
Constructing 2D Isotropic harmonic oscillator orbits using phasor plots
Examples with x-y phase lag: $\alpha_{xy} = \alpha_x - \alpha_y = 15^\circ, 30^\circ, \text{and } \pm 75^\circ$
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Earth matter vs nuclear matter:

Introducing the “neutron starlet” and “Black-Hole-Earth”
Coulomb force vanishes inside-spherical shell \((\text{Gauss-law})\)

Shell mass element
\[
m = (\text{solid-angle factor } A) \, d^2
\]

Gravity at \(R\) due to shell mass elements
\[
\frac{G \, M}{D^2} - \frac{G \, m}{d^2} = \frac{A \, d\Omega}{\sin \Theta}
\]
\[
(D^2 - \frac{d^2}{d^2}) A = 0
\]

Cancellation of Shell mass element
\[
M = (\text{solid-angle factor } A)D^2
\]

Coulomb force inside-spherical body due to stuff below you, only.

Gravitational force at \(r_<\) is just that of planet \(m_c\) below \(r_<\)

\(M_<\) \(r_<\)

Unit 1
Fig. 9.6
Coulomb force vanishes inside-spherical shell (Gauss-law)

Shell mass element
\[ m = (\text{solid-angle factor } A) \, d^2 \]

Gravity at \( r \) due to shell mass elements
\[
\frac{G \, M}{D^2} - \frac{G \, m}{d^2} = A = \frac{d \Omega}{\sin \Theta}
\]
\[
\frac{(D^2 - d^2)}{D^2} A = 0
\]

Cancellation of Shell mass element
\[ M = (\text{solid-angle factor } A) D^2 \]

Coulomb force inside-spherical body due to stuff below you, only.

Gravitational force at \( r_\prec \) is just that of planet \( m_\prec \) below \( r_\prec \)

\[
F_{\text{inside}}(r_\prec) = G \frac{m M_\prec}{r_\prec^2} = G m \frac{4\pi}{3} \frac{M_\prec}{r_\prec} = G m \frac{4\pi}{3} \rho_\oplus r_\prec = m g \frac{r_\prec}{R_\oplus} \equiv m g \cdot x
\]

Note:
Hooke’s (linear) force law for \( r_\prec \) inside uniform body

Unit 1 Fig. 9.6
Coulomb force vanishes inside-spherical shell (Gauss-law)

Shell mass element
\[ m = \text{solid-angle factor } A \times d^2 \]

Gravity at \( r \)
due to shell mass elements
\[ \frac{G M}{D^2} - \frac{G m}{d^2} = \]
\[ \left( \frac{D^2}{d^2} - \frac{d^2}{d^2} \right) A = 0 \]

Cancellation of
Shell mass element
\[ M = \text{solid-angle factor } A \times D^2 \]

Coulomb force inside-spherical body due to stuff below you, only.

Gravitational force at \( r_\prec \) is just that of planet \( M_\prec \) below \( r_\prec \)

\[ F_{\text{inside}}(r_\prec) = G \frac{mM_\prec}{r_\prec^2} = Gm \frac{4\pi}{3} \frac{M_\prec}{r_\prec^3} r_\prec = Gm \frac{4\pi}{3} \rho_\odot r_\prec = mg \frac{r_\prec}{R_\odot} \equiv mg \cdot x \]

Earth surface gravity acceleration: \( g = G \frac{M_\odot}{R_\odot^2} = G \frac{M_\odot}{R_\odot^3} R_\odot = G \frac{4\pi}{3} \frac{M_\odot}{R_\odot^3} R_\odot = G \frac{4\pi}{3} \rho_\odot R_\odot = 9.8 m / s \)

\( G = 6.67384(80) \times 10^{-11} \text{Nm}^2/\text{C}^2 \sim (2/3) \times 10^{-10} \)
Coulomb force vanishes inside-spherical shell (Gauss-law)

Gravity at \( r \) due to shell mass elements
\[
\frac{GM}{D^2} - \frac{Gm}{d^2} = \frac{(D^2 - d^2)}{D^2} A = 0
\]

Cancellation of Shell mass element
\[
M = (\text{solid-angle factor } A) D^2
\]

Coulomb force inside-spherical body due to stuff below you, only.

Gravitational force at \( r_\leq \) is just that of planet \( M_\leq \) below \( r_\leq \)

\[
F_{\text{inside}}(r_\leq) = G \frac{mM_\leq}{r_\leq^2} = Gm \frac{4\pi}{3} \frac{M_\leq}{r_\leq^3} \equiv mg \frac{r_\leq}{R_\oplus} = mg \cdot x
\]

Earth surface gravity acceleration: 
\[
g = G \frac{M_\oplus}{R_\oplus^2} = G \frac{M_\oplus}{R_\oplus^3} R_\oplus = G \frac{4\pi}{3} \frac{M_\oplus}{R_\oplus^3} \rho_\oplus R_\oplus = G \frac{4\pi}{3} \rho_\oplus R_\oplus = 9.8 \text{ m/s}
\]

\[G = 6.67384(80) \times 10^{-11} \text{Nm}^2/\text{C}^2 \sim (2/3)10^{-10}\]

Earth radius: \( R_\oplus = 6.371 \times 10^6 \text{ m} \approx 6.4 \times 10^6 \text{ m} \)

Earth mass: \( M_\oplus = 5.9722 \times 10^{24} \text{ kg} \approx 6.0 \times 10^{24} \text{ kg} \)

Note:
Hooke’s (linear) force law for \( r_\leq \) inside uniform body

Solar radius: \( R_\odot = 6.955 \times 10^8 \text{ m} \approx 7.0 \times 10^8 \text{ m} \)

Solar mass: \( M_\odot = 1.9889 \times 10^{30} \text{ kg} \approx 2.0 \times 10^{30} \text{ kg} \)
Geometry of idealized “Sophomore-physics Earth”

Coulomb field outside  Isotropic Harmonic Oscillator (IHO) field inside

Contact-geometry of potential curve(s)

“Crushed-Earth” models: 3 key energy “steps” and 4 key energy “levels”

Earth matter vs nuclear matter:

Introducing the “neutron starlet” and “Black-Hole-Earth”
The ideal “Sophomore-Physics-Earth” model of geo-gravity

\[ F(x) = -x \] (inside Earth)
\[ U(x) = (x^2 - 3)/2 \]

\[ F(x) = 1/x^2 \] (outside Earth)
\[ U(x) = -1/x \]
...conventional parabolic geometry...carried to extremes...

(Review of Lect. 6 p.29)

Unit 1
Fig. 9.4
Geometry of idealized “Sophomore-physics Earth”

Coulomb field outside Isotropic Harmonic Oscillator (IHO) field inside

Contact-geometry of potential curve(s)

“Crushed-Earth” models: 3 key energy “steps” and 4 key energy “levels”

Earth matter vs nuclear matter:

Introducing the “neutron starlet” and “Black-Hole-Earth”
The “Three (equal) steps from Hell”

- Orbit energy: \( E_\odot = -G \frac{M_\odot}{2R_\odot} \)
- Orbit speed: \( v_\odot = \sqrt{\frac{GM_\odot}{R_\odot}} \)

Surface gravity: \( g = -\frac{GM_\odot}{R_\odot^2} \)

Dissociation threshold: \( PE = 0 \)

Orbit level: \( PE = -G \frac{M_\odot}{2R_\odot} \)

Surface level: \( PE = -G \frac{M_\odot}{R_\odot} \)

Ground level: \( PE = -G \frac{3M_\odot}{2R_\odot} \)

Surface potential: \( PE = -\frac{GM_\odot}{R_\odot} \)

Surface escape speed: \( v_e = \sqrt{\frac{2GM_\odot}{R_\odot}} \)

\( KE = PE \) relation: \( \frac{1}{2}mv_e^2 = \frac{mM_\odot}{R_\odot} \)
If you could crush Earth radius 2-times-smaller...

2 times \( \bigodot \)-orbit energy: \( E_\bigodot = -G \frac{M_\oplus}{2R_\oplus} \)

\( \sqrt{2} \) times \( \bigodot \)-orbit speed: \( v_\bigodot = \sqrt{G \frac{M_\oplus}{R_\oplus}} \)

\( \sqrt{2} \) times surface escape speed: \( v_e = \sqrt{G \frac{2M_\oplus}{R_\oplus}} \)

4 times the surface gravity: \( g = -G \frac{M_\oplus}{R_\oplus^2} \)

1. 2x Crushed Earth
   - 1/2 radius
   - 8 times as dense
   - 1/8 focal distance or \( \lambda \)
   - 1/8 minimum radius of curvature
   - 8 times maximum curvature

2. 2 times surface potential: \( PE = -G \frac{M_\oplus}{R_\oplus} \)
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“Crushed-Earth” models: 3 key energy “steps” and 4 key energy “levels”

Earth matter vs nuclear matter:

Introducing the “neutron starlet” and “Black-Hole-Earth”
Examples of “crushed” matter

Earth matter

*Earth mass*: \( M_\oplus = 5.9722 \times 10^{24} \text{ kg} \approx 6.0 \cdot 10^{24} \text{ kg} \). Density \( \rho_\oplus \approx 6.0 \cdot 10^{24-21} \approx 6 \cdot 10^3 \text{kg/m}^3 \)

*Earth radius*: \( R_\oplus = 6.371 \cdot 10^6 \text{ m} = 6.4 \cdot 10^6 \text{ m} \)

*Earth volume*: \( (4\pi / 3)R_\oplus^3 = 4 \cdot 260 \cdot 10^{18} \approx 10^{21} \text{ m}^3 \)
Examples of “crushed” matter

Earth matter Earth mass: $M_\oplus = 5.9722 \times 10^{24} \text{kg} \approx 6.0 \cdot 10^{24} \text{kg}$. Density $\rho_\oplus \sim 6.0 \cdot 10^{24-21} \approx 6 \cdot 10^3 \text{kg/m}^3$

Earth radius: $R_\oplus = 6.371 \cdot 10^6 \text{m} = 6.4 \cdot 10^6 \text{m}$ Earth volume: $(4\pi/3)R_\oplus^3 = 4 \cdot 260 \cdot 10^{18} \sim 10^{21} \text{m}^3$

Nuclear matter Nucleon mass $= 1.67 \cdot 10^{-27} \text{kg} \sim 2 \cdot 10^{-27} \text{kg}$.

Say a nucleus of atomic weight 50 has a radius of 3 fm, or 50 nucleons each with a mass $2 \cdot 10^{-27} \text{kg}$.

That’s $100 \cdot 10^{-27} = 10^{-25} \text{ kg}$ packed into a volume of $\frac{4\pi}{3}r^3 = \frac{4\pi}{3} (3 \cdot 10^{-15})^3 \text{ m}^3$ or about $10^{-43} \text{ m}^3$. 
Examples of “crushed” matter

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Nuclear matter  Nucleon mass = \( 1.67 \cdot 10^{-27} \text{ kg} \). \( \approx 2 \cdot 10^{-27} \text{ kg} \).

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Nuclear density is \( 10^{-25+43} = 10^{18} \text{ kg/m}^3 \) or a trillion (\( 10^{12} \)) kilograms in the size of a fingertip.

Earth radius crushed by a factor of \( 0.5 \cdot 10^{-5} \) to \( R_{\text{crush}\oplus} \approx 300 \text{ m} \) would approach neutron-star density.
Examples of “crushed” matter

Earth matter  Earth mass: $M_\oplus = 5.9722 \times 10^{24}$ kg $\approx 6.0 \cdot 10^{24}$ kg. Density $\rho_\oplus \approx 6.0 \cdot 10^{24-21} \sim 6 \cdot 10^3$ kg/m$^3$

Earth radius: $R_\oplus = 6.371 \cdot 10^6$ m $\approx 6.4 \cdot 10^6$ m  Earth volume: $(4\pi / 3)R_\oplus^3 = 4 \cdot 260 \cdot 10^{18} \sim 10^{21}$ m$^3$

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Earth radius crushed by a factor of $0.5 \cdot 10^{-5}$ to $R_{\text{crush}} \approx 300$ m  would approach neutron-star density.

Introducing the “Neutron starlet”  1 cm$^3$ of nuclear matter: mass $= 10^{12}$ kg.
Examples of “crushed” matter

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Earth volume: \((4\pi/3)R_\oplus^3 = 4 \cdot 260 \cdot 10^{18} \approx 10^{21} \text{ m}^3\)

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Nuclear density is \(10^{-25+43} = 10^{18} \text{ kg/m}^3\) or a trillion \((10^{12})\) kilograms in the size of a fingertip.

Earth radius crushed by a factor of \(0.5 \cdot 10^{-5}\) to \(R_{\text{crush}} = 300 \text{ m}\) would approach neutron-star density.

Introducing the “Neutron starlet” 1 cm\(^3\) of nuclear matter: mass = \(10^{12} \text{ kg}\).

Introducing the “Black Hole Earth” Suppose Earth is crushed so that its surface escape velocity is the speed of light \(c = 3.0 \cdot 10^{12} \text{ m/s}\).

\[
c = \sqrt{\frac{2GM}{R_\oplus}}
\]

\[
R_\oplus = 2GM/c^2 = 9 \text{ mm} \approx 1 \text{ cm}
\] (fingertip size!)
Isotopic harmonic oscillator dynamics in 1D, 2D, and 3D

Sinusoidal space-time dynamics derived by geometry

Isotropic harmonic oscillator orbits in 1D and 2D (You get 3D for free!)

Constructing 2D Isotropic harmonic oscillator orbits using phasor plots

Examples with x-y phase lag: $\alpha_{x-y} = \alpha_x - \alpha_y = 15^\circ, 30^\circ, \text{and } \pm 75^\circ$
Isotropic Harmonic Oscillator phase dynamics in uniform-body

I.H.O. Force law

\[ F = -x \quad (1\text{-Dimension}) \]

\[ F = -r \quad (2 \text{ or } 3\text{-Dimensions}) \]

Each dimension \( x, y, \) or \( z \) obeys the following:

\[
\text{Total } E = KE + PE = \frac{1}{2} mv^2 + U(x) = \frac{1}{2} mv^2 + \frac{1}{2} kx^2 = \text{const.}
\]

Equations for \( x \)-motion \([x(t) \text{ and } v_x = v(t)]\) are given first. They apply as well to dimensions \([y(t) \text{ and } v_y = v(t)]\) and \([z(t) \text{ and } v_z = v(t)]\) in the ideal isotropic case.
Isotropic Harmonic Oscillator phase dynamics in uniform-body

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\[ 1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = \left( \frac{v}{\sqrt{2E/m}} \right)^2 + \left( \frac{x}{\sqrt{2E/k}} \right)^2 \]

Another example of the old “scale-a-circle” trick...

\[ 1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos \theta)^2 + (\sin \theta)^2 \]

Let : \(1\) \( v = \sqrt{2E/m} \cos \theta \), and : \(2\) \( x = \sqrt{2E/k} \sin \theta \)
Isotropic Harmonic Oscillator phase dynamics in uniform-body

I.H.O. Force law

\[ F = -x \quad (1\text{-Dimension}) \]

\[ F = -r \quad (2\text{ or } 3\text{-Dimensions}) \]

Each dimension \( x, y, \) or \( z \) obeys the following:

Total \( E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{const}. \)

Equations for \( x \)-motion \([x(t) \text{ and } v_x = v(t)]\) are given first. They apply as well to dimensions \([y(t) \text{ and } v_y = v(t)]\) and \([z(t) \text{ and } v_z = v(t)]\) in the ideal isotropic case.

Let: (1) \( v = \sqrt{2E/m} \cos \theta \), and: (2) \( x = \sqrt{2E/k} \sin \theta \)

\[ \sqrt{\frac{2E}{m}} \cos \theta = \frac{dx}{dt} = \frac{d\theta}{dt} \frac{dx}{d\theta} = \omega \frac{dx}{d\theta} = \omega \sqrt{\frac{2E}{k}} \cos \theta \]

Unit 1

Fig. 9.10

(Paths are always 2-D ellipses if viewed right!)
**Isotropic Harmonic Oscillator phase dynamics in uniform-body**

**I.H.O. Force law**

\( F = -x \) (1-Dimension)

\( F = -r \) (2 or 3-Dimensions)

Each dimension \( x, y, \) or \( z \) obeys the following:

Total \( E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{const}. \)

Equations for \( x \)-motion \([x(t) \text{ and } v_x=v(t)]\) are given first. They apply as well to dimensions \([y(t) \text{ and } v_y=v(t)]\) and \([z(t) \text{ and } v_z=v(t)]\) in the ideal isotropic case.

\[
1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = \left( \frac{v}{\sqrt{2E/m}} \right)^2 + \left( \frac{x}{\sqrt{2E/k}} \right)^2
\]

Let: \( (1) \ v = \sqrt{2E/m} \cos \theta \), and: \( (2) \ x = \sqrt{2E/k} \sin \theta \)

\[
\sqrt{\frac{2E}{m}} \cos \theta = v = \frac{dx}{dt} = \frac{d\theta}{dt} \frac{dx}{d\theta} = \omega \frac{dx}{d\theta} = \omega \sqrt{\frac{2E}{k}} \cos \theta
\]

\[
\omega = \frac{d\theta}{dt} = \sqrt{\frac{k}{m}}
\]

\[
\text{by def. (3)} \quad \text{by (1)/ (2)}
\]

Another example of the old “scale-a-circle” trick...

Unit 1

Fig. 9.10

(Paths are always 2-D ellipses if viewed right!)
Isotropic Harmonic Oscillator phase dynamics in uniform-body

I.H.O. Force law

\( F = -x \) \text{ (1-Dimension)}

\( F = -r \) \text{ (2 or 3-Dimensions)}

Each dimension \( x, y, \) or \( z \) obeys the following:

Total \( E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{const.} \)

Equations for \( x \)-motion\n\[ [x(t) \text{ and } v_x = v(t)] \]
are given first. They apply as well to dimensions\n\[ [y(t) \text{ and } v_y = v(t)] \] and\n\[ [z(t) \text{ and } v_z = v(t)] \] in the ideal isotropic case.

Let : \( 1 \) \( v = \sqrt{\frac{2E}{m}} \cos \theta \), and : \( 2 \) \( x = \sqrt{\frac{2E}{k}} \sin \theta \)

\[ \sqrt{\frac{2E}{m}} \cos \theta = v = \frac{dx}{dt} = \frac{d\theta}{dt} \frac{dx}{d\theta} = \omega \frac{dx}{d\theta} = \omega \sqrt{\frac{2E}{k}} \cos \theta \]

by def. \( 3 \) \( \omega = \frac{d\theta}{dt} = \sqrt{\frac{k}{m}} \)

\[ \theta = \int \omega \cdot dt = \omega \cdot t + \alpha \]

-- by integration given constant \( \omega \) --

\( \sqrt{\frac{2E}{k}} \sin \theta = \frac{dx}{dt} = \frac{d\theta}{dt} \frac{dx}{d\theta} = \omega \frac{dx}{d\theta} = \omega \sqrt{\frac{2E}{k}} \sin \theta \)

by def. \( 3 \) \( \omega = \frac{d\theta}{dt} = \sqrt{\frac{k}{m}} \)

\[ \theta = \int \omega \cdot dt = \omega \cdot t + \alpha \]

-- by integration given constant \( \omega \) --

Another example of the old “scale-a-circle” trick...

Unit 1
Fig. 9.10

1-D

2-D

(Paths are always 2-D ellipses if viewed right!)
Isotropic harmonic oscillator dynamics in 1D, 2D, and 3D

Sinusoidal space-time dynamics derived by geometry

Isotropic harmonic oscillator orbits in 1D and 2D (You get 3D for free!)

Constructing 2D Isotropic harmonic oscillator orbits using phasor plots

Examples with x-y phase lag: $\alpha_{x-y} = \alpha_x - \alpha_y = 15^\circ, 30^\circ, \text{and } \pm 75^\circ$
Isotropic Harmonic Oscillator phase dynamics in uniform-body

Unit 1
Fig. 9.10

clockwise orbit if x is behind y
Left-handed

counter-clockwise if y is behind x
Right-handed

(a) 1-D Oscillator Phasor Plot
velocity \( v_x/\omega \)
position \( x \)
Phasor goes clockwise by angle \( \omega t \)

(b) 2-D Oscillator Phasor Plot
y-position
(x-Phase \( 45^\circ \) behind the y-Phase)

\( y-v/\omega \)
\( x-v/\omega \)

(0,2) (3,5) (4,6) (5,7) (8,-6) (9,-5)

\( F = -v \)
\( F_x, F_y \)
(a) Phasor Plots for 2-D Oscillator or Two 1D Oscillators (x-Phase 90° behind the y-Phase)

(b) x-Phase 0° behind the y-Phase
(In-phase case)

These are more generic examples with radius of x-phasor differing from that of the y-phasor.
Isotropic harmonic oscillator dynamics in 1D, 2D, and 3D

Sinusoidal space-time dynamics derived by geometry
Isotropic harmonic oscillator orbits in 1D and 2D (You get 3D for free!)
Constructing 2D Isotropic harmonic oscillator orbits using phasor plots

Examples with x-y phase lag: \( \alpha_{x-y} = \alpha_x - \alpha_y = 15^\circ, 30^\circ, \text{ and } \pm 75^\circ \)
\begin{align*}
x(t) &= A_x \cos(\omega \cdot t - \alpha_x) = A_x \cos(\alpha_x - \omega \cdot t) \\
\frac{v_x(t)}{\omega} &= -A_x \sin(\omega \cdot t - \alpha_x) = A_x \sin(\alpha_x - \omega \cdot t)
\end{align*}

\begin{align*}
y(t) &= A_y \cos(\omega \cdot t - \alpha_y) = A_y \cos(\alpha_y - \omega \cdot t) \\
\frac{v_y(t)}{\omega} &= -A_y \sin(\omega \cdot t - \alpha_y) = A_y \sin(\alpha_y - \omega \cdot t)
\end{align*}

Initial phases at \( t=0 \)
\[
x(t) = A_x \cos(\omega \cdot t - \alpha_x) = A_x \cos(\alpha_x - \omega \cdot t)
\]
\[
\frac{v_x(t)}{\omega} = -A_x \sin(\omega \cdot t - \alpha_x) = A_x \sin(\alpha_x - \omega \cdot t)
\]
\[
y(t) = A_y \cos(\omega \cdot t - \alpha_y) = A_y \cos(\alpha_y - \omega \cdot t)
\]
\[
\frac{v_y(t)}{\omega} = -A_y \sin(\omega \cdot t - \alpha_y) = A_y \sin(\alpha_y - \omega \cdot t)
\]

Later phases at time \( t \)
Ellipsometry Contact Plots

vs.

Relative phase $\Delta \alpha = \alpha_y - \alpha_x$

$\Delta \alpha = +15^\circ$

(Left-polarized clockwise case)
Ellipsometry Contact Plots

Relative phase $\Delta \alpha = \alpha_y - \alpha_x$

$\Delta \alpha = \alpha_x - \alpha_y = +30^\circ$

*Left-polarized (clockwise) orbit*

Initial phase $(\alpha_x = 45^\circ, \alpha_y = +15^\circ)$
Ellipsometry Contact Plots

Relative phase $\Delta \alpha = \alpha_y - \alpha_x$

$\Delta \alpha = \alpha_x - \alpha_y = +30^\circ$

*Left-polarized (clockwise) orbit*

Initial phase $(\alpha_x = 45^\circ, \alpha_y = +15^\circ)$

...after time advances by $150^\circ$:

$(\alpha_x = 45^\circ - 150^\circ = -105^\circ, \alpha_y = +15^\circ - 150^\circ = -135^\circ)$
Ellipsometry Contact Plots

vs.

Relative phase $\Delta \alpha = \alpha_y - \alpha_x$

$\Delta \alpha = \alpha_x - \alpha_y = -75^\circ$

Right-polarized (anti-clockwise) orbit

Initial phase ($\alpha_x = 90^\circ, \alpha_y = +165^\circ$)
Ellipsometry Contact Plots

\[ \Delta \alpha = \alpha_y - \alpha_x \]

\[ \Delta \alpha = \alpha_x - \alpha_y = -75^\circ \]

Right-polarized (anti-clockwise) orbit

Initial phase \((\alpha_x = 90^\circ, \alpha_y = +165^\circ)\)

...after time advances by 135°:
\((\alpha_x = 90^\circ - 135^\circ = -45^\circ, \alpha_y = +165^\circ - 135^\circ = +30^\circ)\)
Ellipsometry Contact Plots

Relative phase $\Delta \alpha = \alpha_x - \alpha_y$

$\Delta \alpha = 15^\circ$

(Right-rotated anti-clockwise case)

Initial phase ($\alpha_x = 120^\circ$, $\alpha_y = 135^\circ$

...after time advances by $90^\circ$

($\alpha_x = 120^\circ - 90^\circ = 30^\circ$, $\alpha_y = 135^\circ - 90^\circ = 45^\circ$)

$V_y/\omega$

$V_x$

$V(90^\circ)$
$\alpha_x = f_{12}$

$\Delta \alpha = -15^\circ$

(Right-polarized anti-clockwise case)

Initial phase ($\alpha_x = 120^\circ, \alpha_y = +135^\circ$)

After time advances by $180^\circ$:

($\alpha_x = 120^\circ - 180^\circ = -60^\circ, \alpha_y = +135^\circ - 180^\circ = -45^\circ$)
Ellipsometry Contact Plots

Relative phase $\Delta \alpha = \alpha_x - \alpha_y$

$\Delta \alpha = 15^\circ$

(Right-polarized anti-clockwise case)

Initial phase $x = 120^\circ, x_y = 135^\circ$

... after time advances by $240^\circ$:

$\alpha = 120^\circ - 240^\circ = 120^\circ, x_y = 135^\circ - 240^\circ = 105^\circ$
Ellipsometry Contact Plots

Relative phase \( \Delta \alpha = \alpha_x - \alpha_y \)

\( \Delta \alpha = 15° \)

(Right polarized anti-clockwise case)

Initial phase \( (x = 120°, y = 135°) \)

After time advances by 300°:

\( (x = 120° - 300° = 180°, y = 135° - 300° = 165°) \)
Ellipsometry Contact Plots

\[ \Delta \alpha = \alpha_x - \alpha_y \]

\[ \Delta \alpha = 15^\circ \]

(Right polarized anti-clockwise case)

Initial phase \( (\alpha_x = 120^\circ, \alpha_y = 135^\circ) \)

... after time advances by 330°:

\( (\alpha_x = 120^\circ, 300^\circ = -120^\circ, 150^\circ, \alpha_y = +135^\circ, -330^\circ = -195^\circ = +165^\circ) \)