Axiomatic development of classical mechanics
(Ch. 1 and Ch. 2 of Unit 1)

Geometry of momentum conservation axiom
- Totally Inelastic “ka-runch” collisions*
- Perfectly Elastic “ka-bong” and Center Of Momentum (COM) symmetry*

Geometry of Galilean translation symmetry
- Time reversal symmetry
  ...of COM collisions

Algebra, Geometry, and Physics of momentum conservation axiom
- Vector algebra of collisions
- Matrix or tensor algebra of collisions
- Deriving Energy Conservation Theorem

* Download Superball Collision Simulator
  http://www.uark.edu/rso/modphys/animations/BounceItWeb.html
A problem in \textit{space-time}:

\textit{Before collision.....}

\textit{After collision...what velocities?}

\textbf{(b) Collision!}
A problem in **space-time**: (60 mph Cell-faxing 4ton SUV rear-ends 10 mph 1ton VW)

Before collision.....

After collision...what velocities?

Conventional solution: Get out formulas:

\[ \Sigma mV_{\text{before}} = \Sigma mV_{\text{before}} \] [momentum conservation]

\[ \Sigma mV^2_{\text{before}} = \Sigma mV^2_{\text{before}} \] [energy conservation]

etc.
A problem in **space-time**: *(60 mph Cell-faxing 4-ton SUV rear-ends 10 mph 1-ton VW)*

**Before collision.....**

**After collision...what velocities?**

- Perfectly elastic case
- Totally inelastic case

Conventional solution:
Get out formulas:
- \[ \Sigma mV_{(\text{before})} = \Sigma mV_{(\text{after})} \] [momentum conservation]
- \[ \Sigma mV^2_{(\text{before})} = \Sigma mV^2_{(\text{after})} \] [energy conservation]
- etc.

*But an UNconventional way is quicker and slicker.....*

..... *(Just have to draw 2 lines! ... (and a circle...)*
A problem in **space-time**: (60 mph Cell-faxing 4-ton SUV rear-ends 10 mph 1-ton VW)

**Before collision...**

**Velocity-velocity Plot**

- **100 mph**
- **90**
- **80**
- **70**
- **60**
- **50**
- **40**
- **30**
- **20**
- **10**
- **0**

- **VW**
- **50 mph**
- **40**
- **30**
- **20**
- **10**
- **0**

- **V**
- **100 mph**
- **90**
- **80**
- **70**
- **60**
- **50**
- **40**
- **30**
- **20**
- **10**
- **0**

**After collision...what velocities?**

- **Perfectly elastic case**
- **Totally inelastic case**

**Conventional solution:**

Get out formulas:

\[ \Sigma mV(\text{before}) = \Sigma mV(\text{after}) \]  [momentum conservation]

\[ \Sigma mV^2(\text{before}) = \Sigma mV^2(\text{after}) \]  [energy conservation]

etc.
A problem in \textit{space-time}: (60 mph Cell-faxing 4ton SUV rear-ends 10 mph 1ton VW)

Before collision.....

After collision...what velocities?

Conventional solution:
Get out formulas:
\[ \Sigma mV(\text{before})=\Sigma mV(\text{before}) \] \text{[momentum conservation]}
\[ \Sigma mV^2(\text{before})=\Sigma mV^2(\text{before}) \] \text{[energy conservation]}

\[ M_{\text{SUV}}V_{\text{SUV}}+M_{\text{VW}}V_{\text{VW}} = \text{constant is Axiom #1} \]
A problem in **space-time** : (60 mph Cell-faxing 4ton SUV rear-ends 10 mph 1ton VW)

Before collision.....

After collision...what velocities?

Conventional solution:
Get out formulas:
\[ \sum mV_{\text{before}} = \sum mV_{\text{after}} \] [momentum conservation]
\[ \sum mV^2_{\text{before}} = \sum mV^2_{\text{after}} \] [energy conservation]
e tc.

\[ M_{\text{SUV}} V_{\text{SUV}} + M_{\text{VW}} V_{\text{VW}} \]

**constant is Axiom #1**
A problem in **space-time**: (60 mph Cell-faxing 4ton SUV rear-ends 10 mph 1ton VW)

Before collision.....

After collision...what velocities?

Conventional solution:
Get out formulas:
\[ \Sigma mV_{(\text{before})} = \Sigma mV_{(\text{before})} \]  [momentum conservation]
\[ \Sigma mV^2_{(\text{before})} = \Sigma mV^2_{(\text{before})} \]  [energy conservation]
etc.

\[ M_{SUV}V_{SUV} + M_{VW}V_{VW} = \text{constant is Axiom #1} \]

**Velocity-velocity Plot**

**Velocity-velocity Plot**

**Velocity-velocity Plot**

**Velocity-velocity Plot**
Geometry of momentum conservation axiom

Totally Inelastic “ka-runch” collisions

Perfectly Elastic “ka-bong” and Center Of Momentum (COM) symmetry
A problem in **space-time**:

(60 mph Cell-faxing 4ton SUV rear-ends 10 mph 1ton VW)

Before collision.....

1 mile

-1 -0.8 -0.6 -0.4 -0.2 0


V_{SUV} 60 mph 10 mph

After collision...what velocities?

Perfectly elastic case

Totally inelastic case

(b) Collision!

Conventional solution:
Get out formulas:

\[ \Sigma mV_{(before)} = \Sigma mV_{(after)} \] [momentum conservation]

\[ \Sigma mV^2_{(before)} = \Sigma mV^2_{(after)} \] [energy conservation]

etc.

\[ M_{SUV} V_{SUV} + M_{VW} V_{VW} = \text{constant is Axiom} \ #1 \]

Velocity-velocity Plot

100 mph

100 mph

10 mph

10 mph

V_{VW} 50 mph 10 mph

V_{VW} 50 mph 10 mph

INITIAL

FINAL

Ka-runch!

Ka-bong!

BINGO!
Geometry of momentum conservation axiom

Totally Inelastic “ka-runch” collisions

Perfectly Elastic “ka-bong” and Center Of Momentum (COM) symmetry
A problem in *space-time*: (60 mph Cell-faxing 4ton SUV rear-ends 10 mph 1ton VW)

Before collision.....

After collision...what velocities?

Perfectly elastic case

Totally inelastic case

Conventional solution:
Get out formulas:
\[ \Sigma mV_{\text{before}} = \Sigma mV_{\text{after}} \] [momentum conservation]
\[ \Sigma mV^2_{\text{before}} = \Sigma mV^2_{\text{after}} \] [energy conservation]

etc.

\[ M_{\text{SUV}}V_{\text{SUV}} + M_{\text{VW}}V_{\text{VW}} = \text{constant is Axiom #1} \]

Velocity-velocity Plot

100 mph 90 80 70 60 50 40 30 20 10 0

10 mph 0 10 20 30 40 50 60 70 80 90 100 mph

100 mph 90 80 70 60 50 40 30 20 10 0

10 mph 0 10 20 30 40 50 60 70 80 90 100 mph

DOUBBLE BINGO!
A problem in **space-time**:

Before collision.....

1 mile

-1 -0.8 -0.6 -0.4 -0.2 0


60 mph

10 mph

After collision...what velocities?

Perfectly elastic case

Totally inelastic case

Conventional solution:

Get out formulas:

\[ \Sigma mV_{\text{before}} = \Sigma mV_{\text{after}} \text{ [momentum conservation]} \]

\[ \Sigma mV^2_{\text{before}} = \Sigma mV^2_{\text{after}} \text{ [energy conservation]} \]

etc.

\[ M_{\text{SUV}} V_{\text{SUV}} + M_{\text{VW}} V_{\text{VW}} = \text{constant is Axiom #1} \]

Velocity-velocity Plot

100 mph

90

80

70

60

50

40

30

20

10

0

100 mph

50 mph

40

30

20

10

0

100 mph

50 mph

40

30

20

10

0

Superball Collision Simulator

http://www.uark.edu/rsa/modphys/animations/BounceItWeb.html

DOUBLE BINGO!

FINAL Ka-bong!

FINAL Ka-runch!

\[ \frac{4}{-1} = \frac{M_{\text{SUV}}}{-m_{\text{VW}}} = \frac{\Delta V_{\text{VW}}}{\Delta V_{\text{SUV}}} \]

Slope = -4

100 mph

90

80

70

60

50

40

30

20

10

0

100 mph

50 mph

40

30

20

10

0
A problem in \textit{space-time} : (60\text{mph} \text{ Cell-faxing 4ton SUV} \text{ rear-ends} \ 10\text{mph} \text{ 1ton VW})

Before collision.....

After collision...what velocities?

Conventional solution:
Get out formulas:
\[ \Sigma mV(\text{before})=\Sigma mV(\text{before}) \text{ [momentum conservation]} \]
\[ \Sigma mV^2(\text{before})=\Sigma mV^2(\text{before}) \text{ [energy conservation]} \]

\[ M_{\text{SUV}} V_{\text{SUV}} + M_{\text{VW}} V_{\text{VW}} = \text{constant is Axiom} \ #1 \]

\[ \text{slope} = -4 \]

\[ "\text{Ka-Runch!}" \]
\[ \text{(Extreme inelastic collision)} \]

\[ (V_{\text{SUV}})^{\text{FIN}} = 50\text{mph} \]
\[ (V_{\text{VW}})^{\text{FIN}} = 50\text{mph} \]

\[ (V_{\text{SUV}})^{\text{IN}} = 60\text{mph} \]
\[ (V_{\text{VW}})^{\text{IN}} = 10\text{mph} \]

Fig. 2.1 in Unit 1
A problem in space-time: (60 mph Cell-faxing 4ton SUV rear-ends 10 mph 1ton VW)

Before collision.....

After collision...what velocities?

Perfectly elastic case

Totally inelastic case

Conventional solution: Get out formulas:

$\Sigma mV(\text{before}) = \Sigma mV(\text{before})$ [momentum conservation]

$\Sigma mV^2(\text{before}) = \Sigma mV^2(\text{before})$ [energy conservation] etc.

$M_{\text{SUV}} V_{\text{SUV}} + M_{\text{VW}} V_{\text{VW}} =$ constant is Axiom #1

slope = -4

Notice “Ka-Bong”

Figure 2.2 scaling (ft./min. is more realistic)

“Ka-Bong!” (Ideal elastic collision)

$V_{\text{SUV}}^{\text{FIN}} = 40 \text{ ft per min}$

$V_{\text{VW}}^{\text{FIN}} = 90 \text{ ft per min}$

$V_{\text{SUV}}^{\text{COM}} = 50 \text{ ft per min}$

$V_{\text{VW}}^{\text{COM}} = 50 \text{ ft per min}$

$V_{\text{SUV}}^{\text{IN}} = 60 \text{ ft per min}$

$V_{\text{VW}}^{\text{IN}} = 10 \text{ ft per min}$

Fig. 2.2 in Unit 1

“Ka-Runch!” (Extreme inelastic collision)

$V_{\text{SUV}}^{\text{FIN}} = 50 \text{ mph}$

$V_{\text{VW}}^{\text{FIN}} = 50 \text{ mph}$

$V_{\text{SUV}}^{\text{IN}} = 60 \text{ mph}$

$V_{\text{VW}}^{\text{IN}} = 10 \text{ mph}$

Fig. 2.1 in Unit 1
Geometry of Galilean translation symmetry

45° shift in \((V_1,V_2)\)-space
Time reversal symmetry
...of COM collisions
A problem in \textbf{space-time}: (60 mph Cell-faxing 4ton SUV rear-ends 10 mph 1ton VW)

\textit{Geometry of Galilean translation (A \textcolor{red}{symmetry transformation})}

If you increase your velocity by 50 mph,...
...the rest of the world appears to be 50 mph \textcolor{green}{slower}

(a) Galileo transforms to \textit{COM frame}

\textbf{Fig. 2.5a}
in Unit 1
A problem in **space-time**: (60 mph Cell-faxing 4ton SUV rear-ends 10 mph 1ton VW)

**Geometry of Galilean translation (A symmetry transformation)**
If you increase your velocity by 50 mph,...
...the rest of the world appears to be 50 mph **slower**

(a) Galileo transforms to **COM** frame

(b) ... and to five or six other reference frames

**Fig. 2.5a**
in Unit 1

**Fig. 2.5b**
in Unit 1
Geometry of Galilean translation symmetry

45° shift in $(V_1, V_2)$-space

Time reversal symmetry

...of COM collisions
A problem in **space-time**: (60 mph Cell-faxing 4ton SUV rear-ends 10 mph 1ton VW)

**Geometry of Galilean translation (A symmetry transformation)**

*If you increase your velocity by 50 mph,...
...the rest of the world appears to be 50 mph slower*

(a) Galileo transforms to COM frame

(b) ... and to five or six other reference frames

**Time-reversal (F-I)**

**symmetry pairs**

(Four examples)
Geometry of Galilean translation symmetry

45\degree shift in \((V_1,V_2)\)-space

Time reversal symmetry

...of COM collisions
A problem in **space-time** : (60 mph Cell-faxing 4ton SUV rear-ends 10 mph 1ton VW)

Geometry of Galilean translation (A symmetry transformation)
If you increase your velocity by 50 mph,...
...the rest of the world appears to be 50 mph **slower**

(a) Galileo transforms to COM frame

(b) ... and to five or six other reference frames

Fig. 2.5a in Unit 1

Fig. 2.5b in Unit 1

THE
COM Time-reversal
symmetry pair
(Just 1 case)
Algebra, Geometry, and Physics of momentum conservation axiom

Vector algebra of collisions
Matrix or tensor algebra of collisions
Deriving Energy Conservation Theorem
Energy Ellipse geometry
Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

\[(M_{SU} + M_{VW}) V^{COM} = M_{SU} V_{SU}^{IN} + M_{VW} V_{VW}^{IN} = M_{SU} V_{SU}^{FIN} + M_{VW} V_{VW}^{FIN}\]

\[M_{SU} = 4\]

slope \(\frac{M_{SU}}{m_{VW}}\)

\[m_{VW} = 1\]

\[V_{VW}^{IN} + V_{VW}^{FIN} / 2\]

\[V_{SU}^{IN}\]
Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

\[(M_{SU} + M_{VW})V^{COM} = M_{SU}V_{SU}^{IN} + M_{VW}V_{VW}^{IN} = M_{SU}V_{SU}^{FIN} + M_{VW}V_{VW}^{FIN}\]

Mass weighted average velocity at anytime is Center of Mass velocity \(V^{COM}\):

\[const. = V^{COM} = \frac{M_{SU}V_{SU}^{IN} + M_{VW}V_{VW}^{IN}}{(M_{SU} + M_{VW})} = \frac{M_{SU}V_{SU}^{FIN} + M_{VW}V_{VW}^{FIN}}{(M_{SU} + M_{VW})}\]
Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

\[(M_{SU} + M_{VW}) V_{COM} = M_{SU} V_{SU} + M_{VW} V_{VW} \]

Mass weighted average velocity at anytime is Center of Mass velocity \( V_{COM} \):

\[
\text{const.} = V_{COM} = \frac{M_{SU} V_{SU} + M_{VW} V_{VW}}{(M_{SU} + M_{VW})}
\]

Express this using velocity vectors:

\[
V_{IN} = \begin{pmatrix} V_{SU} \\ V_{VW} \end{pmatrix}
\]

\[
V_{FIN} = \begin{pmatrix} V_{SU} \\ V_{VW} \end{pmatrix}
\]

\[
V_{COM} = \begin{pmatrix} V_{COM} \\ V_{COM} \end{pmatrix} = V_{COM} \begin{pmatrix} 1 \\ 1 \end{pmatrix}
\]

\[
= V_{COM} u, \quad \text{Define funny-unit vector:} \quad u = \begin{pmatrix} 1 \\ 1 \end{pmatrix}
\]
Algebra, Geometry, and Physics of momentum conservation axiom

Vector algebra of collisions
Matrix or tensor algebra of collisions
Deriving Energy Conservation Theorem
Energy Ellipse geometry
**Algebra, Geometry, and Physics of Momentum Conservation Axiom**

**Conservation of momentum line:**

\[(M_{SU} + M_{VW}) V_{COM} = M_{SU} V_{SU}^{IN} + M_{VW} V_{VW}^{IN} = M_{SU} V_{SU}^{FIN} + M_{VW} V_{VW}^{FIN}\]

Mass weighted average velocity at any time is **Center of Mass velocity** \(V_{COM}\):

\[
\text{const.} = V_{COM} = \frac{M_{SU} V_{SU}^{IN} + M_{VW} V_{VW}^{IN}}{(M_{SU} + M_{VW})} = M_{SU} V_{SU}^{FIN} + M_{VW} V_{VW}^{FIN}
\]

Express this using velocity vectors:

\[
V_{IN} = \begin{pmatrix} V_{SU}^{IN} \\ V_{VW}^{IN} \end{pmatrix}
\]

\[
V_{FIN} = \begin{pmatrix} V_{SU}^{FIN} \\ V_{VW}^{FIN} \end{pmatrix}
\]

\[
V_{COM} = \begin{pmatrix} V_{SU}^{COM} \\ V_{VW}^{COM} \end{pmatrix} = V_{COM} \begin{pmatrix} 1 \\ 1 \end{pmatrix}
\]

Define funny-unit vector:

\[
u = \begin{pmatrix} 1 \\ 1 \end{pmatrix}
\]

...and matrix operators:

\[
M = \begin{pmatrix} M_{SU} & 0 \\ 0 & M_{VW} \end{pmatrix}
\]

...that give momentum vector:

\[
P = M \cdot V = \begin{pmatrix} M_{SU} & 0 \\ 0 & M_{VW} \end{pmatrix} \begin{pmatrix} V_{SU} \\ V_{VW} \end{pmatrix} = \begin{pmatrix} P_{SU} \\ P_{VW} \end{pmatrix} = \begin{pmatrix} M_{SU} V_{SU} \\ M_{VW} V_{VW} \end{pmatrix}
\]
Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

\[(M_{SV}+M_{VW})V^{\text{COM}} = M_{SV}V_{SV}^{\text{IN}} + M_{VW}V_{VW}^{\text{IN}} = M_{SV}V_{SV}^{\text{FIN}} + M_{VW}V_{VW}^{\text{FIN}}\]

Mass weighted average velocity at anytime is Center of Mass velocity \(V^{\text{COM}}\):

\[\text{const.} = V^{\text{COM}} = \frac{M_{SV}V_{SV}^{\text{IN}} + M_{VW}V_{VW}^{\text{IN}}}{M_{SV}+M_{VW}} = \frac{M_{SV}V_{SV}^{\text{FIN}} + M_{VW}V_{VW}^{\text{FIN}}}{M_{SV}+M_{VW}}\]

Express this using velocity vectors:

\[V^{\text{IN}} = \begin{pmatrix} V_{SV}^{\text{IN}} \\ V_{VW}^{\text{IN}} \end{pmatrix}, \quad V^{\text{FIN}} = \begin{pmatrix} V_{SV}^{\text{FIN}} \\ V_{VW}^{\text{FIN}} \end{pmatrix}\]

\[V^{\text{COM}} = \begin{pmatrix} V_{SV}^{\text{COM}} \\ V_{VW}^{\text{COM}} \end{pmatrix} = V^{\text{COM}} (1)

Define funny-unit vector:

\[u = \begin{pmatrix} 1 \\ 1 \end{pmatrix}\]

...and matrix operators:

\[M = \begin{pmatrix} M_{SV} & 0 \\ 0 & M_{VW} \end{pmatrix}\]

...that give momentum vector: \(P = M \cdot V = \begin{pmatrix} M_{SV} & 0 \\ 0 & M_{VW} \end{pmatrix} \begin{pmatrix} V_{SV} \\ V_{VW} \end{pmatrix} = \begin{pmatrix} P_{SV} \\ P_{VW} \end{pmatrix} = \begin{pmatrix} M_{SV}V_{SV} \\ M_{VW}V_{VW} \end{pmatrix}\)

whose sum of components is constant.

\[\text{const.} = u \cdot P = P_{SV} + P_{VW} = M_{SV}V_{SV} + M_{VW}V_{VW} = u \cdot M \cdot V = u \cdot M \cdot V^{\text{IN}} = u \cdot M \cdot V^{\text{FIN}}\]
Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:
\[(M_{SUv}+M_{vW})V^{COM}=M_{SUv}V_{SUv,IN}+M_{vW}V_{vW,IN}\]

Mass weighted average velocity at anytime is Center of Mass velocity \(V^{COM}\):
\[\text{const.} = V^{COM}=\frac{M_{SUv}V_{SUv,IN}+M_{vW}V_{vW,IN}}{(M_{SUv}+M_{vW})}\]

Express this using velocity vectors:
\[V_{IN} = \begin{pmatrix} V_{SUv,IN} \\ V_{vW,IN} \end{pmatrix} \]
\[V_{FIN} = \begin{pmatrix} V_{SUv,FIN} \\ V_{vW,FIN} \end{pmatrix} \]
\[V^{COM} = \begin{pmatrix} V^{COM}_{SUv} \\ V^{COM}_{vW} \end{pmatrix} = V^{COM} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \]

\[= V^{COM} u \quad \text{Define funny-unit vector:} \quad u = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \]

...and matrix operators:
\[M = \begin{pmatrix} M_{SUv} & 0 \\ 0 & M_{vW} \end{pmatrix} \]

...that give momentum vector: \(P = M \cdot V = \begin{pmatrix} M_{SUv} & 0 \\ 0 & M_{vW} \end{pmatrix} \begin{pmatrix} V_{SUv} \\ V_{vW} \end{pmatrix} = \begin{pmatrix} P_{SUv} \\ P_{vW} \end{pmatrix} = \begin{pmatrix} M_{SUv}V_{SUv} \\ M_{vW}V_{vW} \end{pmatrix}\]

\[\text{const.} = u \cdot P = P_{SUv} + P_{vW} = M_{SUv}V_{SUv} + M_{vW}V_{vW} = u \cdot M \cdot V = u \cdot M \cdot V_{IN} = u \cdot M \cdot V_{FIN} \]

Denote Center of Momentum \(V^{COM}\) with engineer’s centering symbol

Time-Reversal Symmetry Axiom
Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

\[(M_{SU} + M_{yw}) V^{COM} = M_{SU} V_{SU}^{IN} + M_{yw} V_{yw}^{IN} = M_{SU} V_{SU}^{FIN} + M_{yw} V_{yw}^{FIN}\]

Mass weighted average velocity at anytime is Center of Mass velocity \( V^{COM} \):

\[\text{const.} = V^{COM} = \frac{M_{SU} V_{SU}^{IN} + M_{yw} V_{yw}^{IN}}{(M_{SU} + M_{yw})} = \frac{M_{SU} V_{SU}^{FIN} + M_{yw} V_{yw}^{FIN}}{(M_{SU} + M_{yw})}\]

Express this using velocity vectors:

\[V^{IN} = \begin{pmatrix} V_{SU}^{IN} \\ V_{yw}^{IN} \end{pmatrix}, \quad V^{FIN} = \begin{pmatrix} V_{SU}^{FIN} \\ V_{yw}^{FIN} \end{pmatrix}\]

\[V^{COM} = \begin{pmatrix} V^{COM} \\ V^{COM} \end{pmatrix} = V^{COM} \begin{pmatrix} 1 \\ 1 \end{pmatrix}\]

\[= V^{COM} \mathbf{u}\]

...and matrix operators:

\[M = \begin{pmatrix} M_{SU} & 0 \\ 0 & M_{yw} \end{pmatrix}\]

...that give momentum vector: \( P = M \cdot V = \begin{pmatrix} M_{SU} & 0 \\ 0 & M_{yw} \end{pmatrix} \begin{pmatrix} V_{SU} \\ V_{yw} \end{pmatrix} = \begin{pmatrix} P_{SU} \\ P_{yw} \end{pmatrix} = \begin{pmatrix} M_{SU} V_{SU} \\ M_{yw} V_{yw} \end{pmatrix}\]

whose sum of components is constant.

(by \( \mathbf{u} \cdot \mathbf{p} \) product)

\[\text{const.} = \mathbf{u} \cdot P = P_{SU} + P_{yw} = M_{SU} V_{SU} + M_{yw} V_{yw} = \mathbf{u} \cdot M \cdot V = \mathbf{u} \cdot M \cdot V^{IN} = \mathbf{u} \cdot M \cdot V^{FIN}\]
Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

\[(M_{SV} + M_{vw}) V_{COM} = M_{SV} V_{SV}^{IN} + M_{vw} V_{vw}^{IN} = M_{SV} V_{SV}^{FIN} + M_{vw} V_{vw}^{FIN}\]

Mass weighted average velocity at anytime is Center of Mass velocity \(V_{COM}\):

\[\text{const.} = V_{COM} = \frac{M_{SV} V_{SV}^{IN} + M_{vw} V_{vw}^{IN}}{(M_{SV} + M_{vw})} = \frac{M_{SV} V_{SV}^{FIN} + M_{vw} V_{vw}^{FIN}}{(M_{SV} + M_{vw})}\]

Express this using velocity vectors:

\[V_{IN}^{\text{FIN}} = \begin{pmatrix} V_{SV}^{\text{FIN}} \\ V_{vw}^{\text{FIN}} \end{pmatrix}\]

\[V_{FIN}^{\text{FIN}} = \begin{pmatrix} V_{SV}^{\text{FIN}} \\ V_{vw}^{\text{FIN}} \end{pmatrix}\]

\[V_{COM}^{\text{FIN}} = \begin{pmatrix} V_{COM}^{\text{FIN}} \\ V_{COM}^{\text{FIN}} \end{pmatrix} = V_{COM}^{\text{FIN}}(1)\]

...and matrix operators:

\[M = \begin{pmatrix} M_{SV} & 0 \\ 0 & M_{vw} \end{pmatrix}\]

...that give momentum vector:

\[P = M \cdot V = \begin{pmatrix} M_{SV} & 0 \\ 0 & M_{vw} \end{pmatrix} \begin{pmatrix} V_{SV} \\ V_{vw} \end{pmatrix} = \begin{pmatrix} P_{SV} \\ P_{vw} \end{pmatrix} = \begin{pmatrix} M_{SV} V_{SV} \\ M_{vw} V_{vw} \end{pmatrix}\]

whose sum of components is constant. (by \(u \cdot \text{product}\))

\[\text{const.} = u \cdot P = P_{SV} + P_{vw} = M_{SV} V_{SV}^{FIN} + M_{vw} V_{vw}^{FIN} = u \cdot M \cdot V = u \cdot M \cdot V^{IN}\]

Then:

\[V_{COM}^{\text{FIN}} = V_{COM}^{\text{FIN}}(1)\]

\[V_{COM}^{\text{FIN}} = V_{COM}^{\text{FIN}}(1)\]

\[V_{COM}^{\text{FIN}} = V_{COM}^{\text{FIN}}(1)\]

\[V_{COM}^{\text{FIN}} = V_{COM}^{\text{FIN}}(1)\]
Algebra, Geometry, and Physics of momentum conservation axiom

- Vector algebra of collisions
- Matrix or tensor algebra of collisions
- Deriving Energy Conservation Theorem
- Energy Ellipse geometry
Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

\[(M_{SU} + M_{VW})V_{COM} = M_{SU}V_{SUV}^{IN} + M_{VW}V_{VW}^{IN}\]

Mass weighted average velocity at anytime is Center of Mass velocity \( V_{COM} \):

\[
\text{const.} = V_{COM} = \frac{M_{SU}V_{SUV}^{IN} + M_{VW}V_{VW}^{IN}}{(M_{SU} + M_{VW})}
\]

Express this using velocity vectors:

\[V_{IN}^{FIN} = \begin{pmatrix} V_{SUV}^{IN} \\ V_{VW}^{IN} \end{pmatrix}
\]

\[V_{FIN} = \begin{pmatrix} V_{SUV}^{FIN} \\ V_{VW}^{FIN} \end{pmatrix}
\]

\[V_{COM} = \begin{pmatrix} V_{COM}^{FIN} \\ V_{COM}^{IN} \end{pmatrix}
\]

...and matrix operators:

\[M = \begin{pmatrix} M_{SU} & 0 \\ 0 & M_{VW} \end{pmatrix}
\]

...that give momentum vector: \( P = MV \)

\[
\text{const.} = u \cdot P = P_{SU} + P_{VW} = M_{SU}V_{SUV}^{IN} + M_{VW}V_{VW}^{IN} = u \cdot M \cdot V = u \cdot M \cdot V^{IN}
\]

Then: \( V_{COM} = V_{COM}^{IN} \) gives:

\[V_{COM} \cdot M \cdot V_{COM} = V_{COM} \cdot M \cdot V^{IN} = V_{COM} \cdot M \cdot V^{FIN}
\]

By **T-Symmetry Axiom**: \( V_{COM}^{FIN} = V_{IN}^{FIN} + V_{FIN}^{FIN} \). Substituting:

\[V_{COM} \cdot M \cdot V_{COM} = 1/2(V_{IN}^{FIN} + V_{FIN}^{FIN}) \cdot M \cdot V^{IN} = 1/2(V_{IN}^{FIN} + V_{FIN}^{FIN}) \cdot M \cdot V^{FIN} \]
Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

\[(M_{SU} + M_{VW})V_{COM} = M_{SU}V_{SU} + M_{VW}V_{VW}\]

Mass weighted average velocity at anytime is Center of Mass velocity \(V_{COM} \) :

\[
\text{const.} = V_{COM} = M_{SU}V_{SU} + M_{VW}V_{VW}
\]

Express this using velocity vectors:

\[
V_{IN} = \frac{V_{SU} + V_{VW}}{M_{SU} + M_{VW}}
\]

Then:

\[
V_{COM} = V_{COM} \cdot u
\]

...and matrix operators:

\[
M = \begin{bmatrix} M_{SU} & 0 \\ 0 & M_{VW} \end{bmatrix} = M^T
\]

...that give momentum vector: \( P = M \cdot V \):

\[
\text{const.} = u \cdot P = P_{SU} + P_{VW} = M_{SU}V_{SU} + M_{VW}V_{VW}
\]

Then:

\[
V_{COM} = V_{COM} \cdot M = \frac{1}{2}(V_{IN} + V_{FIN})
\]

By T-Symmetry Axiom: \( V_{COM} = (V_{FIN} + V_{FIN}) \cdot M \cdot V_{FIN} \)
Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

\[(M_{SUv} + M_{Vw}) V_{COM} = M_{SUv} V_{SUv} IN + M_{Vw} V_{Vw} IN = M_{SUv} V_{SUv} FIN + M_{Vw} V_{Vw} FIN\]

Mass weighted average velocity at anytime is Center of Mass velocity \( V_{COM} \):

\[
\text{const.} = V_{COM} = \frac{M_{SUv} V_{SUv} IN + M_{Vw} V_{Vw} IN}{(M_{SUv} + M_{Vw})} = M_{SUv} V_{SUv} FIN + M_{Vw} V_{Vw} FIN
\]

Express this using velocity vectors:

\[
V_{IN} = \begin{pmatrix} V_{SUv} \, IN \\ V_{Vw} \, IN \end{pmatrix}
\]

\[
V_{FIN} = \begin{pmatrix} V_{SUv} \, FIN \\ V_{Vw} \, FIN \end{pmatrix}
\]

\[
V_{COM} = \begin{pmatrix} V_{COM} \, V_{COM} \\ V_{SUv} \, V_{SUv} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} V_{SUv} \, V_{SUv} \\ V_{Vw} \, V_{Vw} \end{pmatrix}
\]

...and matrix operators:

\[
M = \begin{pmatrix} M_{SUv} & 0 \\ 0 & M_{Vw} \end{pmatrix} = M^T
\]

By **M-symmetry** \( M = M^T \),

...that give momentum vector:

\[
P = M \cdot V = \begin{pmatrix} M_{SUv} & 0 \\ 0 & M_{Vw} \end{pmatrix} \begin{pmatrix} V_{SUv} \\ V_{Vw} \end{pmatrix} = \begin{pmatrix} P_{SUv} \\ P_{Vw} \end{pmatrix} = \begin{pmatrix} M_{SUv} V_{SUv} \\ M_{Vw} V_{Vw} \end{pmatrix}
\]

whose sum of components is constant.

\[
\text{const.} = u \cdot P = P_{SUv} + P_{Vw} = M_{SUv} V_{SUv} + M_{Vw} V_{Vw} = u \cdot M \cdot V
\]

Then:

\[
V_{COM} = V_{COM} \cdot u
\]

By **T-Symmetry Axiom**:

\[
V_{COM} = 1/2(V_{IN} + V_{FIN})
\]

Substituting:

\[
V_{COM} \cdot M \cdot V = 1/2(V_{IN} + V_{FIN}) \cdot M \cdot V = 1/2 V_{IN} \cdot M \cdot V + 1/2 V_{FIN} \cdot M \cdot V
\]
Algebra, Geometry, and Physics of momentum conservation axiom

Vector algebra of collisions
Matrix or tensor algebra of collisions
Completing derivation of Energy Conservation Theorem
Energy Ellipse geometry
Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

\[(M_{SU}+M_{v})V_{COM}=M_{SU}V_{SU} \text{ IN} + M_{v}V_{v} \text{ IN} = M_{SU}V_{SU} \text{ FIN} + M_{v}V_{v} \text{ FIN}\]

Mass weighted average velocity at anytime is Center of Mass velocity \( V_{COM} \):

\[\text{const.}=V_{COM}=M_{SU}V_{SU} \text{ IN} + M_{v}V_{v} \text{ IN} = M_{SU}V_{SU} \text{ FIN} + M_{v}V_{v} \text{ FIN}\]

Express this using velocity vectors:

\[V_{\text{IN}} = \begin{pmatrix} V_{SU} \text{ IN} \\ V_{v} \text{ IN} \end{pmatrix}, \quad V_{\text{FIN}} = \begin{pmatrix} V_{SU} \text{ FIN} \\ V_{v} \text{ FIN} \end{pmatrix}, \quad V_{\text{COM}} = \begin{pmatrix} V_{\text{COM}} \text{ IN} \\ V_{\text{COM}} \text{ FIN} \end{pmatrix} = V_{\text{COM}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}\]

...and matrix operators:

\[M = \begin{pmatrix} M_{SU} & 0 \\ 0 & M_{v} \end{pmatrix} = M^{\text{Transpose}}\]

\[M\text{-symmetry } M = M^{T}\]

...that give momentum vector: \( P = M \cdot V = \begin{pmatrix} M_{SU} & 0 \\ 0 & M_{v} \end{pmatrix} \begin{pmatrix} V_{SU} \text{ IN} \\ V_{v} \text{ IN} \end{pmatrix} = \begin{pmatrix} P_{SU} \text{ IN} \\ P_{v} \text{ IN} \end{pmatrix} = M_{SU}V_{SU} \)

\[\text{const.}=u \cdot P = P_{SU} \text{ IN} + P_{v} \text{ IN} = M_{SU}V_{SU} + M_{v}V_{v} = u \cdot M \cdot V\]

Then: \( V_{\text{COM}} = V_{\text{COM}} u \) gives:

\[V_{\text{COM}} \cdot M \cdot V_{\text{IN}} = V_{\text{COM}} \cdot M \cdot V_{\text{FIN}}\]

By \(M\text{-symmetry } M=M^{T}\): \( V_{\text{FIN}} \cdot M \cdot V_{\text{IN}} = V_{\text{FIN}} \cdot M \cdot V_{\text{FIN}}\)

this becomes:

\[V_{\text{COM}} \cdot M \cdot V_{\text{COM}} = \frac{1}{2}V_{\text{FIN}} \cdot M \cdot V_{\text{IN}} = \frac{1}{2}V_{\text{FIN}} \cdot M \cdot V_{\text{FIN}}\]

These are equations for energy conservation ellipse:

\[\text{const.} = \frac{1}{2}M_{SU}V_{SU}^{2} + \frac{1}{2}M_{v}V_{v}^{2}\]

\[= \frac{1}{2}M_{SU}V_{SU}^{2} + \frac{1}{2}M_{v}V_{v}^{2}\]

= Kinetic Energy = KE is now defined and proved a constant under T-Symmetry
Algebra, Geometry, and Physics of momentum conservation axiom

Vector algebra of collisions
Matrix or tensor algebra of collisions
Deriving Energy Conservation Theorem
Energy Ellipse geometry
Algebra, Geometry, and Physics of Momentum Conservation Axiom
(being one of $\infty$-many...)

Momentum Conservation Axiom

plus

T-Symmetry Axiom
(M=M$^T$ implied)

gives

Kinetic Energy Conservation Theorem

All lines of slope -$M_{SUV}/m_{VW}$...are bisected by the
(slope=1)-COM line

These are equations for energy conservation ellipse:

\[ KE = \frac{1}{2} M_{SUV} V_{SUV}^2 + \frac{1}{2} m_{VW} V_{VW}^2 \]

\[ \frac{1}{2} = \frac{V_{SUV}^2}{2 \cdot KE} + \frac{V_{VW}^2}{2 \cdot KE} = \frac{x^2}{a^2} + \frac{y^2}{b^2} \]
\[ \begin{align*}
V & = \sqrt{2 \cdot KE} \\
W & = \sqrt{2 \cdot KE} \\
b & = \sqrt{2 \cdot KE_{VW}} \\
a & = \sqrt{2 \cdot KE_{MSUV}} \\
KE & = 7,250 \\
M_{SUV} & = 4 \\
m_{VW} & = 1 \\
M_{SUV} \cdot P_{Total} & = 250
\end{align*} \]
'V' VVW

Elastic Kinetic Energy ellipse (KE=7,250)

$$a = \sqrt{\frac{2 \cdot KE}{M_{SUV}}} = 60.21$$

$$b = \sqrt{\frac{2 \cdot KE}{m_{VW}}} = 120.42$$

Initial-point (60,10)

Final "Ka-Bong"-point $(40,90)$

Inelastic Kinetic Energy ellipse (IE=6,250)

$$a = \sqrt{\frac{2 \cdot IE}{M_{SUV}}} = 55.9$$

$$b = \sqrt{\frac{2 \cdot IE}{m_{VW}}} = 111.8$$

Initial-point (60,10)

Final point (40,90)

Momentum $P_{Total} = 250$

$m_{VW} = 1$

$M_{SUV} = 4$

Fig. 3.1 a in Unit 1

Fig. 3.1 b in Unit 1

$V_{COM}$ point

elastic FIN

inelastic FIN

COM Energy ellipse ($E_{COM} = 1,000$)
As usual in physics, opposite extremes are easier to analyze than the generic “real(er) world” in between!

Fig. 3.2  (This case has Bush era requisite SUV mass of the 6 ton “Hummer”)

Next: *The X-2 pen-launcher*