

Lectures 6 to 8
Tue. 2.9.16 to Thur 2.11.2016

Introducing Lightwave Fourier Analysis

(Comparing wave dynamics to classical behavior in Ch. 3 thru Ch. 5 of Unit 1)

Introducing lightwave Fourier analysis - Pulse Waves (PW) versus Continuous Waves (CW)

Simplest is CW (Continuous Wave, Cosine Wave, Colored Wave, Complex Wave,...)

CW parameters: Wavelength λ and Wave period τ

CW inverse parameters: Wavenumber $\kappa=1/\lambda$ and Wave frequency $\nu=1/\tau$

CW angular parameters: Wavevector $k=2\pi\kappa=2\pi/\lambda$ and angular frequency $\omega=2\pi\nu=2\pi/\tau$

CW wavefunction : $\psi=A \exp[i(kx-\omega t)]=A \cos(kx-\omega t)+iA \sin(kx-\omega t)$

Wave phasors, phasor chain plots, dispersion functions $\omega(k)$, and phase velocity $V_{phase}=\omega(k)/k$

Special case: Lightwave linear dispersion: $V_{phase}=c$ or: $\omega(k)=ck$

Introducing PW (Pulse Wave, Particle-like Wave, Packet Wave,...) archetypes compared to CW

Building PW from CW components using “Fourier Control” app-panel

Fourier PW “box-car” geometric series summed

Animation of PW obeying lightwave linear dispersion $\omega(k)=ck$

Important Evenson axiom for relativity: “All colors go c”

Visualizing PW wave uncertainty relations for space: $\Delta x \cdot \Delta \kappa=1$ and time: $\Delta t \cdot \Delta \nu=1$

PW “wrinkles” go away if Fourier “boxcar” is tapered to a softer “Gaussian”

Opposite-pair CW (colliding $\pm m=\pm 2$) Fourier components trace a Cartesian space-time grid

→ *Introducing lightwave Fourier analysis - Pulse Waves (PW) versus Continuous Waves (CW)*

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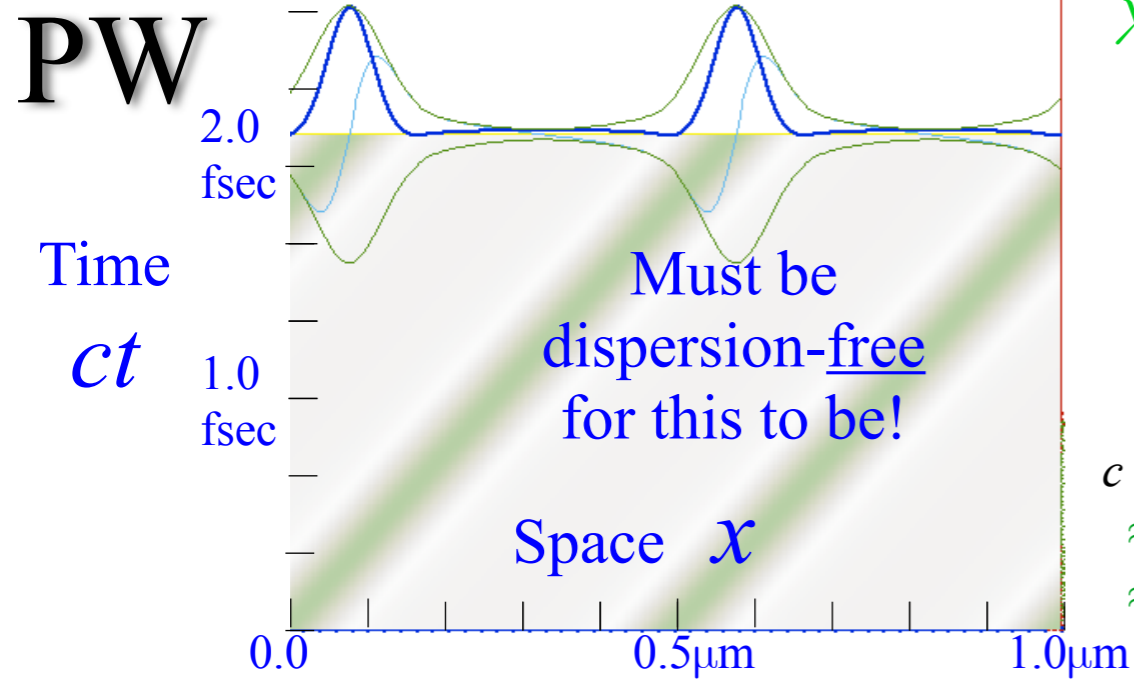
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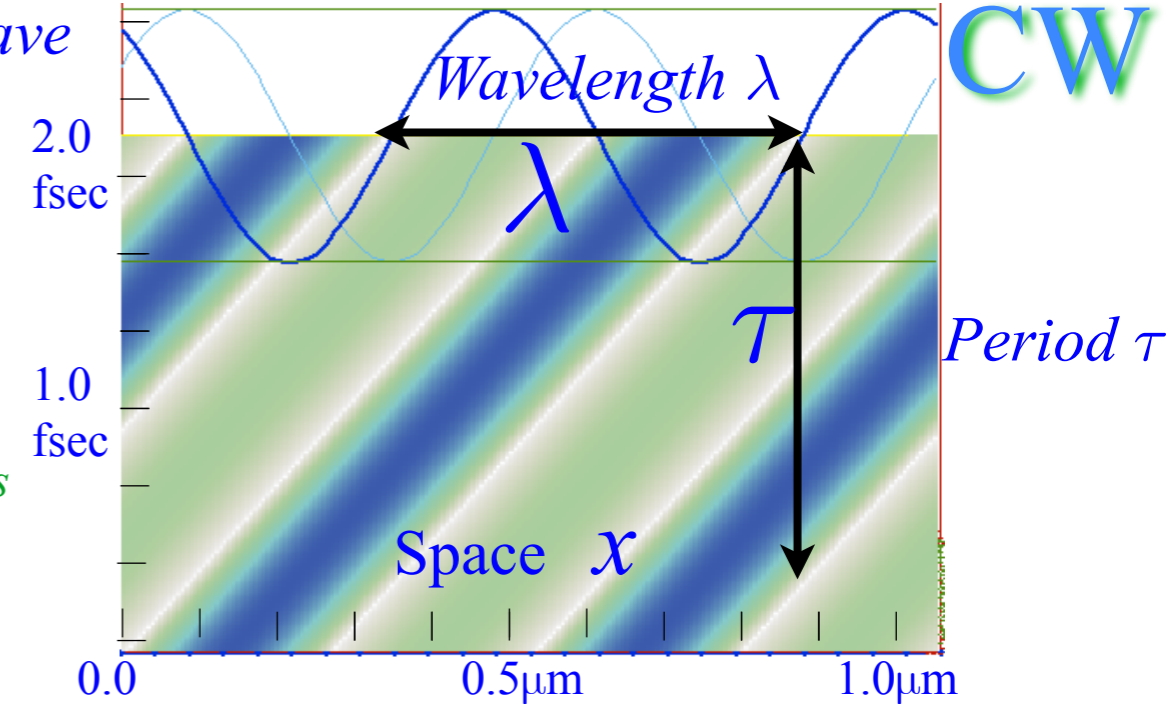


$\lambda = 0.5 \mu\text{m}$ per wave

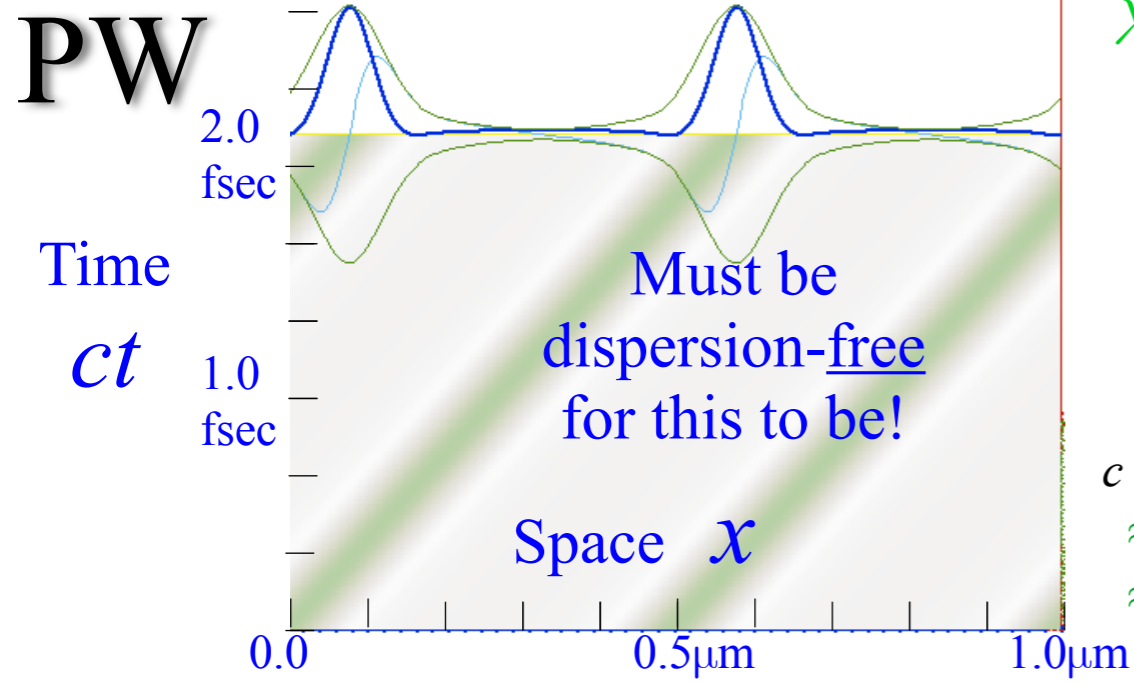
$c = 2.99792458 \cdot 10^8 \text{m/s}$
 $\approx 3 \cdot 10^8 \text{m/s}$
 $\approx 0.3 \mu\text{m/fs} \approx 1 \text{ft/ns}$

Time
 ct

CW



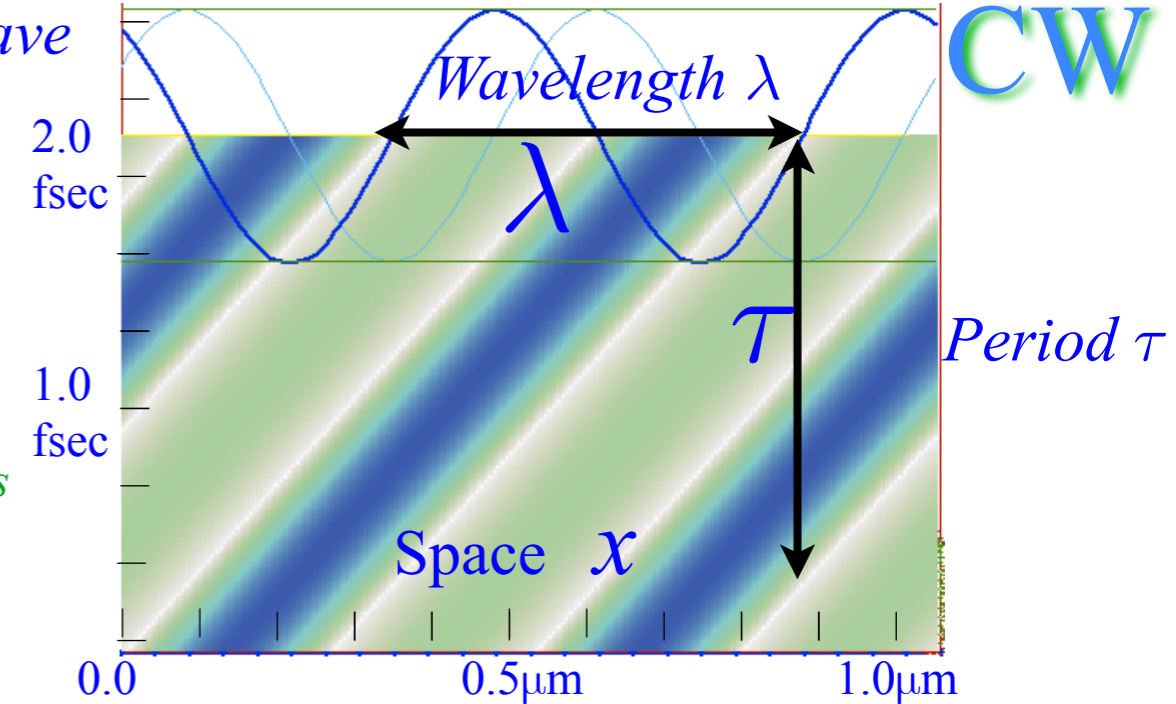
It helps to introduce two *archetypes* of light waves and contrast them.



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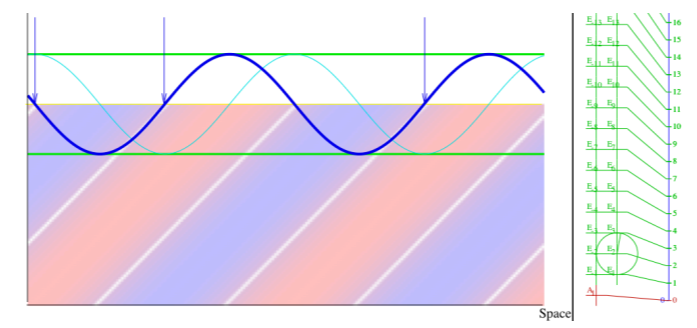
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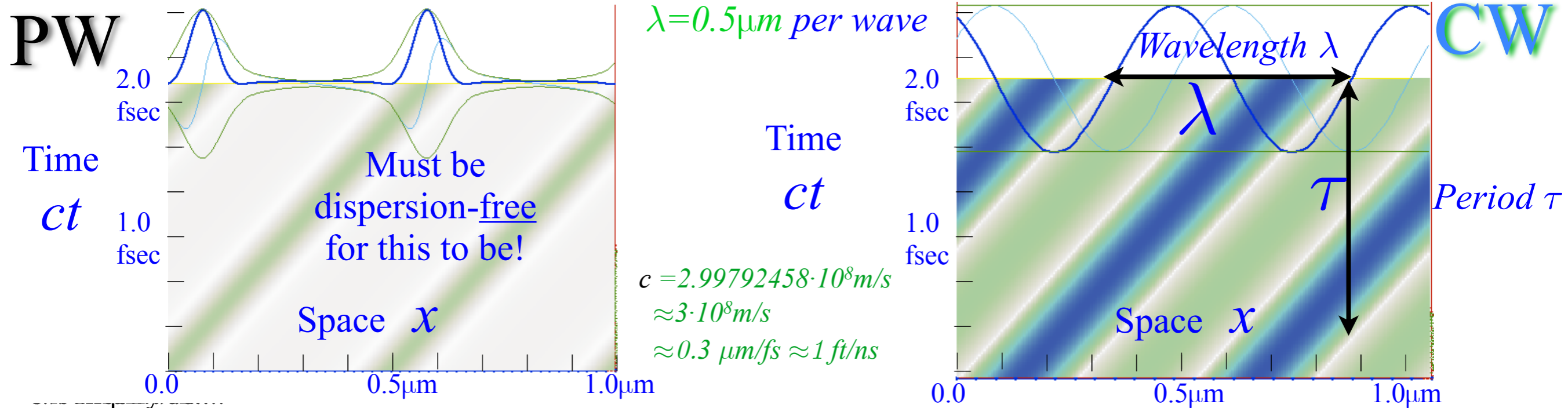
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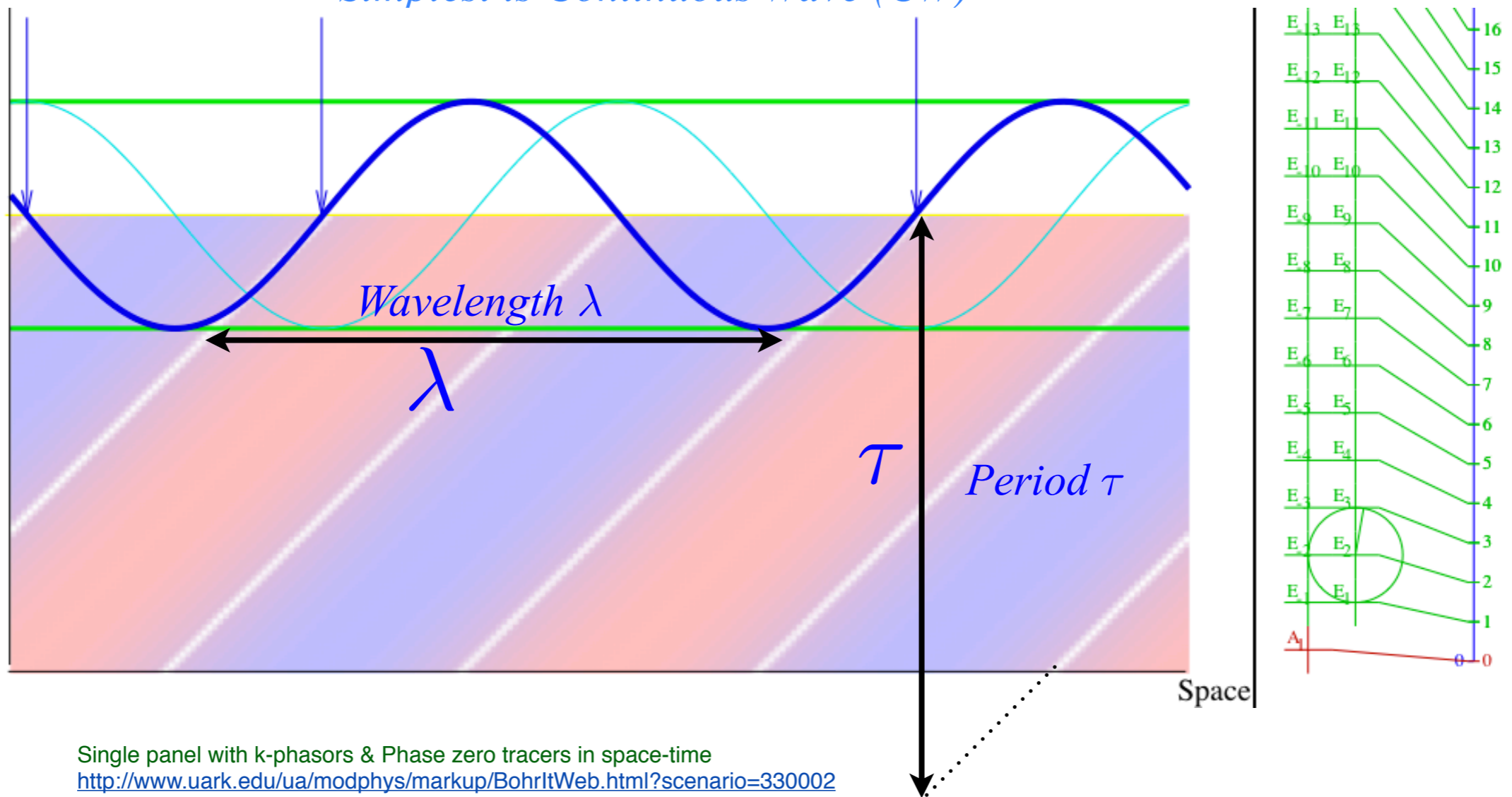
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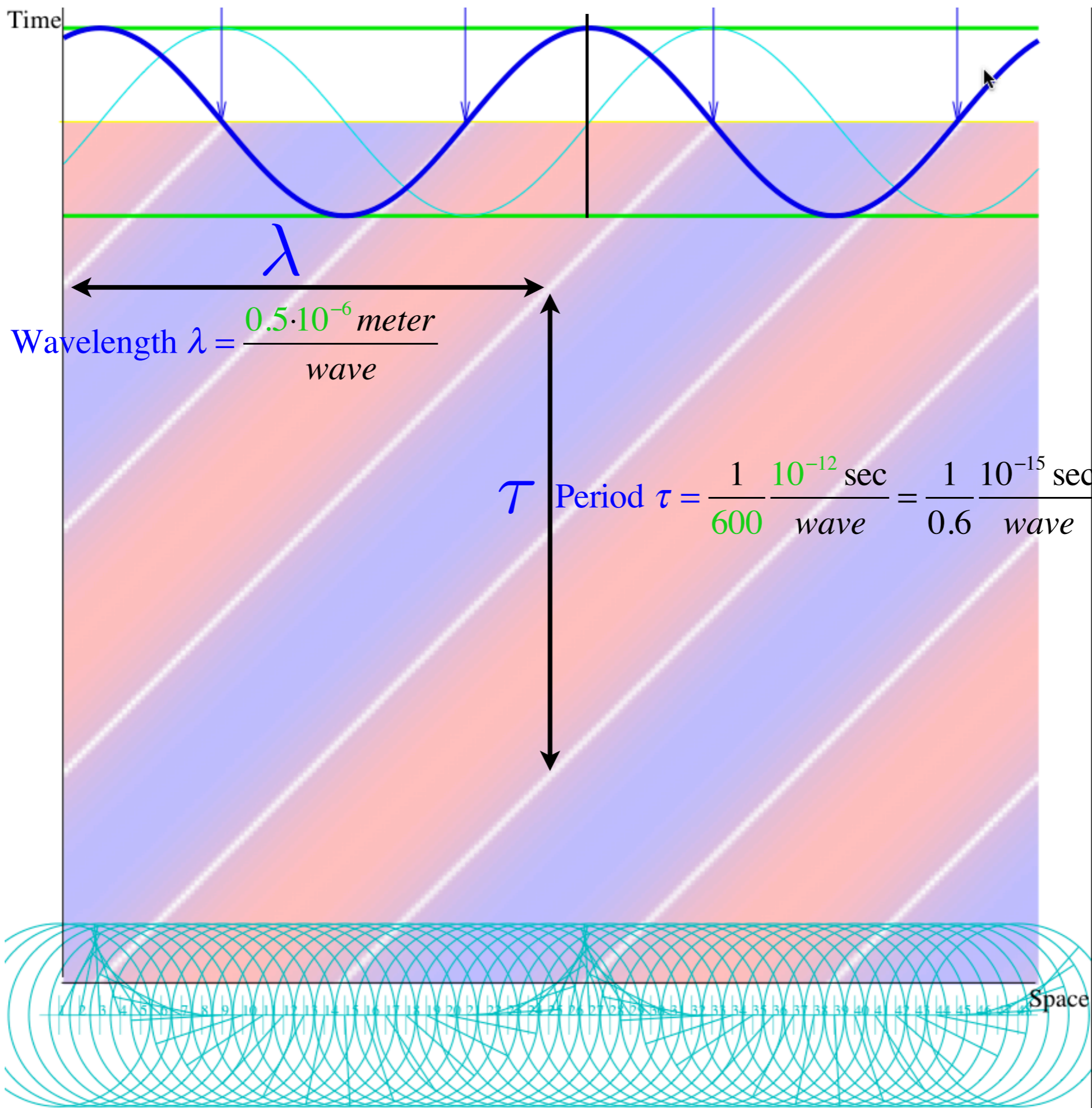
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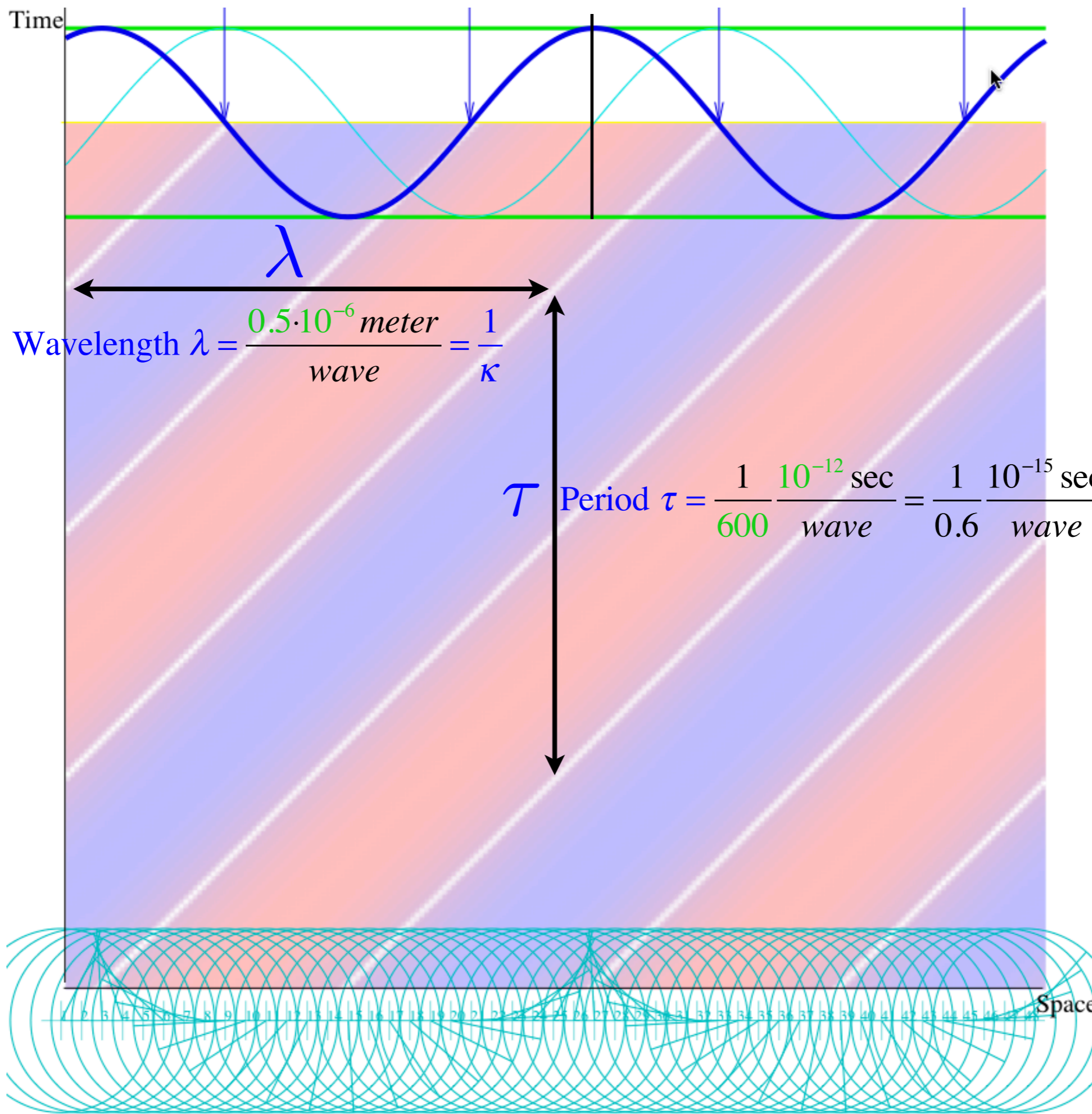
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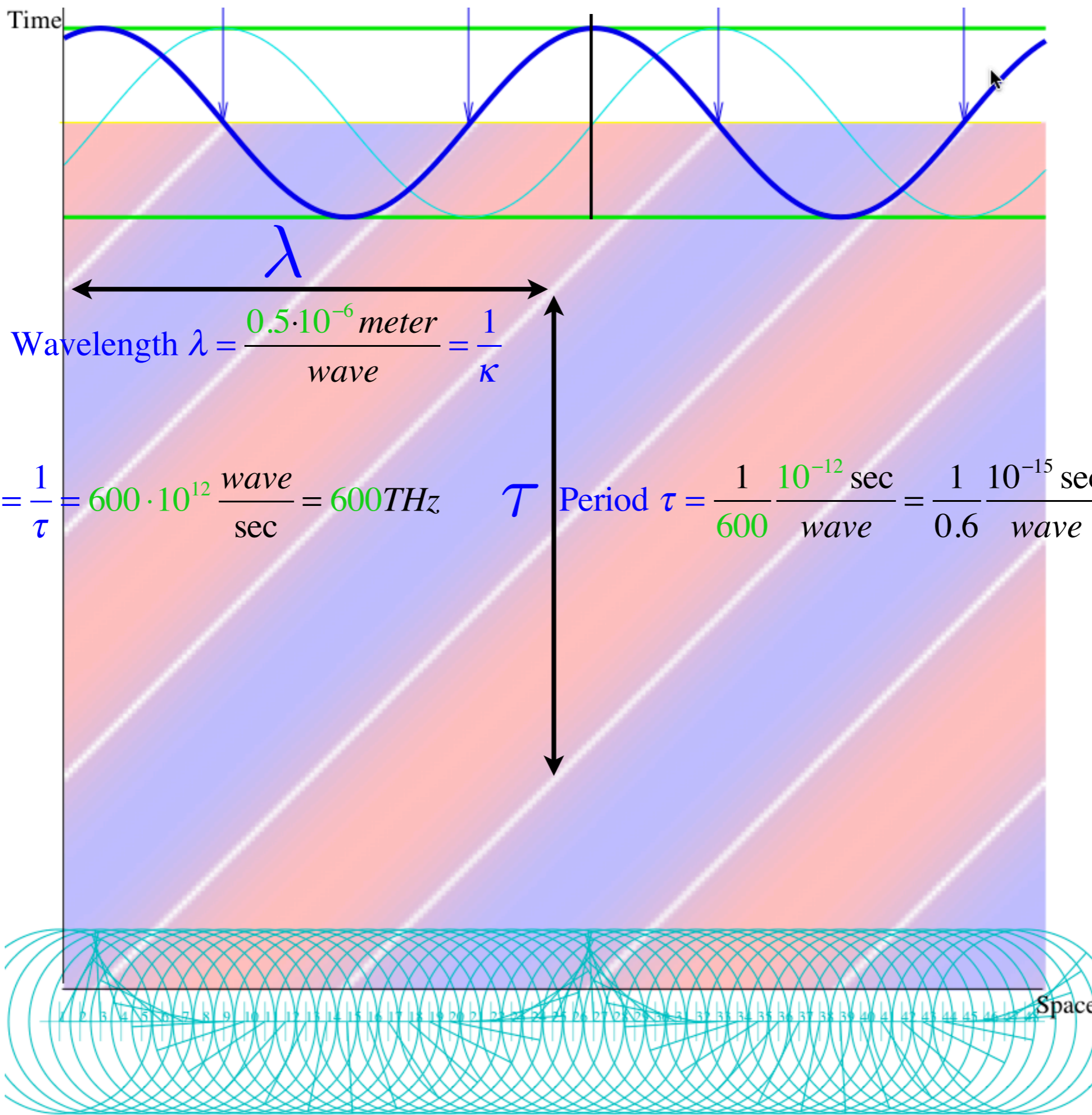
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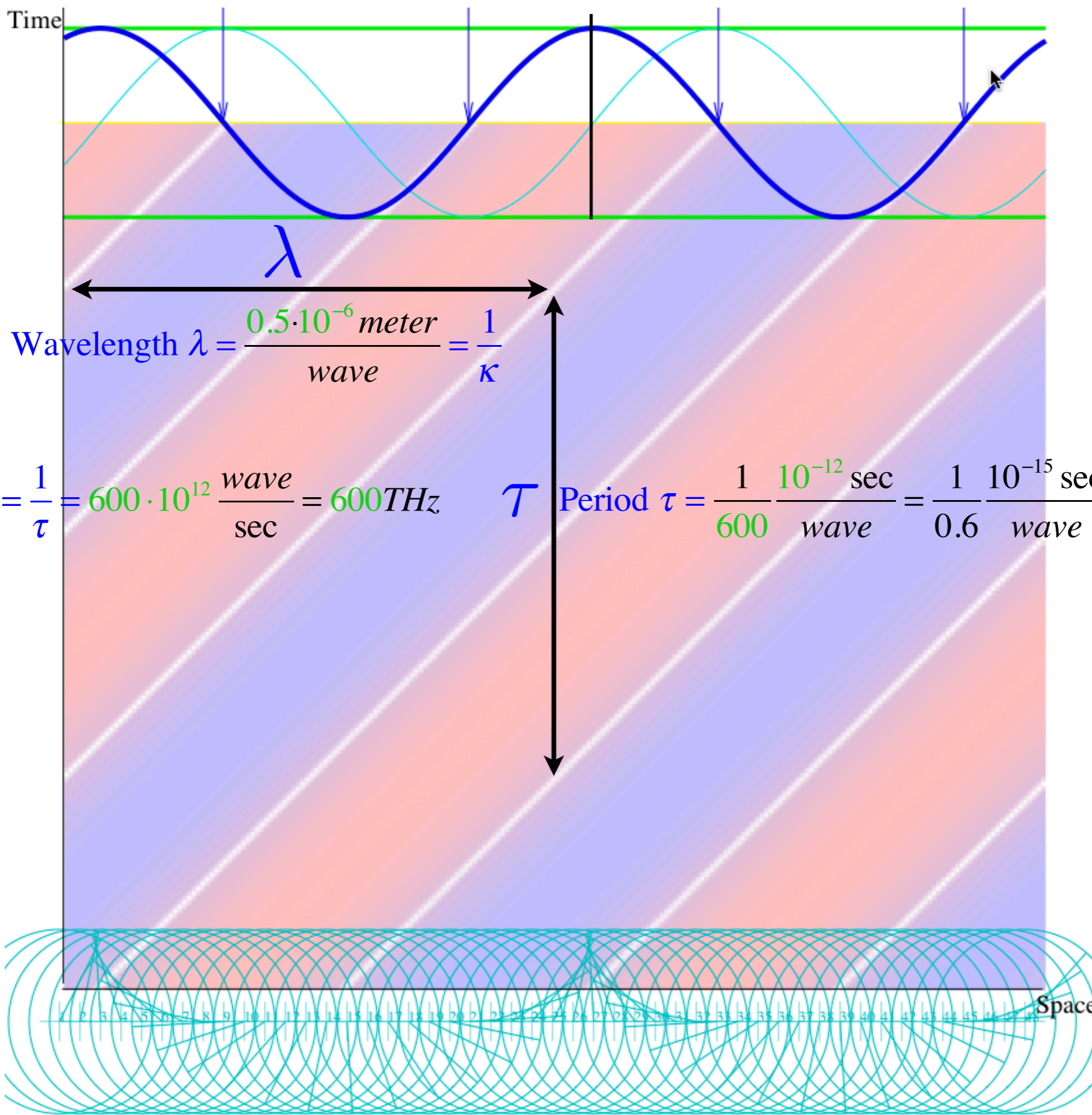
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Wavelength $\lambda = \frac{0.5 \cdot 10^{-6} \text{ meter}}{\text{wave}} = \frac{1}{\kappa}$

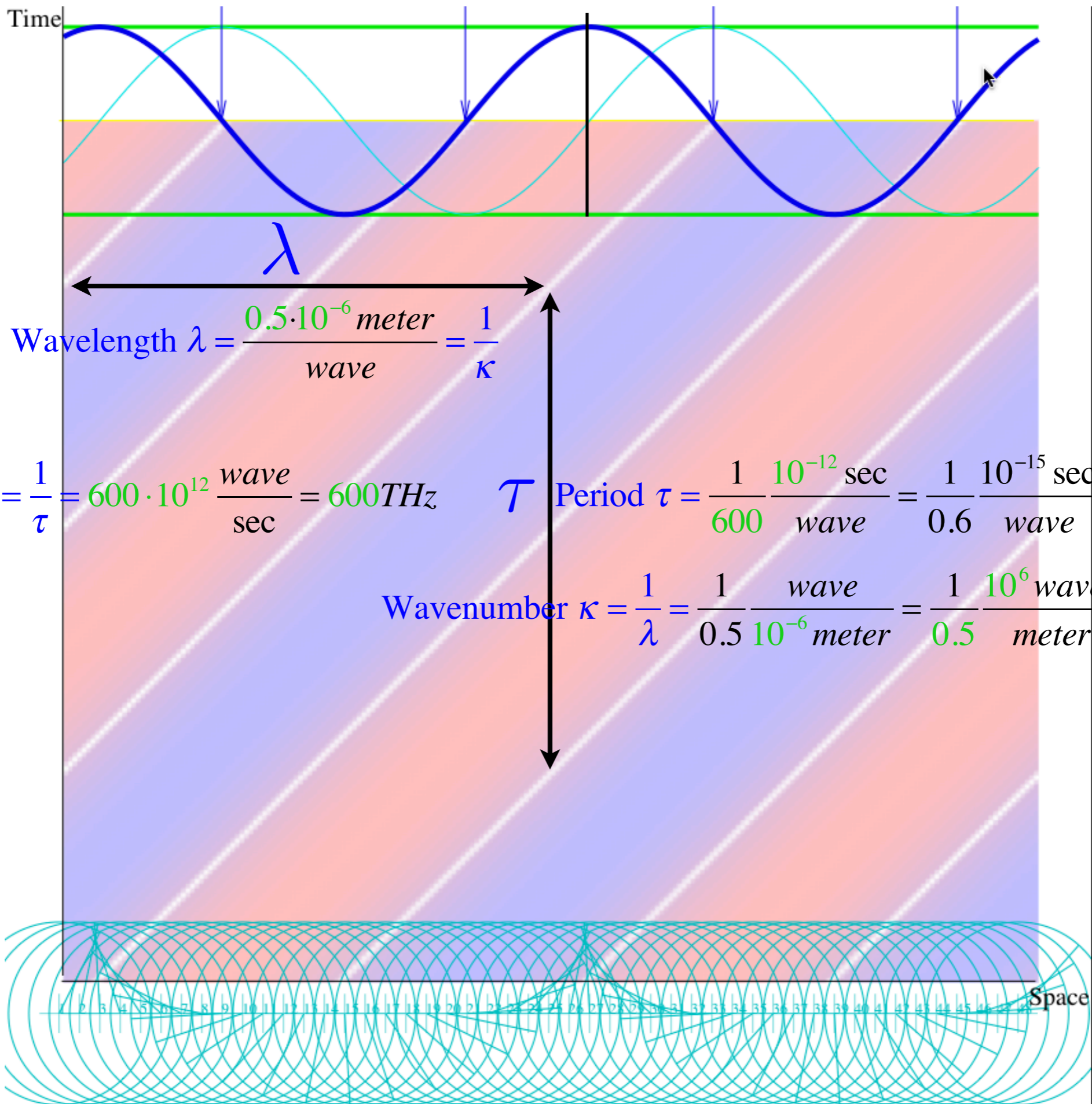
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Heinrich Hertz
1857-1894
1 Hz = 1 sec⁻¹

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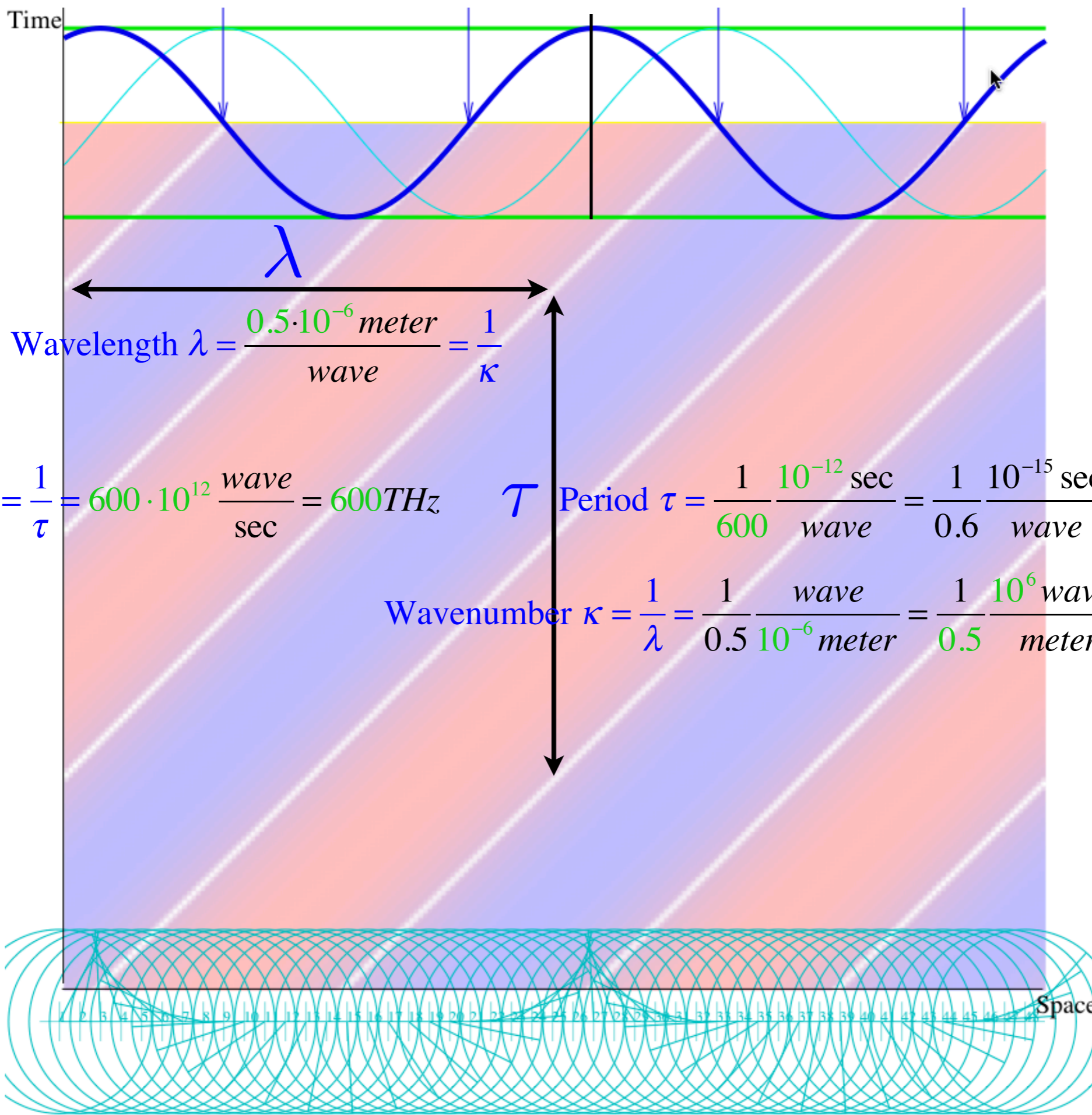
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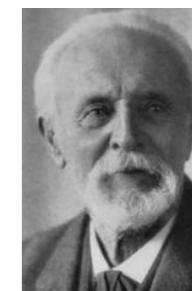
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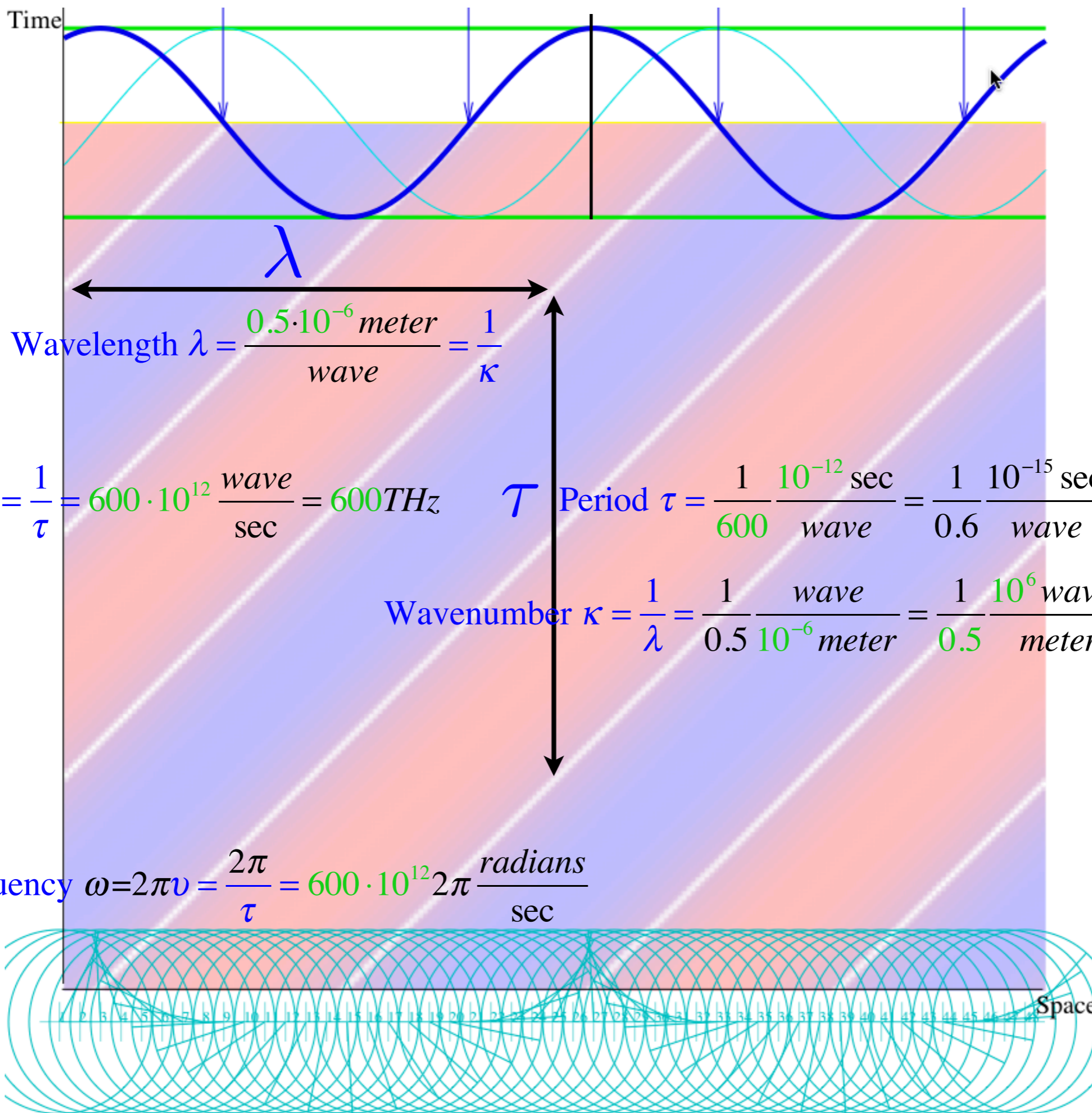
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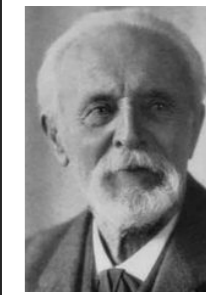
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Angular Frequency $\omega = 2\pi\nu = \frac{2\pi}{\tau} = 600 \cdot 10^{12} 2\pi \frac{\text{radians}}{\text{sec}}$

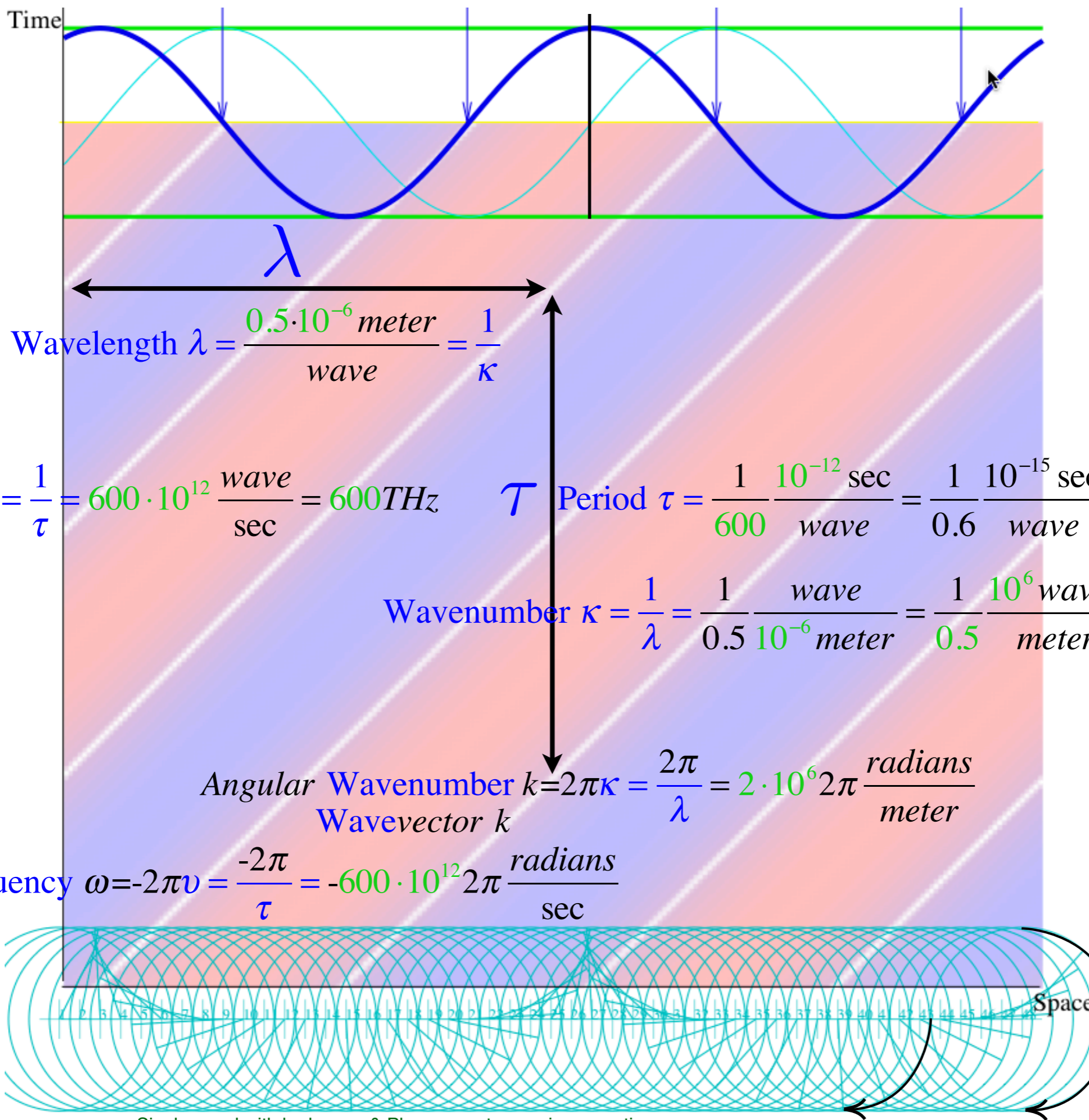


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1Hz=1sec⁻¹



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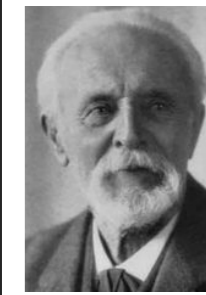
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Angular Wavenumber $k = 2\pi\kappa = \frac{2\pi}{\lambda} = 2 \cdot 10^6 2\pi \frac{\text{radians}}{\text{meter}}$
 Wavevector k

Angular Frequency $\omega = -2\pi\nu = \frac{-2\pi}{\tau} = -600 \cdot 10^{12} 2\pi \frac{\text{radians}}{\text{sec}}$

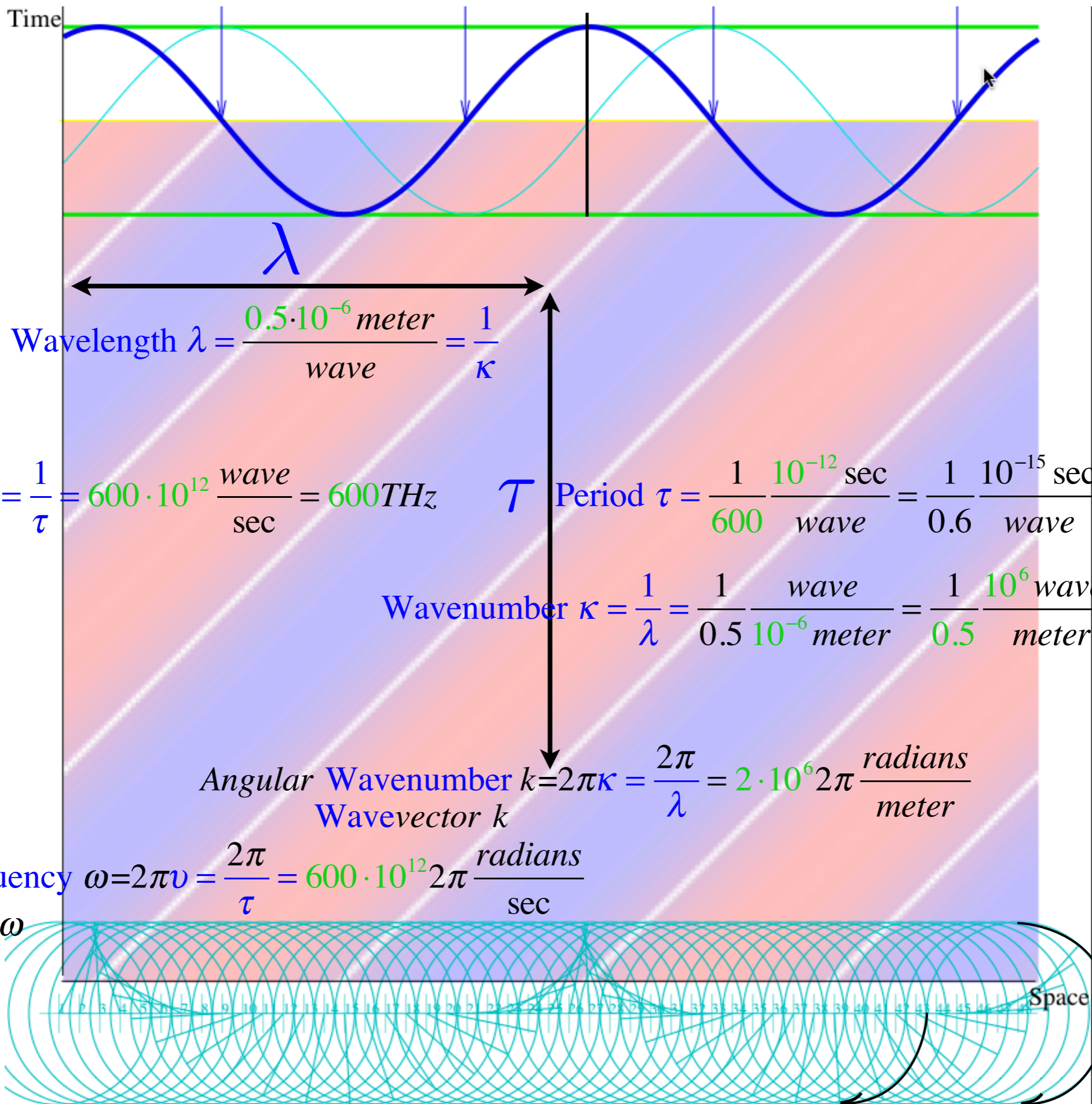


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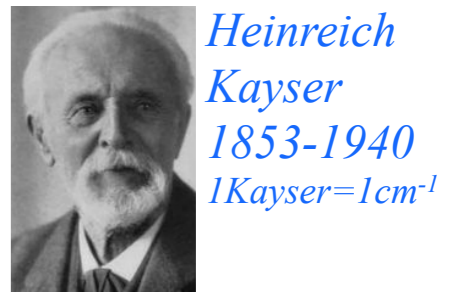
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Wavescalar ω
 Not standard terminology (but should be)



Angular Frequency
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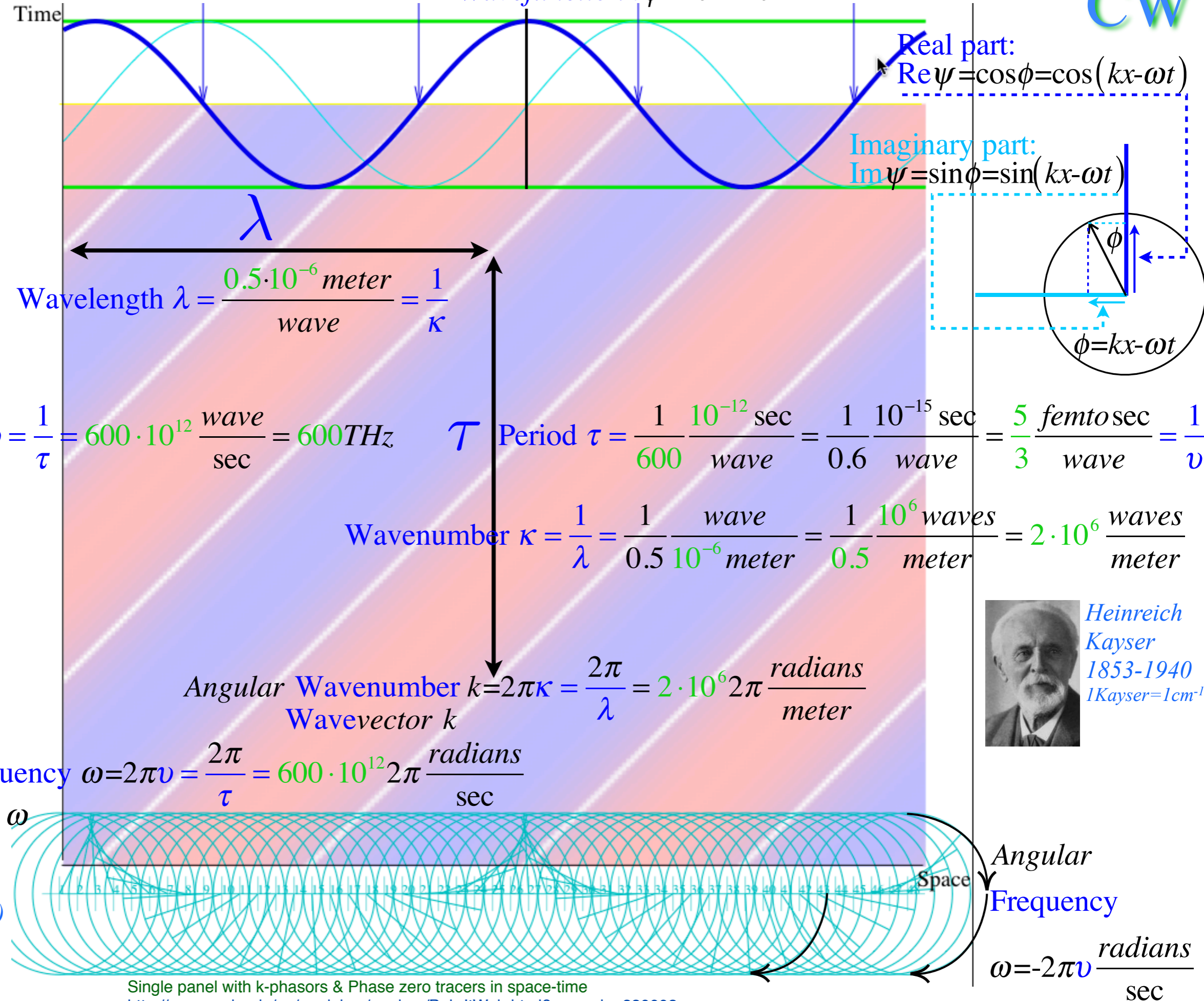
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CW

Wavefunction: $\psi = e^{i\phi} = e^{i(kx-\omega t)}$



Real part:
 $\text{Re}\psi = \cos\phi = \cos(kx-\omega t)$

Imaginary part:
 $\text{Im}\psi = \sin\phi = \sin(kx-\omega t)$

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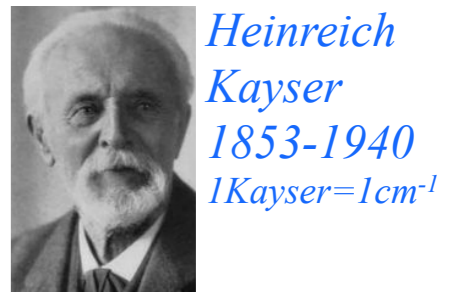
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Angular Frequency
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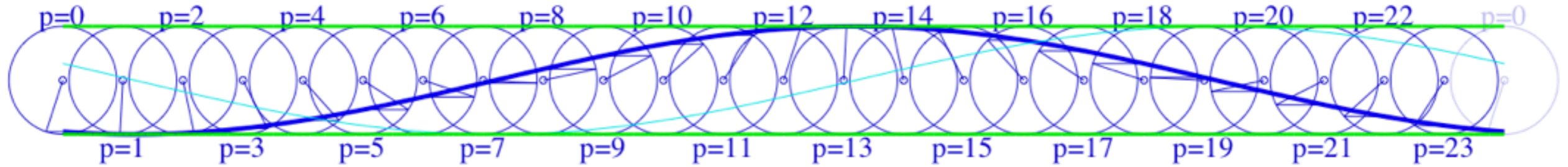
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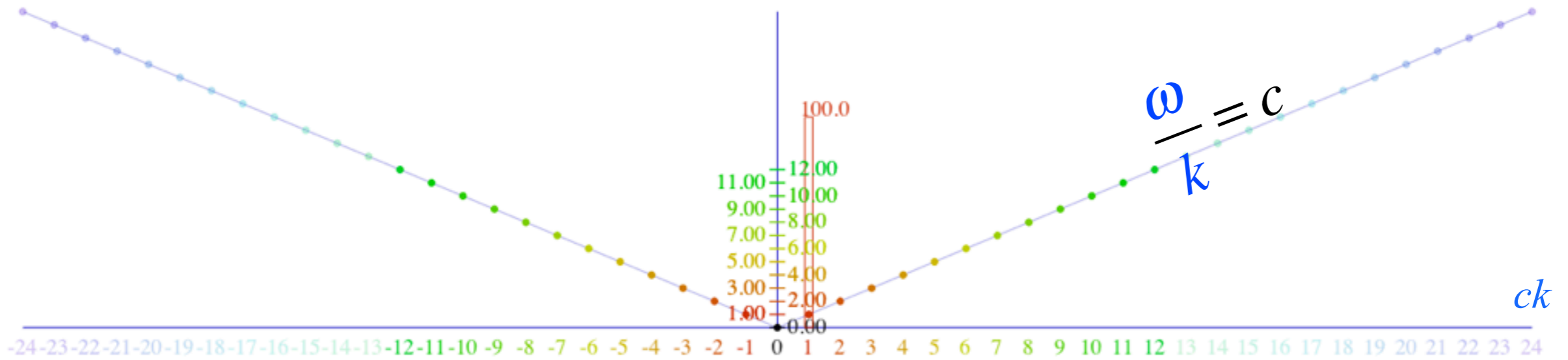
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Wavefunction : $\psi = e^{i\phi} = e^{i(kx - \omega t)}$ Here ($ck = 1, \omega = 1$)

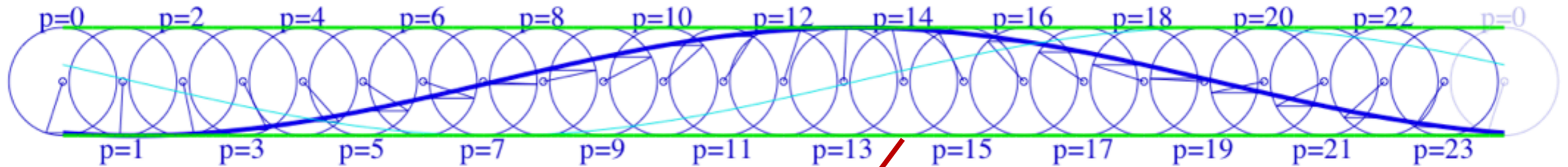


$\omega = ck$



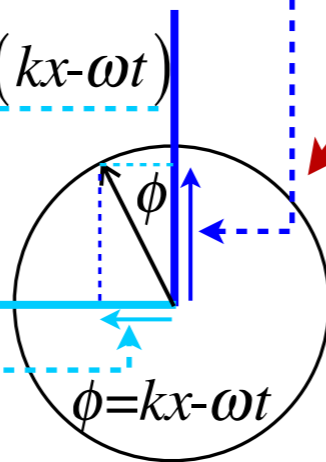
Wavevector k (in units of $2\pi/L$)

Wavefunction : $\psi = e^{i\phi} = e^{i(kx-\omega t)}$ Here ($ck = 1, \omega = 1$)



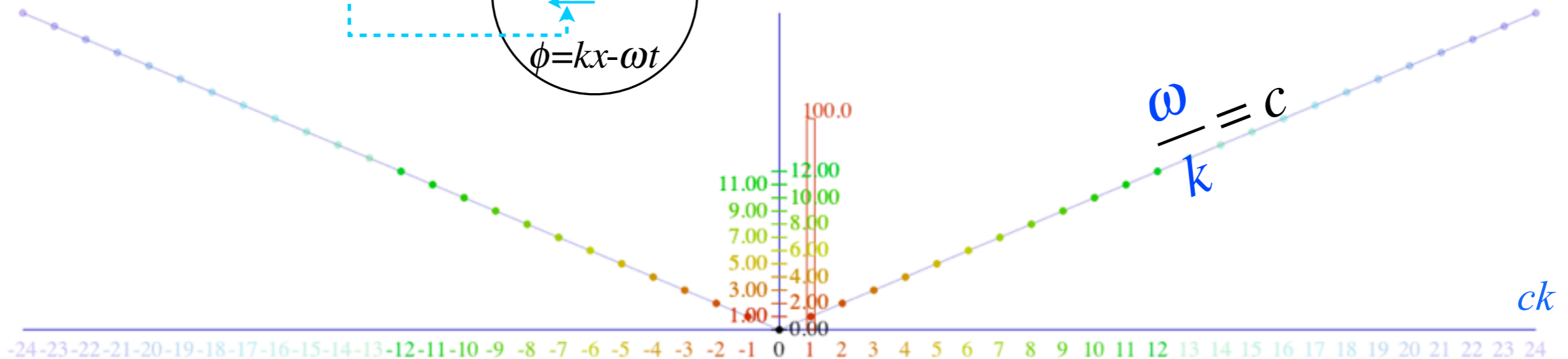
Real part:
 $\text{Re } \psi = \cos \phi = \cos(kx - \omega t)$

Imaginary part:
 $\text{Im } \psi = \sin \phi = \sin(kx - \omega t)$



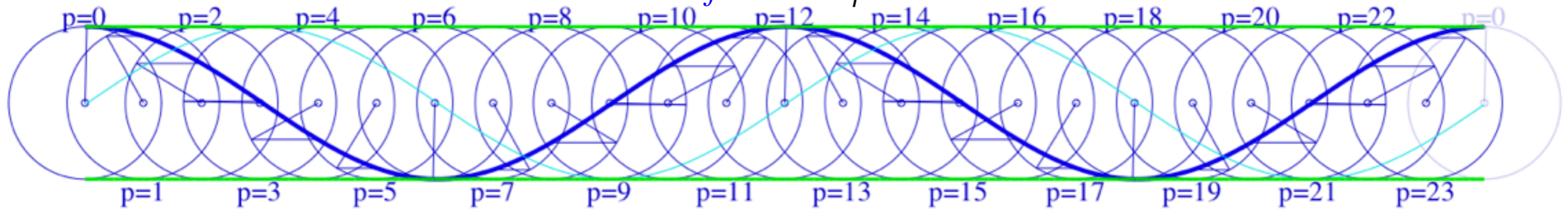
$\omega = ck$

$\frac{\omega}{k} = c$

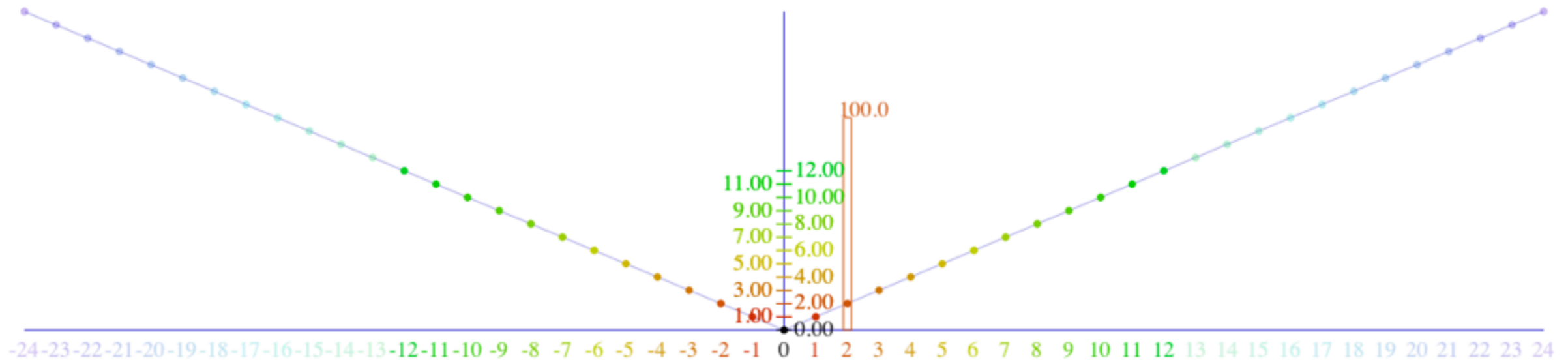


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$\omega =$



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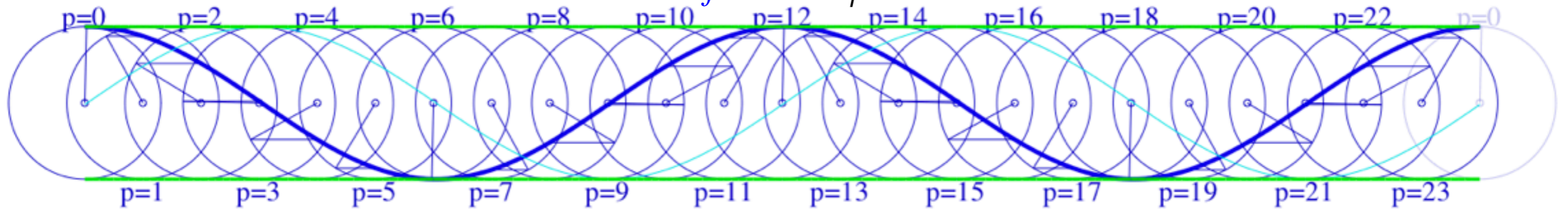
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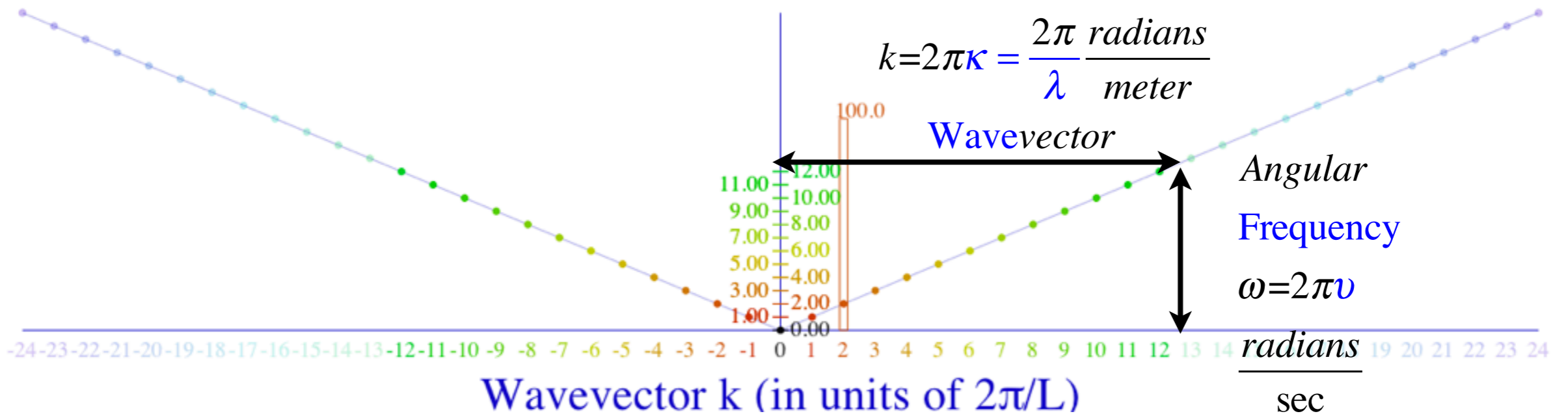


Q: How fast does phase $\phi = kx - \omega t$ go?

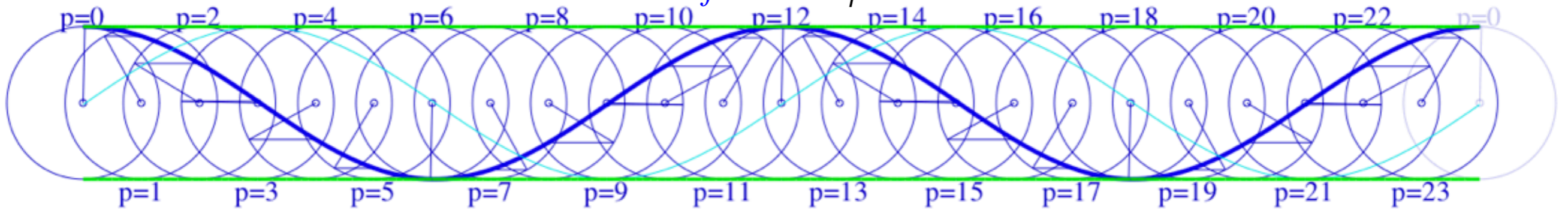


$\omega = 2\pi\nu$

$k = 2\pi\kappa = \frac{2\pi \text{ radians}}{\lambda \text{ meter}}$



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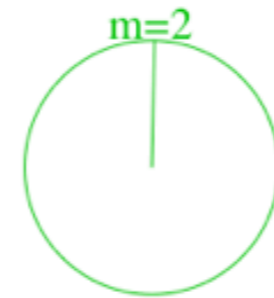


Q: How fast does phase $\phi = kx - \omega t$ go?

A: Solve $\phi = kx - \omega t$ to get: $kx = \omega t + \phi$

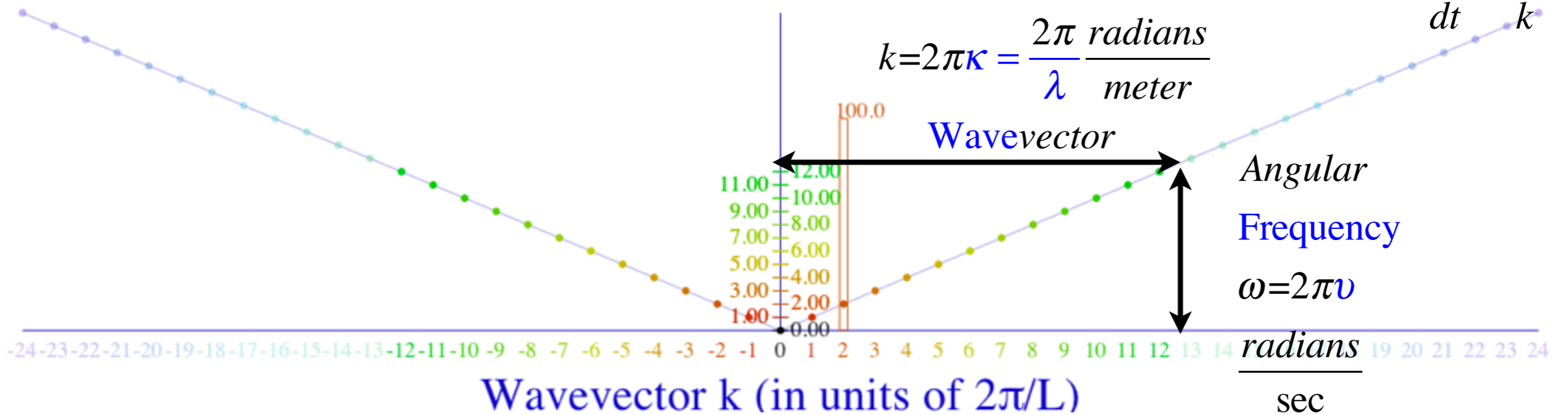
$$x = \frac{\omega}{k}t + \frac{\phi}{k}$$

$$\frac{dx}{dt} = \frac{\omega}{k}$$



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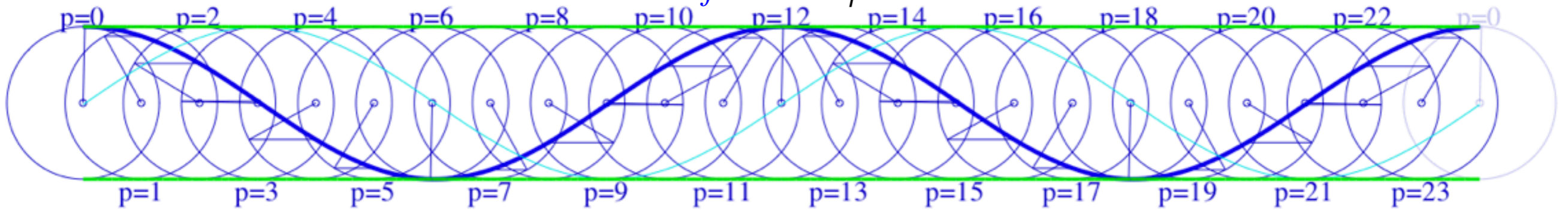
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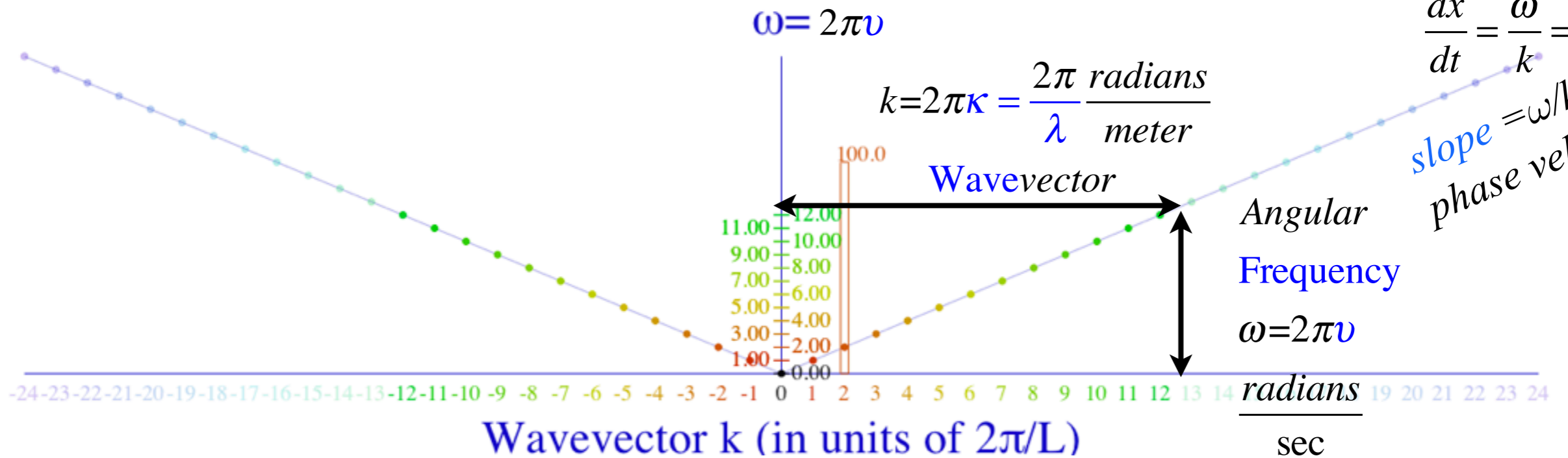


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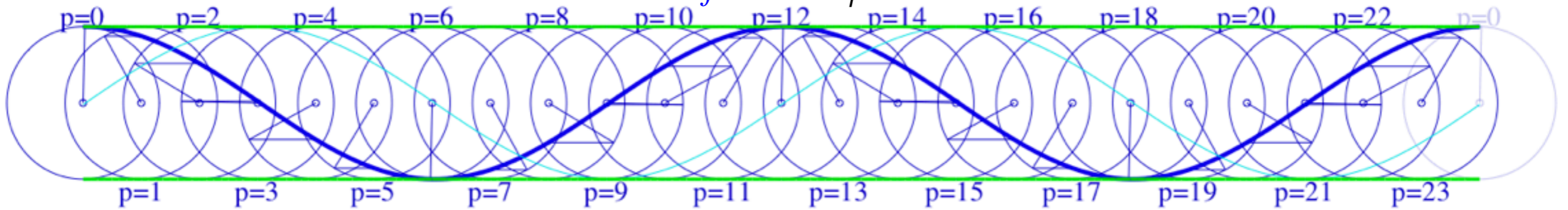
$$x = \frac{\omega}{k}t + \frac{\phi}{k}$$

$$\frac{dx}{dt} = \frac{\omega}{k} = c$$

slope = $\omega/k = c$
phase velocity



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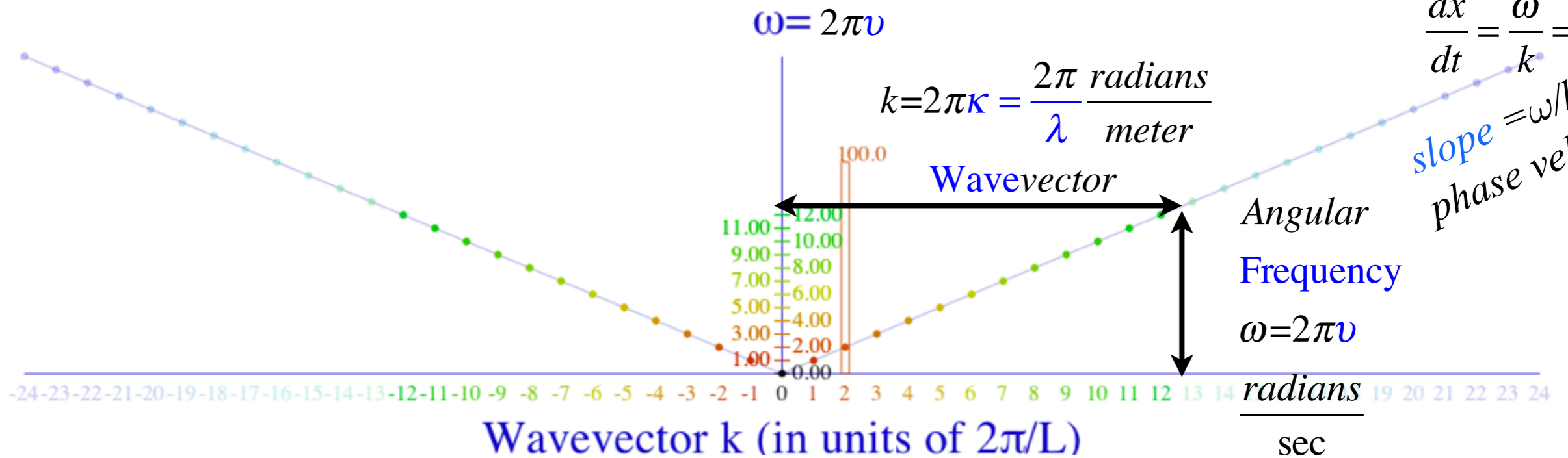


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Usually we plot light waves unit slope $= \omega/ck = 1$ for phase velocity since "All colors go c"

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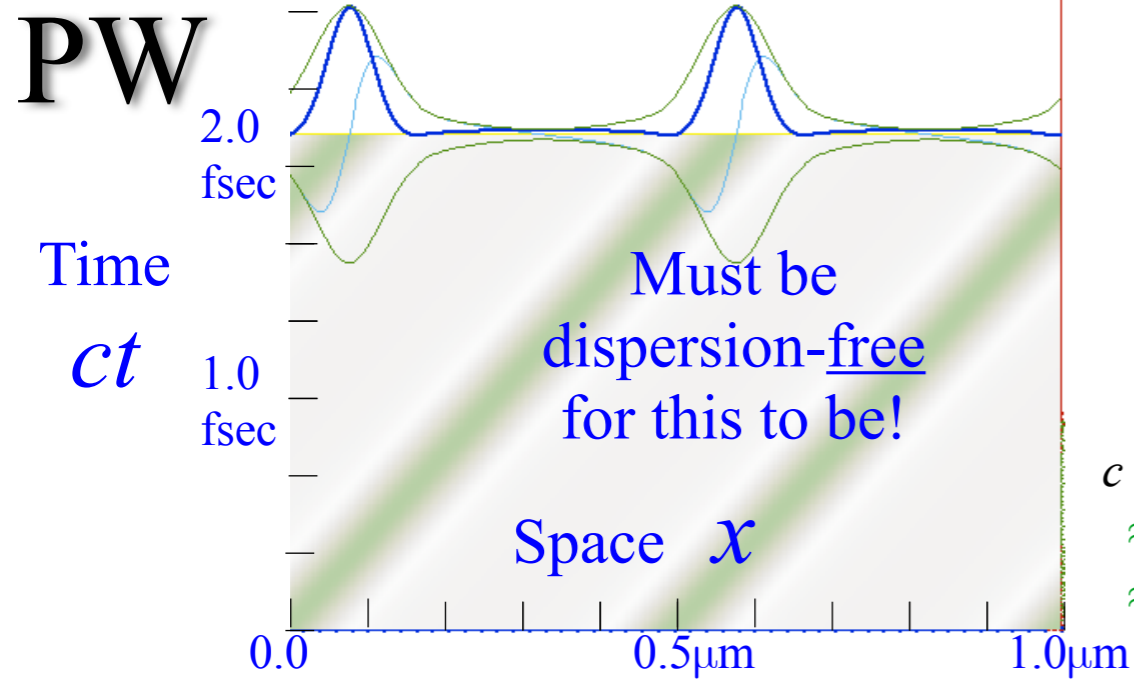
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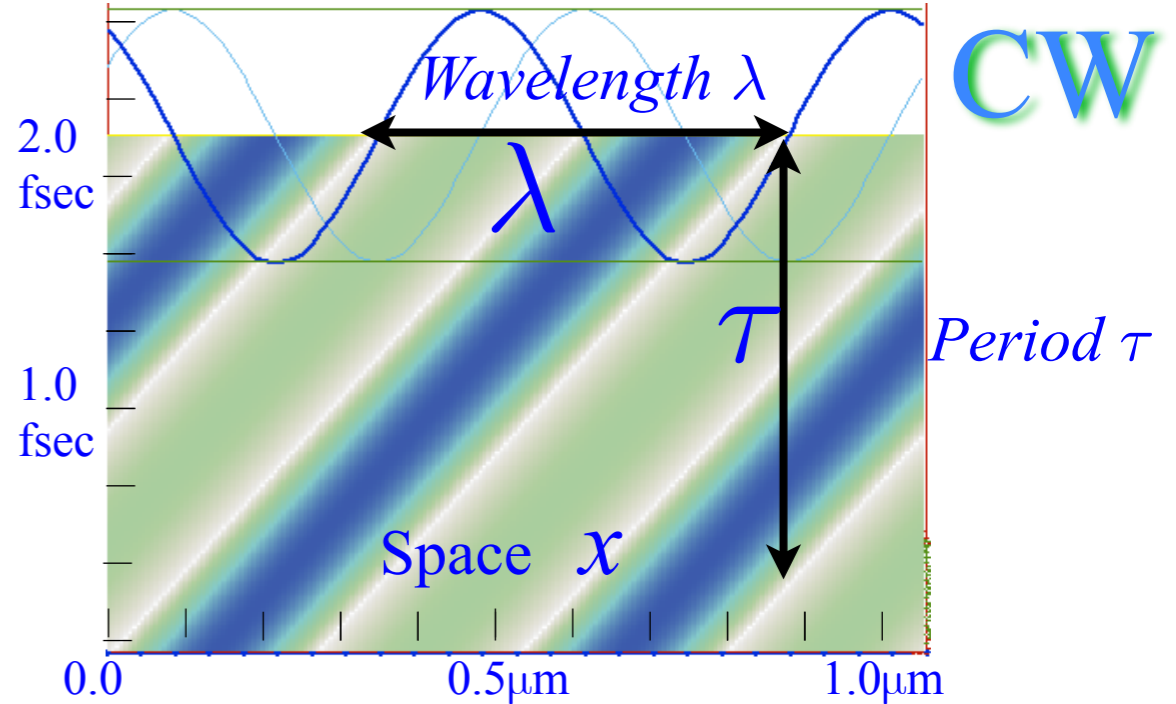
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$c = 2.99792458 \cdot 10^8 \text{ m/s}$
 $\approx 3 \cdot 10^8 \text{ m/s}$
 $\approx 0.3 \text{ } \mu\text{m/fs} \approx 1 \text{ ft/ns}$



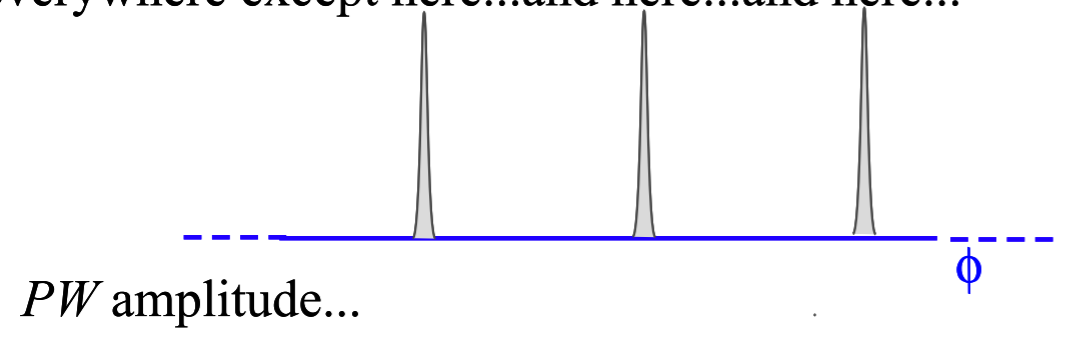
It helps to introduce two *archetypes* of light waves and contrast them.

The first (*PW*) is a *Particle-like Wave* or part of a *Pulse-Wave* train.
 The second (*CW*) is a *Coherent Wave* or part of a *Continuous-Wave* train.

..or *Cosine Wave* ...or *Colored Wave*

(1) *The PW archetype*

PW amplitude is **ZERO** (mostly...) everywhere except here...and here...and here...

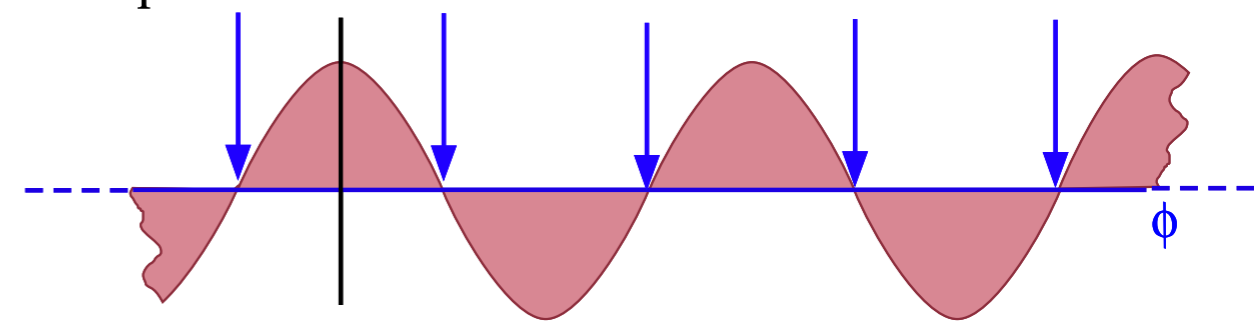


ZEROS.
 ...but has sharp **PEAKS.**
 ...is best defined by where it **IS.**

Ideal *PW* shape is a *Dirac Delta function.*

(2) *The CW archetype*

CW amplitude is **NON-zero** (exactly!) everywhere except here...and here...and here...and here...



...is mostly **NON-zero** with rounded crests and troughs.
 ...but has sharp **ZEROS.**
 ...is best defined by where it **IS NOT.**

Ideal *CW* shape is a *cosine wave* ($\cos(\phi)$)

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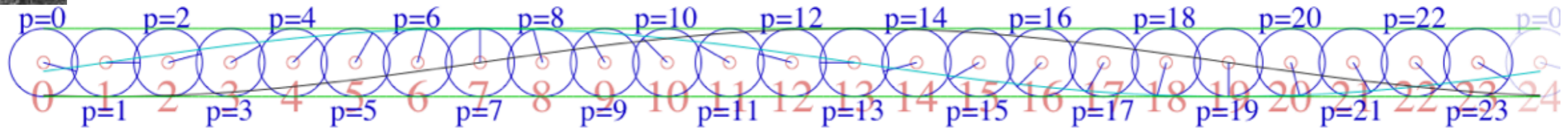
Use WaveIt "Fourier Control" to put in any Fourier components you like!

Launch Local Controls Scenarios Resume Set T=0 Zero Amps T-Scale= 1



Jean-Baptiste Joseph Fourier
1768-1830

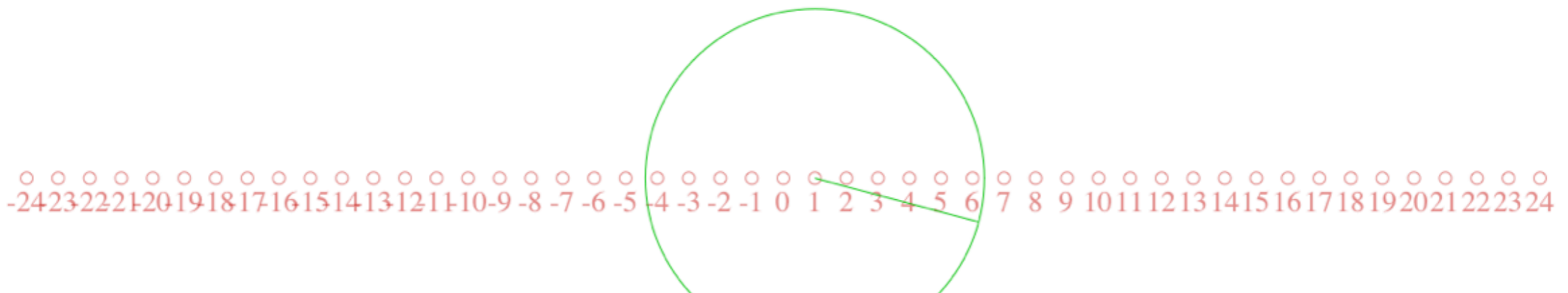
t =



Wave amplitudes vs. position p (p in units of L/24)

Click-Drag from dots to change amplitudes. Click here to zero all:

Wave amplitude vs. wavevector m (m in units of $2\pi/L$)



solo right 1-CW over linear dispersion + k-histogram
http://www.uark.edu/ua/modphys/markup/WaveItWeb.html?scenario=1CW_K+1_2016HP

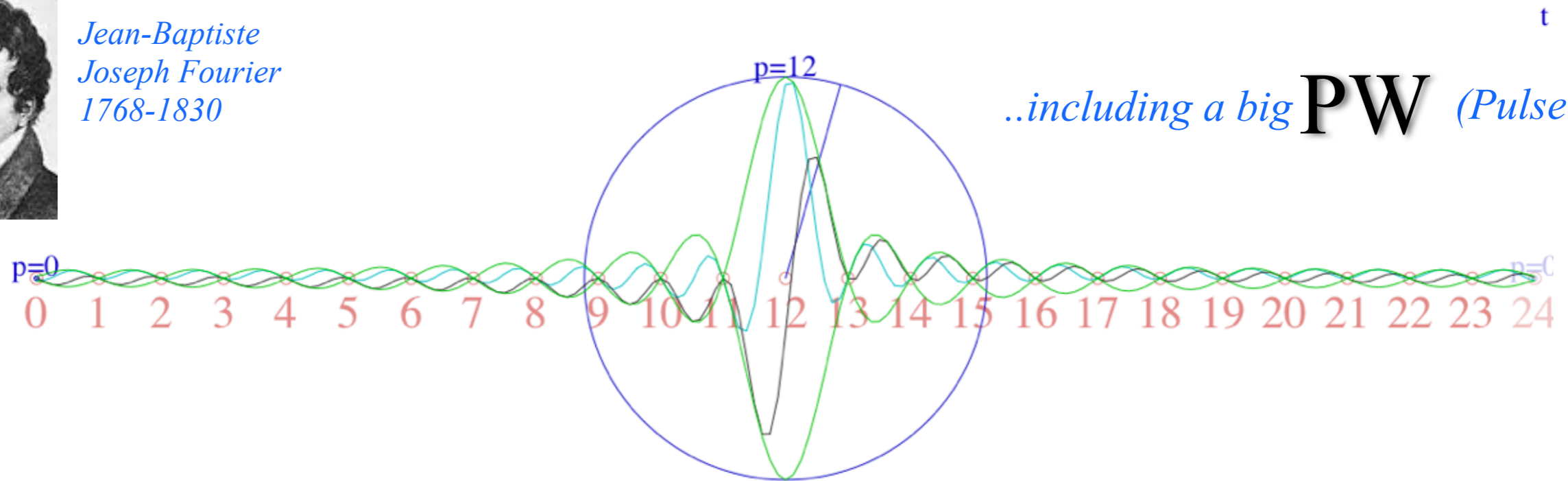
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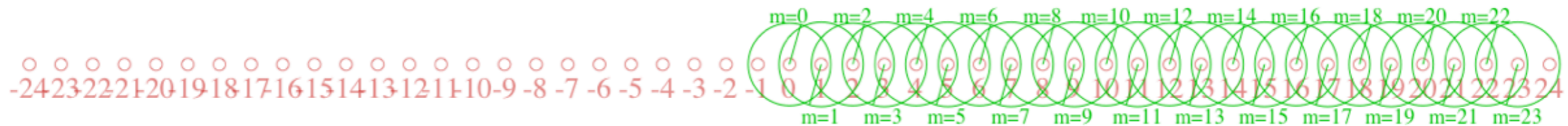


..including a big **PW** (Pulse Wave)

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This is called a "Boxcar" spectrum when each Fourier component-m has same amplitude

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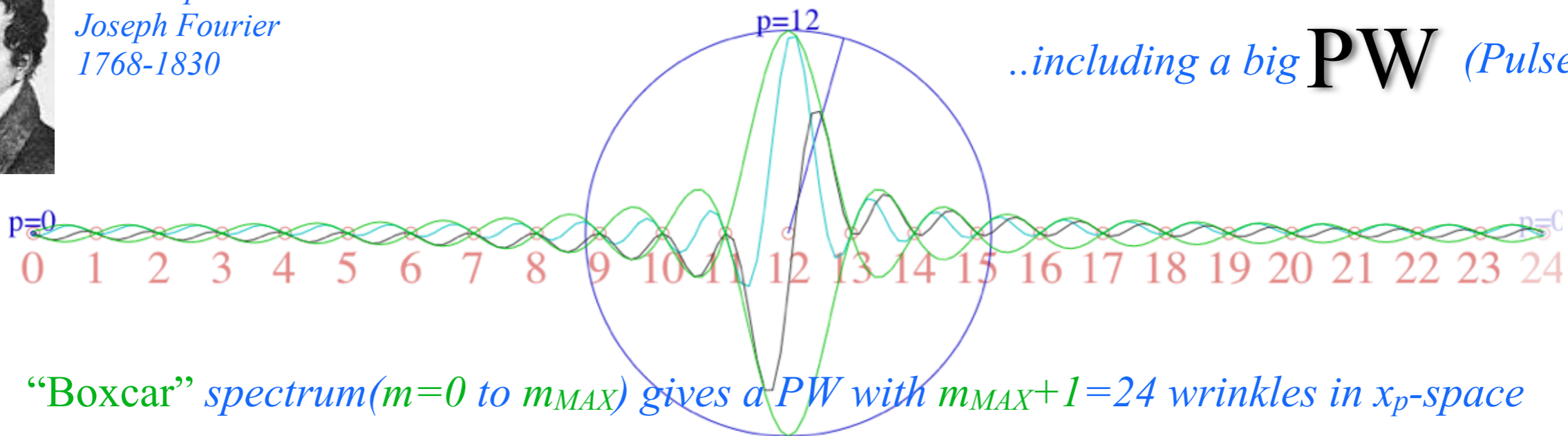
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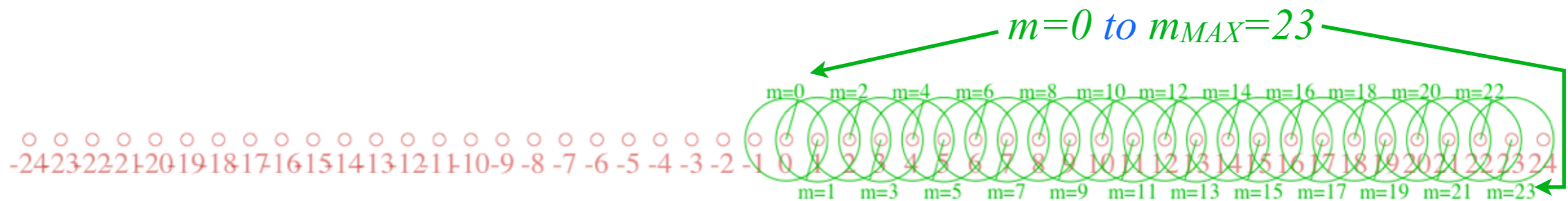
..including a big **PW** (Pulse Wave)

“Boxcar” spectrum ($m=0$ to m_{MAX}) gives a PW with $m_{MAX}+1=24$ wrinkles in x_p -space

Wave amplitudes vs. position p (p in units of $L/24$)

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Wave amplitude vs. wavevector m (m in units of $2\pi/L$)



This is called a “Boxcar” spectrum when each Fourier component- m has same amplitude

(But alternating \pm phases)

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PW forms are also called *Wave Packets (WP)*

since

they are

interfering

sums of

many

CW terms

(10-Color Waves make up this pulse)

CW terms are also called

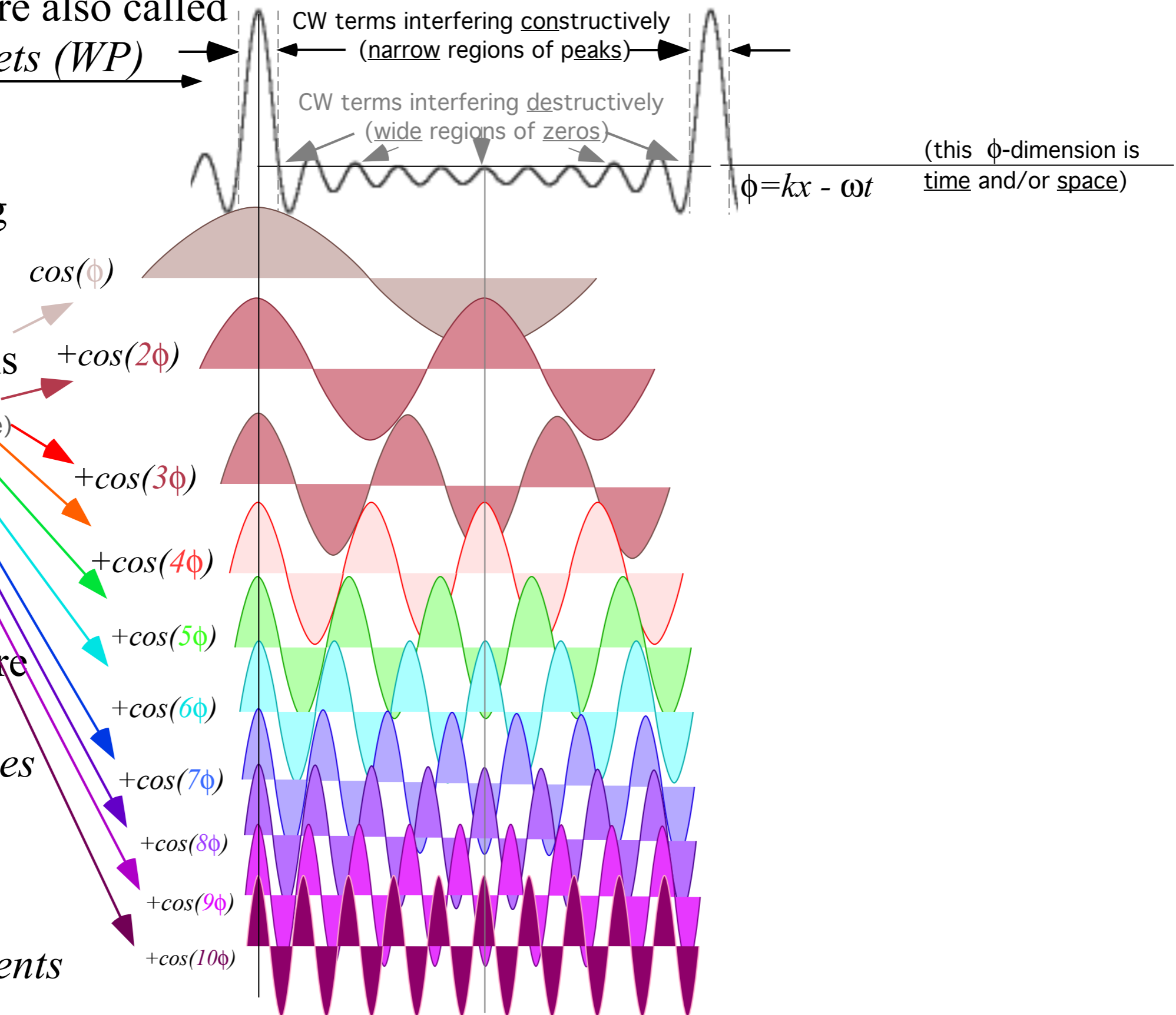
Color Waves

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WaveIt animation: 24 Spectral Components

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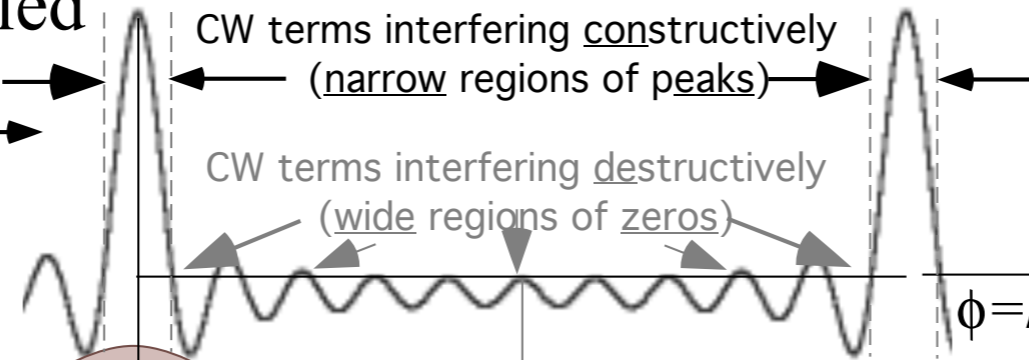
Color Waves

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Sum geometric series: $S = 1 + a + a^2 + a^3 + a^4 \dots + a^n$ for: $a = e^{i\phi}$

$\cos(\phi)$

$+ \cos(2\phi)$

$+ \cos(3\phi)$

$+ \cos(4\phi)$

$+ \cos(5\phi)$

$+ \cos(6\phi)$

$+ \cos(7\phi)$

$+ \cos(8\phi)$

$+ \cos(9\phi)$

$+ \cos(10\phi)$

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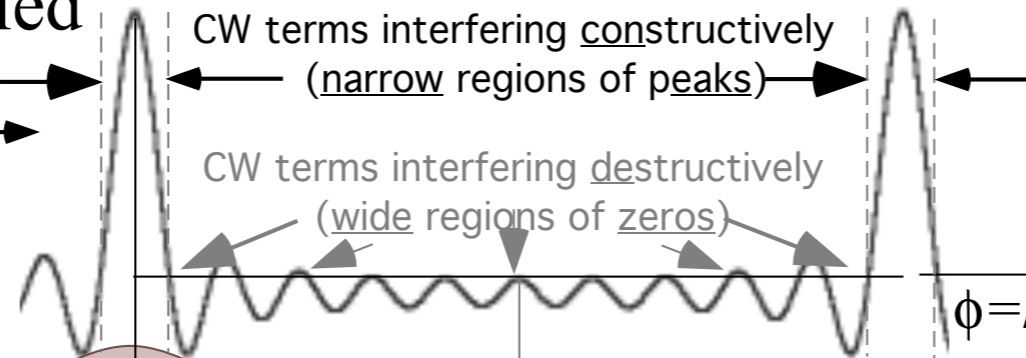
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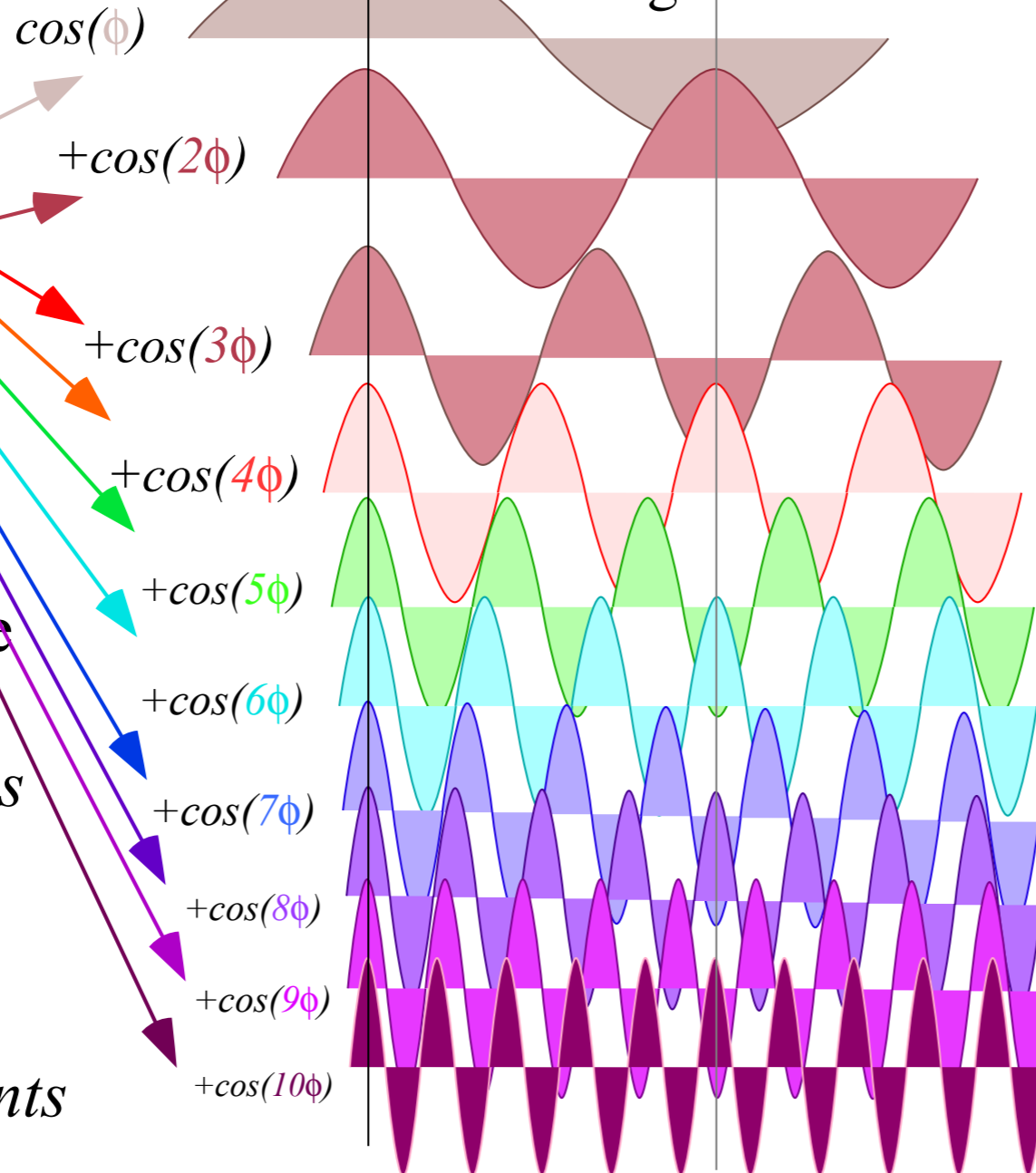
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(this ϕ -dimension is time and/or space)

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$$aS = a+a^2+a^3+a^4\cdots+a^n+a^{n+1}$$



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PW

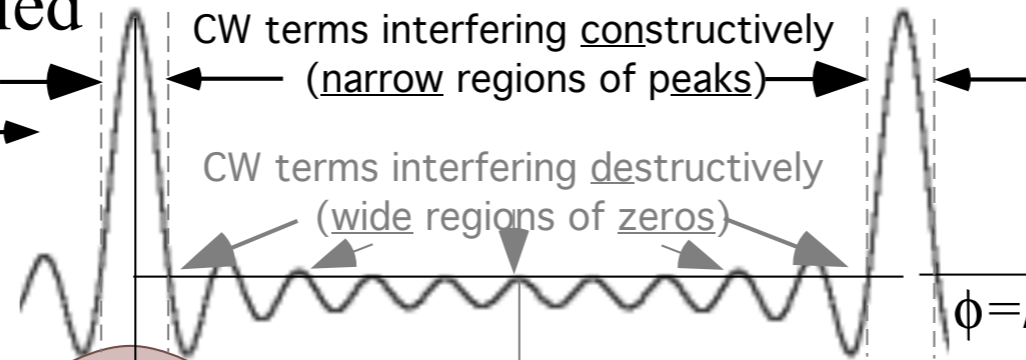
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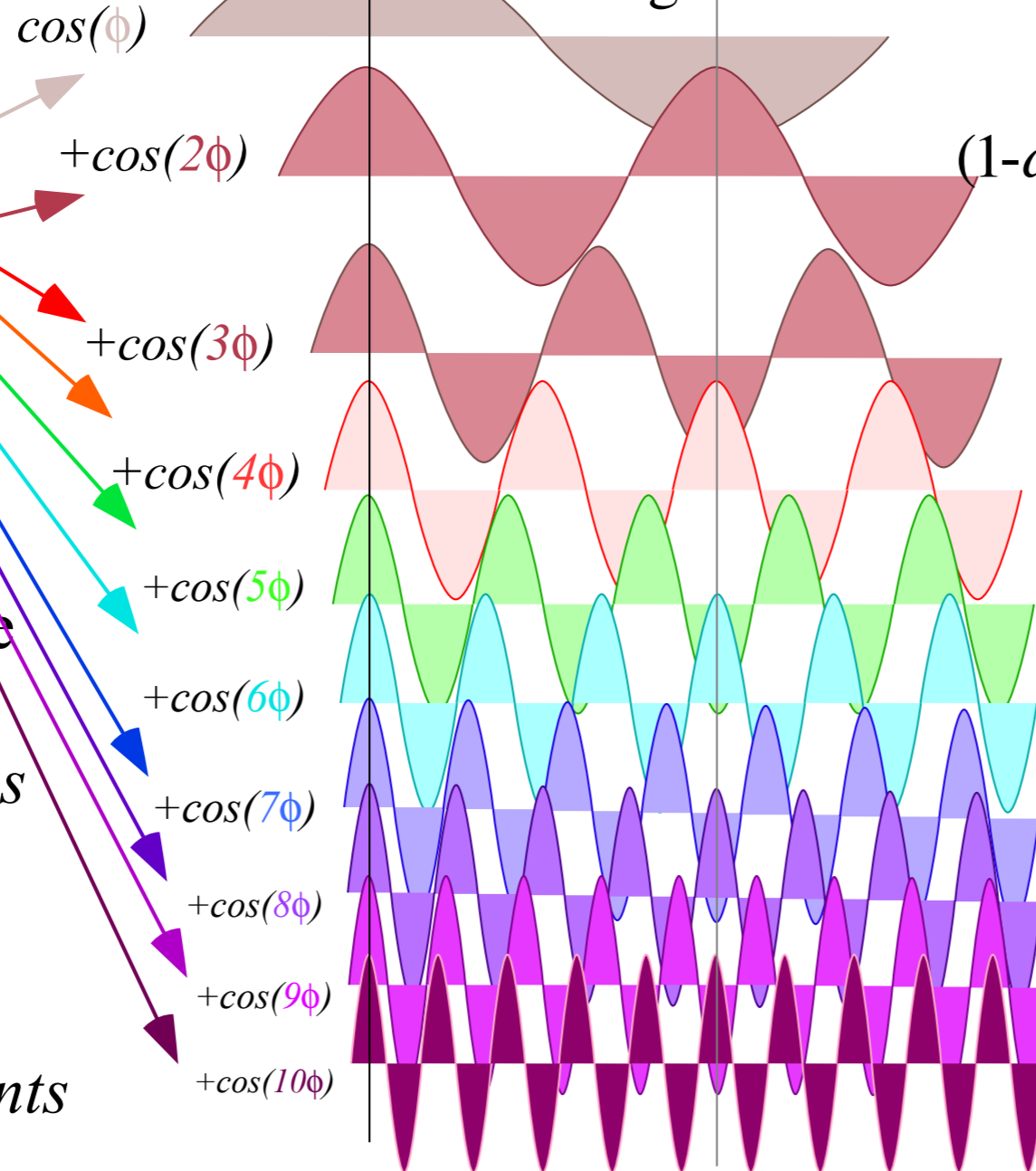
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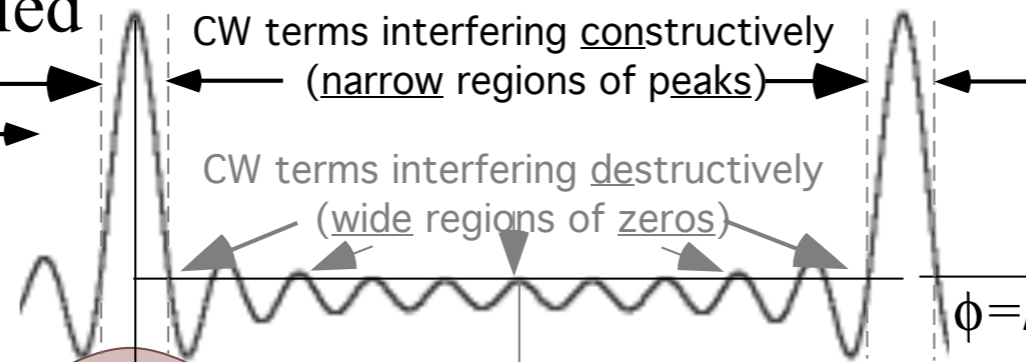
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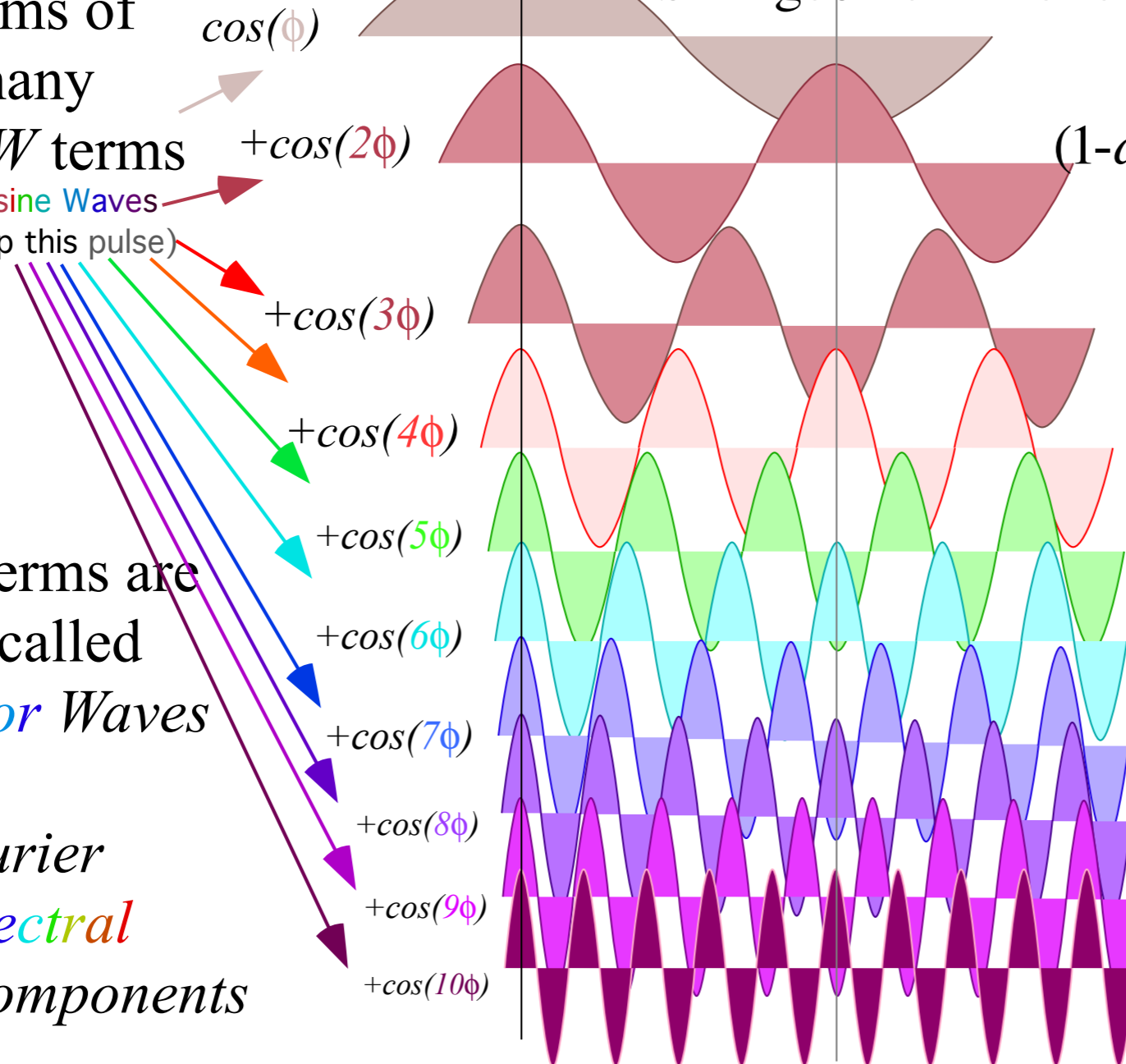


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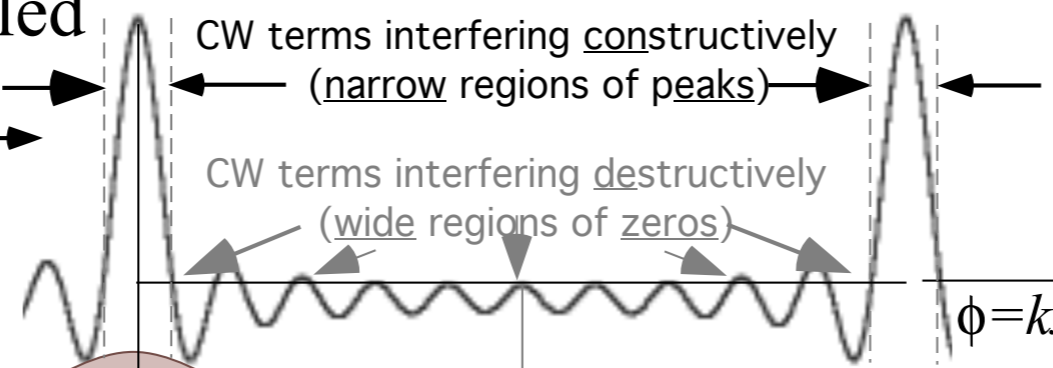
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$$(1-a)S = 1 - a^{n+1}$$

$$S = \frac{1-a^{n+1}}{1-a} = \frac{a^{\frac{n+1}{2}} \left(a^{-\frac{n+1}{2}} - a^{\frac{n+1}{2}} \right)}{a^{\frac{1}{2}} \left(a^{-\frac{1}{2}} - a^{\frac{1}{2}} \right)}$$

$$= e^{i\phi \frac{n}{2}} \left(\frac{e^{-i\phi \frac{n+1}{2}} - e^{i\phi \frac{n+1}{2}}}{e^{-\frac{i\phi}{2}} - e^{\frac{i\phi}{2}}} \right) = e^{i\phi \frac{n}{2}} \frac{\sin \frac{n+1}{2} \phi}{\sin \frac{\phi}{2}}$$

WaveIt animation: 24 Spectral Components

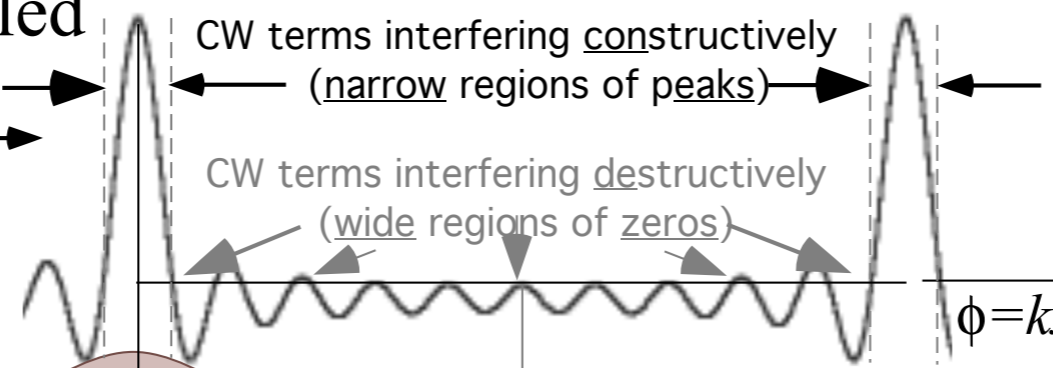
PW

PW forms are also called Wave Packets (WP)

since they are interfering sums of many CW terms

(10-Cosine Waves make up this pulse)

CW terms are also called **Color Waves** or **Fourier Spectral Components**



(this ϕ -dimension is time and/or space)

Sum geometric series: $S=1+a+a^2+a^3+a^4\cdots+a^n$ for: $a = e^{i\phi}$

$$aS = a+a^2+a^3+a^4\cdots+a^n+a^{n+1}$$

$$(1-a)S = 1 - a^{n+1}$$

$$S = \frac{1-a^{n+1}}{1-a} = \frac{a^{\frac{n+1}{2}} \left(a^{-\frac{n+1}{2}} - a^{\frac{n+1}{2}} \right)}{a^{\frac{1}{2}} \left(a^{-\frac{1}{2}} - a^{\frac{1}{2}} \right)}$$

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$$\xrightarrow{as\phi \rightarrow 0} e^{i\phi \frac{n}{2}} \frac{n+1}{2} \frac{\phi}{\phi} \rightarrow n+1$$

WaveIt animation: 24 Spectral Components

Introducing lightwave Fourier analysis - Pulse Waves (PW) versus Continuous Waves (CW)

Simplest is CW (Continuous Wave, Cosine Wave, Colored Wave, Complex Wave,...)

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Wave phasors, phasor chain plots, dispersion functions $\omega(k)$, and phase velocity $V_{phase}=\omega(k)/k$

Special case: Lightwave linear dispersion: $V_{phase}=c=\omega(k)=ck$

Introducing PW (Pulse Wave, Particle-like Wave, Packet Wave,...) archetypes compared to CW

Building PW from CW components using “Fourier Control” app-panel

Fourier PW “box-car” geometric series summed

➔ Animation of PW obeying lightwave linear dispersion $\omega(k)=ck$ ←

Important Evenson axiom for relativity: “All colors go c”

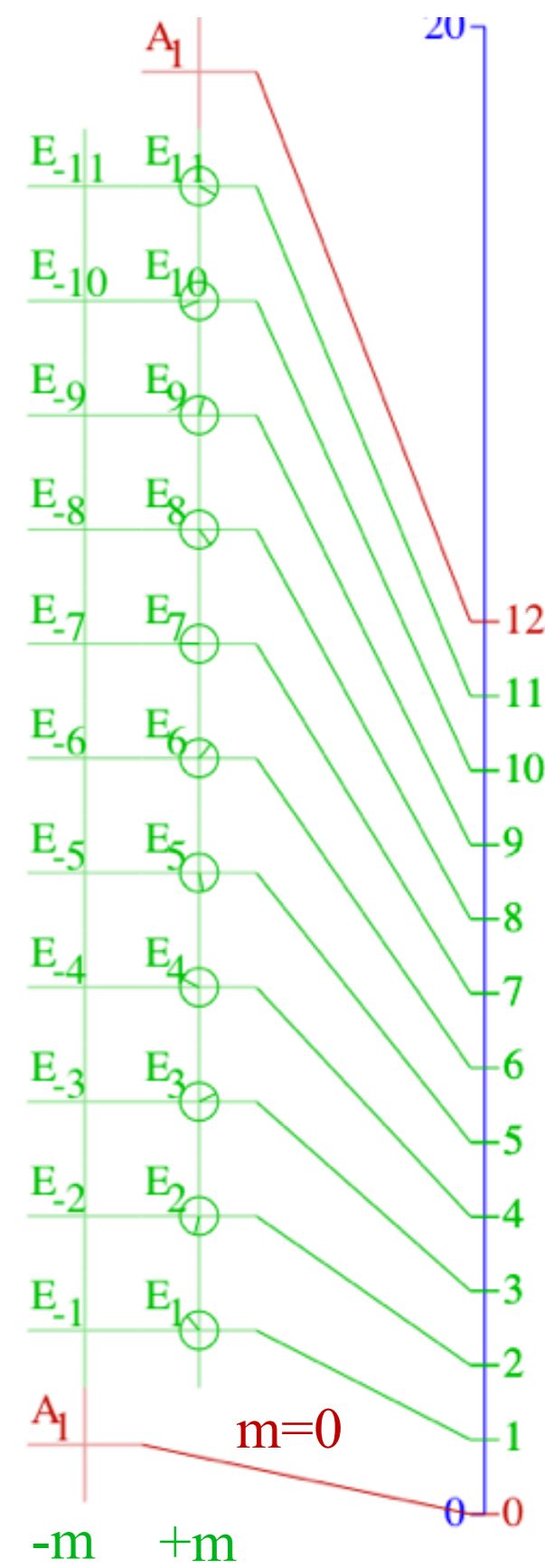
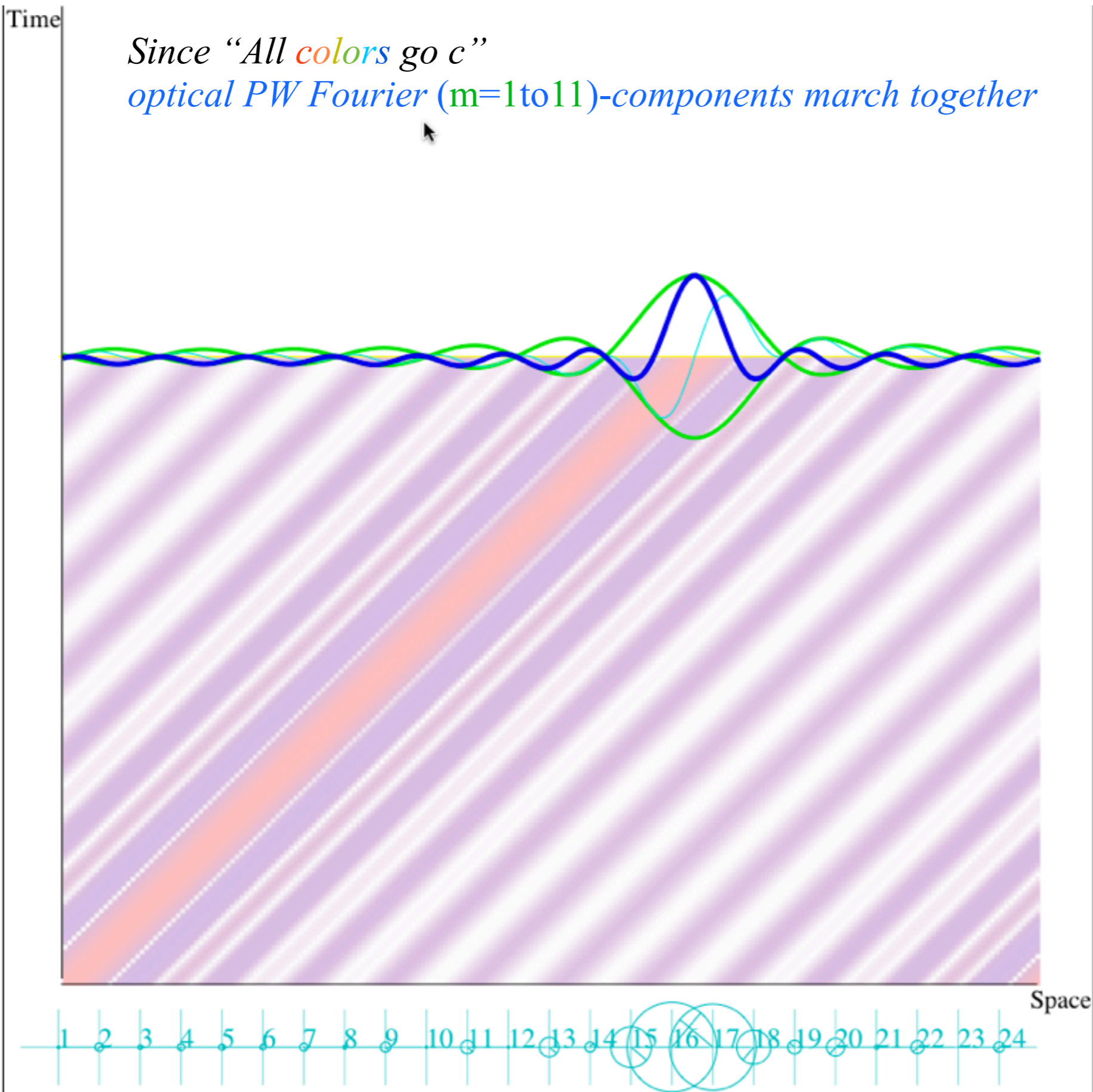
Visualizing PW wave uncertainty relations for space: $\Delta x \cdot \Delta \kappa=1$ and time: $\Delta t \cdot \Delta \nu=1$

PW “wrinkles” go away if Fourier “boxcar” is tapered to a softer “Gaussian”

Opposite-pair CW (colliding $\pm m=\pm 2$) Fourier components trace a Cartesian space-time grid

PW

Since "All colors go c"
 optical PW Fourier (m=1 to 11)-components march together



1-Way Dirac Delta PW +1
<http://www.uark.edu/ua/modphys/markup/BohrItWeb.html?scenario=4002>

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Fourier PW “box-car” geometric series summed

Animation of PW obeying lightwave linear dispersion $\omega(k)=ck$

 *Important Evenson axiom for relativity: “All colors go c”* 

Visualizing PW wave uncertainty relations for space: $\Delta x \cdot \Delta \kappa = 1$ and time: $\Delta t \cdot \Delta \nu = 1$

PW “wrinkles” go away if Fourier “boxcar” is tapered to a softer “Gaussian”

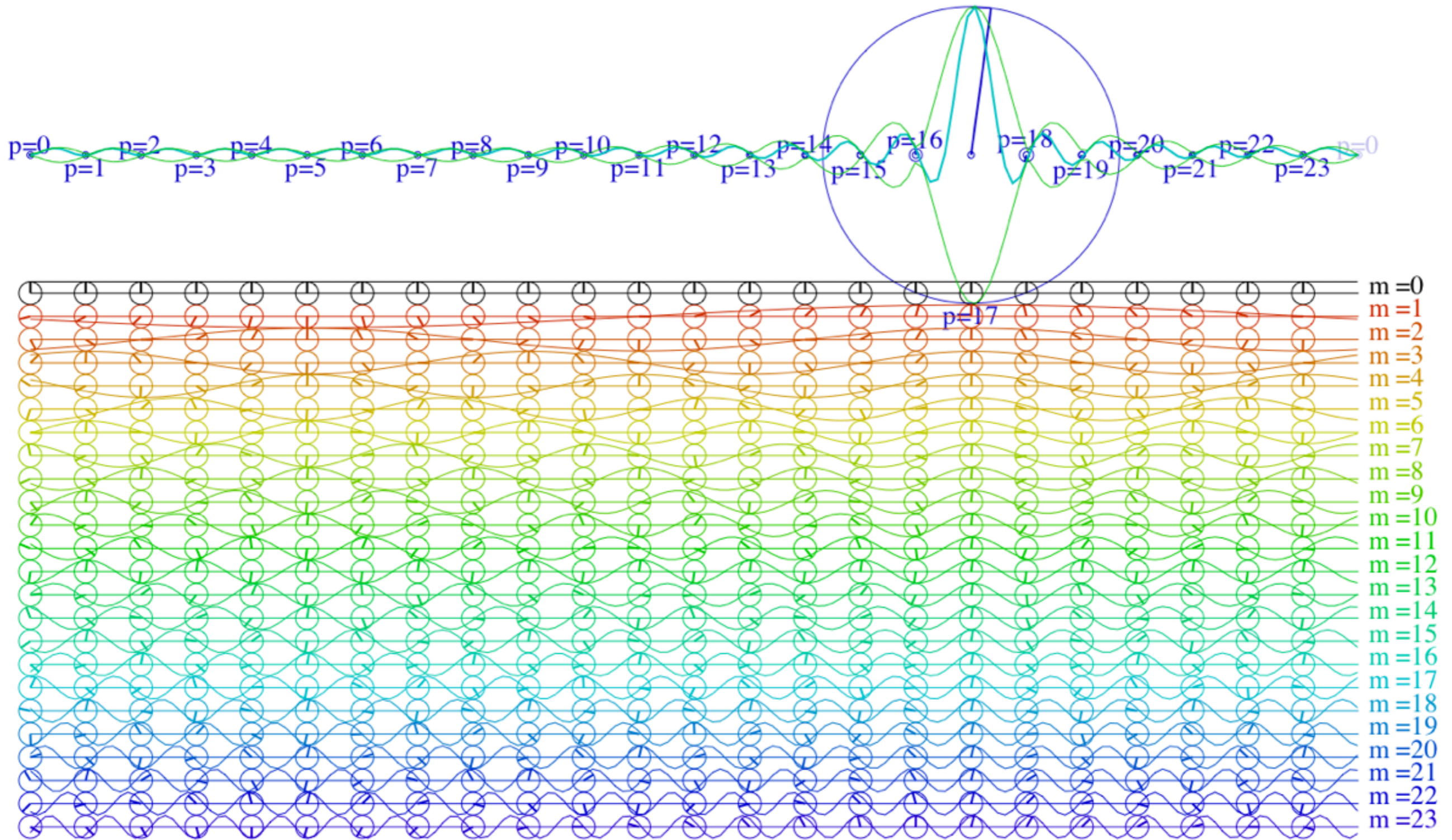
Opposite-pair CW (colliding $\pm m = \pm 2$) Fourier components trace a Cartesian space-time grid

PW *Since "All colors go c" optical PW Fourier CW components march together in lock-step*

Position p (in units of $L/24$)

Fourier Control On

$t = 1.32$



http://www.uark.edu/ua/modphys/markup/WaveltWeb.html?scenario=1PW_R_Stacked_2016HP

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PW “wrinkles” go away if Fourier “boxcar” is tapered to a softer “Gaussian”

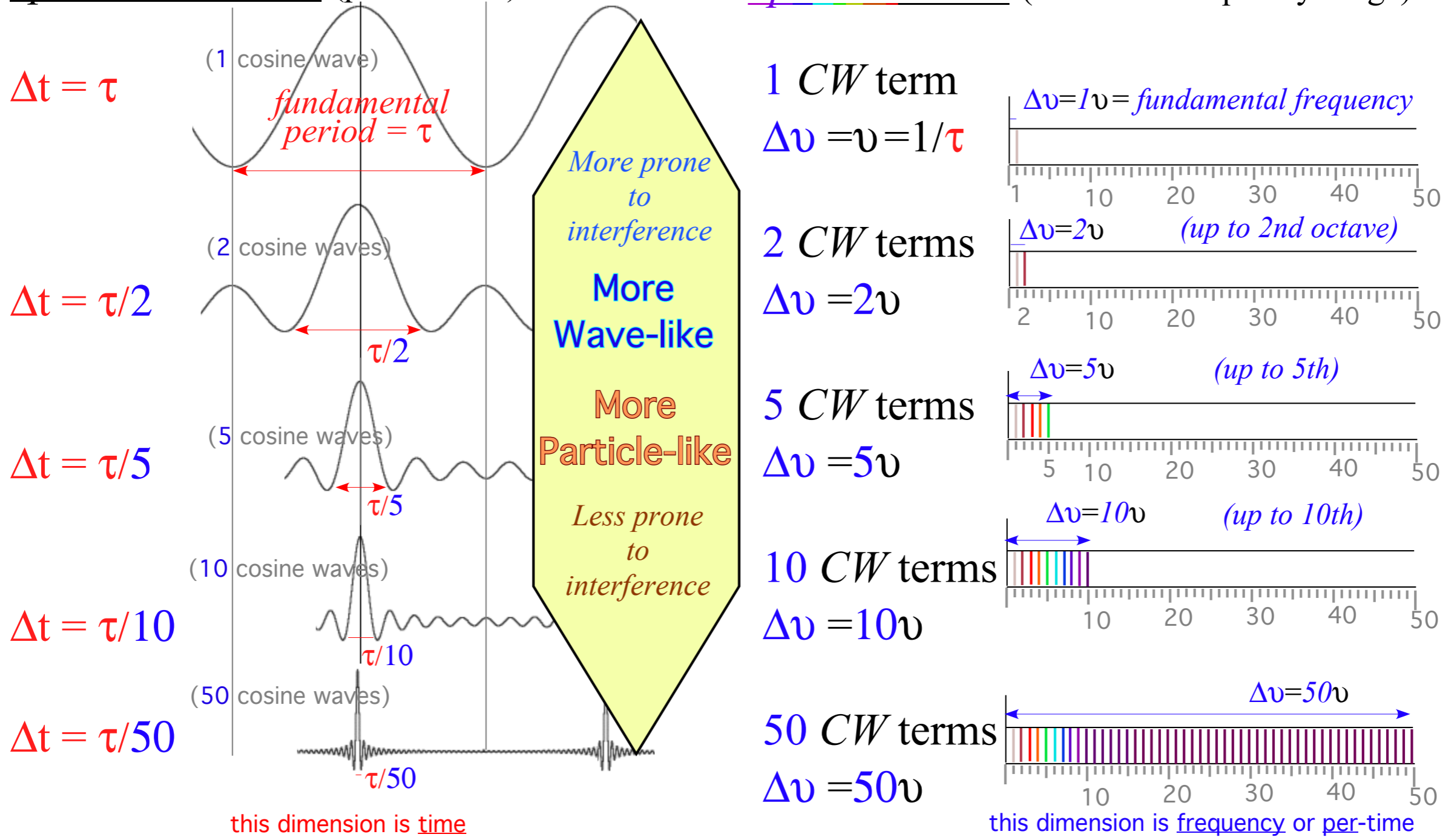
Opposite-pair CW (colliding $\pm m = \pm 2$) Fourier components trace a Cartesian space-time grid

Comparing spacetime uncertainty (Δx or Δt) with per-spacetime bandwidth ($\Delta \kappa$ or $\Delta \nu$)

PW widths reduce proportionally with more CW terms (greater *Spectral* width)

Space-time width (pulse width)

Spectral width (harmonic frequency range)



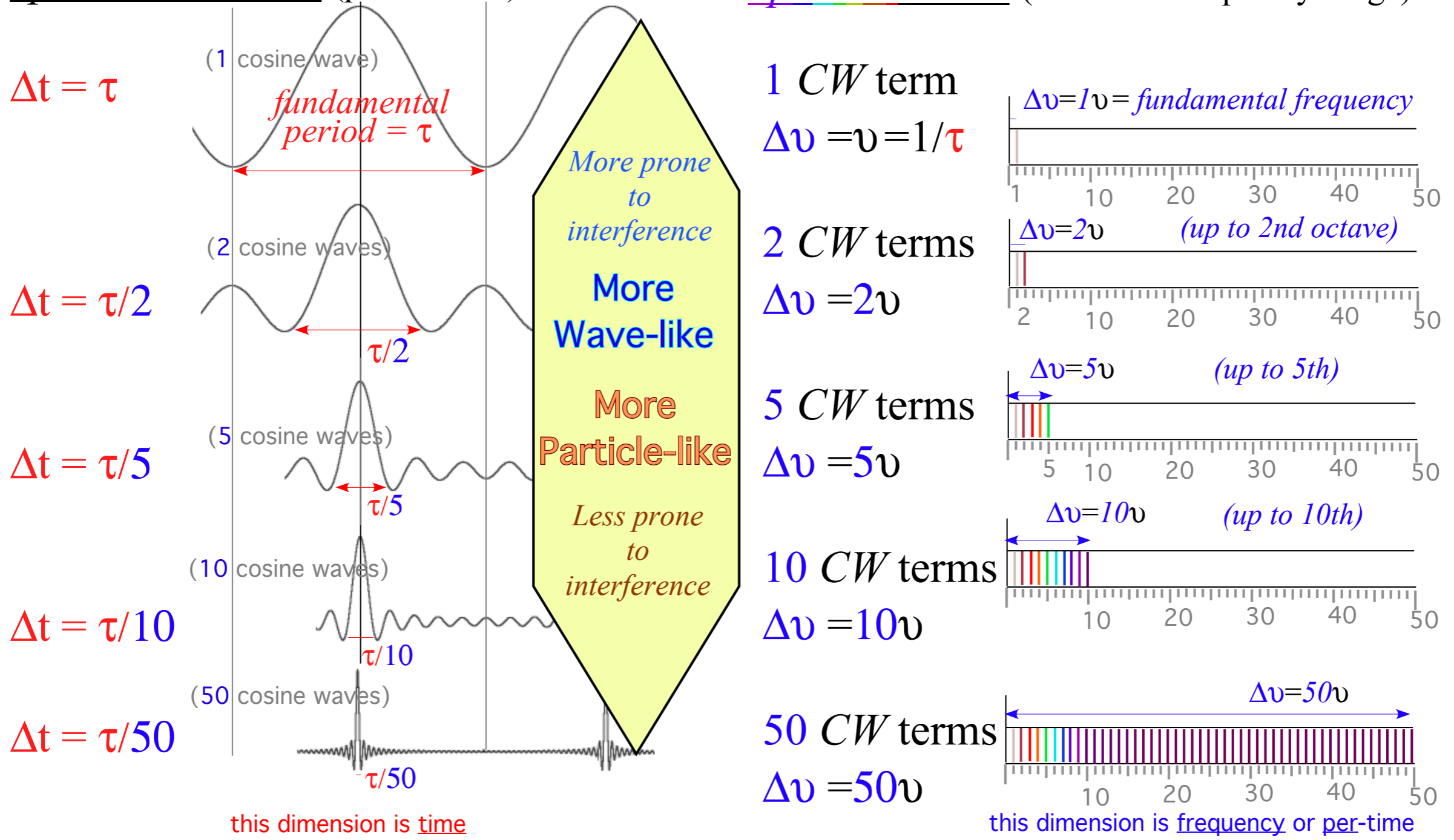
Fourier-Heisenberg product: $\Delta t \cdot \Delta \nu = 1$ (time-frequency uncertainty relation)

Comparing spacetime uncertainty (Δx or Δt) with per-spacetime bandwidth ($\Delta \kappa$ or $\Delta \nu$)

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Space-time width (pulse width)

Spectral width (harmonic frequency range)



Fourier-Heisenberg product: $\Delta t \cdot \Delta \nu = 1$ (time-frequency uncertainty relation)

or this dimension is space...

$\Delta x \cdot \Delta \kappa = 1$

if this dimension is wavenumber or per-space...

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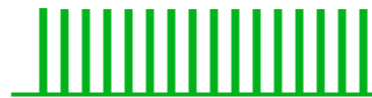
Fourier PW “box-car” geometric series summed

Animation of PW obeying lightwave linear dispersion $\omega(k)=ck$

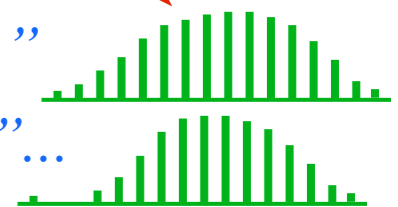
Important *Evenson axiom* for relativity: “All colors go c”

→ Visualizing PW wave uncertainty relations for space: $\Delta x \cdot \Delta \kappa = 1$ and time: $\Delta t \cdot \Delta \nu = 1$ ←

PW “wrinkles” go away if Fourier “boxcar” is tapered to a softer “Gaussian”



or “Poissonian”...



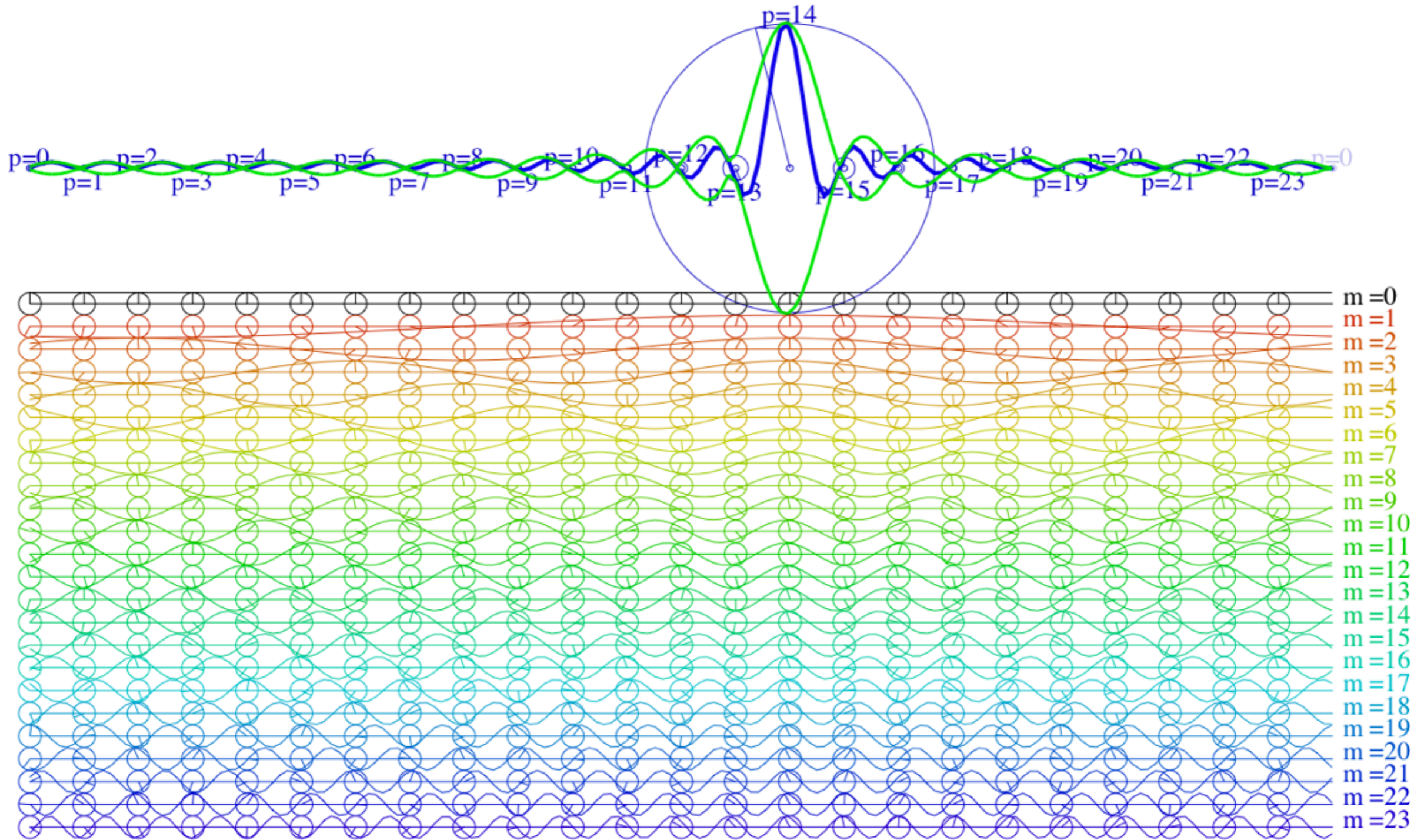
Opposite-pair CW (colliding $\pm m = \pm 2$) Fourier components trace a Cartesian space-time grid

PW “wrinkles” go away if Fourier “boxcar”  is tapered to a softer “Gaussian” 

Position p (in units of $L/24$)

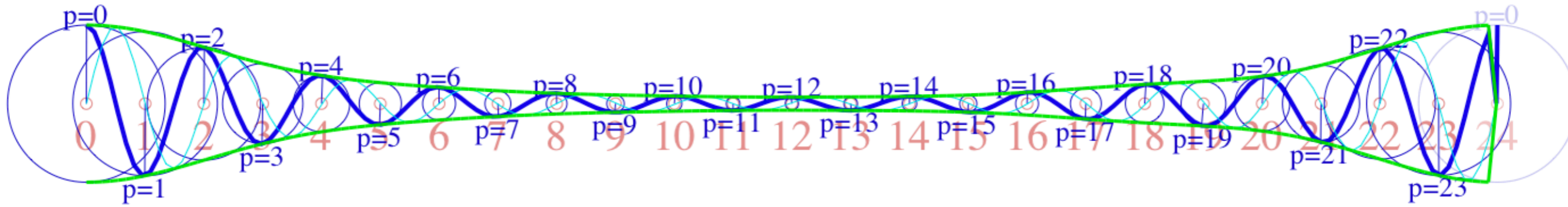
Fourier Control On

$t = 6.79$



http://www.uark.edu/ua/modphys/markup/WaveItWeb.html?scenario=1PW_R_Stacked_2016HP

PW “wrinkles” go away if Fourier “boxcar”  is tapered to a softer “Gaussian” 



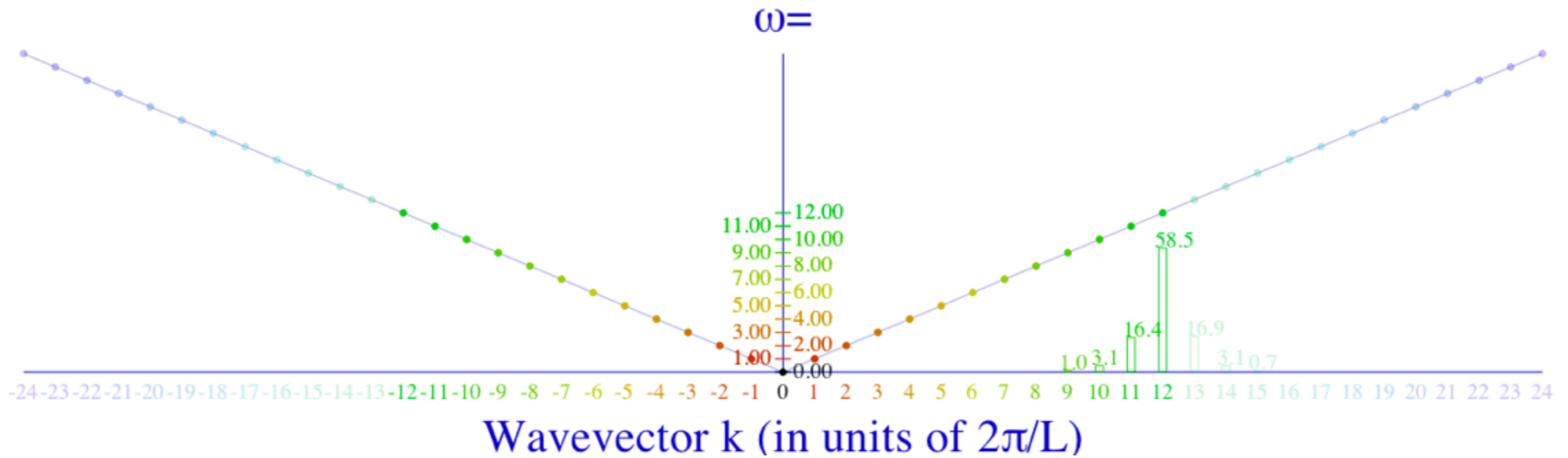
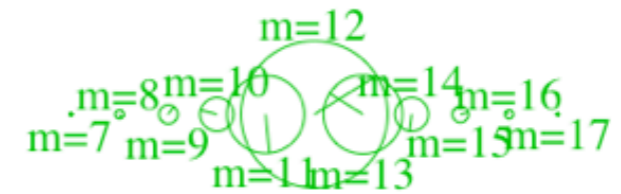
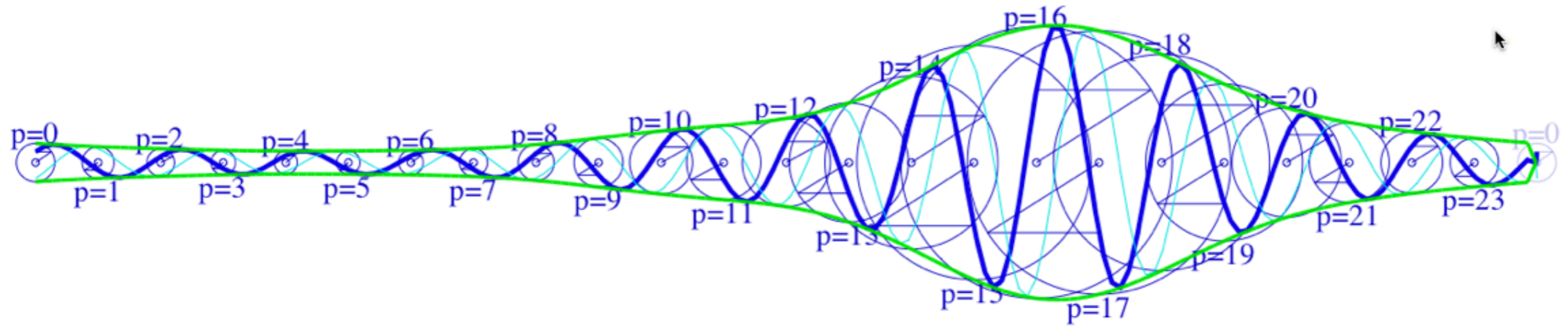
Wave amplitudes vs. position p (p in units of $L/24$)

Click-Drag from dots to change amplitudes. Click here to zero all:

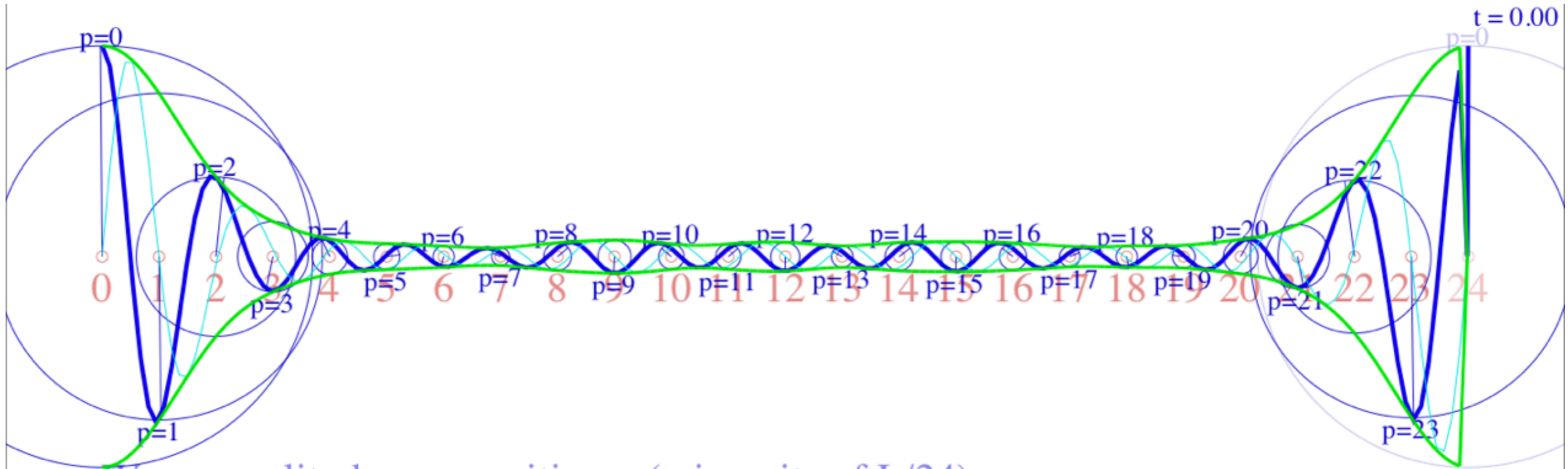
Wave amplitude vs. wavevector m (m in units of $2\pi/L$)



PW “wrinkles” go away if Fourier “boxcar”  is tapered to a softer “Gaussian” 



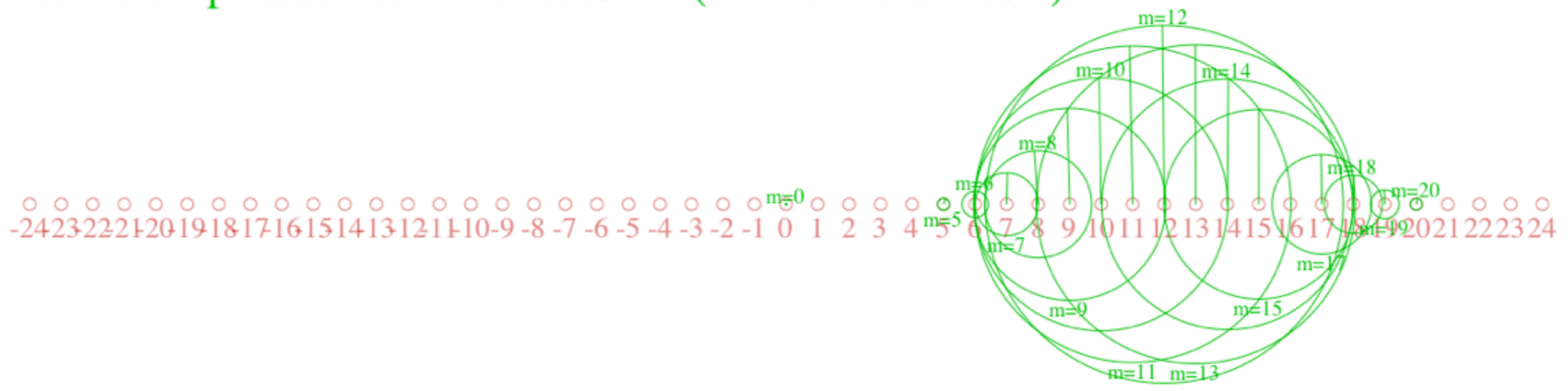
PW “wrinkles” go away if Fourier “boxcar”  is tapered to a softer “Gaussian” 



Wave amplitudes vs. position p (p in units of $L/24$)

Click-Drag from dots to change amplitudes. Click here to zero all:

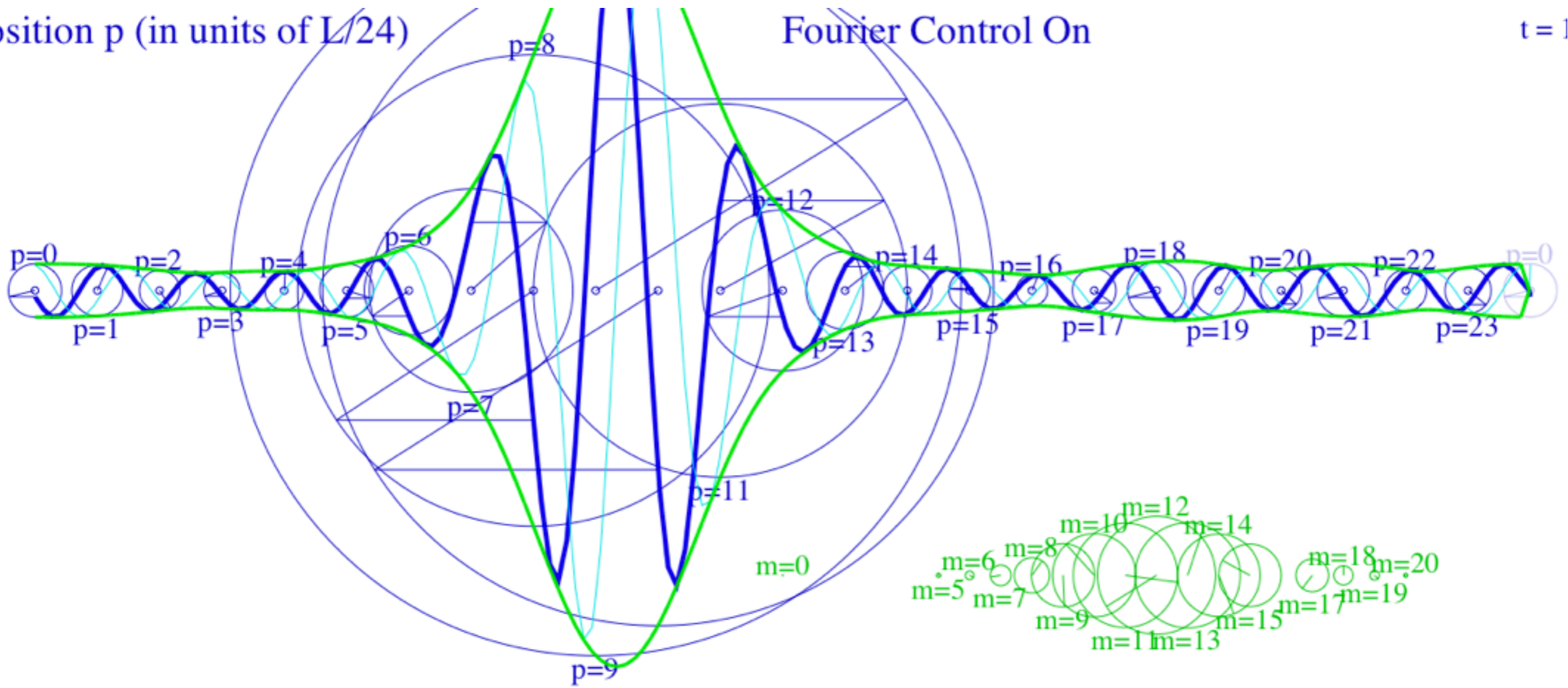
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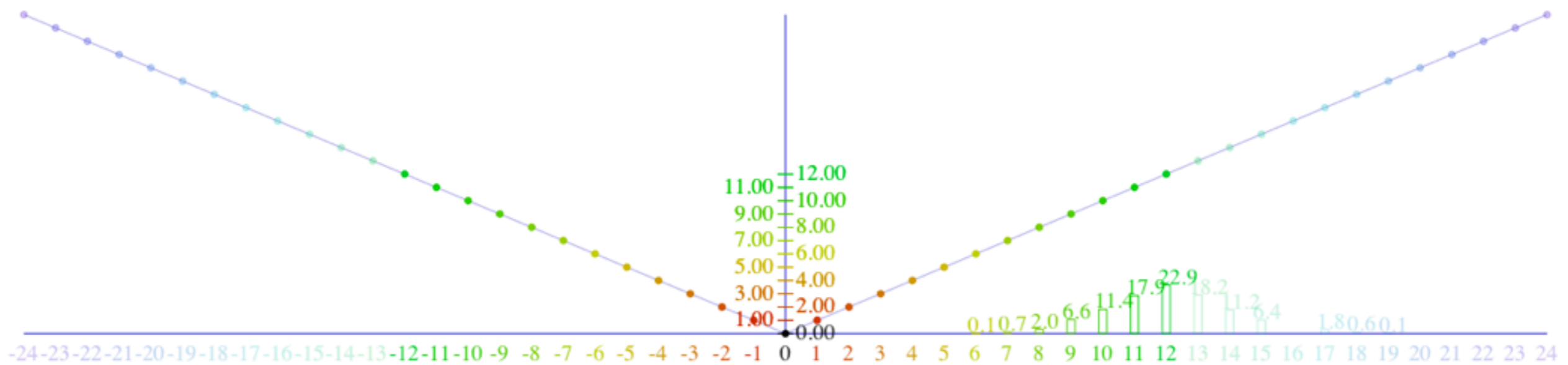
Position p (in units of $L/24$)

Fourier Control On

$t = 1$

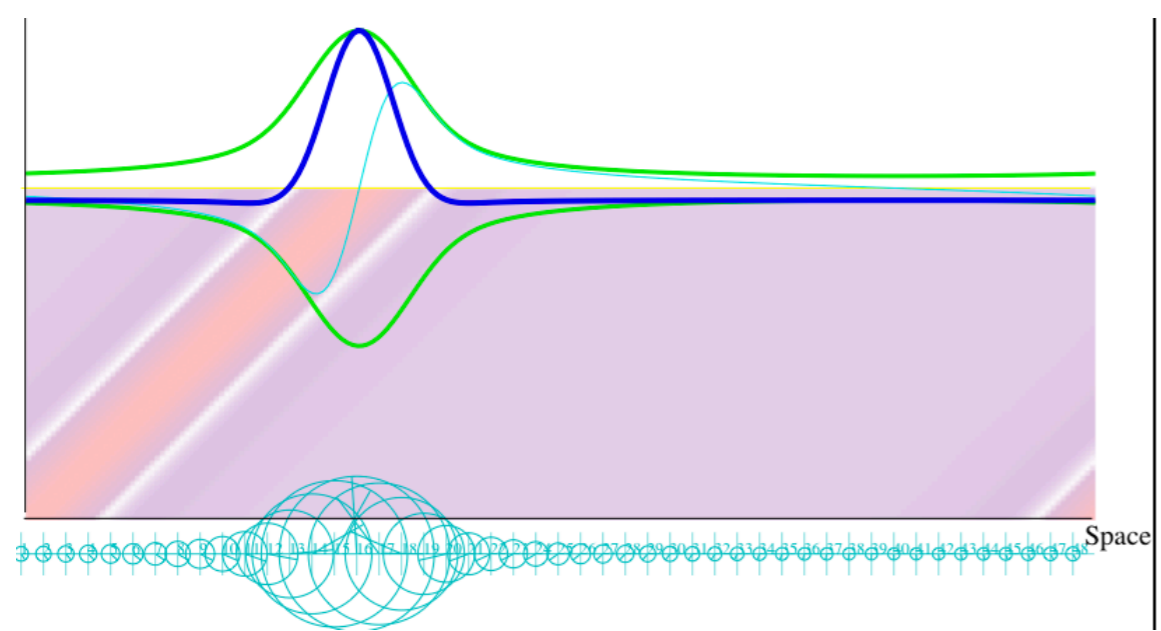


$\omega =$

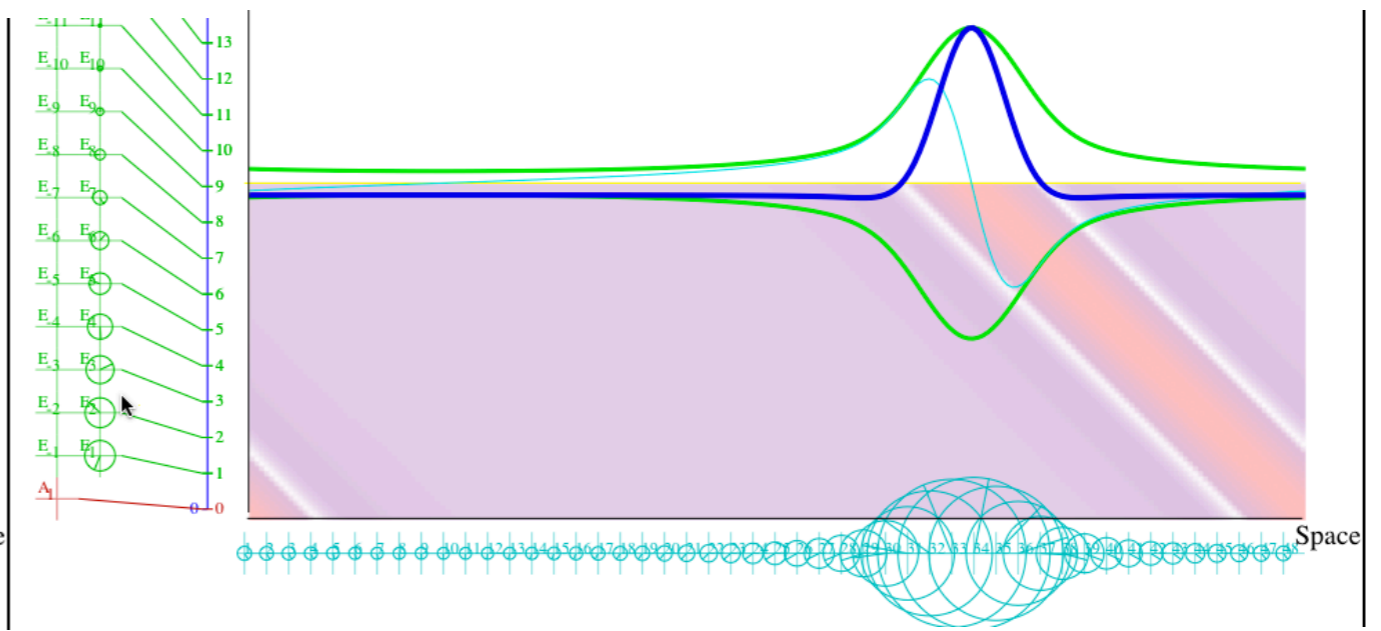


Wavevector k (in units of $2\pi/L$)

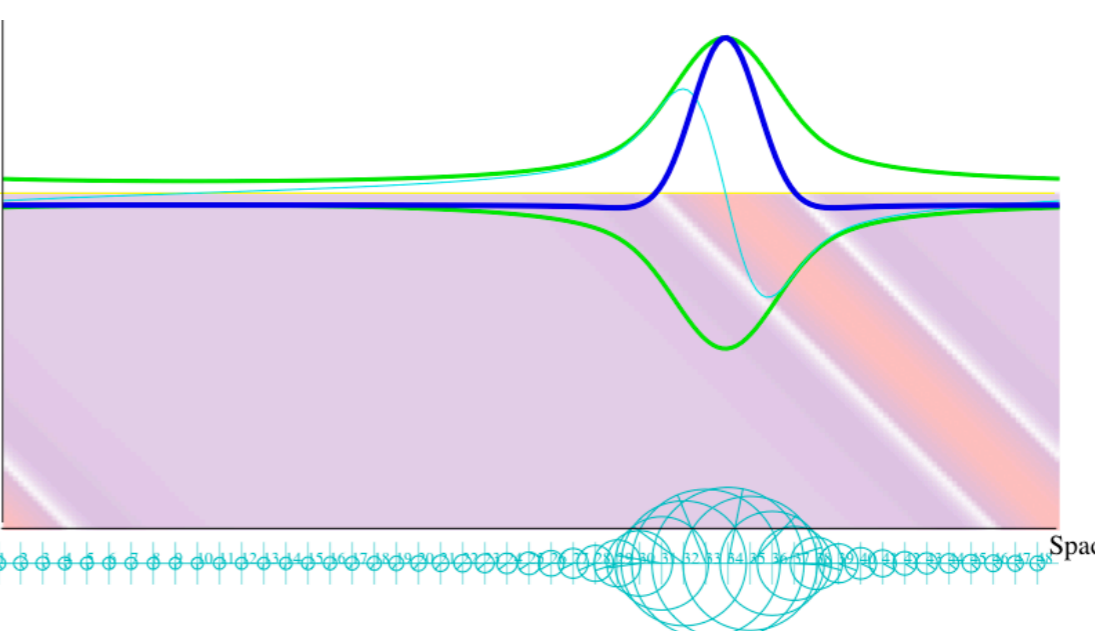
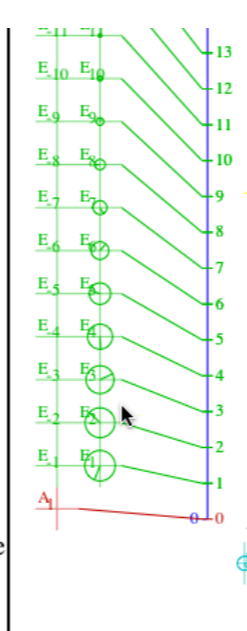
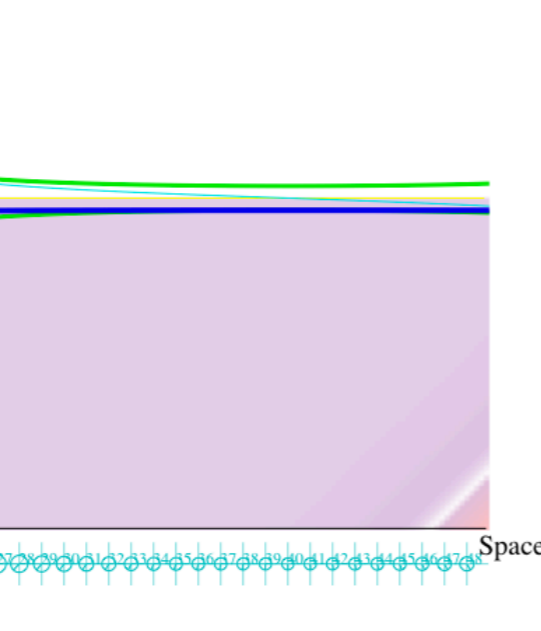
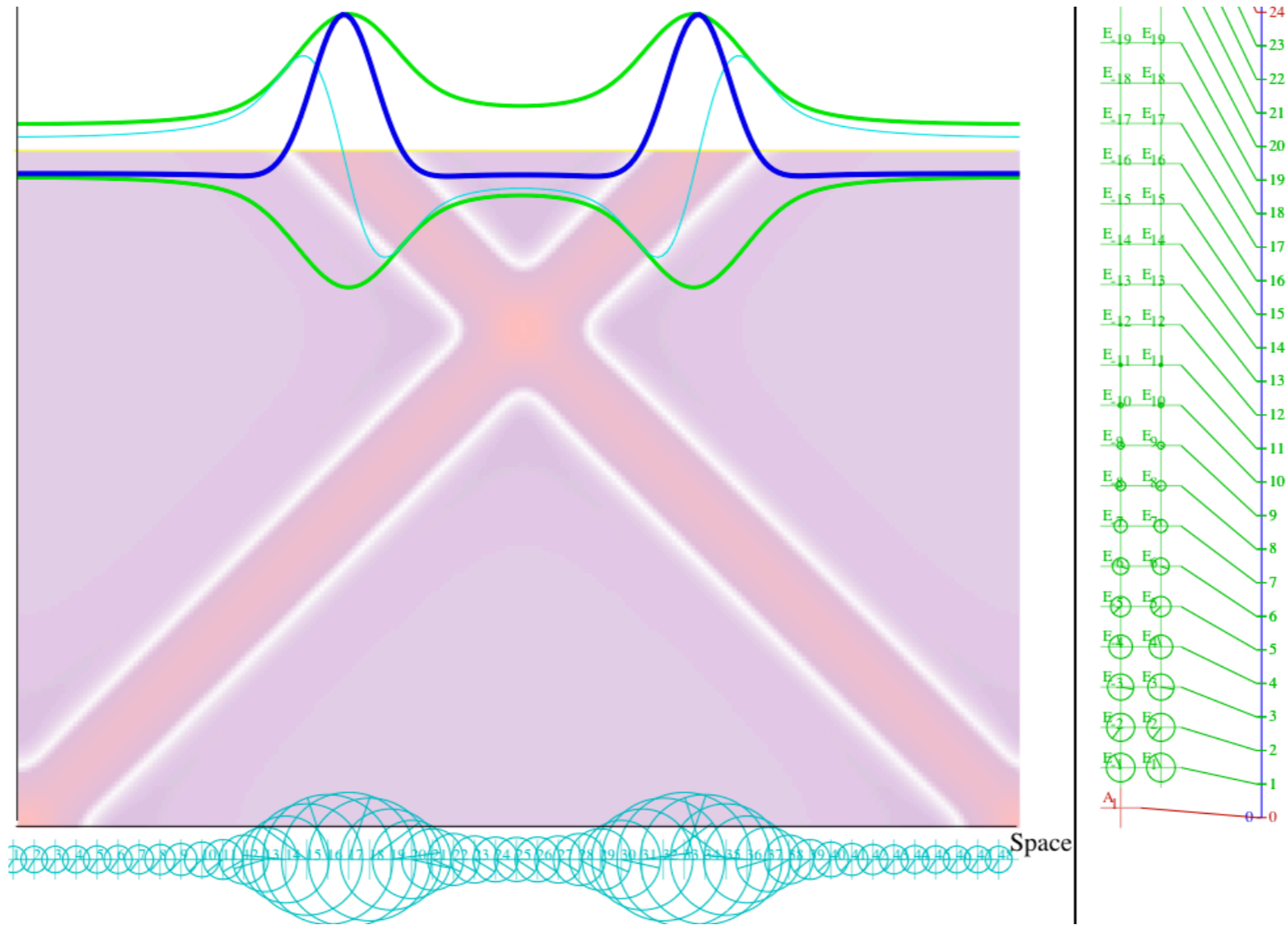
PW “wrinkles” go away if Fourier “boxcar”  is tapered to a softer “Gaussian” 



1-Way Gaussian PW -1
<http://www.uark.edu/ua/modphys/markup/BohrItWeb.html?scenario=5001>



1-Way Gaussian PW +1
<http://www.uark.edu/ua/modphys/markup/BohrItWeb.html?scenario=5002>



1-Way Gaussian PW -1
<http://www.uark.edu/ua/modphys/markup/BohrItWeb.html?scenario=5001>

1-Way Gaussian PW +1
<http://www.uark.edu/ua/modphys/markup/BohrItWeb.html?scenario=5002>

2-Way Gaussian PW ± 1
<http://www.uark.edu/ua/modphys/markup/BohrItWeb.html?scenario=5000>

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Building PW from CW components using “Fourier Control” app-panel

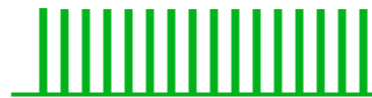
Fourier PW “box-car” geometric series summed

Animation of PW obeying lightwave linear dispersion $\omega(k)=ck$

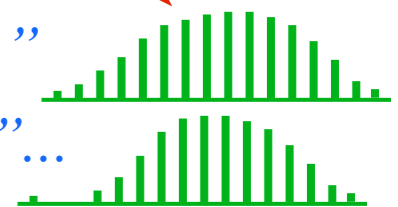
Important *Evenson axiom* for relativity: “All colors go c”

→ Visualizing PW wave uncertainty relations for space: $\Delta x \cdot \Delta \kappa = 1$ and time: $\Delta t \cdot \Delta \nu = 1$ ←

PW “wrinkles” go away if Fourier “boxcar” is tapered to a softer “Gaussian”



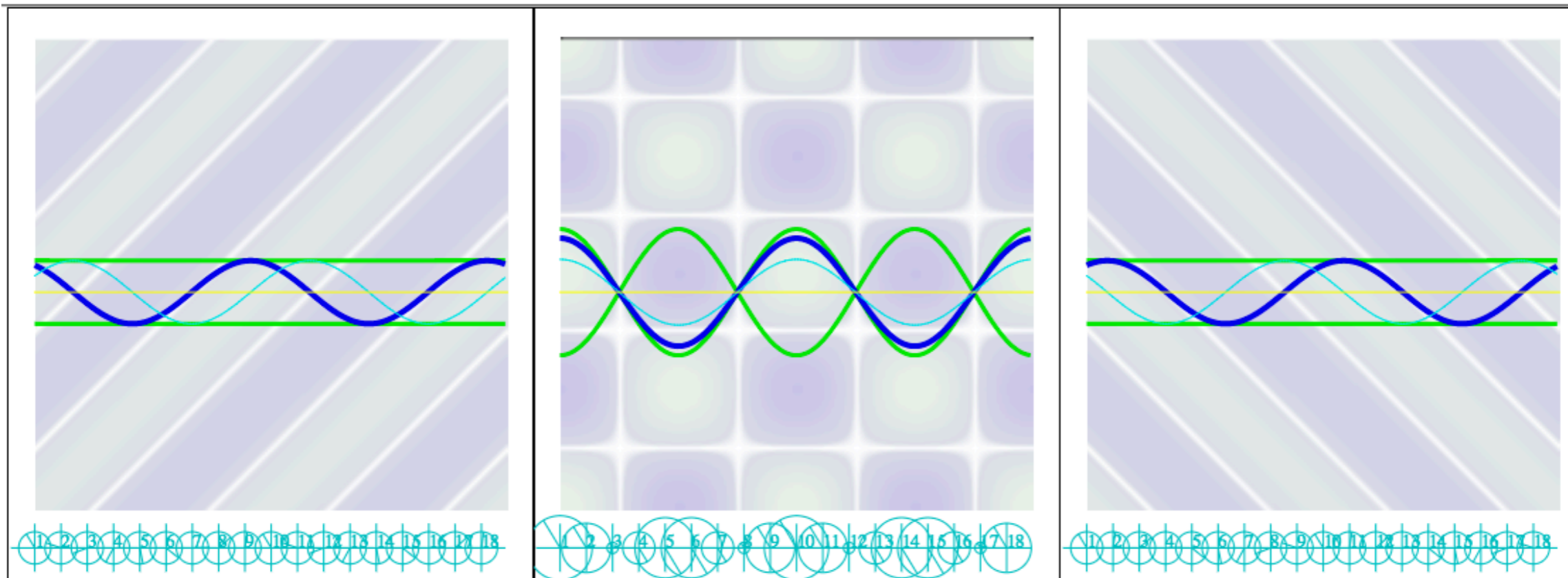
or “Poissonian”...



Opposite-pair CW (colliding $\pm m = \pm 2$) Fourier components trace a Cartesian space-time grid

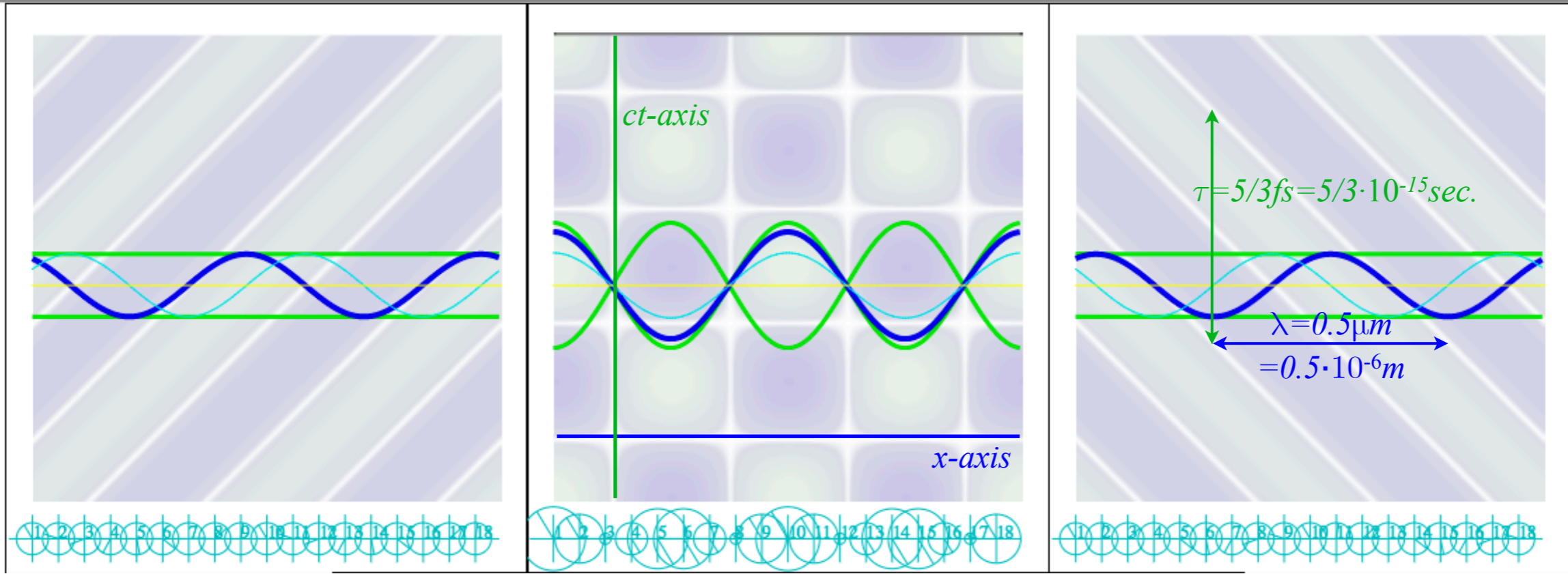
Spacetime animation of head-on collision of two $\nu=600\text{THz}$ CW modes of light

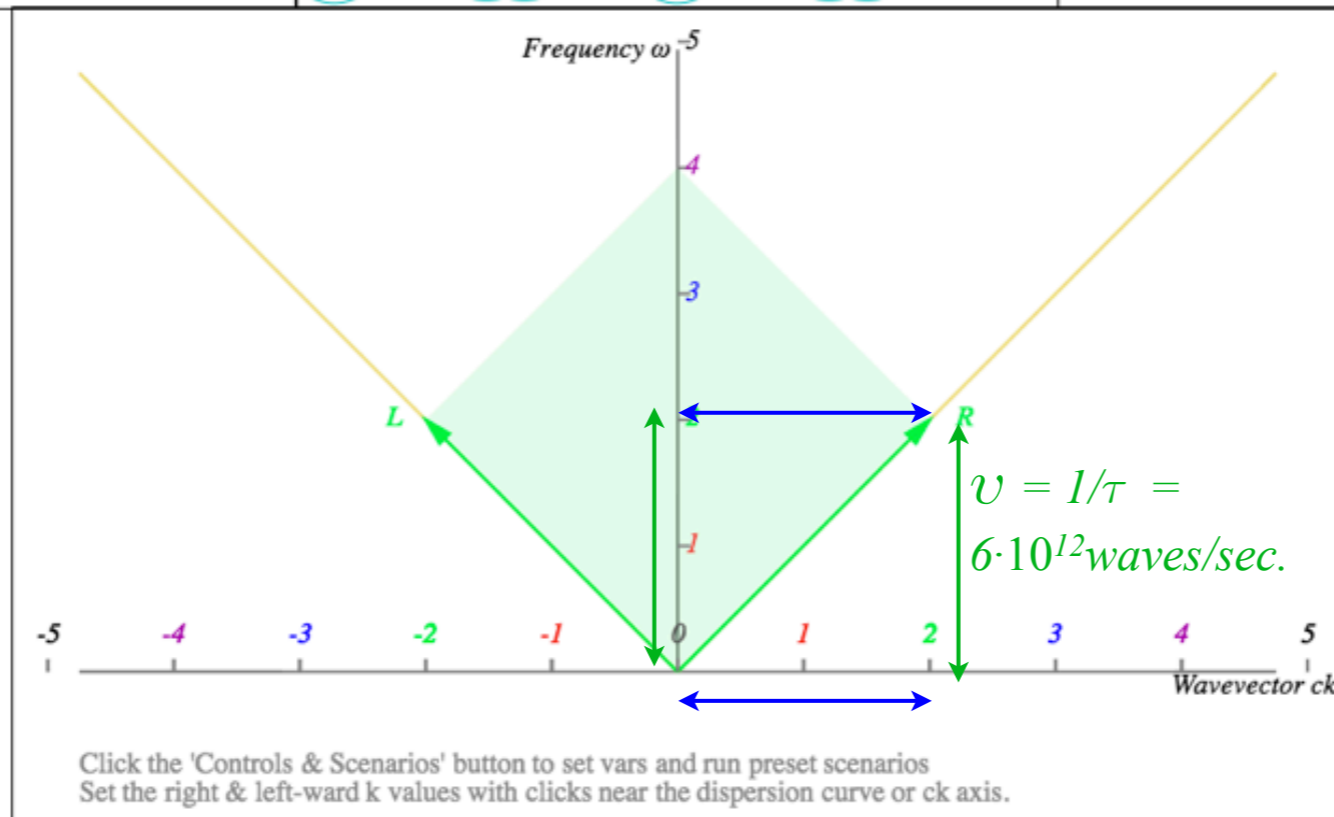
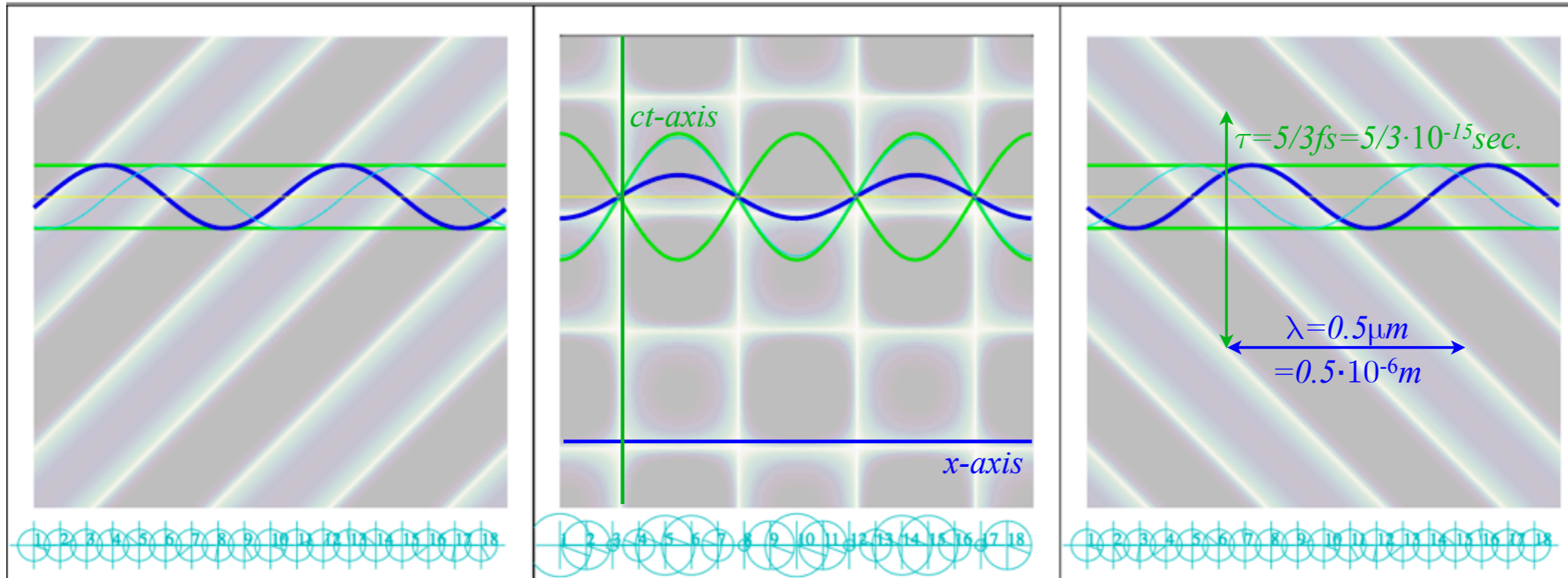
CW



Spacetime animation of head-on collision of two $\nu=600\text{THz}$ CW modes of light

CW





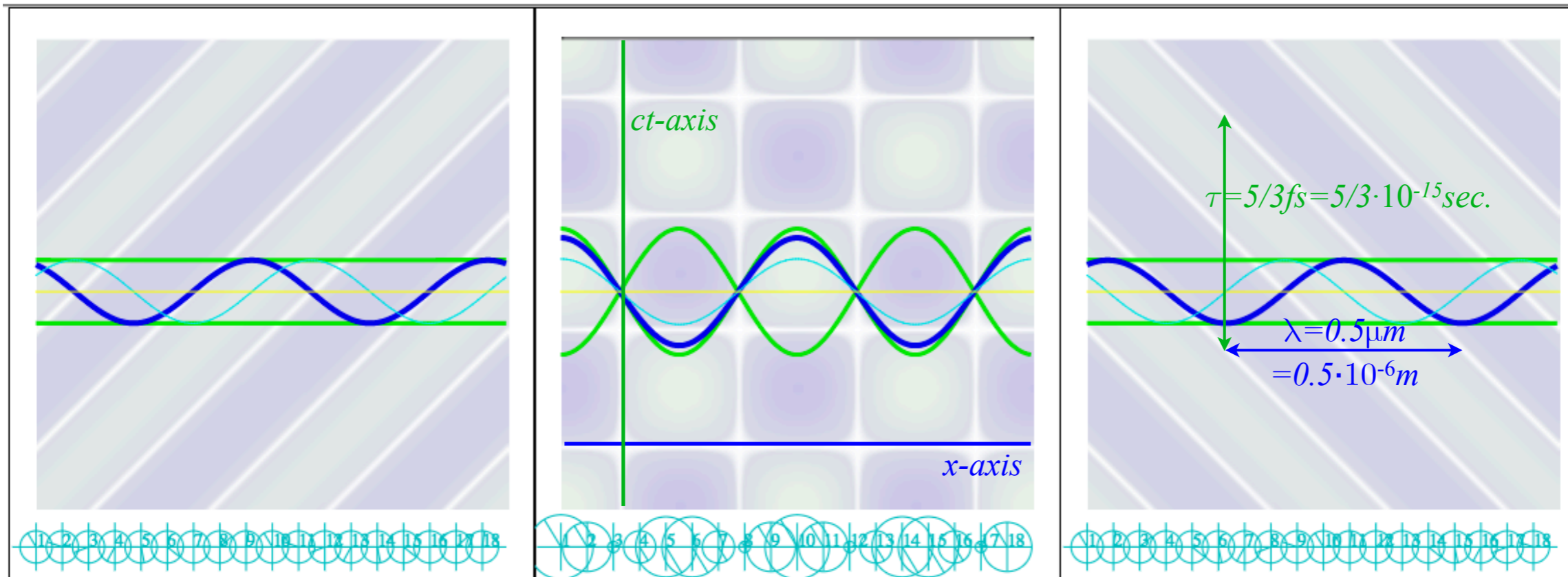
$$\kappa = 1/\lambda = 2 \cdot 10^6 \text{ waves/m}$$

Opposite-pair CW (colliding $\pm m = \pm 2$) Fourier components trace a Cartesian space-time grid

 *Colliding PW) Fourier components trace space-time “baseball diamonds”*

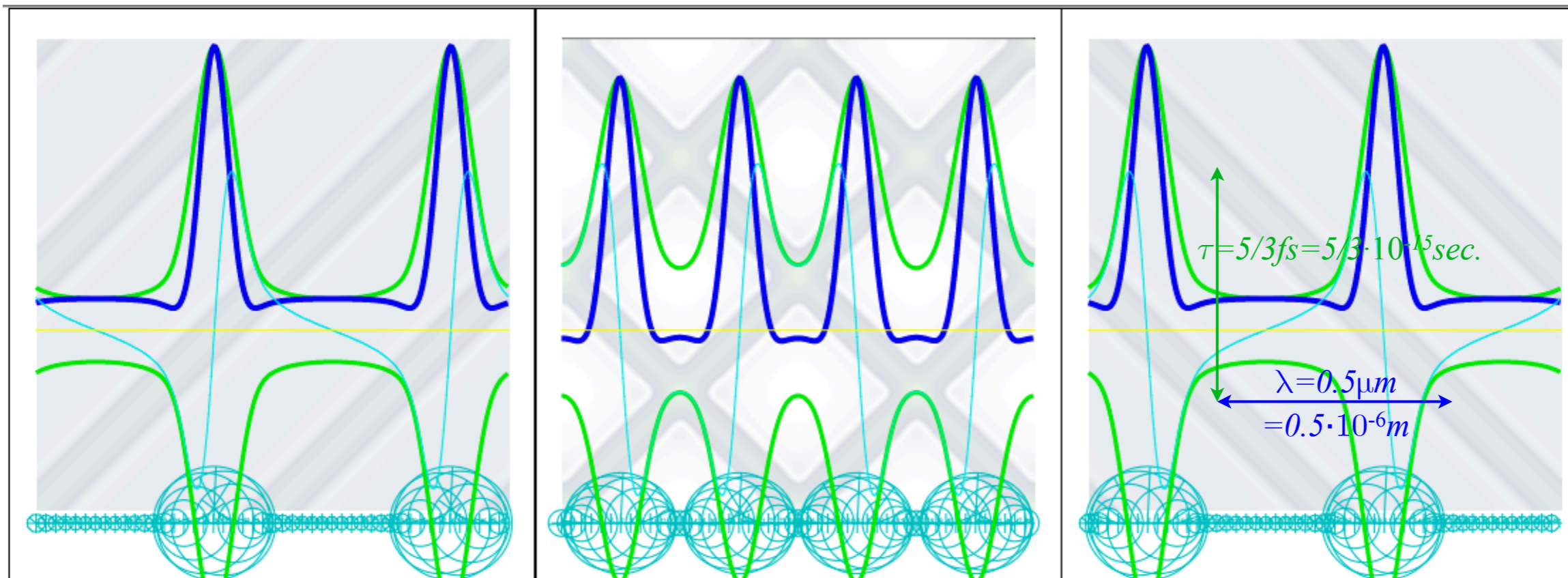
Spacetime animation of head-on collision of two $\nu=600\text{THz}$ CW modes of light


CW



Spacetime animation of head-on collision of two $N\nu=600N$ THz PW modes of light

PW



Opposite-pair CW (colliding $\pm m = \pm 2$) Fourier components trace a Cartesian space-time grid
Colliding PW lightwaves trace space-time “baseball diamonds”
 *Introducing CW (colliding $\pm m = \pm 2$) Doppler shifted to ($m = -1$ and $m = +4$)*

