Lectures 3-4extra
Tue 2.2.2016

Analysis of 1D 2-Body Collisions

(Ch. 2 to Ch. 4 of Unit 1)

Review of elastic Kinetic Energy ellipse geometry

The X2 Superball pen launcher
- *Perfectly elastic "ka-bong" velocity amplification effects (Faux-Flubber)*

Geometry of X2 launcher bouncing in box
- *Independent Bounce Model (IBM)*
- *Geometric optimization and range-of-motion calculation(s)*
- *Integration of (V_1,V_2) data to space-time plots $(y_1(t),t)$ and $(y_2(t),t)$ plots*
- *Integration of (V_1,V_2) data to space-space plots (y_1, y_2)* *Examples $(M_1=7, M_2=1)$ and $(M_1=49, M_2=1)$*

Multiple collisions calculated by matrix operator products
- *Matrix or tensor algebra of 1-D 2-body collisions*

Ellipse rescaling-geometry and reflection-symmetry analysis

- *Rescaling KE ellipse to circle*
  - *How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on*
- *Reflections in the clothing store: "It's all done with mirrors!"*
- *Introducing hexagonal symmetry D_6~C_{6v} (Resulting for $m_1/m_2=3$)*
  - *Group multiplication and product table*
  - *Classical collision paths with D_6~C_{6v} (Resulting from $m_1/m_2=3$)*
    - *Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$ and $(M_1=100, M_2=1)$*

Review of elastic Kinetic Energy ellipse geometry

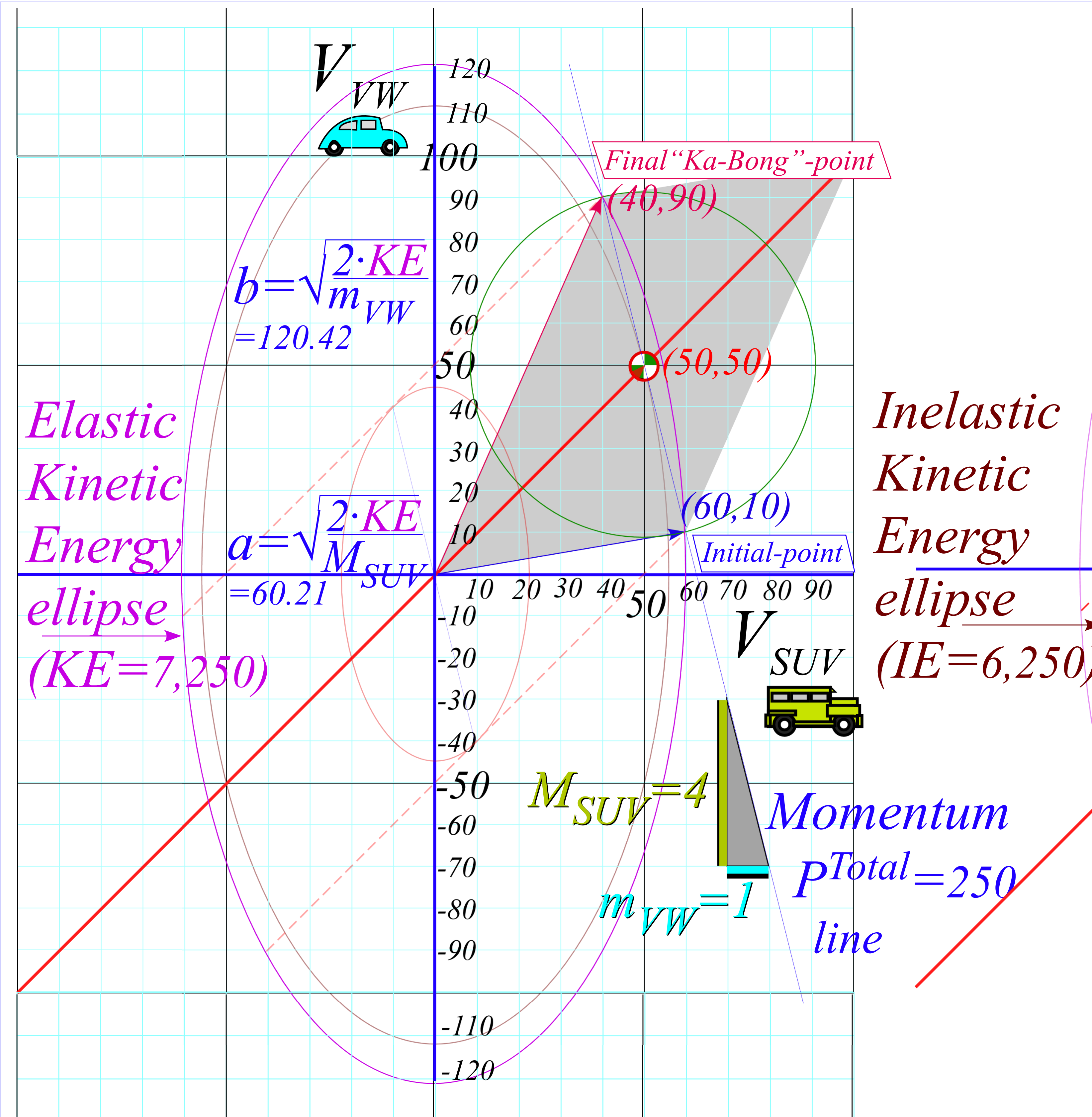


Fig. 2.3a
(Unit 1)

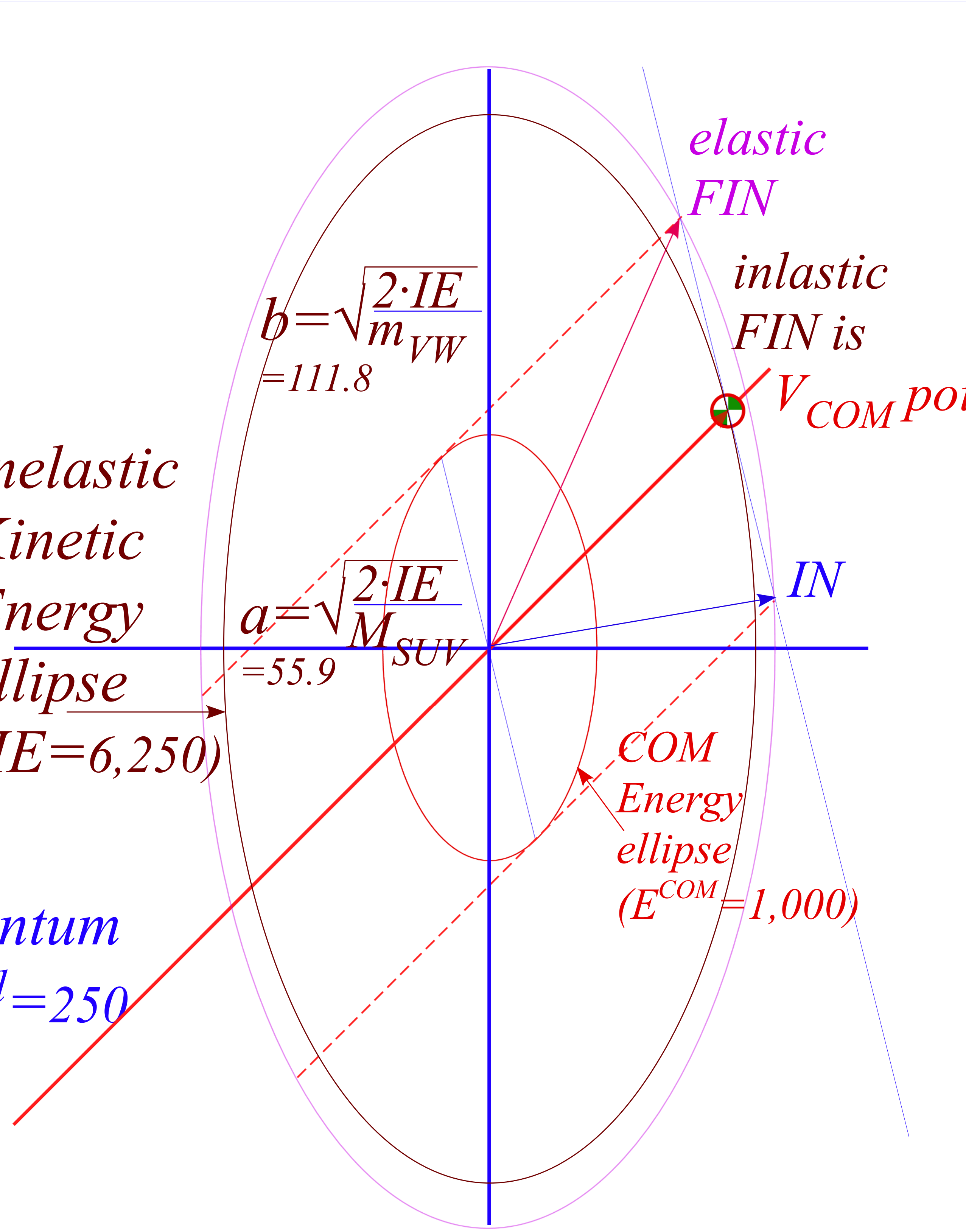


Fig. 2.3b
(Unit 1)

The X-2 Pen launcher and Superball Collision Simulator*

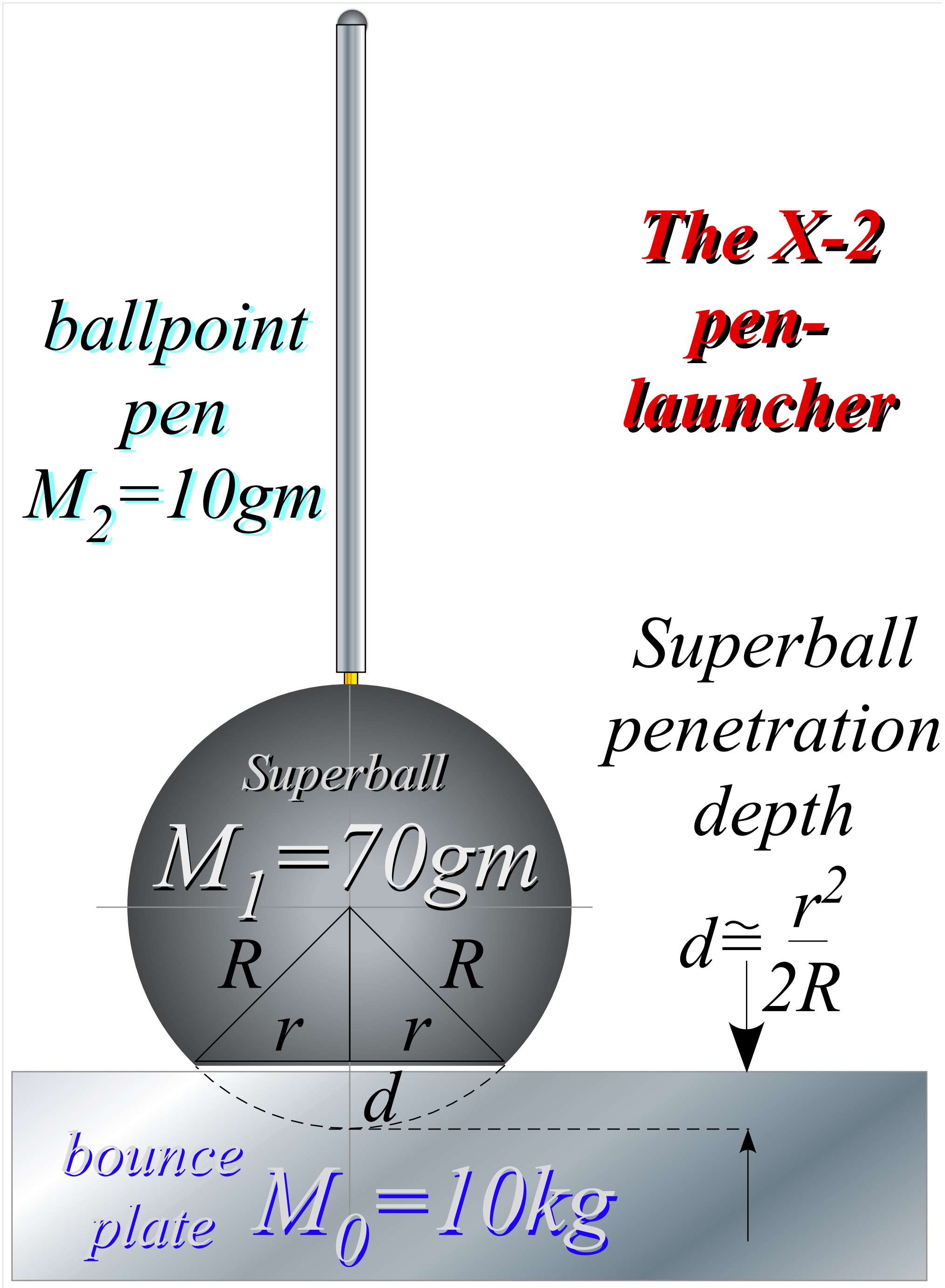
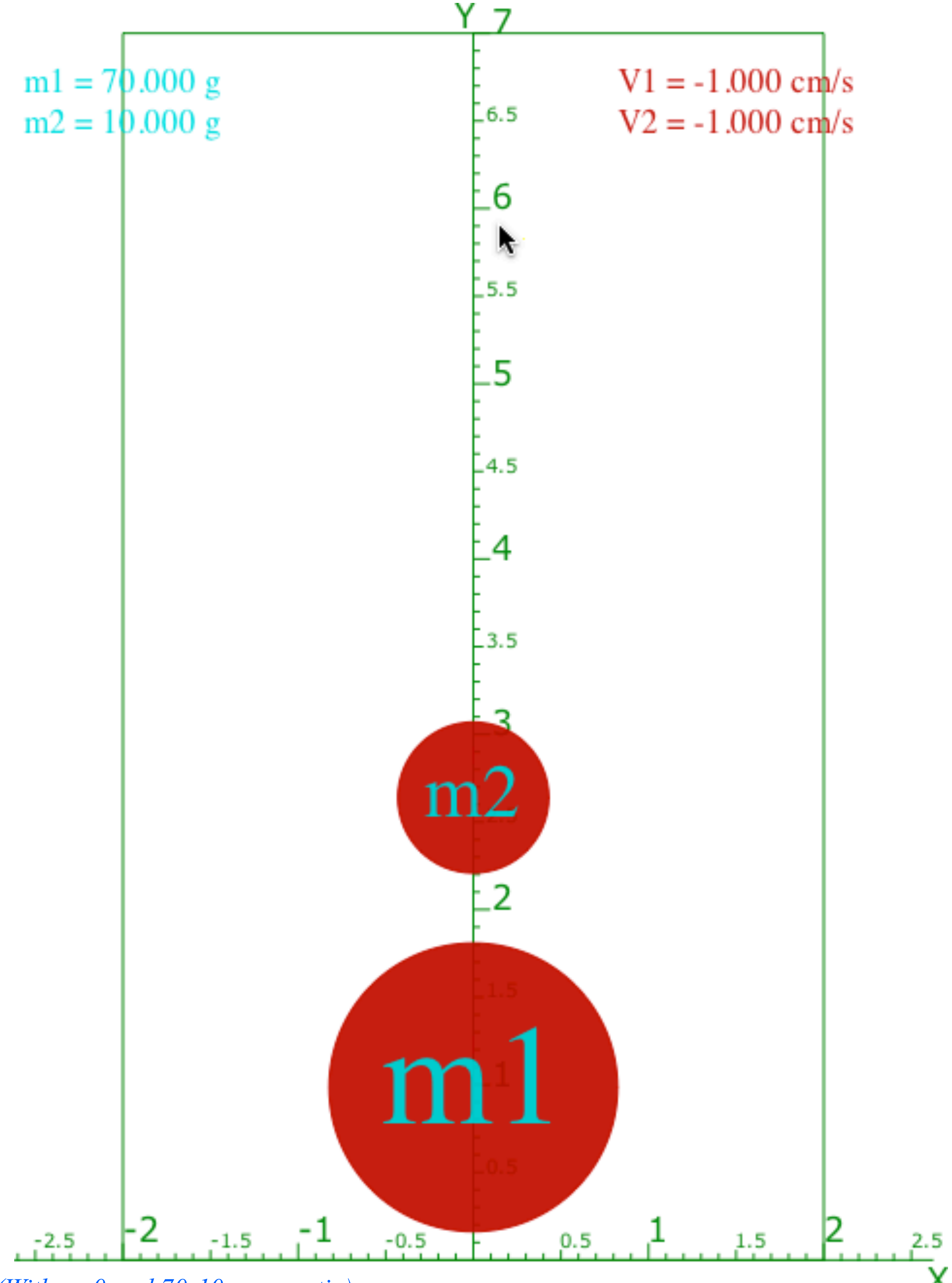


Fig. 3.1 (Unit 1)



(With g=0 and 70:10 mass ratio)

*Launch Generic Superball Collision Web Simulator http://www.uark.edu/ua/modphys/markup/BounceItWeb.html?scenario=1007

*The X-2 Pen launcher and Superball Collision Simulator**

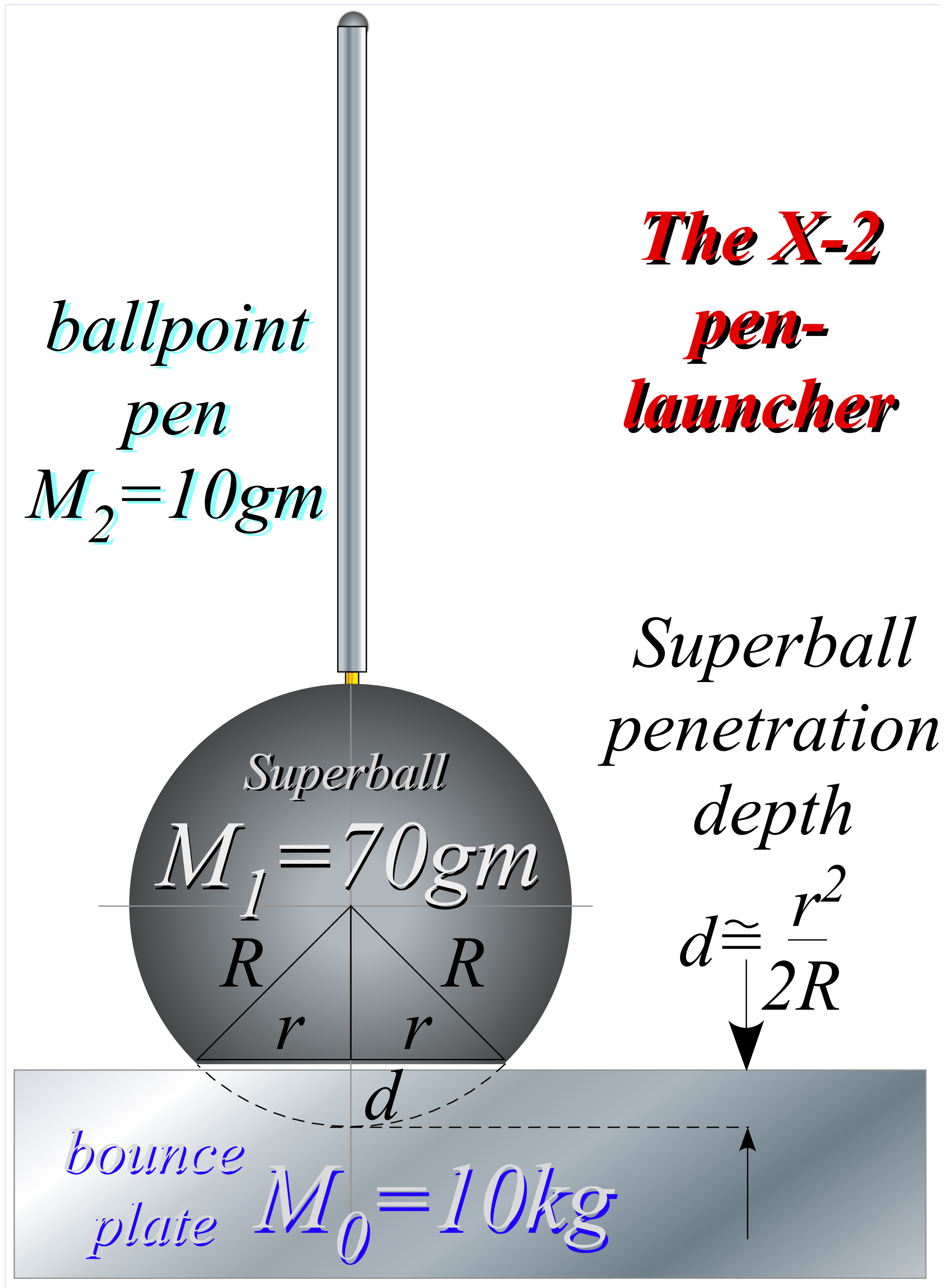


Fig. 3.1 (Unit 1)

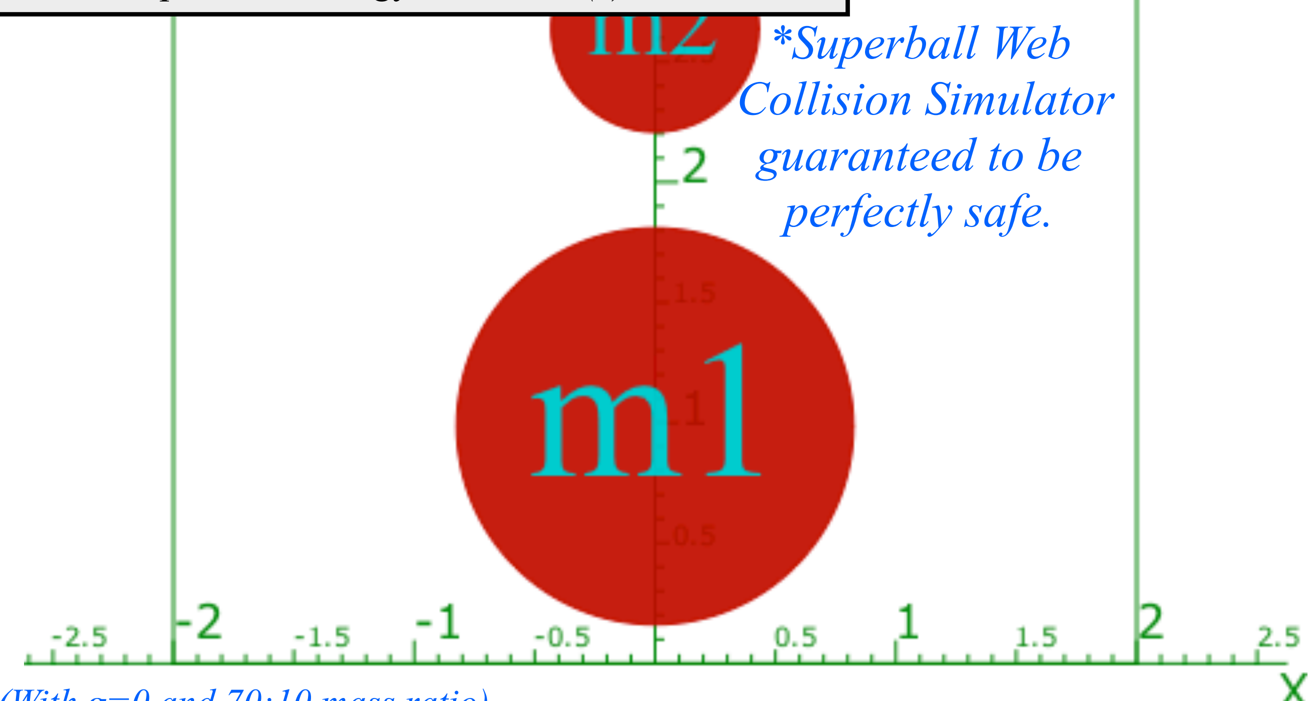
Caution: Product Liablility Disclaimer

This ballpoint pen could be hazardous to your health!

The experiments which are the subject of this discussion are both spectacular and potentially dangerous, and care to protect one's eyes should be taken. The simplest experiment involves sticking a ball point pen into a superball or other hard rubber ball and dropping the two onto a hard floor.

If done correctly the pen will eject the ball with such force it may stick in the ceiling of the room. Obviously you want to be careful with this weapon. And, this goes doubly and triply for the more advanced models that may be developed in the course of studying this stuff. It is recommended that experimenters wear safety glasses when doing these experiments with pens. (We could just say don't use pens, but that's an easy way to do this experiment and probably the way most people will go about it.) Some of the tangential experiments associated with this development are less hazardous. To measure the potential force function of a ball one may simply paint the ball and measure the spot size as a function of drop height *h*.

The saggital approximation $d=r^2/2R$ allows one to quickly convert spot radius *r* to penetration depth *x* for a superball of radius *R* as shown in the figure. Equating this to *Mgh* gives the ball potential energy function *V(x)*.



**Superball Web Collision Simulator guaranteed to be perfectly safe.*

(With g=0 and 70:10 mass ratio)

**Launch Generic Superball Collision Web Simulator* http://www.uark.edu/ua/modphys/markup/BounceItWeb.html?scenario=1007

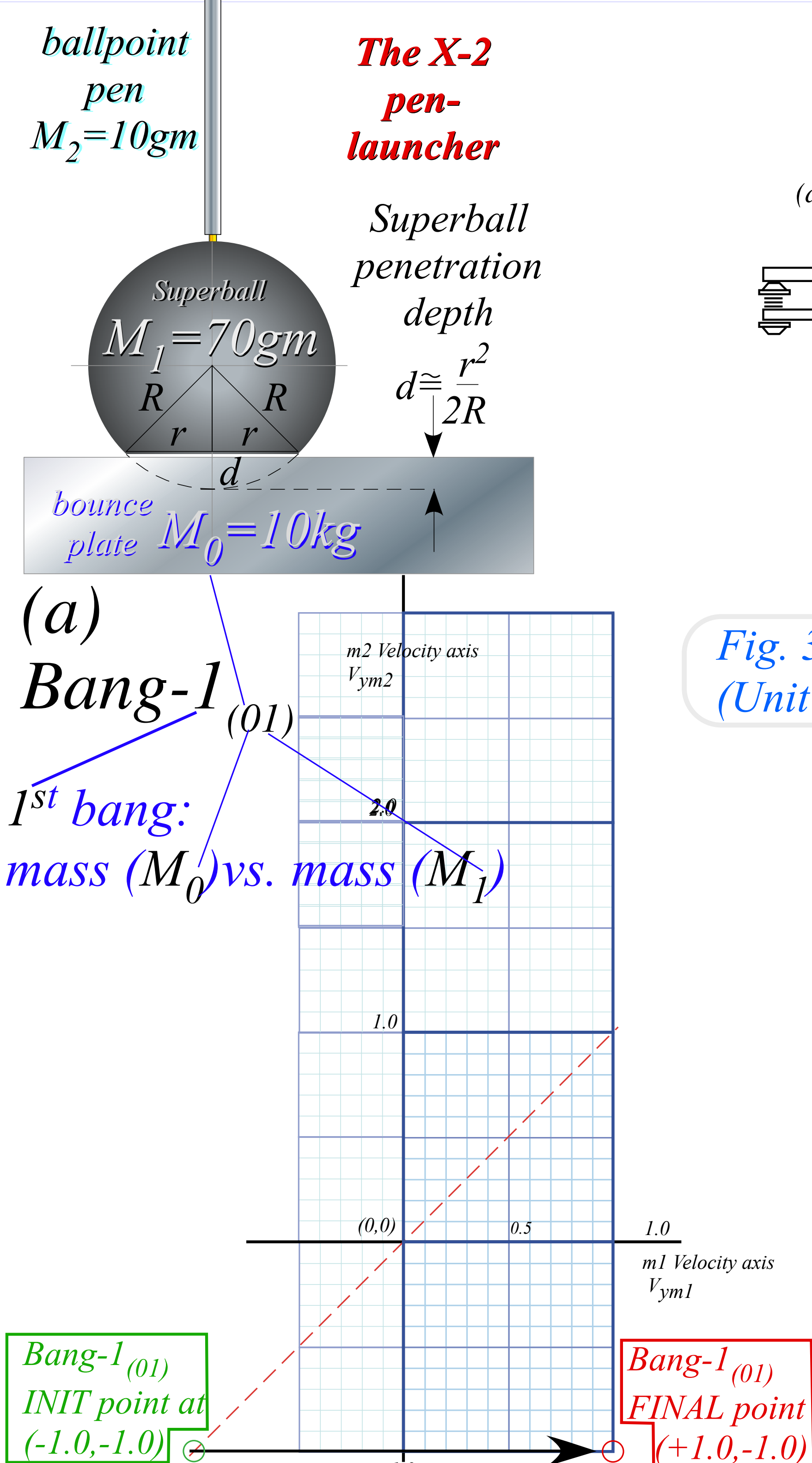
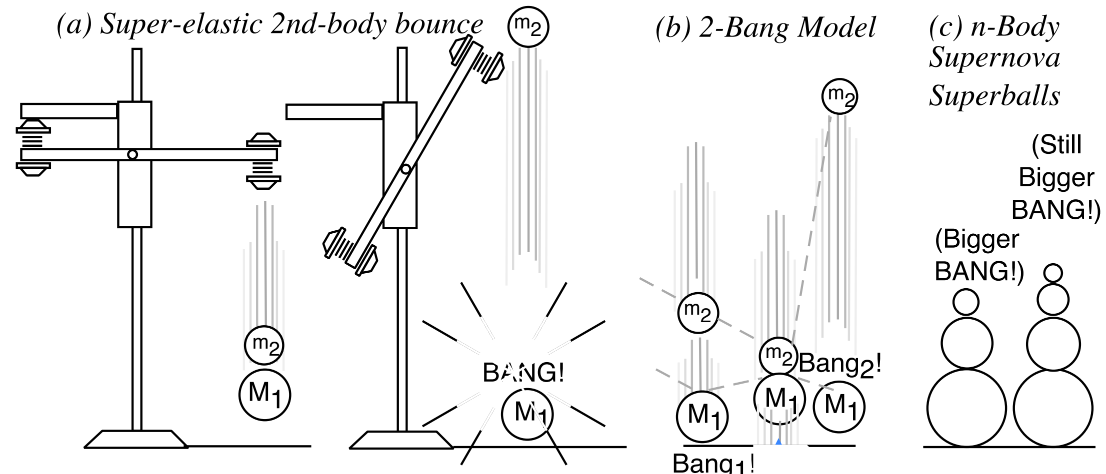


Fig. 3.4 (Unit 1)

This 1st bang is a floor-bounce of M_1 off very massive plate/Earth M_0

Fig. 3.3 (Unit 1)



1st bang: M_1 off floor

(With g=0 and 70:10 mass ratio)

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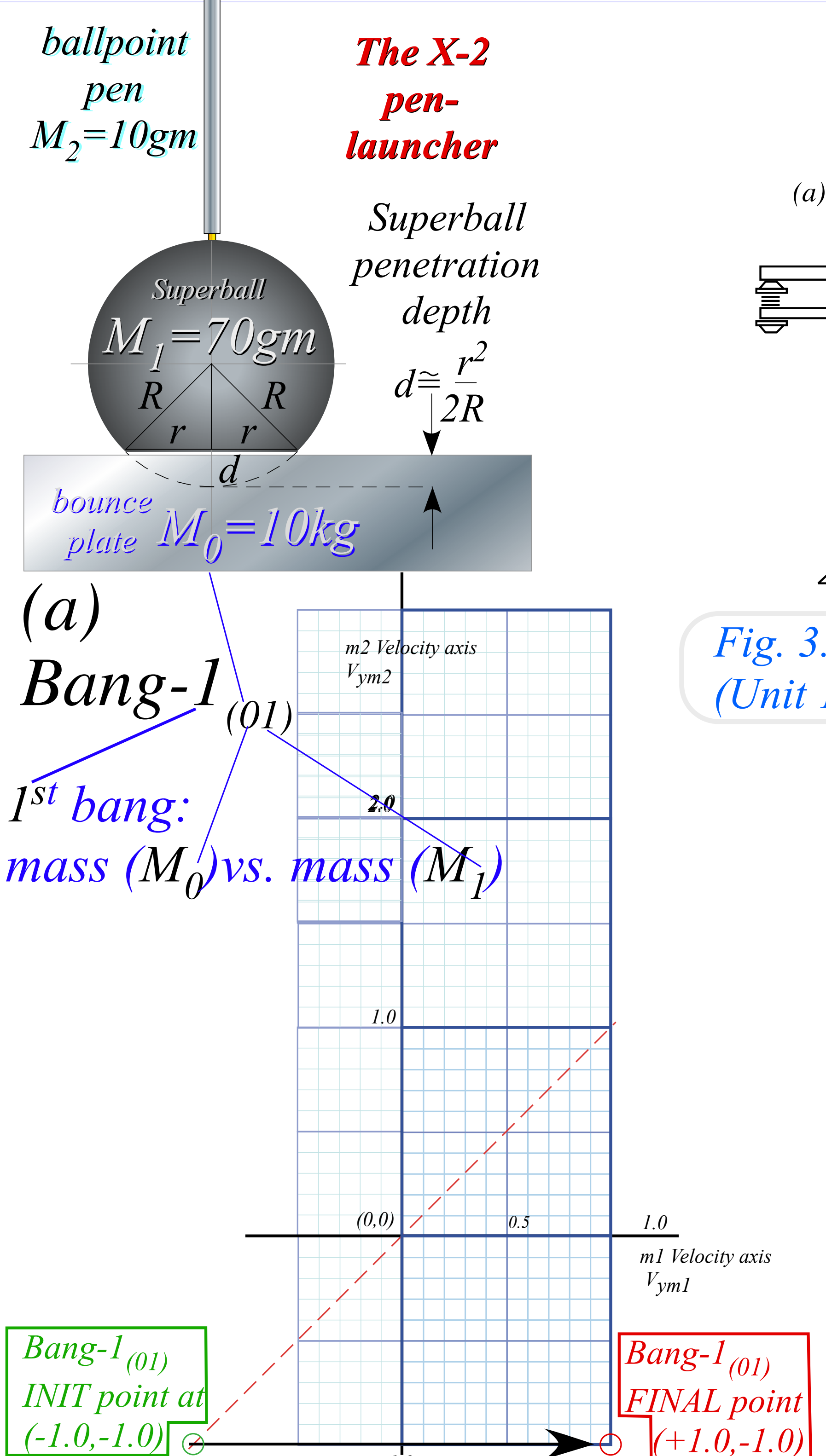

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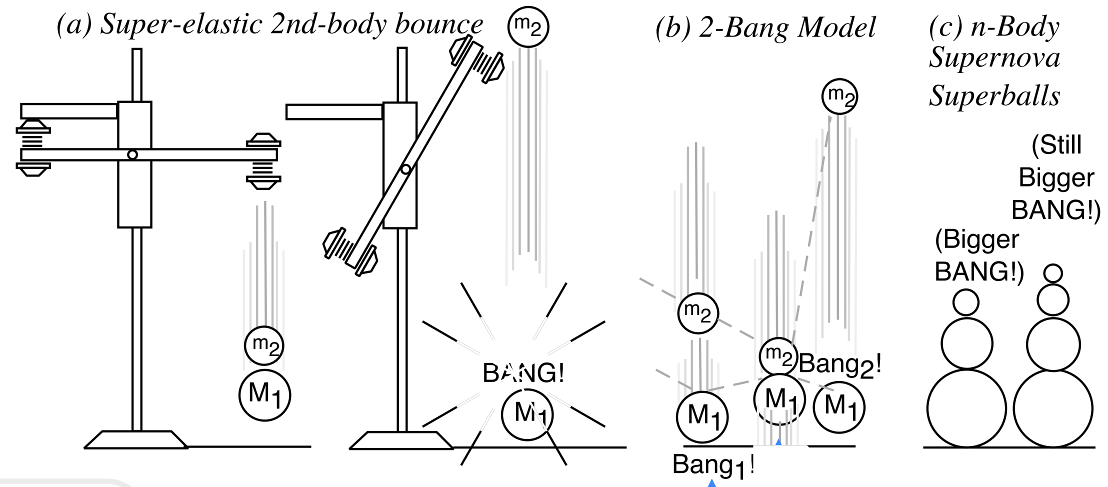
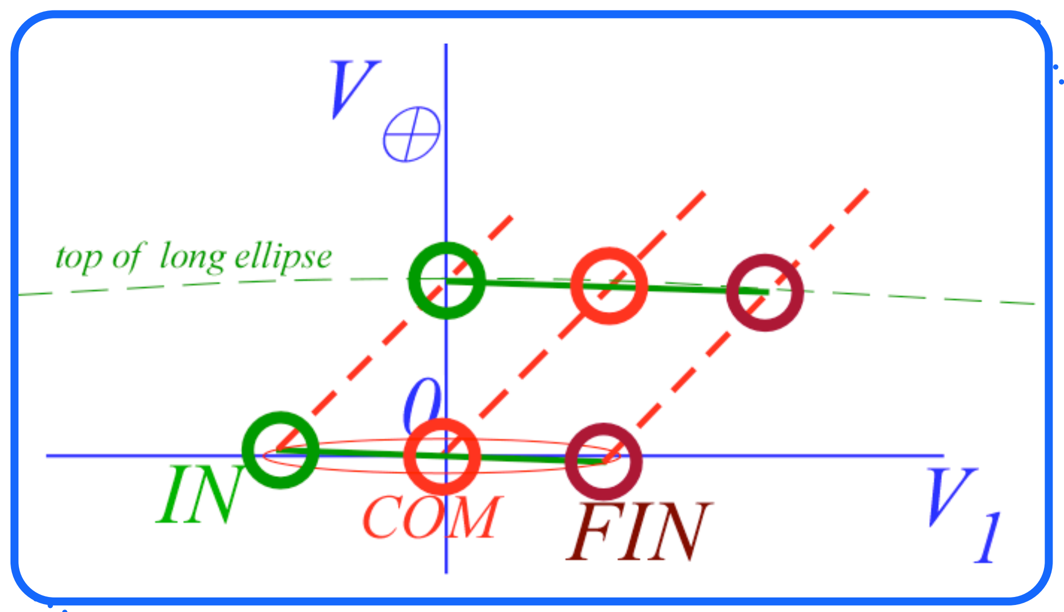
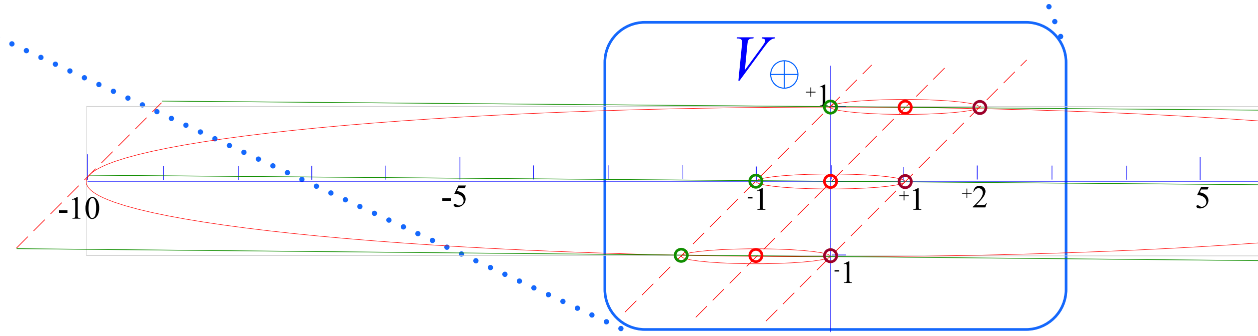


Fig. 3.4 (Unit 1)

1st bang: M_1 off floor

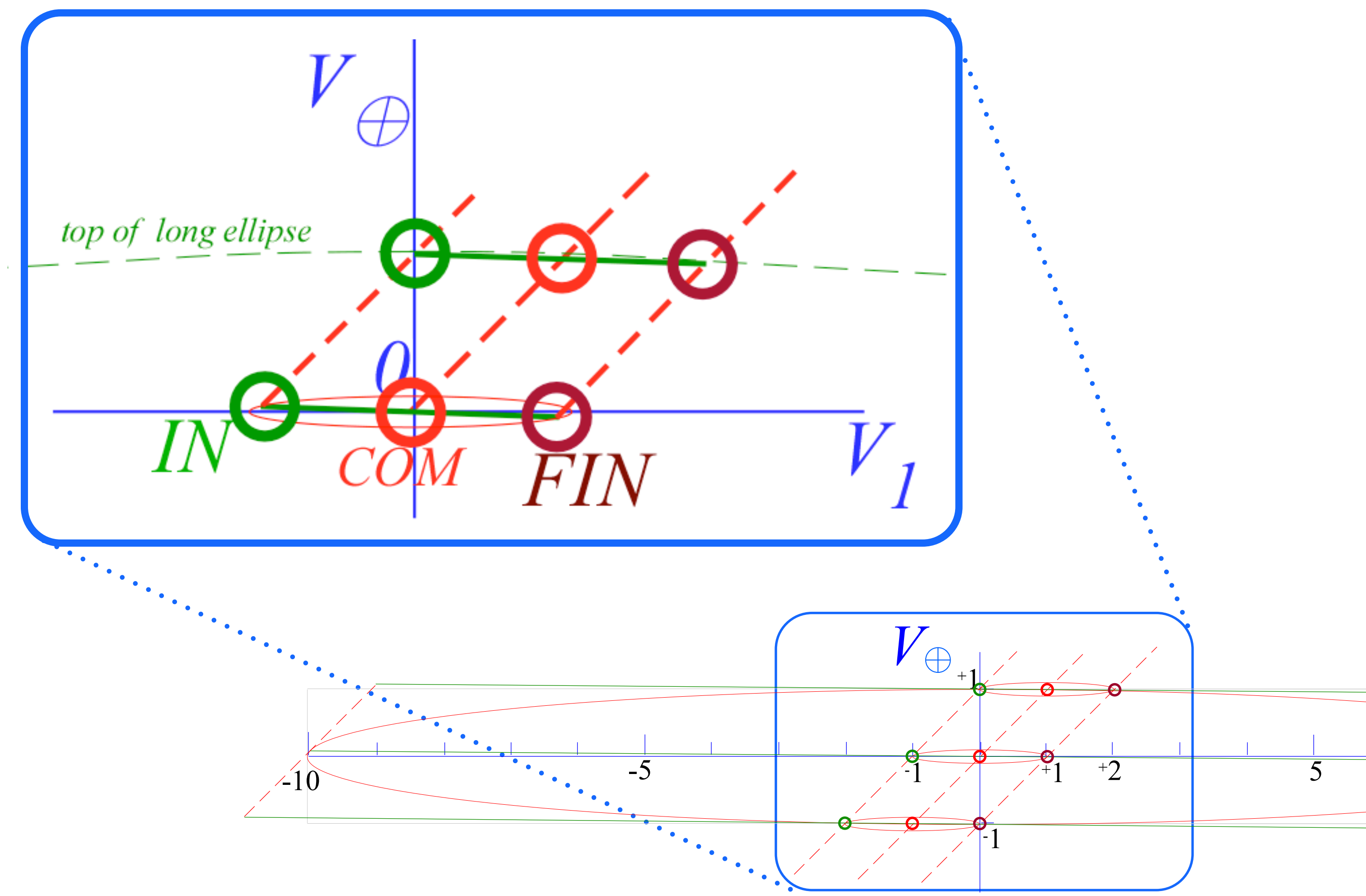


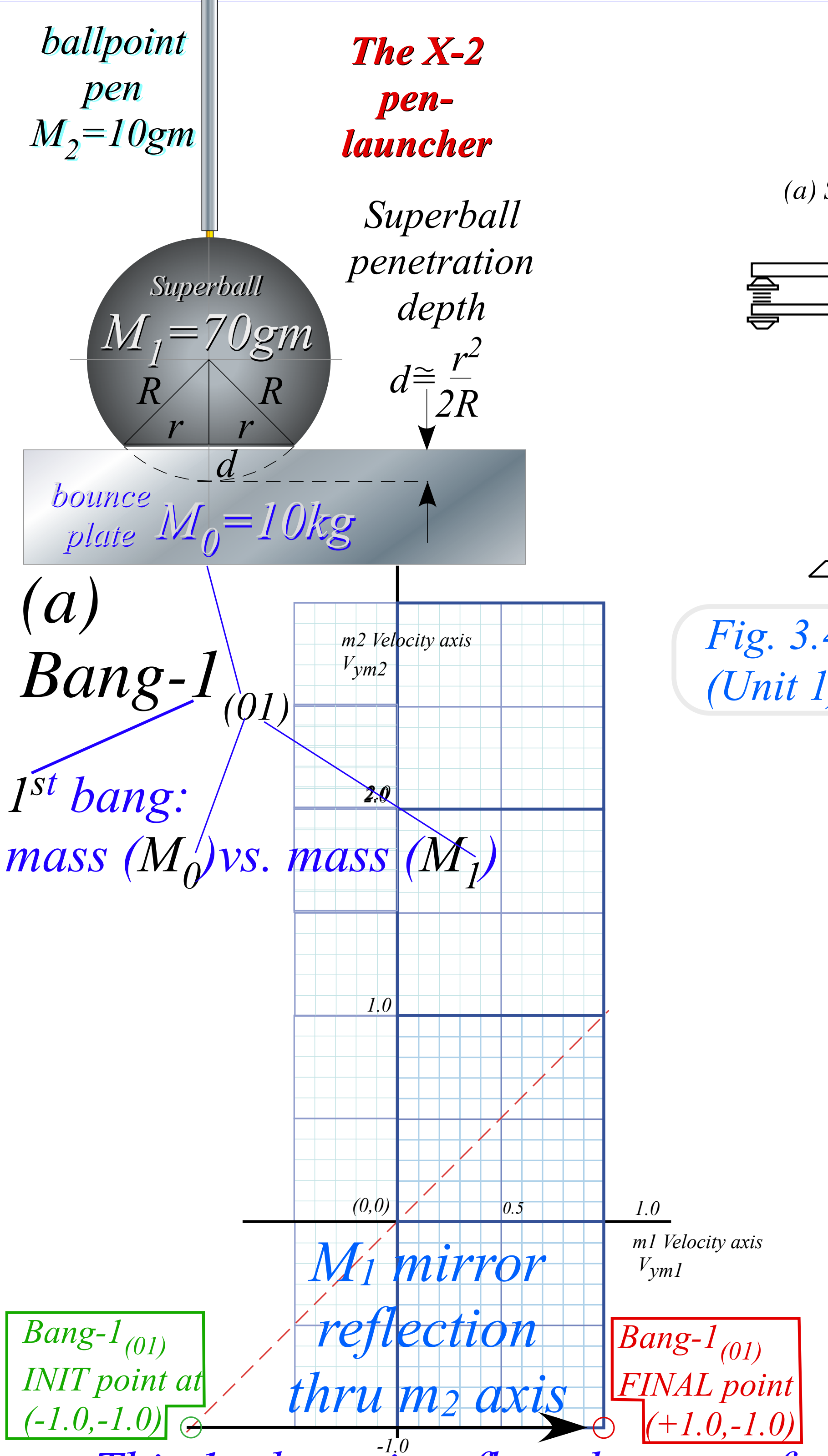


(With g=0 and 70:10 mass ratio)
http://www.uark.edu/ua/modphys/markup/BounceItWeb.html?scenario=1007

(With g and 70:35 mass ratio)
http://www.uark.edu/ua/modphys/testing/markup/BounceItWeb.html

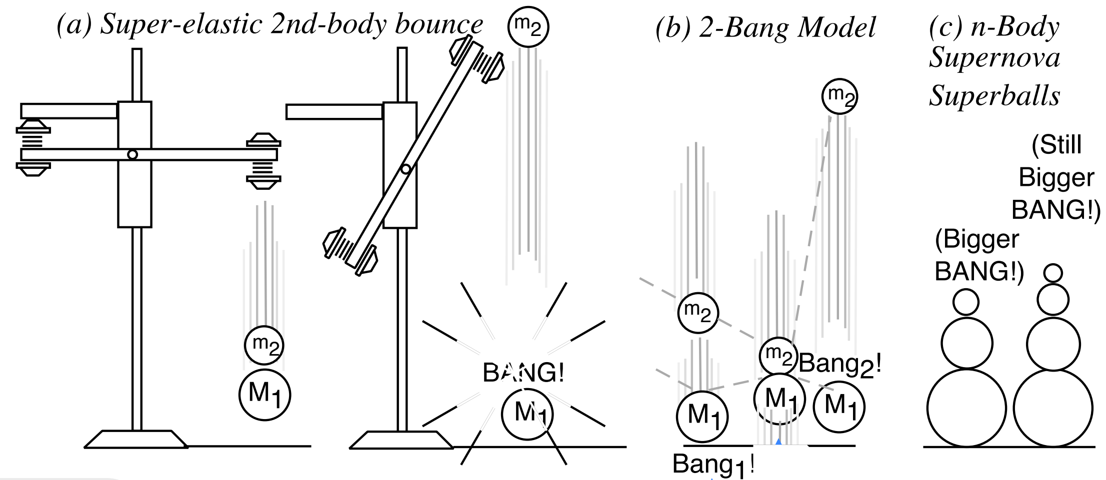
(a) 1st bang of M_1 off
floor plate $M_\oplus$ =100 M_1 along
$(V_1,V_\oplus)$-momentum line of slope
$-M_1/M_\oplus$ =-1/100
from IN-end to COM to FIN-end
of ($a/b = \sqrt{M_\oplus}/\sqrt{M_1} = 10$) ellipse





This 1st bang is a floor-bounce of M_1 off very massive plate/Earth M_0

Fig. 3.3 (Unit 1)



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1st bang: M_1 off floor

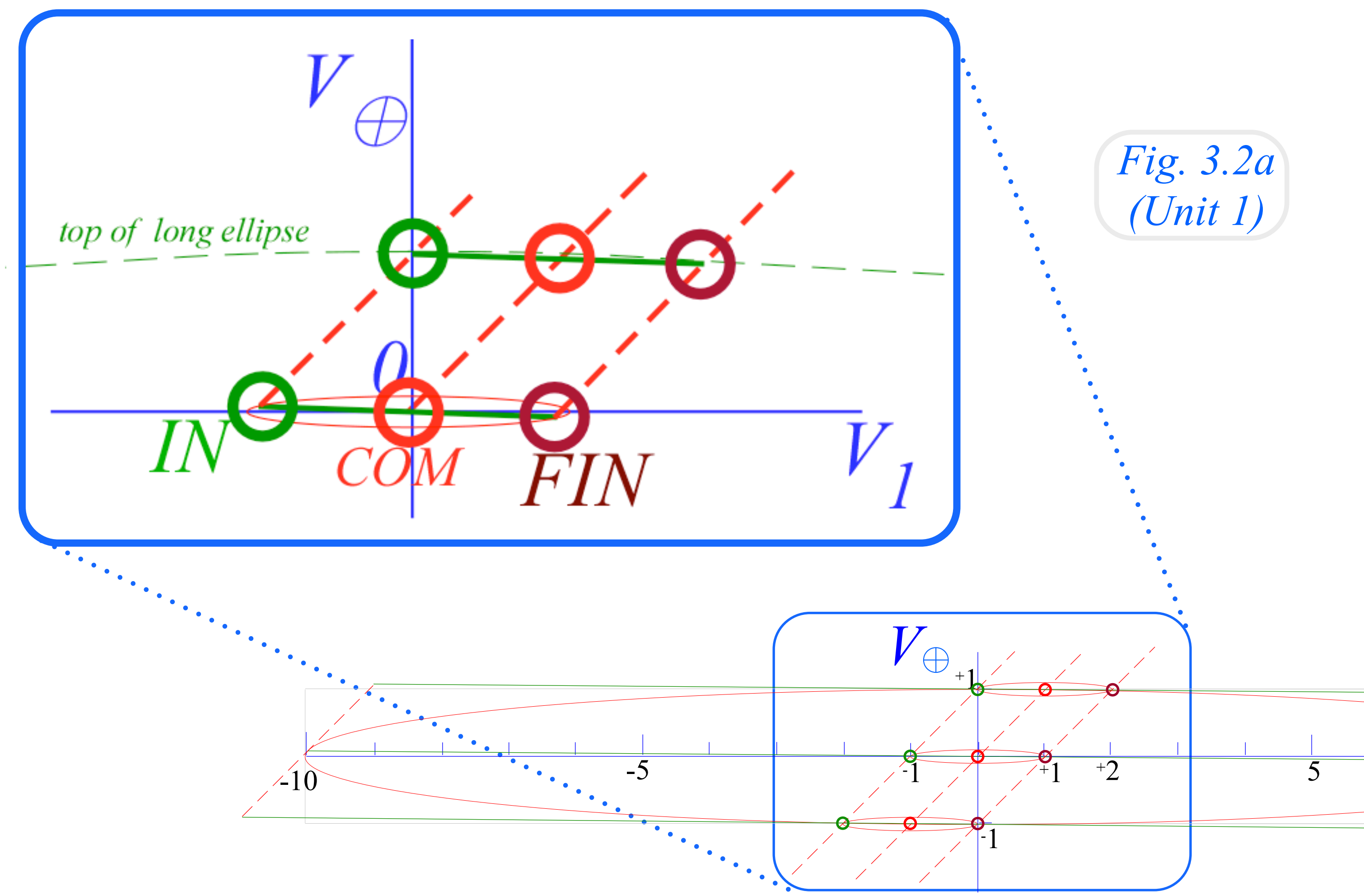


Fig. 3.2a (Unit 1)

(With g=0 and 70:10 mass ratio)
http://www.uark.edu/ua/modphys/markup/BounceItWeb.html?scenario=1007

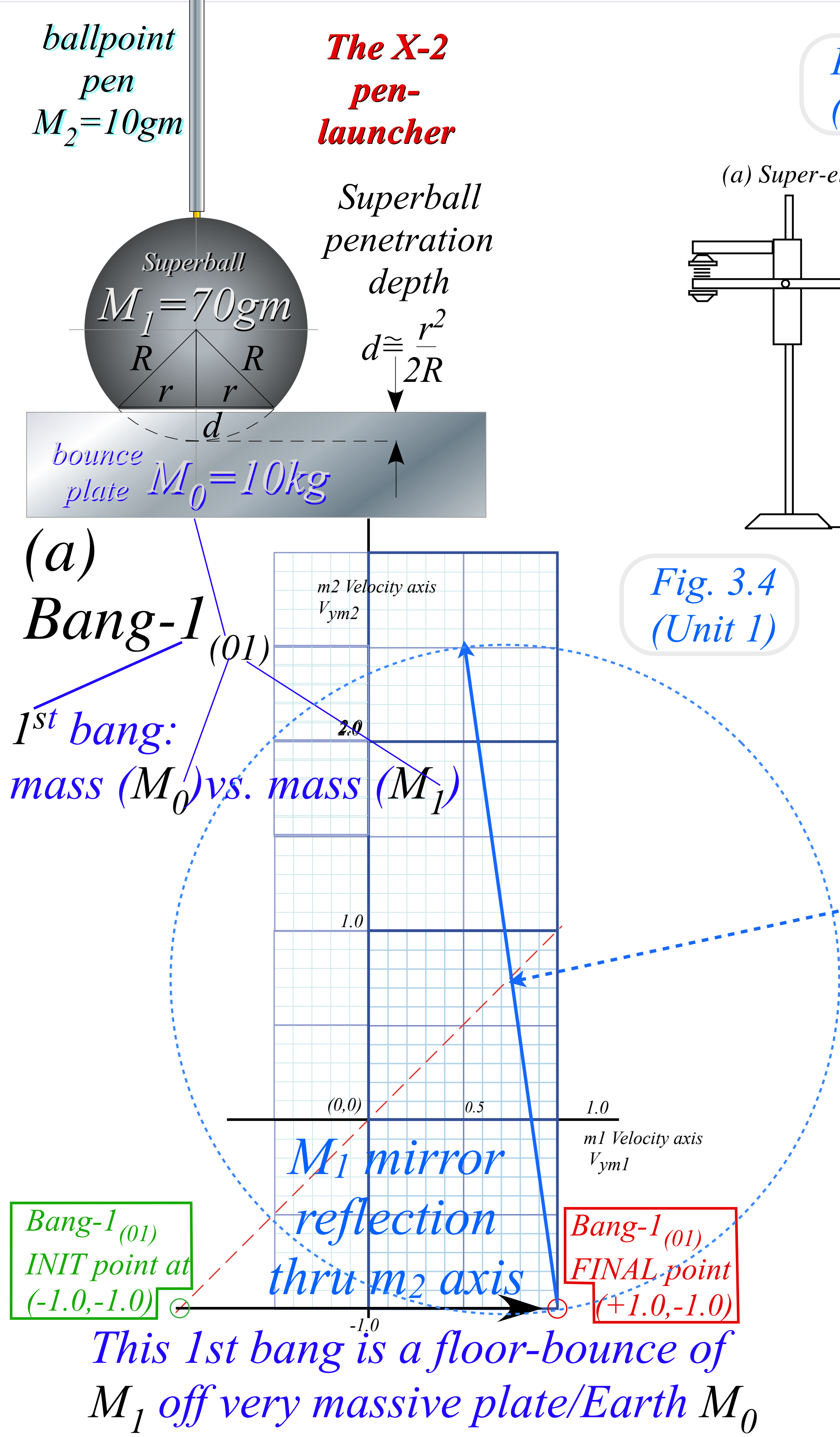
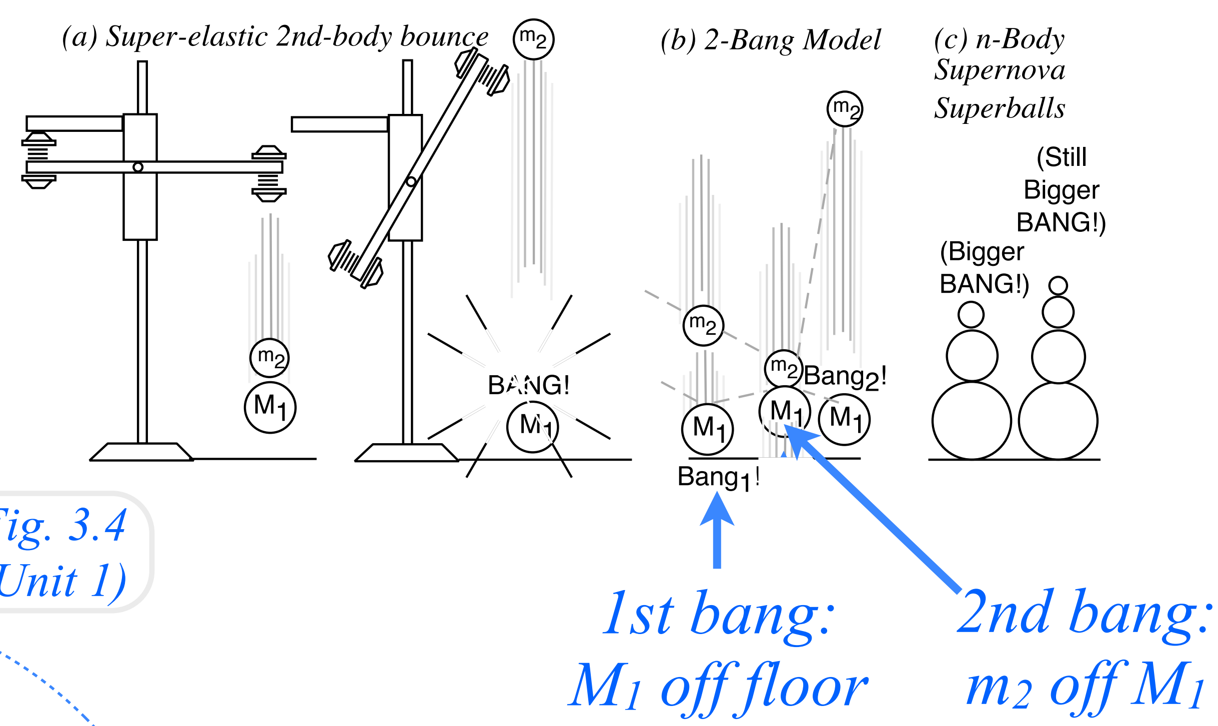


Fig. 3.4 (Unit 1)

Fig. 3.3 (Unit 1)



(With g=0 and 70:10 mass ratio)
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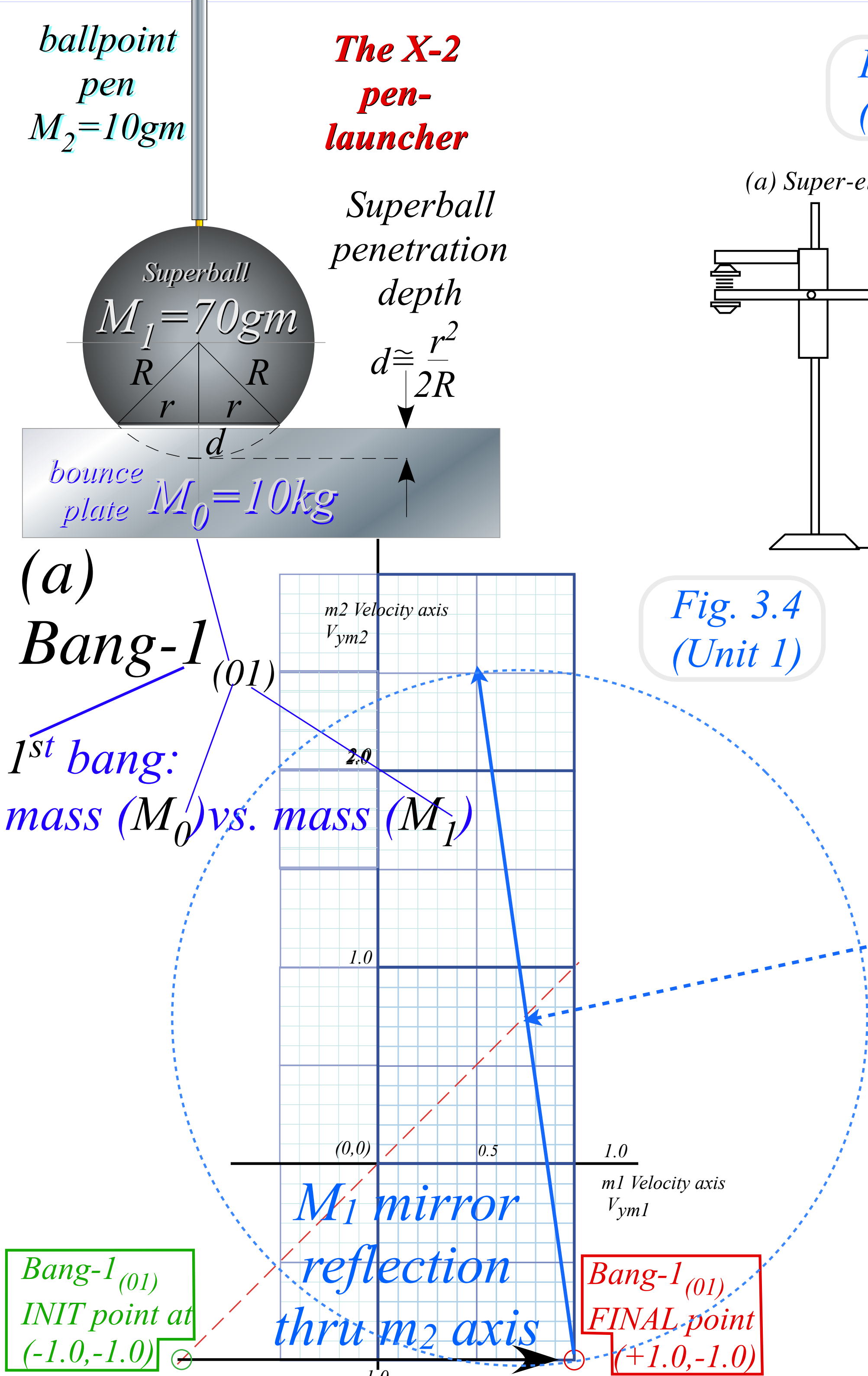
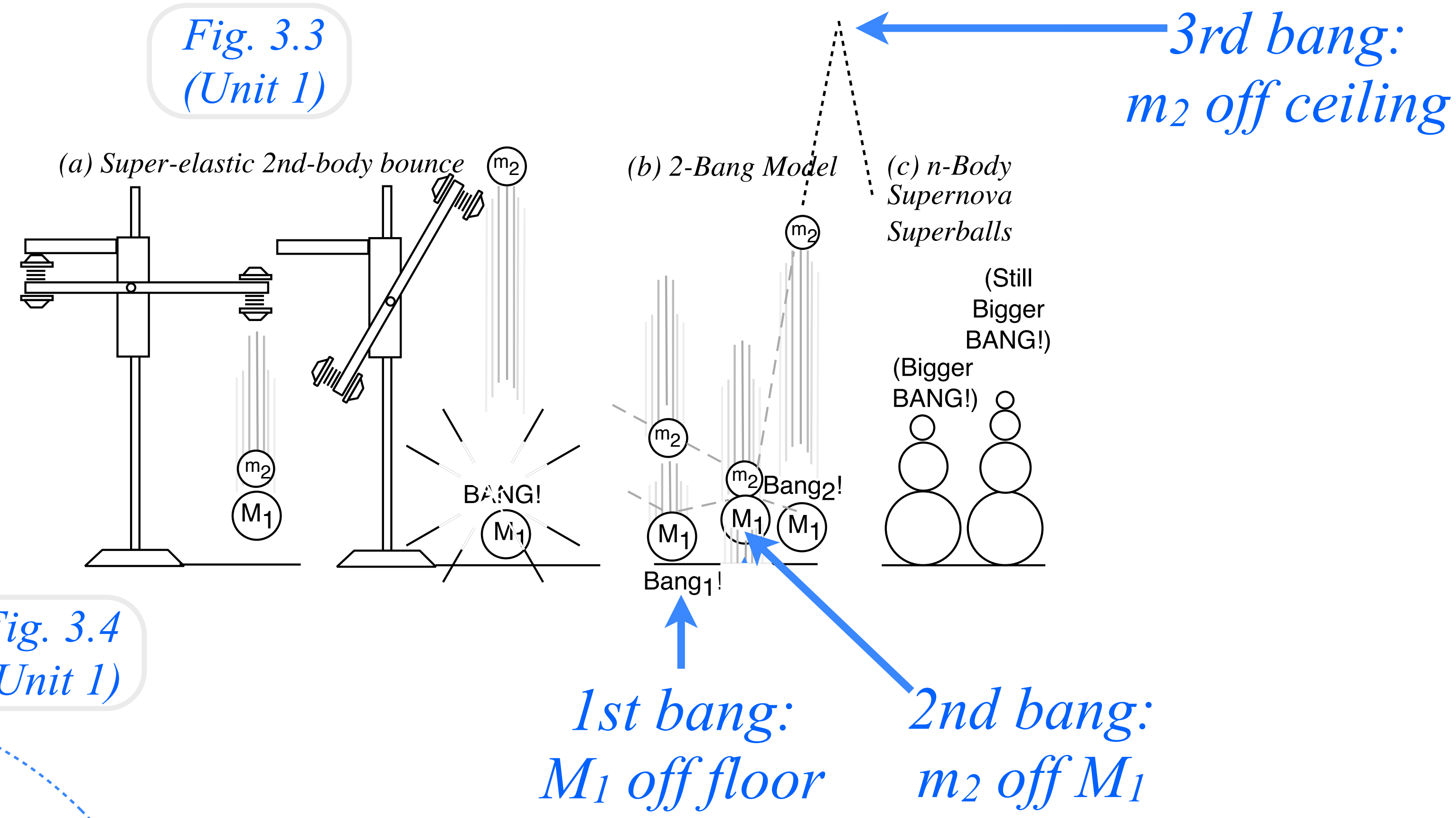


This 1st bang is a floor-bounce of M_1 off very massive plate/Earth M_0



(With g=0 and 70:10 mass ratio)
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(With g and 70:35 mass ratio)
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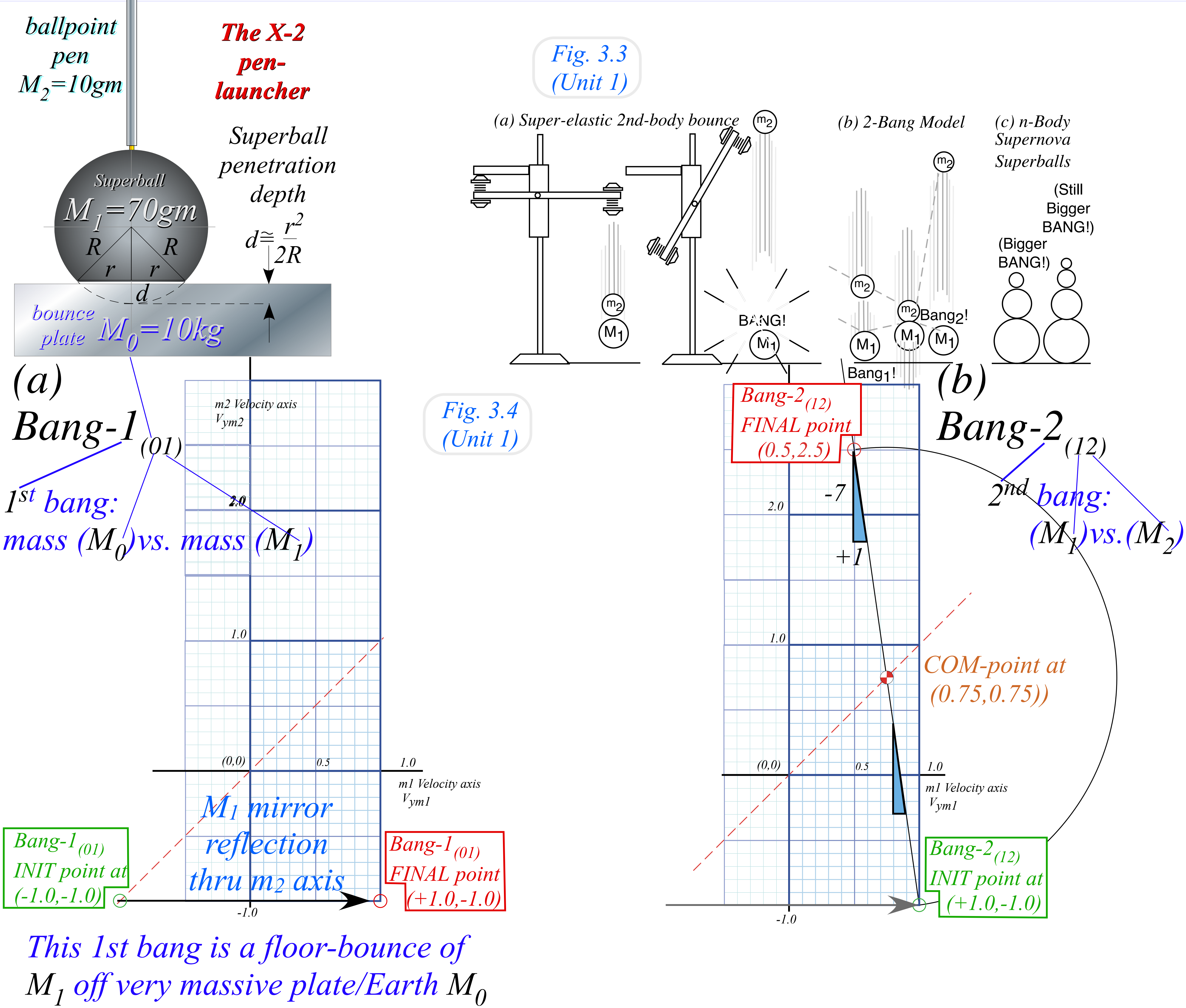
Geometry of X2 launcher bouncing in box

→ *Independent Bounce Model (IBM)*

Geometric optimization and range-of-motion calculation(t)

Integration of (V_1,V_2) data to space-time plots ($y_1(t)$,t) and ($y_2(t)$,t) plots

Integration of (V_1,V_2) data to space-space plots (y_1, y_2)



ballpoint pen M2=10gm
The X-2 pen-launcher
Superball penetration depth
d≅ r2/2R
Superball
M1=70gm
R R
r r
d
bounce plate M0=10kg
(a)
Bang-1(01)
1st bang: mass (M0)vs. mass (M1)
m2 Velocity axis Vym2
2.0
1.0
(0,0)
0.5
1.0
m1 Velocity axis Vym1
-1.0
M1 mirror reflection thru m2 axis
Bang-1(01) INIT point at (-1.0,-1.0)
Bang-1(01) FINAL point (+1.0,-1.0)
This 1st bang is a floor-bounce of M1 off very massive plate/Earth M0
Fig. 3.3 (Unit 1)
(a) Super-elastic 2nd-body bounce
m2
M1
BANG!
(b) 2-Bang Model
Bang2!
Bang1!
(c) n-Body Supernova Superballs
(Still Bigger BANG!)
(Bigger BANG!)
Fig. 3.4 (Unit 1)
(b)
Bang-2(12)
2nd bang: (M1)vs.(M2)
Bang-2(12) FINAL point (0.5,2.5)
-7
+1
2.0
1.0
COM-point at (0.75,0.75))
(0,0)
0.5
1.0
m1 Velocity axis Vym1
-1.0
Bang-2(12) INIT point at (+1.0,-1.0)

Geometry of X2 launcher bouncing in box

Independent Bounce Model (IBM)

→ *Geometric optimization and range-of-motion calculation(s)*

Integration of (V_1,V_2) data to space-time plots $(y_1(t),t)$ and $(y_2(t),t)$ plots

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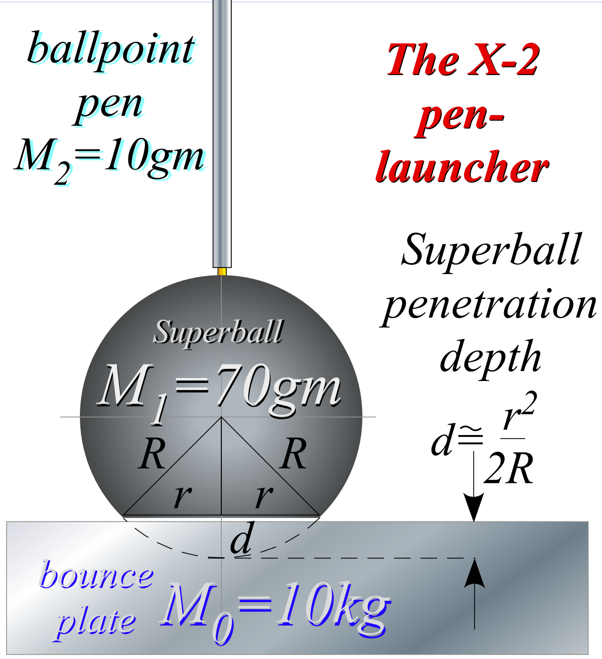
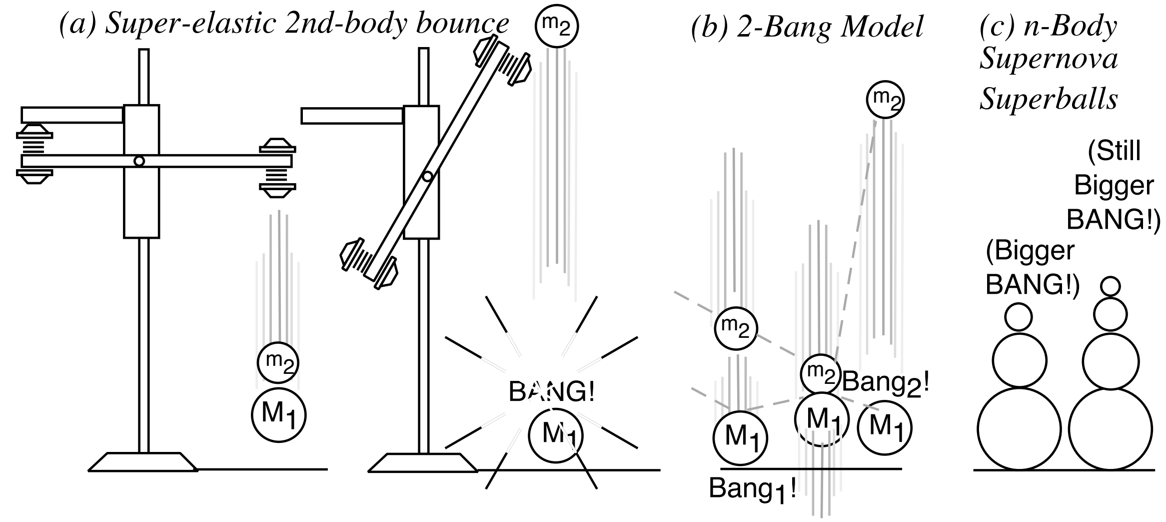


Fig. 3.3 (Unit 1)



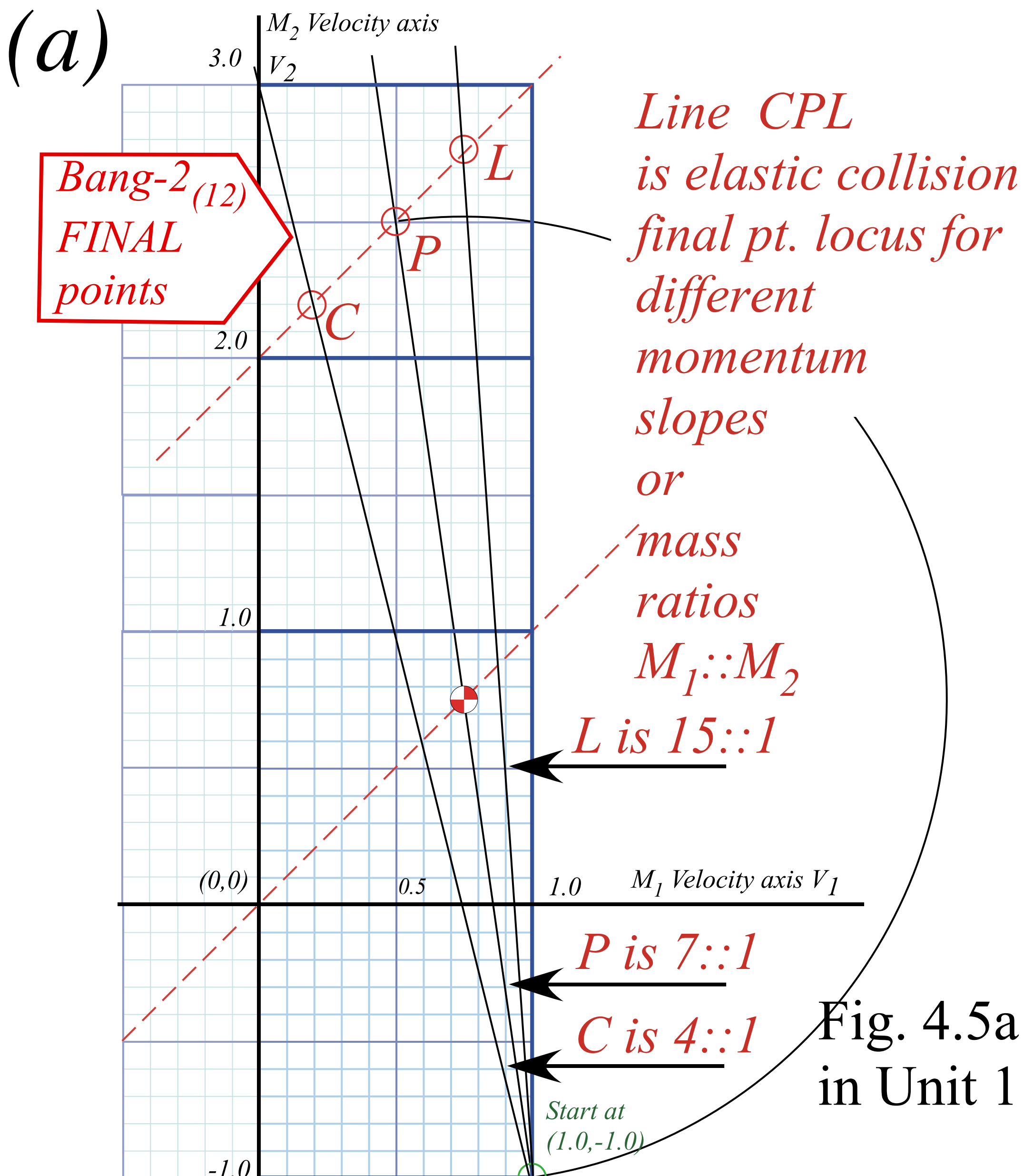


Fig. 4.5a in Unit 1

(With g=0 and 70:10 mass ratio)
http://www.uark.edu/ua/modphys/markup/BounceItWeb.html?scenario=1007

(With g and 70:35 mass ratio)
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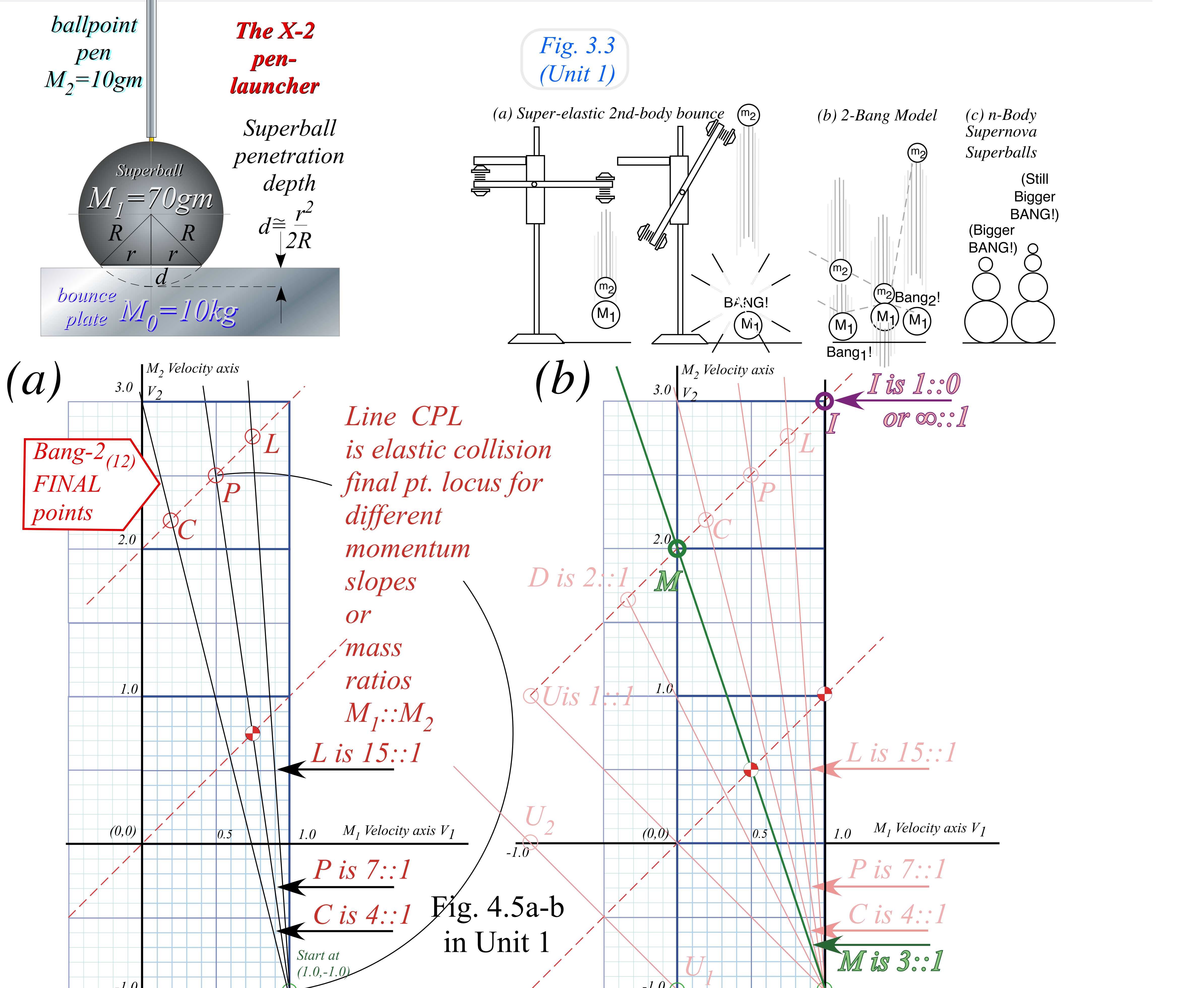

(With g=0 and 70:10 mass ratio)
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(With g and 70:35 mass ratio)
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Geometry of X2 launcher bouncing in box

Independent Bounce Model (IBM)

Geometric optimization and range-of-motion calculation(s)

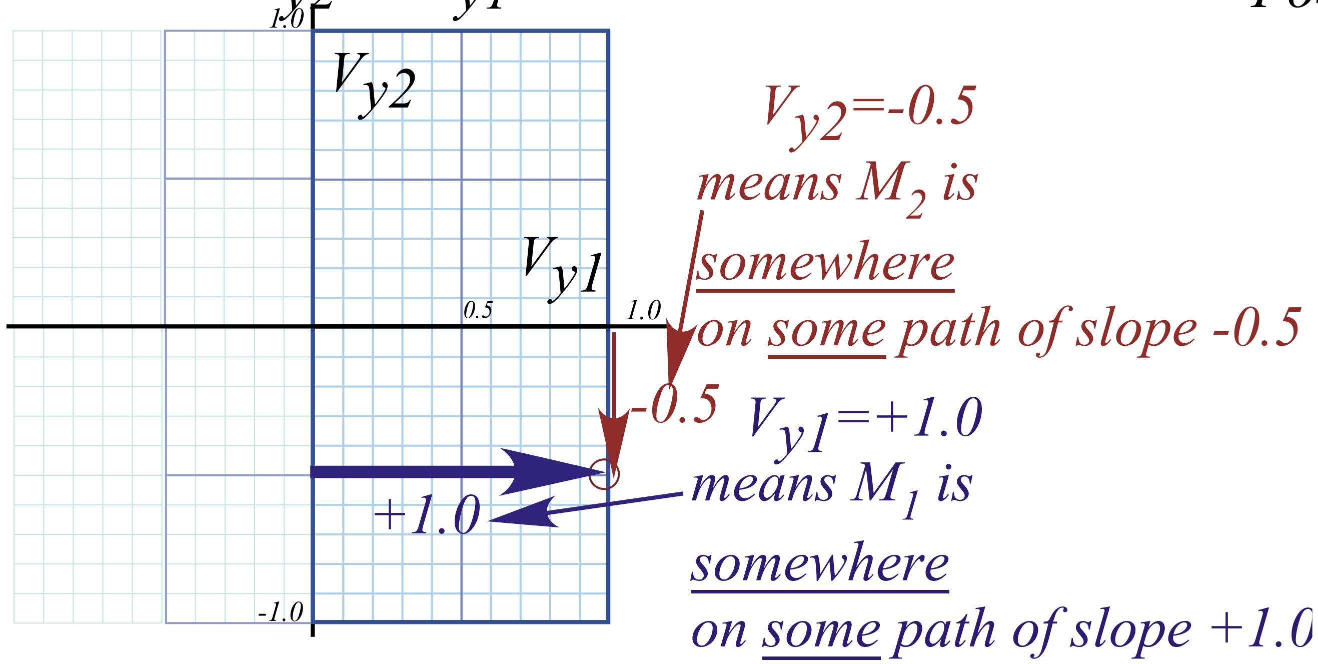
→ *Integration of (V_1,V_2) data to space-time plots ($y_1(t)$,t) and ($y_2(t)$,t) plots*

Integration of (V_1,V_2) data to space-space plots (y_1, y_2) *Examples (M_1=7, M_2=1) and (M_1=49, M_2=1)*

Geometric "Integration" (Converting Velocity data to Spacetime)

Velocity V_{y2} vs. V_{y1} Plot

Position y vs. Time t Plot



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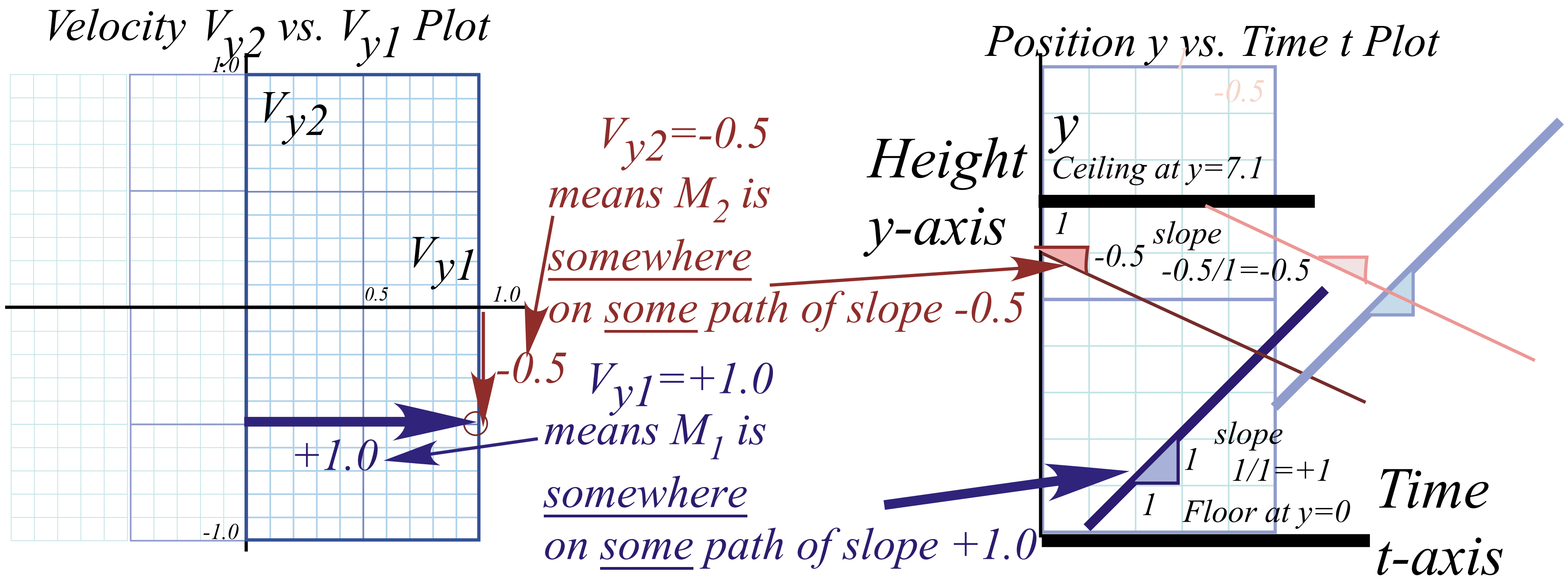

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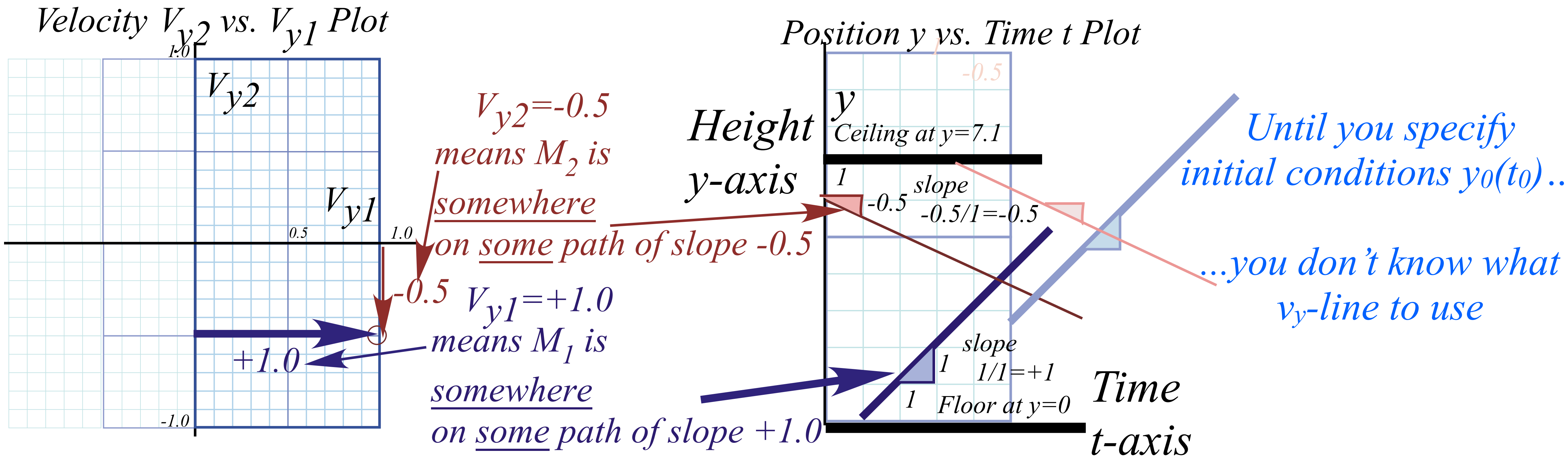

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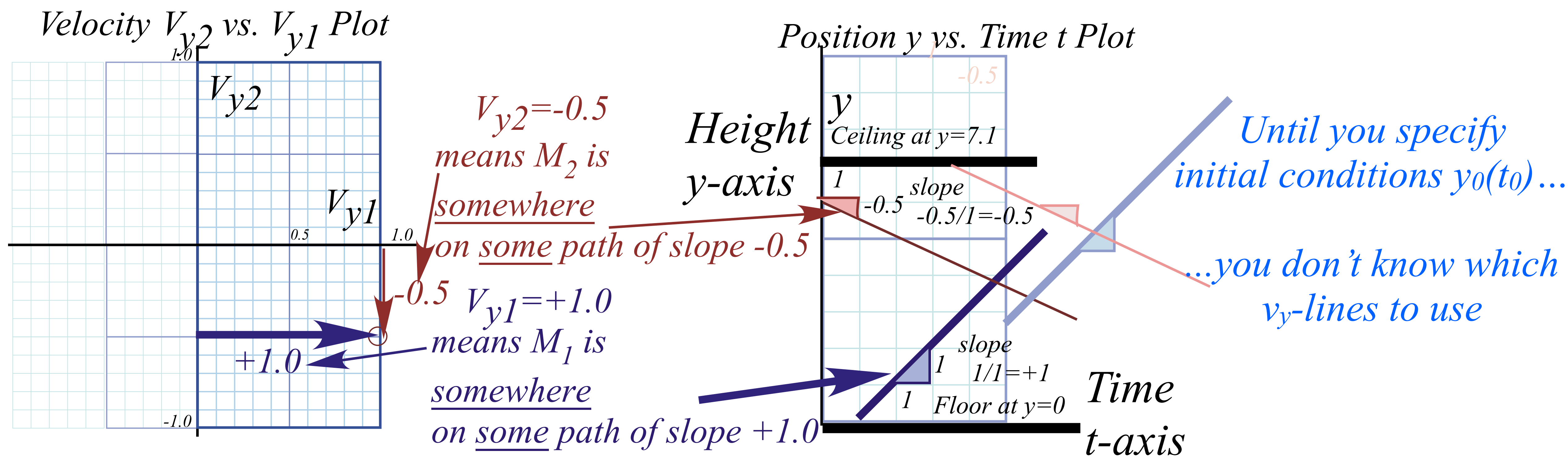
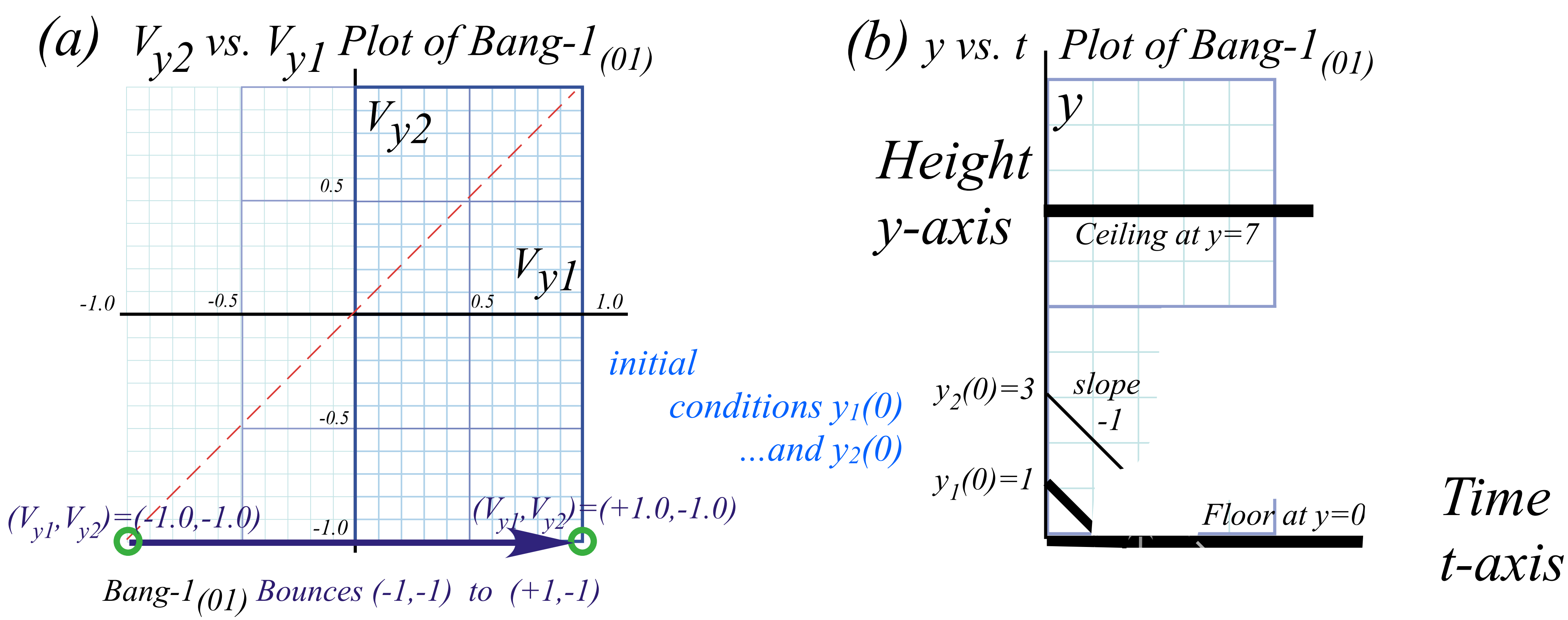
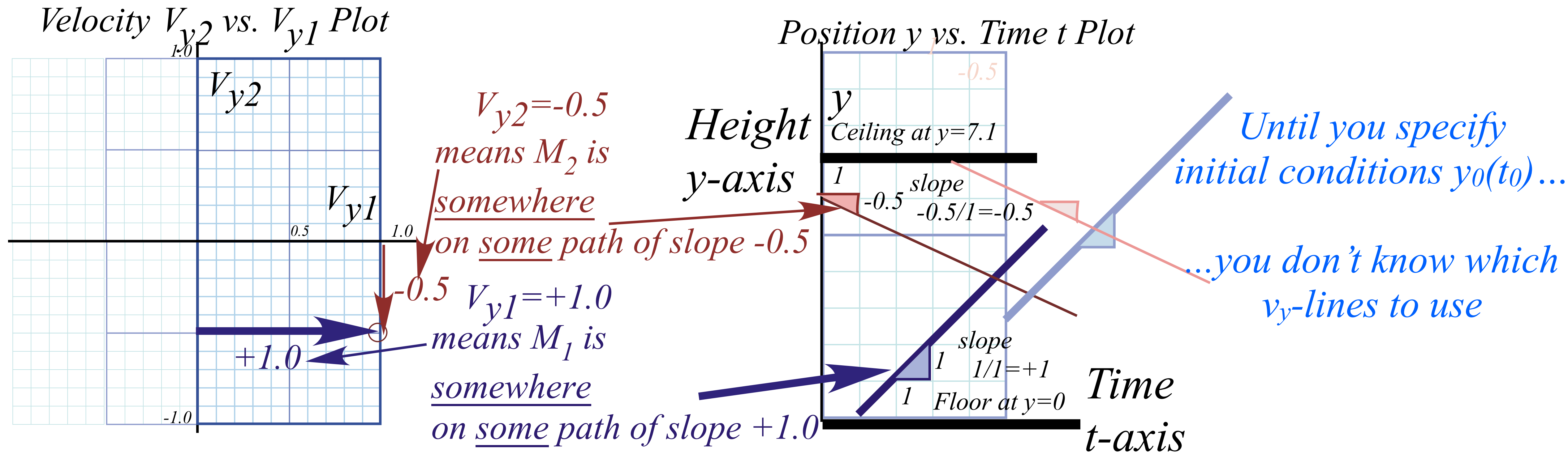


Fig. 3.6 (Unit 1)



Geometric "Integration" (Converting Velocity data to Spacetime)



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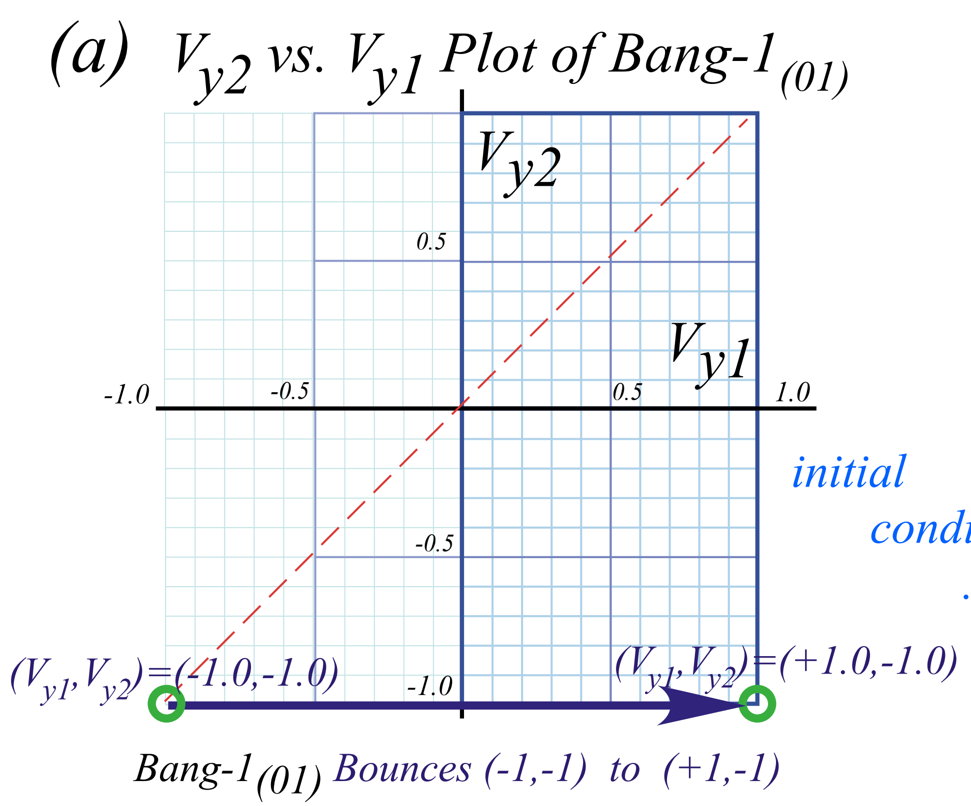
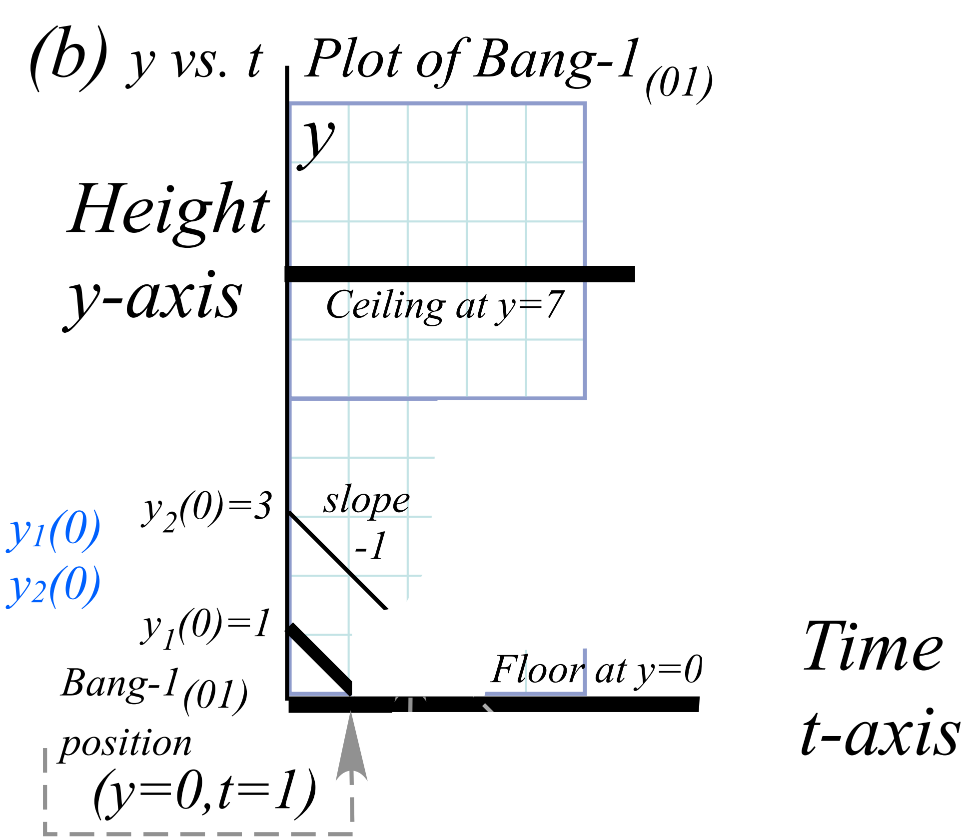




Geometric "Integration" (Converting Velocity data to Spacetime)

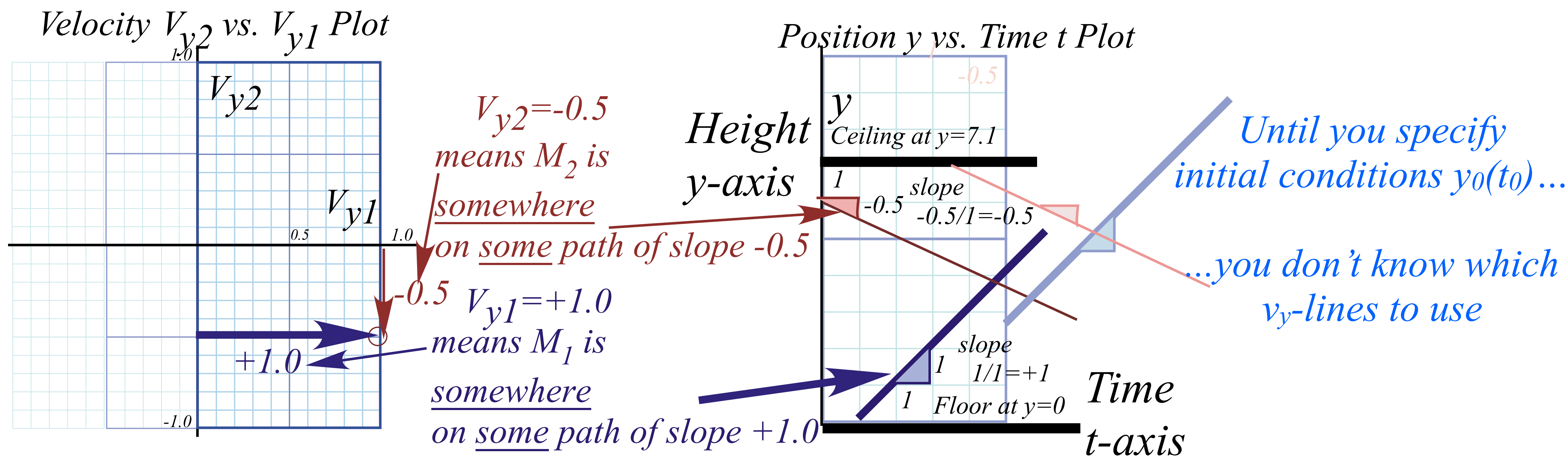
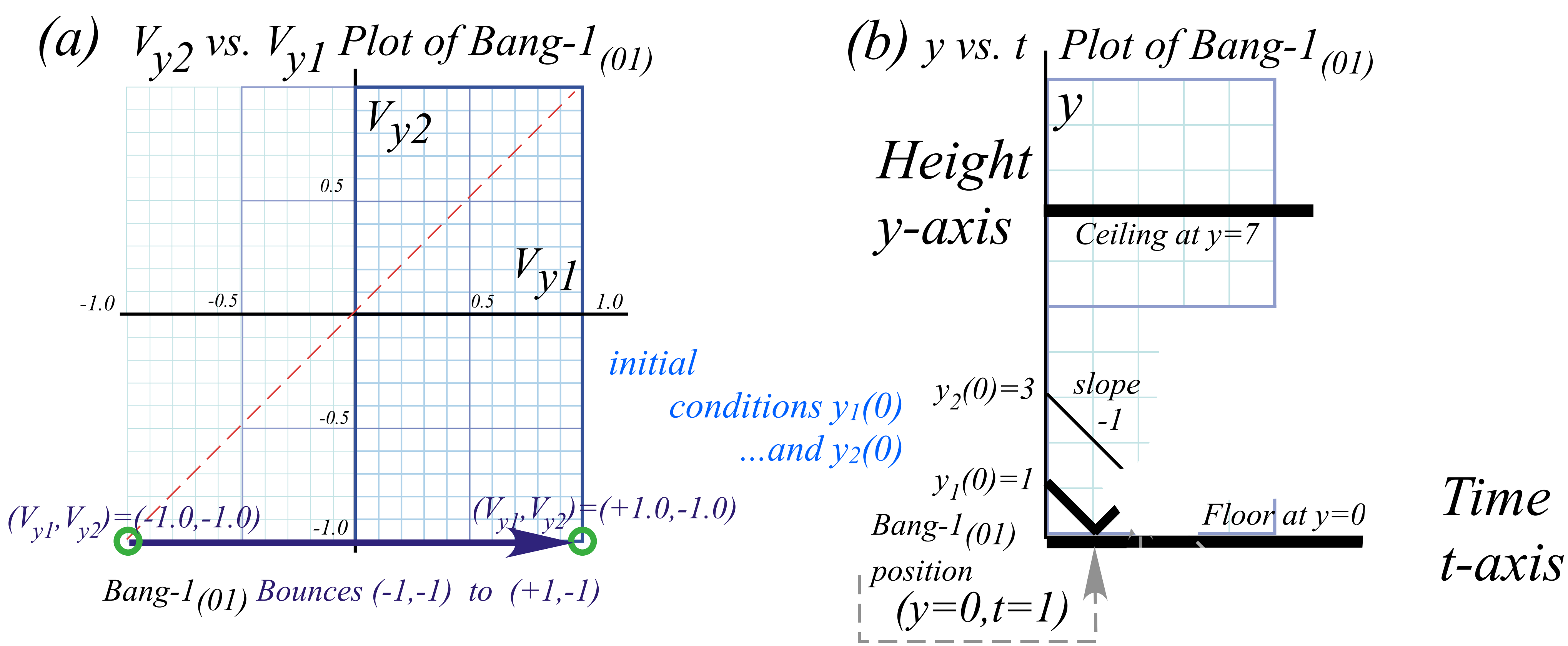


Fig. 3.6
(Unit 1)



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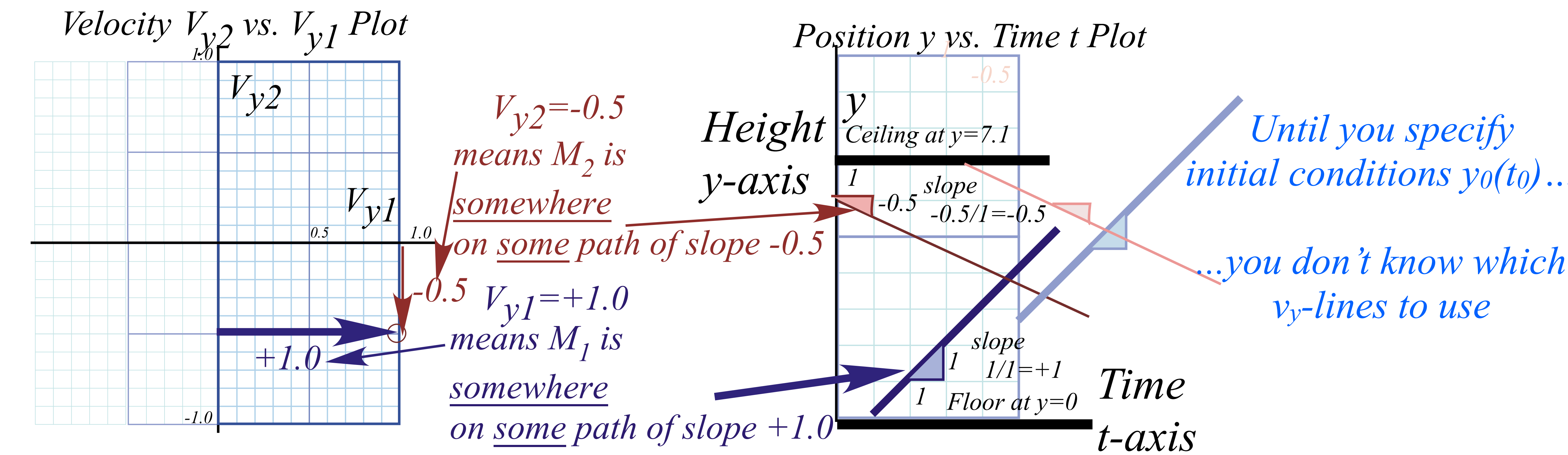

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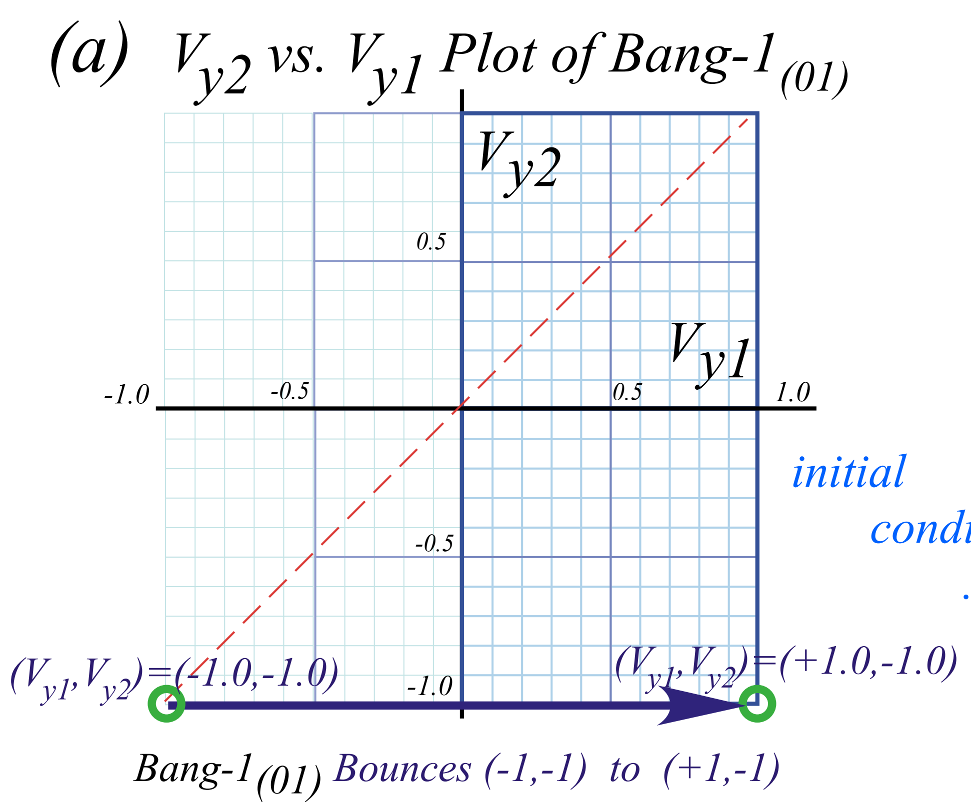

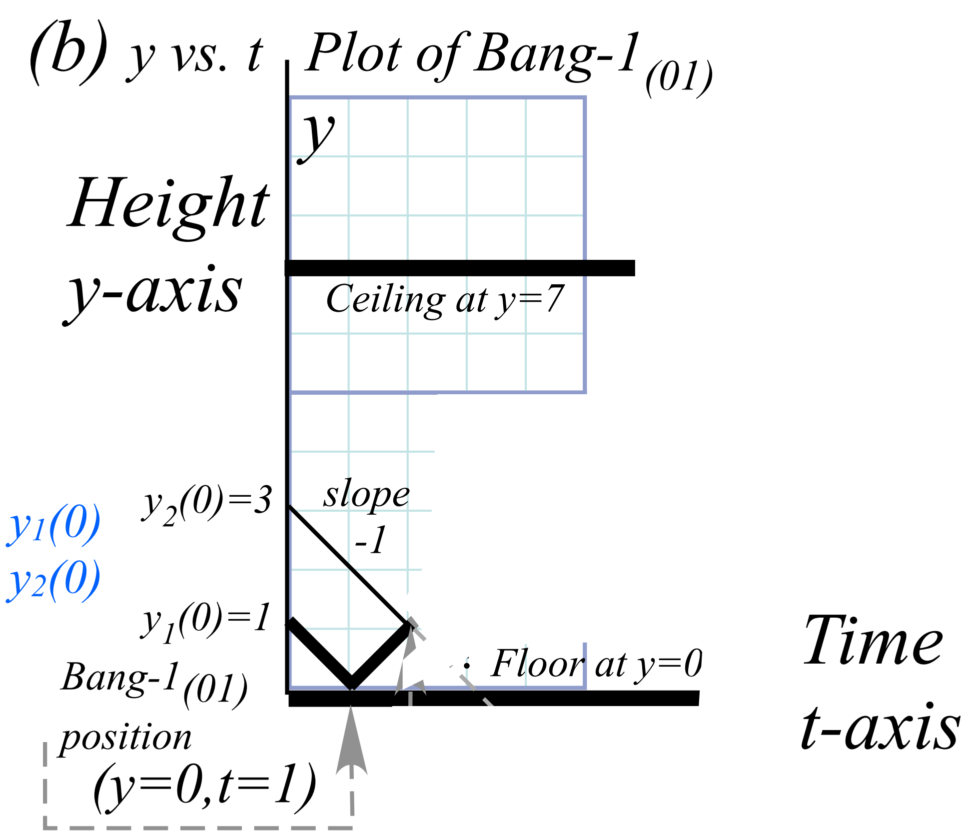

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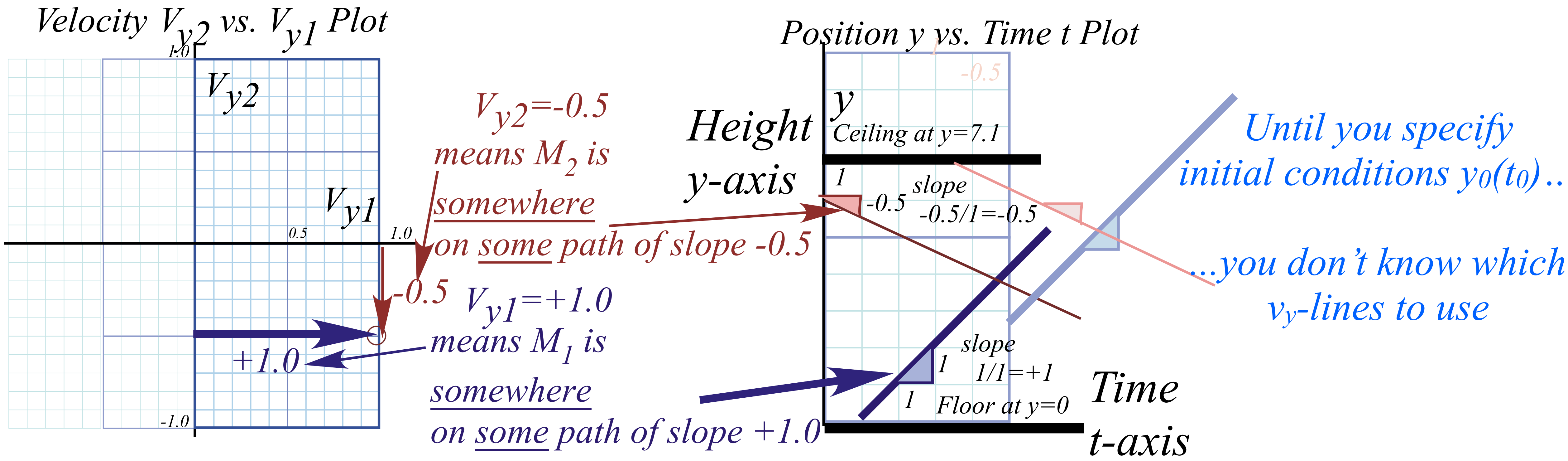

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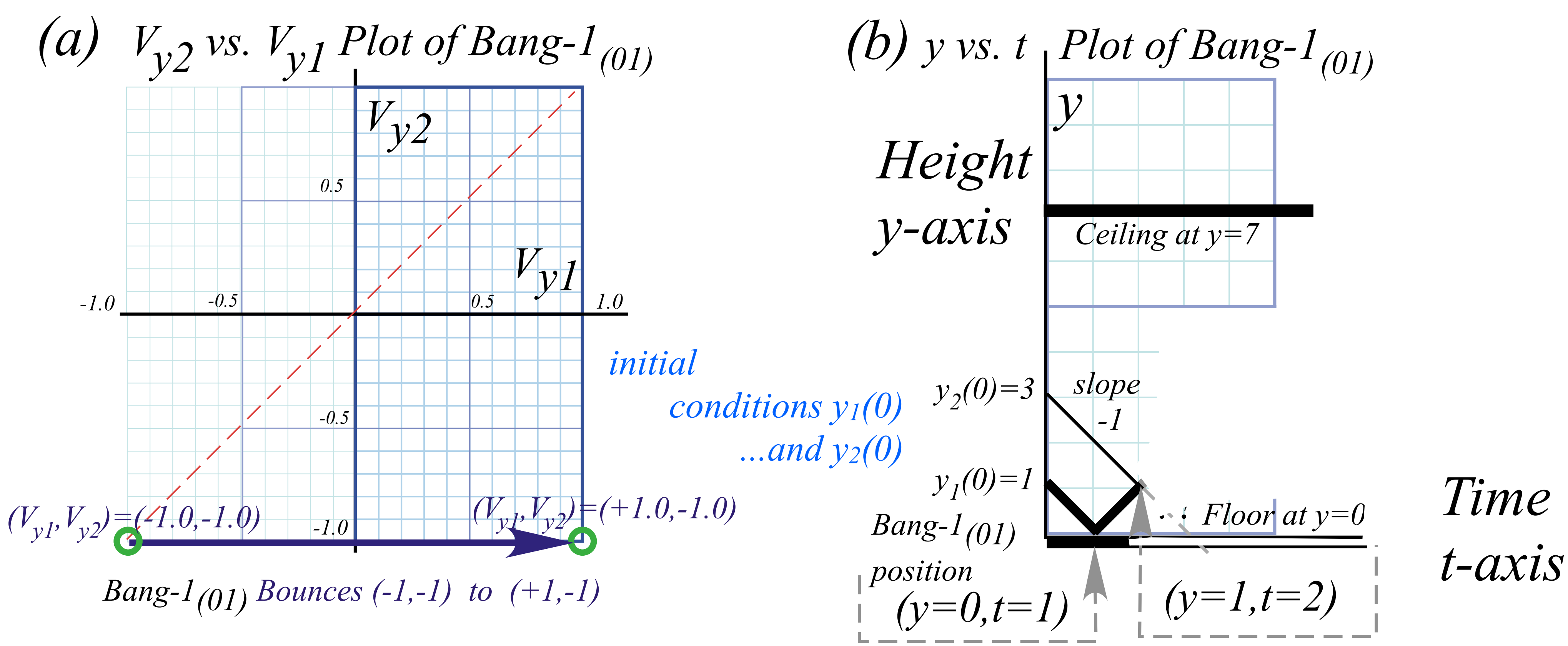

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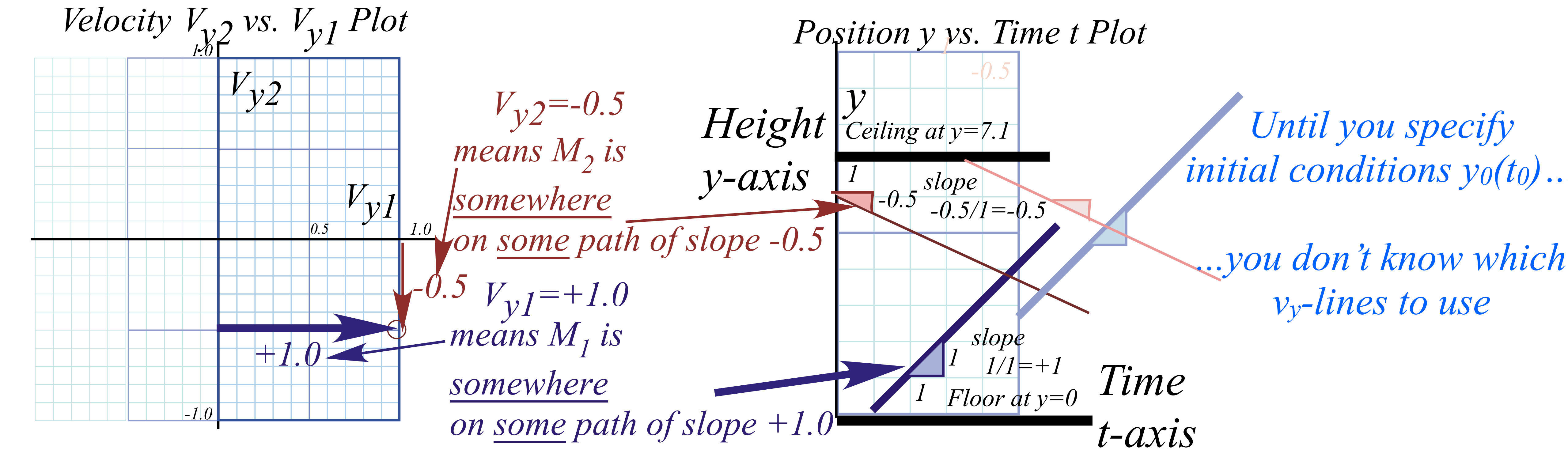

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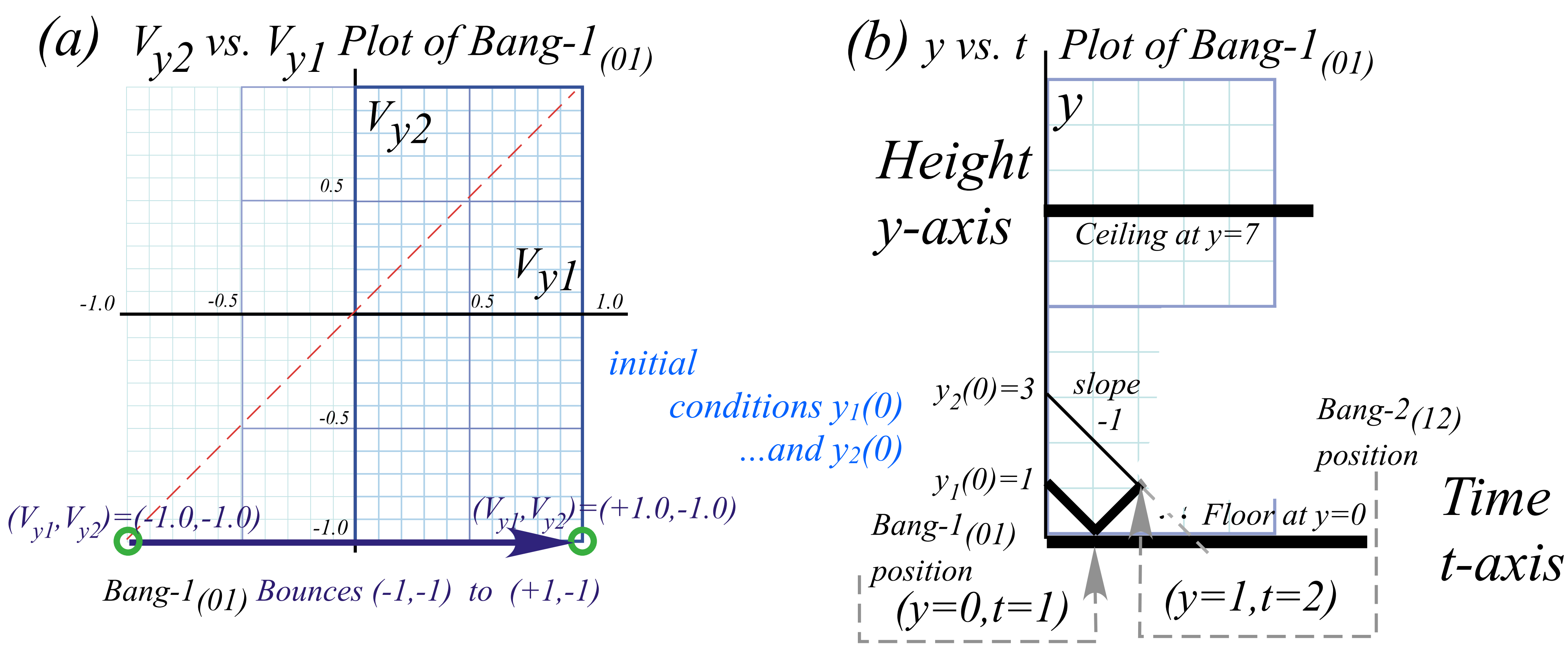
(Unit 1)



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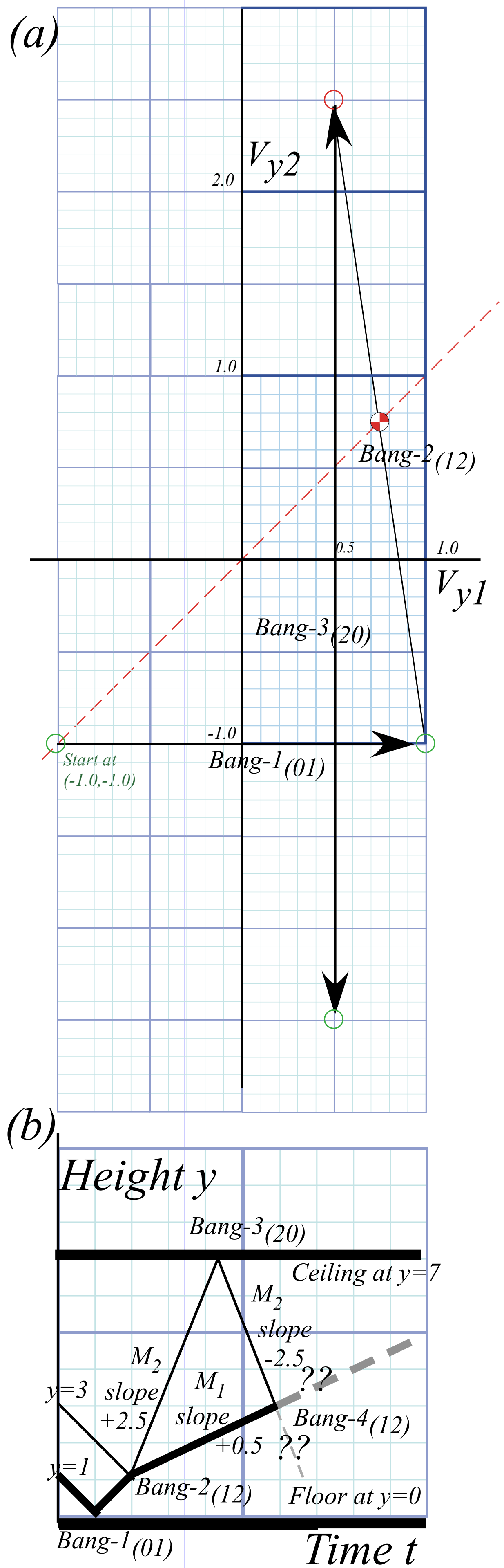
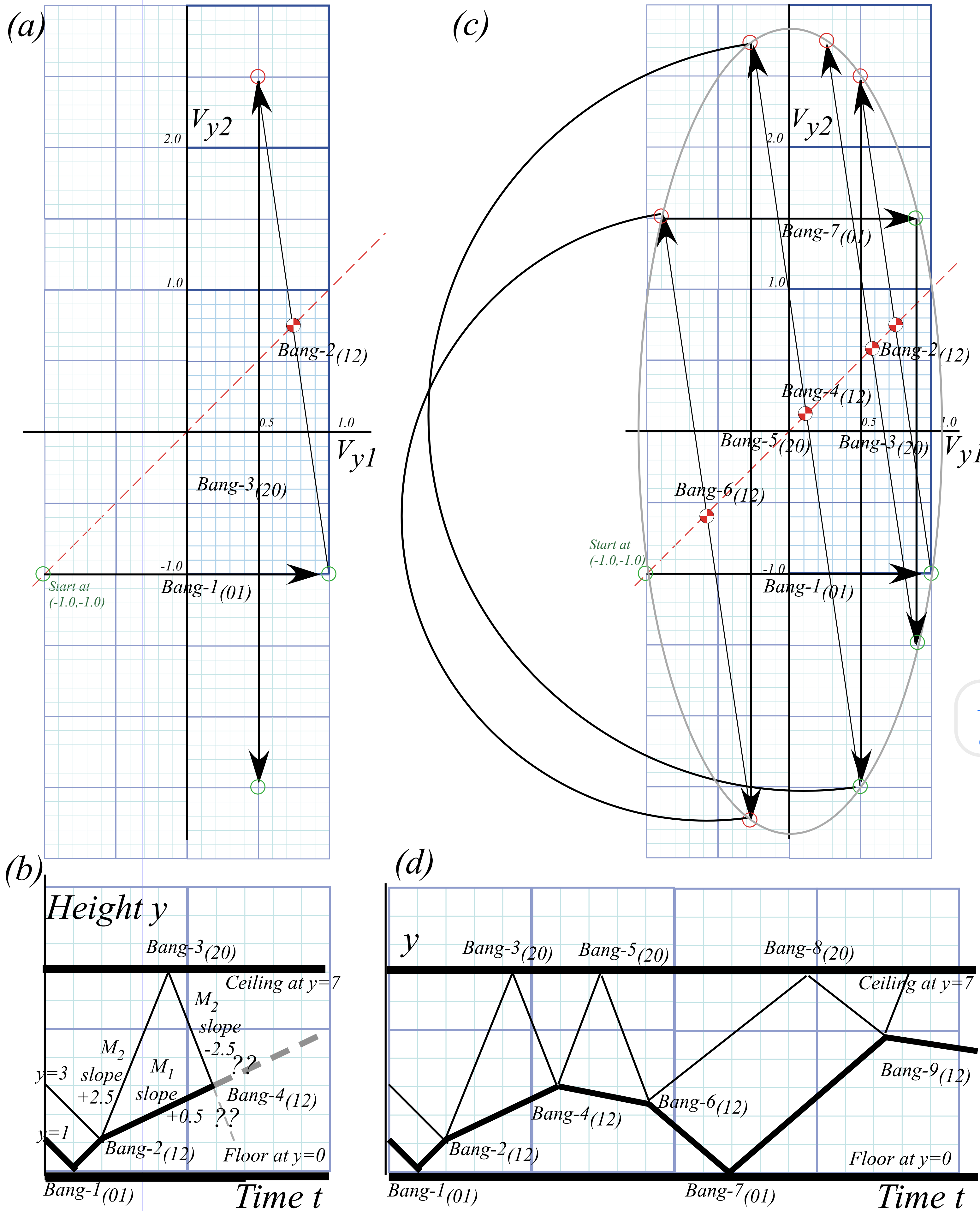


Geometric "Integration" (Converting Velocity data to Spacetime)



Kinetic Energy Ellipse

$$KE = \frac{1}{2}M_1V_1^2 + \frac{1}{2}M_2V_2^2 = \frac{1}{2} + \frac{7}{2} = 4$$

$$1 = \frac{V_1^2}{2KE/M_1} + \frac{V_2^2}{2KE/M_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Fig. 3.7 (Unit 1)

Geometric "Integration" (Converting Velocity data to Spacetime)

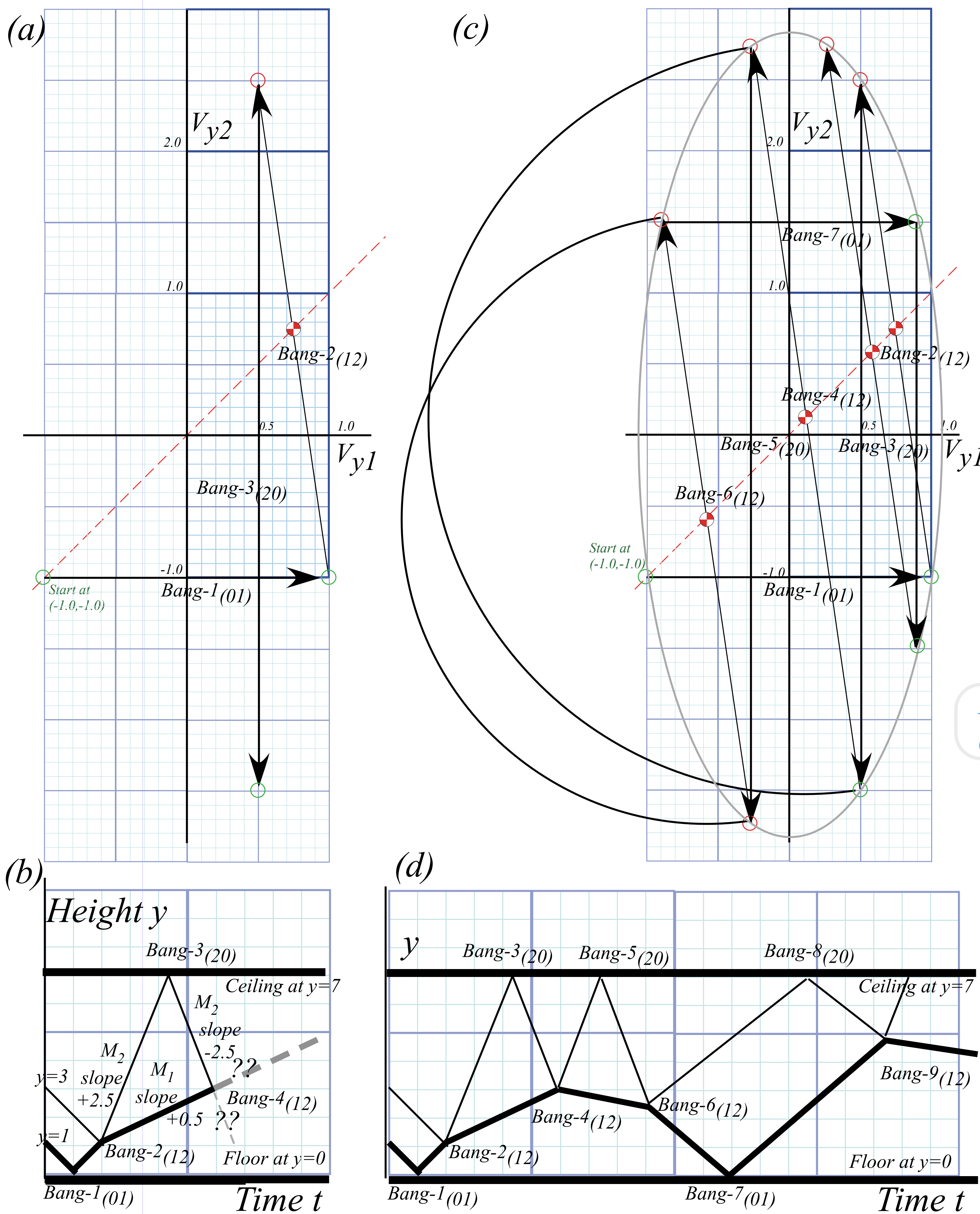


Fig. 3.7 (Unit 1)

Kinetic Energy Ellipse

$$KE = \frac{1}{2}M_1V_1^2 + \frac{1}{2}M_2V_2^2 = \frac{7}{2} + \frac{1}{2} = 4$$

$$1 = \frac{V_1^2}{2KE/M_1} + \frac{V_2^2}{2KE/M_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

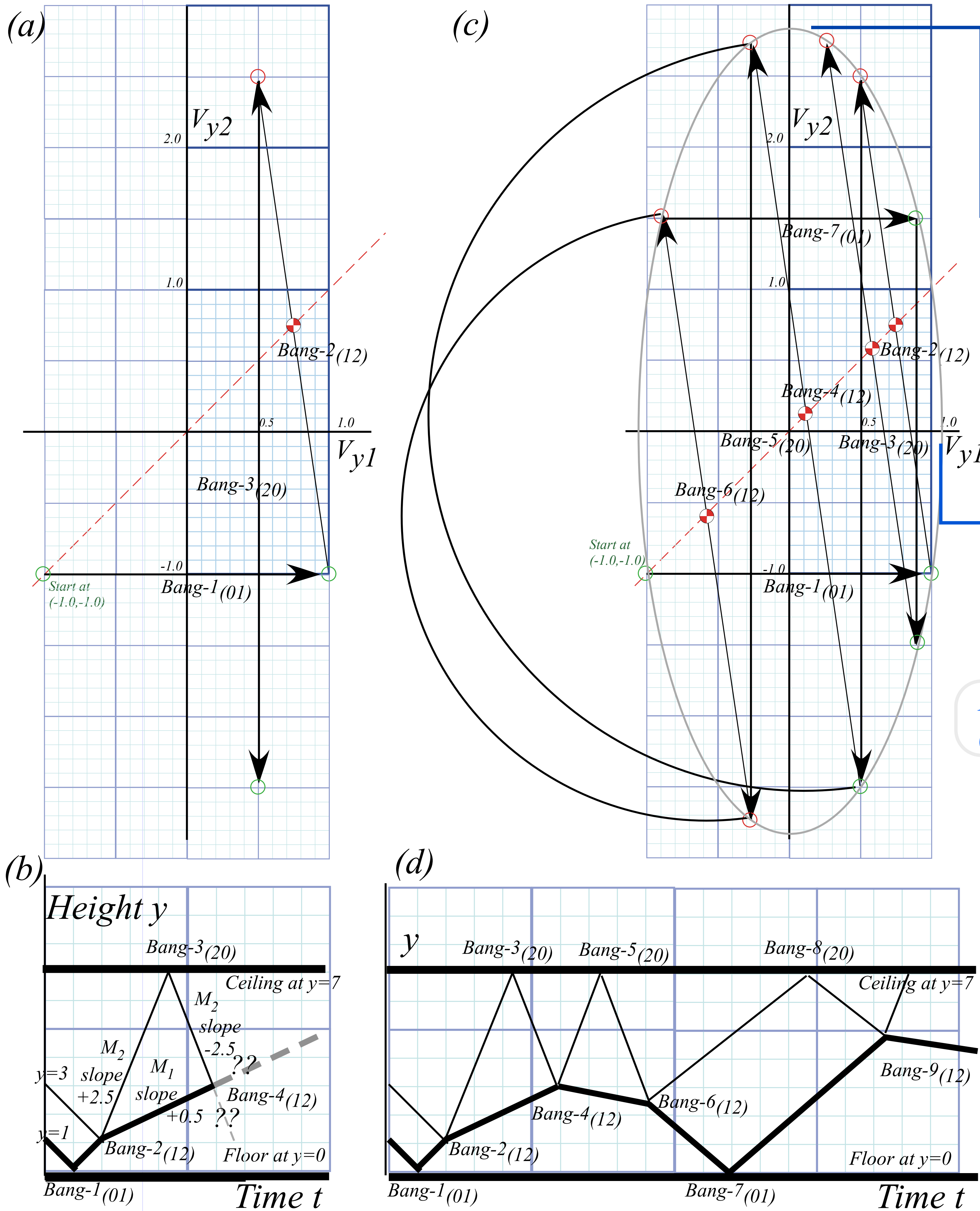
Ellipse radius 1

$$a_1 = \sqrt{2KE/M_1}$$

Ellipse radius 2

$$a_2 = \sqrt{2KE/M_1}$$

Geometric "Integration" (Converting Velocity data to Spacetime)



Kinetic Energy Ellipse

$$KE = \frac{1}{2}M_1V_1^2 + \frac{1}{2}M_2V_2^2 = \frac{7}{2} + \frac{1}{2} = 4$$

$$1 = \frac{V_1^2}{2KE/M_1} + \frac{V_2^2}{2KE/M_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Ellipse radius 1

$$a_1 = \sqrt{2KE/M_1} = \sqrt{2KE/7} = \sqrt{8/7} = 1.07$$

Ellipse radius 2

$$a_2 = \sqrt{2KE/M_1} = \sqrt{2KE/1} = \sqrt{8/1} = 2.83$$

Fig. 3.7 (Unit 1)

Geometric "Integration" (Converting Velocity data to Spacetime)

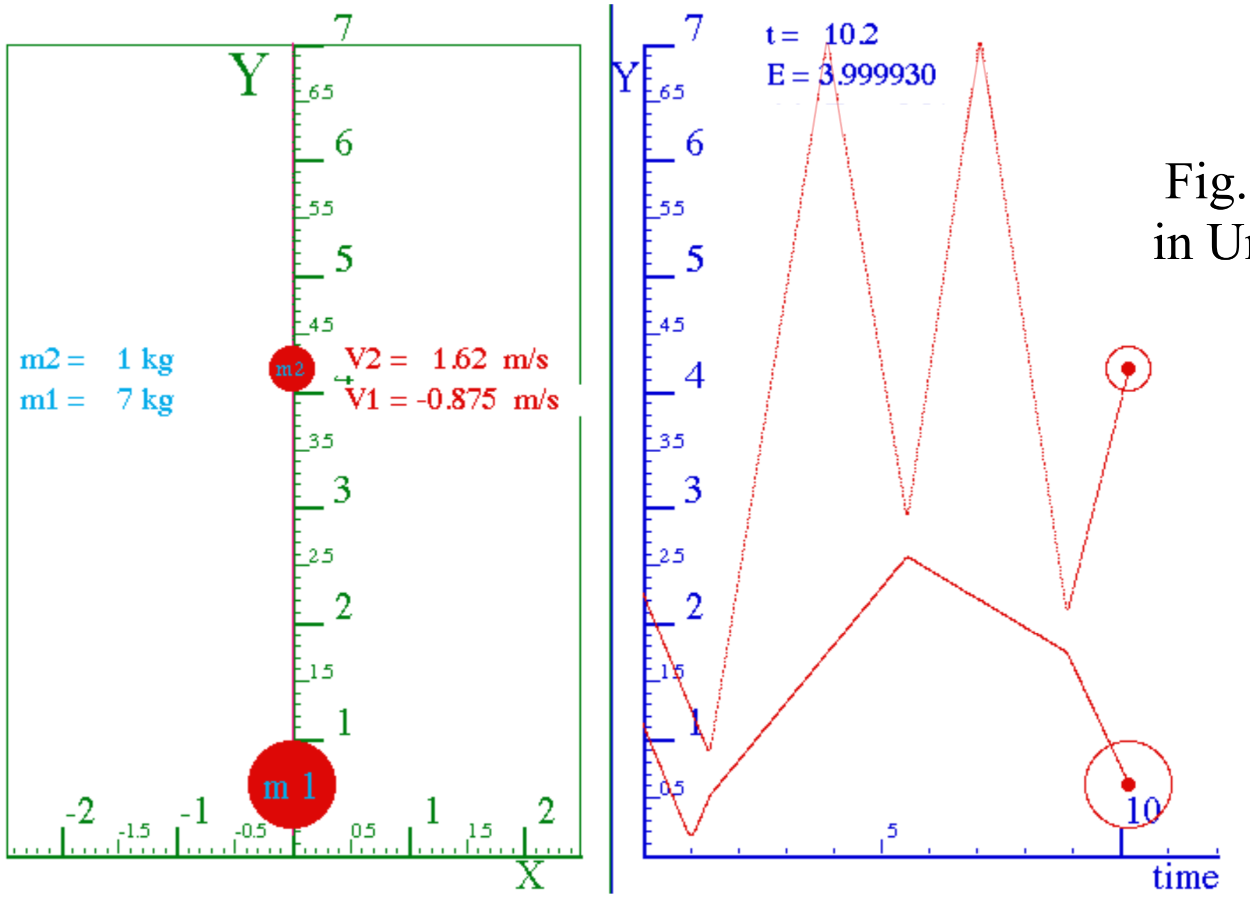


Fig. 4.8 in Unit 1

BounceIt Superball Collision Web Simulator: M_1=70, M_2=10 with Newtonian time plot

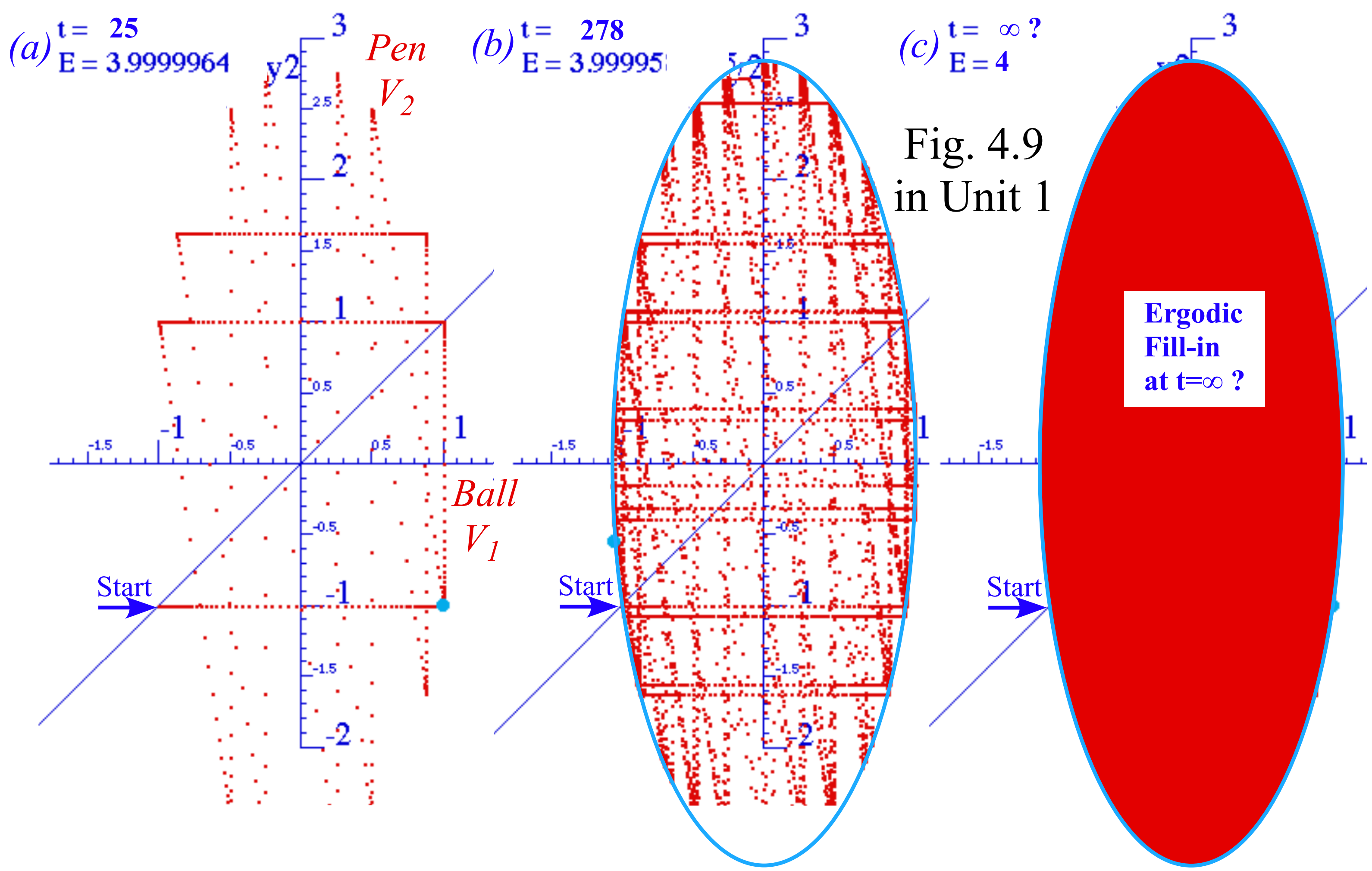


Fig. 4.9 in Unit 1

BounceIt Superball Collision Web Simulator: M_1=70, M_2=10 with V_2 vs V_1 plot

Geometry of X2 launcher bouncing in box

Independent Bounce Model (IBM)

Geometric optimization and range-of-motion calculation(t)

Integration of (V_1,V_2) data to space-time plots $(y_1(t),t)$ and $(y_2(t),t)$ plots

→ *Integration of (V_1,V_2) data to space-space plots (y_1, y_2)* *Examples $(M_1=7, M_2=1)$ and $(M_1=49, M_2=1)$*

Geometric "Integration" (Converting Velocity data to Space-space trajectory)

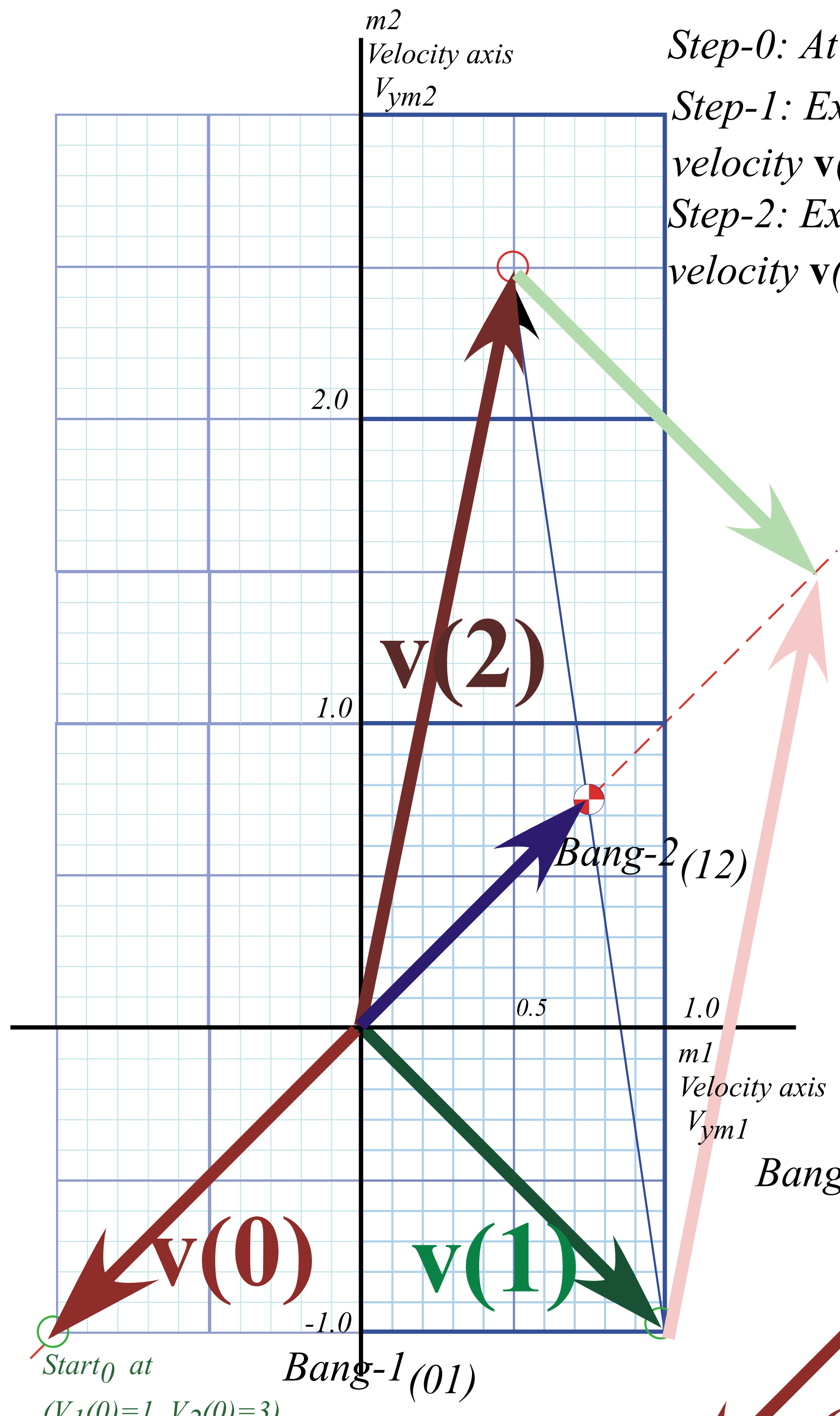
Fig. 3.8
(Unit 1)

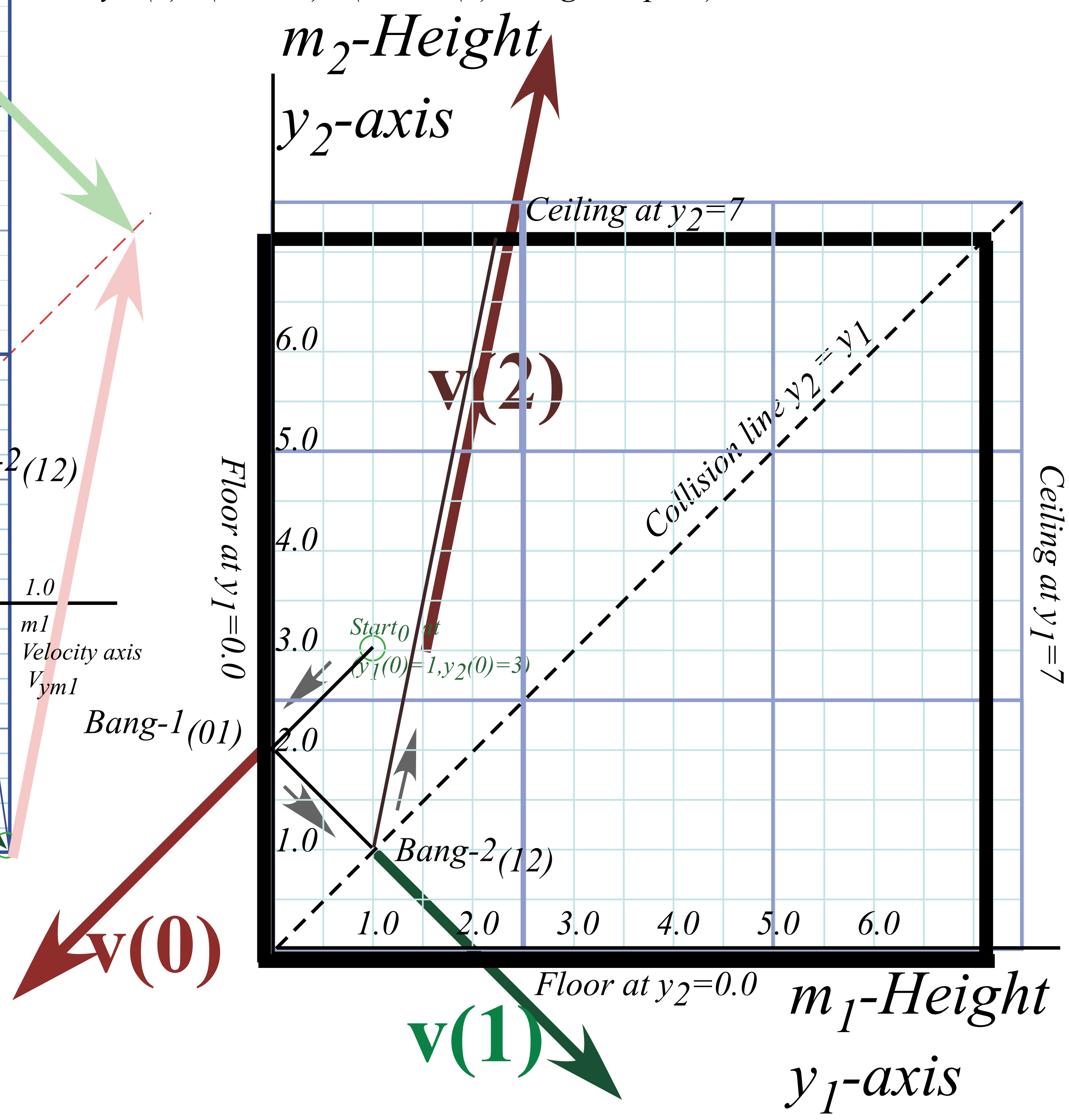
Ellipse radius 1	Ellipse radius 2
$a_1 = \sqrt{2KE/M_1}$	$a_2 = \sqrt{2KE/M_1}$
$= \sqrt{2KE/7}$	$= \sqrt{2KE/1}$
$= \sqrt{8/7}$	$= \sqrt{8/1}$
$= 1.07$	$= 2.83$

Step-0: At starting position $\mathbf{y}(0)=(1,3)$ draw initial velocity $\mathbf{v}(0)=(-1,-1)$ line.

Step-1: Extend $\mathbf{v}(0)$ line to floor point $\mathbf{y}(0)=(0,?)$ and draw Bang-1$_{(01)}$ velocity $\mathbf{v}(1)=(1,-1)$ line. (Find $\mathbf{v}(1)$ using V-V plot.)

Step-2: Extend $\mathbf{v}(1)$ line to collision point $\mathbf{y}(0)=(?,?)$ and draw Bang-2$_{(12)}$ velocity $\mathbf{v}(2)=(0.5,2.5)$. (Find $\mathbf{v}(2)$ using V-V plot.)





Geometric "Integration" (Converting Velocity data to Space-space trajectory)

Fig. 3.9
(Unit 1)

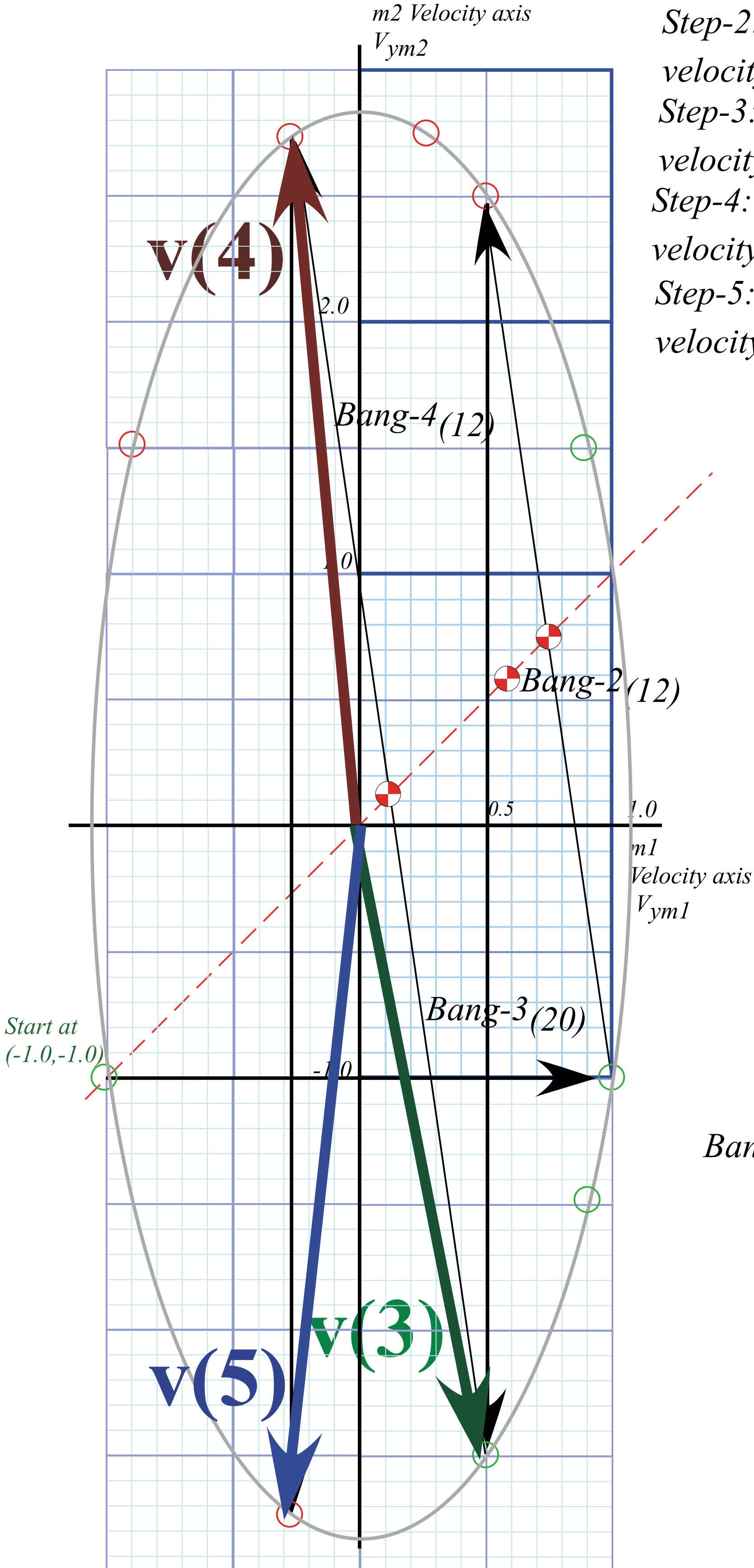
Step-2: Extend **v**(2) line to ceiling point **y**(3)=(?,7.1) and draw Bang-3$_{(20)}$ velocity **v**(3)=(1,-1) line. (Find **v**(3) using V-V plot.)
Step-3: Extend **v**(3) line to collision point **y**(4)=(?,?) and draw Bang-4$_{(12)}$ velocity **v**(4)=(0.5,2.5). (Find **v**(4) using V-V plot.)
Step-4: Extend **v**(4) line to ceiling point **y**(4)=(?,7.1) and draw Bang-5$_{(20)}$ velocity **v**(5)=(1,-1) line. (Find **v**(5) using V-V plot.)
Step-5: Extend **v**(5) line to collision point **y**(6)=(?,?) and draw Bang-6$_{(12)}$ velocity **v**(6)=(0.5,2.5). (Find **v**(6) using V-V plot.)

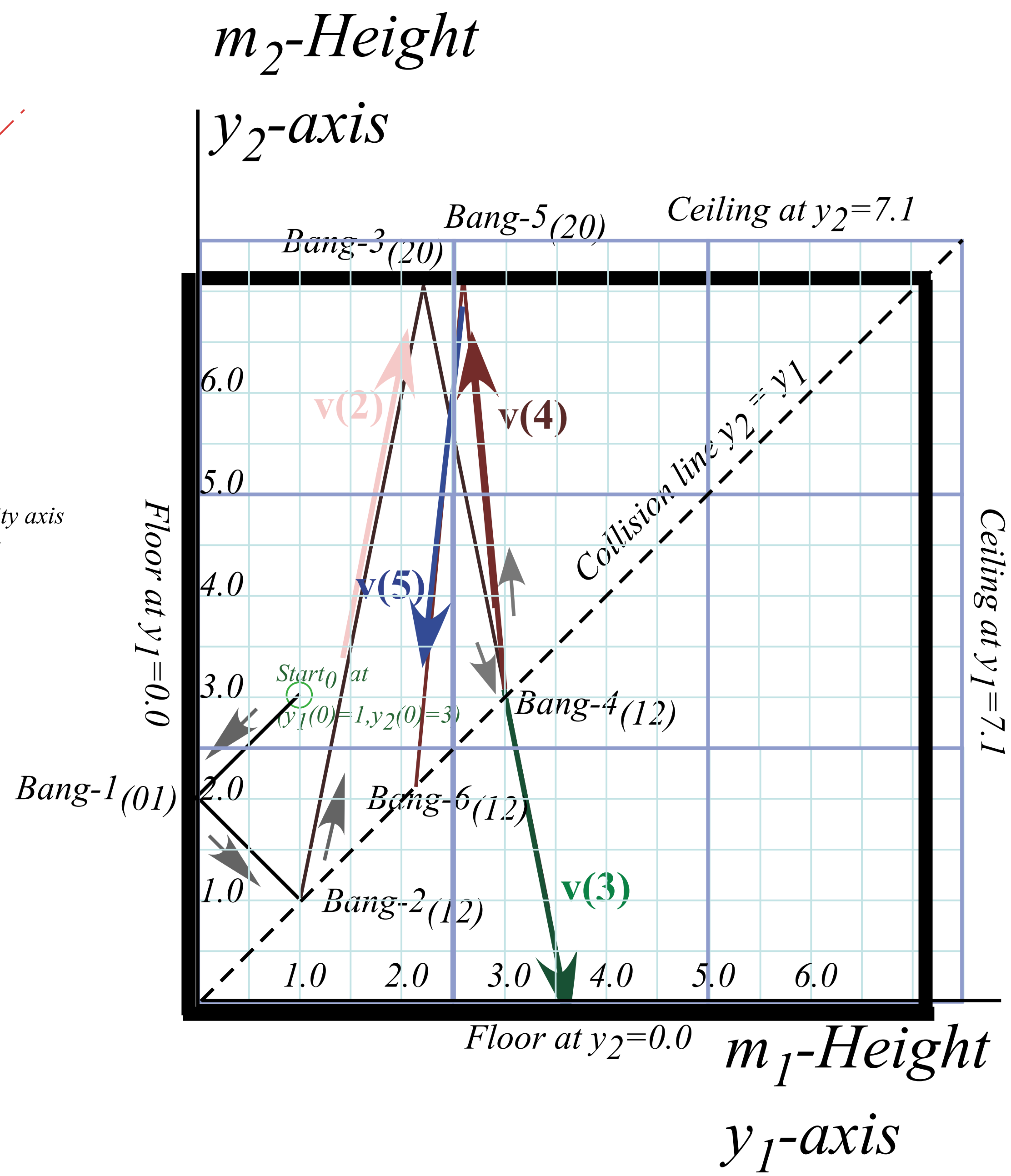
Ellipse radius 1

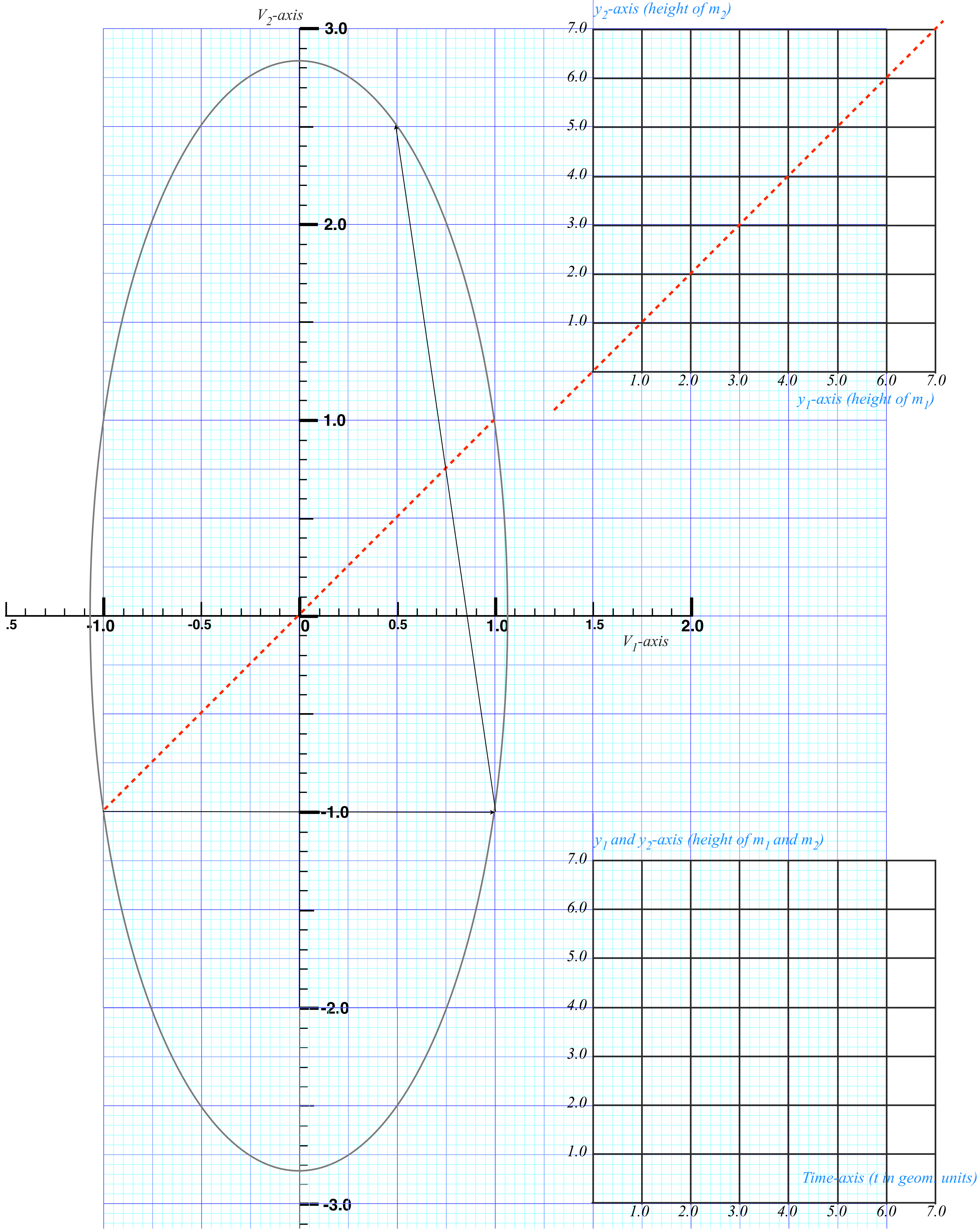
$$a_1 = \sqrt{2KE/M_1} = \sqrt{2KE/7} = \sqrt{8/7} = 1.07$$

Ellipse radius 2

$$a_2 = \sqrt{2KE/M_1} = \sqrt{2KE/1} = \sqrt{8/1} = 2.83$$







Ellipse radius 1

$$a_1 = \sqrt{2KE/M_1} = \sqrt{2KE/7} = \sqrt{8/7} = 1.07$$

Ellipse radius 2

$$a_2 = \sqrt{2KE/M_1} = \sqrt{2KE/1} = \sqrt{8/1} = 2.83$$

Example with masses: $m_1=49$ *and* $m_2=1$

Geometric "Integration" (Converting Velocity data to Space-time trajectory)

Example with masses: m_1=49 and m_2=1

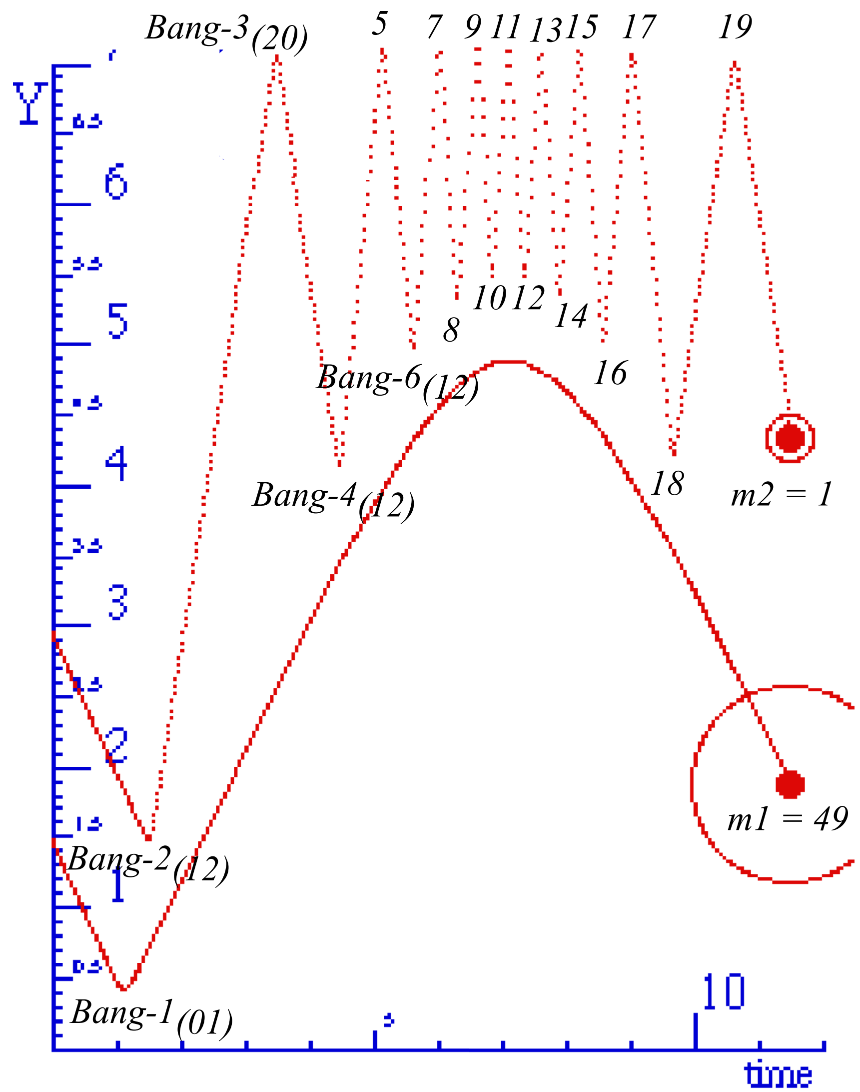
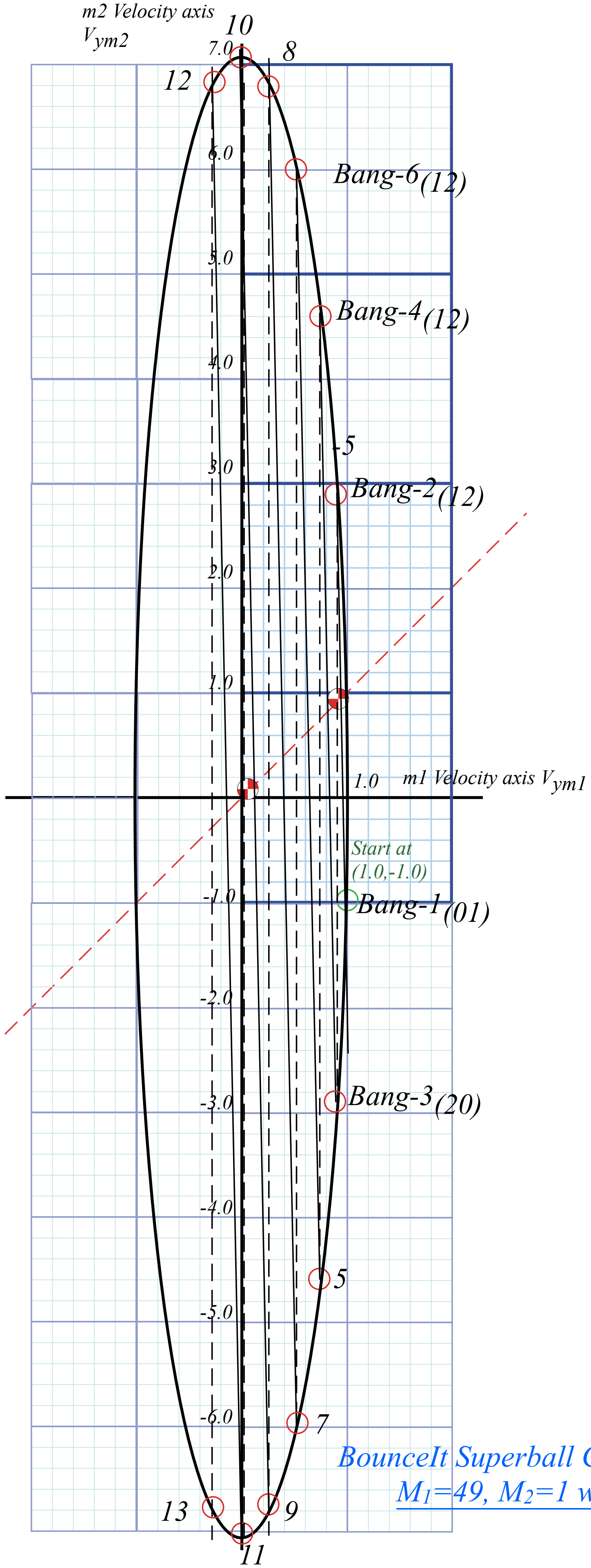


Fig. 4.1 (Unit 1)

BounceIt Superball Collision Web Simulator:
M_1=49, M_2=1 with Newtonian time plot

BounceIt Superball Collision Web Simulator:
M_1=49, M_2=1 with V_2 vs V_1 plot

Geometric "Integration" (Converting Velocity data to Space-time trajectory)



Example with masses: m_1=49 and m_2=1

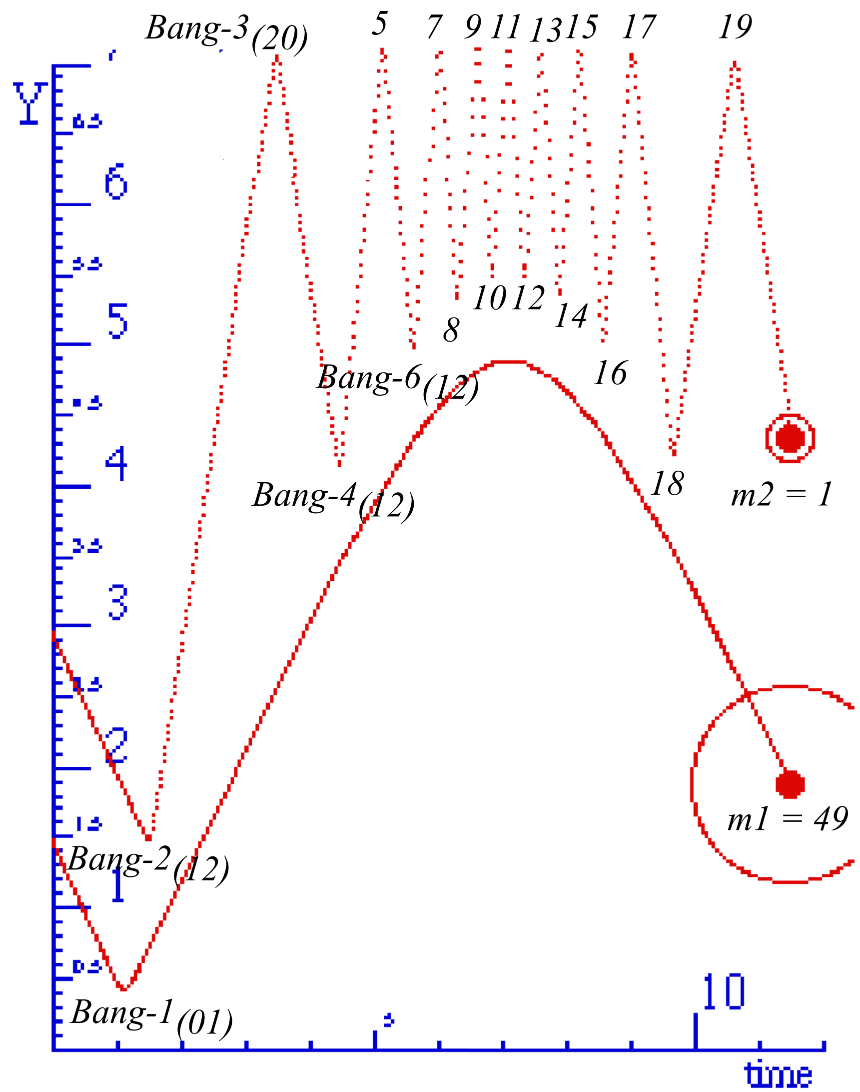
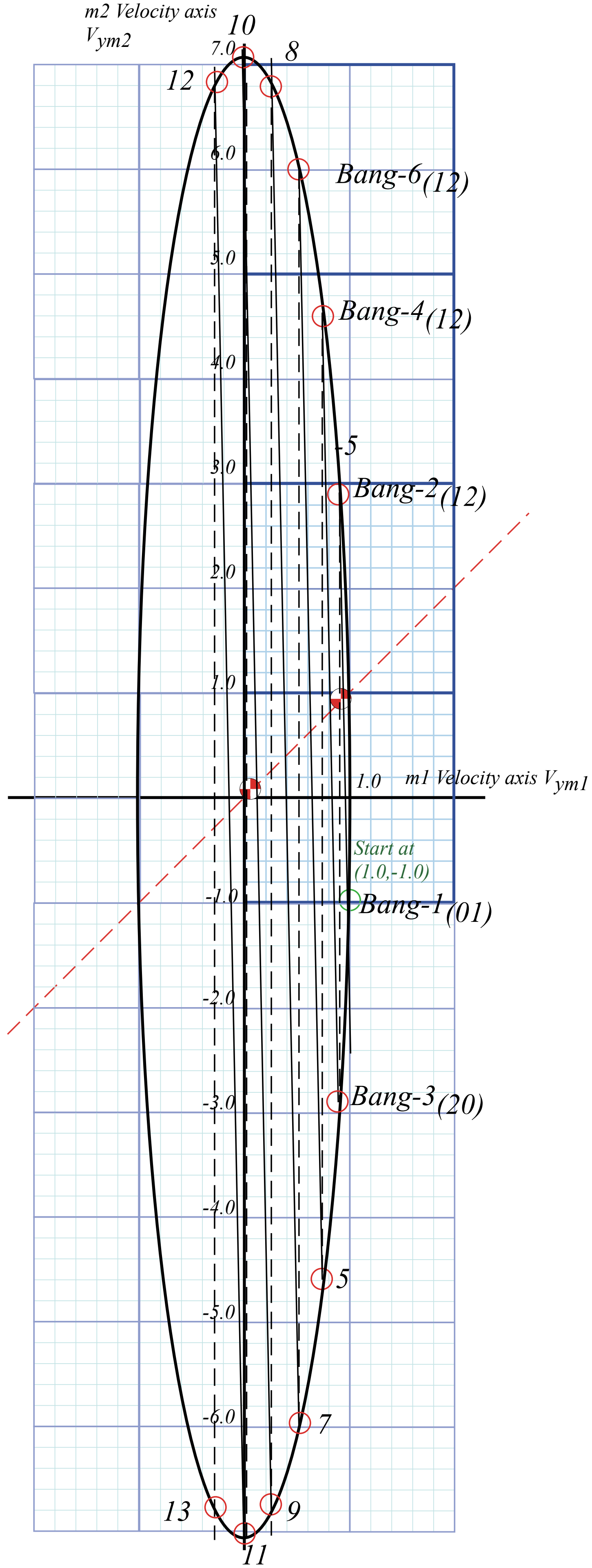


Fig. 4.1 (Unit 1)

BounceIt Superball Collision Web Simulator: M_1=49, M_2=1 with Newtonian time plot

BounceIt Superball Collision Web Simulator: M_1=49, M_2=1 with V_2 vs V_1 plot

Geometric "Integration" (Converting Velocity data to Space-time trajectory)



Example with masses: m_1=49 and m_2=1

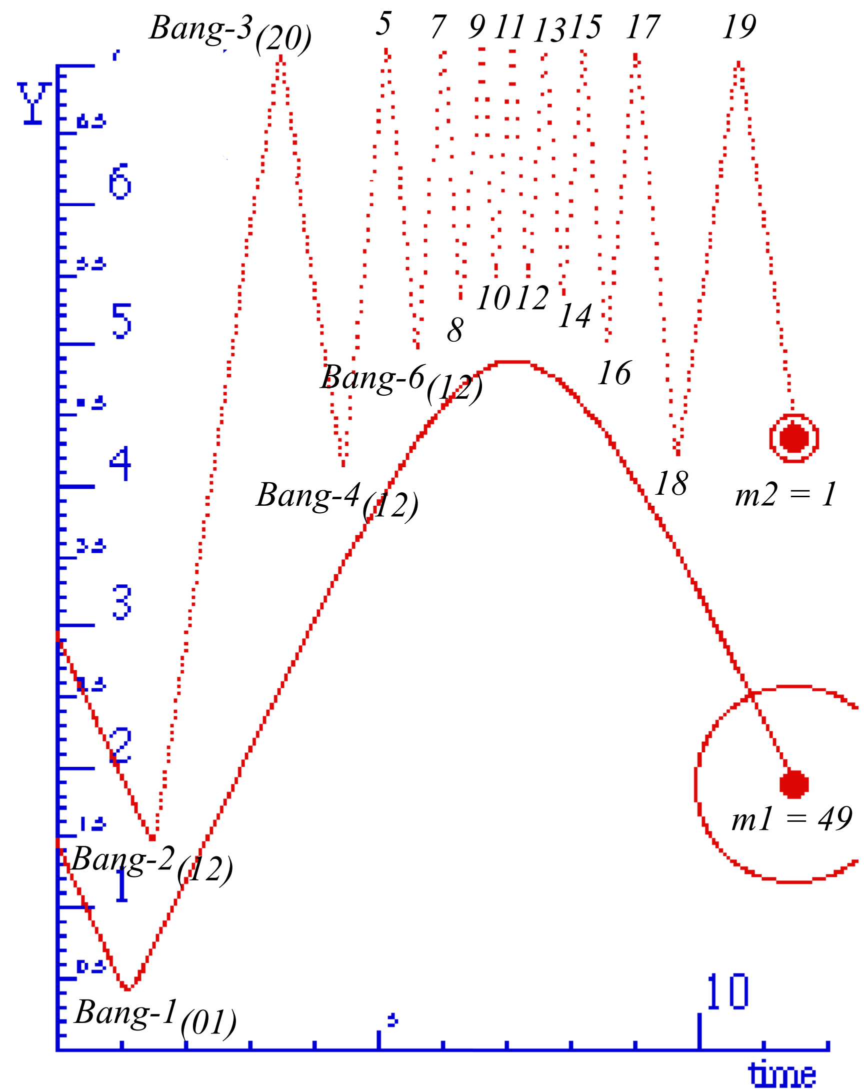


Fig. 4.1
(Unit 1)

Kinetic Energy Ellipse

$$KE = \frac{1}{2}m_1 V_1^2 + \frac{1}{2}m_2 V_2^2 = \frac{49}{2} + \frac{1}{2} = 25$$

$$1 = \frac{V_1^2}{2KE/m_1} + \frac{V_2^2}{2KE/m_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Ellipse radius 1

$$a_1 = \sqrt{2KE/M_1} = \sqrt{2KE/49} = \sqrt{50/49} = 1.01$$

Ellipse radius 2

$$a_2 = \sqrt{2KE/m_2} = \sqrt{2KE/1} = \sqrt{50/1} = 7.07$$

Multiple collisions calculated by matrix operator products

Matrix or tensor algebra of 1-D 2-body collisions

"Mass-bang" matrix **M**, *"Floor-bang" matrix* **F**, *"Ceiling-bang" matrix* **C**.

Geometry and algebra of "ellipse-Rotation" group product: **R**= **C•M**

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $$V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} =$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix}$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \bullet \mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \bullet \mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

Quiz question about linear solution

Linear formula $\mathbf{v}^{FIN} = \mathbf{M} \bullet \mathbf{v}^{IN}$ gives just *one* solution to *quadratic* collision equations.

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} \frac{m_1 - m_2}{m_1 + m_2} & \frac{2m_2}{m_1 + m_2} \\ \frac{2m_1}{m_1 + m_2} & \frac{m_2 - m_1}{m_1 + m_2} \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \bullet \mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

Quiz question about linear solution

Linear formula $\mathbf{v}^{FIN} = \mathbf{M} \bullet \mathbf{v}^{IN}$ gives just *one* solution to *quadratic* collision equations.

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} \frac{m_1 - m_2}{m_1 + m_2} & \frac{2m_2}{m_1 + m_2} \\ \frac{2m_1}{m_1 + m_2} & \frac{m_2 - m_1}{m_1 + m_2} \end{pmatrix}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$$

Q: What is the *second* solution and to what simple process would it correspond?

Example with friction

Multiple collisions calculated by matrix operator products

Matrix or tensor algebra of 1-D 2-body collisions

"Mass-bang" matrix **M**, *"Floor-bang" matrix* **F**, *"Ceiling-bang" matrix* **C**.

Geometry and algebra of "ellipse-Rotation" group product: **R**= **C**•**M**

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \bullet \mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

Matrix operations include...

Floor-bang **F** of m_1:

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \bullet \mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

Matrix operations include...

Floor-bang **F** of m_1:

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Ceiling-bang **C** of m_2:

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \bullet \mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

Matrix operations include...

Floor-bang **F** of m_1:

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Mass-bang **M** of m_1 and m_2 :

$$\mathbf{M} = \begin{pmatrix} \frac{m_1 - m_2}{m_1 + m_2} & \frac{2m_2}{m_1 + m_2} \\ \frac{2m_1}{m_1 + m_2} & \frac{m_2 - m_1}{m_1 + m_2} \end{pmatrix}$$

Ceiling-bang **C** of m_2:

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM}=\frac{V^{FIN}+V^{IN}}{2}=\frac{m_1 v_1+m_2 v_2}{m_1+m_2}$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix}=\begin{pmatrix} 2V^{COM}-v_1^{IN} \\ 2V^{COM}-v_2^{IN} \end{pmatrix}=\begin{pmatrix} 2\frac{m_1 v_1^{IN}+m_2 v_2^{IN}}{m_1+m_2}-v_1^{IN} \\ 2\frac{m_1 v_1^{IN}+m_2 v_2^{IN}}{m_1+m_2}-v_2^{IN} \end{pmatrix}=\frac{\begin{pmatrix} m_1 v_1^{IN}-m_2 v_1^{IN}+2m_2 v_2^{IN} \\ 2m_1 v_1^{IN}+m_2 v_2^{IN}-m_1 v_2^{IN} \end{pmatrix}}{m_1+m_2}=\frac{\begin{pmatrix} m_1-m_2 & 2m_2 \\ 2m_1 & m_2-m_1 \end{pmatrix}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1+m_2}$$

Finally as a matrix operation: $\mathbf{v}^{FIN}=\mathbf{M}\bullet\mathbf{v}^{IN}$...

Matrix operations include...

Floor-bang **F** of m_1:

$$\mathbf{F}=\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Mass-bang **M** of m_1 and m_2 :

$$\mathbf{M}=\begin{pmatrix} \frac{m_1-m_2}{m_1+m_2} & \frac{2m_2}{m_1+m_2} \\ \frac{2m_1}{m_1+m_2} & \frac{m_2-m_1}{m_1+m_2} \end{pmatrix}$$

Ceiling-bang **C** of m_2:

$$\mathbf{C}=\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Let: $m_1=49$ *and* $m_2=1$

$$\mathbf{M}=\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}$$

Multiple collisions calculated by matrix operator products

Matrix or tensor algebra of 1-D 2-body collisions

"Mass-bang" matrix **M**, *"Floor-bang" matrix* **F**, *"Ceiling-bang" matrix* **C**.

Geometry and algebra of "ellipse-Rotation" group product: **R**= **C•M**

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN}+V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1+m_2}$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M}\bullet\mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1+m_2} - v_1^{IN} \\ 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1+m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1+m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1+m_2}$$

Matrix operations include...

Floor-bang **F** of m_1:

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Mass-bang **M** of m_1 and m_2 :

$$\mathbf{M} = \begin{pmatrix} \frac{m_1 - m_2}{m_1+m_2} & \frac{2m_2}{m_1+m_2} \\ \frac{2m_1}{m_1+m_2} & \frac{m_2 - m_1}{m_1+m_2} \end{pmatrix}$$

Ceiling-bang **C** of m_2:

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Let: $m_1=49$ and $m_2=1$

$$\mathbf{M} = \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}$$

Define *"ellipse-Rotation"* **R** as *group product*: $\mathbf{R} = \mathbf{C}\bullet\mathbf{M} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}$

$$\left|FIN^{9}\right\rangle = \mathbf{C} \bullet \mathbf{M} \bullet \mathbf{C} \bullet \mathbf{M} \bullet \mathbf{C} \bullet \mathbf{M} \bullet \mathbf{C} \bullet \mathbf{M} \bullet \mathbf{F} \left|IN^{0}\right\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \begin{pmatrix} v_1^{IN} = -1 \\ v_2^{IN} = -1 \end{pmatrix}_{(\text{INITIAL }(0))}$$

$$\left|FIN^9\right\rangle = \mathbf{C} \bullet \mathbf{M} \bullet \mathbf{C} \bullet \mathbf{M} \bullet \mathbf{C} \bullet \mathbf{M} \bullet \mathbf{C} \bullet \mathbf{M} \bullet \mathbf{F} \left|IN^0\right\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}} \cdot \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \begin{pmatrix} v_1^{IN} = -1 \\ v_2^{IN} = -1 \end{pmatrix}_{\text{(INITIAL (0))}}$$

$$\left|FIN^9\right\rangle = \mathbf{R} \bullet \mathbf{R} \bullet \mathbf{R} \bullet \mathbf{R} \bullet \mathbf{F} \left|IN^0\right\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} v_1 = 1 \\ v_2 = -1 \end{pmatrix}_{\text{(after Bang-1)}}$$

"ellipse-Rotation" group product: **R= C•M**

$$|FIN^9\rangle = \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{F}\, |IN^0\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}} \cdot \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \begin{pmatrix} v_1^{IN} = -1 \\ v_2^{IN} = -1 \end{pmatrix}_{\text{(INITIAL (0))}}$$

$$|FIN^9\rangle = \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{F}\, |IN^0\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} v_1 = 1 \\ v_2 = -1 \end{pmatrix}_{\text{(after Bang-1)}}$$

$$= \begin{pmatrix} v_1 = 0.2925 \\ v_2 = -6.768 \end{pmatrix}_{\text{(after Bang-9)}}$$

"ellipse-Rotation" group product: **R= C•M**

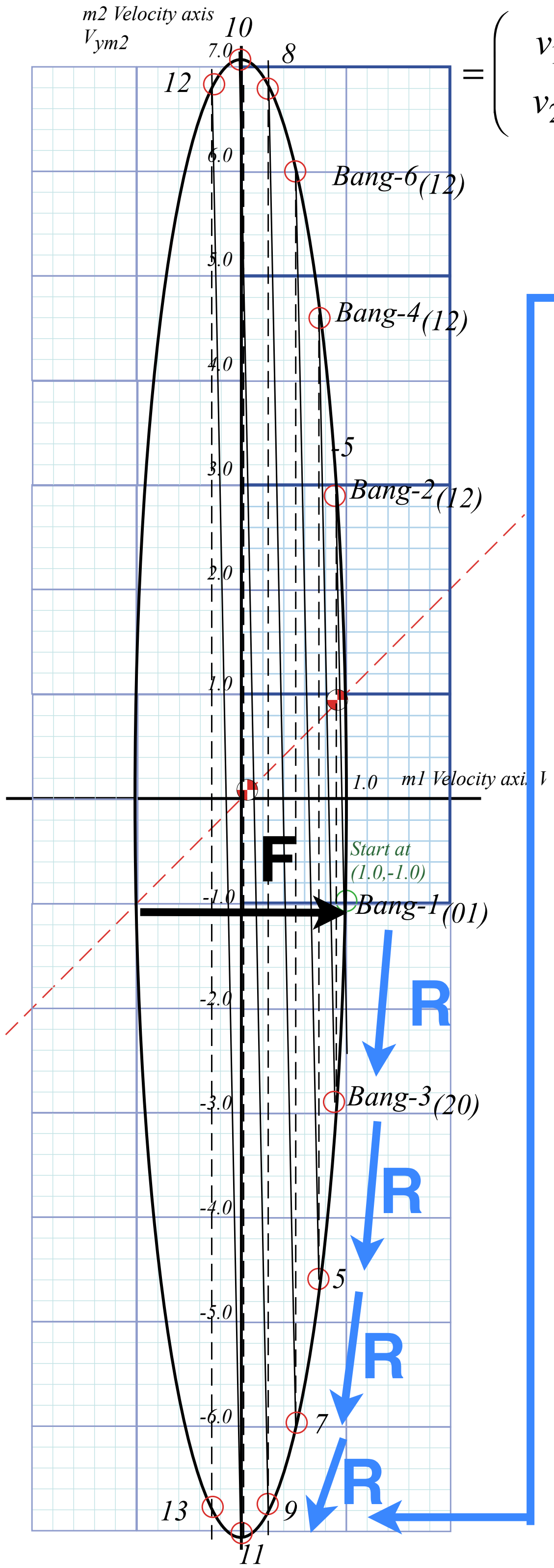
$$|FIN^9\rangle = \mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{F}\,|IN^0\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}\cdot\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}\cdot\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}\cdot\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}\cdot\begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix}\begin{pmatrix} v_1^{IN}=-1 \\ v_2^{IN}=-1 \end{pmatrix}_{\text{(INITIAL (0))}}$$

$$|FIN^9\rangle = \mathbf{R}\cdot\mathbf{R}\cdot\mathbf{R}\cdot\mathbf{R}\cdot\mathbf{F}\,|IN^0\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} v_1=1 \\ v_2=-1 \end{pmatrix}_{\text{(after Bang-1)}}$$

$$= \begin{pmatrix} v_1 = 0.2925 \\ v_2 = -6.768 \end{pmatrix}_{\text{(after Bang-9)}}$$



"ellipse-Rotation" group product: **R**= **C**·**M**

Collisions for mass ratio m_1:m_2= 49:1

Fig. 4.1a-b (revised)

$$\left|FIN^9\right\rangle = \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{F} \left|IN^0\right\rangle$$

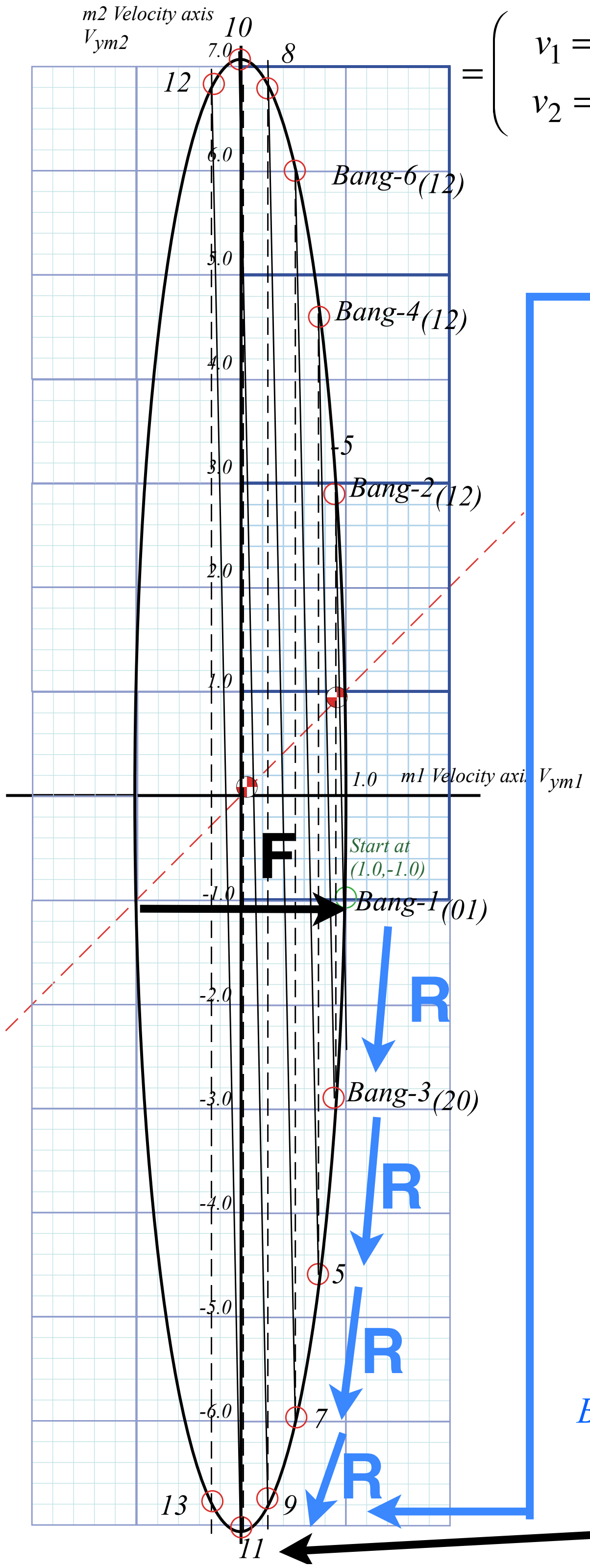
$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}} \cdot \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \begin{pmatrix} v_1^{IN} = -1 \\ v_2^{IN} = -1 \end{pmatrix}_{\text{(INITIAL (0))}}$$

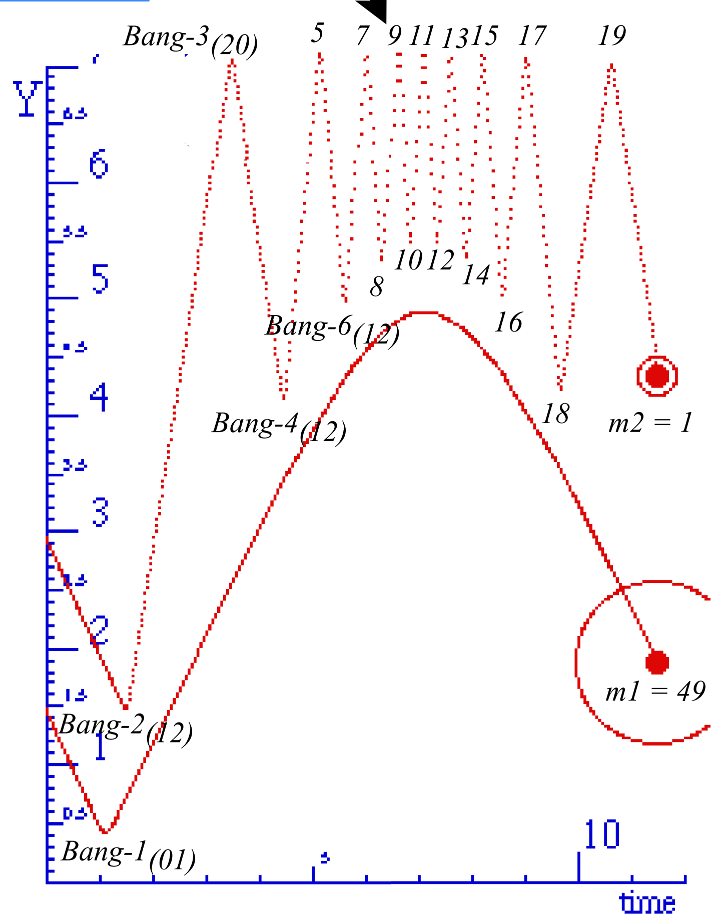
$$\left|FIN^9\right\rangle = \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{F}\left|IN^0\right\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} v_1 = 1 \\ v_2 = -1 \end{pmatrix}_{\text{(after Bang-1)}}$$

$$= \begin{pmatrix} v_1 = 0.2925 \\ v_2 = -6.768 \end{pmatrix}_{\text{(after Bang-9)}}$$

"ellipse-Rotation" group product: **R**= **C**·**M**





$$\begin{pmatrix} v_1^{FIN-11} \\ v_2^{FIN-11} \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix}$$

$$= \begin{pmatrix} v_1 = 0.0100 \\ v_2 = -7.071 \end{pmatrix}_{\text{(after Bang-11)}}$$

Collisions for mass ratio $m_1{:}m_2 = 49{:}1$

<<Under Construction>>
Matrix Collision Web Simulator: M_1=49, M_2=1 V_2 vs V_1 plot

BounceIt Superball Collision Web Simulator: M_1=49, M_2=1 with Newtonian time plot

BounceIt Superball Collision Web Simulator: M_1=49, M_2=1 with V_2 vs V_1 plot

Fig. 4.1a-b
(revised)

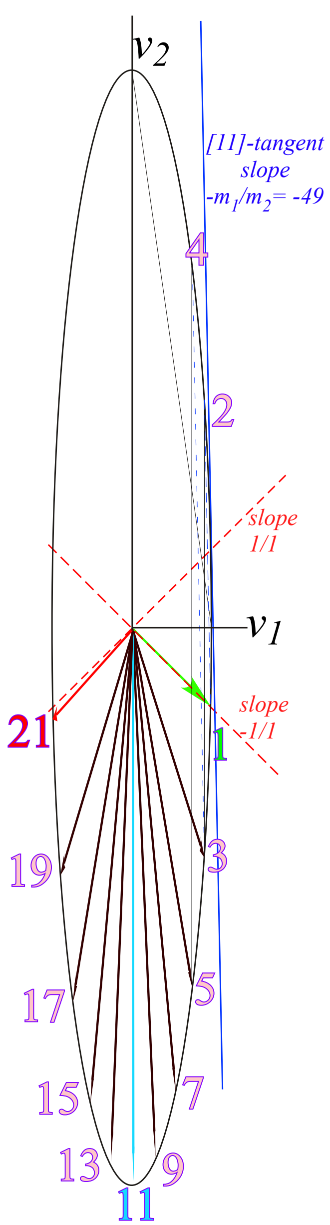
Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $\mathsf{V}_1 = v_1 \cdot \sqrt{m_1}$, $\mathsf{V}_2 = v_2 \cdot \sqrt{m_1}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} \mathsf{V}_1^2 + \frac{1}{2} \mathsf{V}_2^2$



Collisions for mass ratio $m_1 : m_2 = 49:1$

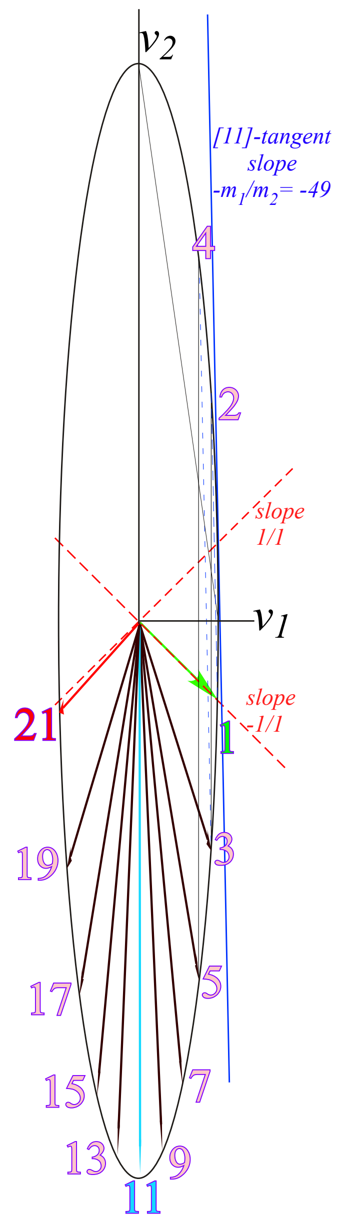
Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_1}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$

$$\begin{pmatrix} v_1^{FIN_1} \\ v_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

becomes:

$$\begin{pmatrix} V_1^{FIN_1} / \sqrt{m_1} \\ V_2^{FIN_1} / \sqrt{m_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 / \sqrt{m_1} \\ V_2 / \sqrt{m_2} \end{pmatrix}$$



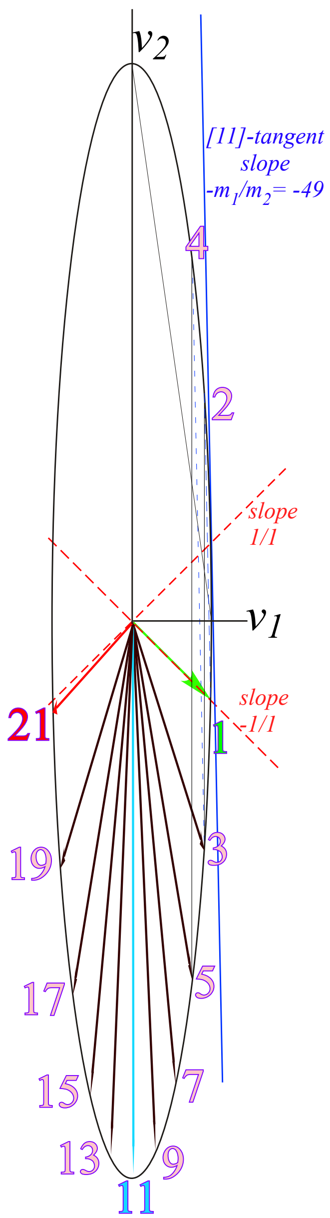
Collisions for mass ratio m_1:m_2= 49:1

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $\mathsf{V}_1 = v_1 \cdot \sqrt{m_1}$, $\mathsf{V}_2 = v_2 \cdot \sqrt{m_1}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} \mathsf{V}_1^2 + \frac{1}{2} \mathsf{V}_2^2$

$$\begin{pmatrix} v_1^{FIN_1} \\ v_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad \text{becomes:} \quad \begin{pmatrix} \mathsf{V}_1^{FIN_1} / \sqrt{m_1} \\ \mathsf{V}_2^{FIN_1} / \sqrt{m_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} \mathsf{V}_1 / \sqrt{m_1} \\ \mathsf{V}_2 / \sqrt{m_2} \end{pmatrix}$$

$$\text{or:} \begin{pmatrix} \mathsf{V}_1^{FIN_1} \\ \mathsf{V}_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ 2\sqrt{m_1 m_2} & m_2 - m_1 \end{pmatrix} \begin{pmatrix} \mathsf{V}_1 \\ \mathsf{V}_2 \end{pmatrix} = \mathsf{M} \bullet \vec{\mathsf{V}} \ , \quad \text{or:} \begin{pmatrix} \mathsf{V}_1^{FIN_2} \\ \mathsf{V}_2^{FIN_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ -2\sqrt{m_1 m_2} & m_1 - m_2 \end{pmatrix} \begin{pmatrix} \mathsf{V}_1 \\ \mathsf{V}_2 \end{pmatrix} = \mathsf{C} \bullet \mathsf{M} \bullet \vec{\mathsf{V}}$$



Collisions for mass ratio m_1:m_2= 49:1

Ellipse rescaling geometry and reflection symmetry analysis

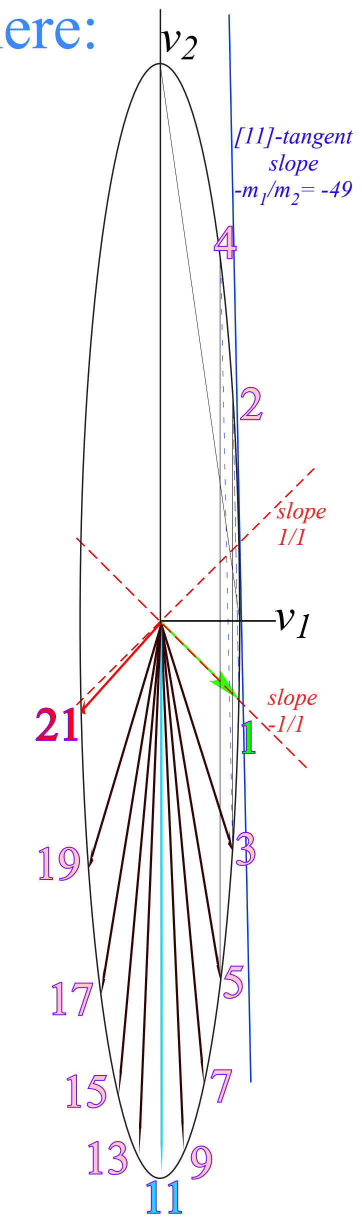
Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_1}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$

$$\begin{pmatrix} v_1^{FIN_1} \\ v_2^{FIN_1} \end{pmatrix} = \frac{1}{M}\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$ becomes: $$\begin{pmatrix} V_1^{FIN_1}/\sqrt{m_1} \\ V_2^{FIN_1}/\sqrt{m_2} \end{pmatrix} = \frac{1}{M}\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix}\begin{pmatrix} V_1/\sqrt{m_1} \\ V_2/\sqrt{m_2} \end{pmatrix}$$

or: $$\begin{pmatrix} V_1^{FIN_1} \\ V_2^{FIN_1} \end{pmatrix} = \frac{1}{M}\begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ 2\sqrt{m_1 m_2} & m_2 - m_1 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \mathbf{M}\bullet\vec{V}$$, or: $$\begin{pmatrix} V_1^{FIN_2} \\ V_2^{FIN_2} \end{pmatrix} = \frac{1}{M}\begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ -2\sqrt{m_1 m_2} & m_1 - m_2 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \mathbf{C}\bullet\mathbf{M}\bullet\vec{V}$$

Then collisions become ***reflections*** $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ and double-collisions become ***rotations*** $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

where: $\cos\theta \equiv \left(\frac{m_1 - m_2}{m_1 + m_2}\right)$ and: $\sin\theta \equiv \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2}\right)$ with: $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 + \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2}\right)^2 = 1$



Collisions for mass ratio m_1:m_2= 49:1

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $\mathsf{V}_1 = v_1 \cdot \sqrt{m_1}$, $\mathsf{V}_2 = v_2 \cdot \sqrt{m_1}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} \mathsf{V}_1^2 + \frac{1}{2} \mathsf{V}_2^2$

$$\begin{pmatrix} v_1^{FIN_1} \\ v_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

becomes:

$$\begin{pmatrix} \mathsf{V}_1^{FIN_1} / \sqrt{m_1} \\ \mathsf{V}_2^{FIN_1} / \sqrt{m_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} \mathsf{V}_1 / \sqrt{m_1} \\ \mathsf{V}_2 / \sqrt{m_2} \end{pmatrix}$$

or:

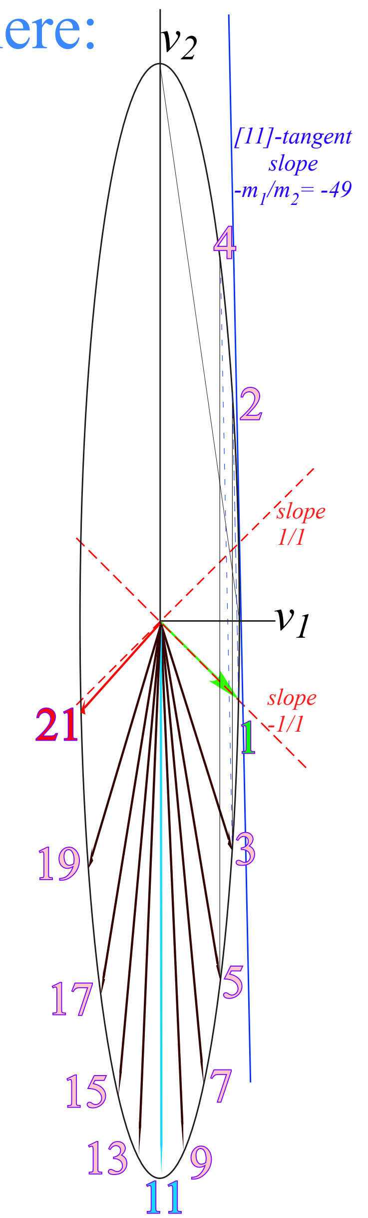
$$\begin{pmatrix} \mathsf{V}_1^{FIN_1} \\ \mathsf{V}_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ 2\sqrt{m_1 m_2} & m_2 - m_1 \end{pmatrix} \begin{pmatrix} \mathsf{V}_1 \\ \mathsf{V}_2 \end{pmatrix} = \mathsf{M} \bullet \vec{\mathsf{V}}$$

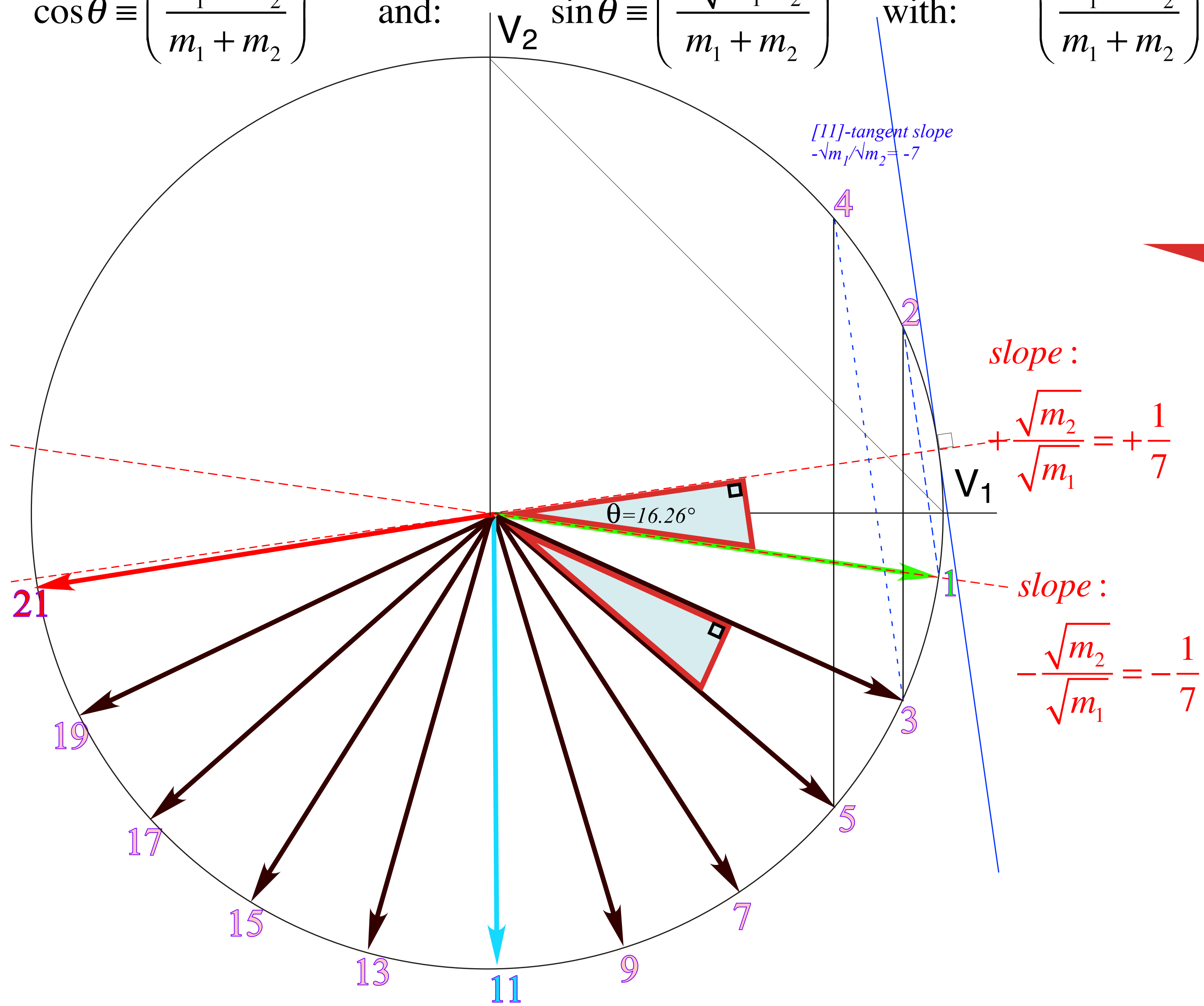
or:

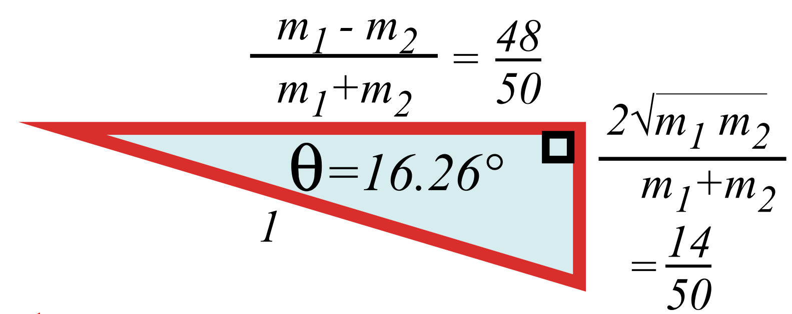
$$\begin{pmatrix} \mathsf{V}_1^{FIN_2} \\ \mathsf{V}_2^{FIN_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ -2\sqrt{m_1 m_2} & m_1 - m_2 \end{pmatrix} \begin{pmatrix} \mathsf{V}_1 \\ \mathsf{V}_2 \end{pmatrix} = \mathsf{C} \bullet \mathsf{M} \bullet \vec{\mathsf{V}}$$

Then collisions become ***reflections*** $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ and double-collisions become ***rotations*** $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

where: $\cos\theta \equiv \left(\frac{m_1 - m_2}{m_1 + m_2} \right)$ and: $\sin\theta \equiv \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right)$ with: $\left(\frac{m_1 - m_2}{m_1 + m_2} \right)^2 + \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right)^2 = 1$







Collisions for mass ratio $m_1 : m_2 = 49:1$

Fig. 4.2a-c

(revised)

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_1}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$

$$\begin{pmatrix} v_1^{FIN_1} \\ v_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

becomes:

$$\begin{pmatrix} V_1^{FIN_1} / \sqrt{m_1} \\ V_2^{FIN_1} / \sqrt{m_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 / \sqrt{m_1} \\ V_2 / \sqrt{m_2} \end{pmatrix}$$

or:

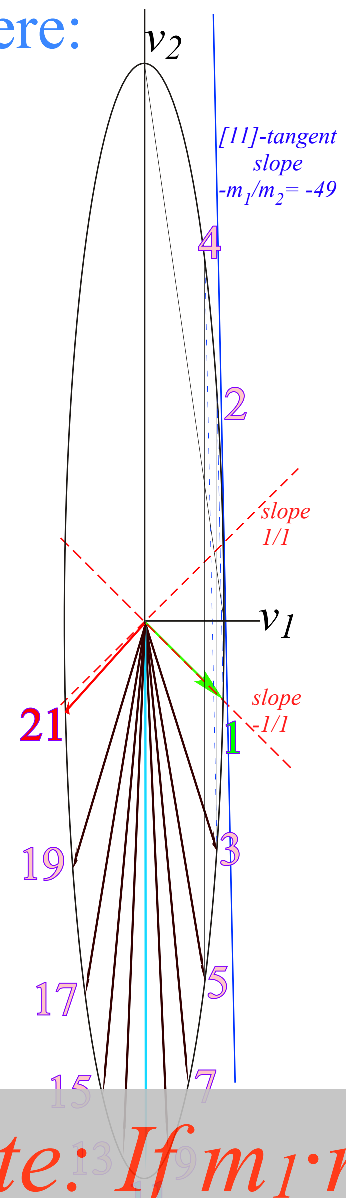
$$\begin{pmatrix} V_1^{FIN_1} \\ V_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ 2\sqrt{m_1 m_2} & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \mathbf{M} \bullet \vec{V}$$

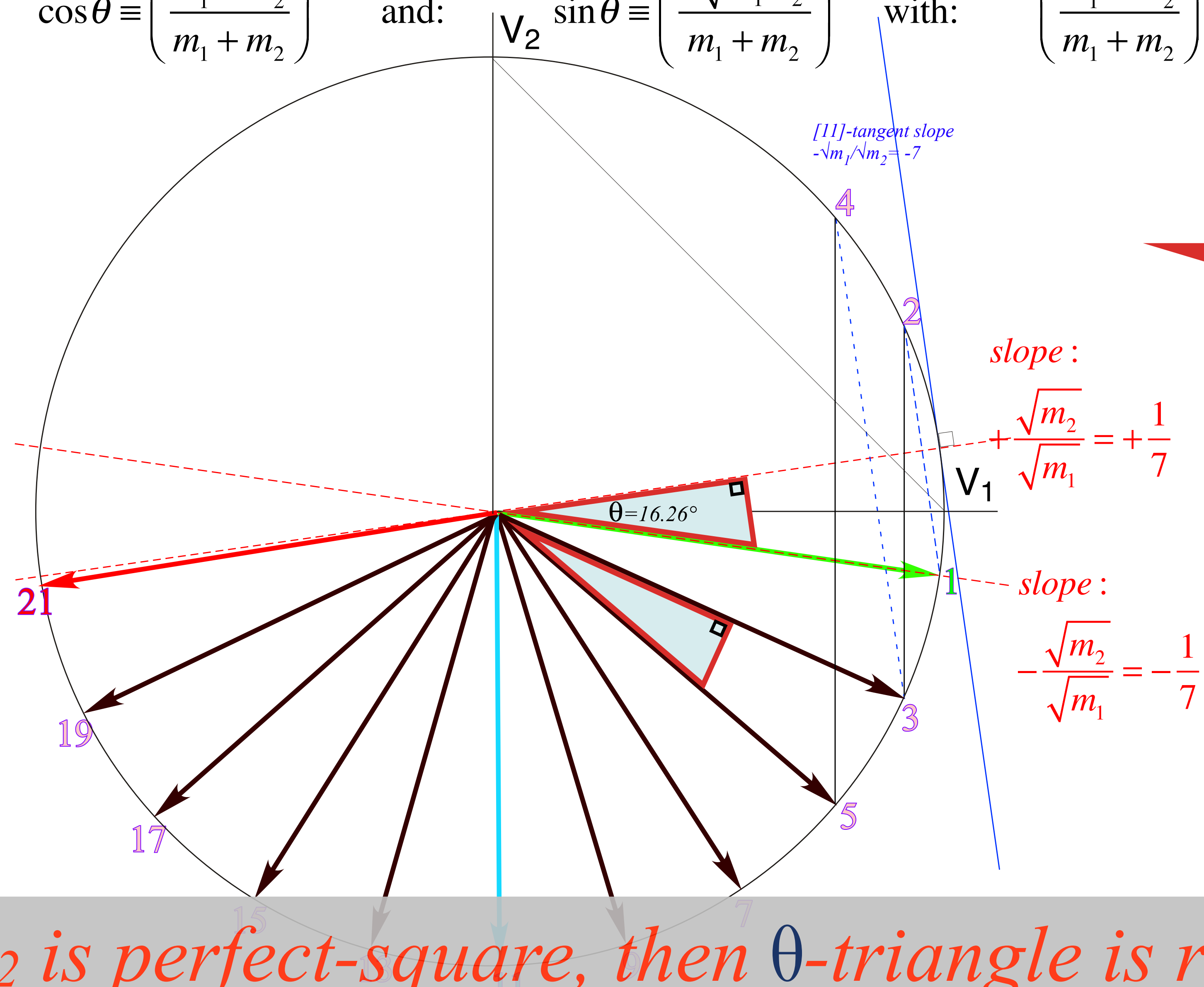
or:

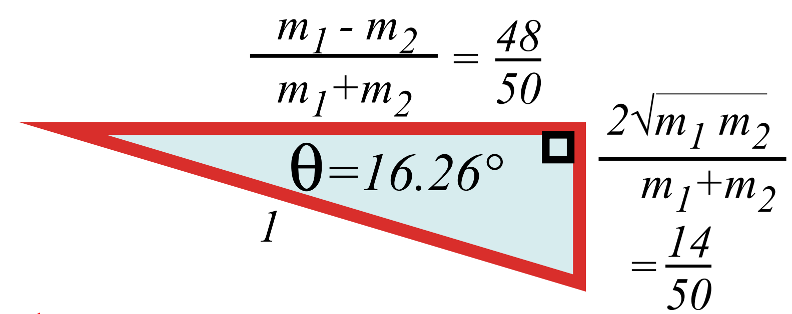
$$\begin{pmatrix} V_1^{FIN_2} \\ V_2^{FIN_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ -2\sqrt{m_1 m_2} & m_1 - m_2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \mathbf{C} \bullet \mathbf{M} \bullet \vec{V}$$

Then collisions become ***reflections*** $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ and double-collisions become ***rotations*** $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

where: $\cos\theta \equiv \left(\frac{m_1 - m_2}{m_1 + m_2} \right)$ and: $\sin\theta \equiv \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right)$ with: $\left(\frac{m_1 - m_2}{m_1 + m_2} \right)^2 + \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right)^2 = 1$







Collisions for mass ratio $m_1 : m_2 = 49:1$

Fig. 4.2a-c

(revised)

Note: If $m_1 \cdot m_2$ is perfect-square, then θ-triangle is rational ($3^2+4^2=5^2$, etc.)

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_1}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$

Then collisions become *reflections* $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ and double-collisions become *rotations*

where: $\cos\theta \equiv \left(\frac{m_1 - m_2}{m_1 + m_2}\right)$ and: $\sin\theta \equiv \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2}\right)$ with: $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 + \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2}\right)^2 = 1$

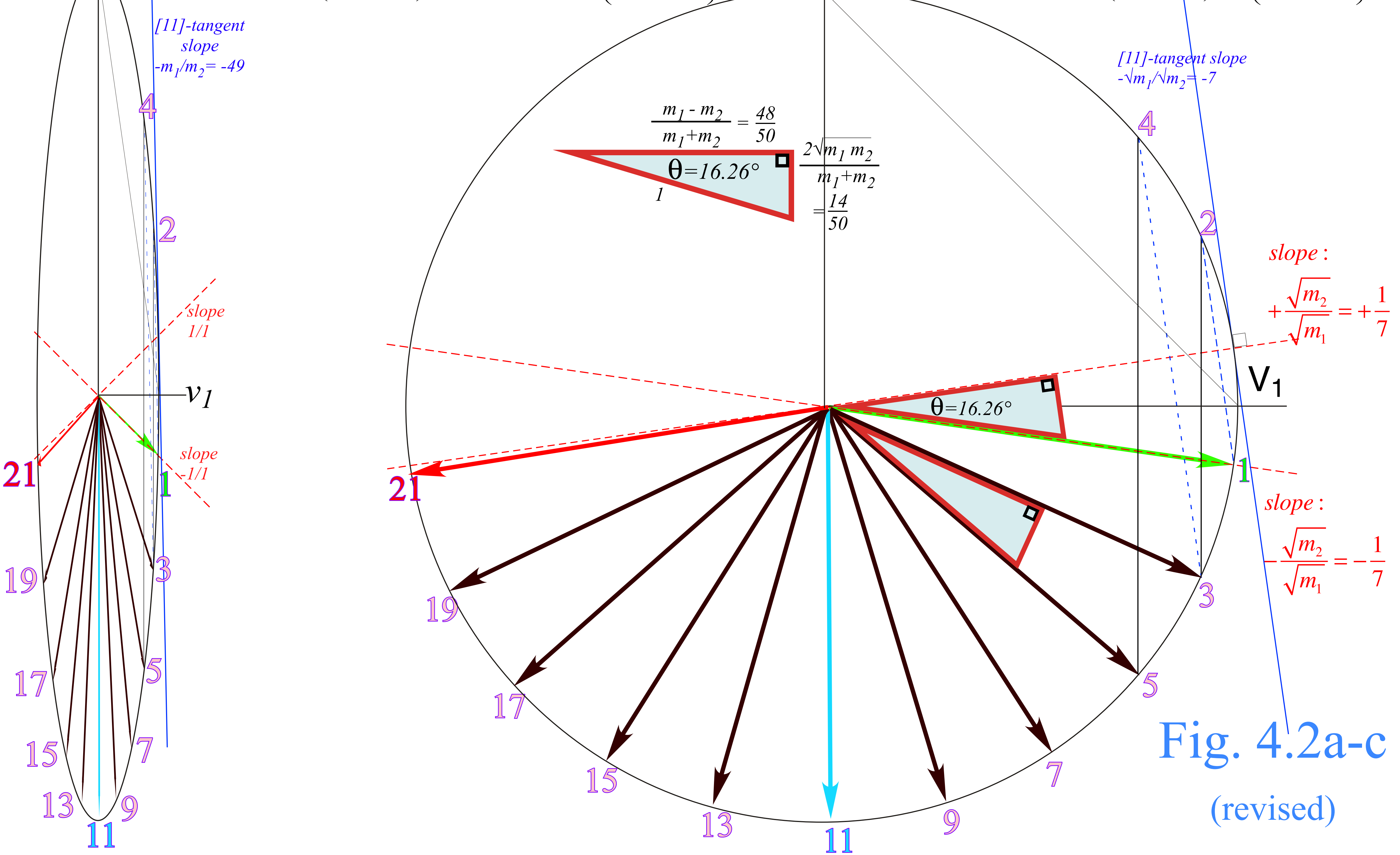


Fig. 4.2a-c

(revised)

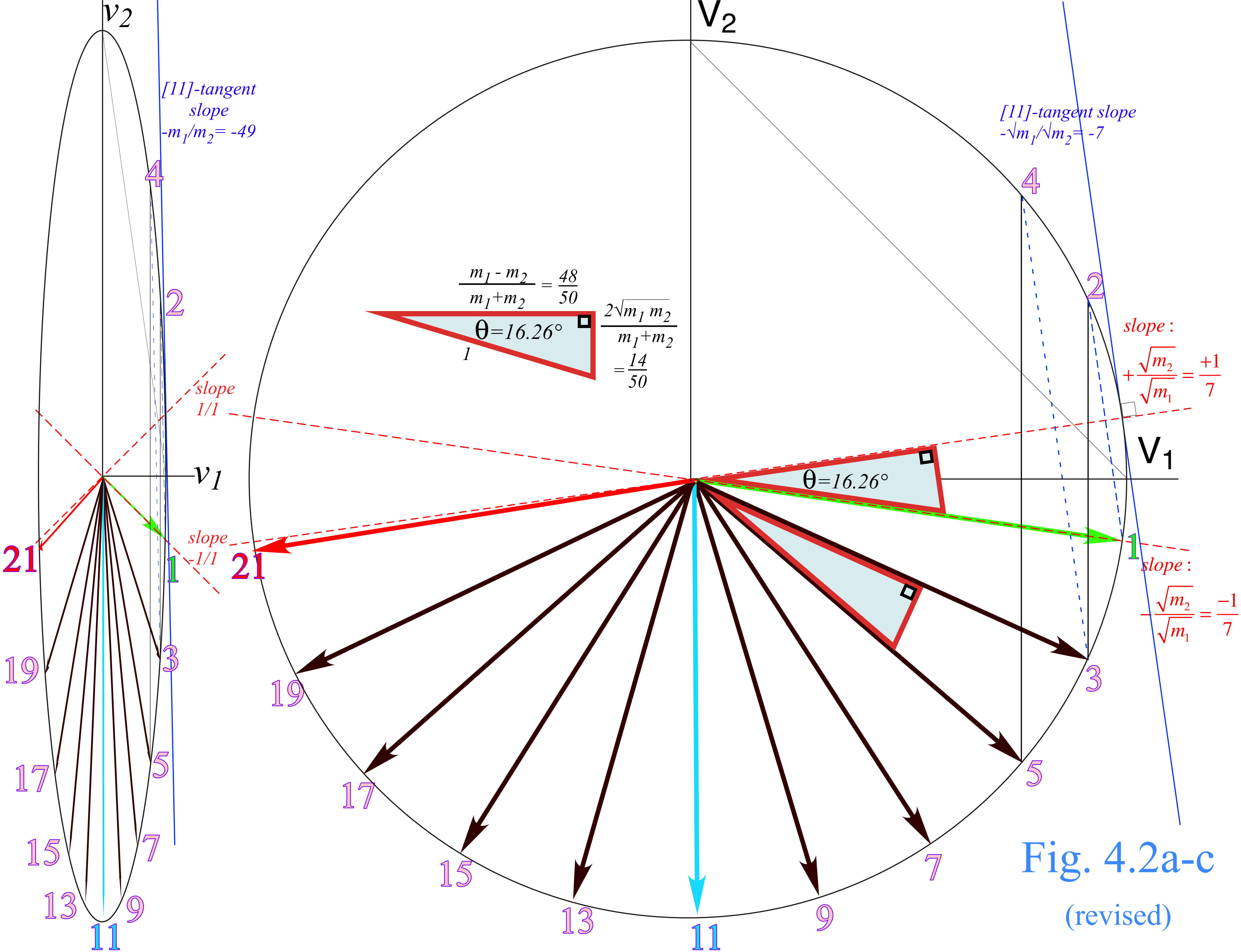


Fig. 4.2a-c

(revised)

Ellipse rescaling-geometry and reflection-symmetry analysis

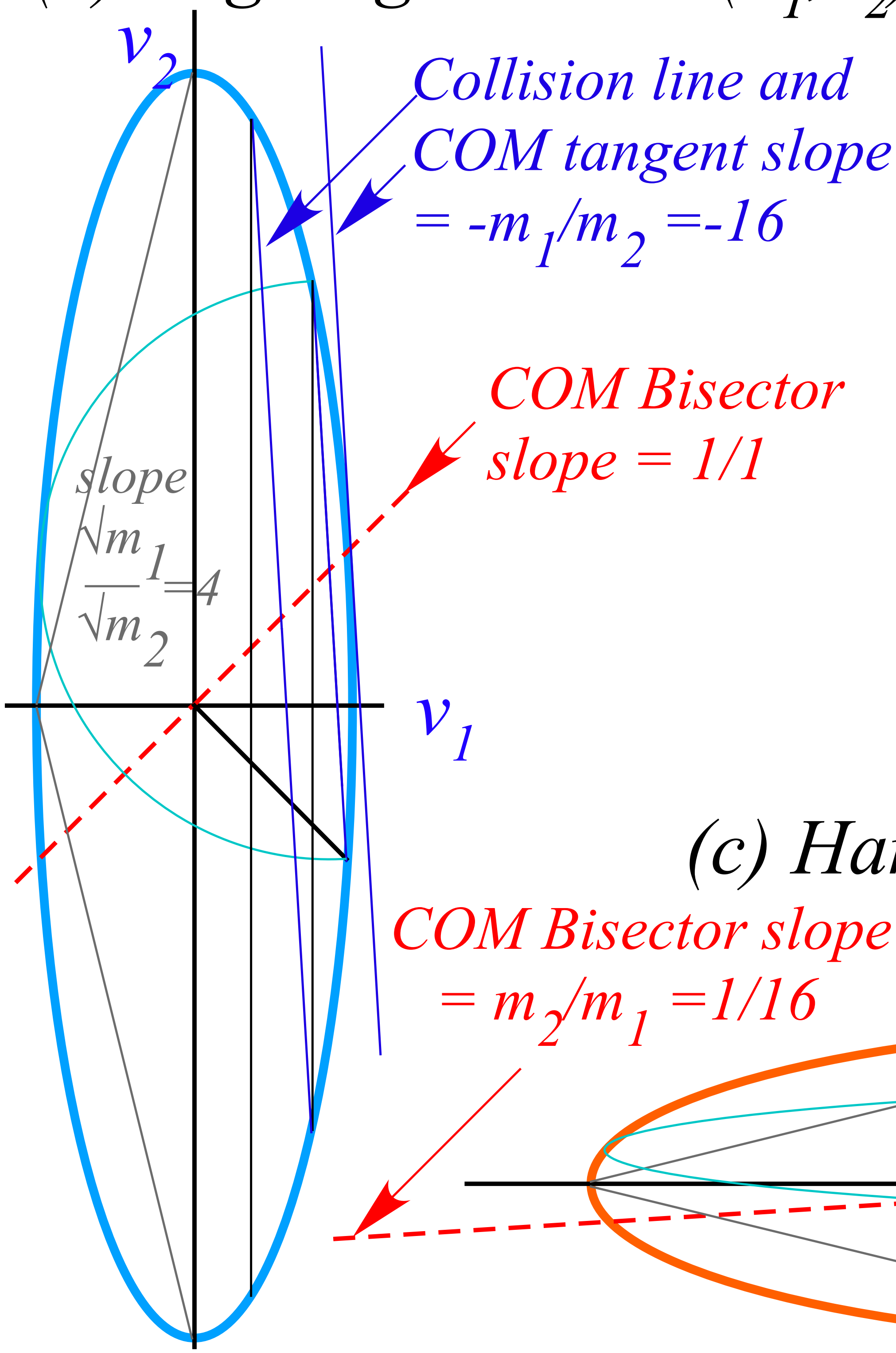
- *Rescaling KE ellipse to circle*
  - *How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on*
- *Reflections in the clothing store: "It's all done with mirrors!"*
- *Introducing hexagonal symmetry $D_6 \sim C_{6v}$ (Resulting for $m_1/m_2=3$)*
  - *Group multiplication and product table*
  - *Classical collision paths with $D_6 \sim C_{6v}$ (Resulting from $m_1/m_2=3$)*
    - *Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$*

What ellipse rescaling leads to...

Fig. 10.1? (Unit 1)

How this relates to Lagrangian, and Hamiltonian mechanics later on

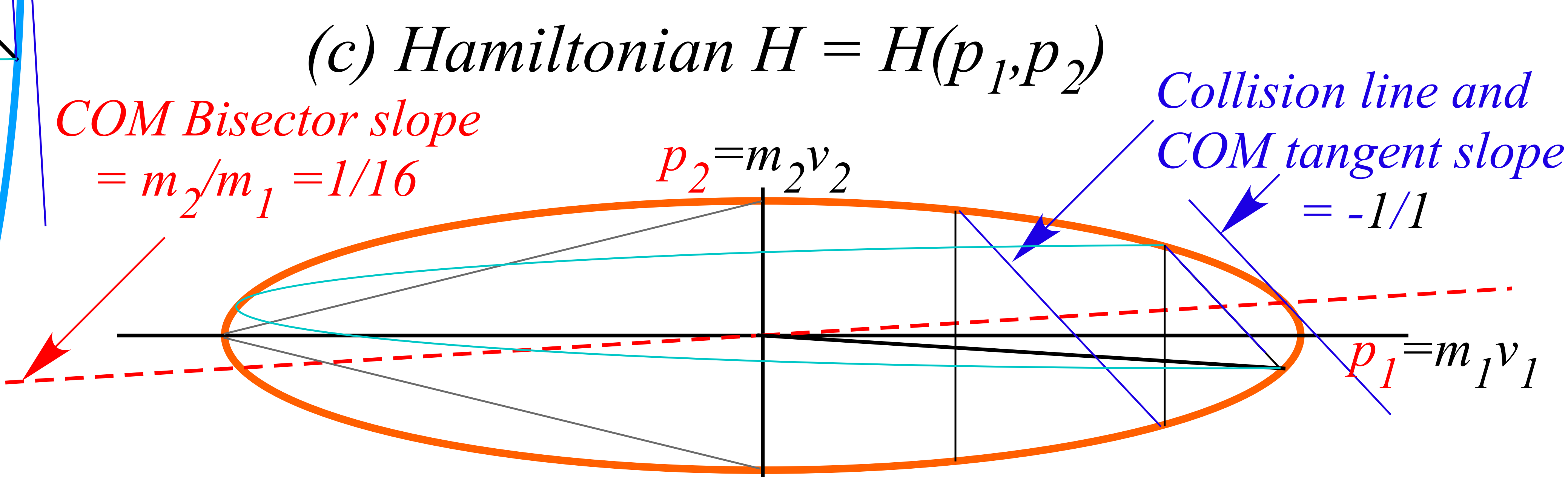
(a) Lagrangian $L = L(v_1, v_2)$



velocity v_1 rescaled to *momentum*: $p_1 = m_1 v_1$

velocity v_2 rescaled to *momentum*: $p_2 = m_2 v_2$

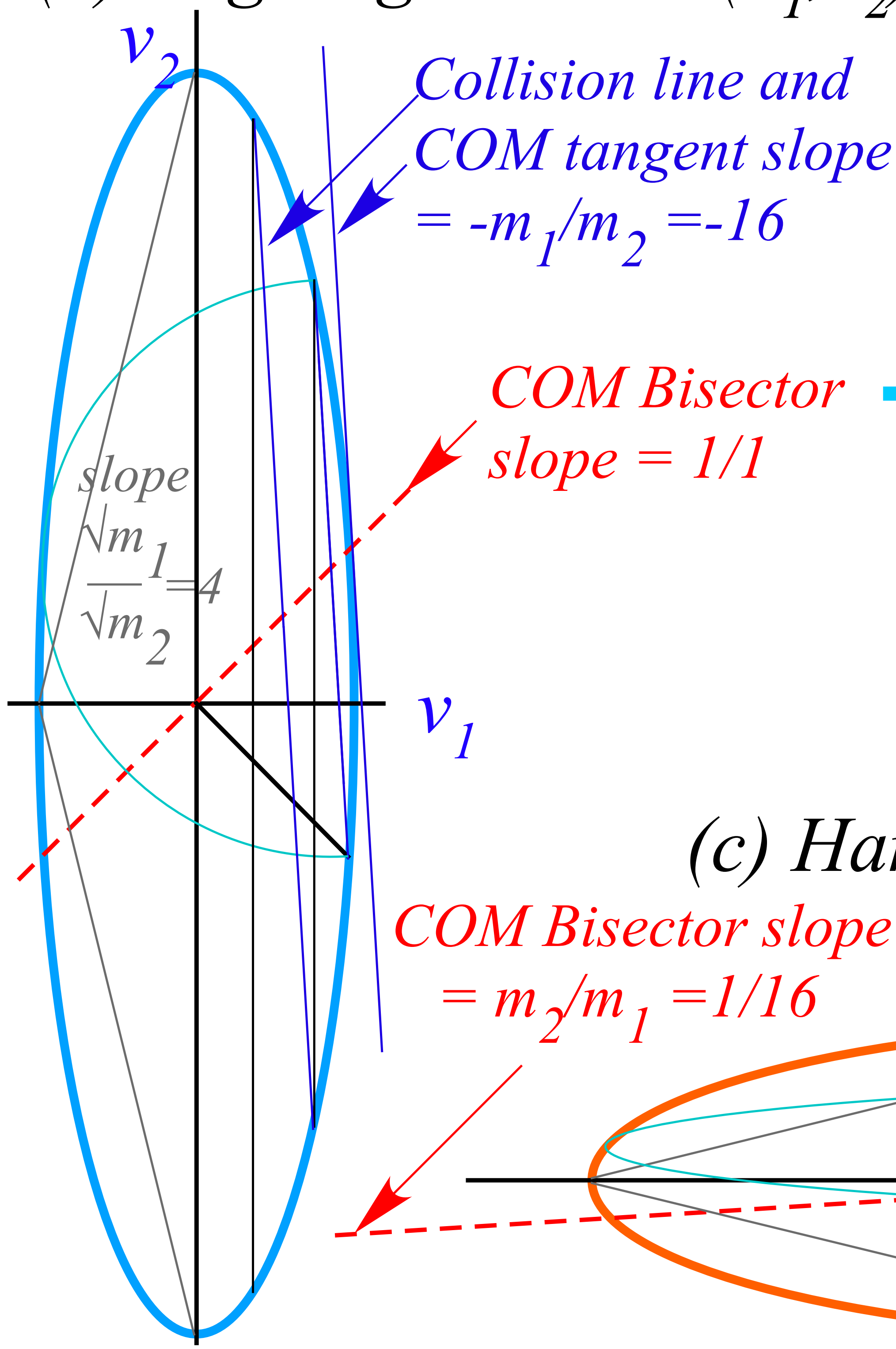
(c) Hamiltonian $H = H(p_1, p_2)$



What ellipse rescaling leads to...

How this relates to Lagrangian, and Hamiltonian mechanics later on

(a) Lagrangian $L = L(v_1, v_2)$



velocity v_1 rescaled to *momentum*: $p_1 = m_1 v_1$

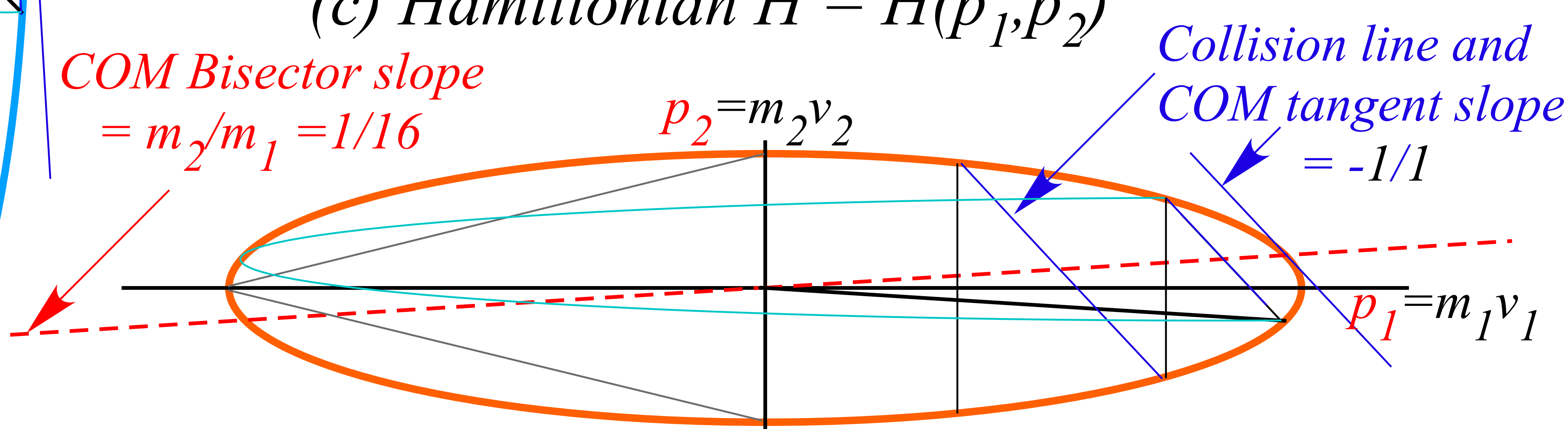
velocity v_2 rescaled to *momentum*: $p_2 = m_2 v_2$

Lagrangian $L(v_1, v_2) = KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

rescaled to

Hamiltonian $H(p_1, p_2) = KE = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2}$

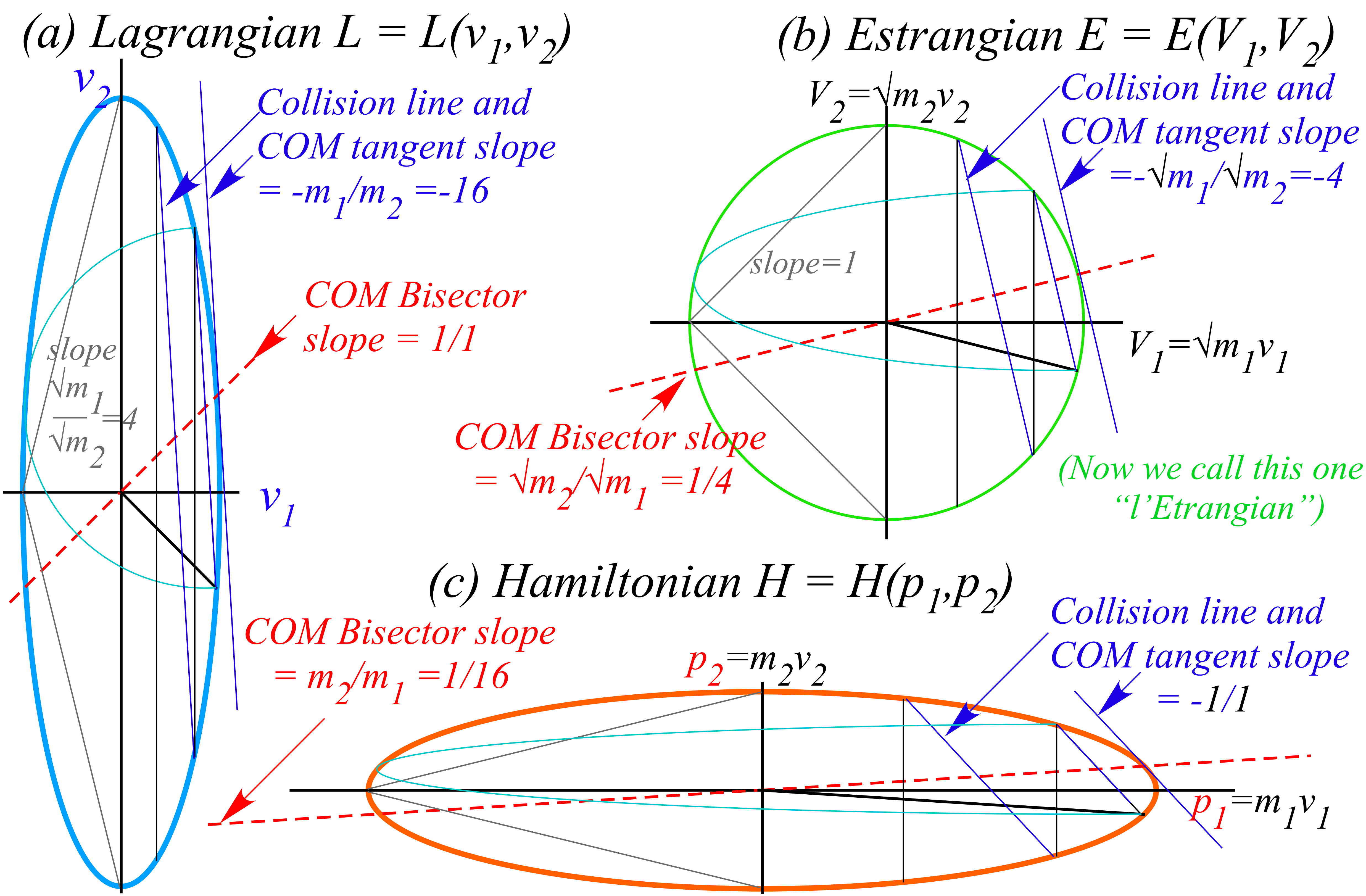
(c) Hamiltonian $H = H(p_1, p_2)$



What ellipse rescaling leads to...

Fig. 10.1? (Unit 1)

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on



Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on

→ *Reflections in the clothing store: "It's all done with mirrors!"*

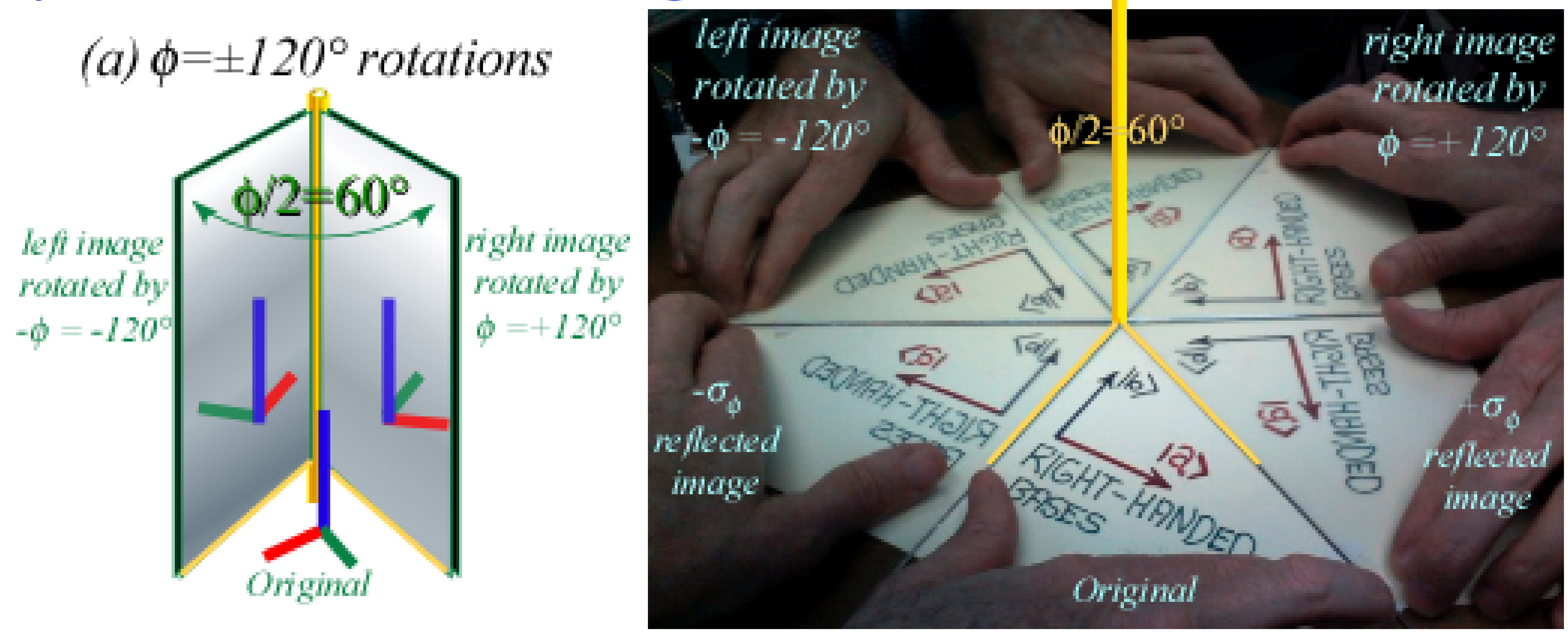
Introducing hexagonal symmetry $D_6 \sim C_{6v}$ (Resulting for $m_1/m_2=3$)

Group multiplication and product table

Classical collision paths with $D_6 \sim C_{6v}$ (Resulting from $m_1/m_2=3$)

Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$

Reflections in clothing store mirrors



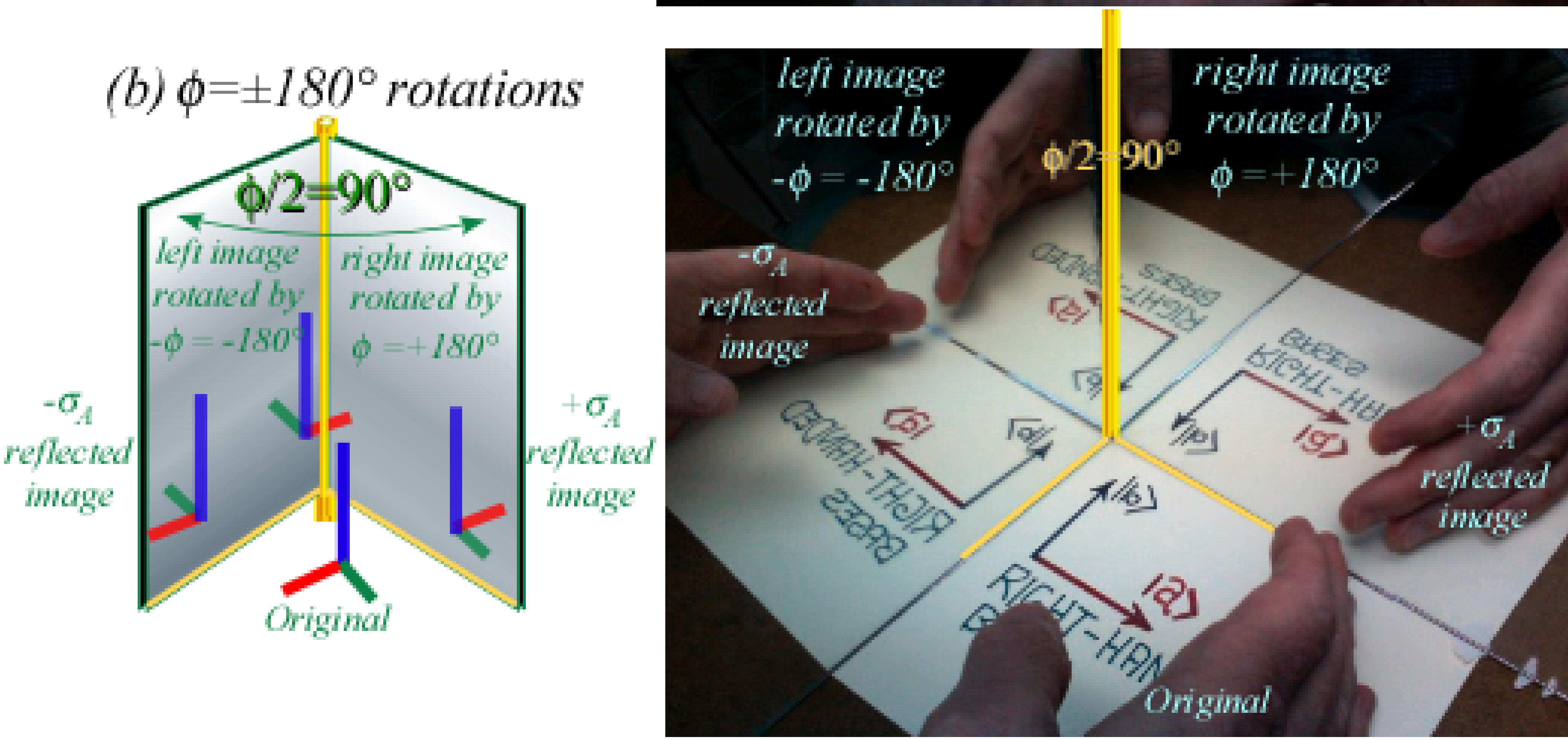


Fig. 5.4a-b

Symmetry: It's all done with mirrors!

(a)Reflections $\sigma_A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $-\sigma_A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ (b)Reflections $\sigma_B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $-\sigma_B = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

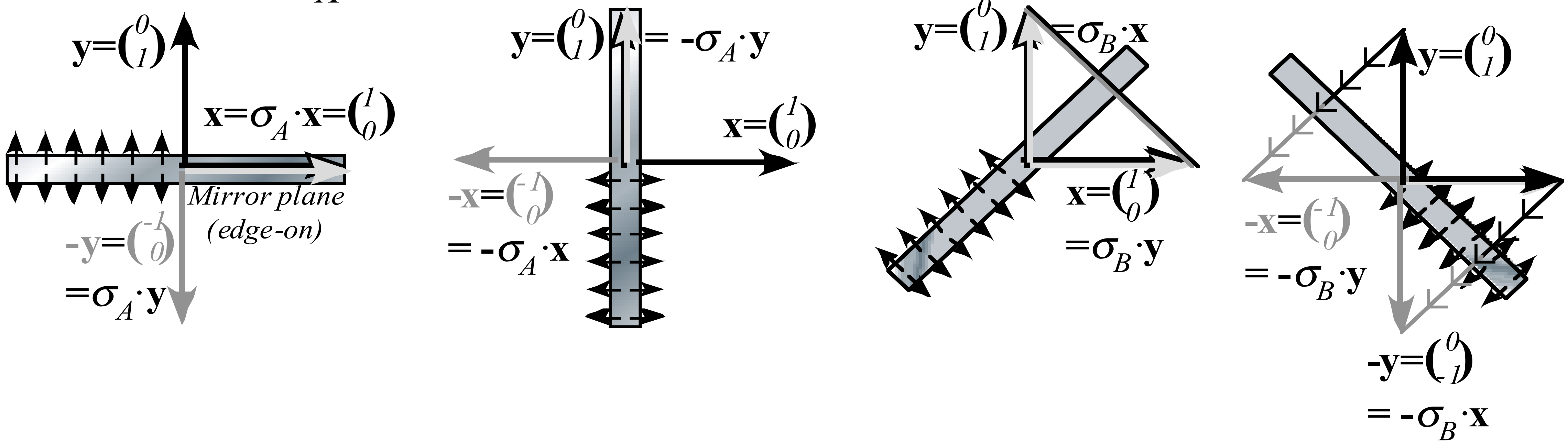
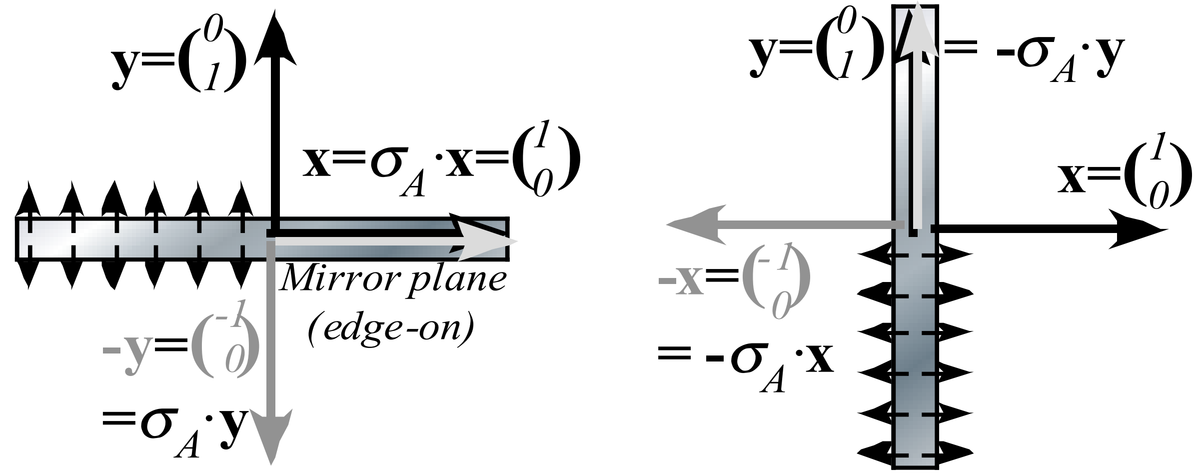


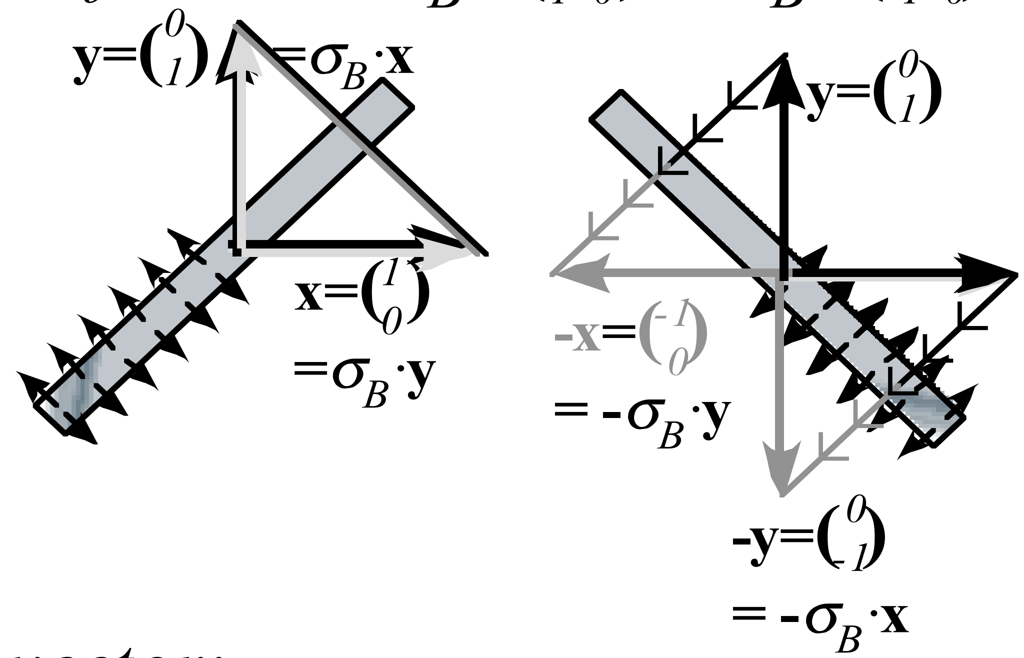
Fig. 4.3
(Unit 1)

Symmetry: It's all done with mirrors!

(a)Reflections $\sigma_A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $-\sigma_A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

(b)Reflections $\sigma_B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $-\sigma_B = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$





(c) σ_ϕ *reflection* $\begin{pmatrix} \cos\phi & \sin\phi \\ \sin\phi & -\cos\phi \end{pmatrix}$

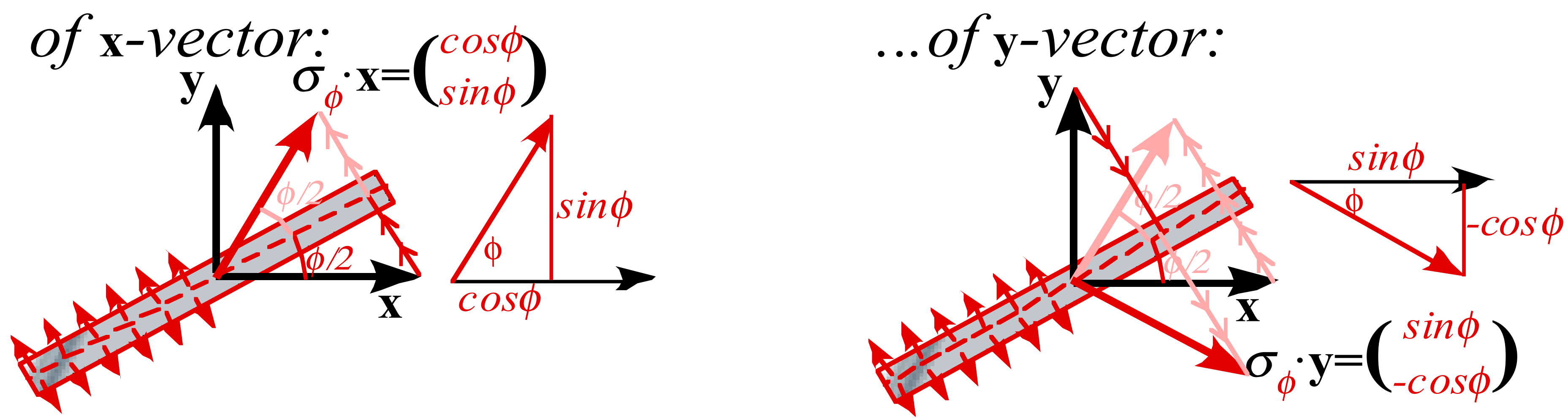


Fig. 4.3
(Unit 1)

Symmetry: It's all done with mirrors!

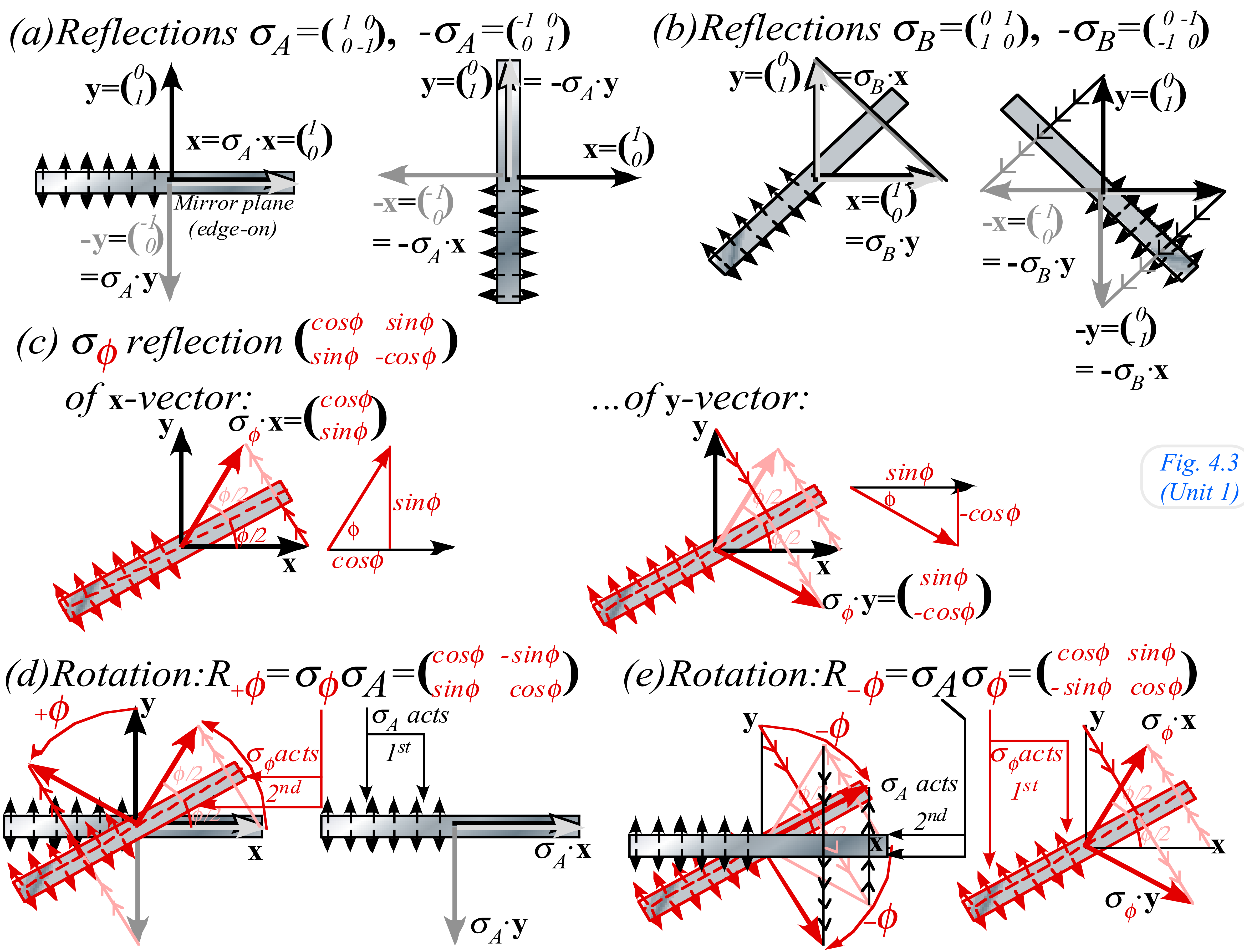


Fig. 4.3 (Unit 1)

Why reflections underlie all symmetry analyses

They work in 1D, 2D, 3D,.....,ND

Product of odd number of reflections is a reflection

... even number of reflections is a rotation (or unit-op **1***)*

Product of rotations just give rotations

Classical objects are semi-rigid and rotate easily
Waves patterns are non-rigid and reflect easily

Why reflections underlie all symmetry analyses

They work in 1D, 2D, 3D,.....,ND

Product of odd number of reflections is a reflection

... even number of reflections is a rotation (or unit-op **1***)*

Product of rotations just give rotations

Classical objects are semi-rigid and rotate easily
Waves patterns are non-rigid and reflect easily

∴ *...wave reflections underlie modern physics*

Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on

Reflections in the clothing store: "It's all done with mirrors!"

Introducing hexagonal symmetry $D_6 \sim C_{6v}$ (Resulting for $m_1/m_2=3$)

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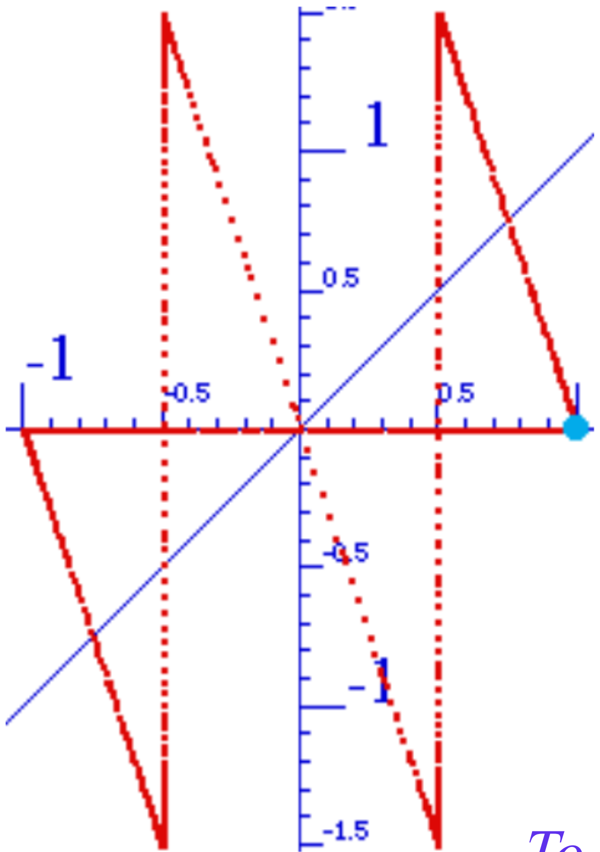
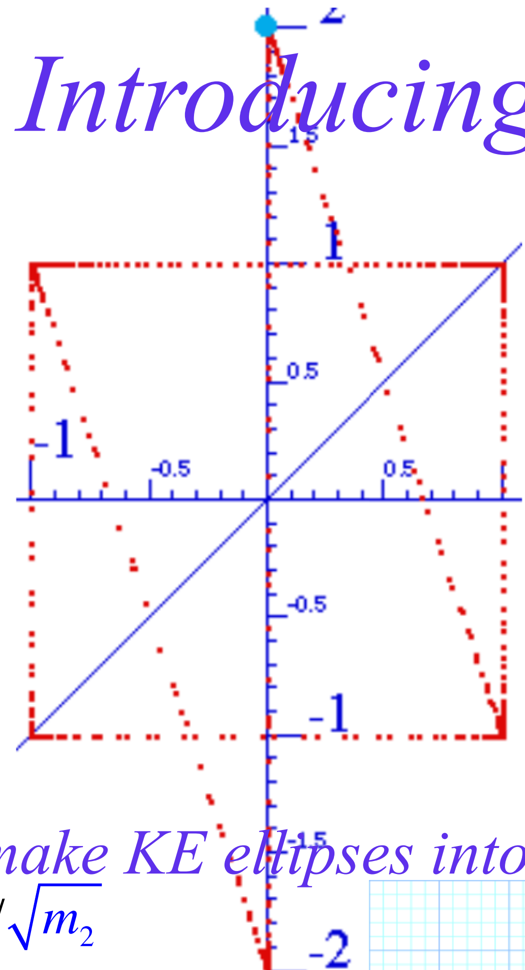
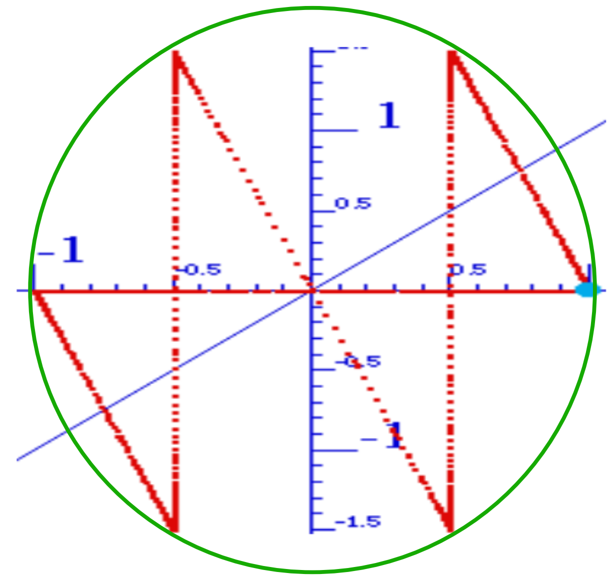
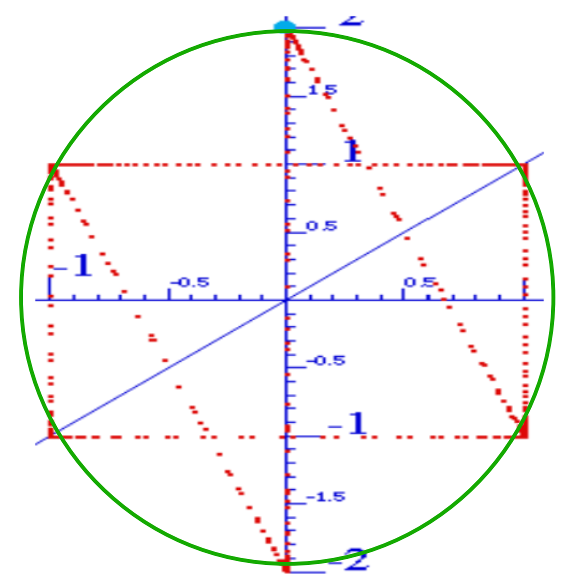
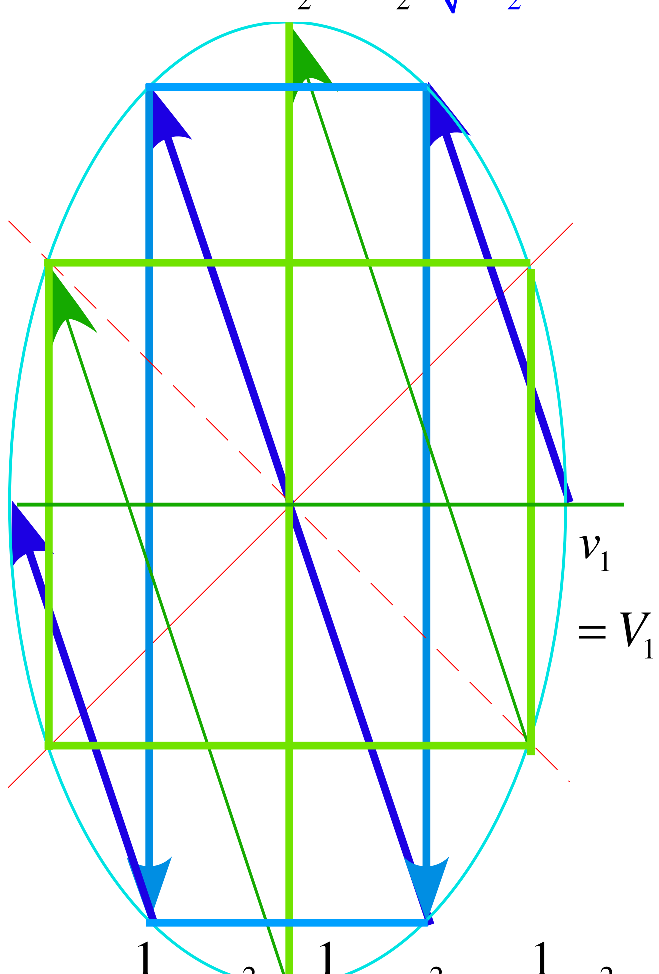
Introducing Symmetry Operators

Collisions for mass ratio m_1:m_2= 3:1

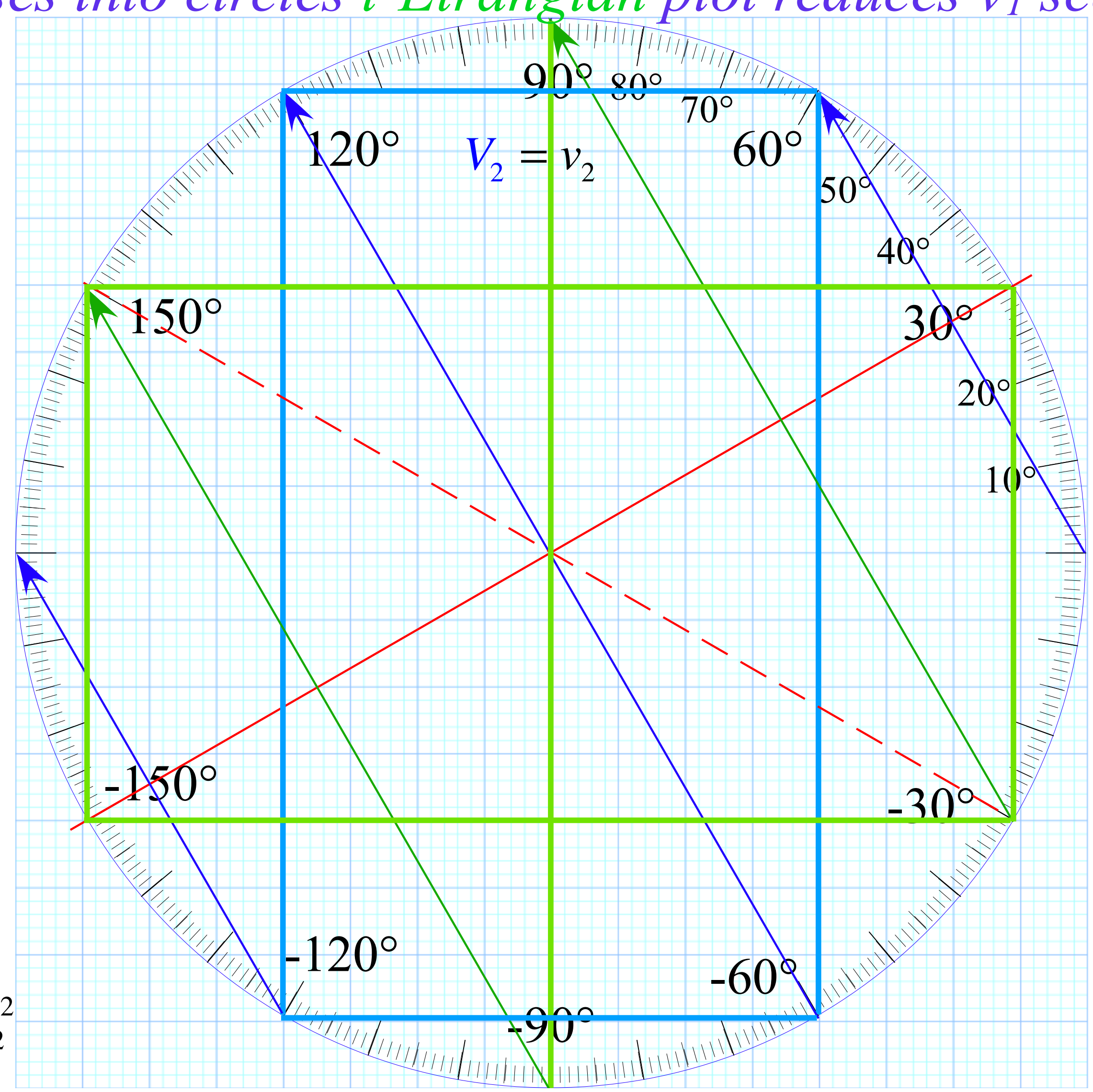





To make KE ellipses into circles l'Etrangian plot reduces v_1 scale by $1/\sqrt{m_1}$,etc.

$v_2 = V_2/\sqrt{m_2}$



$v_1 = V_1/\sqrt{m_1}$



$V_2 = v_2$

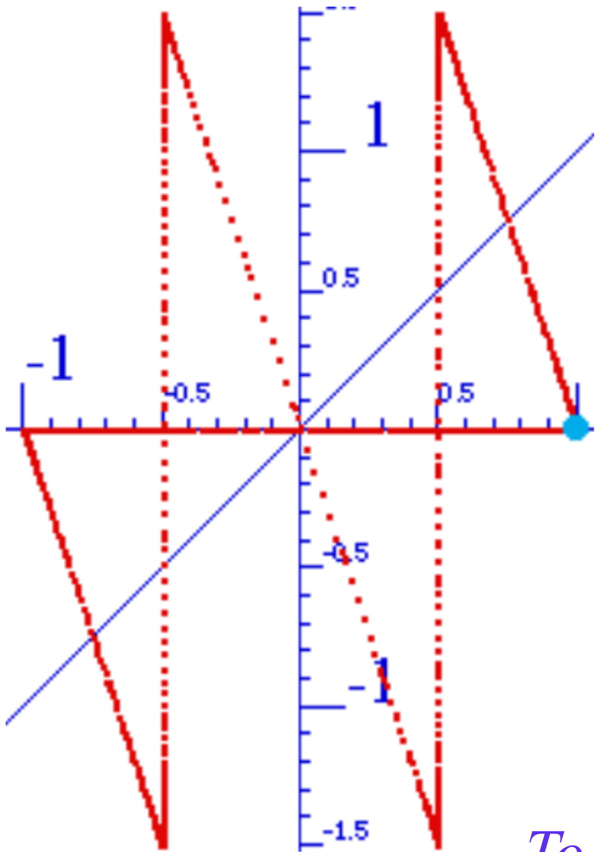
$V_1 = v_1\sqrt{m_1} = v_1\sqrt{3}$

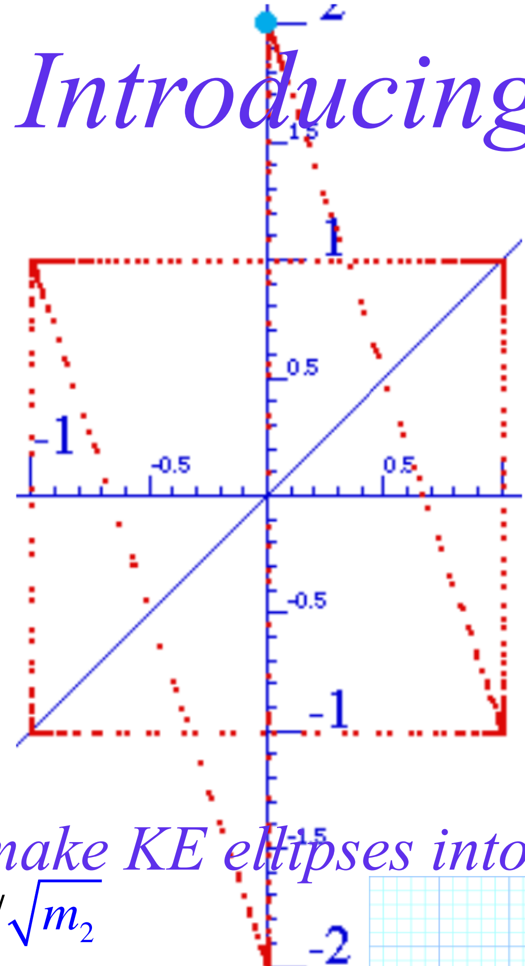
Here:
$1/\sqrt{m_1}=1/\sqrt{3}=0.577$
$1/\sqrt{m_2}=1/\sqrt{1}=1.0$

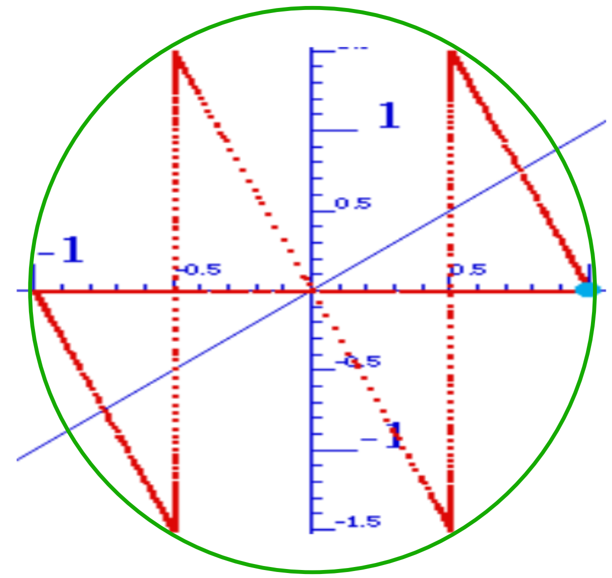
$$KE = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 \quad = \frac{1}{2}V_1^2 \quad + \frac{1}{2}V_2^2$$

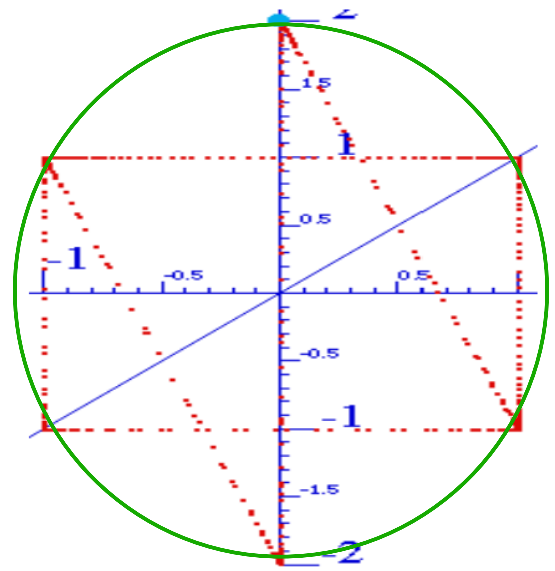
Introducing Symmetry Operators

Collisions for mass ratio m_1:m_2= 3:1









To make KE ellipses into circles l'Etrangian plot reduces v_1 scale by $1/\sqrt{m_1}$,etc.

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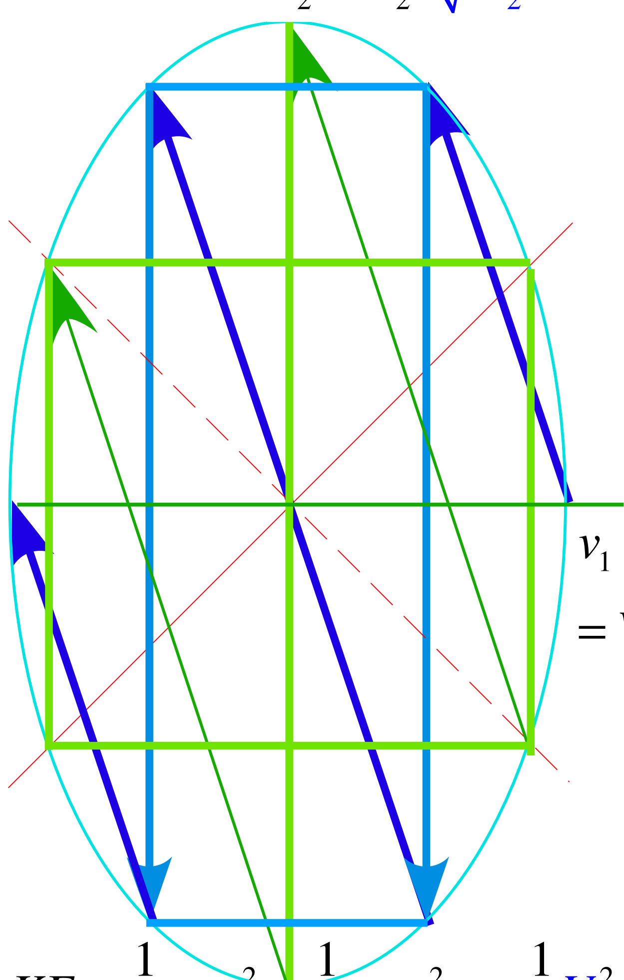
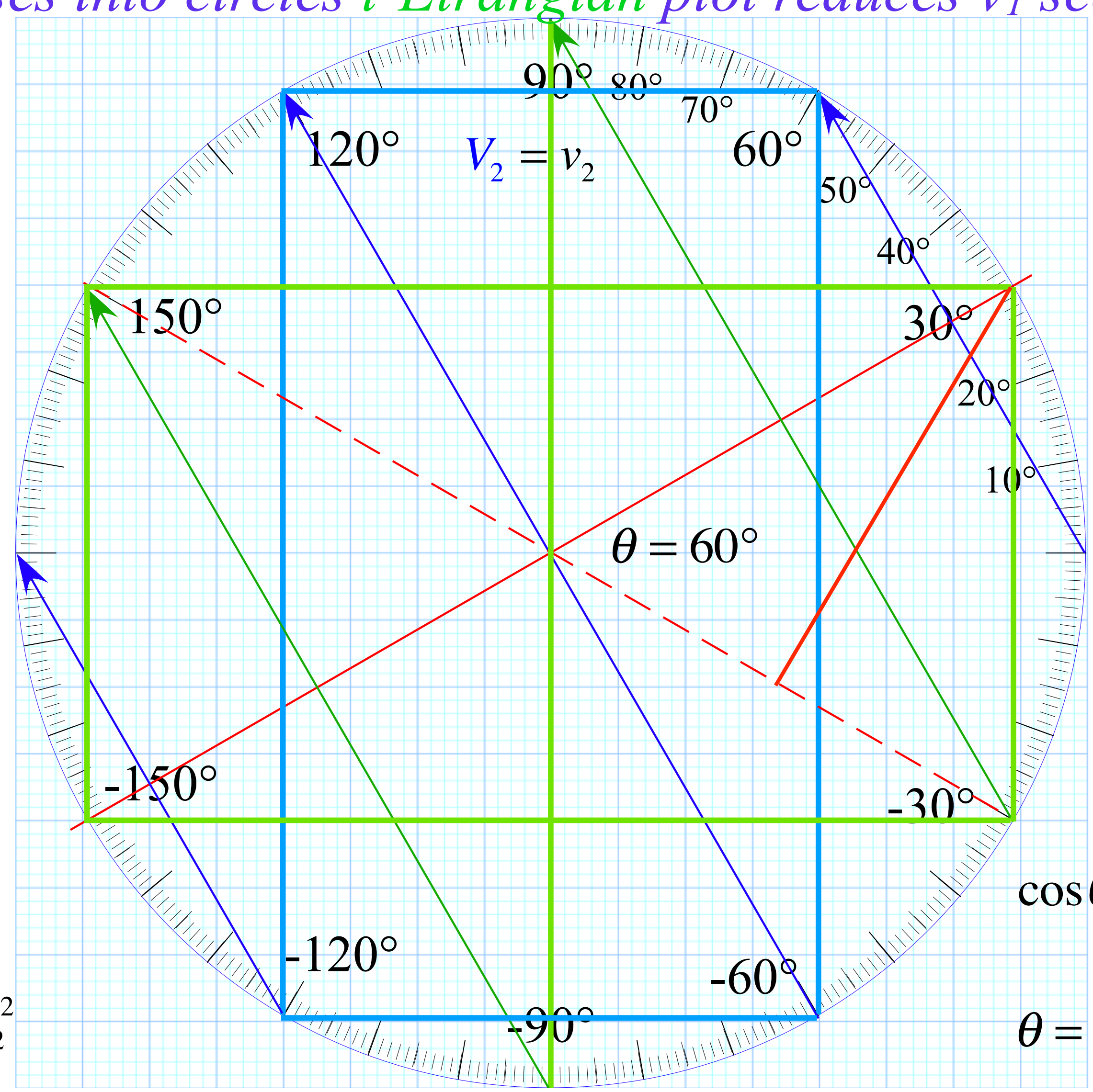


v_1

$= V_1/\sqrt{m_1}$



Here:

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$V_1 = v_1\sqrt{m_1} = v_1\sqrt{3}$

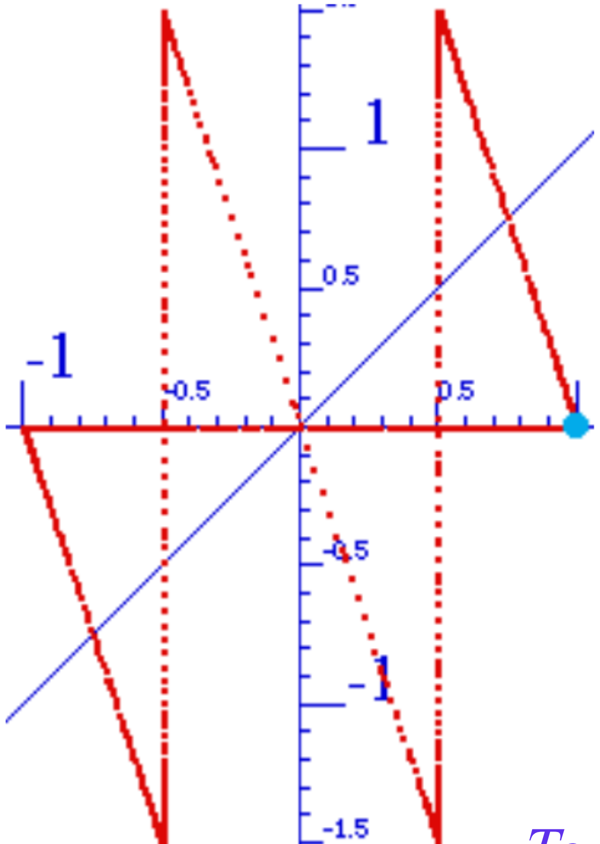
$$\cos\theta = \frac{m_1 - m_2}{m_1 + m_2} = \frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1} = \frac{2}{4} = \frac{1}{2}$$

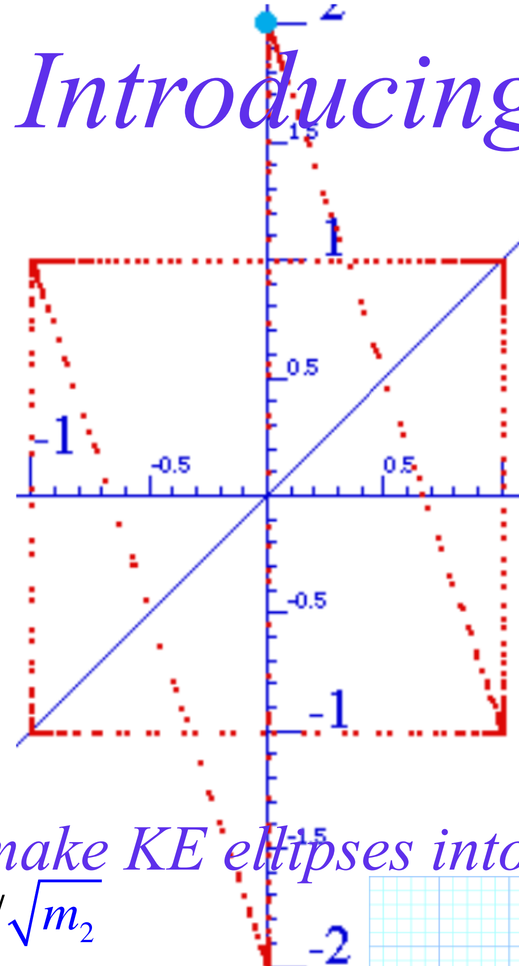
$\theta = 60°$

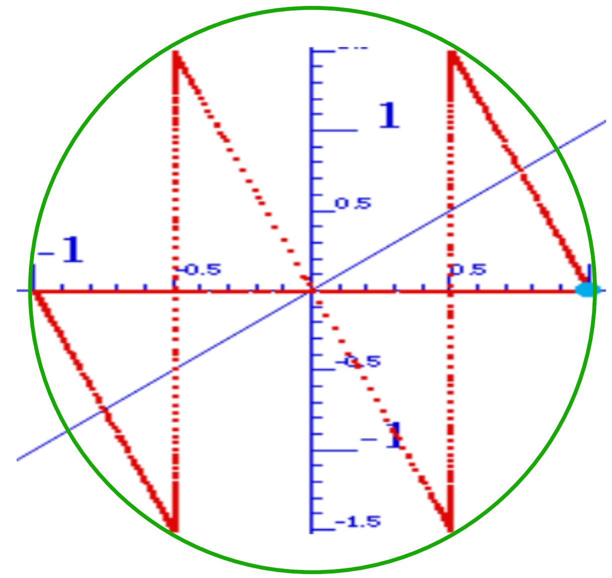
$$KE = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 \quad = \frac{1}{2}V_1^2 \quad + \frac{1}{2}V_2^2$$

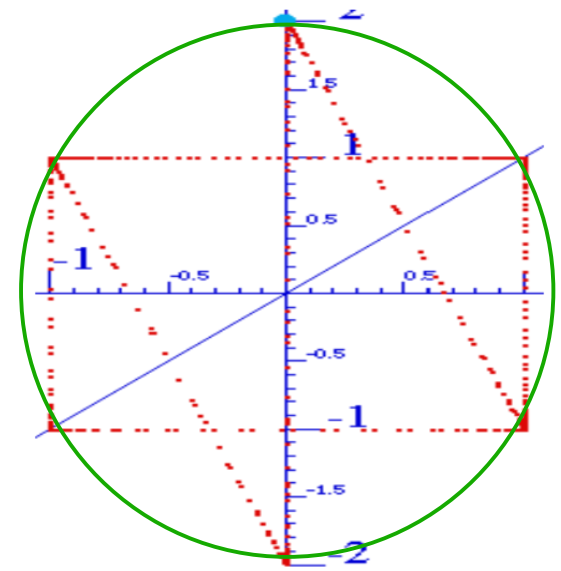
Introducing Symmetry Operators

Collisions for mass ratio m_1:m_2= 3:1

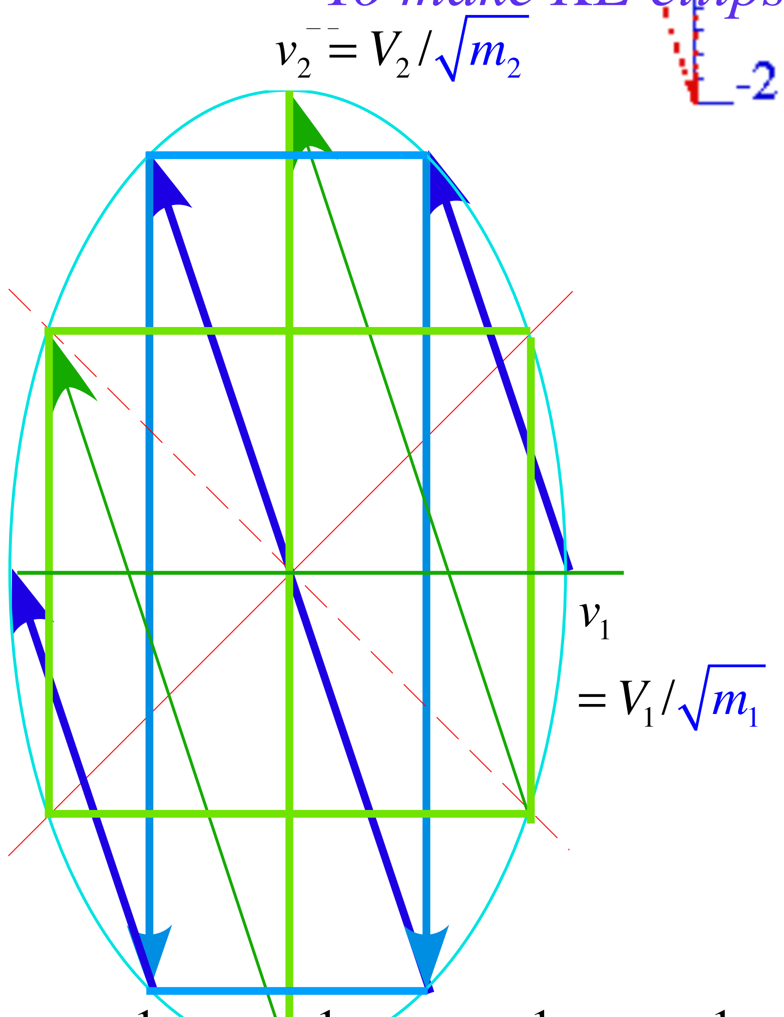




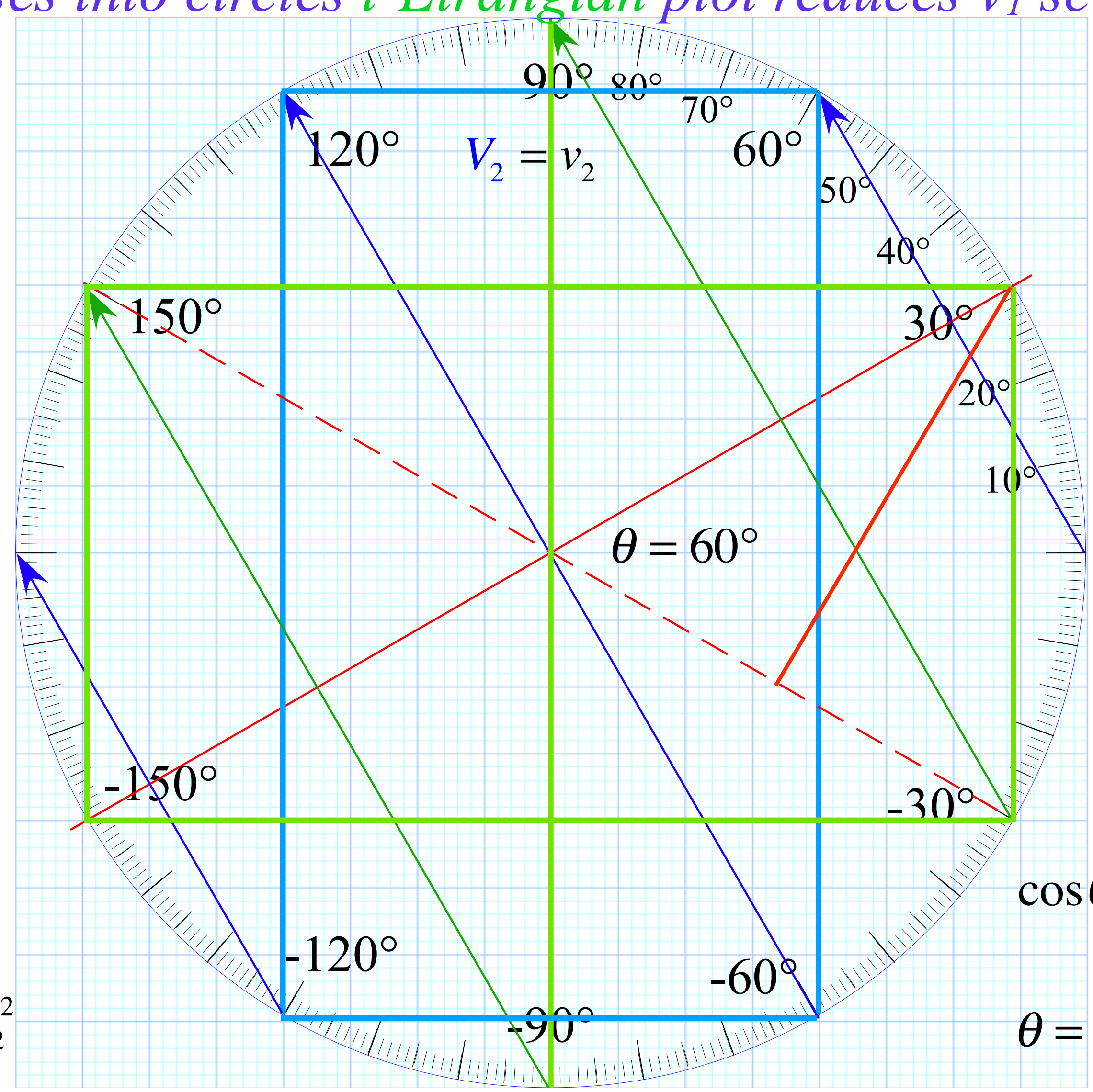




To make KE ellipses into circles l'Etrangian plot reduces v_1 scale by $1/\sqrt{m_1}$,etc.



$$KE = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 \quad = \frac{1}{2}V_1^2 \quad + \frac{1}{2}V_2^2$$



Here:

$1/\sqrt{m_1}=1/\sqrt{3}=0.577$

$1/\sqrt{m_2}=1/\sqrt{1}=1.0$

$$\frac{m_1}{m_2} = \frac{1+\cos\theta}{1-\cos\theta} = \frac{3/2}{1/2} = 3$$

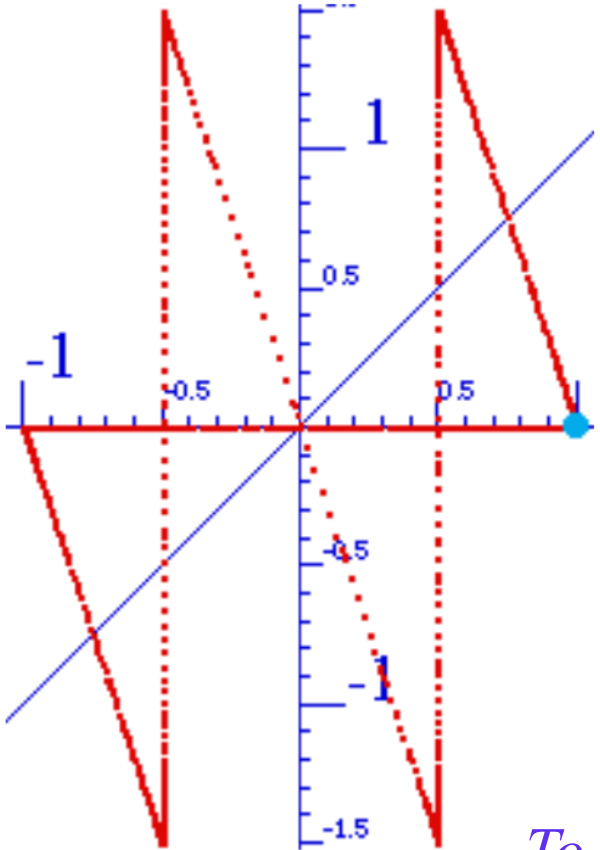
$$V_1 = v_1\sqrt{m_1} = v_1\sqrt{3}$$

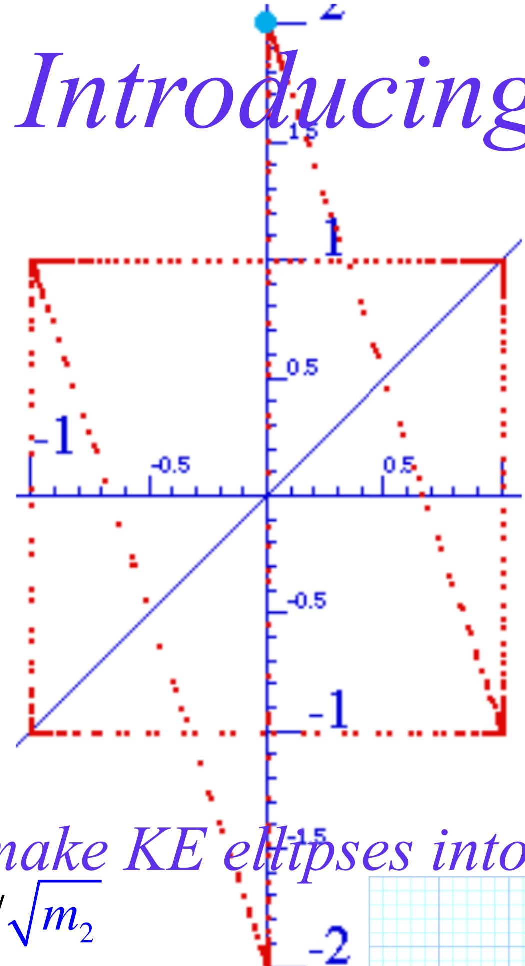
$$\cos\theta = \frac{m_1 - m_2}{m_1 + m_2} = \frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1} = \frac{2}{4} = \frac{1}{2}$$

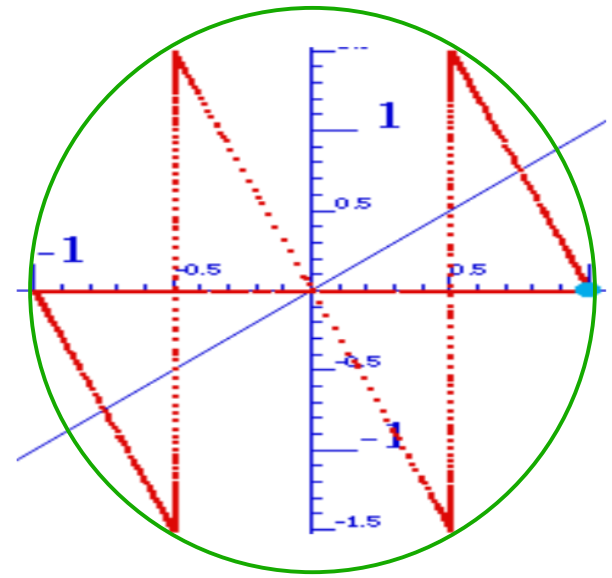
$$\theta = 60°$$

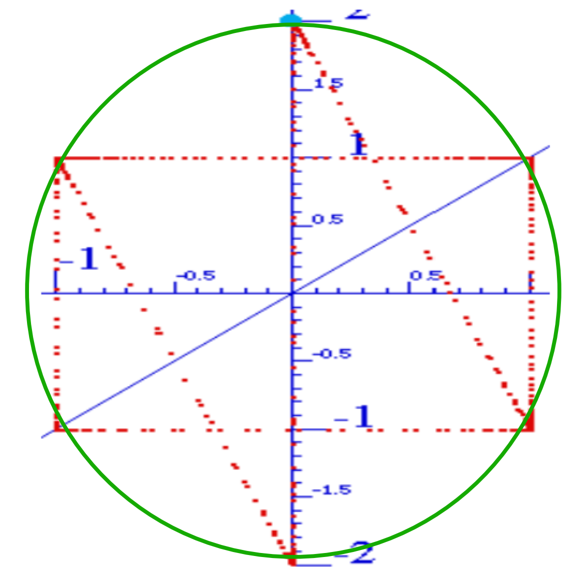
Introducing Symmetry Operators

Collisions for mass ratio m_1:m_2= 3:1



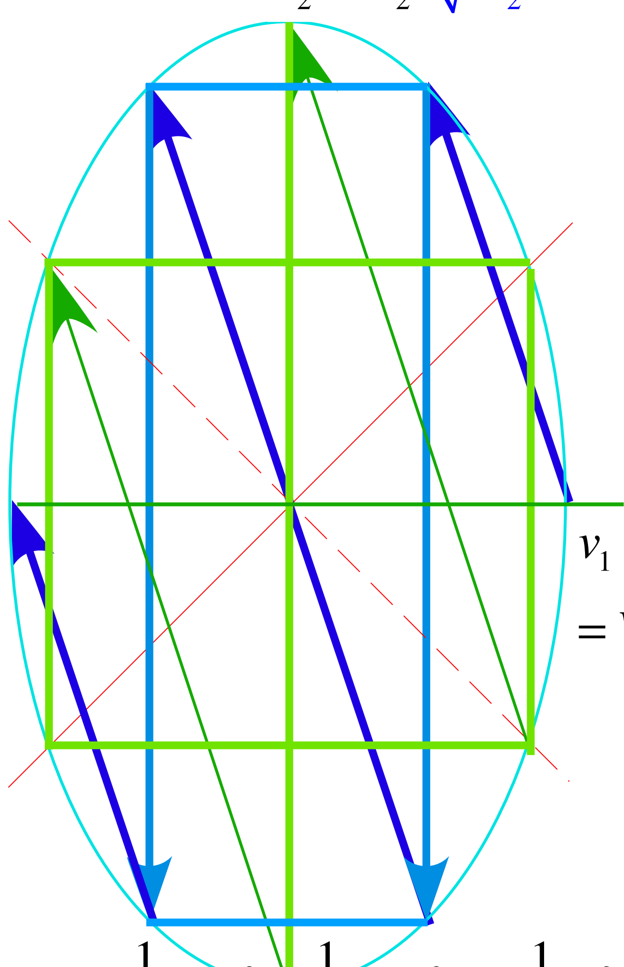






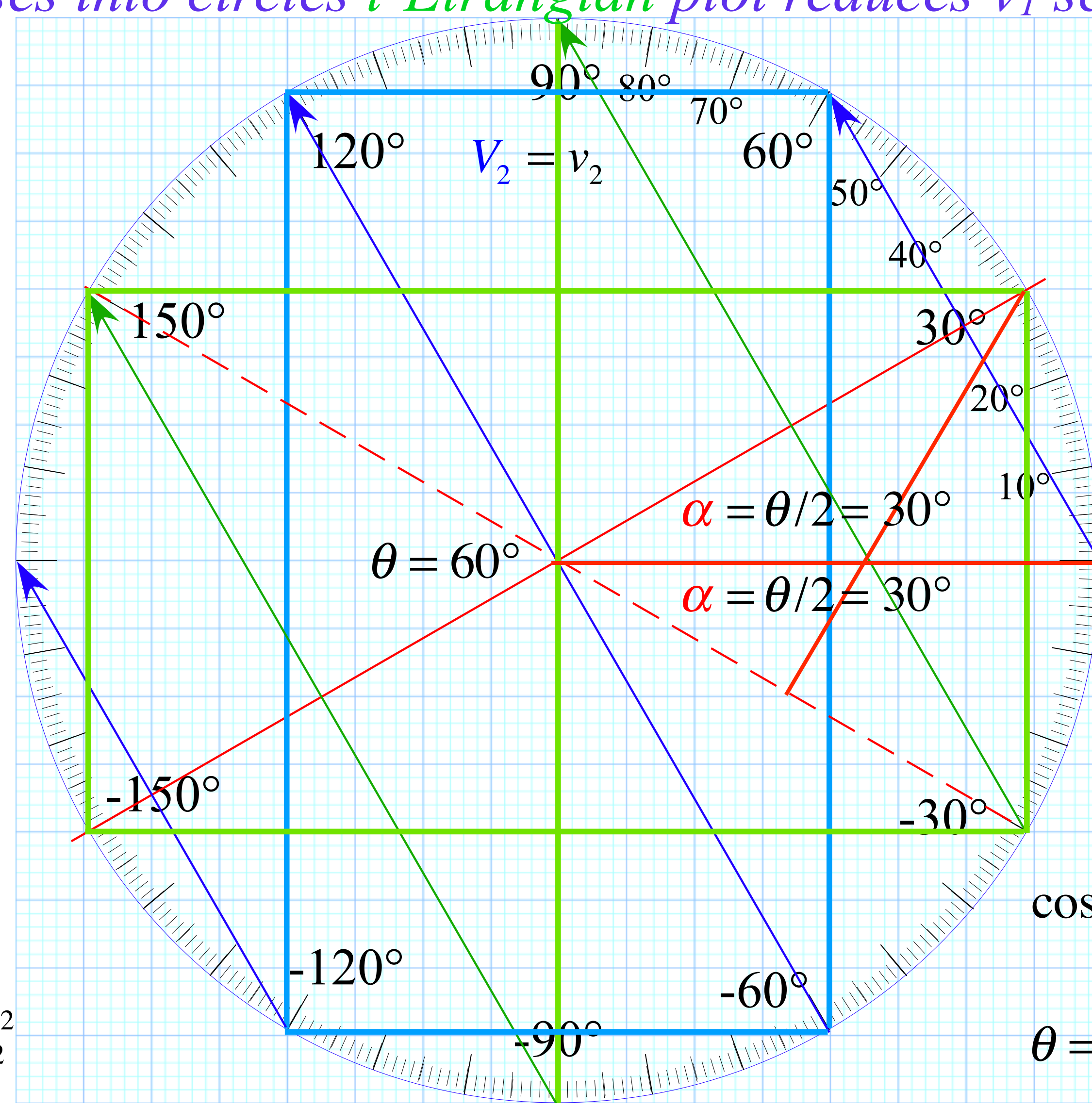
To make KE ellipses into circles l'Etrangian plot reduces v_1 scale by $1/\sqrt{m_1}$,etc.

$v_2 = V_2/\sqrt{m_2}$



$v_1 = V_1/\sqrt{m_1}$

$$KE = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 \quad = \frac{1}{2}V_1^2 \quad + \frac{1}{2}V_2^2$$



$V_1 = v_1\sqrt{m_1} = v_1\sqrt{3}$

Here:

$1/\sqrt{m_1} = 1/\sqrt{3} = 0.577$

$1/\sqrt{m_2} = 1/\sqrt{1} = 1.0$

$$\frac{m_1}{m_2} = \frac{1+\cos\theta}{1-\cos\theta} = \frac{3/2}{1/2} = 3$$

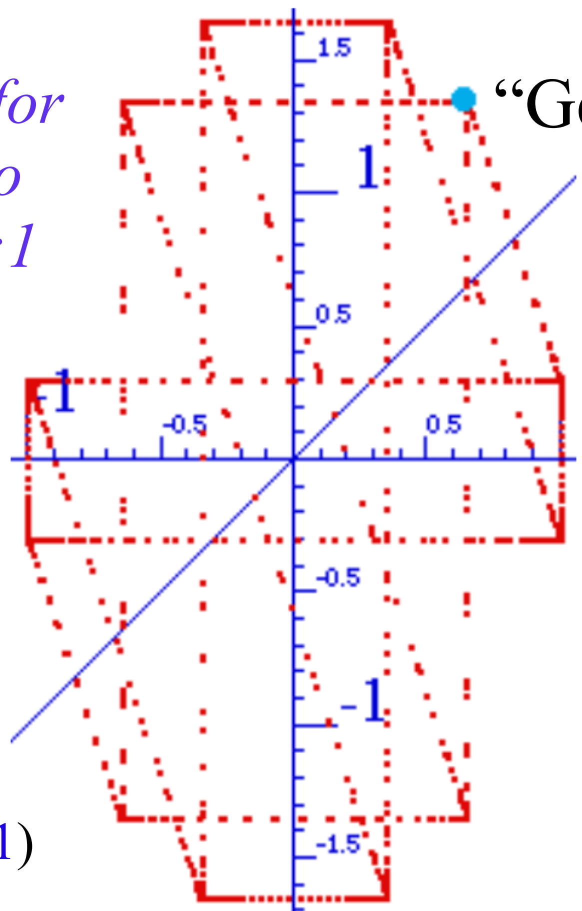
$$\frac{m_1}{m_2} = \frac{1+\cos\theta}{1-\cos\theta} = \frac{\sin^2\alpha}{\cos^2\alpha} = \tan^2\alpha$$

$\alpha = \theta/2$

$$\cos\theta = \frac{m_1 - m_2}{m_1 + m_2} = \frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1} = \frac{2}{4} = \frac{1}{2}$$

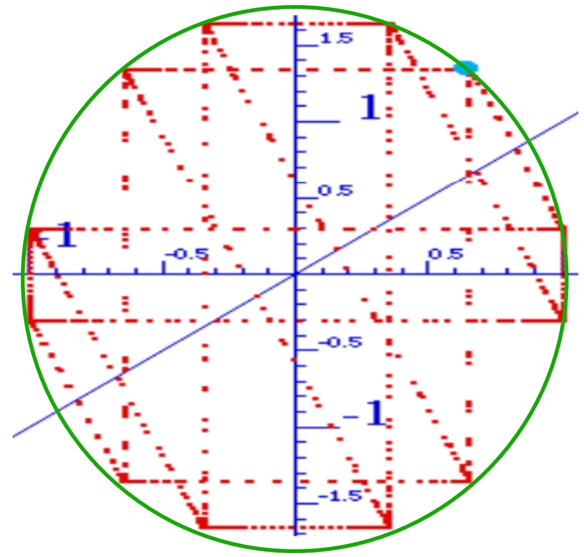
$\theta = 60°$

Collisions for mass ratio m_1:m_2= 3:1

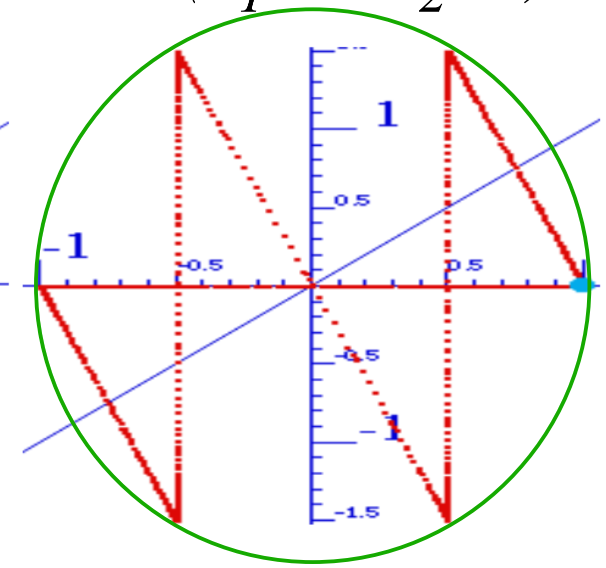


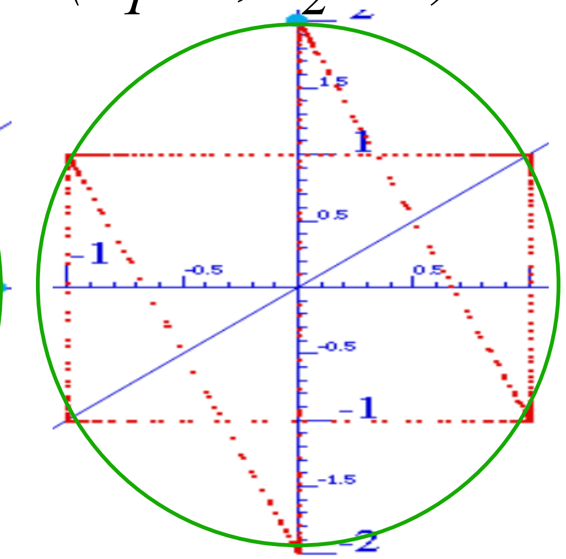
$m_1/m_2=(3)/(1)$

"Generic" initial velocity
$(v_1=1.0, v_2=0.1)$

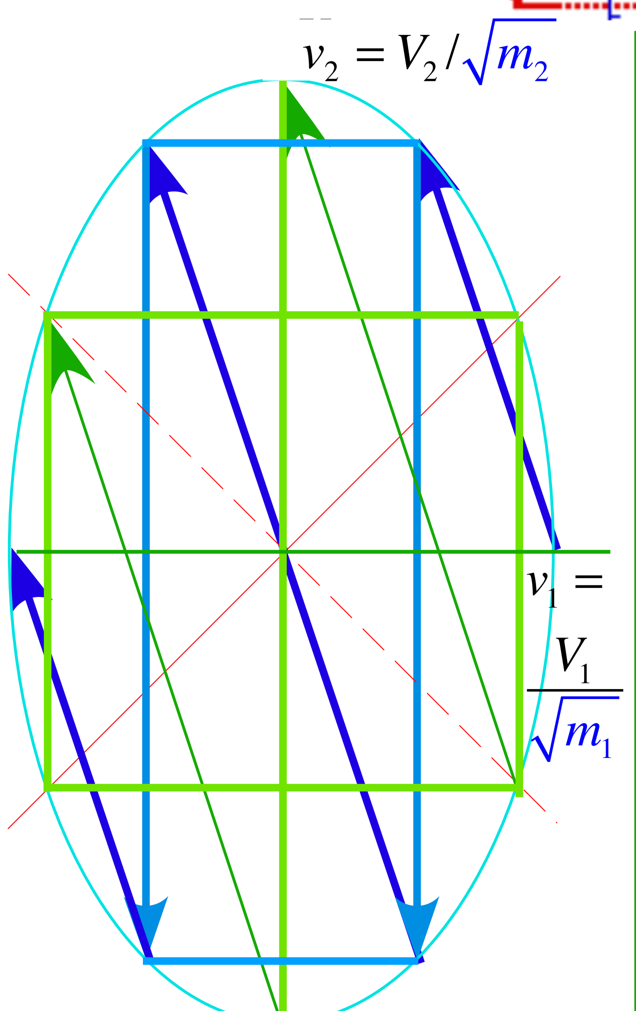


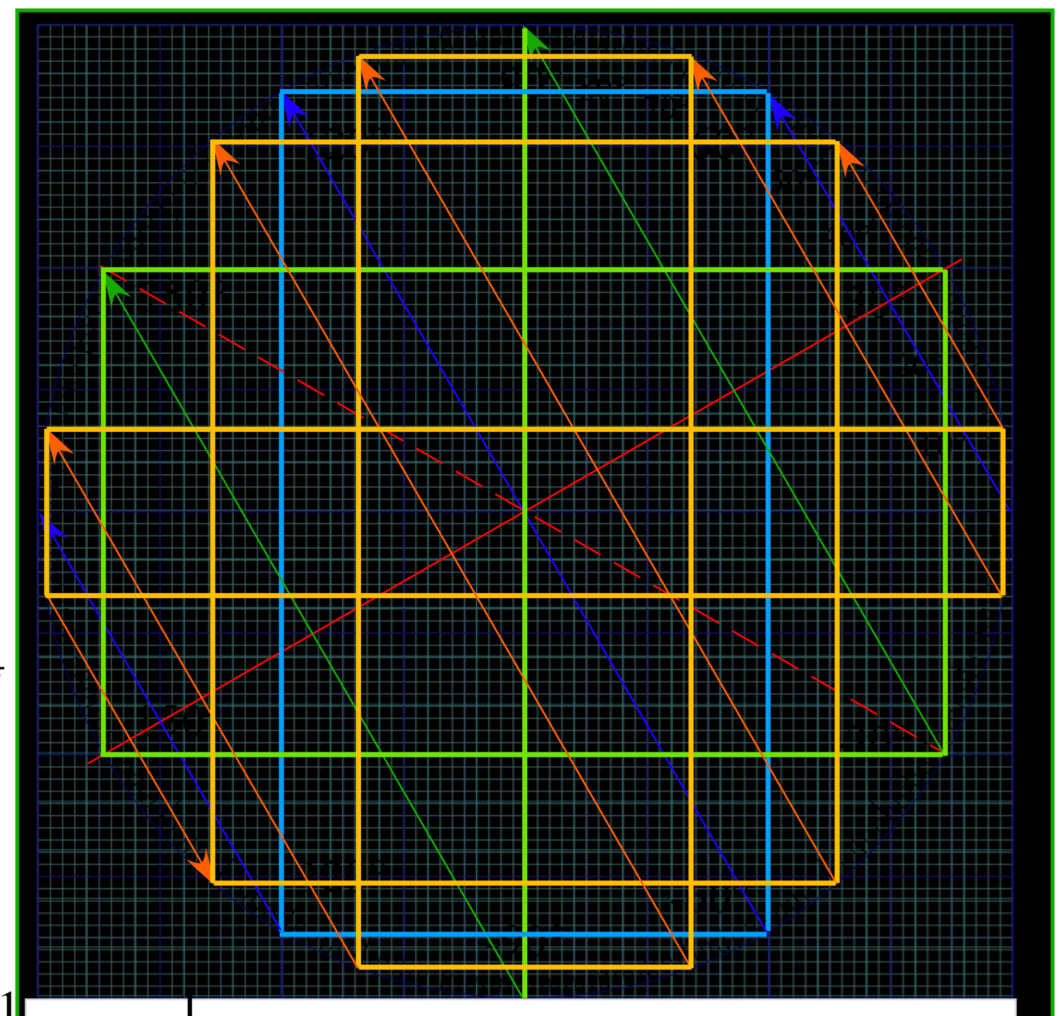
"Symmetric" initial velocity
$(v_1=1, v_2=0)$ or $(v_1=1, v_2=-1)$

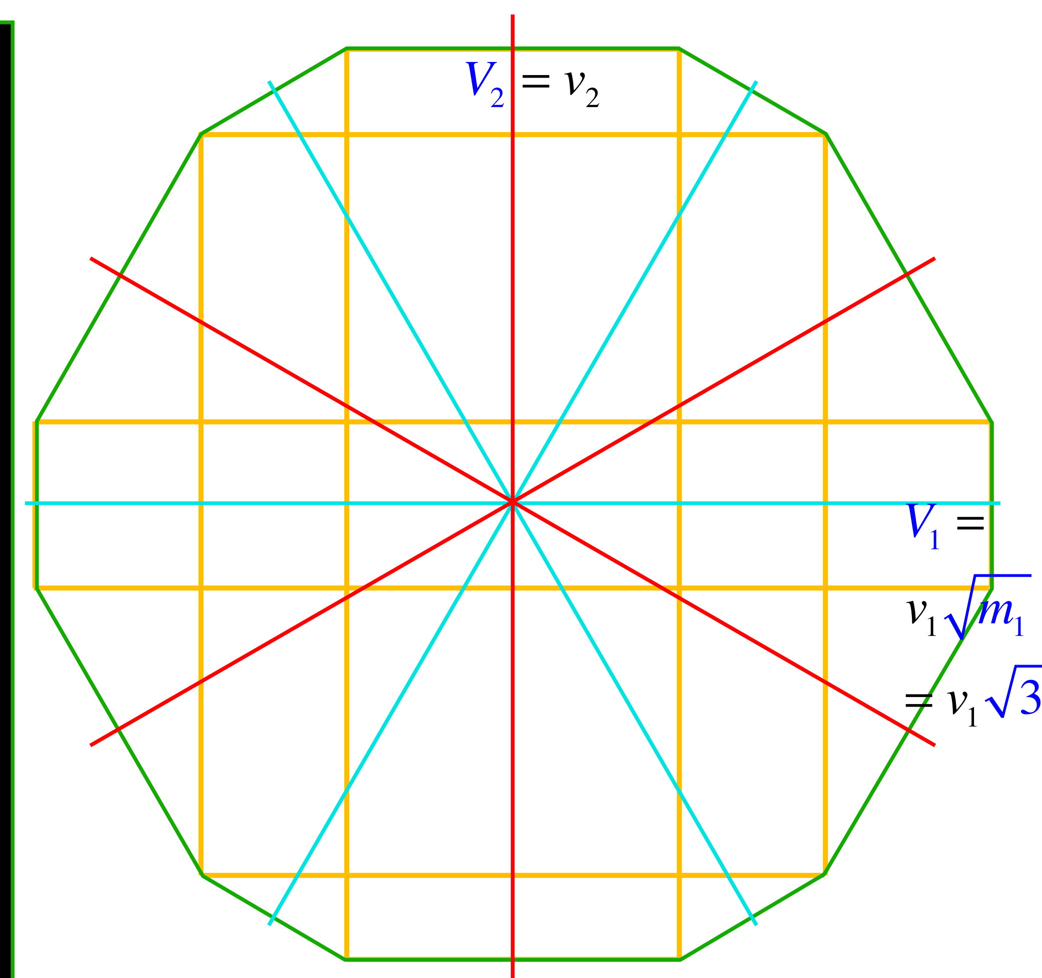




reduce v_1 scale by $1/\sqrt{m_1} = 1/\sqrt{3}=0.577$







$$KE = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}V_1^2 + \frac{1}{2}V_2^2$$

Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on

Reflections in the clothing store: "It's all done with mirrors!"

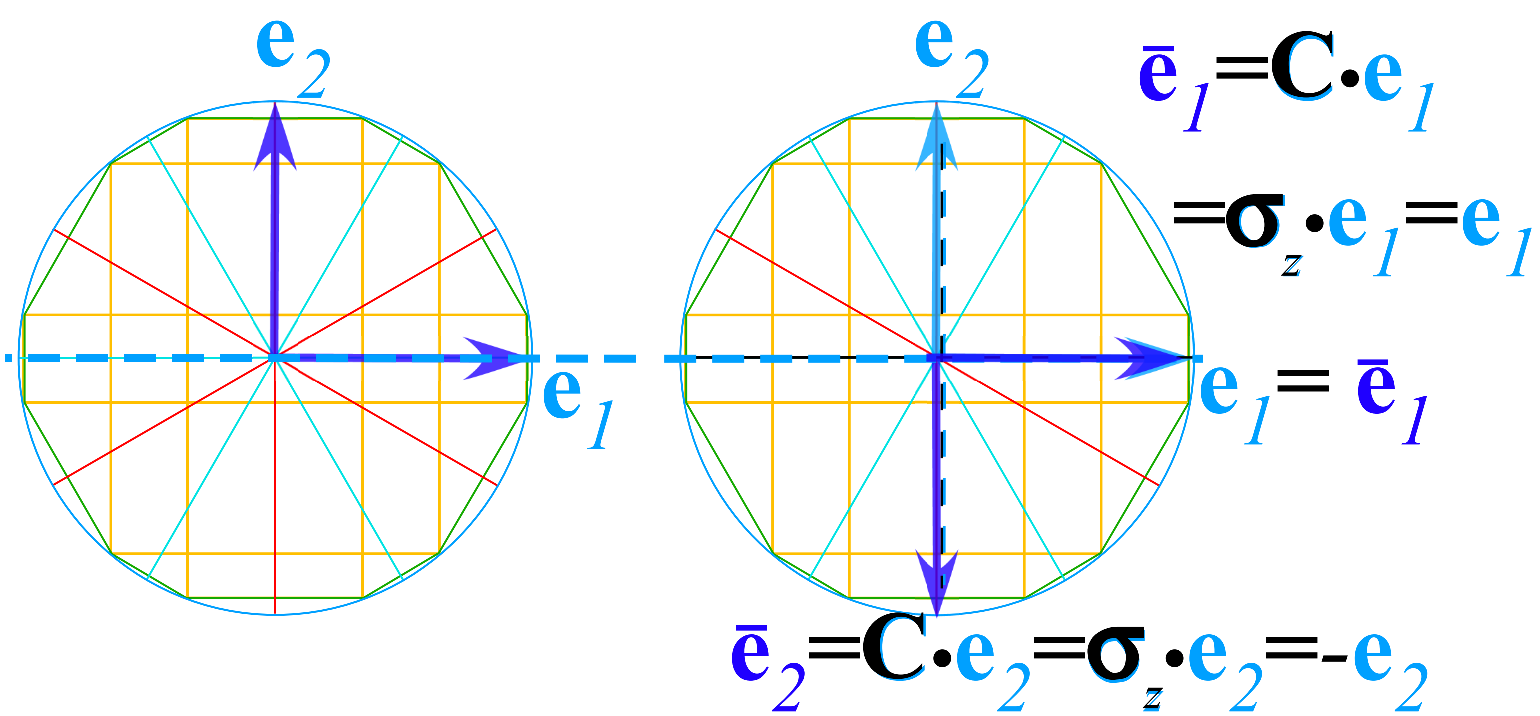
Introducing hexagonal symmetry $D_6 \sim C_{6v}$ (Resulting for $m_1/m_2=3$)

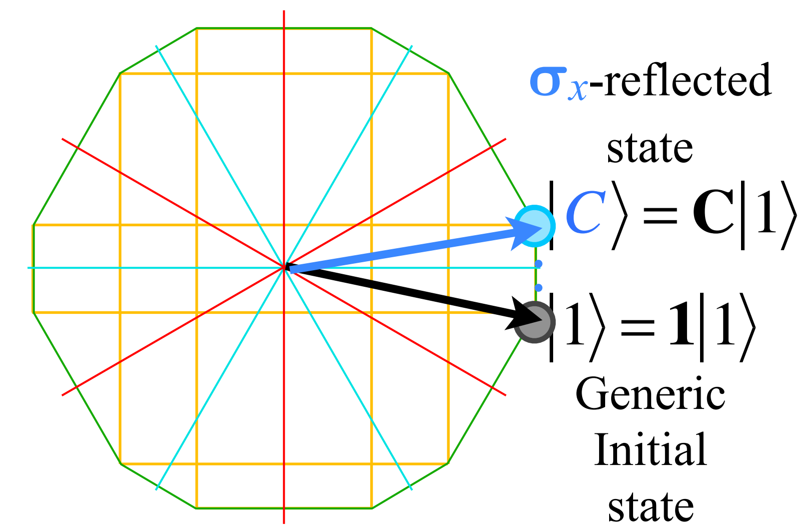
→ *Group multiplication and product table*

Classical collision paths with $D_6 \sim C_{6v}$ (Resulting from $m_1/m_2=3$)

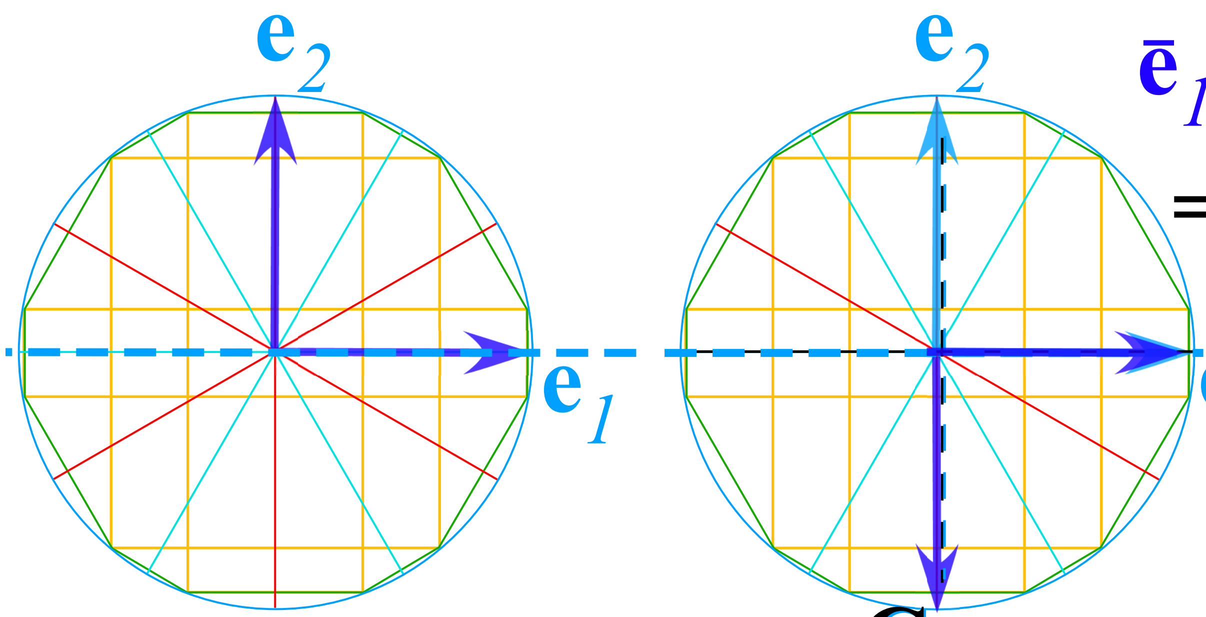
Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$

Effects of Ceiling Bang Matrix $\mathbf{C}=\boldsymbol{\sigma}_z = \begin{pmatrix} \mathbf{e}_1\cdot\mathbf{C}\cdot\mathbf{e}_1 & \mathbf{e}_1\cdot\mathbf{C}\cdot\mathbf{e}_2 \\ \mathbf{e}_2\cdot\mathbf{C}\cdot\mathbf{e}_1 & \mathbf{e}_2\cdot\mathbf{C}\cdot\mathbf{e}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{e}_1\cdot\bar{\mathbf{e}}_1 & \mathbf{e}_1\cdot\bar{\mathbf{e}}_2 \\ \mathbf{e}_2\cdot\bar{\mathbf{e}}_1 & \mathbf{e}_2\cdot\bar{\mathbf{e}}_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$





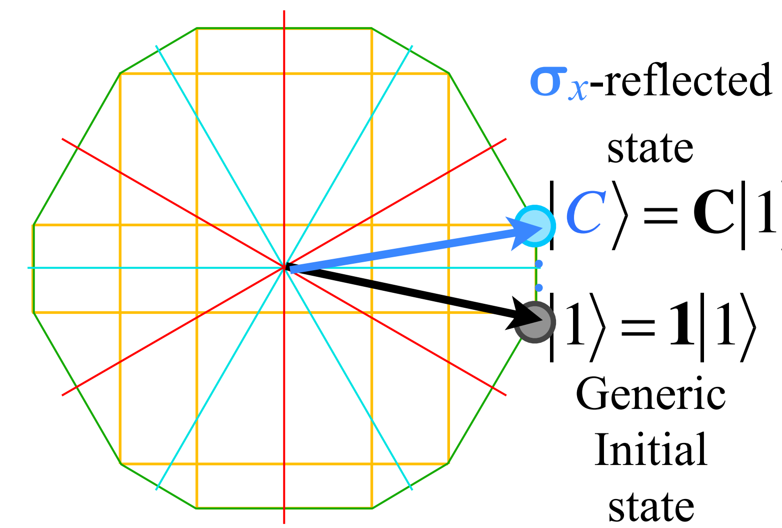
Effects of Ceiling Bang Matrix $\mathbf{C}=\sigma_z = \begin{pmatrix} e_1\cdot\mathbf{C}\cdot e_1 & e_1\cdot\mathbf{C}\cdot e_2 \\ e_2\cdot\mathbf{C}\cdot e_1 & e_2\cdot\mathbf{C}\cdot e_2 \end{pmatrix} = \begin{pmatrix} e_1\cdot\bar{e}_1 & e_1\cdot\bar{e}_2 \\ e_2\cdot\bar{e}_1 & e_2\cdot\bar{e}_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$



$$\bar{e}_1=\mathbf{C}\cdot e_1 =\sigma_z\cdot e_1 = e_1$$

$$e_1 = \bar{e}_1$$

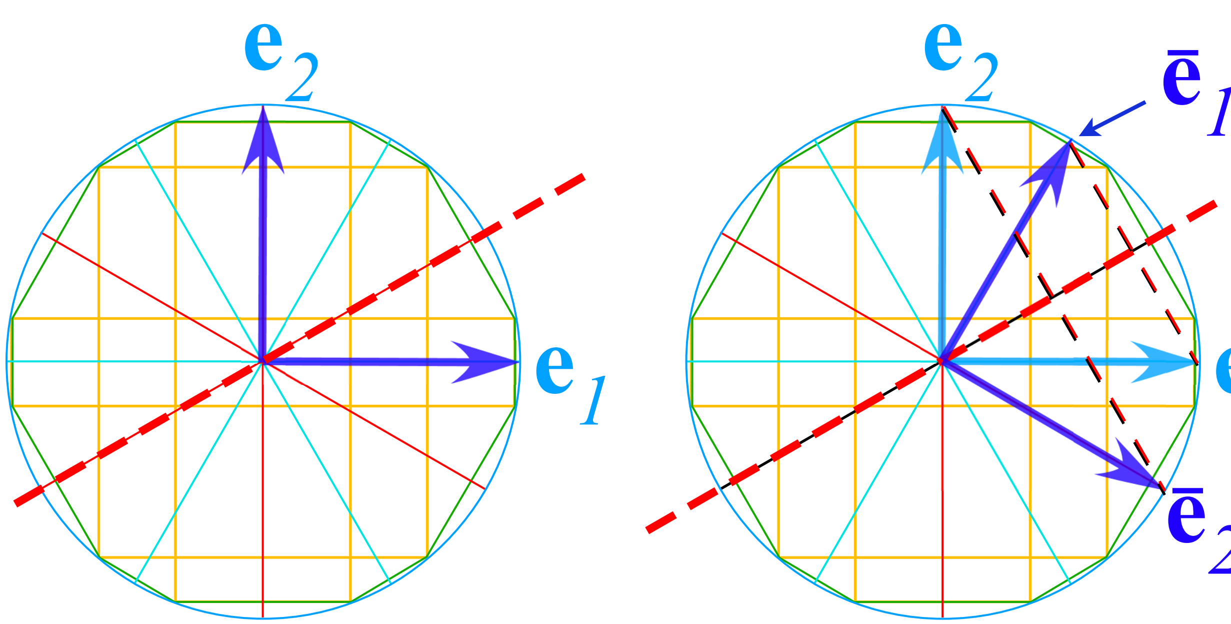
$$\bar{e}_2=\mathbf{C}\cdot e_2=\sigma_z\cdot e_2=-e_2$$



Known as *matrix elements* or *components*

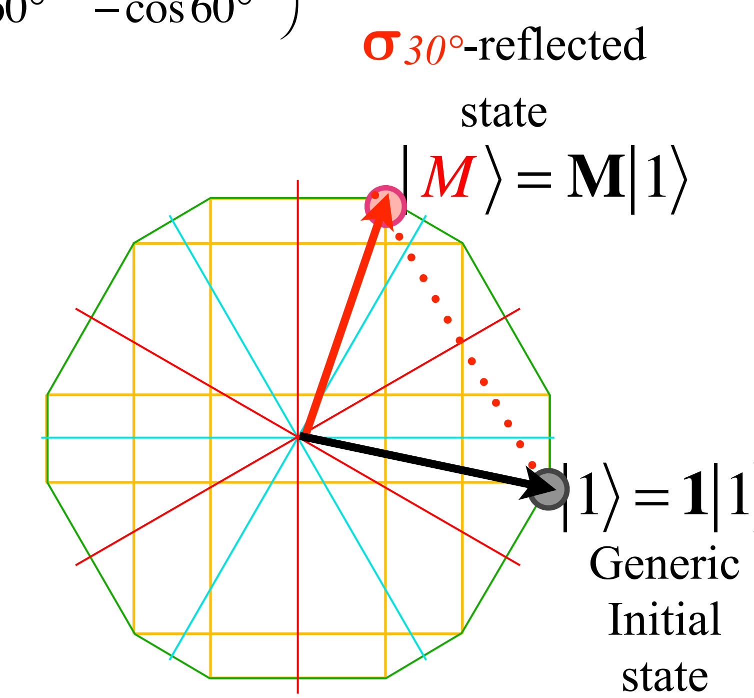
Known as *relative direction cosines*

Effects of Mass Bang Matrix $\mathbf{M}=\sigma_{30^\circ} = \begin{pmatrix} e_1\cdot\mathbf{M}\cdot e_1 & e_1\cdot\mathbf{M}\cdot e_2 \\ e_2\cdot\mathbf{M}\cdot e_1 & e_2\cdot\mathbf{M}\cdot e_2 \end{pmatrix} = \begin{pmatrix} e_1\cdot\bar{e}_1 & e_1\cdot\bar{e}_2 \\ e_2\cdot\bar{e}_1 & e_2\cdot\bar{e}_2 \end{pmatrix} = \begin{pmatrix} \cos 60^\circ & \sin 60^\circ \\ \sin 60^\circ & -\cos 60^\circ \end{pmatrix}$



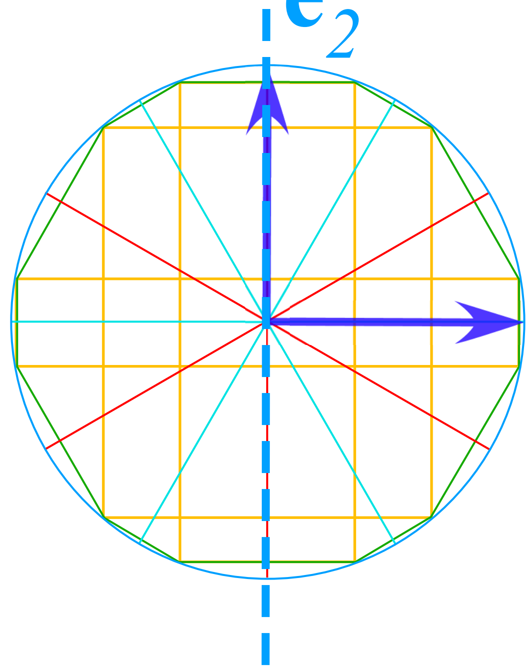
$$\bar{e}_1=\mathbf{M}\cdot e_1 =\sigma_{30^\circ}\cdot e_1$$

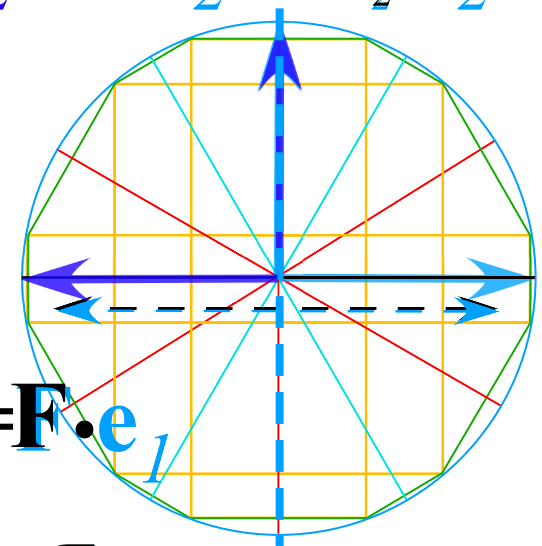
$$\bar{e}_2=\mathbf{M}\cdot e_2 =\sigma_{30^\circ}\cdot e_2$$



Effects of Floor Bang Matrix $\mathbf{F}=-\sigma_z = \begin{pmatrix} \mathbf{e}_1\cdot\mathbf{F}\cdot\mathbf{e}_1 & \mathbf{e}_1\cdot\mathbf{F}\cdot\mathbf{e}_2 \\ \mathbf{e}_2\cdot\mathbf{F}\cdot\mathbf{e}_1 & \mathbf{e}_2\cdot\mathbf{F}\cdot\mathbf{e}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{e}_1\cdot\bar{\mathbf{e}}_1 & \mathbf{e}_1\cdot\bar{\mathbf{e}}_2 \\ \mathbf{e}_2\cdot\bar{\mathbf{e}}_1 & \mathbf{e}_2\cdot\bar{\mathbf{e}}_2 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

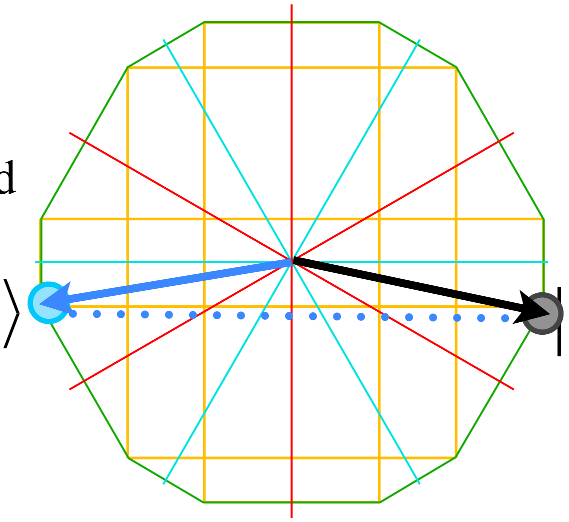
$\bar{\mathbf{e}}_2=\mathbf{F}\cdot\mathbf{e}_2=-\sigma_z\cdot\mathbf{e}_2=+\mathbf{e}_2$



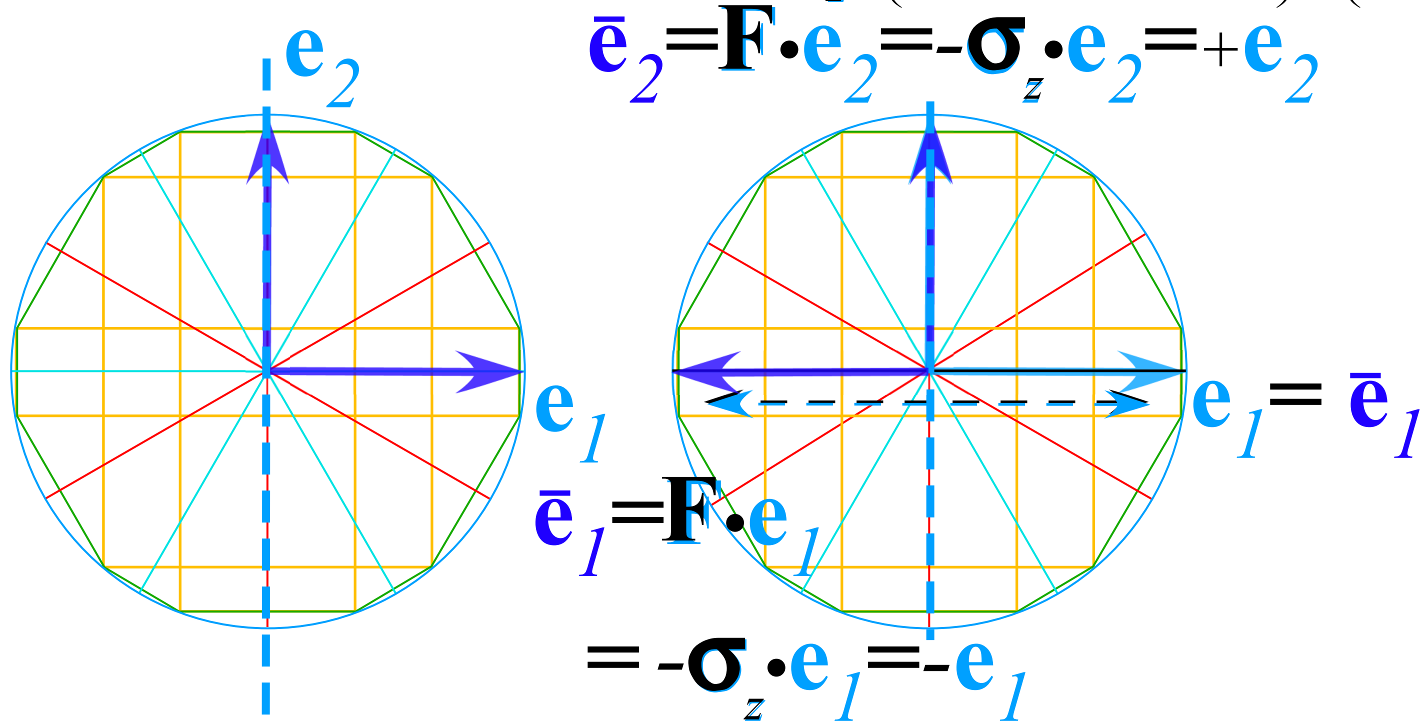


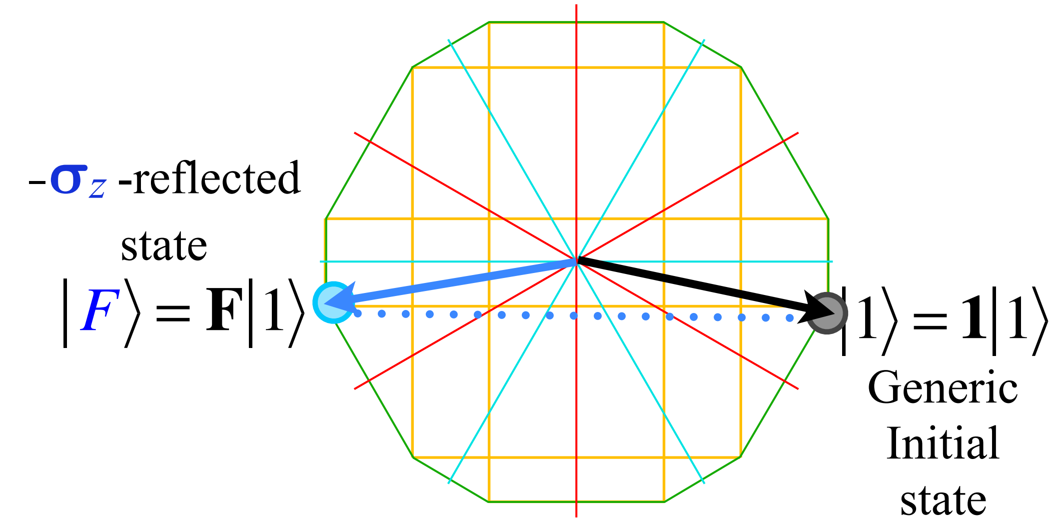
$\bar{\mathbf{e}}_1=\mathbf{F}\cdot\mathbf{e}_1$

$=-\sigma_z\cdot\mathbf{e}_1=-\mathbf{e}_1$

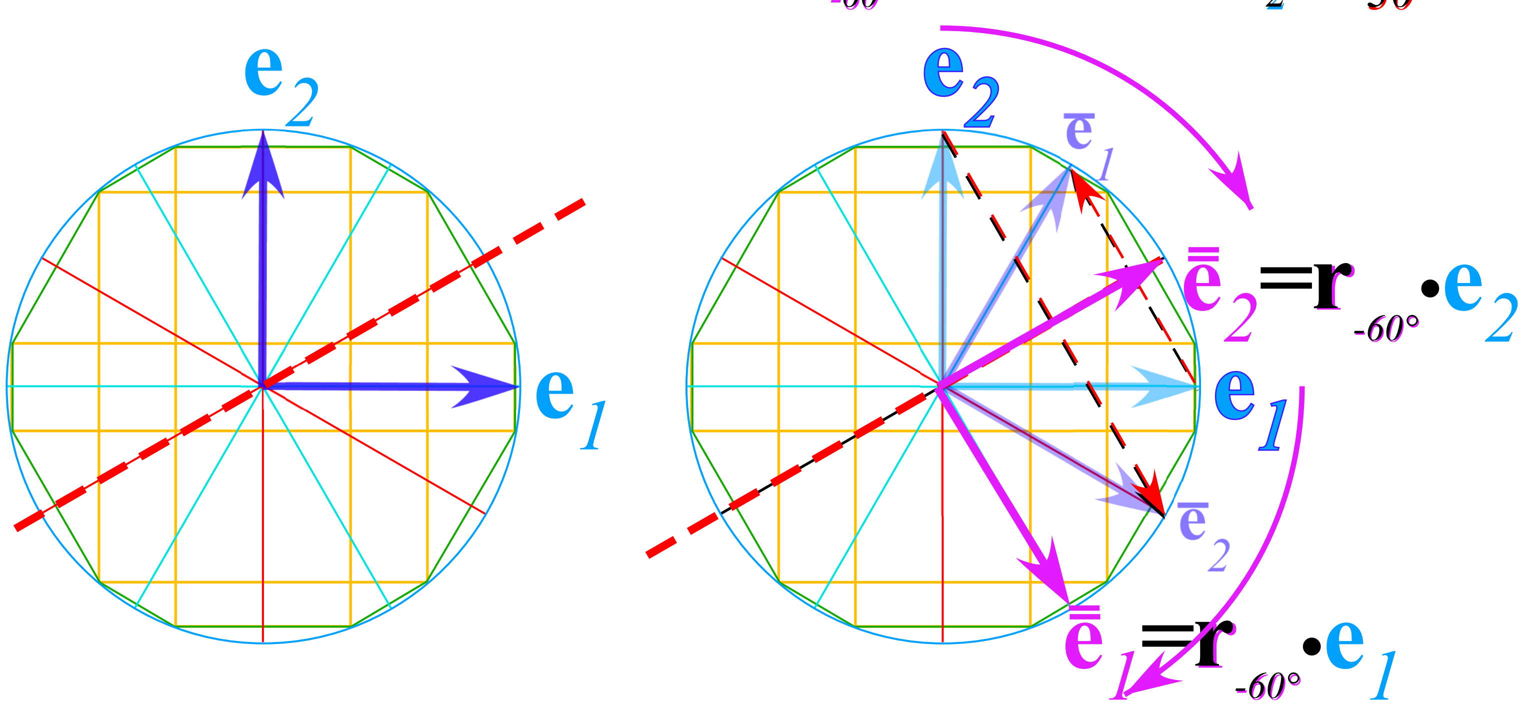


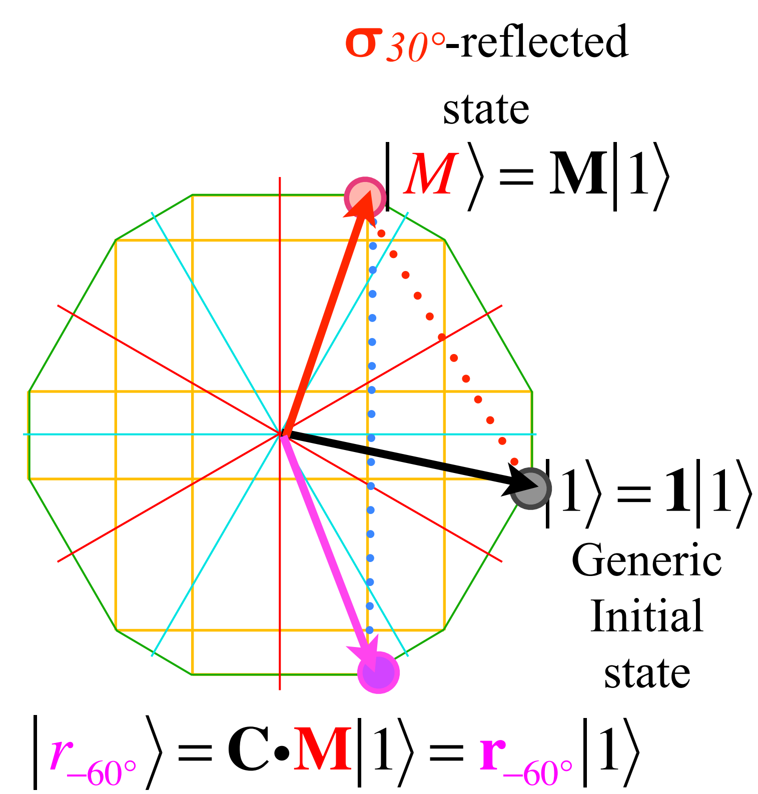
Effects of Floor Bang Matrix $\mathbf{F}=-\sigma_z=\begin{pmatrix} e_1\cdot\mathbf{F}\cdot e_1 & e_1\cdot\mathbf{F}\cdot e_2 \\ e_2\cdot\mathbf{F}\cdot e_1 & e_2\cdot\mathbf{F}\cdot e_2 \end{pmatrix}=\begin{pmatrix} e_1\cdot\bar{e}_1 & e_1\cdot\bar{e}_2 \\ e_2\cdot\bar{e}_1 & e_2\cdot\bar{e}_2 \end{pmatrix}=\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$





Effects of Ceiling **C** after Bang **M**: $\mathbf{r}_{-60°}=\mathbf{C}\cdot\mathbf{M}=\sigma_z\cdot\sigma_{30°}$





$\sigma_{30°}$ $\sigma_{30°}$-reflected state is a $\mathbf{r}_{-60°}$-rotated state

Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

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Reflections in the clothing store: "It's all done with mirrors!"

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→ *Group multiplication and product table* ←

Classical collision paths with $D_6 \sim C_{6v}$ (Resulting from $m_1/m_2=3$)

Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$

D_6	1	r_{120}	$\bar{r}_{120}$	σ_{60}	$\bar{\sigma}_{60}$	σ_z	I	$\bar{r}_{60}$	r_{60}	$\bar{\sigma}_{30}$	σ_{30}	$\bar{\sigma}_z$
1	1											
$\bar{r}_{120}$		1										
r_{120}			1									
σ_{60}				1								
$\bar{\sigma}_{60}$					1							
σ_z						1					$\bar{r}_{60}$	
I							1					
r_{60}								1				
$\bar{r}_{60}$									1			
$\bar{\sigma}_{30}$										1		
σ_{30}						r_{60}					1	
$\bar{\sigma}_z$												1

Note: $\bar{r}_{60} = I r_{120} = r_{120} I = r_{-60}$ and: $I = r_{\pm 180}$

$\bar{r}_{120} = I r_{60} = r_{60} I = r_{-120}$ and: $I^2 = 1$

$\sigma_{60} = I\bar{\sigma}_{30} = \bar{\sigma}_{30} I$

$\bar{\sigma}_{60} = I\sigma_{30} = \sigma_{30} I$

$\bar{\sigma}_z = I\sigma_z = \sigma_z I$

Easy to make hexagonal (D_6) symmetry group table:

Example 1: Find $\boldsymbol{\sigma}_{30°}\bullet\boldsymbol{\sigma}_{-60°}$ = ______?

Solution: Find $\boldsymbol{\sigma}_{30°}$*-plane and state-*$|\boldsymbol{\sigma}_{-60°}\rangle$

Operate former on latter to get: $\boldsymbol{\sigma}_{30°}|\boldsymbol{\sigma}_{-60°}\rangle = |\mathbf{I}\rangle$

That gives answer: $\boldsymbol{\sigma}_{30°}\bullet\boldsymbol{\sigma}_{-60°} = \mathbf{I}$.

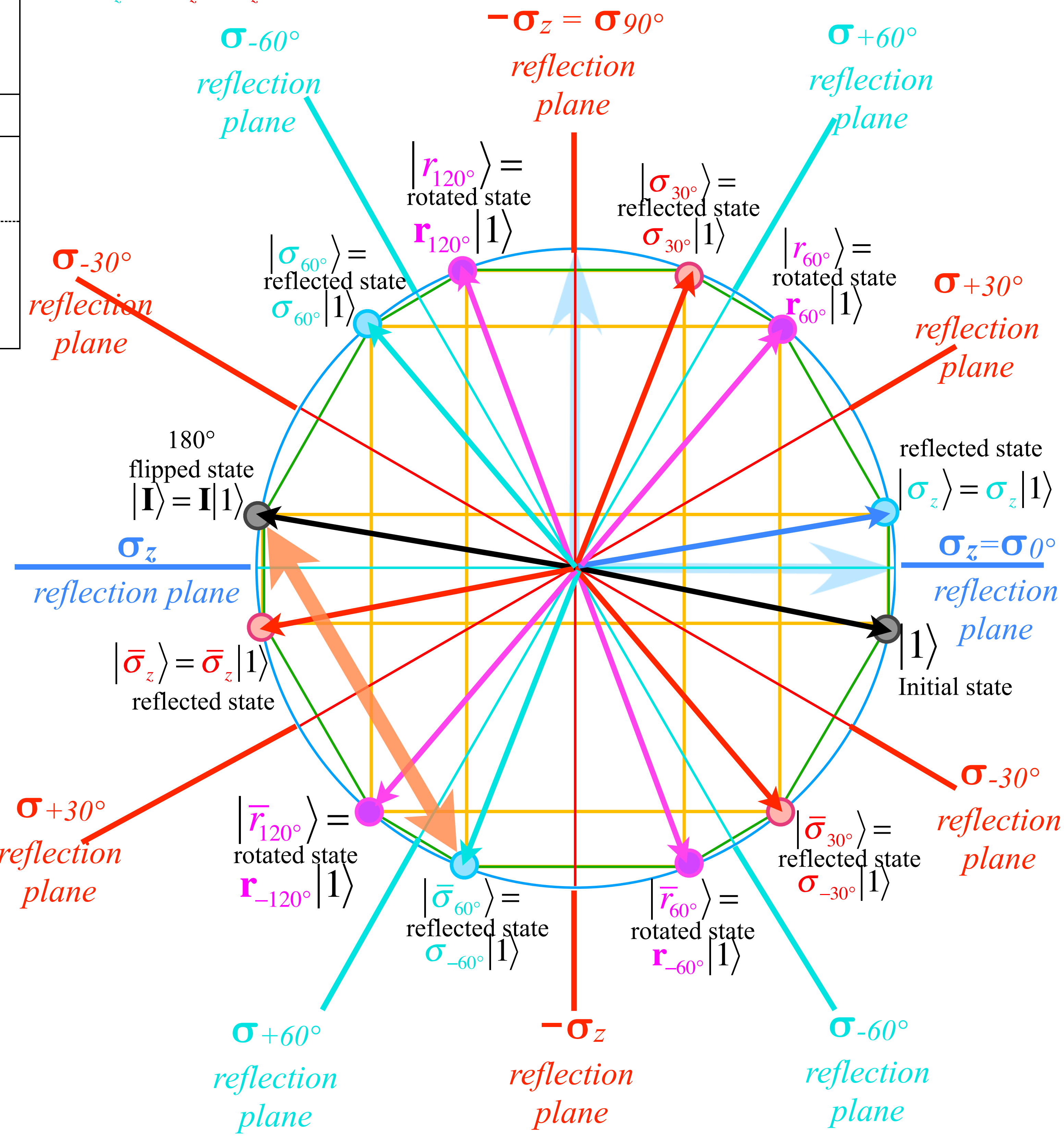
Rest of $\boldsymbol{\sigma}_{30°}$ *row follows:*

11^{th} row	1	r_{120}	$\bar{r}_{120}$	σ_{60}	$\bar{\sigma}_{60}$	σ_z	I	$\bar{r}_{60}$	r_{60}	$\bar{\sigma}_{30}$	σ_{30}	$\bar{\sigma}_z$
σ_{30}	σ_{30}	$\bar{\sigma}_{30}$	$\bar{\sigma}_z$	$\bar{r}_{60}$	I	r_{60}	$\bar{\sigma}_{60}$	σ_{60}	σ_z	r_{120}	1	$\bar{r}_{120}$

Example 2: Find $\mathbf{r}_{60°}\bullet\boldsymbol{\sigma}_{-60°}$ = ______?

Solution: Do $\mathbf{r}_{60°}$*-rotation* $\mathbf{r}_{60°}|\boldsymbol{\sigma}_{-60°}\rangle = |\boldsymbol{\sigma}_{-30°}\rangle$

That gives answer: $\mathbf{r}_{60°}\bullet\boldsymbol{\sigma}_{-60°} = \boldsymbol{\sigma}_{-30°}$



Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on

Reflections in the clothing store: "It's all done with mirrors!"

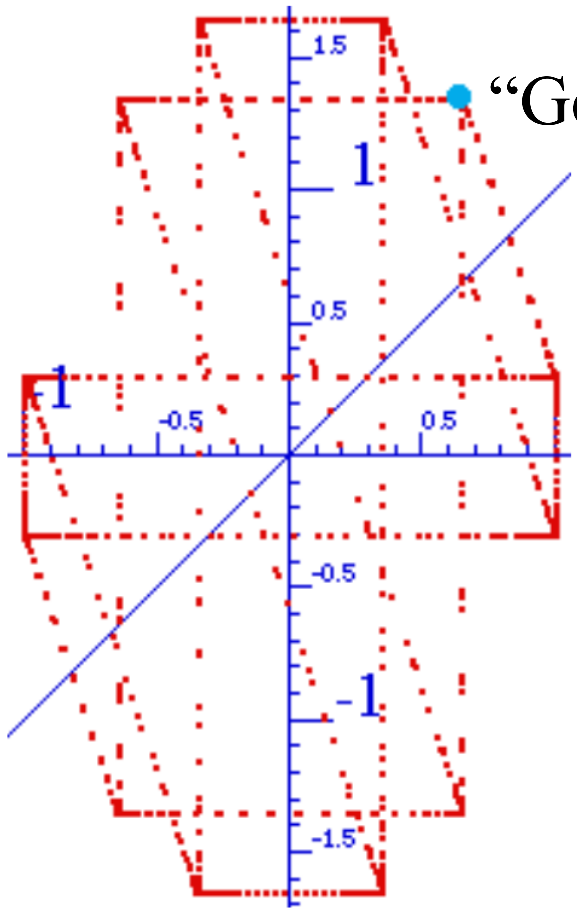
Introducing hexagonal symmetry D_6~C_{6v} (Resulting for m_1/m_2=3)

Group multiplication and product table

→ *Classical collision paths with D_6~C_{6v} (Resulting from m_1/m_2=3)*

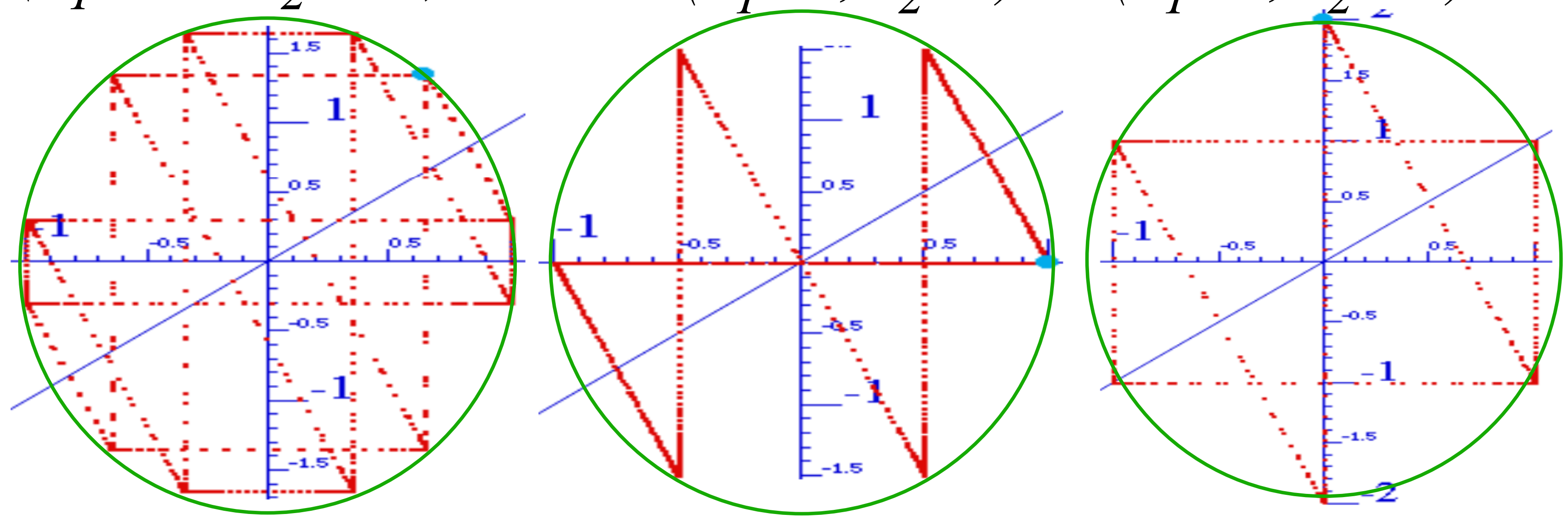
Other not-so-symmetric examples: m_1/m_2=4 and m_1/m_2=7

Collisions for mass ratio $m_1:m_2 = 3:1$

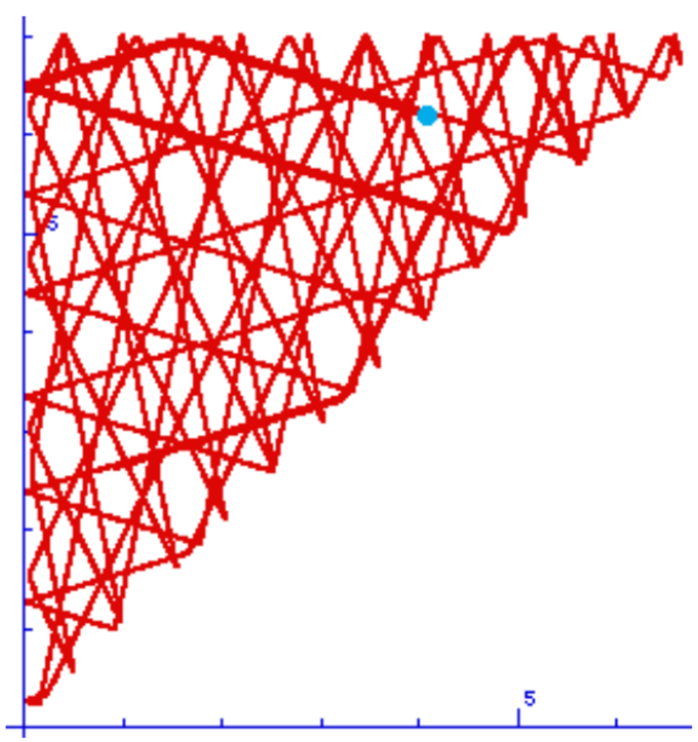


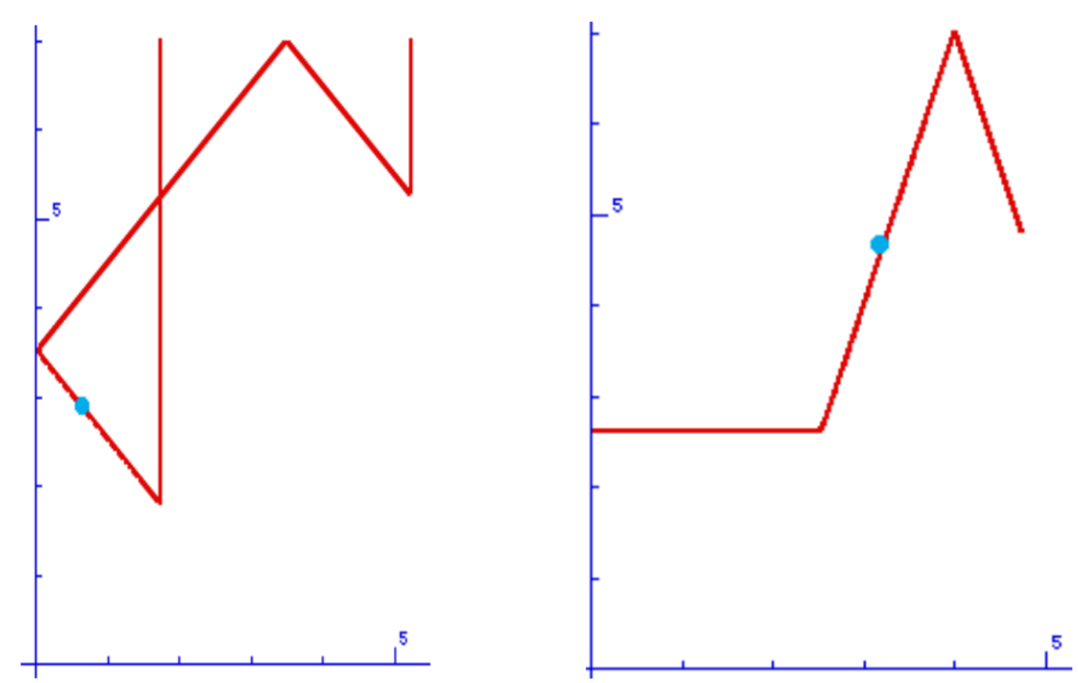
"Generic" initial velocity ($v_1=1.0, v_2=0.1$)

"Symmetric" initial velocity ($v_1=1, v_2=0$) or ($v_1=1, v_2=-1$)

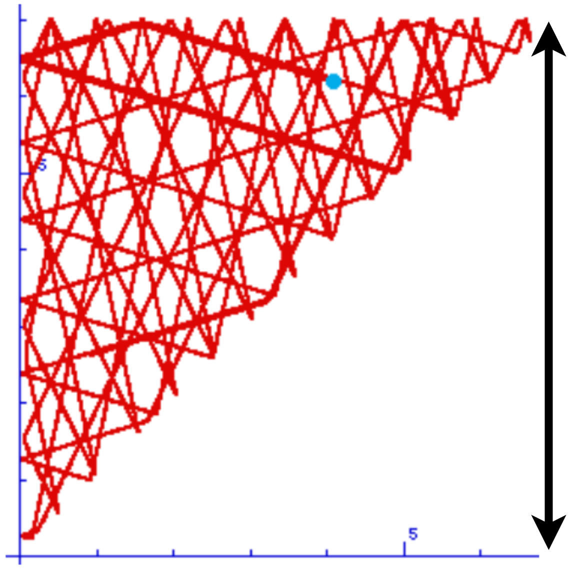


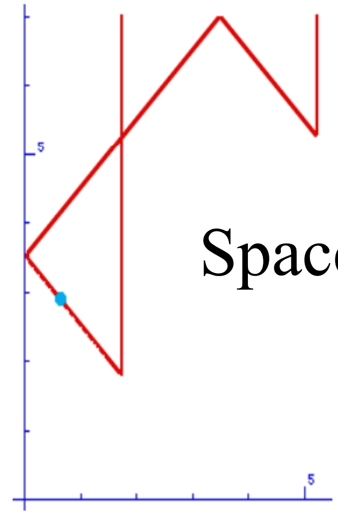
Corresponding space-space (y_1, y_2) paths



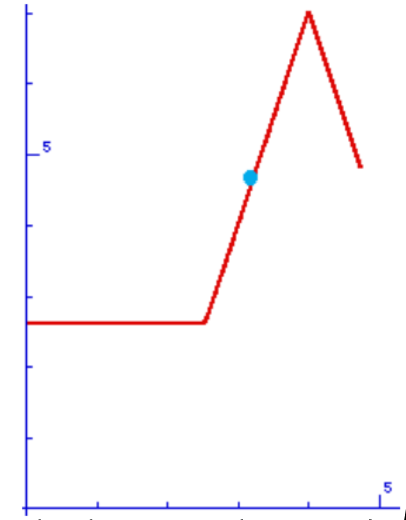


Collisions for mass ratio $m_1:m_2= 3:1$



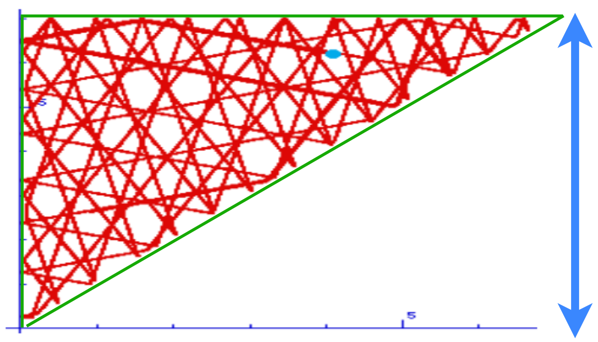


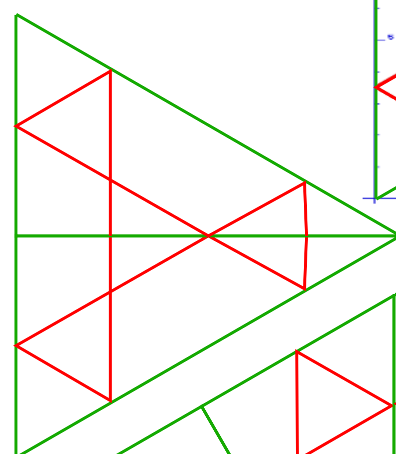
Space-space (y_1, y_2) paths

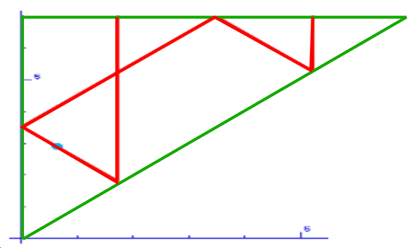


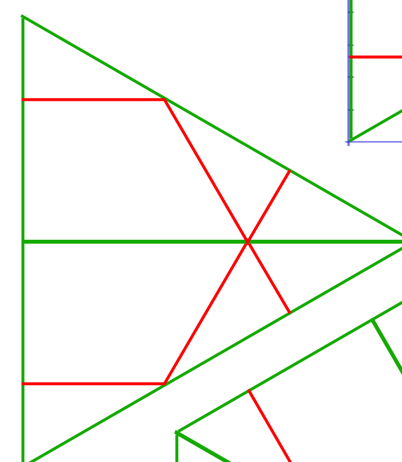
Space-space (y_1, y_2) paths scaled down by $1/\sqrt{3}$...

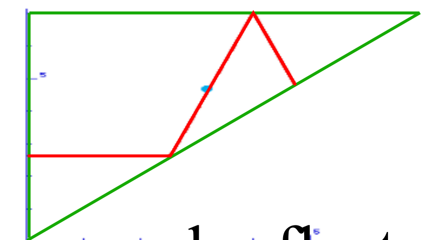
Scaled y down by $1/\sqrt{3}=0.577$





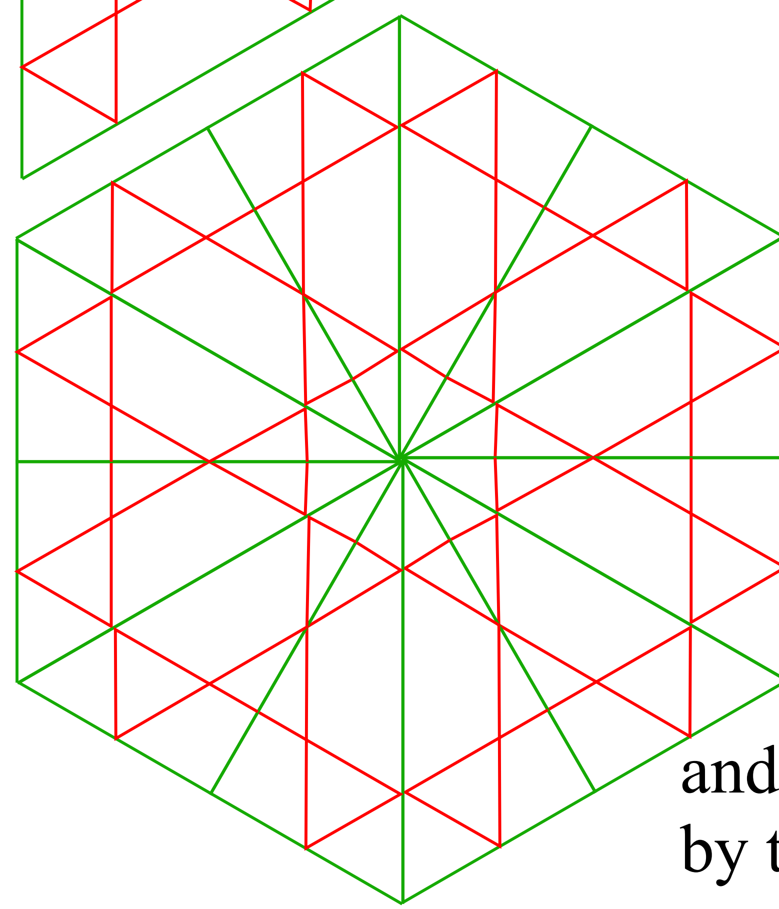


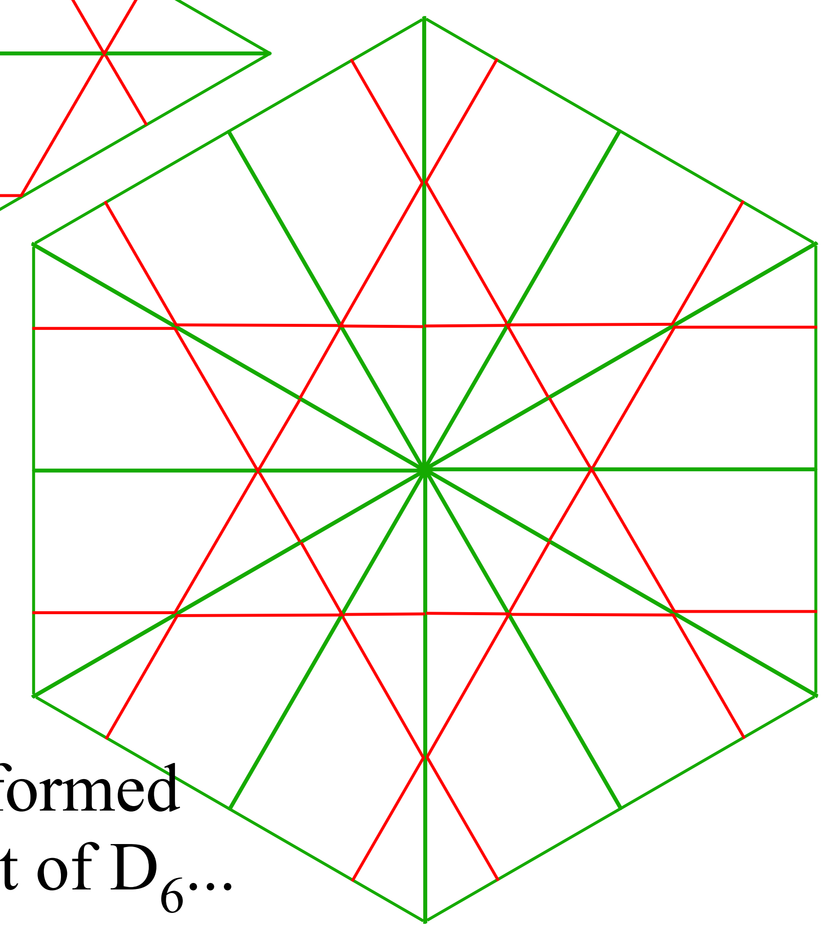




and reflected...

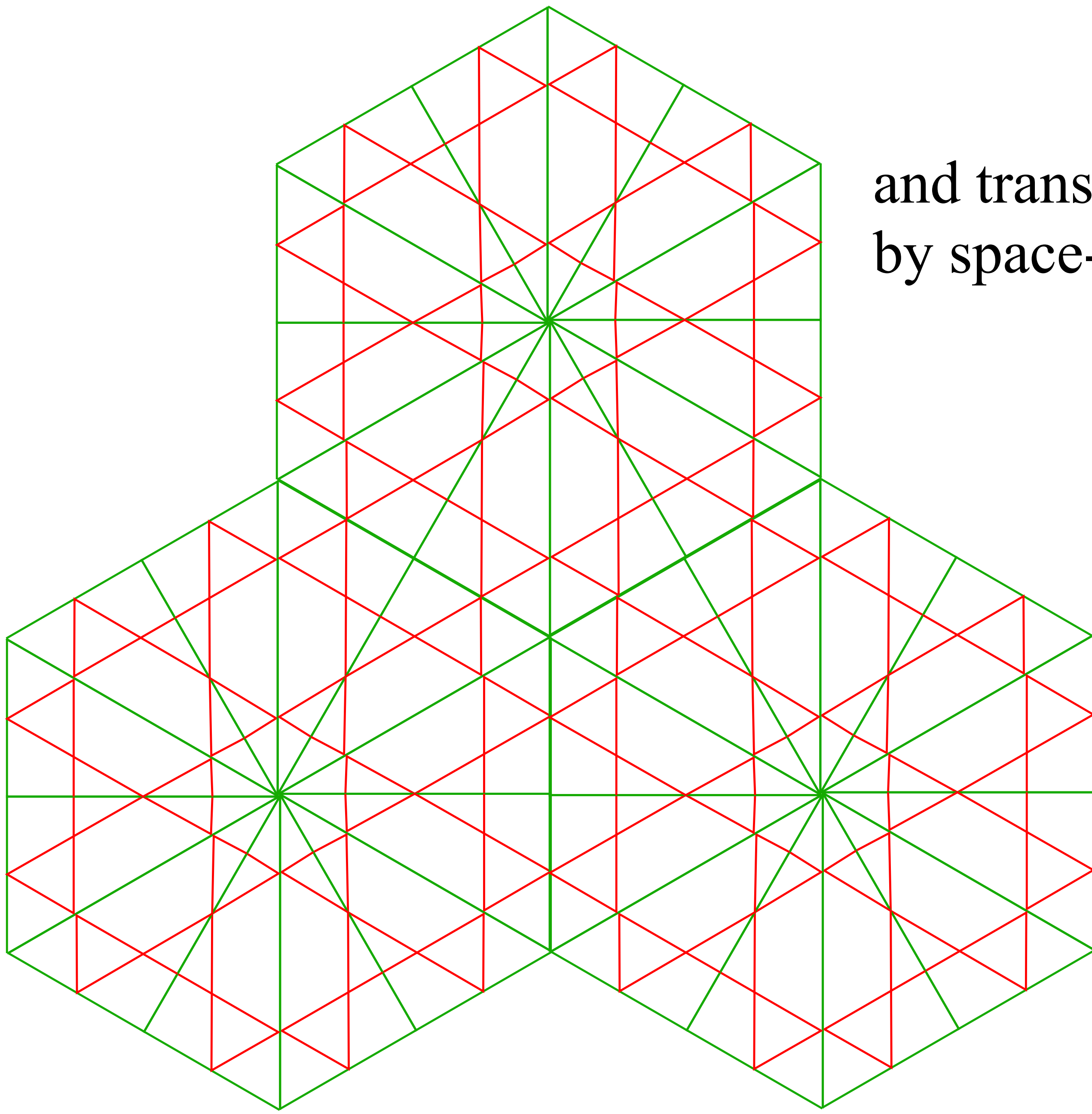
..or could have scaled x up by $\sqrt{3}=1.732$

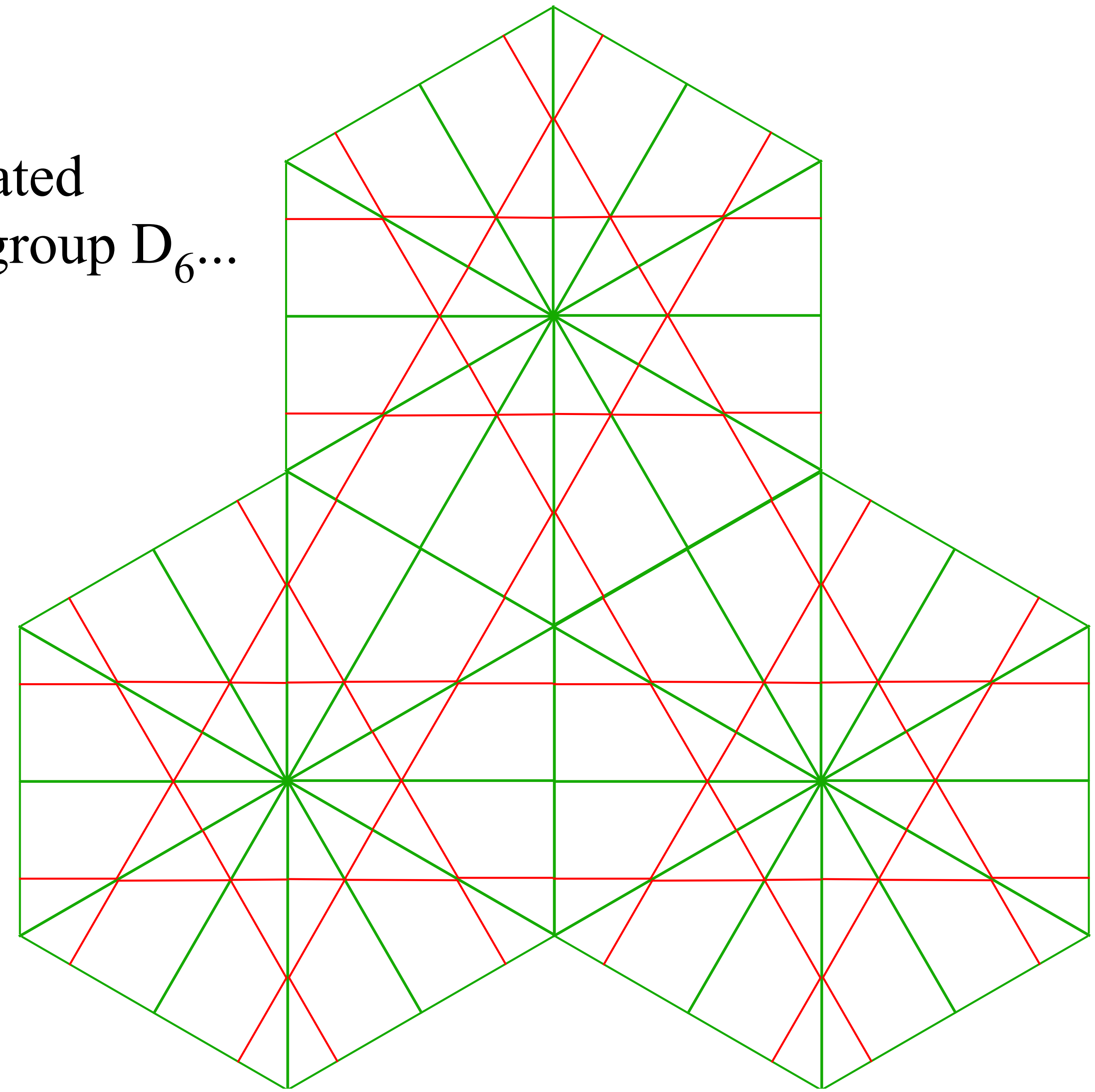




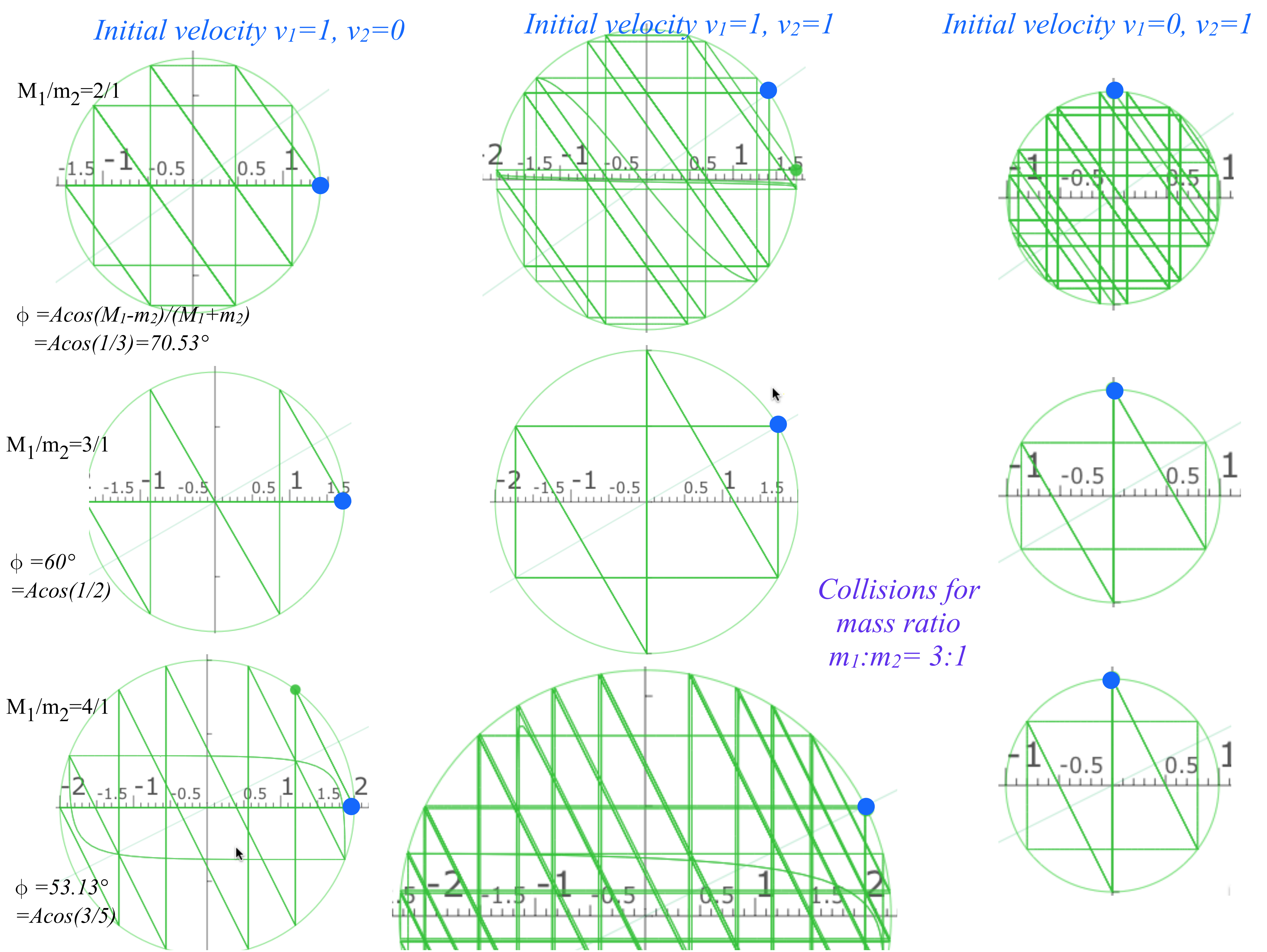
and transformed by the rest of D_6...

and translated
by space-group D_6...





...they're just straight lines going forever.


Initial velocity v1=1, v2=0
Initial velocity v1=1, v2=1
Initial velocity v1=0, v2=1
M1/m2=2/1
ϕ =Acos(M1-m2)/(M1+m2)
=Acos(1/3)=70.53°
M1/m2=3/1
ϕ =60°
=Acos(1/2)
M1/m2=4/1
ϕ =53.13°
=Acos(3/5)
Collisions for
mass ratio
m1:m2= 3:1

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Group multiplication and product table

Classical collision paths with $D_6 \sim C_{6v}$ (Resulting from $m_1/m_2=3$)

→ *Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$* ←

Geometric "Integration" (Converting Velocity data to Spacetime)

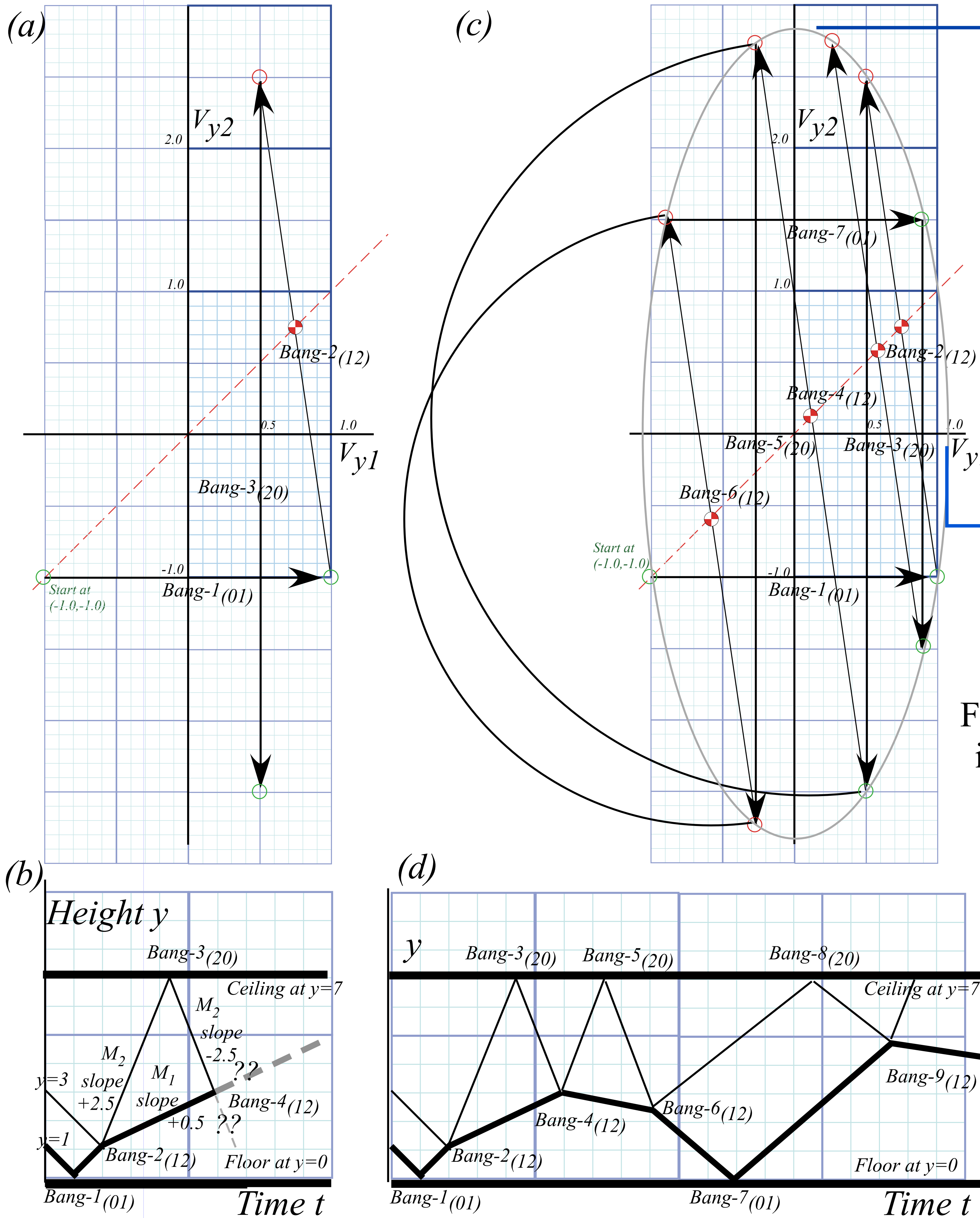


Fig. 4.7a-d in Unit 1

Kinetic Energy Ellipse

$$KE = \frac{1}{2}M_1V_1^2 + \frac{1}{2}M_2V_2^2 = \frac{7}{2} + \frac{1}{2} = 4$$

$$1 = \frac{V_1^2}{2KE/M_1} + \frac{V_2^2}{2KE/M_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Ellipse radius 1

$$a_1 = \sqrt{2KE/M_1} = \sqrt{2KE/7} = \sqrt{8/7} = 1.07$$

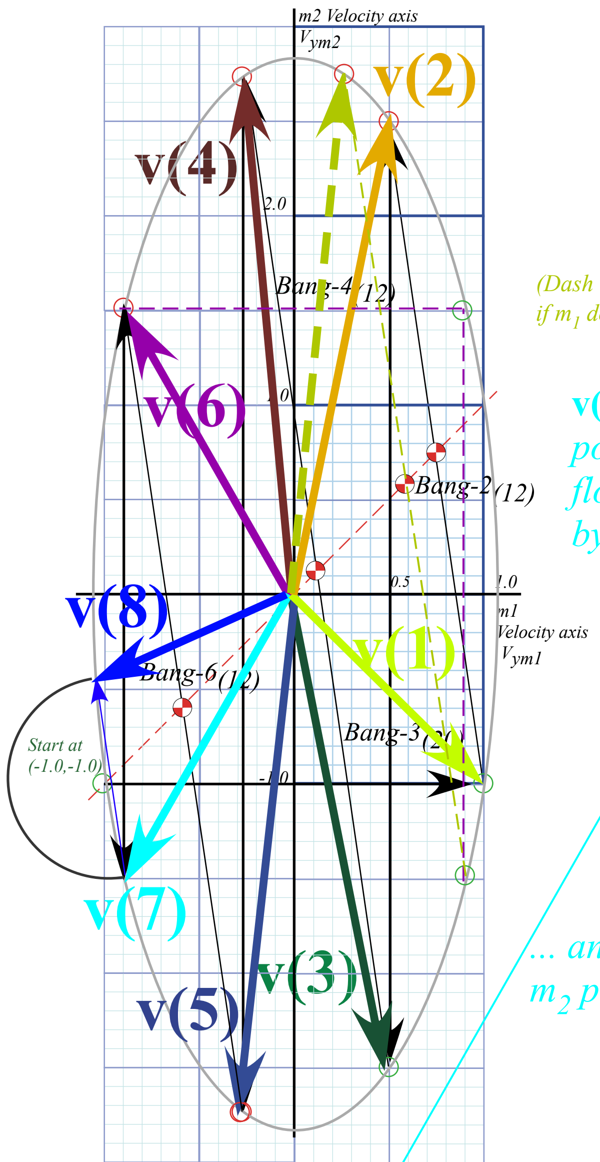
Ellipse radius 2

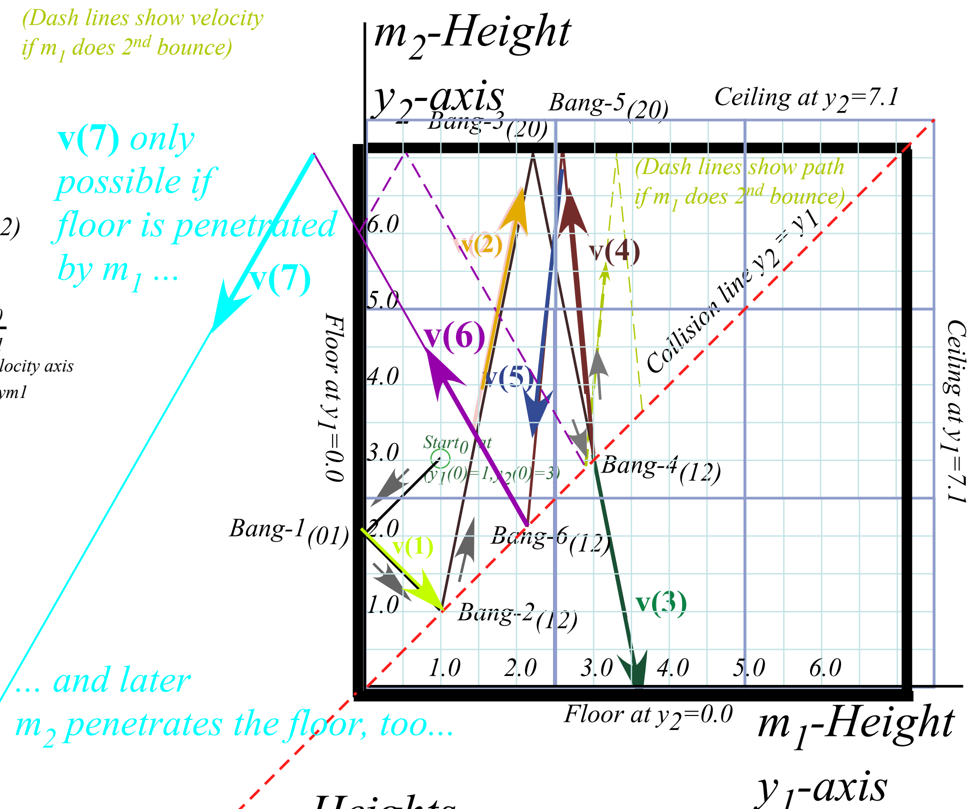
$$a_2 = \sqrt{2KE/M_1} = \sqrt{2KE/1} = \sqrt{8/1} = 2.83$$

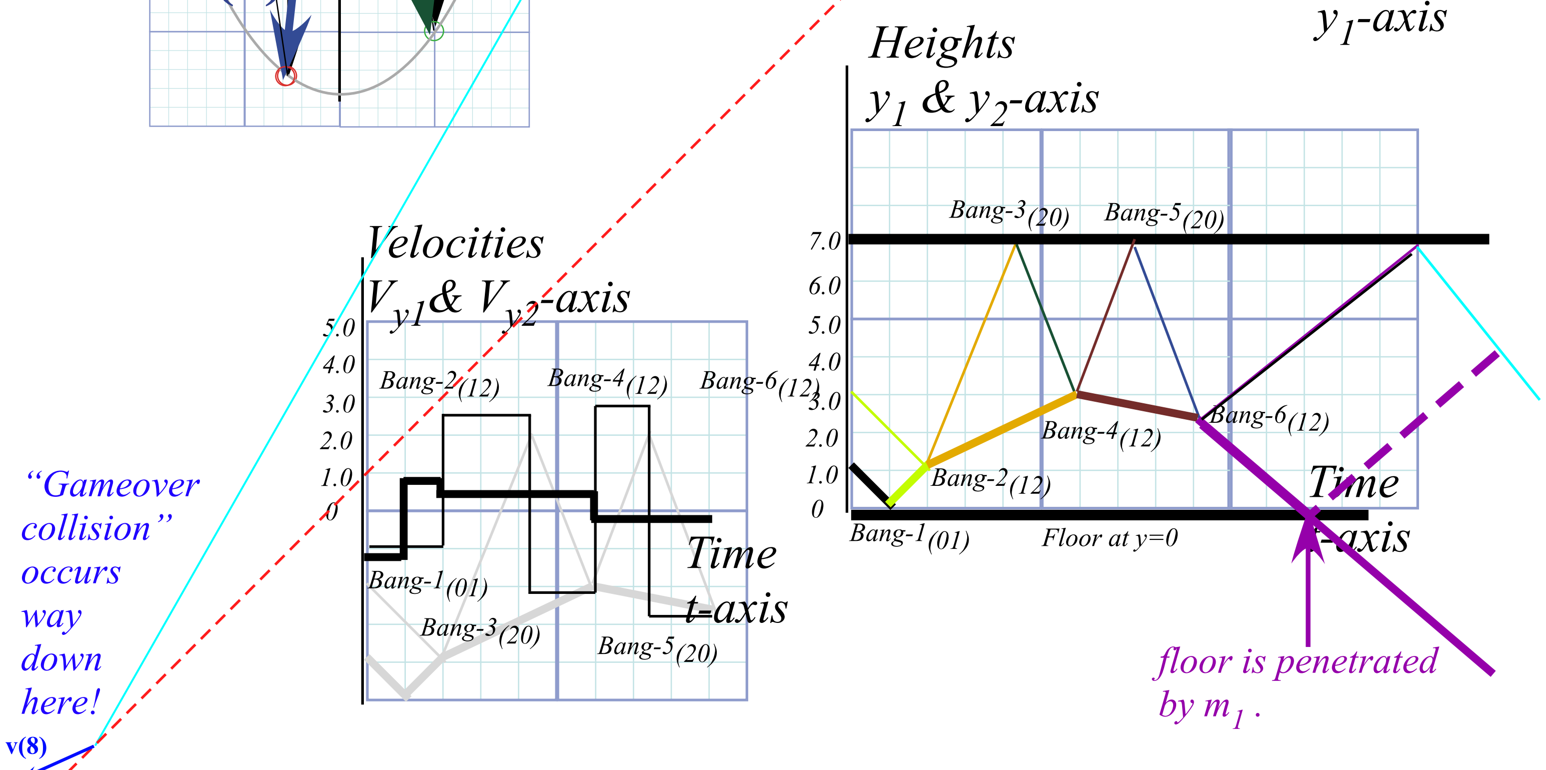
Collisions for mass ratio $m_1 : m_2 = 7:1$

Collisions for mass ratio $m_1{:}m_2 = 7{:}1$

Step-2: Extend **v**(2) *line to ceiling point* **y**(3)=(?,7.1) *and draw Bang-3*$_{(20)}$
velocity **v**(3)=(1,-1) *line. (Find* **v**(3) *using V-V plot.)*
Step-3: Extend **v**(3) *line to collision point* **y**(4)=(?,?) *and draw Bang-4*$_{(12)}$
velocity **v**(4)=(0.5,2.5). *(Find* **v**(4) *using V-V plot.)*
Step-4: Extend **v**(4) *line to ceiling point* **y**(4)=(?,7.1) *and draw Bang-5*$_{(20)}$
velocity **v**(5)=(1,-1) *line. (Find* **v**(5) *using V-V plot.)*
Step-5: Extend **v**(5) *line to collision point* **y**(6)=(?,?) *and draw Bang-6*$_{(12)}$
velocity **v**(6)=(0.5,2.5). *(Find* **v**(6) *using V-V plot.)*







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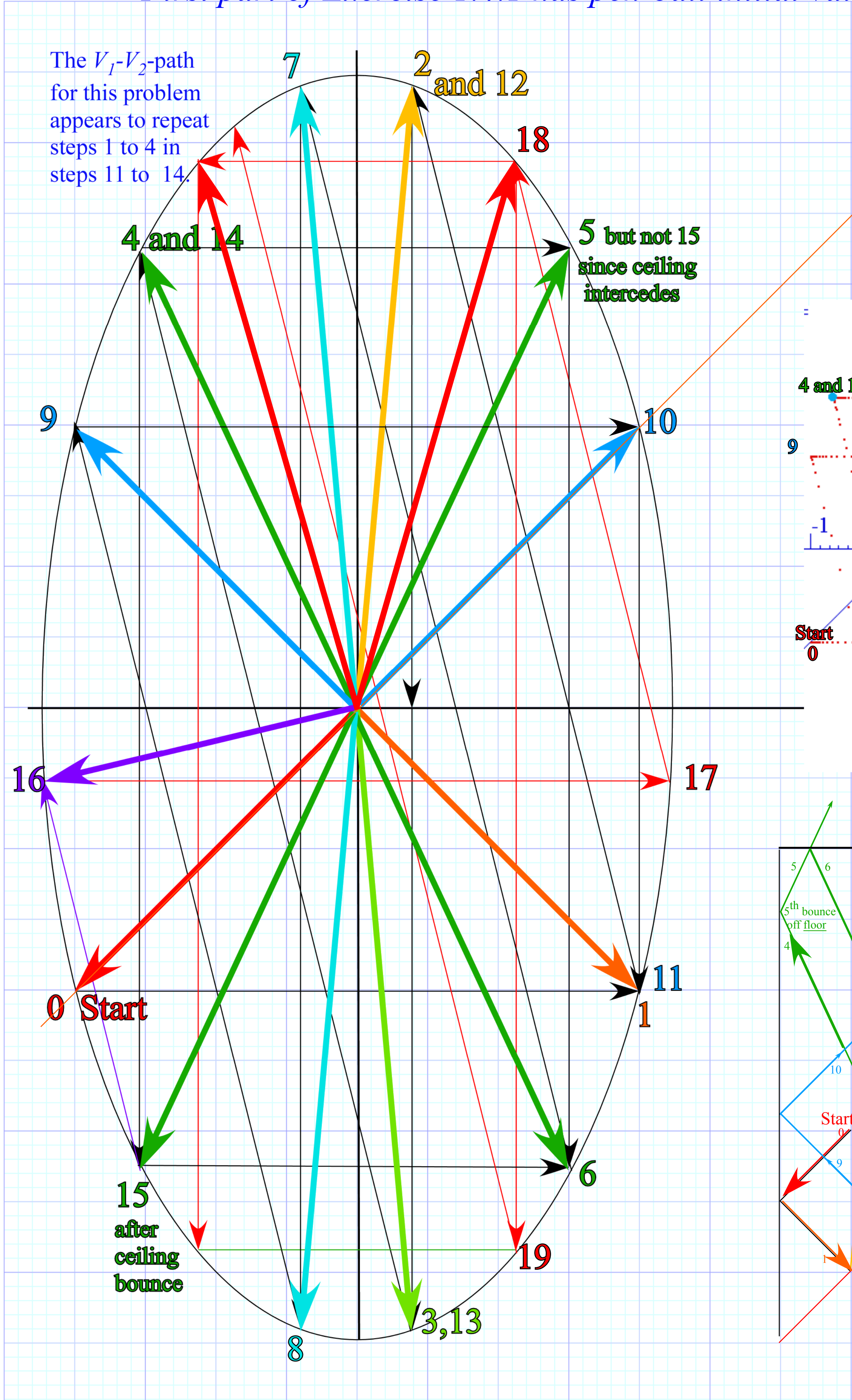
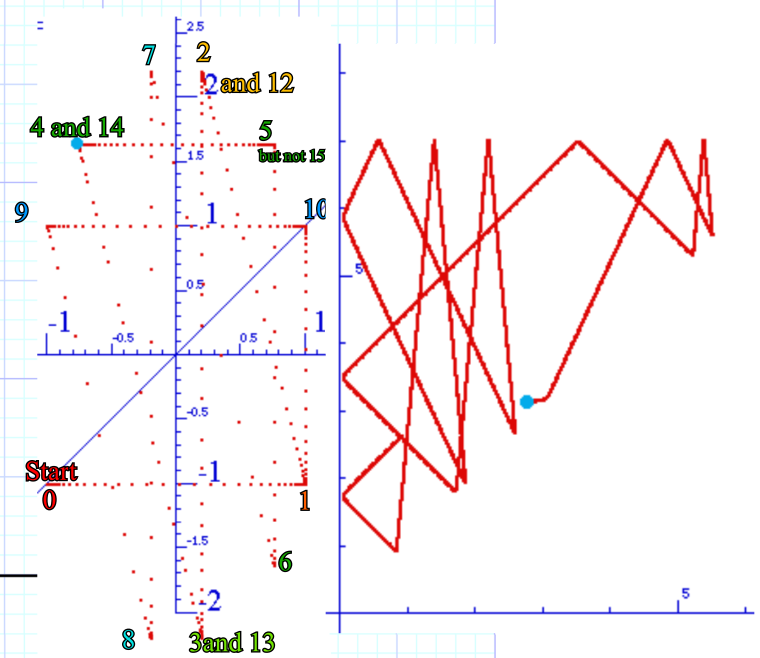
Classical collision paths with $D_6 \sim C_{6v}$ (Resulting from $m_1/m_2=3$)

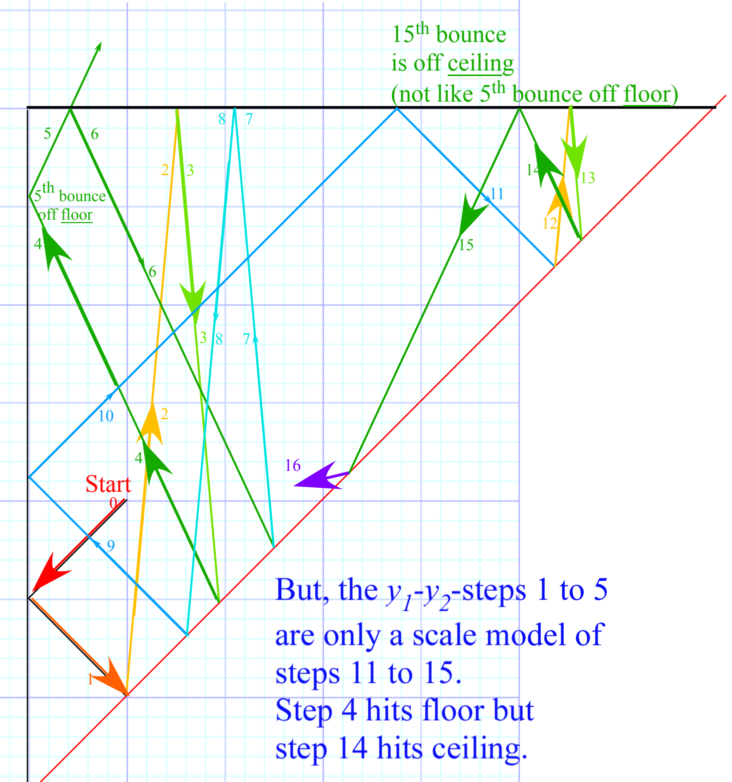
Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$

First part of Exercise 1.4.1 has pen-ball initial values $v_1(0)=-1=v_2(0)$

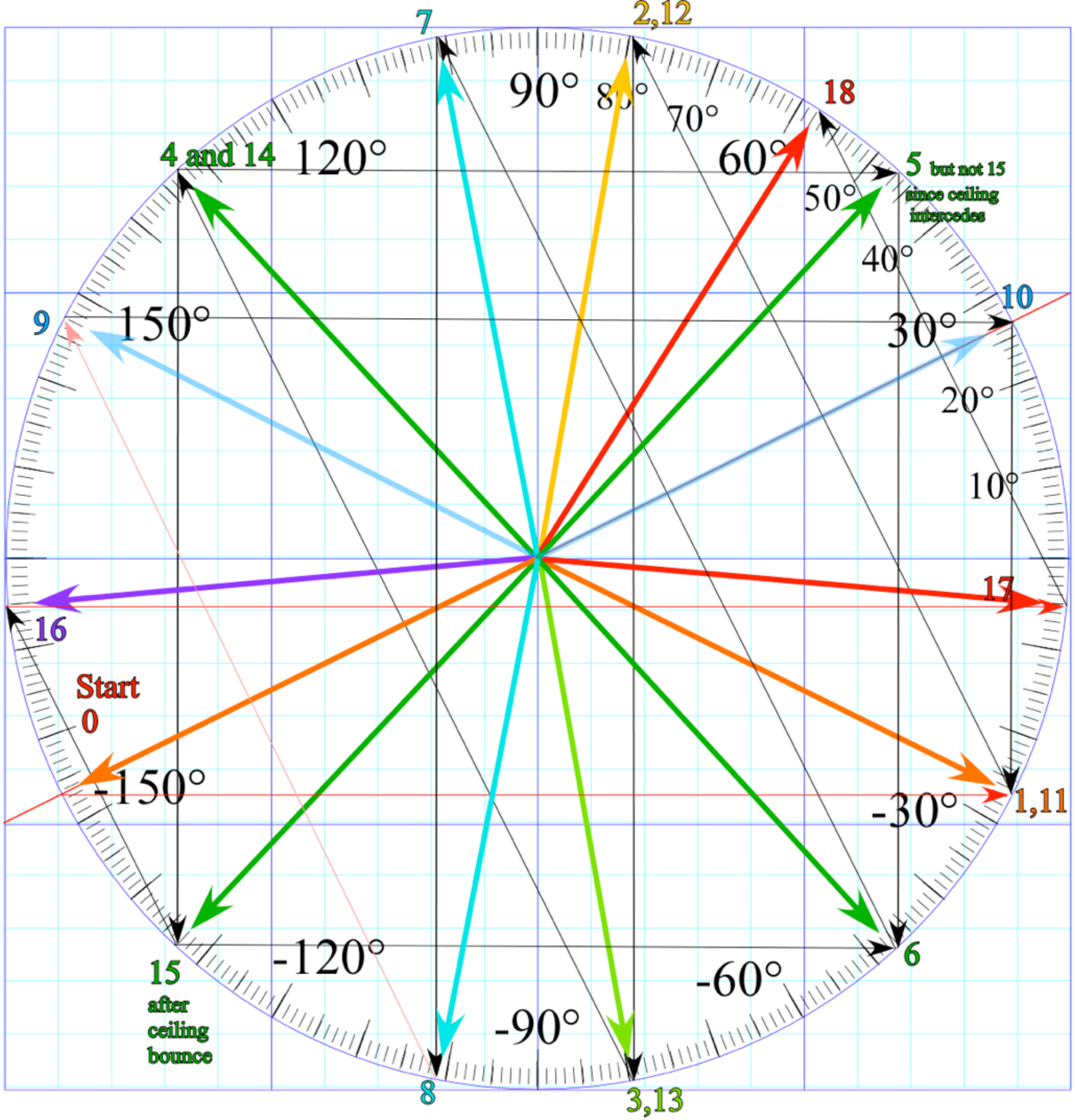
Collisions for mass ratio $m_1:m_2= 4:1$







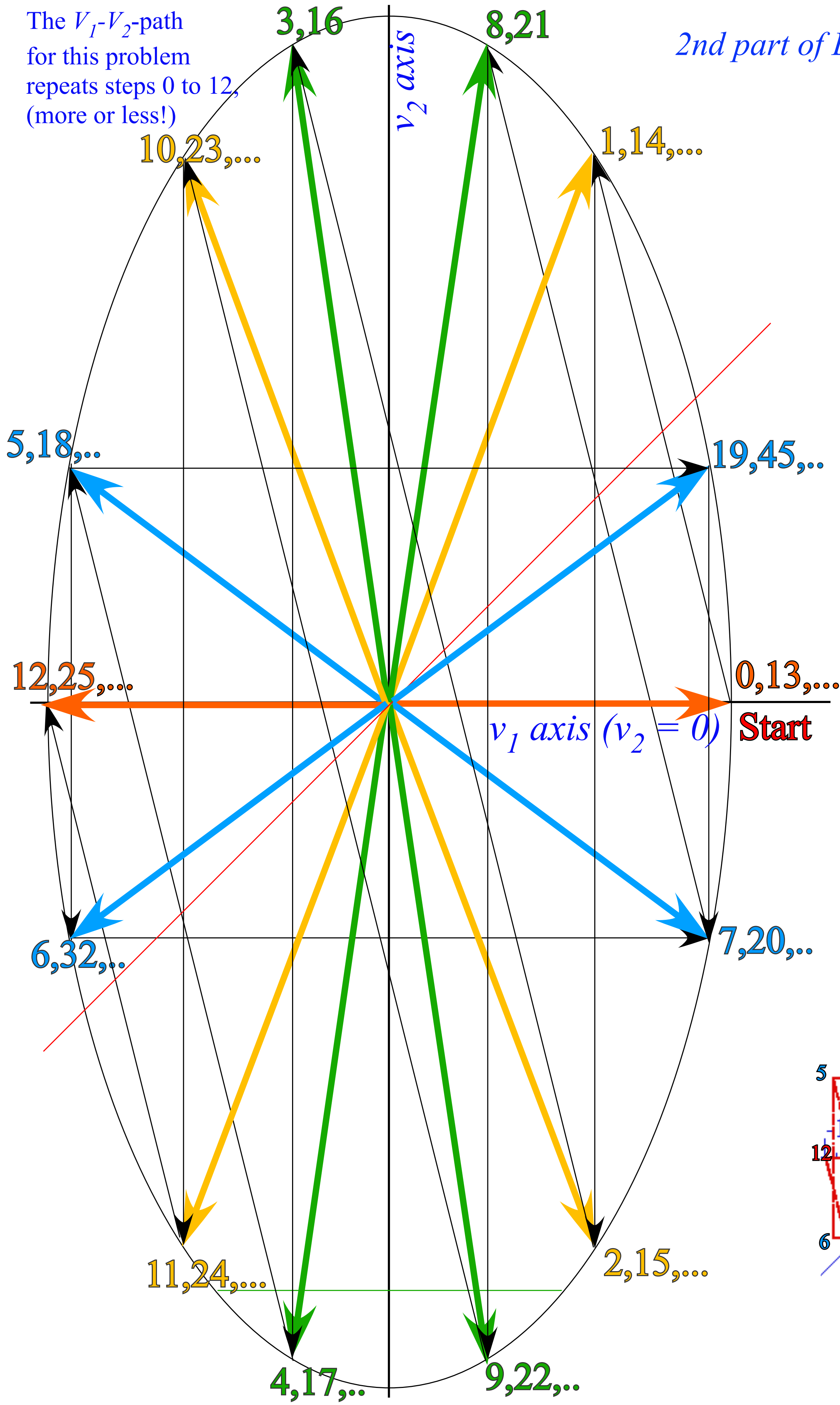
But, the y_1-y_2-steps 1 to 5 are only a scale model of steps 11 to 15.
Step 4 hits floor but step 14 hits ceiling.

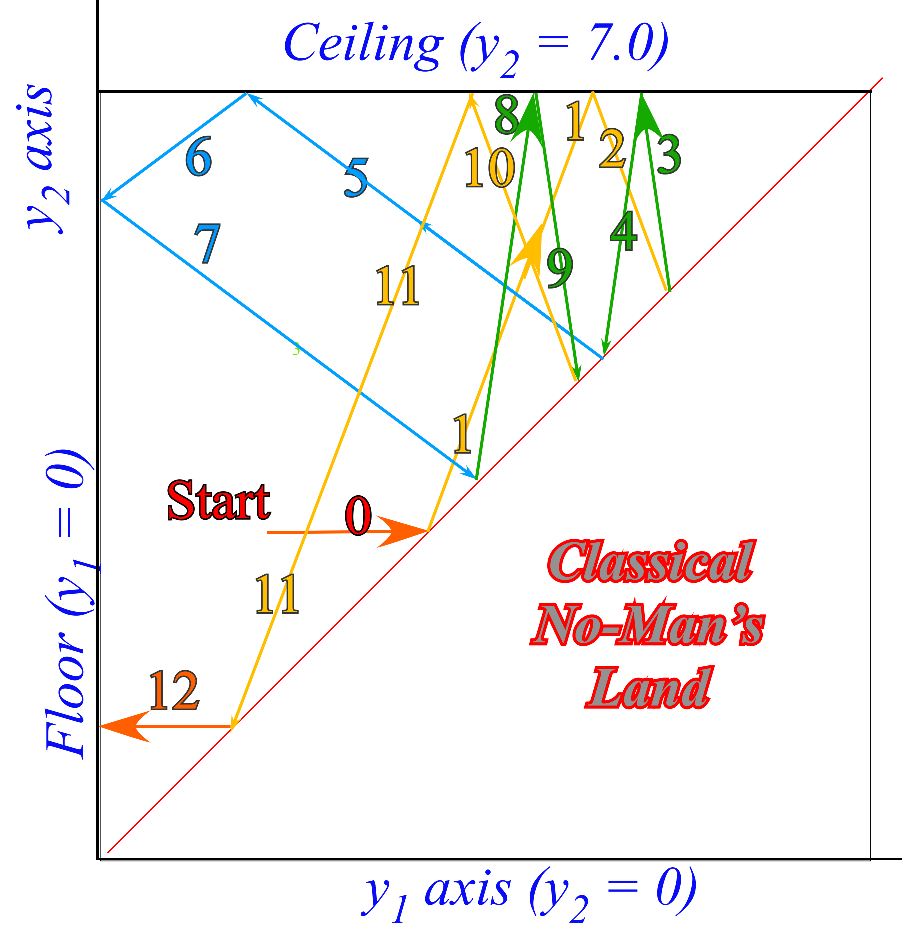


Collisions for mass ratio m_1:m_2= 4:1

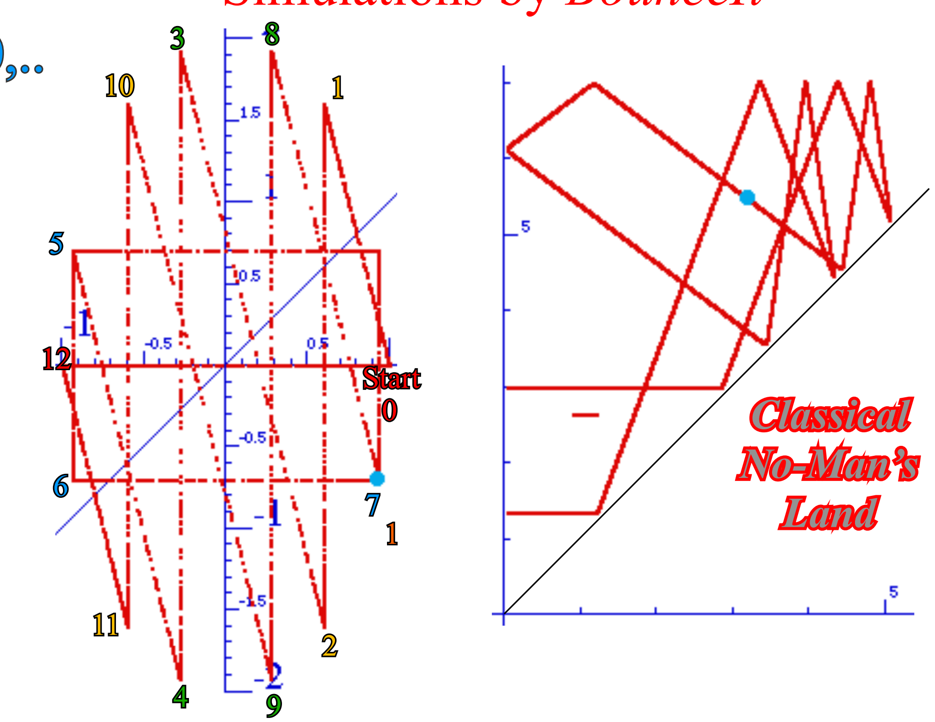
2nd part of Exercise 1.4.1 has pen-ball initial velocity values $v_1(0)=1$ and $v_2(0)=0$

at: $x_1(0)=1.5$ and $x_2(0)=3.0$

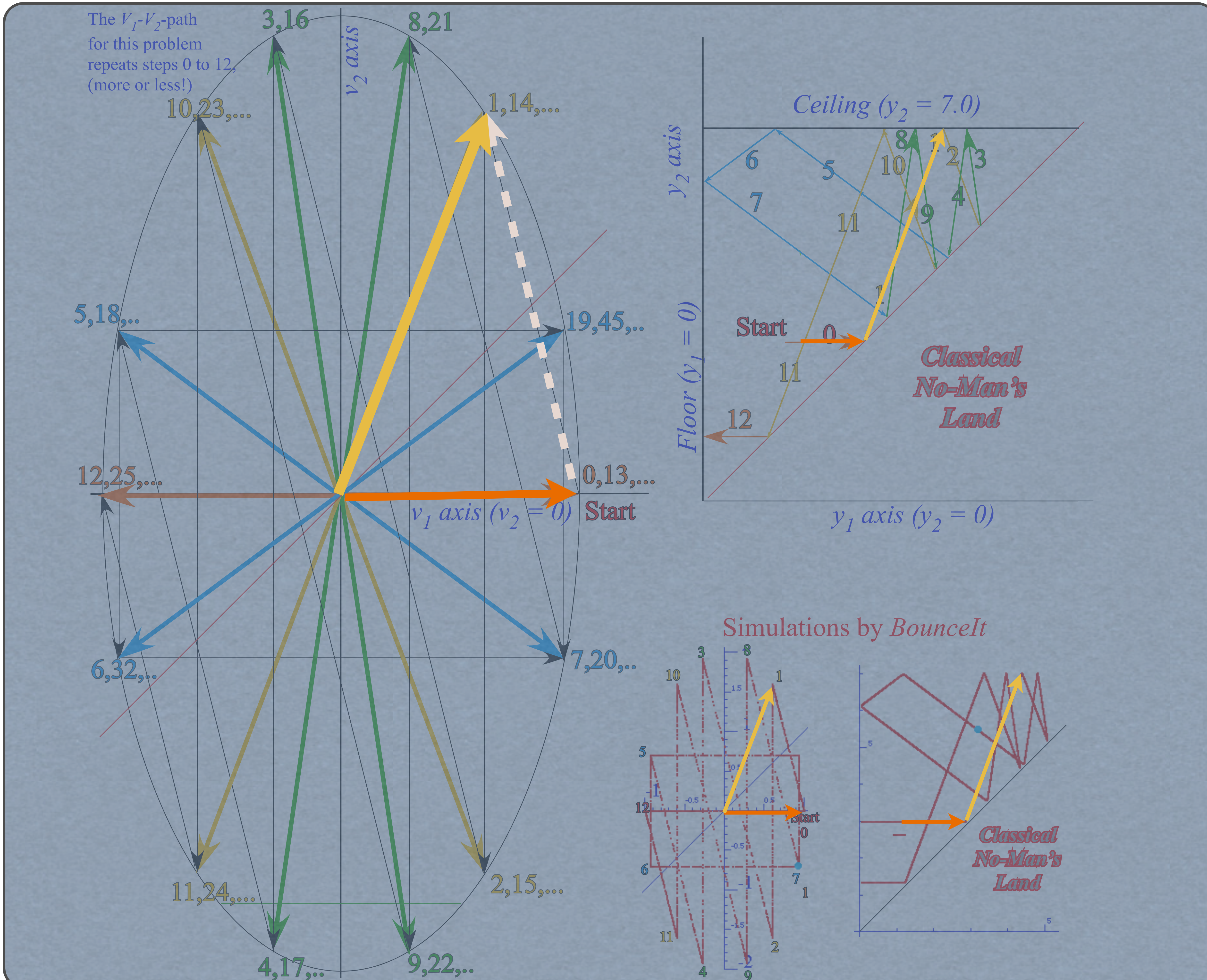


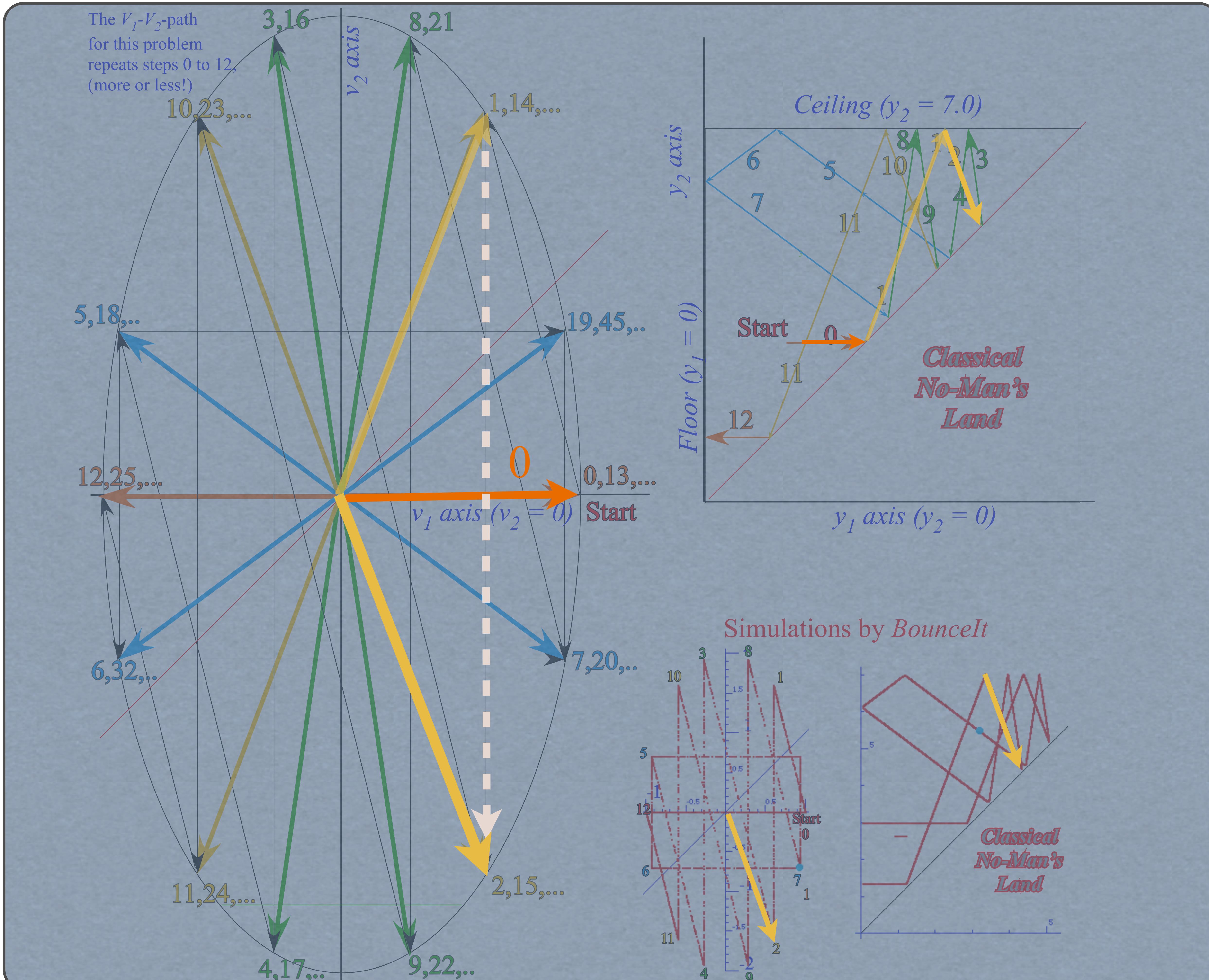


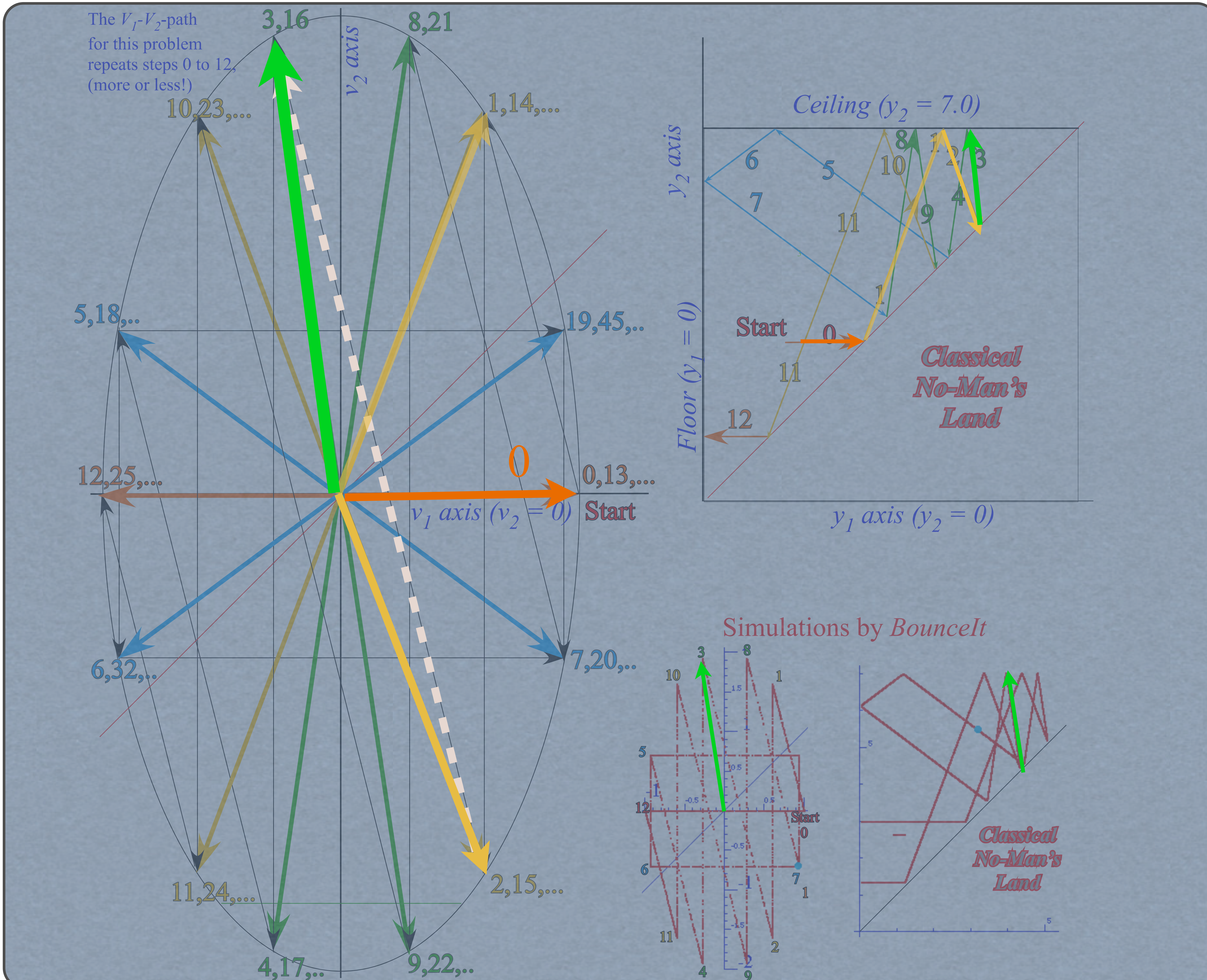
Simulations by *BounceIt*

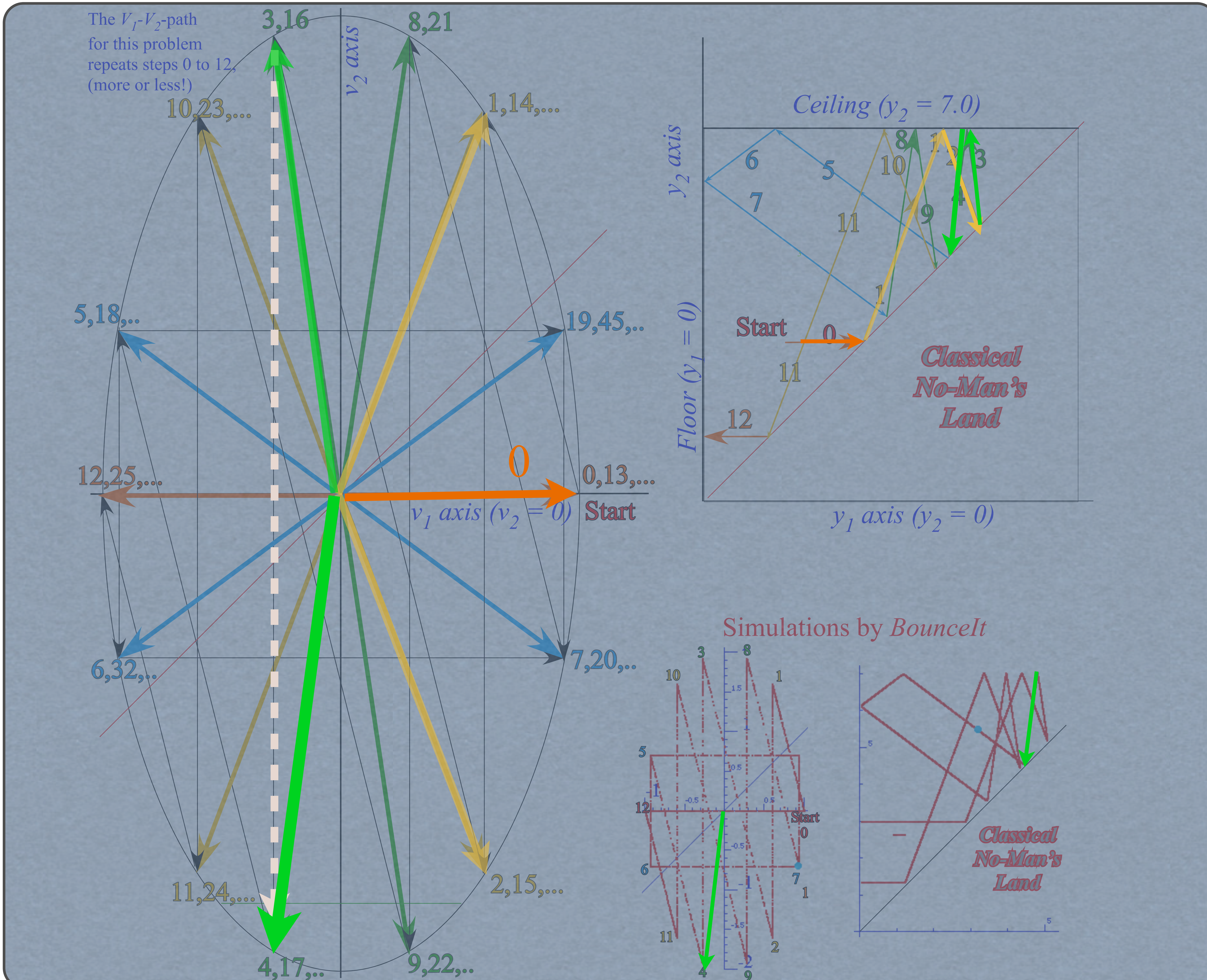


Collisions for mass ratio $m_1:m_2 = 4:1$


The V_1-V_2-path
for this problem
repeats steps 0 to 12,
(more or less!)
v_2 axis
3,16
8,21
1,14,...
10,23,...
5,18,..
19,45,..
12,25,...
0,13,...
Start
v_1 axis (v_2 = 0)
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
Start
Classical
No-Man's
Land
y_1 axis (y_2 = 0)
Simulations by *BounceIt*
Start
Classical
No-Man's
Land


The V_1-V_2-path
for this problem
repeats steps 0 to 12,
(more or less!)
v_2 axis
3,16
8,21
1,14,...
10,23,...
5,18,..
19,45,..
12,25,...
0
0,13,...
Start
v_1 axis (v_2 = 0)
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
y_1 axis (y_2 = 0)
Start
Classical
No-Man's
Land
Simulations by *BounceIt*
Classical
No-Man's
Land


The V1-V2-path
for this problem
repeats steps 0 to 12,
(more or less!)
3,16
8,21
v2 axis
1,14,...
10,23,...
5,18,..
19,45,..
0
12,25,...
0,13,...
v1 axis (v2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y2 = 7.0)
y2 axis
Floor (y1 = 0)
Start
Classical
No-Man's
Land
y1 axis (y2 = 0)
Simulations by BounceIt
Classical
No-Man's
Land


The V_1-V_2-path
for this problem
repeats steps 0 to 12,
(more or less!)
v_2 axis
3,16
8,21
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1,14,...
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v_1 axis (v_2 = 0)
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11,24,...
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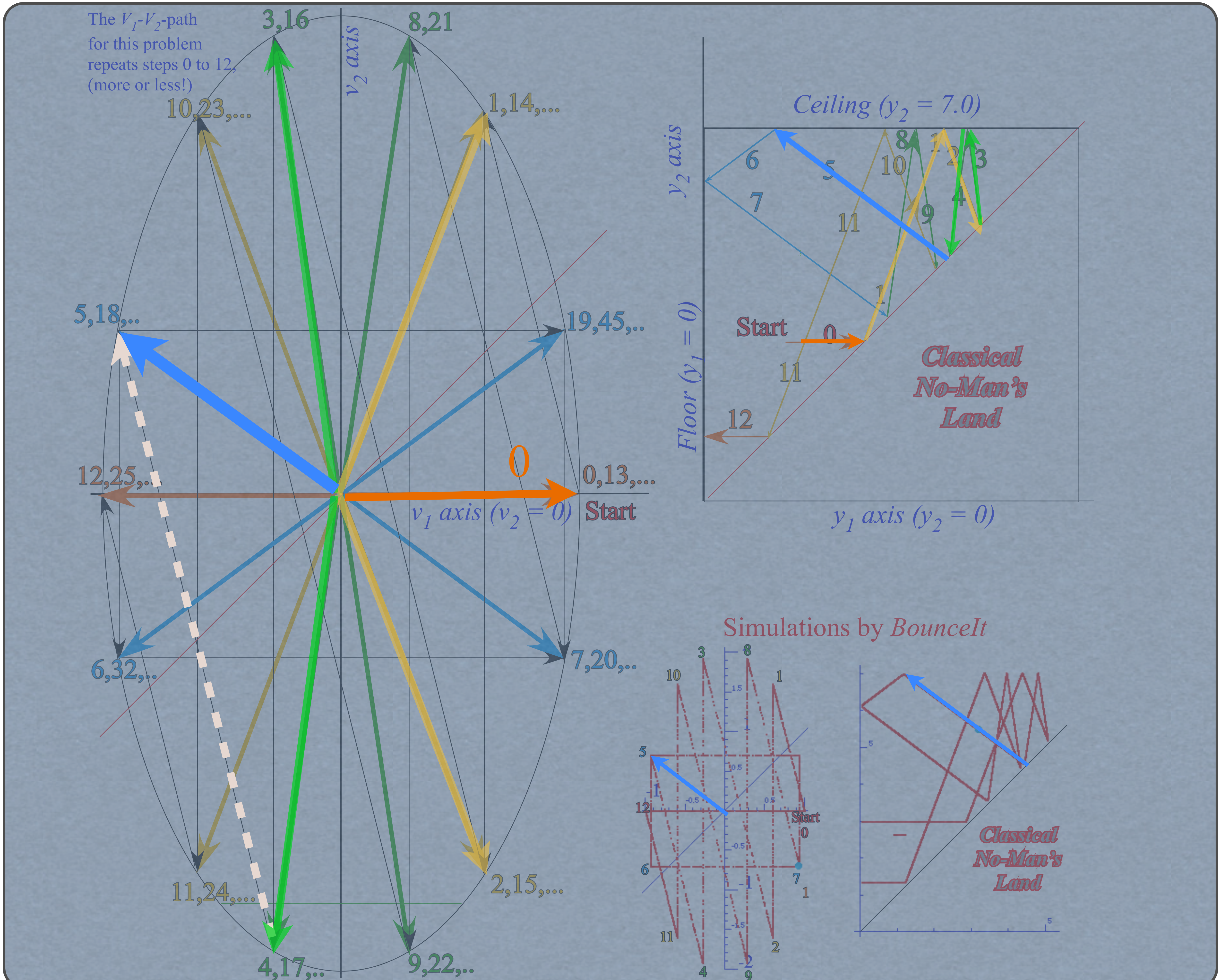
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
y_1 axis (y_2 = 0)
Start
Classical
No-Man's
Land
Simulations by *BounceIt*
Start
Classical
No-Man's
Land


The V_1-V_2-path
for this problem
repeats steps 0 to 12,
(more or less!)
v_2 axis
3,16
8,21
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Ceiling (y_2 = 7.0)
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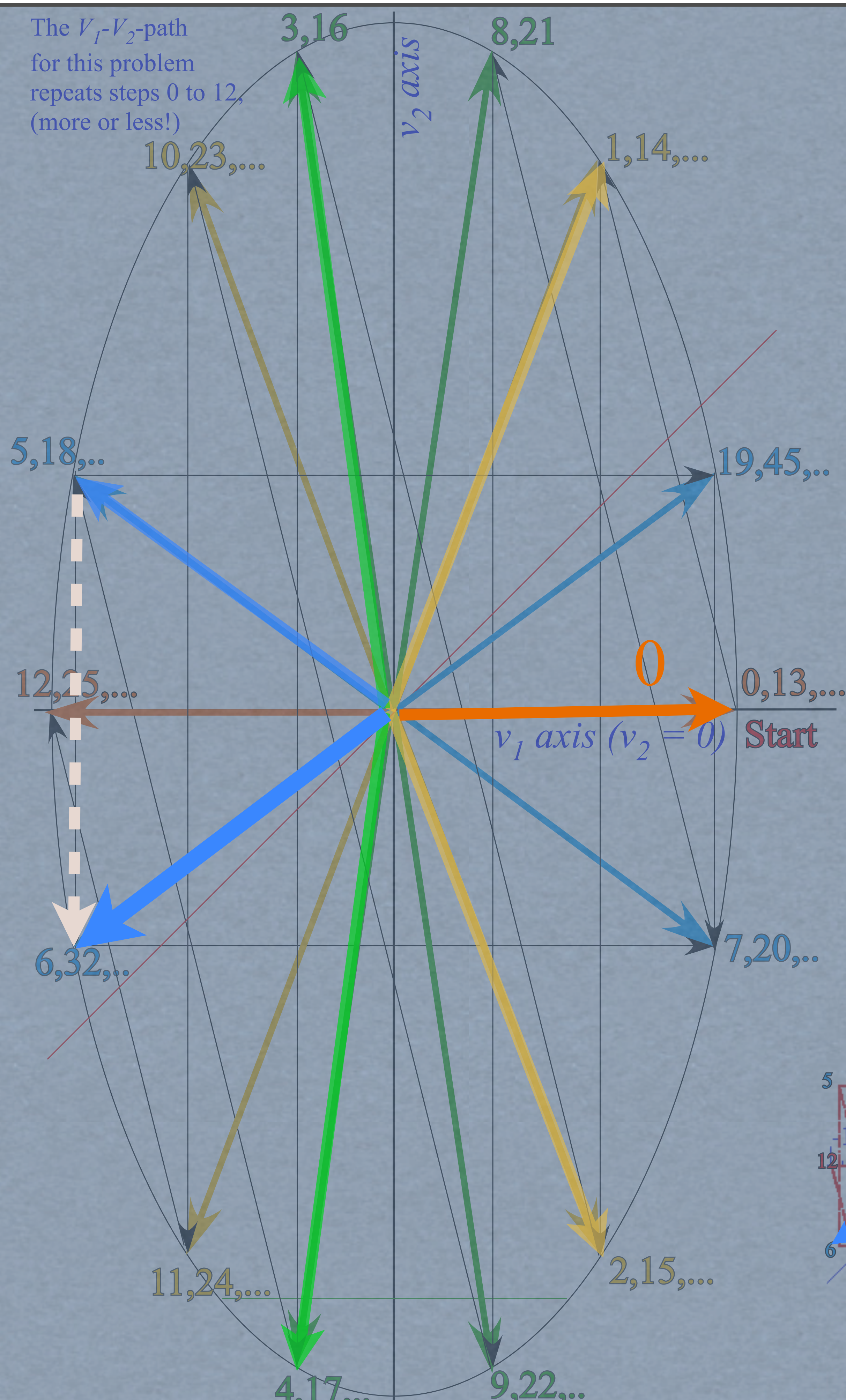
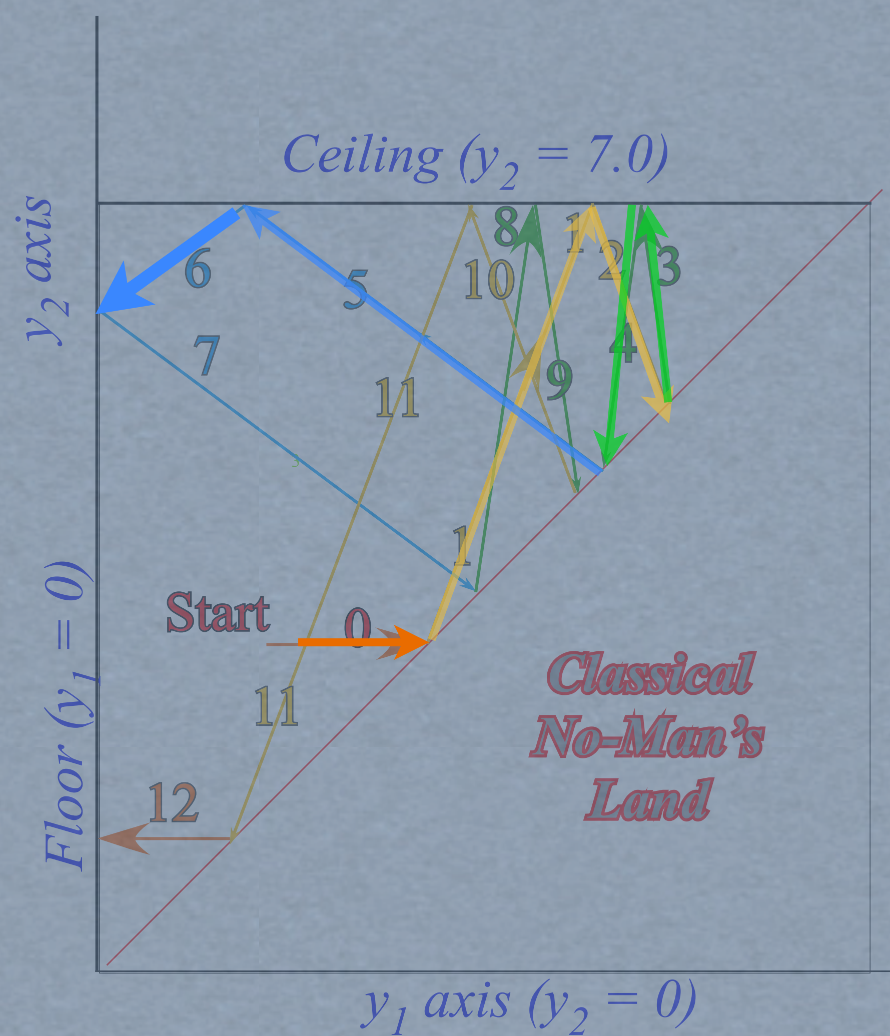
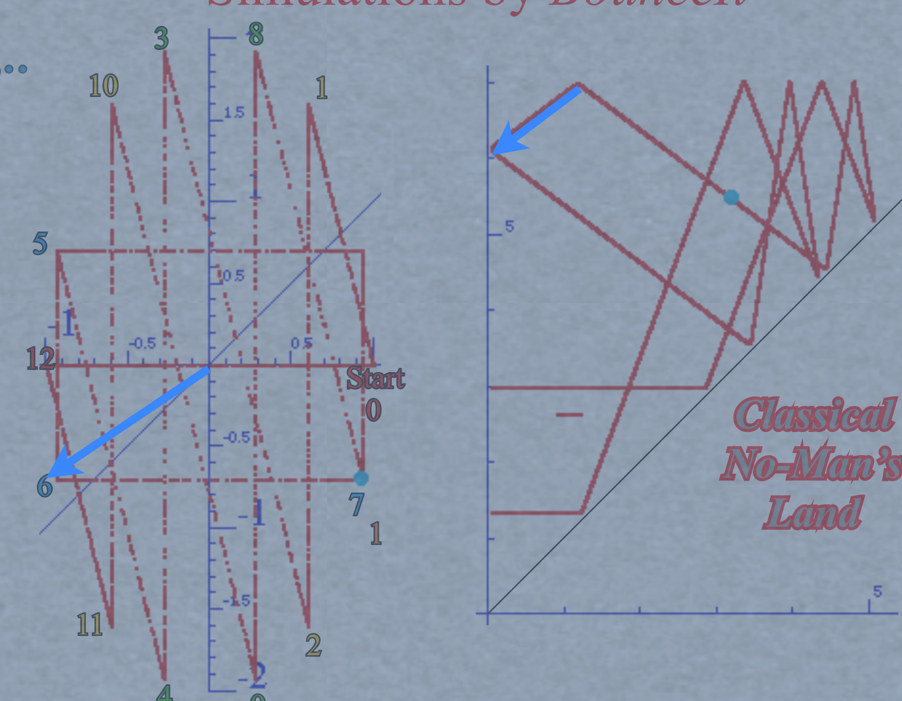
Classical
No-Man's
Land
y_1 axis (y_2 = 0)
Simulations by *BounceIt*
Classical
No-Man's
Land

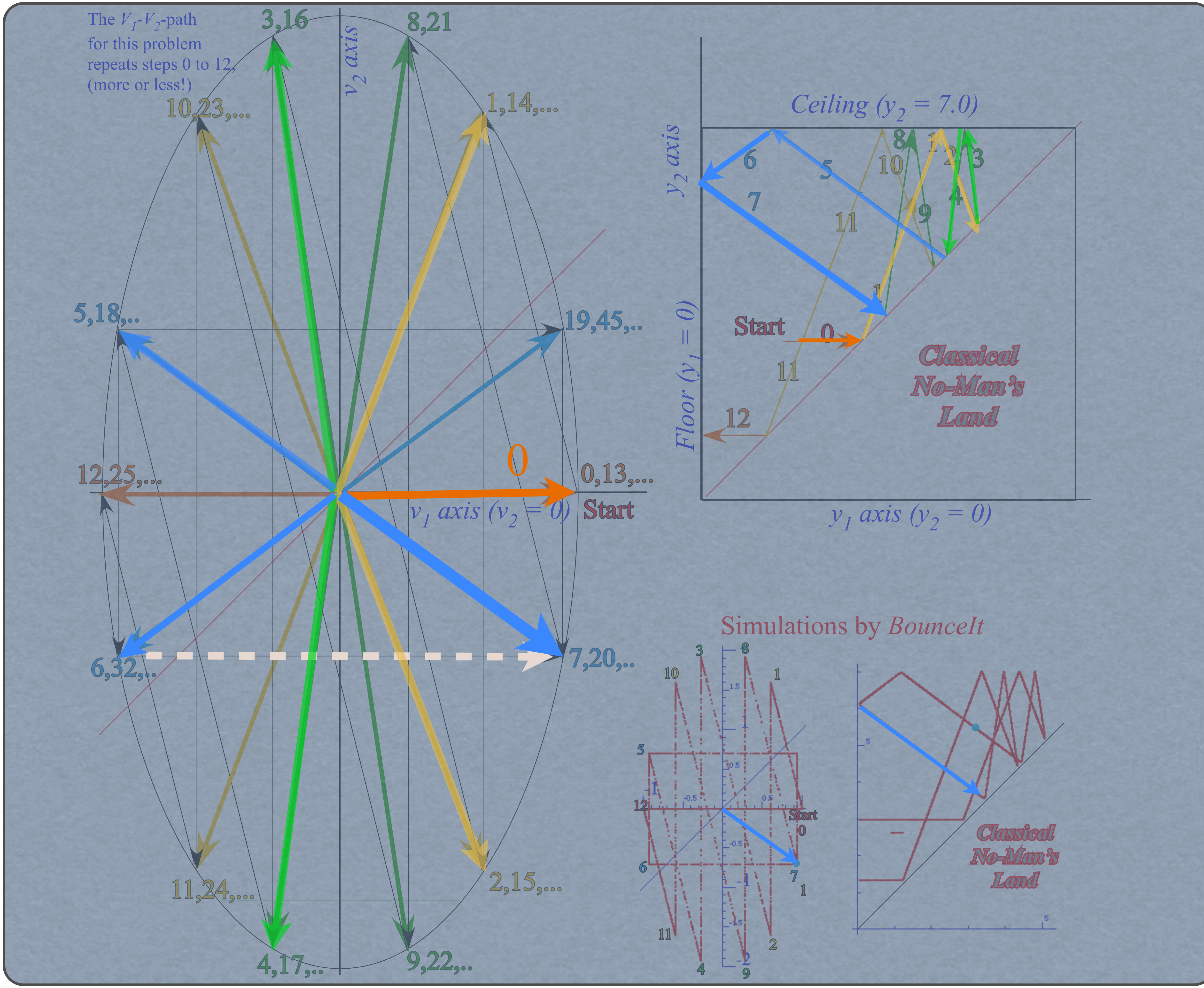
The V_1-V_2-path for this problem repeats steps 0 to 12, (more or less!)





Simulations by *BounceIt*




The V_1-V_2-path
for this problem
repeats steps 0 to 12,
(more or less!)
3,16
8,21
v_2 axis
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Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
Start
Classical
No-Man's
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y_1 axis (y_2 = 0)
Simulations by *BounceIt*
Classical
No-Man's
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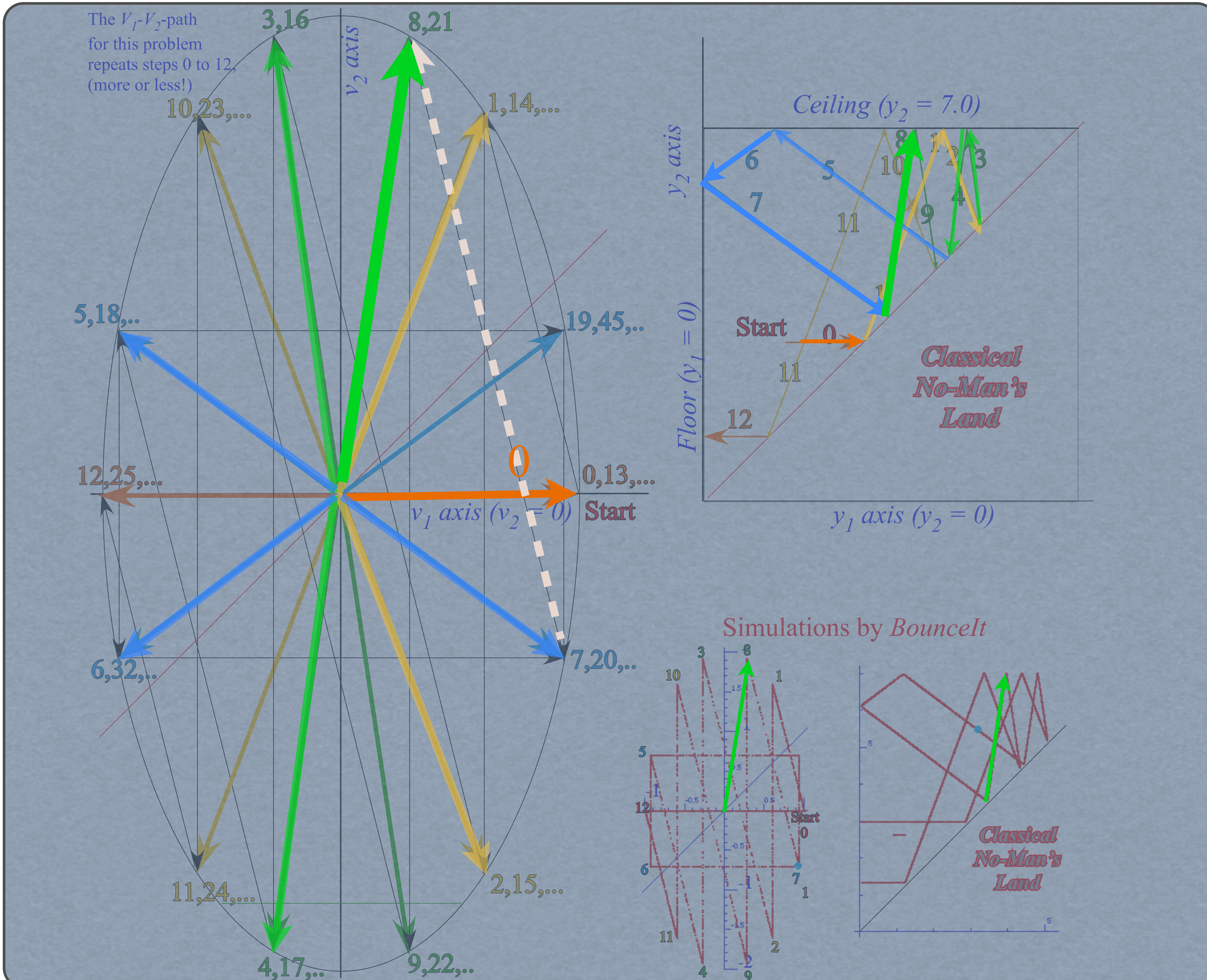

The V_1-V_2-path for this problem repeats steps 0 to 12, (more or less!)
3,16
8,21
v_2 axis
1,14,...
10,23,...
5,18,..
19,45,..
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12,25,...
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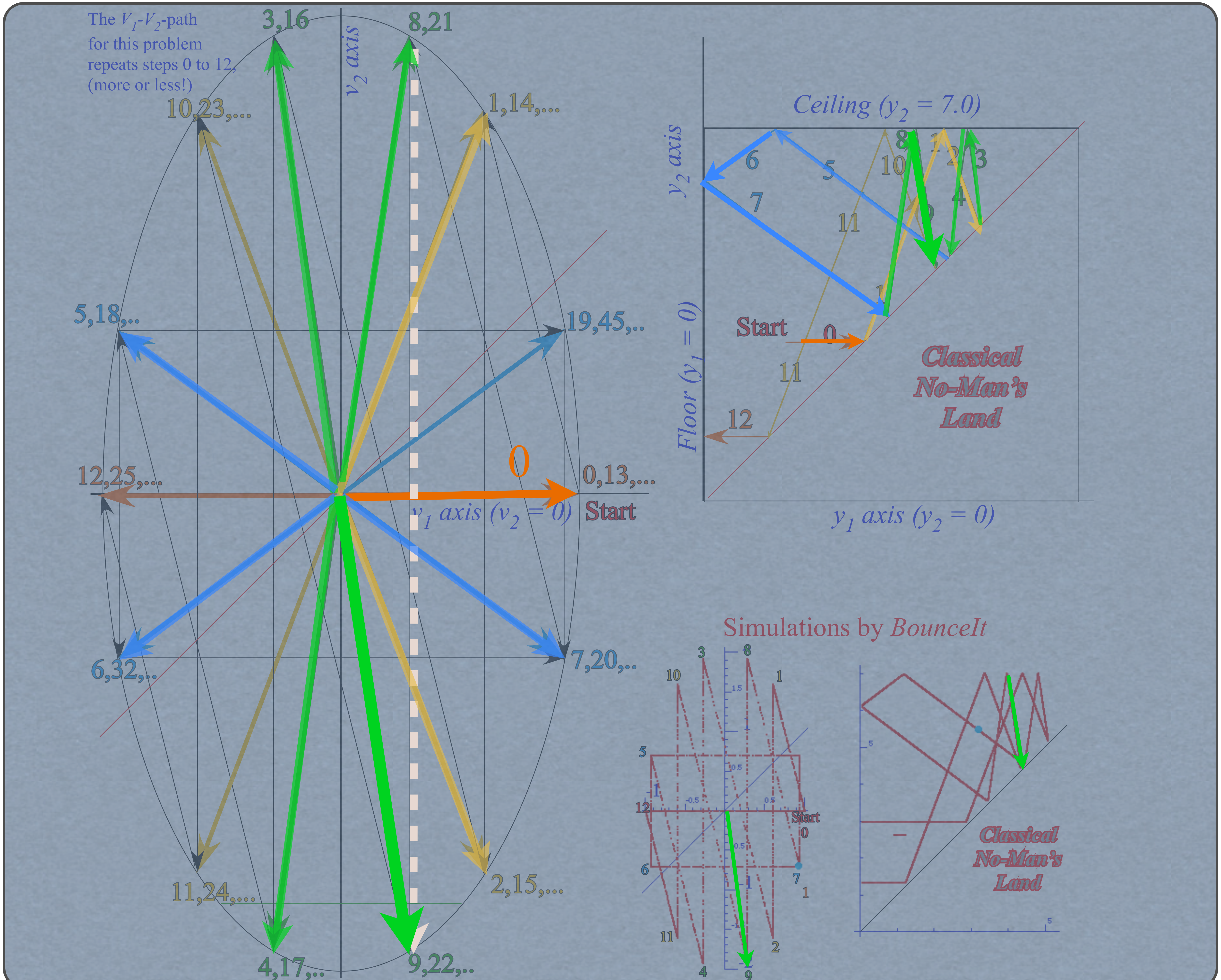
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
y_1 axis (y_2 = 0)
Start
Classical No-Man's Land
Simulations by *BounceIt*
Classical No-Man's Land


The V_1-V_2-path
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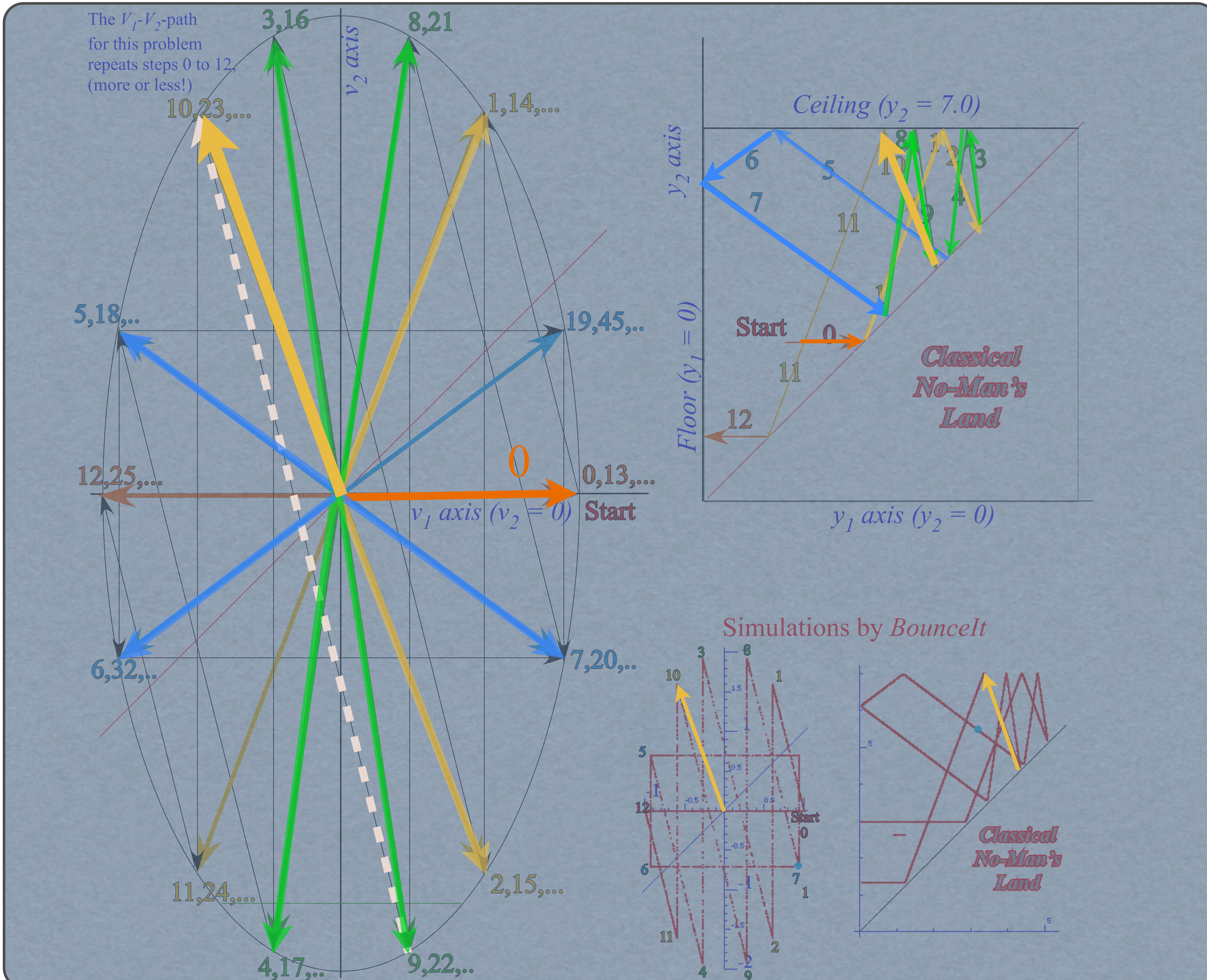
No-Man's
Land



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3,16
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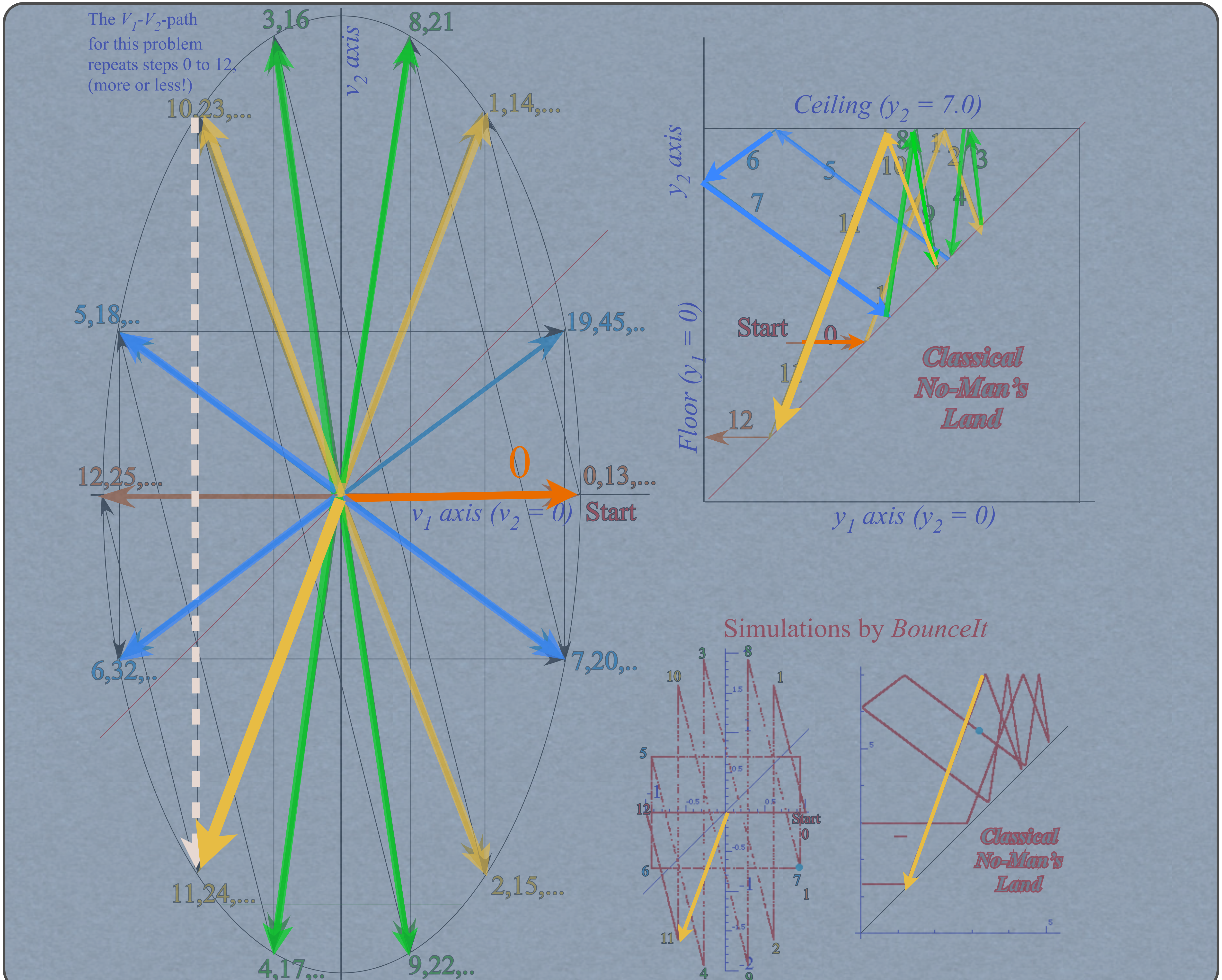
Classical No-Man's Land
y_1 axis (y_2 = 0)
Simulations by *BounceIt*
Classical No-Man's Land


The V_1-V_2-path for this problem repeats steps 0 to 12, (more or less!)
3,16
8,21
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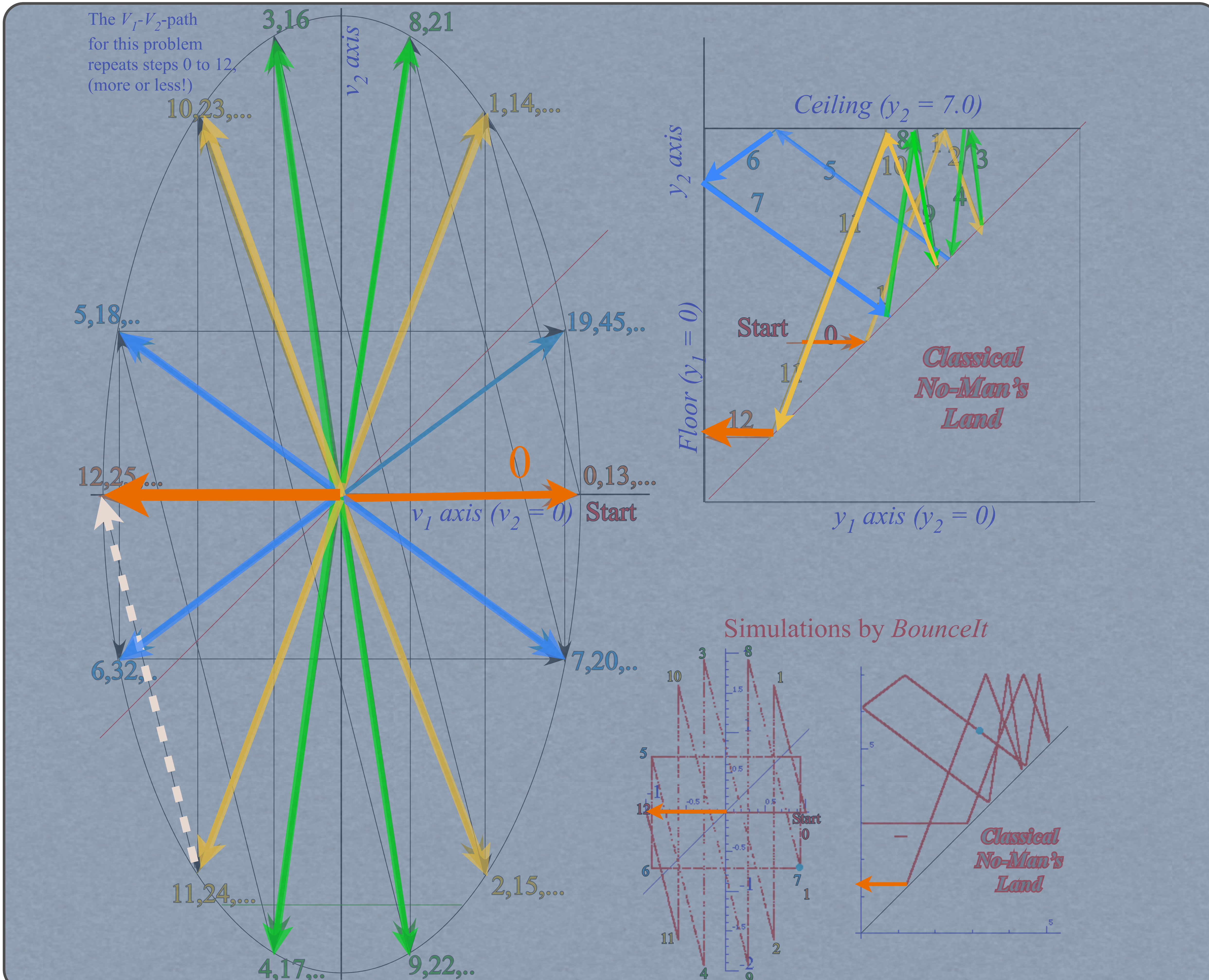
Simulations by *BounceIt*
Classical No-Man's Land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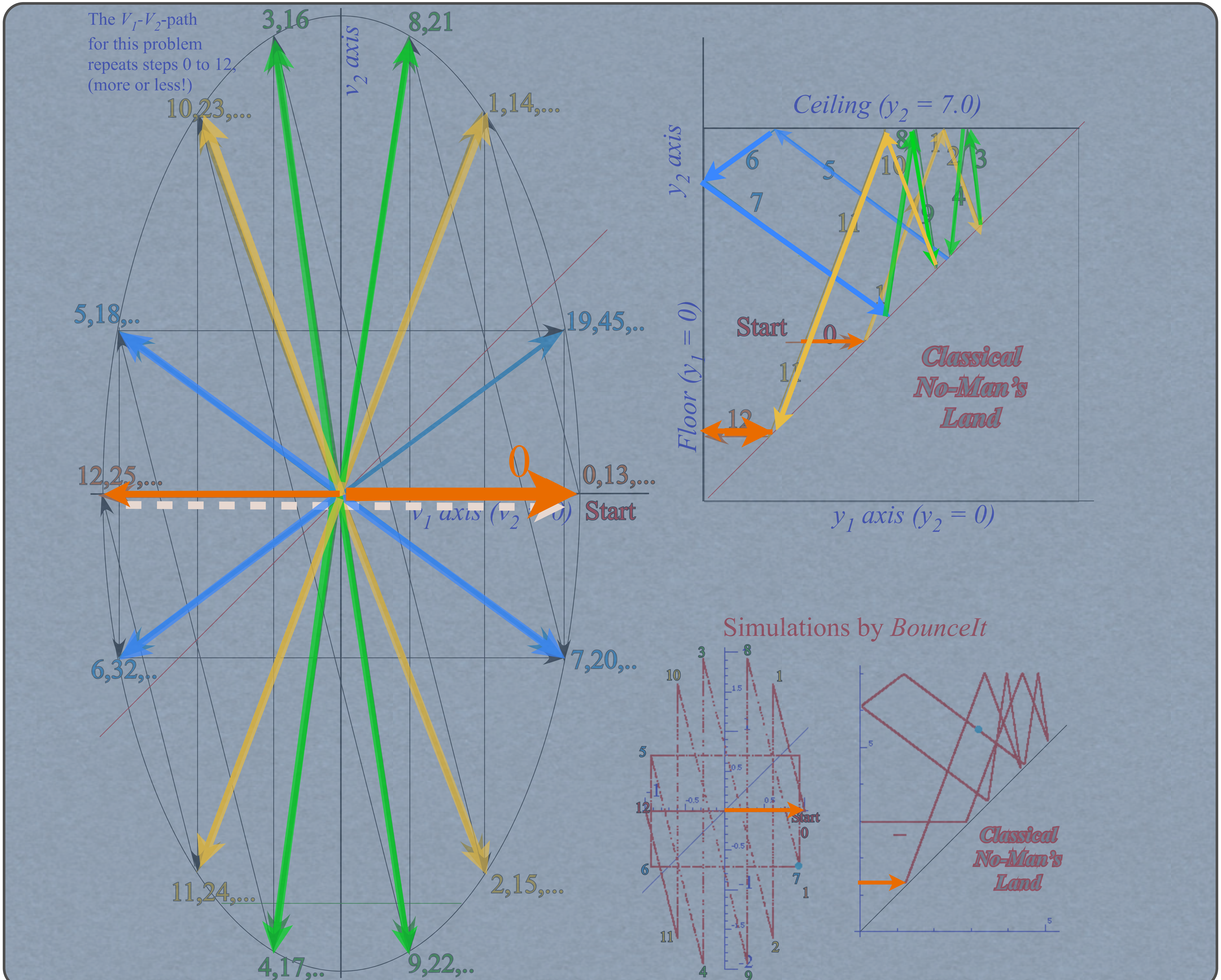

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8,21
v_2 axis
10,23,...
1,14,...
5,18,..
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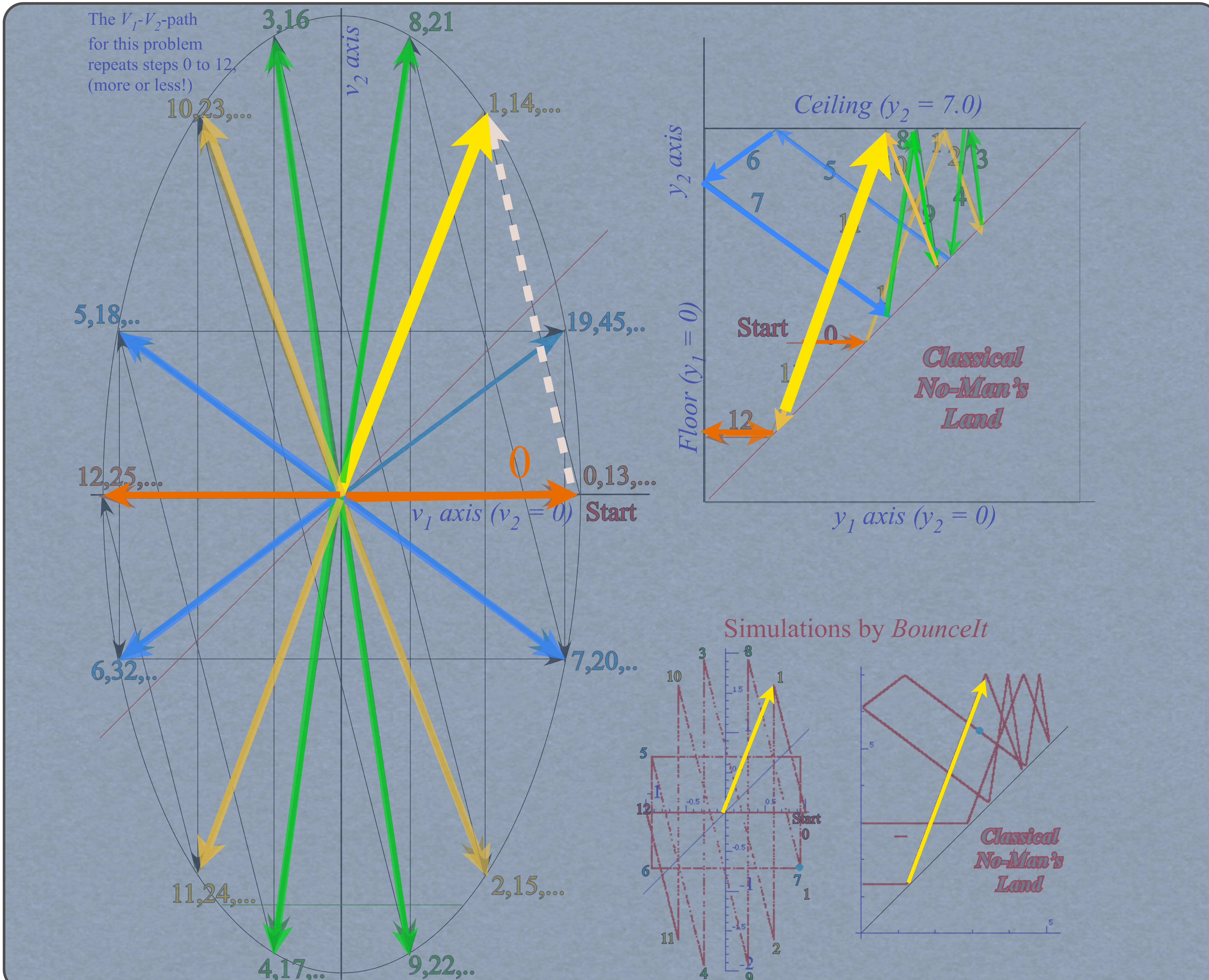
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
Start
Classical No-Man's Land
y_1 axis (y_2 = 0)
Simulations by *BounceIt*
Start
Classical No-Man's Land


The V_1-V_2-path
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repeats steps 0 to 12,
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v_2 axis
1,14,...
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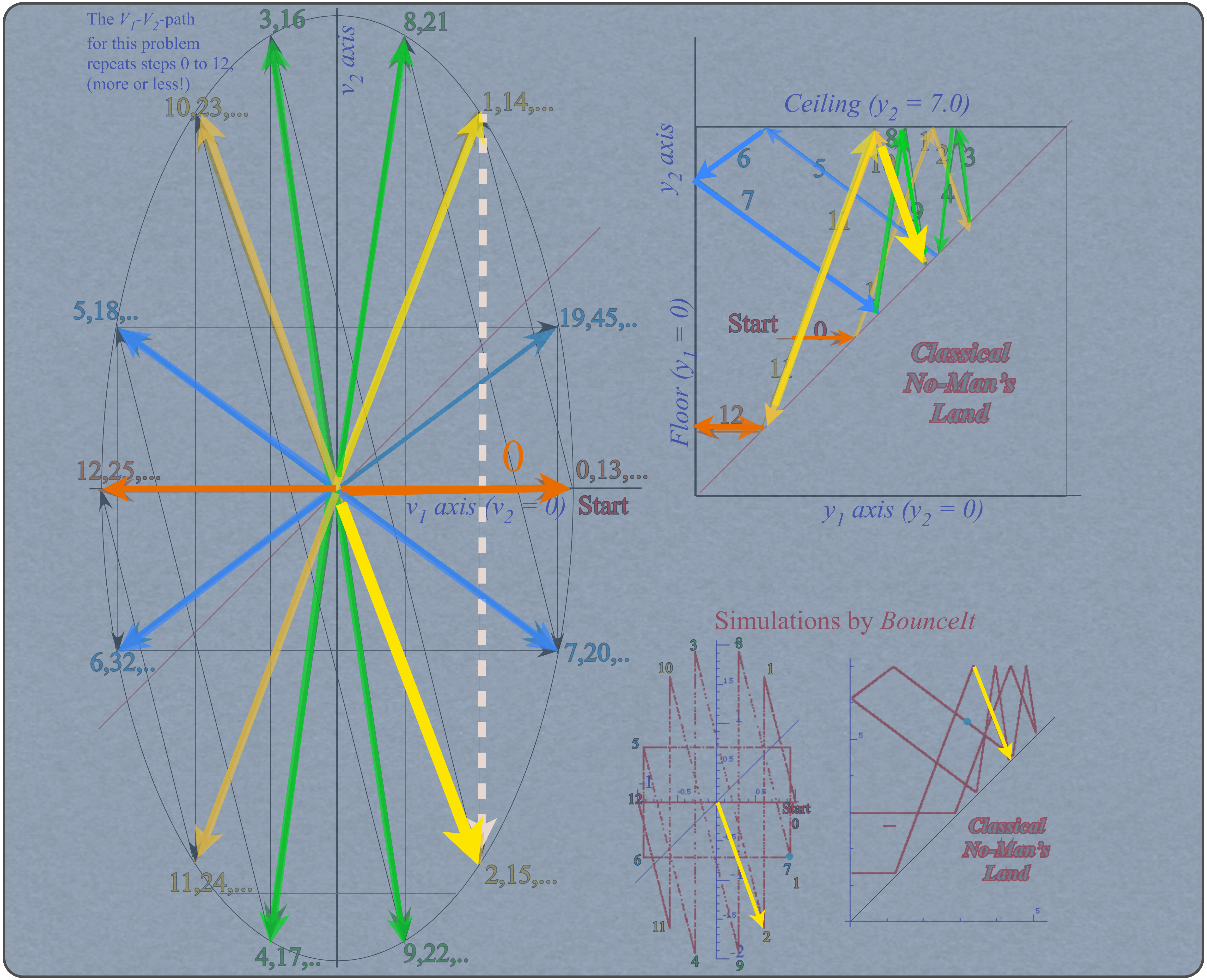
The V_1-V_2-path
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y_1 axis (y_2 = 0)
Simulations by *BounceIt*
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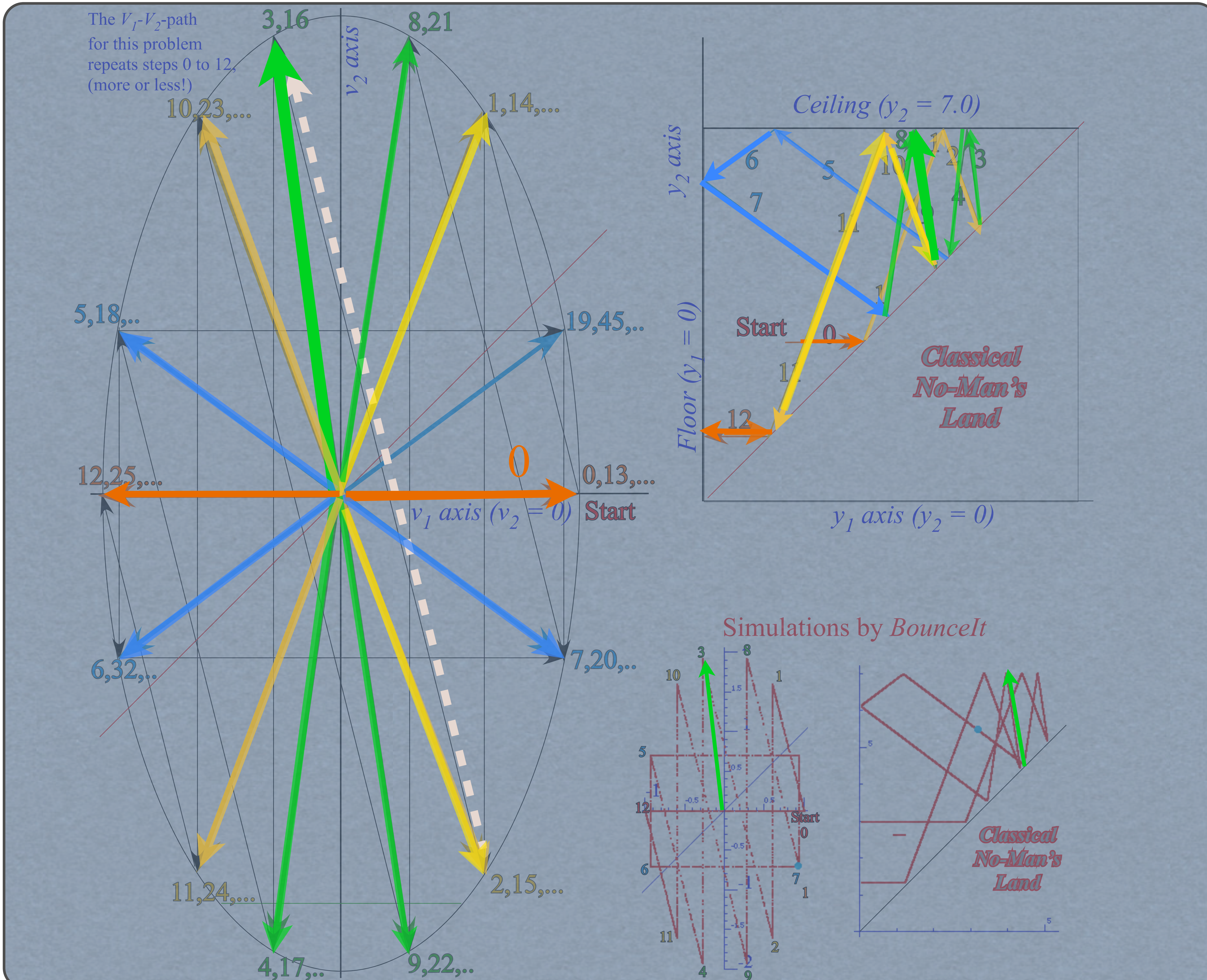
Land


The V_1-V_2-path
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y_2 axis
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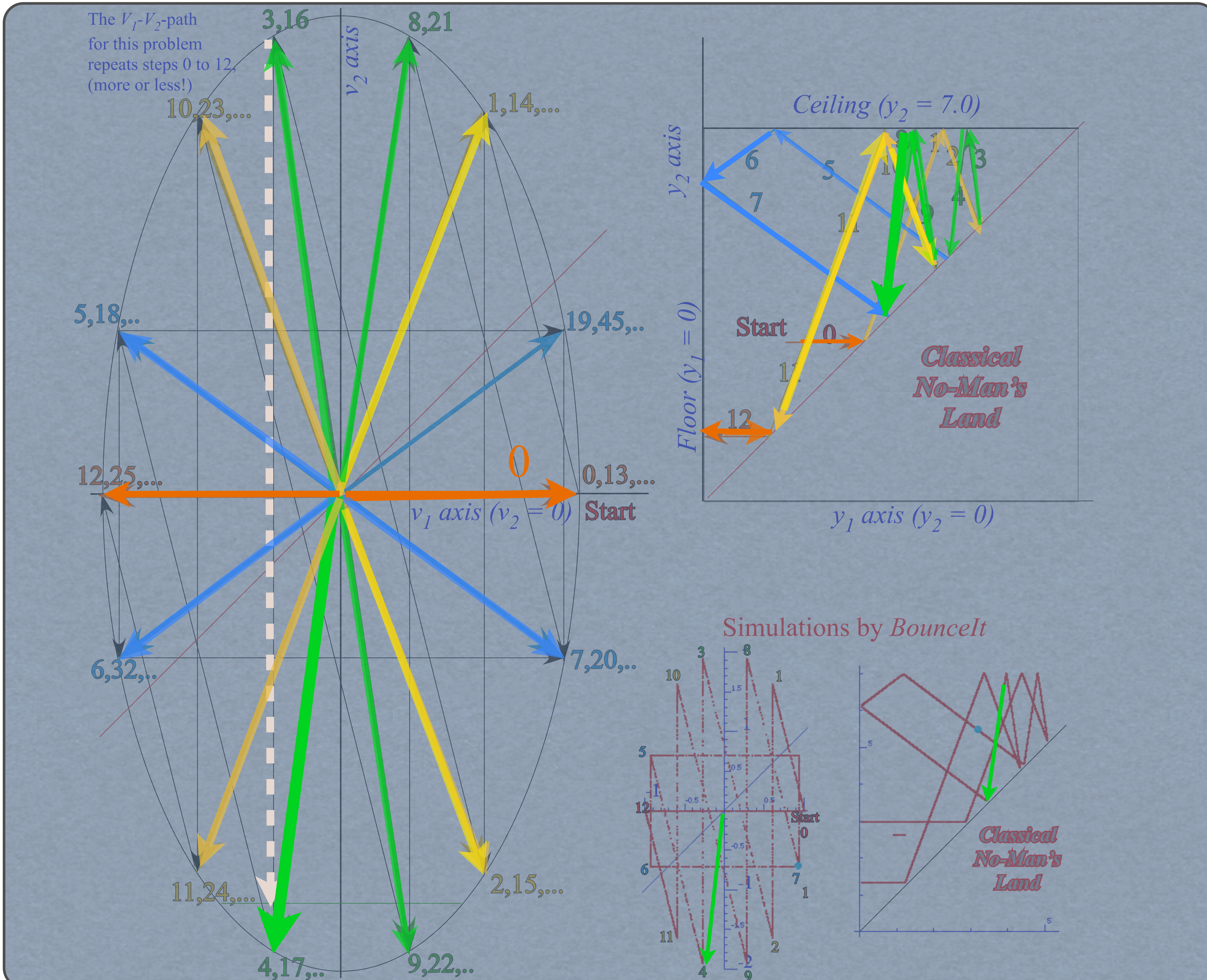
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8,21
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0,13,...
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Start
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Ceiling (y_2 = 7.0)
y_2 axis
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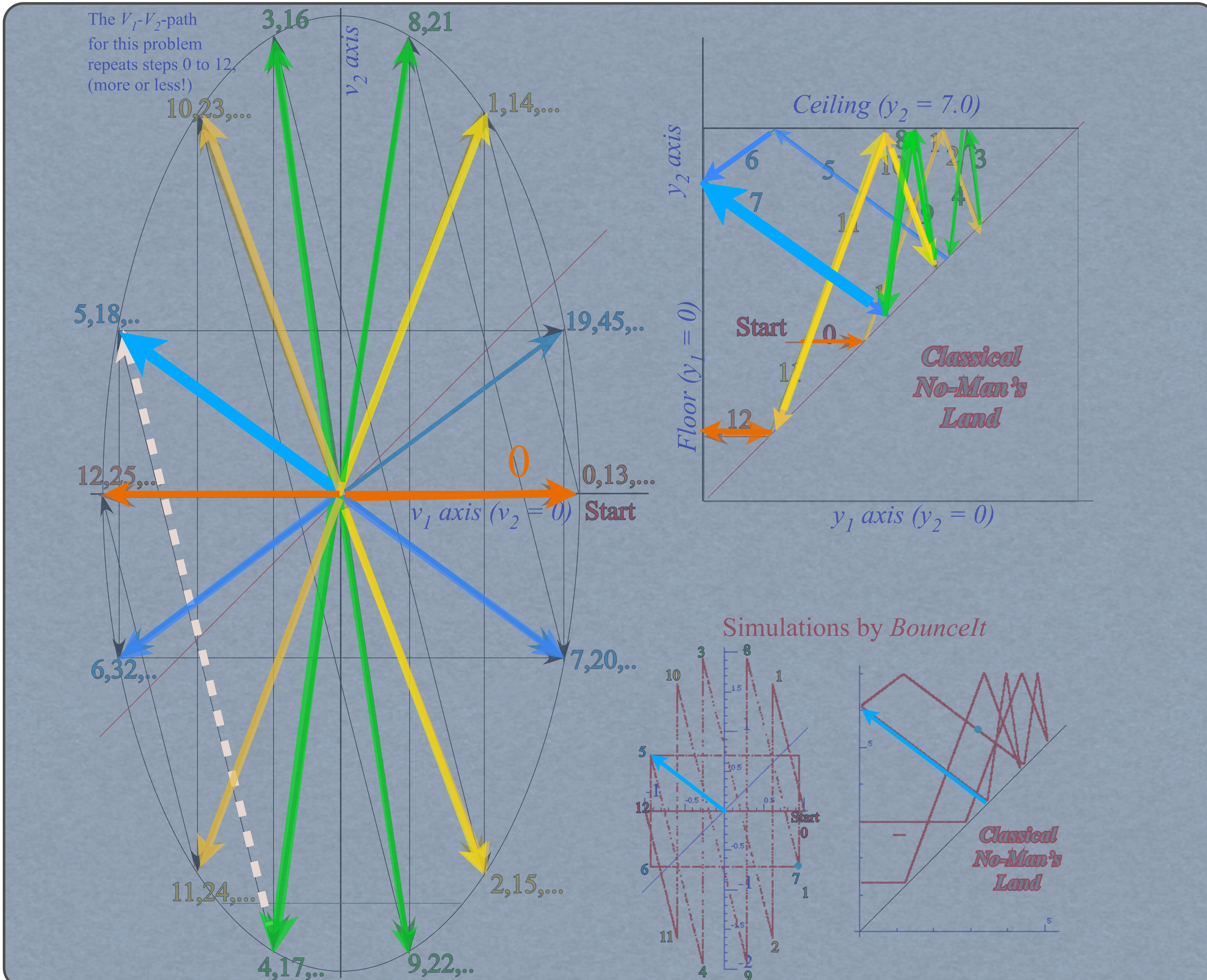

The V_1-V_2-path
for this problem
repeats steps 0 to 12,
(more or less!)
3,16
8,21
v_2 axis
1,14,...
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Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
Start
Classical
No-Man's
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y_1 axis (y_2 = 0)
Simulations by *BounceIt*
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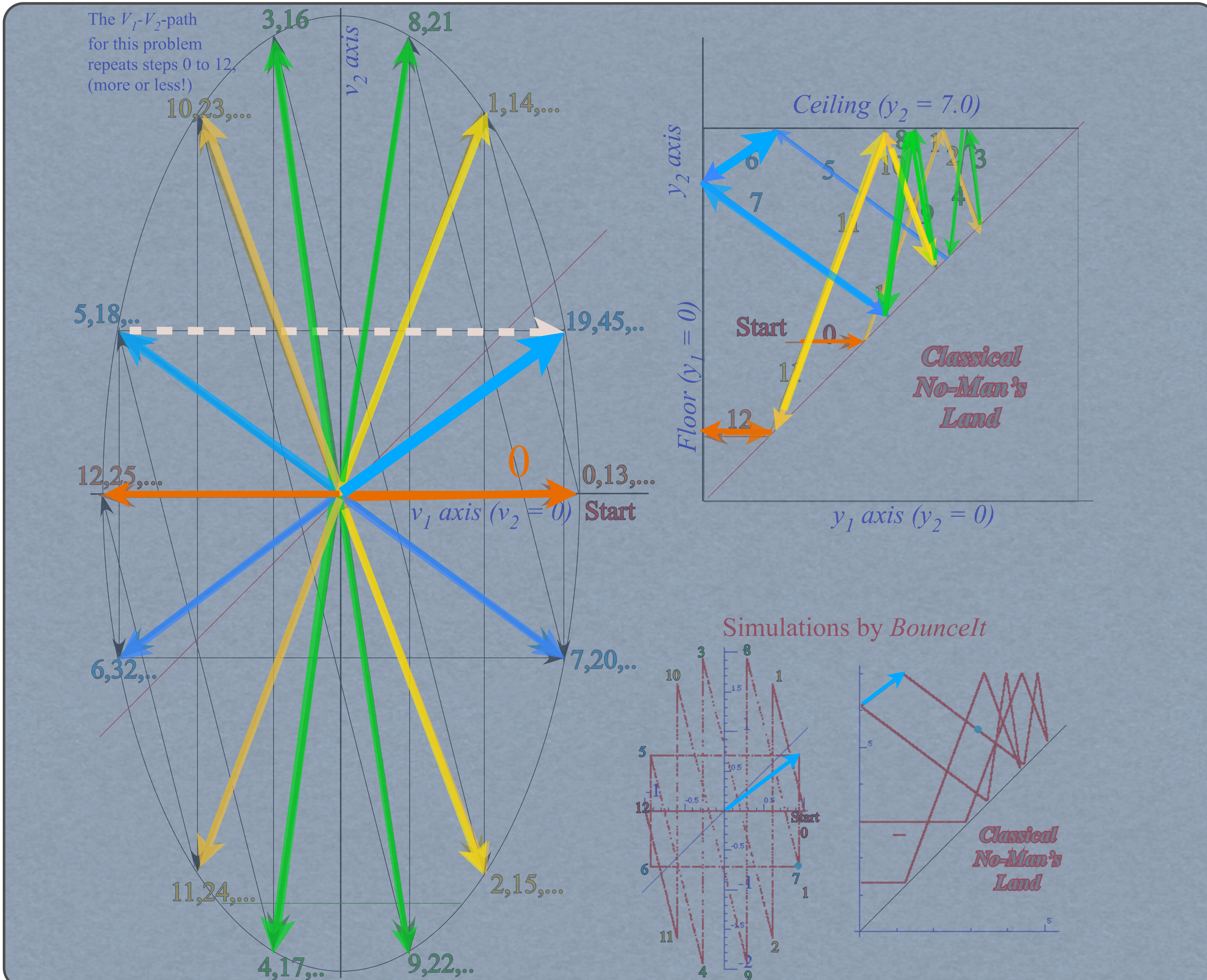
The V_1-V_2-path
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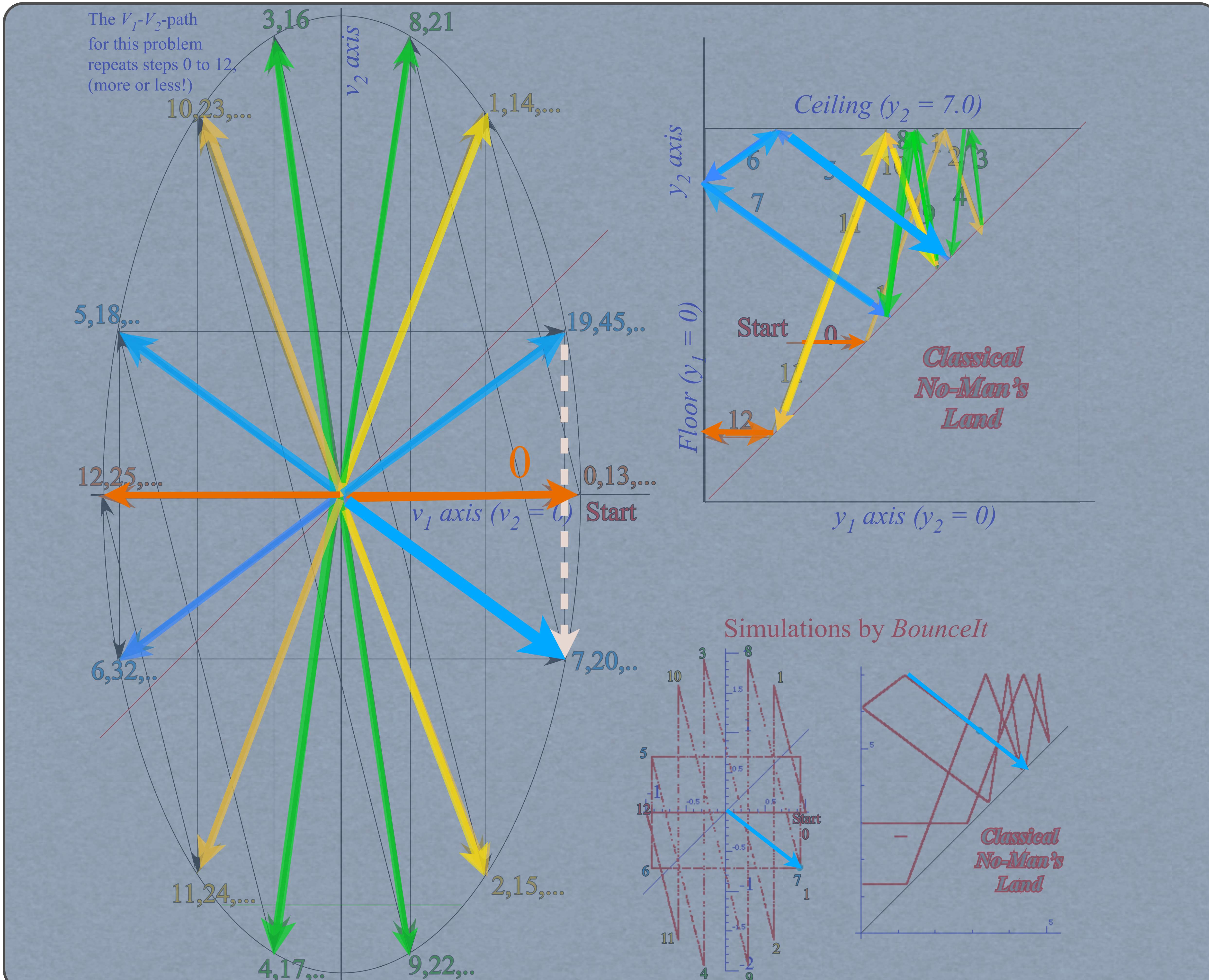
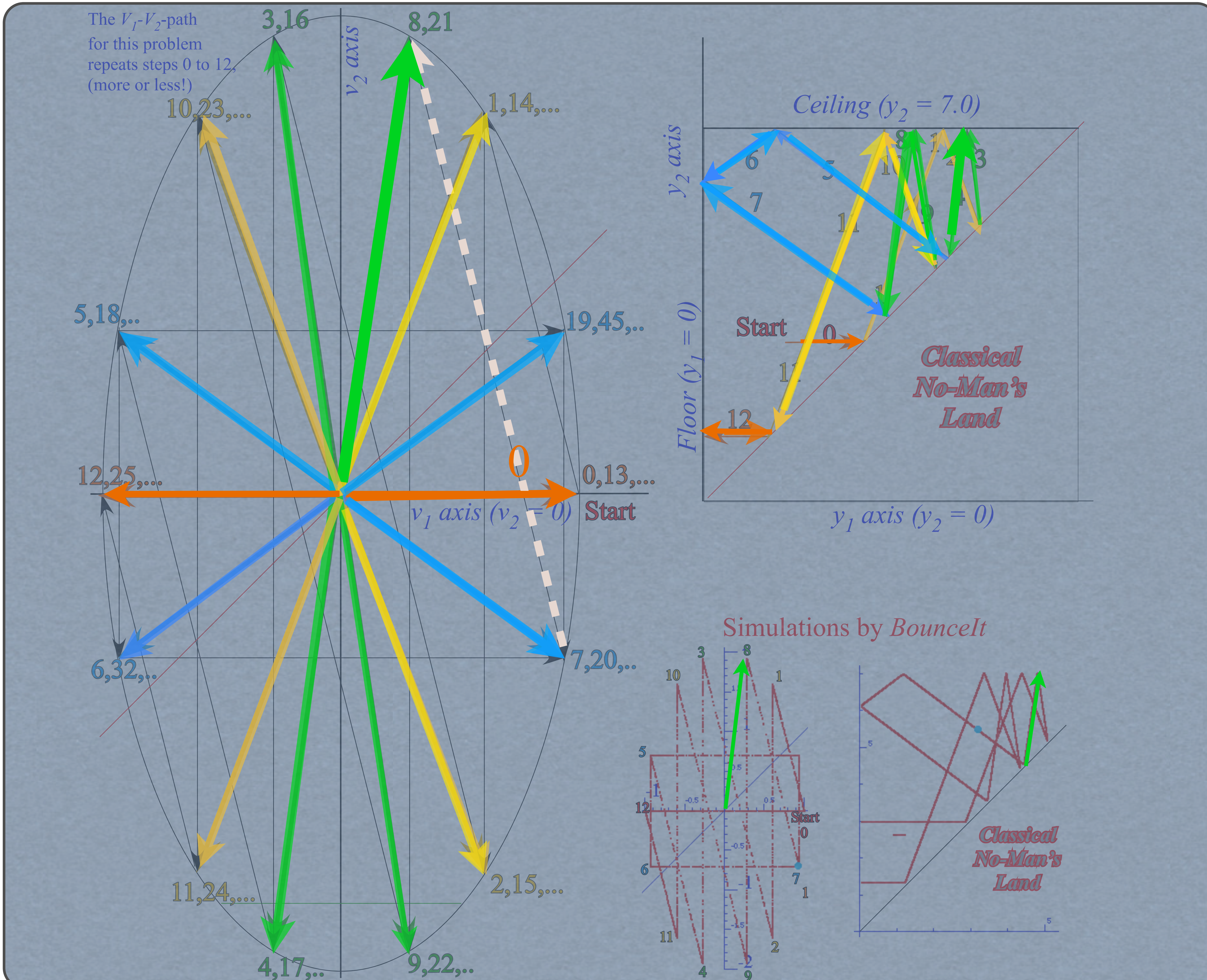
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
Start
Classical
No-Man's
Land
y_1 axis (y_2 = 0)
Simulations by *BounceIt*
Classical
No-Man's
Land


The V_1-V_2-path for this problem repeats steps 0 to 12, (more or less!)
3,16
8,21
v_2 axis
1,14,...
10,23,...
5,18,..
19,45,..
0
12,25,...
0,13,...
Start
v_1 axis ($v_2 = 0$)
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling ($y_2 = 7.0$)
y_2 axis
6
7
Start
0
Floor ($y_1 = 0$)
12
Classical No-Man's Land
y_1 axis ($y_2 = 0$)
Simulations by *BounceIt*
Start
Classical No-Man's Land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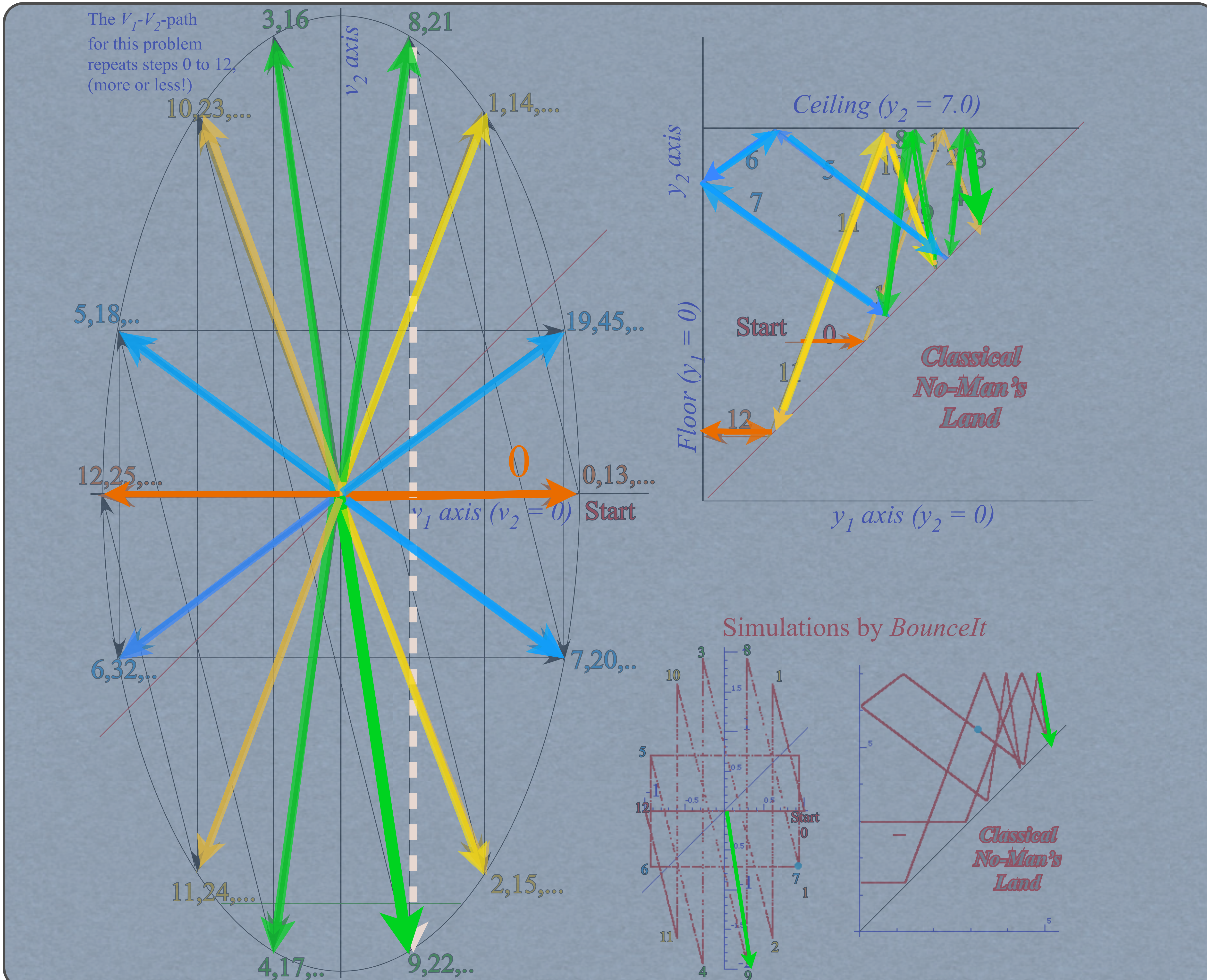
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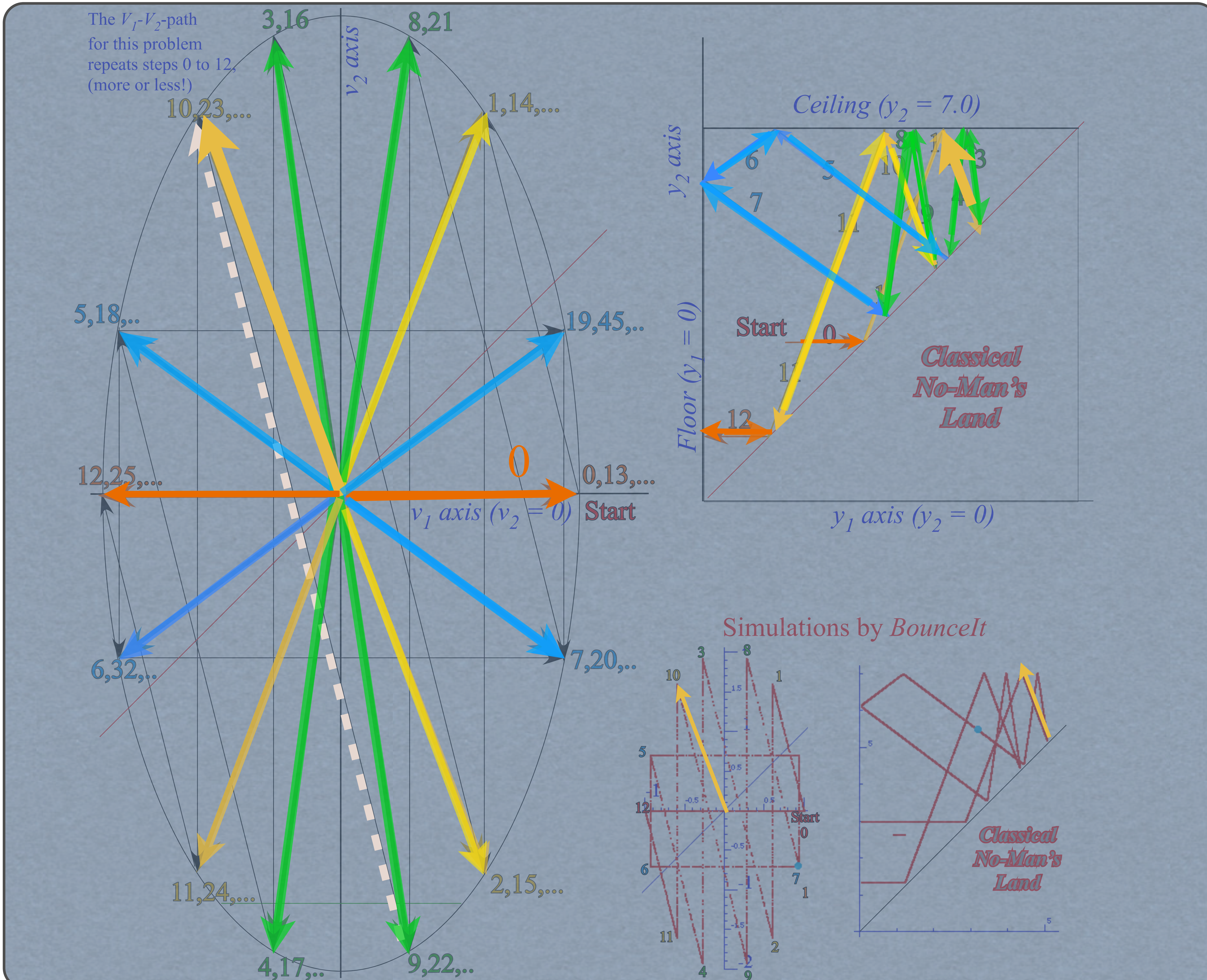
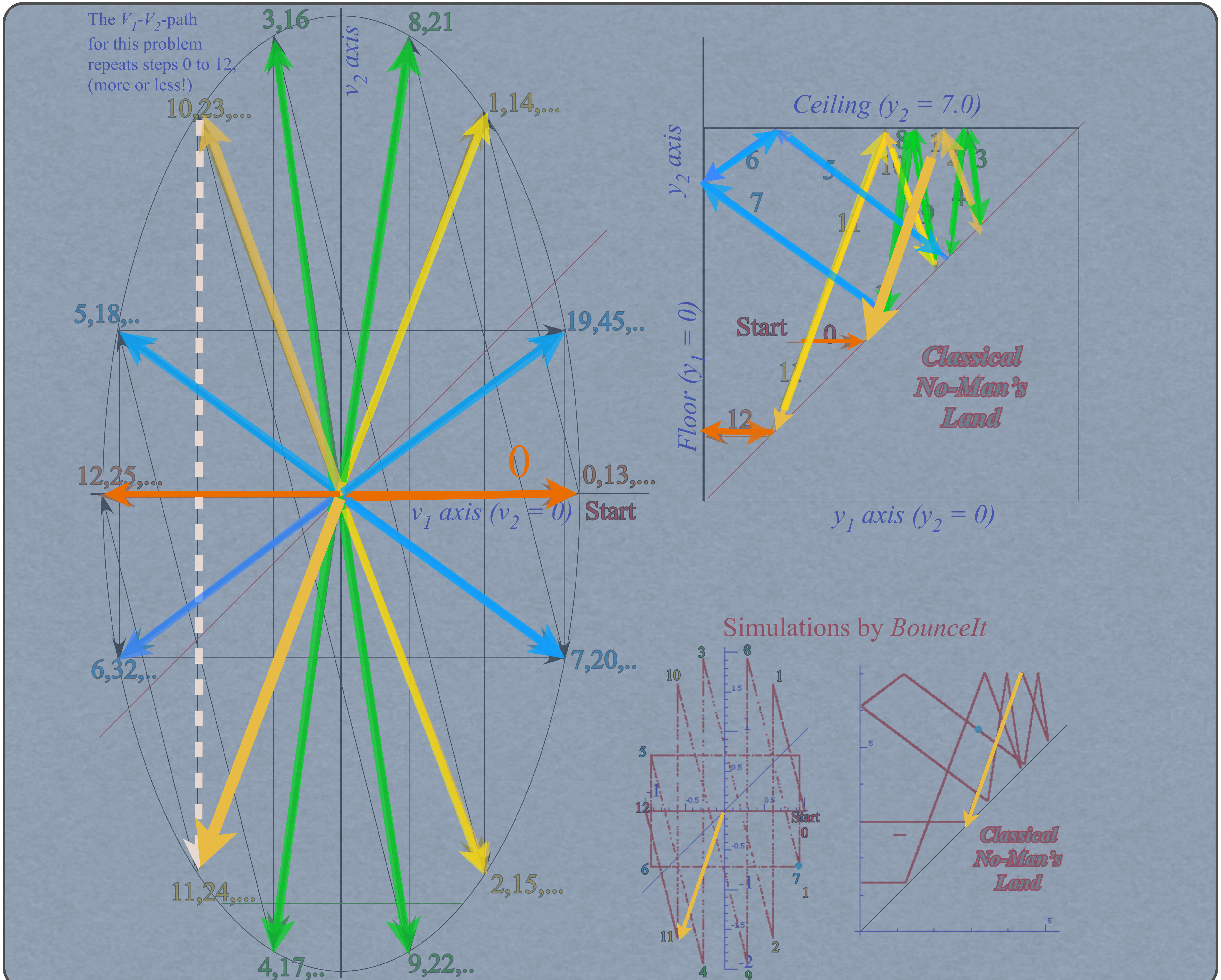
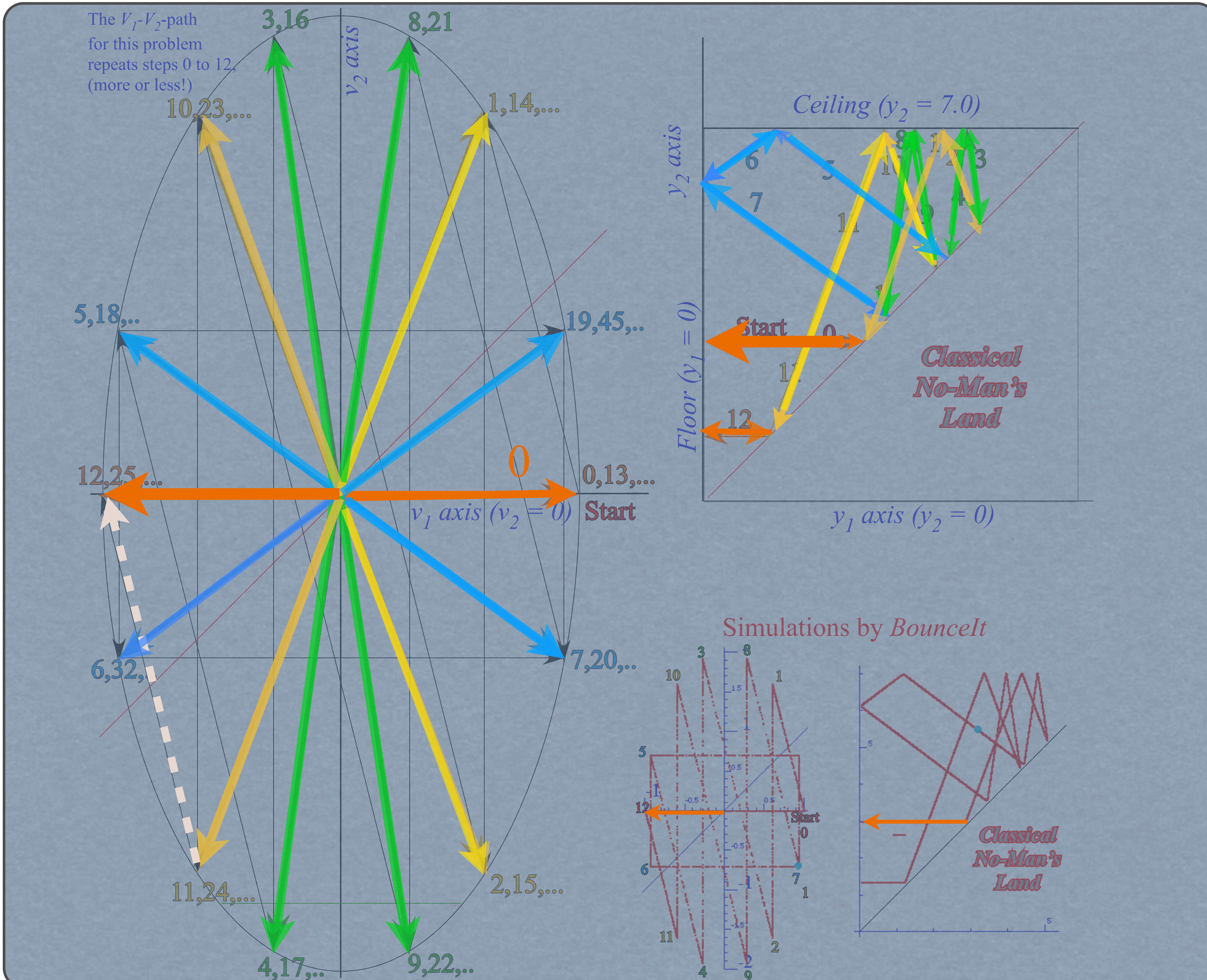
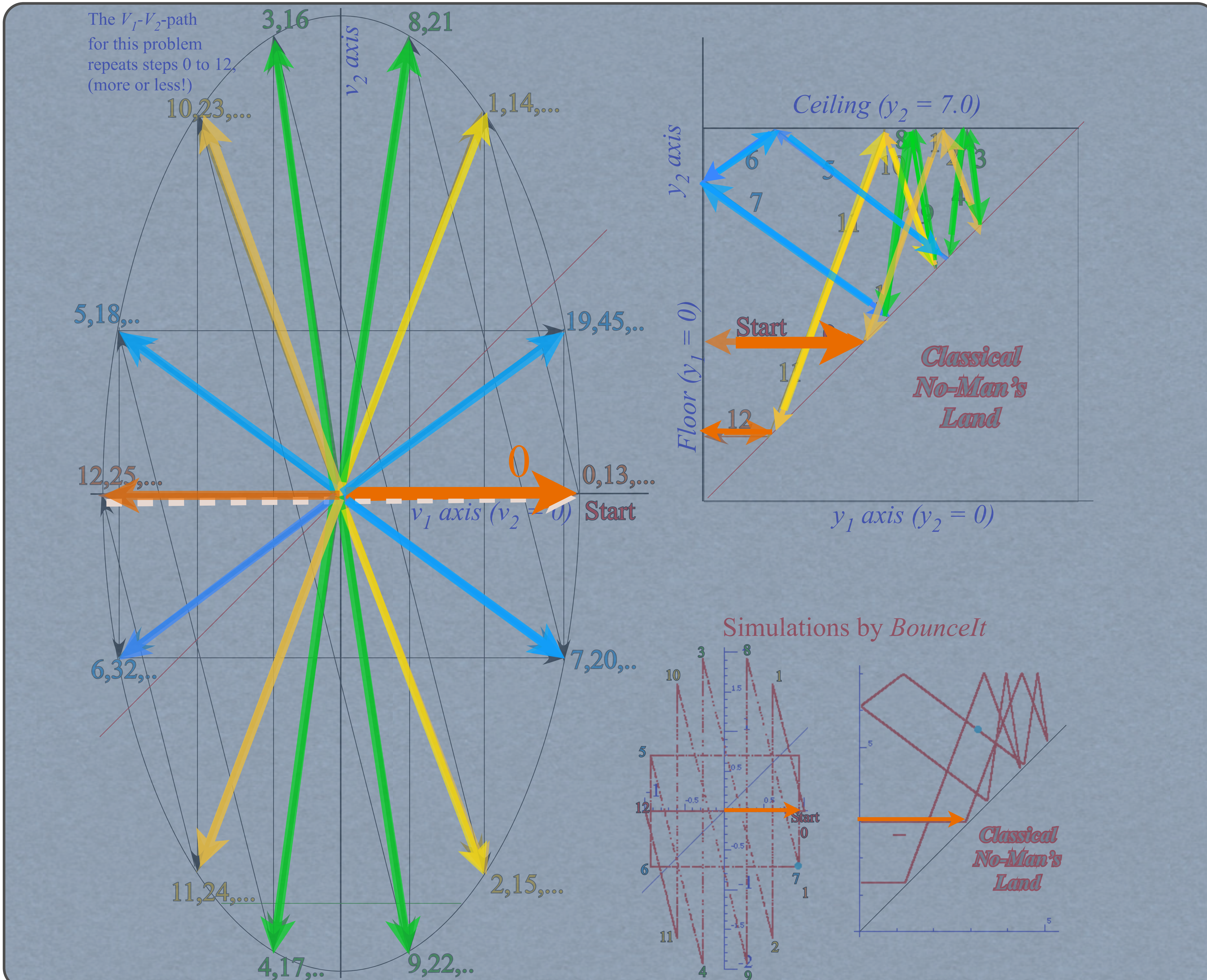
No-Man's
Land


The V_1-V_2-path for this problem repeats steps 0 to 12, (more or less!)
3,16
8,21
1,14,...
10,23,...
5,18,..
19,45,..
12,25,...
0,13,...
Start
v_2 axis
v_1 axis (v_2 = 0)
0
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
y_1 axis (y_2 = 0)
Start
0
12
Classical No-Man's Land
Simulations by *BounceIt*
Classical No-Man's Land

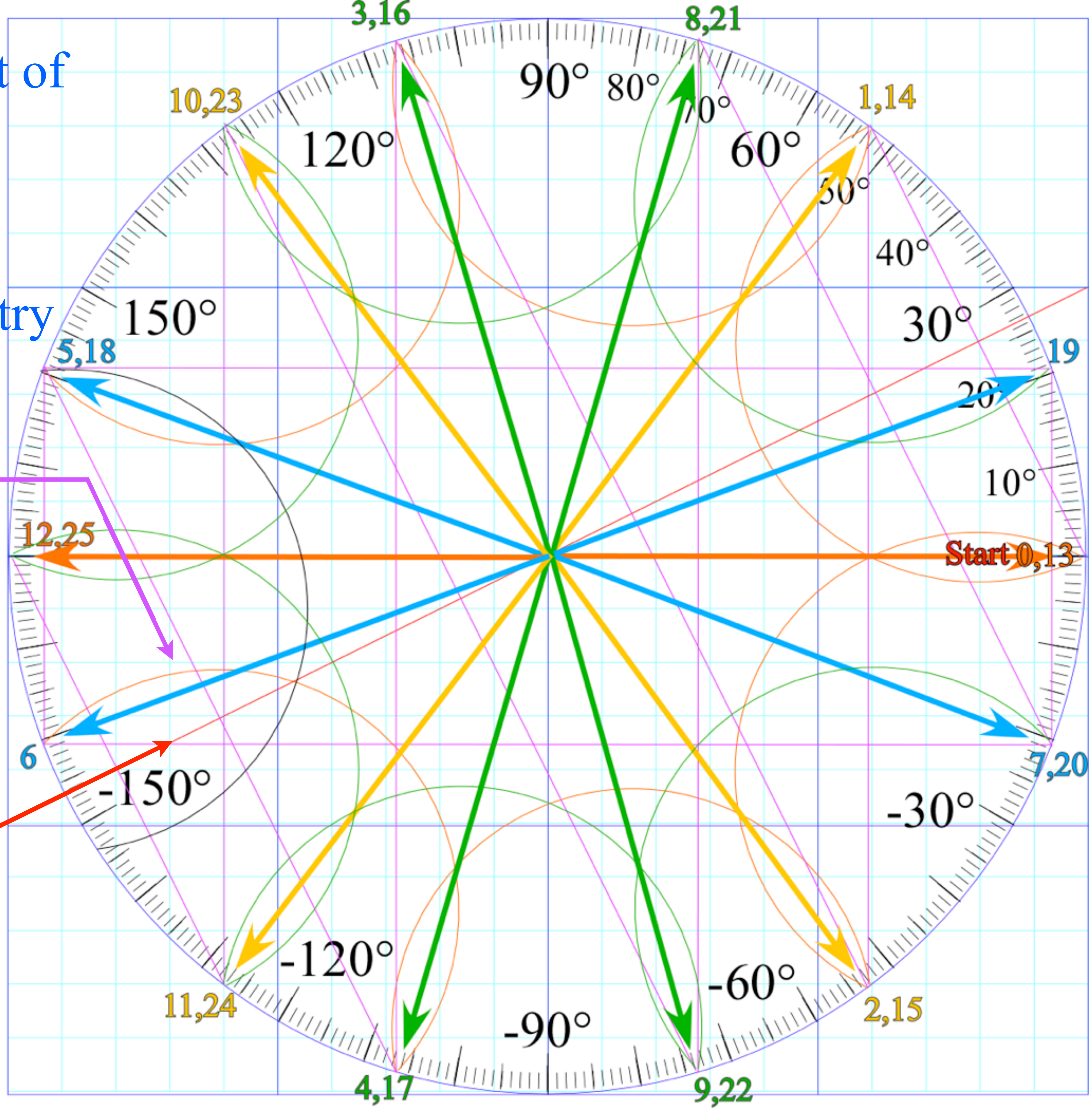

The V1-V2-path
for this problem
repeats steps 0 to 12,
(more or less!)
v2 axis
3,16
8,21
10,23,...
1,14,...
5,18,..
19,45,..
12,25,...
0
0,13,...
Start
v1 axis (v2 = 0)
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y2 = 7.0)
y2 axis
Floor (y1 = 0)
y1 axis (y2 = 0)
Start
Classical
No-Man's
Land
Simulations by BounceIt
Classical
No-Man's
Land


The V_1-V_2-path for this problem repeats steps 0 to 12, (more or less!)
3,16
8,21
v_2 axis
1,14,...
10,23,...
5,18,..
19,45,..
0
12,25,...
0,13,...
Start
v_1 axis (v_2 = 0)
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
y_1 axis (y_2 = 0)
Start
Classical No-Man's Land
Simulations by *BounceIt*
Classical No-Man's Land


The V_1-V_2-path
for this problem
repeats steps 0 to 12,
(more or less!)
v_2 axis
3,16
8,21
10,23,...
1,14,...
5,18,..
19,45,..
12,25,...
0
0,13,...
v_1 axis (v_2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
Start
12
Classical
No-Man's
Land
y_1 axis (y_2 = 0)
Simulations by *BounceIt*
Start
Classical
No-Man's
Land

Estrangian plot of $m_1/m_2=4/1$ collision sequence shows symmetry

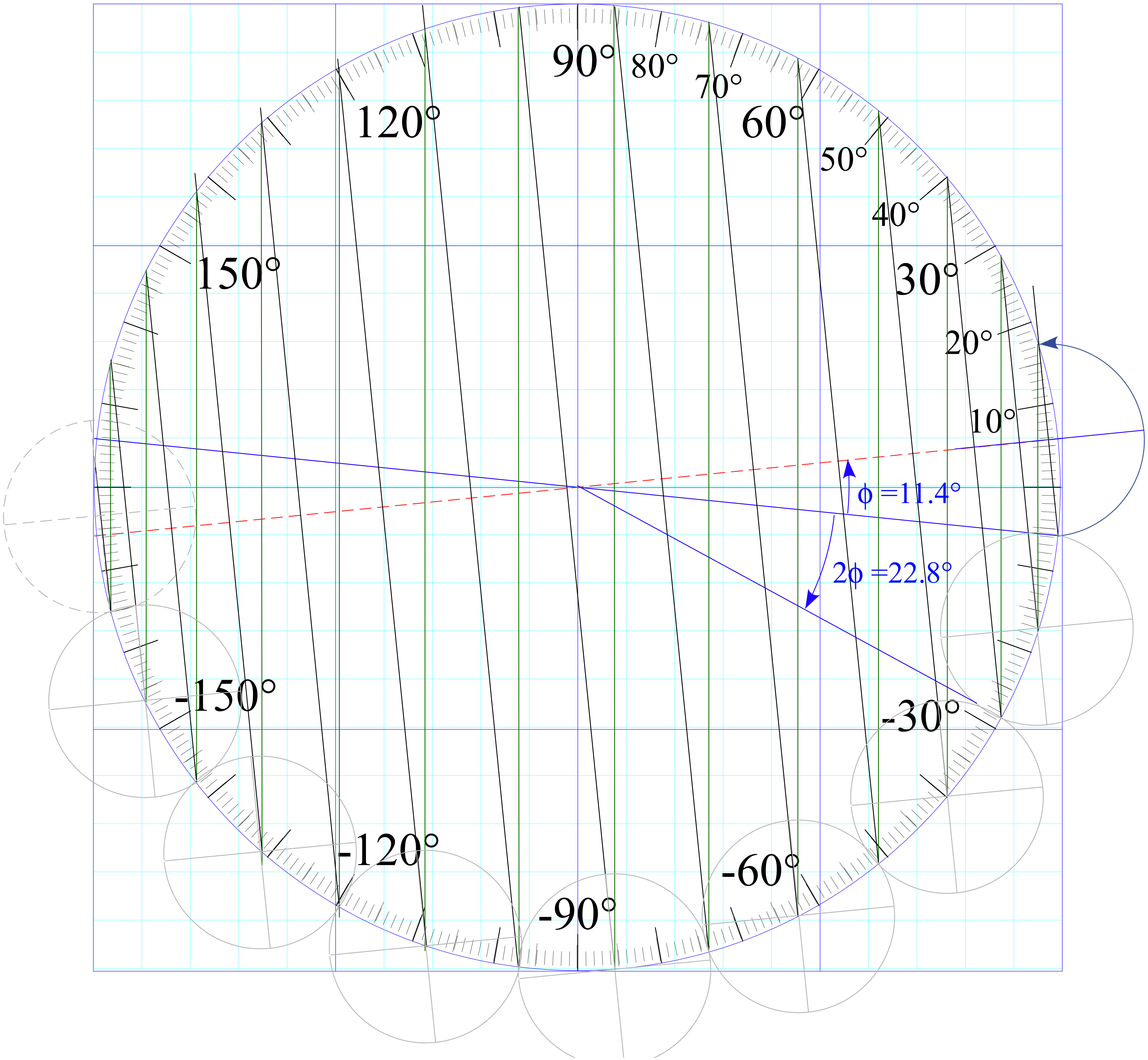
(sort of)



c.o.m. lines
(cons. of mom.)
have slope
$-\sqrt{m_2}/\sqrt{m_1}=-2/1$

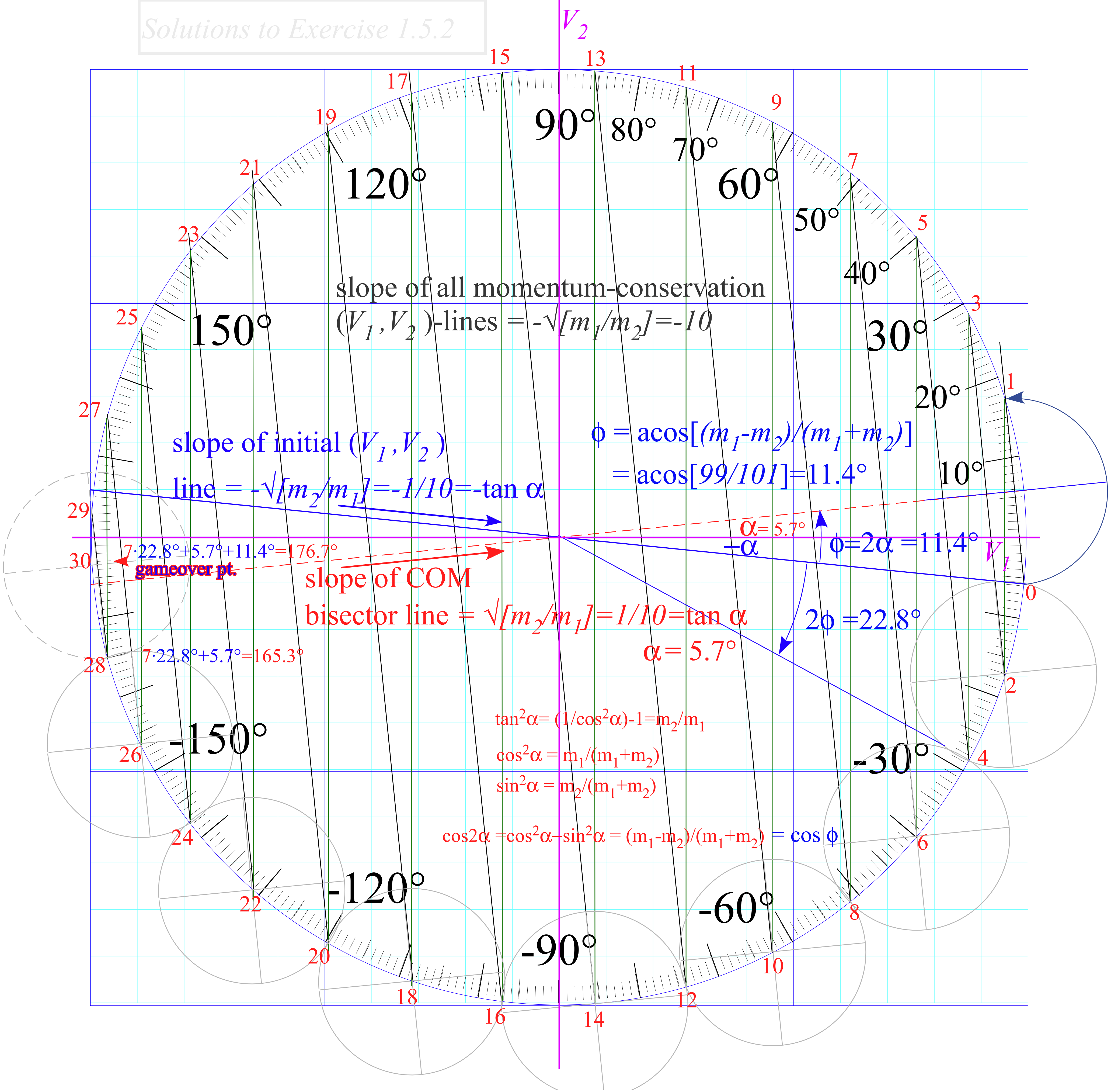
COM line
has slope
$\sqrt{m_2}/\sqrt{m_1}=1/2$

$\phi = \text{acos}[(m_1 - m_2)/(m_1 + m_2)] = \text{acos}[99/101] = 11.4°$

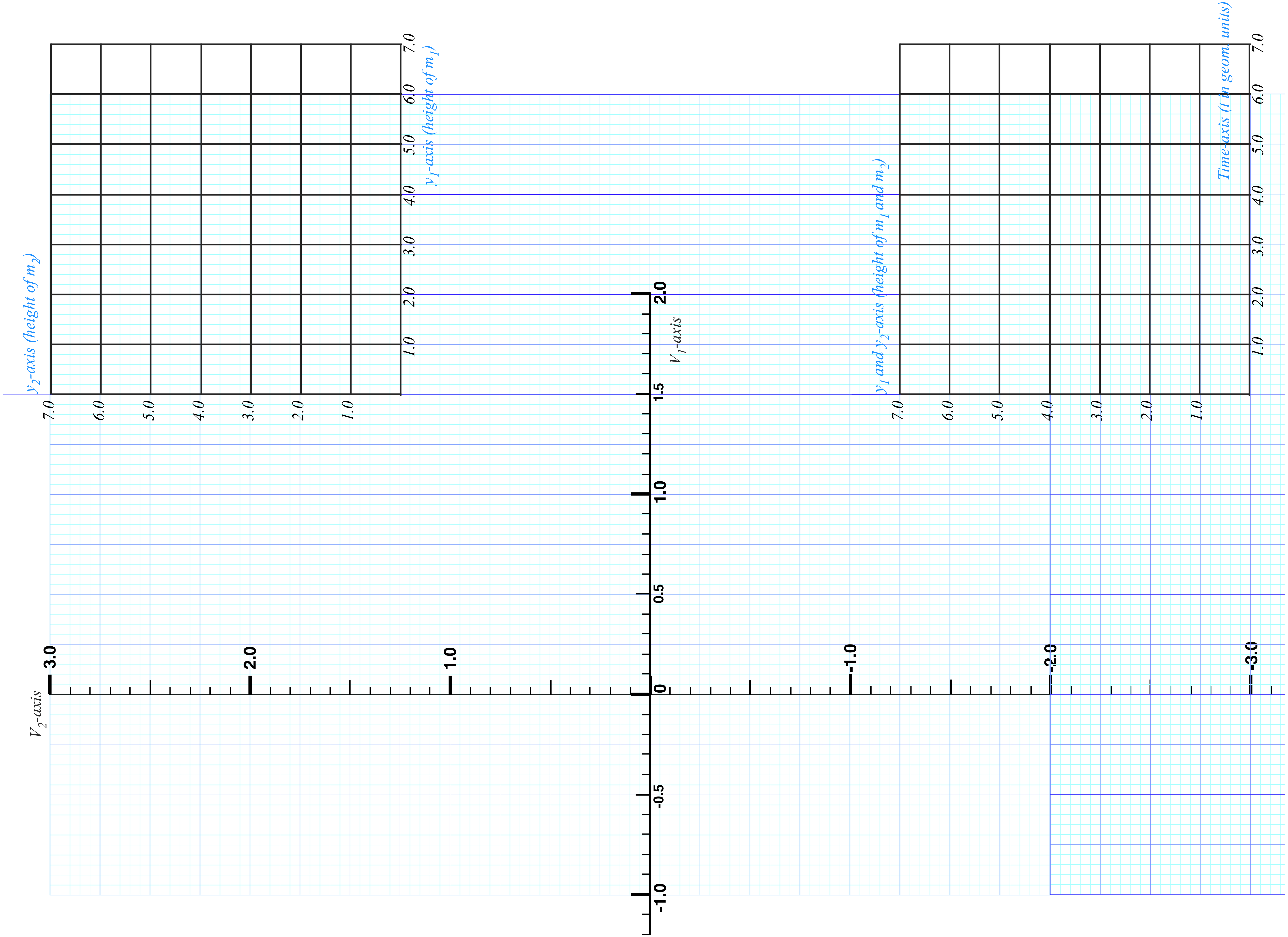


Collisions for mass ratio $m_1{:}m_2 = 100{:}1$

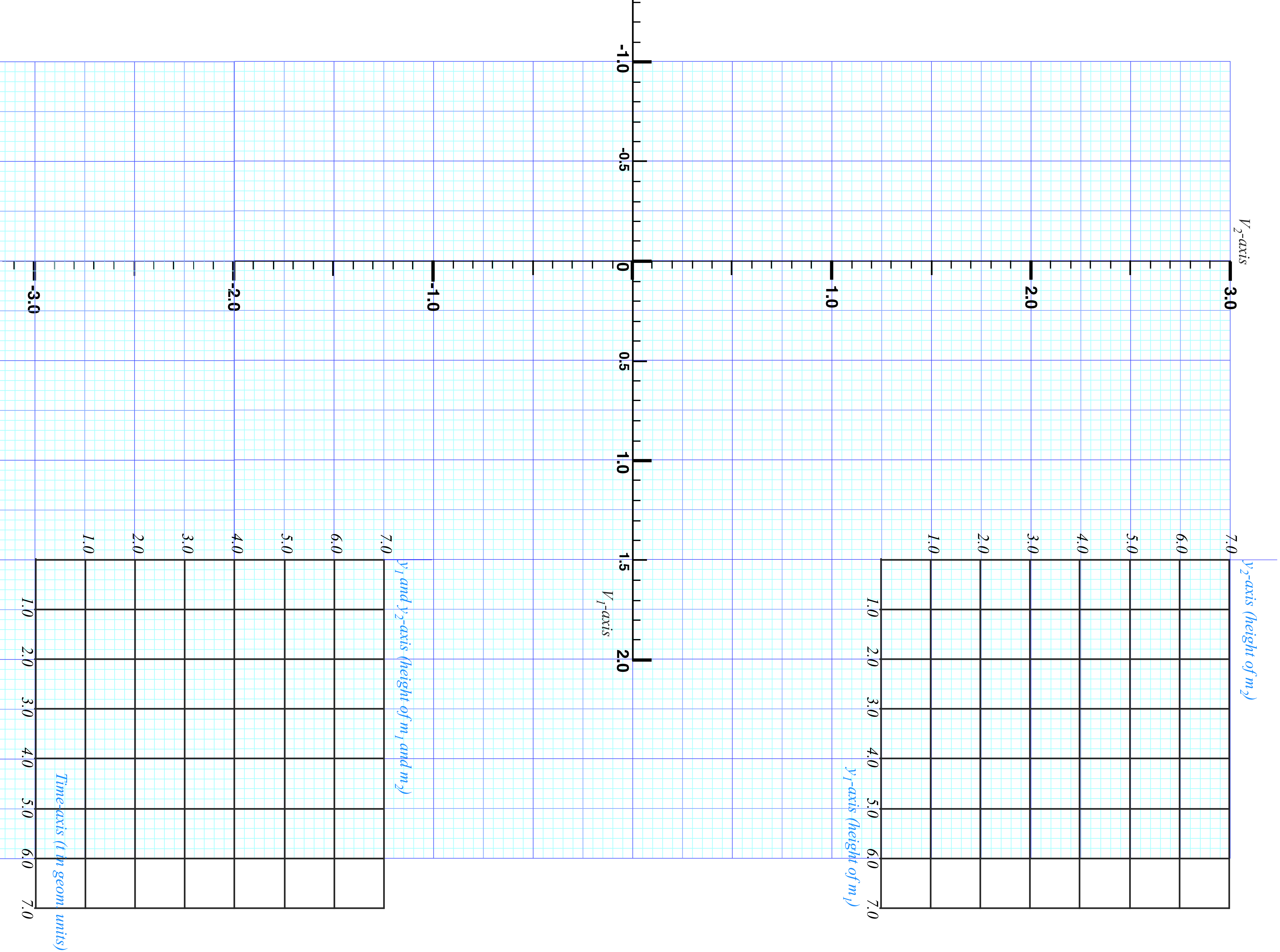
Solutions to Exercise 1.5.2

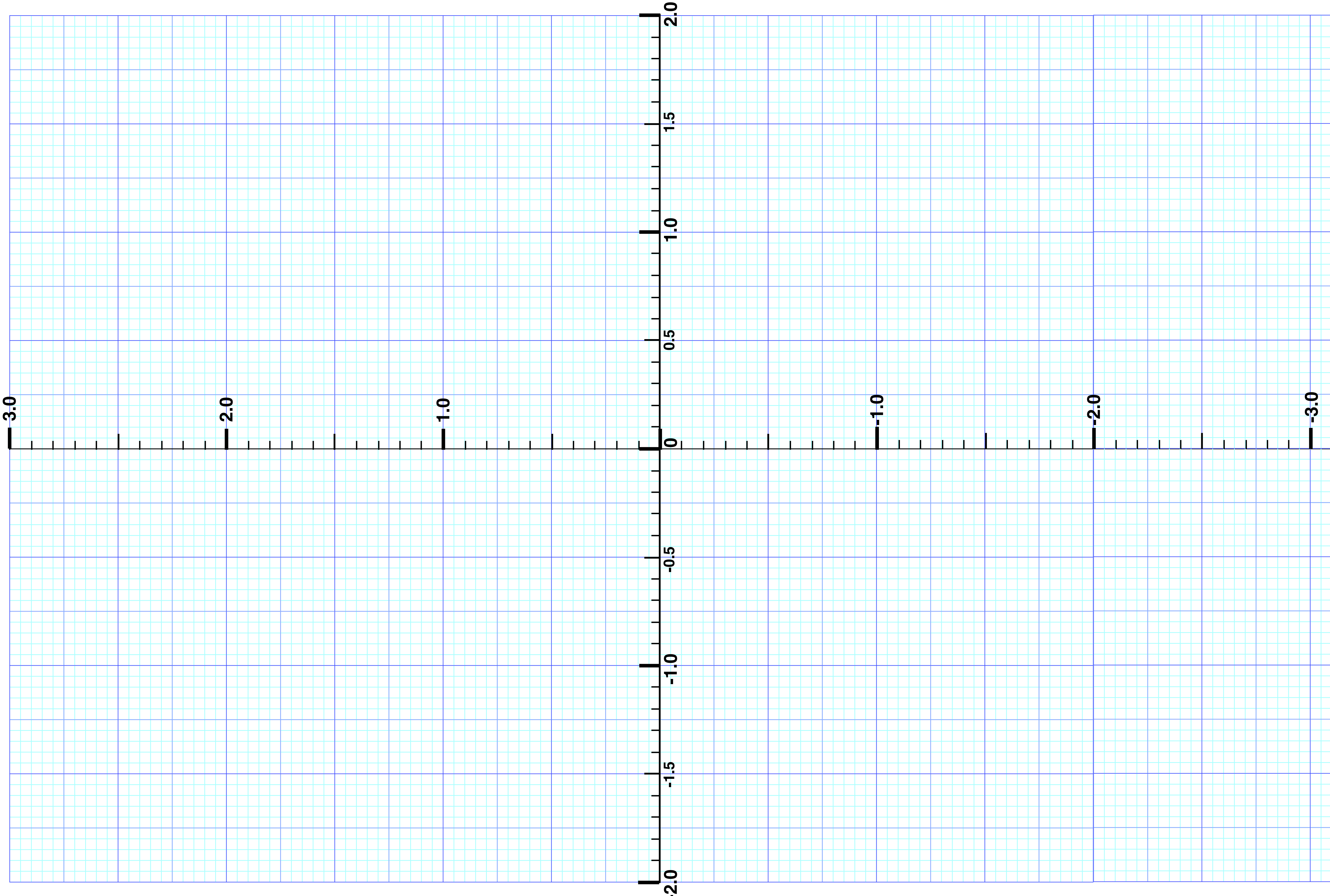


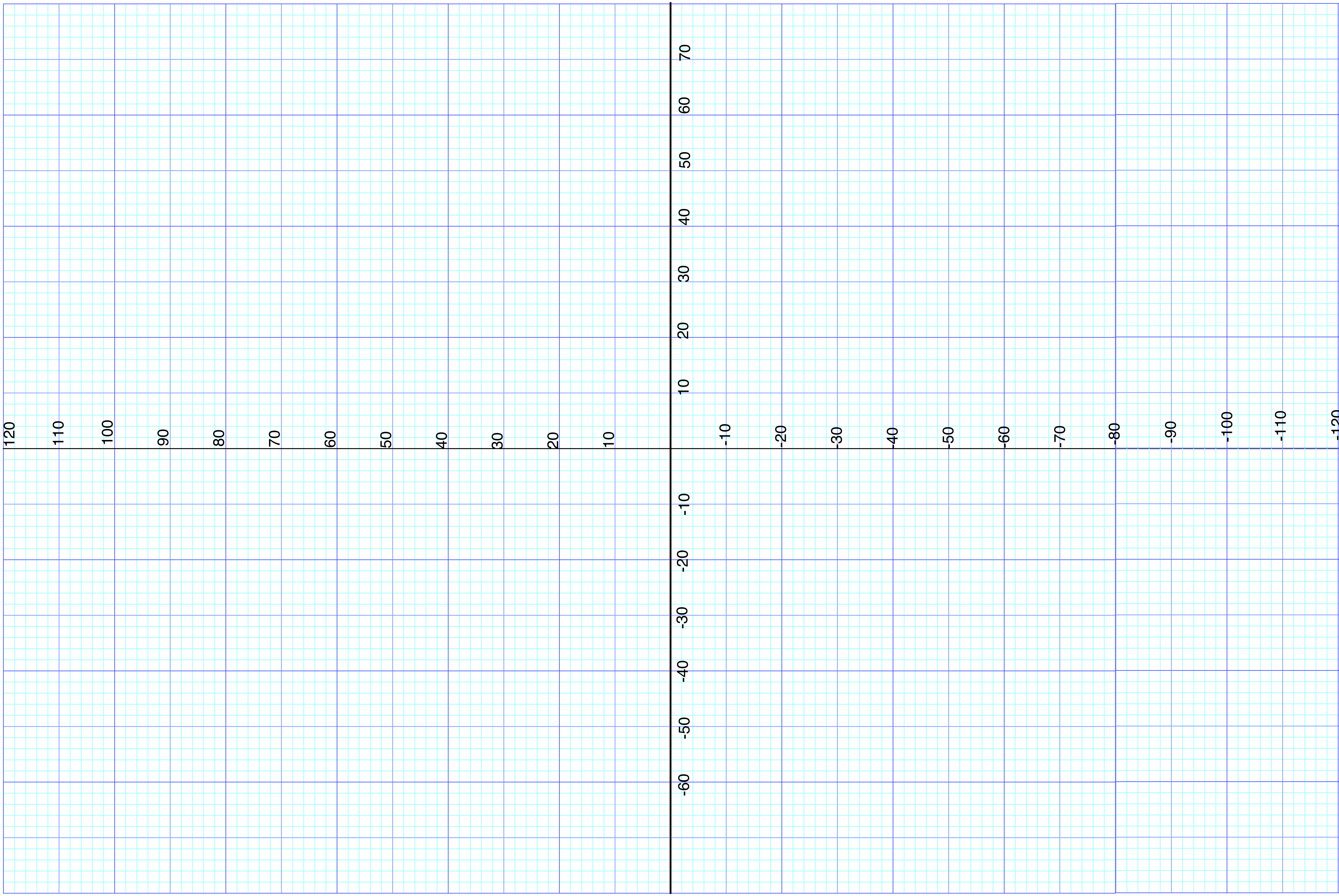
Collisions for mass ratio $m_1{:}m_2 = 100{:}1$

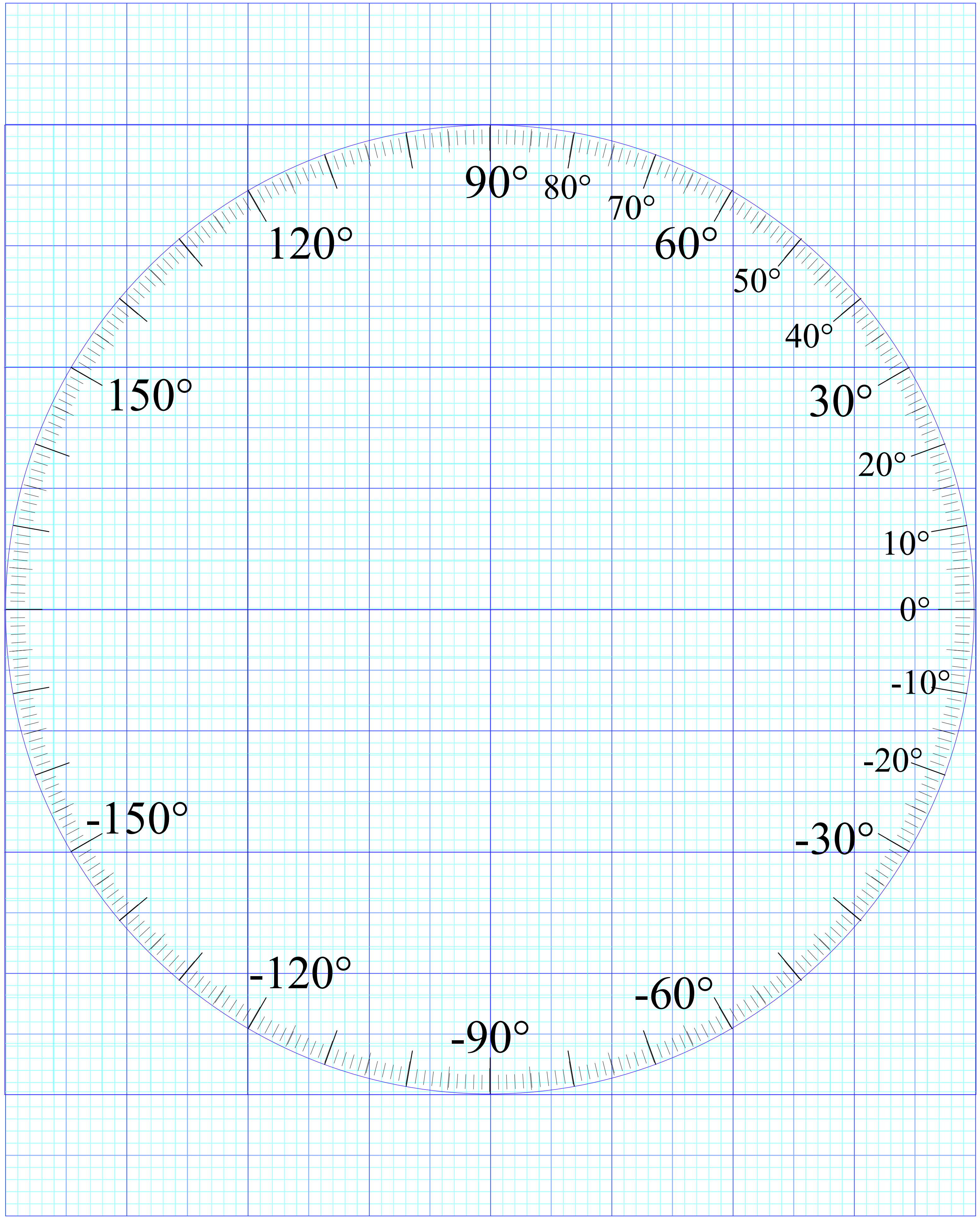


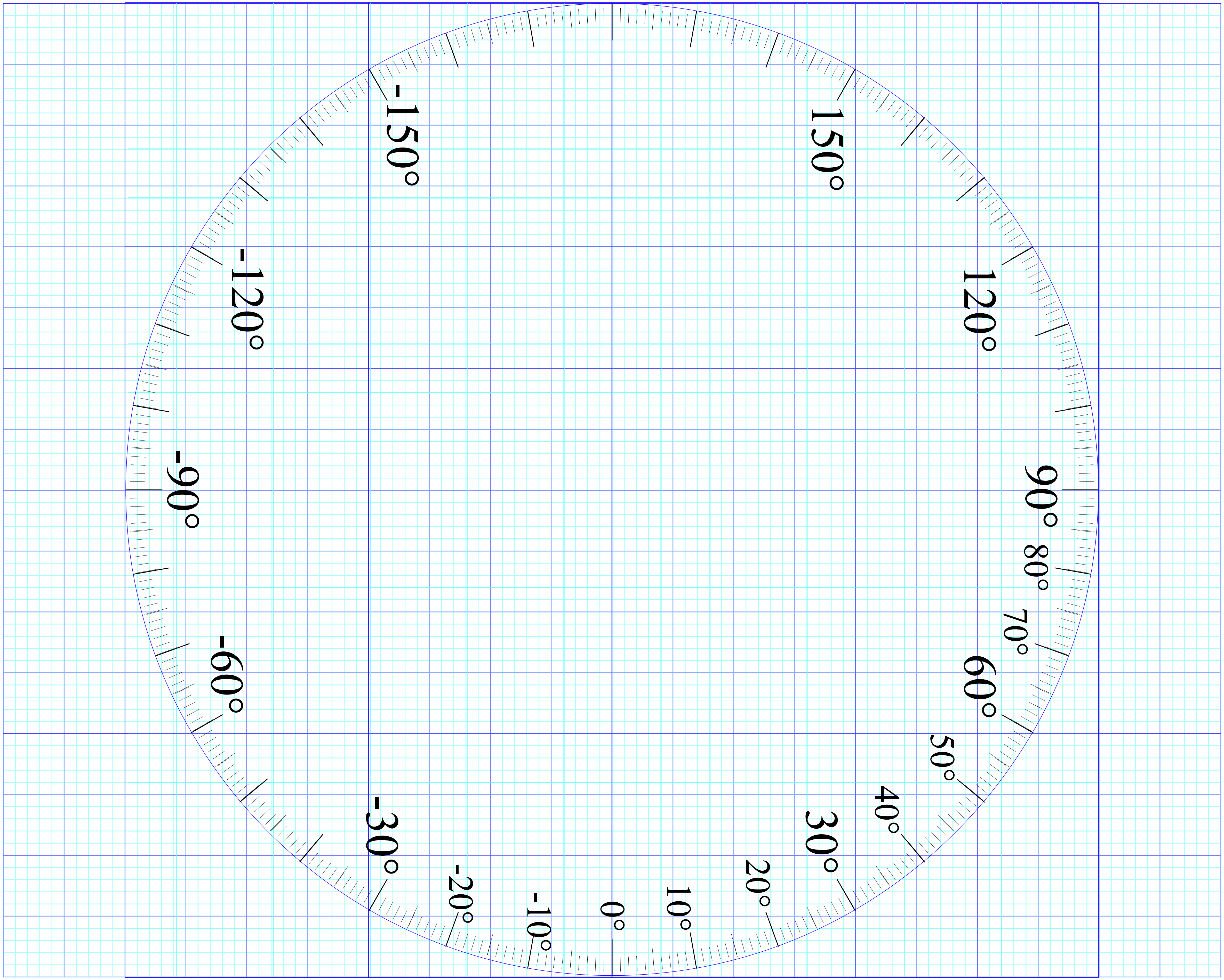
y_2-axis (height of m_2)
y_1-axis (height of m_1)
V_1-axis
y_1 and y_2-axis (height of m_1 and m_2)
Time-axis (t in geom. units)
V_2-axis


V_2-axis
V_1-axis
y_1 and y_2-axis (height of m_1 and m_2)
Time-axis (t in geom. units)
y_2-axis (height of m_2)
y_1-axis (height of m_1)


3.0
2.0
1.0
0
-1.0
-2.0
-3.0
2.0
1.5
1.0
0.5
-0.5
-1.0
-1.5
-2.0


120
110
100
90
80
70
60
50
40
30
20
10
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
-110
-120
70
60
50
40
30
20
10
-10
-20
-30
-40
-50
-60


90°
80°
70°
60°
50°
40°
30°
20°
10°
0°
-10°
-20°
-30°
-60°
-90°
-120°
-150°
150°
120°


0°
10°
20°
30°
40°
50°
60°
70°
80°
90°
120°
150°
-10°
-20°
-30°
-60°
-90°
-120°
-150°