Relawavity: Quantizing wave variables of phase and amplitude
(Unit 3 p.45-64)

Review of wave parameters used to develop relativistic quantum theory
- Bohr-Schrodinger (BS) approximation throws out $Mc^2$ (Is frequency really relative?)
- Effect on group velocity (None) and phase velocity (Absurd)

1st Quantization: Quantizing phase variables $k_m$ and $\omega(k_m)$
- Understanding how quantum dynamics and transitions involve “mixed” states
  - Square well example of mixing unequal frequencies
  - Circle well or ring example of mixing equal or unequal frequencies

Mixing unequal amplitudes makes “Galloping” wave: Analogy of $(SWR, SWQ)$ to $(V_{\text{group}}, V_{\text{phase}})$
- Analogy with optical polarization geometry and Kepler orbits
  - Super-luminal speed and Feynman-Wheeler pair-creation switchbacks

2nd Quantization: Quantizing wave amplitudes $A_N$ and invariance of photon number
- Analogy 1: Many CW (Continuous Waves) add up to make PW (Pulse Waves)
- Analogy 2: Many Photon-Number-Modes add up to make Coherent-Laser-Modes

Heisenberg $\Delta \psi \cdot \Delta t \sim 1 \sim \Delta k \cdot \Delta x$ analogous to $\Delta N \cdot \Delta \text{phase} \sim 1$ uncertainty relations

Electromagnetic wave mode energy: Maxwell vs. Planck-Einstein
- 1st quantization for wave phase variables and classical energy of $E$, $B$, and $A$ fields
- 2nd quantization for wave and Planck quantum energy of $E$, $B$, and $A$ fields
- Scaling $E$-waves to mime quantum $\Psi$-waves and $\psi$-waves

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Relativistic effects on charge, current, and Maxwell Fields
Using (some) wave parameters to develop relativistic quantum theory

\[ u = \frac{\tanh \rho}{c} \approx \rho \quad \text{(for } u \ll c) \]

Energy density, photon number \( N \), and normalization discussed p.74-81

\[ \psi \text{ phase} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \quad \text{(for } u \ll c) \]
\[ c \kappa \text{ phase} = B \sinh \rho \approx B \rho \quad \text{(for } u \ll c) \]
\[ \cos \rho \approx 1 + \frac{1}{2} \rho^2 \approx 1 + \frac{1}{2} \frac{u^2}{c^2} \]
\[ \sinh \rho \approx \rho \approx \frac{u}{c} \]

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\[ h \nu \text{ phase} = hB \cosh \rho = Mc^2 \cosh \rho \]
\[ h \kappa \text{ phase} = hB \sinh \rho = Mc^2 \sinh \rho \]

\[ \text{Planck (1900) } E = \text{ Total Energy: } E = \frac{Me^2}{\sqrt{1-u^2/c^2}} \]
\[ \text{Einstein (1905) } \]

\[ h \nu \text{ phase} = hB \cosh \rho = Mc^2 \cosh \rho \]
\[ h \kappa \text{ phase} = hB \sinh \rho = Mc^2 \sinh \rho \]

\[ c \rho = \frac{Mc}{\sqrt{1-u^2/c^2}} \]

\[ \text{Momentum: } h \kappa \text{ phase} = p = \frac{Mu}{\sqrt{1-u^2/c^2}} \]

\[ \text{DeBroglie (1921) } \]
Using (some) wave parameters to develop relativistic quantum theory

\begin{align*}
\nu_{\text{phase}} &= B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \quad (\text{for } u \ll c) \\
\kappa_{\text{phase}} &= B \sinh \rho \approx B \rho \quad (\text{for } u \ll c)
\end{align*}

At low speeds:

\begin{align*}
\nu_{\text{phase}} &\approx B + \frac{1}{2} \frac{B}{c^2} u^2 \\
\nu_{\text{phase}} \text{ and } \kappa_{\text{phase}} &\approx \frac{B}{c^2} u
\end{align*}

Rescale \( \nu_{\text{phase}} \) by \( h \):

\begin{align*}
M &\approx \frac{hB}{c^2} \quad \text{or: } hB = Mc^2
\end{align*}

\( \eta \nu_{\text{phase}} \approx hB + \frac{1}{2} \frac{hB}{c^2} u^2 \quad (\text{for } u \ll c) \Rightarrow \)

\begin{align*}
\eta \nu_{\text{phase}} &\approx Mc^2 + \frac{1}{2} Mu^2 \\
\eta \nu_{\text{phase}} &\approx Mu
\end{align*}

Lect. 30
p. 3 to 29

\begin{align*}
\nu_{\text{phase}} &= B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \\
\kappa_{\text{phase}} &= B \sinh \rho \approx B \rho
\end{align*}

\begin{align*}
\cosh \rho &\approx 1 + \frac{\rho^2}{2} \\
\sinh \rho &\approx \frac{\rho}{c}
\end{align*}

Need to replace \( \hbar \) with \( \hbar N \) to match e.m. energy density

\begin{align*}
\epsilon, E = h\nu_{\text{phase}}
\end{align*}

Max Planck
1858-1947

Louis DeBroglie
1892-1987

\( h\kappa_{\text{phase}} = hB \sinh \rho = Mc^2 \sinh \rho \)

\( \nu_{\text{phase}} \) and \( \kappa_{\text{phase}} \) resemble

\begin{align*}
U_{\text{phase}} \quad \text{and} \quad K_{\text{phase}}
\end{align*}

(Expensive)

...Try exact \( U_{\text{phase}} \) and \( K_{\text{phase}} \)...

\( \text{Planck (1900)} \)

\( \text{Einstein (1905)} \)

\begin{align*}
E &= \frac{Mc^2}{\sqrt{1 - u^2/c^2}}
\end{align*}

\begin{align*}
\hbar \kappa_{\text{phase}} &= hB \sinh \rho = Mc^2 \sinh \rho
\end{align*}

\begin{align*}
\eta \nu_{\text{phase}} &= hB \cosh \rho = Mc^2 \cosh \rho
\end{align*}

\( \text{DeBroglie (1921)} \)

\begin{align*}
\hbar \kappa_{\text{phase}} &= hB \sinh \rho = Mc^2 \sinh \rho
\end{align*}

\begin{align*}
\eta \nu_{\text{phase}} &= hB \cosh \rho = Mc^2 \cosh \rho
\end{align*}

\begin{align*}
\frac{c p}{\mu} &= \frac{Mu}{\sqrt{1 - u^2/c^2}}
\end{align*}

\begin{align*}
\text{Momentum: } \hbar \kappa_{\text{phase}} &= p = \frac{Mu}{\sqrt{1 - u^2/c^2}}
\end{align*}
Using (some) wave parameters to develop relativistic quantum theory

\[ v_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \quad \text{(for } u \ll c) \]

\[ c \kappa_{\text{phase}} = B \sinh \rho \approx B \rho \quad \text{(for } u \ll c) \]

\[ \frac{u}{c} = \tanh \rho \approx \rho \quad \text{At low speeds:} \]

\[ v_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \quad \leftrightarrow \text{for } (u \ll c) \Rightarrow \]

\[ \kappa_{\text{phase}} \approx \frac{B}{c^2} u \]

Rescale \( v_{\text{phase}} \) by \( h \) so: \( M = \frac{hB}{c^2} \) or: \( hB = Mc^2 \)

\[ h \nu_{\text{phase}} \approx hB + \frac{1}{2} \frac{hB}{c^2} u^2 \quad \leftrightarrow \text{for } (u \ll c) \Rightarrow \]

\[ h \kappa_{\text{phase}} \approx \frac{hB}{c^2} u \]

\[ h \nu_{\text{phase}} \approx Mc^2 + \frac{1}{2} Mu^2 \quad \leftrightarrow \text{for } (u \ll c) \Rightarrow \]

\[ h \kappa_{\text{phase}} \approx Mu \]

\( B = \nu_A \) \( B = \nu_A = c \kappa_A \)

Lect. 30
p. 3 to 29

\( \nu_{\text{phase}} \) and \( \kappa_{\text{phase}} \) resemble

formulæ for Newton’s kinetic energy \( \frac{1}{2} Mu^2 \) and momentum \( Mu \).

So attach scale factor \( h \) (or \( hN \)) to match units.

\( \text{Natural wave conspiracy} \? \) \( \text{Expensive cheap trick} ?? \)

...Try exact \( \nu_{\text{phase}} \) and \( \kappa_{\text{phase}} \)...

\[ h \nu_{\text{phase}} = hB \cosh \rho = Mc^2 \cosh \rho \]

\[ h \kappa_{\text{phase}} = hB \sinh \rho = Mc^2 \sinh \rho \]

\[ \text{Planck (1900)} \]

\[ = \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1-u^2/c^2}} \]

\[ \text{Einstein (1905)} \]

\[ \text{Momentum: } h \kappa_{\text{phase}} = p = \frac{Mu}{\sqrt{1-u^2/c^2}} \]

\( \text{DeBroglie (1921)} \)

Need to replace \( h \) with \( hN \) to match e.m. energy density \( \varepsilon, E \star E = h N \nu_{\text{phase}} \)

This motivates the “particle” normalization

\[ \int \Psi^* \Psi \, dV = N \quad \Psi = \frac{\sqrt{E}}{h \nu} \]

Planck (1900)

\[ \frac{1}{\sqrt{\beta^2-1}} = \frac{1}{\sqrt{\frac{u^2}{c^2}} - 1} \quad (\text{old-fashioned notation}) \]

\[ \beta = \frac{u}{c} \]

\[ \beta = \frac{u}{c} \]

\[ \frac{1}{2} = 0.5 \quad 3/5 = 0.6 \quad 3/4 = 0.75 \quad 4/5 = 0.8 \quad 5/4 = 1.25 \quad 4/3 = 1.33 \quad 5/3 = 1.67 \quad 2/1 = 2.0 \]
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Relativistic effects on charge, current, and Maxwell Fields
Bohr-Schroedinger (BS) approximation throws out $Mc^2$ (Is frequency really relative?)

\[
E = \hbar \omega = \frac{Mc^2}{\sqrt{1-u^2/c^2}} = Mc^2 \cosh \rho
\]

given:

\[
cp = \hbar c k
\]

\[
\frac{Mc}{\sqrt{1-u^2/c^2}} = Mc^2 \sinh \rho
\]

Some details concerning
Lect. 30 - slide 31

Using (some) wave coordinates for relativistic quantum theory

(a) Exact Einstein-Plank Dispersion
(b) Bohr-Schroedinger Approximation

Relativity Web Simulation
Relativistic Terms - Einstein-Plank Dispersion
Bohr-Schrodinger (BS) approximation throws out $M c^2$ (Is frequency really relative?)

\[
E = \hbar \omega \\
= \frac{M c^2}{\sqrt{1 - u^2 / c^2}} = M c^2 \cosh \rho = M c^2 \sqrt{1 + \sinh^2 \rho} = \sqrt{(M c^2)^2 + (cp)^2}
\]

given:

\[
cp = \hbar c k \\
\frac{M c u}{\sqrt{1 - u^2 / c^2}} = M c^2 \sinh \rho
\]

Some details concerning
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Using (some) wave coordinates for relativistic quantum theory

(a) Exact Einstein-Planck Dispersion

(b) Bohr-Schrodinger Approximation

RelaWavity Web Simulation
Relativistic Terms - Einstein-Plank Dispersion
Bohr-Schrodinger (BS) approximation throws out $Mc^2$  

\[ E = \hbar \omega \]
\[ = \frac{Mc^2}{\sqrt{1-u^2/c^2}} = Mc^2 \cosh \rho = Mc^2 \sqrt{1+\sinh^2 \rho} = \sqrt{(Mc^2)^2 + (cp)^2} \]
\[ E = \left[ (Mc^2)^2 + (cp)^2 \right]^{1/2} \approx Mc^2 + \frac{1}{2M} p^2 \]

BS- binomial approximation  
(a+b)^n = a^n + na^{n-1}b + ...  
replaces hyperbola:  
\[ E = \left[ (Mc^2)^2 + (cp)^2 \right]^{1/2} = Mc^2 + \frac{1}{2} \frac{e^2p^2}{Mc^2} + ... \]
with parabola:  
\[ E = Mc^2 + \frac{1}{2M} p^2 \]
Bohr-Schrodinger (BS) approximation throws out $Mc^2$ (Is frequency really relative?)

$$E = \hbar \omega$$

$$= \frac{Mc^2}{\sqrt{1-u^2/c^2}} = Mc^2 \cosh \rho = Mc^2 \sqrt{1+\sinh^2 \rho} = \sqrt{(Mc^2)^2 + (cp)^2}$$

$$E = \left[\left(\frac{Mc^2}{\sqrt{1-u^2/c^2}}\right)^2 + (cp)^2\right]^{1/2} \approx \frac{1}{2M} p^2 \rightarrow \frac{1}{2M} p^2$$

The BS claim: may shift energy origin $(E=Mc^2, \ cp=0)$ to $(E=0, \ cp=0)$. (Frequency is relative!)

BS- binomial approximation $(a+b)^n = a^n + na^{n-1}b + \ldots$ replaces hyperbola: 

$$E = \left[\left(\frac{Mc^2}{\sqrt{1-u^2/c^2}}\right)^2 + (cp)^2\right]^{1/2} = Mc^2 + \frac{1}{2} \frac{c^2 p^2}{Mc^2} + \ldots$$

with parabola: 

$$E = Mc^2 + \frac{1}{2M} p^2$$

RelaWavity Web Simulation
Relativistic Terms - Einstein-Plank Dispersion
Review of wave parameters used to develop relativistic quantum theory

Bohr-Schroedinger (BS) approximation throws out $Mc^2$ (*Is frequency really relative?*)

Effect on group velocity (*None*) and phase velocity (*Absurd*)

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Relativistic effects on charge, current, and Maxwell Fields
Bohr-Schroedinger (BS) approximation throws out $Mc^2$ (Is frequency really relative?)

Given:

$cp = \hbar c k$

$Mc = M c \sqrt{1 - u^2 / c^2} = M c^2 \sinh \rho$

$E = \hbar \omega$

$$E = \frac{Mc^2}{\sqrt{1 - u^2 / c^2}} = Mc^2 \cosh \rho = Mc^2 \sqrt{1 + \sinh^2 \rho} = \sqrt{(Mc^2)^2 + (cp)^2}$$

$$E = \left[ (Mc^2)^2 + (cp)^2 \right]^{1/2} \approx Mc^2 + \frac{1}{2M} p^2 \quad \text{BS-approx} \quad \frac{1}{2M} p^2$$

The BS claim: May shift energy origin $(E=Mc^2, cp=0)$ to $(E=0, cp=0)$. (Frequency is relative!)

BS- binomial approximation

$$(a+b)^n = a^n + na^{n-1}b + \ldots$$

replaces hyperbola:

$$E = \left[ (Mc^2)^2 + (cp)^2 \right]^{1/2} = Mc^2 + \frac{1}{2} \frac{c^2 p^2}{Mc^2} + \ldots$$

with parabola:

$$E = Mc^2 + \frac{1}{2M} p^2$$

This affects wave phase dynamics only:
Bohr-Schrodinger (BS) approximation throws out $M c^2$ (Is frequency really relative?)

\[
E = \hbar \omega \\
= \frac{M c^2}{\sqrt{1-u^2/c^2}} = M c^2 \cosh \rho = M c^2 \sqrt{1+\sinh^2 \rho} = \sqrt{(M c^2)^2 + (cp)^2}
\]

\[
E = \left[ (M c^2)^2 + (cp)^2 \right]^{1/2} \approx M c^2 + \frac{1}{2M} p^2 \quad \text{BS-approx} \quad \frac{1}{2M} p^2
\]

The BS claim: may shift energy origin $(E=M c^2, \ cp=0)$ to $(E=0, \ cp=0)$. (Frequency is relative!)

BS-binomial approximation $(a+b)^n = a^n + n a^{n-1} b + ...$

replaces hyperbola: $E = \left[ (M c^2)^2 + (cp)^2 \right]^{1/2} = M c^2 + \frac{1}{2} \frac{c^2 p^2}{M c^2} + ...$

with parabola: $E = M c^2 + \frac{1}{2M} p^2$

This affects wave phase dynamics only:

Group velocity $u = V_{\text{group}} = \frac{d \omega}{dk}$ is a differential quantity unaffected by origin shift.

But, Phase velocity $\frac{\omega}{k} = V_{\text{phase}}$ is greatly reduced by deleting $M c^2$ from $E = \hbar \omega$. 

Relativistic Terms - Einstein-Plank Dispersion

RelaWavity Web Simulation
**Bohr-Schrodinger (BS) approximation throws out** $Mc^2$ (Is frequency really relative?)

\[ E = \hbar \omega \]

\[ E = \frac{Mc^2}{\sqrt{1-u^2/c^2}} = Mc^2 \cosh \rho = Mc^2 \sqrt{1+\sinh^2 \rho} = \sqrt{(Mc^2)^2 + (cp)^2} \]

\[ E = \left[ \left( Mc^2 \right)^2 + (cp)^2 \right]^{1/2} \approx Mc^2 + \frac{1}{2M} p^2 \quad \text{BS-approx} \rightarrow \frac{1}{2M} p^2 \]

The BS claim: may shift energy origin $(E=Mc^2, \ cp=0)$ to $(E=0, \ cp=0)$. *(Frequency is relative!)*

**BS- binomial approximation** \( (a+b)^n = a^n + na^{n-1}b + \ldots \)

replaces hyperbola: \[ E = \left[ \left( Mc^2 \right)^2 + (cp)^2 \right]^{1/2} = Mc^2 + \frac{1}{2} \frac{c^2 p^2}{Mc^2} + \ldots \]

with parabola: \[ E = Mc^2 + \frac{1}{2M} p^2 \]

This affects wave phase dynamics only:

Group velocity \( u = V_{\text{group}} = \frac{d\omega}{dk} \) is a **differential** quantity unaffected by origin shift.

But, Phase velocity \( \frac{\omega}{k} = V_{\text{phase}} \) is greatly reduced by deleting $Mc^2$ from $E=\hbar \omega$.

It slows from super-luminal $V_{\text{phase}}=c/u$ to a sedate sub-luminal speed of $V_{\text{group}}/2$.

\[ \omega_{BS}(k) = \frac{k^2}{2M} \quad \text{gives:} \quad V_{\text{phase}} = \frac{\omega_{BS}}{k} = \frac{k}{2M} \]

and: \[ V_{\text{group}} = \frac{d\omega_{BS}}{dk} = \frac{k}{M} \]
**Bohr-Schrodinger (BS) approximation throws out Mc**²

\[ E = \hbar \omega \]

\[ E = \frac{M_c^2}{\sqrt{1-u^2/c^2}} \approx M_c^2 \cosh \rho = M_c^2 \sqrt{1+\sinh^2 \rho} = \sqrt{\left(M_c^2\right)^2 + (cp)^2} \]

\[ E = \left( M_c^2 \right)^2 + (cp)^2 \]

\[ \approx M_c^2 + \frac{1}{2M} p^2 \]

**BS-approx**

\[ \frac{1}{2M} p^2 \]

**The BS claim:** may shift energy origin \((E=Mc^2, cp=0)\) to \((E=0, cp=0)\). *(Frequency is relative!)*

**BS- binomial approximation**

\[ \left(a+b\right)^n = a^n + na^{n-1}b + \ldots \]

replaces hyperbola:

\[ E = \left[ \left( M_c^2 \right)^2 + (cp)^2 \right]^{1/2} = Mc^2 + \frac{1}{2} \frac{c^2 p^2}{M_c^2} + \ldots \]

with parabola:

\[ E = Mc^2 + \frac{1}{2M} p^2 \]

This affects wave phase dynamics only:

Group velocity \( u = V_{\text{group}} = \frac{d\omega}{dk} \) is a *differential* quantity unaffected by origin shift.

But, Phase velocity \( \frac{\omega}{k} = V_{\text{phase}} \) is greatly reduced by deleting \( Mc^2 \) from \( E = \hbar \omega \).

It slows from super-luminal \( V_{\text{phase}} = c^2/u \) to a sedate sub-luminal speed of \( V_{\text{group}}/2 \).

\[ \omega_{BS} (k) = \frac{k^2}{2M} \]

\[ V_{phase} = \frac{\omega_{BS}}{k} = \frac{k}{2M} \]

and:

\[ V_{group} = \frac{d\omega_{BS}}{dk} = \frac{k}{M} \]

**1st quantization:**

Restrict wave-number \( \kappa \) to integer or \( \frac{1}{2} \)-integer quanta *(to fit in cavity)*
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Relativistic effects on charge, current, and Maxwell Fields
Here is a rare but important case where \( \frac{d\omega(k)}{dk} \) equals \( \frac{\Delta \omega}{\Delta k} \). (Usually not so unless limit \( \Delta k \to 0 \) exists.)

Standard formula for classical group velocity is

\[
V_{\text{group}} = \frac{d\omega(k)}{dk}
\]

But this may fail if \( \omega(k) \) is quantized and thus discrete \( \omega(k_m) \).

Then we need to use an exact quantum form:

\[
V_{\text{group}} = \frac{\Delta \omega}{\Delta k} = \frac{\omega(k_1) - \omega(k_2)}{k_1 - k_2}
\]

Here the exact discrete value is:

\[
V_{\text{group}} = \frac{\Delta \omega}{\Delta k} = \frac{\omega(k_1) - \omega(k_2)}{k_1 - k_2} = \frac{Be^\rho - Be^{-\rho}}{Be^\rho - (-Be^{-\rho})} = \frac{e^\rho - e^{-\rho}}{e^\rho + Be^{-\rho}} = \frac{\sinh \rho}{\cosh \rho} = \tanh \rho
\]

This time it matches calculus value:

\[
V_{\text{group}} = \frac{d\omega(k)}{dk} = \frac{d(cosh \rho)}{d(k)} = \frac{\sinh \rho}{\cosh \rho} = \tanh \rho
\]
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Quantized $\omega$ and $k$ \textit{Counting wave kink numbers}

If everything is made of waves then we expect \textit{quantization} of everything because waves only thrive if \textit{integral} numbers $n$ of their “kinks” fit into whatever structure (box, ring, etc.) they’re supposed to live. The numbers $n$ are called \textit{quantum numbers}.

\begin{tabular}{c c c c}
OK box quantum numbers: & $n=1$ & $n=2$ & $n=3$ & $n=4$
\end{tabular}

\begin{tabular}{c c c c}
(+ integers only) & 1 half-wave & 2 half-waves & 3 half-waves & 4 half-waves
\end{tabular}

Some \textit{NOT OK} numbers: $n=0.67$ & $n=1.7$ & $n=2.59$ & $n=4$

too fat! & too thin! & wrong color again! & \null

\textit{NOTE:} We’re using “false-color” here.

\textit{This doesn’t mean a system’s energy can’t vary continuously between “OK” values $E_1$, $E_2$, $E_3$, $E_4$, ...}
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OK box quantum numbers:  

$\begin{align*}
\text{n = 1} & : 1 \text{ half-wave} \\
\text{n = 2} & : 2 \text{ half-waves} \\
\text{n = 3} & : 3 \text{ half-waves} \\
\text{n = 4} & : 4 \text{ half-waves}
\end{align*}$

(+ integers only)

Some NOT OK numbers: $n = 0.67$

$n = 0.67$

too fat!

$n = 1.7$

too thin!

$n = 2.59$

wrong color again!

$n = 4$

misfits...

...not tolerated!

NOTE: We’re using “false-color” here.

This doesn’t mean a system’s energy can’t vary continuously between “OK” values $E_1, E_2, E_3, E_4,$…

In fact its state can be a linear combination of any of the “OK” waves $|E_1>, |E_2>, |E_3>, |E_4>, ...$
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**OK box quantum numbers:**

\[
\begin{align*}
    n &= 1 & & n = 2 & & n = 3 & & n = 4 \\
    1 \text{ half-wave} & & 2 \text{ half-waves} & & 3 \text{ half-waves} & & 4 \text{ half-waves}
\end{align*}
\]

(*+ integers only*)

Some **NOT OK numbers:**

\[
\begin{align*}
    n &= 0.67 & & n = 1.7 & & n = 2.59 & & n = 4
\end{align*}
\]

- too fat!  
- too thin!  
- misfits...  
- wrong color again!  
- ...not tolerated!

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*This doesn’t mean a system’s energy can’t vary *continuously* between “OK” values \( E_1, E_2, E_3, E_4, \ldots \). In fact its state can be a linear combination of any of the “OK” waves \( |E_1>, |E_2>, |E_3>, |E_4>, \ldots \).*

That’s the only way you get any light in or out of the system to “see” it.

\[
\begin{align*}
    \omega_{32} &= (E_3 - E_2)/\hbar \\
    \omega_{21} &= (E_2 - E_1)/\hbar
\end{align*}
\]
Quantized $\omega$ and $k$  

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OK box quantum numbers: 

<table>
<thead>
<tr>
<th>$n$</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 half-wave</td>
</tr>
<tr>
<td>2</td>
<td>2 half-waves</td>
</tr>
<tr>
<td>3</td>
<td>3 half-waves</td>
</tr>
<tr>
<td>4</td>
<td>4 half-waves</td>
</tr>
</tbody>
</table>

(All integers only)

Some NOT OK numbers: 

<table>
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<td>misfits...</td>
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$$|E_4>$$

$$|E_3>$$

$$|E_2>$$

$$|E_1>$$

These eigenstates are just the ways the wavy system can “play dead”...
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Consider two lowest $E$-states by themselves

$|E_2\rangle$

$|E_1\rangle$

By Inter-Ship and University of Arkansas Physics
Elegant Educational Tools Since 2001
Consider two lowest $E$-states by themselves in time

$$e^{-i\omega_2 t} |E_2\rangle$$

$$e^{-i\omega_1 t} |E_1\rangle$$
Consider two lowest E-states by themselves in time.

Now combine (add) them:

\[ |E_1\rangle + |E_2\rangle \]

\[ = \frac{|E_1\rangle + |E_2\rangle}{\sqrt{2}} \]
Consider two lowest E-states by themselves in time

\[ e^{-i\omega_1 t} |E_1\rangle \]

\[ e^{-i\omega_2 t} |E_2\rangle \]

Now combine (add) them and let time roll!

\[ \left( e^{-i\omega_1 t} |E_1\rangle + e^{-i\omega_2 t} |E_2\rangle \right) / \sqrt{2} \]
Consider two lowest E-states by themselves in time

\[ e^{-i\omega_1 t} |E_1\rangle \]

and

\[ e^{-i\omega_2 t} |E_2\rangle \]

Now combine (add) them and let time roll!

\[ (e^{-i\omega_1 t} |E_1\rangle + e^{-i\omega_2 t} |E_2\rangle) / \sqrt{2} \]
Consider two lowest $E$-states by themselves in time

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---

**OK box quantum numbers:**

- \( n=1 \)
- \( n=2 \)
- \( n=3 \)
- \( n=4 \)

- (+ integers only)

---

Some **NOT OK numbers:**

- \( n=0.67 \)  
  too fat!

- \( n=1.7 \)  
  too thin!

- \( n=2.59 \)  
  wrong color again!

- \( n=4 \)  
  ...not tolerated!

---

**NOTE:** We’re using “false-color” here.

---

**Rings tolerate a zero** *(kinkless)* quantum wave but require \( \pm \) *integral* wave number.

---

**OK ring quantum numbers:**

- \( m=0 \)
- \( m=\pm 1 \)
- \( m=\pm 2 \)
- \( m=3 \)

---

*(\( \pm \) integral number of wavelengths)*

---

**Bohr’s models of atomic spectra (1913-1923)** are beginnings of *quantum wave mechanics* built on *Planck-Einstein* (1900-1905) relation \( E=\hbar \nu \).  *DeBroglie* relation \( p=\hbar/\lambda \) comes around 1923.
Consider two lowest $E$-states by themselves

$|E_{m=+1}\rangle$

$|E_{m=0}\rangle$

By Heter-st and University of Arkansas Physics  Elegant Educational Tools Since 2001
Consider two lowest $E$-states by themselves

\[ |E_{m=0}\rangle \]

\[ |E_{m=+1}\rangle \]

Now combine (add) them and let time roll!

\[ \left( e^{-i\omega_0 t} |E_0\rangle + e^{-i\omega_{+1} t} |E_{+1}\rangle \right) / \sqrt{2} \]
Consider two lowest $E$-states by themselves

$$|E_{m=0}\rangle$$

Now combine (add) them and let time roll!

$$\left( e^{-i\omega_0 t} |E0\rangle + e^{-i\omega_{+1} t} |E_{+1}\rangle \right)/\sqrt{2}$$

(Just moves forward rigidly)
Consider two degenerate $E$-states by themselves

$|E_{m=+1}\rangle$

$|E_{m=-1}\rangle$

0th Fourier
1st Fourier:

10 15 20

0th Fourier
1st Fourier:

10 15 20
Consider two degenerate \( E \)-states by themselves

\[
|E_{m=+1}\rangle
\]

Now combine (add) them and let time roll!

\[
\frac{e^{-i\omega_{-1}t}|E_{-1}\rangle + e^{-i\omega_{+1}t}|E_{+1}\rangle}{\sqrt{2}}
\]

Group wave is stationary if \( \omega_{-1} = \omega_{+1} \) but phase can move or "gallop" faster than light!

If \( \omega_{-1} < \omega_{+1} \) then \( V_{\text{group}} < 0 \)

If \( \omega_{-1} > \omega_{+1} \) then \( V_{\text{group}} > 0 \)

Nothing CAN go faster than light
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2-CW dynamics has two 1-CW amplitudes $A_\to$ and $A_\gets$ that may be unmatched. ($A_\to \neq A_\gets$)

$$A_\to e^{i(k_\to x - \omega_\to t)} + A_\gets e^{i(k_\gets x - \omega_\gets t)} = e^{i(k_\Sigma x - \omega_\Sigma t)} [A_\to e^{i(k_\Delta x - \omega_\Delta t)} + A_\gets e^{-i(k_\Delta x - \omega_\Delta t)}]$$

Waves have half-sum mean-phase rates $(k_\Sigma, \omega_\Sigma)$ and half-difference group rates $(k_\Delta, \omega_\Delta)$.

$$k_\Sigma = (k_\to + k_\gets) / 2$$
$$k_\Delta = (k_\to - k_\gets) / 2$$
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\[
A_\rightarrow e^{i(k_\rightarrow x - \omega_\rightarrow t)} + A_\leftarrow e^{i(k_\leftarrow x - \omega_\leftarrow t)} = e^{i(k_\Sigma x - \omega_\Sigma t)} [A_\rightarrow e^{i(k_\Delta x - \omega_\Delta t)} + A_\leftarrow e^{-i(k_\Delta x - \omega_\Delta t)}]
\]

Waves have half-sum mean-phase rates $(k_\Sigma, \omega_\Sigma)$ and half-difference group rates $(k_\Delta, \omega_\Delta)$.

\[
k_\Sigma = (k_\rightarrow + k_\leftarrow) / 2 \quad k_\Delta = (k_\rightarrow - k_\leftarrow) / 2
\]
\[
\omega_\Sigma = (\omega_\rightarrow + \omega_\leftarrow) / 2 \quad \omega_\Delta = (\omega_\rightarrow - \omega_\leftarrow) / 2
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Now consider amplitude mean $A_\Sigma = (A_\rightarrow + A_\leftarrow) / 2$ and amplitude half-difference $A_\Delta = (A_\rightarrow - A_\leftarrow) / 2$. 
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$$A_{\rightarrow}e^{i(k_{\rightarrow}x-\omega_{\rightarrow}t)} + A_{\leftarrow}e^{i(k_{\leftarrow}x-\omega_{\leftarrow}t)} = e^{i(k_{\Sigma}x-\omega_{\Sigma}t)}[A_{\rightarrow}e^{i(k_{\Delta}x-\omega_{\Delta}t)} + A_{\leftarrow}e^{-i(k_{\Delta}x-\omega_{\Delta}t)}]$$

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Detailed wave motion depends on standing-wave-ratio SWR or the inverse standing-wave-quotient SWQ.

$$\frac{Envelope-Min.}{Envelope-Max.} = SWR = \frac{(A_{\rightarrow} - A_{\leftarrow})}{(A_{\rightarrow} + A_{\leftarrow})}$$
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$$\frac{\text{Envelope Min.}}{\text{Envelope Max.}} = \text{SWR} = \frac{(A_\rightarrow - A_\leftarrow)}{(A_\rightarrow + A_\leftarrow)}$$

Envelope Maximum

$$2A_\Sigma = (A_\rightarrow + A_\leftarrow)$$

Envelope Minimum

$$2A_\Delta = (A_\rightarrow - A_\leftarrow)$$

$$\text{SWQ} = \frac{(A_\rightarrow + A_\leftarrow)}{(A_\rightarrow - A_\leftarrow)} = \frac{1}{\text{SWR}}$$
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They’re analogous to group velocity $V_{\text{group}} < c$ frequency ratios and inverse phase velocity $V_{\text{phase}} > c$ ratios.

$$V_{\text{group}} = \frac{\omega_\Delta}{k_\Delta} = \frac{(\omega_\rightarrow - \omega_\leftarrow)}{(k_\rightarrow - k_\leftarrow)} = c \frac{(\omega_\rightarrow - \omega_\leftarrow)}{(\omega_\rightarrow + \omega_\leftarrow)}$$

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$$V_{\text{group}} = \frac{\omega_\Delta}{ck_\Delta} = \frac{(\omega_\rightarrow - \omega_\leftarrow)}{c(k_\rightarrow - k_\leftarrow)} = \frac{(\omega_\rightarrow - \omega_\leftarrow)}{(\omega_\rightarrow + \omega_\leftarrow)}$$

$$V_{\text{phase}} = \frac{\omega_\Sigma}{ck_\Sigma} = \frac{(\omega_\rightarrow + \omega_\leftarrow)}{c(k_\rightarrow + k_\leftarrow)} = \frac{(\omega_\rightarrow + \omega_\leftarrow)}{(\omega_\rightarrow - \omega_\leftarrow)} = \frac{c}{V_{\text{group}}}$$
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\[
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\[
\omega_\Sigma = (\omega_\rightarrow + \omega_\leftarrow)/2 \quad \omega_\Delta = (\omega_\rightarrow - \omega_\leftarrow)/2
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\[
\frac{\text{Envelope-Min.}}{\text{Envelope-Max.}} = \frac{A_\rightarrow - A_\leftarrow}{A_\rightarrow + A_\leftarrow} = \text{SWR} = \frac{A_\rightarrow + A_\leftarrow}{A_\rightarrow - A_\leftarrow} = 1
\]

They’re analogous to group velocity \( V_{\text{group}} <c \) frequency ratios and inverse phase velocity \( V_{\text{phase}}>c \) ratios.

\[
V_{\text{group}} = \frac{\omega_\Delta}{k_\Delta} = \frac{(\omega_\rightarrow - \omega_\leftarrow)}{(k_\rightarrow - k_\leftarrow)} = \frac{c(\omega_\rightarrow - \omega_\leftarrow)}{(\omega_\rightarrow + \omega_\leftarrow)}
\]

\[
V_{\text{group}} = \frac{\omega_\Delta}{ck_\Delta} = \frac{(\omega_\rightarrow - \omega_\leftarrow)}{c(k_\rightarrow - k_\leftarrow)} = \frac{c(\omega_\rightarrow - \omega_\leftarrow)}{(\omega_\rightarrow + \omega_\leftarrow)}
\]

\[
V_{\text{phase}} = \frac{\omega_\Sigma}{k_\Sigma} = \frac{(\omega_\rightarrow + \omega_\leftarrow)}{(k_\rightarrow + k_\leftarrow)} = \frac{c(\omega_\rightarrow + \omega_\leftarrow)}{(\omega_\rightarrow - \omega_\leftarrow)}
\]

\[
V_{\text{phase}} = \frac{\omega_\Sigma}{ck_\Sigma} = \frac{(\omega_\rightarrow + \omega_\leftarrow)}{c(k_\rightarrow + k_\leftarrow)} = \frac{c(\omega_\rightarrow + \omega_\leftarrow)}{(\omega_\rightarrow - \omega_\leftarrow)}
\]

\[
\frac{V_{\text{group}}}{c} = \frac{c}{V_{\text{phase}}} \quad \text{is analogous to:} \quad \text{SWR} = \frac{1}{\text{SWQ}}
\]
Review of wave parameters used to develop relativistic quantum theory
Bohr-Schroedinger (BS) approximation throws out $Mc^2$ (*Is frequency really relative?*)
Effect on group velocity (*None*) and phase velocity (*Absurd*)

1st Quantization: Quantizing phase variables $k_m$ and $\omega(k_m)$
Understanding how quantum dynamics and transitions involve “mixed” states
Square well example of mixing unequal frequencies
Circle well or ring example of mixing equal or unequal frequencies

Mixing unequal amplitudes makes “Galloping” wave: Analogy of ($SWR$, $SWQ$) to ($V_{\text{group}}$, $V_{\text{phase}}$)
Analogy with optical polarization geometry and Kepler orbits
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2nd Quantization: Quantizing wave amplitudes $A_N$ and invariance of photon number
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Relativistic effects on charge, current, and Maxwell Fields
Two extremes for Standing Wave Ratio

SWR = 1

SWR = 0

SWR = -1/5

SWR = -3/5

(not shown in (x,ct) plots)

SWR = 1

SWR = +3/5

SWR = +1/5

SWR = 0

SWR = -1/5

SWR = -3/5

(from: Fig. 4.5.2)

(from: Fig. 8.6.3)

QTforCA

CMwBang!

Unit 2 Ch.4

Unit 8 Ch.6

BohrIt Web Simulations Links
(embedded in corners)

Fig. 6.1 Monochromatic (1-frequency) 2-CW wave space-time patterns.
Two extremes for Standing Wave Ratio

SWR = 0

SWR = 1

SWR = -3/5

SWR = +3/5

SWR = -1/5

SWR = +1/5

SWR = 1

BohrIt Web Simulations Links (embedded in corners)
Two extremes for Standing Wave Ratio

SWR=0

...and

SWR=-1

SWR=1

1-frequency cases

\( E_{\phi} = 0.2 \)
\( E_{\varphi} = 0.8 \)

SWR=+3/5

SWR=0

SWR=+1/5

\( E_{\phi} = 0.4 \)
\( E_{\varphi} = 0.6 \)

SWR=+1/5

SWR=0

SWR=-1/5

\( E_{\phi} = 0.5 \)
\( E_{\varphi} = 0.5 \)

\( \omega_{\varphi} = 2c, \ k_{\varphi} = 2 \)

\( \omega_{\phi} = 2c, \ k_{\phi} = -2 \)

\( u_{\text{GROUP}} = 0 \)
\( u_{\text{PHASE}} = \infty \)

SWR=-1/5

SWR=-3/5

\( E_{\phi} = 0.6 \)
\( E_{\varphi} = 0.4 \)

SWR=-3/5

SWR=-1

\( E_{\phi} = 0.8 \)
\( E_{\varphi} = 0.2 \)

SWR=-1

(not shown)

(not shown in (x,ct) plots)

from: Fig. 4.5.2
QTforCA
Unit 2 Ch.4

from: Fig. 8.6.3
CMwBang!
Unit 8 Ch.6

---

Fig. 6.1 Monochromatic (1-frequency) 2-CW wave space-time patterns.
Two extremes for Standing Wave Ratio

1-frequency cases

(a) $E_x = 0.2$
$E_y = 0.8$
$\omega_x = 2c$, $k_x = 2$
$\omega_y = 2c$, $k_y = -2$
$u_{GROUP} = 0$
$u_{PHASE} = \infty$

(b) $E_x = 0.4$
$E_y = 0.6$
$\omega_x = 4c$, $k_x = 4$
$\omega_y = 2c$, $k_y = -2$
$u_{GROUP}/c = 3/5$
$u_{PHASE}/c = 5/3$

(c) $E_x = 0.5$
$E_y = 0.5$
$\omega_x = 1c$, $k_x = 1$
$\omega_y = 2c$, $k_y = -2$
$u_{GROUP}/c = 3/5$
$u_{PHASE}/c = 5/3$

2-frequency cases

(d) $E_x = 0.6$
$E_y = 0.4$
$\omega_x = 4c$, $k_x = 4$
$\omega_y = 2c$, $k_y = -2$
$u_{GROUP}/c = 3/5$
$u_{PHASE}/c = 5/3$

SWR=0

Same SWR cases viewed at $u/c = 3/5$

Same SWR cases viewed at $u/c = 3/5$

SWR=+1/5

SWR=+3/5

SWR=+1/5

$u_{GROUP}/c = 3/5$
$u_{PHASE}/c = 5/3$

BohrIt Web Simulations Links (embedded in corners)

SWR=-3/5

SWR=-1

BohrIt Web Simulations Links (embedded in corners)

SWR=-3/5

SWR=-1

SWR=-3/5

SWR=-1

Same SWR cases viewed at $u/c = 3/5$

SWR=0

Two extremes for Standing Wave Ratio

SWR=+3/5

SWR=+1/5

SWR=0

SWR=-1

SWR=-3/5

SWR=-1

Same SWR cases viewed at $u/c = 3/5$

SWR=0

Two extremes for Standing Wave Ratio

SWR=+3/5

SWR=+1/5

SWR=0

SWR=-1

SWR=-3/5

SWR=-1

Same SWR cases viewed at $u/c = 3/5$

SwR=0

Two extremes for Standing Wave Ratio

from: Fig. 4.5.2
from: Fig. 8.6.3
QTforCA
CMwBang!
Unit 2 Ch.4
Unit 8 Ch.6

www.uark.edu/ua/pirelli/html/amplitude_probability_2.html
1-frequency cases

(a) $E_\phi=0.2$
$E_\rho=0.8$

(b) $E_\phi=0.4$
$E_\rho=0.6$

(c) $E_\phi=0.5$
$E_\rho=0.5$

$\omega_\phi=2c$, $k_\phi=2$
$\omega_\rho=2c$, $k_\rho=-2$

$u_{\text{GROUP}}=0$
$u_{\text{PHASE}}=\infty$

2-frequency cases

(a) $E_\phi=0.2$
$E_\rho=0.8$

(b) $E_\phi=0.4$
$E_\rho=0.6$

(c) $E_\phi=0.5$
$E_\rho=0.5$

$\omega_\phi=4c$, $k_\phi=4$
$\omega_\rho=1c$, $k_\rho=-1$

$u_{\text{GROUP}}/c=3/5$
$u_{\text{PHASE}}/c=5/3$

(d) $E_\phi=0.6$
$E_\rho=0.4$

(e) $E_\phi=0.8$
$E_\rho=0.2$

Fig. 6.1 Monochromatic (1-frequency)
2-CW wave space-time patterns.

Fig. 6.2 Dichromatic (2-frequency)
2-CW wave space-time patterns.

Fig. 6.3 (a-g) Elliptic polarization ellipses relate to galloping waves in Fig. 6.1. (h-i) Kepler anomalies.

polarization analogy

(a) $b/a=+1/1$

right circular polarization

right moving wave

(b) $b/a=+3/5$

$r$-elliptical polarization

(c) $b/a=+1/5$

right galloping waves

(d) $b/a=0$

$x$-plane polarization

(e) $b/a=-1/5$

left galloping waves

(f) $b/a=-3/5$

SWR = -3/5

(g) $b/a=-1/1$

left circular polarization

left moving wave

SWR = -1

Fig. 6.3 (a-g) Elliptic polarization ellipses relate to galloping waves in Fig. 6.1. (h-i) Kepler anomalies.
Fig. 6.3 (a-g) Elliptic polarization ellipses relate to galloping waves in Fig. 6.1. (h-i) Kepler anomalies.

**polarization analogy**

- **Elliptic oscillator orbit**
  \[ SWR = b/a = 1/5 \]
  - Kepler anomaly
    \[ \tan \phi(t) = \frac{y}{b \sin \omega t} = \frac{a \cos \omega t}{x} \]
  - Elliptical Orbit
    - Relativity Web Simulation
      - Elliptic Polarization animation
    - With C-type rotation
  - BohrIt Web Simulations Links (embedded in corners)

**Fig. 6.1** Monochromatic (1-frequency) 2-CW wave space-time patterns.

**Fig. 6.2** Dichromatic (2-frequency) 2-CW wave space-time patterns.
Review of wave parameters used to develop relativistic quantum theory
Bohr-Schrodinger (BS) approximation throws out $Mc^2$ (Is frequency really relative?)
Effect on group velocity (None) and phase velocity (Absurd)

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Relativistic effects on charge, current, and Maxwell Fields

Tuesday, May 3, 2016
Super-luminal speeds and Feynman-Wheeler switchbacks.

Fig. 2.B.10 Lighthouse plot of two Happenings

Before

Brief faster-than-light travel
Super-luminal speeds and Feynman-Wheeler switchbacks

Fig. 2.B.10 Lighthouse plot of two Happenings

Fig. 2.B.11 Ship plot of two Happenings
Waves that go back in time - The Feynman-Wheeler Switchback

Minkowski Zero-Grids are Spacetime Switchbacks for \(-u_{\text{GROUP}} < \text{SWR} < 0\)

\[
\begin{array}{c|c}
\omega & \omega \\ \hline
\rightarrow & = 4c \\
\leftarrow & = 1c \\
\end{array}
\]

\[
\begin{array}{c|c}
k & k \\ \hline
\rightarrow & = 4 \\
\leftarrow & = -1 \\
\end{array}
\]

\[
\begin{array}{c|c}
u_{\text{GROUP}} & u_{\text{PHASE}} \\ \hline
c3/5 & c5/3 \\
\end{array}
\]

Group zero speed

\[
u_{\text{GROUP}} = c3/5
\]

Phase zero speed

\[
u_{\text{PHASE}} = c5/3
\]

Group zero speed limit

\[
\frac{u_{\text{GROUP}} + \text{SWR}}{1 + u_{\text{GROUP}} \cdot \frac{\text{SWR}}{c^2}} = \frac{5}{11}
\]

Phase "anti-zero" going "back-in-time"

\[
\begin{array}{c}
\frac{3}{5} + \frac{-1}{5} = \frac{2}{22} = \frac{5}{11}
\end{array}
\]

Wave zero-anti-zero annihilation and creation occur together at the same spacetime point for \(\text{SWR} = 0\)

Wave zero-anti-zero annihilation and creation occur separately at different spacetime points for \(-u_{\text{GROUP}} < \text{SWR} < 0\)
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Relativistic effects on charge, current, and Maxwell Fields
Quantized Amplitude Counting “photon” number (2nd-Quantization)

Planck’s relation $E = N\hbar\nu$ began as a tenative axiom to explain low-T light. Then he tried to disavow it! Einstein picked it up in his 1905 paper. Since then its use has grown enormously and continues to amaze, amuse (or bewilder) all who study it. A current view is that it represents the quantization of optical field amplitude. We picture this below as $N$-photon wave states for each box-mode of $m$ wave kinks.

Quantum field definitions have been called “2nd quantization” or “wave-waves”

NOTE: We’re using “false-color” here.

These are the fundamental “zero-point” or “vacuum” levels

1st Quantization

Quantized Wavenumber (“kink” or momentum number)
Quantized Amplitude Counting “photon” number (2nd-Quantization)

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Quantized Amplitude Counting “photon” number \((2^{nd}\text{-Quantization})\)

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Quantum field definitions have been called “2nd quantization” or “wave-waves”

\[ E = N\hbar \nu \]

\[ N = 0, 1, 2, \ldots \]

\[ m = 0, 1, 2, \ldots \]

\[ m = 1 \]

\[ m = 2 \]

\[ m = 3 \]

\[ m = 4 \]

\[ N = 0 \]

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These are the 1st excited or fundamental transition levels

These are the 2nd excited levels

Quantum field definitions have been called “2nd quantization” or “wave-waves”

**NOTE:** We’re using “false-color” here.

These are the fundamental “zero-point” or “vacuum” levels

Quantized Wavenumber (“kink” or momentum number)
Quantum numbers $N$ of field or $n, m, \ldots$ of modes are invariants and not changed by boosting velocity. Each mode fundamental frequency $\nu_n = n\nu_1$ and its $N$-photon multiples belong to invariant hyperbolas.

Boosted observers see distorted frequencies and lengths, but will agree on the numbers $n$ and $N$ of mode nodes and photons.

This is how light waves can “fake” some of the properties of classical “things” such as invariance or object permanence.

It takes at least TWO CW’s to achieve such invariance. One CW is not enough and cannot have non-zero invariant $N$. Invariance is an interference effect that needs at least two-to-tango!
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Relativistic effects on charge, current, and Maxwell Fields
**Coherent States: Oscillator Amplitude Packets** analogous to **Wave Packets**

We saw how adding *CW’s* (Continuous Waves $m=1, 2, 3, ...$) can make *PW* (Pulse Wave) or *WP* (Wave Packet) that is more like a classical “thing” with more localization in space $x$ and time $t$.

http://www.uark.edu/ua/pirelli/html/coherent_vs_photon_1.html
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Scaling E-waves to mime quantum $\Psi$-waves and $\psi$-waves

Relativistic effects on charge, current, and Maxwell Fields
**Coherent States: Oscillator Amplitude Packets** analogous to **Wave Packets**

We saw how adding CW’s (Continuous Waves $m=1,2,3...$) can make PW (Pulse Wave) or WP (Wave Packet) that is more like a classical “thing” with more localization in space $x$ and time $t$.

![Diagram showing Coherent States and Wave Packets](http://www.uark.edu/ua/pirelli/html/head_on_3.html)

**Analogy:**
Adding photons (Quantized amplitude $N=0,1,2...$) can make a CS (Coherent State) or OAP (Oscillator Amplitude Packet) that is more like a classical wave oscillation with more localization in field amplitude.

![Diagram showing Coherent States and Oscillator Amplitude Packets](http://www.uark.edu/ua/pirelli/html/coherent_vs_photon_1.html)
Coherent States: Oscillator Amplitude Packets analogous to Wave Packets

We saw how adding CW’s (Continuous Waves $m=1,2,3...$) can make PW (Pulse Wave) or WP (Wave Packet) that is more like a classical “thing” with more localization in space $x$ and time $t$.

| $m=1$ | PLUS | $m=2$ | PLUS | $m=3$ etc. | EQUALS | $|PW\rangle$

![Image](http://www.uark.edu/ua/pirelli/html/head_on_3.html)

**Analogy:**
Adding photons (Quantized amplitude $N=0,1,2...$) can make a CS (Coherent State) or OAP (Oscillator Amplitude Packet) that is more like a classical wave oscillation with more localization in field amplitude.

| $N=0$ | PLUS | $N=1$ | PLUS | $N=2$ etc. | EQUALS | $|OAP\rangle$

![Image](http://www.uark.edu/ua/pirelli/html/coherent_vs_photon_1.html)

Pure photon states have localized (certain) $N$ but delocalized (uncertain) amplitude and phase.

OAP states have delocalized (uncertain) $N$ but more localized (certain) amplitude and phase.
Review of wave parameters used to develop relativistic quantum theory
Bohr-Schrodinger (BS) approximation throws out $Mc^2$ (*Is frequency really relative?*)
Effect on group velocity (*None*) and phase velocity (*Absurd*)

1st Quantization: Quantizing phase variables $k_m$ and $\omega(k_m)$
Understanding how quantum dynamics and transitions involve “mixed” states
Square well example of mixing unequal frequencies
Circle well or ring example of mixing equal or unequal frequencies

Mixing unequal amplitudes makes “Galloping” wave: Analogy of $(SWR, SWQ)$ to $(V_{group}, V_{phase})$
Analogy with optical polarization geometry and Kepler orbits
Super-luminal speed and Feynman-Wheeler pair-creation switchbacks

2nd Quantization: Quantizing wave amplitudes $A_N$ and invariance of photon number
Analogy 1: Many CW (Continuous Waves) add up to make PW (Pulse Waves)
Analogy 2: Many Photon-Number-Modes add up to make Coherent-Laser-Modes
Heisenberg $\Delta \psi \cdot \Delta t \sim 1 \sim \Delta k \cdot \Delta x$ analogous to $\Delta N \cdot \Delta phase \sim 1$ uncertainty relations

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Coherent States (contd.) Spacetime wave grid is impossible without coherent states

Pure photon number $N$-states would make useless spacetime coordinates

Total uncertainty of amplitude and phase makes the count pattern a wash. To see grids some $N$-uncertainty is necessary!
Coherent States (contd.) \textit{Spacetime wave grid is impossible without coherent states}

Pure photon number $N$-states would make useless spacetime coordinates

Coherent-$\alpha$-states are defined by continuous amplitude-packet parameter $\alpha$ whose square is average photon number $\bar{N}=|\alpha|^2$. Coherent-states make better spacetime coordinates for larger $\bar{N}=|\alpha|^2$.

Coherent-state uncertainty in photon number (and mass) varies with amplitude parameter $\Delta N \sim \alpha \sim \sqrt{N}$ so a coherent state with $\bar{N}=|\alpha|^2=10^6$ only has a 1-in-1000 uncertainty $\Delta N \sim \alpha \sim \sqrt{N}=1000$. 
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Electromagnetic wave mode energy: Maxwell vs Planck-Einstein

1st Quantization conditions for wave phase variables (mode-fits-cavity $\ell$)

Wave vector: $2\pi k_n = \frac{2\pi}{\lambda_n} = n \frac{2\pi}{\ell}$, 
Frequency: $2\pi \nu = \omega_n = c k_n = c n \frac{2\pi}{\ell}$

Vector potential of standing wave mode: $A = e_1 2\left| a \right| \sin (k \cdot r - \omega t + \phi)$

$E = -\frac{\partial A}{\partial t}$ (Electric field)
$E_0 e_1 = 2\left| a \right| \omega e_1$

$B = \nabla \times A$ (Magnetic field)

$B_0 (k \times e_1) = e_2 2\left| a \right| k$

Maxwell equations

$E_0 e_1 = 2\left| a \right| \omega e_1$

Speed of Light:
$2.99792458 \text{ m/sec.} = \frac{\omega}{k} = \frac{1}{c} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$

$\varepsilon_0 = (8.854) \times 10^{-7} \frac{Nm^2}{C^2}$
$\mu_0 = 4\pi \times 10^{-7} \frac{N}{A^2}$
Electromagnetic wave mode energy: Maxwell vs Planck-Einstein

1st Quantization conditions for wave phase variables (mode-fits-cavity \( \ell \))

Wave vector : \( 2\pi k_n = \frac{2\pi}{\lambda_n} = n \frac{2\pi}{\ell} \)
Frequency : \( 2\pi \nu = \omega_n = c k_n = c n \frac{2\pi}{\ell} \)

Vector potential of standing wave mode : \( A = e_1 2a |\omega| e_1 \sin(k \cdot r - \omega t + \phi) \)

Maxwell equations

\[
\begin{align*}
\mathbf{E} &= -\frac{\partial A}{\partial t} \quad \text{(Electric field)} \\
E_0 e_1 &= 2 |\omega| e_1 \\
\mathbf{B} &= \nabla \times \mathbf{A} \quad \text{(Magnetic field)} \\
B_0 (k \times e_1) &= e_2 2 |\omega| k \\
\end{align*}
\]

Electromagnetic mean energy density \( U \) and total Energy in volume \( V \)

\[
\langle U \rangle V = \frac{\varepsilon_0}{2} E \cdot E + \frac{1}{2 \mu_0} B \cdot B \bigg|_V = \frac{\varepsilon_0}{2} |A|^2 \omega^2 + \frac{|A|^2}{2 \mu_0} k^2 \bigg( \cos^2 (k \cdot r - \omega t + \phi) \bigg)
\]

\[
= \frac{\varepsilon_0}{2} \omega^2 |A|^2 V = \frac{1}{2 \mu_0} k^2 |A|^2 V \\
given: \langle \cos^2 (k \cdot r - \omega t + \phi) \rangle = \frac{1}{2}
\]

Speed of Light:
\[
2.99792458 \text{ m/sec} = \frac{\omega}{k} = c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}
\]

\[
\varepsilon_0 = (8.854) \times 10^{-7} \text{ Nm}^2 \text{C}^{-2}
\]

\[
\mu_0 = 4\pi 10^{-7} \text{ N} \text{m}^2 \text{A}^{-2}
\]

\[\text{Tuesday, May 3, 2016}\]
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Relativistic effects on charge, current, and Maxwell Fields

* * *

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Electromagnetic wave mode energy: Maxwell vs Planck-Einstein

1st Quantization conditions for wave phase variables (mode-fits-cavity $\ell$)

- Wave vector: \[ 2\pi \kappa = \kappa_n = n \frac{2\pi}{\lambda_n} = n \frac{2\pi}{\ell}, \]
- Frequency: \[ 2\pi \nu = \omega_n = c \kappa_n = c n \frac{2\pi}{\ell} \]

Vector potential of standing wave mode: \[ \mathbf{A} = \mathbf{e}_1 2|a| \sin (k \cdot \mathbf{r} - \omega t + \phi) \]

Maxwell equations:

- Electric field: \[ \mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} \]
- Magnetic field: \[ \mathbf{B} = \nabla \times \mathbf{A} \]

- \[ \mathbf{E}_0 \mathbf{e}_1 = 2|a| \omega \mathbf{e}_1 \]
- \[ \mathbf{B} = (\mathbf{k} \times \mathbf{e}_1) B_0 \cos (k \cdot \mathbf{r} - \omega t + \phi) \]
- \[ B_0 (\mathbf{k} \times \mathbf{e}_1) = \mathbf{e}_2 2|a|k \]

Electromagnetic mean energy density $U$ and total energy in volume $V$

- \[ \langle U \rangle V = \left( \frac{\varepsilon_0}{2} \mathbf{E} \cdot \mathbf{E} + \frac{1}{2\mu_0} \mathbf{B} \cdot \mathbf{B} \right) V = V \left( \frac{\varepsilon_0}{2} |\mathbf{A}|^2 \omega^2 + \frac{|\mathbf{A}|^2}{2\mu_0} k^2 \right) \cos^2 (k \cdot \mathbf{r} - \omega t + \phi) \]
- \[ \langle U \rangle V = \frac{\varepsilon_0}{2} \omega^2 |\mathbf{A}|^2 V = \frac{1}{2\mu_0} k^2 |\mathbf{A}|^2 V \]
- \[ \text{E-energy} = \text{B-energy} \]
- Given: \[ \langle \cos^2 (k \cdot \mathbf{r} - \omega t + \phi) \rangle = \frac{1}{2} \]

2nd Quantization conditions for wave amplitudes (action fits HO phase space)

Equating total energy $\langle U \rangle V$ to Planck’s $E_N(\omega) = \hbar N \omega$ axiom gives mean square field amplitudes

- \[ \langle U \rangle V = \hbar N \omega = \frac{\varepsilon_0}{2} \omega^2 |\mathbf{A}|^2 V = \frac{\varepsilon_0}{2} |\mathbf{E}|^2 V \]

Tuesday, May 3, 2016
Electromagnetic wave mode energy: Maxwell vs Planck-Einstein

1st Quantization conditions for wave phase variables (mode-fits-cavity $\ell$)

\[
\text{wave vector} : 2\pi k = n = \frac{2\pi}{\lambda_n} = \frac{2\pi}{\ell}, \quad \text{frequency} : 2\pi \nu = \omega \mathbf{n} = c k_n = c n \frac{2\pi}{\ell}
\]

Vector potential of standing wave mode: \( \mathbf{A} = e_1 2|a| \sin(k \cdot r - \omega t + \phi) \)

\[
\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} \quad (\text{Electric field})
= e_1 E_0 \cos(k \cdot r - \omega t + \phi)
E_0 e_1 = 2|a| \omega e_1
\]

Maxwell equations

\[
\mathbf{B} = \nabla \times \mathbf{A} \quad (\text{Magnetic field})
= (k \times e_1) B_0 \cos(k \cdot r - \omega t + \phi)
B_0 (k \times e_1) = e_2 2|a|k
\]

Electromagnetic mean energy density \( U \) and total Energy in volume \( V \)

\[
\langle U \rangle V = \left( \frac{\varepsilon_0}{2} \mathbf{E} \cdot \mathbf{E} + \frac{1}{2\mu_0} \mathbf{B} \cdot \mathbf{B} \right) V = V \left( \frac{\varepsilon_0}{2} |\mathbf{A}|^2 \omega^2 + \frac{|\mathbf{A}|^2}{2\mu_0} k^2 \right) \langle \cos^2(k \cdot r - \omega t + \phi) \rangle
\]

\[
= \frac{\varepsilon_0}{2} \omega^2 |\mathbf{A}|^2 V = \frac{1}{2\mu_0} k^2 |\mathbf{A}|^2 V \quad \text{given:} \quad \langle \cos^2(k \cdot r - \omega t + \phi) \rangle = \frac{1}{2}
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Equating total Energy \( \langle U \rangle V \) to Planck’s \( E_N(\omega) = \hbar N \omega \) axiom gives mean square field amplitudes

\[
\langle U \rangle V = \hbar N \omega = \frac{\varepsilon_0}{2} \omega^2 |\mathbf{A}|^2 V = \frac{\varepsilon_0}{2} |\mathbf{E}|^2 V \quad |\mathbf{A}|^2 = \frac{2\hbar N}{\varepsilon_0 \omega V}, \quad \omega^2 |\mathbf{A}|^2 = |\mathbf{E}|^2 = \frac{2\hbar N \omega}{\varepsilon_0 V}
\]

\[
|\mathbf{A}| = \sqrt{\frac{2\hbar N}{\varepsilon_0 \omega V}}, \quad |\mathbf{E}| = \omega |\mathbf{A}| = \sqrt{\frac{2\hbar N \omega}{\varepsilon_0 V}}
\]

Tuesday, May 3, 2016
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1st Quantization conditions for wave phase variables (mode-fits-cavity $\ell$)

$\text{wave vector: } 2\pi \kappa = k_n = \frac{2\pi}{\lambda_n} = n \frac{2\pi}{\ell}$, $\text{frequency: } 2\pi \nu = \omega_n = c k_n = c n \frac{2\pi}{\ell}$

Vector potential of standing wave mode: $A = e_1 2|a| \sin(k \cdot r - \omega t + \phi)$

B = $\nabla \times A$ (Magnetic field)

$B_0(k \cdot e_1) = e_2 2|a| k$

Electromagnetic mean energy density $U$ and total Energy in volume $V$

$\langle U \rangle V = \left( \frac{\varepsilon_0}{2} E \cdot E + \frac{1}{2 \mu_0} B \cdot B \right) V$

$= \frac{\varepsilon_0}{2} |A|^2 \omega^2 + \frac{|A|^2}{2 \mu_0} k^2 \langle \cos^2(k \cdot r - \omega t + \phi) \rangle$

E-energy = B-energy

$= \frac{\varepsilon_0}{2} \omega^2 |A|^2 V = \frac{1}{2 \mu_0} k^2 |A|^2 V$

given: $\langle \cos^2(k \cdot r - \omega t + \phi) \rangle = \frac{1}{2}$

2nd Quantization conditions for wave amplitudes (action fits HO phase space)

Equating total Energy $\langle U \rangle V$ to Planck’s $E_N(\omega) = \hbar N \omega$ axiom gives mean square field amplitudes

$\langle U \rangle V = \hbar N \omega = \frac{\varepsilon_0}{2} \omega^2 |A|^2 V = \frac{\varepsilon_0}{2} |E|^2 V$

$|A|^2 = \frac{2\hbar N}{\varepsilon_0 \omega V}$, $\omega^2 |A|^2 = |E|^2 = \frac{2\hbar N \omega}{\varepsilon_0 V}$

$N$ and $\omega$ are both frequencies for quantum wave so E-field has Doppler $e^{\pm \rho}$-shifts just like $N$ and $\omega$

Now we see how Planck’s $E_N(\omega) = \hbar N \omega$ axiom has the classical quadratic $\omega^2 |A|^2$ oscillator energy
Review of wave parameters used to develop relativistic quantum theory
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Relativistic effects on charge, current, and Maxwell Fields
Making electromagnetic E-waves have quantum cavity $\Psi$-wave and $\psi$-wave properties

Previous equations for energy $U$ per volume

$$
\langle U \rangle V = \hbar N \omega = \frac{\varepsilon_0}{2} \omega^2 |A|^2 V = \frac{\varepsilon_0}{2} |E|^2 V
$$

$$
|A| = \sqrt{\frac{2\hbar N}{\varepsilon_0 \omega V}}, \quad |E| = \omega |A| = \sqrt{\frac{2\hbar N \omega}{\varepsilon_0 V}}
$$

$$
\langle U \rangle = \frac{\hbar N \omega}{V} = \frac{\varepsilon_0}{2} \omega^2 |A|^2 = \frac{\varepsilon_0}{2} |E|^2
$$

$$
\frac{\langle U \rangle}{\hbar \omega} = N = \frac{\varepsilon_0}{2\hbar} \omega |A|^2 = \frac{\varepsilon_0}{2\hbar \omega} |E|^2
$$
Making electromagnetic E-waves have quantum cavity $\Psi$-wave and $\psi$-wave properties

Previous equations for energy $U$ per volume

$$\langle U \rangle V = \hbar N \omega = \frac{\varepsilon_0}{2} \omega^2 |A|^2 V = \frac{\varepsilon_0}{2} |E|^2 V$$

$$|A| = \sqrt{\frac{2\hbar N}{\varepsilon_0 \omega V}}, \quad |E| = \omega |A| = \sqrt{\frac{2\hbar N \omega}{\varepsilon_0 V}}$$

$$\langle U \rangle = \frac{\hbar N \omega}{V} = \frac{\varepsilon_0}{2} \omega^2 |A|^2 = \frac{\varepsilon_0}{2} |E|^2$$

$$\frac{\langle U \rangle}{\hbar \omega} = \frac{N}{V} = \frac{\varepsilon_0}{2\hbar} \omega |A|^2 = \frac{\varepsilon_0}{2\hbar} |E|^2$$

Rescale $E$ by $s = \sqrt{\frac{\varepsilon_0}{2\hbar \omega}}$ to get $x$ and $y$ component wave function

$$\tilde{\Psi} = \begin{pmatrix} \Psi_x \\ \Psi_y \end{pmatrix} = sE = \sqrt{\frac{\varepsilon_0}{2\hbar \omega}} E$$

whose volume integral

$$\iiint_V dV \tilde{\Psi}^* \cdot \tilde{\Psi} = \iiint_V dV \left( |\Psi_x|^2 + |\Psi_y|^2 \right) \propto \iiint_V dV \frac{N}{V} = N$$

is

$$\iiint_V dV \left( |\Psi_x|^2 + |\Psi_y|^2 \right) = N$$

(Initially normalized to particle number $N$.)

Poynting flux $S$ is scaled to get counts per area·second.

$$S = cU = c\varepsilon_0 |E|^2 = \hbar n \omega \quad \text{where:} \quad n = Nc/V \text{(per m}^2\text{per sec.)}$$
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Relativistic effects on charge, current, and Maxwell Fields
Relativistic effects on charge, current, and Maxwell Fields

(+): Charge fixed
(-): Charge moving to right (Negative current density)
(+): Charge density is Equal to the (-) Charge density

Observer velocity is zero relative to (+) line of charge
wire appears neutral
Relativistic effects on charge, current, and Maxwell Fields

(+, -) Charge fixed (-) Charge moving to right (Negative current density $\vec{j}(x,t)$)
(+, -) Charge density is Equal to the (-) Charge density ($\rho(x,t) = 0$)

Observer velocity is zero relative to (+) line of charge

wire appears neutral
Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz asynchrony

Asynchrony due to off-diagonal $\sinh \rho$ (a 1\textsuperscript{st}-order effect)

in Lorentz transform: \[
\begin{pmatrix}
\cosh \rho & \sinh \rho \\
\sinh \rho & \cosh \rho
\end{pmatrix} \sim \begin{pmatrix} 1 & v/c \\ v/c & 1 \end{pmatrix}
\]

(+): Charge fixed
(-): Charge moving to right
(Negative current density $\vec{j}(x,t)$)
(+): Charge density is Greater than (-) Charge density
(Positive $\rho(x,t) > 0$)

Observer velocity is $+v$ relative to (+) line of charge
wire appears positive (+) (repulsive to observer $q[+]$)

Observer has $q[+]$ “test-charge”

Observer has $q[+]$ “test-charge”
Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz asynchrony

Asynchrony due to off-diagonal \( \begin{pmatrix} \cosh \rho & \sinh \rho \\ \sinh \rho & \cosh \rho \end{pmatrix} \) (a 1st-order effect)

in Lorentz transform:
\[
\begin{pmatrix} \cosh \rho & \sinh \rho \\ \sinh \rho & \cosh \rho \end{pmatrix} \sim \begin{pmatrix} 1 & \frac{v}{c} \\ \frac{v}{c} & 1 \end{pmatrix}
\]

Observer has \( q^+ \) “test-charge”

Observer velocity is \(+v\) relative to line of charge

(+): Charge fixed (-): Charge moving to right (Negative current density \( j(x,t) \))

(+): Charge density is Greater than (-): Charge density (Positive \( \rho(x,t) > 0 \))

wire appears positive (+) (repulsive to observer \( q^+ \))
Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz **asynchrony**

Asynchrony due to off-diagonal \( \sinh \rho \) (a 1\(^{st}\)-order effect)

in Lorentz transform:

\[
\begin{pmatrix}
\cosh \rho & \sinh \rho \\
\sinh \rho & \cosh \rho
\end{pmatrix}
\sim
\begin{pmatrix}
1 & \frac{v}{c} \\
\frac{v}{c} & 1
\end{pmatrix}
\]

Observer has \( q[+] \)

“test-charge”

Observer velocity is \(-v\) relative to (+) line of charge

wire appears negative (-) (attractive to observer \( q[+] \))

(+) Charge fixed (-) Charge moving to right (**Negative current density** \( \mathbf{j}(x,t) \))

(+) Charge density is **Less** than (-) Charge density (**Negative** \( \rho(x,t)<0 \))
Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz asynchrony

Asynchrony due to off-diagonal (a 1st-order effect)

Observer has

$q^{[+]}

“test-charge”

Observer velocity is $-v$ relative to

$(+)$ line of charge

Wire appears negative (-) (attractive to

observer $q^{[+]})$

$(+)$ Charge fixed (-) Charge moving to right ($\text{Negative current density } \mathbf{j}(x,t)$)

$(+)$ Charge density is $\text{Less than}$ (-) Charge density ($\text{Negative } \rho(x,t)<0$)
Simple 1\textsuperscript{st}-order relativistic geometry of magnetism

If Black is moving to Left

Before \textbf{red} starts moving to right
Black sees same number of \textbf{red} and \textbf{blue}

After \textbf{red} starts moving to right
Black sees more \textbf{red} than \textbf{blue}

If Black is moving to Right

Before \textbf{red} starts moving to right
Black sees same number of \textbf{red} and \textbf{blue}

After \textbf{red} starts moving to right
Black sees more \textbf{blue} than \textbf{red}
Magnetic B-field is relativistic $\sinh \rho$ 1st order-effect

\[
\frac{\rho(-)}{\rho(+)} = \frac{(+) \text{ charge separation}}{(-) \text{ charge separation}} = \frac{x(+) + x(-)}{x(-)}
\]

\[
\frac{\rho(-)}{\rho(+)} = \frac{x(+)}{x(-)} + 1 = \frac{uv}{c^2} + 1
\]

\[
\rho(+) - \rho(-) = \rho(+) \left(1 - \frac{\rho(-)}{\rho(+)}\right) = -\frac{uv}{c^2} \rho(+)
\]

Unit square: $(u/c) / 1 = x(+) / y$
$(v/c) / 1 = y / x(-)$

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Magnetic B-field is relativistic $\sinh \rho$ 1st order effect

\[
\frac{\rho(-)}{\rho(+)} = \frac{(+ \text{ charge separation}}{(- \text{ charge separation}} = \frac{x(+) + x(-)}{x(-)}
\]

\[
\frac{\rho(-)}{\rho(+)} = \frac{x(+) + 1}{x(-) + \frac{uv}{c^2} + 1}
\]

\[
\rho(+ - \rho(-)) = \rho(+)
\left(1 - \frac{\rho(-)}{\rho(+)}\right) = -\frac{uv}{c^2} \rho(+)
\]

Using 4-vectors to EL Transform (charge-current) = ($c\rho$, j)

\[
\begin{pmatrix}
  c\rho' \\
  j_x' \\
  j_y' \\
  j_z'
\end{pmatrix} =
\begin{pmatrix}
  \cosh \rho & \sinh \rho & \cdots \\
  \sinh \rho & \cosh \rho & \cdots \\
  \cdots & \cdots & 1 \\
  \cdots & \cdots & \cdots & 1
\end{pmatrix}
\begin{pmatrix}
  c\rho \\
  j_x \\
  j_y \\
  j_z
\end{pmatrix}
\]

Unit square: \((u/c)/1 = x(+) / y\)  
\((v/c)/1 = y/x(-)\)
The electric force field \( \mathbf{E} \) of a charged line varies inversely with radius. The Gauss formula for force in mks units:

\[
F = qE = q \left[ \frac{1}{4\pi \varepsilon_0} \frac{2\rho}{r} \right], \quad \text{where:} \quad \frac{1}{4\pi \varepsilon_0} = 9 \times 10^9 \, \frac{N \cdot m^2}{Coul}.
\]

\[
F = qE = q \left[ \frac{1}{4\pi \varepsilon_0} \frac{2}{r} \left( -\frac{uv}{c^2} \rho(+) \right) \right] = -\frac{2qv}{4\pi \varepsilon_0 c^2} \frac{\rho(+)u}{r} = -2 \times 10^{-7} \frac{I_q I_\rho}{r}
\]

\[
\frac{1}{4\pi \varepsilon_0} = 9 \times 10^9 \quad c^2 = 9 \times 10^{-16}
\]

\[
\frac{1}{(4\pi \varepsilon_0 c^2)} = 10^{-7}
\]

\[
I \rho < 0 \quad F \quad I q > 0 \quad F (\text{repels})
\]

**I see excess (+) charge up there. Yuk!**

\[
I \rho < 0 \quad F
\]

\[
I q < 0 \quad + \quad F (\text{attracts})
\]

**I see excess (-) charge up there. Yum!**

Magnetic B-field is relativistic \( \sinh \rho \) 1st order-effect
The electric force field \( \mathbf{E} \) of a charged line varies inversely with radius. The Gauss formula for force in mks units:

\[
F = qE = q \left[ \frac{1}{4\pi \varepsilon_0} \frac{2\rho}{r} \right], \quad \text{where:} \quad \frac{1}{4\pi \varepsilon_0} = 9 \times 10^9 \ \text{N} \cdot \text{m}^2 / \text{Coul}.
\]

\[
F = qE = q \left[ \frac{1}{4\pi \varepsilon_0} \frac{2}{r} \left( -\frac{u v}{c^2} \rho(+) \right) \right] = -\frac{2}{4\pi \varepsilon_0 c^2} \frac{q v}{r} \rho(+) = -2 \times 10^{-7} \frac{I_q I_\rho}{r}.
\]

\[
1/4\pi \varepsilon_0 = 9 \cdot 10^9
\]
\[
c^2 = 9 \cdot 10^{-16}
\]
\[
1/(4\pi \varepsilon_0 c^2) = 10^{-7}
\]

I see excess (+) charge up there. Yuk!

(Suppose (+) carriers)

I see excess (-) charge up there. Yum!

(Suppose (+) carriers)
Relating photons to Maxwell energy density and Poynting flux
Relativistic variation and invariance of frequency ($\omega,k$) and amplitudes
How probability $\psi$-waves and flux $\psi$-waves evolved
Properties of amplitude $\psi^*\psi$-squares
More on unmatched amplitudes AND unmatched frequencies AND unmatched quanta
Light Energy and Flux 2-CW vs 1-CW-light
What if head-on CW’s $\nu_A=1200\text{THz}$ and $\nu_B=300\text{THz}$ pair-up in a 2-CW-light beam?

They form a rest frame going $u=c\frac{\nu_A-\nu_B}{\nu_A+\nu_B}=3c/5$ with a mean or base color $\nu_0=\sqrt{\nu_A\nu_B}$
($\nu_0=B=600\text{THz}$ is green here. Neither has this singly.)
Light Energy and Flux $2$-$CW$ vs $1$-$CW$-light

What if head-on CW’s $\nu_A=1200\text{THz}$ and $\nu_B=300\text{THz}$ pair-up in a $2$-$CW$-light beam?

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Light Energy and Flux 2-CW vs 1-CW-light

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Relating Planck’s \( E \) to Maxwell’s Density \( U=E/V \)

Maxwell field energy \( E \), a product of mean-square electric field \( \langle E^2 \rangle \), volume of cavity \( V \), and constant \( \varepsilon_0=8.854\cdot10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2 \), approximates Planck’s energy \( \bar{N}\hbar\nu_0 \).

\[
E = \langle E^2 \rangle \varepsilon_0 = \bar{N}\hbar\nu_0 \quad \text{Maxwell-Planck Energy} \\
U = \langle E^2 \rangle \varepsilon_0 = \bar{N}\hbar\nu_0/V \quad \text{Maxwell-Planck Density}
\]

Field Energy \( =|\mathbf{E}|^2\varepsilon_0 \) \( 1/4\pi\varepsilon_0 = 9\cdot10^9 \)
Light Energy and Flux 2-CW vs 1-CW-light

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$$E=\langle E^2 \rangle \varepsilon_0 = \bar{N}\hbar\nu_0 \quad \text{Maxwell-Planck Energy}$$

$$U=\langle E^2 \rangle \varepsilon_0 = \bar{N}\hbar\nu_0/V \quad \text{Maxwell-Planck Density}$$

Example: Let a $\frac{1}{4}\text{\mu m}$-cube cavity (Half-wave at 600THz) have $\bar{N}=10^{10}$ photons in volume $V=\left(\frac{1}{4} \times 10^{-6}\text{m}\right)^3$.

Energy per photon: $\hbar\nu_0=4\cdot 10^{-19}\text{J}=2.5\text{ eV}$

Energy of $\bar{N}$ photons: $\bar{N}\hbar\nu_0=4\cdot 10^{-9}\text{J}=25\text{GeV}$

E-field per photon: $E_1=\sqrt{(\hbar\nu_0/\varepsilon_0)}=7.6\cdot 10^3\text{V/m}$

E-field of $\bar{N}$ photons: $E_{\bar{N}}=7.6\cdot 10^{13}\text{V/m}$
Energy and Flux (contd) 2-CW vs 1-CW-light

Planck $E = N\hbar \nu$ relation allows us to interpret our $N$-quantized 2-CW mode as a box or cavity of $N_{(\text{more-or-less}}\uparrow)$ photons where $N$ is invariant to speed $u$ of box.

† depends on how we set the mode’s coherent state.
Energy and Flux (contd) 2-CW vs 1-CW-light

Planck $E = N\hbar \nu$ relation allows us to interpret our $N$-quantized 2-CW mode as a box or cavity of $N_{\text{(more-or-less)}}$ photons where $N$ is invariant to speed $u$ of box.

If we open the box our 2-CW mode “divorces” into two separate 1-CW beams of $N/2_{\text{(more-or-less)}}$ photons. Each beam has NO rest frame and NO numbers invariant to $u$. 
Energy and Flux (contd) 2-CW vs 1-CW-light

Planck $E=N\hbar\nu$ relation allows us to interpret our $N$-quantized 2-CW mode as a box or cavity of $N$ (more-or-less†) photons where $N$ is invariant to speed $u$ of box.

If we open the box our 2-CW mode “divorces” into two separate 1-CW beams of $N/2$ (more-or-less) photons. Each beam has NO rest frame and NO numbers invariant to $u$.

Relating Poynting’s Intensity $S=cU$ to Planck’s Flux

Poynting intensity $S$ is a product of $c=2.99792458 m/s$ and density $U$. It approximates Planck’s energy $E=N\hbar\nu$ times $c$ and divided by cavity volume $V$.

$$S=cU=(Nc/V)\hbar\nu = n\hbar\nu$$

Poynting-Planck Flux (Watts per square meter)

The photon-count rate is $n=Nc/V$ (per square meter per second) and $\hbar\nu$ is energy (per count).
Relating photons to Maxwell energy density and Poynting flux

Relativistic variation and invariance of frequency \((\omega,k)\) and amplitudes

How probability \(\psi\)-waves and flux \(\psi\)-waves evolved

Properties of amplitude \(\psi^*\psi\)-squares

More on unmatched amplitudes AND unmatched frequencies AND unmatched quanta
Frequency and Amplitude Variance 2-CW-light vs 1-CW-light

2-CW modes have invariance

Maxwell-Planck energy $E$ is photon number $N(m^{-3})$ times 2-CW-frequency $\nu_1$.

$$E = \langle U \rangle \cdot V = \varepsilon_0 \langle E^2 \rangle \cdot V = \varepsilon_0 \langle E_{2-CW} \rangle \cdot V = hN\nu_1 = h\nu_N$$

 Photon number $N$ and rest-frame frequencies $\nu_1...\nu_N$ are invariant to rapidity $\rho$ and occupy $(\omega,ck)$-hyperbolas in per-spacetime.
Frequency and Amplitude Variance 2-CW-light vs 1-CW-light

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Maxwell-Planck energy $E$ is photon number $N(m^{-3})$ times 2-CW-frequency $\nu_1$.

$$E = \langle U \rangle \cdot V = \varepsilon_0 \langle E^2 \rangle \cdot V = \varepsilon_0 \langle E_{2-CW} * E_{2-CW} \rangle \cdot V = hN\nu_1 = h\nu_N$$

Photon number $N$ and rest-frame frequencies $\nu_1...\nu_N$ are invariant to rapidity $\rho$ and occupy $(\omega,ck)$-hyperbolas in per-spacetime.

1-CW beams lack invariance (have “variance” ala’ Doppler)

Planck-Poynting flux $S$ is count rate $n=\frac{Nc}{V(m^{-2}s^{-1})}$ times 1-CW-frequency $\nu_\rightarrow$ or $\nu_\leftarrow$.

Count rate $n$ and frequency $\nu$ Doppler shift by $b=e^{\pm \rho}$ factors and occupy $(\omega=\pm ck)$-baselines.

Note: $E_{1-CW}^{\rightarrow \leftarrow}(\sqrt{c\varepsilon_0/h}) = \sqrt{(n_\leftarrow \nu_\leftarrow)}$ is geometric mean of amplitude frequency $n_\rightarrow$ and phase frequency $\nu_\rightarrow$. 
Important result below:

**Amplitudes of 1-CW “exponentiate” just like frequency, and intensity does at twice the rate**

(A double-double whammy!)

---

**1-CW beams lack invariance** (have “variance” ala’ Doppler)

Planck-Poynting flux $S$ is count rate $n = Nc/V(m^{-2}s^{-1})$ times 1-CW-frequency $\nu$ or $\nu'$.

Count rate $n$ and frequency $\nu$ Doppler shift by $b = e^\pm \rho$ factors and occupy ($\omega = \pm cK$)-baselines.

Shifts by $b = e^{+2\rho}$

$S \rightarrow = cU \rightarrow = c\varepsilon_0 \langle E^2 \rangle = c\varepsilon_0 \langle E_{\rightarrow 1-CW} \ast E_{\rightarrow 1-CW} \rangle = hn \rightarrow \nu$

$S \leftarrow = cU \leftarrow = c\varepsilon_0 \langle E^2 \rangle = c\varepsilon_0 \langle E_{\leftarrow 1-CW} \ast E_{\leftarrow 1-CW} \rangle = hn \leftarrow \nu$

Each blue shifts by $b = e^\rho$

Each red shifts by $r = e^{-\rho}$

Note: $E_{\rightarrow 1-CW} \sqrt{(c\varepsilon_0/\hbar)} = \sqrt{(n \nu)}$ is geometric mean of amplitude frequency $n \leftrightarrow$ and phase frequency $\nu \leftrightarrow$. 

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Relating photons to Maxwell energy density and Poynting flux

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How Probability Amplitudes $\psi$ or $\bar{\psi}$ Come About (An optical view)

Maxwell-Planck-Poynting flux $S = c U = c \varepsilon_0 |E|^2 = c \varepsilon_0 E^* E = n h \nu$ has count rate $n = N c / V (m^{-2} s^{-1})$

If each $E$-field amplitude factor is scaled by a factor $\sqrt{\varepsilon_0 / h \nu} = \sqrt{\varepsilon_0 / h \kappa}$ the result is a flux probability amplitude $\psi = E \sqrt{\varepsilon_0 / h \nu}$ whose square equals flux count rate $n (m^{-2} s^{-1})$.

$$\psi^* \psi = n \ (m^{-2} s^{-1})$$

A fixed probability amplitude $\psi = E \sqrt{\varepsilon_0 / h \nu}$ has square equal to $N/V$ (particles per volume).

$$\psi^* \psi = N/V \ (m^3)$$
How Probability Amplitudes $\psi$ or $\psi$ Come About (An optical view)

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$$\psi^*\psi = N/V \quad (m^3)$$

Here’s how to answer Planck’s worry about photons

**Q:** How can classical oscillator energy $(Amplitude)^2(frequency)^2$ jive with linear Planck law $S = n\hbar\nu$?

**A:** Let amplitude $\psi$ or $\psi$ contain inverse square root of frequency: $\psi = E\sqrt{\frac{c\varepsilon_0}{\hbar\nu}}$ the “quantum amplitude”

Energy $\sim |A|^2 \nu^2$ where vector potential $A$ defines electric field: $E = \frac{\partial A}{\partial t} = i\omega A = 2\pi i\nu A$

$$\text{Energy} \sim |A|^2 \nu^2 = |A\sqrt{\nu}|^2 \nu = \left| \frac{E}{2\pi \nu} \right|^2 \nu = \left| \frac{E}{2\pi \sqrt{\nu}} \right|^2 \nu \sim E\sqrt{\frac{c\varepsilon_0}{\hbar\nu}} = n\hbar\nu$$
How Probability Amplitudes $\psi$ or $\Psi$ Come About (An optical view)

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If each $E$-field amplitude factor is scaled by a factor $\sqrt{c e_0} / h\nu = \sqrt{\frac{e_0}{h\kappa}}$ the result is a flux probability amplitude $\psi = E \sqrt{\frac{e_0}{h\nu}}$ whose square equals flux count rate $n (m^{-2} s^{-1})$.

$$\psi^* \psi = n \quad (m^{-2} s^{-1})$$

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$$\psi^* \psi = N / V \quad (m^3)$$

Probability Waves $\psi(x,t)$ (More optical views)

Optical $E$-field amplitudes like $E(x,t) = E_0 e^{i(kx - \omega t)}$ vary with space $x$ and time $t$. So do scaled $\psi(x,t)$ amplitudes whose sum-$\Sigma$ (integral-$\int$) over cells $\Delta V$ (or $dV$) must be particle number $N$. For 1-particle systems ($N = 1$) this is the unit norm rule.

$$\Sigma_j \psi(x_j,t)^* \psi(x_j,t) \Delta V_j = N \quad \text{or:} \quad \int \psi(x,t)^* \psi(x,t) dV = N$$
How Probability Amplitudes $\psi$ or $\bar{\psi}$ Come About (An optical view)
Maxwell-Planck-Poynting flux $S = cU = c\varepsilon_0|E|^2 = c\varepsilon_0E^\ast E = n\hbar\nu$ has count rate $n = Nc/V(m^{-2}s^{-1})$
If each $E$-field amplitude factor is scaled by a factor $\sqrt{\frac{c\varepsilon_0}{\hbar\nu}} = \sqrt{\frac{\varepsilon_0}{h_\nu}}$ the result is a flux probability amplitude $\psi = E\sqrt{\frac{\varepsilon_0}{\hbar\nu}}$ whose square equals flux count rate $n(m^{-2}s^{-1})$.

$$\psi^\ast\psi = n \quad (m^{-2}s^{-1})$$

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$$\Sigma_j\psi(x_j,t)^\ast\psi(x_j,t)\Delta V_j = N \quad \text{or:} \quad \int\psi(x,t)^\ast\psi(x,t)dV = N$$

Born interpreted $\psi(x,t)^\ast\psi(x,t)$ as probable expectation of particle count. Schrodinger objected to the probability wave interpretation that is now accepted and called the Schrodinger theory. A relativistic wave view lends merit to his objections.
**Doppler Transformation of 2-CW Modes**

Doppler shift of opposite-\(k\) 1-CW beams. As derived before phases are invariant: \((k'x'-\omega't'=kx-\omega t)\)

E-wave: \[
E = E \rightarrow e^{i(k \cdot x - \omega t)} + E \leftarrow e^{i(k \cdot x - \omega t)}
\]

- **blue shift**
  \[
  E' \rightarrow = bE \rightarrow = e^{+\rho}E
  \]

- **red shift**
  \[
  E' \leftarrow = rE \leftarrow = e^{-\rho}E
  \]

\[\Psi\text{-wave}: \Psi = \psi \rightarrow e^{i(k \cdot x - \omega t)} + \psi \leftarrow e^{i(k \cdot x - \omega t)} \]

- **scaled blue shift**
  \[
  \psi' \rightarrow = \sqrt{b} \psi \rightarrow = e^{+\rho/2}\psi
  \]

- **scaled red shift**
  \[
  \psi' \leftarrow = \sqrt{r} \psi \leftarrow = e^{-\rho/2}\psi
  \]
Doppler Transformation of 2-CW Modes

Doppler shift of opposite-k 1-CW beams. As derived before phases are invariant: \((k'x'-\omega't'=kx-\omega t)\)

E-wave: \(E = E_+ e^{i(k_x x - \omega t)} + E_- e^{i(k_x x - \omega t)}\)

\(\text{blue shift}\) \[
E' \rightarrow = b \ E \rightarrow
= e^{\rho} E
\]

\(\text{red shift}\) \[
E' \leftarrow = r \ E \leftarrow
= e^{-\rho} E
\]

\(\Psi\)-wave: \(\Psi = \Psi_+ e^{i(k_x x - \omega t)} + \Psi_- e^{i(k_x x - \omega t)}\)

\(\text{scaled blue shift}\) \[
\Psi' \rightarrow = \sqrt{b} \ \Psi \rightarrow
= e^{\rho/2} \Psi
\]

\(\text{scaled red shift}\) \[
\Psi' \leftarrow = \sqrt{r} \ \Psi \leftarrow
= e^{-\rho/2} \Psi
\]

Parameters related to relative velocity \(u\):

\[
\beta = u/c = \tanh \rho = \frac{\sinh \rho}{\cosh \rho} = \frac{e^{\rho} - e^{-\rho}}{e^{\rho} + e^{-\rho}} = \frac{b^2 - 1}{b^2 + 1}
\]

\[
b^2 = \frac{1 + \beta}{1 - \beta} = \frac{1 + \tanh \rho}{1 - \tanh \rho}
\]
Doppler Transformation of 2-CW Modes

Doppler shift of opposite-k 1-CW beams. As derived before phases are invariant: \((k'x'−\omega' t')=kx−\omega t\)

\[ E\text{-wave: } E = E e^{i(k\cdot x − \omega t)} + E e^{i(k\cdot x − \omega t)} \]

\[ \text{blue shift} \]
\[ E' \rightarrow b E \rightarrow e^{+\rho} E \]
\[ E' \leftarrow e^{-\rho} E \leftarrow \]

\[ \text{red shift} \]
\[ E' \rightarrow r E \rightarrow e^{+\rho} E \]
\[ E' \leftarrow e^{-\rho} E \leftarrow \]

\[ \Psi\text{-wave: } \Psi = \Psi e^{i(k\cdot x − \omega t)} + \Psi e^{i(k\cdot x − \omega t)} \]

\[ \text{scaled blue shift} \]
\[ \Psi' \rightarrow \sqrt{b} \Psi \rightarrow e^{+\rho/2} \Psi \]
\[ \Psi' \leftarrow e^{-\rho/2} \Psi \leftarrow \]

\[ \text{scaled red shift} \]
\[ \Psi' \rightarrow \sqrt{r} \Psi \rightarrow e^{+\rho/2} \Psi \]
\[ \Psi' \leftarrow e^{-\rho/2} \Psi \leftarrow \]

Parameters related to relative velocity \(u\):

\[ \beta = \frac{u}{c} = \tanh \rho = \frac{e^{+\rho}−e^{-\rho}}{e^{+\rho}+e^{-\rho}} = \frac{b^2−1}{b^2+1} \]

\[ b^2 = \frac{1+\beta}{1−\beta} = \frac{1+\tanh \rho}{1−\tanh \rho} \]

Transformation of SWR (or SWQ) and \(u_{\text{GROUP}}\) (or \(u_{\text{PHASE}}\)) is a non-linear transformation

\[ \text{SWR' = } \frac{\text{SWR}+\beta}{1+\text{SWR}\cdot \beta} = \frac{\text{SWR}+u/c}{1+\text{SWR}\cdot u/c} \]

\[ u'_{\text{GROUP}}/c = \frac{u_{\text{GROUP}}/c+\beta}{1+u_{\text{GROUP}}\cdot \beta/c} = \frac{(u_{\text{GROUP}}+u)/c}{1+u_{\text{GROUP}}\cdot u/c^2} \]
**Doppler Transformation of 2-CW Modes**

**Doppler shift of opposite-k 1-CW beams.** As derived before phases are invariant: \((k'x'−ω't'=kx−ωt)\)

**E-wave:** \(E = \mathbf{E} \rightarrow e^{i(k \cdot x - \omega \cdot t)} + \mathbf{E} \leftarrow e^{i(k \cdot x - \omega \cdot t)}\)

- **blue shift**
  \[E' \rightarrow b \mathbf{E} \rightarrow e^{+\rho \mathbf{E}}\]
- **red shift**
  \[E' \leftarrow r \mathbf{E} \leftarrow e^{-\rho \mathbf{E}}\]

**Parameters related to relative velocity \(u\):**

\[\beta = \frac{u}{c} = \tanh \rho = \frac{\sinh \rho}{\cosh \rho} = \frac{e^+ - e^-}{e^+ + e^-} = \frac{b^2 - 1}{b^2 + 1}\]

\[b^2 = \frac{1 + \beta}{1 - \beta} = \frac{1 + \tanh \rho}{1 - \tanh \rho}\]

Transformation of **SWR** (or **SWQ**) and \(u_{\text{GROUP}}\) (or \(u_{\text{PHASE}}\)) is a **non-linear** transformation

\[\text{SWR'} = \frac{\text{SWR} + \beta}{1 + \text{SWR} \cdot \beta} = \text{SWR} + \frac{u}{c}\]

\[\text{u'_{\text{GROUP}}} = \frac{u_{\text{GROUP}} + \beta}{1 + u_{\text{GROUP}} \cdot \beta / c} = \frac{(u_{\text{GROUP}} + u) / c}{1 + u_{\text{GROUP}} \cdot u / c^2}\]

Both are restatements of hyperbolic trig identity: \(\tanh(a+b) = \frac{\tanh(a) + \tanh(b)}{1 + \tanh(a) \cdot \tanh(b)}\)

Last term is ignorable if both \(a\) and \(b\) are small

**Velocity addition is non-linear but rapidity addition is always linear:** \(\rho_{a+b} = \rho_a + \rho_b\)

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**www.uark.edu/ua/pirelli/php/amplitude/probability_5.php**

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**Tuesday, May 3, 2016**
**Special Relativity and Quantum Mechanics by Ruler and Compass I**

The simplest molecules—2. The lasers form Minkowski Space-time (and Reciprocity-related) Frame Coordinates

- **Special Relativity and Quantum Mechanics are very much a story of the geometry of light-wave motion**
- **How do we measure space and time with light waves?**
  - Use ICP laser-beam for a space-based theory
  - Use ICP laser-beam and wave interference geometry
- **Relativity—Using light's own wave-like nature to better understand special relativity and quantum mechanics**

Galileo's Revenge (Part 2)

Rapidity adds just like Galilean velocity

**Link to pdf version of Part I online**

Note: When printed at their optimal resolution, each poster is 7 feet across!
Using wave parameters to quickly derive Planck (1900), Einstein (1905), and DeBroglie (1921) formulation.

Energy: $E = mc^2$.

Velocity: $v = \frac{c}{\sqrt{1 - \frac{v^2}{c^2}}}$.

Momentum: $p = \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}}$.

Lagrangian Legendre transform to Lagrangian.

Hamiltonian Legendre transform to Hamiltonian.

Comparing Lagrangian (velocity) with Hamiltonian (momentum).

Recoil from emitting $\gamma$-ray or $\nu$-ray is analogous to Doppler effect, but for high-energy particles.

The "Rocket Source" of relativistic optical emission.

Discrete (h*ν) versus (h/λ) plot of Compton scattering.

Discrete time (t) versus space (x) plot of constant acceleration paths.

Definition(s) of mass for relativity and quantum theory.

Legendre transforms to Legendre transforms.

More common definitions using group velocity.

$\phi = \theta = \frac{1}{2} \ln \left[ \frac{\lambda}{\lambda_0} \right]$ - Recoil from emitting high-energy photon.

$K_{in} = K_{out}$ - Conservation of momentum.

$\phi = \theta = \frac{1}{2} \ln \left[ \frac{\lambda}{\lambda_0} \right]$ - Recoil from emitting high-energy photon.

All-rational-fraction lattice (Feynman path integrals defined).

Recoil from emitting high-energy photon. (Don't object - he's dead.)

 Exact vs Approximate (hω) versus (h/λ) plot.

The “Rocket Source” of relativistic optical emission.

Discrete (hω) versus (h/λ) plot of Compton scattering.