Lecture 31 Relawavity-Dynamics
Tuesday 5.03.2016

Relawavity: Quantizing wave variables of phase and amplitude

(Unit 3 p.45-64)

➔ Review of wave parameters used to develop relativistic quantum theory
- Bohr-Schrodinger (BS) approximation throws out Mc^2 *(Is frequency really relative?)*
  - Effect on group velocity *(None)* and phase velocity *(Absurd)*

1^{st} Quantization: Quantizing phase variables k_m and $\omega(k_m)$
- Understanding how quantum dynamics and transitions involve "mixed" states
  - Square well example of mixing unequal frequencies
  - Circle well or ring example of mixing equal or unequal frequencies

Mixing unequal amplitudes makes "Galloping" wave: Analogy of (*SWR*, *SWQ*) to (V_{group},V_{phase})
- Analogy with optical polarization geometry and Kepler orbits
  - Super-luminal speed and Feynman-Wheeler pair-creation switchbacks

2^{nd} Quantization: Quantizing wave amplitudes A_N and invariance of photon number
- Analogy 1: Many CW (Continuous Waves) add up to make PW (Pulse Waves)
- Analogy 2: Many Photon-Number-Modes add up to make Coherent-Laser-Modes
- Heisenberg $\Delta\upsilon\cdot\Delta t \sim 1 \sim \Delta\kappa\cdot\Delta x$ analogous to $\Delta N\cdot\Delta phase \sim 1$ uncertainty relations

Electromagnetic wave mode energy: Maxwell vs. Planck-Einstein
- 1^{st} quantization for wave phase variables and classical energy of **E**, **B** , and **A** fields
- 2^{nd} quantization for wave and Planck quantum energy of **E**, **B** , and **A** fields
  - Scaling **E**-waves to mime quantum Ψ-waves and ψ-waves

Relativistic effects on charge, current, and Maxwell Fields

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Using (some) wave parameters to develop relativistic quantum theory

$\upsilon_{phase} = B\cosh\rho \approx B + \frac{1}{2}B\rho^2$ (for $u \ll c$)

$c\kappa_{phase} = B\sinh\rho \approx B\rho$ (for $u \ll c$)

$\frac{u}{c} = \tanh\rho \approx \rho$ (for $u \ll c$)

$\cosh\rho \approx 1 + \frac{1}{2}\rho^2 \approx 1 + \frac{1}{2}\frac{u^2}{c^2}$

$\sinh\rho \approx \rho \approx \frac{u}{c}$

Lect. 30
p. 3 to 29


Max Planck
1858-1947


Louis DeBroglie
1892-1987

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

At low speeds:

$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \Leftarrow$ for $(u \ll c) \Rightarrow \kappa_{phase} \approx \frac{B}{c^2}u$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$ (The famous Mc^2 shows up here!)

$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \Leftarrow$ for $(u \ll c) \Rightarrow h\kappa_{phase} \approx \frac{hB}{c^2}u$

So attach scale factor h (or hN) to match units.

~~Natural wave conspiracy~~ ~~Lucky coincidences??~~ Expensive ~~Cheap~~ trick??

$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \Leftarrow$ for $(u \ll c) \Rightarrow h\kappa_{phase} \approx Mu$

...Try exact υ_{phase} and κ_{phase}...

Energy density, photon number N, and normalization discussed p.74-81

Need to replace h with hN to match e.m. energy density $\varepsilon_0 \mathbf{E}^*\!\cdot\!\mathbf{E} = hN\upsilon_{phase}$

$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$

Planck (1900)

$=$ Total Energy: $E = \frac{Mc^2}{\sqrt{1-u^2/c^2}}$

Einstein (1905)

This motivates the "particle" normalization $\int \Psi^*\Psi\, dV = N$ $\quad \Psi = \sqrt{\frac{\varepsilon_0}{h\upsilon}}E$

$hc\kappa_{phase} = hB\sinh\rho = Mc^2\sinh\rho$

$\frac{1}{\sqrt{\beta^{-2}-1}} = \frac{\frac{u}{c}}{\sqrt{1-\frac{u^2}{c^2}}}$ (old-fashioned notation)

$cp = \frac{Muc}{\sqrt{1-u^2/c^2}}$

Momentum: $h\kappa_{phase} = p = \frac{Mu}{\sqrt{1-u^2/c^2}}$

DeBroglie (1921)

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	
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rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$			
stellar angle $\forall\ \sigma$	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta = \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

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$$hc\kappa_{phase} = hB\sinh\rho = Mc^2\sinh\rho$$

$$cp = \frac{Muc}{\sqrt{1 - u^2/c^2}}$$

$$\frac{1}{\sqrt{\beta^{-2} - 1}} = \frac{\frac{u}{c}}{\sqrt{1 - \frac{u^2}{c^2}}} \quad \text{(old-fashioned notation)}$$

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Bohr-Schrodinger (BS) approximation throws out Mc^2 (Is frequency really relative?)

given:

$E = \hbar\omega$ ENERGY

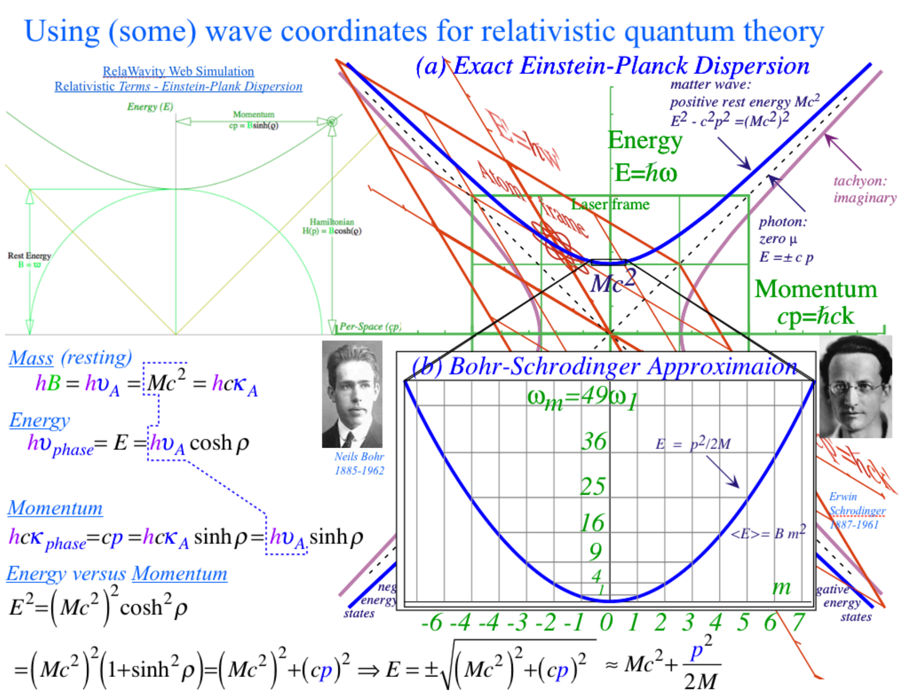
$$= \frac{Mc^2}{\sqrt{1-u^2/c^2}} = Mc^2 \cosh\rho$$

$cp = \hbar ck$ MOMENTUM

$$= \frac{Mcu}{\sqrt{1-u^2/c^2}} = Mc^2 \sinh\rho$$

RelaWavity Web Simulation
Relativistic *Terms - Einstein-Plank Dispersion*

Some details concerning
Lect. 30 - slide 31



Bohr-Schrodinger (BS) approximation throws out Mc^2

(Is frequency really relative?)

ENERGY

$$E = \hbar\omega = \frac{Mc^2}{\sqrt{1-u^2/c^2}} = Mc^2\cosh\rho = Mc^2\sqrt{1+\sinh^2\rho} = \sqrt{\left(Mc^2\right)^2 + (cp)^2}$$

given: MOMENTUM

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RelaWavity Web Simulation
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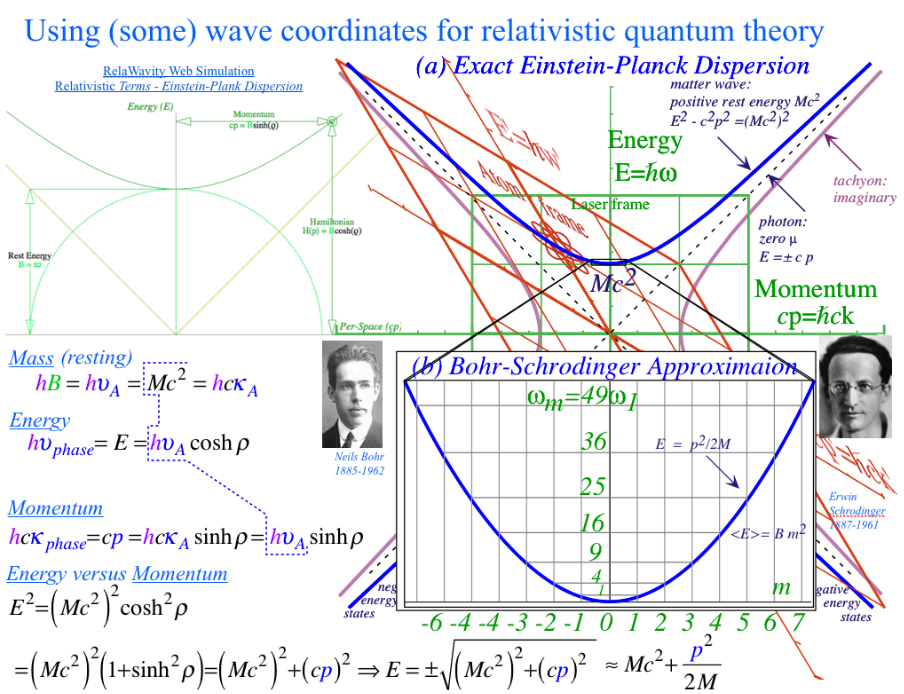

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RelaWavity Web Simulation
Relativistic *Terms - Einstein-Plank Dispersion*

$$E = \left[\left(Mc^2\right)^2 + (cp)^2\right]^{1/2} \approx \cancel{Mc^2} + \frac{1}{2M}p^2 \xrightarrow{BS-approx} \frac{1}{2M}p^2$$

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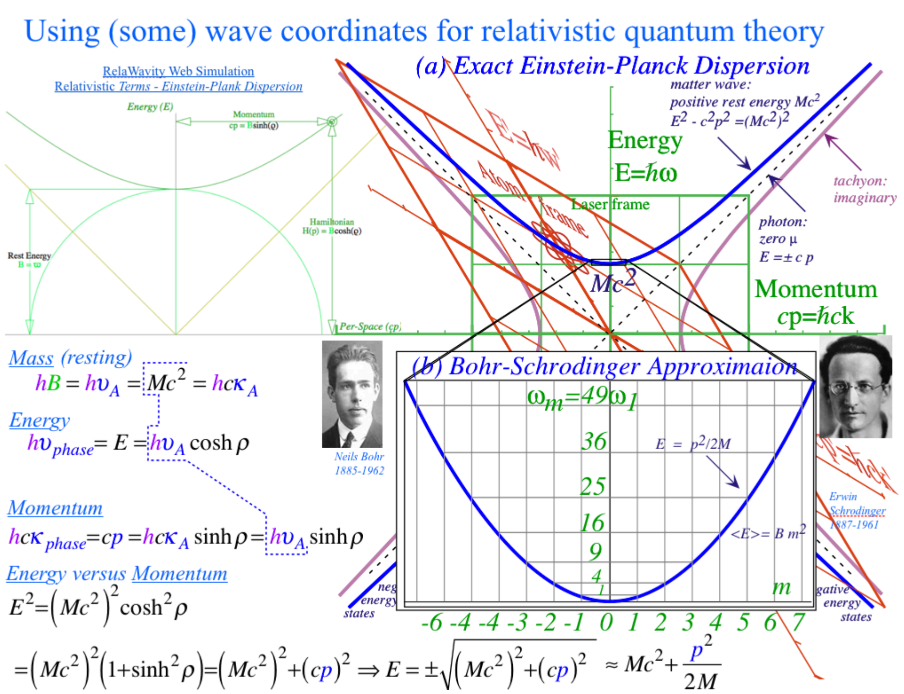

BS- binomial approximation $(a+b)^n = a^n + na^{n-1}b + \ldots$

replaces hyperbola: $E = \left[\left(Mc^2\right)^2 + (cp)^2\right]^{\frac{1}{2}} = Mc^2 + \frac{1}{2}\frac{c^2p^2}{Mc^2} + \ldots$

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$E=\hbar\omega$ ENERGY

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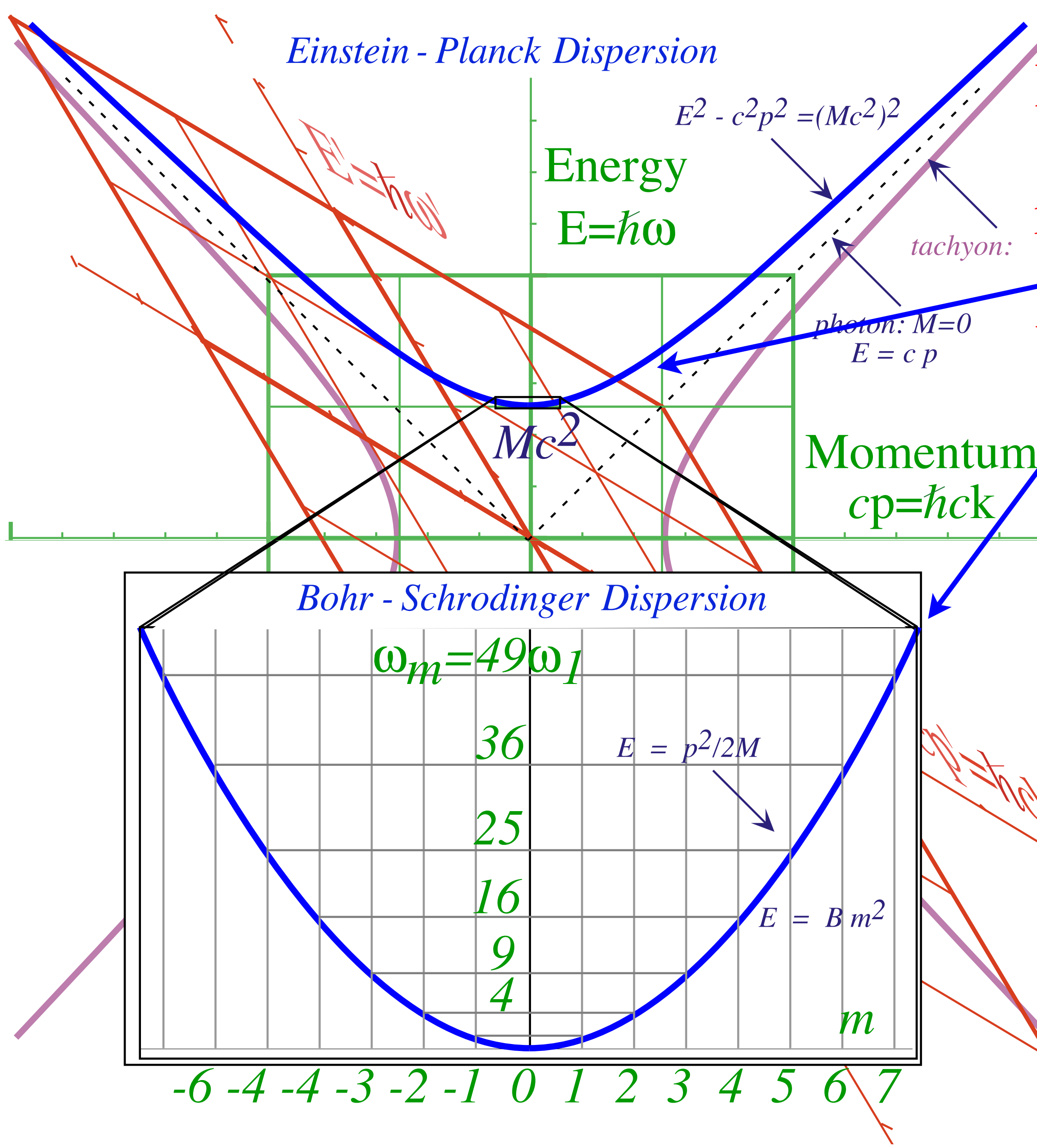

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RelaWavity Web Simulation
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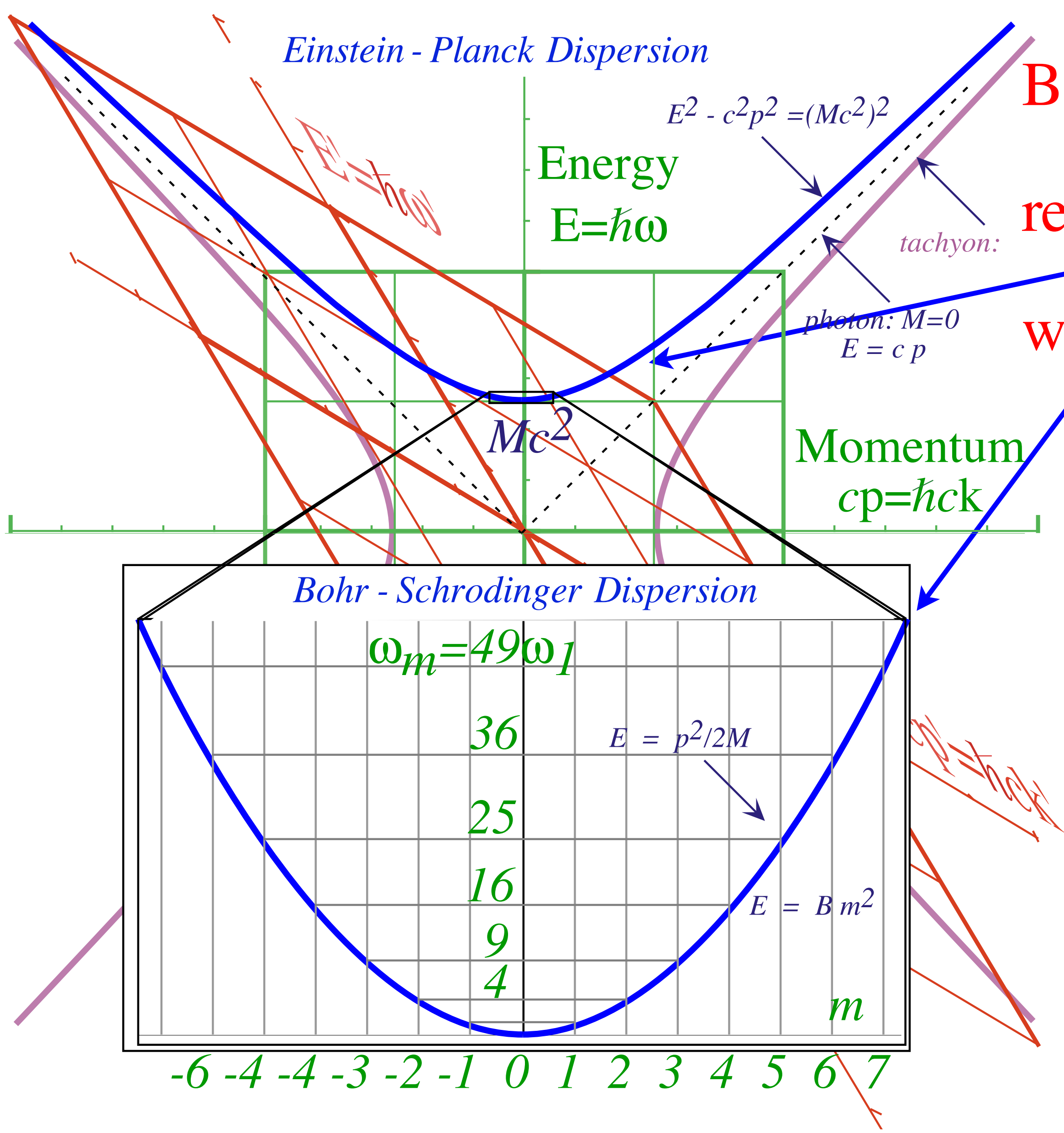

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This affects wave phase dynamics only:

Bohr-Schrodinger (BS) approximation throws out Mc^2 *(Is frequency really relative?)*

ENERGY: $E = \hbar\omega$

given: MOMENTUM: $cp = \hbar ck$

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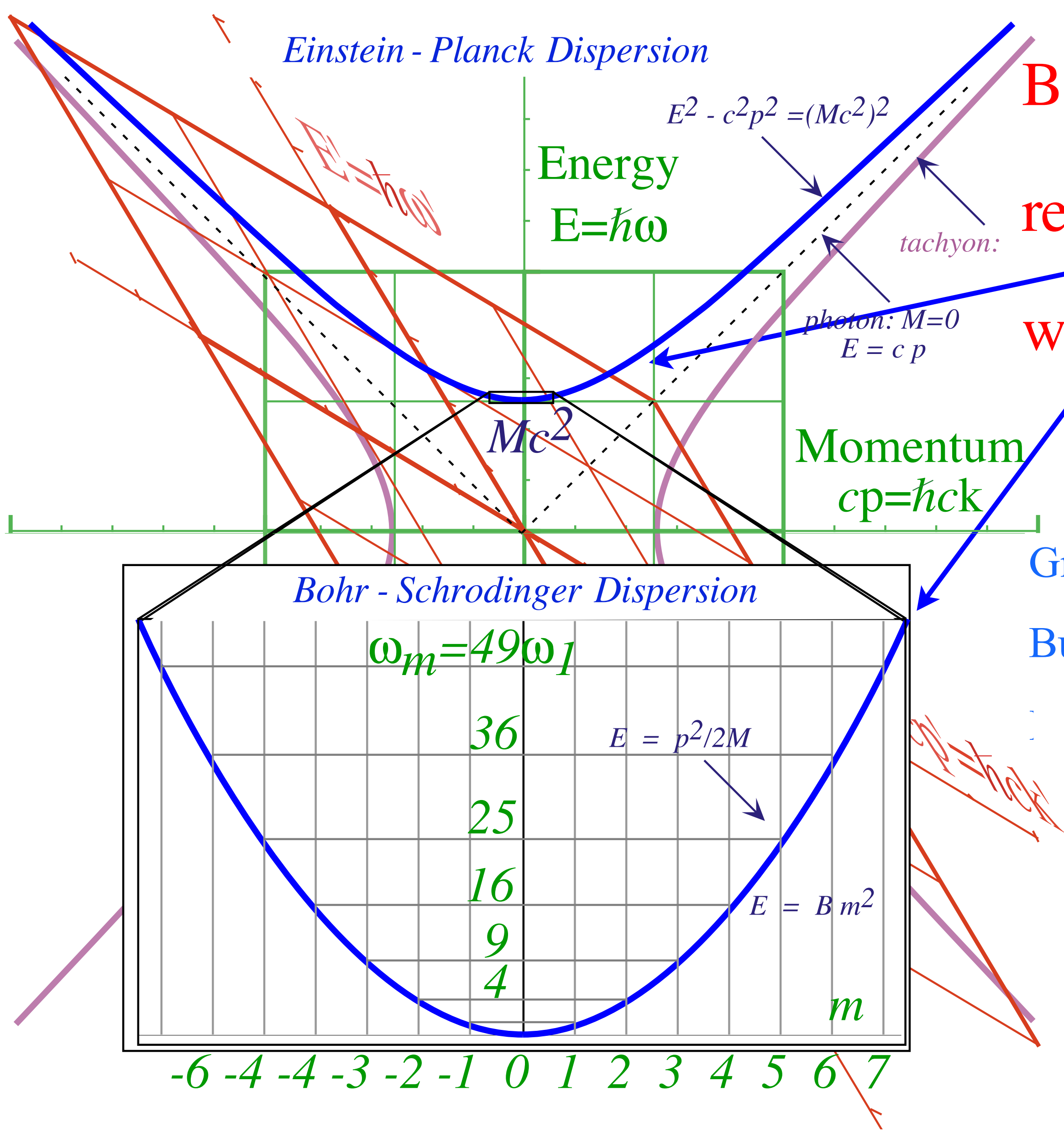

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This affects wave phase dynamics only:

Group velocity $u = V_{group} = \frac{d\omega}{dk}$ is a *differential* quantity unaffected by origin shift.

But, Phase velocity $\frac{\omega}{k} = V_{phase}$ is greatly reduced by deleting Mc^2 from $E=\hbar\omega$.

Bohr-Schrodinger (BS) approximation throws out Mc^2 (Is frequency really relative?)

ENERGY

$$E=\hbar\omega=\frac{Mc^2}{\sqrt{1-u^2/c^2}}=Mc^2\cosh\rho=Mc^2\sqrt{1+\sinh^2\rho}=\sqrt{(Mc^2)^2+(cp)^2}$$

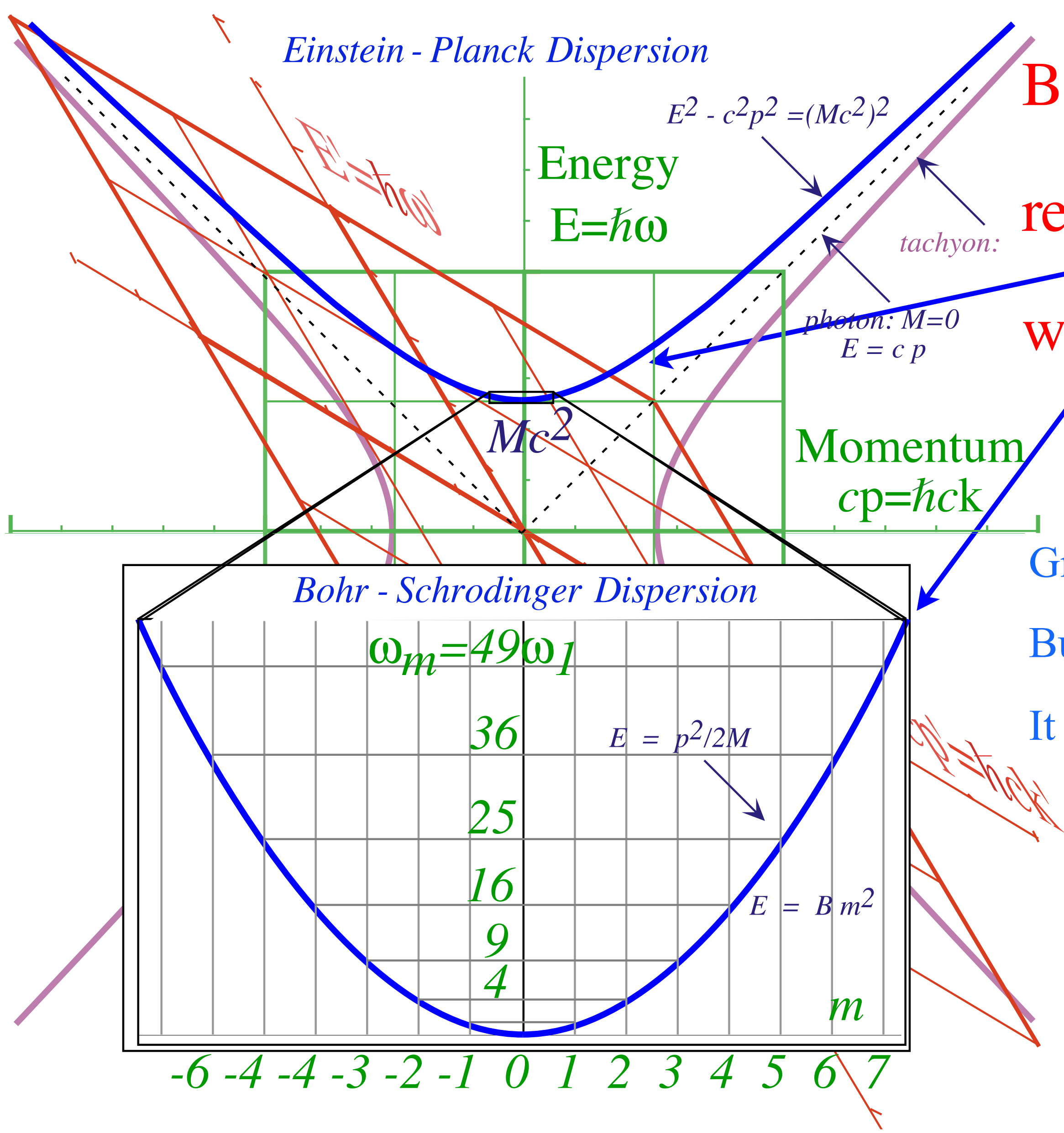
given: MOMENTUM

$$cp=\hbar ck=\frac{Mcu}{\sqrt{1-u^2/c^2}}=Mc^2\sinh\rho$$

RelaWavity Web Simulation
Relativistic *Terms - Einstein-Plank Dispersion*

$$E=\left[(Mc^2)^2+(cp)^2\right]^{1/2}\approx \cancel{Mc^2}+\frac{1}{2M}p^2 \xrightarrow{BS-approx} \frac{1}{2M}p^2$$

The BS claim: may shift energy origin ($E=Mc^2$, $cp=0$) to ($E=0$, $cp=0$). (*Frequency is relative!*)



BS- binomial approximation $(a+b)^n=a^n+na^{n-1}b+\ldots$

replaces hyperbola: $E=\left[(Mc^2)^2+(cp)^2\right]^{\frac{1}{2}}=Mc^2+\frac{1}{2}\frac{c^2p^2}{Mc^2}+\ldots$

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$$\omega_{BS}(k)=\frac{k^2}{2M} \quad \text{gives:} \quad V_{phase}=\frac{\omega_{BS}}{k}=\frac{k}{2M}$$

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$E = \hbar\omega$ ENERGY

$= \dfrac{Mc^2}{\sqrt{1-u^2/c^2}} = Mc^2 \cosh\rho = Mc^2\sqrt{1+\sinh^2\rho} = \sqrt{(Mc^2)^2 + (cp)^2}$

given: $cp = \hbar ck$ MOMENTUM

$= \dfrac{Mcu}{\sqrt{1-u^2/c^2}} = Mc^2 \sinh\rho$

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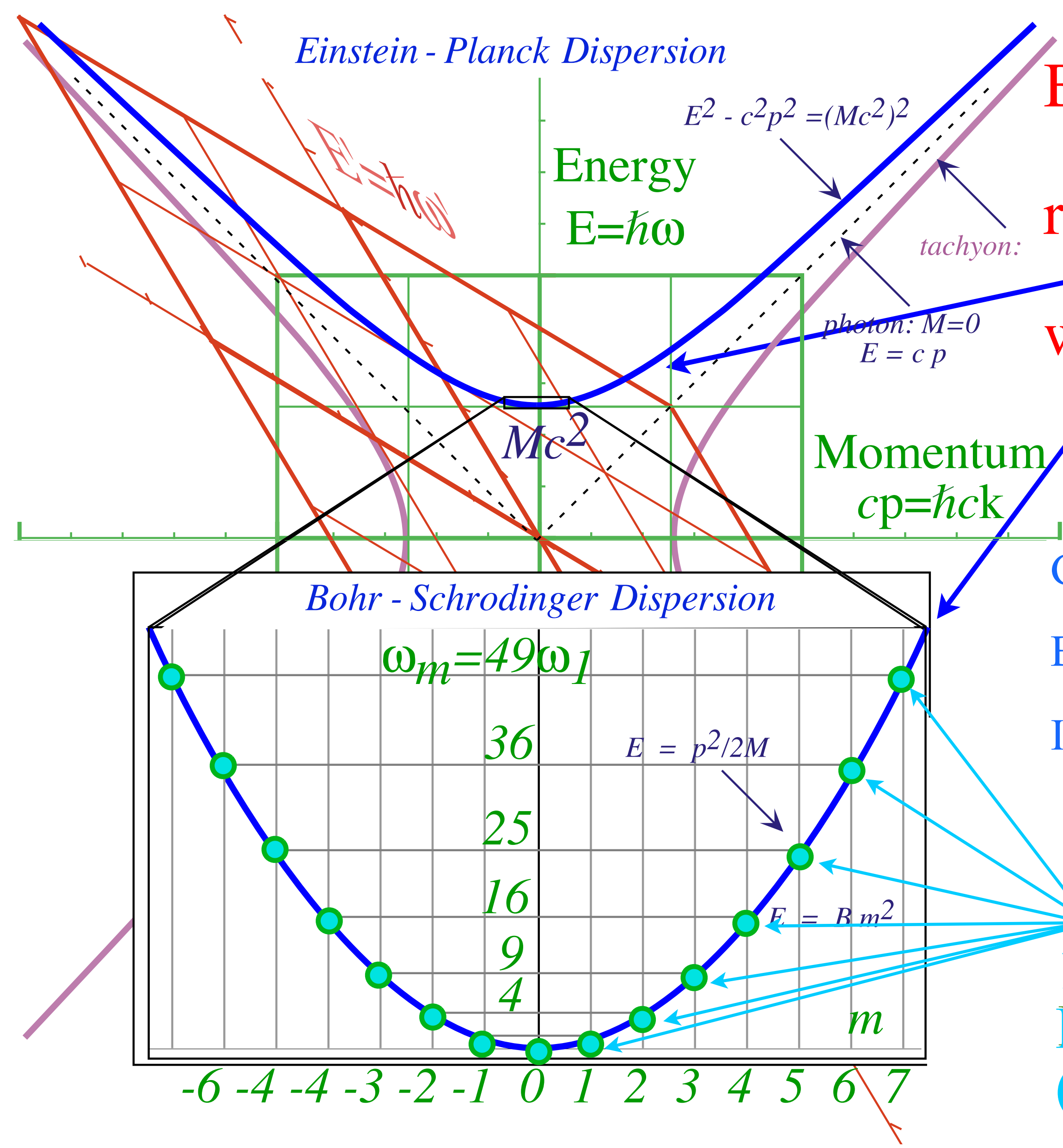

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1st quantization:
Restrict wave-number κ to integer or ½-integer quanta
(to fit in cavity)

Review of wave parameters used to develop relativistic quantum theory

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Effect on group velocity *(None)* and phase velocity *(Absurd)*

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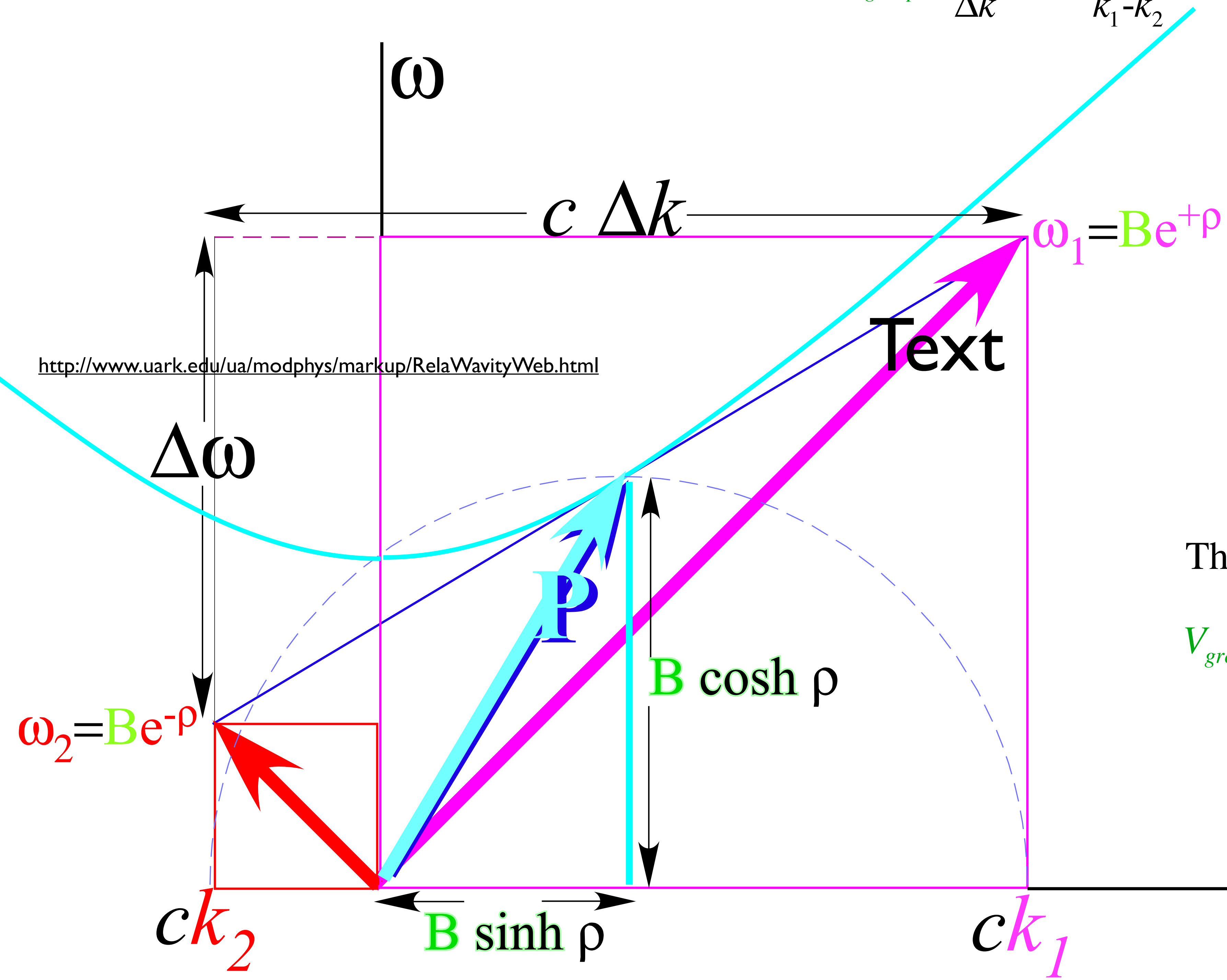
Relativistic effects on charge, current, and Maxwell Fields

Here is a rare but important case where $\frac{d\omega(k)}{dk}$ equals $\frac{\Delta\omega}{\Delta k}$. (Usually not so unless limit $\Delta k \to 0$ exists.)

Standard formula for classical group velocity is $V_{group} = \frac{d\omega(k)}{dk}$

But this may fail if $\omega(k)$ is quantized and thus discrete $\omega(k_m)$.

Then we need to use an exact quantum form: $V_{group} = \frac{\Delta\omega}{\Delta k} = \frac{\omega(k_1)\text{-}\omega(k_2)}{k_1\text{-}k_2}$



Here the exact discrete value is:

$$V_{group} = \frac{\Delta\omega}{\Delta k} = \frac{\omega(k_1) - \omega(k_2)}{k_1 - k_2}$$

$$= \frac{Be^{+\rho} - Be^{-\rho}}{Be^{+\rho} - (-Be^{-\rho})}$$

$$= \frac{e^{+\rho} - e^{-\rho}}{e^{+\rho} + Be^{-\rho}} = \frac{\sinh\rho}{\cosh\rho} = \tanh\rho$$

This time it matches calculus value:

$$V_{group} = \frac{d\omega(k)}{dk} = \frac{d(\cosh\rho)}{d(\sinh\rho)}$$

$$= \frac{\sinh\rho}{\cosh\rho} = \tanh\rho$$

Review per-space-time

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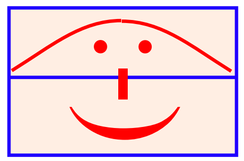
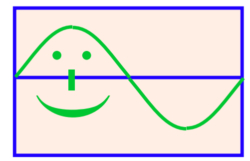

1 half-wave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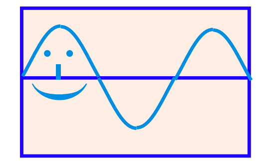
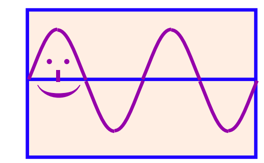

3 half-waves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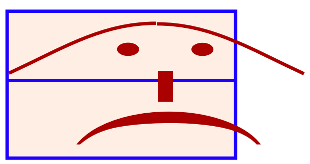

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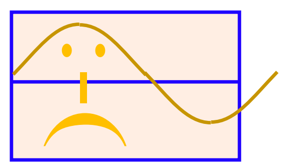

wrong color again!

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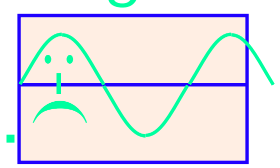


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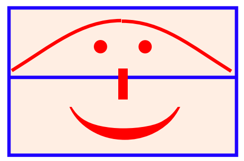

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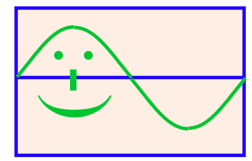

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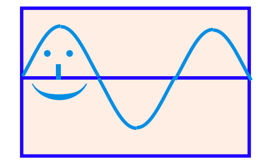

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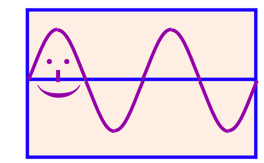

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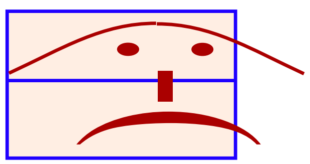

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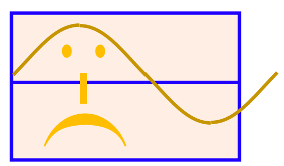

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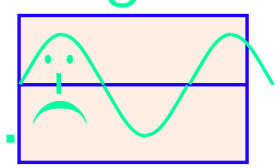

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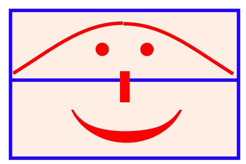

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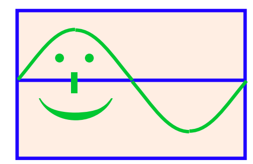

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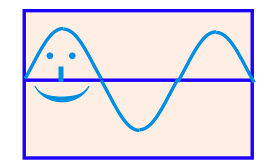

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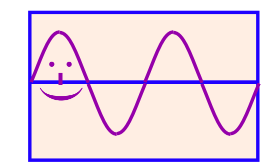

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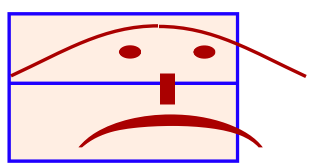

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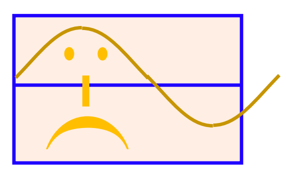

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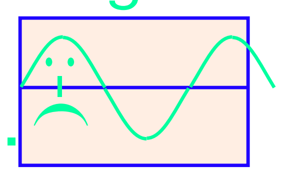

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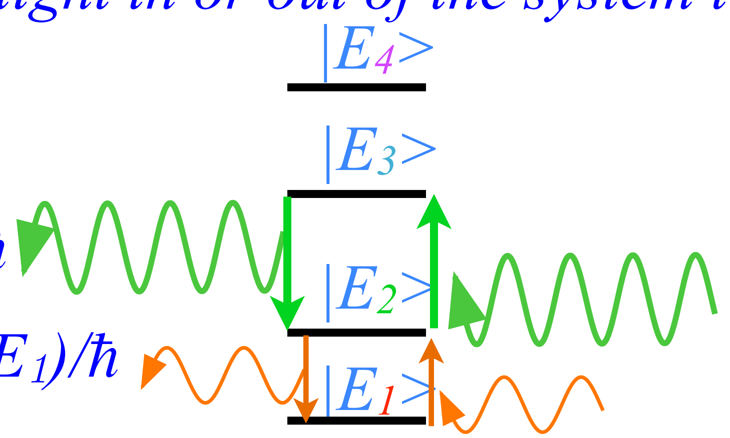

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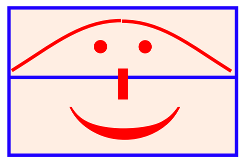
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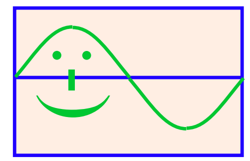
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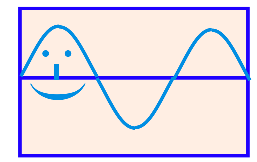
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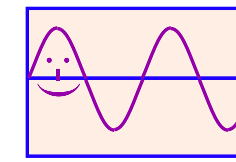
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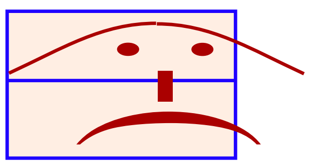

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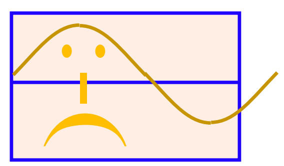

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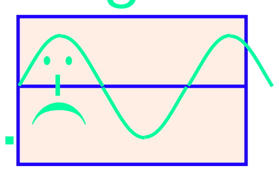

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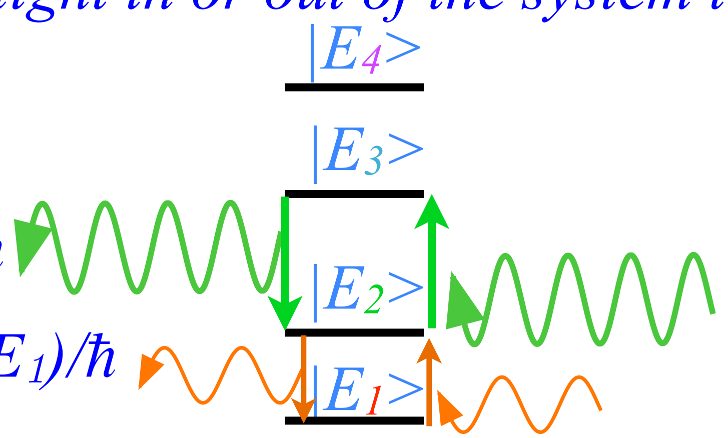

These eigenstates are just the ways the wavy system can "play dead"...

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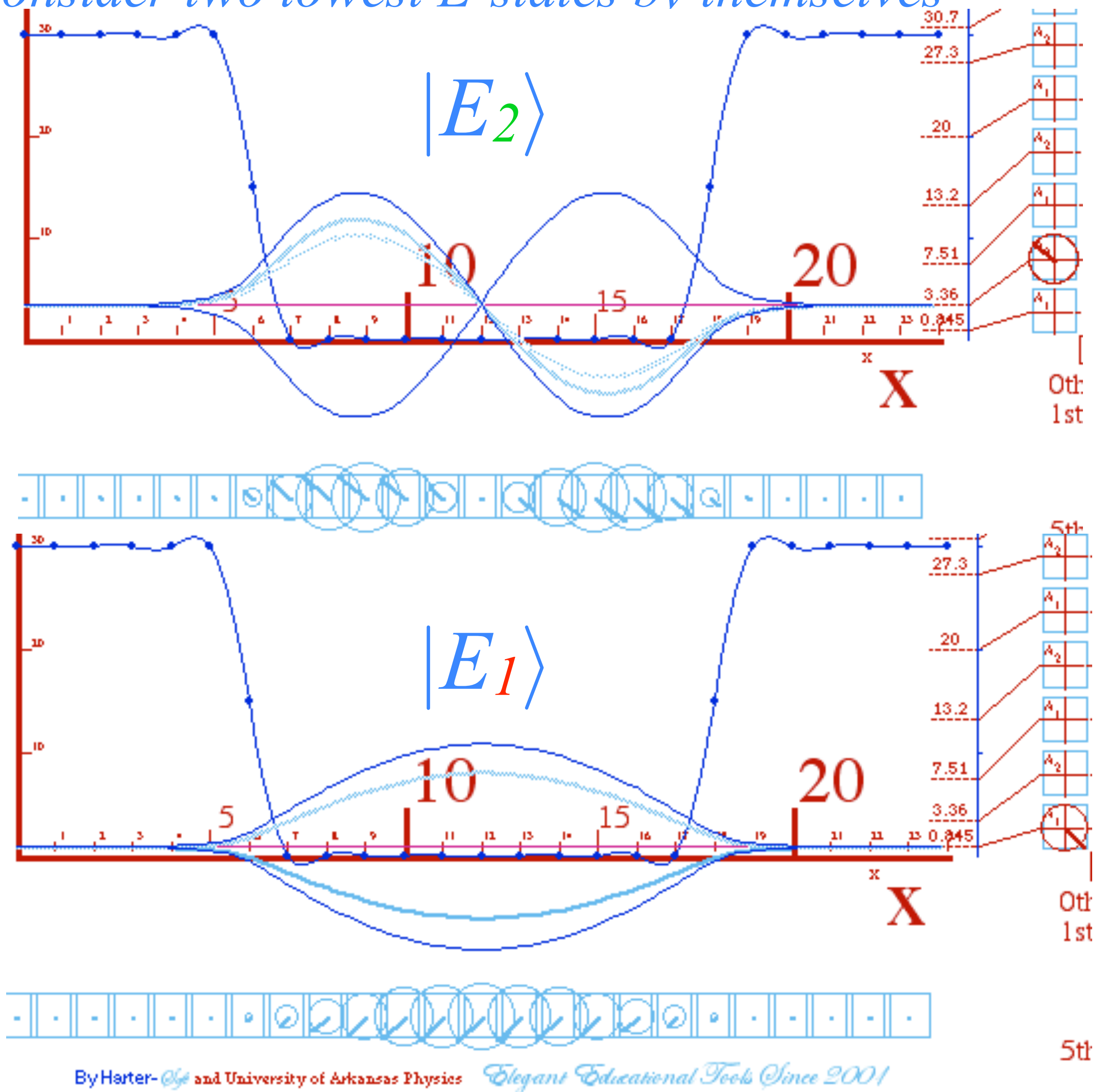
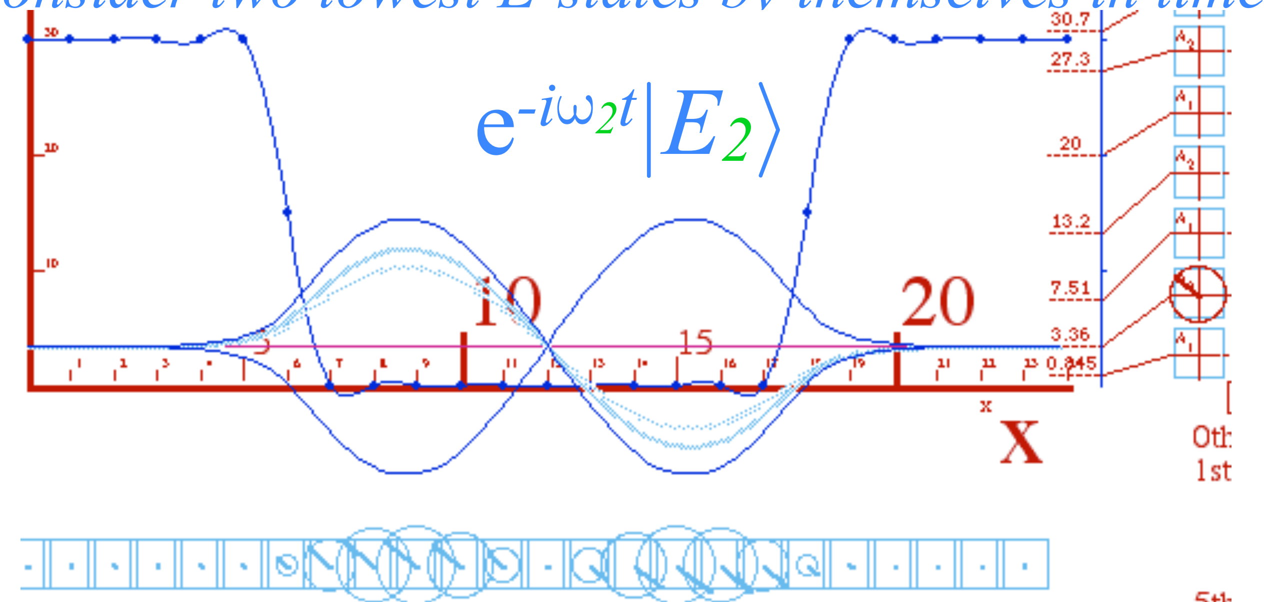
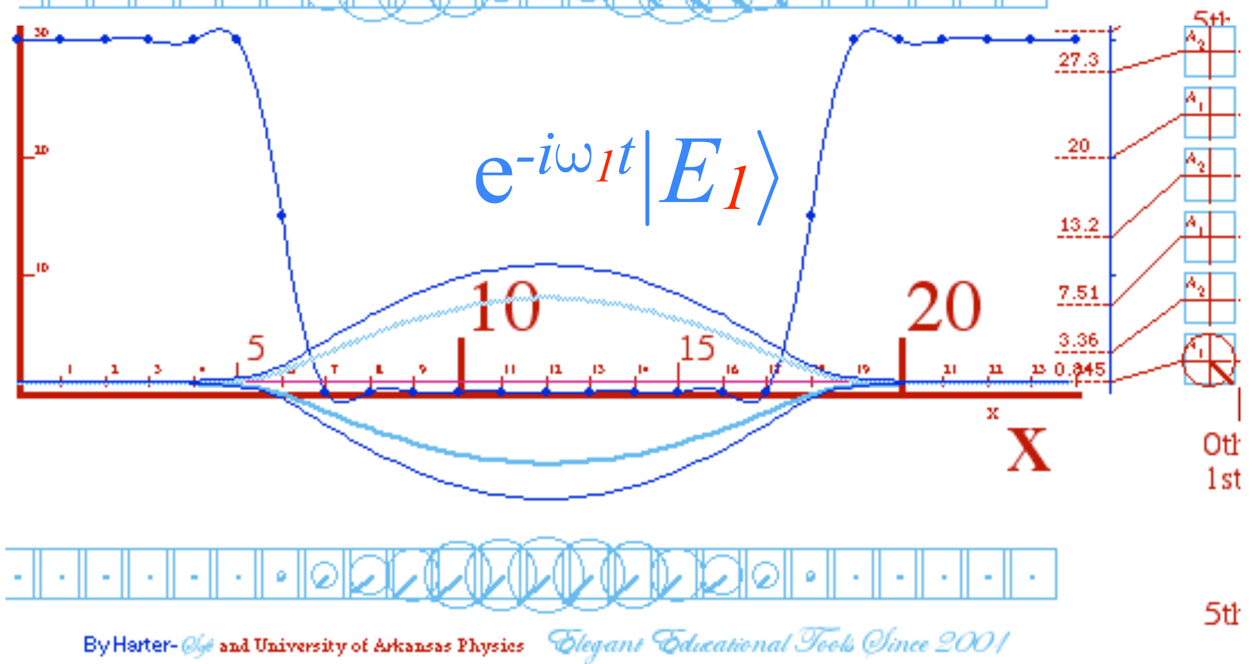
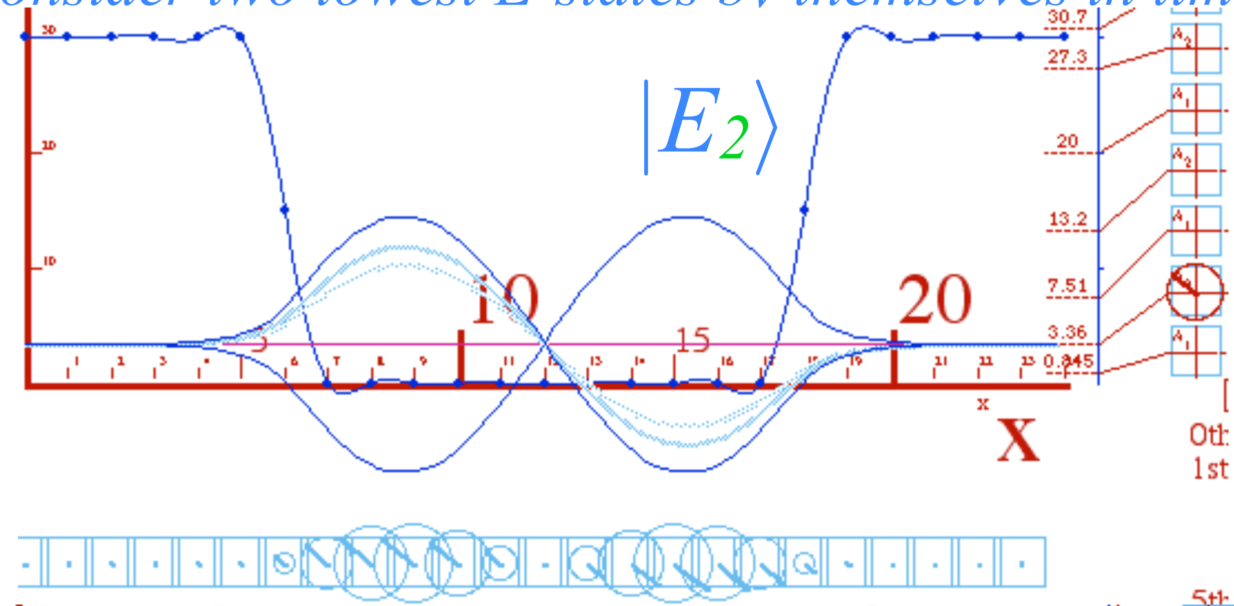


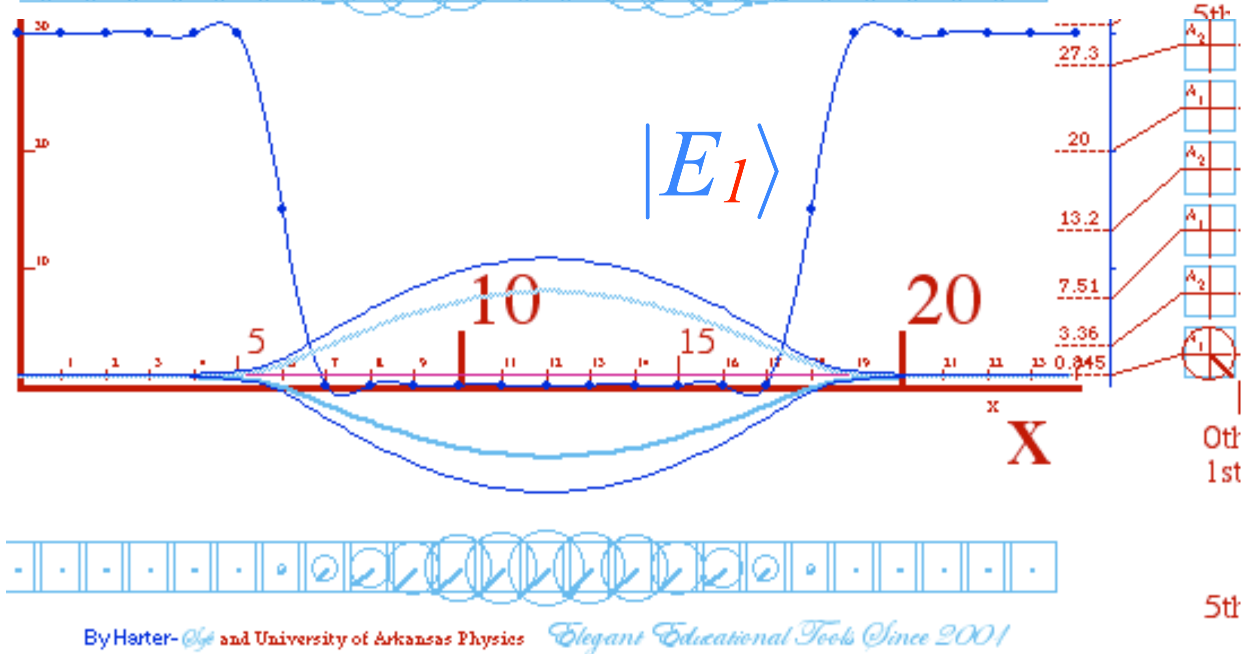
Consider two lowest E-states by themselves in time

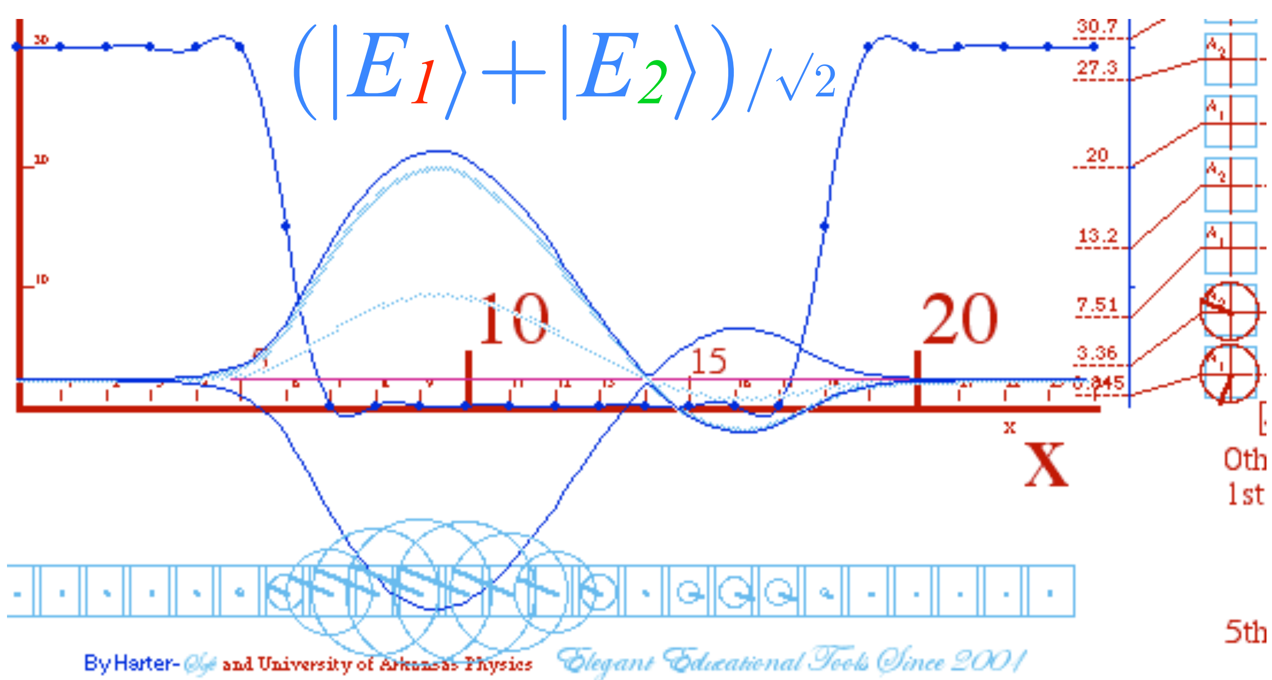




Consider two lowest E-states by themselves in time *Now combine (add) them*







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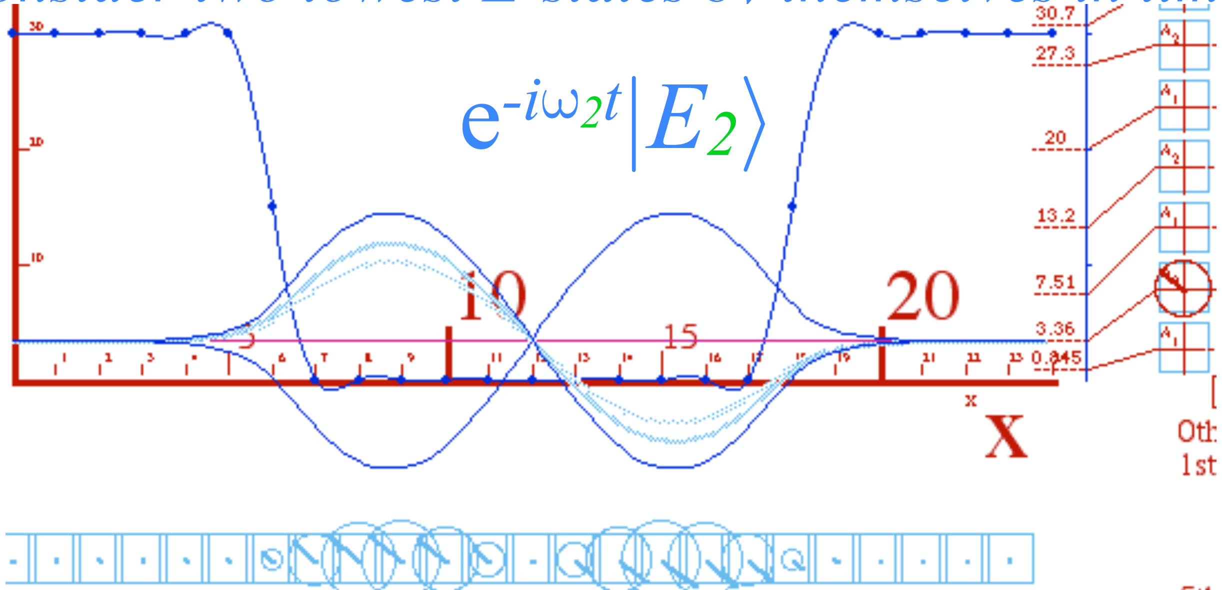
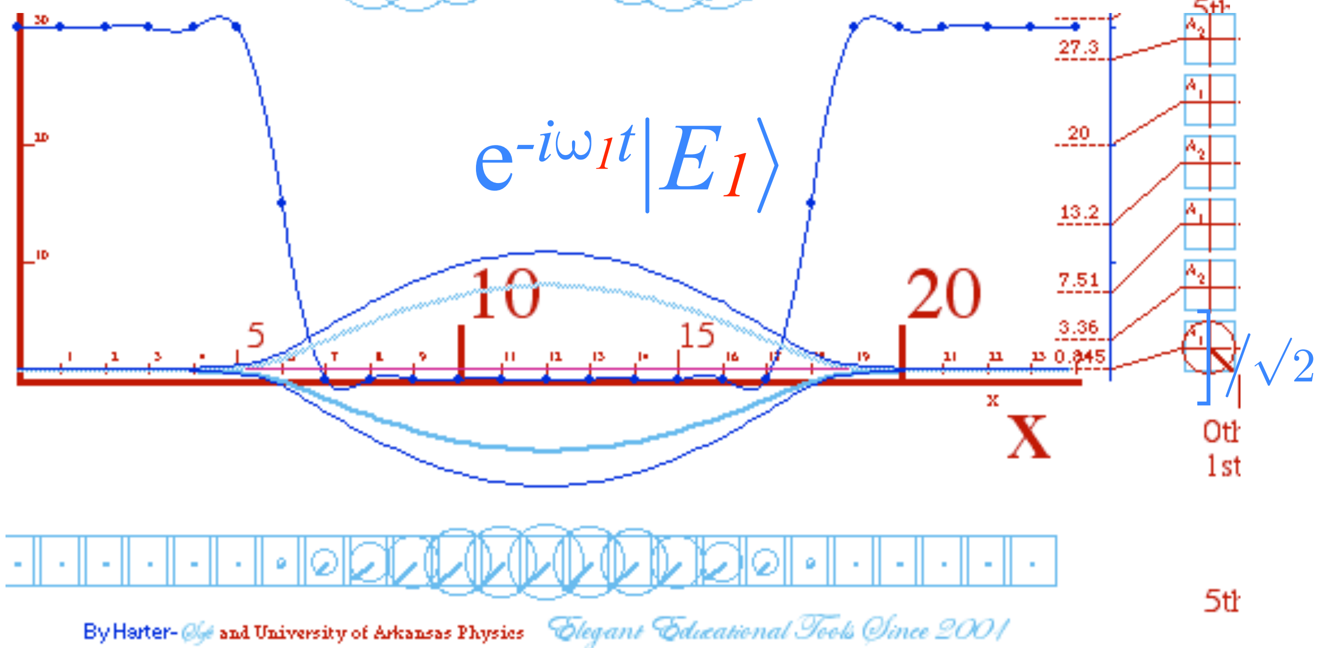
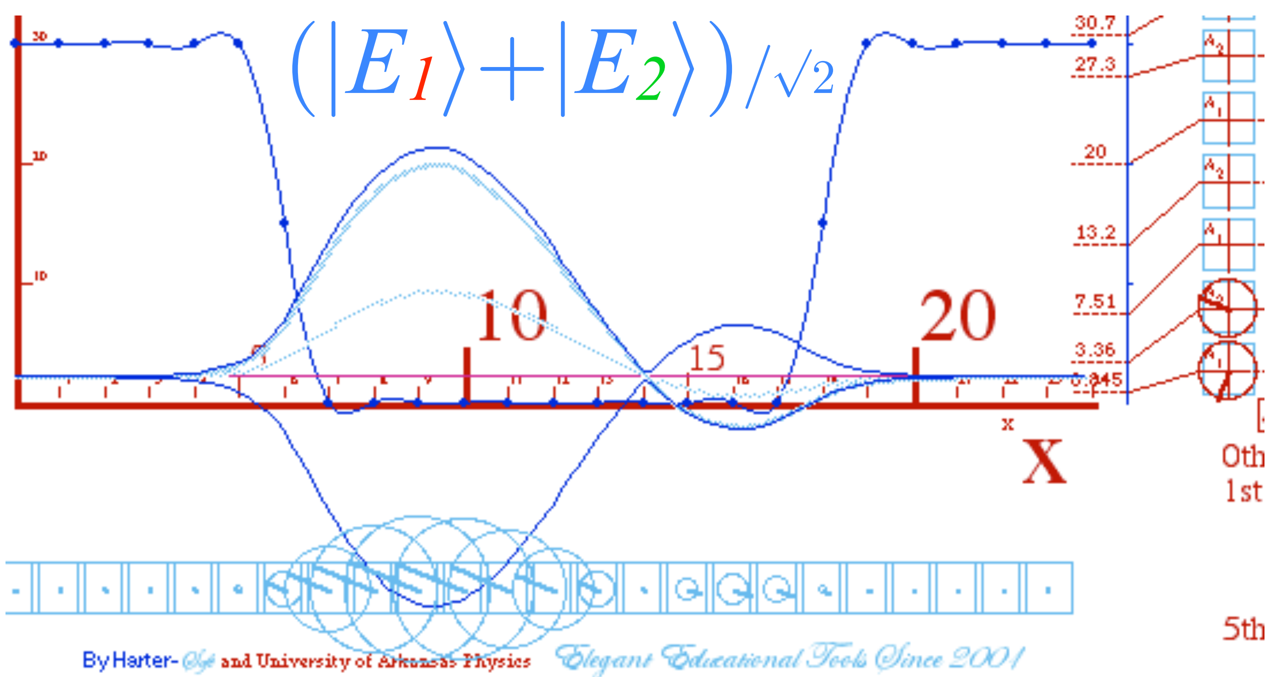
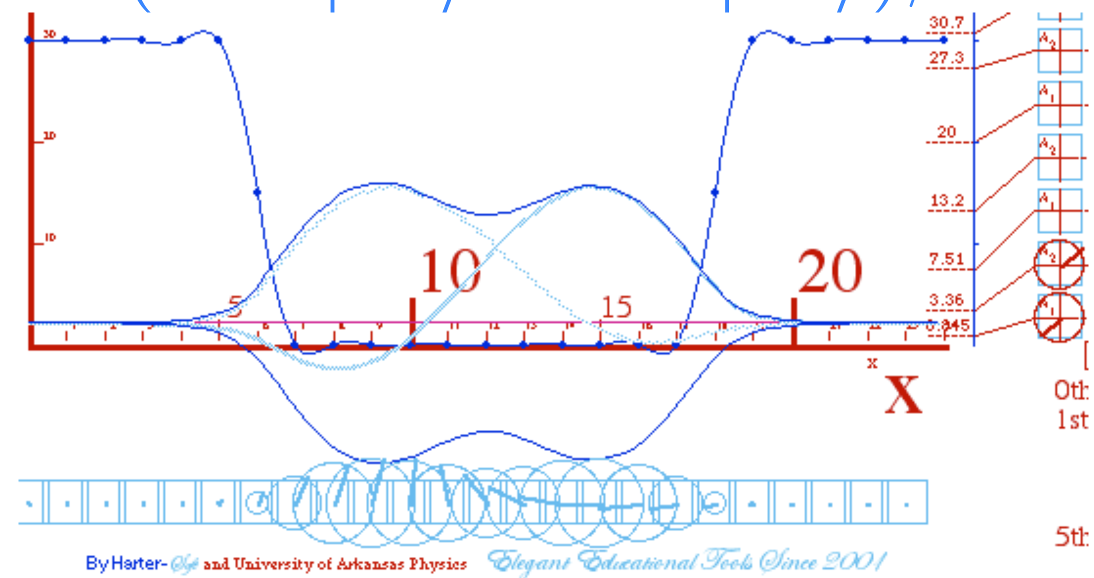






Now combine (add) them and let time roll!

$$\left(e^{-i\omega_1 t}|E_1\rangle + e^{-i\omega_2 t}|E_2\rangle\right)/\sqrt{2}$$



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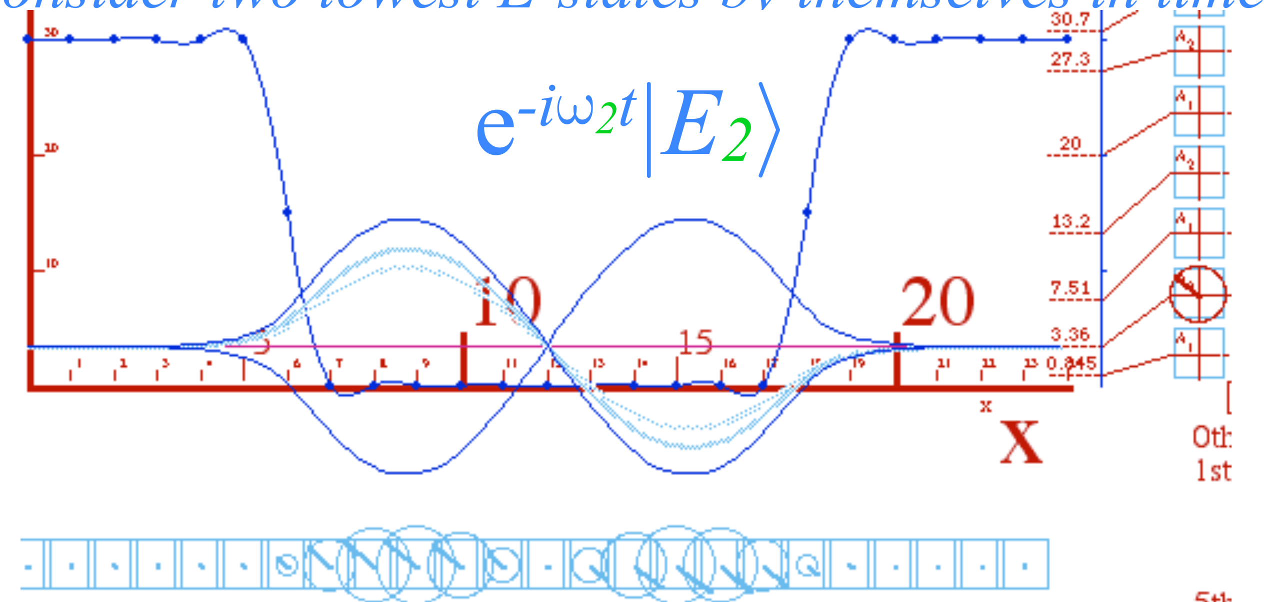

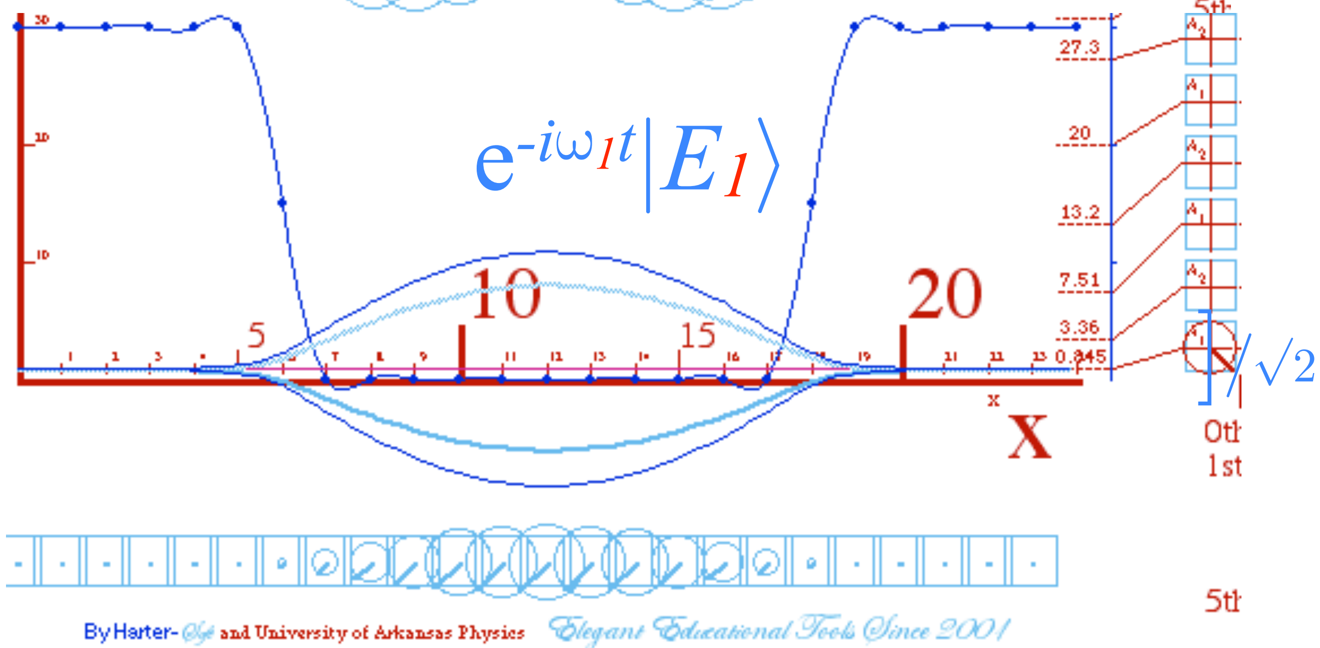

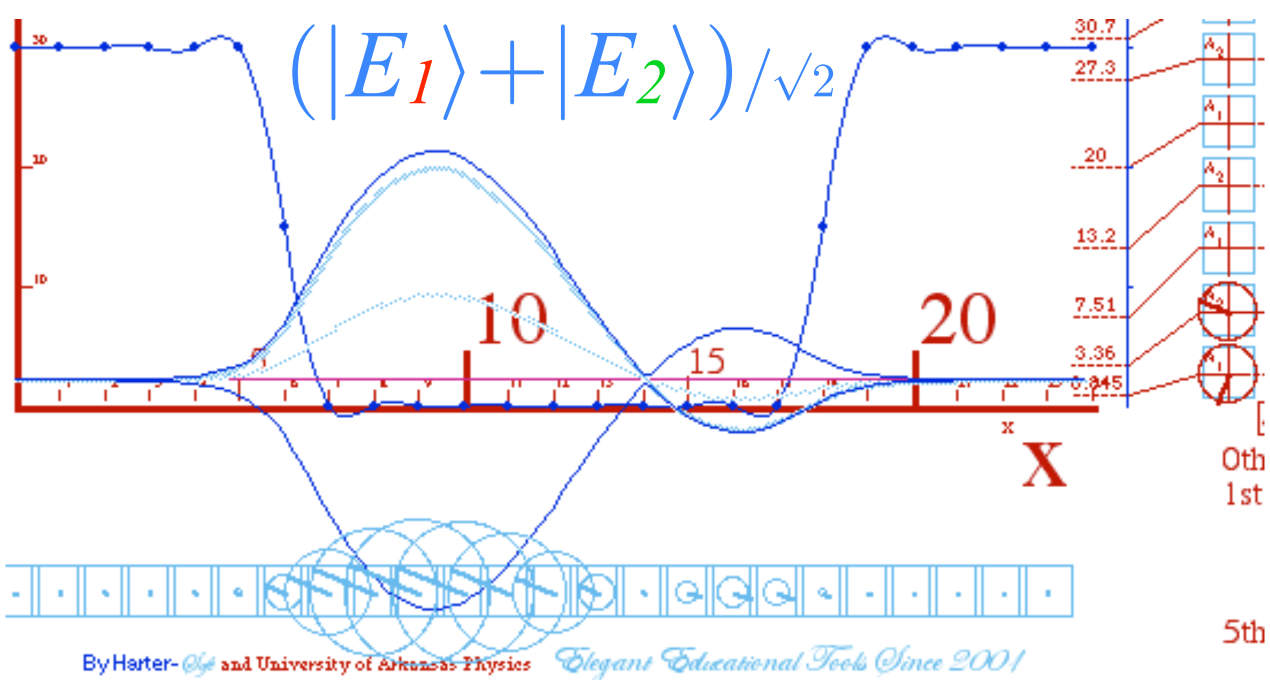

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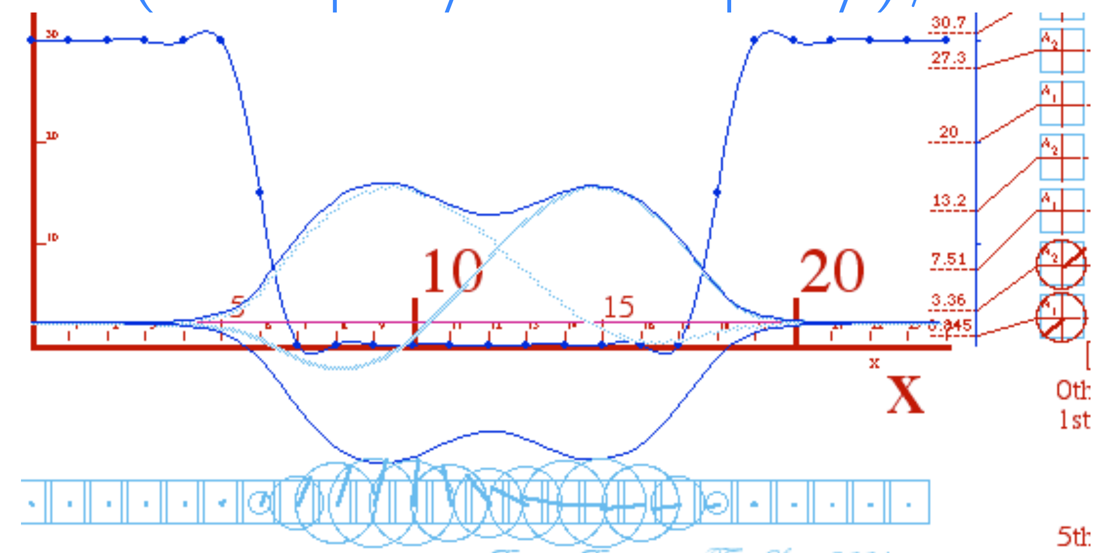

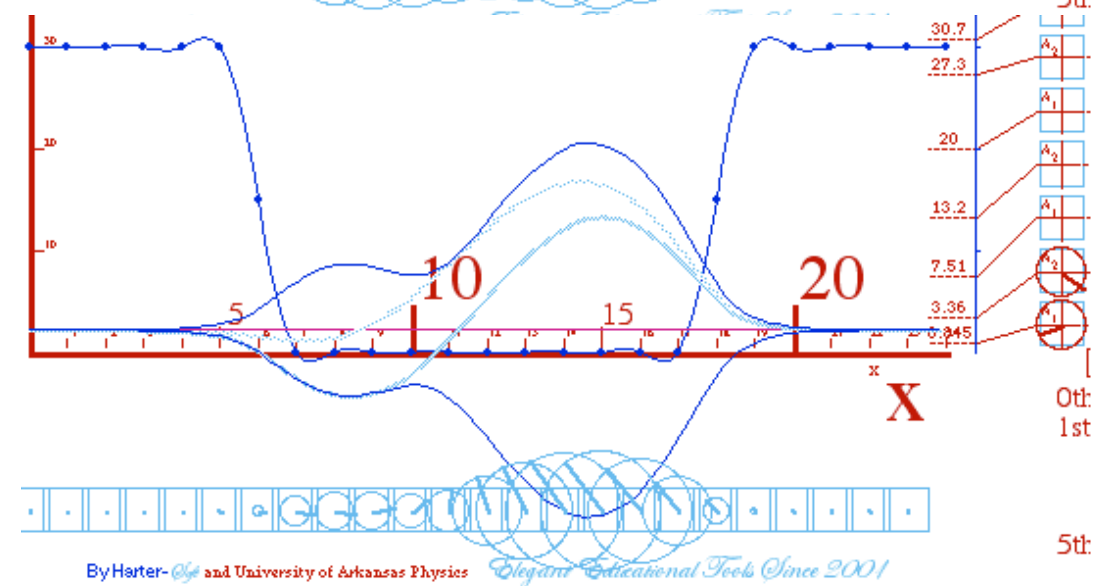

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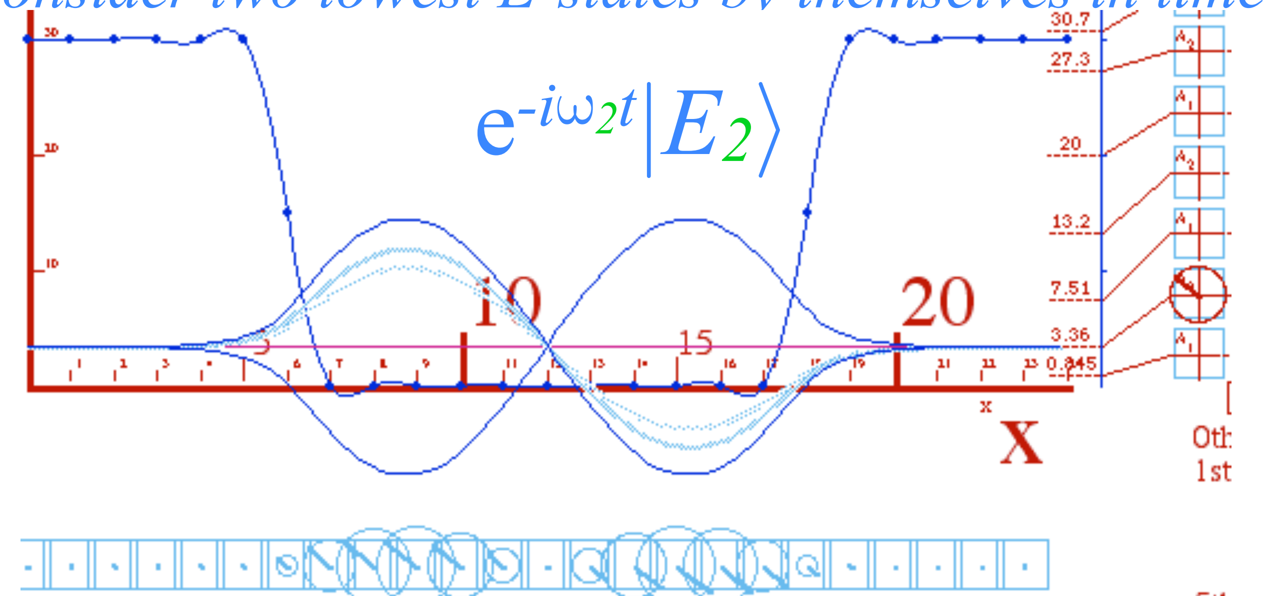

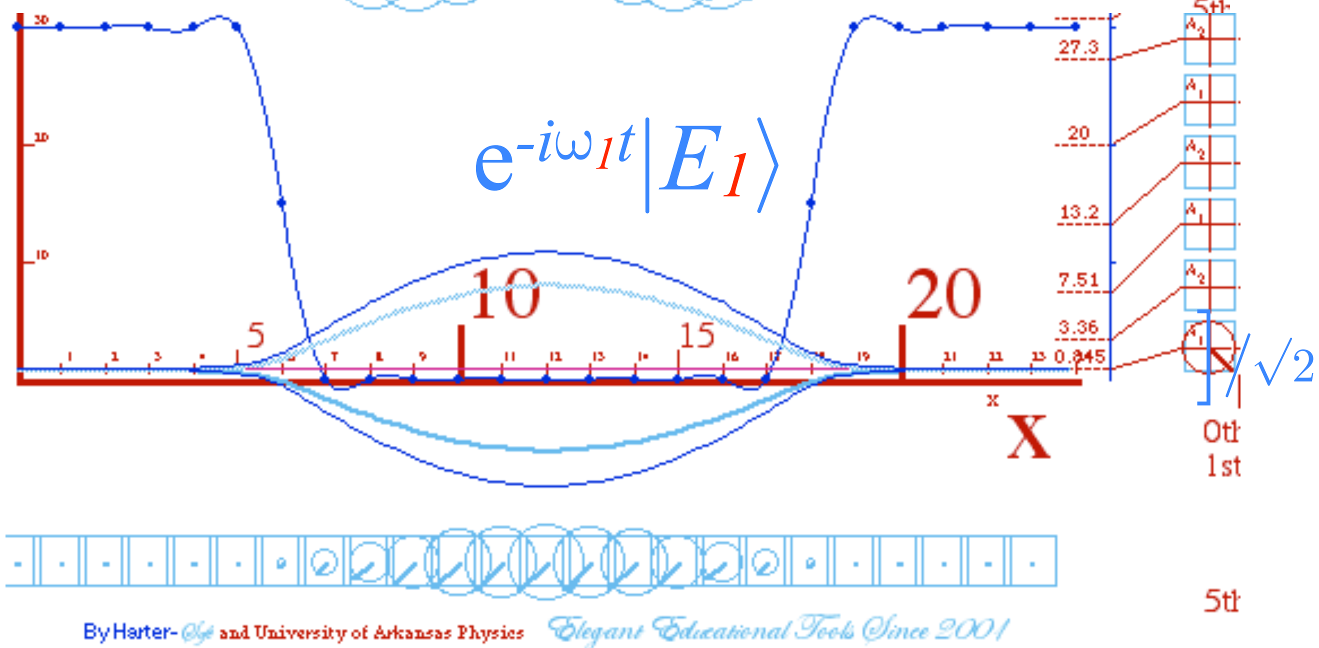

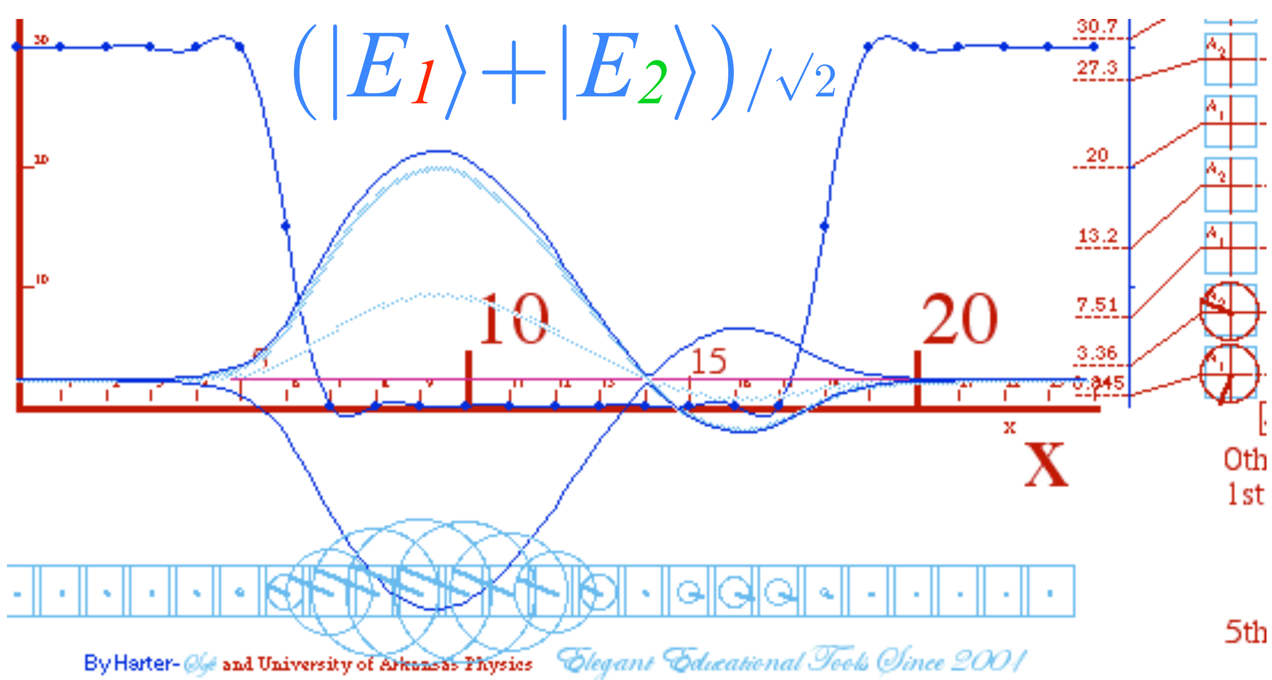

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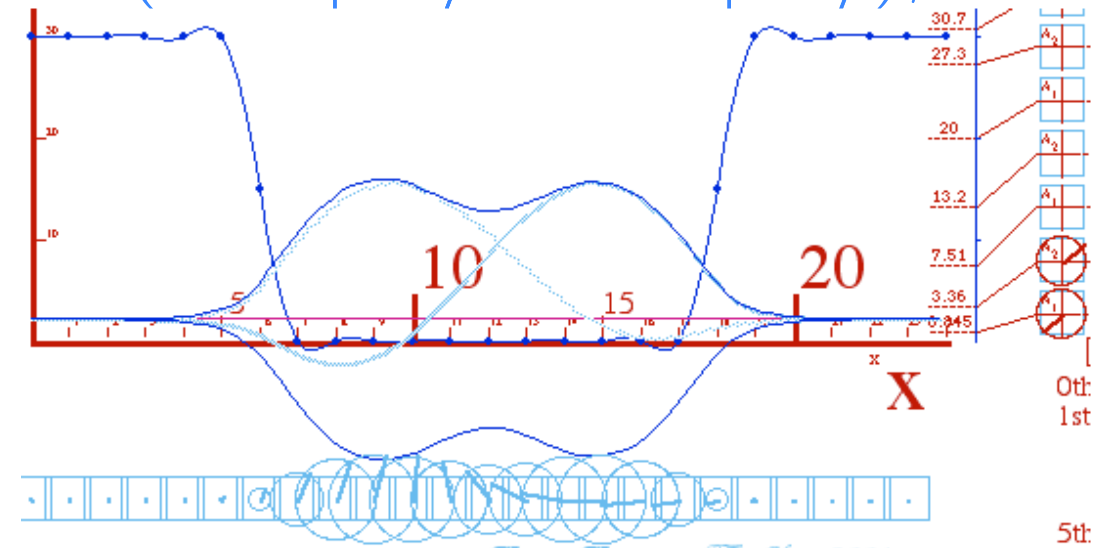
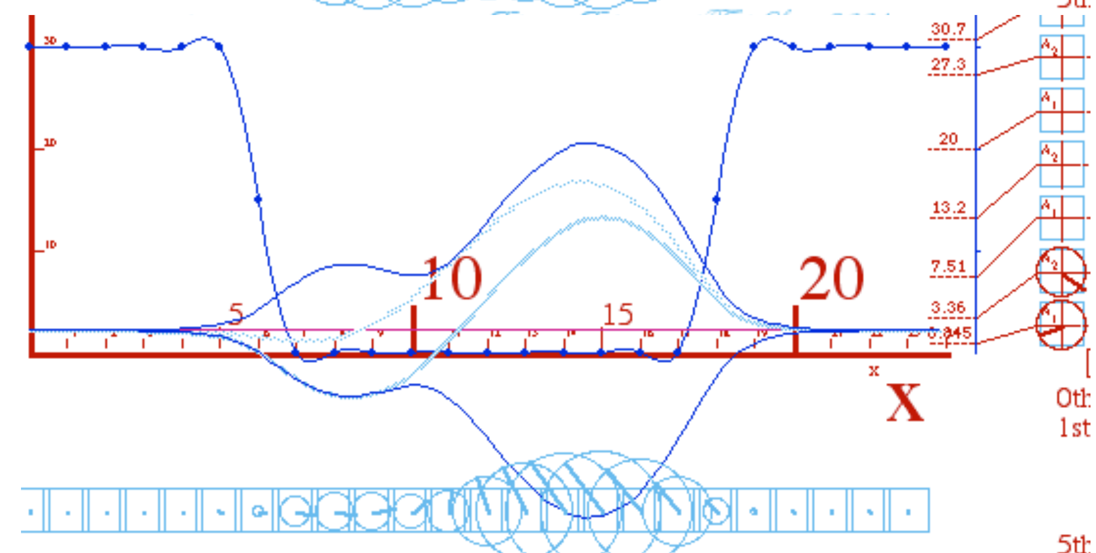
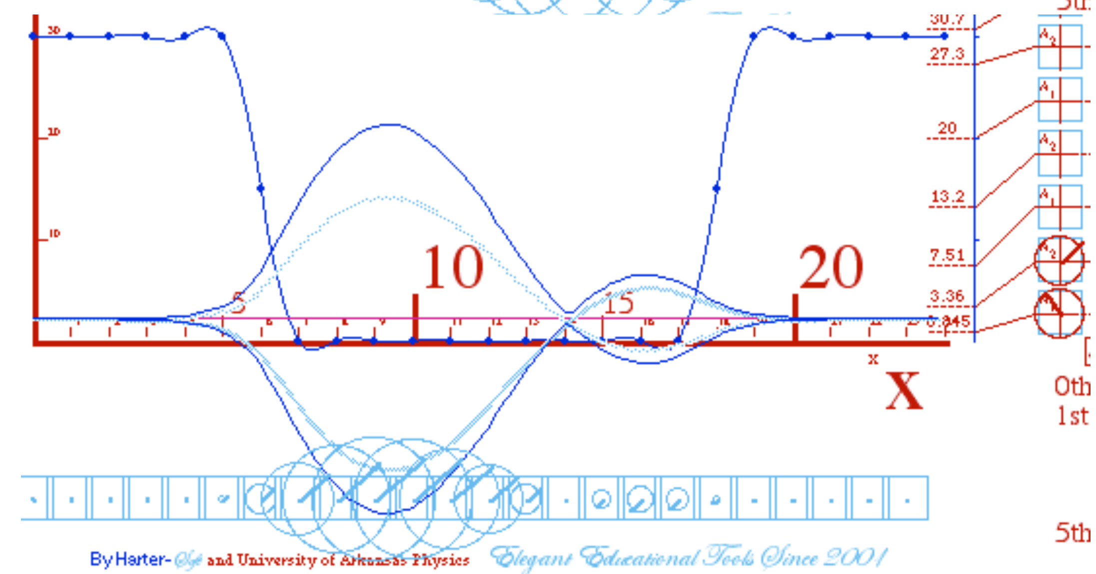






Review of wave parameters used to develop relativistic quantum theory

Bohr-Schrodinger (BS) approximation throws out Mc^2 *(Is frequency really relative?)*

Effect on group velocity *(None)* and phase velocity *(Absurd)*

1st Quantization: Quantizing phase variables k_m and $\omega(k_m)$

Understanding how quantum dynamics and transitions involve "mixed" states

Square well example of mixing unequal frequencies

Circle well or ring example of mixing equal or unequal frequencies

Mixing unequal amplitudes makes "Galloping" wave: Analogy of (*SWR*, *SWQ*) to (V_{group},V_{phase})

Analogy with optical polarization geometry and Kepler orbits

Super-luminal speed and Feynman-Wheeler pair-creation switchbacks

2nd Quantization: Quantizing wave amplitudes A_N and invariance of photon number

Analogy 1: Many CW (Continuous Waves) add up to make PW (Pulse Waves)

Analogy 2: Many Photon-Number-Modes add up to make Coherent-Laser-Modes

Heisenberg $\Delta\upsilon\Delta t$~1~$\Delta\kappa\Delta x$ analogous to $\Delta N \Delta phase$~1 uncertainty relations

Electromagnetic wave mode energy: Maxwell vs. Planck-Einstein

1st quantization for wave phase variables and classical energy of **E**, **B** , and **A** fields

2nd quantization for wave and Planck quantum energy of **E**, **B** , and **A** fields

Scaling **E**-waves to mime quantum Ψ-waves and ψ-waves

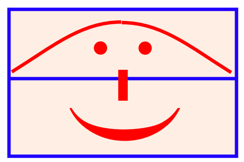
Relativistic effects on charge, current, and Maxwell Fields

Quantized ω and *k* *Counting wave kink numbers*

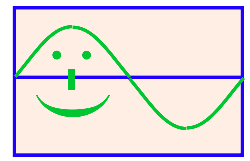
If everything is made of waves then we expect *quantization* of everything because waves only thrive if *integral* numbers *n* of their "kinks" fit into whatever structure (box, ring, etc.) they're supposed to live. The numbers *n* are called *quantum numbers*.

OK box quantum numbers: *n=1* *n=2* *n=3* *n=4*

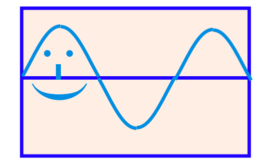
(+ integers only)



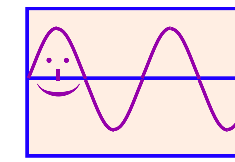
1 half-wave



2 half-waves



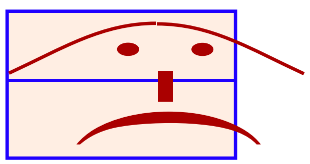
3 half-waves



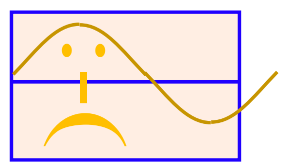
4 ha

Some NOT OK numbers: *n=0.67* *n=1.7* *n=2.59* *n=4*

too fat!

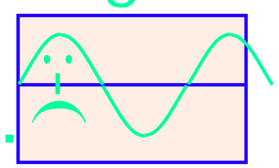


too thin!



wrong color again!

misfits...



...not tolerated !

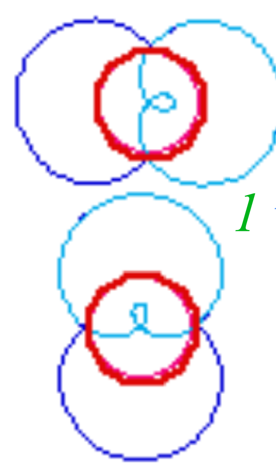
NOTE: We're using "false-color" here.

Rings tolerate a *zero* (kinkless) quantum wave but require ±*integral wave number.*

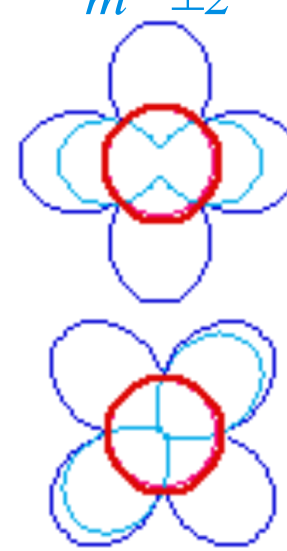
OK ring quantum numbers: *m=0* *m=±1* *m=±2* *m=3*

(± integral number of wavelengths)

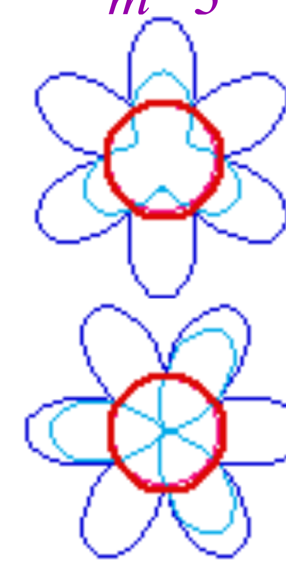
0 waves



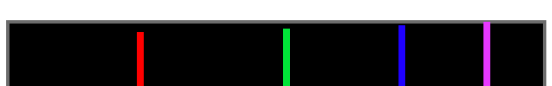
1 waves



2 waves



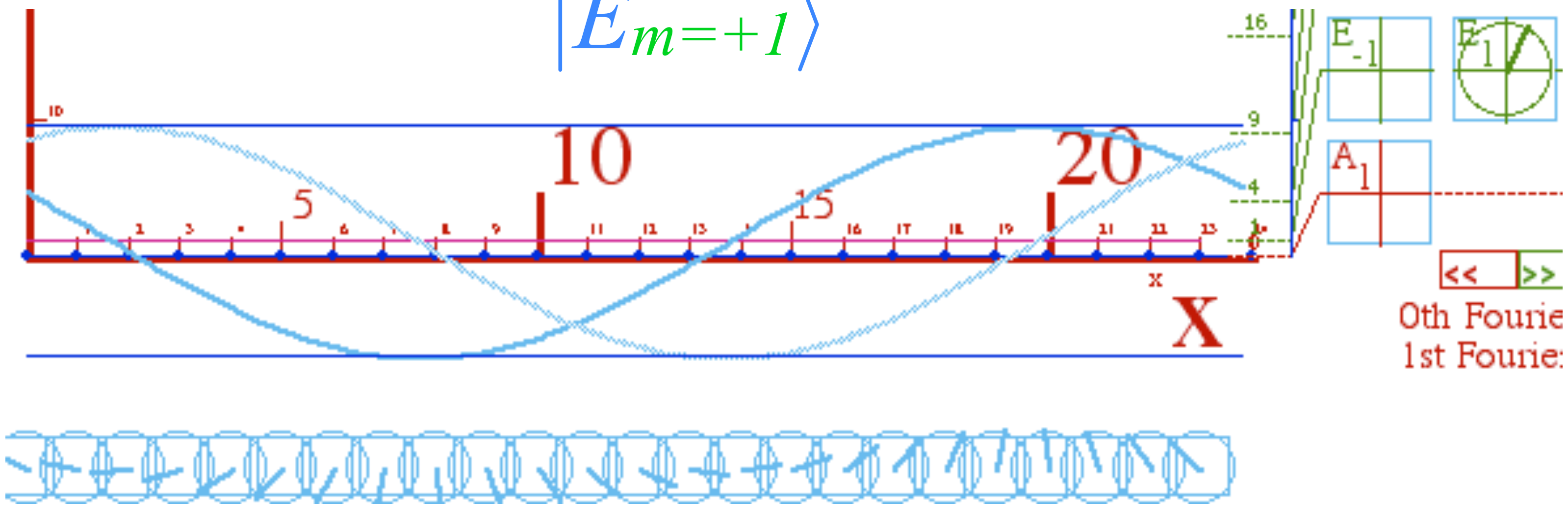
3 waves



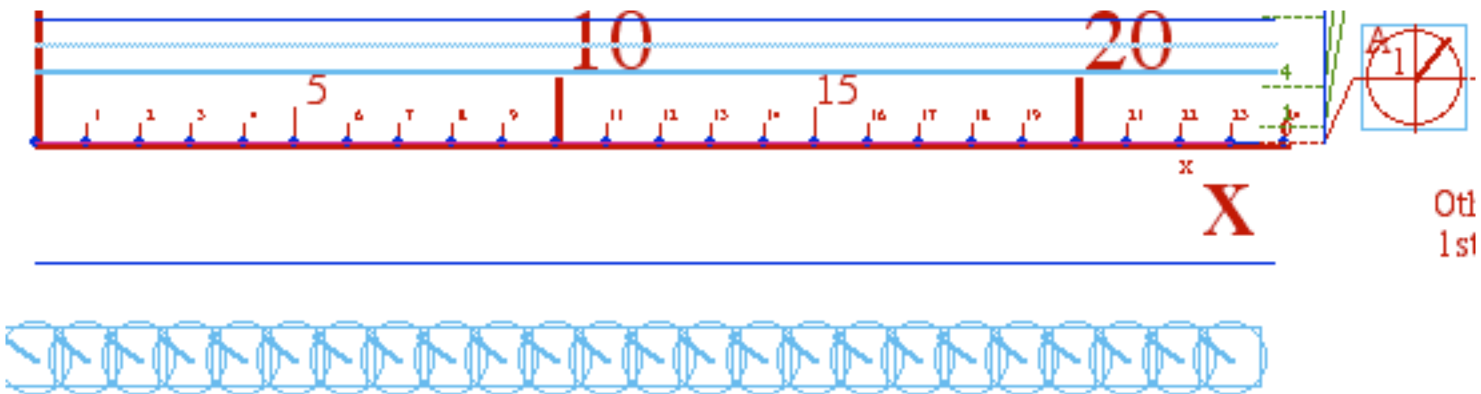
Bohr's models of *atomic spectra (1913-1923)* are beginnings of *quantum wave mechanics* built on *Planck-Einstein (1900-1905)* relation $E=h\upsilon$. *DeBroglie* relation $p=h/\lambda$ comes around *1923*.

Consider two lowest E-states by themselves

$$\left|E_{m=+1}\right\rangle$$



$$\left|E_{m=0}\right\rangle$$



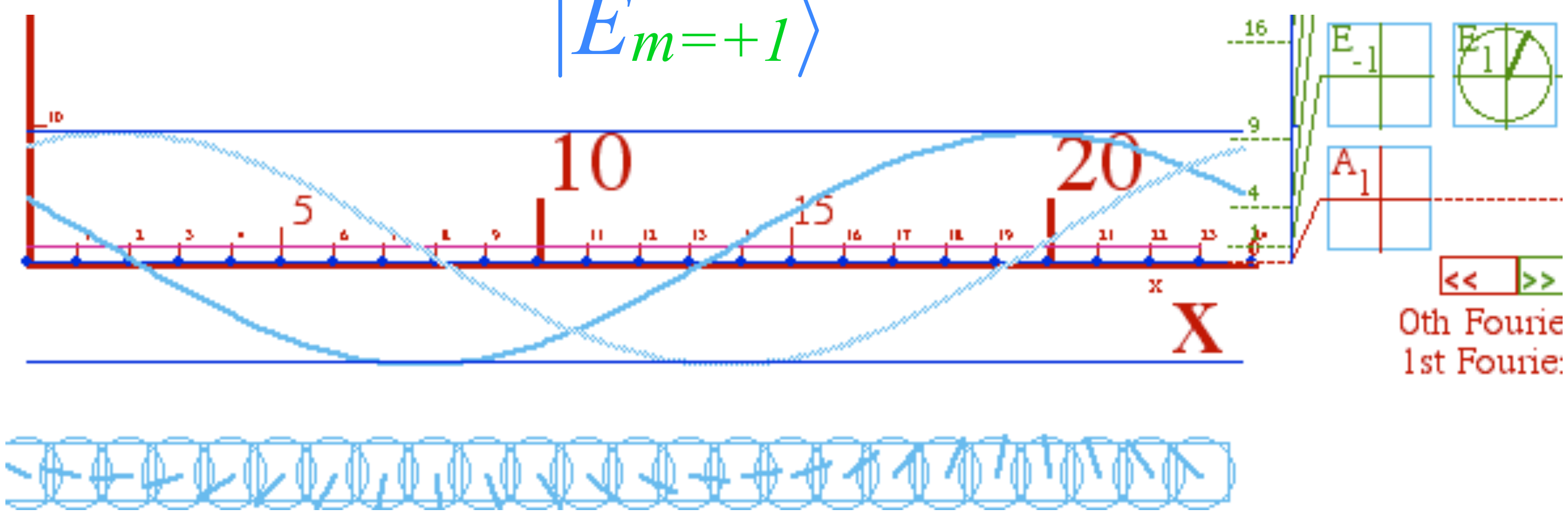
By Harter-SoftElegant Educational Tools Since 2001 and University of Arkansas Physics *Elegant Educational Tools Since 2001*

Consider two lowest E-states by themselves

Now combine (add) them and let time roll!

$$\left(e^{-i\omega_0 t}|E_0\rangle + e^{-i\omega_{+1} t}|E_{+1}\rangle\right)/\sqrt{2}$$

$|E_{m=+1}\rangle$



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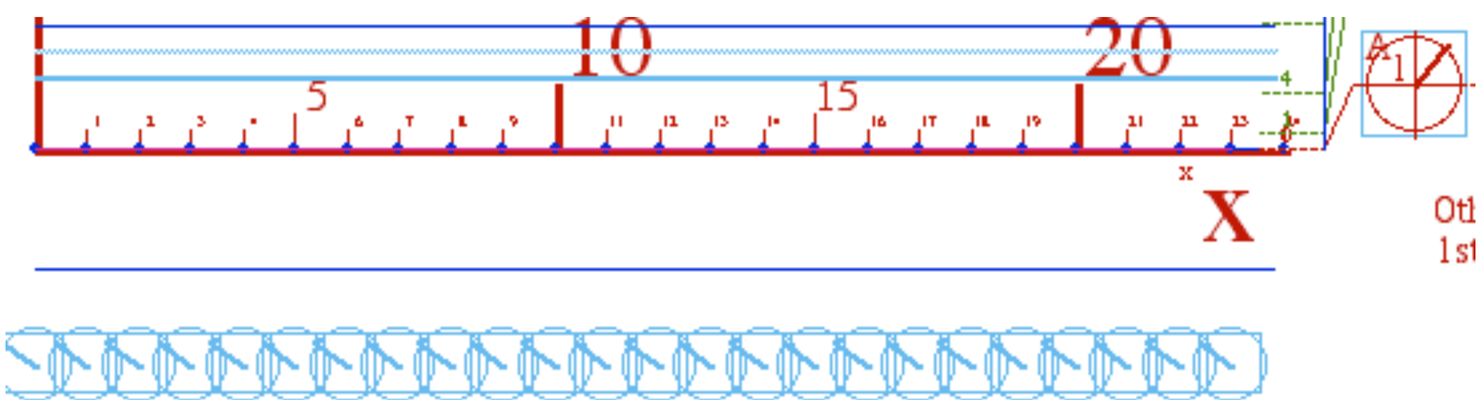

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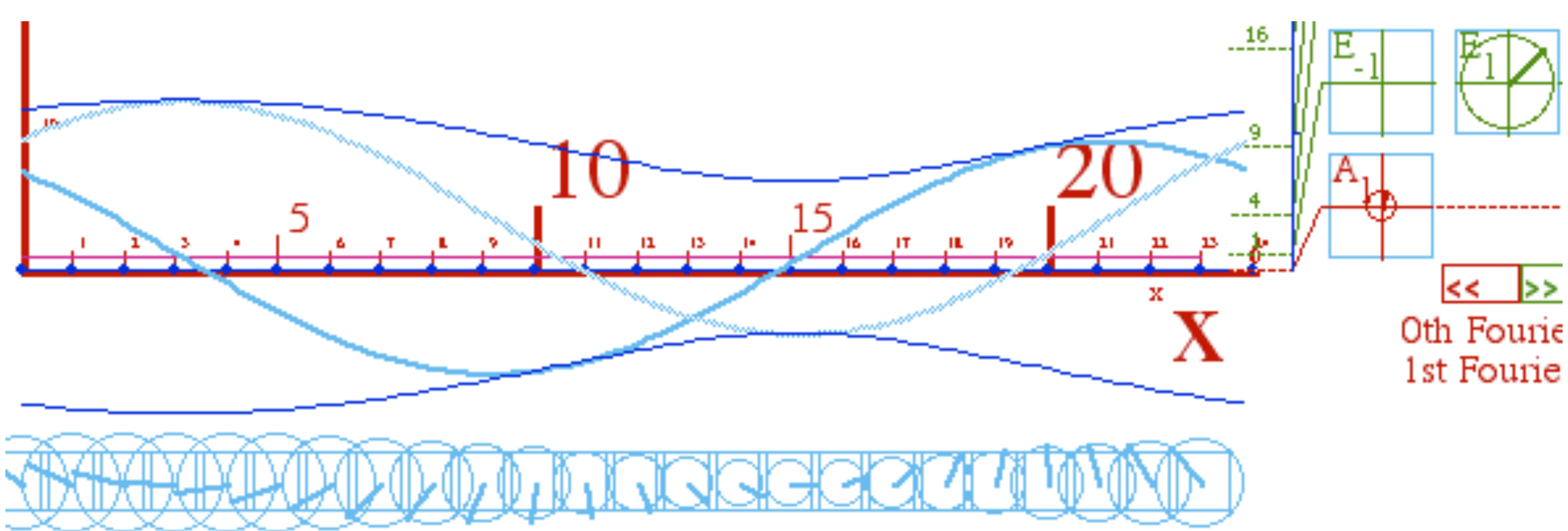

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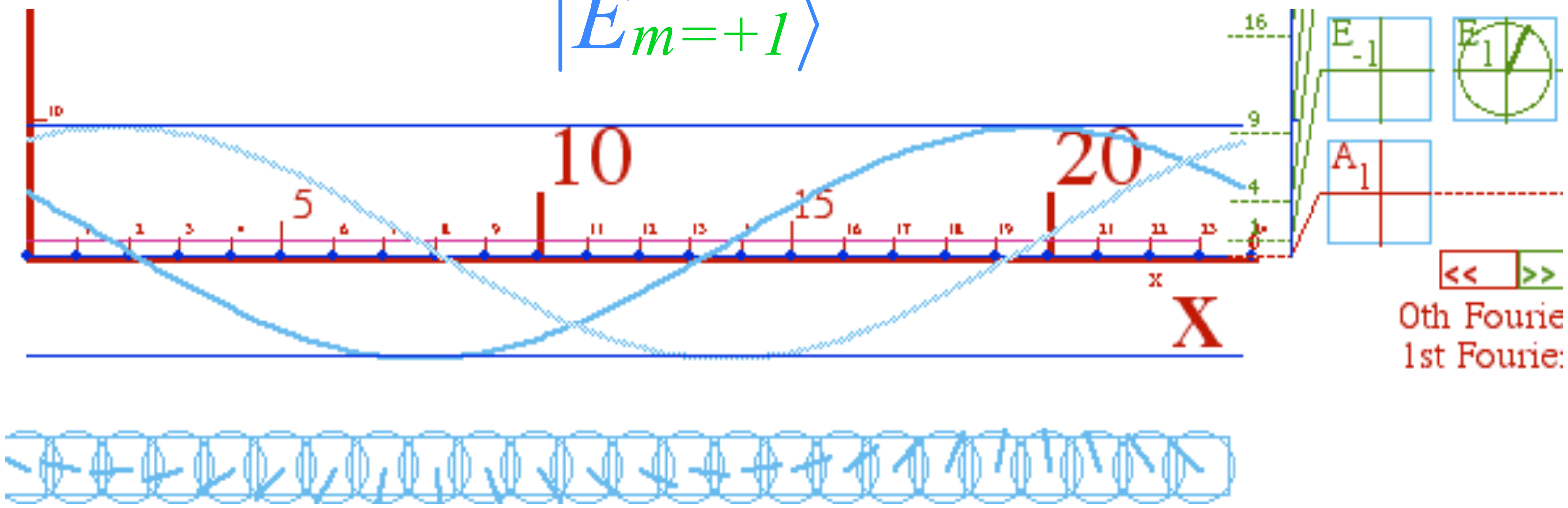

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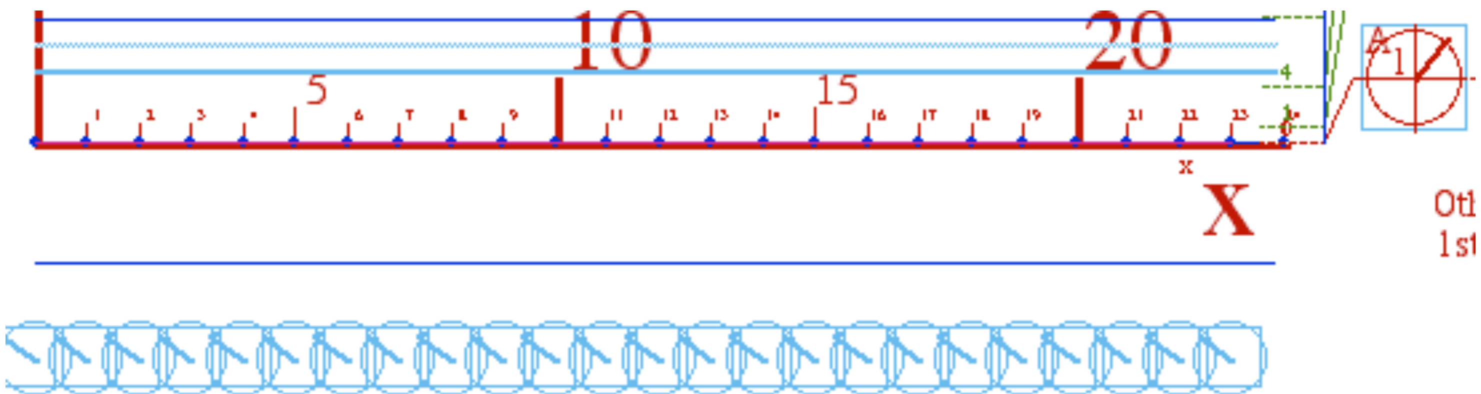

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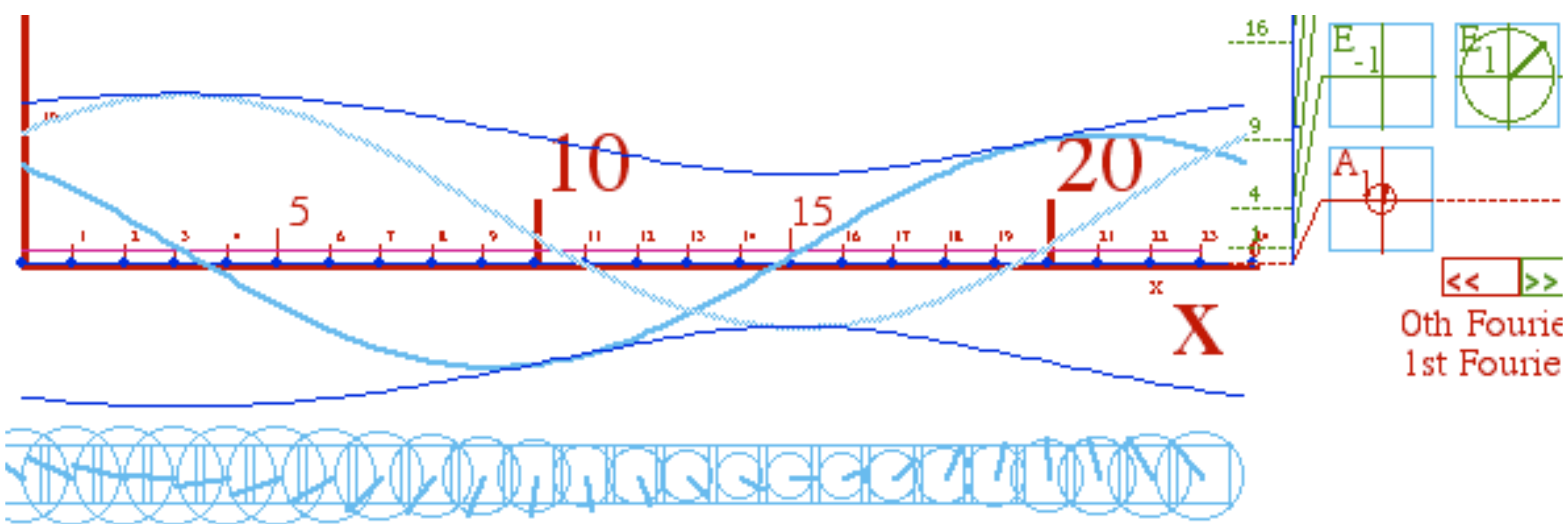
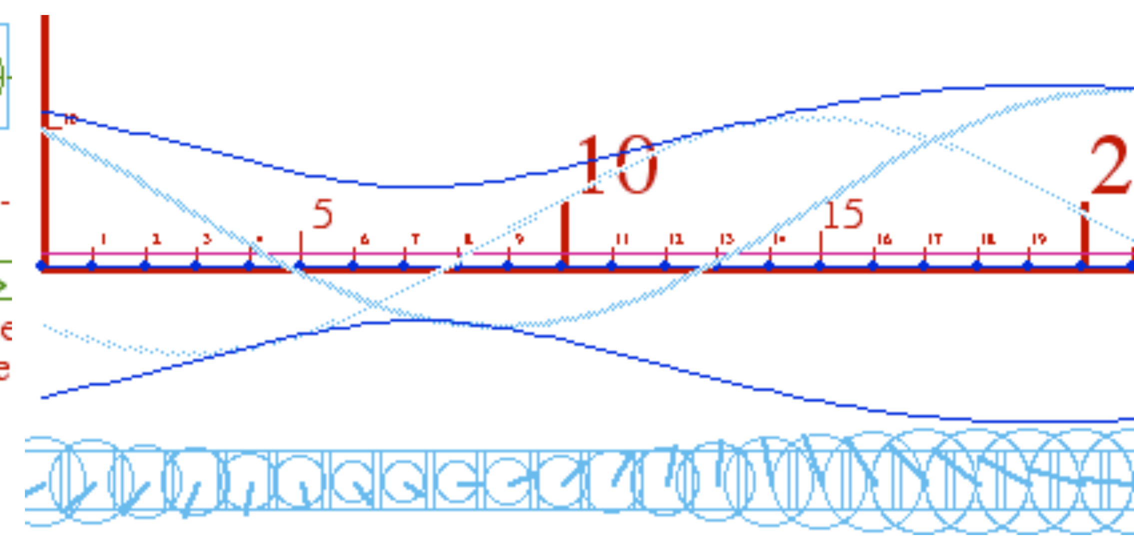
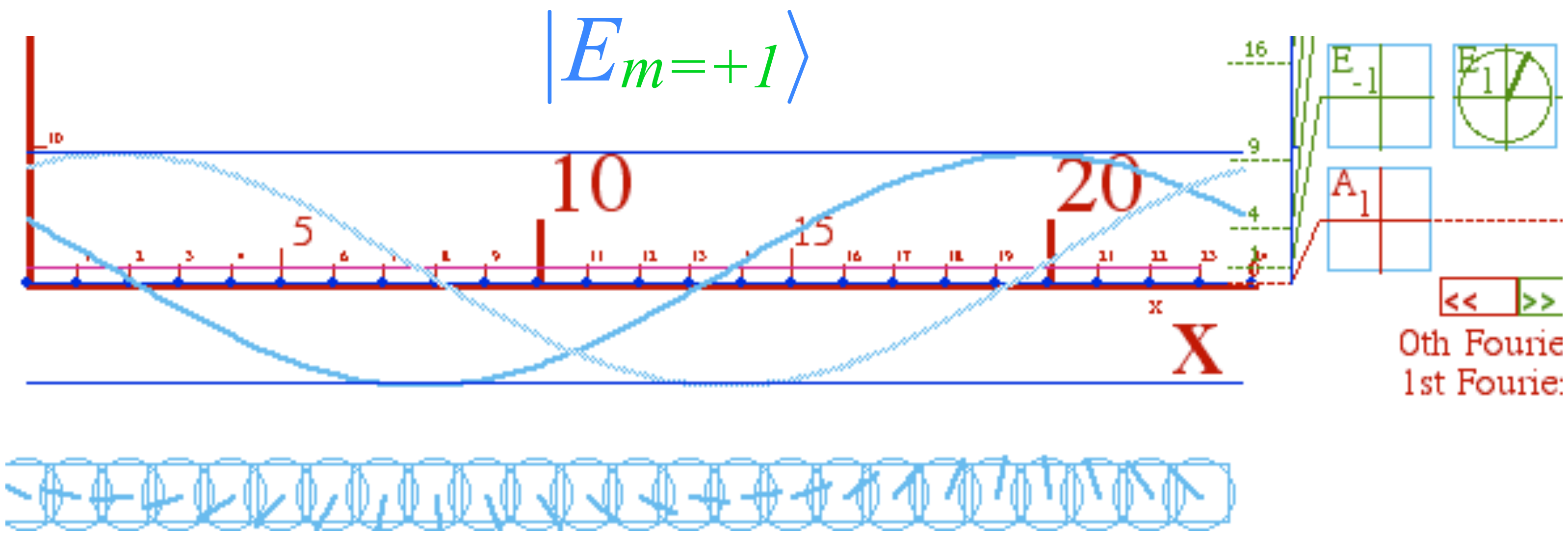
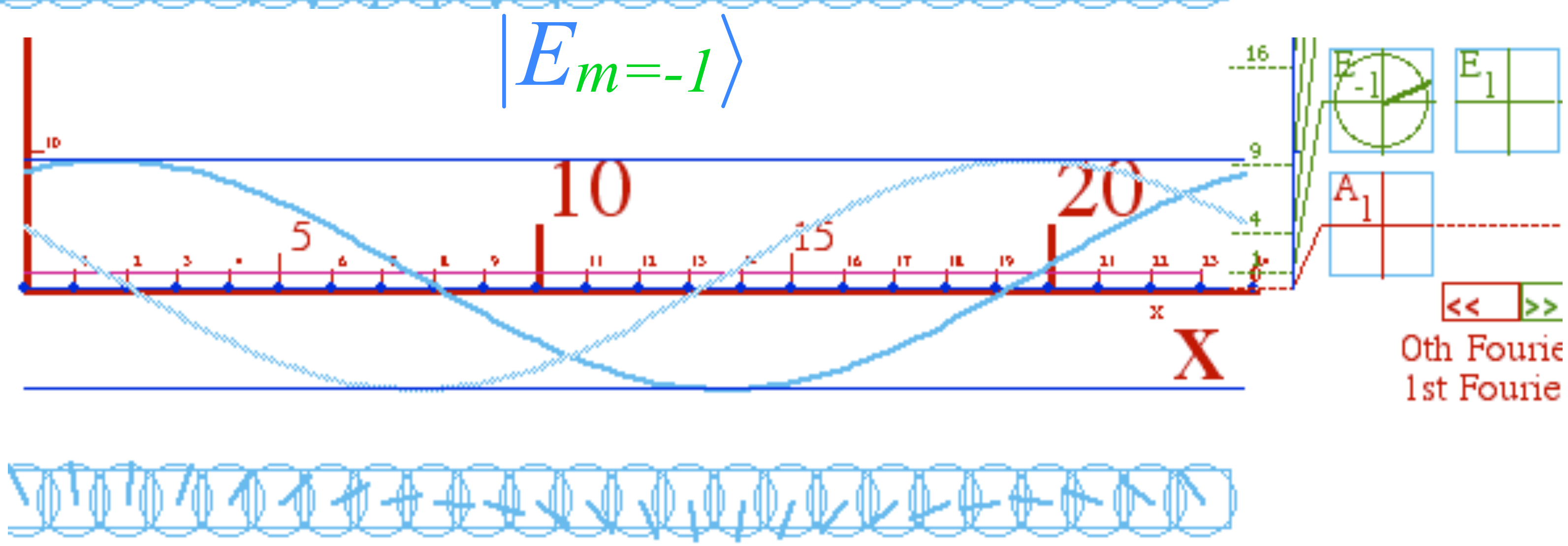
(Just moves forward rigidly)





Consider two degenerate E-states by themselves





Consider two degenerate E-states by themselves

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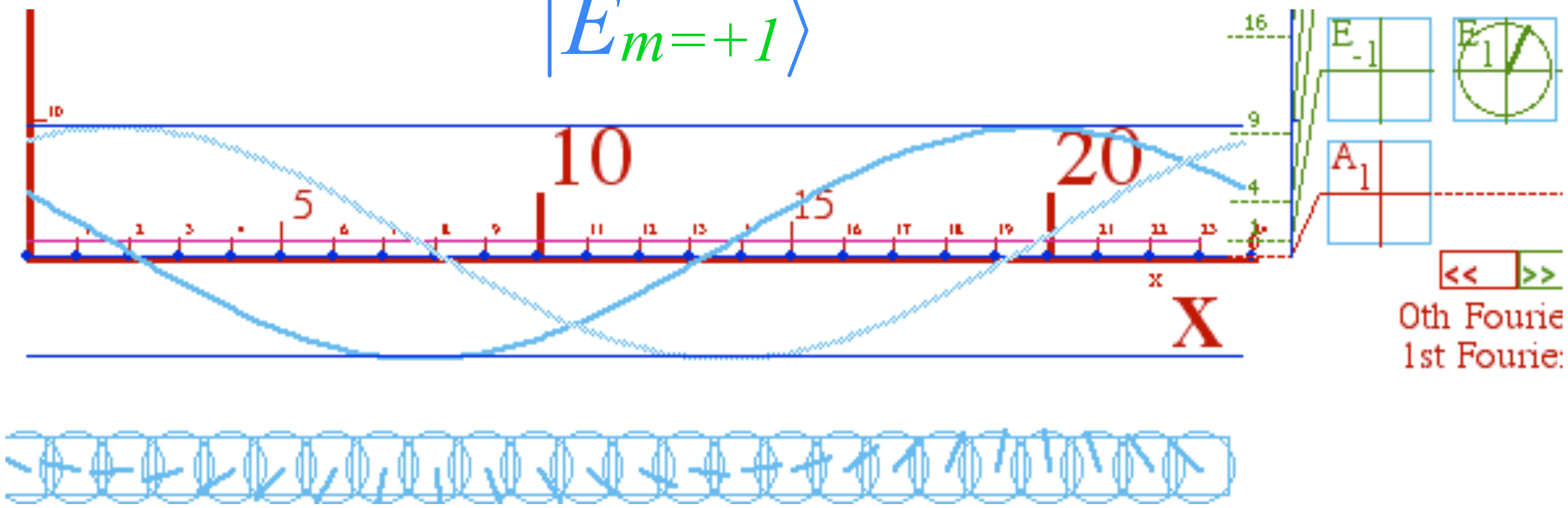
$$\left(e^{-i\omega_{-1}t}\left|E_{-1}\right\rangle + e^{-i\omega_{+1}t}\left|E_{+1}\right\rangle \right) / \sqrt{2}$$

$\left|E_{m=+1}\right\rangle$



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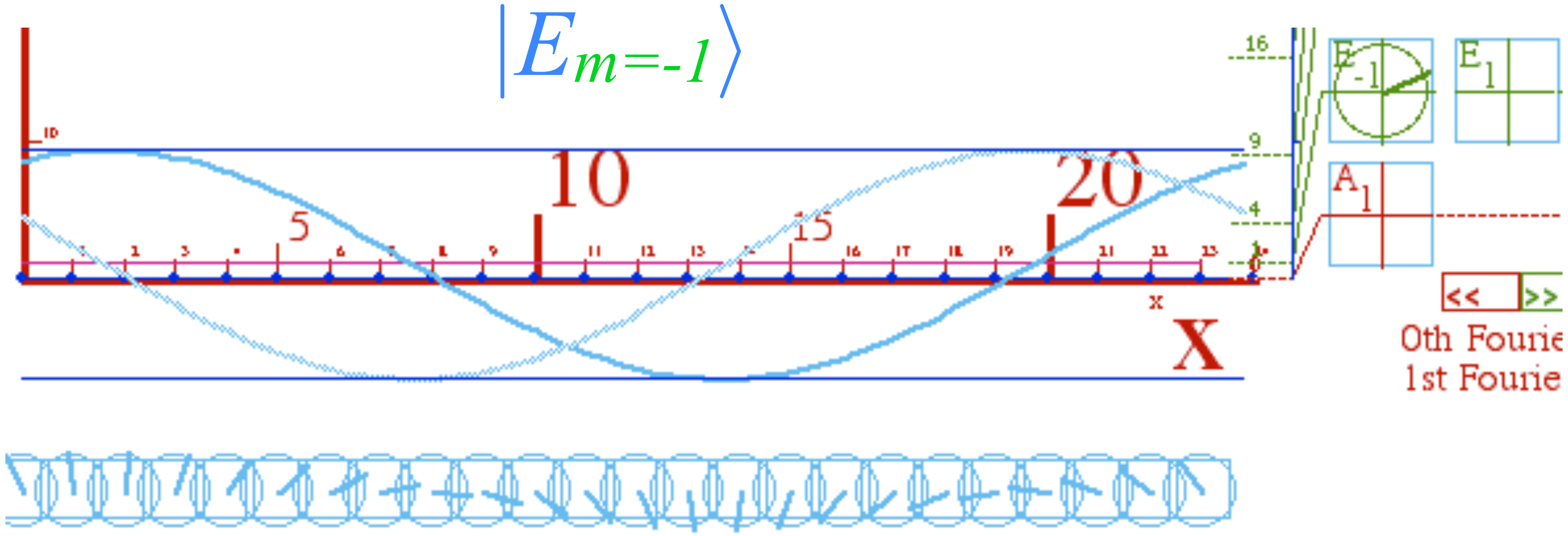
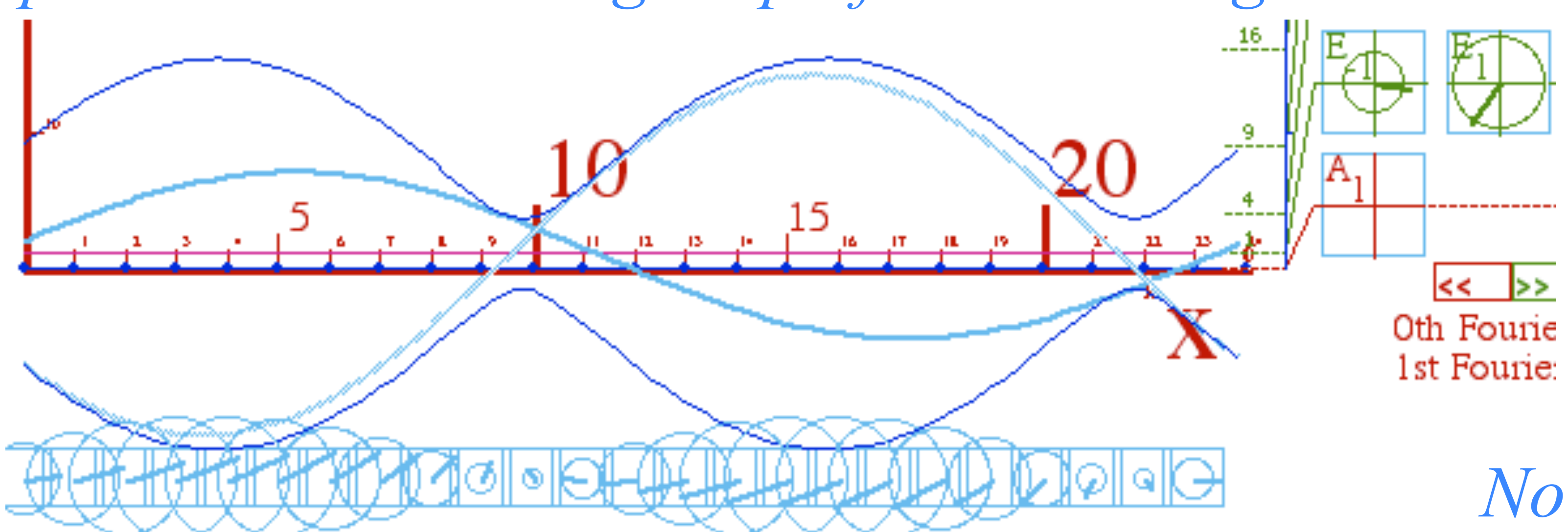


Group wave is stationary if $\omega_{-1}=\omega_{+1}$ but phase can move or "gallop" faster than light!

If $\omega_{-1}<\omega_{+1}$ then $V_{group}<0$

If $\omega_{-1}>\omega_{+1}$ then $V_{group}>0$



Nothing CAN go faster than light

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Galloping waves due to unmatched 2-CW amplitudes

2-CW dynamics has two 1-CW amplitudes $A_{\rightarrow}$ and $A_{\leftarrow}$ that may be *un*matched. $(A_{\rightarrow} \neq A_{\leftarrow})$

$$A_{\rightarrow} e^{i(k_{\rightarrow} x - \omega_{\rightarrow} t)} + A_{\leftarrow} e^{i(k_{\leftarrow} x - \omega_{\leftarrow} t)} = e^{i(k_{\Sigma} x - \omega_{\Sigma} t)} [A_{\rightarrow} e^{i(k_{\Delta} x - \omega_{\Delta} t)} + A_{\leftarrow} e^{-i(k_{\Delta} x - \omega_{\Delta} t)}]$$

Waves have half-sum mean-phase rates $(k_{\Sigma}, \omega_{\Sigma})$ and half-difference group rates $(k_{\Delta}, \omega_{\Delta})$.

$$k_{\Sigma} = (k_{\rightarrow} + k_{\leftarrow})/2 \qquad k_{\Delta} = (k_{\rightarrow} - k_{\leftarrow})/2$$

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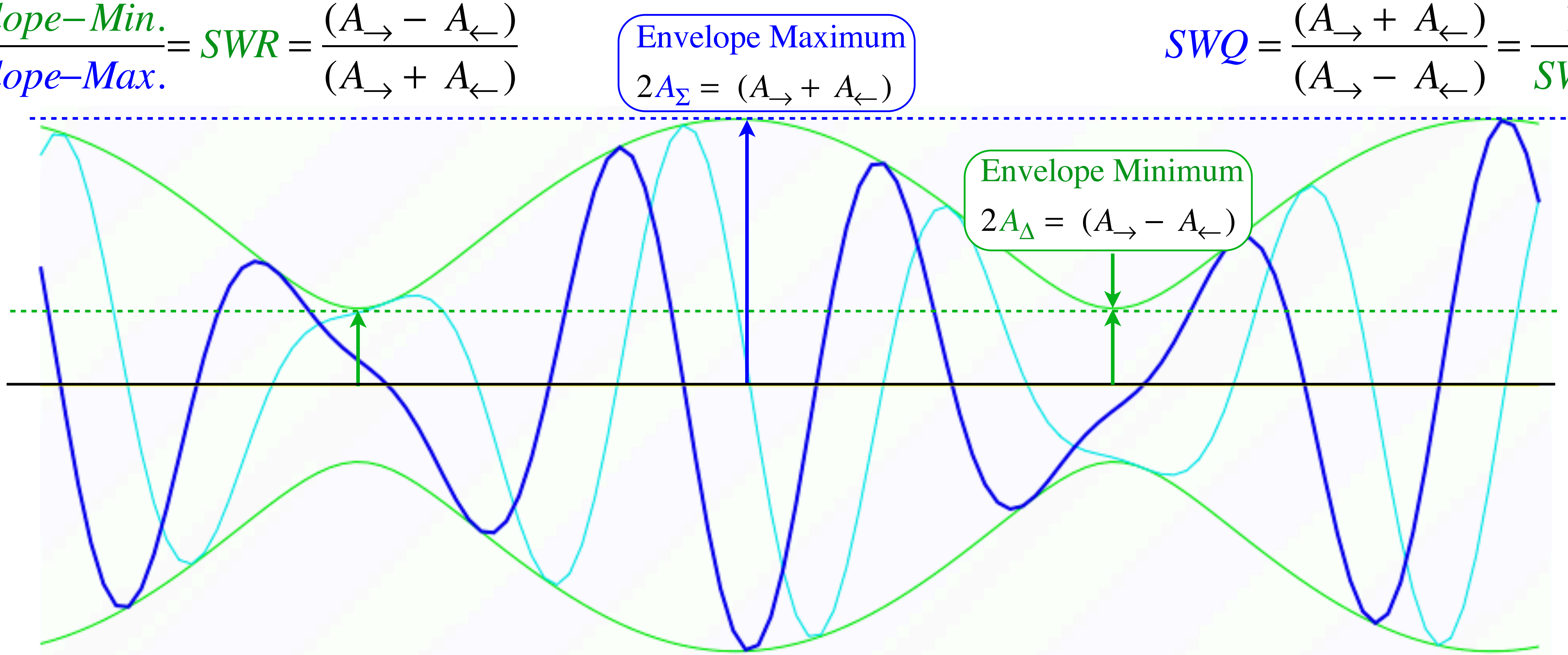

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Waves have half-sum mean-phase rates $(k_\Sigma, \omega_\Sigma)$ and half-difference group rates $(k_\Delta, \omega_\Delta)$.

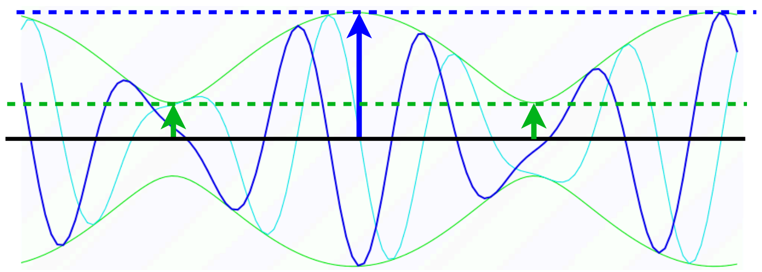
$$k_\Sigma = (k_\rightarrow + k_\leftarrow)/2 \qquad k_\Delta = (k_\rightarrow - k_\leftarrow)/2$$

$$\omega_\Sigma = (\omega_\rightarrow + \omega_\leftarrow)/2 \qquad \omega_\Delta = (\omega_\rightarrow - \omega_\leftarrow)/2$$

Now consider amplitude mean $A_\Sigma = (A_\rightarrow + A_\leftarrow)/2$ and amplitude half-difference $A_\Delta = (A_\rightarrow - A_\leftarrow)/2$.

Detailed wave motion depends on standing-wave-ratio SWR or the inverse standing-wave-quotient SWQ.

$$\frac{Envelope-Min.}{Envelope-Max.} = SWR = \frac{(A_\rightarrow - A_\leftarrow)}{(A_\rightarrow + A_\leftarrow)}$$



$$SWQ = \frac{(A_\rightarrow + A_\leftarrow)}{(A_\rightarrow - A_\leftarrow)} = \frac{1}{SWR}$$

They're analogous to group velocity $V_{group} < c$ frequency ratios and inverse phase velocity $V_{phase} > c$ ratios.

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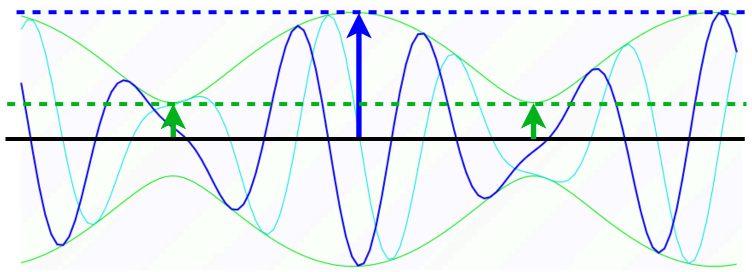

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$$\frac{V_{group}}{c} = \frac{\omega_{\Delta}}{ck_{\Delta}} = \frac{(\omega_{\rightarrow} - \omega_{\leftarrow})}{c(k_{\rightarrow} - k_{\leftarrow})} = \frac{(\omega_{\rightarrow} - \omega_{\leftarrow})}{(\omega_{\rightarrow} + \omega_{\leftarrow})}$$

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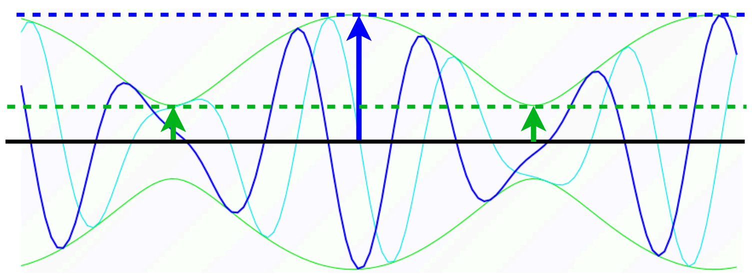

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$\frac{V_{group}}{c} = \frac{c}{V_{phase}}$ is analogous to: $SWR = \frac{1}{SWQ}$

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Effect on group velocity *(None)* and phase velocity *(Absurd)*

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(a) $E_{\leftarrow}=0.2$ $E_{\rightarrow}=0.8$

(b) $E_{\leftarrow}=0.4$ $E_{\rightarrow}=0.6$

1-frequency cases

(c) $E_{\leftarrow}=0.5$ $E_{\rightarrow}=0.5$

$\omega_{\rightarrow}=2c,\ k_{\rightarrow}=2,$

$\omega_{\leftarrow}=2c,\ k_{\leftarrow}=-2$

$u_{GROUP}=0$

$u_{PHASE}=\infty$

(d) $E_{\leftarrow}=0.6$ $E_{\rightarrow}=0.4$

(e) $E_{\leftarrow}=0.8$ $E_{\rightarrow}=0.2$

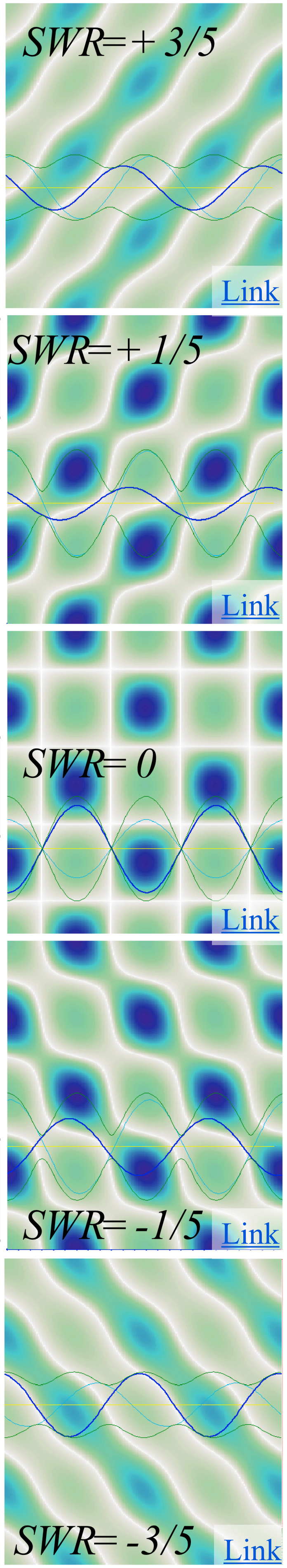
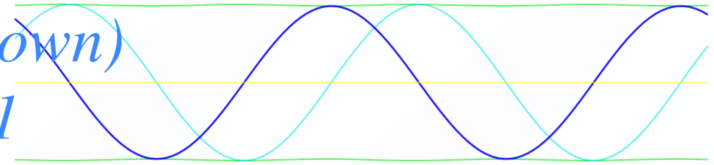


Fig. 6.1 Monochromatic (1-frequency) 2-CW wave space-time patterns.

(not shown)
SWR=1



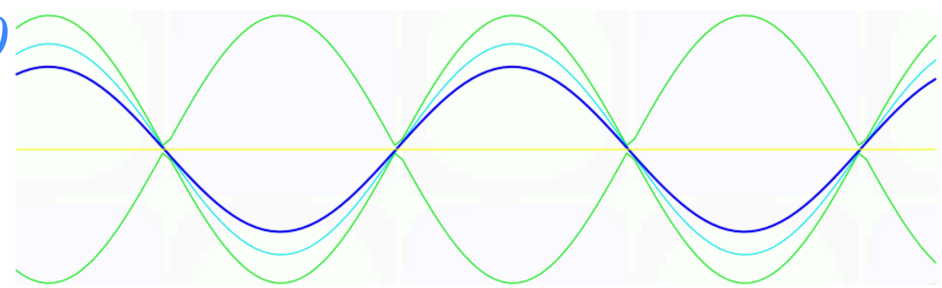
(not shown in (x,ct) plots)

SWR=+3/5

SWR=+1/5

SWR=1
Two extremes for Standing Wave Ratio
SWR=0

SWR=0



SWR=-1/5

SWR=-3/5

BohrIt Web Simulations Links
(embedded in corners)

from:Fig. 4.5.2 *QTforCA* *Unit 2 Ch.4*

from:Fig. 8.6.3 *CMwBang!* *Unit 8 Ch.6*

(a) $E_{\leftarrow}=0.2$ $E_{\rightarrow}=0.8$

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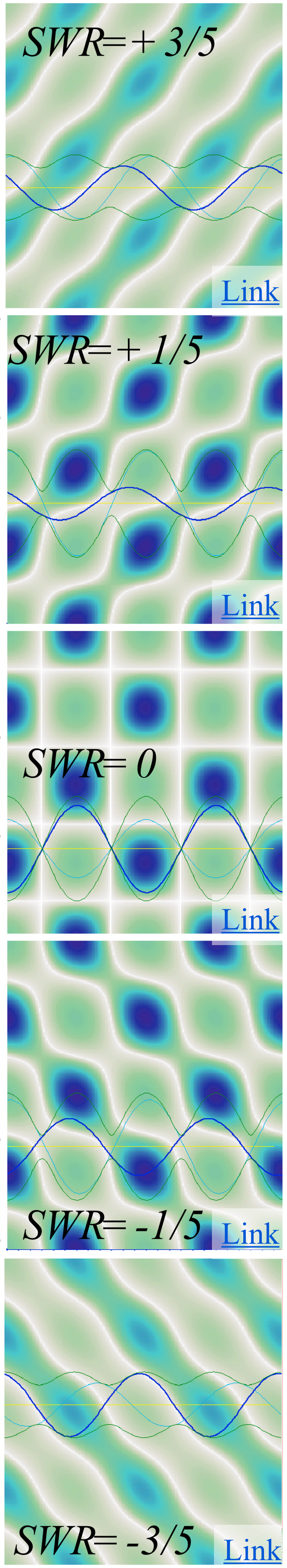


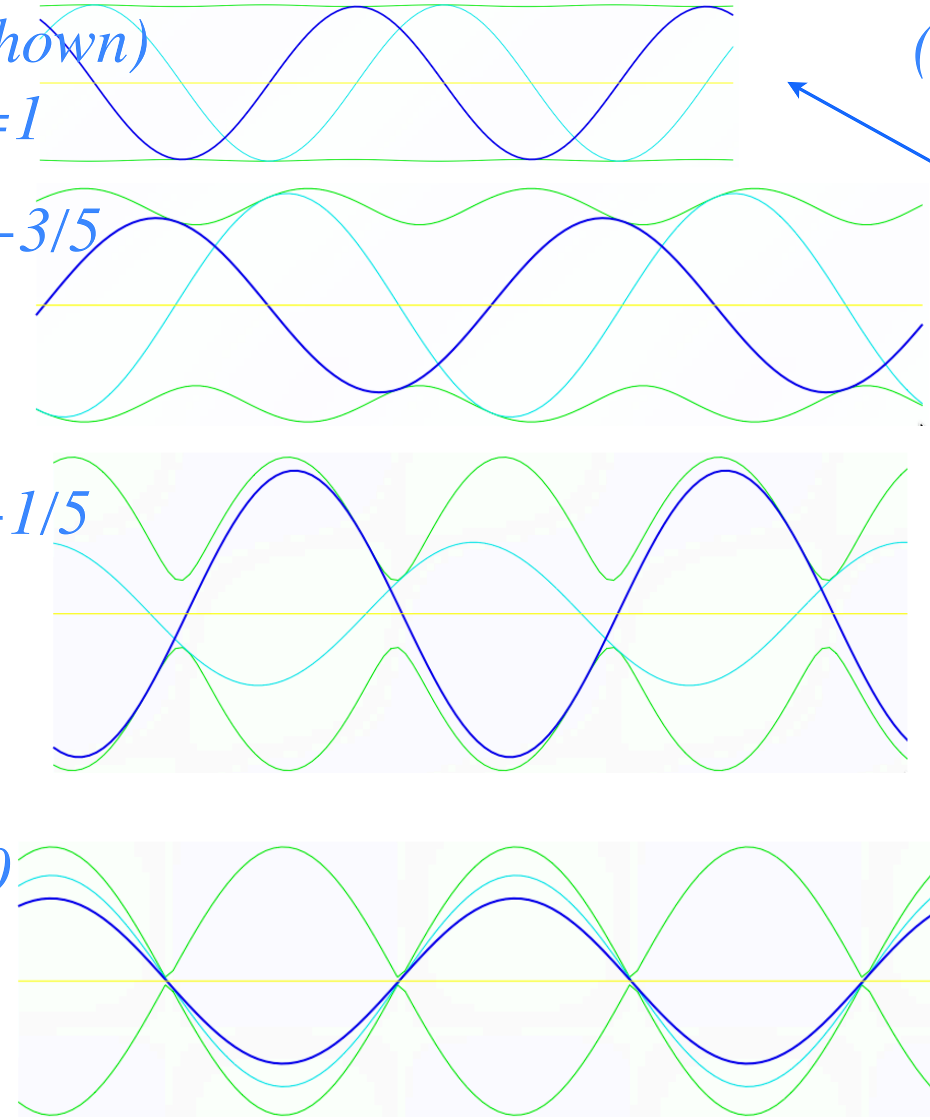
Fig. 6.1 Monochromatic (1-frequency) 2-CW wave space-time patterns.

(not shown) *SWR=1*

SWR=+3/5

SWR=+1/5

SWR=0



SWR=-1/5

SWR=-3/5

(not shown in (x,ct) plots)

SWR=1
Two extremes for Standing Wave Ratio
SWR=0

BohrIt Web Simulations Links
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from:Fig. 4.5.2
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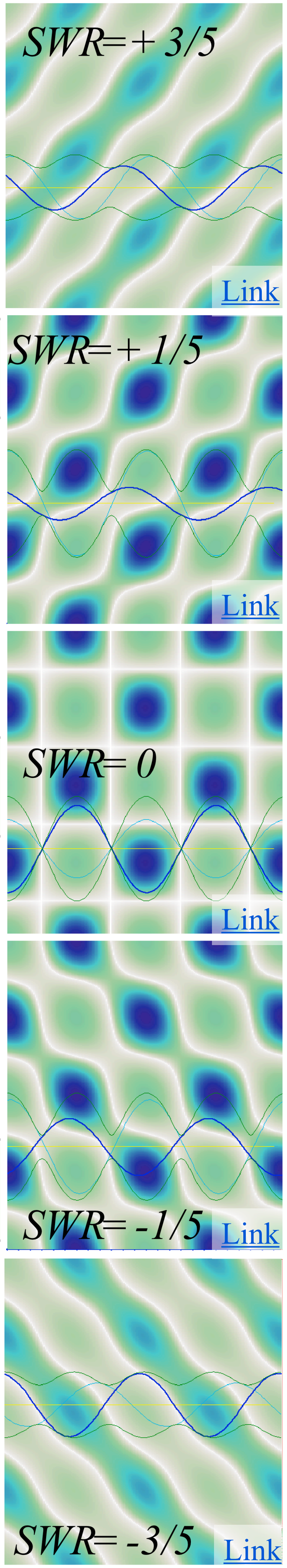

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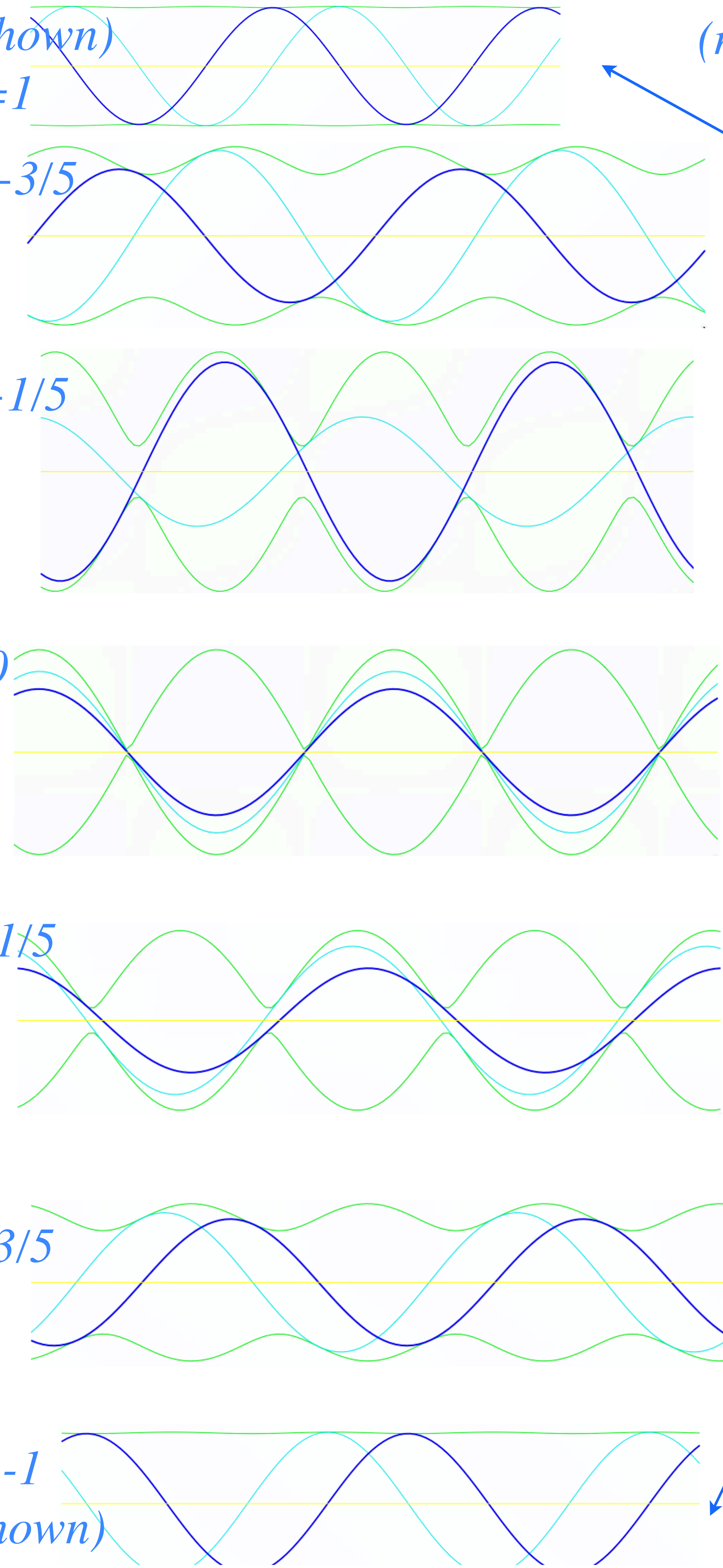
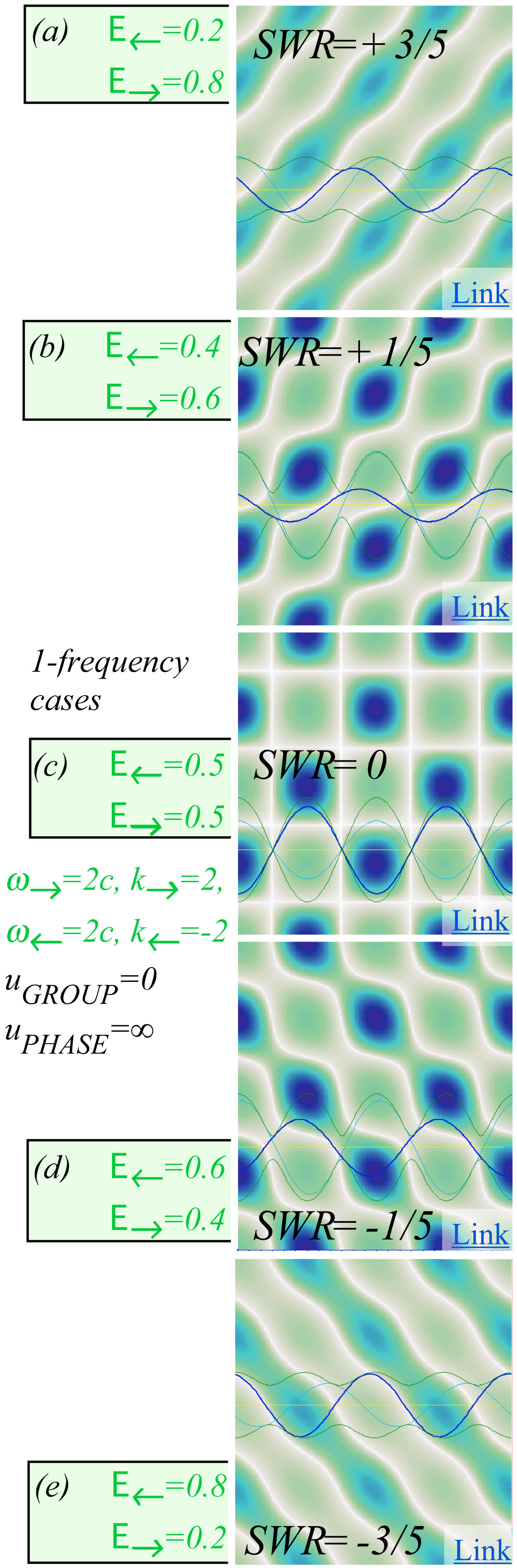


(not shown in (x,ct) plots)

SWR=1
Two extremes for Standing Wave Ratio
SWR=0 ...and
SWR=-1

from:Fig. 4.5.2 QTforCA Unit 2 Ch.4

from:Fig. 8.6.3 CMwBang! Unit 8 Ch.6



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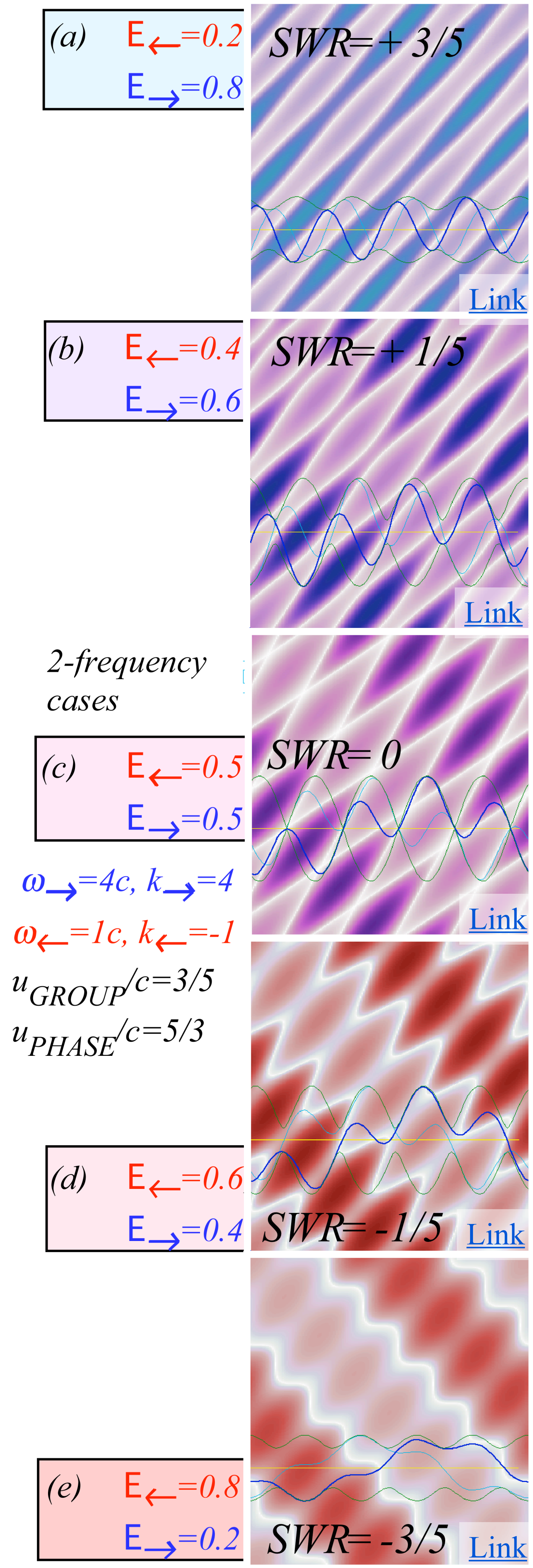


Fig. 6.2 Dichromatic (2-frequency) 2-CW wave space-time patterns.

SWR=1 (not shown)

SWR=+3/5

SWR=+1/5

SWR=0

SWR=-1/5

SWR=-3/5

SWR=-1 (not shown)

Same SWR cases viewed at u/c = 3/5

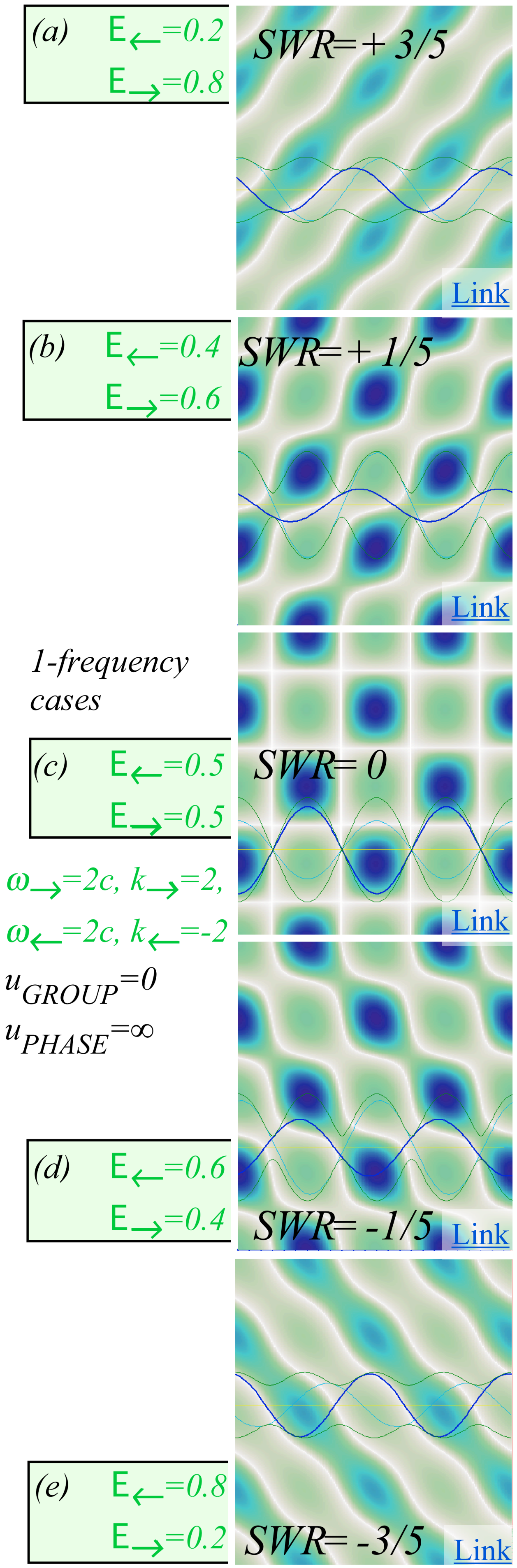
SWR=1
Two extremes for Standing Wave Ratio
SWR=0 ...and
SWR=-1

BohrIt Web Simulations Links
(embedded in corners)

from:Fig. 4.5.2 QTforCA Unit 2 Ch.4

from:Fig. 8.6.3 CMwBang! Unit 8 Ch.6

www.uark.edu/ua/pirelli/html/amplitude_probability_2.html



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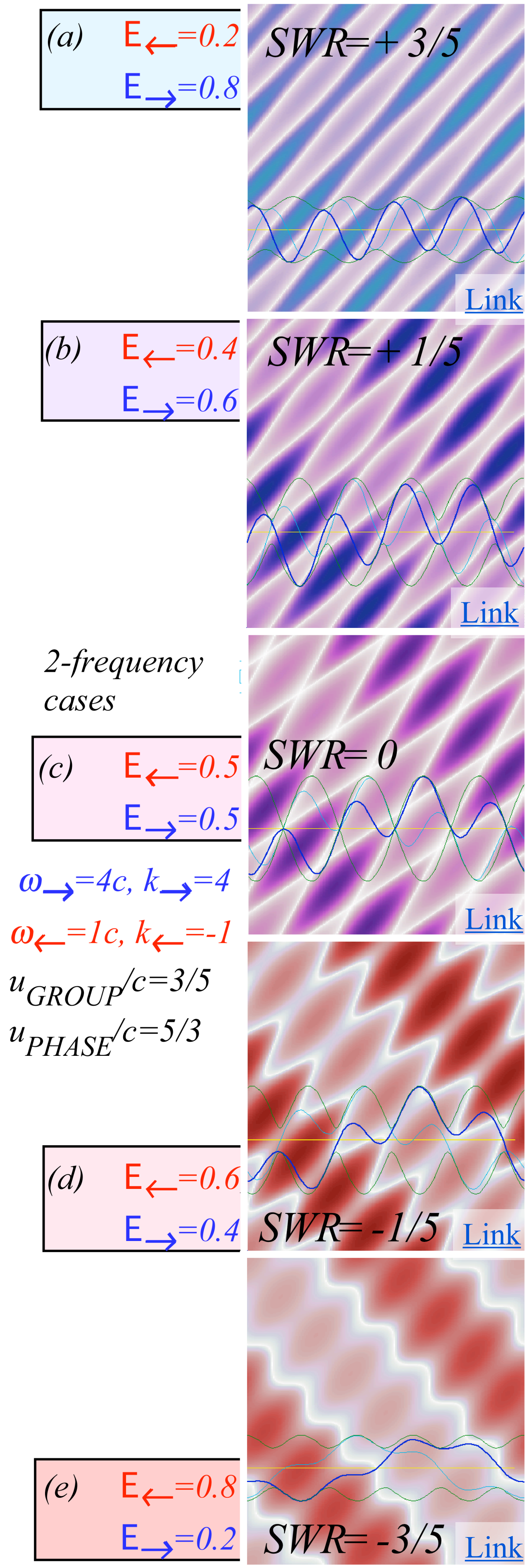


Fig. 6.2 Dichromatic (2-frequency) 2-CW wave space-time patterns.

polarization analogy

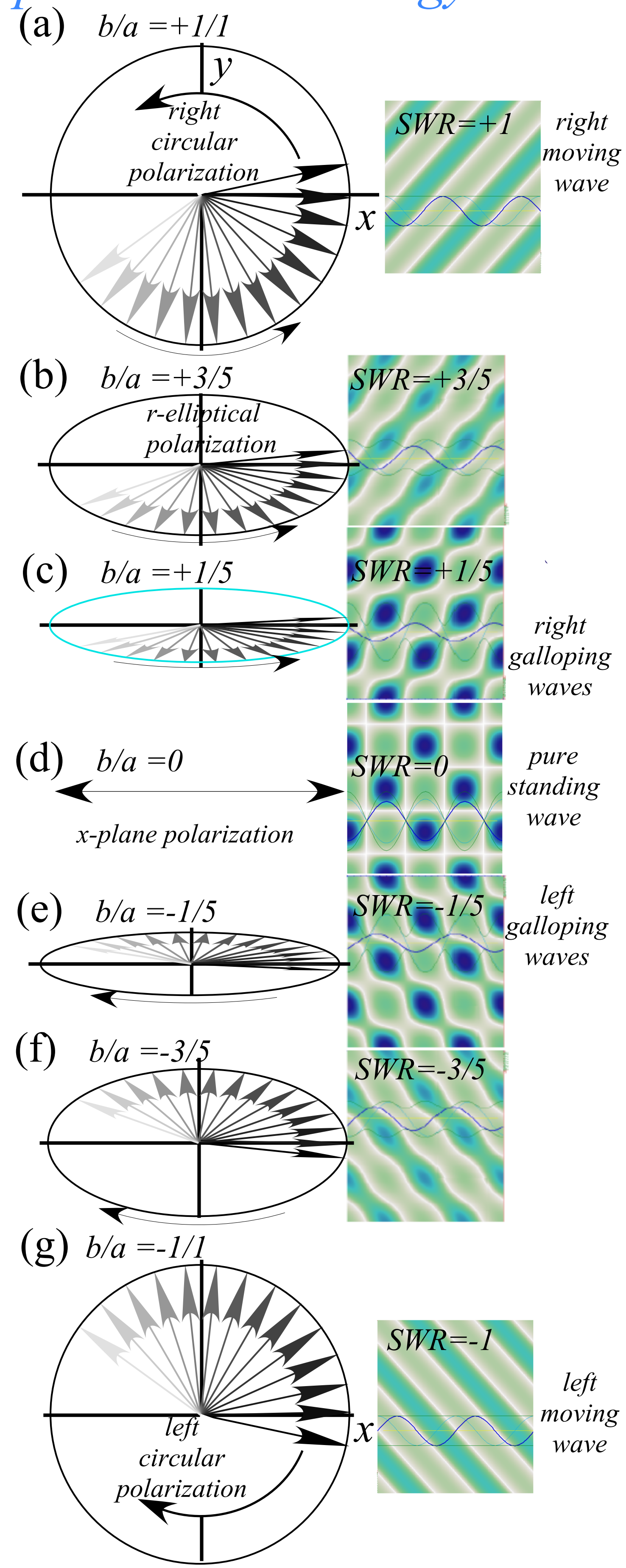
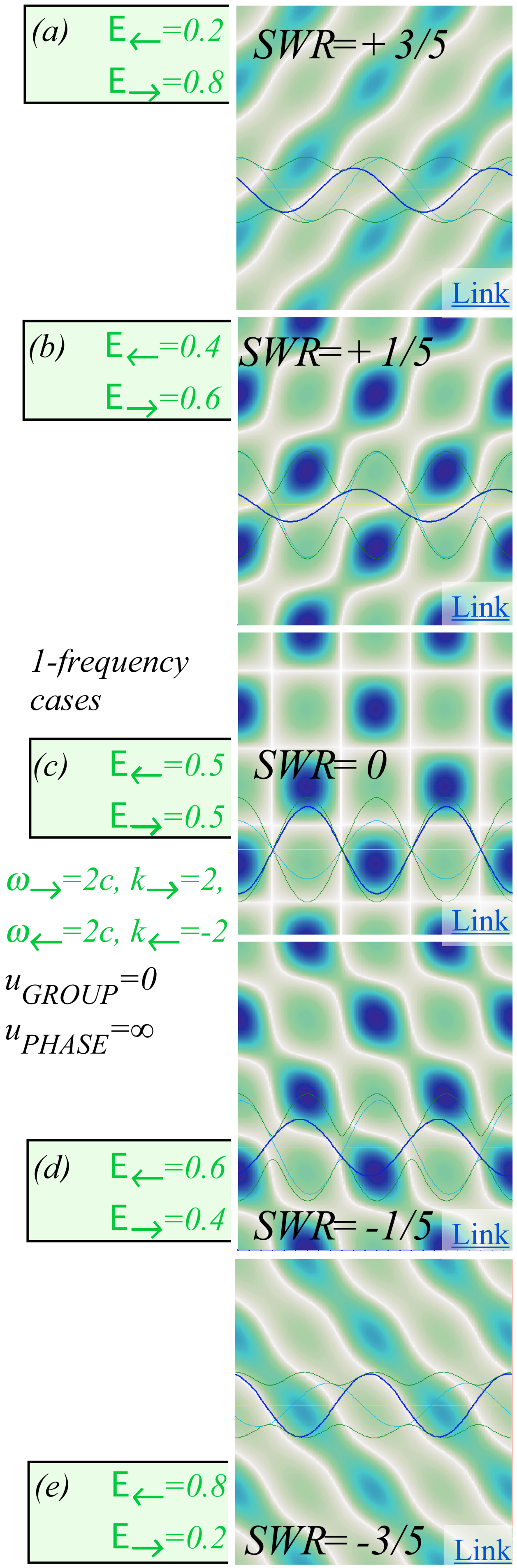


Fig. 6.3 (a-g) Elliptic polarization ellipses relate to galloping waves in Fig. 6.1. (h-i) Kepler anomalies.



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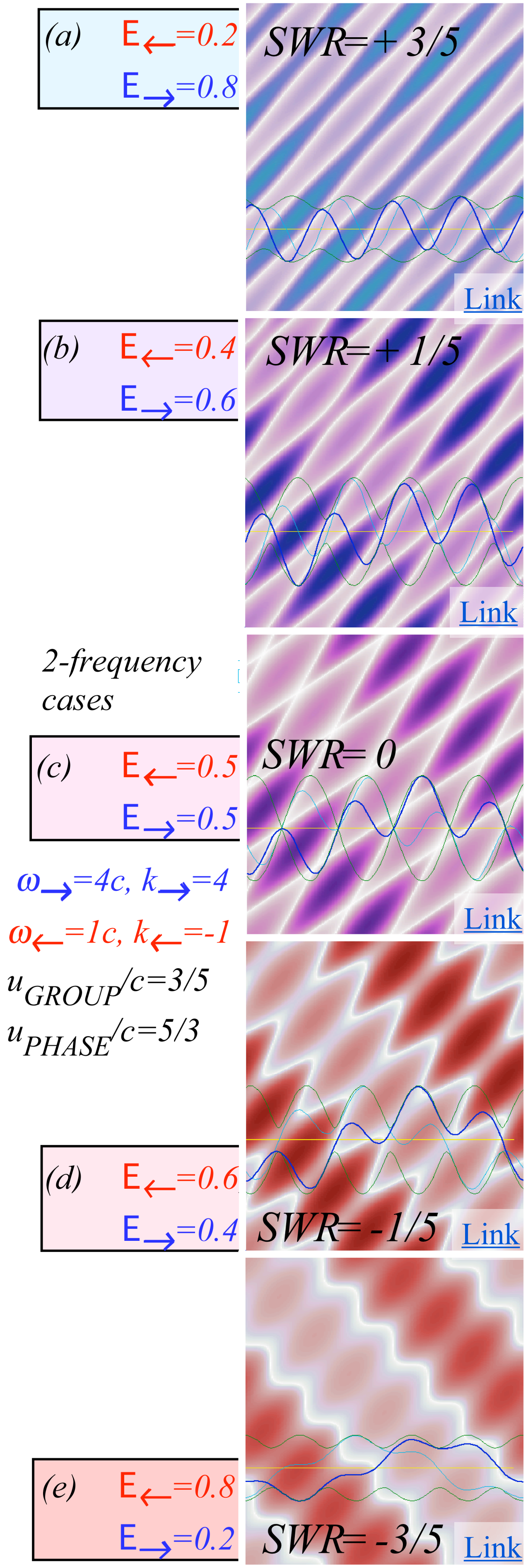

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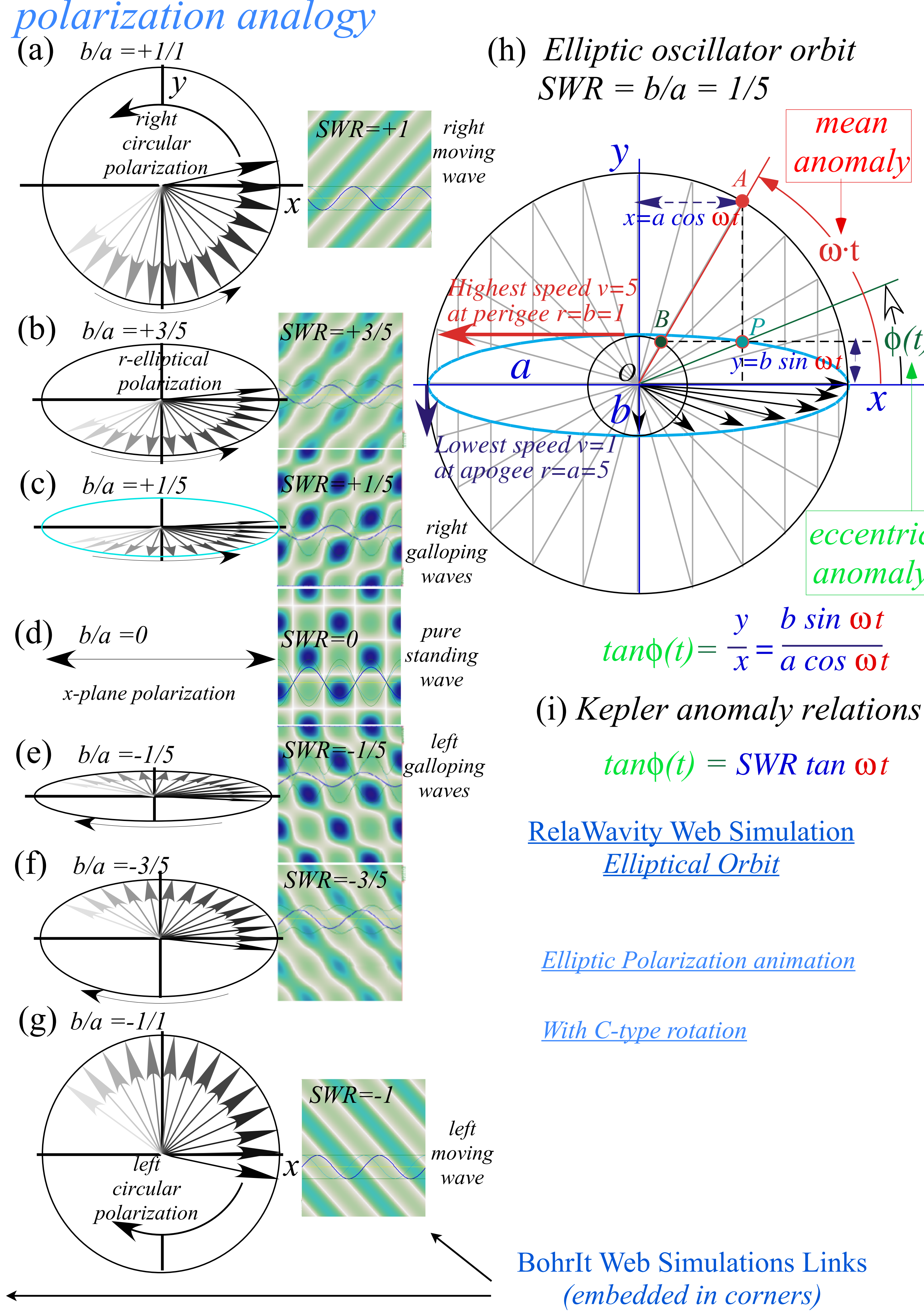

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http://www.uark.edu/ua/modphys/markup/BoxItWeb.html?AU2=1.0&BU2=0.0&CU2=0.0&DU2=1.0&xInitial=1.0&yInitial=0.0&pxInitial=0.0&pyInitial=0.4&wantBoxLines=1

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Bohr-Schrodinger (BS) approximation throws out Mc^2 *(Is frequency really relative?)*

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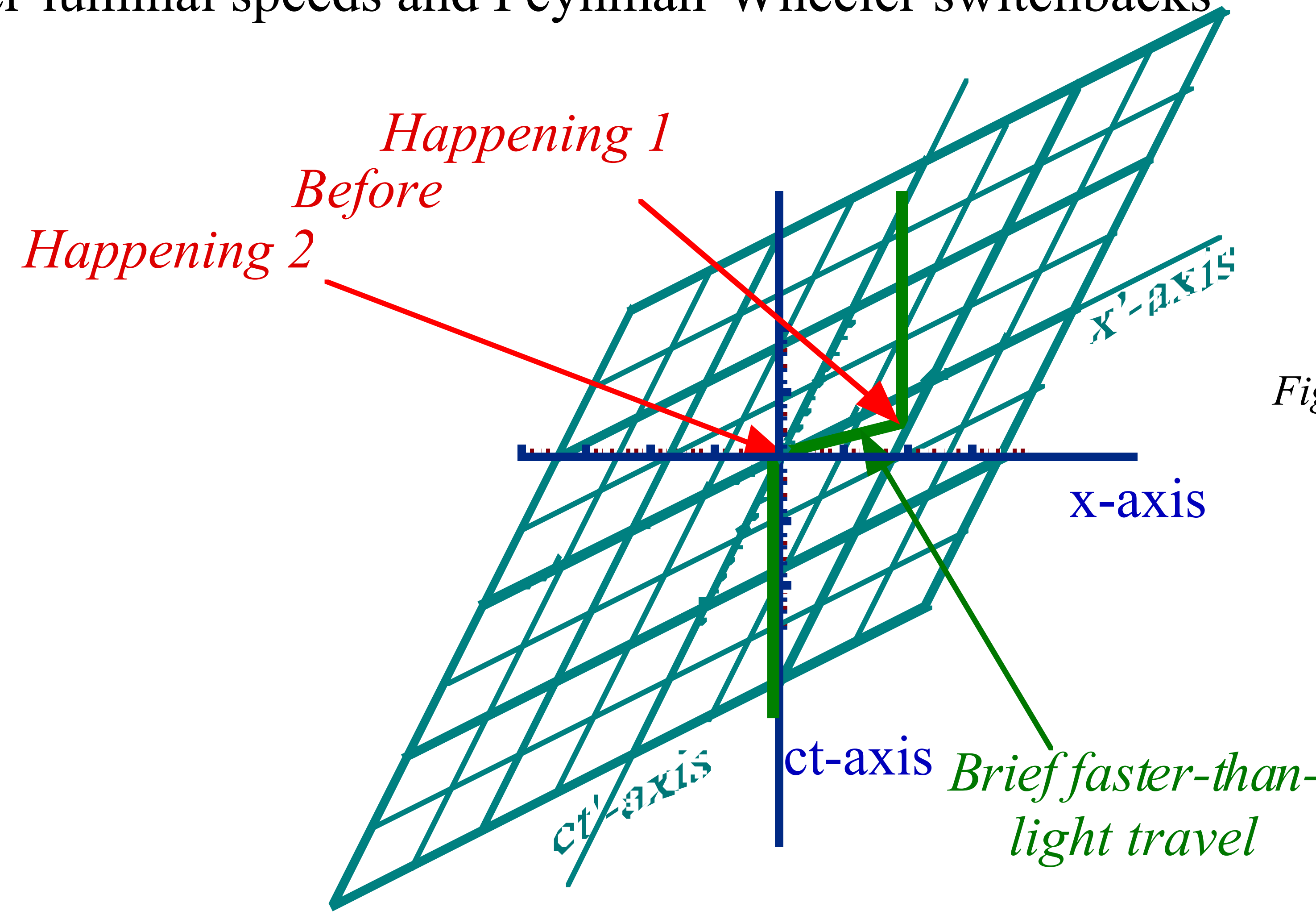
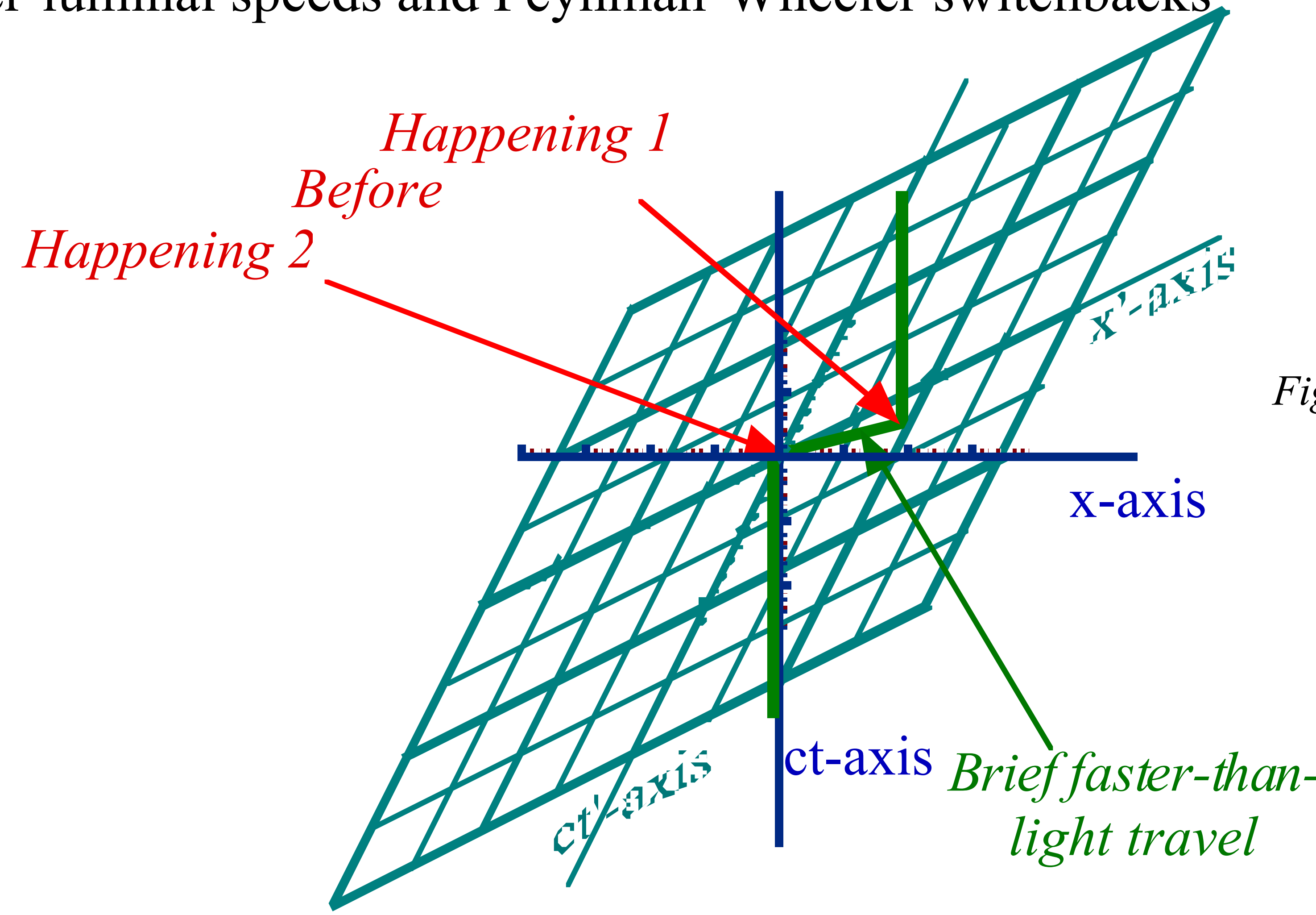


Fig. 2.B.10 Lighthouse plot of two Happenings

Super-luminal speeds and Feynman-Wheeler switchbacks



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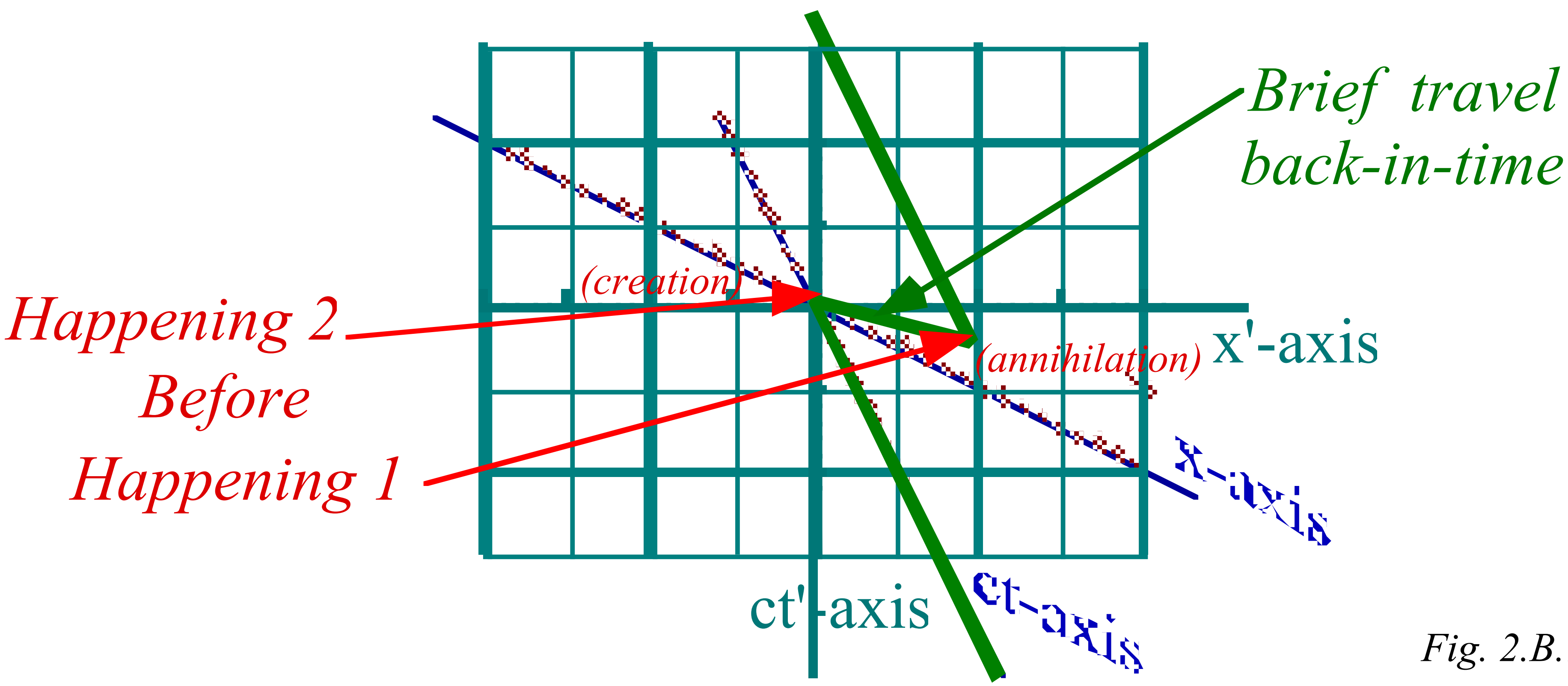


Fig. 2.B.11 Ship plot of two Happenings

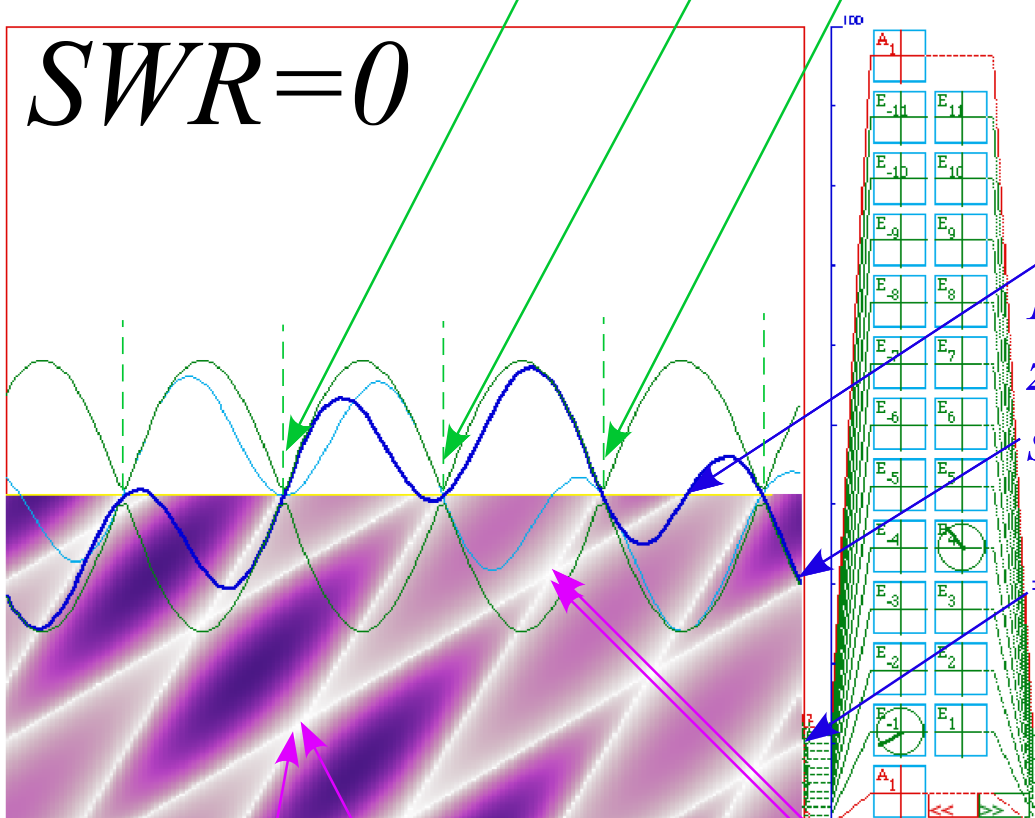
Minkowski Zero-Grids are Spacetime *Switchbacks* for $-u_{GROUP}<SWR<0$

$\omega_{\rightarrow}=4c$	$\omega_{\leftarrow}=1c$
$k_{\rightarrow}=4,$	$k_{\leftarrow}=-1$
$u_{GROUP}=c3/5$	$u_{PHASE}=c5/3$

Group-zero speed $u_{GROUP}=c3/5$

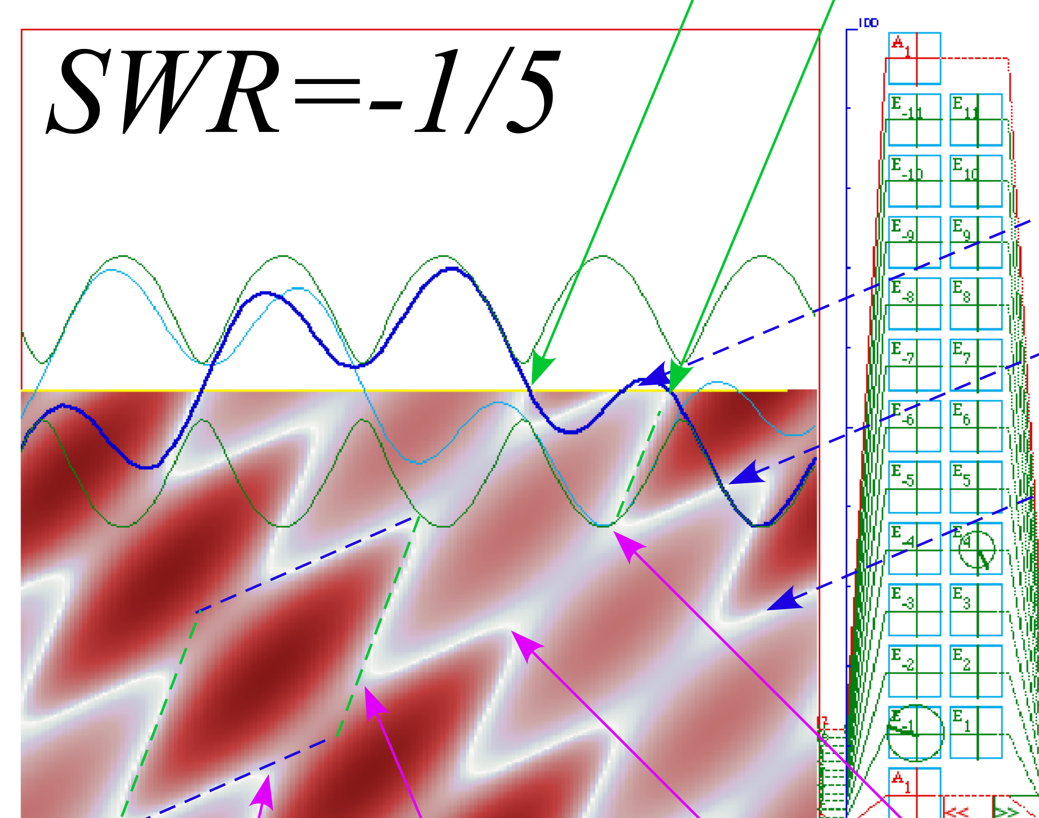
Group zero speed limit
$$\frac{u_{GROUP}+SWR}{1+u_{GROUP}\cdot\frac{SWR}{c^2}}=5c/11$$

$$\frac{\frac{3}{5}+\frac{-1}{5}}{1+\frac{3}{5}\frac{-1}{5}}=\frac{\frac{2}{5}}{\frac{22}{25}}=\frac{5}{11}$$



Phase zero speed $u_{PHASE}=c5/3$

$E_{\leftarrow}=0.5$, $E_{\rightarrow}=0.5$



Phase "anti-zero" going "back-in-time"

Phase zero speed limit
$$\frac{u_{PHASE}+SWR}{1+u_{PHASE}\cdot\frac{SWR}{c^2}}=11c/5$$

$$\frac{\frac{5}{3}+\frac{-1}{5}}{1+\frac{5}{3}\frac{-1}{5}}=\frac{\frac{22}{15}}{\frac{10}{25}}=\frac{11}{5}$$

$E_{\leftarrow}=0.6$, $E_{\rightarrow}=0.4$

Wave zero-anti-zero annihilation *and* creation occur *together* at *the same* spacetime point for $SWR=0$

Wave zero-anti-zero annihilation and creation occur *separately* at *different* spacetime points for $-u_{GROUP}<SWR<0$

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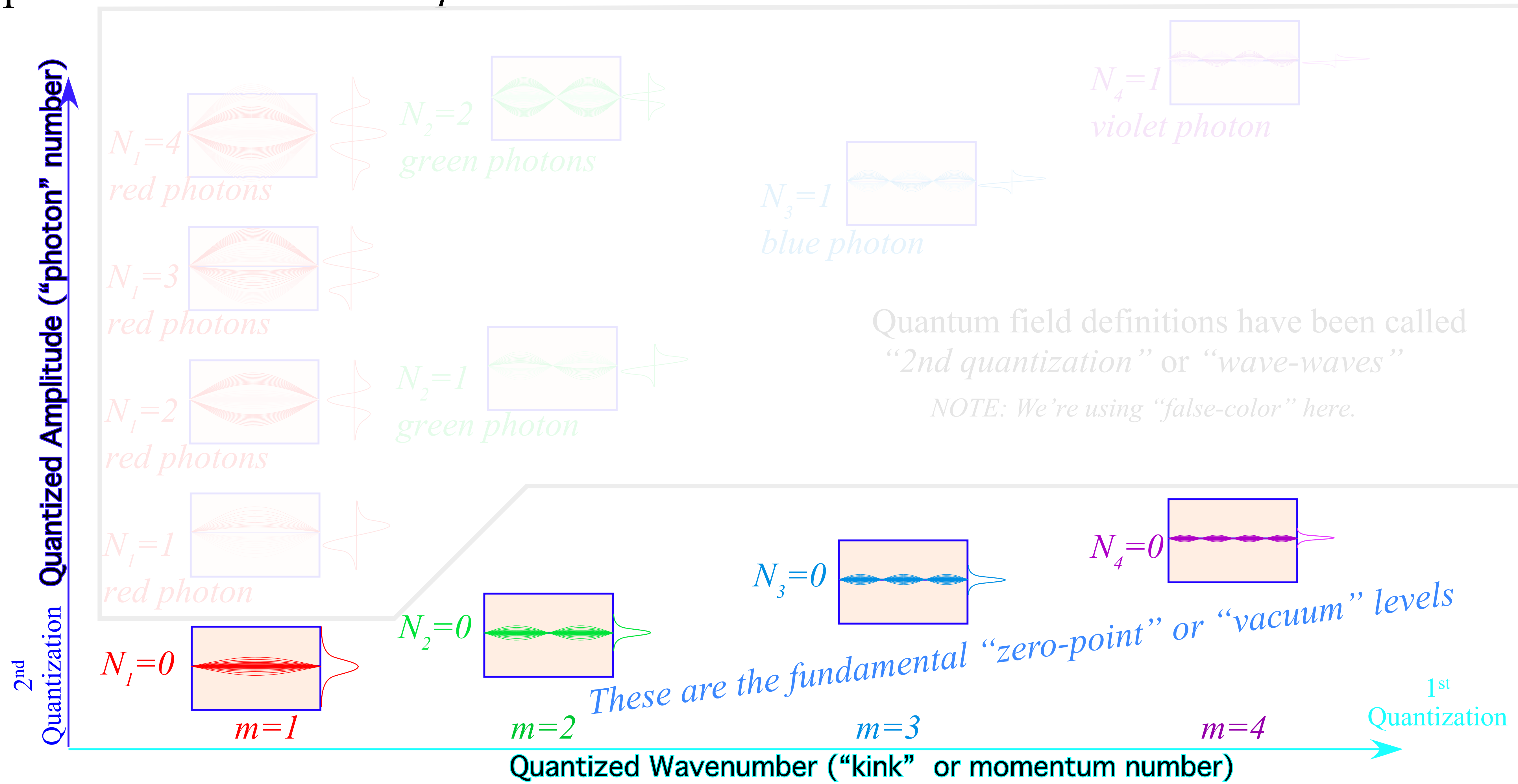
Quantized *Amplitude* *Counting "photon" number* *(2nd-Quantization)*

Planck's relation $E=Nh\upsilon$ began as a tenative axiom to explain low-T light. Then he tried to disavow it! Einstein picked it up in his 1905 paper. Since then its use has grown enormously and continues to amaze, amuse (or bewilder) all who study it.

A current view is that it represents the *quantization* of optical field *amplitude*. We picture this below as *N-photon* wave states for each box-mode of *m* wave kinks.



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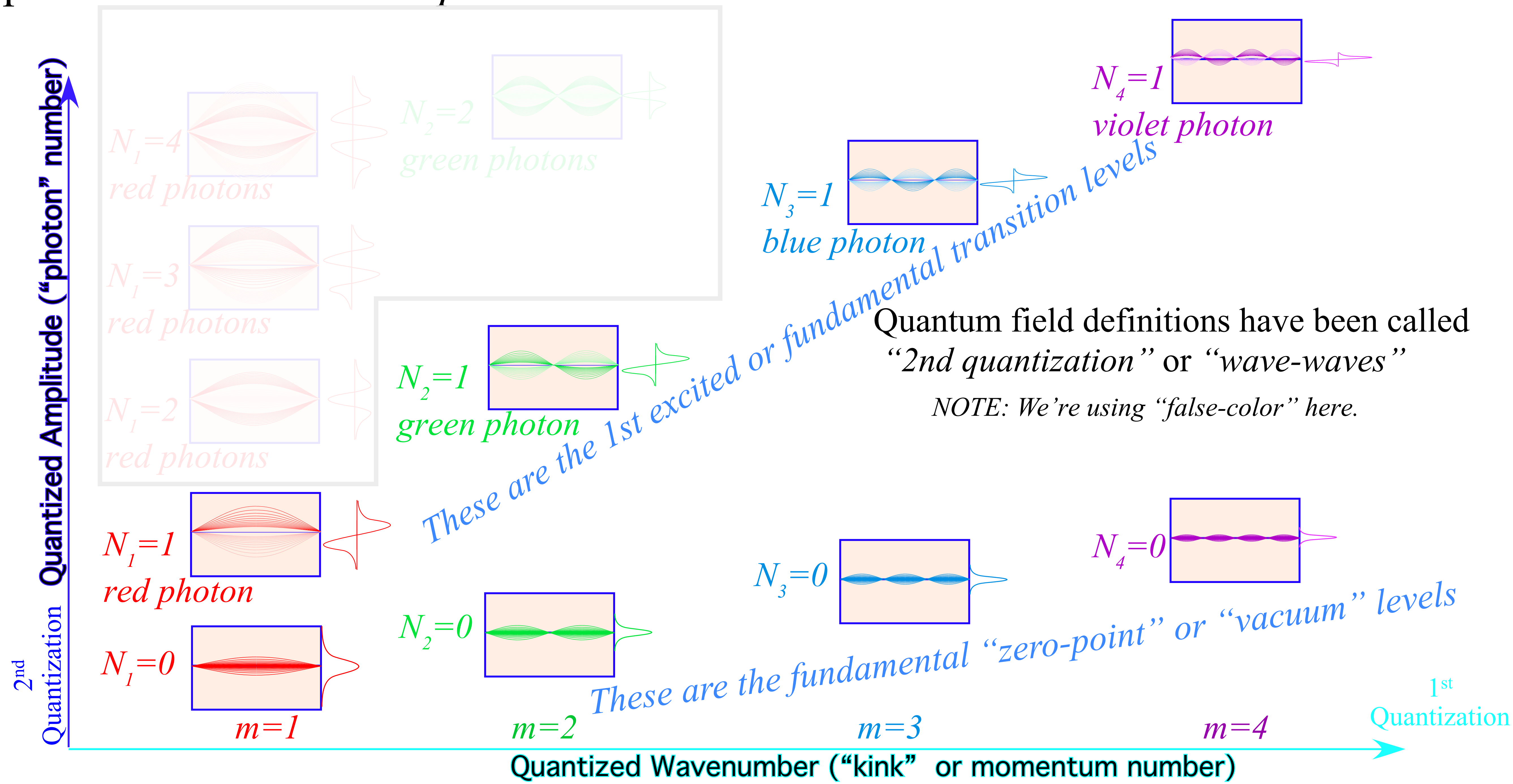

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Planck's relation $E=Nh\upsilon$ began as a tenative axiom to explain low-T light. Then he tried to disavow it! Einstein picked it up in his 1905 paper. Since then its use has grown enormously and continues to amaze, amuse (or bewilder) all who study it.

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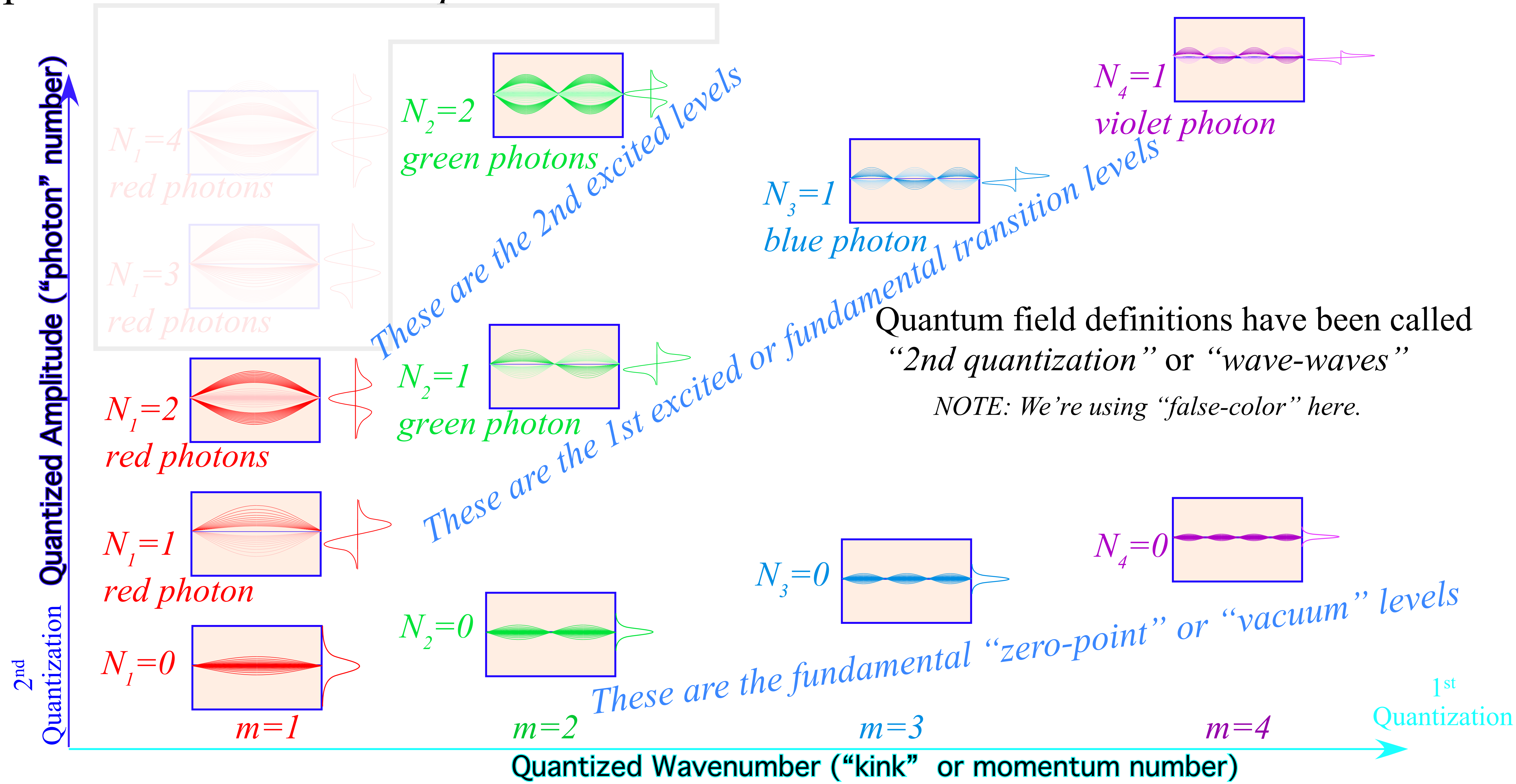

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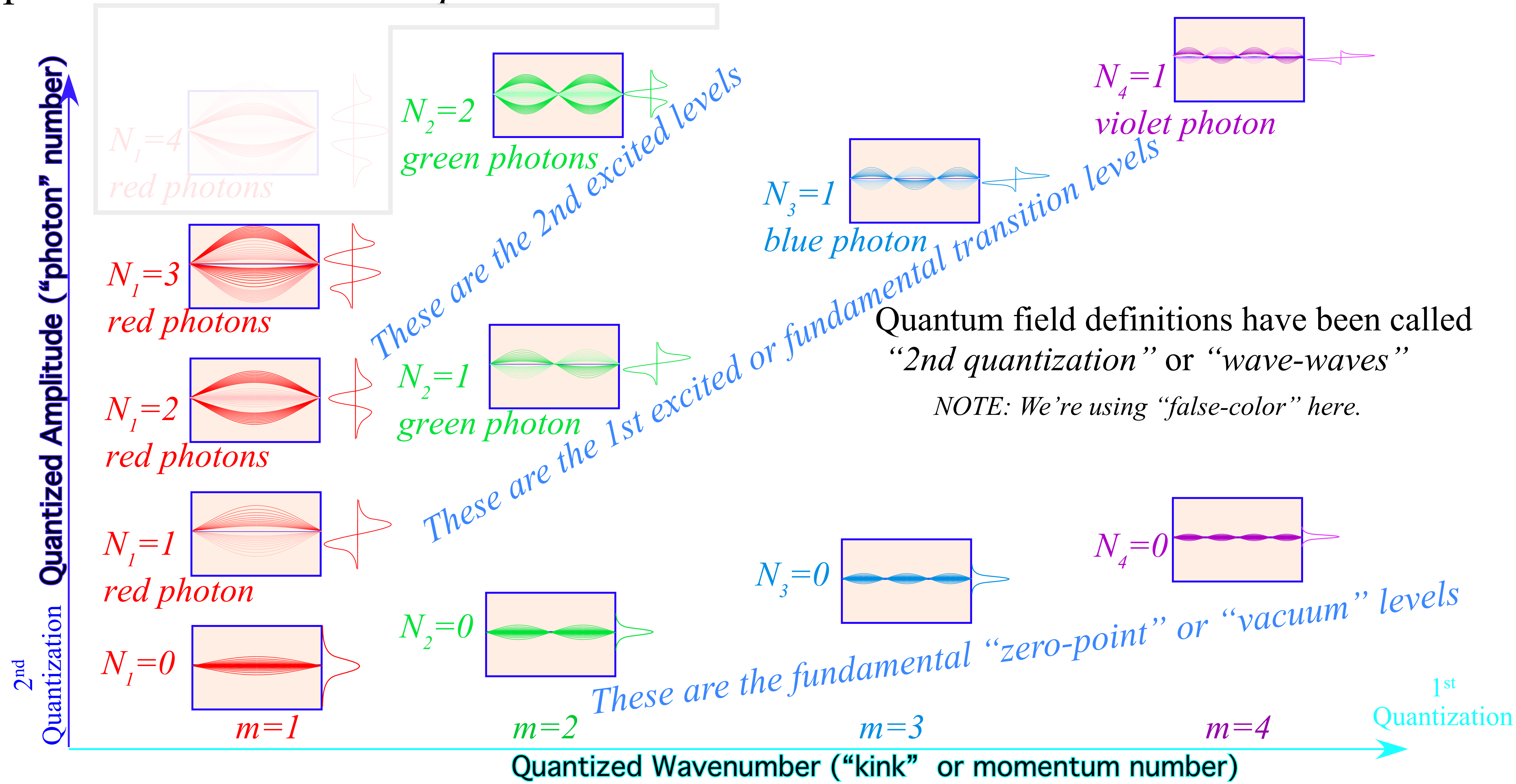

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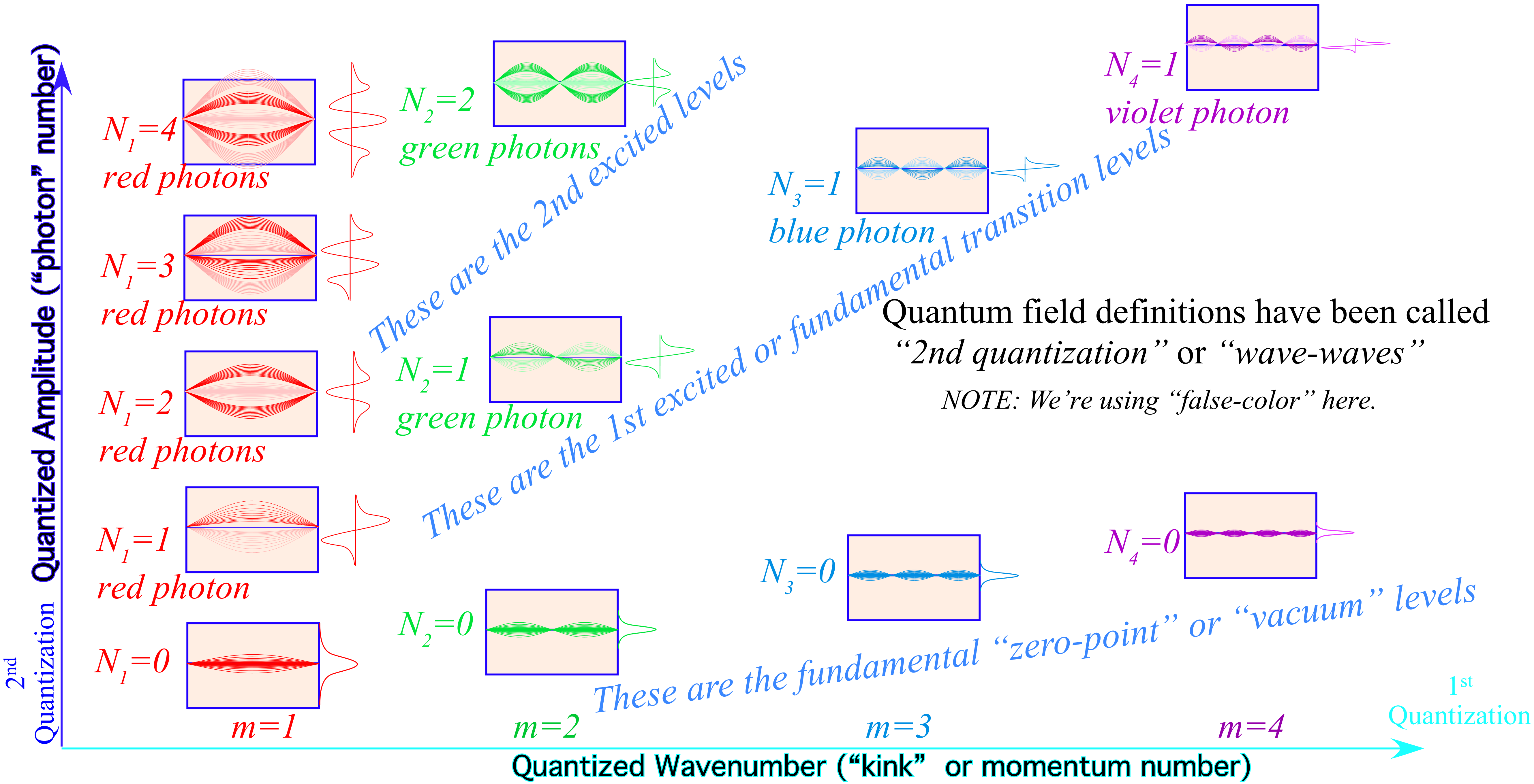

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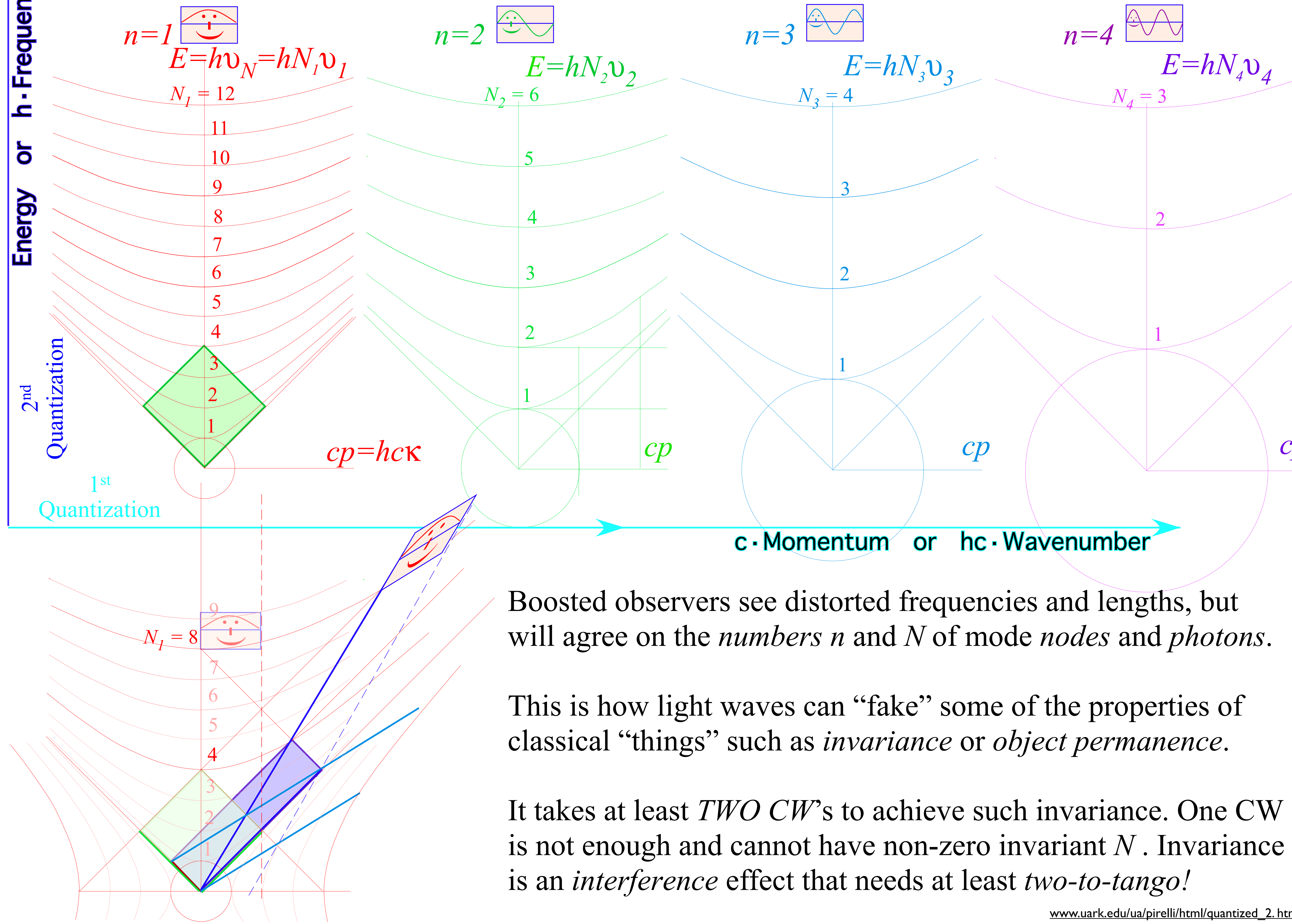
Quantum numbers *N* of field or *n, m,..* of modes are *invariants* and *not* changed by boosting velocity.
Each mode fundamental frequency $\upsilon_n = n\upsilon_1$ and its *N*-photon multiples belong to invariant hyperbolas.



Boosted observers see distorted frequencies and lengths, but will agree on the *numbers n* and *N* of mode *nodes* and *photons*.

This is how light waves can “fake” some of the properties of classical “things” such as *invariance* or *object permanence*.

It takes at least *TWO CW*’s to achieve such invariance. One CW is not enough and cannot have non-zero invariant *N* . Invariance is an *interference* effect that needs at least *two-to-tango!*

www.uark.edu/ua/pirelli/html/quantized_2.html

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Effect on group velocity *(None)* and phase velocity *(Absurd)*

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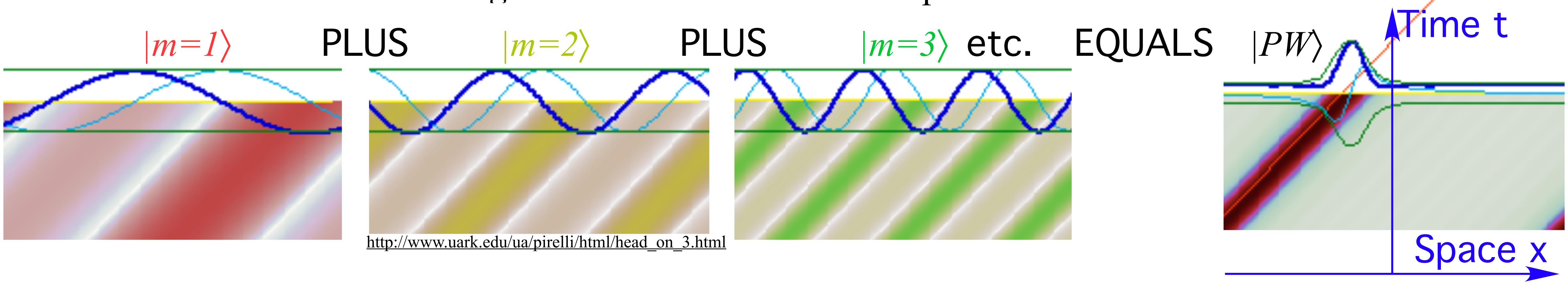
We saw how adding *CW*'s *(Continuous Waves m=1,2,3...)* can make *PW (Pulse Wave)* or *WP (Wave Packet)* that is more like a classical *"thing"* with more *localization* in space *x* and time *t*.



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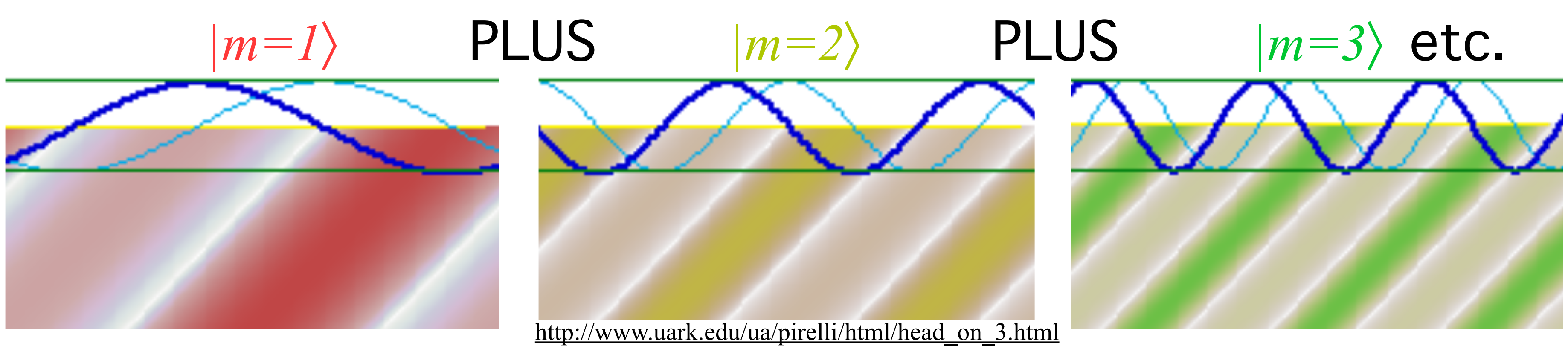
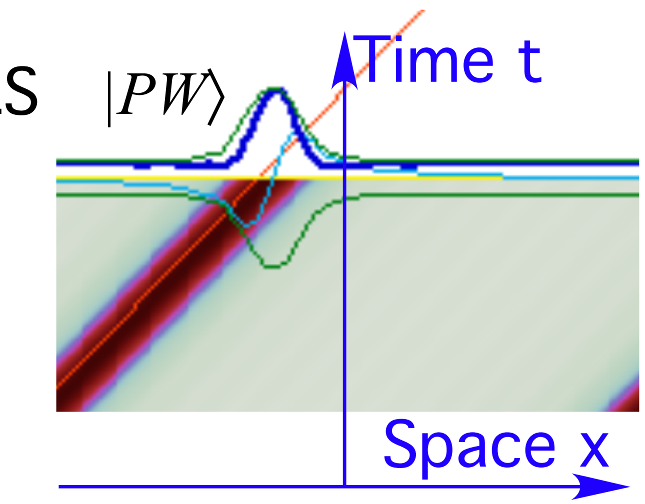
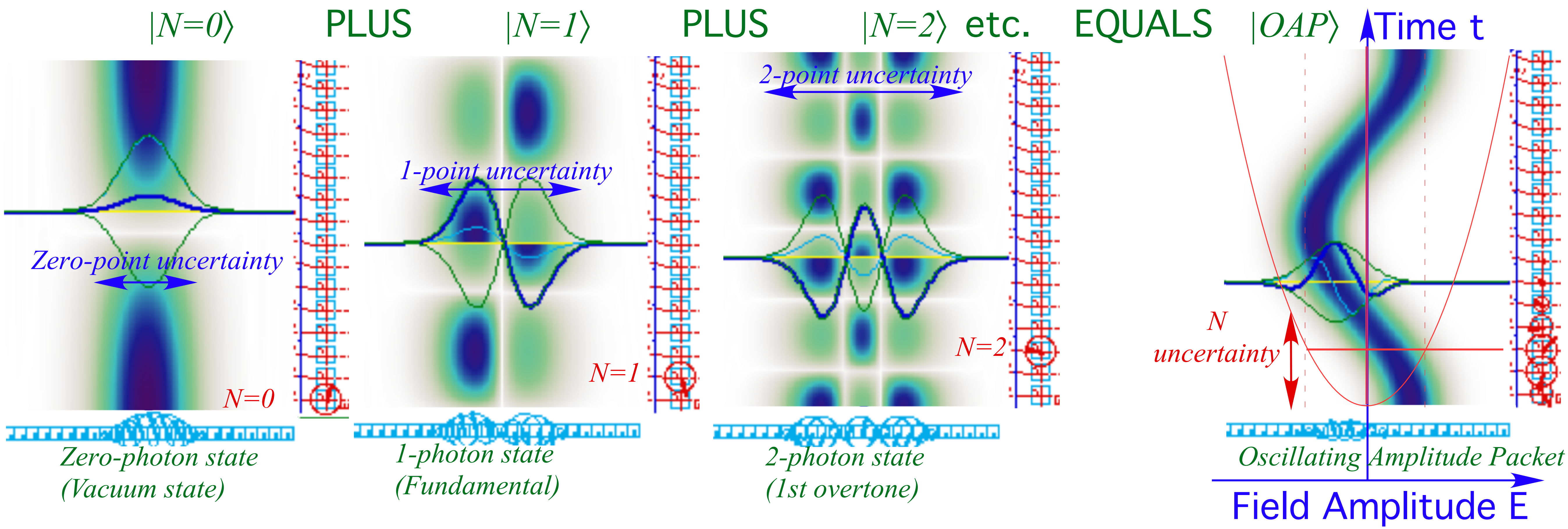




Analogy:

Adding *photons (Quantized amplitude N=0,1,2...)* can make a *CS (Coherent State)* or *OAP (Oscillator Amplitude Packet)* that is more like a *classical wave oscillation* with more *localization* in field amplitude.



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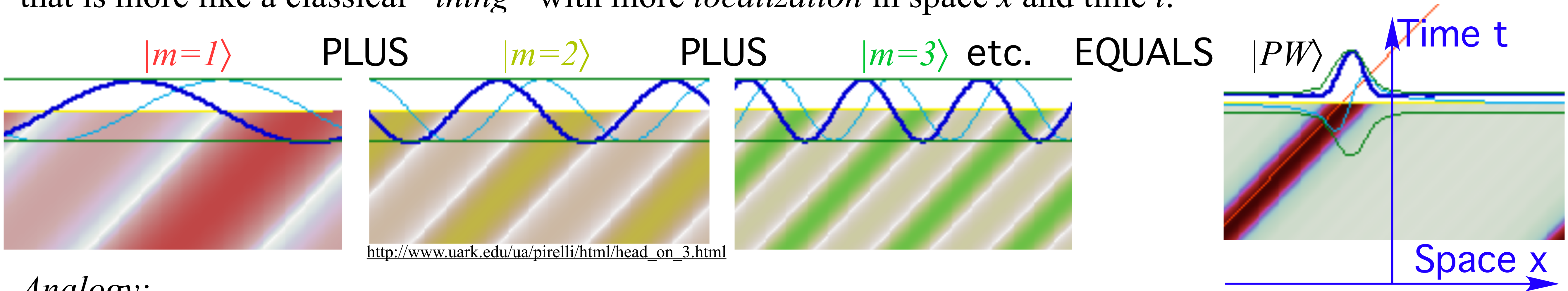

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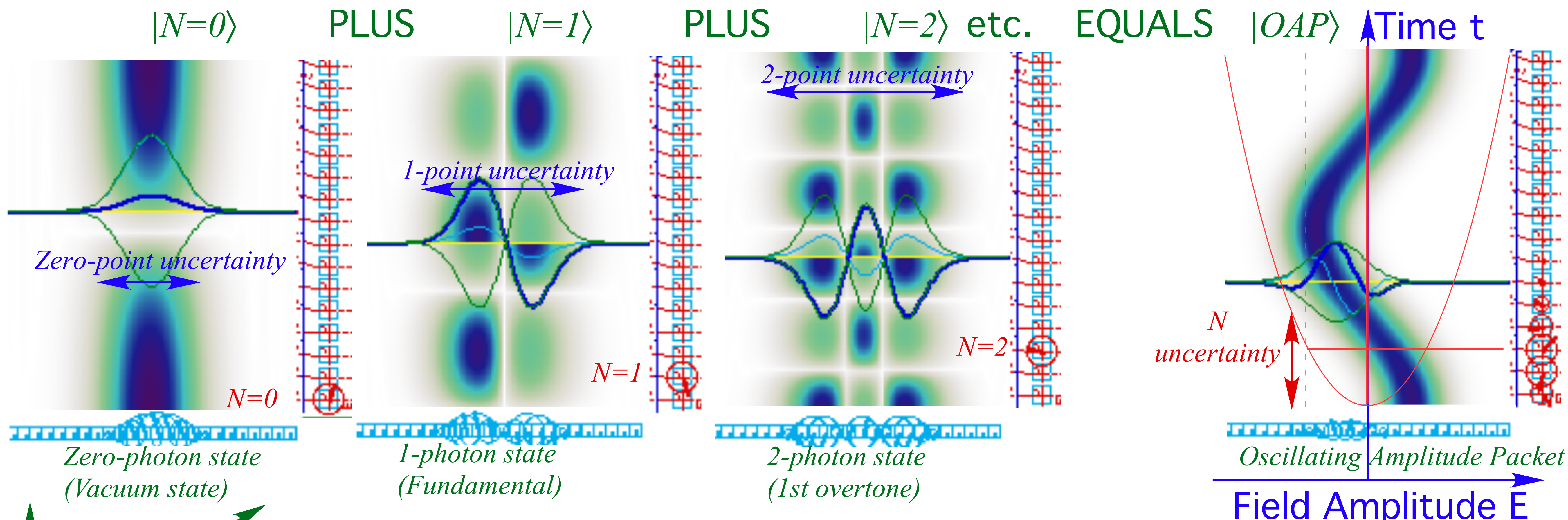

Pure photon states have localized (certain) *N* but *de*localized (*un*certain) amplitude and phase.

OAP states have *de*localized (*un*certain) *N* but more localized (certain) amplitude and phase.

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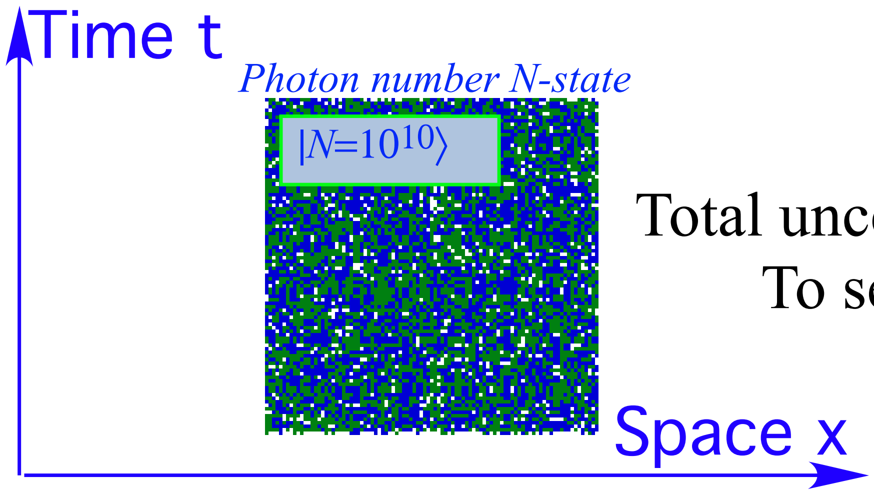
Pure photon number *N*-states would make useless spacetime coordinates



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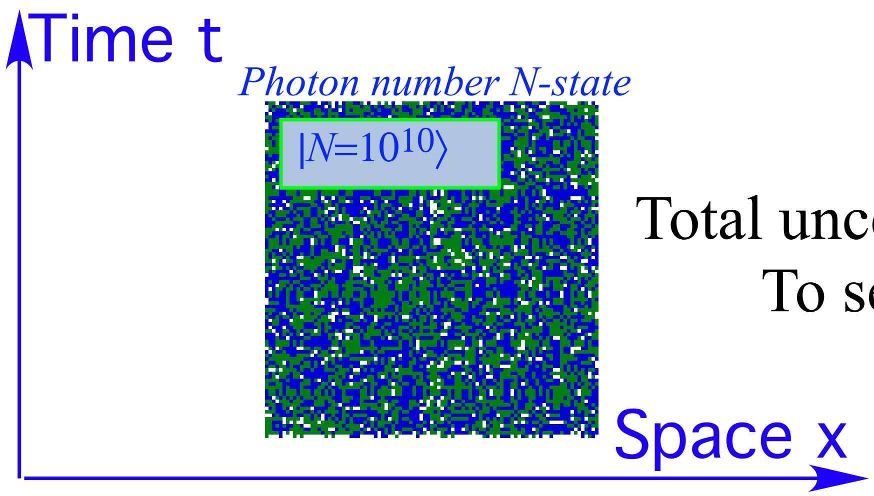

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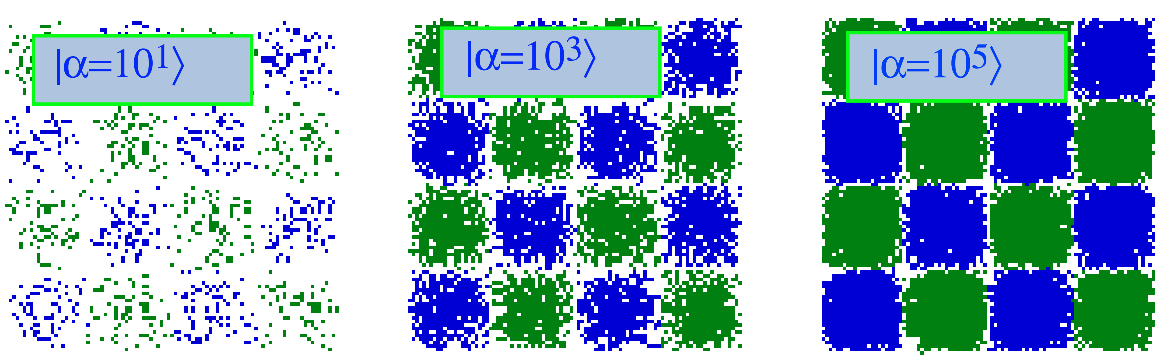
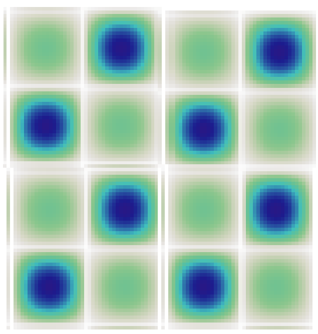
Coherent-α-states are defined by continuous amplitude-packet parameter α whose square is average photon number $\bar{N}=|\alpha|^2$. Coherent-states make better spacetime coordinates for larger $\bar{N}=|\alpha|^2$.

Quantum field coherent α-states



$\bar{N}=100$ $\Delta N=10$ | $\bar{N}=10^6$ $\Delta N=10^3$ | $\bar{N}=10^{10}$ $\Delta N=10^5$

Classical limit



Coherent-state uncertainty in photon number (and mass) varies with amplitude parameter $\Delta N \sim \alpha \sim \sqrt{N}$ so a coherent state with $\bar{N}=|\alpha|^2=10^6$ only has a 1-in-1000 uncertainty $\Delta N \sim \alpha \sim \sqrt{N}=1000$.

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Vector potential of standing wave mode : $\mathbf{A} = \mathbf{e}_1 2|a|\sin(\mathbf{k}\cdot\mathbf{r} - \omega t + \phi)$

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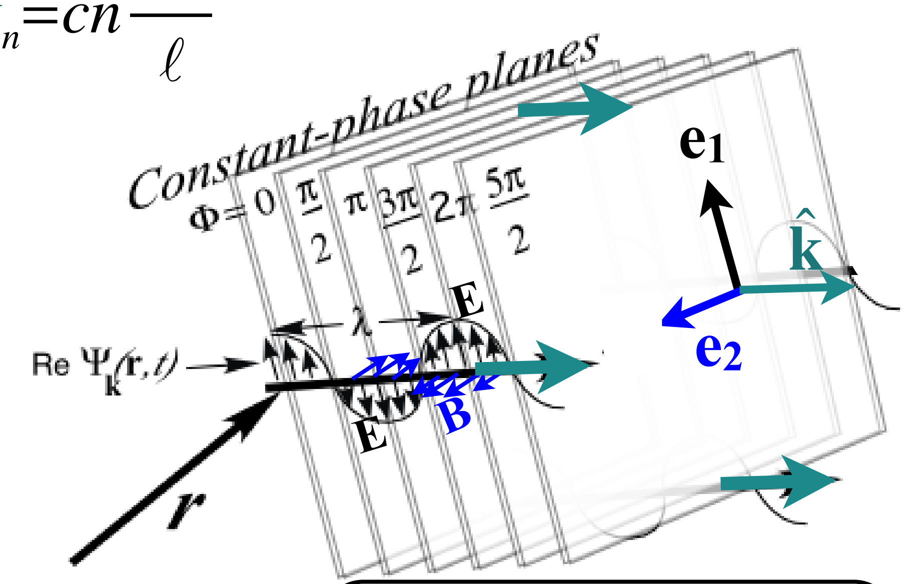

Speed of Light:
2.99792458 m/sec.=

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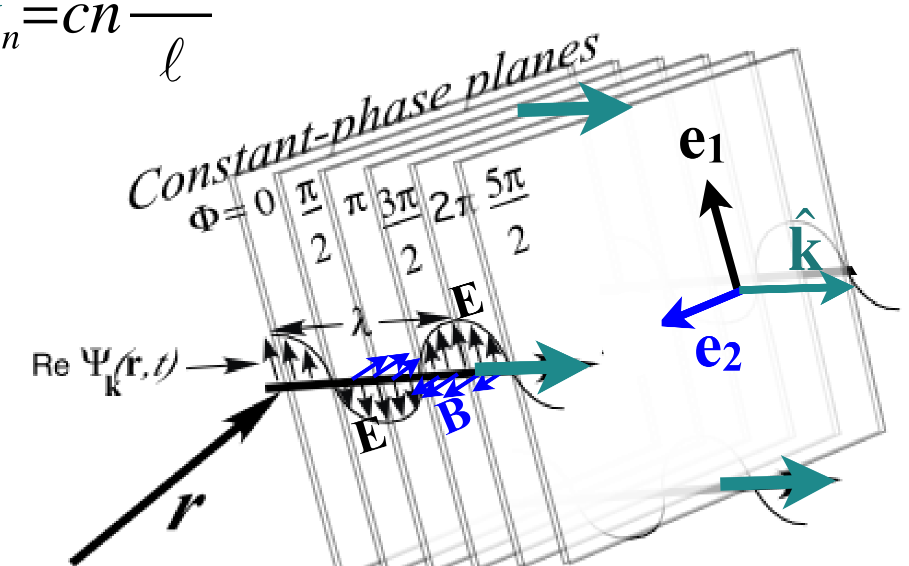

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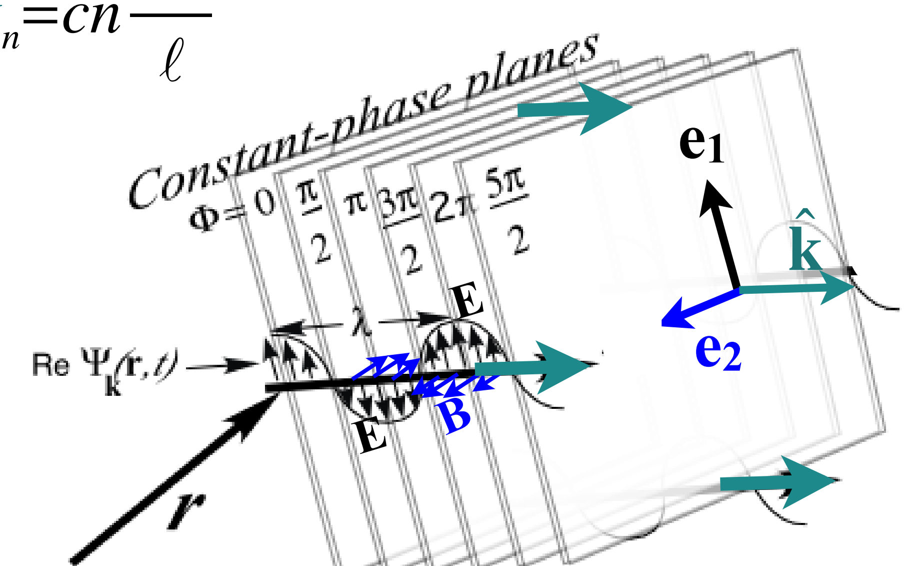

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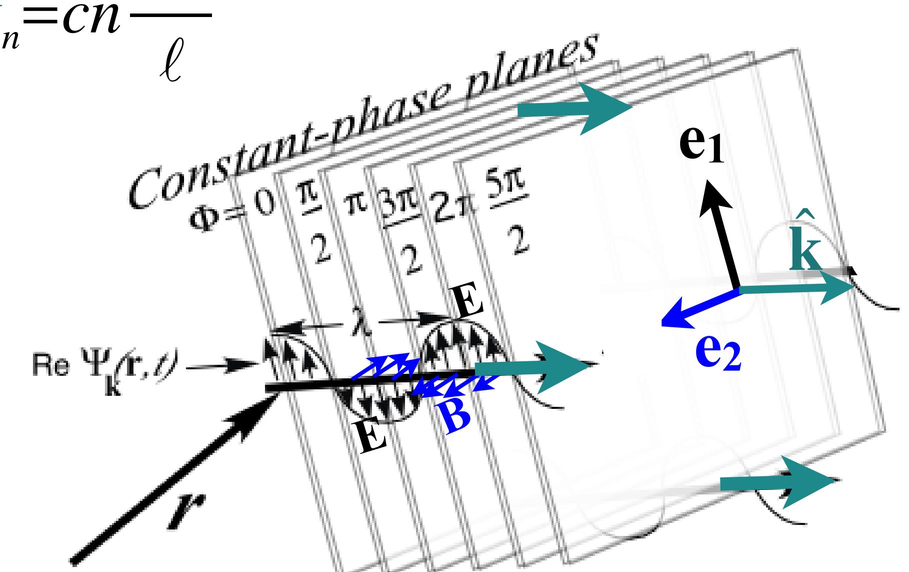

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$$\langle U\rangle V = \hbar N\omega = \frac{\varepsilon_0}{2}\omega^2|\mathbf{A}|^2 V = \frac{\varepsilon_0}{2}|\mathbf{E}|^2 V \qquad |\mathbf{A}|^2 = \frac{2\hbar N}{\varepsilon_0\omega V}, \quad \omega^2|\mathbf{A}|^2 = |\mathbf{E}|^2 = \frac{2\hbar N\omega}{\varepsilon_0 V}$$

QUANTITY
photon counts per sec.

QUALITY
cycles per sec.

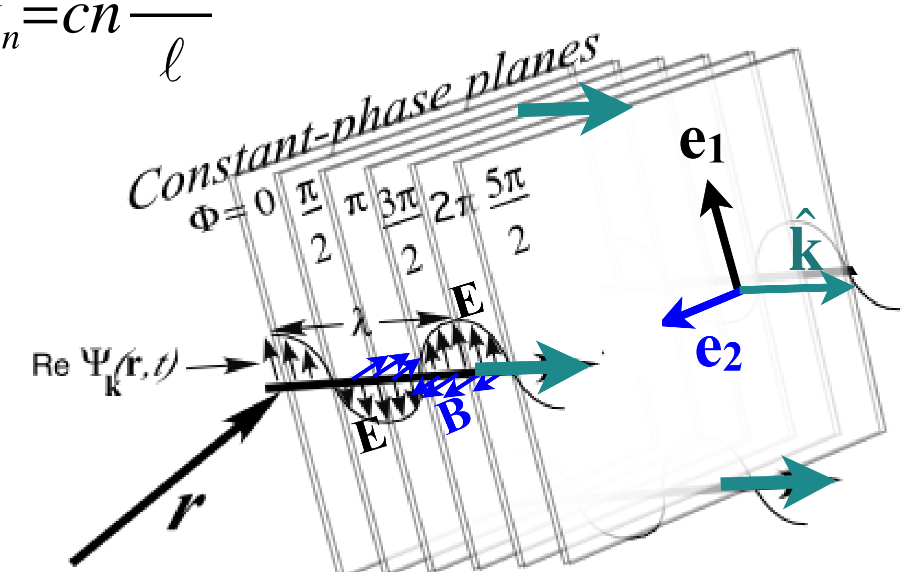
$$|\mathbf{A}| = \sqrt{\frac{2\hbar N}{\varepsilon_0\omega V}}, \quad |\mathbf{E}| = \omega|\mathbf{A}| = \sqrt{\frac{2\hbar N\omega}{\varepsilon_0 V}}$$

Electromagnetic wave mode energy: Maxwell *vs* Planck-Einstein

1[st] Quantization conditions for wave phase variables (mode-fits-cavity ℓ)

$$\text{wave vector}: 2\pi\kappa = k_n = \frac{2\pi}{\lambda_n} = n\frac{2\pi}{\ell}, \qquad \text{frequency}: 2\pi\upsilon = \omega_n = ck_n = cn\frac{2\pi}{\ell}$$

Vector potential of standing wave mode : $\mathbf{A} = \mathbf{e}_1 2|a|\sin(\mathbf{k}\cdot\mathbf{r} - \omega t + \phi)$



$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} \quad \text{(Electric field)}$$
$$= \mathbf{e}_1 E_0 \cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \phi)$$
$$E_0\mathbf{e}_1 = 2|a|\,\omega\,\mathbf{e}_1$$

Maxwell equations

$$\mathbf{B} = \nabla \times \mathbf{A} \quad \text{(Magnetic field)}$$
$$= (\mathbf{k}\times\mathbf{e}_1) B_0 \cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \phi)$$
$$B_0(\mathbf{k}\times\mathbf{e}_1) = \mathbf{e}_2 2|a|k$$

Electromagnetic mean energy density U and total *Energy* in volume V

$$\langle U\rangle V = \left\langle \frac{\varepsilon_0}{2}\mathbf{E}\cdot\mathbf{E} + \frac{1}{2\mu_0}\mathbf{B}\cdot\mathbf{B}\right\rangle V = V\left(\frac{\varepsilon_0}{2}|\mathbf{A}|^2\omega^2 + \frac{|\mathbf{A}|^2}{2\mu_0}k^2\right)\langle\cos^2(\mathbf{k}\cdot\mathbf{r} - \omega t + \phi)\rangle$$

E-*energy*=**B**-*energy*

$$= \frac{\varepsilon_0}{2}\omega^2|\mathbf{A}|^2 V = \frac{1}{2\mu_0}k^2|\mathbf{A}|^2 V \qquad \text{given: } \langle\cos^2(\mathbf{k}\cdot\mathbf{r} - \omega t + \phi)\rangle = \frac{1}{2}$$

Speed of Light:
2.99792458 m/sec.=

$$\frac{\omega}{k} = c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$$
$$\varepsilon_0 = (8.854)\cdot 10^{-7}\frac{Nm^2}{C^2}$$
$$\mu_0 = 4\pi 10^{-7}\frac{N}{A^2}$$

2[nd] Quantization conditions for wave amplitudes (action fits HO phase space)

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QUANTITY
photon counts per sec.

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cycles per sec.

$$|\mathbf{A}| = \sqrt{\frac{2\hbar N}{\varepsilon_0\omega V}}, \quad |\mathbf{E}| = \omega|\mathbf{A}| = \sqrt{\frac{2\hbar N\omega}{\varepsilon_0 V}}$$

N *and* ω are both *frequencies for quantum wave so*
E-*field has Doppler* $e^{\pm\rho}$-*shifts just like* N *and* ω

Now we see how Planck's $E_N(\omega) = \hbar N\omega$ axiom has the classical quadratic $\omega^2|A|^2$ oscillator energy

Review of wave parameters used to develop relativistic quantum theory

Bohr-Schrodinger (BS) approximation throws out Mc^2 *(Is frequency really relative?)*

Effect on group velocity *(None)* and phase velocity *(Absurd)*

1st Quantization: Quantizing phase variables k_m and $\omega(k_m)$

Understanding how quantum dynamics and transitions involve "mixed" states

Square well example of mixing unequal frequencies

Circle well or ring example of mixing equal or unequal frequencies

Mixing unequal amplitudes makes "Galloping" wave: Analogy of (*SWR*, *SWQ*) to (V_{group},V_{phase})

Analogy with optical polarization geometry and Kepler orbits

Super-luminal speed and Feynman-Wheeler pair-creation switchbacks

2nd Quantization: Quantizing wave amplitudes A_N and invariance of photon number

Analogy 1: Many CW (Continuous Waves) add up to make PW (Pulse Waves)

Analogy 2: Many Photon-Number-Modes add up to make Coherent-Laser-Modes

Heisenberg $\Delta\upsilon\Delta t\sim1\sim\Delta\kappa\Delta x$ analogous to $\Delta N\Delta phase\sim1$ uncertainty relations

Electromagnetic wave mode energy: Maxwell vs. Planck-Einstein

1st quantization for wave phase variables and classical energy of **E**, **B** , and **A** fields

2nd quantization for wave and Planck quantum energy of **E**, **B** , and **A** fields

Scaling **E**-waves to mime quantum Ψ-waves and ψ-waves

Relativistic effects on charge, current, and Maxwell Fields

Making electromagnetic E-waves have quantum cavity Ψ-wave and ψ-wave properties

Previous equations for energy U per volume

$$\langle U \rangle V = \hbar N \omega = \frac{\varepsilon_0}{2}\omega^2 |\mathbf{A}|^2 V = \frac{\varepsilon_0}{2}|\mathbf{E}|^2 V \qquad |\mathbf{A}| = \sqrt{\frac{2\hbar N}{\varepsilon_0 \omega V}} \;,\; |\mathbf{E}| = \omega |\mathbf{A}| = \sqrt{\frac{2\hbar N \omega}{\varepsilon_0 V}}$$

$$\langle U \rangle = \frac{\hbar N \omega}{V} = \frac{\varepsilon_0}{2}\omega^2 |\mathbf{A}|^2 = \frac{\varepsilon_0}{2}|\mathbf{E}|^2$$

$$\frac{\langle U \rangle}{\hbar \omega} = \frac{N}{V} = \frac{\varepsilon_0}{2\hbar}\omega |\mathbf{A}|^2 = \frac{\varepsilon_0}{2\hbar\omega}|\mathbf{E}|^2$$

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$$\langle U\rangle = \frac{\hbar N\omega}{V} = \frac{\varepsilon_0}{2}\omega^2|\mathbf{A}|^2 = \frac{\varepsilon_0}{2}|\mathbf{E}|^2$$

$$\frac{\langle U\rangle}{\hbar\omega} = \frac{N}{V} = \frac{\varepsilon_0}{2\hbar}\omega|\mathbf{A}|^2 = \frac{\varepsilon_0}{2\hbar\omega}|\mathbf{E}|^2$$

Rescale $\mathbf{E}$ by $s = \sqrt{\frac{\varepsilon_0}{2\hbar\omega}}$ to get x and y component wave function

$$\vec{\Psi} = \begin{pmatrix} \Psi_x \\ \Psi_y \end{pmatrix} = s\mathbf{E} = \sqrt{\frac{\varepsilon_0}{2\hbar\omega}}\mathbf{E}$$

whose volume integral $\displaystyle\oiiint_V dV\,\vec{\Psi}^*\bullet\vec{\Psi} = \oiiint_V dV\left(\left|\Psi_x\right|^2 + \left|\Psi_y\right|^2\right) \propto \oiiint_V dV\,\frac{N}{V} = N$

is $\displaystyle\oiiint_V dV\left(\left|\Psi_x\right|^2 + \left|\Psi_y\right|^2\right) = N$ (It is normalized to particle number N.)

Poynting flux S is scaled to get counts per area·second.

$$S = cU = c\varepsilon_0|\mathbf{E}|^2 = \hbar n\omega \quad \text{where:} \quad n = Nc/V\,(\text{per } m^2 \text{ per } sec.)$$

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Relativistic effects on charge, current, and Maxwell Fields

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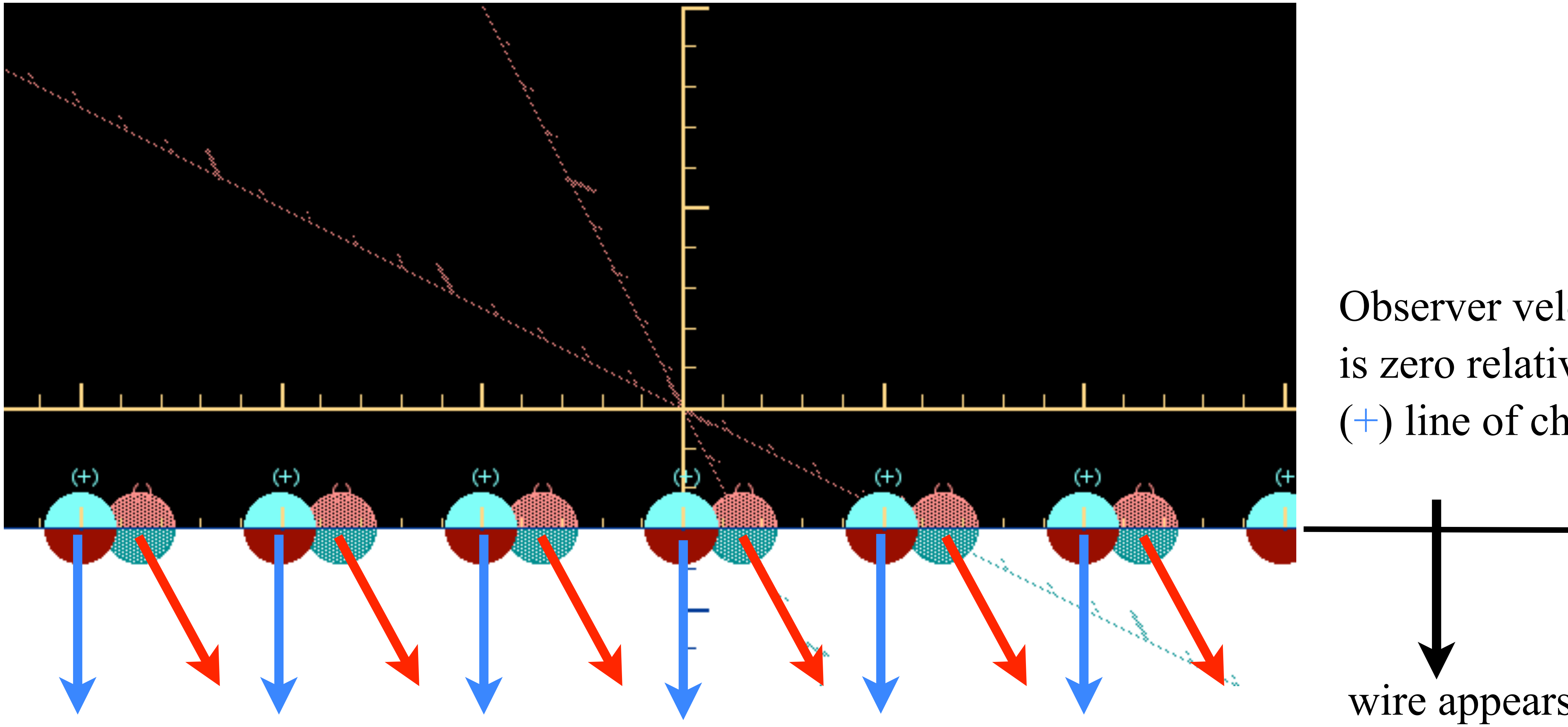


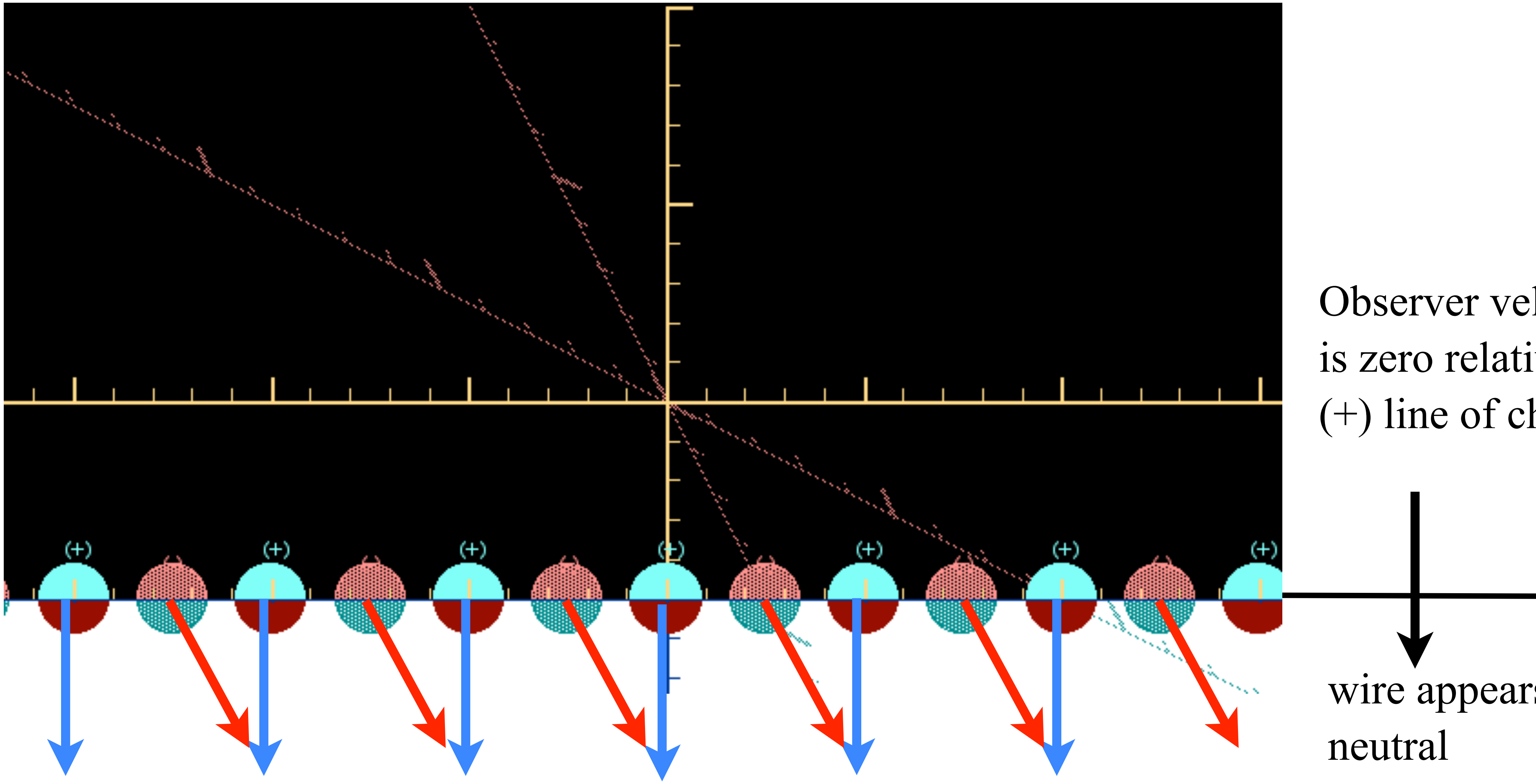
Observer velocity is zero relative to (+) line of charge

wire appears neutral

(+) Charge fixed (-) Charge moving to right (*Negative current density)*

(+) Charge density is Equal to the (-) Charge density

Relativistic effects on charge, current, and Maxwell Fields



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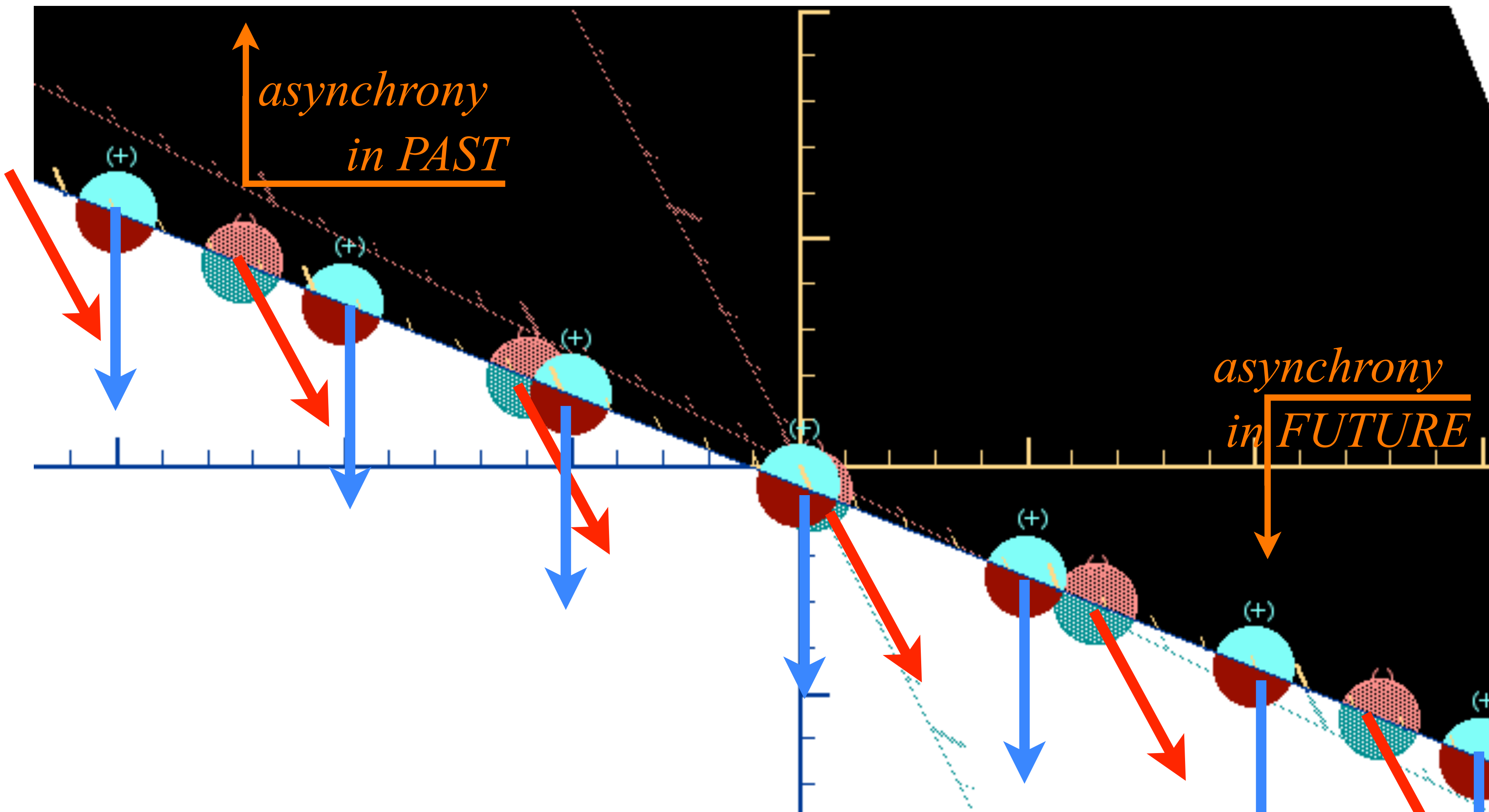
(+) Charge density is Equal to the (-) Charge density (*Zero* $\rho(x,t)=0$)

Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz asynchrony

Asynchrony due to off-diagonal $\sinh\rho$ (a 1st-order effect)

in Lorentz tranform : $\begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \sim \begin{pmatrix} 1 & v/c \\ v/c & 1 \end{pmatrix}$



observer has
$q_{[+]}$
"test-charge"

Observer velocity
is $+v$ relative to
(+) line of charge

(+) Charge fixed (-) Charge moving to right (*Negative current density* $\vec{\mathbf{j}}(x,t)$)

(+) Charge density is *Greater* than (-) Charge density (*Positive* $\rho(x,t)>0$)

wire appears
postive (+)
(repulsive to
observer $q_{[+]}$)

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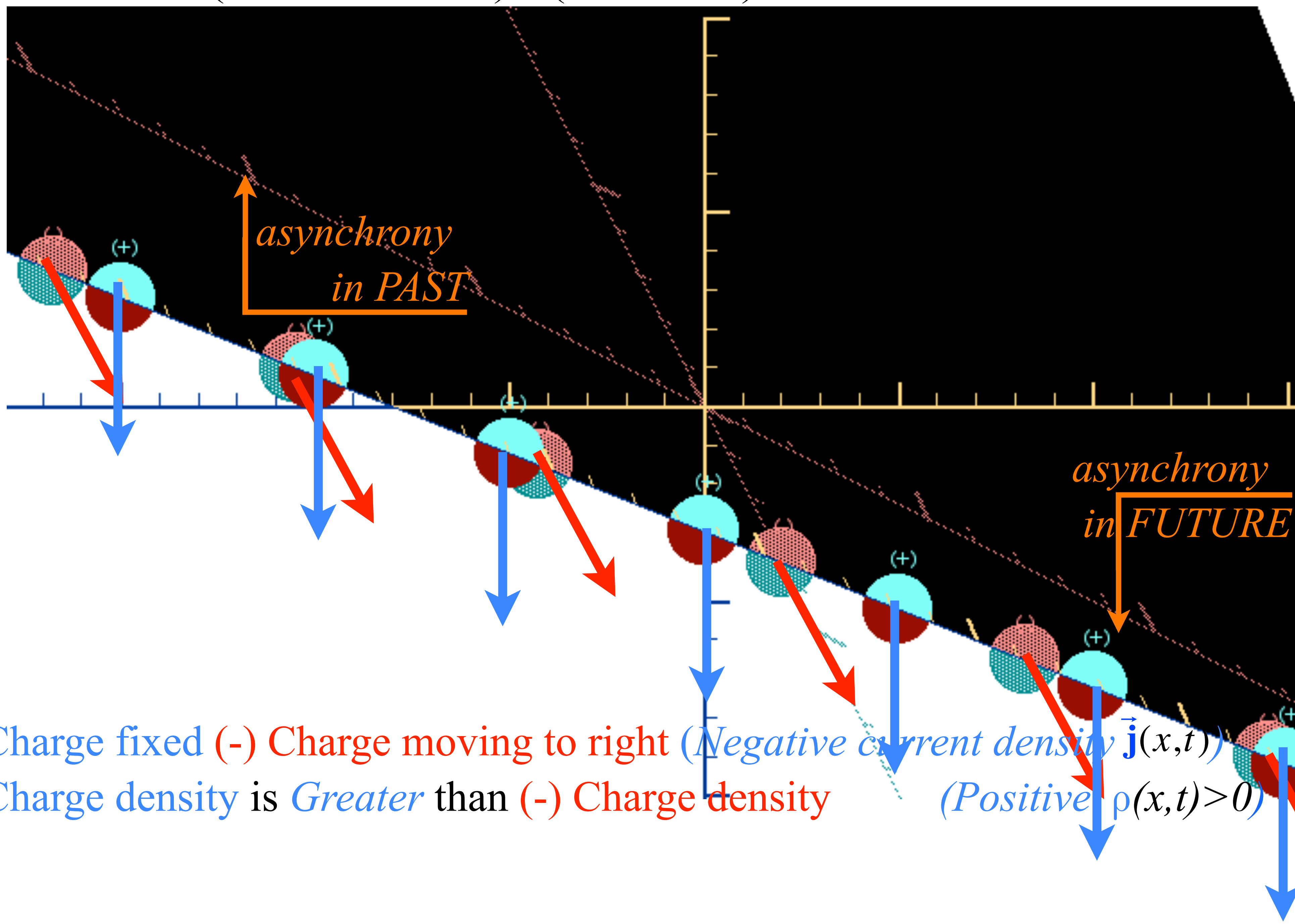

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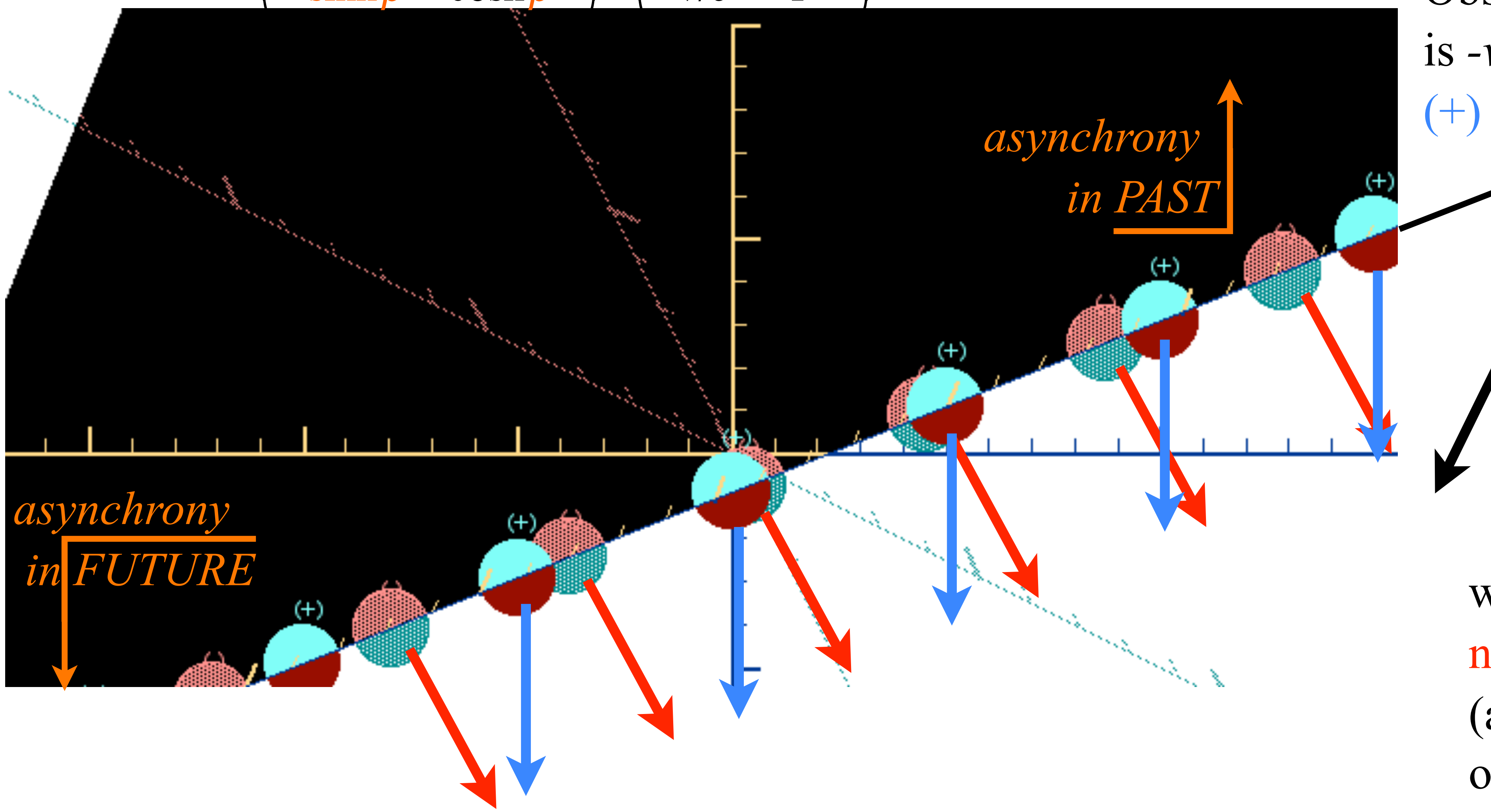
observer has $q_{[+]}$ "test-charge"

Observer velocity is $-v$ relative to (+) line of charge



wire appears negative (-) (attractive to observer $q_{[+]}$)

(+) Charge fixed (-) Charge moving to right (*Negative current density* $\vec{\mathbf{j}}(x,t)$)

(+) Charge density is *Less* than (-) Charge density (*Negative* $\rho(x,t)<0$)

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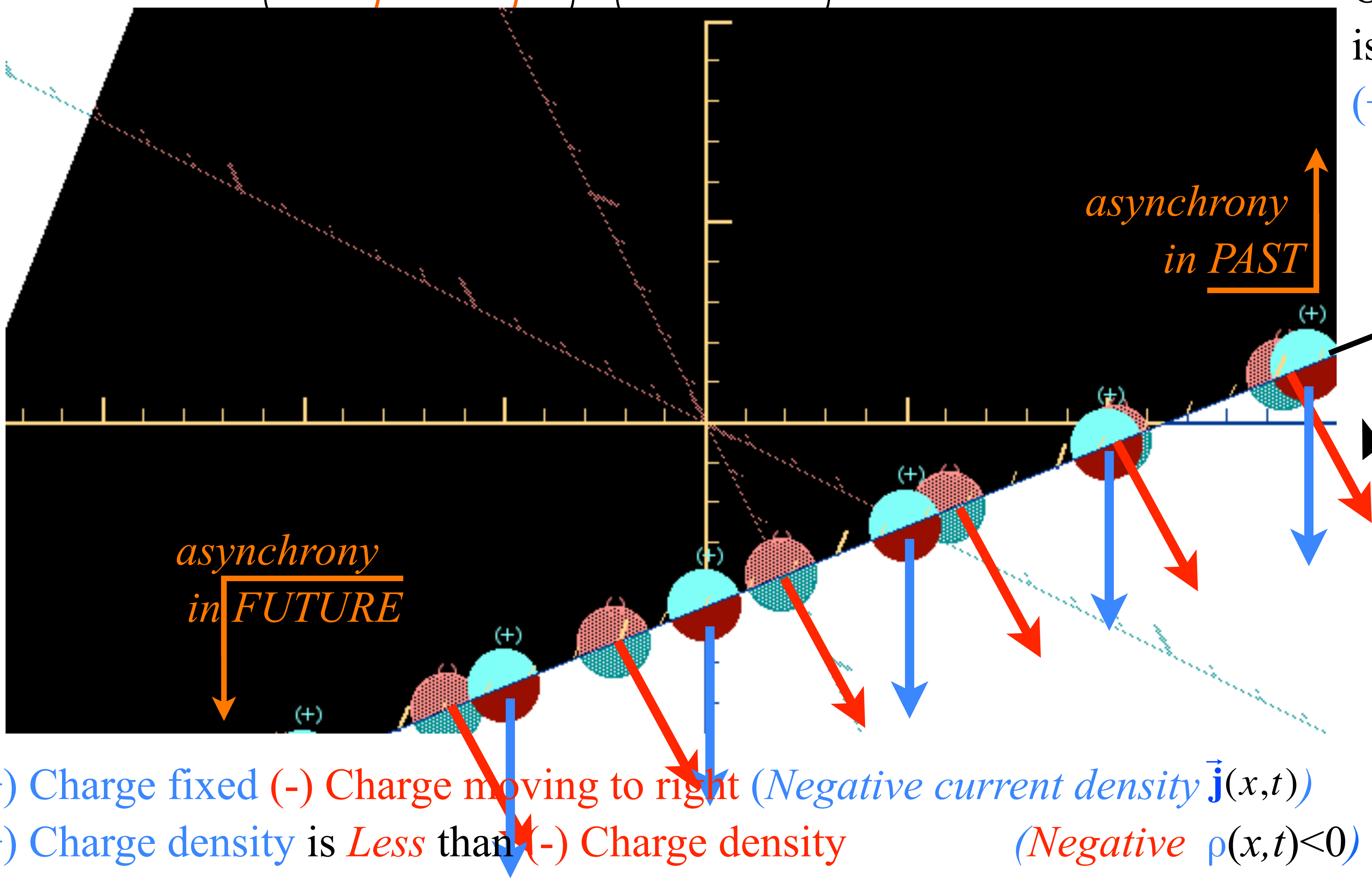

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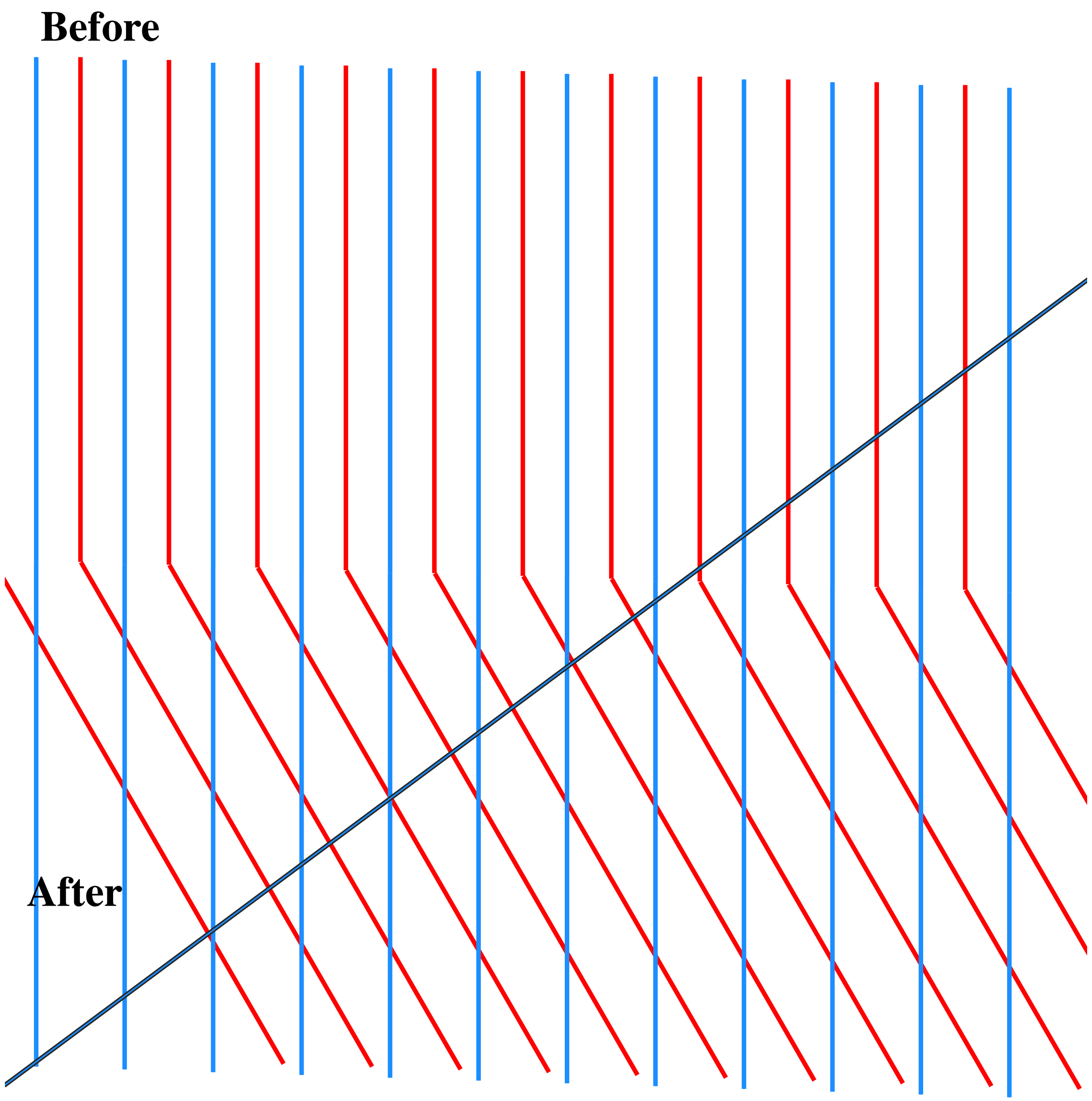
Simple 1st-order relativistic geometry of magnetism



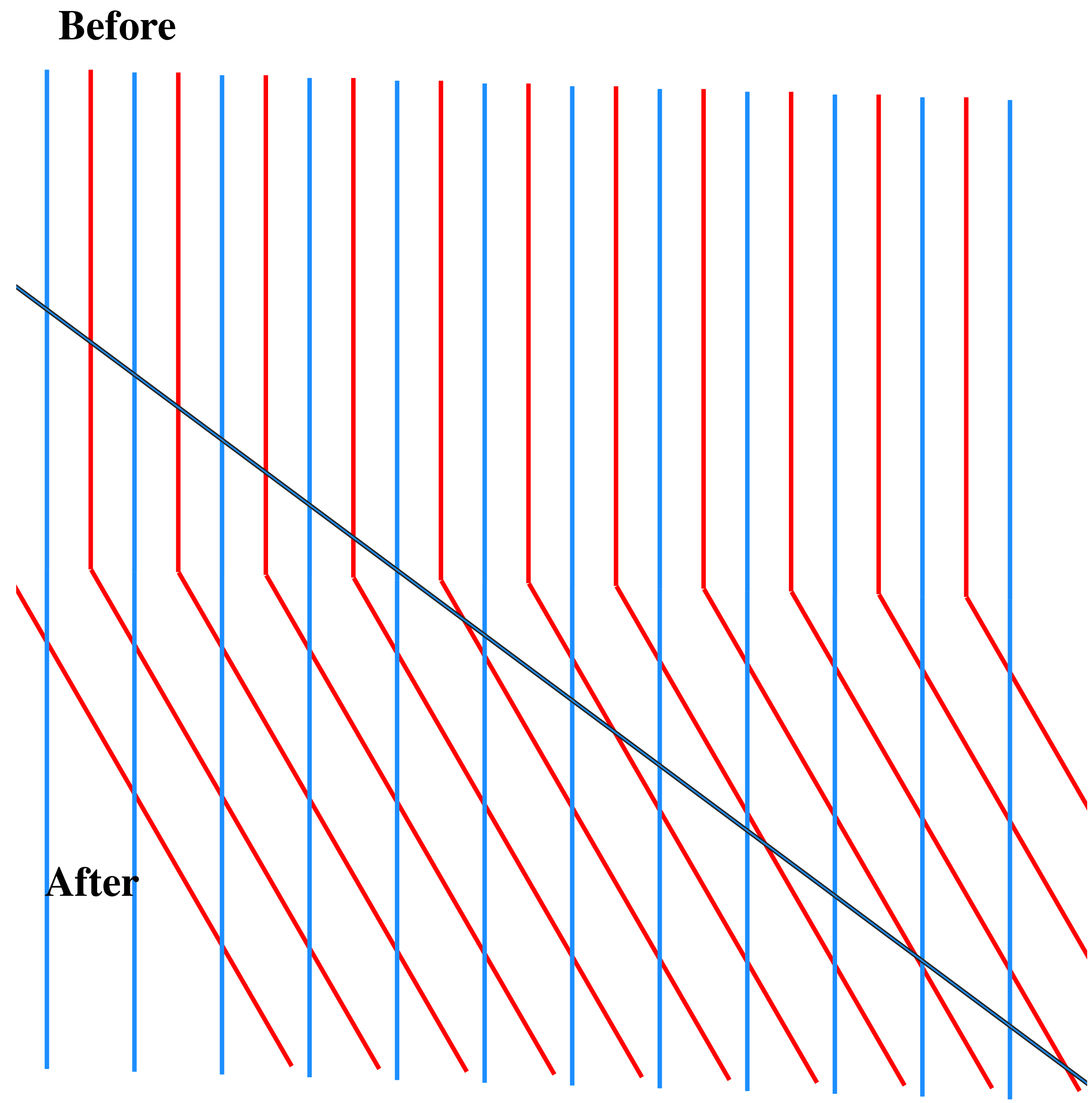
If Black is moving to Left

Before red starts moving to right

Black sees same number of red and blue

After red starts moving to right

Black sees more red than blue



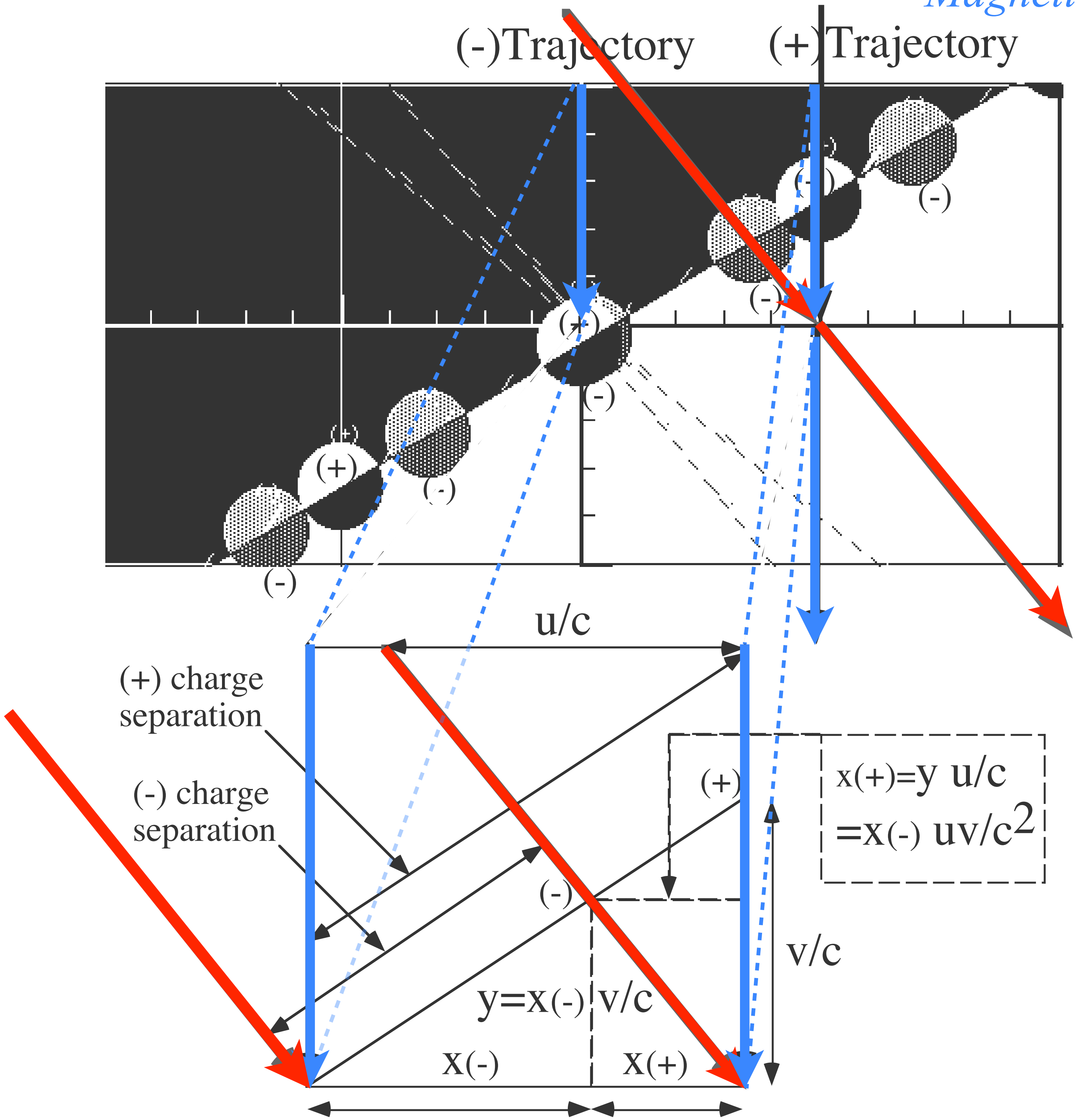
If Black is moving to Right

Before red starts moving to right

Black sees same number of red and blue

After red starts moving to right

Black sees more blue than red



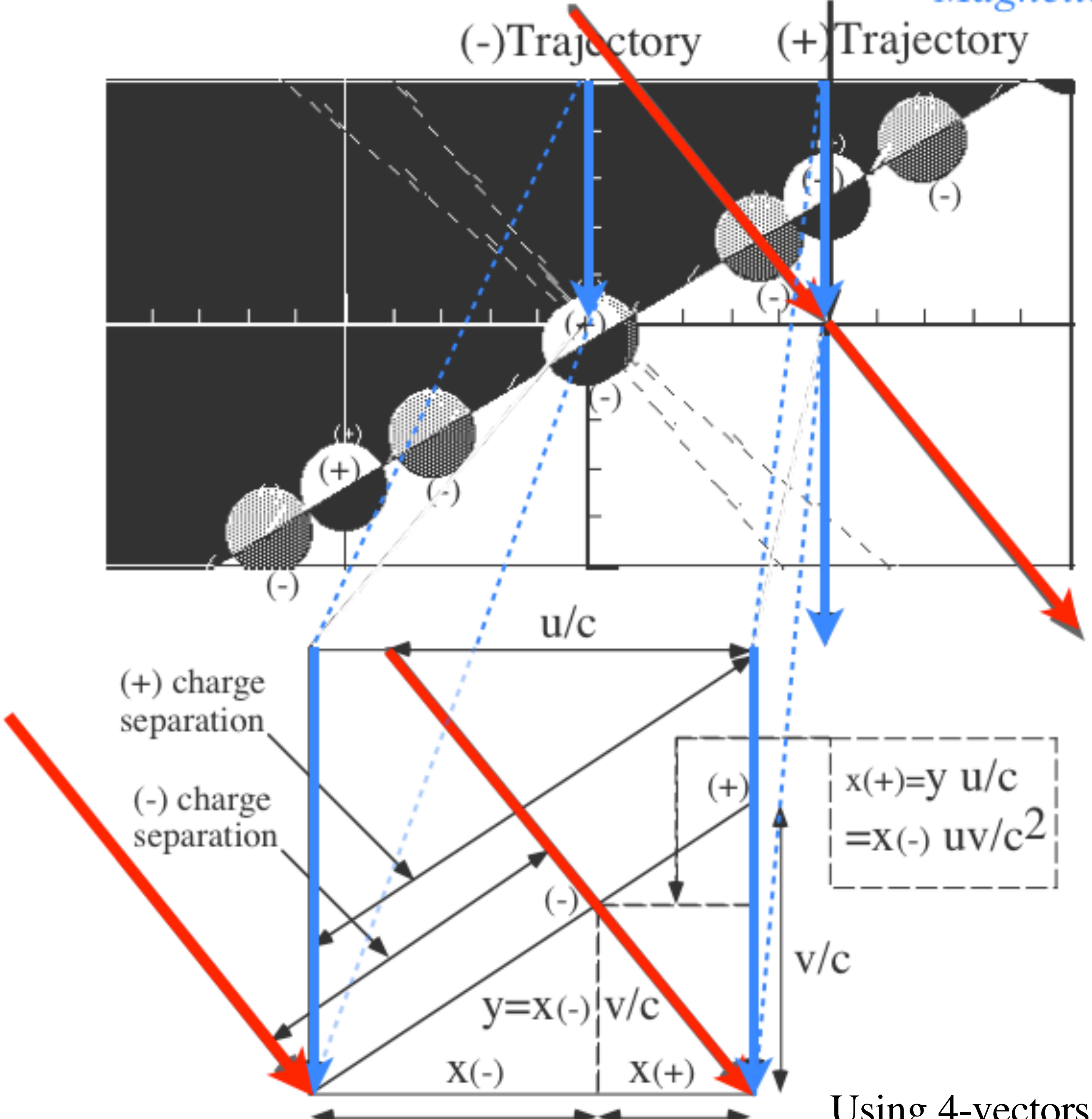
$$\frac{\rho(-)}{\rho(+)}=\frac{(+)\text{ charge separation}}{(-)\text{ charge separation}}=\frac{x(+)+x(-)}{x(-)}$$

$$\frac{\rho(-)}{\rho(+)}=\frac{x(+)}{x(-)}+1=\frac{uv}{c^2}+1$$

$$\rho(+)-\rho(-)=\rho(+)\left(1-\frac{\rho(-)}{\rho(+)}\right)=-\frac{uv}{c^2}\rho(+)$$

Unit square: (u/c) /1 = x(+)/y

(v/c) /1 = y/x(-)



$$\frac{\rho(-)}{\rho(+)} = \frac{(+)\text{ charge separation}}{(-)\text{ charge separation}} = \frac{x(+)+x(-)}{x(-)}$$

$$\frac{\rho(-)}{\rho(+)} = \frac{x(+)}{x(-)} + 1 = \frac{uv}{c^2} + 1$$

$$\rho(+) - \rho(-) = \rho(+)\left(1 - \frac{\rho(-)}{\rho(+)}\right) = -\frac{uv}{c^2}\rho(+)$$

Unit square: (u/c) /1 = x(+)/y

(v/c) /1 = y/x(-)

Using 4-vectors to EL Transform (charge-current)=$(c\rho, \mathbf{j})$

$$\begin{pmatrix} c\rho' \\ j_{x'} \\ j_{y'} \\ j_{z'} \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho & \cdot & \cdot \\ \sinh\rho & \cosh\rho & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & 1 \end{pmatrix} \begin{pmatrix} c\rho \\ j_x \\ j_y \\ j_z \end{pmatrix}$$

The electric force field **E** of a charged line varies inversely with radius. The Gauss formula for force in mks units :

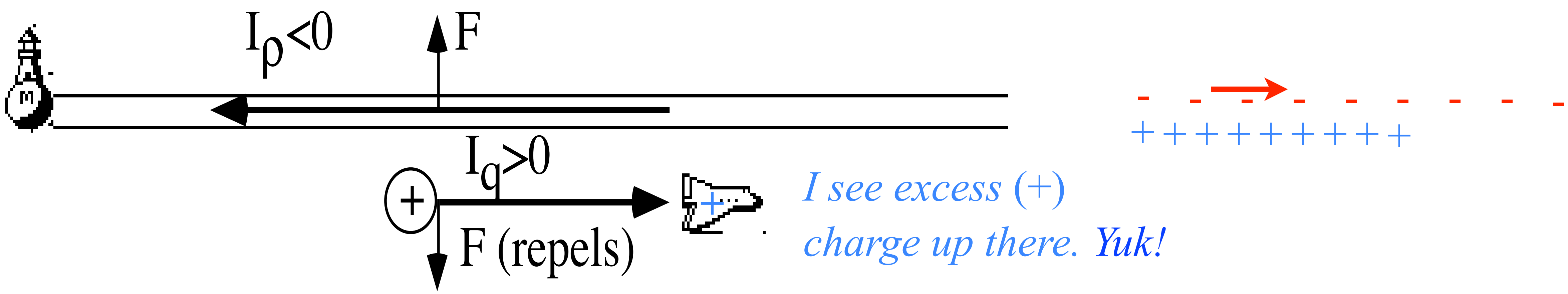
$$F = qE = q\left[\frac{1}{4\pi\varepsilon_0}\frac{2\rho}{r}\right] \quad , \quad \text{where:} \quad \frac{1}{4\pi\varepsilon_0} = 9\times 10^9 \frac{N\cdot m^2}{Coul.}$$

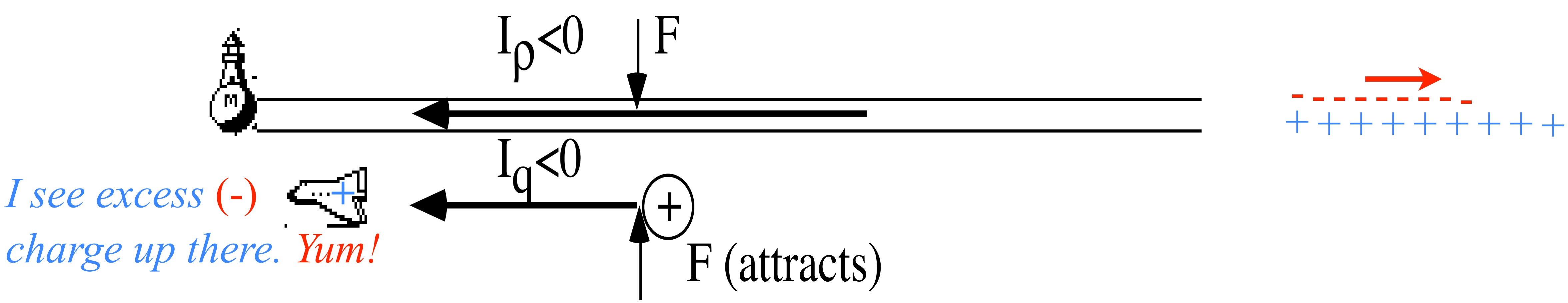
$$F = qE = q\left[\frac{1}{4\pi\varepsilon_0}\frac{2}{r}\left(-\frac{uv}{c^2}\rho(+)\right)\right] = -\frac{2\,qv\,\rho(+)u}{4\pi\varepsilon_0 c^2\, r} = -2\times 10^{-7}\frac{I_q\, I_\rho}{r}$$

$1/4\pi\varepsilon_0 = 9\cdot 10^9$

$c^2 = 9\cdot 10^{-16}$

$1/(4\pi\varepsilon_0 c^2) = 10^{-7}$





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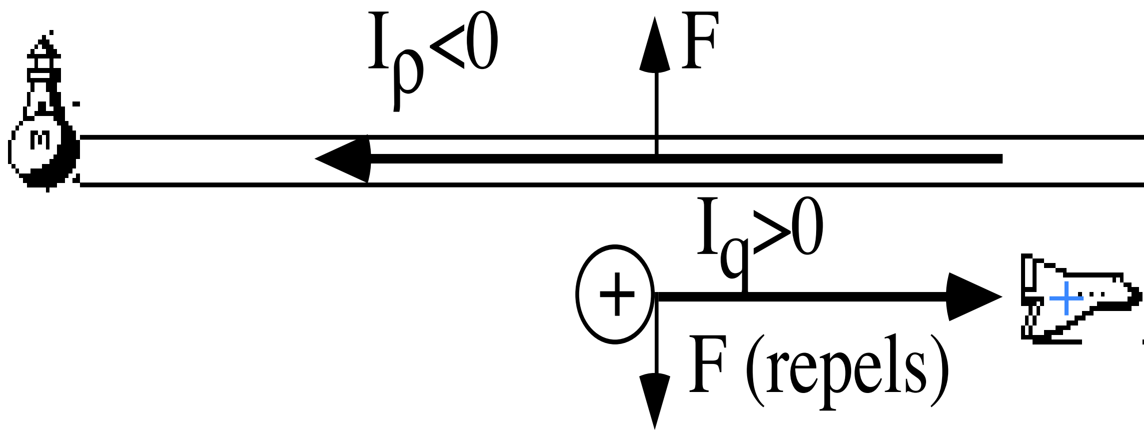
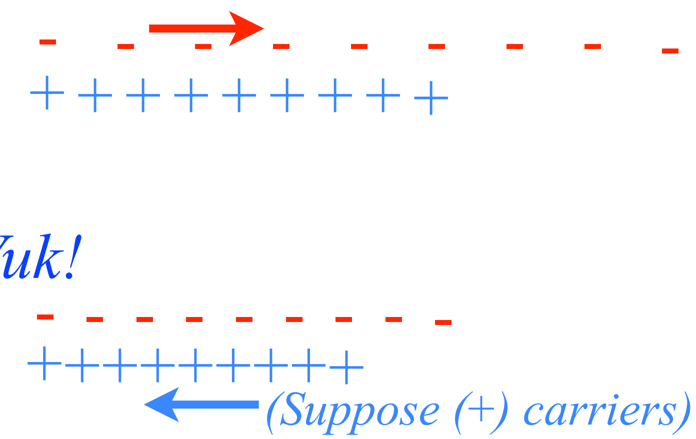
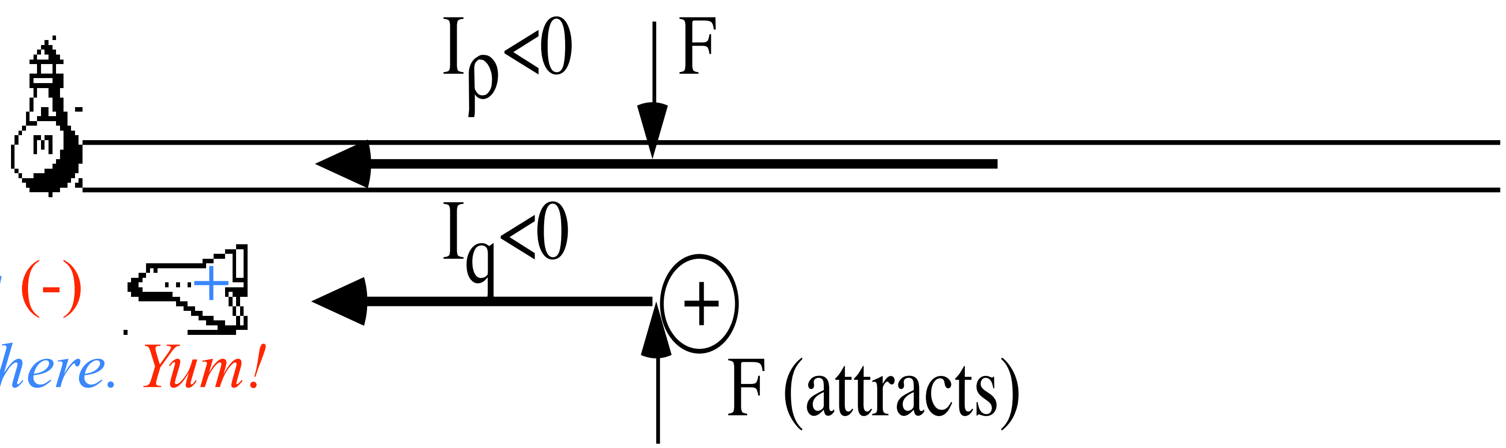
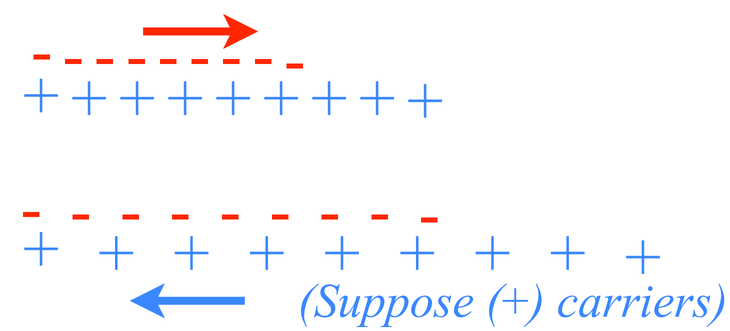


I see excess (+) charge up there. Yuk!





I see excess (-) charge up there. Yum!



2005 Pirelli treatments

➡ *Relating photons to Maxwell energy density and Poynting flux*

Relativistic variation and invariance of frequency (ω,k) and amplitudes

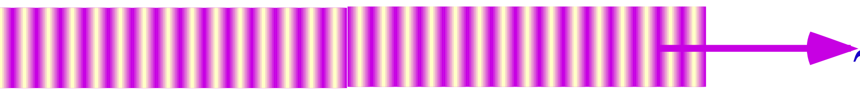
How probability ψ-waves and flux ψ-waves evolved

*Properties of amplitude ψ*ψ-squares*

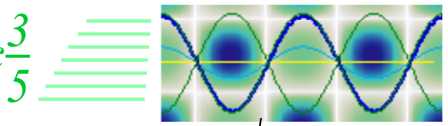
More on unmatched amplitudes AND unmatched frequencies AND unmatched quanta

Light Energy and Flux *2-CW vs 1-CW-light*

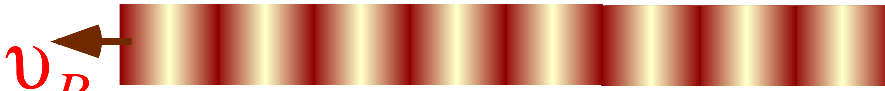
What if head-on CW's υ_A *=1200THz* and υ_B *=300THz* pair-up in a *2-CW*-light beam?



Group velocity $u=c\frac{\upsilon_A-\upsilon_B}{\upsilon_A+\upsilon_B}=c\frac{3}{5}$



Green 600THz $=\upsilon_0=\sqrt{(\upsilon_A\ \upsilon_B)}$



They form a *rest frame* going $u=c\frac{\upsilon_A-\upsilon_B}{\upsilon_A+\upsilon_B}=3c/5$ with a *mean* or *base* color $\upsilon_0=\sqrt{(\upsilon_A\ \upsilon_B)}$

($\upsilon_0=B=600THz$ is *green* here. Neither has this singly.)

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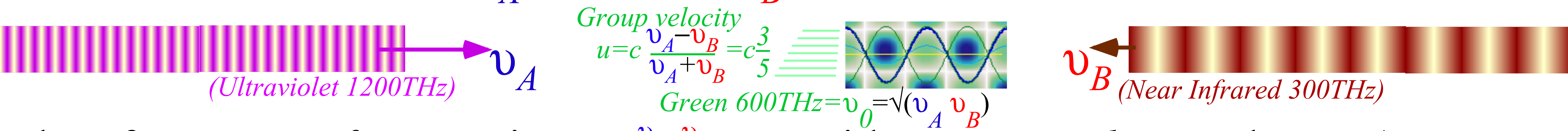

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Relating Planck's E to Maxwell's Density U=E/V

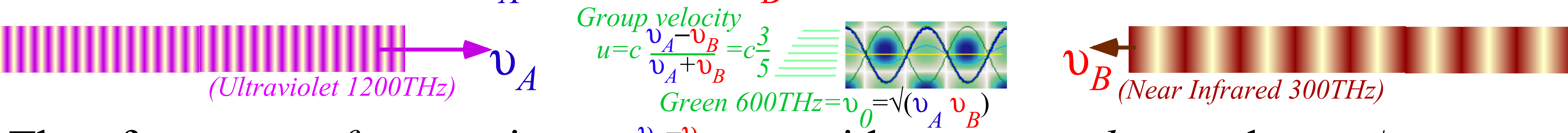
Maxwell field energy *E*, a product of mean-square electric field $\langle \mathsf{E}^2\rangle$, volume of cavity *V*, and constant $\varepsilon_0=8.854\cdot10^{-12}C^2/N\cdot m^2$, approximates Planck's energy $\bar{N}h\upsilon_0$.

$E=\langle \mathsf{E}^2\rangle V\varepsilon_0=\bar{N}h\upsilon_0$ *Maxwell-Planck Energy* $U=\langle \mathsf{E}^2\rangle\varepsilon_0=\bar{N}h\upsilon_0/V$ *Maxwell-Planck Density*

Field Energy $=|\mathsf{E}|^2\varepsilon_0$ $1/4\pi\varepsilon_0=9\cdot10^9$

Light Energy and Flux *2-CW vs 1-CW-light*

What if head-on CW's $\upsilon_{A=1200THz}$ and $\upsilon_{B=300THz}$ pair-up in a *2-CW*-light beam?



They form a *rest frame* going $u=c\frac{\upsilon_A-\upsilon_B}{\upsilon_A+\upsilon_B}=3c/5$ with a *mean* or *base* color $\upsilon_0=\sqrt{(\upsilon_A\,\upsilon_B)}$ ($\upsilon_0=B=600THz$ is *green* here. Neither has this singly.) *All* observers agree on υ_0 since *all* shift-products $b\upsilon_A\, r\upsilon_B$ equal $(\upsilon_0)^2$ due to Doppler-time-symmetry (b=1/r). Single *CW*'s get *invariant* properties if they pair-up. The υ_A-υ_B pairing above makes a number $\bar{N}$ of *invariant mass quanta* $M_1=h\upsilon_0/c^2=4.42\cdot 10^{-36}kg$ where the number $\bar{N}$ is invariant, too. $\bar{N}$ is Planck's *photon number* for the cavity rest energy $E=\bar{N}h\upsilon_0$.

Relating Planck's E to Maxwell's Density U=E/V

Maxwell field energy *E*, a product of mean-square electric field $\langle \mathsf{E}^2\rangle$, volume of cavity *V*, and constant $\varepsilon_0=8.854\cdot 10^{-12}C^2/N\cdot m^2$, approximates Planck's energy $\bar{N}h\upsilon_0$.

$E=\langle \mathsf{E}^2\rangle V\varepsilon_0=\bar{N}h\upsilon_0$ *Maxwell-Planck Energy* $U=\langle \mathsf{E}^2\rangle\varepsilon_0=\bar{N}h\upsilon_0/V$ *Maxwell-Planck Density*

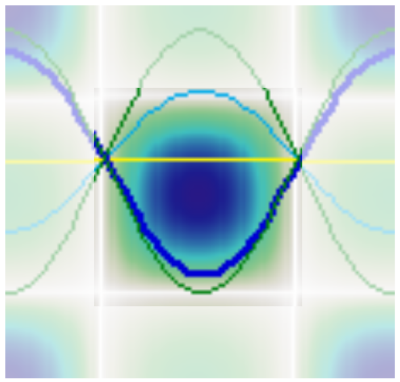
Example: Let a $\frac{1}{4}\mu m$-cube cavity *(Half-wave at 600Thz)* have $\bar{N}=10^{10}$ photons in volume $V=(\frac{1}{4}10^{-6}m)^3$.

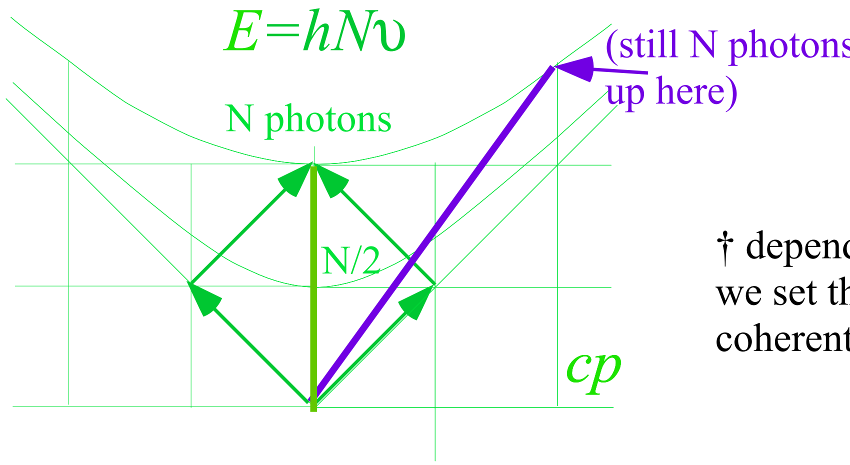
Energy per photon: $h\upsilon_0=4\cdot 10^{-19}$J=2.5 eV Energy of $\bar{N}$ photons: $\bar{N}h\upsilon_0=4\cdot 10^{-9}$J=25GeV

E-field per photon: $\mathsf{E}_1=\sqrt{(h\upsilon_0/V\varepsilon_0)}=7.6\cdot 10^3$V/m E-field of $\bar{N}$ photons: $\mathsf{E}_N=7.6\cdot 10^{13}$V/m

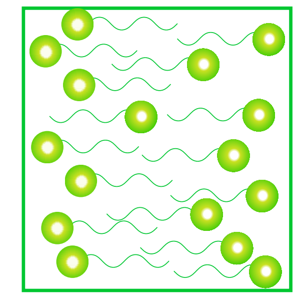
Energy and Flux (contd) *2-CW- vs 1-CW-light*

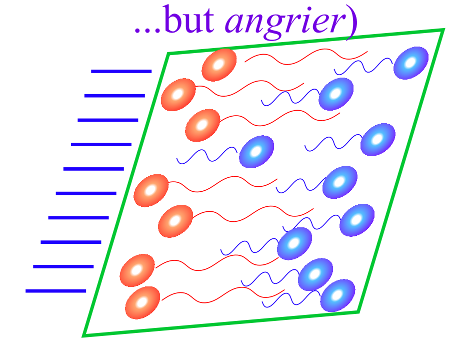
Planck $E=Nh\upsilon$ relation allows us to interpret our N-quantized *2-CW* mode as a box or *cavity* of $N_{\text{(more-or-less†)}}$ photons where N ***is invariant*** to speed u of box.





† depends on how we set the mode's coherent state.





Energy and Flux (contd) *2-CW- vs 1-CW-light*

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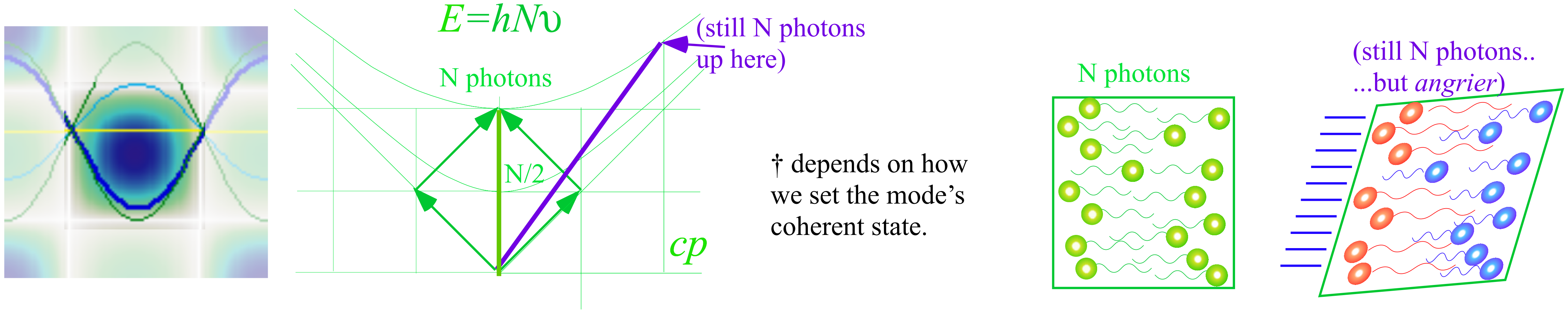
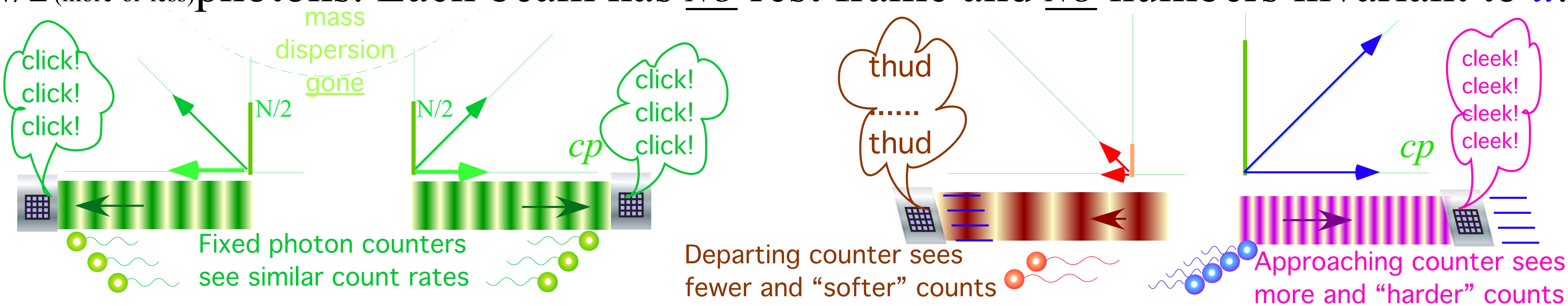


If we open the box our *2-CW* mode "divorces" into two separate *1-CW* beams of *N/2*(more-or-less) photons. Each beam has NO rest frame and NO numbers invariant to *u*.



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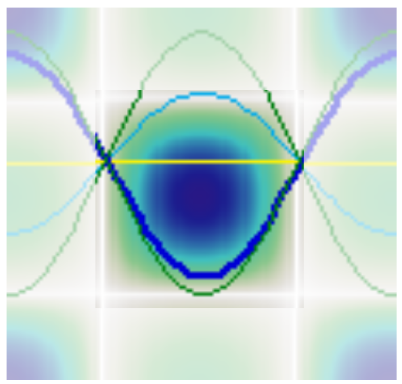

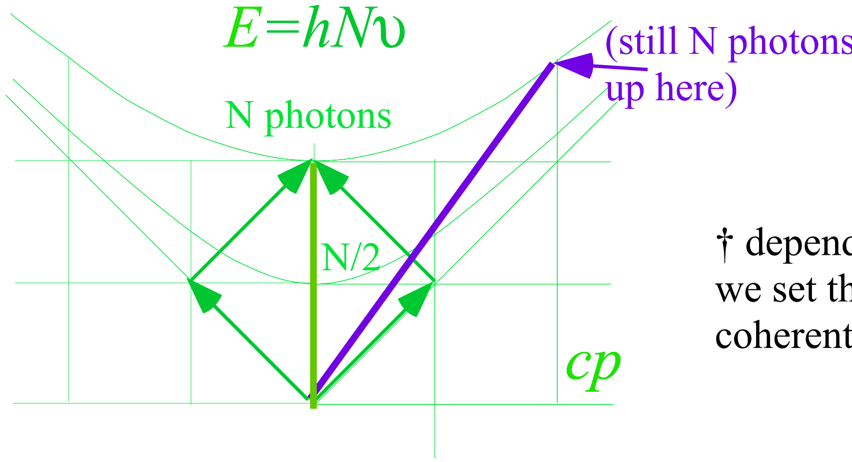

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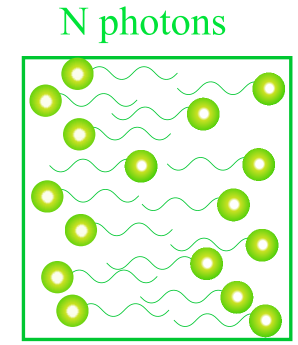

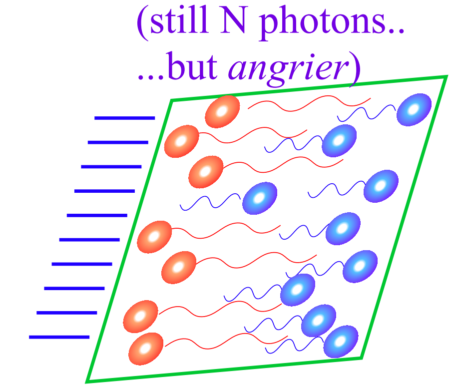

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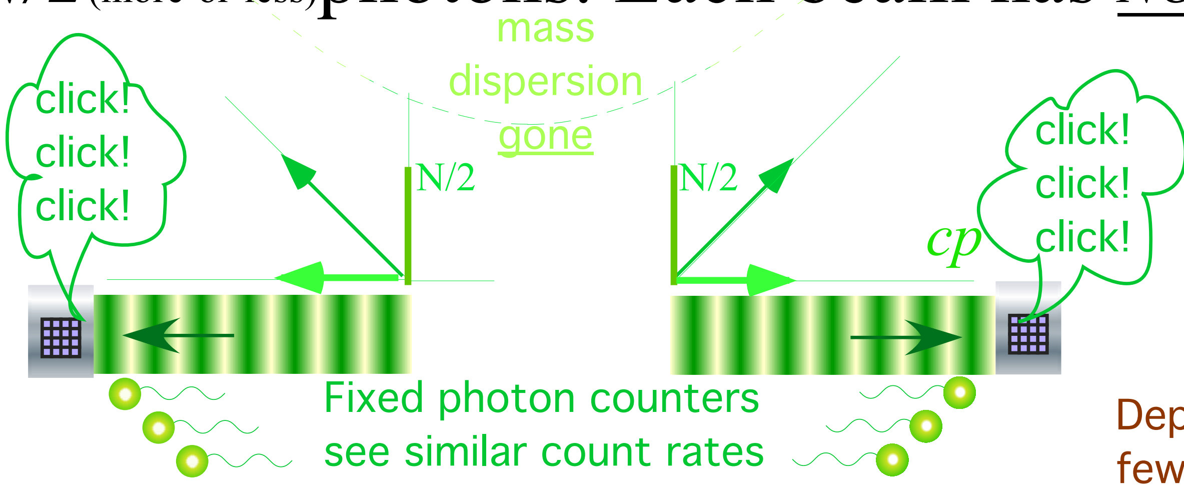
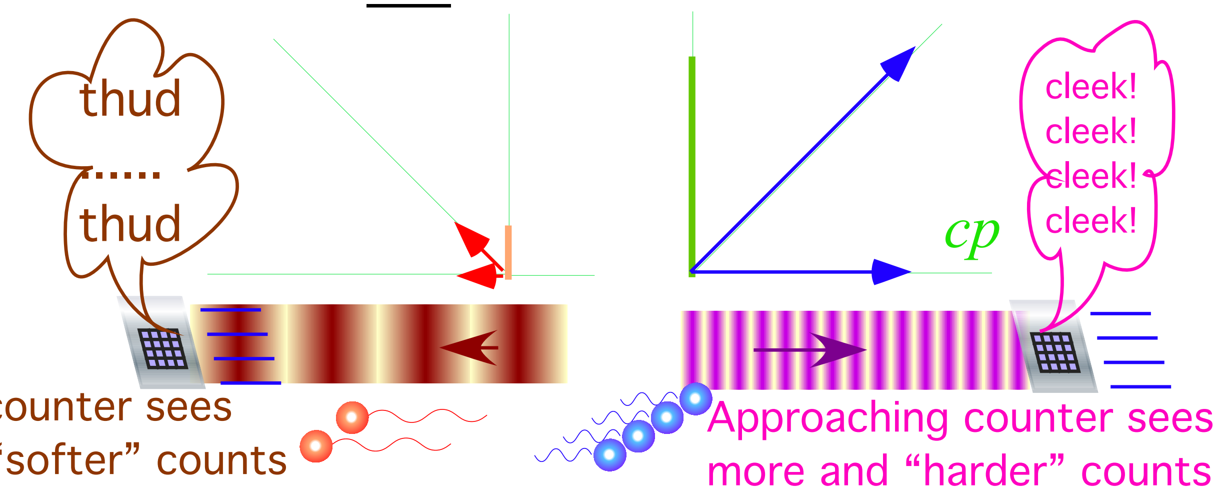




Relating Poynting's Intensity S=cU to Planck's Flux

Poynting intensity *S* is a product of $c=2.99792458 m/s$ and density *U*. It approximates Planck's energy $E=Nh\upsilon$ times *c* and divided by cavity volume *V*.

$$S=cU=(Nc/V)h\upsilon = n\, h\upsilon \quad \textit{Poynting-Planck Flux} \quad \textit{(Watts per square meter)}$$

The *photon-count rate* is $n=Nc/V$ *(per square meter per second)* and $h\upsilon$ is energy *(per count)*.

Relating photons to Maxwell energy density and Poynting flux

➡ *Relativistic variation and invariance of frequency (ω,k) and amplitudes*

How probability ψ-waves and flux ψ-waves evolved

*Properties of amplitude ψ*ψ-squares*

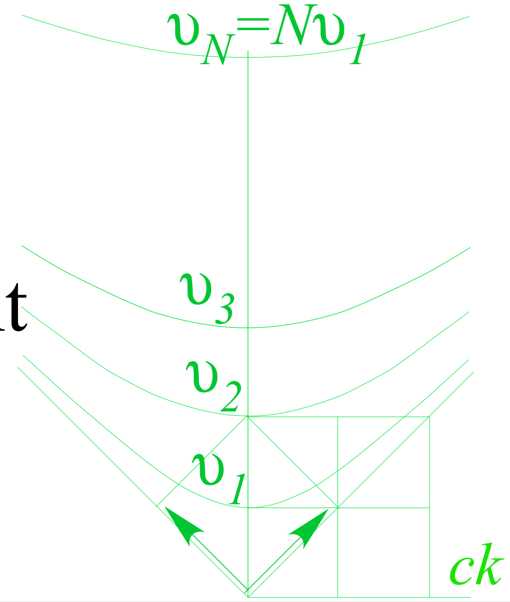
More on unmatched amplitudes AND unmatched frequencies AND unmatched quanta

Frequency and Amplitude Variance *2-CW-light vs 1-CW-light*
2-CW modes have invariance

Maxwell-Planck energy E is *photon number* $N(m^{-3})$ times *2-CW-frequency* υ_1.

Invariant to ρ — Each is ρ-invariant

$$E=\langle U\rangle\cdot V=\varepsilon_0\langle \mathsf{E}^2\rangle\cdot V=\varepsilon_0\langle \mathsf{E}_{2\text{-}CW}{}^*\mathsf{E}_{2\text{-}CW}\rangle\cdot V=hN\upsilon_1=h\upsilon_N$$



Photon number N and rest-frame frequencies $\upsilon_1 ... \upsilon_N$ are invariant to rapidity ρ and occupy *(ω,ck)-hyperbolas* in per-spacetime.

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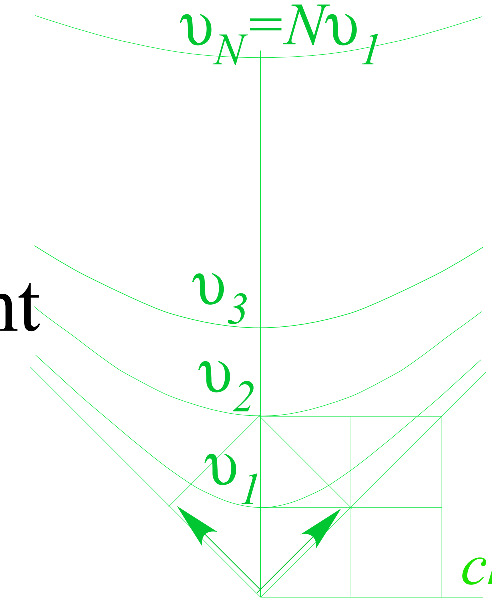

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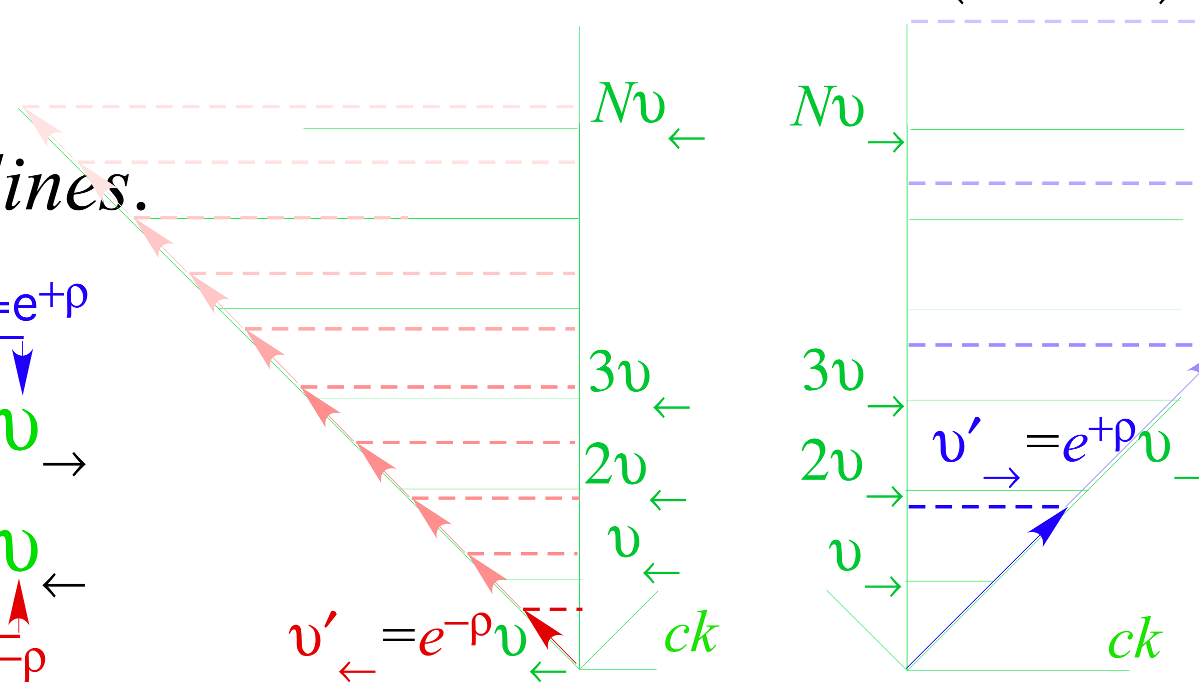
1-CW beams lack invariance (have *"variance"* ala′ Doppler)

Planck-Poynting flux S is *count rate* $n=Nc/V(m^{-2}s^{-1})$ times *1-CW-frequency* $\upsilon_{\leftarrow}$ or $\upsilon_{\rightarrow}$.

Count rate n and frequency υ Doppler shift by $b=e^{\pm\rho}$ factors and occupy *(ω=±ck)-baselines.*



Shifts by $b=e^{+2\rho}$ — Each blue shifts by $b=e^{+\rho}$

$$S_{\rightarrow}=cU_{\rightarrow}=c\varepsilon_0\langle \mathsf{E}^2\rangle=c\varepsilon_0\langle \mathsf{E}^{\rightarrow}_{1\text{-CW}}{}^*\mathsf{E}^{\rightarrow}_{1\text{-CW}}\rangle=hn_{\rightarrow}\upsilon_{\rightarrow}$$

$$S_{\leftarrow}=cU_{\leftarrow}=c\varepsilon_0\langle \mathsf{E}^2\rangle=c\varepsilon_0\langle \mathsf{E}^{\leftarrow}_{1\text{-CW}}{}^*\mathsf{E}^{\leftarrow}_{1\text{-CW}}\rangle=hn_{\leftarrow}\upsilon_{\leftarrow}$$

Shifts by $r=e^{-2\rho}$ — Each red shifts by $r=e^{-\rho}$

Note: $\mathsf{E}^{\leftrightarrow}_{1\text{-CW}}\sqrt{(c\varepsilon_0/h)}=\sqrt{(n_{\leftrightarrow}\upsilon_{\leftrightarrow})}$ is geometric mean of *amplitude frequency* $n_{\leftrightarrow}$ and *phase frequency* $\upsilon_{\leftrightarrow}$.

Important result below:

Amplitudes of 1-CW "exponentiate" just like frequency, and intensity does at twice the rate

(A double-double whammy!)

<u>*1-CW beams lack invariance*</u> (have *"variance"* ala′ Doppler)

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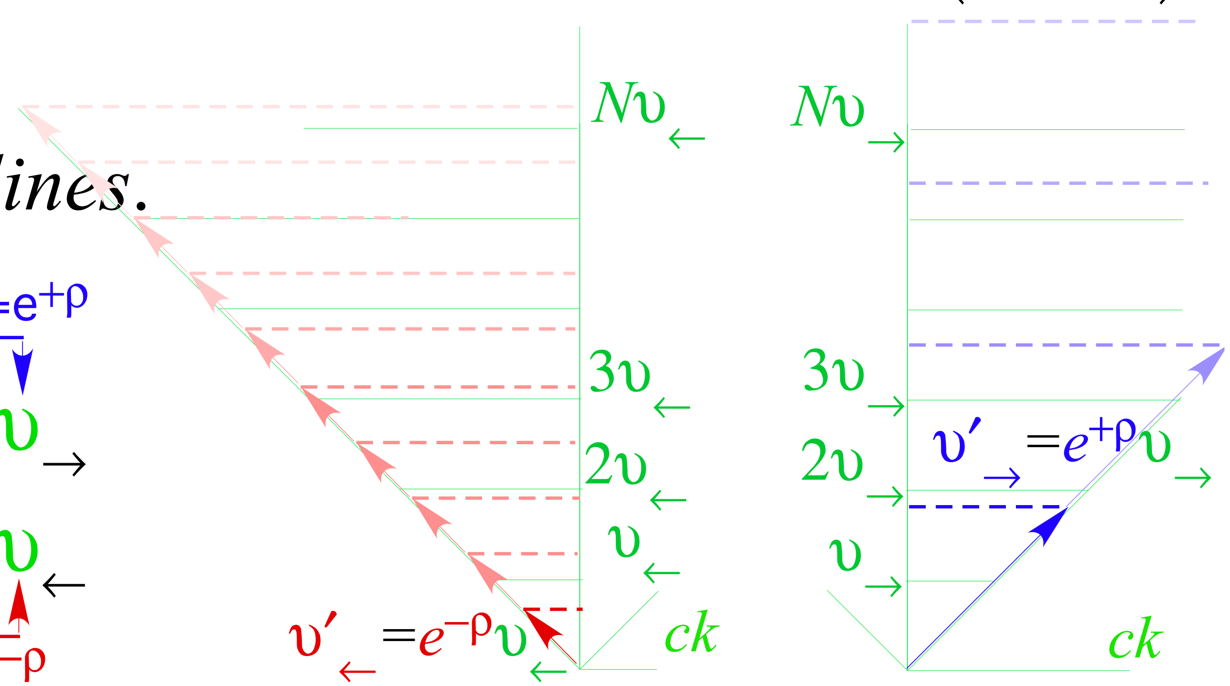

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Relating photons to Maxwell energy density and Poynting flux

Relativistic variation and invariance of frequency (ω,k) and amplitudes

➡ *How probability ψ-waves and flux ψ-waves evolved*

Properties of amplitude $\psi^*\psi$-squares

More on unmatched amplitudes AND unmatched frequencies AND unmatched quanta

How Probability Amplitudes ψ *or* ψ *Come About* *(An optical view)*

Maxwell-Planck-Poynting flux $S = cU = c\varepsilon_0 |\mathsf{E}|^2 = c\varepsilon_0 \mathsf{E}^*\mathsf{E} = nh\upsilon$ has count rate $n = Nc/V (m^{-2}s^{-1})$

If each E-field amplitude factor is scaled by a factor $\sqrt{\frac{c\varepsilon_0}{h\upsilon}} = \sqrt{\frac{\varepsilon_0}{h\kappa}}$ the result is a *flux probability amplitude* $\psi = \mathsf{E}\sqrt{\frac{c\varepsilon_0}{h\upsilon}}$ whose square equals flux count rate $n (m^{-2}s^{-1})$.

$$\psi^*\psi = n \quad (m^{-2}s^{-1})$$

A *fixed probability amplitude* $\psi = \mathsf{E}\sqrt{\frac{\varepsilon_0}{h\upsilon}}$ has square equal to N/V *(particles per volume)*.

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Here's how to answer Planck's worry about photons

Q: How can classical oscillator energy (Amplitude)^2(frequency)^2 jive with linear Planck law S=nhυ?

A: Let amplitude ψ *or* ψ *contain inverse square root of frequency:* $\psi = E\sqrt{\frac{c\varepsilon_0}{h\upsilon}}$ *the "quantum amplitude"*

$$Energy \sim |A|^2\upsilon^2 \text{ where vector potential } \mathbf{A} \text{ defines electric field: } \mathbf{E}=\frac{\partial\mathbf{A}}{\partial t}=i\omega\mathbf{A}=2\pi i\upsilon\mathbf{A}$$

$$Energy \sim |A|^2\upsilon^2 = \left|A\sqrt{\upsilon}\right|^2\upsilon = \left|\frac{E}{2\pi\upsilon}\sqrt{\upsilon}\right|^2\upsilon = \left|\frac{E}{2\pi\sqrt{\upsilon}}\right|^2\upsilon \sim \left|E\sqrt{\frac{c\varepsilon_0}{h\upsilon}}\right|^2 = nh\upsilon$$

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Probability Waves $\psi(x,t)$ *(More optical views)*

Optical E-field amplitudes like $\mathsf{E}(x,t)=\mathsf{E}_0 e^{i(kx-\omega t)}$ vary with space x and time t. So do scaled $\psi(x,t)$ ampliudes whose sum-Σ (integral-$\int$) over cells ΔV (or dV) must be particle number N. For 1-particle systems $(N=1)$ this is the *unit norm* rule.

$$\Sigma_j \psi(x_j,t)^*\psi(x_j,t)\Delta V_j=N \qquad \text{or:} \qquad \int\psi(x,t)^*\psi(x,t)dV=N$$

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Born interpreted $\psi(x,t)^*\psi(x,t)$ as *probable expectation* of particle count. Schrodinger objected to the *probability wave* interpretation that is now accepted and called the Schrodinger theory. A relativistic wave view lends merit to his objections.

Doppler Transformation of 2-CW Modes

Doppler shift of *opposite-k 1-CW* beams. As derived before phases are invariant: $(k'x'-\omega't'=kx-\omega t)$

E-*wave:* $E=E_{\rightarrow}e^{i(k_{\rightarrow}x-\omega_{\rightarrow}t)}+E_{\leftarrow}e^{i(k_{\leftarrow}x-\omega_{\leftarrow}t)}$

Ψ-*wave:* $\Psi=\psi_{\rightarrow}e^{i(k_{\rightarrow}x-\omega_{\rightarrow}t)}+\psi_{\leftarrow}e^{i(k_{\leftarrow}x-\omega_{\leftarrow}t)}$

blue shift

$$E'_{\rightarrow}=b\,E_{\rightarrow}=e^{+\rho}E_{\rightarrow}$$

red shift

$$E'_{\leftarrow}=r\,E_{\leftarrow}=e^{-\rho}E_{\leftarrow}$$

$$\psi=E\sqrt{\frac{\varepsilon_0}{h\upsilon}}$$

scaled blue shift

$$\psi'_{\rightarrow}=\sqrt{b}\;\psi_{\rightarrow}=e^{+\rho/2}\psi_{\rightarrow}$$

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Parameters related to *relative velocity* u:

$$\beta=u/c=\tanh\rho=\frac{\sinh\rho}{\cosh\rho}=\frac{e^{+\rho}-e^{-\rho}}{e^{+\rho}+e^{-\rho}}=\frac{b^2-1}{b^2+1}$$

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Transformation of *SWR* (or *SWQ*) and u_{GROUP} (or u_{PHASE}) is a *non-linear* transformation

$$SWR'=\frac{E'_{\rightarrow}-E'_{\leftarrow}}{E'_{\rightarrow}+E'_{\leftarrow}}=\frac{b^2E_{\rightarrow}-E_{\leftarrow}}{b^2E_{\rightarrow}+E_{\leftarrow}}=\frac{(1+\beta)E_{\rightarrow}-(1-\beta)E_{\leftarrow}}{(1+\beta)E_{\rightarrow}+(1-\beta)E_{\leftarrow}}=\frac{(E_{\rightarrow}-E_{\leftarrow})+\beta(E_{\rightarrow}+E_{\leftarrow})}{(E_{\rightarrow}+E_{\leftarrow})+\beta(E_{\rightarrow}-E_{\leftarrow})}=\frac{SWR+\beta}{1+\beta\cdot SWR}$$

SWR (or *SWQ*) *Transformation*

$$SWR'=\frac{SWR+\beta}{1+SWR\cdot\beta}=\frac{SWR+u/c}{1+SWR\cdot u/c}$$

u_{GROUP} (or u_{PHASE}) *Transformation*

$$u'_{GROUP}/c=\frac{u_{GROUP}/c+\beta}{1+u_{GROUP}\cdot\beta/c}=\frac{(u_{GROUP}+u)/c}{1+u_{GROUP}\cdot u/c^2}$$

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$$\psi=E\sqrt{\frac{\varepsilon_0}{h\upsilon}}$$

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SWR (or *SWQ*) *Transformation*

$$SWR'=\frac{SWR+\beta}{1+SWR\cdot\beta}=\frac{SWR+u/c}{1+SWR\cdot u/c}$$

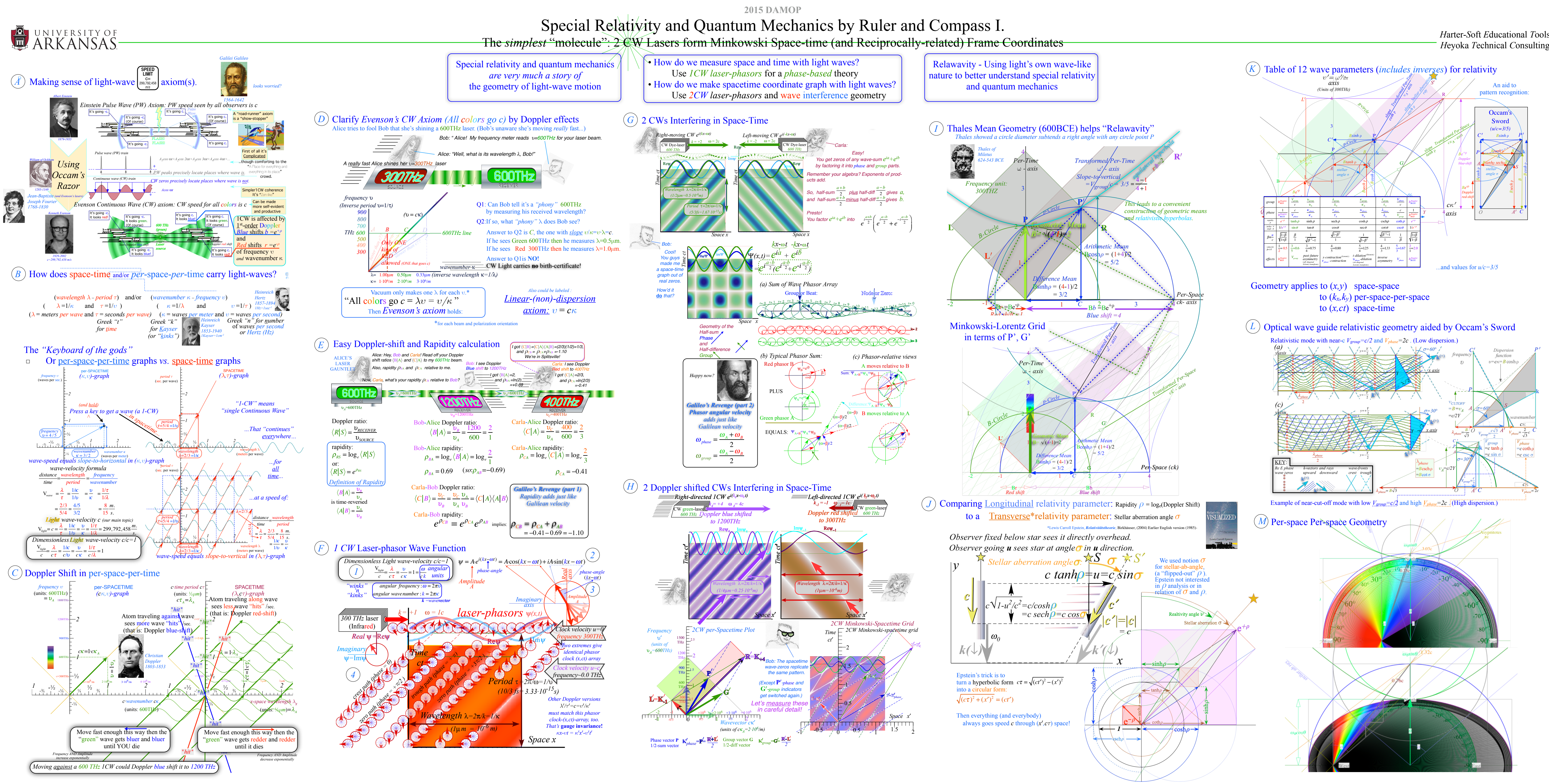
u_{GROUP} (or u_{PHASE}) *Transformation*

$$u'_{GROUP}/c=\frac{u_{GROUP}/c+\beta}{1+u_{GROUP}\cdot\beta/c}=\frac{(u_{GROUP}+u)/c}{1+u_{GROUP}\cdot u/c^2}$$

Both are restatements of hyperbolic trig identity: $\tanh(a+b)=\frac{\tanh(a)+\tanh(b)}{1+\tanh(a)\cdot\tanh(b)}$

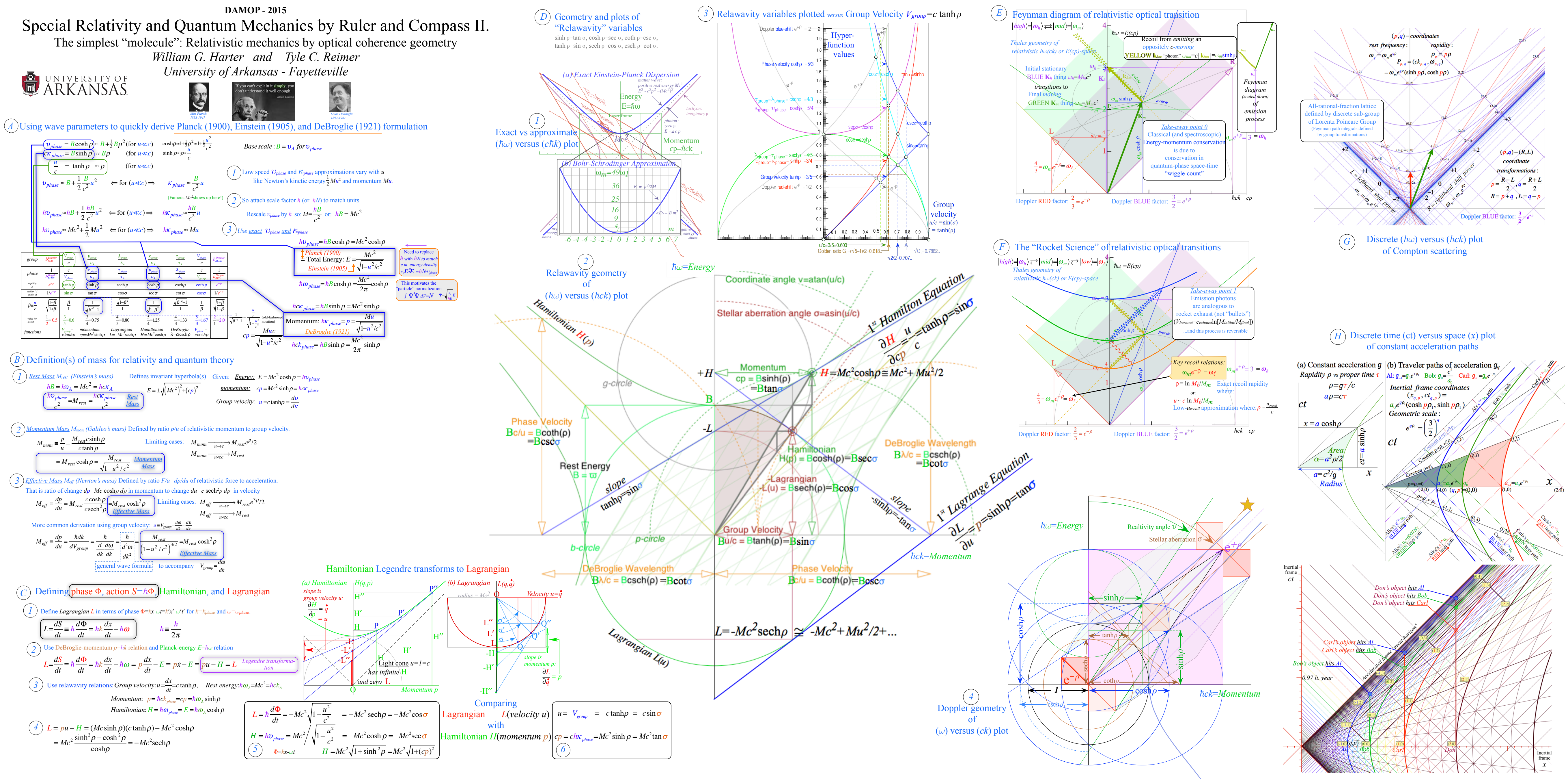
last term is ignorable if both a and b are small

Velocity addition is *non-linear* but rapidity addition is always *linear*: $\rho_{a+b}=\rho_a+\rho_b$



[Link to pdf version of *Part I* online]

Note: When printed at their optimal resolution, each poster is *7 feet across!*



[Link to pdf version of *Part II* online]

Note: When printed at their optimal resolution, each poster is *7 feet across!*