

Lecture 30 *Relativity*-Applications 4

Thursday 4.28.2016

Relativity: Relativistic quantum mechanics I Basic theory

(Unit 3 p.45-61 - 4.26.16)

- ➔ Using (some) wave parameters to develop relativistic quantum theory
 - Low rapidity approximations to v_{phase} and $c\kappa_{phase}$ match to Newtonian KE and momentum
 - How Mc^2 pops right up
 - Exact v_{phase} gives exact Planck-Einstein energy formulas (1900-1905)
 - Exact $c\kappa_{phase}$ gives exact Bohr momentum and dispersion formulas (1921-1927)
 - Bohr-Schrodinger approximation to dispersion (*Who threw away the Mc^2 ?!!*)
- “*What’s the Matter with Mass?*” Definition(s) of relativistic and quantum mechanical mass
 - (1) Einsteinian rest mass (2) Galilean momentum mass (3) Newtonian effective mass
 - Three Faces of Eve: A photon’s split-mass personality
- Relativistic action S and Lagrangian-Hamiltonian relations: How invariant phase works
 - The Legendre transformation relations
 - Deriving Lagrangian and Hamiltonian functions
 - Geometry of 1st Lagrangian and 1st Hamiltonian equations
 - Poincare invariant action differential
 - Hamilton-Jacobi equations
 - How Hamilton-Jacobi derives Schrodinger-op equations
 - How Huygens contact transformations determine motion

Using (some) wave parameters to develop relativistic quantum theory

➔ Low rapidity approximations to ν_{phase} and $c\kappa_{phase}$ match to Newtonian KE and momentum

How Mc^2 pops right up

Exact ν_{phase} gives exact Planck-Einstein energy formulas (1900-1905)

Exact $c\kappa_{phase}$ gives exact Bohr momentum and dispersion formulas (1921-1927)

Bohr-Schrodinger approximation to dispersion (*Who threw away the Mc^2 ?!!*)

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(1) Einsteinian rest mass (2) Galilean momentum mass (3) Newtonian effective mass

Three Faces of Eve: A photon’s split personality

Relativistic action S and Lagrangian-Hamiltonian relations: How invariant phase works

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Hamilton-Jacobi equations

How Hamilton-Jacobi derives Schrodinger-op equations

How Huygens contact transformations determine motion

Using (some) wave parameters to develop relativistic quantum theory

$$v_{phase} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \text{ (for } u \ll c \text{)}$$

$$cK_{phase} = B \sinh \rho \approx B \rho \text{ (for } u \ll c \text{)}$$

$$\cosh \rho \approx 1 + \frac{1}{2} \rho^2$$

$$\sinh \rho \approx \rho$$

$$B = v_A$$

$$B = v_A = cK_A$$

At low speeds: ...

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{v_{group}}{v_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{K_{group}}{K_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{K_{phase}}{K_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{v_{phase}}{v_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh \rho$	$\sinh \rho$	$\operatorname{sech} \rho$	$\cosh \rho$	$\operatorname{csch} \rho$	$\operatorname{coth} \rho$	$e^{+\rho}$
stellar ∇ angle σ	$1/e^{+\rho}$	$\sin \sigma$	$\tan \sigma$	$\cos \sigma$	$\sec \sigma$	$\cot \sigma$	$\csc \sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\frac{\sqrt{1-\beta}}{\sqrt{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^2-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^2-1}}{1}$	$\frac{1}{\beta}$	$\frac{\sqrt{1+\beta}}{\sqrt{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2} = 0.5$	$\frac{3}{5} = 0.6$	$\frac{3}{4} = 0.75$	$\frac{4}{5} = 0.80$	$\frac{5}{4} = 1.25$	$\frac{4}{3} = 1.33$	$\frac{5}{3} = 1.67$	$\frac{2}{1} = 2.0$

[RelaWavity Web Simulation](#)
 Relativistic Terms (Dual plot w/expanded table)

Using (some) wave parameters to develop relativistic quantum theory

$$v_{phase} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \quad (\text{for } u \ll c)$$

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phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{K_{phase}}{K_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{v_{phase}}{v_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
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$\beta \equiv \frac{u}{c}$	$\frac{\sqrt{1-\beta}}{\sqrt{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\frac{\sqrt{1+\beta}}{\sqrt{1-\beta}}$
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$\beta \equiv \frac{u}{c}$	$\frac{\sqrt{1-\beta}}{\sqrt{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\frac{\sqrt{1+\beta}}{\sqrt{1-\beta}}$
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At low speeds:

$$v_{phase} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad K_{phase} \approx \frac{B}{c^2} u$$

time	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{v_{group}}{v_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{v_{phase}}{v_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
space	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{K_{phase}}{K_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{K_{group}}{K_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh \rho$	$\sinh \rho$	$\operatorname{sech} \rho$	$\cosh \rho$	$\operatorname{csch} \rho$	$\operatorname{coth} \rho$	$e^{+\rho}$
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At low speeds:
 \Leftarrow for $(u \ll c) \Rightarrow$

$$K_{phase} \approx \frac{B}{c^2} u$$

v_{phase} and K_{phase} resemble formulae for Newton's kinetic energy and momentum

Resembles: $const. + \frac{1}{2} Mu^2$

Resembles: Mu

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{v_{group}}{v_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{K_{group}}{K_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{K_{phase}}{K_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{v_{phase}}{v_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
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⇐ for ($u \ll c$) ⇒

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v_{phase} and K_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2} Mu^2$ and momentum Mu .

Rescale v_{phase} by h so: $M = \frac{hB}{c^2}$

Resembles: $const. + \frac{1}{2} Mu^2$

Resembles: Mu

time	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{v_{group}}{v_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{v_{phase}}{v_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
space	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{K_{phase}}{K_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{K_{group}}{K_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh \rho$	$\sinh \rho$	$\operatorname{sech} \rho$	$\cosh \rho$	$\operatorname{csch} \rho$	$\operatorname{coth} \rho$	$e^{+\rho}$
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$$B = v_A$$

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At low speeds:

$$v_{phase} \approx B + \frac{1}{2} \frac{B}{c^2} u^2$$

⇐ for ($u \ll c$) ⇒

$$K_{phase} \approx \frac{B}{c^2} u$$

v_{phase} and K_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2} Mu^2$ and momentum Mu .

Rescale v_{phase} by h so: $M = \frac{hB}{c^2}$

So attach scale factor h to match units.

Resembles: $const. + \frac{1}{2} Mu^2$

Resembles: Mu

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{v_{group}}{v_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{K_{group}}{K_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{K_{phase}}{K_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{v_{phase}}{v_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
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Rescale v_{phase} by h so: $M = \frac{hB}{c^2}$

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Resembles: $const. + \frac{1}{2} Mu^2$

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phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{K_{phase}}{K_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{v_{phase}}{v_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh \rho$	$\sinh \rho$	$\operatorname{sech} \rho$	$\cosh \rho$	$\operatorname{csch} \rho$	$\operatorname{coth} \rho$	$e^{+\rho}$
stellar ∇ angle σ	$1/e^{+\rho}$	$\sin \sigma$	$\tan \sigma$	$\cos \sigma$	$\sec \sigma$	$\cot \sigma$	$\csc \sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\frac{\sqrt{1-\beta}}{\sqrt{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\frac{\sqrt{1+\beta}}{\sqrt{1-\beta}}$
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Resembles: $const. + \frac{1}{2} Mu^2$

Resembles: Mu

So attach scale factor h to match units.

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phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{K_{phase}}{K_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{v_{phase}}{v_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh \rho$	$\sinh \rho$	$\operatorname{sech} \rho$	$\cosh \rho$	$\operatorname{csch} \rho$	$\operatorname{coth} \rho$	$e^{+\rho}$
stellar ∇ angle σ	$1/e^{+\rho}$	$\sin \sigma$	$\tan \sigma$	$\cos \sigma$	$\sec \sigma$	$\cot \sigma$	$\csc \sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\frac{\sqrt{1-\beta}}{\sqrt{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^2-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^2-1}}{1}$	$\frac{1}{\beta}$	$\frac{\sqrt{1+\beta}}{\sqrt{1-\beta}}$
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rapidity ρ	$e^{-\rho}$	$\tanh \rho$	$\sinh \rho$	$\operatorname{sech} \rho$	$\cosh \rho$	$\operatorname{csch} \rho$	$\operatorname{coth} \rho$	$e^{+\rho}$
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$\beta \equiv \frac{u}{c}$	$\frac{\sqrt{1-\beta}}{\sqrt{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^2-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^2-1}}{1}$	$\frac{1}{\beta}$	$\frac{\sqrt{1+\beta}}{\sqrt{1-\beta}}$
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Using (some) wave parameters to develop relativistic quantum theory

Low rapidity approximations to ν_{phase} and $c\kappa_{phase}$ match to Newtonian KE and momentum

How Mc^2 pops right up

➔ Exact ν_{phase} gives exact Planck-Einstein energy formulas (1900-1905)

Exact $c\kappa_{phase}$ gives exact Bohr momentum and dispersion formulas (1921-1927)

Bohr-Schrodinger approximation to dispersion (*Who threw away the Mc^2 ?!!*)

“*What’s the Matter with Mass?*” Definition(s) of relativistic and quantum mechanical mass

(1) Einsteinian rest mass (2) Galilean momentum mass (3) Newtonian effective mass

Three Faces of Eve: A photon’s split personality

Relativistic action S and Lagrangian-Hamiltonian relations: How invariant phase works

The Legendre transformation relations

Deriving Lagrangian and Hamiltonian functions

Geometry of 1st Lagrangian and 1st Hamiltonian equations

Poincare invariant action differential

Hamilton-Jacobi equations

How Hamilton-Jacobi derives Schrodinger-op equations

How Huygens contact transformations determine motion

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(old-fashioned notation)

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$$= \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1-u^2/c^2}}$$

Planck (1900) \uparrow
 Einstein (1905) \uparrow

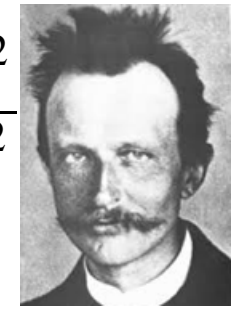
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rapidity ρ	$e^{-\rho}$	$\tanh \rho$	$\sinh \rho$	$\text{sech } \rho$	$\cosh \rho$	$\text{csch } \rho$	$\text{coth } \rho$	$e^{+\rho}$
stellar angle σ	$1/e^{+\rho}$	$\sin \sigma$	$\tan \sigma$	$\cos \sigma$	$\sec \sigma$	$\cot \sigma$	$\csc \sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\frac{\sqrt{1-\beta}}{\sqrt{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^2-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^2-1}}{1}$	$\frac{1}{\beta}$	$\frac{\sqrt{1+\beta}}{\sqrt{1-\beta}}$
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Max Planck
1858-1947

Using (some) wave parameters to develop relativistic quantum theory



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$$B = v_A$$

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$$\cosh \rho \approx 1 + \frac{1}{2} \rho^2 \approx 1 + \frac{1}{2} \frac{u^2}{c^2}$$

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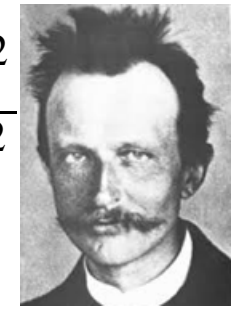
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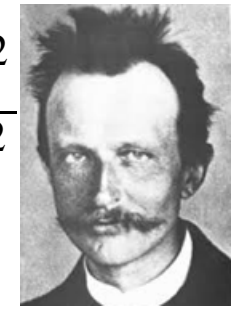
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For more visit the Pirelli Challenge Site
[Quantized amplitude](#)

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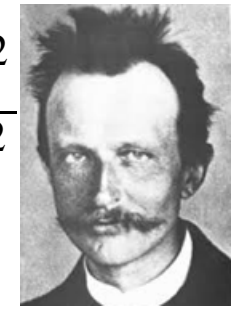
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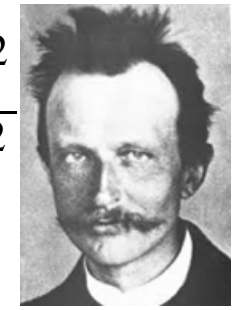
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Resolution and dirty secret: E , N , and v_{phase} are all frequencies!

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Low rapidity approximations to ν_{phase} and $c\kappa_{phase}$ match to Newtonian KE and momentum

How Mc^2 pops right up

Exact ν_{phase} gives exact Planck-Einstein energy formulas (1900-1905)

➔ Exact $c\kappa_{phase}$ gives exact Bohr momentum and dispersion formulas (1921-1927)

Bohr-Schrodinger approximation to dispersion (*Who threw away the Mc^2 ?!!*)

“*What’s the Matter with Mass?*” Definition(s) of relativistic and quantum mechanical mass

(1) Einsteinian rest mass (2) Galilean momentum mass (3) Newtonian effective mass

Three Faces of Eve: A photon’s split personality

Relativistic action S and Lagrangian-Hamiltonian relations: How invariant phase works

The Legendre transformation relations

Deriving Lagrangian and Hamiltonian functions

Geometry of 1st Lagrangian and 1st Hamiltonian equations

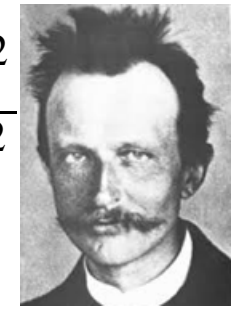
Poincare invariant action differential

Hamilton-Jacobi equations

How Hamilton-Jacobi derives Schrodinger-op equations

How Huygens contact transformations determine motion

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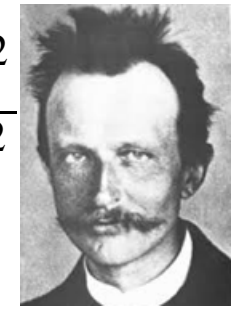
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value for $\beta=3/5$	$\frac{1}{2} = 0.5$	$\frac{3}{5} = 0.6$	$\frac{3}{4} = 0.75$	$\frac{4}{5} = 0.80$	$\frac{5}{4} = 1.25$	$\frac{4}{3} = 1.33$	$\frac{5}{3} = 1.67$	$\frac{2}{1} = 2.0$

Using (some) wave parameters to develop relativistic quantum theory



Max Planck
1858-1947

$$B = v_A$$

$$B = v_A = cK_A$$

$$\cosh \rho \approx 1 + \frac{1}{2} \rho^2 \approx 1 + \frac{1}{2} \frac{u^2}{c^2}$$

$$\sinh \rho \approx \rho \approx \frac{u}{c}$$

$$v_{phase} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \quad (\text{for } u \ll c)$$

$$cK_{phase} = B \sinh \rho \approx B \rho \quad (\text{for } u \ll c)$$

$$\frac{u}{c} = \tanh \rho \approx \rho \quad (\text{for } u \ll c)$$

At low speeds:

$$v_{phase} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow$$

$$K_{phase} \approx \frac{B}{c^2} u$$

v_{phase} and K_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2} Mu^2$ and momentum Mu .

Rescale v_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

$$hv_{phase} \approx hB + \frac{1}{2} \frac{hB}{c^2} u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow$$

$$hK_{phase} \approx \frac{hB}{c^2} u$$

So attach scale factor h (or hN) to match units.

$$hv_{phase} \approx Mc^2 + \frac{1}{2} Mu^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow$$

$$hK_{phase} \approx Mu$$

Lucky coincidences?? Cheap trick??
...Try exact v_{phase} and K_{phase} ...

Need to replace h with hN to match e.m. energy density $\epsilon_0 E \cdot E = hN v_{phase}$

$$hv_{phase} = hB \cosh \rho = Mc^2 \cosh \rho$$

Planck (1900)

$$= \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1-u^2/c^2}}$$

Einstein (1905)

$$hK_{phase} = hB \sinh \rho = Mc^2 \sinh \rho$$

$$\frac{1}{\sqrt{\beta^2 - 1}} = \frac{\frac{u}{c}}{\sqrt{1 - \frac{u^2}{c^2}}} \quad (\text{old-fashioned notation})$$

$$cp = \frac{Mc u}{\sqrt{1 - u^2/c^2}}$$

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{v_{group}}{v_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{K_{group}}{K_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$
phase	$b_{BLUE}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{K_{phase}}{K_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{v_{phase}}{v_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$
rapidity ρ	$e^{-\rho}$	$\tanh \rho$	$\sinh \rho$	$\text{sech } \rho$	$\cosh \rho$	$\text{csch } \rho$	$\text{coth } \rho$
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phase	$b_{BLUE}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{K_{phase}}{K_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{v_{phase}}{v_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$b_{RED}^{Doppler}$
rapidity ρ	$e^{-\rho}$	$\tanh \rho$	$\sinh \rho$	$\text{sech } \rho$	$\cosh \rho$	$\text{csch } \rho$	$\text{coth } \rho$	$e^{+\rho}$
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DeBroglie (1921)

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Einstein (1905)

This motivates the "particle" normalization $\int \Psi^* \Psi dV = N$ $\Psi = \sqrt{\frac{\epsilon_0}{hv}} E$

$$hK_{phase} = hB \sinh \rho = Mc^2 \sinh \rho$$

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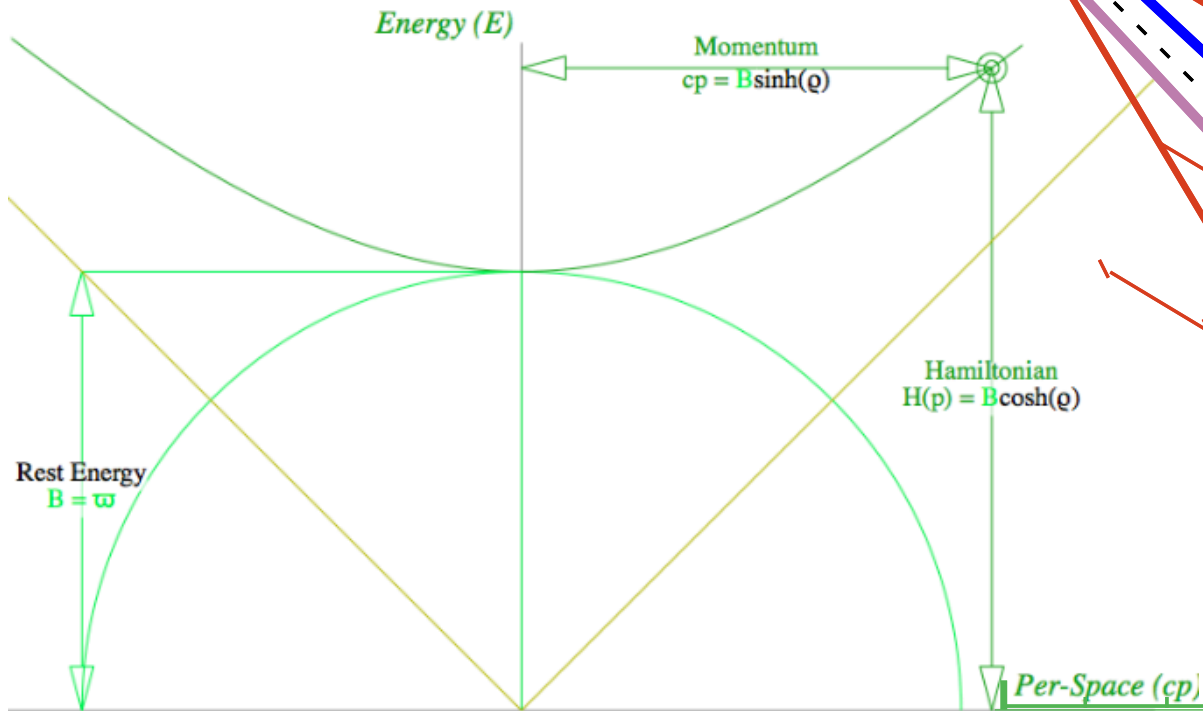
Hamilton-Jacobi equations

How Hamilton-Jacobi derives Schrodinger-op equations

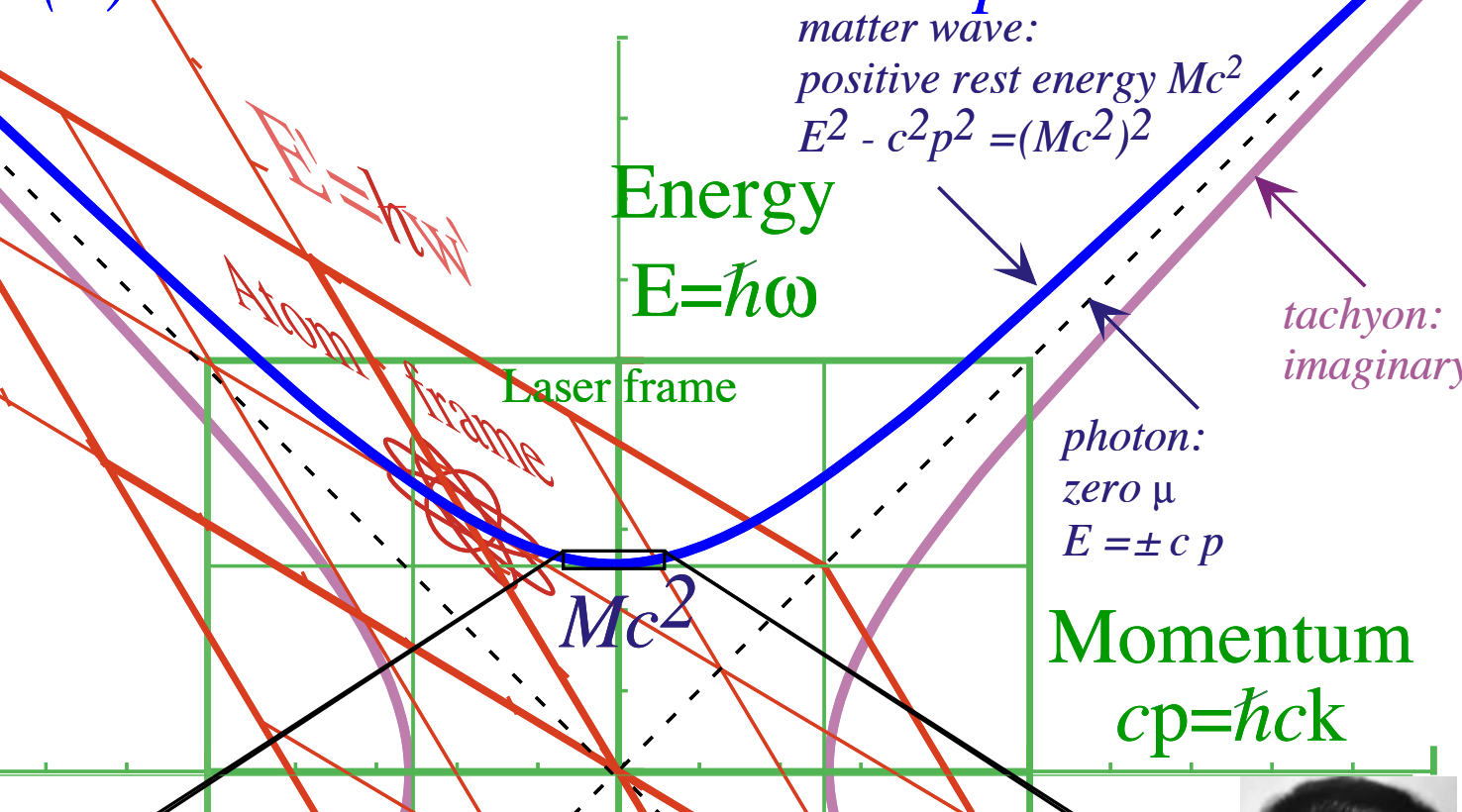
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RelaWavity Web Simulation
 Relativistic Terms - Einstein-Planck Dispersion



(a) Exact Einstein-Planck Dispersion



Mass (resting)

$$hB = h\nu_A = Mc^2 = hck_A$$

Energy

$$h\nu_{phase} = E = h\nu_A \cosh \rho$$

Momentum

$$hck_{phase} = cp = hck_A \sinh \rho = h\nu_A \sinh \rho$$

Energy versus Momentum

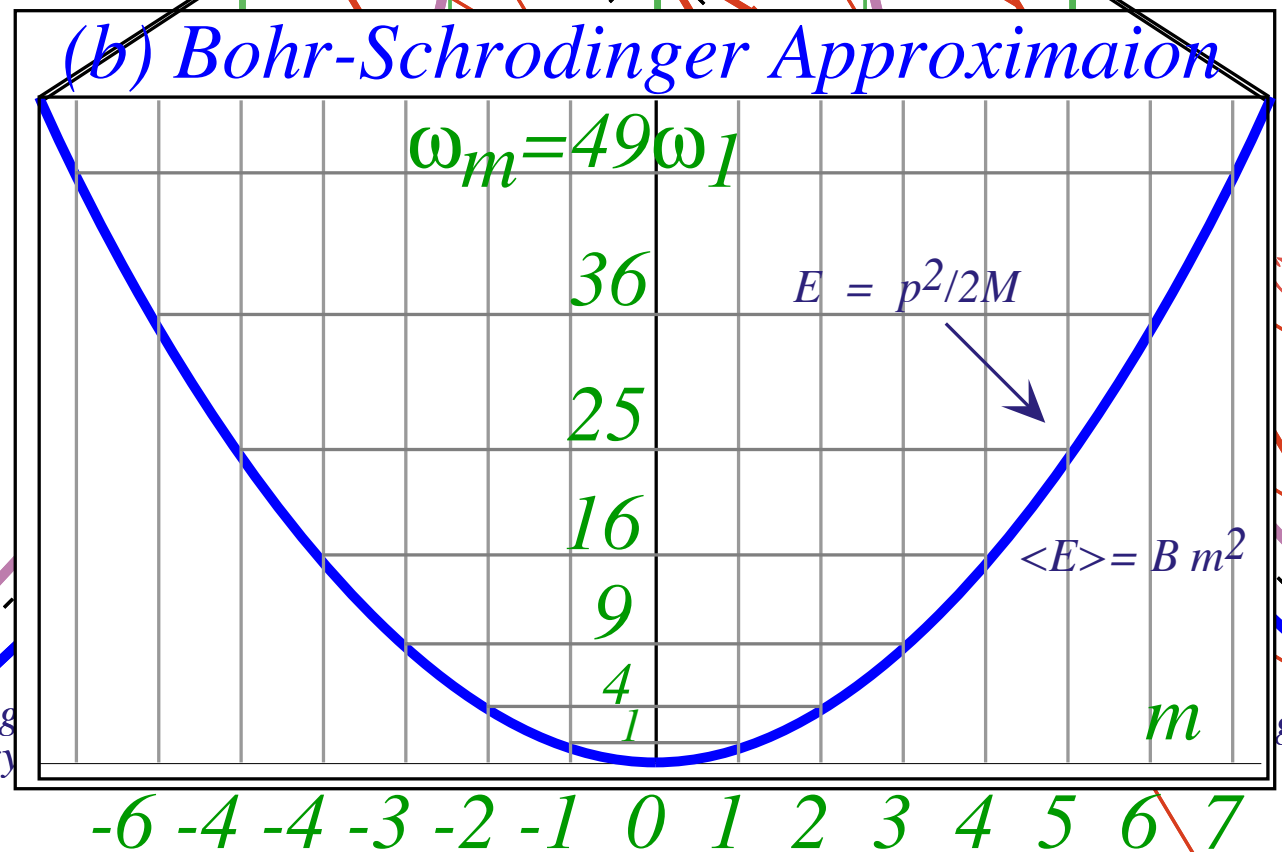
$$E^2 = (Mc^2)^2 \cosh^2 \rho$$

$$= (Mc^2)^2 (1 + \sinh^2 \rho) = (Mc^2)^2 + (cp)^2 \Rightarrow E = \pm \sqrt{(Mc^2)^2 + (cp)^2} \approx Mc^2 + \frac{p^2}{2M}$$



Niels Bohr
1885-1962

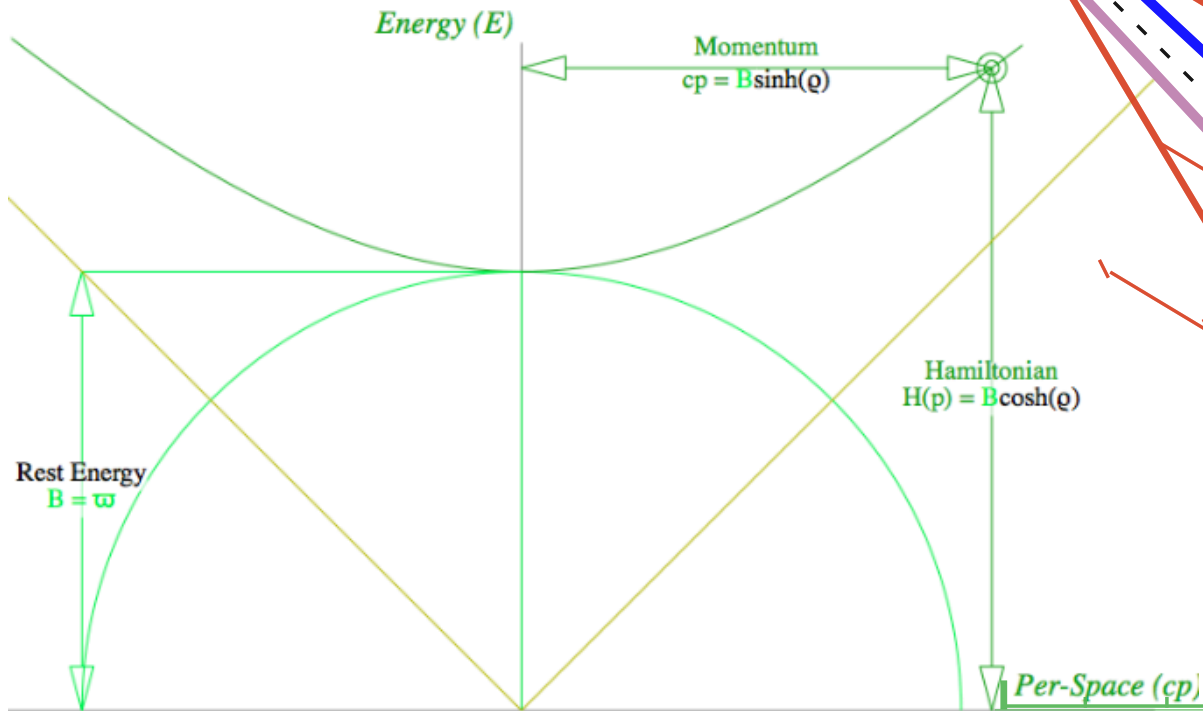
(b) Bohr-Schrodinger Approximation



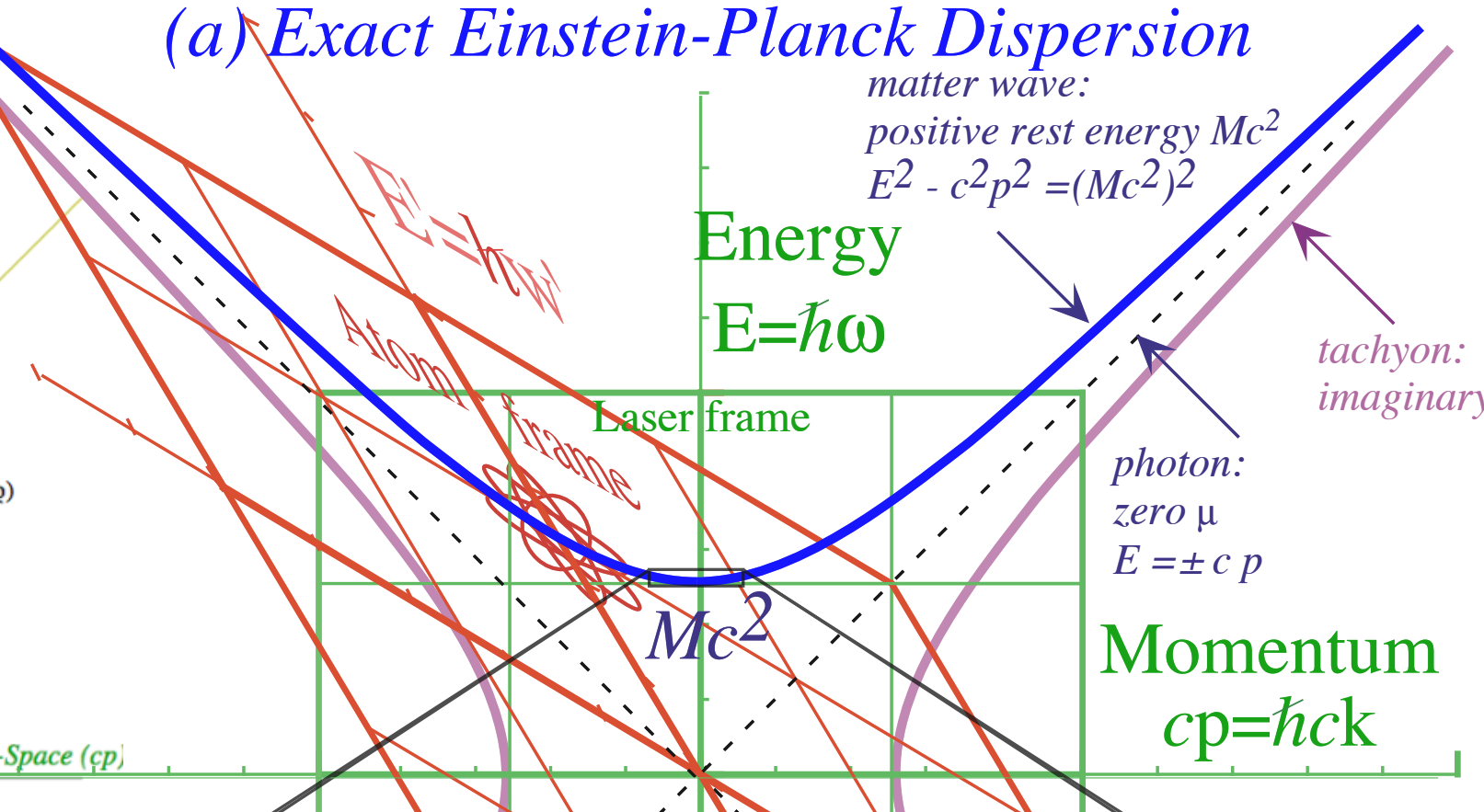
Erwin Schrodinger
1887-1961

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Mass (resting)

$$hB = h\nu_A = Mc^2 = hc\kappa_A$$

Energy

$$h\nu_{phase} = E = h\nu_A \cosh \rho$$

Momentum

$$hc\kappa_{phase} = cp = hc\kappa_A \sinh \rho = h\nu_A \sinh \rho$$

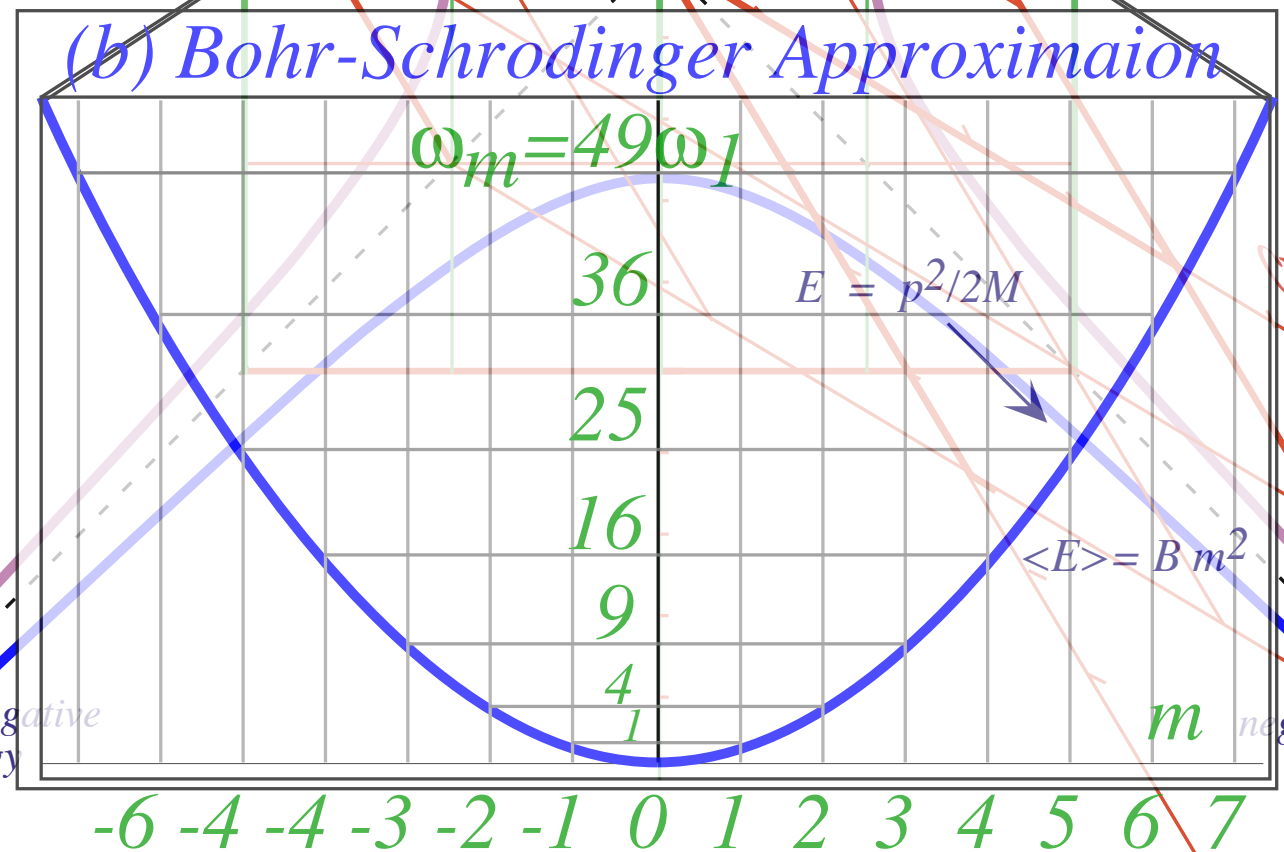
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low speed approximation

(b) Bohr-Schrodinger Approximaion



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velocity: $u = c \tanh \rho = \frac{d\nu}{d\kappa}$

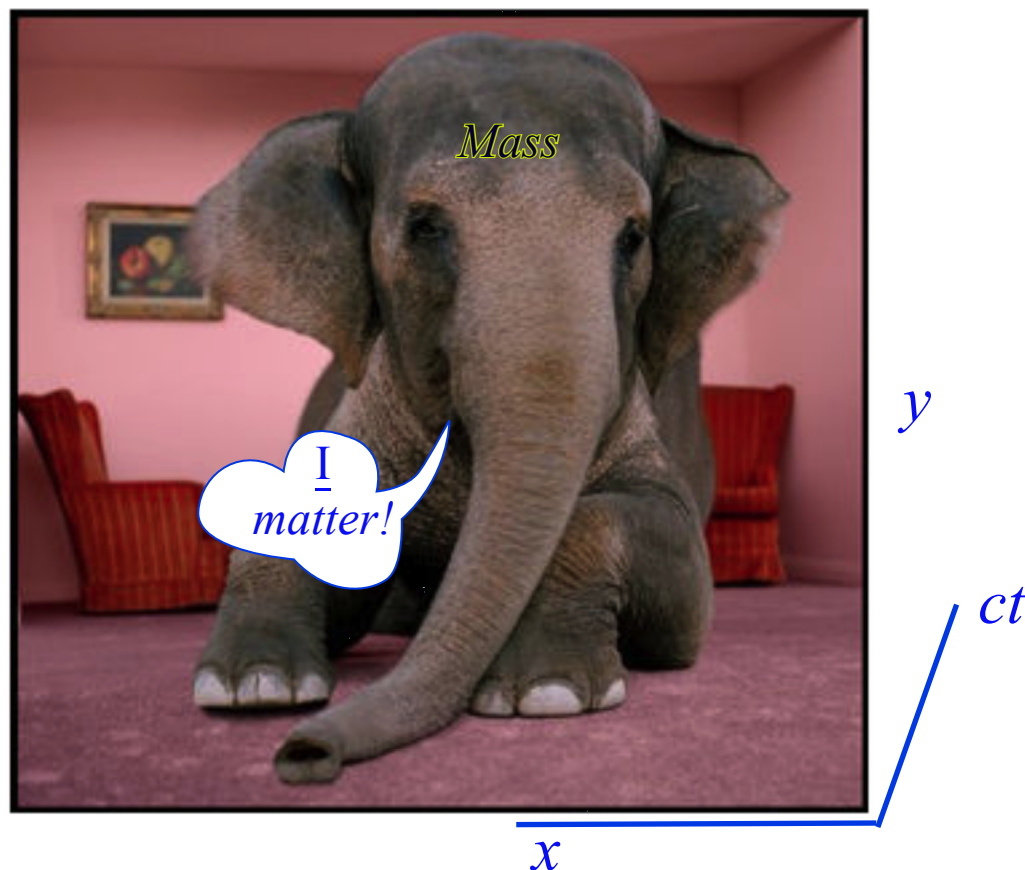
Rest Mass M_{rest} (Einstein's mass)

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Defines invariant hyperbola(s)

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- *What's the matter with Mass?*



Shining some light on the elephant in the spacetime room

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velocity: $u = c \tanh \rho = \frac{d\nu}{dK}$

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Effective Mass M_{eff} (Newton's mass) Defined by ratio $F/a = dp/du$ of relativistic force to acceleration.

Definition(s) of mass for relativity/quantum

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$= h\nu_{phase}$

Rest Mass M_{rest} (Einstein's mass)

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momentum: $cp = Mc^2 \sinh \rho$

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$= h c \kappa_{phase}$

$\frac{h\nu_{phase}}{c^2} = M_{rest} = \frac{h c \kappa_{phase}}{c^2}$ Rest Mass

velocity: $u = c \tanh \rho = \frac{d\nu}{d\kappa}$

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$M_{mom} \equiv \frac{p}{u} = \frac{M_{rest} c \sinh \rho}{c \tanh \rho}$

Limiting cases: $M_{mom} \xrightarrow{u \rightarrow c} M_{rest} e^{\rho} / 2$

$M_{mom} \xrightarrow{u \ll c} M_{rest}$

$= M_{rest} \cosh \rho = \frac{M_{rest}}{\sqrt{1 - u^2 / c^2}}$ Momentum Mass

Effective Mass M_{eff} (Newton's mass) Defined by ratio $F/a = dp/du$ of relativistic force to acceleration.

That is ratio of change $dp = Mc \cosh \rho d\rho$ in momentum to change $du = c \operatorname{sech}^2 \rho d\rho$ in velocity

Definition(s) of mass for relativity/quantum

Given: Energy: $E = Mc^2 \cosh \rho$

$= h\nu_{phase}$

momentum: $cp = Mc^2 \sinh \rho$

$= hc\kappa_{phase}$

velocity: $u = c \tanh \rho = \frac{d\nu}{d\kappa}$

Defines invariant hyperbola(s)

$$E = \pm \sqrt{(Mc^2)^2 + (cp)^2}$$

Rest Mass M_{rest} (Einstein's mass)

$$hB = h\nu_A = Mc^2 = hc\kappa_A$$

$$\frac{h\nu_{phase}}{c^2} = M_{rest} = \frac{hc\kappa_{phase}}{c^2} \quad \frac{\text{Rest}}{\text{Mass}}$$

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$\frac{h\nu_{phase}}{c^2} = M_{rest} = \frac{hcK_{phase}}{c^2}$ Rest Mass

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Definition(s) of mass for relativity/quantum

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general wave formula

to accompany

$$V_{group} = \frac{d\omega}{dk}$$

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Effective Mass M_{eff} (Newton's mass) Defined by $F/a = dp/du$

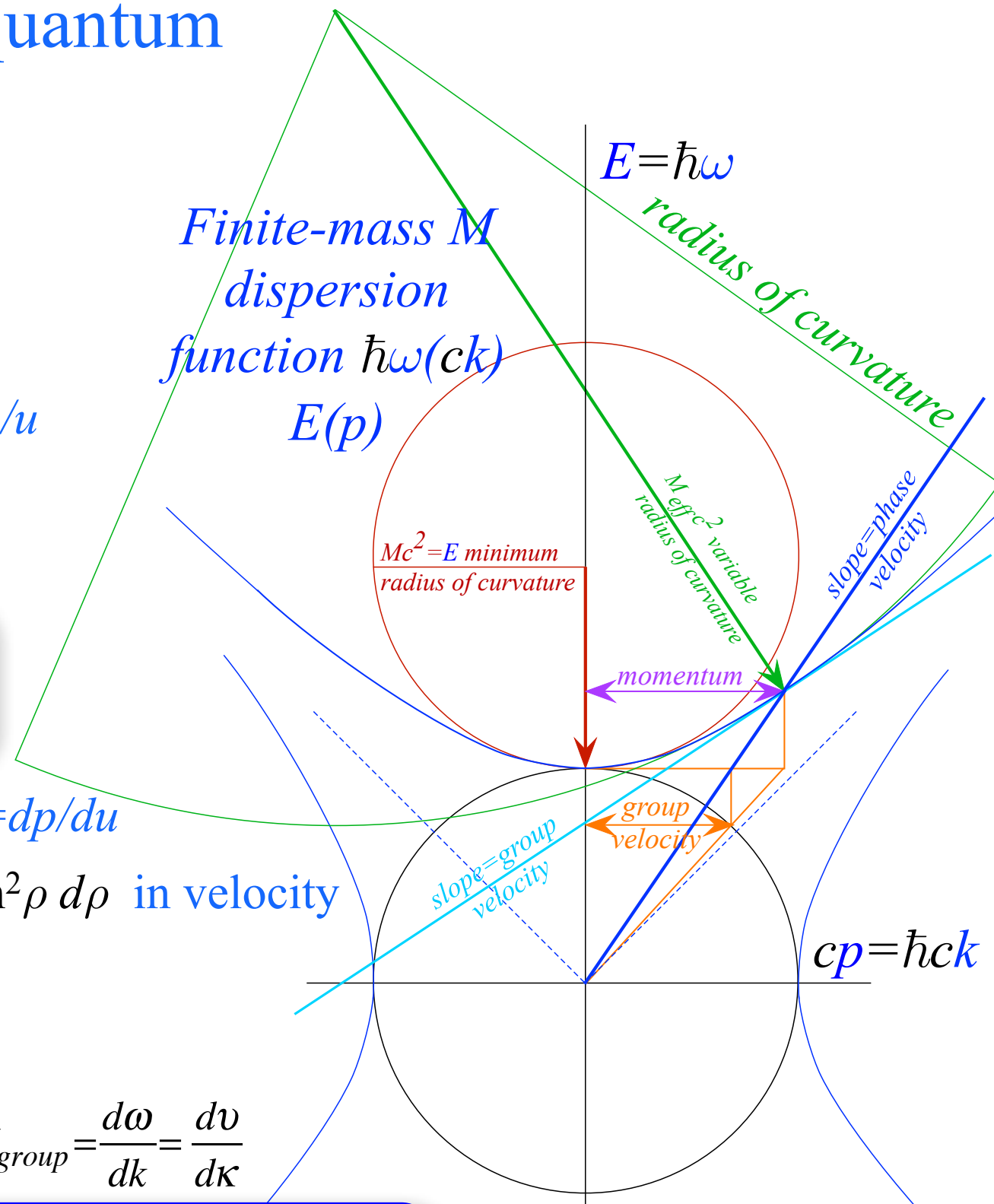
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general wave formula to accompany $V_{group} = \frac{d\omega}{dk}$



Effective mass is proportional to the radius of curvature of $\omega(k)$ dispersion.

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How much mass does a γ -photon have?

Rest Mass (a) γ -rest mass: $M_{rest}^{\gamma} = 0$,

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Effective Mass (c) γ -effective mass: $M_{eff}^{\gamma} = \infty$.

Definition(s) of mass for relativity/quantum

How much mass does a γ -photon have?

$$M_{mom}^{\gamma} = \frac{h\nu}{c^2} = \nu(1.2 \cdot 10^{-51}) \text{kg} \cdot \text{s}$$
$$= 4.5 \cdot 10^{-36} \text{kg} \quad (\text{for: } \nu=600\text{THz})$$

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Newton complained about his “corpuscles” of light having “fits” (going crazy).

(For Newton these 3 mass values would be evidence of triple Schizophrenia.)

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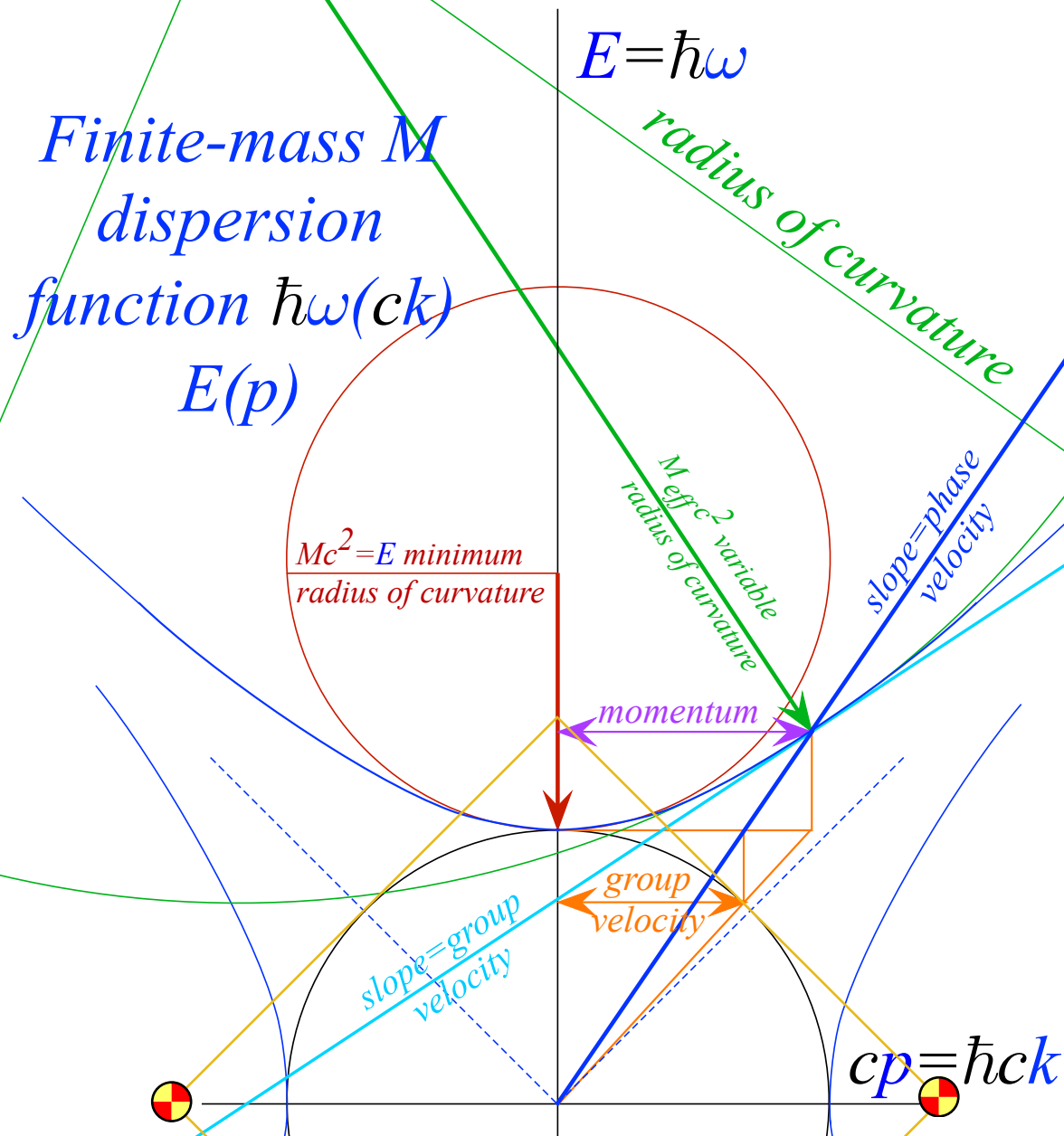
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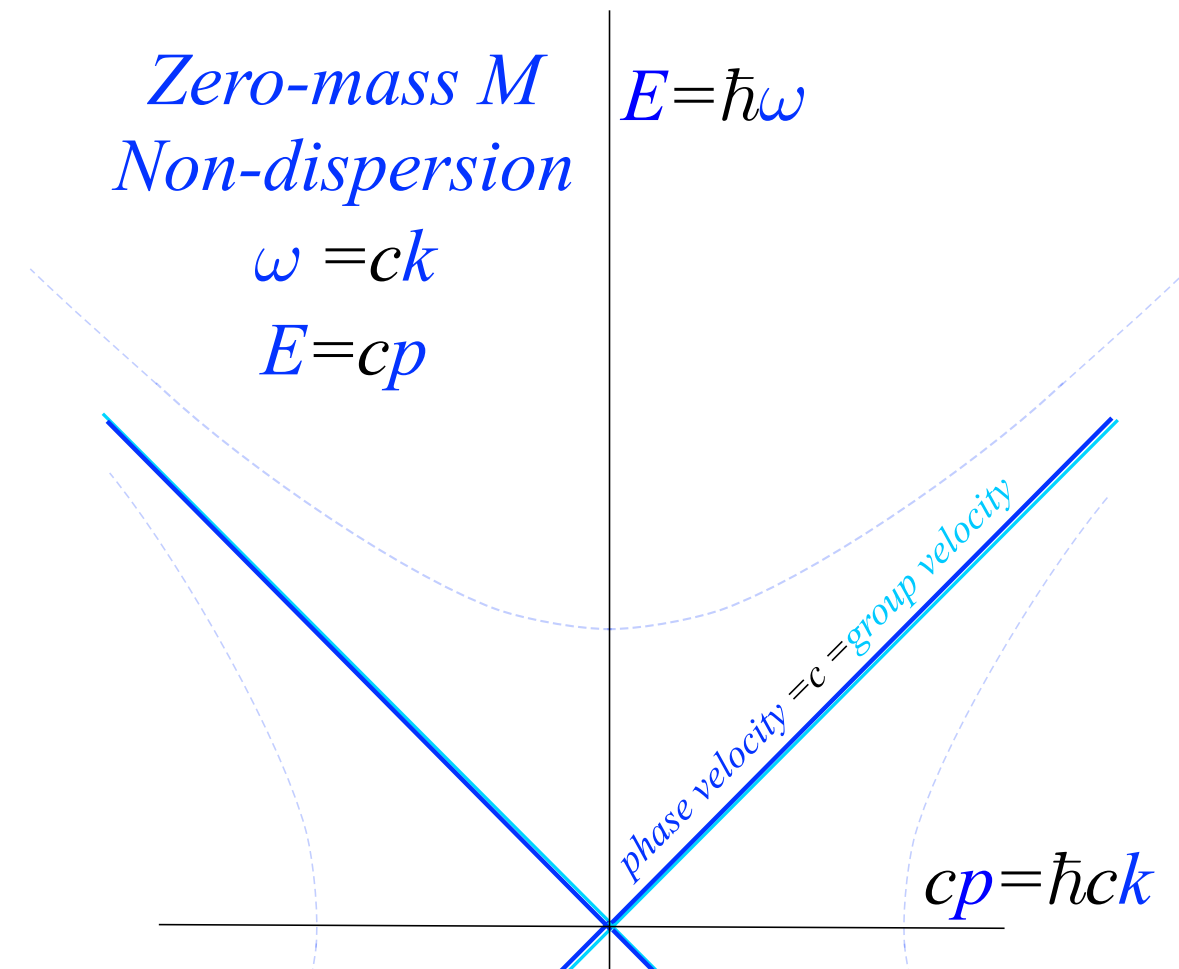
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γ -(non)-dispersion has
INFINITE
radius of curvature

Zero-mass M
Non-dispersion
 $\omega = ck$
 $E = cp$



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Define *Lagrangian* L using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega't'$ for wave of $k = k_{phase}$ and $\omega = \omega_{phase}$.

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega$$

$$\hbar \equiv \frac{h}{2\pi}$$

$$\begin{aligned} h\nu_A &= Mc^2 = hc\kappa_A \\ h\nu_{phase} &= E = h\nu_A \cosh \rho \\ hc\kappa_{phase} &= cp = h\nu_A \sinh \rho \end{aligned}$$

Prior wave relations

← linear Hz format angular phasor →
format format

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$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega = p \frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L \quad \text{Legendre transformation}$$

$$p = \hbar k = Mc \sinh \rho$$

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$$L \text{ is : } Mc^2 \frac{-1}{\cosh \rho} = -Mc^2 \operatorname{sech} \rho$$

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Prior wave relations

← linear Hz
format

angular phasor
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Note: $Mc u = Mc^2 \tanh \rho$

$$= Mc^2 \frac{\sinh^2 \rho - \cosh^2 \rho}{\cosh \rho} = -Mc^2 \operatorname{sech} \rho$$

Compare *Lagrangian* L

$$L = \hbar \dot{\Phi} = -Mc^2 \sqrt{1 - \frac{u^2}{c^2}} = -Mc^2 \operatorname{sech} \rho$$

$$\hbar \omega_A = Mc^2 = \hbar c k_A$$

Prior wave relations

$$\hbar \omega_{phase} = E = \hbar \omega_A \cosh \rho$$

← linear Hz format

angular phasor →

$$\hbar c k_{phase} = cp = \hbar \omega_A \sinh \rho$$

format

format

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$$\hbar \equiv \frac{h}{2\pi}$$

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$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega = p \frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L$$

Legendre transformation

Use Group velocity: $u = \frac{dx}{dt} = c \tanh \rho$

$$p = \hbar k = Mc \sinh \rho$$

$$E = \hbar \omega = Mc^2 \cosh \rho = H$$

$$L = pu - H = (Mc \sinh \rho)(c \tanh \rho) - Mc^2 \cosh \rho$$

Note: $Mc u = Mc^2 \tanh \rho$

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Also: $cp = Mc^2 \sinh \rho$

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$$\hbar = h/2\pi$$

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$$= Mc^2 \sin \sigma$$

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$$= \hbar ck = Mc^2 \tan \sigma$$

with Hamiltonian $H=E$

$$H = \hbar\omega = Mc^2 / \sqrt{1 - \frac{u^2}{c^2}} = Mc^2 \cosh \rho = Mc^2 \sec \sigma$$

$$= Mc^2 \sqrt{1 + \sinh^2 \rho} = Mc^2 \sqrt{1 + (cp)^2}$$

Including stellar angle σ

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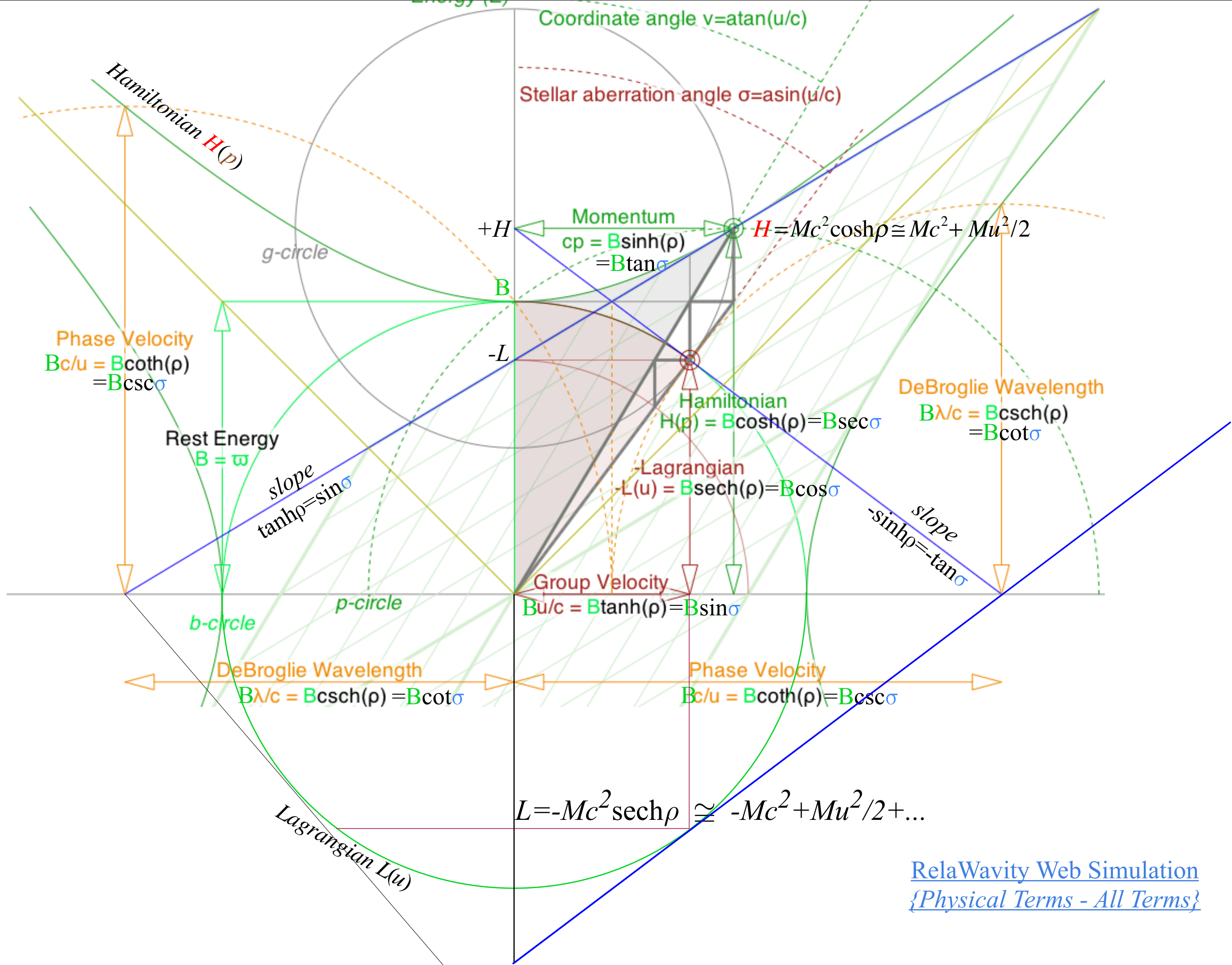
➔ Geometry of 1st Lagrangian and 1st Hamiltonian equations

Poincare invariant action differential

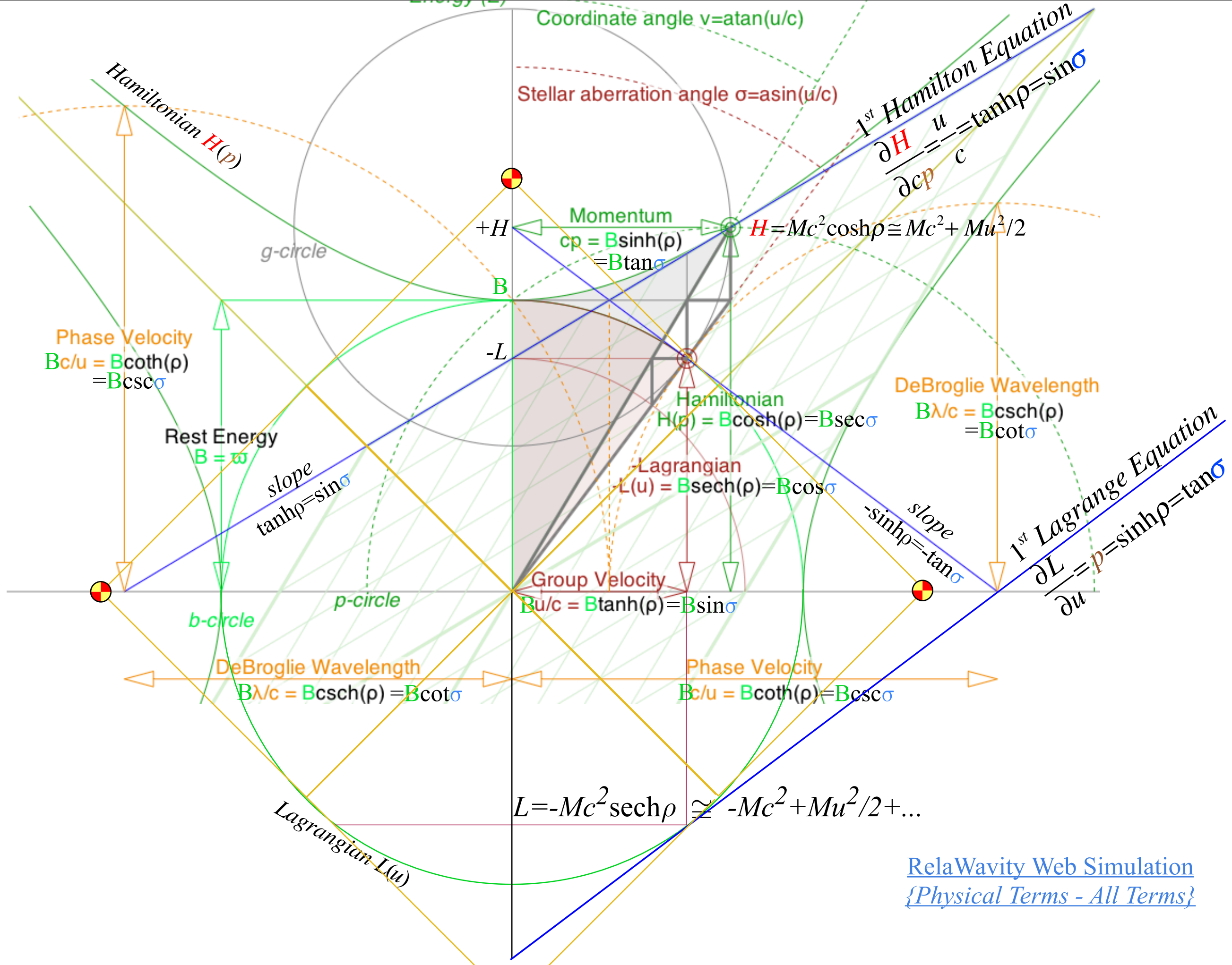
Hamilton-Jacobi equations

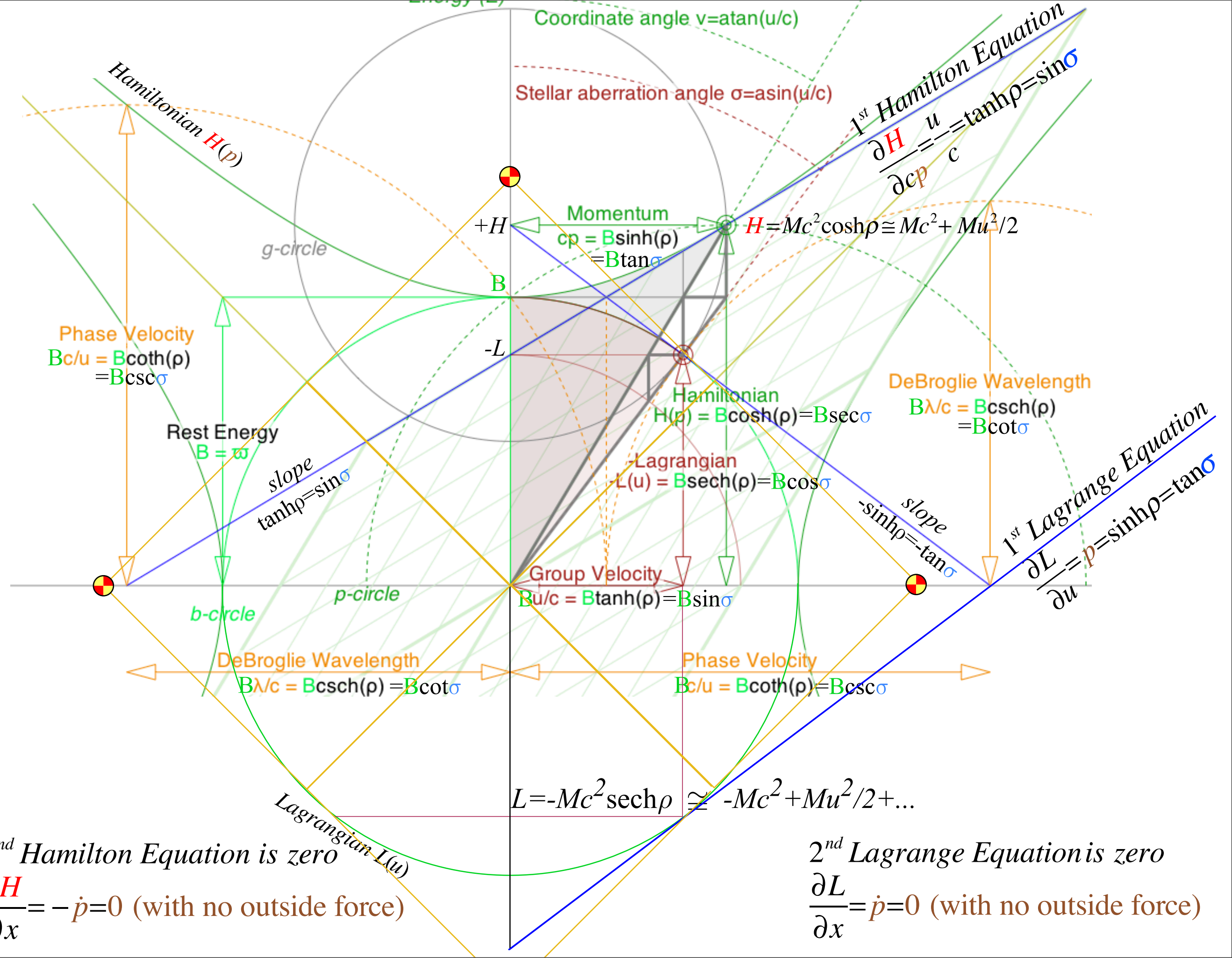
How Hamilton-Jacobi derives Schrodinger-op equations

How Huygens contact transformations determine motion



[RelaWavity Web Simulation](#)
 {Physical Terms - All Terms}





Coordinate angle $v = \text{atan}(u/c)$

Stellar aberration angle $\sigma = \text{asin}(u/c)$

1st Hamilton Equation
 $\frac{\partial H}{\partial cp} = \frac{u}{c} = \tanh \rho = \sin \sigma$

Momentum $cp = B \sinh(\rho) = B \tan \sigma$
 Hamiltonian $H = Mc^2 \cosh \rho \approx Mc^2 + Mu^2/2$

Phase Velocity
 $Bc/u = B \coth(\rho) = B \csc \sigma$

DeBroglie Wavelength
 $B\lambda/c = B \operatorname{csch}(\rho) = B \cot \sigma$

Rest Energy $B = \omega$

Hamiltonian $H(p) = B \cosh(\rho) = B \sec \sigma$

Lagrangian $L(u) = B \operatorname{sech}(\rho) = B \cos \sigma$

1st Lagrange Equation
 $\frac{\partial L}{\partial u} = p = \sinh \rho = \tan \sigma$

slope $\tanh \rho = \sin \sigma$

slope $-\sinh \rho = -\tan \sigma$

Group Velocity $Bu/c = B \tanh(\rho) = B \sin \sigma$

b-circle

p-circle

DeBroglie Wavelength $B\lambda/c = B \operatorname{csch}(\rho) = B \cot \sigma$

Phase Velocity $Bc/u = B \coth(\rho) = B \csc \sigma$

$L = -Mc^2 \operatorname{sech} \rho \approx -Mc^2 + Mu^2/2 + \dots$

2nd Hamilton Equation is zero
 $\frac{\partial H}{\partial x} = -\dot{p} = 0$ (with no outside force)

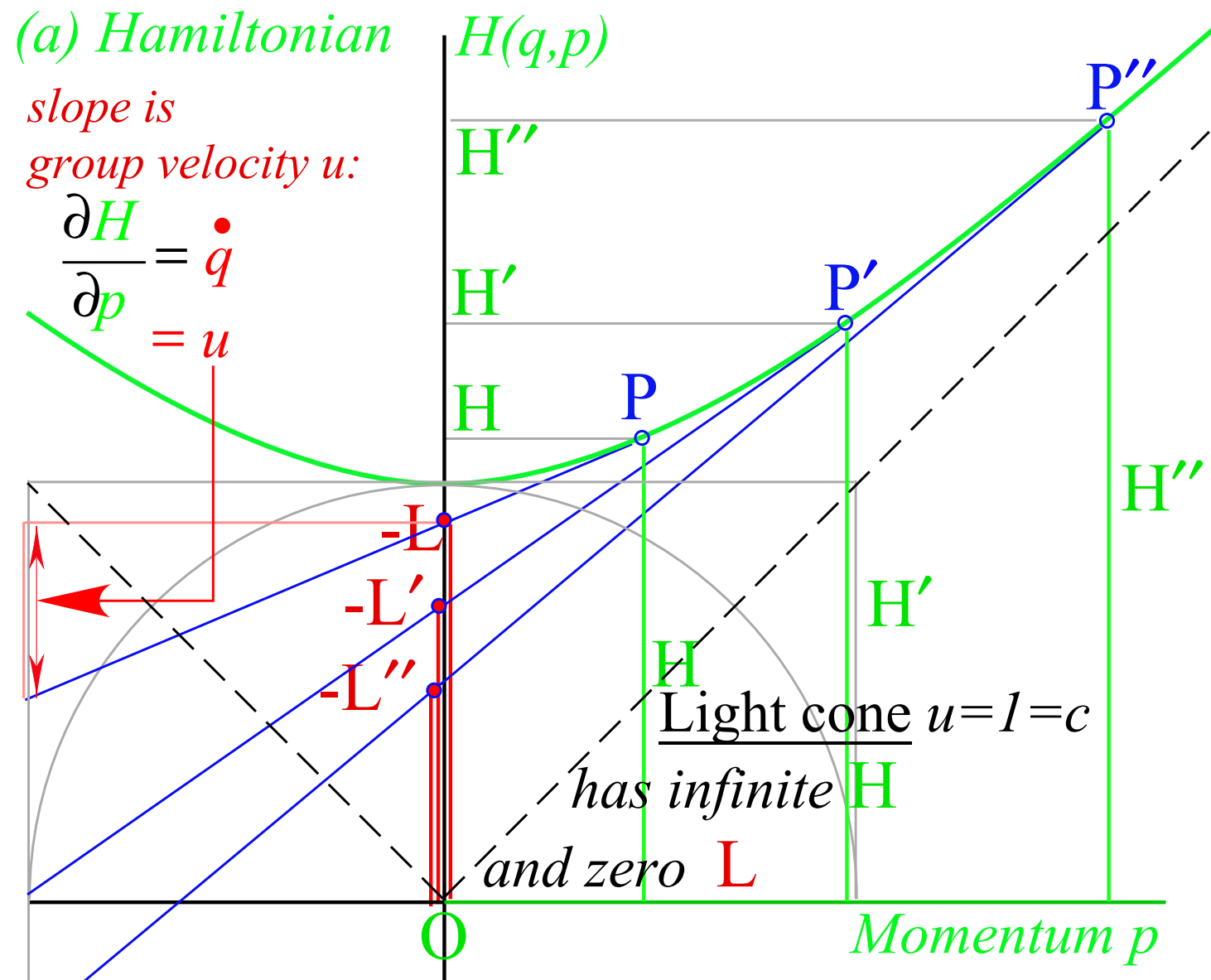
2nd Lagrange Equation is zero
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Poincare Invariant Action $dS=Ldt=p dq-H dt=\hbar d\Phi$ (phase)

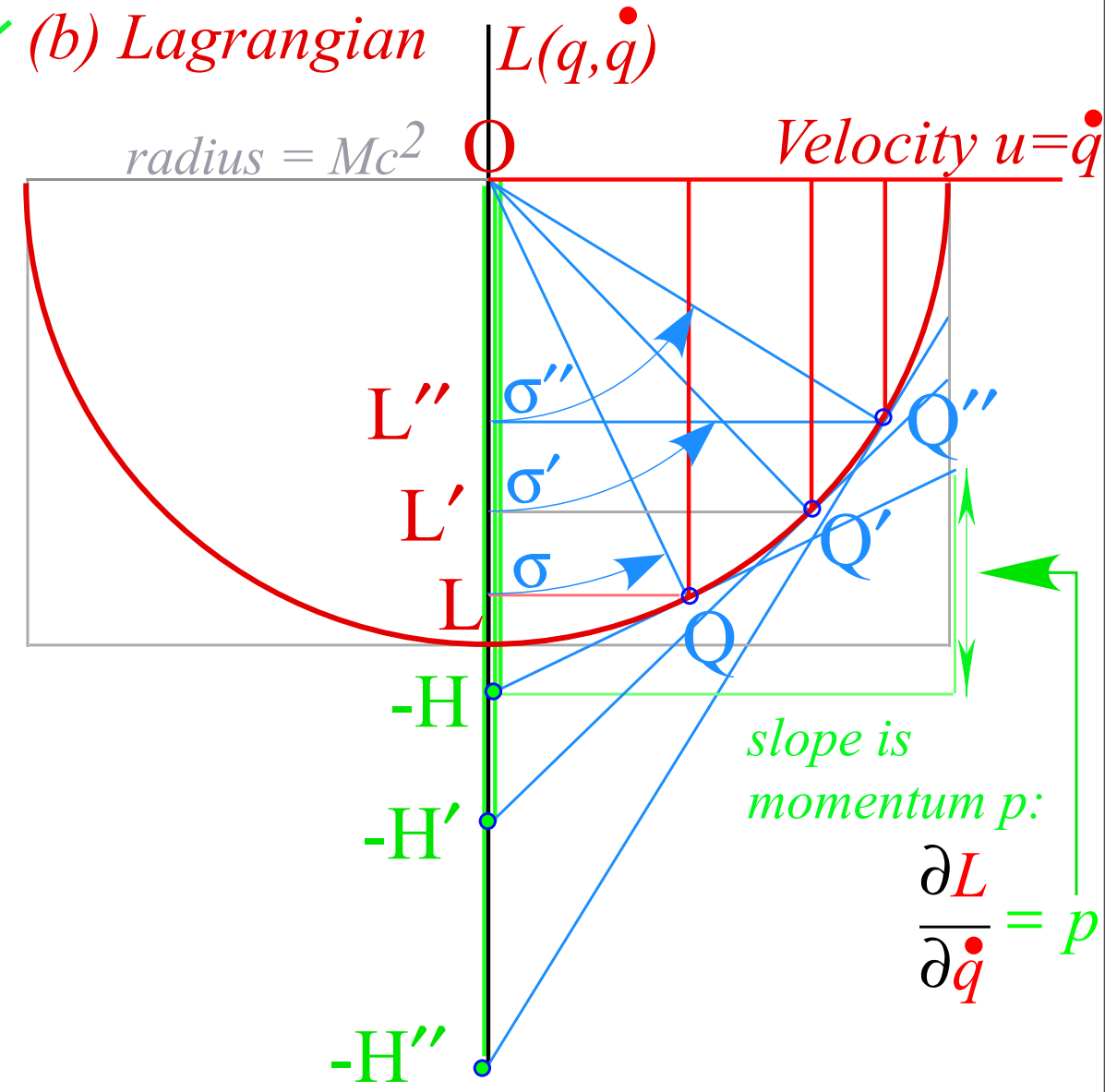
Hamiltonian $H(p,q)=p\dot{q}-L$ vs. Lagrangian $L(\dot{q},q)=p\dot{q}-H$

Contact transformation: (slope, -intercept) of H (or L) tangent determines the (X, Y coordinates) of L (or H).

(Also, called a Legendre contact transformation which is a special case of a Huygens transformation that uses contacting tangent curves instead of lines.)



Here *slope* is group velocity $u=\dot{q}$
 Y-coordinate is *energy* $H=\hbar\omega$



Here *slope* is momentum p
 Y-coordinate is *phase rate* $L=\hbar\Phi$

SRQTbyR&C Unit 3
 Fig. 26

Using (some) wave parameters to develop relativistic quantum theory

Low rapidity approximations to ν_{phase} and $c\kappa_{phase}$ match to Newtonian KE and momentum

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Compare *Lagrangian L*

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Define *Action S = \hbar \Phi*

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Prior wave relations

← linear Hz format angular phasor →
format format

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Poincare Invariant action differential

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$$\frac{\partial S}{\partial t} = -H$$

Hamilton-Jacobi equations

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← linear Hz format angular phasor format →

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How Jacobi-Hamilton derives Schrodinger equations

(Given "quantum wave")

$$\psi(\mathbf{r}, t) = e^{iS/\hbar} = e^{i(\mathbf{p}\cdot\mathbf{r} - H\cdot t)/\hbar} = e^{i(\mathbf{k}\cdot\mathbf{r} - \omega\cdot t)}$$

$$dS \text{ is integrable if: } \frac{\partial S}{\partial \mathbf{r}} = \mathbf{p} \quad \text{and:} \quad \frac{\partial S}{\partial t} = -H$$

These conditions are known as Jacobi-Hamilton equations

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Q: When is the *Action*-differential dS integrable?

A: Differential $dW = f_x(x,y)dx + f_y(x,y)dy$ is *integrable* to a $W(x,y)$ if: $f_x = \frac{\partial W}{\partial x}$ and: $f_y = \frac{\partial W}{\partial y}$

Similar to conditions for integrating work differential $dW = \mathbf{f}\cdot d\mathbf{r}$ to get potential $W(\mathbf{r})$. That condition is **no curl allowed**: $\nabla \times \mathbf{f} = \mathbf{0}$ or ∂ -symmetry of W :

$$\frac{\partial f_x}{\partial y} = \frac{\partial^2 W}{\partial y \partial x} = \frac{\partial^2 W}{\partial x \partial y} = \frac{\partial f_y}{\partial x}$$

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Momentum Operator
or \mathbf{p} -op in \mathbf{r} -basis
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Try 1st t -derivative of wave ψ

$$\frac{\partial}{\partial t} \psi(\mathbf{r}, t) = \frac{\partial}{\partial t} e^{iS/\hbar} = \frac{\partial(iS/\hbar)}{\partial t} e^{iS/\hbar} = (i/\hbar) \frac{\partial S}{\partial t} \psi(\mathbf{r}, t)$$

$$= (i/\hbar)(-H) \psi(\mathbf{r}, t) \text{ or: } i\hbar \frac{\partial}{\partial t} \psi(\mathbf{r}, t) = H \psi(\mathbf{r}, t)$$

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Try 1st \mathbf{r} -derivative of wave ψ

$$\frac{\partial}{\partial \mathbf{r}} \psi(\mathbf{r}, t) = \frac{\partial}{\partial \mathbf{r}} e^{iS/\hbar} = \frac{\partial(iS/\hbar)}{\partial \mathbf{r}} e^{iS/\hbar} = (i/\hbar) \frac{\partial S}{\partial \mathbf{r}} \psi(\mathbf{r}, t)$$

$$\frac{\partial}{\partial \mathbf{r}} \psi(\mathbf{r}, t) = (i/\hbar) \mathbf{p} \psi(\mathbf{r}, t) \text{ or: } \frac{\hbar}{i} \frac{\partial}{\partial \mathbf{r}} \psi(\mathbf{r}, t) = \mathbf{p} \psi(\mathbf{r}, t)$$

Momentum Operator
or \mathbf{p} -op in \mathbf{r} -basis
 $\mathbf{p} \Rightarrow \frac{\hbar}{i} \frac{\partial}{\partial \mathbf{r}}$

Try 1st t -derivative of wave ψ

$$\frac{\partial}{\partial t} \psi(\mathbf{r}, t) = \frac{\partial}{\partial t} e^{iS/\hbar} = \frac{\partial(iS/\hbar)}{\partial t} e^{iS/\hbar} = (i/\hbar) \frac{\partial S}{\partial t} \psi(\mathbf{r}, t)$$

$$= (i/\hbar)(-H) \psi(\mathbf{r}, t) \text{ or: } i\hbar \frac{\partial}{\partial t} \psi(\mathbf{r}, t) = H \psi(\mathbf{r}, t)$$

Schrodinger time equation
 $i\hbar \psi(\mathbf{r}, t) = H \psi(\mathbf{r}, t)$

Using (some) wave parameters to develop relativistic quantum theory

Low rapidity approximations to ν_{phase} and $c\kappa_{phase}$ match to Newtonian KE and momentum

How Mc^2 pops right up

Exact ν_{phase} gives exact Planck-Einstein energy formulas (1900-1905)

Exact $c\kappa_{phase}$ gives exact Bohr momentum and dispersion formulas (1921-1927)

Bohr-Schrodinger approximation to dispersion (*Who threw away the Mc^2 ?!!*)

“*What’s the Matter with Mass?*” Definition(s) of relativistic and quantum mechanical mass

(1) Einsteinian rest mass (2) Galilean momentum mass (3) Newtonian effective mass

Three Faces of Eve: A photon’s split personality

Relativistic action S and Lagrangian-Hamiltonian relations: How invariant phase works

The Legendre transformation relations

Deriving Lagrangian and Hamiltonian functions

Geometry of 1st Lagrangian and 1st Hamiltonian equations

Poincare invariant action differential

Hamilton-Jacobi equations

How Hamilton-Jacobi derives Schrodinger-op equations

➔ How Huygens contact transformations determine motion

Huygen's contact transformations enforce minimum action

Each point \mathbf{r}_k on a wavefront "broadcasts" in all directions.

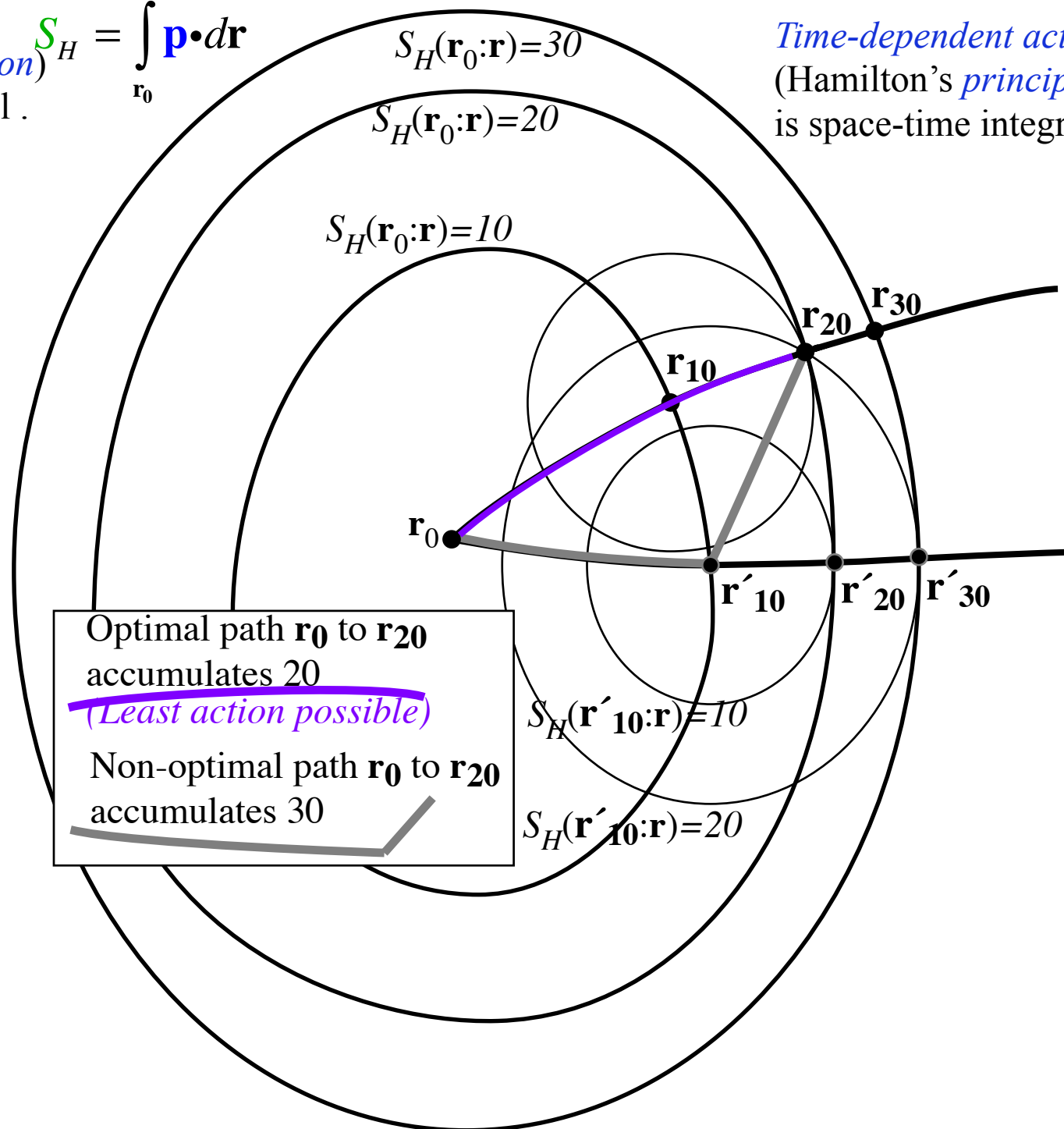
Only **minimum action** path interferes constructively

Time-independent action
(Hamilton's *reduced action*)
is a purely spatial integral .

$$S_H = \int_{\mathbf{r}_0}^{\mathbf{r}_1} \mathbf{p} \cdot d\mathbf{r}$$

Time-dependent action
(Hamilton's *principle action*)
is space-time integral .

$$S_p = \int_{\mathbf{r}_0 t_0}^{\mathbf{r}_1 t_1} (\mathbf{p} \cdot d\mathbf{r} - H \cdot dt)$$



Optimal path \mathbf{r}_0 to \mathbf{r}_{20}
accumulates 20
(Least action possible)

Non-optimal path \mathbf{r}_0 to \mathbf{r}_{20}
accumulates 30

CMwBang! Unit 1
Fig. 12.12

Huygen's contact transformations enforce minimum action

Each point \mathbf{r}_k on a wavefront "broadcasts" in all directions.

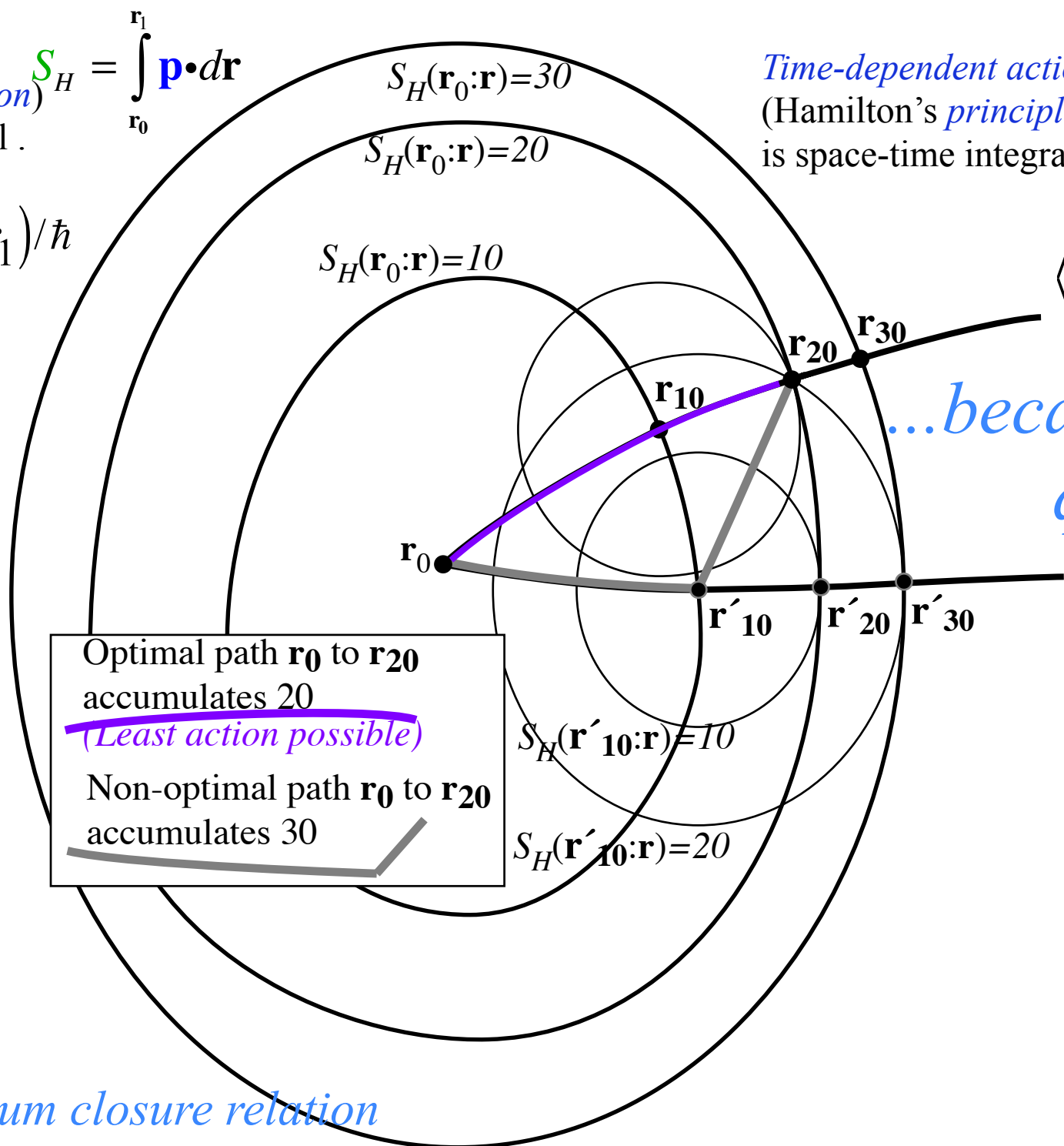
Only **minimum action** path interferes constructively

Time-independent action (Hamilton's *reduced action*) $S_H = \int_{\mathbf{r}_0}^{\mathbf{r}_1} \mathbf{p} \cdot d\mathbf{r}$ is a purely spatial integral.

Time-dependent action $S_p = \int_{\mathbf{r}_0, t_0}^{\mathbf{r}_1, t_1} (\mathbf{p} \cdot d\mathbf{r} - H \cdot dt)$ (Hamilton's *principle action*) is space-time integral.

$$\langle \mathbf{r}_1 | \mathbf{r}_0 \rangle = e^{i S_H(\mathbf{r}_0 : \mathbf{r}_1) / \hbar}$$

$$\langle \mathbf{r}_1, t_1 | \mathbf{r}_0, t_0 \rangle = e^{i S(\mathbf{r}_0, t_0 : \mathbf{r}_1, t_1) / \hbar}$$



Optimal path \mathbf{r}_0 to \mathbf{r}_{20} accumulates 20
 (Least action possible)
 Non-optimal path \mathbf{r}_0 to \mathbf{r}_{20} accumulates 30

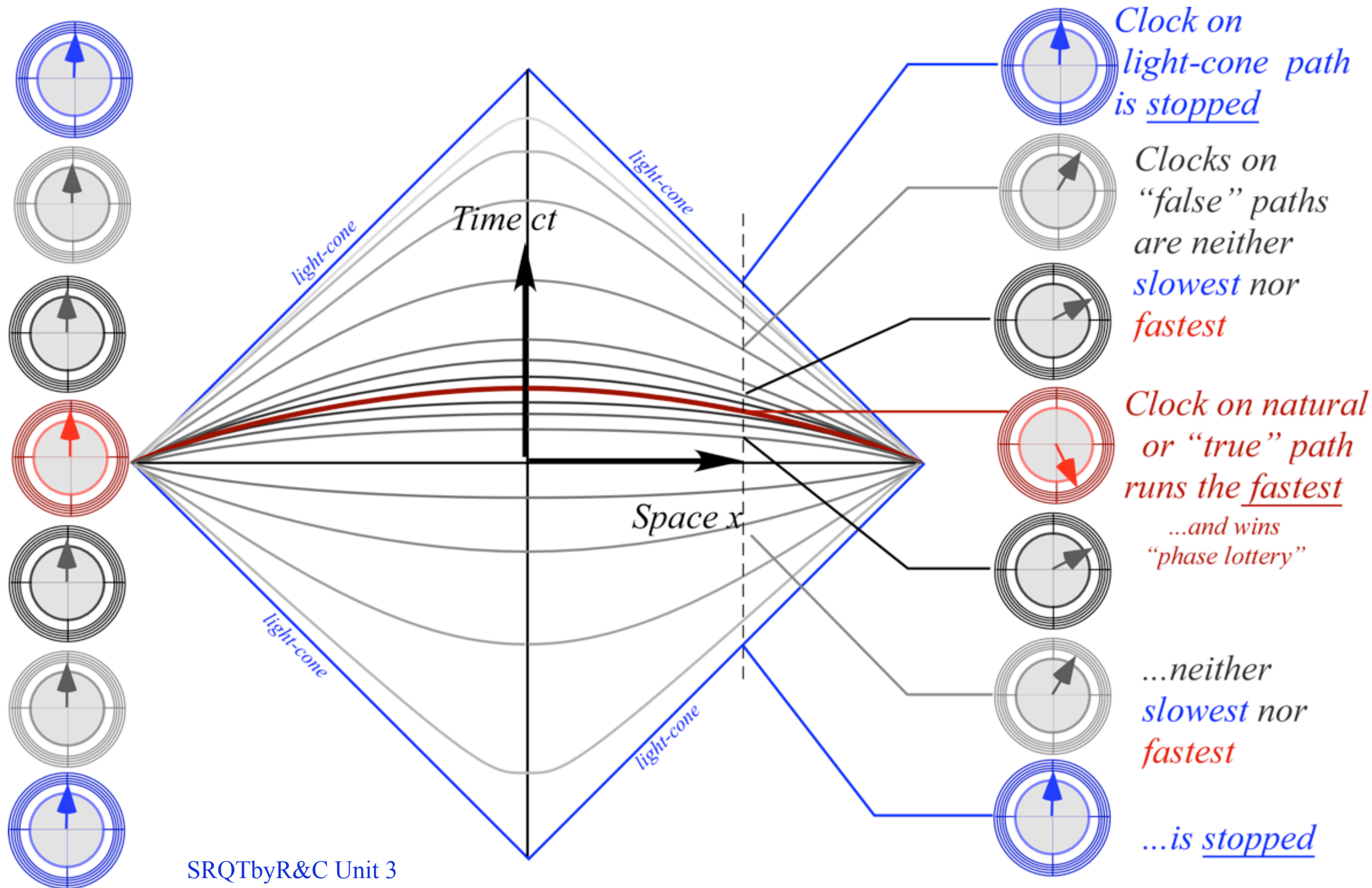
...because action is quantum wave phase

CMwBang! Unit 1
 Fig. 12.12

Feynman's path-sum closure relation

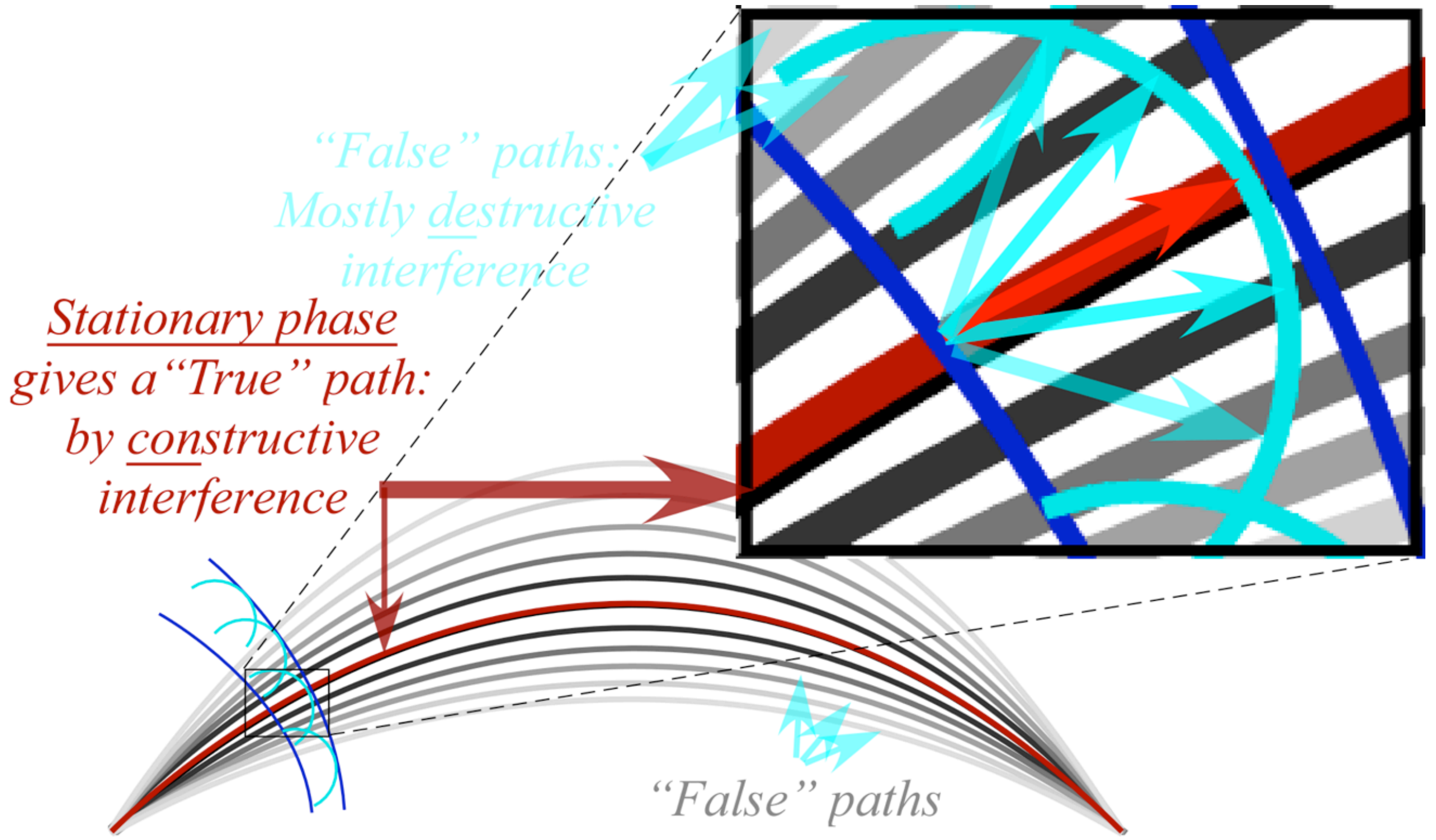
$$\sum_{\mathbf{r}'} \langle \mathbf{r}_1 | \mathbf{r}' \rangle \langle \mathbf{r}' | \mathbf{r}_0 \rangle \equiv \sum_{\mathbf{r}'} e^{i(S_H(\mathbf{r}_0 : \mathbf{r}') + S_H(\mathbf{r}' : \mathbf{r}_1)) / \hbar} = e^{i S_H(\mathbf{r}_0 : \mathbf{r}_1) / \hbar} = \langle \mathbf{r}_1 | \mathbf{r}_0 \rangle$$

Huygen's contact transformations enforce minimum action



SRQTbyR&C Unit 3
Fig. 28

Huygen's contact transformations enforce minimum action



SRQTbyR&C Unit 3
Fig. 29

Special Relativity and Quantum Mechanics by Ruler and Compass I.

The simplest "molecule": 2 CW Lasers form Minkowski Space-time (and Reciprocally-related) Frame Coordinates

A Making sense of light-wave axiom(s).

Using Occam's Razor

Evenson Continuous Wave (CW) axiom: CW speed for all colors is c

1CW is affected by 1st-order Doppler and Red shifts $c = c'$ of frequency ν and wavenumber k

B How does space-time and per-space-per-time carry light-waves?

$(\text{wavelength } \lambda = \text{period } \tau)$ and/or $(\text{wavenumber } k = \text{frequency } \nu)$

$(\lambda = 1/k \text{ and } \tau = 1/\nu)$ and $(k = 1/\lambda \text{ and } \nu = 1/\tau)$

$(\lambda = \text{meters per wave and } \tau = \text{seconds per wave})$ $(k = \text{waves per meter and } \nu = \text{waves per second})$

Greek "k" for wavenumber for Kayser (or "kinks")

The "Keyboard of the gods" Or per-space-per-time graphs vs. space-time graphs

Wave-speed equals slope-to-vertical in (c, \nu)-graph

Wave-speed equals slope-to-horizontal in (c, k)-graph

Dimensionless Light wave-velocity $c/c = 1$

C Doppler Shift in per-space-per-time

Atom traveling along wave sees less wave "hits"/sec. (that is: Doppler red-shift)

Atom traveling against wave sees more wave "hits"/sec. (that is: Doppler blue-shift)

Move fast enough this way then the "green" wave gets bluer and bluer until YOU die

Move fast enough this way then the "green" wave gets redder and redder until it dies

D Clarify Evenson's CW Axiom (All colors go c) by Doppler effects

Special relativity and quantum mechanics are very much a story of the geometry of light-wave motion

How do we measure space and time with light waves? Use 1CW laser-phasers for a phase-based theory

How do we make spacetime coordinate graph with light waves? Use 2CW laser-phasers and wave interference geometry

Relativity - Using light's own wave-like nature to better understand special relativity and quantum mechanics

Thales Mean Geometry (600BCE) helps "Relativity"

Minkowski-Lorentz Grid in terms of P', G'

Comparing Longitudinal relativity parameter: Rapidity $\rho = \log(\text{Doppler Shift})$ to a Transverse*relativity parameter: Stellar aberration angle σ

Per-space Per-space Geometry

E Easy Doppler-shift and Rapidity calculation

Doppler ratio: $(R/S) = \frac{v_{\text{observer}}}{v_{\text{source}}}$

rapidity: $\rho_{RS} = \log_e(R/S)$

Definition of Rapidity: $(R/S) = e^{\rho_{RS}}$ is time-reversed $(A/B) = \frac{v_A}{v_B}$

Bob-Alice Doppler ratio: $(B/A) = \frac{v_B}{v_A} = \frac{1}{(A/B)}$

Bob-Alice rapidity: $\rho_{BA} = \log_e(B/A) = \log_e \frac{1}{(A/B)}$

Carla-Alice Doppler ratio: $(C/A) = \frac{v_C}{v_A} = \frac{400}{600} = \frac{2}{3}$

Carla-Alice rapidity: $\rho_{CA} = \log_e(C/A) = \log_e \frac{2}{3}$

Carla-Bob Doppler ratio: $(B/C) = \frac{v_B}{v_C} = \frac{v_B}{v_A} \cdot \frac{v_A}{v_C} = (A/B)(A/C)$

Carla-Bob rapidity: $\rho_{CB} = \rho_{CA} + \rho_{AB}$ implies $\rho_{CB} = \rho_{CA} + \rho_{AB} = -1.10$

F 1 CW Laser-phaser Wave Function

Dimensionless Light wave-velocity $c/c = 1$

Wave function: $\psi = A e^{i(kx - \omega t)} = A \cos(kx - \omega t) + i A \sin(kx - \omega t)$

Wave vector $k = 2\pi/\lambda$

Angular frequency $\omega = 2\pi\nu$

Phase angle $\phi = kx - \omega t$

Amplitude A

Imaginary axis i

Real axis $\text{Re } \psi = \text{Re } \psi$

Imaginary $\text{Im } \psi = \text{Im } \psi$

Wavelength $\lambda = 2\pi/k = 1/k$

Period $T = 2\pi/\omega = 1/\nu$

Other Doppler versions $\lambda'/\lambda = c'/c = \nu/\nu'$ must match this phasor clock/clock-array, too. That's gauge invariance! $c'x' = c^2 t'$

G 2 CWs Interfering in Space-Time

Right-moving CW $e^{i(kx - \omega t)}$

Left-moving CW $e^{-i(kx - \omega t)}$

Sum of Wave Phasor Array

Group Beat

Carrier Wave

Geometry of the High-Frequency Phase and Half-Difference Group

Typical Phasor Sum

Phasor-relative views

Galileo's Revenge (part 2) Phasor angular velocity adds just like Galilean velocity

Galileo's Revenge (part 1) Rapidity adds just like Galilean velocity

H 2 Doppler shifted CWs Interfering in Space-Time

Right-directed ICW $e^{i(kx - \omega t)}$

Left-directed ICW $e^{-i(kx - \omega t)}$

Doppler blue shifted to 1200THz

Doppler red shifted to 300THz

2CW per-Spacetime Plot

2CW Minkowski-Spacetime grid

Phase vector \mathbf{P} 1.2-nm vector

Group vector \mathbf{G} 1.2-dB vector

I Thales Mean Geometry (600BCE) helps "Relativity"

Minkowski-Lorentz Grid in terms of P', G'

Comparing Longitudinal relativity parameter: Rapidity $\rho = \log(\text{Doppler Shift})$ to a Transverse*relativity parameter: Stellar aberration angle σ

Observer fixed below star sees it directly overhead. Observer going u sees star at angle σ in u direction.

Stellar aberration angle σ

Reality angle ν

Stellar aberration σ

Epstein's trick is to turn a hyperbolic form $ct = \sqrt{(ct')^2 - (x')^2}$ into a circular form: $\sqrt{(ct')^2 + (x')^2} = (ct')$

Then everything (and everybody) always goes speed c through (x', ct') space!

J Table of 12 wave parameters (includes inverses) for relativity

Geometry applies to (x, y) space-space to (k_x, k_y) per-space-per-space to (x, ct) space-time

Optical wave guide relativistic geometry aided by Occam's Sword

Relativistic mode with near- c $V_{\text{group}} = c/2$ and $V_{\text{phase}} = 2c$ (Low dispersion)

Example of near-cut-off mode with low $V_{\text{group}} = c/3$ and high $V_{\text{phase}} = 2c$ (High dispersion)

Per-space Per-space Geometry

Per-space Per-space Geometry

Stellar aberration angle σ

Reality angle ν

Stellar aberration σ

Epstein's trick is to turn a hyperbolic form $ct = \sqrt{(ct')^2 - (x')^2}$ into a circular form: $\sqrt{(ct')^2 + (x')^2} = (ct')$

Then everything (and everybody) always goes speed c through (x', ct') space!

Per-space Per-space Geometry

Stellar aberration angle σ

Reality angle ν

Stellar aberration σ

Epstein's trick is to turn a hyperbolic form $ct = \sqrt{(ct')^2 - (x')^2}$ into a circular form: $\sqrt{(ct')^2 + (x')^2} = (ct')$

Then everything (and everybody) always goes speed c through (x', ct') space!

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Note: When printed at their optimal resolution, each poster is 7 feet across!

Special Relativity and Quantum Mechanics by Ruler and Compass II.

The simplest "molecule": Relativistic mechanics by optical coherence geometry

William G. Harter and Tyle C. Reimer
University of Arkansas - Fayetteville



DAMOP - 2015



A Using wave parameters to quickly derive Planck (1900), Einstein (1905), and DeBroglie (1921) formulation

Base scale: $B = v_A$ for v_{phase}

$v_{phase} = B \cosh \rho = B + \frac{1}{2} B \rho^2$ (for $u \ll c$)
 $k_{phase} = B \sinh \rho = B \rho$ (for $u \ll c$)
 $\frac{u}{c} = \tanh \rho = \rho$ (for $u \ll c$)

Low speed v_{phase} and k_{phase} approximations vary with u like Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu .

So attach scale factor h (or \hbar) to match units
 Rescale k_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$

Use exact v_{phase} and k_{phase}

$h v_{phase} = h B \cosh \rho = Mc^2 \cosh \rho$
 Planck (1900) Total Energy: $E = \frac{Mc^2}{\sqrt{1-u^2/c^2}}$
 Einstein (1905) $E = \sqrt{1-u^2/c^2} Mc^2$
 DeBroglie (1921) Momentum: $h k_{phase} = p = \frac{Mu}{\sqrt{1-u^2/c^2}}$

group	$\frac{h v_{phase}}{c}$	$\frac{h k_{phase}}{c}$	$\frac{h v_{phase}}{c}$	$\frac{h k_{phase}}{c}$	$\frac{h v_{phase}}{c}$	$\frac{h k_{phase}}{c}$	$\frac{h v_{phase}}{c}$	$\frac{h k_{phase}}{c}$	$\frac{h v_{phase}}{c}$	$\frac{h k_{phase}}{c}$
rest	1	0	1	0	1	0	1	0	1	0
low speed	$1 + \frac{1}{2} \rho^2$	ρ	$1 + \frac{1}{2} \rho^2$	ρ	$1 + \frac{1}{2} \rho^2$	ρ	$1 + \frac{1}{2} \rho^2$	ρ	$1 + \frac{1}{2} \rho^2$	ρ
relativistic	$\cosh \rho$	$\sinh \rho$	$\cosh \rho$	$\sinh \rho$	$\cosh \rho$	$\sinh \rho$	$\cosh \rho$	$\sinh \rho$	$\cosh \rho$	$\sinh \rho$

B Definition(s) of mass for relativity and quantum theory

Rest Mass M_{rest} (Einstein's mass) Defines invariant hyperbola(s) Given: Energy: $E = Mc^2 \cosh \rho = h v_{phase}$
 momentum: $cp = Mc^2 \sinh \rho = h c k_{phase}$
 Group velocity: $u = c \tanh \rho = \frac{dE}{dc}$

2 Momentum Mass M_{mom} (Galileo's mass) Defined by ratio p/v of relativistic momentum to group velocity.

Limiting cases: $M_{mom} \xrightarrow{u \rightarrow 0} M_{rest} e^{\rho/2}$
 $M_{mom} \xrightarrow{u \rightarrow c} M_{rest}$

3 Effective Mass M_{eff} (Newton's mass) Defined by ratio $F/a = dp/du$ of relativistic force to acceleration.

Limiting cases: $M_{eff} \xrightarrow{u \rightarrow 0} M_{rest} e^{3\rho/2}$
 $M_{eff} \xrightarrow{u \rightarrow c} M_{rest}$

More common derivation using group velocity: $u = v_{group} = \frac{dE}{dp}$

C Defining phase Φ , action $S = \hbar \Phi$, Hamiltonian, and Lagrangian

1 Define Lagrangian L in terms of phase $\Phi = \int k dx - \int \omega dt$ for $k = k_{phase}$ and $\omega = \omega_{phase}$.

$L = \frac{dS}{dt} = \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega$ $\hbar = \frac{h}{2\pi}$

2 Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar \omega$ relation

$L = \frac{dS}{dt} = \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega = p \frac{dx}{dt} - E \equiv p u - E \equiv p u - H = L$ Legendre transformation

3 Use relativity relations: Group velocity: $u = \frac{dE}{dp} = c \tanh \rho$. Rest energy: $\omega_0 = Mc^2 = \hbar c k_0$

Momentum: $p = \hbar k_{phase} = c p = \hbar \omega_0 \sinh \rho$
 Hamiltonian: $H = \hbar \omega_{phase} = E = \hbar \omega_0 \cosh \rho$

$L = p u - H = (Mc \sinh \rho)(c \tanh \rho) - Mc^2 \cosh \rho = Mc^2 \frac{\sinh^2 \rho}{\cosh \rho} - Mc^2 \cosh \rho = -Mc^2 \operatorname{sech} \rho$

$L = \hbar \frac{d\Phi}{dt} = -Mc^2 \sqrt{1 - \frac{u^2}{c^2}} = -Mc^2 \operatorname{sech} \rho = -Mc^2 \cos \sigma$

$H = \hbar \omega_{phase} = Mc^2 \sqrt{1 + \frac{u^2}{c^2}} = Mc^2 \cosh \rho = Mc^2 \sec \sigma$

$H = Mc^2 \sqrt{1 + \sinh^2 \rho} = Mc^2 \sqrt{1 + (cp)^2}$

$\Phi = \int k dx - \int \omega dt$

$u = v_{group} = c \tanh \rho = c \sin \sigma$

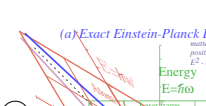
$cp = \hbar k_{phase} = Mc^2 \sinh \rho = Mc^2 \tan \sigma$

$H = Mc^2 \sqrt{1 + \sinh^2 \rho} = Mc^2 \sqrt{1 + (cp)^2}$

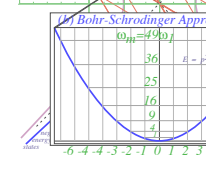
$\Phi = \int k dx - \int \omega dt$

D Geometry and plots of "Relativity" variables

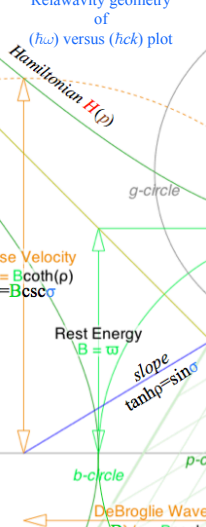
$\sinh \rho = \tan \sigma$, $\cosh \rho = \sec \sigma$, $\coth \rho = \csc \sigma$,
 $\tanh \rho = \sin \sigma$, $\operatorname{sech} \rho = \cos \sigma$, $\operatorname{csch} \rho = \cot \sigma$.



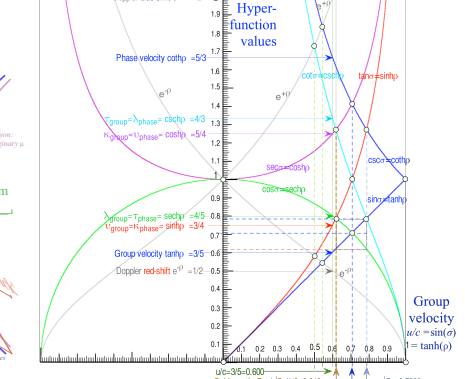
1 Exact vs approximate (h*omega) versus (h*k) plot



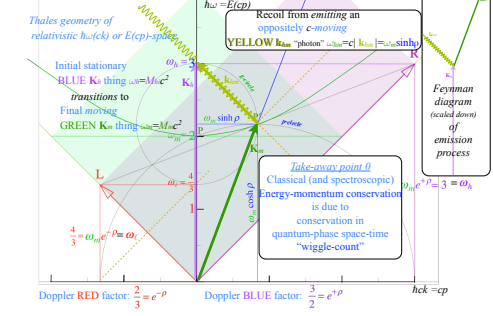
2 Relativity geometry of (h*omega) versus (h*k) plot



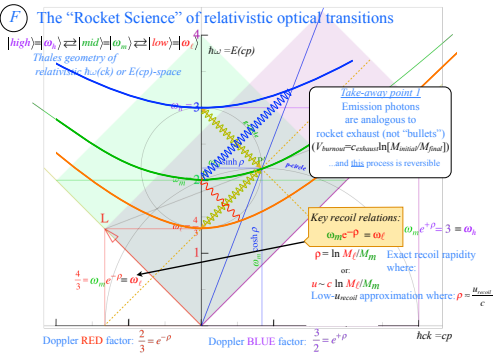
3 Relativity variables plotted versus Group Velocity $v_{group} = c \tanh \rho$



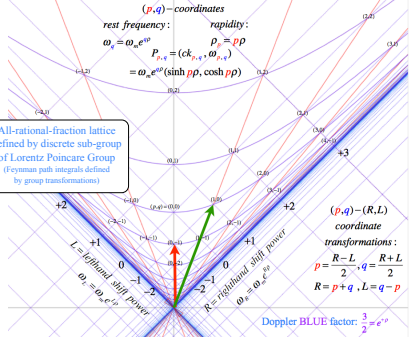
E Feynman diagram of relativistic optical transition



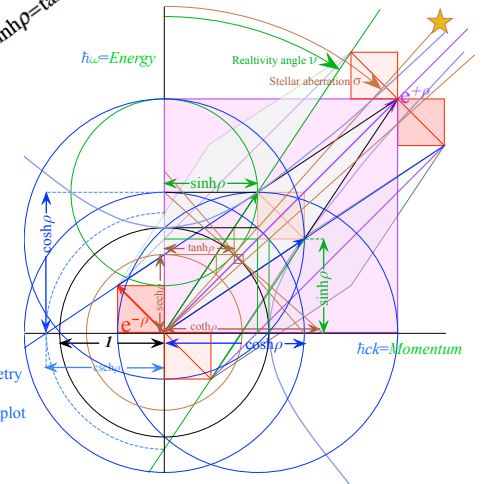
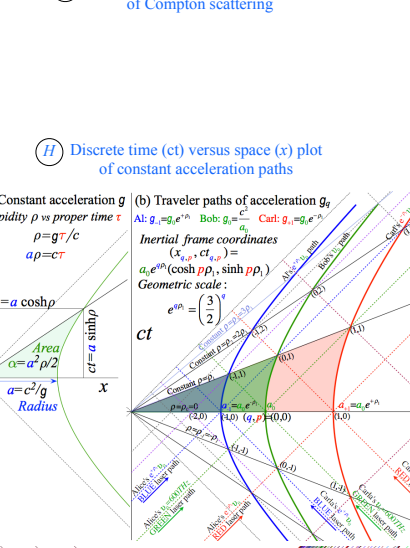
F The "Rocket Science" of relativistic optical transitions



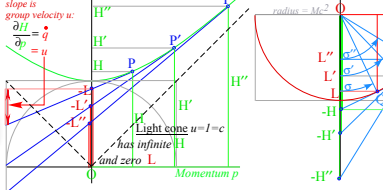
G Discrete (h*omega) versus (h*k) plot of Compton scattering



H Discrete time (ct) versus space (x) plot of constant acceleration paths



Hamiltonian Legendre transforms to Lagrangian



Comparing Lagrangian L(velocity u) with Hamiltonian H(momentum p)

$L = p u - H = (Mc \sinh \rho)(c \tanh \rho) - Mc^2 \cosh \rho = -Mc^2 \operatorname{sech} \rho$
 $H = \hbar \omega_{phase} = Mc^2 \sqrt{1 + \frac{u^2}{c^2}} = Mc^2 \cosh \rho = Mc^2 \sec \sigma$
 $\Phi = \int k dx - \int \omega dt$
 $u = v_{group} = c \tanh \rho = c \sin \sigma$
 $cp = \hbar k_{phase} = Mc^2 \sinh \rho = Mc^2 \tan \sigma$
 $H = Mc^2 \sqrt{1 + \sinh^2 \rho} = Mc^2 \sqrt{1 + (cp)^2}$

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