Relawavity: Relativistic wave mechanics V. Velocity geometry

A neo-liberal trigonometry lesson (sine, tangent, and secant) functions of angular sector area $\sigma$

- Complimentary functions (... cosine, and cotangent, cosecant)

- Hyper-trigonometry of (tanh $\rho$, sinh $\rho$, and cosh $\rho$, sech $\rho$, and csch $\rho$, coth $\rho$)

- Functions of hyper-angular sector area $\rho$ related to functions of $\sigma$

- Each circular trig function has a hyperbolic “country-cousin” function

- ...and big-party fun was had by all!

Pattern recognition aids and “Occam-sword” geometry

- Relating velocity parameters $\beta = \frac{u}{c}$ to rapidity $\rho$ to k-angle $\sigma$ to u/c-angle $\nu$

- Relating wave dimensional parameters of phase wave and group wave

- Parameter-space symmetry points

Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration k-angle $\sigma$

- Review of proper time relations and basis of Epstein’s cosmic speedometer

- Epstein geometry for relativistic parameters

Spectral details of per-spacetime 4-vector $(\omega_0, \omega_x, \omega_y, \omega_z) = (\omega, ck_x, ck_y, ck_z)$ transformation
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A neo-liberal trigonometry lesson (sine, tangent, and secant) (\(\sin\sigma, \tan\sigma, \text{ and } \sec\sigma\))

Circular Functions
- \(m\angle(\sigma) = 0.4805\)
- Length(\(\sigma\)) = 0.4805
- Area(\(\sigma\)) = 0.4805

\[
\sin(\sigma) = 0.4622 \\
\tan(\sigma) = 0.5212 \\
\sec(\sigma) = 1.1277
\]

For unit circle \(OA = 1\)

Angle \(\sigma = 0.48\) radians

\[
\text{Arc}AB \cdot 1 = 0.48 \text{ cm}
\]

Total Area \(ABOA'B'\)

\[
\sigma \cdot 1^2 = 0.48 \text{ cm}^2
\]

RelaWavity Web Simulation

Unit Circle

Thursday, April 21, 2016
A neo-liberal trigonometry lesson (sine, tangent, and secant) 
( \sin \sigma, \tan \sigma, \text{ and } \sec \sigma)

A One-Radian angle \( \sigma \) has arc-length equal to radius 
\( \sigma = 1 \Leftrightarrow OA = AB = OB \)

A One-Radian angle \( \sigma \) also has a diameter-swept area equal to its radius squared 
\( \sigma = 1 \Leftrightarrow (OA)^2 = (AB)^2 = (OB)^2 \)

For unit circle \( OA = 1 \)

Angle \( \sigma = 1.00 \) radians
Arc \( AB \) \( \sigma \cdot 1 = 1.00 \) cm
Total Area \( ABOA'B' \) \( \sigma \cdot l^2 = 1.00 \) cm\(^2\)
A neo-liberal trigonometry lesson (sine, tangent, and secant) 
(sin$\sigma$, tan$\sigma$, and sec$\sigma$)

For unit circle $OA=1$

Angle $\sigma = 1.19$ radians 
$ArcAB \sigma \cdot 1 = 1.19$ cm 
Total Area $ABOA'B'$ 
$\sigma \cdot 1^2 = 1.19$ cm$^2$
A neo-liberal trigonometry lesson \((\text{sine}, \tan, \text{ and } \sec)\)

\(\sin \sigma, \tan \sigma, \text{ and } \sec \sigma\)

For unit circle \(OA = 1\)

- Angle \(\sigma = 2.60 \text{ radians}\)
- \(\text{Arc} AB \cdot 1 = 2.60 \text{ cm}\)
- Total Area \(ABOA'B'\)
  \(\sigma \cdot l^2 = 2.60 \text{ cm}^2\)

\(\text{RelaWavity Web Simulation}\)

\(\text{Unit Circle}\)
A neo-liberal trigonometry lesson (sine, tangent, and secant) 

\( \sin \sigma, \tan \sigma, \text{ and } \sec \sigma \)

For unit circle \( OA = 1 \)

Angle \( \sigma = 3.14 \) radians

\( \text{Arc} AB \cdot 1 = 3.14 \) cm

Total Area \( ABOA'B' \)

\( \sigma \cdot l^2 = 3.14 \) cm²
A neo-liberal trigonometry lesson (sine, tangent, and secant) functions of angular sector area $\sigma$

- Complimentary functions (... cosine, and cotangent, cosecant)
- Hyper-trigonometry of (tanh$\rho$, sinh$\rho$, and cosh$\rho$, sech$\rho$, and csch$\rho$, coth$\rho$)

Functions of hyper-angular sector area $\rho$ related to functions of $\sigma$

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A neo-liberal trigonometry (*sine*, *tangent*, and *secant*, *cosine*)

Circular Functions
- $m\angle(\sigma) = 0.8541$
- Length(\(\sigma\)) = 0.8541
- Area(\(\sigma\)) = 0.8541
- $\sin(\sigma) = 0.7540$
- $\tan(\sigma) = 1.1477$
- $\sec(\sigma) = 1.5223$
- $\cos(\sigma) = 0.6569$

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*Unit Circle*
A neo-liberal trigonometry (sine, tangent, and secant, cosine, and cotangent)

Circular Functions
mL(σ) = 0.8541
Length(σ) = 0.8541
Area(σ) = 0.8541
sin(σ) = 0.7540
tan(σ) = 1.1477
sec(σ) = 1.5223
cos(σ) = 0.6569
cot(σ) = 0.8713

RelaWavity Web Simulation
6 Fundamental Trigonometric Functions

Thursday, April 21, 2016
A neo-liberal trigonometry (sine, tangent, and secant, cosine, and cotangent, cosecant)

Circular Functions
m∠(σ) = 0.8534
Length(σ) = 0.8534
Area(σ) = 0.8534
sin(σ) = 0.7535
tan(σ) = 1.1461
sec(σ) = 1.5210
cos(σ) = 0.6575
cot(σ) = 0.8725
csc(σ) = 1.3271

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6 Fundamental Trigonometric Functions
A neo-liberal trigonometry lesson (sine, tangent, and secant) functions of angular sector area $\sigma$
Complimentary functions (... cosine, and cotangent, cosecant)

Hyper-trigonometry of ( $\tanh \rho$, $\sinh \rho$, and $\cosh \rho$, $\sech \rho$, and $\csch \rho$, $\coth \rho$ )
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A Hyper-trigonometry

Hyper-(sine, tangent, and secant, cosine, and cotangent, cosecant)

\(\sin \sigma, \tan \sigma, \text{ and } \sec \sigma, \cos \sigma, \text{ and } \cot \sigma, \csc \sigma\)

\(\tanh \rho, \sinh \rho, \text{ and } \cosh \rho, \text{sech} \rho, \text{ and } \csch \rho, \coth \rho\)

Circular Functions

\(m \angle (\sigma) = 0.8534\)
\(\text{Length}(\sigma) = 0.8534\)
\(\text{Area}(\sigma) = 0.8534\)

\(\sin(\sigma) = 0.7535\)
\(\tan(\sigma) = 1.1461\)
\(\sec(\sigma) = 1.5210\)

\(\cos(\sigma) = 0.6575\)
\(\cot(\sigma) = 0.8725\)
\(\csc(\sigma) = 1.3271\)

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*Transition to Hyperbolic Functions*
A Hyper-trigonometry Hyper-\((\sin, \tan, \text{ and secant}, \cos, \text{ and cotangent}, \csc)\) \((\sin_\sigma, \tan_\sigma, \text{ and sec}_\sigma, \cos_\sigma, \text{ and cot}_\sigma, \csc_\sigma)\) \((\tanh_\rho, \sinh_\rho, \text{ and cosh}_\rho, \text{ sech}_\rho, \text{ and csch}_\rho, \coth_\rho)\)

<table>
<thead>
<tr>
<th>Circular Functions</th>
<th>Hyperbolic Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>(mL(\sigma) = 0.8534)</td>
<td>(\varrho = 0.9810)</td>
</tr>
<tr>
<td>Length((\sigma) = 0.8534)</td>
<td>Area((\varrho) = 0.9810)</td>
</tr>
<tr>
<td>Area((\sigma) = 0.8534)</td>
<td>(\sin(\varrho) = 0.7535)</td>
</tr>
<tr>
<td>(\sin(\sigma) = 0.7535)</td>
<td>(\tan(\varrho) = 0.7535)</td>
</tr>
<tr>
<td>(\tan(\sigma) = 1.1461)</td>
<td>(\sin(\rho) = 1.1461)</td>
</tr>
<tr>
<td>(\sec(\sigma) = 1.5210)</td>
<td>(\cos(\rho) = 1.5210)</td>
</tr>
<tr>
<td>(\cos(\sigma) = 0.6575)</td>
<td>(\sech(\varrho) = 0.6575)</td>
</tr>
<tr>
<td>(\cot(\sigma) = 0.8725)</td>
<td>(\csch(\varrho) = 0.8725)</td>
</tr>
<tr>
<td>(\csc(\sigma) = 1.3271)</td>
<td>(\coth(\varrho) = 1.3271)</td>
</tr>
</tbody>
</table>
A Hyper-trigonometry

Hyper-(sine, tangent, and secant, cosine, and cotangent, cosecant)

\[
\begin{align*}
\sin(\sigma), \tan(\sigma), \text{ and } \sec(\sigma), \\
\cos(\sigma), \text{ and } \cot(\sigma), \\
\csc(\sigma)
\end{align*}
\]

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Dual parameterization - $\rho$ and $\sigma$
A neo-liberal trigonometry lesson (sine, tangent, and secant) functions of angular sector area $\sigma$

Complimentary functions (... cosine, and cotangent, cosecant)

Hyper-trigonometry of (tanh$\rho$, sinh$\rho$, and cosh$\rho$, sech$\rho$, and csch$\rho$, coth$\rho$)

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Spectral details of per-spacetime 4-vector $(\omega_0, \omega_x, \omega_y, \omega_z) = (\omega, ck_x, ck_y, ck_z)$ transformation
The straight scoop on “hyper-angle” and “rapidity” (They’re area!)

\[ y/x = \tanh \theta = \frac{v}{c} \]

\[ y = \sinh \rho \]

\[ x = \cosh \rho \]

\[ A = \frac{1}{2} \text{base} \times \text{altitude} - \text{area under curve} = \frac{1}{2} xy - \int y \, dx \]

The “Area” being calculated is the total Gray Area between hyperbola pairs, X axis, and sloping u-line.
The straight scoop on “hyper-angle” and “rapidity” (They’re area!)

The “Area” being calculated is the **total** Gray Area between **hyperbola** pairs, X axis, and sloping u-line

Useful hyperbolic identities

\[
\sinh^2 \rho = \left( \frac{e^\rho - e^{-\rho}}{2} \right)^2 = \frac{1}{4} \left( e^{2\rho} + e^{-2\rho} - 2 \right) = \frac{\cosh 2\rho - 1}{2}
\]

\[
\sinh \rho \cosh \rho = \left( \frac{e^\rho - e^{-\rho}}{2} \right) \left( \frac{e^\rho + e^{-\rho}}{2} \right) = \frac{1}{4} \left( e^{2\rho} - e^{-2\rho} \right) = \frac{1}{2} \sinh 2\rho
\]
The straight scoop on “hyper-angle” and “rapidity” (They’re area!)

Useful hyperbolic identities

\[ \sinh^2 \rho = \left( \frac{e^\rho - e^{-\rho}}{2} \right)^2 = \frac{1}{4} (e^{2\rho} + e^{-2\rho} - 2) = \frac{\cosh 2\rho - 1}{2} \]

\[ \sinh \theta \cosh \theta = \left( \frac{e^\theta - e^{-\theta}}{2} \right) \left( \frac{e^\theta + e^{-\theta}}{2} \right) = \frac{1}{4} (e^{2\theta} - e^{-2\theta}) = \frac{1}{2} \sinh 2\theta \]

\[ \int \cosh a\rho \, d\rho = \frac{1}{a} \sinh a\rho \]

The “Area” being calculated is the total Gray Area between hyperbola pairs, X axis, and sloping u-line

The straight scoop on “hyper-angle” and “rapidity” (They’re area!)

Useful hyperbolic identities

\[ \sinh^2 \rho = \left( \frac{e^\rho - e^{-\rho}}{2} \right)^2 = \frac{1}{4} (e^{2\rho} + e^{-2\rho} - 2) = \frac{\cosh 2\rho - 1}{2} \]

\[ \sinh \theta \cosh \theta = \left( \frac{e^\theta - e^{-\theta}}{2} \right) \left( \frac{e^\theta + e^{-\theta}}{2} \right) = \frac{1}{4} (e^{2\theta} - e^{-2\theta}) = \frac{1}{2} \sinh 2\theta \]

\[ \int \cosh a\rho \, d\rho = \frac{1}{a} \sinh a\rho \]
The straight scoop on “hyper-angle” and “rapidity” (They’re area!)

The “Area” being calculated is the total Gray Area between hyperbola pairs, X axis, and sloping u-line

Useful hyperbolic identities

\[
\sinh^2 \rho = \left( \frac{e^\rho - e^{-\rho}}{2} \right)^2 = \frac{1}{4} \left( e^{2\rho} + e^{-2\rho} - 2 \right) = \frac{1}{2} \cosh 2\rho - 1
\]

\[
\int \cosh a\theta \, d\theta = \frac{1}{a} \sinh a\theta
\]

Amazing result: \( \text{Area} = \rho \) is rapidity
A neo-liberal trigonometry lesson (sine, tangent, and secant) functions of angular sector area \( \sigma \)
Complimentary functions (... cosine, and cotangent, cosecant)
Hyper-trigonometry of (\( tanh \rho, sinh \rho, \) and \( cosh \rho, sech \rho, \) and \( csch \rho, coth \rho \))
Functions of hyper-angular sector area \( \rho \) related to functions of \( \sigma \)

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A Hyper-trigonometry

Hyper- (sine, tangent, and secant, cosine, and cotangent, cosecant)

\[
(sin_\sigma, tan_\sigma, sec_\sigma, cos_\sigma, cot_\sigma, csc_\sigma)
\]

(tanh_\rho, sinh_\rho, cosh_\rho, sech_\rho, csch_\rho, coth_\rho)

Circular \( \sigma \) versus Hyperbolic \( \rho \)

Circular Functions
- \( m_L(\sigma) = 0.8534 \)
- \( \text{Length}(\sigma) = 0.8534 \)
- \( \text{Area}(\sigma) = 0.8534 \)
- \( \sin(\sigma) = 0.7535 \)
- \( \tan(\sigma) = 1.1461 \)
- \( \sec(\sigma) = 1.5210 \)
- \( \cos(\sigma) = 0.6575 \)
- \( \cot(\sigma) = 0.8725 \)
- \( \csc(\sigma) = 1.3271 \)

Hyperbolic Functions
- \( q = 0.9810 \)
- \( \text{Area}(q) = 0.9810 \)
- \( \tanh(q) = 0.7535 \)
- \( \sinh(q) = 1.1461 \)
- \( \cosh(q) = 1.5210 \)
- \( \sech(q) = 0.6575 \)
- \( \csch(q) = 0.8725 \)
- \( \coth(q) = 1.3271 \)

Dual parameterization - \( q \) and \( \rho \)

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A Hyper-trigonometry

Hyper-\((\sin, \tan, \text{and } \sec, \cos, \text{and } \cot, \csc)\)

\((\sin_{\sigma}, \tan_{\sigma}, \text{and } \sec_{\sigma}, \cos_{\sigma}, \text{and } \cot_{\sigma}, \csc_{\sigma})\)

Circular \(\sigma\) versus Hyperbolic \(\rho\)

Circular Functions
- \(mL(\sigma) = 0.8541\)
- \(\text{Length}(\sigma) = 0.8541\)
- \(\text{Area}(\sigma) = 0.8541\)
- \(\sin(\sigma) = 0.7540\)
- \(\tan(\sigma) = 1.1477\)
- \(\sec(\sigma) = 1.5223\)
- \(\cos(\sigma) = 0.6569\)
- \(\cot(\sigma) = 0.8713\)
- \(\csc(\sigma) = 1.3263\)

Hyperbolic Functions
- \(q = 0.9821\)
- \(\text{Area}(q) = 0.9821\)
- \(\tanh(q) = 0.7540\)
- \(\sinh(q) = 1.1477\)
- \(\cosh(q) = 1.5223\)
- \(\sech(q) = 0.6569\)
- \(\csch(q) = 0.8713\)
- \(\coth(q) = 1.3263\)

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\(\sigma\) Relations

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A Hyper-trigonometry Hyper-(sine, tangent, and secant, cosine, and cotangent, cosecant)

(sinσ, tanσ, and secσ, cosσ, and cotσ, cscσ)

(tanhρ, sinhρ, and coshρ, sechρ, and cschρ, cothρ)

Circular σ versus Hyperbolic ρ

RelaWavity Web Simulation ρ Relations
A Hyper-trigonometry

Hyper-(sine, tangent, and secant, cosine, and cotangent, cosecant)

\((\sin\sigma, \tan\sigma, \text{ and } \sec\sigma, \cos\sigma, \text{ and } \cot\sigma, \csc\sigma)\)

\((\tanh\rho, \sinh\rho, \text{ and } \cosh\rho, \sech\rho, \text{ and } \csch\rho, \coth\rho)\)

Circular Functions

- \(mL(\sigma) = 0.8541\)
- \(\text{Length}(\sigma) = 0.8541\)
- \(\text{Area}(\sigma) = 0.8541\)

- \(\sin(\sigma) = 0.7540\)
- \(\tan(\sigma) = 1.1477\)
- \(\sec(\sigma) = 1.5223\)
- \(\cos(\sigma) = 0.6569\)
- \(\cot(\sigma) = 0.8713\)
- \(\csc(\sigma) = 1.3263\)

Hyperbolic Functions

- \(\text{Area}(\rho) = 0.9821\)
- \(\text{Area}(\rho) = 0.9821\)
- \(\tanh(\rho) = 0.7540\)
- \(\sinh(\rho) = 1.1477\)
- \(\cosh(\rho) = 1.5223\)
- \(\sech(\rho) = 0.6569\)
- \(\csch(\rho) = 0.8713\)
- \(\coth(\rho) = 1.3263\)

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Occam’s Swords

Thursday, April 21, 2016
A Hyper-trigonometry

Hyper-(sine, tangent, and secant, cosine, and cotangent, cosecant)

\((\sin \sigma, \tan \sigma, \text{ and } \sec \sigma, \cos \sigma, \text{ and } \cot \sigma, \csc \sigma)\)

\((\tanh \rho, \sinh \rho, \text{ and } \cosh \rho, \sech \rho, \text{ and } \csch \rho, \coth \rho)\)
A Hyper-trigonometry

Hyper-(sine, tangent, and secant, cosine, and cotangent, cosecant)

Circular $\sigma$ versus Hyperbolic $\rho$

Circular Functions
- $mL(\sigma) = 1.2089$
- $\text{Length}(\sigma) = 1.2089$
- $\text{Area}(\sigma) = 1.2089$
- $\sin(\sigma) = 0.9352$
- $\tan(\sigma) = 2.6418$
- $\sec(\sigma) = 2.8247$
- $\cos(\sigma) = 0.3540$
- $\cot(\sigma) = 0.3785$
- $\csc(\sigma) = 1.0692$

Hyperbolic Functions
- $q = 1.6986$
- $\text{Area}(q) = 1.6986$
- $\tanh(q) = 0.9352$
- $\sinh(q) = 2.6418$
- $\cosh(q) = 2.8247$
- $\sech(q) = 0.3540$
- $\csch(q) = 0.3785$
- $\coth(q) = 1.0692$
Circular Functions
- $\sin(\sigma) = 0.9999$
- $\tan(\sigma) = 71.4639$
- $\sec(\sigma) = 71.4709$
- $\cos(\sigma) = 0.0140$
- $\cot(\sigma) = 0.0140$
- $\csc(\sigma) = 1.0001$

Hyperbolic Functions
- $\sinh(\rho) = 71.4639$
- $\cosh(\rho) = 71.4709$
- $\tanh(\rho) = 0.9999$
- $\coth(\rho) = 1.0001$
- $\sech(\rho) = 0.0140$
- $\csch(\rho) = 0.0140$

$u/c = \tanh(\rho) = 0.999901$
for $\rho = 4.96$
A neo-liberal trigonometry lesson (sine, tangent, and secant) functions of angular sector area $\sigma$
Complimentary functions (... cosine, and cotangent, cosecant)
Hyper-trigonometry of ( tanh$\rho$, sinh$\rho$, and cosh$\rho$, sech$\rho$, and csch$\rho$, coth$\rho$ )
Functions of hyper-angular sector area $\rho$ related to functions of $\sigma$
Each circular trig function has a hyperbolic “country-cousin” function
...and big-party fun was had by all!
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A Hyper-trigonometry Hyper-(sine, tangent, and secant, cosine, and cotangent, cosecant)
(sine, tangent, and secant, cosine, and cotangent, cosecant)
(tangent, sine, and cosine, and cotangential, cosecant)
A neo-liberal trigonometry lesson (*sine, tangent, and secant*) functions of angular sector area $\sigma$
Complimentary functions (... *cosine, and cotangent, cosecant*)
Hyper-trigonometry of (*tanh $\rho$, sinh $\rho$, and cosh $\rho$, sech $\rho$, and csch $\rho$, coth $\rho$*)
Functions of hyper-angular sector area $\rho$ related to functions of $\sigma$
Each circular trig function has a hyperbolic “country-cousin” function
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Pattern recognition aids and “Occam-sword” geometry
Relating velocity parameters $\beta=u/c$ to *rapidity $\rho$* to **k-angle $\sigma$** to $u/c$-angle $\nu$
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Pattern recognition: “Occam’s Sword”

Fig. 5.10 CW cosmic speedometer.
Geometry of Lorentz boost of counter-propagating waves.
Pattern recognition aid: “Occam’s Sword”

\[ \sinh \rho = \tan \sigma \]

\[ \tanh \rho = \sin \sigma = \frac{u}{c} = \tan \nu \]

*Fig. 5.10 CW cosmic speedometer. Geometry of Lorentz boost of counter-propagating waves.*
Pattern recognition aid: “Occam’s Sword”

Fig. 5.5
Relativistic wave mechanics geometry.
(a) Overview.
(b-d) Details of contacting tangents.

(c) Basic construction given \( u/c = 45/53 \)

(d) \( u/c = 3/5 \)

\[ H = \frac{53}{28} \]

\[ H = \frac{5}{4} \]

\[ e^\rho = 2/7 \]

\[ e^\rho = 1/2 \]

\[ \sigma \]

\[ \sigma \]
A neo-liberal trigonometry lesson (sine, tangent, and secant) functions of angular sector area $\sigma$
Complimentary functions (... cosine, and cotangent, cosecant)
Hyper-trigonometry of (tanh$\rho$, sinh$\rho$, and cosh$\rho$, sech$\rho$, and csch$\rho$, coth$\rho$)
Functions of hyper-angular sector area $\rho$ related to functions of $\sigma$
Each circular trig function has a hyperbolic “country-cousin” function
...and big-party fun was had by all!

Pattern recognition aids and “Occam-sword” geometry

Relating velocity parameters $\beta=\frac{u}{c}$ to rapidity $\rho$ to k-angle $\sigma$ to $u/c$-angle $\nu$
Relating wave dimensional parameters of phase wave and group wave
Parameter-space symmetry points

Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration k-angle $\sigma$
Review of proper time relations and basis of Epstein’s cosmic speedometer
Epstein geometry for relativistic parameters

Spectral details of per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) = (\omega,ck_x,ck_y,ck_z)$ transformation
Relating **Longitudinal** relativity parameter: Rapidity $\rho = \log_e(\text{Doppler Shift})$

to **Transverse** relativity parameter: Stellar aberration angle $\sigma$

(a) Circular Functions

$\sin(\sigma) = 0.6000$
$\tan(\sigma) = 0.7500$
$\sec(\sigma) = 1.2500$

**Relativity Web Simulation**

*Geometry of Stellar Aberration Angle*
Relating **Longitudinal** relativity parameter: Rapidity \( \rho = \log_e(\text{Doppler Shift}) \)

to **Transverse** relativity parameter: Stellar aberration angle \( \sigma \)

\[
\sinh \rho = \tan \sigma
\]

\[
\tanh \rho = \sin \sigma = \frac{u}{c} = \tan \nu
\]
Relating **Longitudinal** relativity parameter: Rapidity \( \rho = \log_e(\text{Doppler Shift}) \)

to **Transverse** relativity parameter: Stellar aberration angle \( \sigma \)

\[
\sinh \rho = \tan \sigma \\
\tanh \rho = \sin \sigma = \frac{u}{c} = \beta = \tan \nu
\]
A neo-liberal trigonometry lesson \((\text{sine, tangent, and secant})\) functions of angular sector area \(\sigma\).
Complimentary functions \(...) \text{cosine, and cotangent, cosecant}\)
Hyper-trigonometry of \((\text{tanh}\,\rho, \text{sinh}\,\rho, \text{and cosh}\,\rho, \text{sech}\,\rho, \text{and csch}\,\rho, \text{coth}\,\rho)\)
Functions of hyper-angular sector area \(\rho\) related to functions of \(\sigma\).
Each \text{circular} trig function has a \text{hyperbolic} “country-cousin” function.
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Yet another view: The Epstein space-proper-time approach to SR uses \text{stellar aberration} \text{k-angle} \(\sigma\)
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Spectral details of per-spacetime 4-vector \((\omega_0, \omega_x, \omega_y, \omega_z) = (\omega, ck_x, ck_y, ck_z)\) transformation
Summary of optical wave parameters for relativity and QM

...and their geometry

\[ v' = \frac{\omega'}{2\pi} \]

axis

(Units of 300THz)

An aid to pattern recognition:

Occam's Sword

\( (u/c=3/5) \)

RelaWavity Web Simulation

{perSpace - perTime All}
$v' = \omega'/2\pi$

axis

(Units of 300THz)

Table of 12 wave parameters
(includes inverses) for relativity
...and values for $u/c=3/5$

RelaWavity Web Simulation
Relativistic Terms (Dual plot w/expanded table)
A neo-liberal trigonometry lesson (*sine*, *tangent*, and *secant*) functions of angular sector area $\sigma$

Complimentary functions (... *cosine*, *and* *cotangent*, *cosecant*)

Hyper-trigonometry of (*tanh* $\rho$, *sinh* $\rho$, *and* *cosh* $\rho$, *sech* $\rho$, *and* *csch* $\rho$, *coth* $\rho$)

Functions of hyper-angular sector area $\rho$ related to functions of $\sigma$

Each **circular** trig function has a **hyperbolic** “country-cousin” function...

...and big-party fun was had by all!

**Pattern recognition aids and “Occam-sword” geometry**

Relating velocity parameters $\beta=u/c$ to *rapidity* $\rho$ to *k-angle* $\sigma$ to *u/c-angle* $\nu$

Relating wave dimensional parameters of phase wave and group wave

→ Parameter-space symmetry points

Yet another view: The Epstein space-proper-time approach to SR uses **stellar aberration** *k-angle* $\sigma$

Review of proper time relations and basis of Epstein’s cosmic speedometer

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Spectral details of per-spacetime 4-vector $(\omega_0, \omega_x, \omega_y, \omega_z) = (\omega, ck_x, ck_y, ck_z)$ transformation
Parameter-space symmetry points

Hyper-function values

<table>
<thead>
<tr>
<th>group</th>
<th>$b_{Doppler}^{Doppler}$</th>
<th>$V_{group}/c$</th>
<th>$v_{group}/v_A$</th>
<th>$\lambda_{group}/\lambda_A$</th>
<th>$\kappa_{group}/\kappa_A$</th>
<th>$\tau_{group}/\tau_A$</th>
<th>$c/V_{phase}$</th>
<th>$b_{Doppler}^{Doppler}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>phase</td>
<td>$1/V_{phase}$</td>
<td>$\kappa_{phase}/\kappa_A$</td>
<td>$\tau_{phase}/\tau_A$</td>
<td>$v_{phase}/v_A$</td>
<td>$\lambda_{phase}/\lambda_A$</td>
<td>$V_{phase}/c$</td>
<td>$1/V_{phase}$</td>
<td></td>
</tr>
<tr>
<td>rapidity $\rho$</td>
<td>$e^{-\rho}$</td>
<td>$\tanh \rho$</td>
<td>$\sinh \rho$</td>
<td>$\sech \rho$</td>
<td>$\cosh \rho$</td>
<td>$\csch \rho$</td>
<td>$\coth \rho$</td>
<td>$e^{\rho}$</td>
</tr>
<tr>
<td>stellar V angle $\sigma$</td>
<td>$1/e^{-\rho}$</td>
<td>$\sin \sigma$</td>
<td>$\tan \sigma$</td>
<td>$\cos \sigma$</td>
<td>$\sec \sigma$</td>
<td>$\cot \sigma$</td>
<td>$\csc \sigma$</td>
<td>$1/e^{-\rho}$</td>
</tr>
<tr>
<td>$\beta = \frac{u}{c}$</td>
<td>$\frac{1-\beta}{1+\beta}$</td>
<td>$\frac{\beta}{1}$</td>
<td>$\frac{1}{\sqrt{\beta^2 - 1}}$</td>
<td>$\frac{1}{1}$</td>
<td>$\frac{1}{\sqrt{\beta^2 - 1}}$</td>
<td>$\frac{1}{1}$</td>
<td>$\frac{1}{\beta}$</td>
<td>$\frac{1+\beta}{1-\beta}$</td>
</tr>
</tbody>
</table>
| value for $\beta < 1$ | $\frac{1}{2}$ | $0.5$ | $\frac{3}{5}$ | $0.6$ | $\frac{3}{4}$ | $0.75$ | $\frac{4}{5}$ | $0.80$ | $\frac{5}{4}$ | $1.25$ | $\frac{4}{3}$ | $1.33$ | $\frac{5}{3}$ | $1.67$ | $\frac{2}{1}$ | $2.0$

Group velocity $w/c = \sin(\sigma)$

Golden ratio $G = (\sqrt{5} - 1)/2 = 0.618..$

$\sqrt{2}/2 = 0.707..$
A neo-liberal trigonometry lesson (sine, tangent, and secant) functions of angular sector area $\sigma$

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Comparing **Longitudinal** relativity parameter: Rapidity $\rho = \log_e(\text{Doppler Shift})$

to a **Transverse** relativity parameter: Stellar aberration angle $\sigma^*$


We used notion $\sigma$ for stellar-ab-angle, (a “flipped-out” $\rho$).

Epstein seemed resistent to $\rho$ analysis or relations between $\sigma$ and $\rho$. 

Purchase at: 

---

Thursday, April 21, 2016
Comparing **Longitudinal relativity parameter:** Rapidity $\rho = \log_e(\text{Doppler Shift})$

to a **Transverse relativity parameter:** Stellar aberration angle $\sigma$*


Observer fixed below star sees it directly overhead.
Observer going $u$ sees star at angle $\sigma$ in $u$ direction.

Stellar aberration angle $\sigma$:

$$c \tanh \rho = u = c \sin \sigma$$

Epstein seemed resistant to $\rho$ analysis or relations between $\sigma$ and $\rho$.

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Spectral details of per-spacetime 4-vector $(\omega_0, \omega_x, \omega_y, \omega_z) = (\omega, c k_x, c k_y, c k_z)$ transformation
Review of Proper time $\tau_0$ and proper frequency $\omega_0$

\[
\begin{pmatrix}
  ck \\
  c \tau_0
\end{pmatrix}
= \begin{pmatrix}
  \cosh \rho & \sinh \rho \\
  \sinh \rho & \cosh \rho
\end{pmatrix}
\begin{pmatrix}
  ck' \\
  x'
\end{pmatrix}
\]

\[
\begin{pmatrix}
  x \\
  ct
\end{pmatrix}
= \begin{pmatrix}
  \cosh \rho & \sinh \rho \\
  \sinh \rho & \cosh \rho
\end{pmatrix}
\begin{pmatrix}
  x' \\
  ct'
\end{pmatrix}
\]

**Hyperbolic invariants to Lorentz transformation**

*Per-space-time invariant:*

\[
\omega_0^2 = \omega^2 - (ck)^2 = \omega'^2 - (ck')^2
\]

$\omega_0$ is called “proper frequency” or rate of “aging”

\[
\omega_0 = \omega \sqrt{1 - \frac{c^2k^2}{\omega^2}} = \omega' \sqrt{1 - \frac{c^2k'^2}{\omega'^2}}
\]

\[
= \omega \sqrt{1 - \frac{c^2}{V_{\text{phase}}^2}} = \omega' \sqrt{1 - \frac{c^2}{V'_{\text{phase}}^2}}
\]

\[
\omega = \frac{\omega_0}{\sqrt{1 - \frac{k^2}{(c\omega)^2}}}
\]

\[
\omega' = \frac{\omega_0}{\sqrt{1 - \frac{k'^2}{(c\omega')^2}}}
\]

*Space-time invariant:*

\[
(c\tau_0)^2 = (ct)^2 - x^2 = (ct')^2 - (x')^2
\]

$\tau_0$ is called “proper time” or “age”:

\[
\tau_0 = t \sqrt{1 - \frac{x^2}{(ct)^2}} = t' \sqrt{1 - \frac{x'^2}{(ct')^2}}
\]

\[
= t \sqrt{1 - \frac{u^2}{c^2}} = t' \sqrt{1 - \frac{u'^2}{c^2}}
\]

Coordinate time $t$ dilates to greater than $\tau_0$
Comparing **Longitudinal relativity parameter:** Rapidity $\rho = \log_e($Doppler Shift$)$ to a **Transverse relativity parameter:** Stellar aberration angle $\sigma^*$


*Observer fixed below star sees it directly overhead.*
*Observer going $u$ sees star at angle $\sigma$ in $u$ direction.*

Stellar aberration angle $\sigma$:

$$c \tanh \rho = u = \frac{c}{\sin \sigma}$$

Epstein seemed resistant to $\rho$ analysis or relations between $\sigma$ and $\rho$.

We used notion $\sigma$ for stellar-ab-angle, (a “flipped-out” $\rho$).

Purchase at: Amazon
Comparing **Longitudinal relativity parameter:** Rapidity $\rho = \log_e(\text{Doppler Shift})$

to a **Transverse relativity parameter:** Stellar aberration angle $\sigma$*


Observer fixed below star sees it directly overhead.
Observer going $\mathbf{u}$ sees star at angle $\sigma$ in $\mathbf{u}$ direction.

We used notion $\sigma$ for stellar-ab-angle, (a “flipped-out” $\rho$).

Epstein’s trick is to turn a hyperbolic form into a circular form:

$c \sqrt{1-u^2/c^2} = c/cosh \rho$

$= c \ sech \rho = c \ cos \sigma$

$|c'| = |c| = c$

$\sqrt{(c\tau)^2 + (x')^2} = (ct')$

*(for Proper time)*
Comparing **Longitudinal** relativity parameter: Rapidity $\rho = \log_e(\text{Doppler Shift})$

to a **Transverse** relativity parameter: Stellar aberration angle $\sigma^*$


**Observer fixed below star sees it directly overhead.**
**Observer going u sees star at angle $\sigma$ in u direction.**

Stellar aberration angle $\sigma$:

$$c \tanh \rho = u = c \sin \sigma$$

We used notion $\sigma$ for stellar-observation angle (a “flipped-out” $\rho$).

Epstein’s trick is to turn a hyperbolic form $c \tau = \sqrt{(ct')^2 - (x')^2}$ (for Proper time)
into a circular form: $\sqrt{(c\tau)^2 + (x')^2} = (ct')$

Then everything (and everybody) always goes speed $c$ through $(x', c\tau)$ space! *Purchase at:*
A neo-liberal trigonometry lesson (*sine*, *tangent*, and *secant*) functions of angular sector area $\sigma$

Complimentary functions (… *cosine*, and *cotangent*, *cosecant*)

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Relating wave dimensional parameters of phase wave and group wave

Parameter-space symmetry points

Yet another view: The Epstein space-proper-time approach to SR uses *stellar aberration* *k-angle* $\sigma$

Review of proper time relations and basis of Epstein’s cosmic speedometer

⇒ Epstein geometry for relativistic parameters

Spectral details of per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z)=(\omega,ck_x,ck_y,ck_z)$ transformation
Relating **Longitudinal** relativity parameter: Rapidity \( \rho = \log_e(\text{Doppler Shift}) \)

to **Transverse** relativity parameter: Stellar aberration angle \( \sigma \)


**Proper time** \( c\tau \) vs. **coordinate space** \( x \) - (L. C. Epstein’s “Cosmic Speedometer”)

Particles \( P \) and \( P' \) have speed \( u \) in \((x', ct')\) and speed \( c \) in \((x, c\tau)\)

**Proper time** \( c\tau \)

\[
ct = \sqrt{(ct')^2 - (x')^2}
\]

**Coordinate** \( x' = (u/c)ct' = ut' \)

**Einstein time dilation:**

\[
ct' = c\tau \sec \sigma = c\tau \cosh \rho = c\tau / \sqrt{1-u^2/c^2}
\]

**Lorentz length contraction:**

\[
L' = L \operatorname{sech} \rho = L \cos \sigma = L \cdot \sqrt{1-u^2/c^2}
\]

**Proper Time asimultaneity:**

\[
c\Delta\tau = L' \sinh \rho = L \cos \sigma \sinh \rho = L \cos \sigma \tan \sigma = L \sin \sigma = L / \sqrt{c^2/u^2-1} \sim L \ u/c
\]

Epstein’s trick is to turn a hyperbolic form \( c\tau = \sqrt{(ct')^2 - (x')^2} \) into a circular form: \( \sqrt{(c\tau)^2 + (x')^2} = (ct') \)

Then everything (and everybody) always goes speed \( c \) through \((x', c\tau)\) space!
Relating **Longitudinal** relativity parameter: \( \rho = \log_e(\text{Doppler Shift}) \)

to **Transverse** relativity parameter: Stellar aberration angle \( \sigma \)


**Proper time** \( c\tau \) **vs. coordinate space** \( x \) - (L. C. Epstein’s “Cosmic Speedometer”)

Particles \( P \) and \( P' \) have speed \( u \) in \((x', c\tau')\) and speed \( c \) in \((x, c\tau)\)

Proper time \( C\tau \)

\[
c\tau = \sqrt{(c\tau')^2 - (x')^2}
\]

Coordinate \( x' = (u/c)c\tau' = u' \)

Einstein time dilation:

\[
ct' = c\tau \sec \sigma = c\tau \cosh \rho = c\tau / \sqrt{1-u^2/c^2}
\]

Lorentz length contraction:

\[
L' = L \sech \rho = L \cos \sigma = L \sqrt{1-u^2/c^2}
\]

Proper Time simultaneity:

\[
c \Delta \tau = L' \sinh \rho = L \cos \sigma \sinh \rho
\]

\[
= L \cos \sigma \tan \sigma
\]

\[
= L \sin \sigma = L / \sqrt{\cosh^2 \rho - 1} \sim L u/c
\]

Epstein’s trick is to turn a hyperbolic form \( c\tau = \sqrt{(c\tau')^2 - (x')^2} \) (for Proper time)

into a circular form: \( \sqrt{(c\tau)^2 + (x')^2} = (c\tau') \)

Then everything (and everybody) always goes speed \( c \) through \((x', c\tau)\) space!

**RelativIt Epstein Plots**

In development!

*Litehouse-centric*  
*Ship-centric*  
*Non-co-moving observer*
A neo-liberal trigonometry lesson (sine, tangent, and secant) functions of angular sector area $\sigma$
Complimentary functions (... cosine, and cotangent, cosecant)
Hyper-trigonometry of ( $\tanh \rho$, $\sinh \rho$, and $\cosh \rho$, $\sech \rho$, and $\csch \rho$, $\coth \rho$ )
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Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration k-angle $\sigma$
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Epstein geometry for relativistic parameters

$\Rightarrow$ Spectral details of per-spacetime 4-vector $(\omega_0, \omega_x, \omega_y, \omega_z) = (\omega, ck_x, ck_y, ck_z)$ transformation
Spectral details of Lorentz boost of North-South-East-West plane-wave 4-vectors \((\omega_0,\omega_x,\omega_y,\omega_z)\)

\[ \sigma = 30^\circ = 0.524 \]
\[ \rho = 0.549 \]
\[ e^\rho = \sqrt{3} \]
\[ e^{-\rho} = 1/\sqrt{3} \]

\(\omega_0\tan\sigma = \omega_0\sinh\rho = 1/\sqrt{3}\)

\(u/c = \sin \sigma = 1/2\)
\(u/c = \tanh \rho = 1/2\)

South starlight in lighthouse frame is straight down \(x\)-axis:
\[ (\omega_\downarrow, ck_{\downarrow}, ck_{\downarrow}, ck_{\downarrow}) = (\omega_0, -\omega_0, 0, 0) \]

\[ + \rho_z\]-rapidity ship frame sees starlight Lorentz transformed to:
\[ (\omega'_\downarrow, ck'_{\downarrow}, ck'_{\downarrow}, ck'_{\downarrow}) = (\omega_0 \cosh \rho_z, -\omega_0, 0, -\omega_0 \sinh \rho_z) \]

\[
\begin{pmatrix}
\omega'_\downarrow \\
ck'_{\downarrow} \\
ck'_{\downarrow} \\
ck'_{\downarrow}
\end{pmatrix}
= \begin{pmatrix}
\cosh \rho_z & -\sinh \rho_z \\
1 & 0 \\
1 & 0 \\
-\sinh \rho_z & \cosh \rho_z
\end{pmatrix}
\begin{pmatrix}
\omega_\downarrow \\
ck_{\downarrow} \\
ck_{\downarrow} \\
ck_{\downarrow}
\end{pmatrix}
= \begin{pmatrix}
\cosh \rho_z & -\sinh \rho_z \\
1 & 0 \\
1 & 0 \\
-\sinh \rho_z & \cosh \rho_z
\end{pmatrix}
\begin{pmatrix}
\omega_0 \\
-\omega_0 \\
0 \\
-\omega_0 \sinh \rho_z
\end{pmatrix}
= \begin{pmatrix}
\omega_0 \cosh \rho_z \\
-\omega_0 \\
0 \\
-\omega_0 \sinh \rho_z
\end{pmatrix}
= \begin{pmatrix}
\omega_0 \sec \sigma \\
-\omega_0 \\
0 \\
-\omega_0 \tan \sigma
\end{pmatrix}
\]
Lecture 27 discusses Lorentz boost of North-South-East-West plane-wave 4-vectors \((\omega_0, \omega_x, \omega_y, \omega_z)\)

\[
\begin{align*}
\sigma &= 30^\circ = 0.524 \\
\rho &= 0.549 \\
e^\rho &= \sqrt{3} \\
e^{-\rho} &= 1/\sqrt{3}
\end{align*}
\]

\[
\begin{align*}
u/c &= \sin \sigma = 1/2 \\
u/c &= \tanh \rho = 1/2
\end{align*}
\]

\[
\omega_0 \tan \sigma = \omega_0 \sinh \rho = \omega_0 / \sqrt{3}
\]

For ship going \(u = c \tanh \rho\) along \(z\)-axis

\[
\begin{align*}
\text{West starlight } (\omega_0, 0, 0, -\omega_0) \text{ is blue shifted by } e^\rho &= \cosh \rho + \sinh \rho \\
&= \left( \begin{array}{c} \cosh \rho_z + \sinh \rho_z \\
0 \\
0 \\
-\sinh \rho_z - \cosh \rho_z \end{array} \right) \\
&= \omega_0 e^{+\rho_z} \\
&= \left( \begin{array}{c} \omega_0 e^{+\rho_z} \\
0 \\
0 \\
-\omega_0 e^{+\rho_z} \end{array} \right)
\end{align*}
\]

\[
\text{Blue shift factor is } e^\rho = \cosh \rho + \sinh \rho = \sec \sigma + \tan \sigma
\]

\[
\begin{align*}
\text{East starlight } (\omega_0, 0, 0, +\omega_0) \text{ is red shifted by } e^{-\rho} &= \cosh \rho - \sinh \rho \\
&= \left( \begin{array}{c} \cosh \rho_z - \sinh \rho_z \\
0 \\
0 \\
-\sinh \rho_z + \cosh \rho_z \end{array} \right) \\
&= \omega_0 e^{-\rho_z} \\
&= \left( \begin{array}{c} \omega_0 e^{-\rho_z} \\
0 \\
0 \\
-\omega_0 e^{-\rho_z} \end{array} \right)
\end{align*}
\]

\[
\text{Red shift factor is } e^{-\rho} = \cosh \rho - \sinh \rho = \sec \sigma - \tan \sigma
\]
Faster Lorentz boost of North-South-East-West plane-wave 4-vectors \((\omega_0, \omega_x, \omega_y, \omega_z)\)

Lorentz boost by \(\sigma=60^\circ\) or \(e^\rho=2+\sqrt{3}\)

Lighthouse view \((\omega, c\mathbf{k})\) of wave-vectors

Ship-frame view \((\omega', c\mathbf{k}')\) of wave-vectors

\[ u/c = \sin \sigma = \sqrt{3}/2 \]

\[ u/c = \tanh \rho = \sqrt{3}/2 \]

\[ \sigma = 60^\circ = 1.047 \]

\[ \rho = 1.317 \]

\[ e^\rho = 2+\sqrt{3} \]

\[ e^{-\rho} = 2-\sqrt{3} \]
How does Lorentz boost affect vector of arbitrary \( \theta \)?

Faster Lorentz boost of North-South-East-West plane-wave 4-vectors \((\omega_0, \omega_x, \omega_y, \omega_z)\)

\[\sigma = 60^\circ = 1.047 \quad \rho = 1.317 \quad e^\rho = 2 + \sqrt{3} \quad e^{-\rho} = 2 - \sqrt{3}\]

\( \omega_0 \tan \sigma = \omega_0 \sinh \rho \)

\( \omega_0 \sec \sigma = \omega_0 \cosh \rho \)

Blue shift

\( \omega_0 e^\rho = \omega_0 (2 + \sqrt{3}) \)

Red shift

\( \omega_0 e^{-\rho} = \omega_0 (2 - \sqrt{3}) \)

Lighthouse view \((\omega, c\mathbf{k})\) of wave-vectors

Ship-frame view \((\omega', c\mathbf{k}')\) of wave-vectors

\( u/c = \sin \sigma = \sqrt{3}/2 \)

\( u/c = \tanh \rho = \sqrt{3}/2 \)
Faster Lorentz boost of North-South-East-West plane-wave 4-vectors \((\omega_0, \omega_x, \omega_y, \omega_z)\)

Lorentz boost by \(\sigma=60^\circ\) or \(e^{i\rho} = 2 + \sqrt{3}\)

\[
\sigma = \frac{\sqrt{3}}{2}
\]

\[
u/c = \sin \sigma = \sqrt{3}/2
\]

\[
u/c = \tanh \rho = \sqrt{3}/2
\]

\[
\sigma = 60^\circ = 1.047
\]

\[
\rho = 1.317
\]

\[
e^\rho = 2 + \sqrt{3}
\]

\[
e^{-\rho} = 2 - \sqrt{3}
\]

How does Lorentz boost affect vector of arbitrary \(\theta\)?

Lighthouse view \((\omega, c\mathbf{k})\) of wave-vectors

Ship-frame view \((\omega', c\mathbf{k}')\) of wave-vectors

Let lab starlight ray at polar angle \(\theta\) have \(\mathbf{k}^\uparrow \theta = \omega_0 (1, \cos \theta, 0, -\sin \theta)\). Then ship going \(\mathbf{u}\)-along z-axis sees:

\[
\begin{pmatrix}
\omega'_{\mathbf{k}} \\
ck'_{x,\mathbf{k}} \\
ck'_{y,\mathbf{k}} \\
ck'_{z,\mathbf{k}}
\end{pmatrix} = 
\begin{pmatrix}
\cosh \rho_z & -\sinh \rho_z \\
1 & 1 \\
-\sinh \rho_z & \cosh \rho_z
\end{pmatrix} 
\begin{pmatrix}
\omega_0 \\
\omega_0 \cos \theta \\
0 \\
-\omega_0 \sin \theta
\end{pmatrix} = 
\begin{pmatrix}
\cosh \rho_z + \sinh \rho_z \sin \theta \\
\omega_0 \cos \theta \\
0 \\
-\sinh \rho_z - \cosh \rho_z \sin \theta
\end{pmatrix} = 
\begin{pmatrix}
\sec \sigma + \tan \sigma \sin \theta \\
\cos \theta \\
0 \\
-\tan \sigma - \sec \sigma \sin \theta
\end{pmatrix}
\]
Faster Lorentz boost of North-South-East-West plane-wave 4-vectors \((\omega,\omega_x,\omega_y,\omega_z)\)

\[ \sigma = 60^\circ \text{ or } e^{i\sigma} = 2 + \sqrt{3} \]

How does Lorentz boost affect vector of arbitrary \(\theta\)?

Lighthouse view \((\omega,ck)\) of wave-vectors

Ship-frame view \((\omega',ck')\) of wave-vectors

Let lab starlight ray at polar angle \(\theta\) have \(k^\uparrow = \omega_0 (1, \cos \theta, 0, -\sin \theta)\). Then ship going \(u\) along \(z\)-axis sees:

\[
\begin{pmatrix}
\omega'_{\theta} \\
ck'_{x \theta} \\
ck'_{y \theta} \\
ck'_{z \theta}
\end{pmatrix} =
\begin{pmatrix}
\cosh \rho_z & - \sinh \rho_z \\
0 & 1 & 0 \\
- \sinh \rho_z & \cosh \rho_z \\
0 & 0 & - \omega_0 \sin \theta
\end{pmatrix}
\begin{pmatrix}
\omega_0 \\
\omega_0 \cos \theta \\
0 \\
- \omega_0 \sin \theta
\end{pmatrix} =
\begin{pmatrix}
\cosh \rho_z + \sinh \rho_z \sin \theta \\
\omega_0 \cos \theta \\
0 \\
- \sinh \rho_z - \cosh \rho_z \sin \theta
\end{pmatrix} =
\begin{pmatrix}
\sec \sigma + \tan \sigma \sin \theta \\
\omega_0 \cos \theta \\
0 \\
- \tan \sigma - \sec \sigma \sin \theta
\end{pmatrix}
\]
Faster Lorentz boost of North-South-East-West plane-wave 4-vectors ($\omega_0, \omega_x, \omega_y, \omega_z$)

Lorentz boost by $\sigma=60^\circ$ or $e^{i\rho}=2+\sqrt{3}$

How does Lorentz boost affect vector of arbitrary $\theta$?

Lighthouse view ($\omega, c\mathbf{k}$) of wave-vectors

Ship-frame view ($\omega', c\mathbf{k}'$) of wave-vectors

Let lab starlight ray at polar angle $\theta$ have $\mathbf{k}^\uparrow \theta = \omega_0 (1, \cos \theta, 0, -\sin \theta)$. Then ship going $\mathbf{u}$ along $z$-axis sees:

$$
\begin{pmatrix}
\omega'_{\theta} \\
ck'_x \theta \\
ck'_y \theta \\
ck'_z \theta
\end{pmatrix} =
\begin{pmatrix}
\cosh \rho_z & -\sinh \rho_z \\
-1 & 0 \\
\sinh \rho_z & \cosh \rho_z
\end{pmatrix}
\begin{pmatrix}
\omega_0 \\
\omega_0 \cos \theta \\
0
\end{pmatrix} =
\omega_0
\begin{pmatrix}
\cosh \rho_z + \sinh \rho_z \sin \theta \\
\cos \theta \\
-\sinh \rho_z - \cosh \rho_z \sin \theta
\end{pmatrix} =
\omega_0
\begin{pmatrix}
\sec \sigma + \tan \sigma \sin \theta \\
\cos \theta \\
-\tan \sigma - \sec \sigma \sin \theta
\end{pmatrix}
$$
Faster Lorentz boost of North-South-East-West plane-wave 4-vectors \((\omega_0,\omega_x,\omega_y,\omega_z)\)

Lorentz boost by \(\sigma=60^\circ\) or \(e^{+\rho}=2+\sqrt{3}\)

How does Lorentz boost affect vector of arbitrary \(\theta\)?

Let lab starlight ray at polar angle \(\theta\) have \(k^\uparrow = \omega_0 (1,\cos \theta,0,-\sin \theta)\). Then ship going \(u\) along \(z\)-axis sees:

\[
egin{pmatrix}
\omega'_{\theta} \\
ck'_{x\theta} \\
ck'_{y\theta} \\
ck'_{z\theta}
\end{pmatrix} =
\begin{pmatrix}
\cosh \rho_z & -\sinh \rho_z \\
0 & 1 \\
-\sinh \rho_z & \cosh \rho_z
\end{pmatrix}
\begin{pmatrix}
\omega_0 \\
\omega_0 \cos \theta \\
0
\end{pmatrix}
= \omega_0 
\begin{pmatrix}
\cosh \rho_z + \sinh \rho_z \sin \theta \\
\cos \theta \\
0
\end{pmatrix} = \omega_0 
\begin{pmatrix}
\sec \sigma + \tan \sigma \sin \theta \\
\cos \theta \\
0
\end{pmatrix}
= \omega_0 
\begin{pmatrix}
-\tan \sigma - \sec \sigma \sin \theta \\
0 \\
-\sec \sigma \sin \theta
\end{pmatrix}
\]
Faster Lorentz boost of North-South-East-West plane-wave 4-vectors \( (\omega_0,\omega_x,\omega_y,\omega_z) \)

Lorentz boost by \( \sigma=60^\circ \) or \( e^{i\rho}=2+\sqrt{3} \)

How does Lorentz boost affect vector of arbitrary \( \theta \) ?

Let lab starlight ray at polar angle \( \theta \) have \( k^\uparrow_{\theta} = \omega_0 (1,\cos \theta,0,-\sin \theta) \). Then ship going \( u \) along \( z \)-axis sees:

\[
\begin{pmatrix}
\omega'_{\theta}
\end{pmatrix} =
\begin{pmatrix}
\cosh \rho_z & -\sinh \rho_z \\
1 & 1 \\
-\sinh \rho_z & \cosh \rho_z \\
\end{pmatrix}
\begin{pmatrix}
\omega_0 \\
\omega_0 \cos \theta \\
-\omega_0 \sin \theta \\
\end{pmatrix} =
\begin{pmatrix}
\cosh \rho_z + \sinh \rho_z \sin \theta \\
\cos \theta \\
-\sinh \rho_z - \cosh \rho_z \sin \theta \\
\end{pmatrix} =
\begin{pmatrix}
\sec \sigma + \tan \sigma \sin \theta \\
\cos \theta \\
-\tan \sigma - \sec \sigma \sin \theta \\
\end{pmatrix}
\]

Lighthouse view \( (\omega,ck) \) of wave-vectors

Ship-frame view \( (\omega',ck') \) of wave-vectors
Fig. 2.B.5 Space-Space-Time plot of world lines for Lighthouses. North Lighthouse blink waves trace light cones.