Lecture 28 Relawavity-Applications 2
Thursday 4.21.2016

Relawavity: Relativistic wave mechanics V. Velocity geometry

(Unit 3 p.28-42 - 4.21.16)

A neo-liberal trigonometry lesson (*sine*, *tangent*, and *secant*) functions of angular sector area σ

➔ Complimentary functions (... *cosine,and cotangent, cosecant*)

Hyper-trigonometry of (*tanh*ρ*, sinh*ρ*, and cosh*ρ*, sech*ρ*, and csch*ρ*, coth*ρ)

Functions of hyper-angular sector area ρ related to functions of σ

Each circular trig function has a hyperbolic "country-cousin" function

...and big-party fun was had by all!

Pattern recognition aids and "Occam-sword" geometry

Relating velocity parameters $\beta=u/c$ to *rapidity* ρ to **k**-*angle* σ to *u/c-angle* ν

Relating wave dimensional parameters of phase wave and group wave

Parameter-space symmetry points

Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration **k**-angle σ

Review of proper time relations and basis of Epstein's cosmic speedometer

Epstein geometry for relativistic parameters

Spectral details of per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) =(\omega,ck_x,ck_y,ck_z)$ transformation

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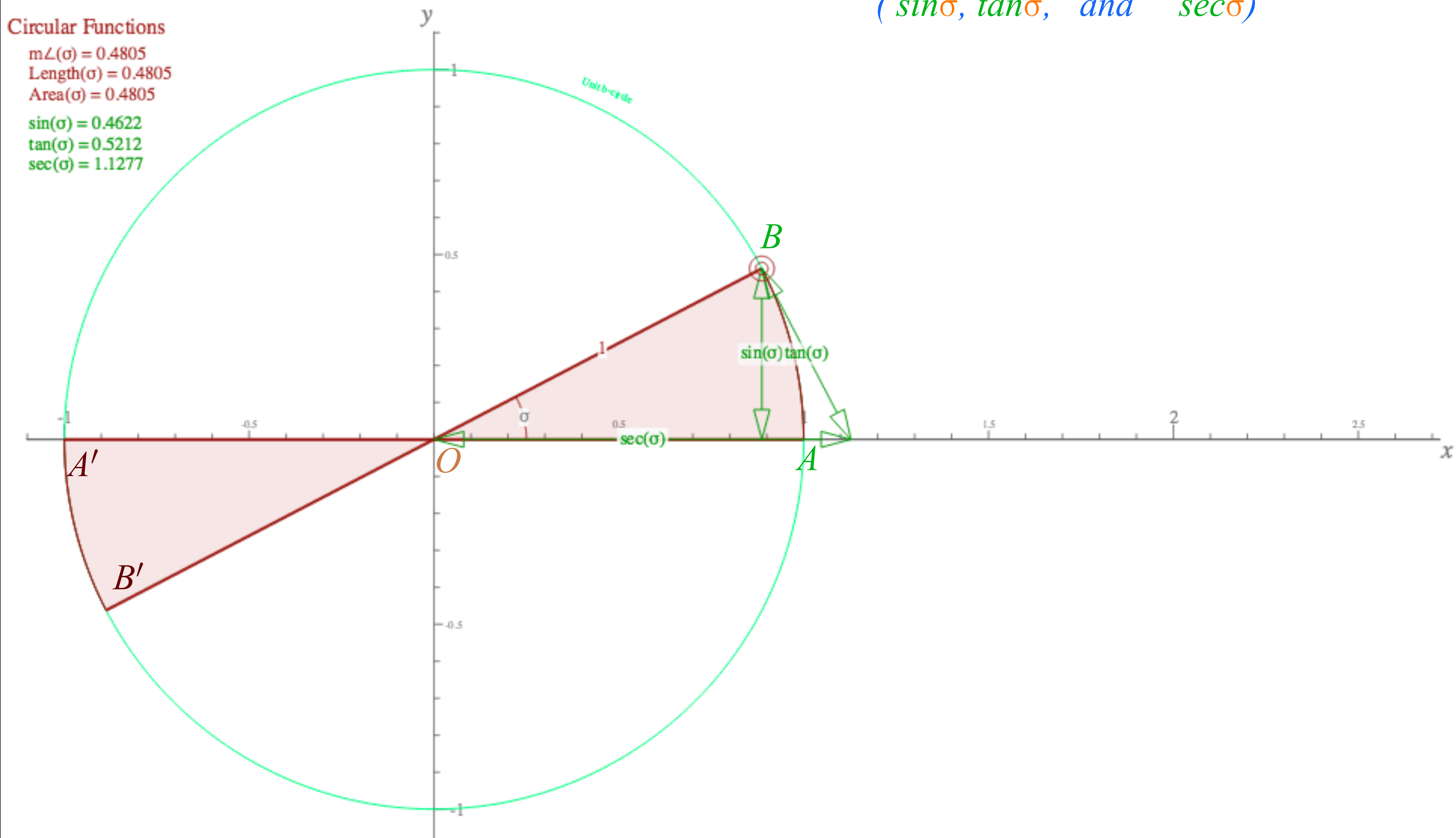
A neo-liberal trigonometry lesson (sine, tangent, and secant)
(sinσ, tanσ, and secσ)



For unit circle $OA=1$

Angle $\sigma = 0.48$ radians

Arc AB $\sigma \cdot 1 = 0.48$ cm

Total Area $ABOA'B'$

$\sigma \cdot 1^2 = 0.48\ cm^2$

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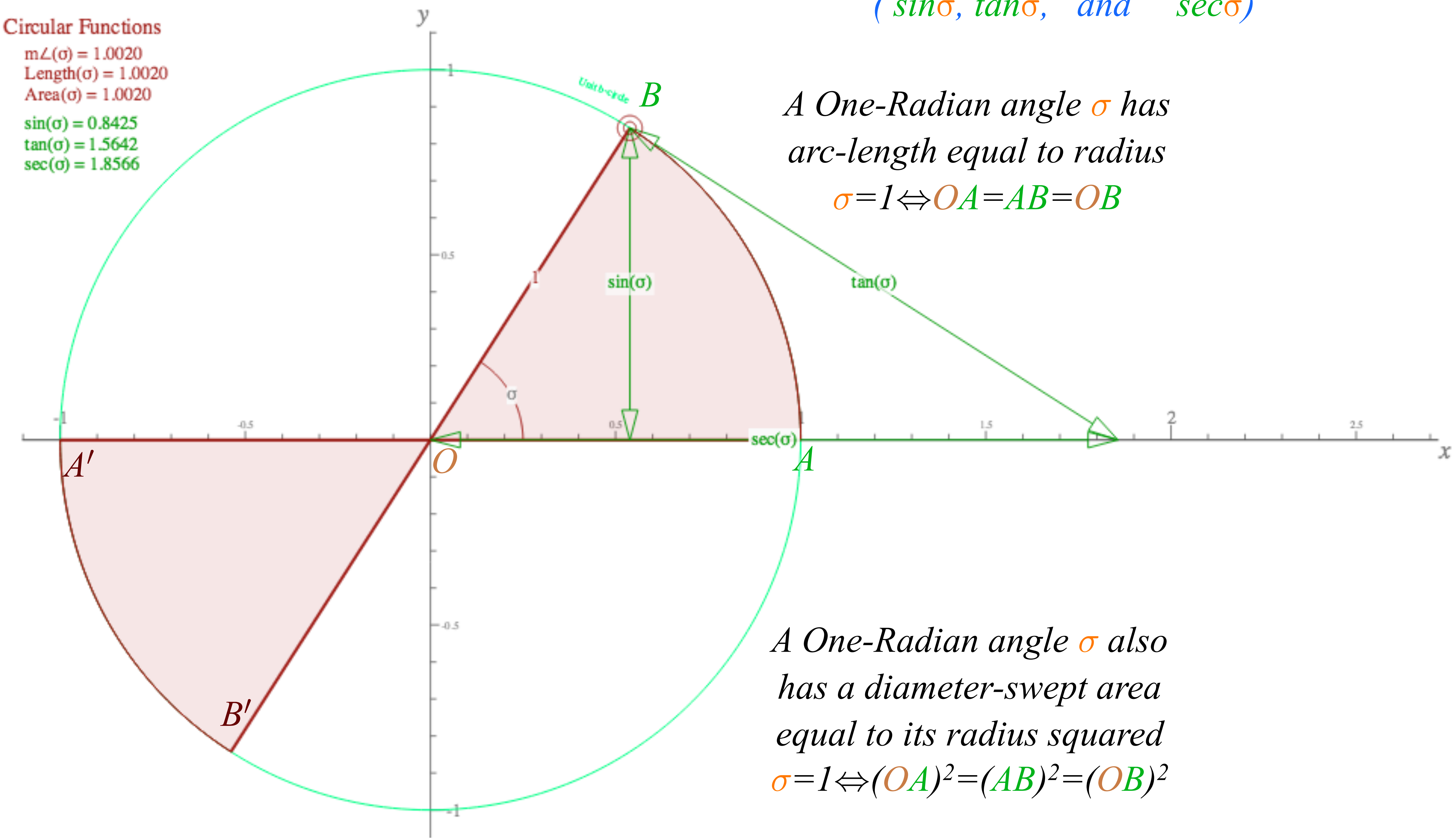
Unit Circle

A neo-liberal trigonometry lesson *(sine, tangent, and secant)*
(sinσ, tanσ, and secσ)



A One-Radian angle σ has arc-length equal to radius
σ=1⇔OA=AB=OB

A One-Radian angle σ also has a diameter-swept area equal to its radius squared
σ=1⇔(OA)²=(AB)²=(OB)²

For unit circle OA=1

Angle σ =1.00 radians
ArcAB σ·1=1.00 cm
Total Area ABOA′B′
σ·1² =1.00 cm²

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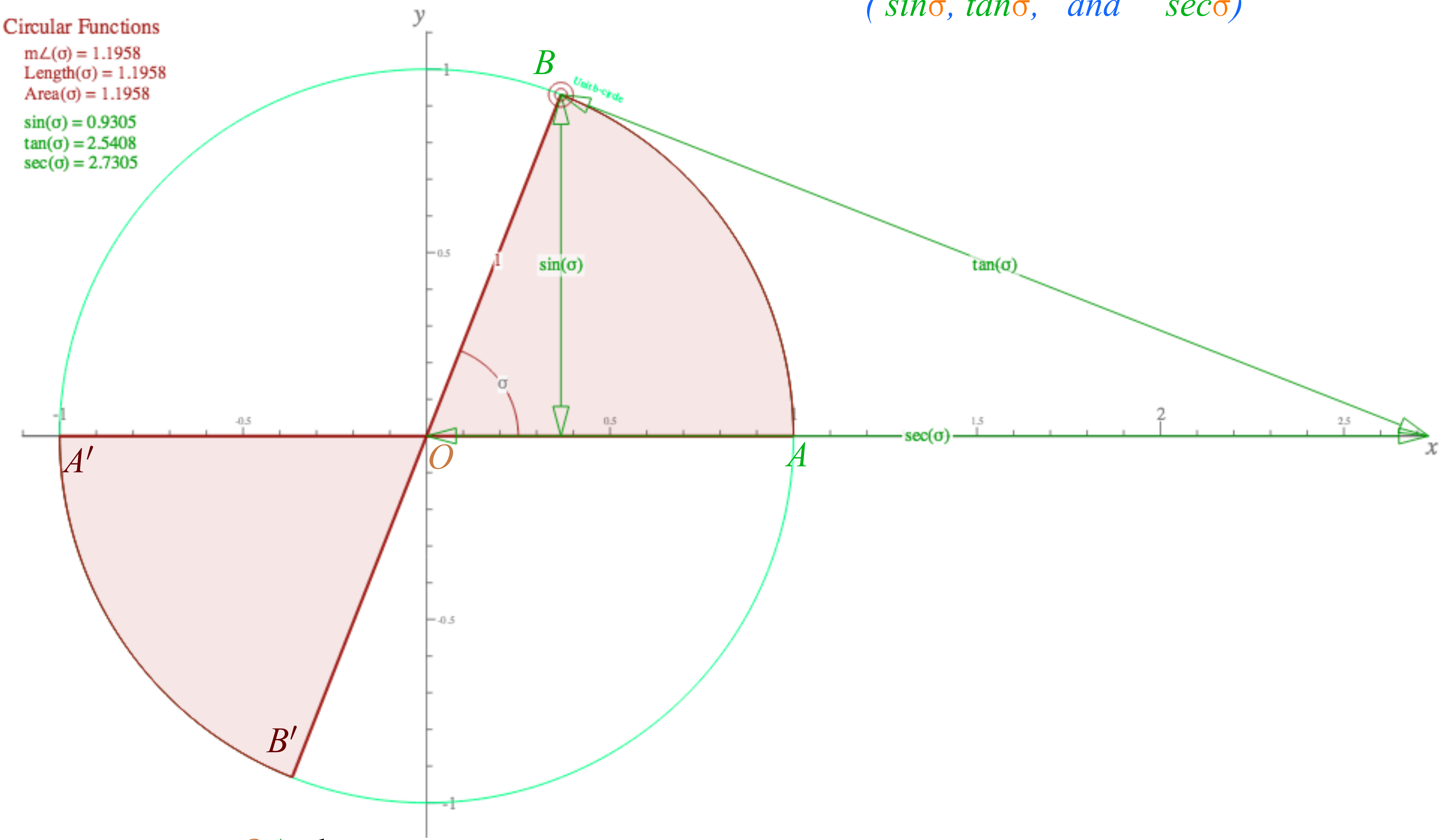
[*Unit Circle*](#)

A neo-liberal trigonometry lesson (sine, tangent, and secant)

(sinσ, tanσ, and secσ)



For unit circle $OA=1$

Angle σ =1.19 radians

ArcAB $\sigma\cdot 1$=1.19 cm

Total Area $ABOA'B'$

$\sigma\cdot 1^2$ =1.19 cm^2

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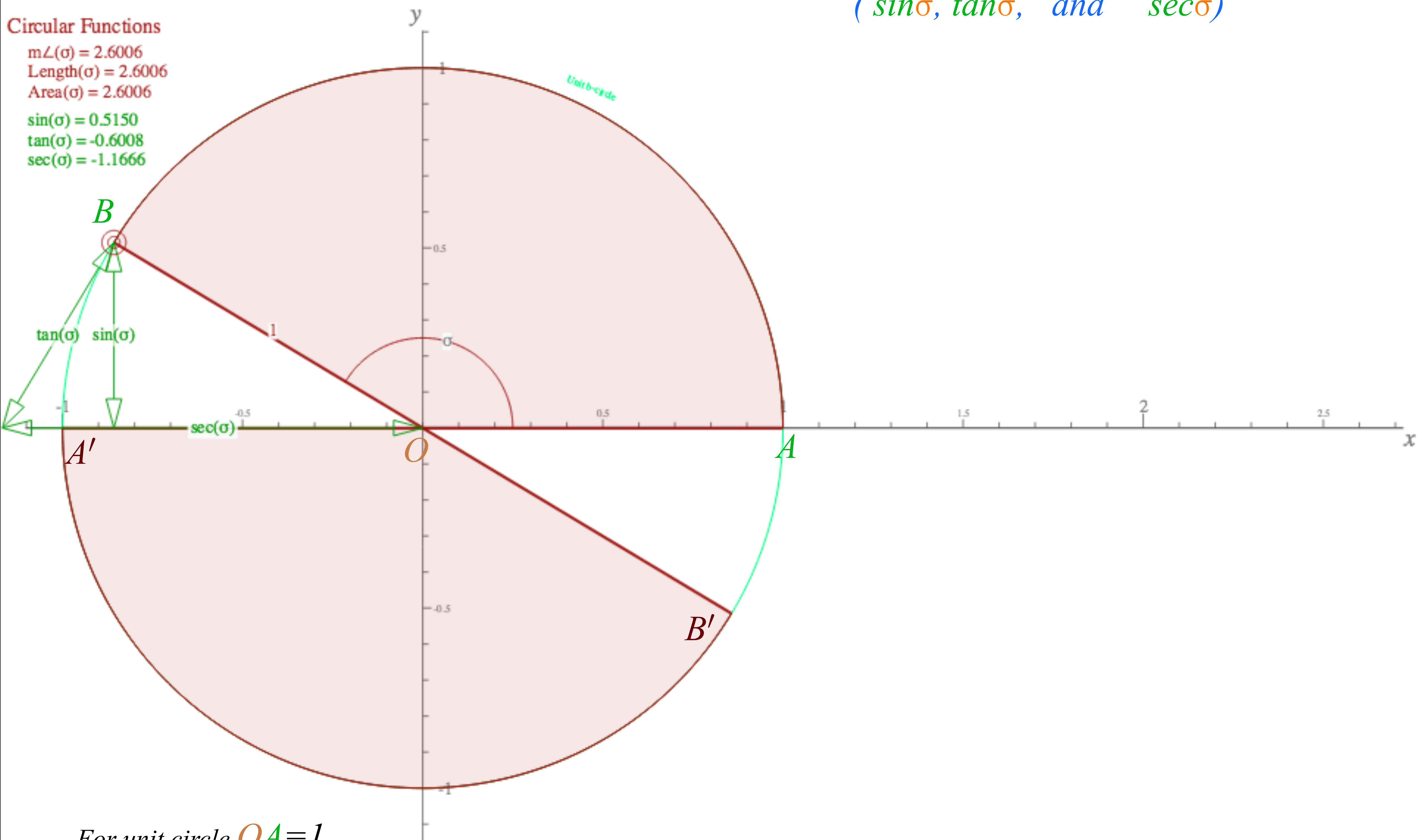
Unit Circle

A neo-liberal trigonometry lesson *(sine, tangent, and secant)*

(sinσ, tanσ, and secσ)



For unit circle OA=1

Angle σ =2.60 radians

ArcAB σ·1=2.60 cm

Total Area ABOA′B′

$\sigma \cdot 1^2 = 2.60\ cm^2$

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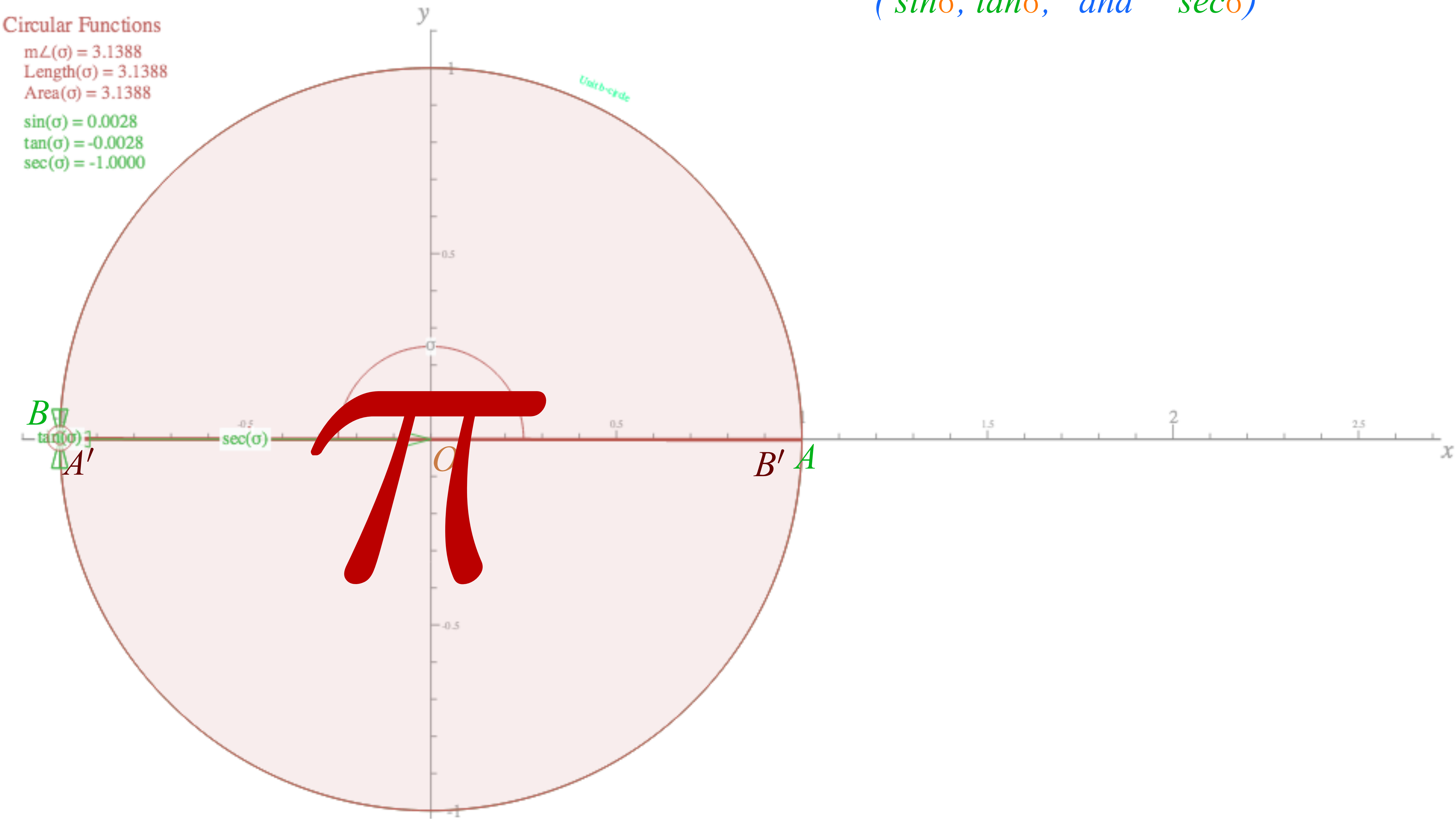
(sinσ, tanσ, and secσ)



For unit circle OA=1

Angle σ =3.14 radians

ArcAB σ·1=3.14 cm

Total Area ABOA′B′

$\sigma \cdot 1^2 = 3.14\ cm^2$

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[*Unit Circle*](#)

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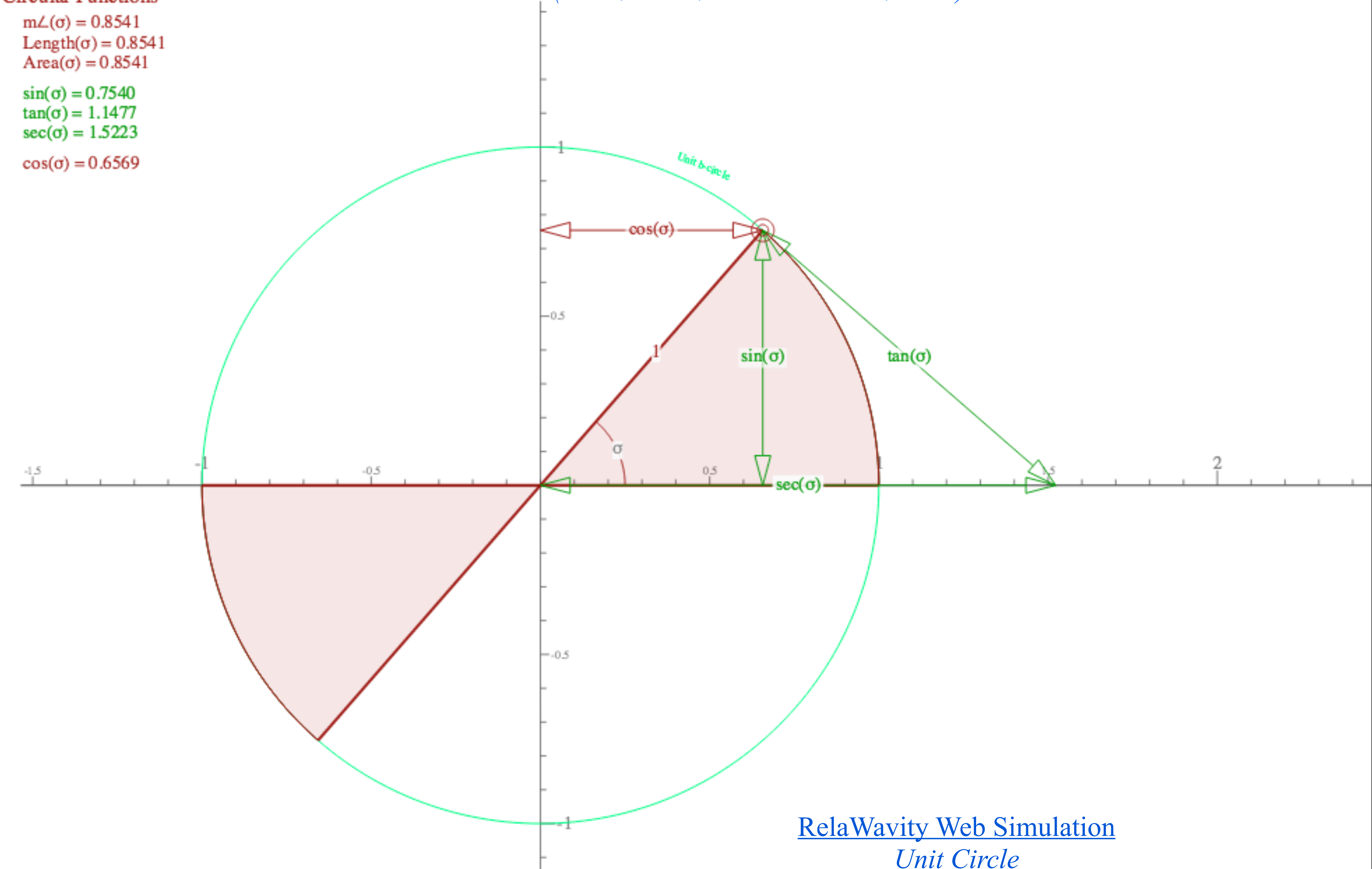
(sinσ, tanσ, and secσ,cosσ)



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[Unit Circle](#)

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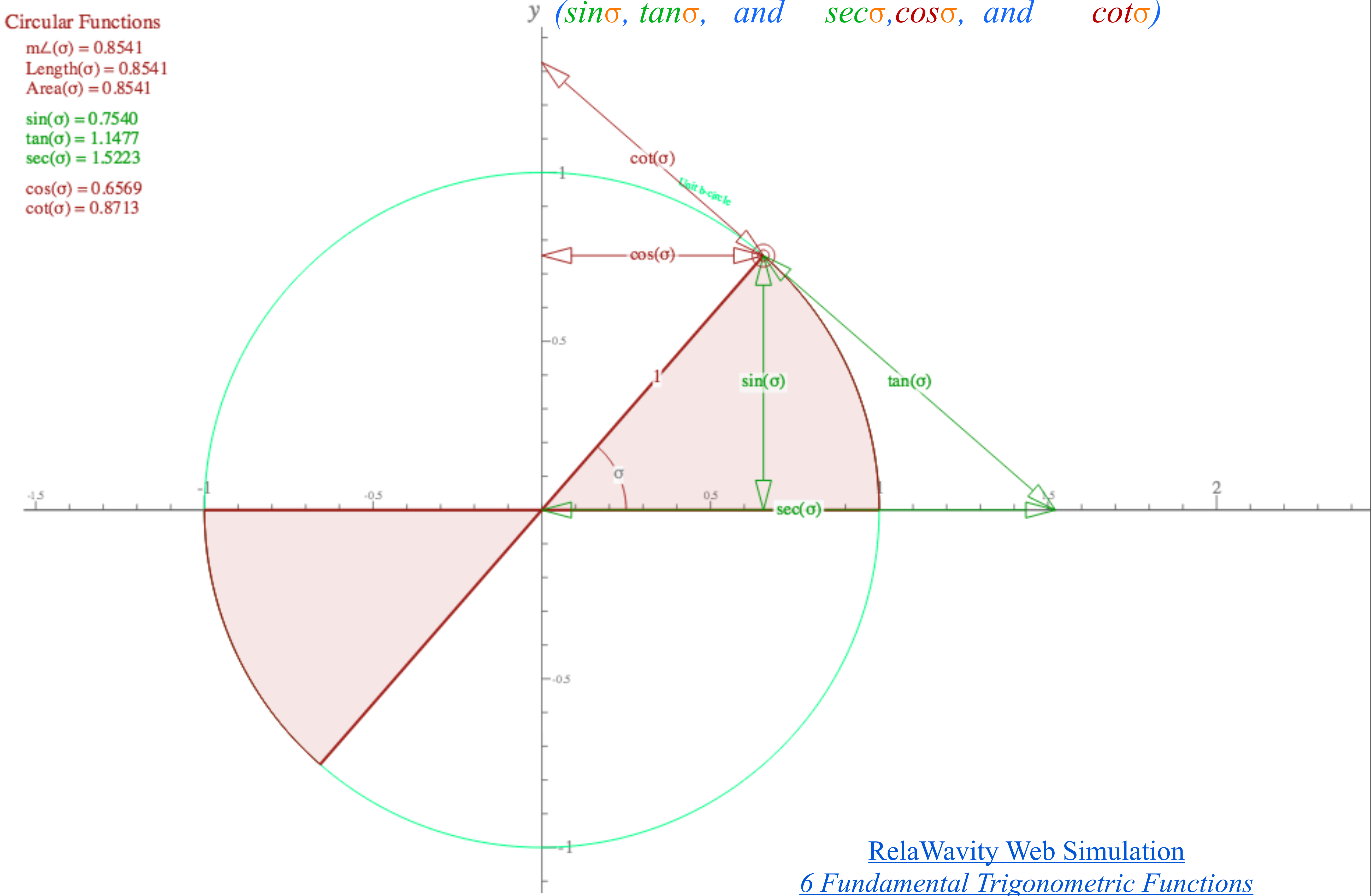

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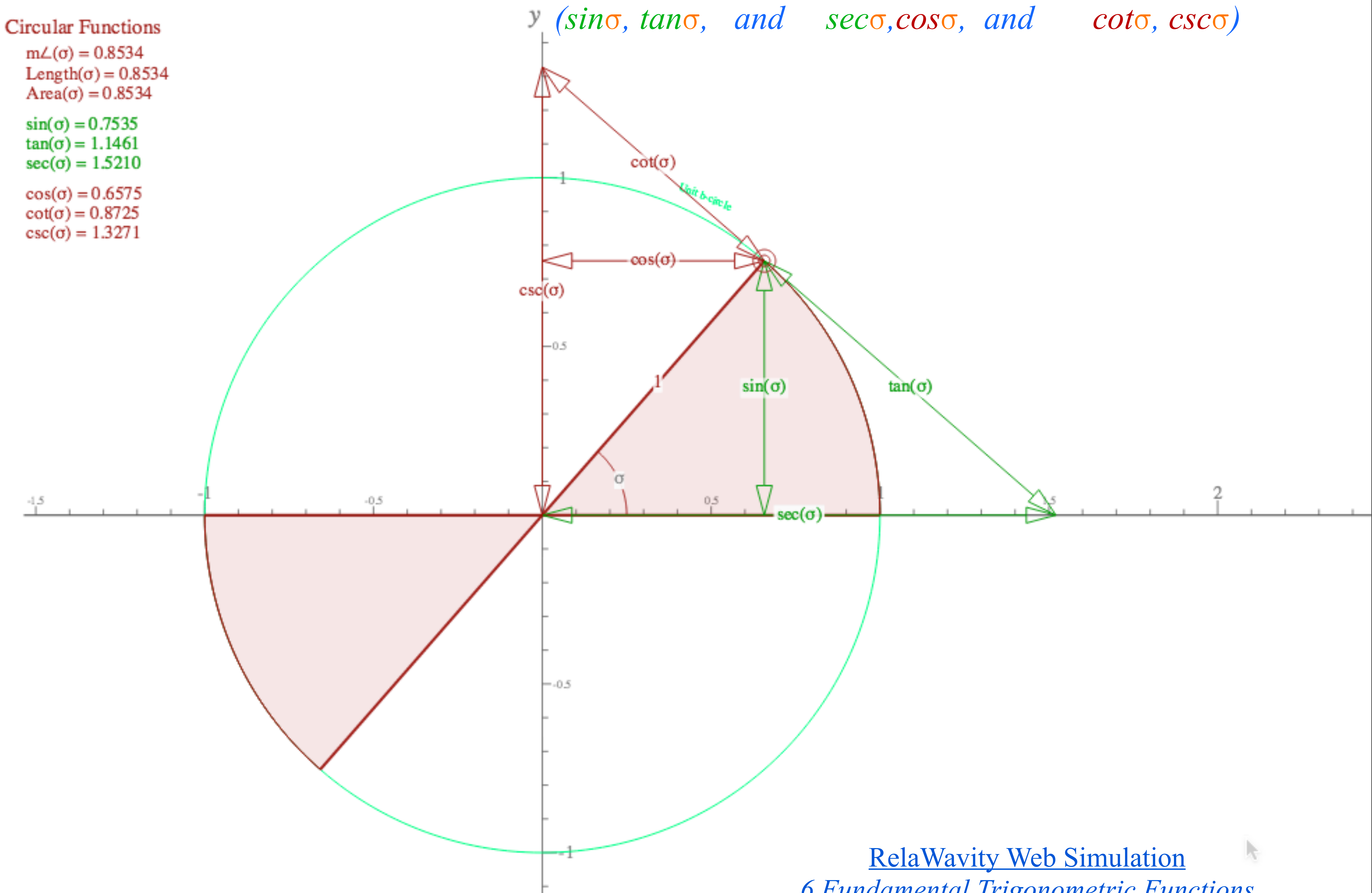
[*6 Fundamental Trigonometric Functions*](#)

A neo-liberal trigonometry *(sine, tangent, and secant, cosine, and cotangent, cosecant)*

(sinσ, tanσ, and secσ, cosσ, and cotσ, cscσ)



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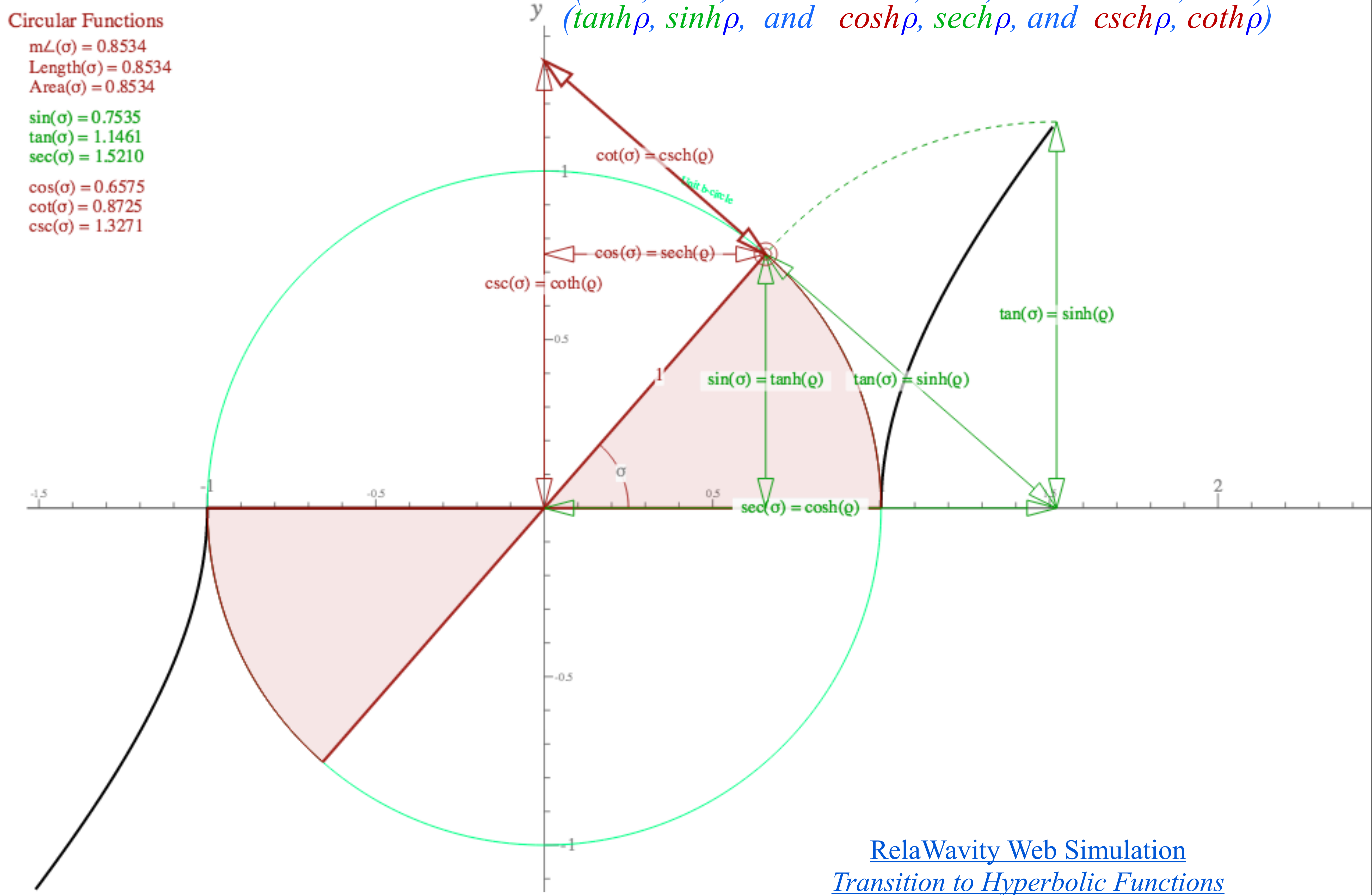
(tanhρ, sinhρ, and coshρ, sechρ, and cschρ, cothρ)



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[Transition to Hyperbolic Functions](#)

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(sinσ, tanσ, and secσ, cosσ, and cotσ, cscσ)

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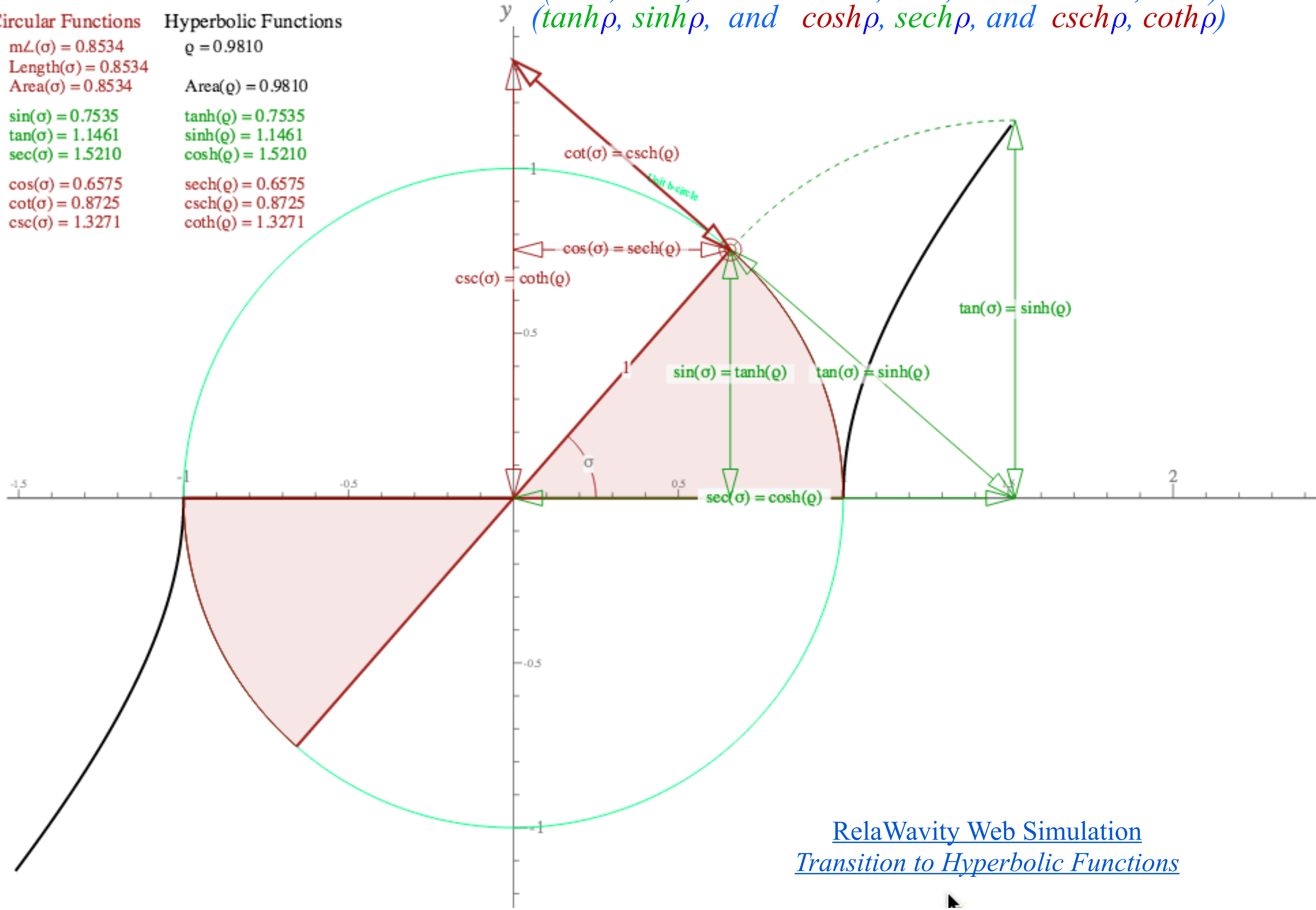
Circular Functions	Hyperbolic Functions
m∠(σ) = 0.8534	ϱ = 0.9810
Length(σ) = 0.8534	
Area(σ) = 0.8534	Area(ϱ) = 0.9810
sin(σ) = 0.7535	tanh(ϱ) = 0.7535
tan(σ) = 1.1461	sinh(ϱ) = 1.1461
sec(σ) = 1.5210	cosh(ϱ) = 1.5210
cos(σ) = 0.6575	sech(ϱ) = 0.6575
cot(σ) = 0.8725	csch(ϱ) = 0.8725
csc(σ) = 1.3271	coth(ϱ) = 1.3271



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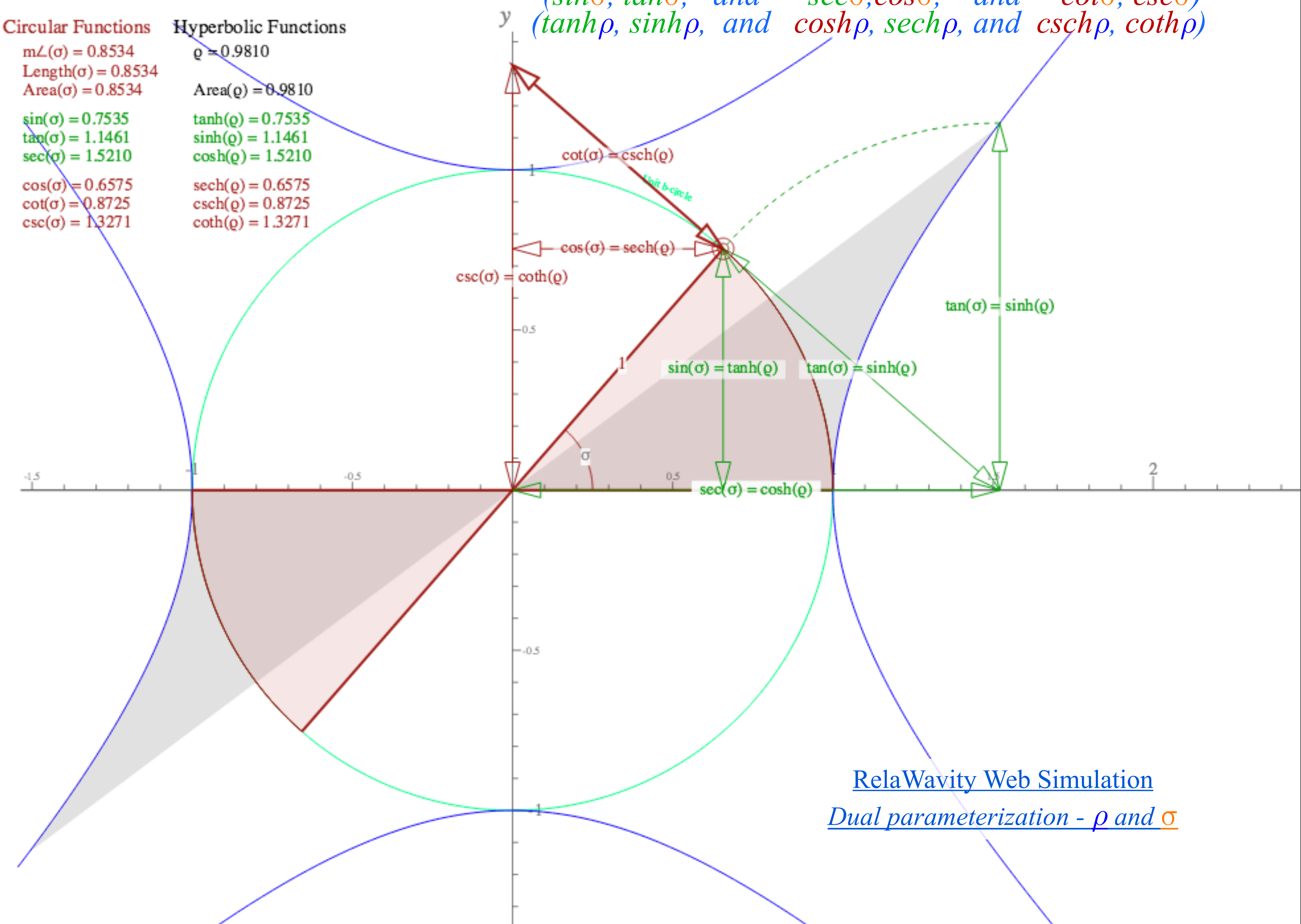

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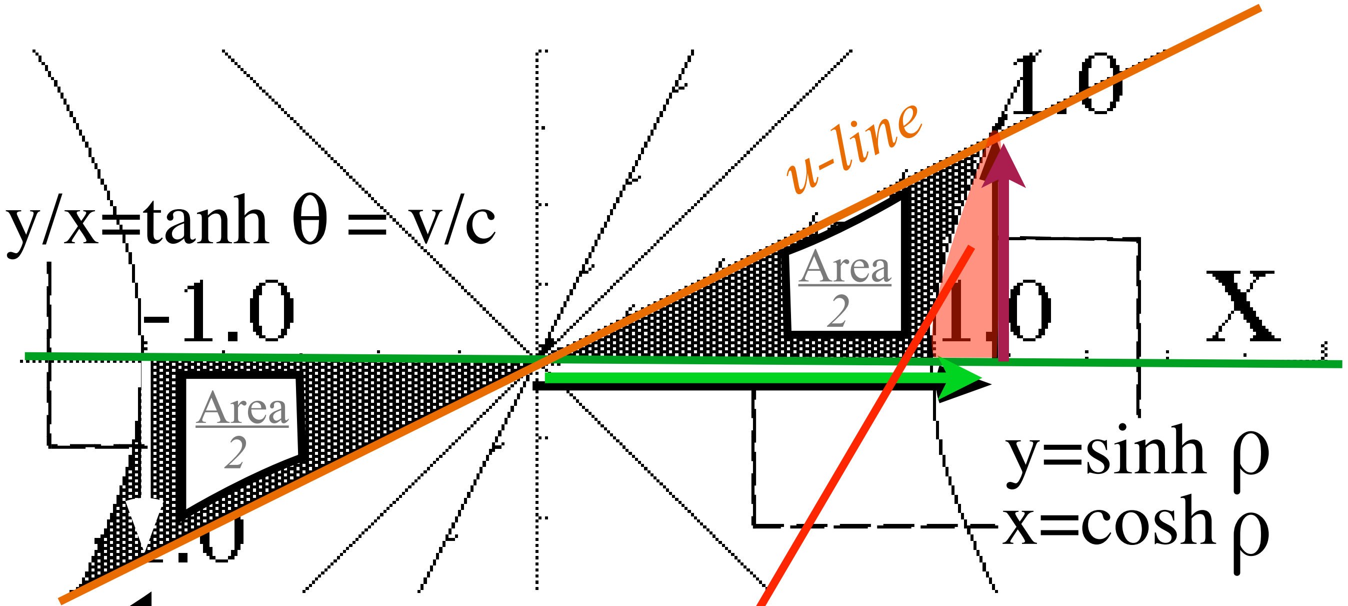
The straight scoop on "hyper-angle" and "rapidity" (They're area!)



$$\frac{\text{Area}}{2} = \frac{1}{2}\,\textit{base} \cdot \textit{altitude} - \text{area under curve} = \frac{1}{2}xy - \int y\,dx$$

The "Area" being calculated is the *<u>total</u>* **Gray Area** *between hyperbola pairs, X axis, and sloping u-line*

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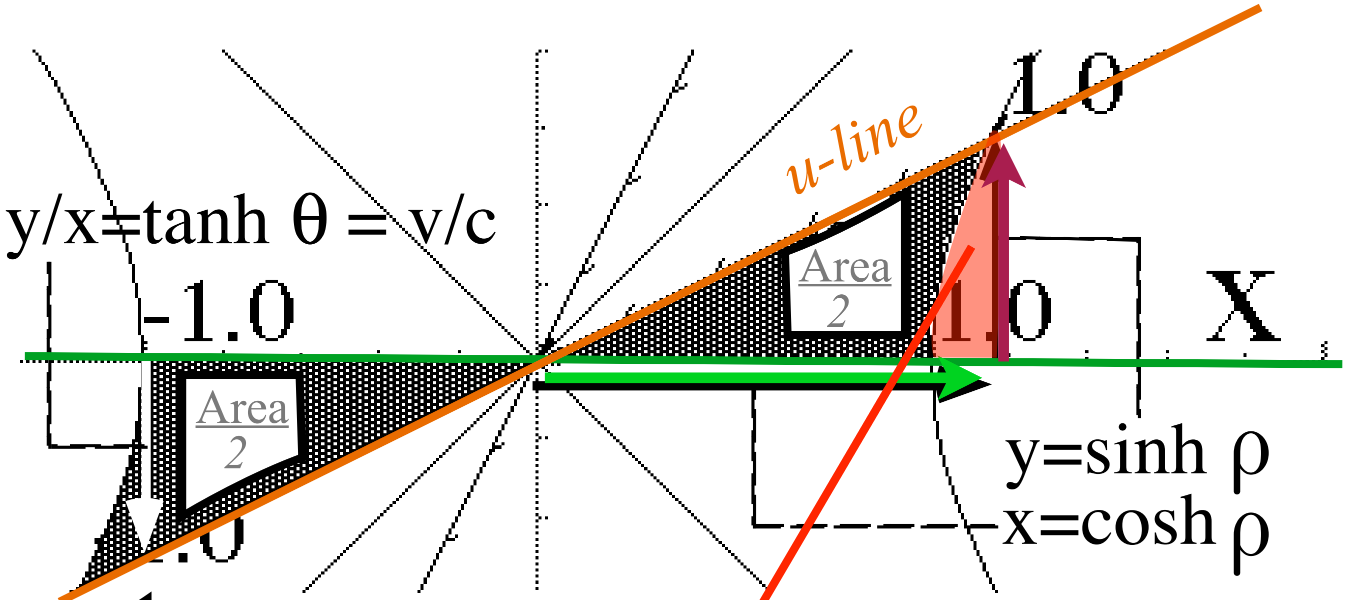

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$$\frac{Area}{2} = \frac{1}{2}\,base \cdot altitude - \text{area under curve} = \frac{1}{2}xy - \int y\,dx$$

$$\frac{Area}{2} = \frac{1}{2}\sinh\rho\cosh\rho - \int \sinh\rho\, d(\cosh\rho)$$

$$d(\cosh\rho) = \sinh\rho\, d\rho$$

Useful hyperbolic identities

$$\sinh^2\rho = \left(\frac{e^{\rho}-e^{-\rho}}{2}\right)^2 = \frac{1}{4}\left(e^{2\rho}+e^{-2\rho}-2\right) = \frac{\cosh 2\rho - 1}{2}$$

$$\sinh\rho\cosh\rho = \left(\frac{e^{\rho}-e^{-\rho}}{2}\right)\left(\frac{e^{\rho}+e^{-\rho}}{2}\right) = \frac{1}{4}\left(e^{2\rho}-e^{-2\rho}\right) = \frac{1}{2}\sinh 2\rho$$

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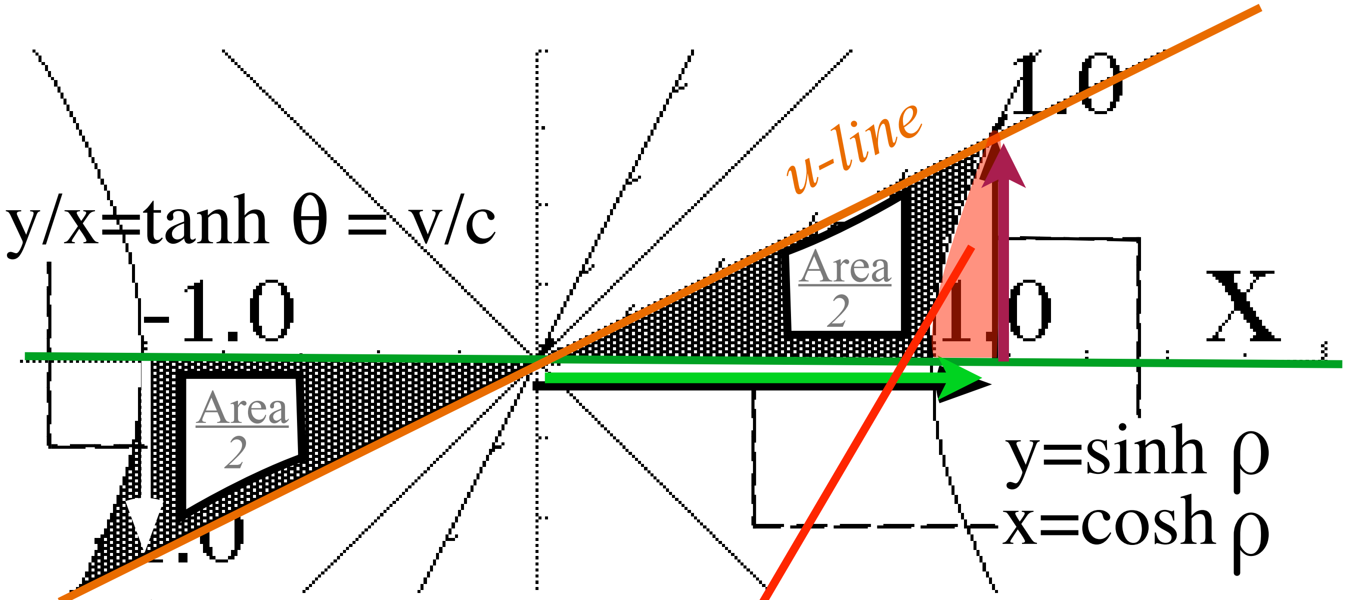

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$$\frac{Area}{2} = \frac{1}{2} \sinh\rho \cosh\rho - \int \sinh\rho \, d(\cosh\rho)$$

$$d(\cosh\rho) = \sinh\rho \, d\rho$$

$$\frac{Area}{2} = \frac{1}{2} \sinh\rho \cosh\rho - \int \sinh^2\rho \, d\rho = \frac{1}{4} \sinh 2\rho - \int \frac{\cosh 2\rho - 1}{2} d\rho$$

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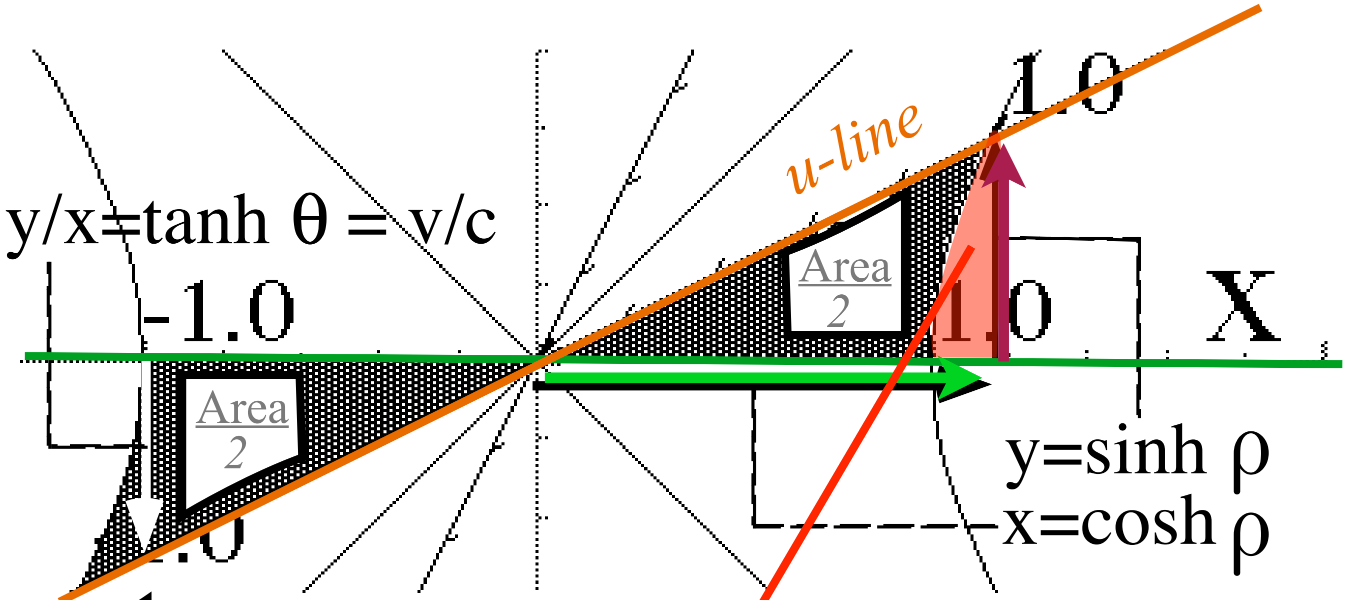
$$\sinh^2\rho = \left(\frac{e^{\rho} - e^{-\rho}}{2}\right)^2 = \frac{1}{4}\left(e^{2\rho} + e^{-2\rho} - 2\right) = \frac{\cosh 2\rho - 1}{2}$$

$$\sinh\theta \cosh\theta = \left(\frac{e^{\theta} - e^{-\theta}}{2}\right)\left(\frac{e^{\theta} + e^{-\theta}}{2}\right) = \frac{1}{4}\left(e^{2\theta} - e^{-2\theta}\right) = \frac{1}{2}\sinh 2\theta$$

$$\int \cosh a\rho \, d\rho = \frac{1}{a} \sinh a\rho$$

The straight scoop on "hyper-angle" and "rapidity" (They're area!)



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$$\int\cosh a\theta\, d\theta = \frac{1}{a}\sinh a\theta$$

$$\frac{Area}{2} = \frac{1}{2}\sinh\rho\cosh\rho - \int\sinh^2\rho\, d\rho = \frac{1}{4}\sinh 2\rho - \int\frac{\cosh 2\rho - 1}{2}\, d\rho$$

$$= \frac{1}{4}\sinh 2\rho - \frac{1}{4}\sinh 2\rho + \int\frac{1}{2}\, d\rho$$

$$= \frac{\rho}{2}$$

Amazing result: Area = ρ is rapidity

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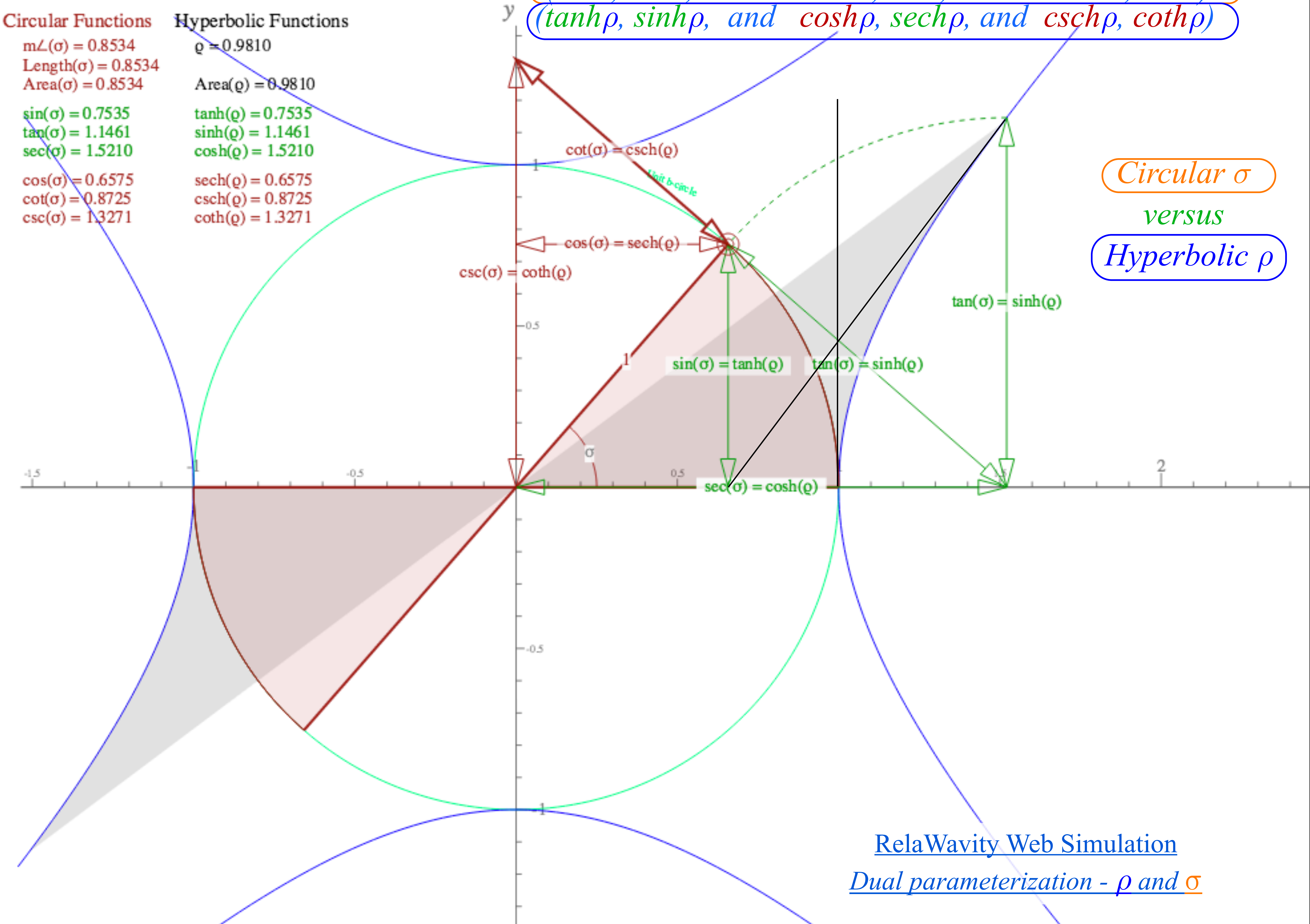

Circular σ

versus

Hyperbolic ρ

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Dual parameterization - ρ and σ

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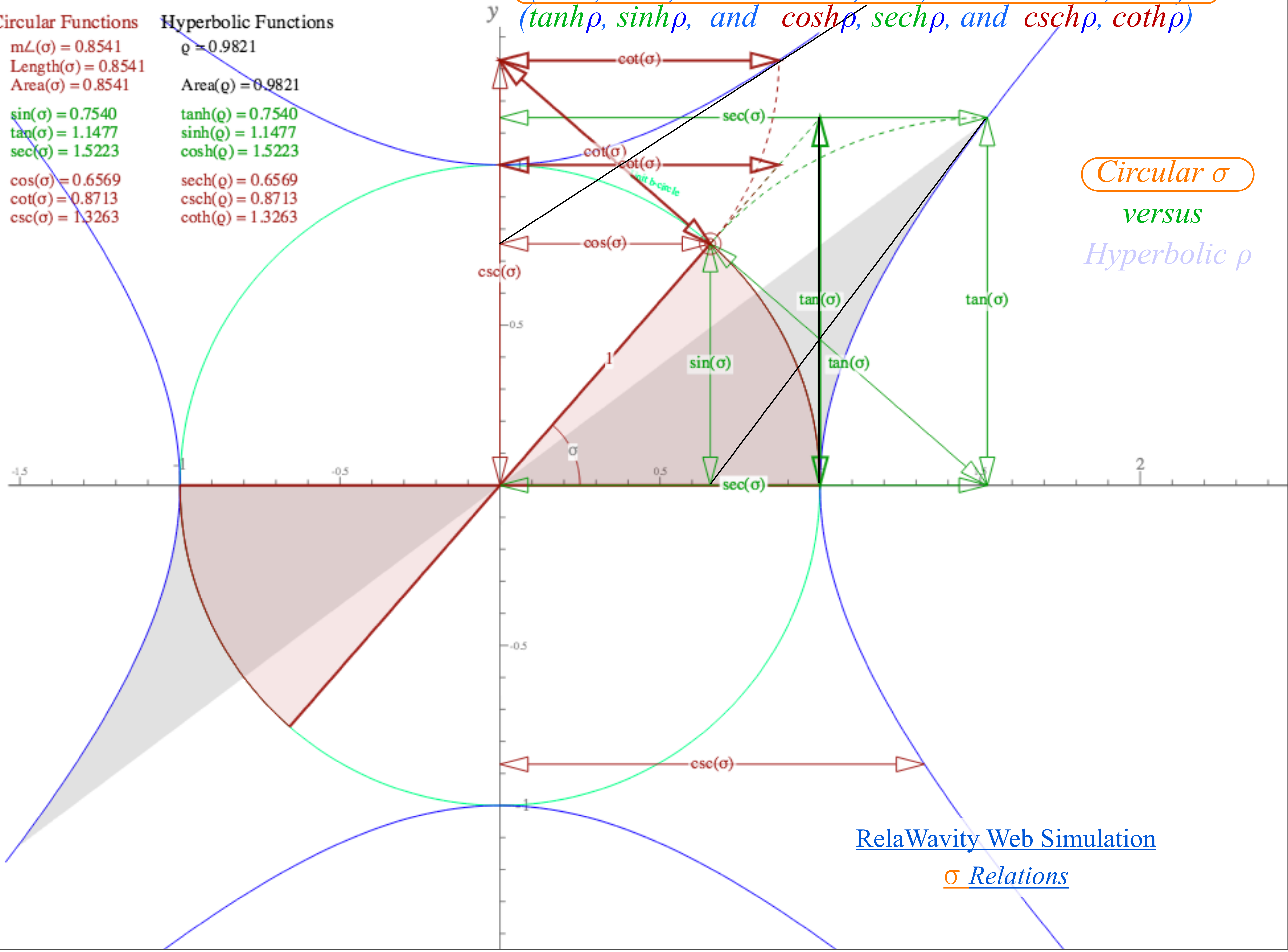

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σ Relations

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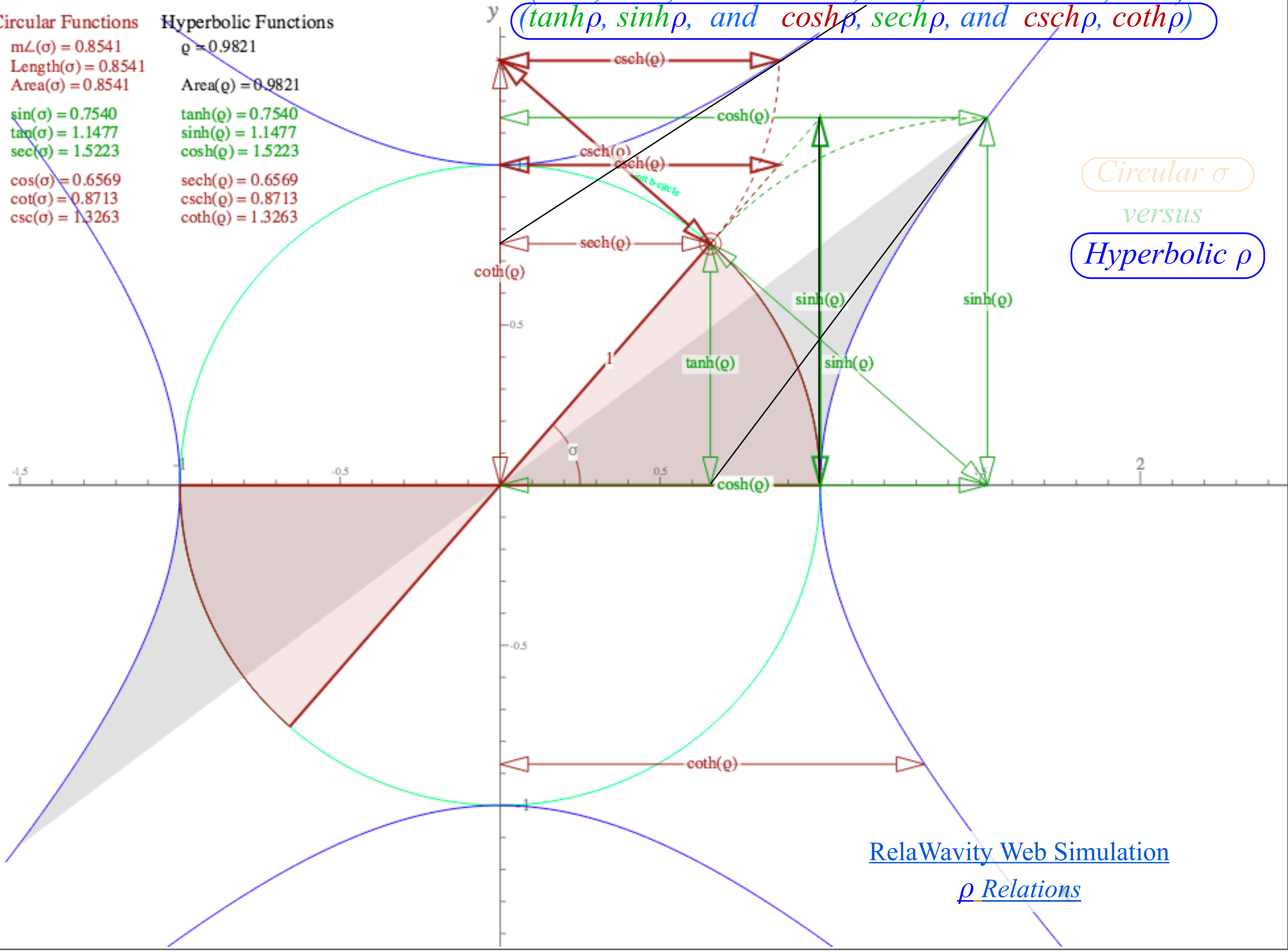

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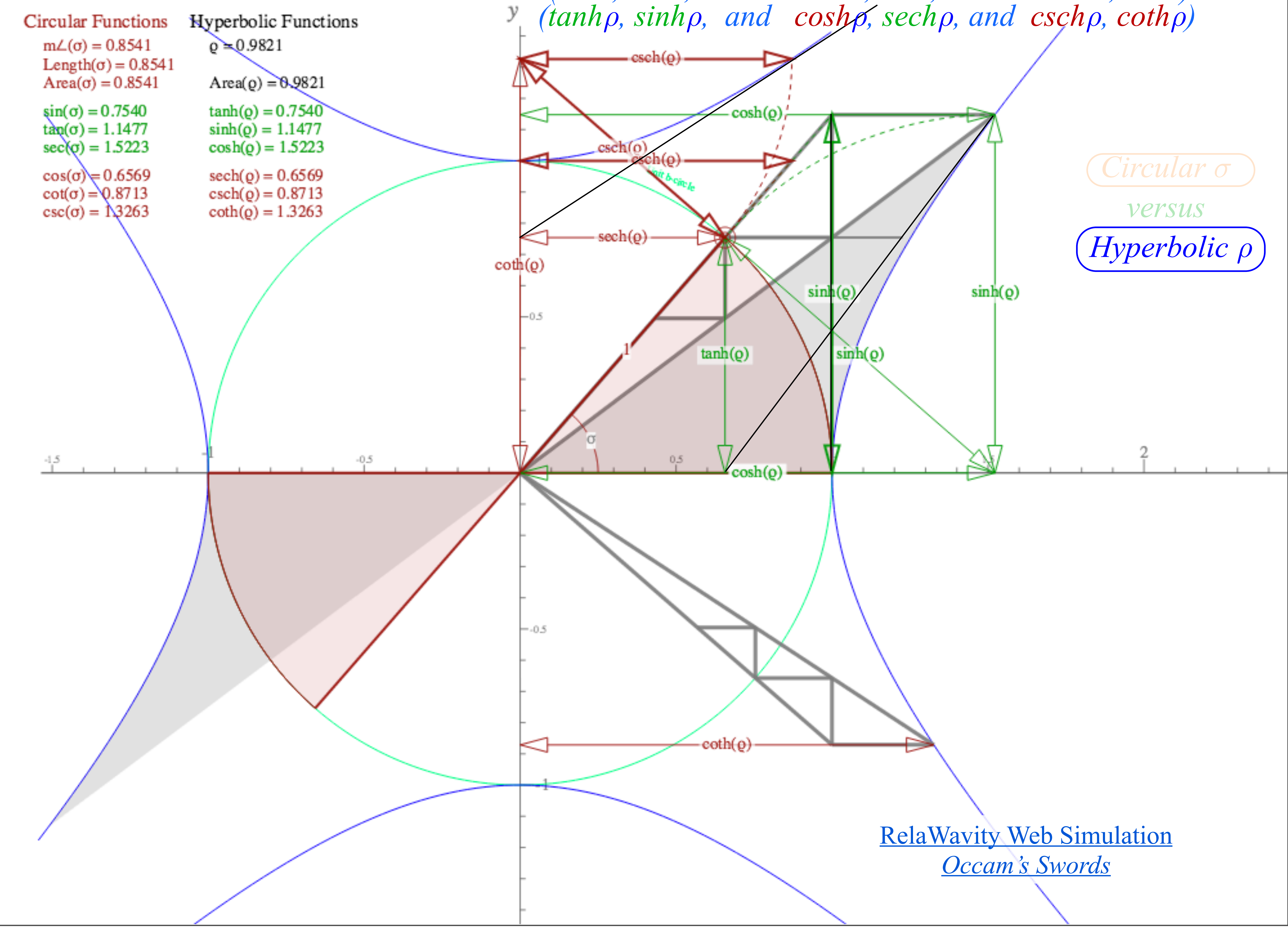

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Occam's Swords

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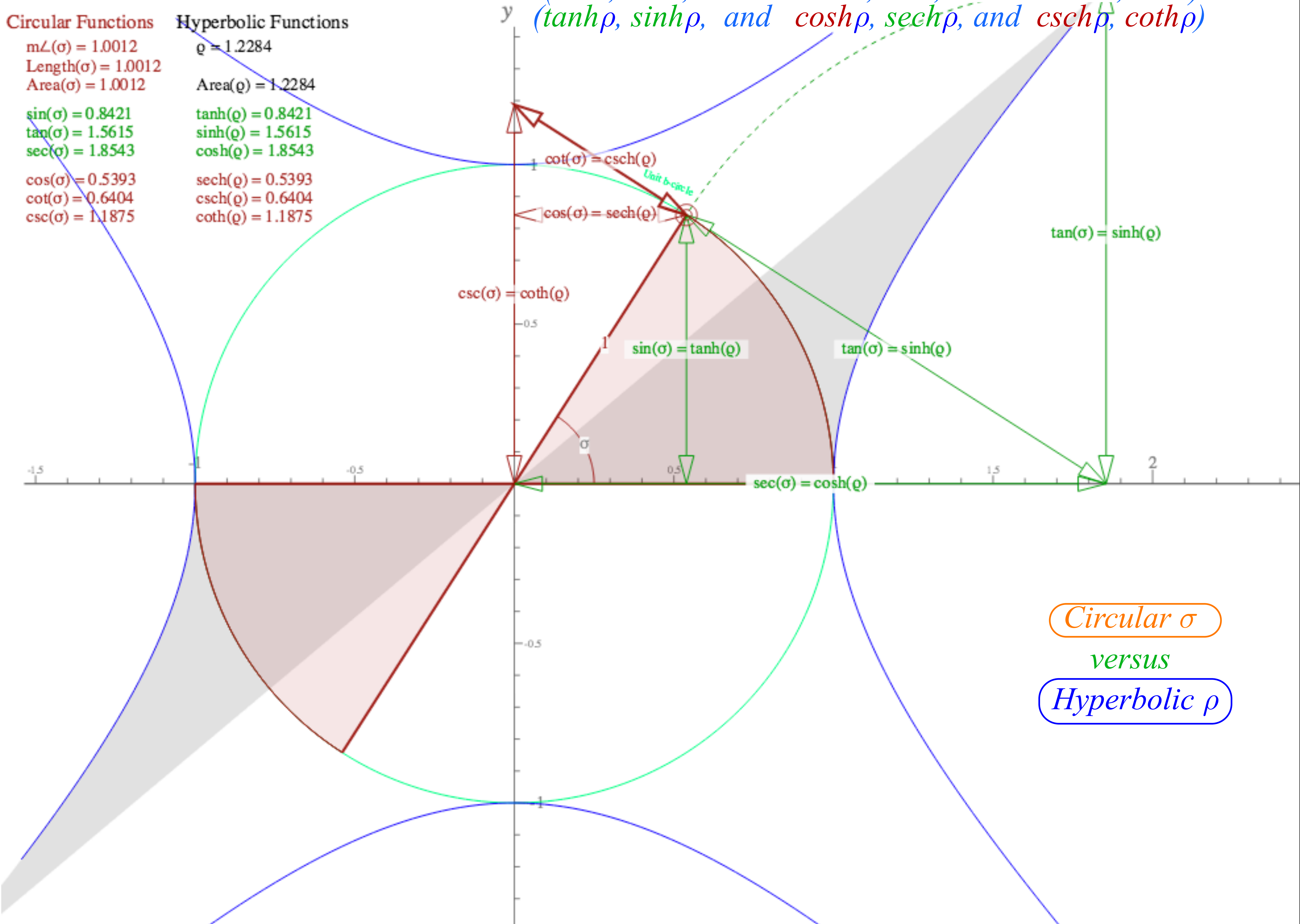

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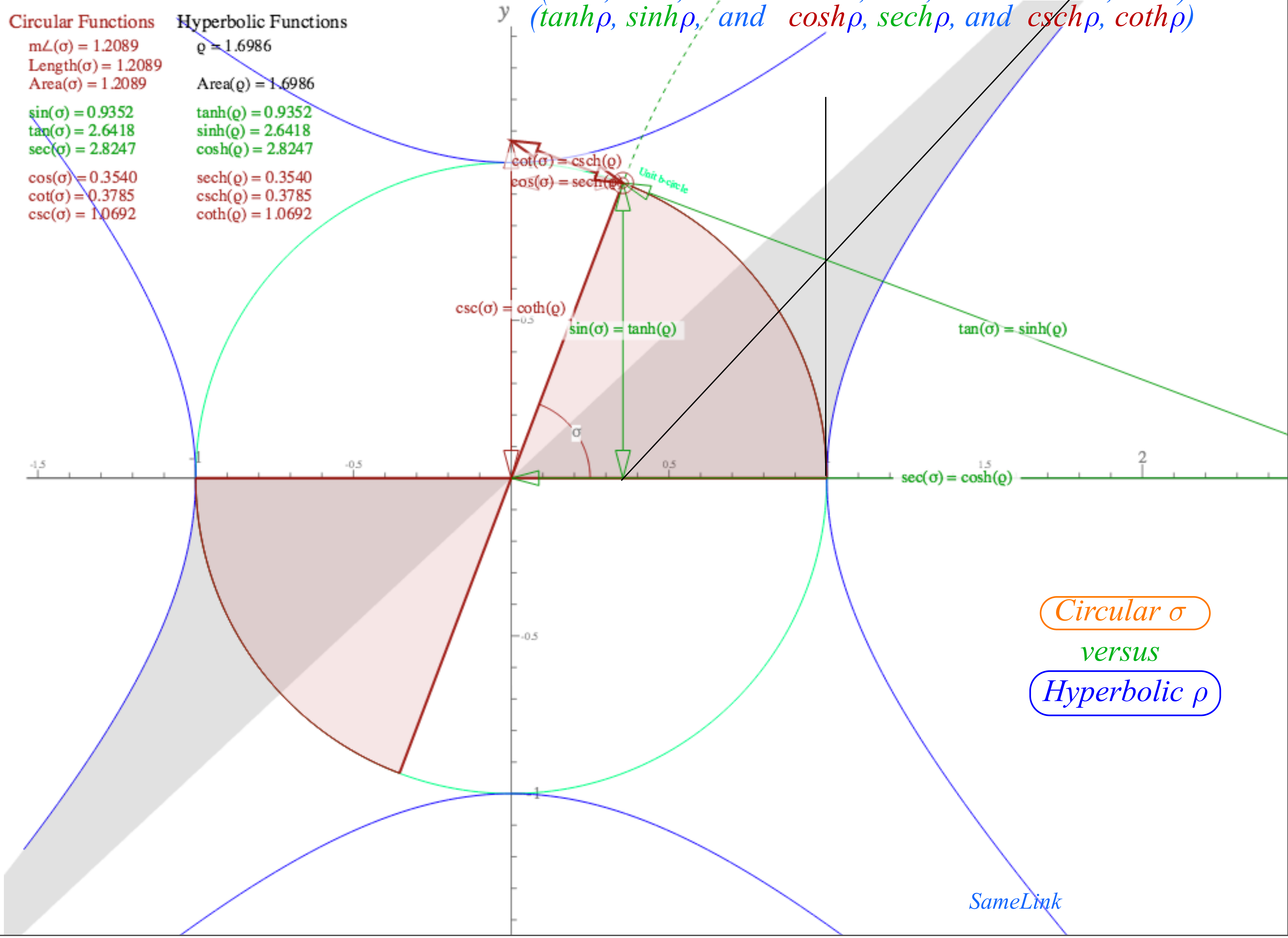

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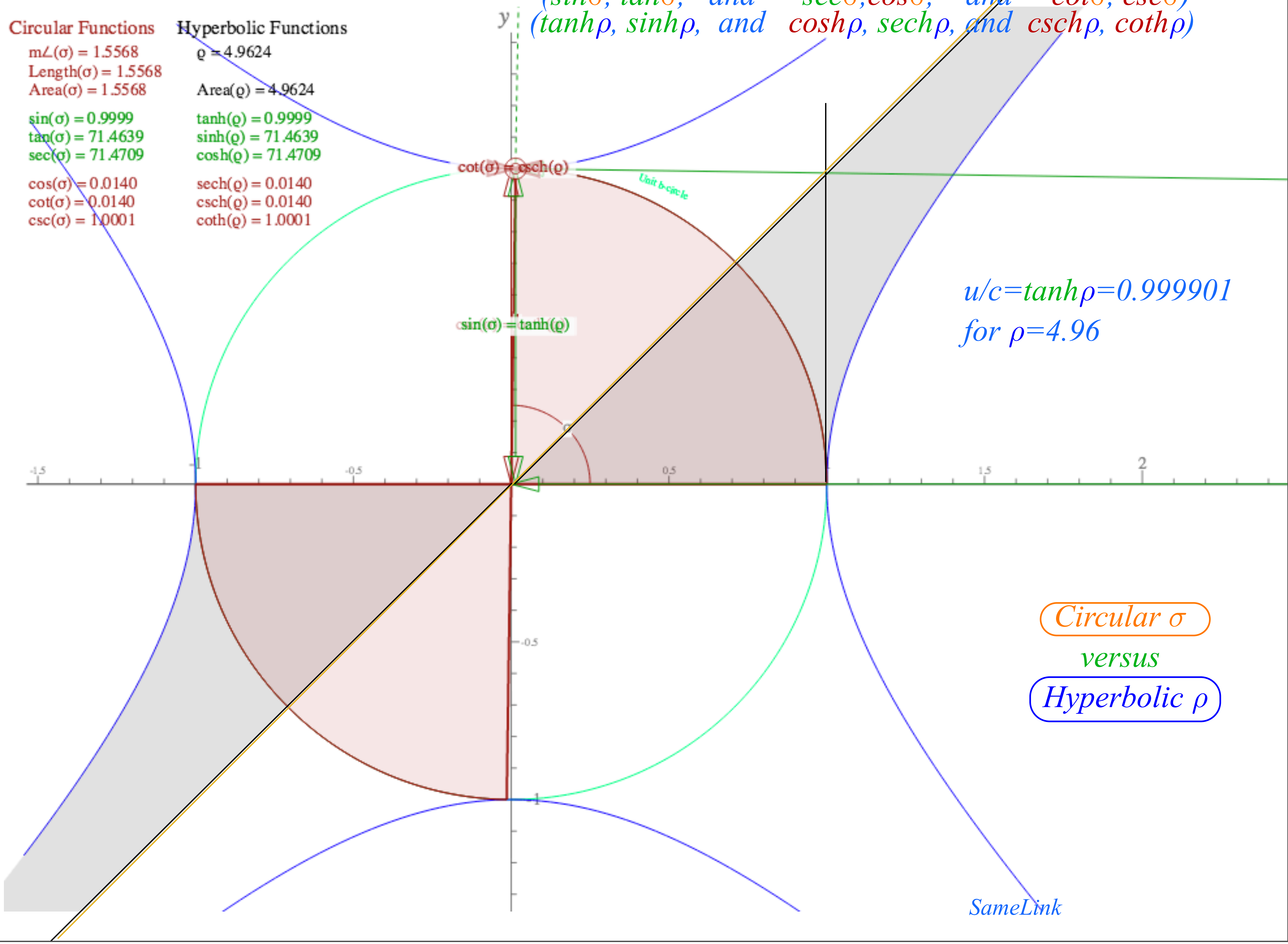

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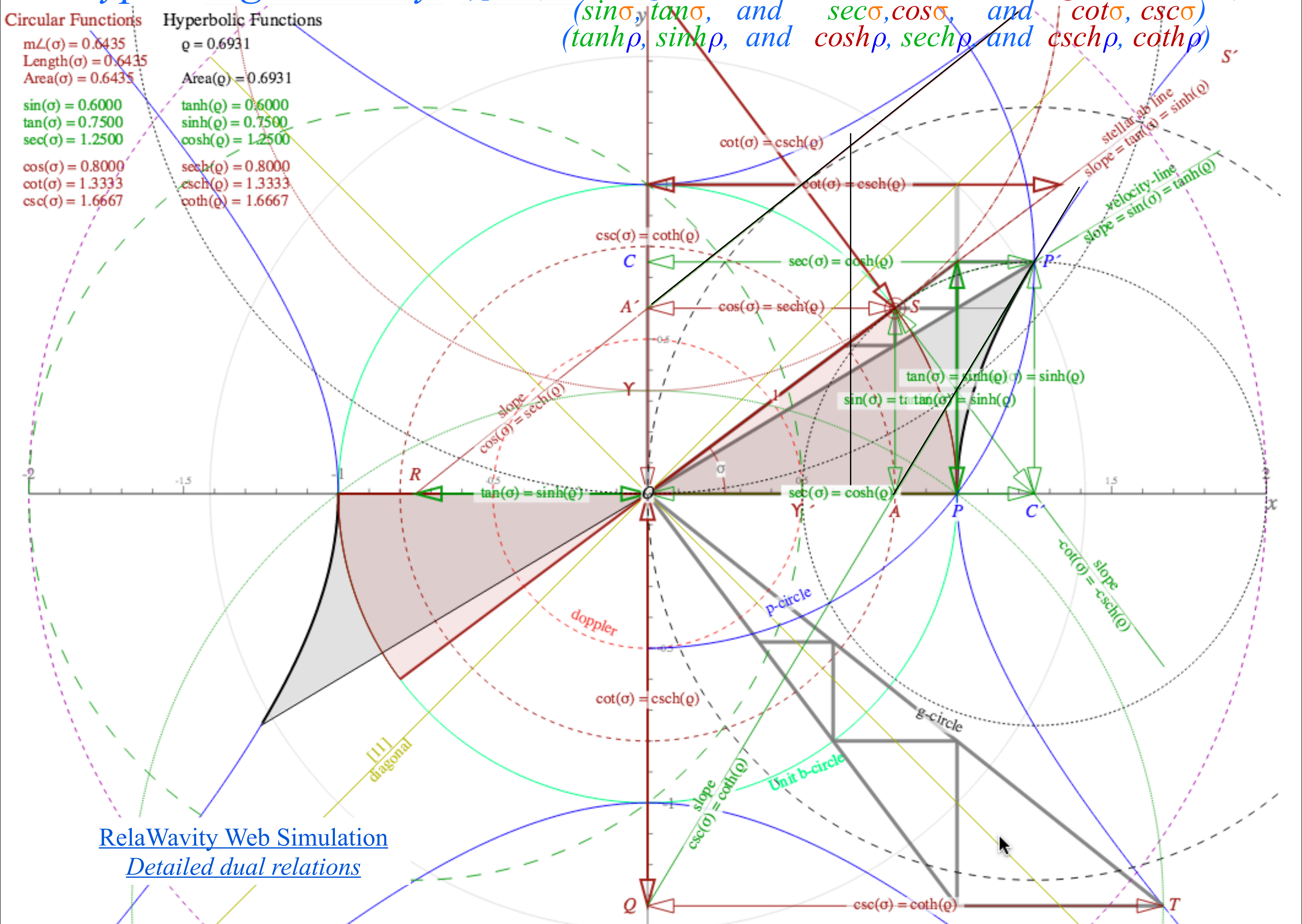
(sinσ, tanσ, and secσ, cosσ, and cotσ, cscσ)

(tanhρ, sinhρ, and coshρ, sechρ, and cschρ, cothρ)



[RelaWavity Web Simulation](#)

[Detailed dual relations](#)

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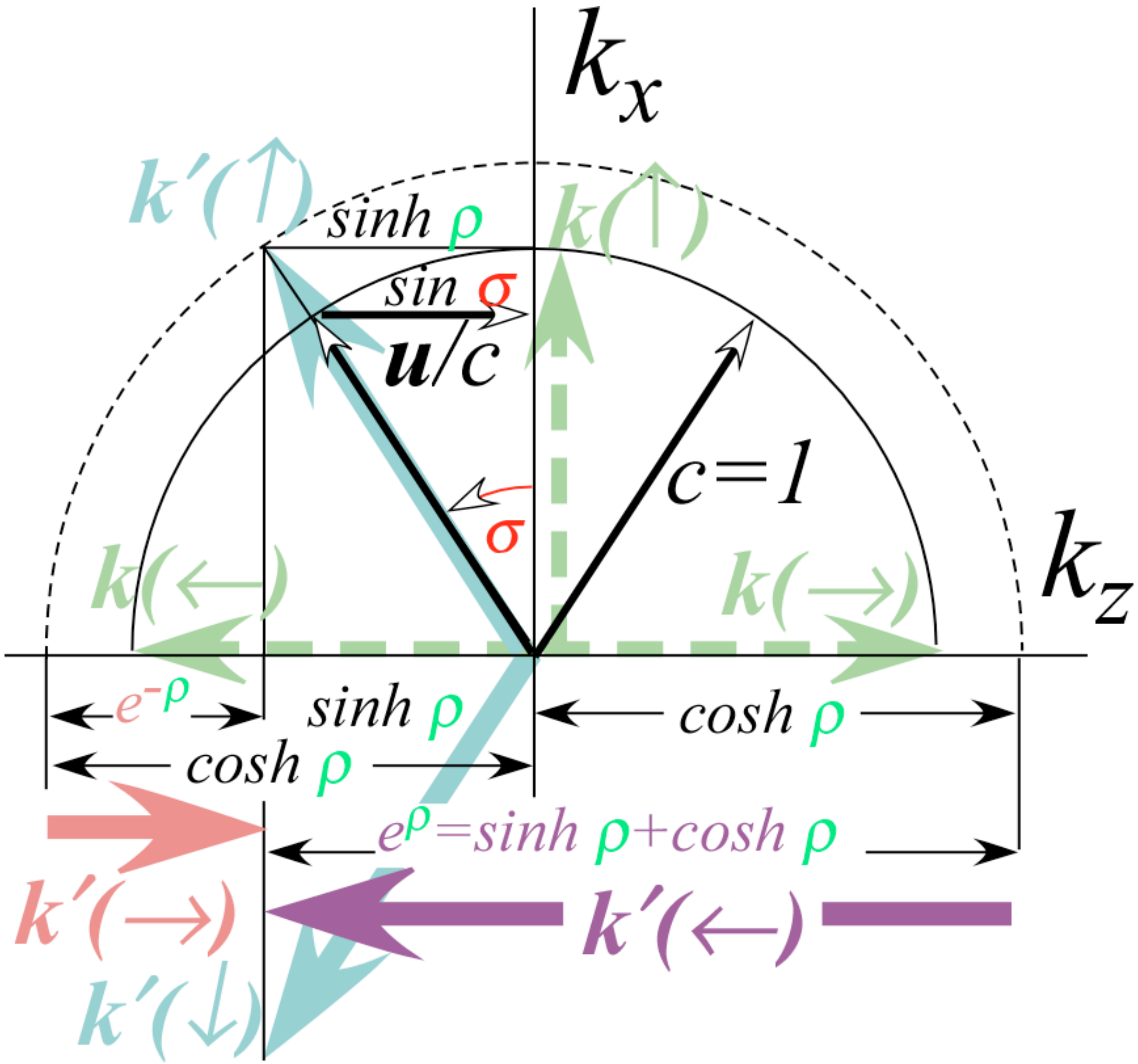
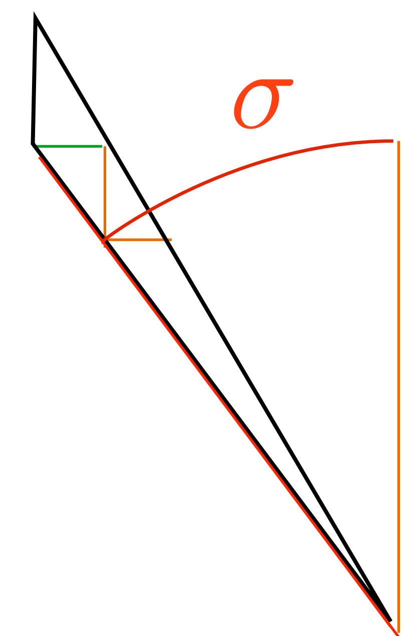
Pattern recognition: "Occam's Sword"

Fig. 5.10 CW cosmic speedometer.

Geometry of Lorentz boost of counter-propagating waves.





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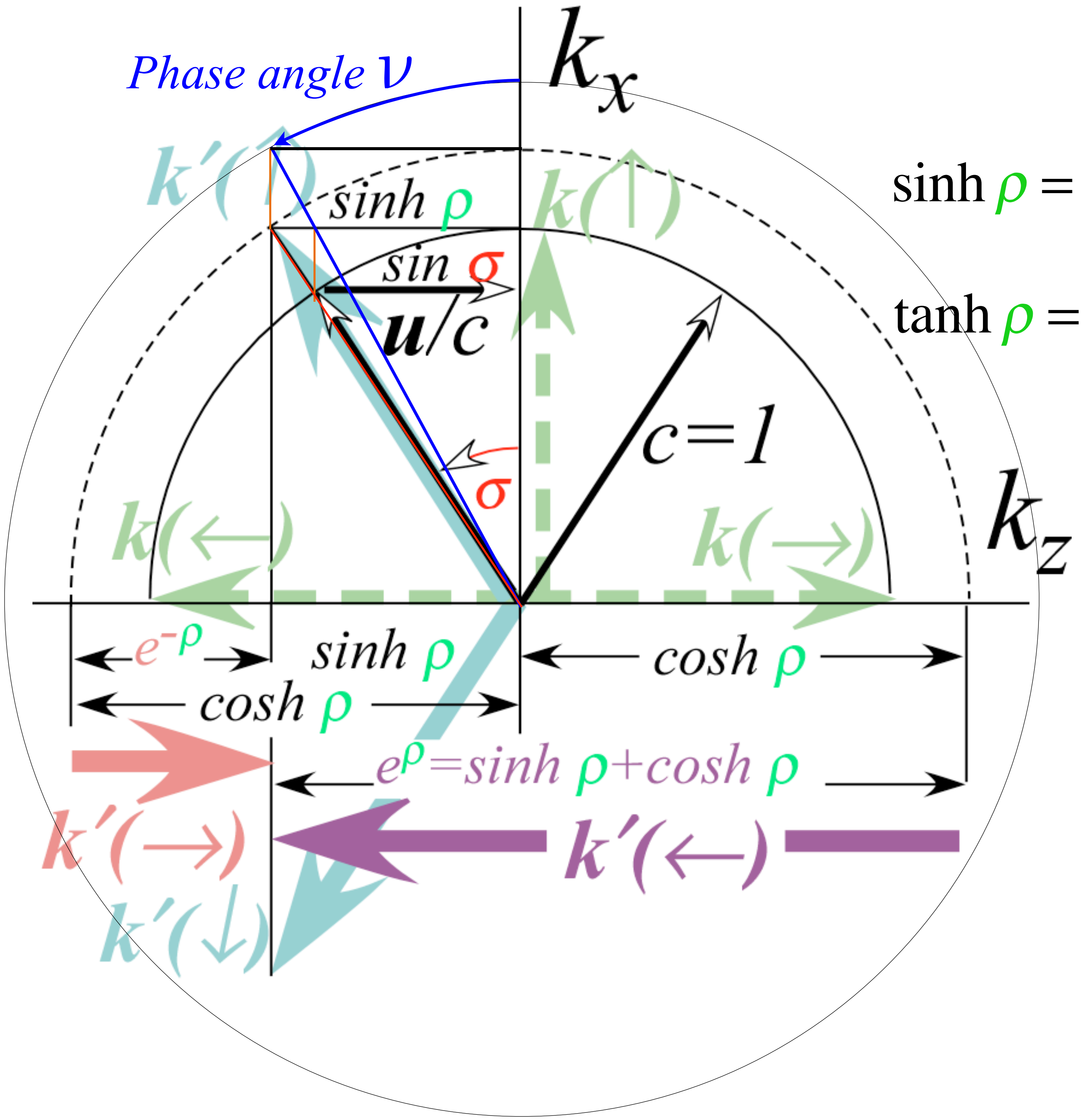
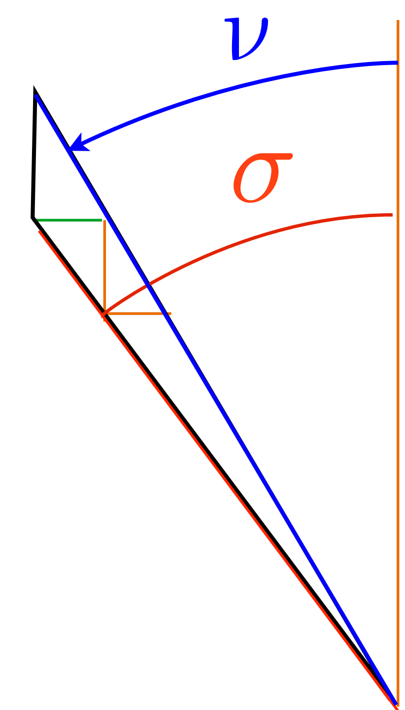


$$\sinh\rho = \tan\sigma$$

$$\tanh\rho = \sin\sigma = \frac{u}{c} = \tan\nu$$



Pattern recognition aid: "Occam's Sword"

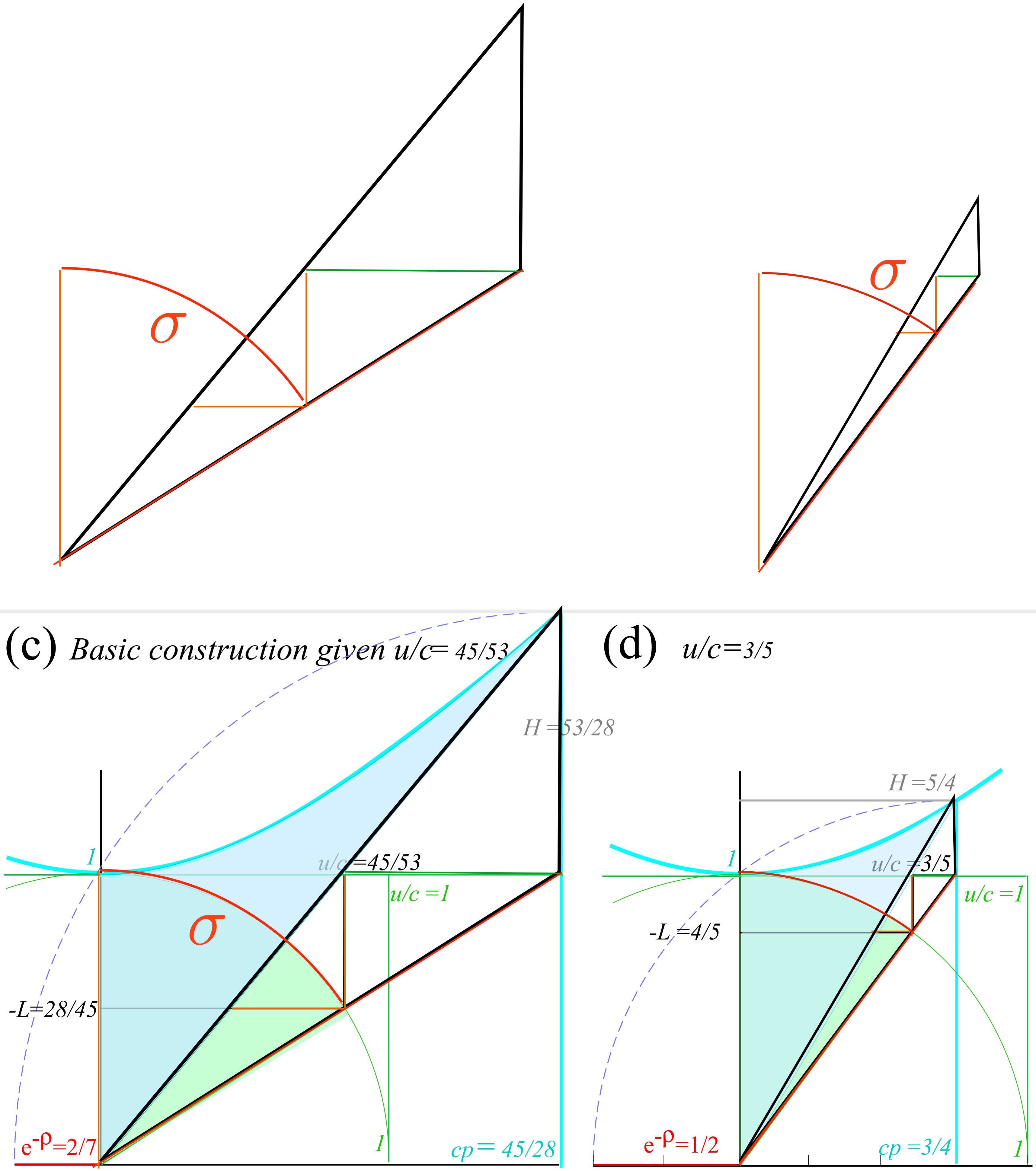


Fig. 5.5

Relativistic wave mechanics geometry.

(a) Overview.

(b-d) Details of contacting tangents.

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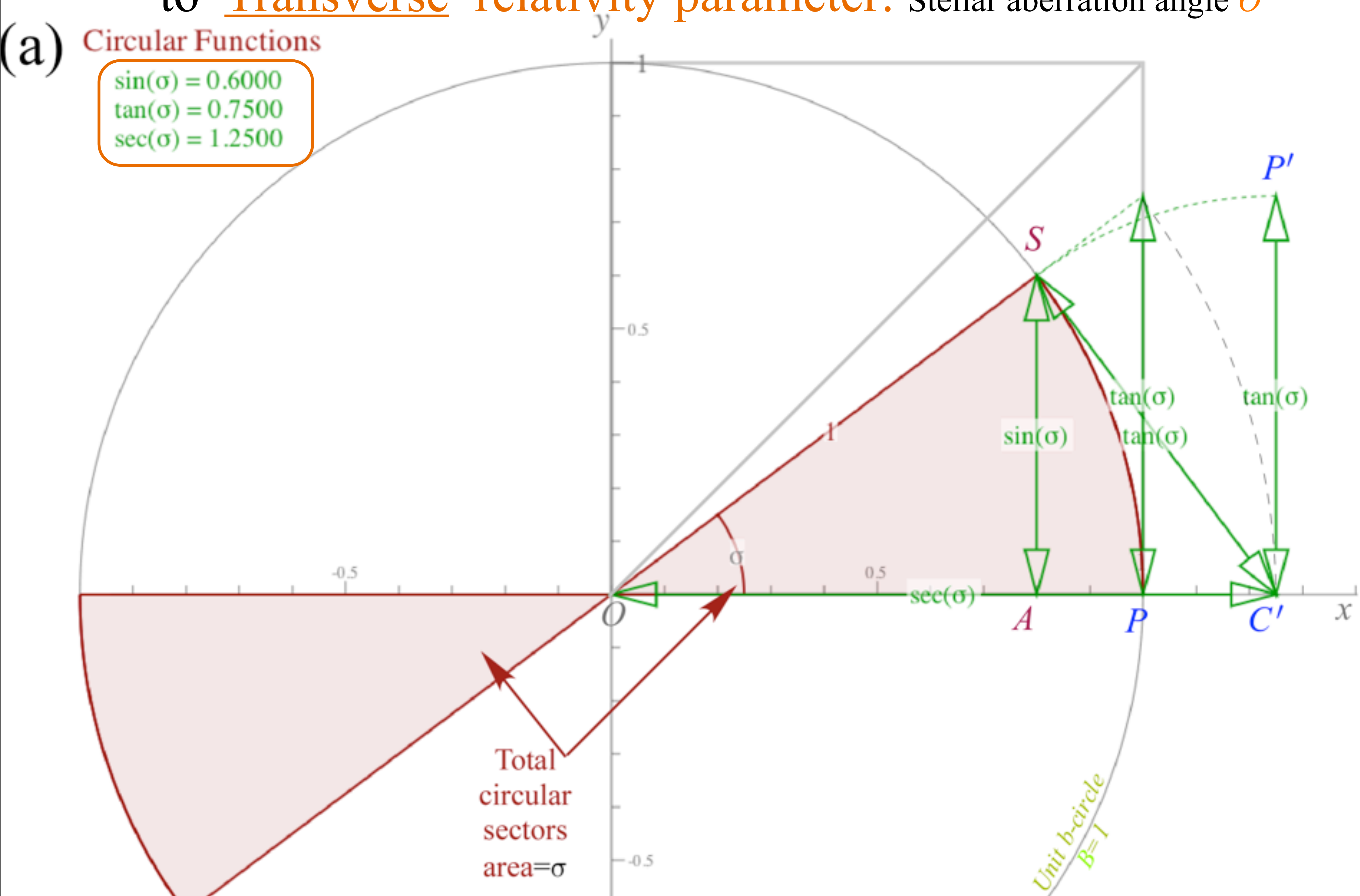
to Transverse relativity parameter: Stellar aberration angle σ



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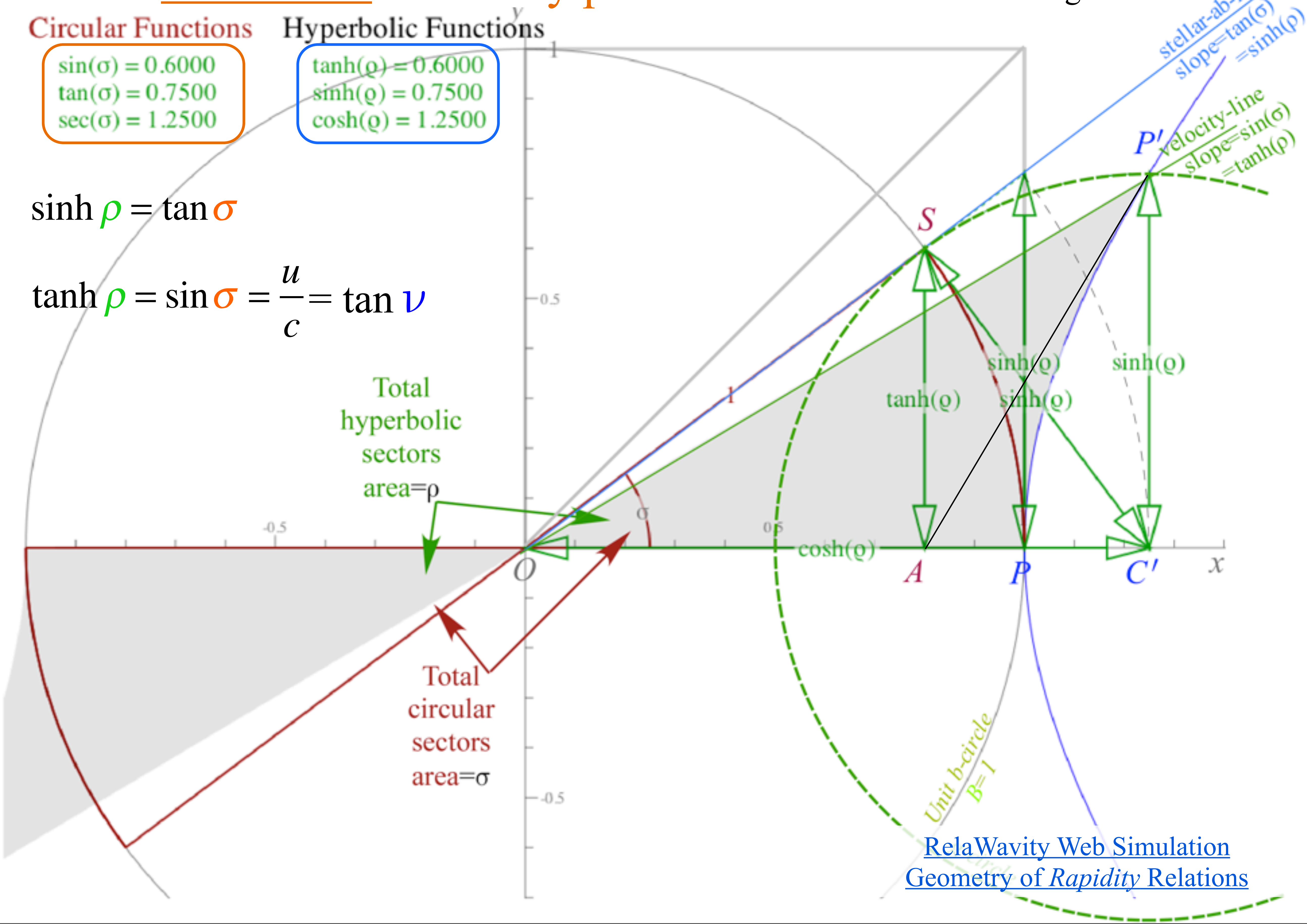
Geometry of Stellar Aberation Angle

Relating Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)
to Transverse relativity parameter: Stellar aberration angle σ

$$\sinh \rho = \tan \sigma$$

$$\tanh \rho = \sin \sigma = \frac{u}{c} = \tan \nu$$



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Geometry of *Rapidity* Relations

Relating Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

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Circular Functions

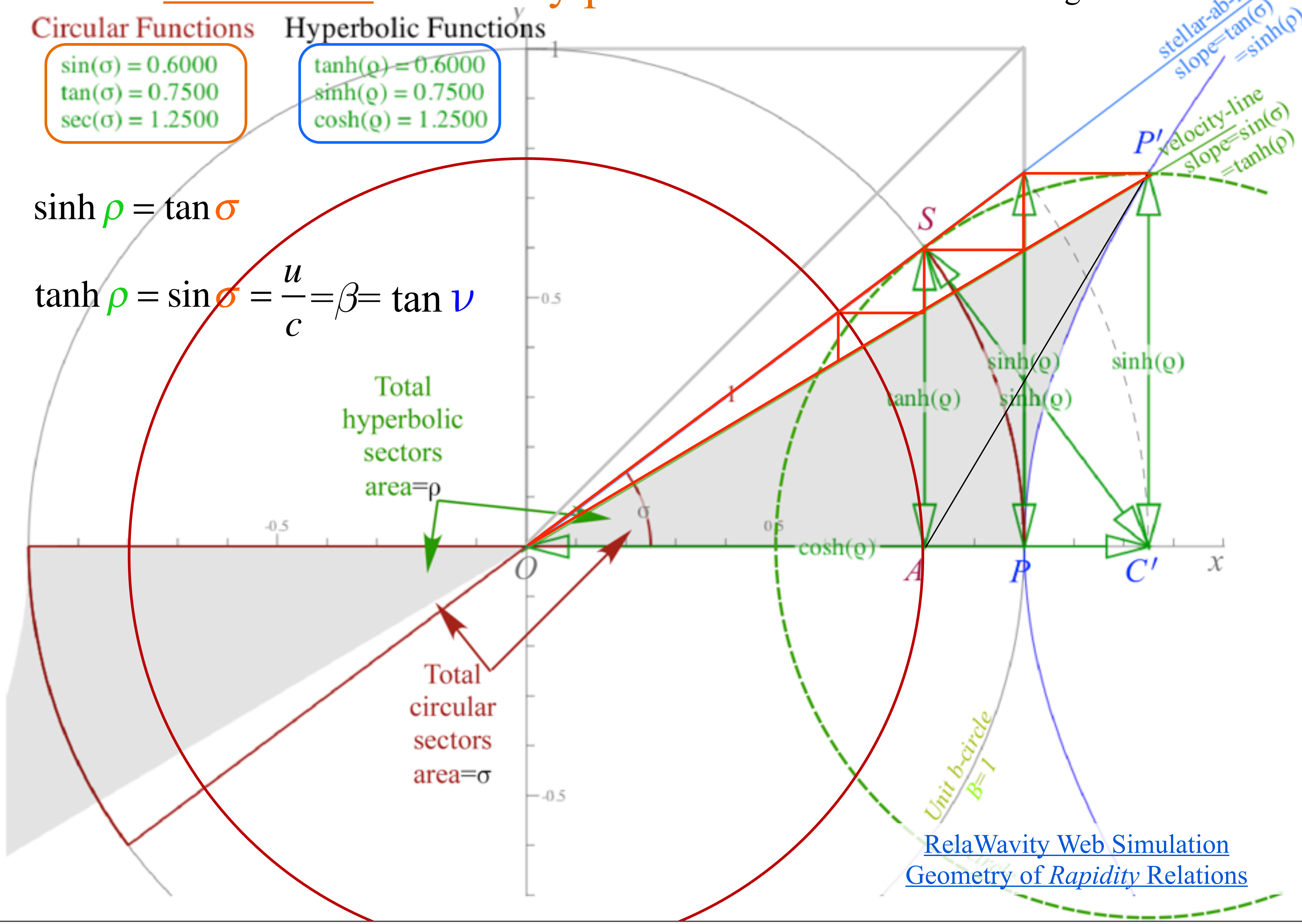
$\sin(\sigma) = 0.6000$
$\tan(\sigma) = 0.7500$
$\sec(\sigma) = 1.2500$

Hyperbolic Functions

$\tanh(\varrho) = 0.6000$
$\sinh(\varrho) = 0.7500$
$\cosh(\varrho) = 1.2500$

$$\sinh \rho = \tan \sigma$$

$$\tanh \rho = \sin \sigma = \frac{u}{c} = \beta = \tan \nu$$



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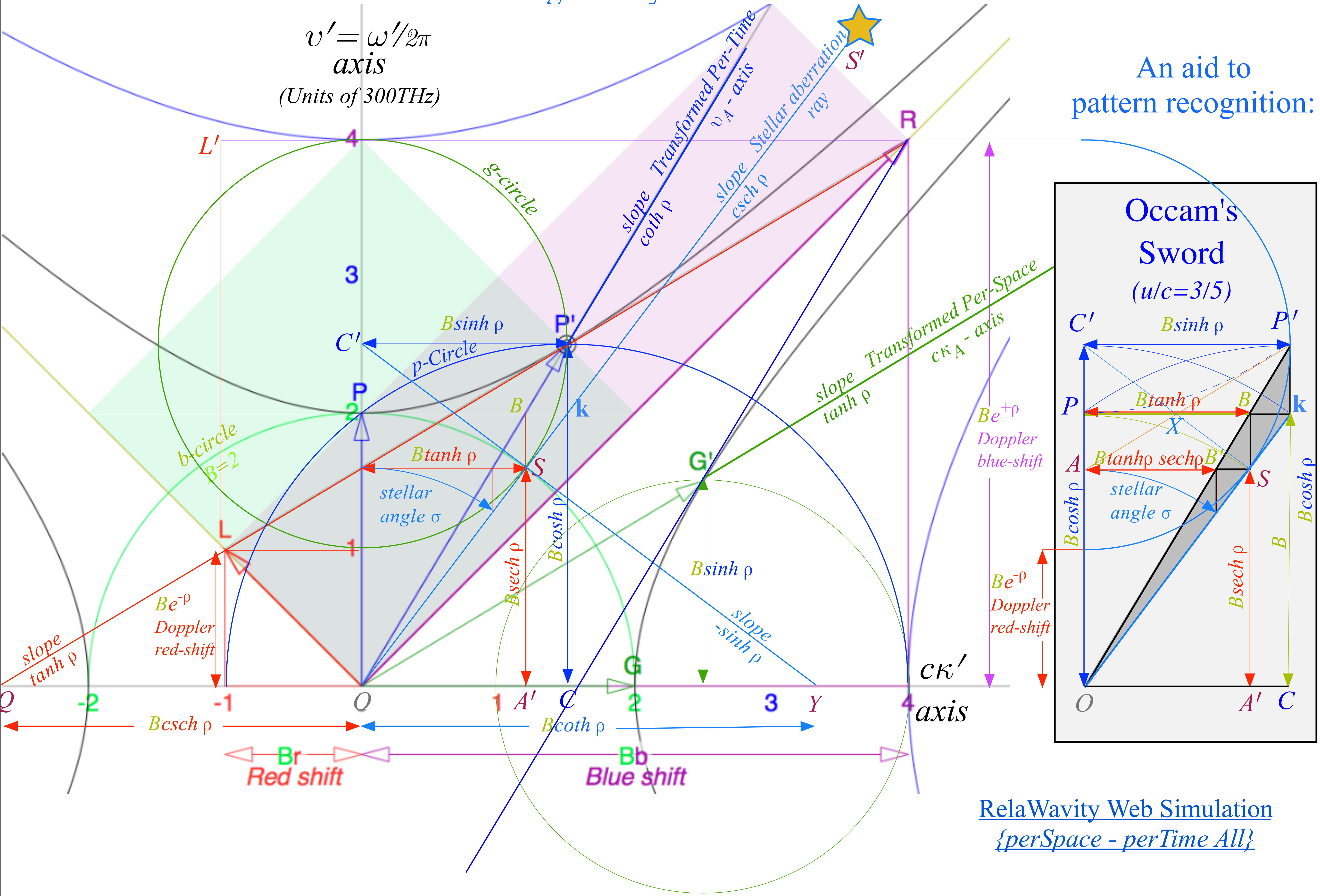
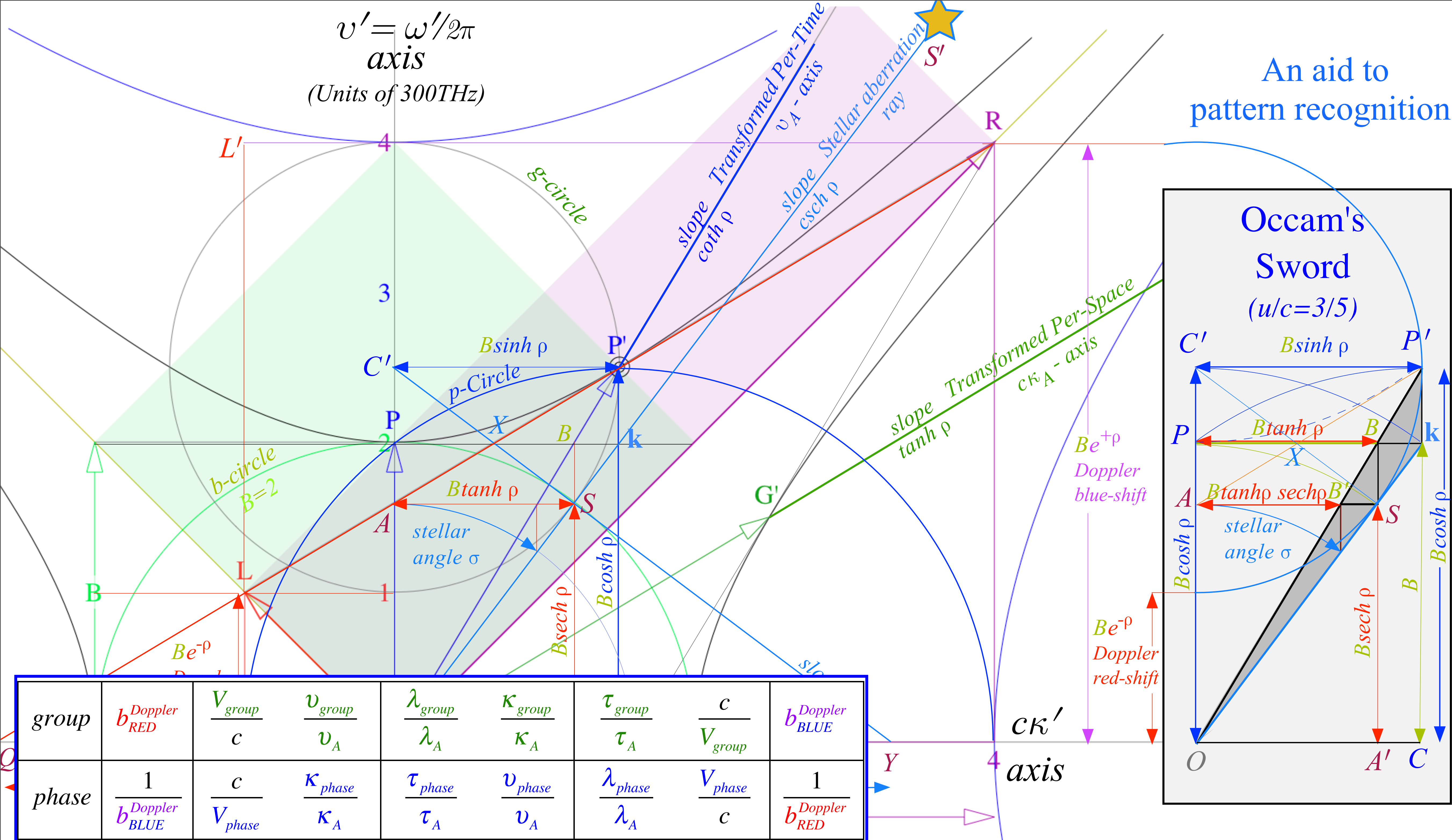
Summary of optical wave parameters for relativity and QM

...and their geometry



RelaWavity Web Simulation
{perSpace - perTime All}



group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Table of 12 wave parameters (includes inverses) for relativity

...and values for *u/c=3/5*

RelaWavity Web Simulation
Relativistic *Terms (Dual plot w/expanded table)*

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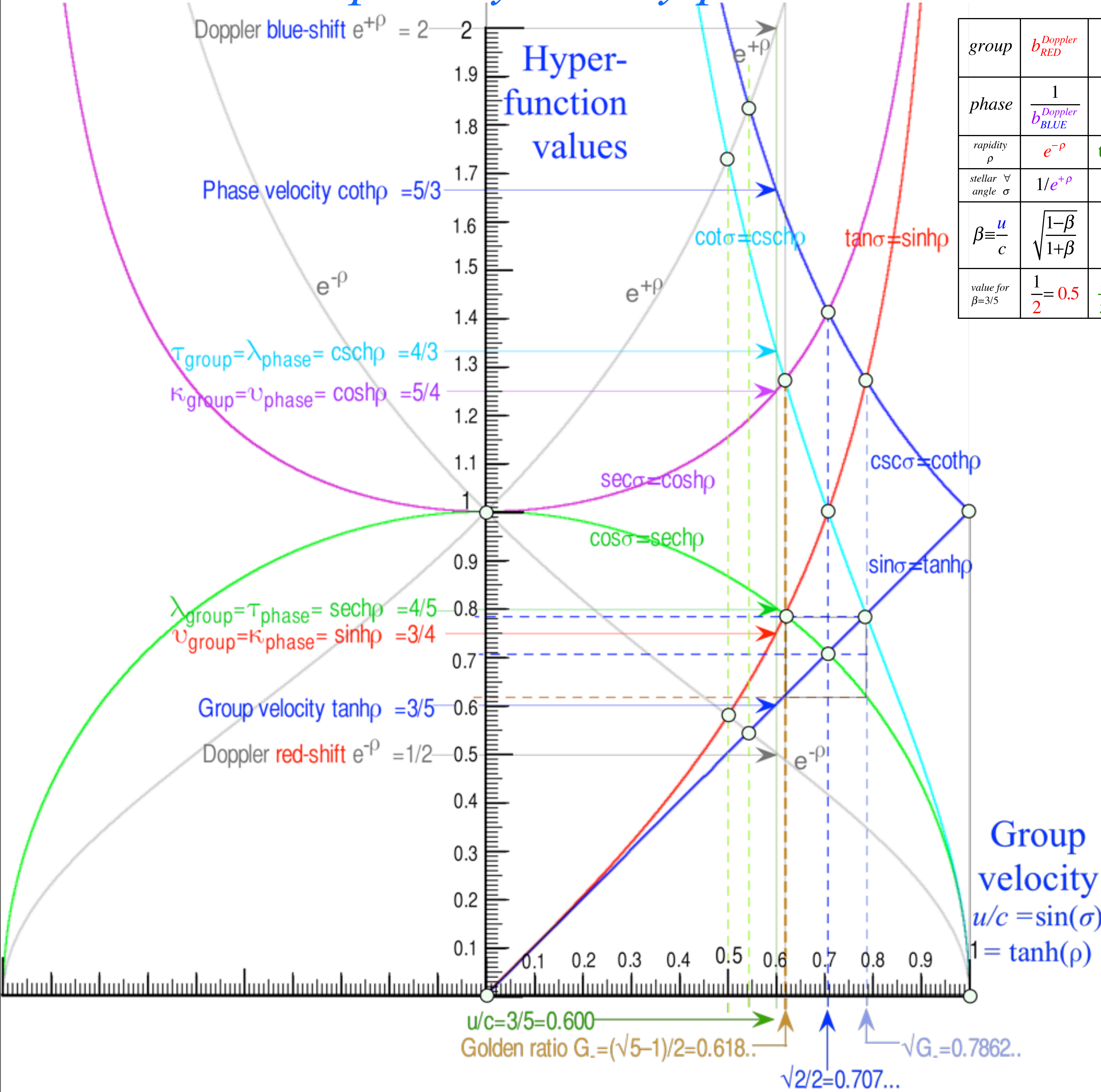

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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b^{Doppler}_{RED}}$
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stellar angle $\forall\ \sigma$	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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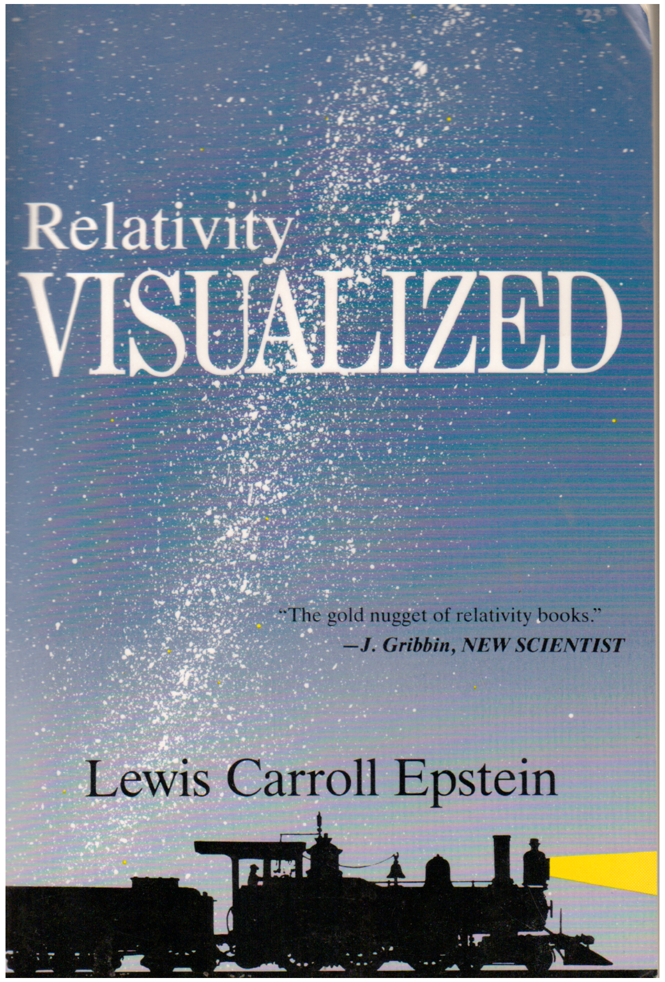

Epstein seemed resistent to ρ analysis or relations between σ and ρ.

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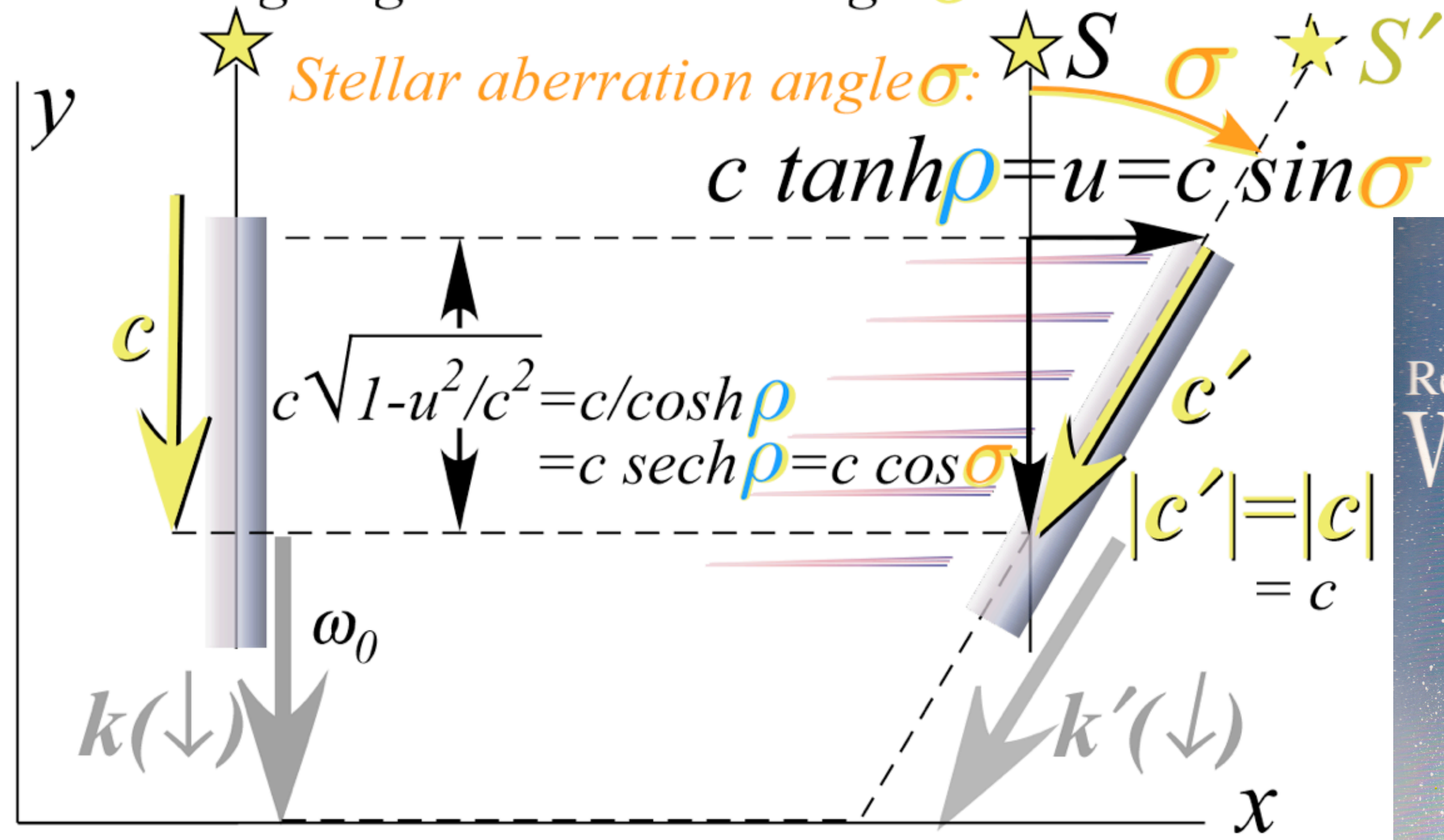

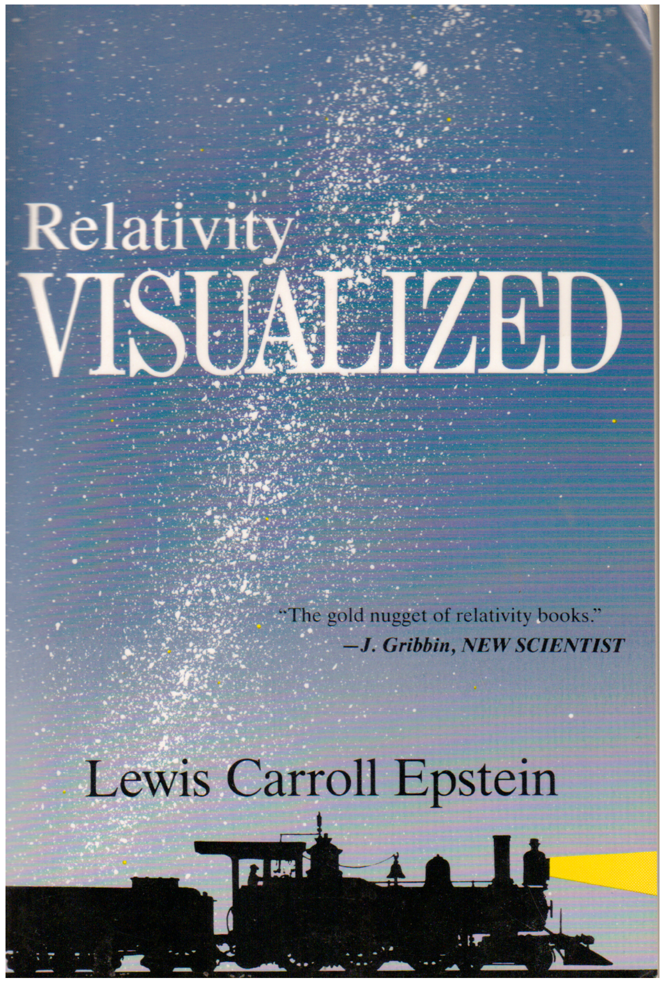

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Review of Proper time τ_0 and proper frequency ω_0

$$\begin{pmatrix} ck \\ \omega \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} ck' \\ \omega' \end{pmatrix}$$

$$\begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

Hyperbolic invariants to Lorentz transformation

Per-space-time invariant:

$$\omega_0^2 = \omega^2 - (ck)^2 = \omega'^2 - (ck')^2$$

ω_0 is called "proper frequency" or rate of "aging"

$$\omega_0 = \omega\sqrt{1-\frac{c^2k^2}{\omega^2}} = \omega'\sqrt{1-\frac{c^2k'^2}{\omega'^2}}$$

$$= \omega\sqrt{1-\frac{c^2}{V_{phase}^2}} = \omega'\sqrt{1-\frac{c^2}{V'^2_{phase}}}$$

$$\omega = \frac{\omega_0}{\sqrt{1-\frac{k^2}{(c\omega)^2}}} = \frac{\omega_0}{\sqrt{1-\frac{c^2}{V_{phase}^2}}}$$

$$\omega' = \frac{\omega_0}{\sqrt{1-\frac{k'^2}{(c\omega')^2}}} = \frac{\omega_0}{\sqrt{1-\frac{c^2}{V'^2_{phase}}}}$$

Space-time invariant:

$$(c\tau_0)^2 = (ct)^2 - x^2 = (ct')^2 - (x')^2$$

τ_0 is called "proper time" or "age":

$$\tau_0 = t\sqrt{1-\frac{x^2}{(ct)^2}} = t'\sqrt{1-\frac{x'^2}{(ct')^2}}$$

$$= t\sqrt{1-\frac{u^2}{c^2}} = t'\sqrt{1-\frac{u'^2}{c^2}}$$

Coordinate time t dilates to greater than τ_0

Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

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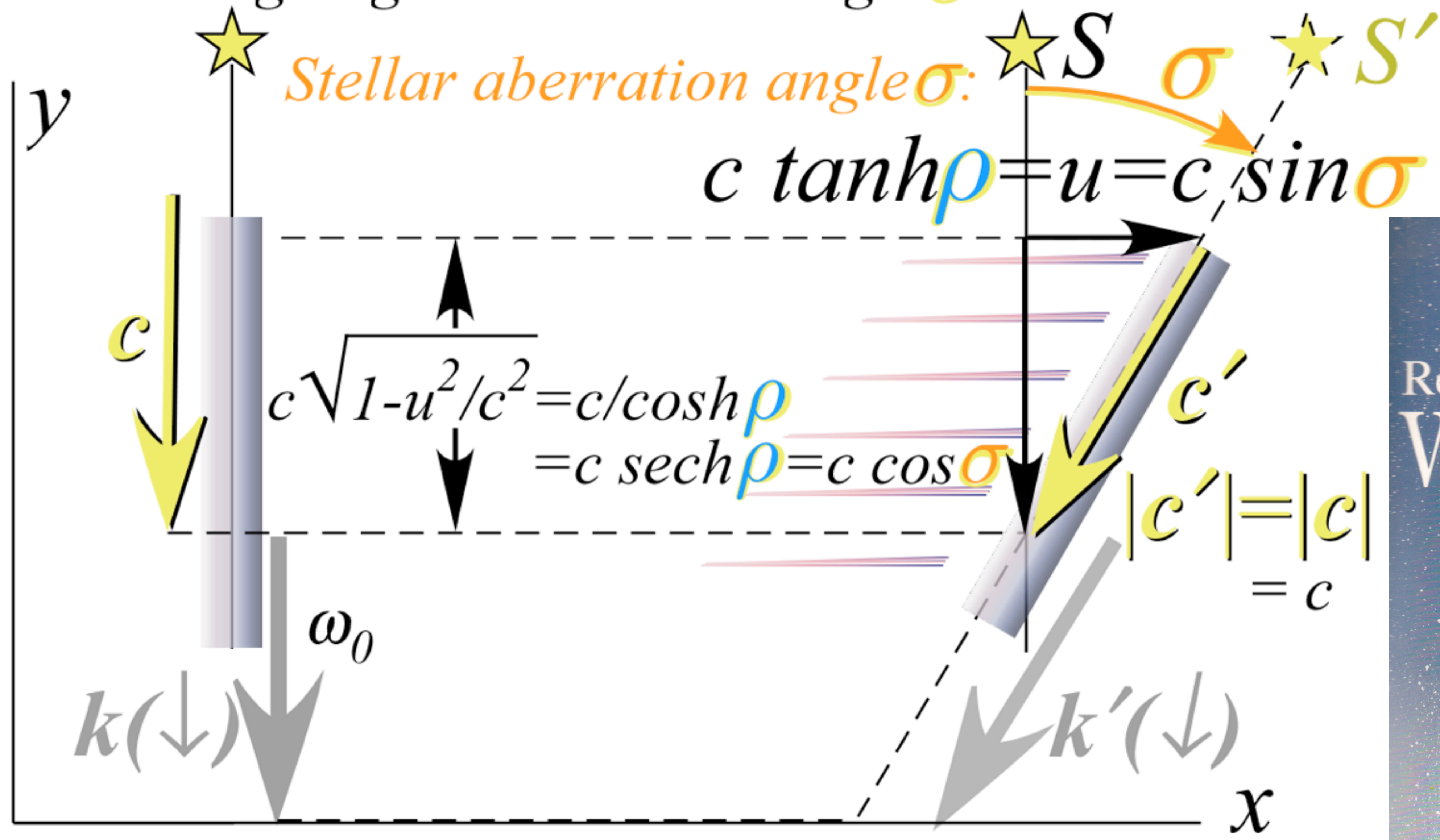

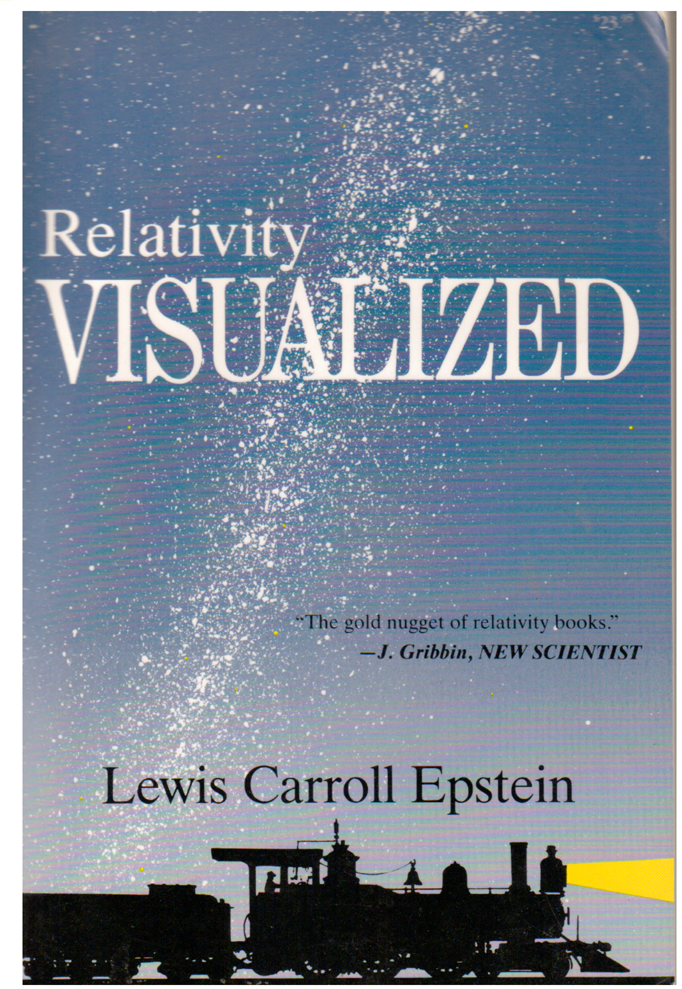

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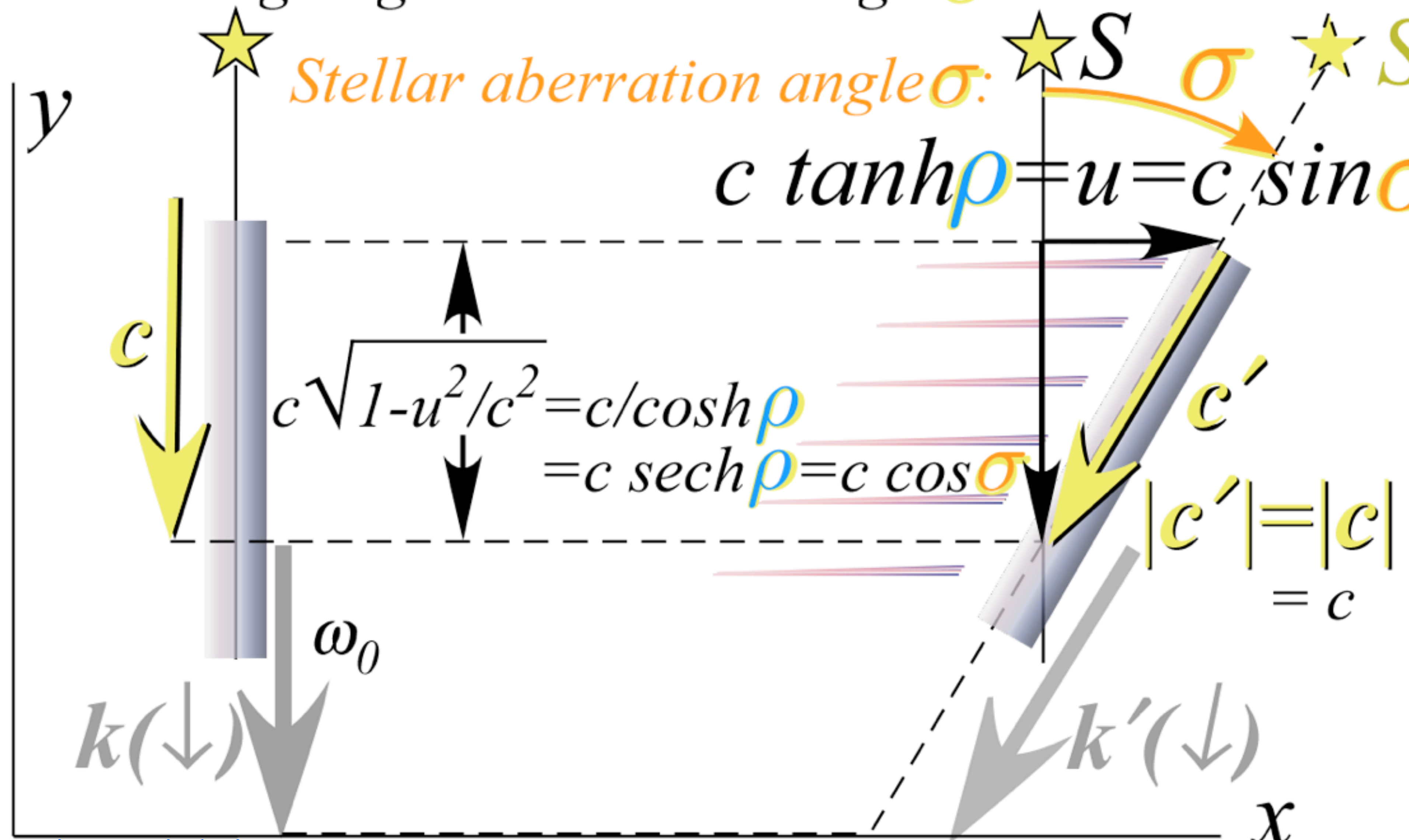

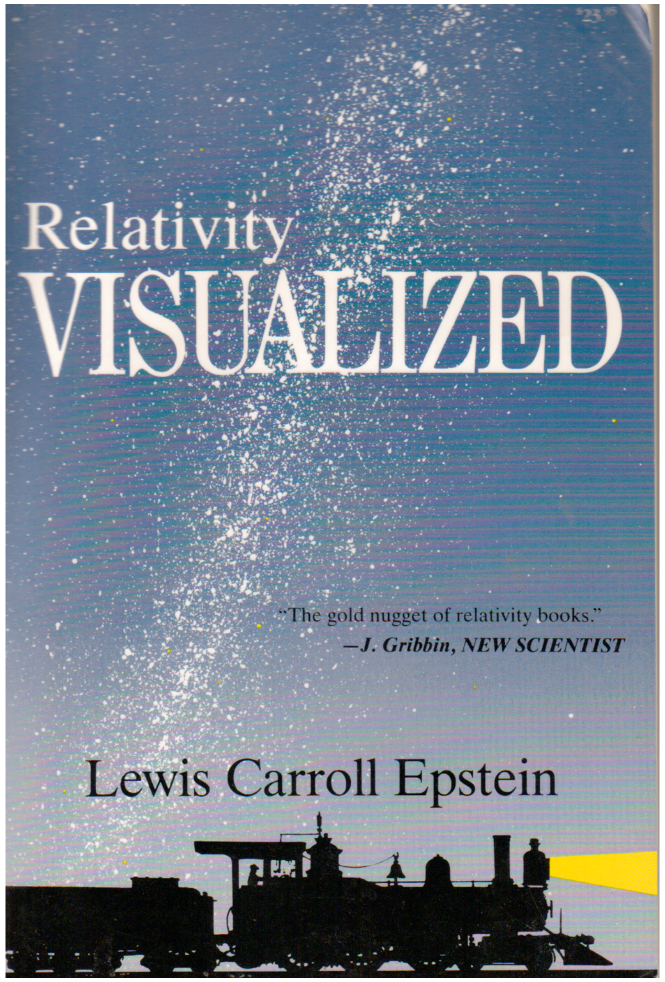

Epstein's trick is to turn a hyperbolic form $c\tau = \sqrt{(ct')^2 - (x')^2}$ (for Proper time)

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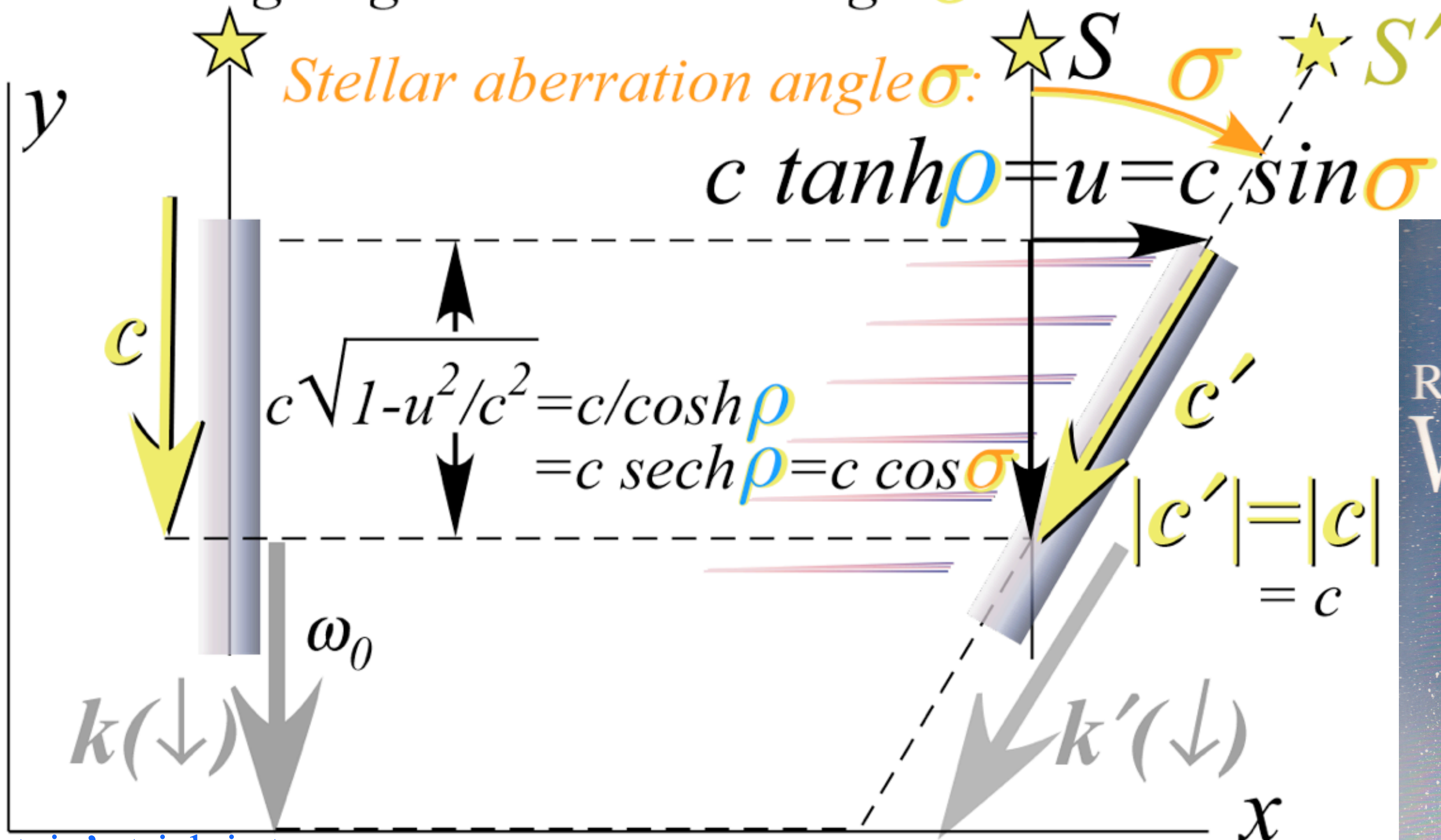

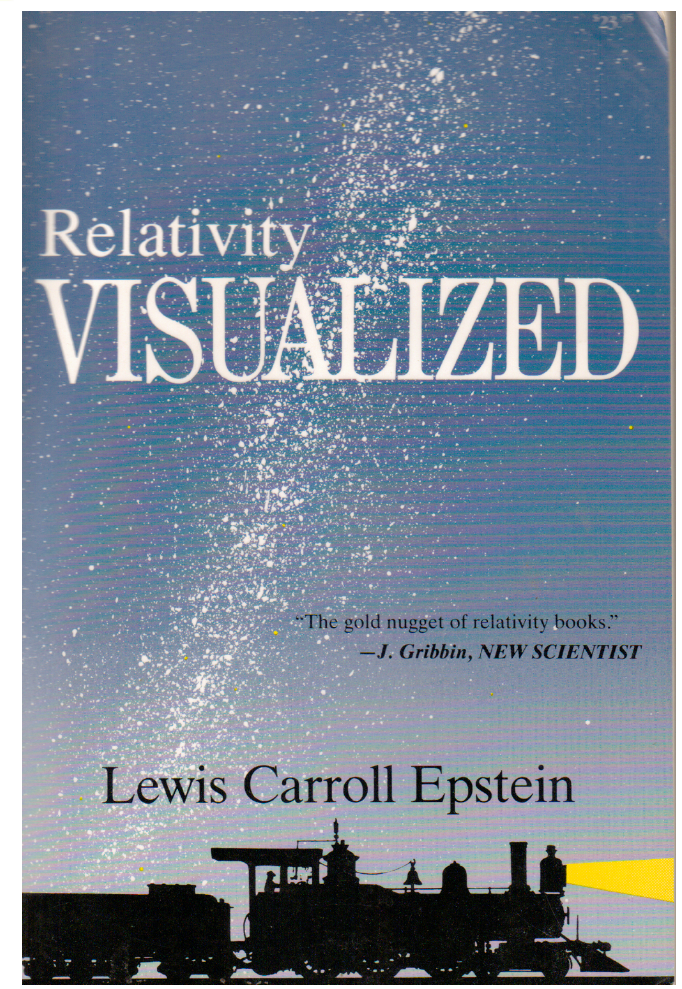

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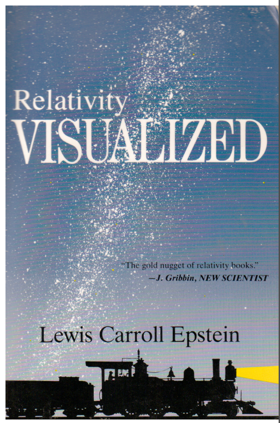
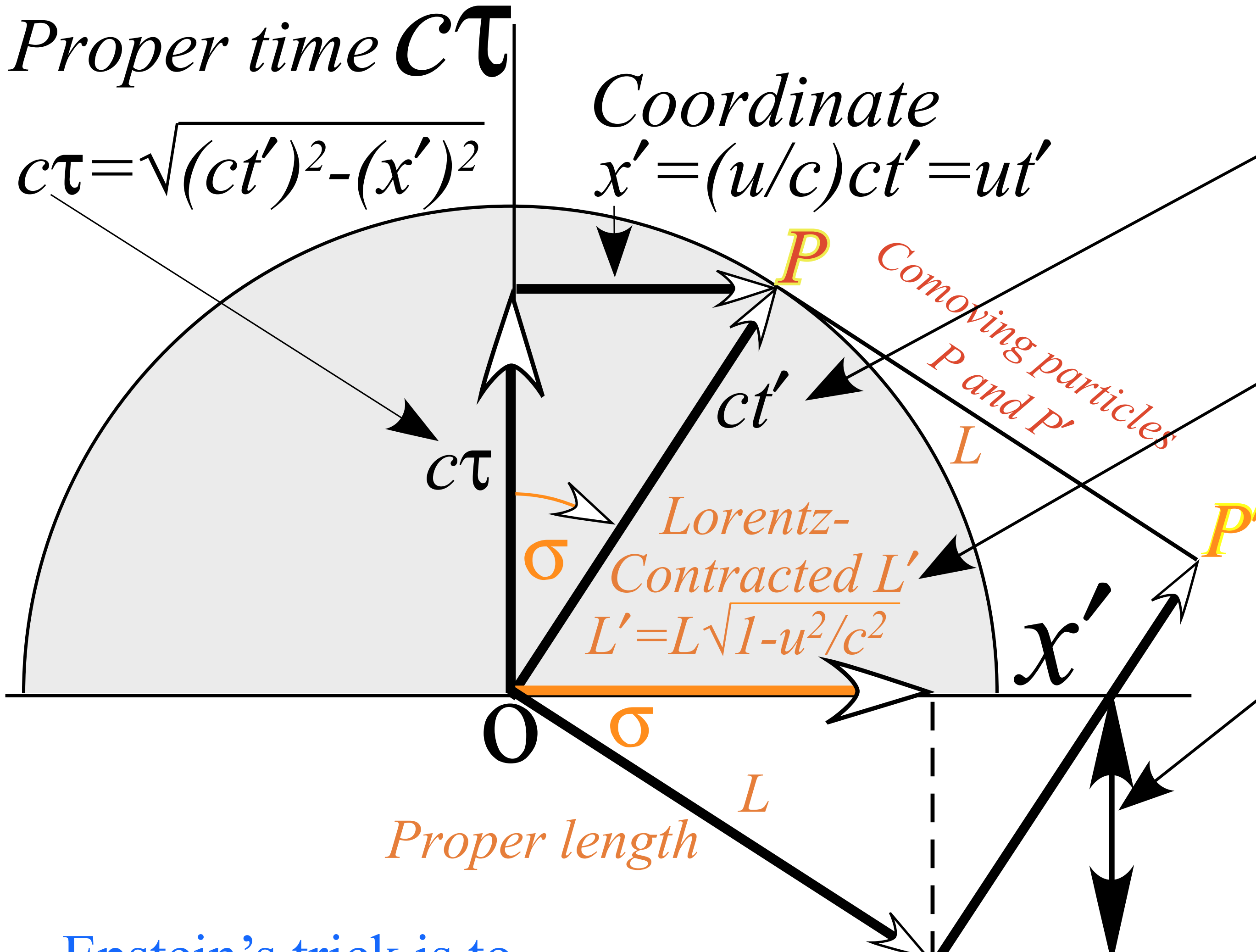


Proper time $c\tau$ *vs. coordinate space* x *- (L. C. Epstein's "Cosmic Speedometer")*

Particles P and P′ have speed u *in* (x', ct') *and speed* c *in* $(x, c\tau)$



Einstein time dilation:

$$ct' = c\tau \sec\sigma = c\tau \cosh\rho = c\tau/\sqrt{1-u^2/c^2}$$

Lorentz length contraction:

$$L' = L\,\mathrm{sech}\rho = L\cos\sigma = L\cdot\sqrt{1-u^2/c^2}$$

Proper Time asimultaneity:

$$c\,\Delta\tau = L'\sinh\rho = L\cos\sigma\sinh\rho$$

$$= L\cos\sigma\tan\sigma$$

$$= L\sin\sigma \quad = L/\sqrt{c^2/u^2-1} \sim L\,u/c$$

(for Proper time)

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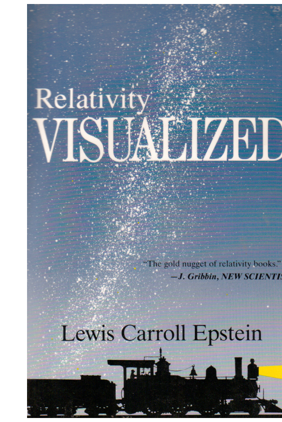
into a circular form: $\sqrt{(c\tau)^2 + (x')^2} = (ct')$

Then everything (and everybody) always goes speed c through $(x', c\tau)$ space!

Relating Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

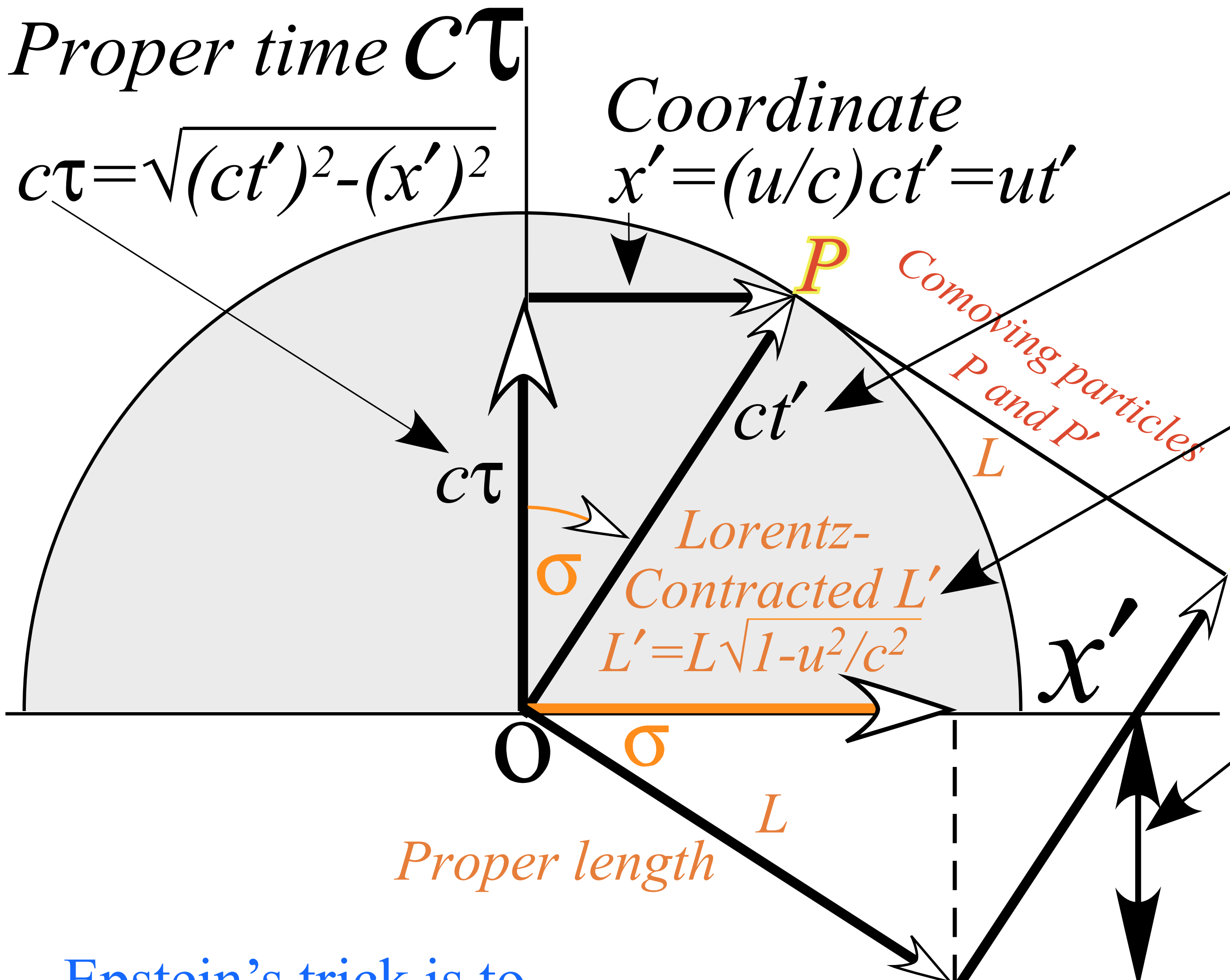
to Transverse relativity parameter: Stellar aberration angle σ

*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-



Proper time cτ vs. coordinate space x - (L. C. Epstein's "Cosmic Speedometer")

Particles P and P′ have speed u in (x′,ct′) and speed c in (x, cτ)



Einstein time dilation:

$$ct'=c\tau\ \sec\sigma=c\tau\ \cosh\rho\ =c\tau/\sqrt{1-u^2/c^2}$$

Lorentz length contraction:

$$L'=L\ \text{sech}\rho\ =L\cos\sigma\ =L\cdot\sqrt{1-u^2/c^2}$$

Proper Time asimultaneity:

$$c\,\Delta\tau=L'\sinh\rho\ =L\cos\sigma\ \sinh\rho$$
$$=L\cos\sigma\ \tan\sigma$$
$$=L\sin\sigma\quad =L/\sqrt{c^2/u^2-1}\sim L\,u/c$$

(for Proper time)

Epstein's trick is to
turn a hyperbolic form $c\tau=\sqrt{(ct')^2-(x')^2}$
into a circular form: $\sqrt{(c\tau)^2+(x')^2}=(ct')$
Then everything (and everybody) always goes speed c through $(x',c\tau)$ space!

RelativIt *Epstein Plots*
In developement!
Litehouse-centic
Ship-centic
Non-co-moving observer

A neo-liberal trigonometry lesson (*sine*, *tangent*, and *secant*) functions of angular sector area σ

Complimentary functions (... *cosine*, *and cotangent*, *cosecant*)

Hyper-trigonometry of ($\tanh\rho$, $\sinh\rho$, *and* $\cosh\rho$, $\text{sech}\rho$, *and* $\text{csch}\rho$, $\coth\rho$)

Functions of hyper-angular sector area ρ related to functions of σ

Each circular trig function has a hyperbolic "country-cousin" function

...and big-party fun was had by all!

Pattern recognition aids and "Occam-sword" geometry

Relating velocity parameters $\beta=u/c$ to *rapidity* ρ to **k**-*angle* σ to *u/c-angle* ν

Relating wave dimensional parameters of phase wave and group wave

Parameter-space symmetry points

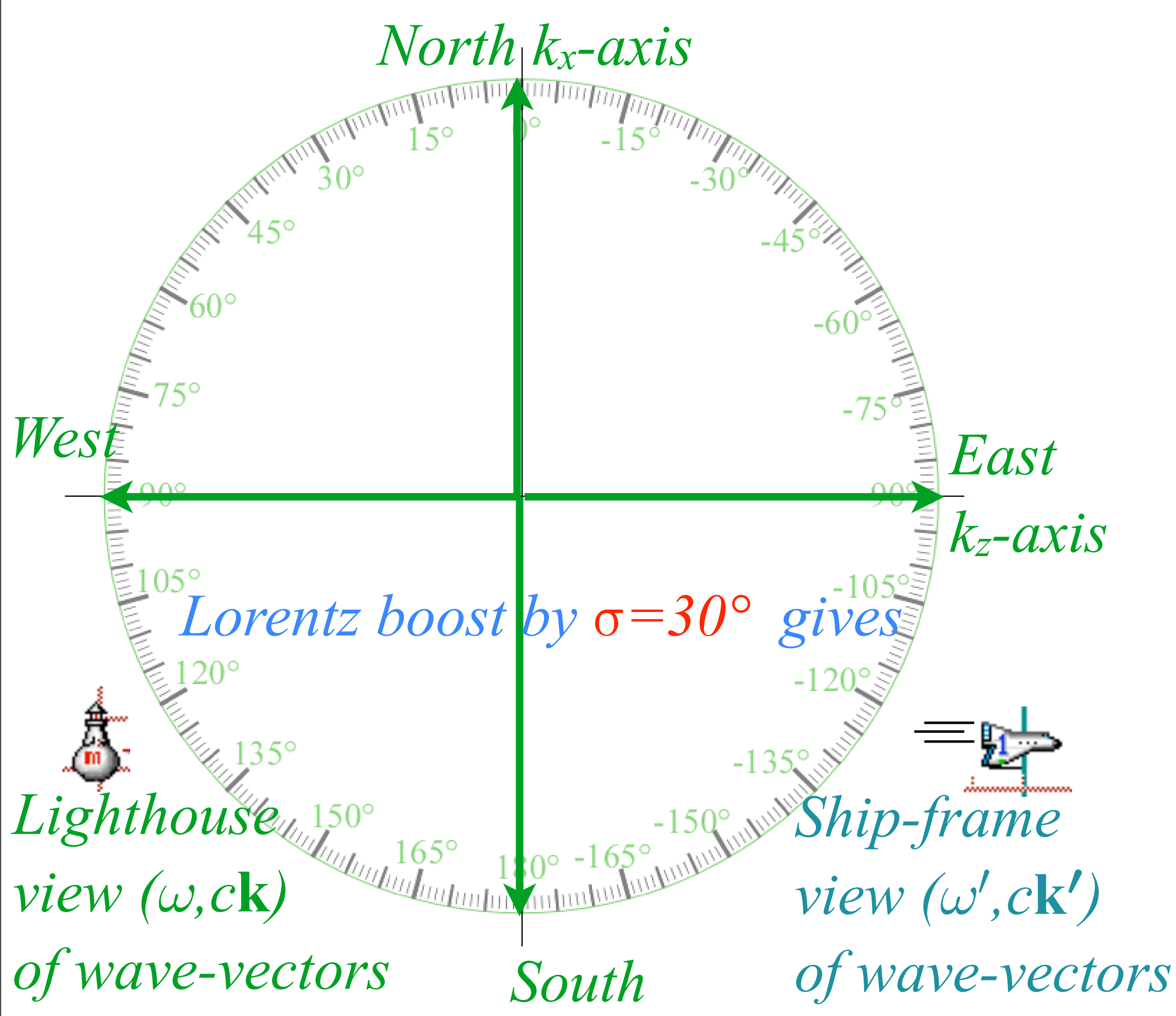
Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration **k**-angle σ

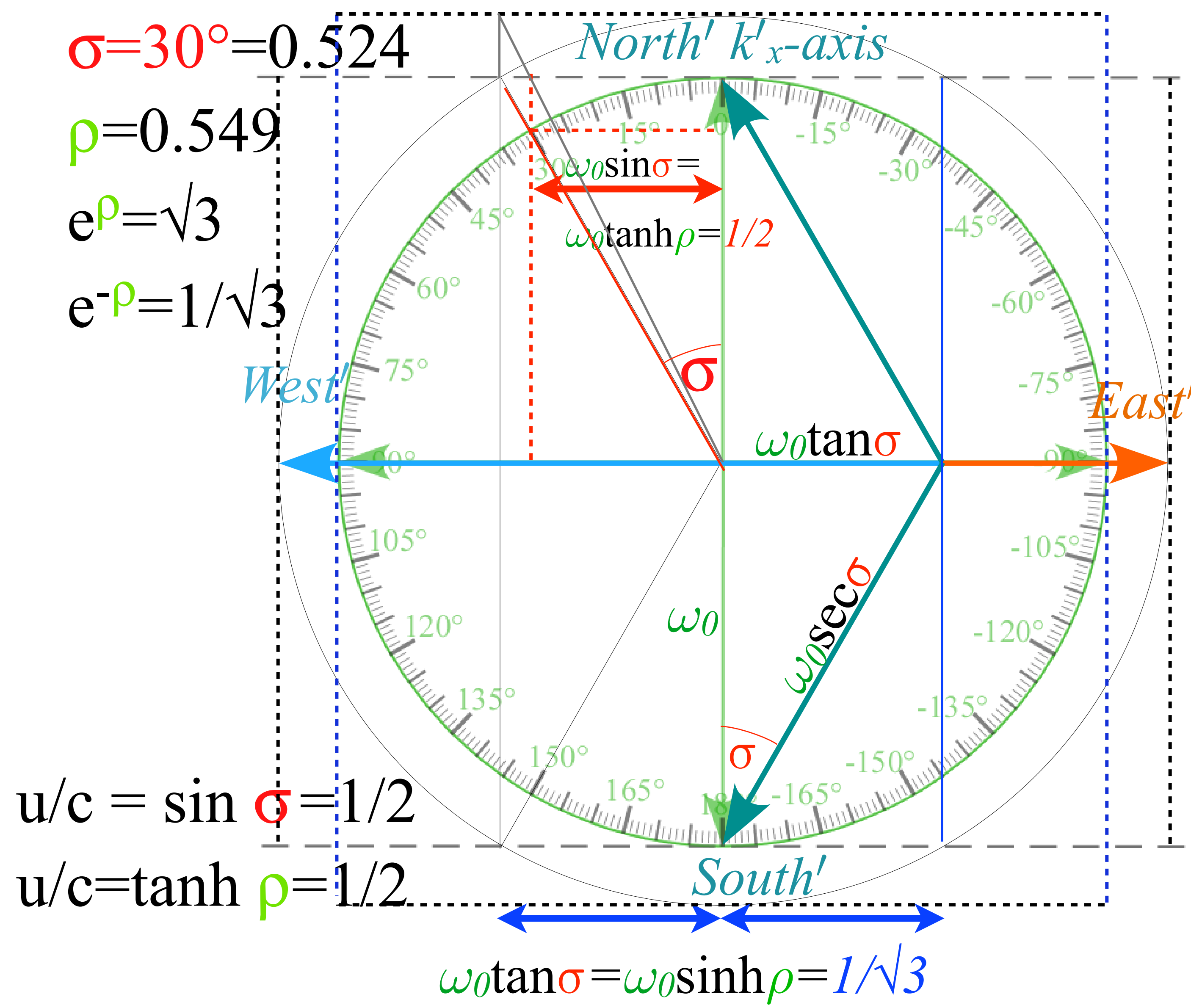
Review of proper time relations and basis of Epstein's cosmic speedometer

Epstein geometry for relativistic parameters

➔ Spectral details of per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) = (\omega, ck_x, ck_y, ck_z)$ transformation

Spectral details of Lorentz boost of North-South-East-West plane-wave 4-vectors $(\omega_0,\omega_x,\omega_y,\omega_z)$



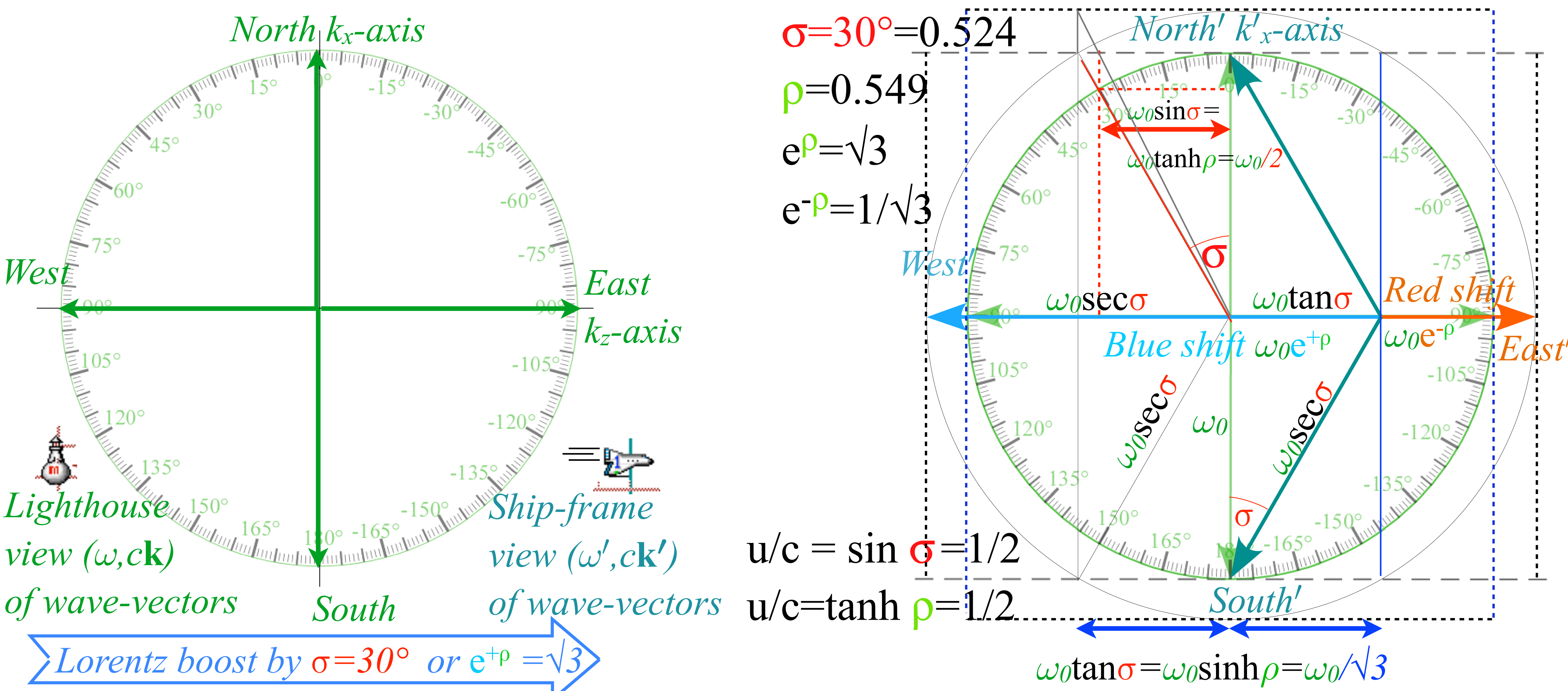


South starlight in lighthouse frame is straight down x-axis : $\left(\omega_{\downarrow},ck_{x\downarrow},ck_{y\downarrow},ck_{z\downarrow}\right)=\left(\omega_0,-\omega_0,0,0\right)$

$+\rho_z$-rapidity ship frame sees starlight Lorentz transformed to :$\left(\omega'_{\downarrow},ck'_{x\downarrow},ck'_{y\downarrow},ck'_{z\downarrow}\right)=\left(\omega_0\cosh\rho_z,-\omega_0,0,-\omega_0\sinh\rho_z\right)$

$$\begin{pmatrix}\omega'_{\downarrow}\\ ck'_{x\downarrow}\\ ck'_{y\downarrow}\\ ck'_{z\downarrow}\end{pmatrix}=\begin{pmatrix}\cosh\rho_z & \cdot & \cdot & -\sinh\rho_z\\ \cdot & 1 & \cdot & \cdot\\ \cdot & \cdot & 1 & \cdot\\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z\end{pmatrix}\begin{pmatrix}\omega_{\downarrow}\\ ck_{x\downarrow}\\ ck_{y\downarrow}\\ ck_{z\downarrow}\end{pmatrix}=\begin{pmatrix}\cosh\rho_z & \cdot & \cdot & -\sinh\rho_z\\ \cdot & 1 & \cdot & \cdot\\ \cdot & \cdot & 1 & \cdot\\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z\end{pmatrix}\begin{pmatrix}\omega_0\\ -\omega_0\\ 0\\ 0\end{pmatrix}=\begin{pmatrix}\omega_0\cosh\rho_z\\ -\omega_0\\ 0\\ -\omega_0\sinh\rho_z\end{pmatrix}=\begin{pmatrix}\omega_0\sec\sigma\\ -\omega_0\\ 0\\ -\omega_0\tan\sigma\end{pmatrix}$$

Lecture 27 discusses Lorentz boost of North-South-East-West plane-wave 4-vectors $(\omega_0,\omega_x,\omega_y,\omega_z)$



For ship going $u=c\tanh\rho$ along z-axis

West starlight $(\omega_0,0,0,-\omega_0)$ is *blue shifted by* $e^{+\rho}=\cosh\rho+\sinh\rho$

$$\begin{pmatrix}\omega'_{\leftarrow}\\ ck'_{x\leftarrow}\\ ck'_{y\leftarrow}\\ ck'_{z\leftarrow}\end{pmatrix}=\omega_0\begin{pmatrix}\cosh\rho_z+\sinh\rho_z\\ 0\\ 0\\ -\sinh\rho_z-\cosh\rho_z\end{pmatrix}=\begin{pmatrix}\omega_0 e^{+\rho_z}\\ 0\\ 0\\ -\omega_0 e^{+\rho_z}\end{pmatrix}$$

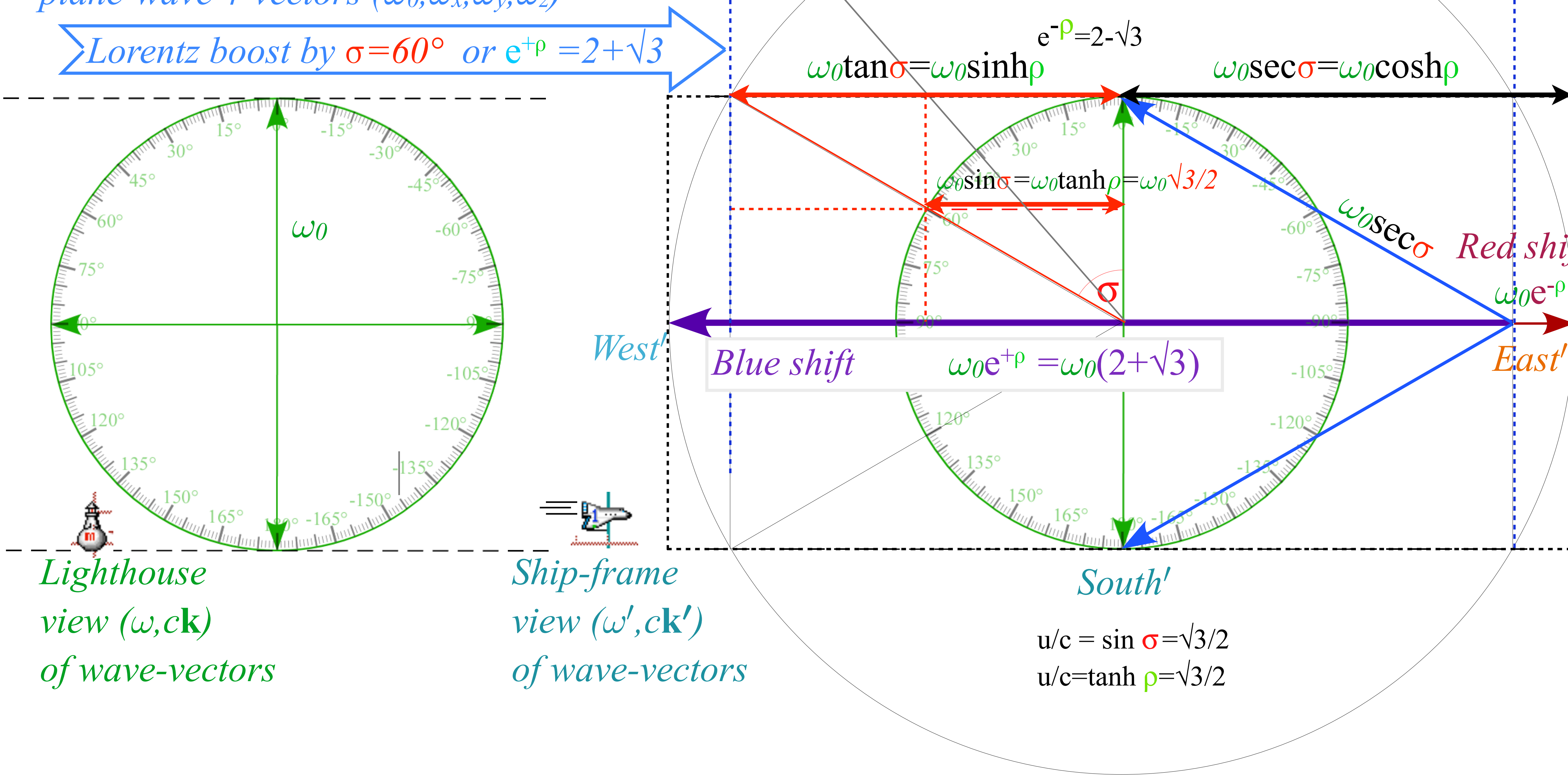
and *East* starlight $(\omega_0,0,0,+\omega_0)$ is *red shifted by* $e^{-\rho}=\cosh\rho-\sinh\rho$

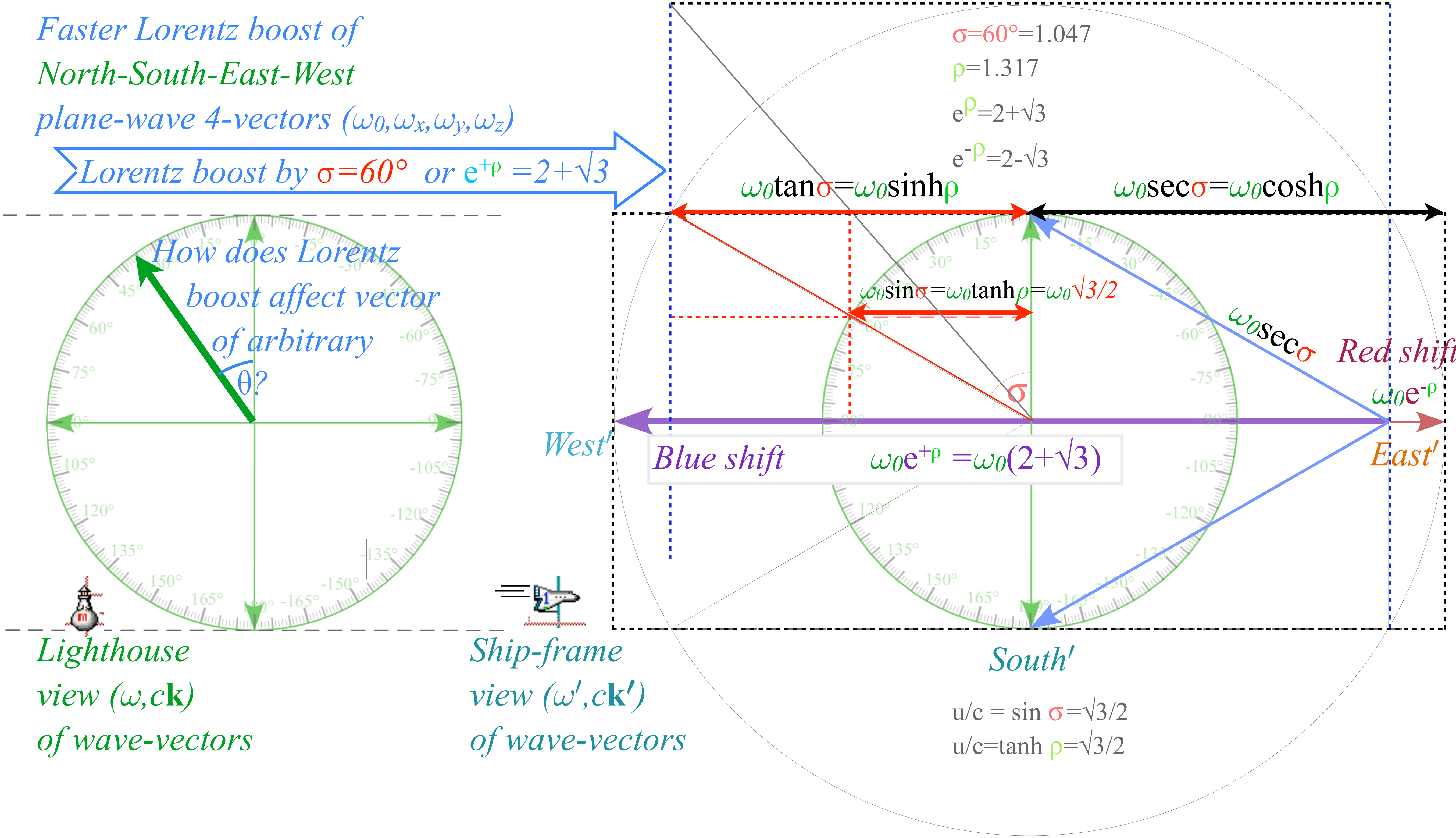
$$\begin{pmatrix}\omega'_{\rightarrow}\\ ck'_{x\rightarrow}\\ ck'_{y\rightarrow}\\ ck'_{z\rightarrow}\end{pmatrix}=\omega_0\begin{pmatrix}\cosh\rho_z-\sinh\rho_z\\ 0\\ 0\\ -\sinh\rho_z+\cosh\rho_z\end{pmatrix}=\begin{pmatrix}\omega_0 e^{-\rho_z}\\ 0\\ 0\\ -\omega_0 e^{-\rho_z}\end{pmatrix}$$

Blue shift factor is $e^{+\rho}=\cosh\rho+\sinh\rho=\sec\sigma+\tan\sigma$

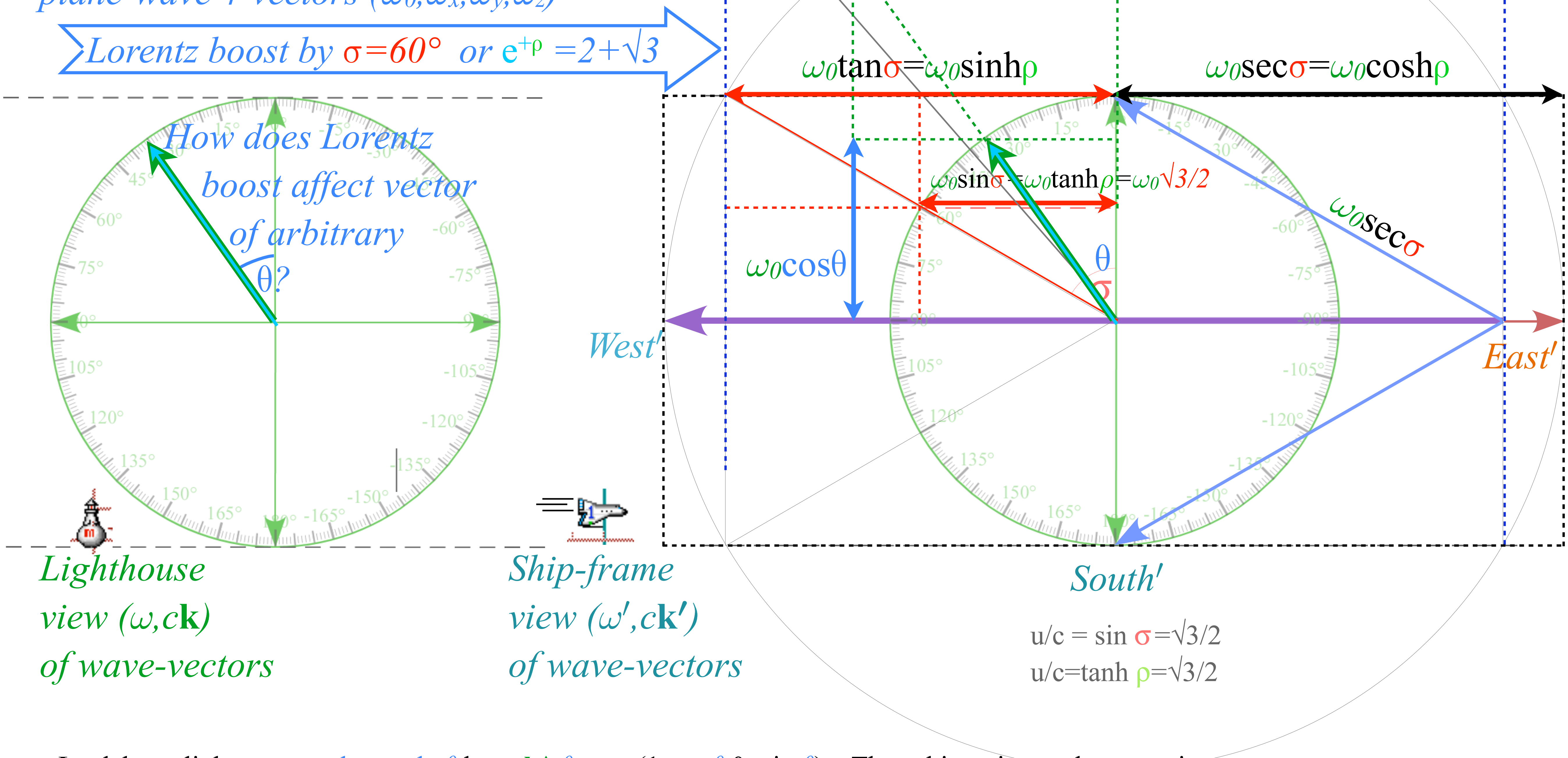
Red shift factor is $e^{-\rho}=\cosh\rho-\sinh\rho=\sec\sigma-\tan\sigma$

Faster Lorentz boost of North-South-East-West plane-wave 4-vectors $(\omega_0,\omega_x,\omega_y,\omega_z)$




Faster Lorentz boost of
North-South-East-West
plane-wave 4-vectors ($\omega_0,\omega_x,\omega_y,\omega_z$)
Lorentz boost by σ=60° or $e^{+\rho}$ =2+√3
How does Lorentz boost affect vector of arbitrary θ?
σ=60°=1.047
ρ=1.317
e^{ρ}=2+√3
$e^{-\rho}$=2-√3
$\omega_0$tanσ=$\omega_0$sinhρ
$\omega_0$secσ=$\omega_0$coshρ
$\omega_0$sinσ=$\omega_0$tanhρ=ω_0√3/2
$\omega_0$secσ
Red shift
$\omega_0 e^{-\rho}$
West′
Blue shift
$\omega_0 e^{+\rho}$ =ω_0(2+√3)
East′
South′
u/c = sin σ=√3/2
u/c=tanh ρ=√3/2
Lighthouse view (ω,ck) of wave-vectors
Ship-frame view (ω′,ck′) of wave-vectors

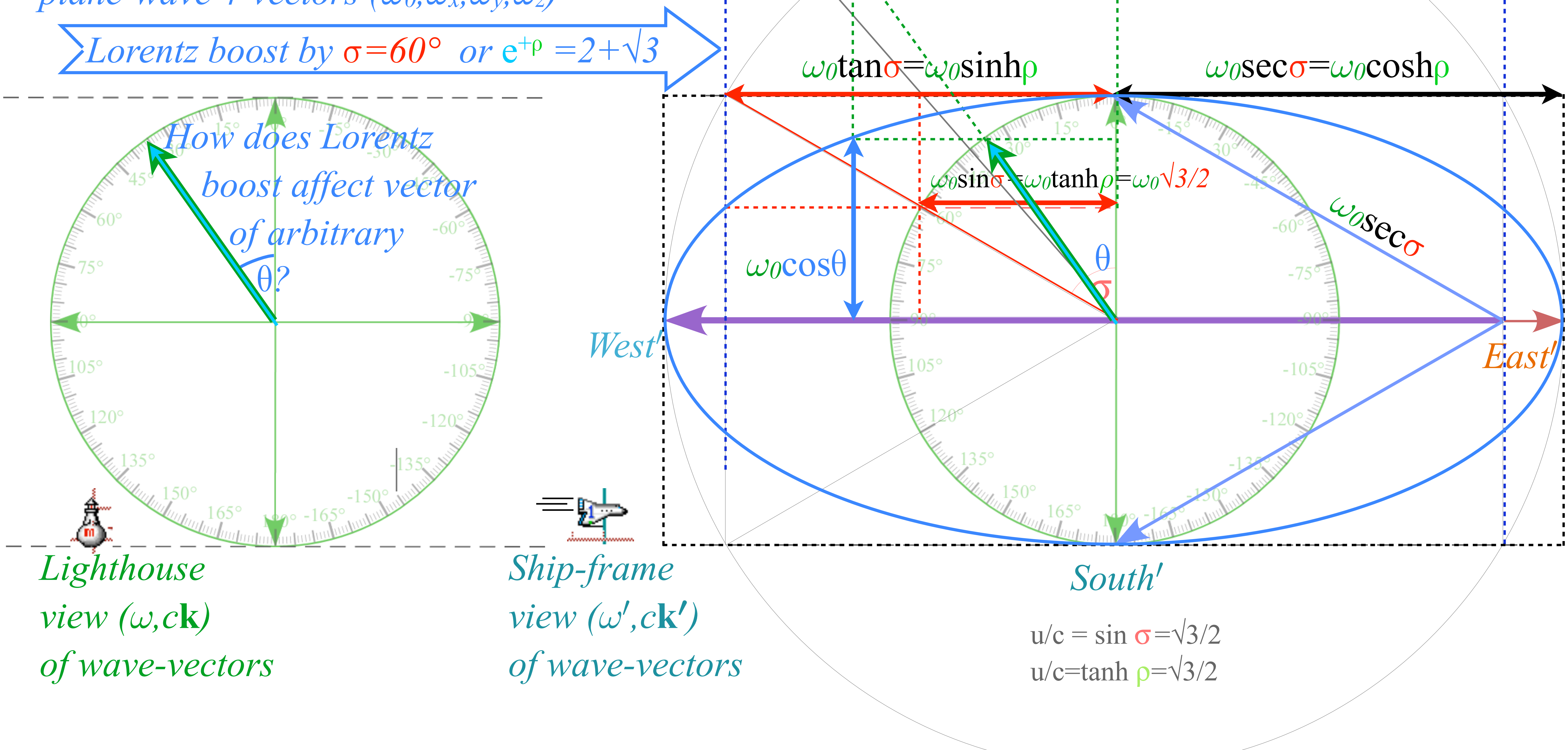
Faster Lorentz boost of North-South-East-West plane-wave 4-vectors $(\omega_0,\omega_x,\omega_y,\omega_z)$



Let lab starlight ray at polar angle θ have $\mathbf{k}\uparrow\theta = \omega_0\,(1,\cos\theta,0,-\sin\theta)$. Then ship going $\mathbf{u}$ along z-axis sees :

$$\begin{pmatrix}\omega'_{\uparrow\theta}\\ ck'_{x\uparrow\theta}\\ ck'_{y\uparrow\theta}\\ ck'_{z\uparrow\theta}\end{pmatrix}=\begin{pmatrix}\cosh\rho_z & \cdot & \cdot & -\sinh\rho_z\\ \cdot & 1 & \cdot & \cdot\\ \cdot & \cdot & 1 & \cdot\\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z\end{pmatrix}\begin{pmatrix}\omega_0\\ \omega_0\cos\theta\\ 0\\ -\omega_0\sin\theta\end{pmatrix}=\omega_0\begin{pmatrix}\cosh\rho_z+\sinh\rho_z\sin\theta\\ \cos\theta\\ 0\\ -\sinh\rho_z-\cosh\rho_z\sin\theta\end{pmatrix}=\omega_0\begin{pmatrix}\sec\sigma+\tan\sigma\sin\theta\\ \cos\theta\\ 0\\ -\tan\sigma-\sec\sigma\sin\theta\end{pmatrix}$$

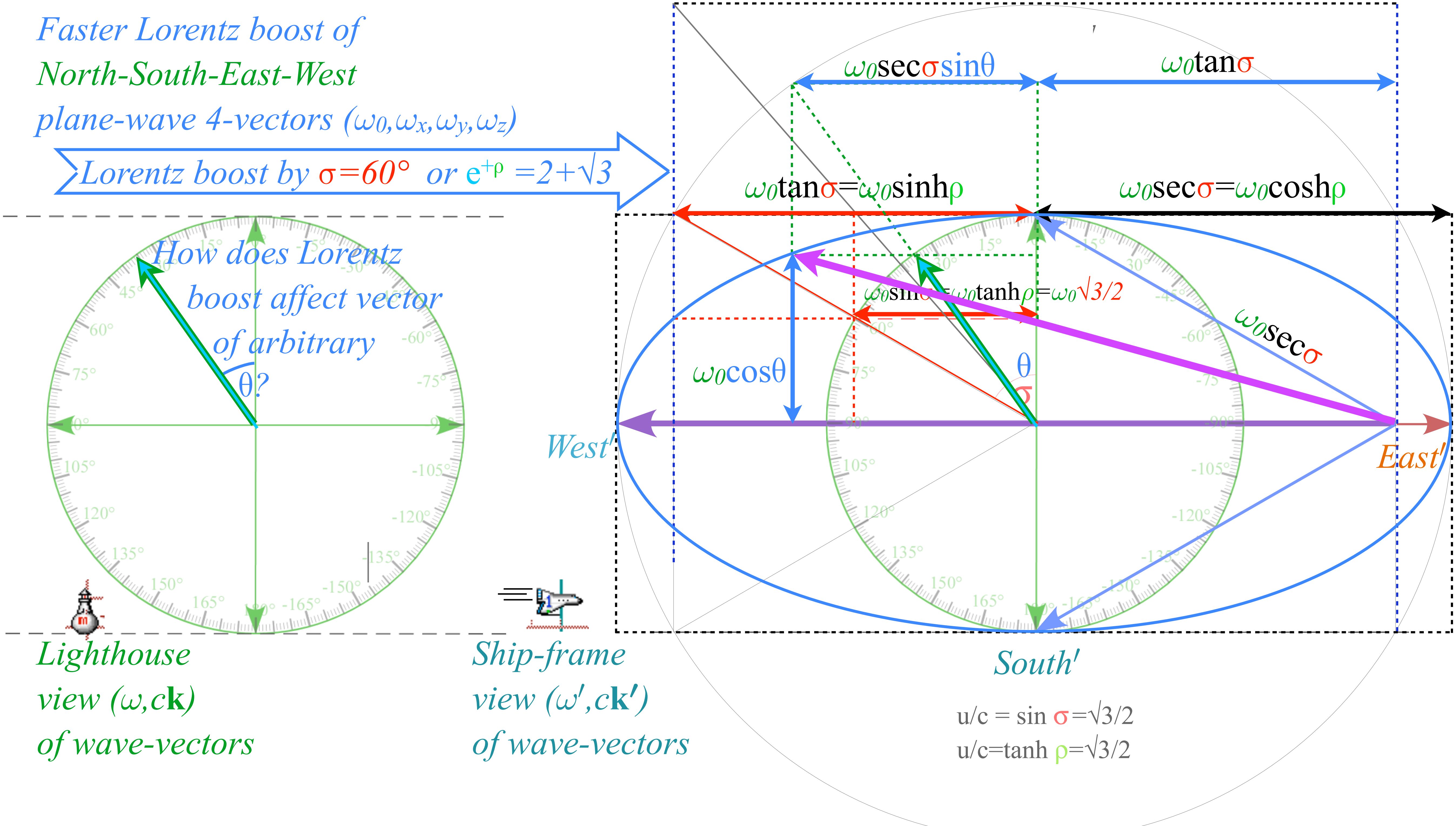
Faster Lorentz boost of
North-South-East-West
plane-wave 4-vectors $(\omega_0,\omega_x,\omega_y,\omega_z)$



Let lab starlight ray at polar angle θ have $\mathbf{k}\uparrow\theta = \omega_0\,(1,\cos\theta,0,-\sin\theta)$. Then ship going **u** along z-axis sees :

$$\begin{pmatrix} \omega'_{\uparrow\theta} \\ ck'_{x\uparrow\theta} \\ ck'_{y\uparrow\theta} \\ ck'_{z\uparrow\theta} \end{pmatrix} = \begin{pmatrix} \cosh\rho_z & \cdot & \cdot & -\sinh\rho_z \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z \end{pmatrix} \begin{pmatrix} \omega_0 \\ \omega_0\cos\theta \\ 0 \\ -\omega_0\sin\theta \end{pmatrix} = \omega_0 \begin{pmatrix} \cosh\rho_z + \sinh\rho_z\sin\theta \\ \cos\theta \\ 0 \\ -\sinh\rho_z - \cosh\rho_z\sin\theta \end{pmatrix} = \omega_0 \begin{pmatrix} \sec\sigma + \tan\sigma\sin\theta \\ \cos\theta \\ 0 \\ -\tan\sigma - \sec\sigma\sin\theta \end{pmatrix}$$

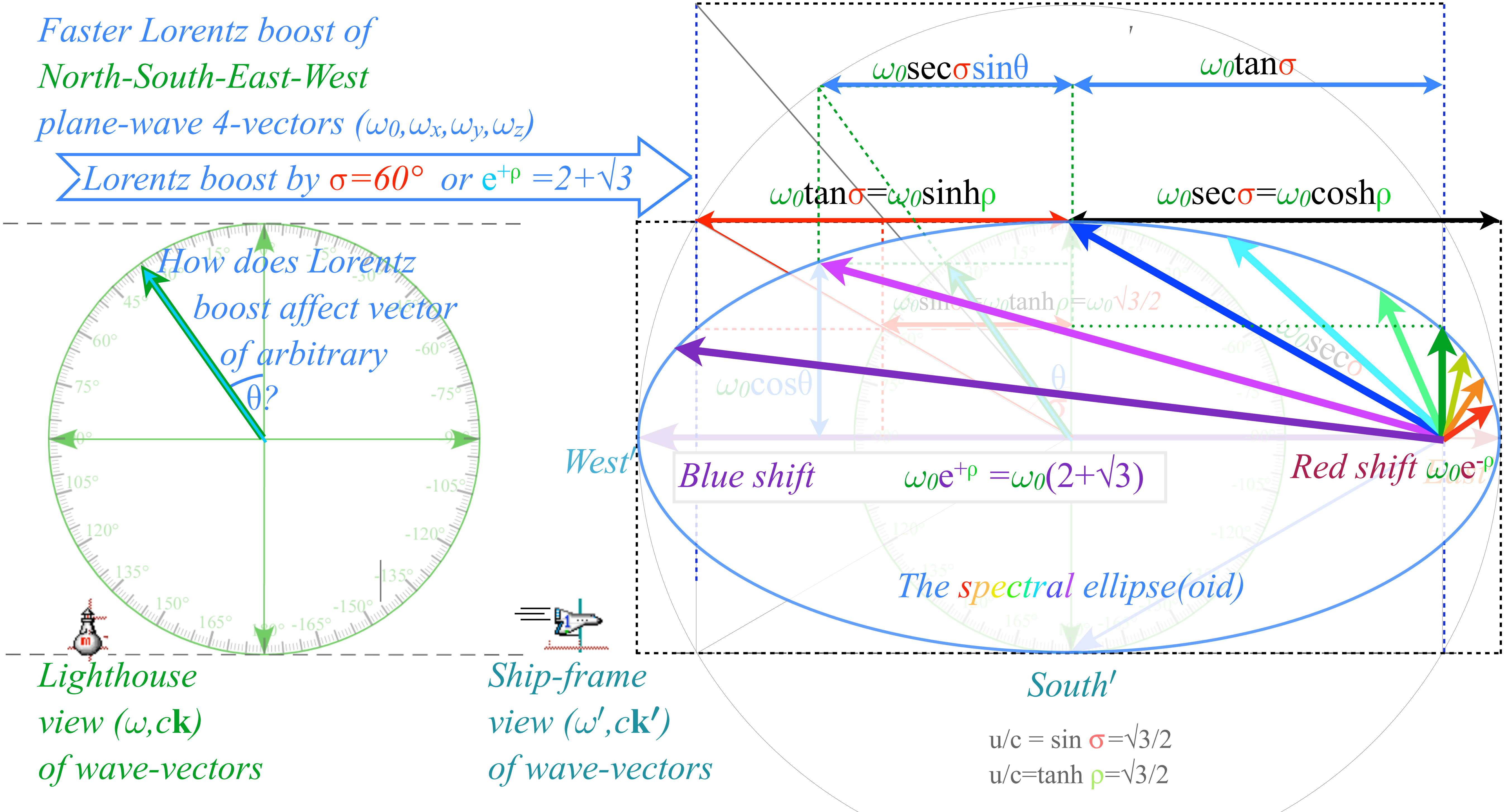
Faster Lorentz boost of North-South-East-West plane-wave 4-vectors $(\omega_0,\omega_x,\omega_y,\omega_z)$



Let lab starlight ray at polar angle θ have $\mathbf{k}\uparrow\theta = \omega_0(1,\cos\theta,0,-\sin\theta)$. Then ship going $\mathbf{u}$ along z-axis sees :

$$\begin{pmatrix} \omega'_{\uparrow\theta} \\ ck'_{x\uparrow\theta} \\ ck'_{y\uparrow\theta} \\ ck'_{z\uparrow\theta} \end{pmatrix} = \begin{pmatrix} \cosh\rho_z & \cdot & \cdot & -\sinh\rho_z \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z \end{pmatrix} \begin{pmatrix} \omega_0 \\ \omega_0\cos\theta \\ 0 \\ -\omega_0\sin\theta \end{pmatrix} = \omega_0 \begin{pmatrix} \cosh\rho_z + \sinh\rho_z\sin\theta \\ \cos\theta \\ 0 \\ -\sinh\rho_z - \cosh\rho_z\sin\theta \end{pmatrix} = \omega_0 \begin{pmatrix} \sec\sigma + \tan\sigma\sin\theta \\ \cos\theta \\ 0 \\ -\tan\sigma - \sec\sigma\sin\theta \end{pmatrix}$$

Faster Lorentz boost of North-South-East-West plane-wave 4-vectors $(\omega_0,\omega_x,\omega_y,\omega_z)$

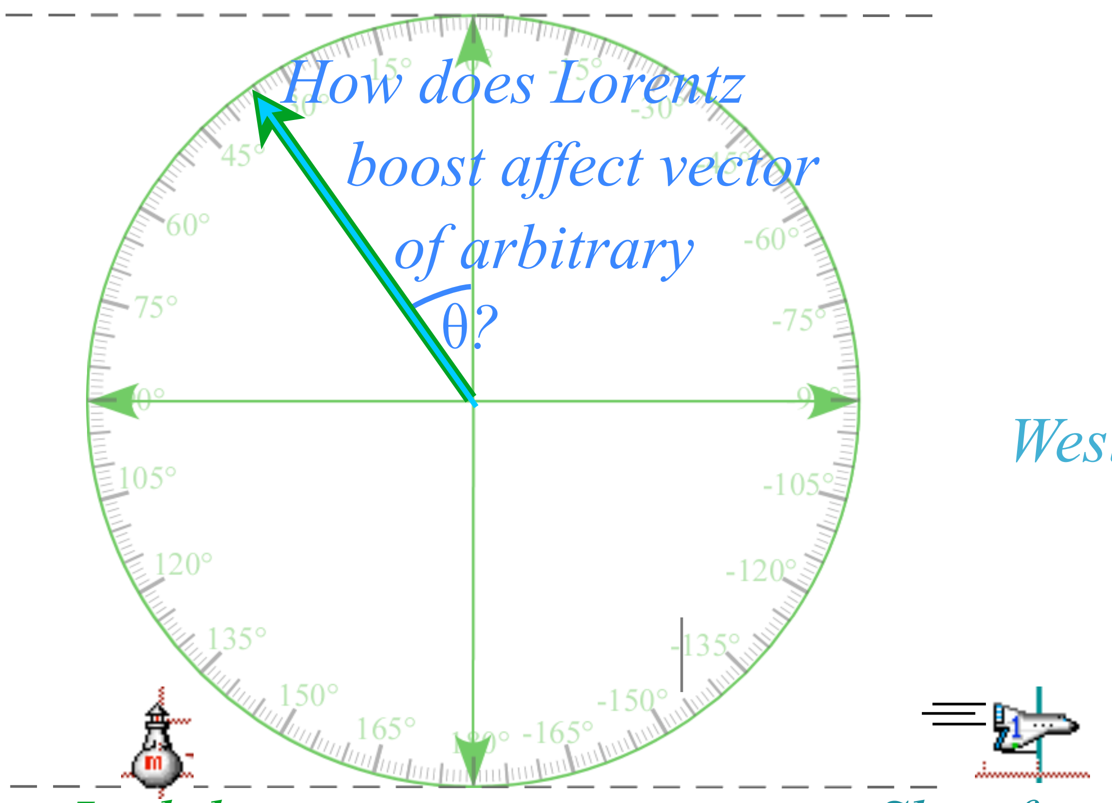


Let lab starlight ray at polar angle θ have $\mathbf{k}\uparrow\theta = \omega_0\,(1,\cos\theta,0,-\sin\theta)$. Then ship going $\mathbf{u}$ along z-axis sees :

$$\begin{pmatrix}\omega'_{\uparrow\theta}\\ ck'_{x\uparrow\theta}\\ ck'_{y\uparrow\theta}\\ ck'_{z\uparrow\theta}\end{pmatrix} = \begin{pmatrix}\cosh\rho_z & \cdot & \cdot & -\sinh\rho_z\\ \cdot & 1 & \cdot & \cdot\\ \cdot & \cdot & 1 & \cdot\\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z\end{pmatrix}\begin{pmatrix}\omega_0\\ \omega_0\cos\theta\\ 0\\ -\omega_0\sin\theta\end{pmatrix} = \omega_0\begin{pmatrix}\cosh\rho_z+\sinh\rho_z\sin\theta\\ \cos\theta\\ 0\\ -\sinh\rho_z-\cosh\rho_z\sin\theta\end{pmatrix} = \omega_0\begin{pmatrix}\sec\sigma+\tan\sigma\sin\theta\\ \cos\theta\\ 0\\ -\tan\sigma-\sec\sigma\sin\theta\end{pmatrix}$$

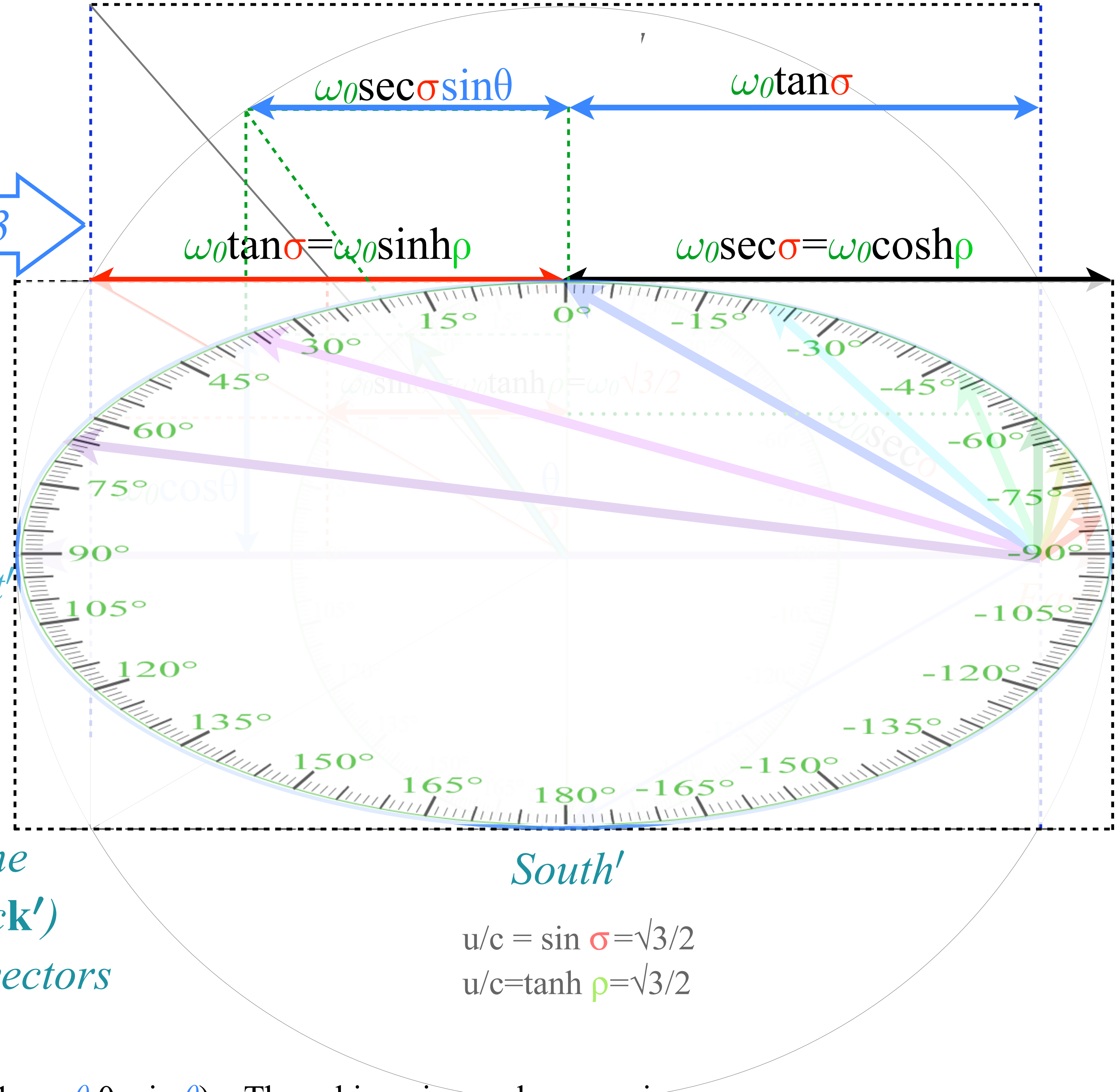
Faster Lorentz boost of
North-South-East-West
plane-wave 4-vectors $(\omega_0,\omega_x,\omega_y,\omega_z)$

Lorentz boost by $\sigma=60°$ *or* $e^{+\rho}=2+\sqrt{3}$



Lighthouse view $(\omega,c\mathbf{k})$ *of wave-vectors*

Ship-frame view $(\omega',c\mathbf{k}')$ *of wave-vectors*



Let lab starlight ray at polar angle θ have $\mathbf{k}\!\uparrow\theta=\omega_0\,(1,\cos\theta,0,-\sin\theta)$. Then ship going $\mathbf{u}$ along z-axis sees :

$$\begin{pmatrix}\omega'_{\uparrow\theta}\\ ck'_{x\uparrow\theta}\\ ck'_{y\uparrow\theta}\\ ck'_{z\uparrow\theta}\end{pmatrix}=\begin{pmatrix}\cosh\rho_z & \cdot & \cdot & -\sinh\rho_z\\ \cdot & 1 & \cdot & \cdot\\ \cdot & \cdot & 1 & \cdot\\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z\end{pmatrix}\begin{pmatrix}\omega_0\\ \omega_0\cos\theta\\ 0\\ -\omega_0\sin\theta\end{pmatrix}=\omega_0\begin{pmatrix}\cosh\rho_z+\sinh\rho_z\sin\theta\\ \cos\theta\\ 0\\ -\sinh\rho_z-\cosh\rho_z\sin\theta\end{pmatrix}=\omega_0\begin{pmatrix}\sec\sigma+\tan\sigma\sin\theta\\ \cos\theta\\ 0\\ -\tan\sigma-\sec\sigma\sin\theta\end{pmatrix}$$

Light-cone-sections are hyperbolas

Main Lighthouse on (x=0,y=0) time line

North Lighthouse-1 on (x=0,y=1) time line

Main Lighthouse blinks trace x=±ct "V"-lines thru each time tic

North Lighthouse-1 blinks trace $x^2-(ct)^2=1$ hyperbolas thru each tic

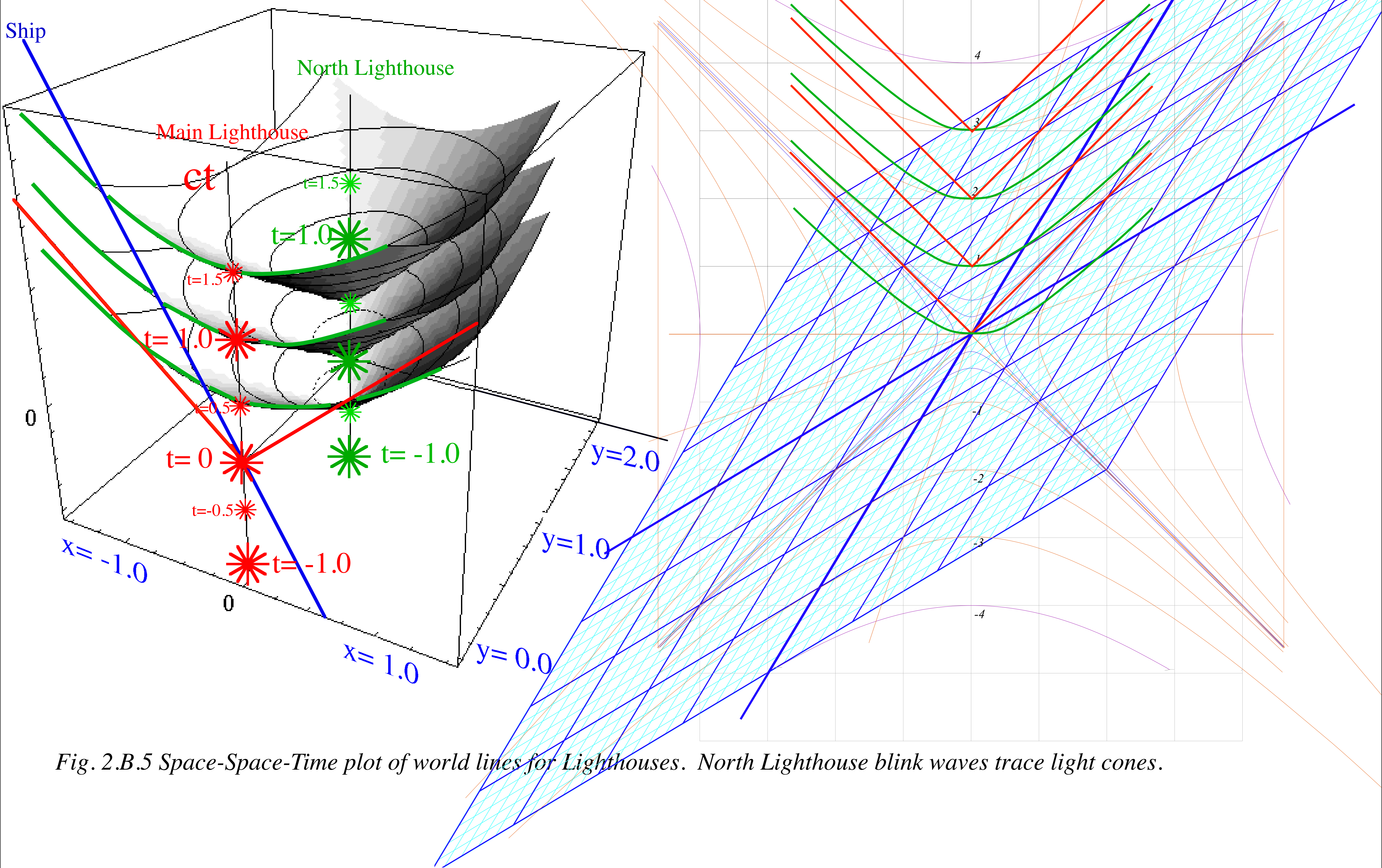
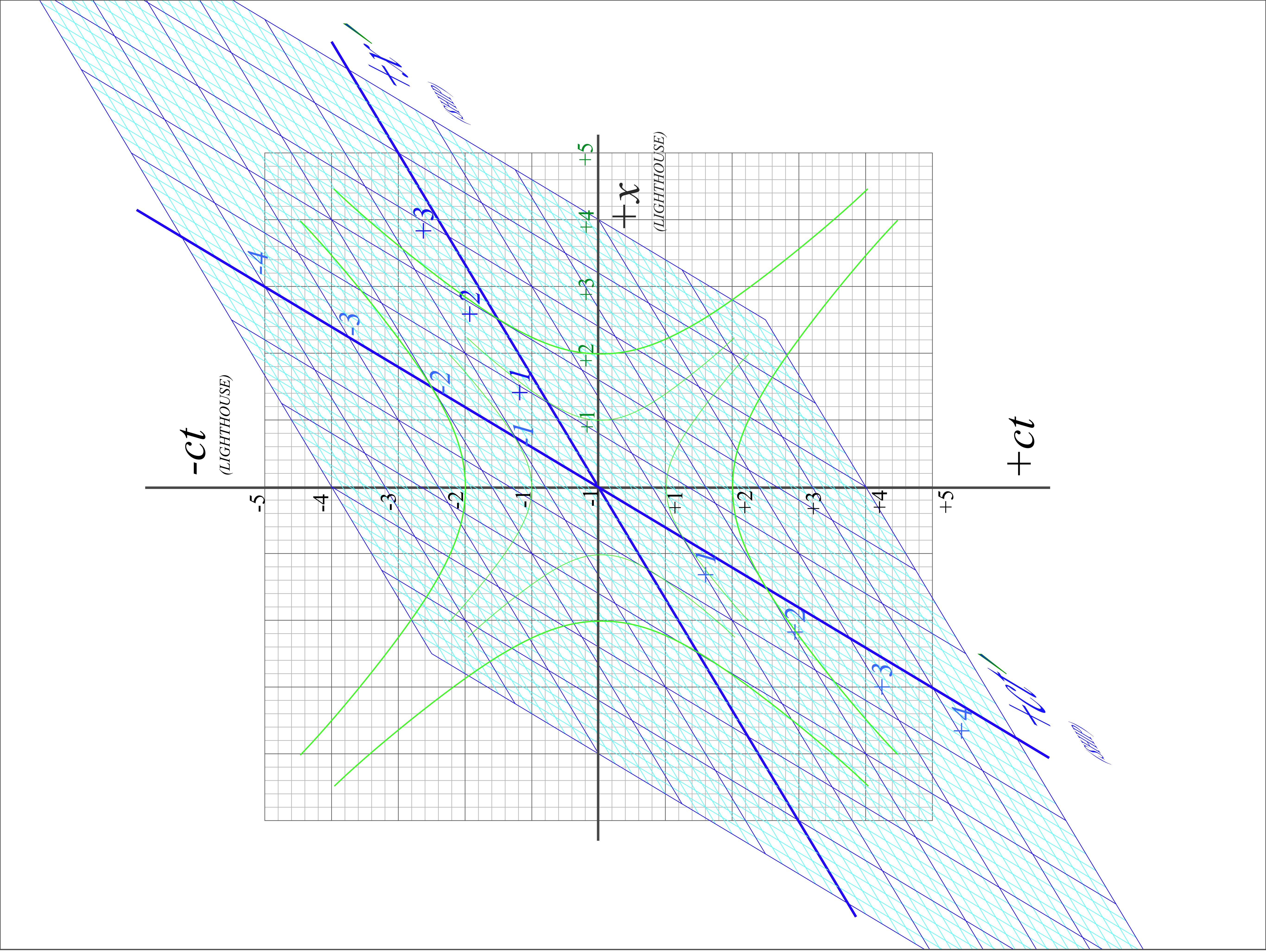


Fig. 2.B.5 Space-Space-Time plot of world lines for Lighthouses. North Lighthouse blink waves trace light cones.


+x
(LIGHTHOUSE)
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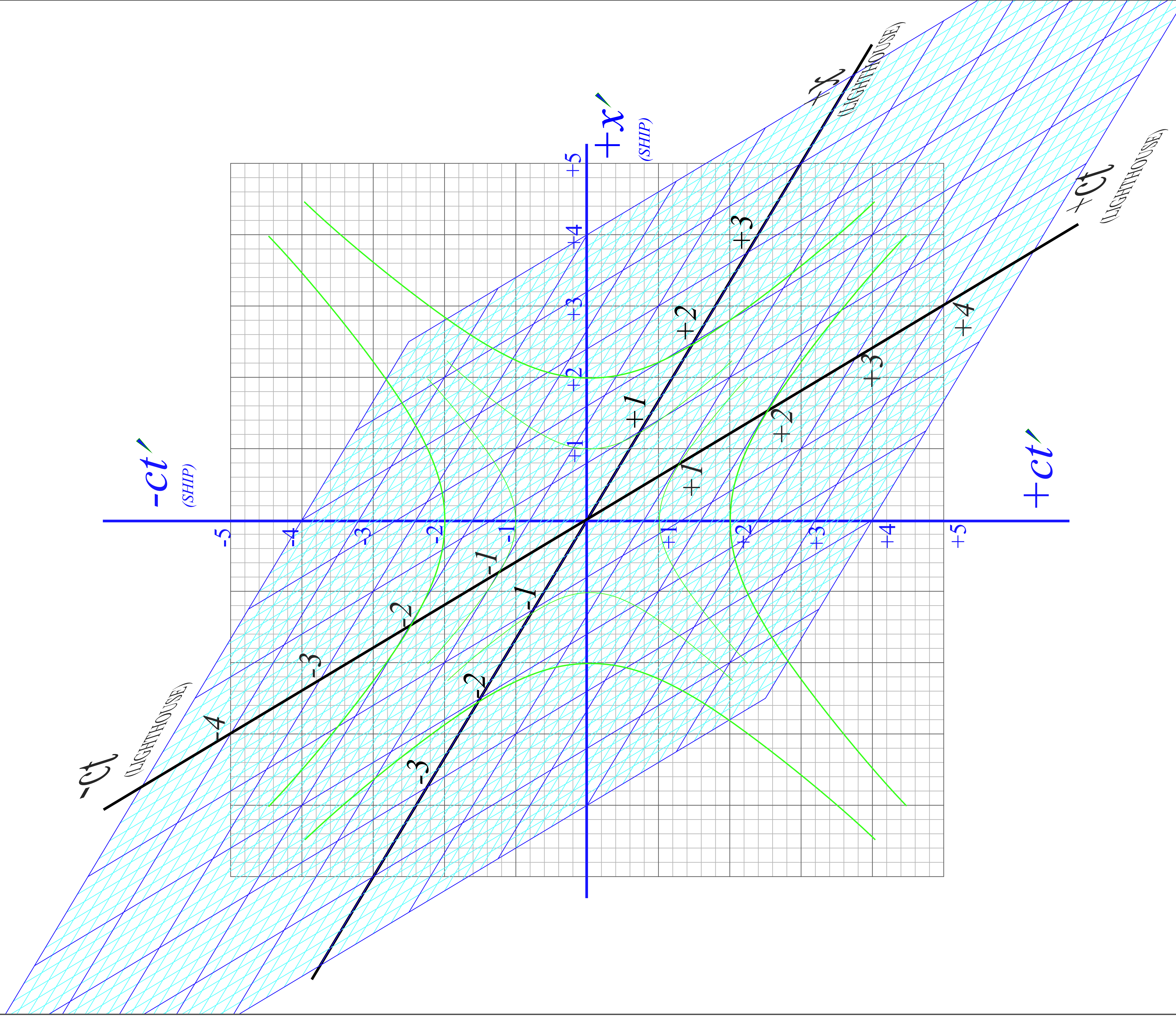

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