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Un-concentric derivation of stellar aberration \(k\)-angle \(\sigma\)

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Circular arc-area \(\sigma\) vs. hyperbolic arc-area \(\rho\)

Each circular trig function has a hyperbolic “country-cousin” function

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2016 animations of lighthouses and ships in \((x,y)\) scenarios and Minkowski \((x,ct)\) plots

### RelativIt Web Simulation

#### Relativistic Events in Main Lighthouse’s Frame

<table>
<thead>
<tr>
<th>Event</th>
<th>Description</th>
<th>Lighthouse (space)</th>
<th>Lighthouse (time)</th>
<th>Ship (space)</th>
<th>Ship (time)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Event 0:</td>
<td>Ship passes Main Lighthouse.</td>
<td>(x = 0)</td>
<td>(ct = 0)</td>
<td>(x' = 0)</td>
<td>(ct' = 0)</td>
</tr>
<tr>
<td>Event 1:</td>
<td>Ship gets hit by first blink from Main Lighthouse.</td>
<td>(x = \frac{1}{\sqrt{3}})</td>
<td>(ct = 0)</td>
<td>(x' = \frac{1}{\sqrt{3}})</td>
<td>(ct' = 0)</td>
</tr>
<tr>
<td>Event 2:</td>
<td>Main Lighthouse blinks second time.</td>
<td>(x = \frac{1}{\sqrt{3}})</td>
<td></td>
<td>(x' = \frac{1}{\sqrt{3}})</td>
<td></td>
</tr>
</tbody>
</table>

To check whether to use matrix \(\begin{pmatrix} \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{pmatrix}\) or else \(\begin{pmatrix} \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{pmatrix}\) just check that \(x' = 0 = 2x + 1ct\) or: \(x = -ct/2\) gives correct path.

\[
\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{3}} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \frac{2}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} 1.155 \\ 0.577 \end{pmatrix} \]
2015 animations of lighthouses and ships in \((x,y)\) scenarios and Minkowski \((x,ct)\) plots

Relativistic Events in Main Lighthouse’s Frame

\[
\begin{pmatrix}
x' \\
ct'
\end{pmatrix} = \begin{pmatrix}
\cosh \rho & \sinh \rho \\
\sinh \rho & \cosh \rho
\end{pmatrix} \begin{pmatrix}
x \\
ct
\end{pmatrix} = \begin{pmatrix}
\frac{1}{\sqrt{1-\beta^2}} & \frac{\beta}{\sqrt{1-\beta^2}} \\
\frac{\beta}{\sqrt{1-\beta^2}} & \frac{1}{\sqrt{1-\beta^2}}
\end{pmatrix} \begin{pmatrix}
x \\
ct
\end{pmatrix} = \begin{pmatrix}
\frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\
\frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}}
\end{pmatrix} \begin{pmatrix}
x \\
ct
\end{pmatrix} = \begin{pmatrix}
1.155 & 0.577 \\
0.577 & 1.155
\end{pmatrix} \begin{pmatrix}
x \\
ct
\end{pmatrix}
\]

Lighthouse Graph
Ref time \(t = 0.25\) sec. \(v/c = -0.50\) litesec/sec.

\(\beta = \frac{1}{2}\) \(e^\rho = \sqrt{3}\)
\[\cosh \rho = \frac{1}{\sqrt{1-\beta^2}} = \frac{2}{\sqrt{3}} = 1.155 = \Delta\]
\[\sinh \rho = \frac{\beta}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{3}} = 0.577\]

Click & Drag at bottom to control animation position

Event 0: Ship passes Main Lighthouse.
Event 1: Ship gets hit by first blink from Main Lighthouse.
Event 2: Main Lighthouse blinks second time.

\[
\text{(Main Lighthouse space)} \quad x = 0 \\
\text{(Main Lighthouse time)} \quad ct = 0
\]

\[
\text{(Ship space)} \quad x' = 0 \\
\text{(Ship time)} \quad ct' = 0
\]

http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=101
2015 animations of lighthouses and ships in \((x,y)\) scenarios and Minkowski \((x,ct)\) plots

\[
\begin{pmatrix}
    x' \\
    ct'
\end{pmatrix}
=
\begin{pmatrix}
    \cosh \rho & \sinh \rho \\
    \sinh \rho & \cosh \rho
\end{pmatrix}
\begin{pmatrix}
    x \\
    ct
\end{pmatrix}
=
\begin{pmatrix}
    \frac{1}{\sqrt{1-\beta^2}} \frac{1}{\sqrt{1-\beta^2}} \\
    \frac{2}{\sqrt{3}} \frac{1}{\sqrt{3}}
\end{pmatrix}
\begin{pmatrix}
    x \\
    ct
\end{pmatrix}
=
\begin{pmatrix}
    1.155 & 0.577 \\
    0.577 & 1.155
\end{pmatrix}
\begin{pmatrix}
    x \\
    ct
\end{pmatrix}
\]

\[
(x') = \left( \frac{2}{\sqrt{3}} \frac{1}{\sqrt{3}} \right) (x) = \left( \frac{2}{\sqrt{3}} \frac{1}{\sqrt{3}} \right) \left( \frac{2}{\sqrt{3}} \frac{1}{\sqrt{3}} \right) (x)
\]

for:

\[
\beta = \frac{1}{2} \quad \text{or:} \quad e^\rho = \sqrt{3}
\]

\[
\cosh \rho = \frac{1}{\sqrt{1-\beta^2}} = \frac{2}{\sqrt{3}} = 1.155 = \Delta
\]

\[
\sinh \rho = \frac{\beta}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{3}} = 0.577
\]

Event 0:
Ship passes Main Lighthouse.

\[
\begin{align*}
(x & = 0) \\
(ct & = 0)
\end{align*}
\]

Event 1: Ship gets hit by first blink from Main Lighthouse.

\[
\begin{align*}
(x & = -1.00) \\
(ct & = 2.00)
\end{align*}
\]

Event 2: Main Lighthouse blinks second time.

\[
\begin{align*}
(x & = 0) \\
(ct & = 2.00)
\end{align*}
\]

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RelativIt Web Simulation
Relativistic Events in
Main Lighthouse’s Frame

$\left( \begin{array}{c} x' \\ ct' \end{array} \right) = \left( \begin{array}{cc} \cosh \rho & \sinh \rho \\ \sinh \rho & \cosh \rho \end{array} \right) \left( \begin{array}{c} x \\ ct \end{array} \right) = \left( \begin{array}{c} \frac{1}{\sqrt{1-\beta^2}} \frac{\beta}{\sqrt{1-\beta^2}} \\ \frac{\beta}{\sqrt{1-\beta^2}} \frac{2}{\sqrt{1-\beta^2}} \end{array} \right) \left( \begin{array}{c} x \\ ct \end{array} \right) = \left( \begin{array}{c} 1.155 \\ 0.577 \end{array} \right) \left( \begin{array}{c} x \\ ct \end{array} \right)$

RelativIt Web Simulation
Relativistic Events in
Main Lighthouse’s Frame

Lighthouse time t = 0.756

Ship v/c (Rel.to Lthse.)= -0.500
Ship v/c (Rel.to Obs.)= -0.500
Lthse v/c (Rel.to Obs.)= 0.000

Event 0: Ship passes Main Lighthouse.
Event 1: Ship gets hit by first blink from Main Lighthouse.
Event 2: Main Lighthouse blinks second time.

(Lighthouse space) $x = 0$
(Lighthouse time) $ct = 0$
(Ship space) $x' = 0$
(Ship time) $ct' = 0$

$cosh \rho = \frac{1}{\sqrt{1-\beta^2}} = \frac{2}{\sqrt{3}} = 1.155 = \Delta$
$\sinh \rho = \frac{\beta}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{3}} = 0.577$

Click & Drag at bottom to control animation position

Caution: May be confusing

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Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration \(k\)-angle \(\sigma\).
Two Famous-Name Coefficients

This number is called an: Einstein time-dilation
(dilated by 25% here)

This number is called a: Lorentz length-contraction
(contractied by 20% here)

Old-Fashioned Notation

Relativistic Terms (Dual plot w/expanded table)
Space-time grid intersections mark Lorentz contraction and Einstein time dilation.
2015 animations of lighthouses and ships in \((x,y)\) scenarios and Minkowski \((x,ct)\) plots

**RelativIt Web Simulation**

**Relativistic Events in Main Lighthouse’s Frame**

\[
\begin{pmatrix}
    x' \\
    ct'
\end{pmatrix}
= \begin{pmatrix}
    \cosh \rho & \sinh \rho \\
    \sinh \rho & \cosh \rho
\end{pmatrix}
\begin{pmatrix}
    x \\
    ct
\end{pmatrix}
= \begin{pmatrix}
    \frac{1}{\sqrt{1-\beta^2}} \\
    \frac{\beta}{\sqrt{1-\beta^2}}
\end{pmatrix}
\begin{pmatrix}
    x \\
    ct
\end{pmatrix}
= \begin{pmatrix}
    \frac{2}{\sqrt{3}} \\
    \frac{1}{\sqrt{3}}
\end{pmatrix}
\begin{pmatrix}
    0.577 \\
    1.155
\end{pmatrix}
\begin{pmatrix}
    x \\
    ct
\end{pmatrix}
\]

\[
\begin{pmatrix}
    x' \\
    ct'
\end{pmatrix}
= \begin{pmatrix}
    \frac{2}{\sqrt{3}} \\
    \frac{1}{\sqrt{3}}
\end{pmatrix}
\begin{pmatrix}
    1.003 \\
    0
\end{pmatrix}
= \begin{pmatrix}
    1.155 \\
    0.577
\end{pmatrix}
\begin{pmatrix}
    x \\
    ct
\end{pmatrix}
\]

**Lighthouse Graph**

Ref time \(t = 1.00\) sec.

\(v/c = -0.50\) lites/sec.

**Ship registers 1\(^{st}\) Lighthouse Blink**

\[
\begin{pmatrix}
    x' \\
    ct'
\end{pmatrix}
= \begin{pmatrix}
    \frac{2}{\sqrt{3}} \\
    \frac{1}{\sqrt{3}}
\end{pmatrix}
\begin{pmatrix}
    0 \\
    0
\end{pmatrix}
= \begin{pmatrix}
    \frac{1}{\sqrt{3}} \\
    \frac{2}{\sqrt{3}}
\end{pmatrix}
\begin{pmatrix}
    0.577 \\
    1.155
\end{pmatrix}
\begin{pmatrix}
    \sinh \rho \\
    \cosh \rho
\end{pmatrix}
\]

\[
\frac{c}{\sqrt{1-\beta^2}} = \sqrt{3} = \Delta
\]

\[
\sinh \rho = \frac{1}{\sqrt{1-\beta^2}} = \frac{2}{\sqrt{3}} = 1.155
\]

\[
\cosh \rho = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{3}} = 0.577
\]

**Event 0:**
Ship passes Main Lighthouse.

\[(\text{Lighthouse space}) \quad x = 0 \quad (\text{Lighthouse time}) \quad ct = 0\]

**Event 1:** Ship gets hit by first blink from Main Lighthouse.

\[(\text{Ship space}) \quad x' = 0 \quad (\text{Ship time}) \quad ct' = 0\]

\[
\begin{pmatrix}
    x \\
    ct
\end{pmatrix} = \begin{pmatrix}
    \frac{2}{\sqrt{3}} \\
    \frac{1}{\sqrt{3}}
\end{pmatrix}
\begin{pmatrix}
    x \\
    ct
\end{pmatrix}
\]

**Event 2:** Main Lighthouse blinks second time.

\[(\text{Lighthouse space}) \quad x = 0 \quad (\text{Lighthouse time}) \quad ct = 0\]

\[
\begin{pmatrix}
    x' \\
    ct'
\end{pmatrix} = \begin{pmatrix}
    \frac{2}{\sqrt{3}} \\
    \frac{1}{\sqrt{3}}
\end{pmatrix}
\begin{pmatrix}
    x' \\
    ct'
\end{pmatrix}
\]

Caution: May be confusing

\[http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=101\]

\[http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=102\]
Ship frame: time dilation $\Delta = \cosh \rho = 1.155$ of Lighthouse blinks

Ref time $t = 1.0$ sec.

Event 1
Event 2

Ship registers 1st Lighthouse Blink
at its position

\[
\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{3}} & 1 \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{2}{\sqrt{3}} \end{pmatrix} = \begin{pmatrix} 0.577 \\ 1.155 \end{pmatrix} = \begin{pmatrix} \sinh \rho \\ \cosh \rho \end{pmatrix}
\]
Ship frame: time dilation $\Delta = \cosh \rho = 1.155$ of Lighthouse blinks

RelativIt Web Simulation
Relativistic Events in Main Lighthouse's Frame

$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \cosh \rho & \sinh \rho \\ \sinh \rho & \cosh \rho \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1-\beta^2}} & \frac{\beta}{\sqrt{1-\beta^2}} \\ \frac{-\beta}{\sqrt{1-\beta^2}} & \frac{1}{\sqrt{1-\beta^2}} \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{3}} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \frac{2}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} 1.155 \\ 0.577 \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix}$$

For:
$$\beta = \frac{1}{2} \quad e^\rho = \sqrt{3}$$

$cosh \rho = \frac{1}{\sqrt{1-\beta^2}} = \frac{2}{\sqrt{3}} = 1.155 = \Delta$

$\sinh \rho = \frac{\beta}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{3}} = 0.577$

Event 0:
Ship passes Main Lighthouse.

(Lighthouse space) $x = 0$
(Lighthouse time) $ct = 0$
(Ship space) $x' = 0$
(Ship time) $ct' = 0$

Event 1: Ship gets hit by first blink from Main Lighthouse.

$Lighthouse Graph$
Ref time $t = 1.25$ sec.
$v/c = 0.5$ litesec/sec.

Event 2: Main Lighthouse blinks second time.

$Lighthouse Graph$
Ship 1
$Lighthouse Graph$
Main Lighthouse

http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=101
http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=102

Caution: May be confusing

Tuesday, April 19, 2016
2015 animations of lighthouses and ships in \((x,y)\) scenarios and Minkowski \((x,ct)\) plots

**RelativIt Web Simulation**

**Relativistic Events in Main Lighthouse’s Frame**

\[
\begin{bmatrix}
  x' \\ ct'
\end{bmatrix} = \begin{bmatrix}
  \cosh \rho & \sinh \rho \\
  \sinh \rho & \cosh \rho
\end{bmatrix} \begin{bmatrix}
  x \\ ct
\end{bmatrix} = \begin{bmatrix}
  \frac{1}{\sqrt{1-\beta^2}} \frac{1}{\sqrt{1-\beta^2}} \\
  \frac{\beta}{\sqrt{1-\beta^2}} & \frac{1}{\sqrt{1-\beta^2}}
\end{bmatrix} \begin{bmatrix}
  x \\ ct
\end{bmatrix} = \begin{bmatrix}
  1.155 & 0.577 \\
  0.577 & 1.155
\end{bmatrix} \begin{bmatrix}
  x \\ ct
\end{bmatrix}
\]

**Lighthouse Graph**

Ref time \(t = 1.50\) sec.

\(v/c = -0.50\) litesecs/sec.

---

**Event 0:**
Ship passes Main Lighthouse.

\(\beta = \frac{1}{2}\)
\(\rho = \sqrt{3}\)

\(\cosh \rho = \frac{1}{\sqrt{1-\beta^2}} = \frac{2}{\sqrt{3}} = 1.155 = \Delta\)
\(\sinh \rho = \frac{\beta}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{3}} = 0.577\)

**Event 1:** Ship gets hit by first blink from Main Lighthouse.

\(\text{(Lighthouse space)} \quad x = 0\)
\(\text{(Lighthouse time)} \quad ct = 0\)

**Event 2:** Main Lighthouse blinks second time.

\(\text{(Ship space)} \quad x' = 0\)
\(\text{(Ship time)} \quad ct' = 0\)

Click & Drag at bottom to control animation position.
2015 animations of lighthouses and ships in \((x,y)\) scenarios and Minkowski \((x,ct)\) plots

\[
\begin{pmatrix}
  x' \\
  ct'
\end{pmatrix} = \begin{pmatrix}
  \cosh \rho & \sinh \rho \\
  \sinh \rho & \cosh \rho
\end{pmatrix} \begin{pmatrix}
  x \\
  ct
\end{pmatrix} = \begin{pmatrix}
  \frac{1}{\sqrt{1-\beta^2}} \frac{\beta}{\sqrt{1-\beta^2}} \\
  \frac{\beta}{\sqrt{1-\beta^2}} \frac{1}{\sqrt{1-\beta^2}}
\end{pmatrix} \begin{pmatrix}
  x \\
  ct
\end{pmatrix} = \begin{pmatrix}
  \frac{2}{\sqrt{3}} \frac{1}{\sqrt{3}} \\
  \frac{1}{\sqrt{3}} \frac{2}{\sqrt{3}}
\end{pmatrix} \begin{pmatrix}
  x \\
  ct
\end{pmatrix} = \begin{pmatrix}
  1.155 & 0.577 \\
  0.577 & 1.155
\end{pmatrix} \begin{pmatrix}
  x \\
  ct
\end{pmatrix}
\]

RelativIt Web Simulation
Relativistic Events in Main Lighthouse’s Frame

\( \text{Lighthouse time } t = 1.753 \)
\( \text{Ship v/c (Rel. to Lhse.)} = -0.500 \)
\( \text{Ship v/c (Rel. to Obs.)} = -0.500 \)
\( \text{Lhse v/c (Rel. to Obs.)} = 0.000 \)

\[
\begin{pmatrix}
  x' \\
  ct'
\end{pmatrix} = \begin{pmatrix}
  \frac{2}{\sqrt{3}} \frac{1}{\sqrt{3}} \\
  \frac{1}{\sqrt{3}} \frac{2}{\sqrt{3}}
\end{pmatrix} \begin{pmatrix}
  x \\
  ct
\end{pmatrix}
\]

\[\text{for:} \quad \beta = \frac{1}{2} \quad \text{or:} \quad e^\rho = \sqrt{3}\]
\[\cosh \rho = \frac{1}{\sqrt{1-\beta^2}} = \frac{2}{\sqrt{3}} = 1.155 = \Delta\]
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<th>Event 2: Main Lighthouse blinks second time.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Lighthouse space) ( x = 0 )</td>
<td>( x = )</td>
<td>( x = )</td>
</tr>
<tr>
<td>(Lighthouse time) ( ct = 0 )</td>
<td>( ct = )</td>
<td>( ct = )</td>
</tr>
<tr>
<td>(Ship space) ( x' = 0 )</td>
<td>( x' = )</td>
<td>( x' = )</td>
</tr>
<tr>
<td>(Ship time) ( ct' = 0 )</td>
<td>( ct' = )</td>
<td>( ct' = )</td>
</tr>
</tbody>
</table>
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Simultaneous events in Lighthouse \((x,y)\) frame: Not so in Ship \((x',y')\) frame

\[
\begin{pmatrix}
  x' \\
  c't'
\end{pmatrix} = \begin{pmatrix}
  \cosh \rho & \sinh \rho \\
  \sinh \rho & \cosh \rho
\end{pmatrix} \begin{pmatrix}
  x \\
  c't
\end{pmatrix} = \begin{pmatrix}
  \frac{1}{\sqrt{1-\beta^2}} & \frac{\beta}{\sqrt{1-\beta^2}} \\
  \frac{\beta}{\sqrt{1-\beta^2}} & \frac{1}{\sqrt{1-\beta^2}}
\end{pmatrix} \begin{pmatrix}
  x \\
  c't
\end{pmatrix} = \begin{pmatrix}
  1.155 & 0.577 \\
  0.577 & 1.155
\end{pmatrix} \begin{pmatrix}
  x \\
  c't
\end{pmatrix}
\]

**Lighthouse time** \(t = 2.003\)

**Ship v/c (Rel to Lhse.) = -0.500**

**Ship v/c (Rel to Obs.) = -0.500**

**Lhse v/c (Rel to Obs.) = 0.000**

**Lighthouse Graph**

Ref time \(t = 2.00\) sec.

\(v/c = -0.50\) litesec/sec.

**Event 0:**
Ship passes Main Lighthouse.

<table>
<thead>
<tr>
<th>Lighthouse space</th>
<th>(x = 0)</th>
<th>(x' = 0)</th>
<th>(x' = 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lighthouse time</td>
<td>(ct = 0)</td>
<td>(ct = 2.00)</td>
<td>(ct = 2.00)</td>
</tr>
</tbody>
</table>

**Event 1:** Ship gets hit by first blink from Main Lighthouse.

<table>
<thead>
<tr>
<th>Ship space</th>
<th>(x = -1.00)</th>
<th>(x' = 0)</th>
<th>(x' = 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ship time</td>
<td>(ct = 2.00)</td>
<td>(ct' = 1.73)</td>
<td>(ct' = 1.73)</td>
</tr>
</tbody>
</table>

**Event 2:** Main Lighthouse blinks second time.

<table>
<thead>
<tr>
<th>Ship space</th>
<th>(x = 0)</th>
<th>(x' = 0)</th>
<th>(x' = 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ship time</td>
<td>(ct = 0)</td>
<td>(ct = 2.00)</td>
<td>(ct = 2.00)</td>
</tr>
</tbody>
</table>

Caution: May be confusing

http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=101

http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=102
2015 animations of lighthouses and ships in \((x,y)\) scenarios and Minkowski \((x,ct)\) plots

Relativistic Events in Main Lighthouse’s Frame

\[
\begin{align*}
\begin{pmatrix} x' \\ ct' \end{pmatrix} &= \begin{pmatrix} \cosh \rho & \sinh \rho \\ \sinh \rho & \cosh \rho \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} \\
&= \frac{1}{\sqrt{1-\beta^2}} \begin{pmatrix} 1 & -\beta \\ \beta & 1 \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} \\
&= \begin{pmatrix} \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} 1.155 \\ 0.577 \end{pmatrix} \\
&= \begin{pmatrix} 0.577 \\ 1.155 \end{pmatrix}
\end{align*}
\]

Lighthouse Graph
Ref time \(t = 2.00\) sec.
\(v/c = -0.50\) litesec/sec.

Key point: Any event happening to you has your \(x\)-value set to zero!

Event 0:
Ship passes Main Lighthouse.

Event 1: Ship gets hit by first blink from Main Lighthouse.
(Lighthouse space) \(x = 0\)
(Lighthouse time) \(ct = 0\)
(Ship space) \(x' = 0\)
(Ship time) \(ct' = 0\)

Event 2: Main Lighthouse blinks second time.
\(x = 0\)
\(ct = 2.00\)
\(x' = \) 
\(ct' = 1.73\)

RelativIt Web Simulation
Relativistic Events in Main Lighthouse’s Frame

Lighthouse time \(t = 2.003\)
Ship \(v/c\) (Rel.to Lhse.)\(=0.500\)
Ship \(v/c\) (Rel.to Obs.)\(=-0.500\)
Lhse \(v/c\) (Rel.to Obs.)\(=0.000\)

Click & Drag at bottom to control animation position

\(\beta = \frac{1}{2}\)
\(e^\rho = \sqrt{3}\)

\(\cosh \rho = \frac{1}{\sqrt{1-\beta^2}} = 1.155 = \Delta\)

\(\sinh \rho = \frac{\beta}{\sqrt{1-\beta^2}} = 0.577\)
2015 animations of lighthouses and ships in \((x,y)\) scenarios and Minkowski \((x,ct)\) plots

RelativIt Web Simulation
Relativistic Events in Main Lighthouse’s Frame

\[
\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \cosh \rho & \sinh \rho \\ \sinh \rho & \cosh \rho \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1-\beta^2}} - \frac{\beta}{\sqrt{1-\beta^2}} \\ \frac{\beta}{\sqrt{1-\beta^2}} \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} 1.155 \\ 0.577 \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} 0.577 \\ 1.155 \end{pmatrix}
\]

**Lighthouse Graph**
Ref time \(t = 2.00\) sec.
\(v/c = -0.50\) litesec/sec.

\(\Delta = 2.30\)

Lighthouse time \(t = 2.003\)
Ship \(v/c (Rel.\ Lhse.) = -0.500\)
Ship \(v/c (Rel.\ Obs.) = -0.500\)
Lhse \(v/c (Rel.\ Obs.) = 0.000\)

**Event 1:** Ship passes Main Lighthouse.
**Event 1:** Ship gets hit by first blink from Main Lighthouse.
**Event 2:** Main Lighthouse blinks second time.

\[
\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} 2 \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \frac{1}{\sqrt{3}} \\ \frac{2}{\sqrt{3}} \end{pmatrix} = \begin{pmatrix} 2 \frac{1}{\sqrt{3}} \\ \frac{2}{\sqrt{3}} \end{pmatrix} = \begin{pmatrix} 1.15 \\ 2.30 \end{pmatrix}
\]

\(\Delta = 2.30\)

\(\beta = \frac{1}{2}\)
\(e^\rho = \sqrt{3}\)

\[
\begin{align*}
\cosh \rho &= \frac{1}{\sqrt{1-\beta^2}} = \frac{2}{\sqrt{3}} = 1.155 = \Delta \\
\sinh \rho &= \frac{\beta}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{3}} = 0.577
\end{align*}
\]

**Event 0:**
Ship passes Main Lighthouse.

\[
\begin{align*}
\text{(Lighthouse space)} & \quad x = 0 \\
\text{(Lighthouse time)} & \quad ct = 0
\end{align*}
\]

**Event 1:** Ship gets hit by first blink from Main Lighthouse.

\[
\begin{align*}
\text{(Lighthouse space)} & \quad x = -1.00 \\
\text{(Lighthouse time)} & \quad ct = 2.00
\end{align*}
\]

**Event 2:** Main Lighthouse blinks second time.

\[
\begin{align*}
\text{(Ship space)} & \quad x' = 0 \\
\text{(Ship time)} & \quad ct' = 1.73
\end{align*}
\]

\[
\begin{align*}
\text{(Ship space)} & \quad x' = x - \frac{1}{\sqrt{3}} \\
\text{(Ship time)} & \quad ct' = \frac{2}{\sqrt{3}}
\end{align*}
\]

Tuesday, April 19, 2016
2015 animations of lighthouses and ships in \((x,y)\) scenarios and Minkowski \((x,ct)\) plots

### Relativistic Events in Main Lighthouse’s Frame

\[
\begin{pmatrix}
  x' \\
  ct'
\end{pmatrix} = \begin{pmatrix}
  \cosh \rho & \sinh \rho \\
  \sinh \rho & \cosh \rho
\end{pmatrix} \begin{pmatrix}
  x \\
  ct
\end{pmatrix} = \begin{pmatrix}
  \frac{1}{\sqrt{1-\beta^2}} \frac{\beta}{\sqrt{1-\beta^2}} \\
  \frac{1}{\sqrt{1-\beta^2}} \frac{1}{\sqrt{1-\beta^2}}
\end{pmatrix} \begin{pmatrix}
  x \\
  ct
\end{pmatrix} = \begin{pmatrix}
  1.155 & 0.577 \\
  0.577 & 1.155
\end{pmatrix} \begin{pmatrix}
  x \\
  ct
\end{pmatrix}
\]

\[
\begin{pmatrix}
  x' \\
  ct'
\end{pmatrix} = \begin{pmatrix}
  \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\
  \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}}
\end{pmatrix} \begin{pmatrix}
  0 \\
  2
\end{pmatrix} = \begin{pmatrix}
  2 \frac{1}{\sqrt{3}} \\
  \frac{2}{\sqrt{3}}
\end{pmatrix} = \begin{pmatrix}
  2 \\
  \frac{1}{\sqrt{3}}
\end{pmatrix} = \begin{pmatrix}
  1.15 \\
  2.30
\end{pmatrix}
\]

### Event 0:
Ship passes Main Lighthouse.

\[
\begin{align*}
\beta &= \frac{1}{2}, & e^\rho &= \sqrt{3} \\
\cosh \rho &= \frac{1}{\sqrt{1-\beta^2}} = \frac{2}{\sqrt{3}} = 1.155 = \Delta \\
\sinh \rho &= \frac{\beta}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{3}} = 0.577
\end{align*}
\]

<table>
<thead>
<tr>
<th>Event 0: Ship passes Main Lighthouse.</th>
<th>Event 1: Ship gets hit by first blink from Main Lighthouse.</th>
<th>Event 2: Main Lighthouse blinks second time.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Lighthouse space) (x = 0)</td>
<td>(Lighthouse time) (ct = 0)</td>
<td>(x = 0)</td>
</tr>
<tr>
<td>(Ship space) (x' = 0)</td>
<td>(Ship time) (ct' = 0)</td>
<td>(x' = c \Delta)</td>
</tr>
<tr>
<td>(x = -1.00)</td>
<td>(ct = 2.00)</td>
<td>(ct' = 2\Delta = 2.30)</td>
</tr>
</tbody>
</table>
RelativIt Web Simulation
Relativistic Events in Main Lighthouse’s Frame

\[
\begin{pmatrix}
    x' \\
    ct'
\end{pmatrix}
= \begin{pmatrix}
    \cosh \rho & \sinh \rho \\
    \sinh \rho & \cosh \rho
\end{pmatrix}
\begin{pmatrix}
    x \\
    ct
\end{pmatrix}
\]

15 animations of lighthouses and ships in \((x,y)\) scenarios and Minkowski \((x,ct)\) plots

Event 0:
Ship passes Main Lighthouse.

\[(Lighthouse \, space) \quad x = 0\]
\[(Lighthouse \, time) \quad ct = 0\]

\[x' = 0 \quad \text{for:} \quad \beta = \frac{1}{2} \quad \eta^2 = \sqrt{3}\]
\[
cosh \rho = \frac{1}{\sqrt{1 - \beta^2}} = \frac{2}{\sqrt{3}} = 1.155 = \Delta
\]
\[
\sinh \rho = \frac{\beta}{\sqrt{1 - \beta^2}} = \frac{1}{\sqrt{3}} = 0.577
\]

Event 1: Ship gets hit by first blink from Main Lighthouse.

\[(Lighthouse \, space) \quad x = -1.00\]
\[(Lighthouse \, time) \quad ct = 2.00\]

\[x' = 0 \quad \Delta = 2.30\]

Event 2: Main Lighthouse blinks second time.

\[(Lighthouse \, space) \quad x = 0\]
\[(Lighthouse \, time) \quad ct = 2.00\]

Caution: May be confusing
Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1st RelativIt animations).

2005 and 2016 animations of lighthouses and ships in \((x,y)\) scenarios and Minkowski \((x,ct)\) plots

Lighthouse \((x,y)\) frame: Dual concentric circular wavefronts serve as timing device

Ship frame: time dilation \(\Delta = \cosh \rho = 1.15\) of Lighthouse blinks

Simultaneous events in Lighthouse \((x,y)\) frame: Not so in Ship \((x',y')\) frame

\(\rightarrow\) Lighthouse-square \((x,ct)\) plots correlated with Ship-square \((x',ct')\) plots

Overlapped Lighthouse \((x,ct)\) and Ship \((x',ct')\) frame Minkowski plots correlate inconsistencies

Ship \((x',y')\) frame: Dual un-concentric circular wavefronts map space-time

Pythagorean derivation of time-dilation factor \(\Delta = \cosh \rho\)

Un-concentric derivation of stellar aberration \(k\)-angle \(\sigma\)

Per-spacetime 4-vector \((\omega_0, \omega_x, \omega_y, \omega_z) = (\omega, ck_x, ck_y, ck_z)\) transformation

“Occam-sword” geometry: A pattern recognition aid

Relating velocity parameter \(\beta = u/c\) to rapidity \(\rho\) to \(k\)-angle \(\sigma\) to \(u/c\)-angle \(\nu\)

Circular arc-area \(\sigma\) vs. hyperbolic arc-area \(\rho\)

Each circular trig function has a hyperbolic “country-cousin” function

Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration \(k\)-angle \(\sigma\)
Simple 2015 animations of lighthouses and ships in \((x,y)\) scenarios and Minkowski \((x,ct)\) plots

**RelativIt Web Simulation**

Relativistic Events in Main Lighthouse’s Frame

\[
\begin{pmatrix}
  x' \\
  ct'
\end{pmatrix}
= 
\begin{pmatrix}
  \cosh \rho & \sinh \rho \\
  \sinh \rho & \cosh \rho
\end{pmatrix}
\begin{pmatrix}
  x \\
  ct
\end{pmatrix}
= 
\begin{pmatrix}
  \frac{1}{\sqrt{1-\beta^2}} & \frac{\beta}{\sqrt{1-\beta^2}} \\
  \frac{\beta}{\sqrt{1-\beta^2}} & \frac{1}{\sqrt{1-\beta^2}}
\end{pmatrix}
\begin{pmatrix}
  x \\
  ct
\end{pmatrix}
= 
\begin{pmatrix}
  1.155 & 0.577 \\
  0.577 & 1.155
\end{pmatrix}
\begin{pmatrix}
  x \\
  ct
\end{pmatrix}
\]

**Ship time** \(t' = 0.006\)

**Ship v/c (Rel. to Lthse.) = 0.500**

**Ship v/c (Rel. to Obs.) = 0.000**

**Lthse v/c (Rel. to Obs.) = 0.500**

**Ship Graph**

Ref time \(t = 0.01\) sec.

\(v/c = -0.50\) litesec/sec.

**Event 0:** Ship passes Main Lighthouse.

**Event 1:** Ship gets hit by first blink from Main Lighthouse.

**Event 2:** Main Lighthouse blinks second time.

\[
\begin{pmatrix}
  x' \\
  ct'
\end{pmatrix}
= 
\begin{pmatrix}
  \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\
  \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}}
\end{pmatrix}
\begin{pmatrix}
  0 \\
  2
\end{pmatrix}
= 
\begin{pmatrix}
  2 & \frac{1}{\sqrt{3}} \\
  \frac{2}{\sqrt{3}} & 2
\end{pmatrix}
\begin{pmatrix}
  \frac{2}{4} \\
  \frac{1}{\sqrt{3}}
\end{pmatrix}
= 
\begin{pmatrix}
  1.15 \\
  2.30
\end{pmatrix}
\]

**for:** \(\beta = \frac{1}{2}\)

\(e^\rho = \sqrt{3}\)

\[\cosh \rho = \frac{1}{\sqrt{1-\beta^2}} = \frac{2}{\sqrt{3}} = 1.155 = \Delta\]

\[\sinh \rho = \frac{\beta}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{3}} = 0.577\]

http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=104
Simple 2015 animations of lighthouses and ships in \((x,y)\) scenarios and Minkowski \((x,ct)\) plots

Relativistic Events in
Main Lighthouse’s Frame

\[
\begin{align*}
\begin{pmatrix} x' \\ ct' \end{pmatrix} &= \begin{pmatrix} \cosh \rho & \sinh \rho \\ \sinh \rho & \cosh \rho \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1-\beta^2}} \frac{\beta}{\sqrt{1-\beta^2}} \\ \frac{\beta}{\sqrt{1-\beta^2}} \frac{1}{\sqrt{1-\beta^2}} \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{3}} \frac{2}{\sqrt{3}} \\ \frac{2}{\sqrt{3}} \frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} 1.155 \\ 0.577 \end{pmatrix} = \begin{pmatrix} 1.155 \\ 0.577 \end{pmatrix}, \\
\end{align*}
\]

for: \(\beta = \frac{1}{2}\) \(e^\rho = \sqrt{3}\)

\[
\begin{align*}
\cosh \rho &= \frac{1}{\sqrt{1-\beta^2}} = \frac{2}{\sqrt{3}} = 1.155 = \Delta \\
\sinh \rho &= \frac{\beta}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{3}} = 0.577
\end{align*}
\]

Ship time \(t' = 0.256\)  
Ship \(v/c\) (Rel. to Lhse.) = 0.500  
Ship \(v/c\) (Rel. to Obs.) = 0.000  
Lhse \(v/c\) (Rel. to Obs.) = 0.500

Ship Graph  
Ref time \(t = 0.26\) sec.  
\(v/c = -0.50\) litesec/sec.

Event 0:  
Ship passes Main Lighthouse.  
Event 1: Ship gets hit by first blink from Main Lighthouse.  
Event 2: Main Lighthouse blinks second time.

<table>
<thead>
<tr>
<th>Event 0: Ship passes Main Lighthouse.</th>
<th>Event 1: Ship gets hit by first blink from Main Lighthouse.</th>
<th>Event 2: Main Lighthouse blinks second time.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Lighthouse space) (x = 0)</td>
<td>(x = -1.00)</td>
<td>(x = 0)</td>
</tr>
<tr>
<td>(Lighthouse time) (ct = 0)</td>
<td>(ct = 2.00)</td>
<td>(ct = 2.00)</td>
</tr>
<tr>
<td>(Ship space) (x' = 0)</td>
<td>(x' = 0)</td>
<td>(x' = c \Delta)</td>
</tr>
<tr>
<td>(Ship time) (ct' = 0)</td>
<td>(ct' = 1.73)</td>
<td>(ct' = 2\Delta = 2.30)</td>
</tr>
</tbody>
</table>

http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=104
Simple 2015 animations of lighthouses and ships in \((x,y)\) scenarios and Minkowski \((x,ct)\) plots

RelativIt Web Simulation

Relativistic Events in Main Lighthouse’s Frame

\[
\begin{pmatrix}
  x' \\
  ct'
\end{pmatrix} = \begin{pmatrix}
  \cosh \rho & \sinh \rho \\
  \sinh \rho & \cosh \rho
\end{pmatrix} \begin{pmatrix}
  x \\
  ct
\end{pmatrix} = \begin{pmatrix}
  \frac{1}{\sqrt{1-\beta^2}} & \frac{\beta}{\sqrt{1-\beta^2}} \\
  \frac{\beta}{\sqrt{1-\beta^2}} & \frac{1}{\sqrt{1-\beta^2}}
\end{pmatrix} \begin{pmatrix}
  x \\
  ct
\end{pmatrix} = \begin{pmatrix}
  1.155 & 0.577 \\
  0.577 & 1.155
\end{pmatrix} \begin{pmatrix}
  x \\
  ct
\end{pmatrix}
\]

Ship time \(t' = 0.506\)

Ship \(v/c\) (Rel. to Lhse.) = 0.500

Ship \(v/c\) (Rel. to Obs.) = 0.000

Lhse \(v/c\) (Rel. to Obs.) = 0.500

\[
\begin{pmatrix}
  x' \\
  ct'
\end{pmatrix} = \begin{pmatrix}
  \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\
  \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}}
\end{pmatrix} \begin{pmatrix}
  0 \\
  2
\end{pmatrix} = \begin{pmatrix}
  2 \frac{1}{\sqrt{3}} \\
  2 \frac{2}{\sqrt{3}}
\end{pmatrix} = \left( \frac{2}{4} \right) \frac{1}{\sqrt{3}} = \left( 1.15 \right) \frac{2.30}{2}
\]

For: \(\beta = \frac{1}{2}\)

\(e^\rho = \sqrt{3}\)

\(\cosh \rho = \frac{1}{\sqrt{1-\beta^2}} = \frac{2}{\sqrt{3}} = 1.155 = \Delta\)

\(\sinh \rho = \frac{\beta}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{3}} = 0.577\)

Event 0: Ship passes Main Lighthouse.

Event 1: Ship gets hit by first blink from Main Lighthouse.

Event 2: Main Lighthouse blinks second time.

\[
\begin{array}{c|c|c}
\text{(Lighthouse space)} & x = 0 & x = -1.00 \\
\text{(Lighthouse time)} & ct = 0 & ct = 2.00 \\
\text{(Ship space)} & x' = 0 & x' = 0 \\
\text{(Ship time)} & ct' = 0 & ct' = 1.73 \\
\end{array}
\]

http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=104

Tuesday, April 19, 2016
Simple 2015 animations of lighthouses and ships in \((x,y)\) scenarios and Minkowski \((x,ct)\) plots

\[
\begin{pmatrix}
  x' \\
  ct'
\end{pmatrix} = \begin{pmatrix}
  \cosh \rho & \sinh \rho \\
  \sinh \rho & \cosh \rho
\end{pmatrix} \begin{pmatrix}
  x \\
  ct
\end{pmatrix} = \begin{pmatrix}
  \frac{1}{\sqrt{1-\beta^2}} & \frac{-\beta}{\sqrt{1-\beta^2}} \\
  \frac{\beta}{\sqrt{1-\beta^2}} & \frac{1}{\sqrt{1-\beta^2}}
\end{pmatrix} \begin{pmatrix}
  1.155 \\
  0.577
\end{pmatrix}
\]

RelativIt Web Simulation

Relativistic Events in Main Lighthouse’s Frame

Event 0:
Ship passes Main Lighthouse.

Event 1: Ship gets hit by first blink from Main Lighthouse.

Event 2: Main Lighthouse blinks second time.

(Lighthouse space) \(x = 0\)

(Lighthouse time) \(ct = 0\)

(Ship space) \(x' = 0\)

(Ship time) \(ct' = 0\)

\(\cosh \rho = \frac{1}{\sqrt{1-\beta^2}} = \frac{2}{\sqrt{3}} = 1.155 = \Delta\)

\(\sinh \rho = \frac{\beta}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{3}} = 0.577\)

\(\beta = \frac{1}{2}\)

\(e^\rho = \sqrt{3}\)

\(\text{http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=104}\)

Click & Drag at bottom to control animation speed

Click & Drag at bottom to control animation speed

for:
\(e^\rho = \sqrt{3}\)

or:
\(\beta = \frac{1}{2}\)

\(\cosh \rho = \frac{2}{\sqrt{3}} = 1.155 = \Delta\)

\(\sinh \rho = \frac{1}{\sqrt{3}} = 0.577\)

Ship Graph

Ref time \(t = 0.76\) sec.

\(v/c = -0.50\) litesec/sec.

\(\Delta = 2.30\)

\(\text{http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=104}\)
Simple 2015 animations of lighthouses and ships in $(x,y)$ scenarios and Minkowski $(x,ct)$ plots

RelativIt Web Simulation
Relativistic Events in Main Lighthouse’s Frame

\[
\begin{pmatrix}
  x' \\
  ct'
\end{pmatrix} = \begin{pmatrix}
  \cosh \rho & \sinh \rho \\
  \sinh \rho & \cosh \rho
\end{pmatrix} \begin{pmatrix}
  x \\
  ct
\end{pmatrix} = \begin{pmatrix}
  \frac{1}{\sqrt{1-\beta^2}} & \frac{\beta}{\sqrt{1-\beta^2}} \\
  \frac{\beta}{\sqrt{1-\beta^2}} & \frac{1}{\sqrt{1-\beta^2}}
\end{pmatrix} \begin{pmatrix}
  x \\
  ct
\end{pmatrix} = \begin{pmatrix}
  \frac{2}{\sqrt{3}} \\
  \frac{1}{\sqrt{3}}
\end{pmatrix} \begin{pmatrix}
  1.155 \\
  0.577
\end{pmatrix} = \begin{pmatrix}
  x \\
  ct
\end{pmatrix}
\]

**Event 0:** Ship passes Main Lighthouse.

- **Lighthouse space:** $x = 0$
- **Lighthouse time:** $ct = 0$

**Event 1:** Ship gets hit by first blink from Main Lighthouse.

- **Ship space:** $x' = 0$
- **Ship time:** $ct' = 0$
- **Main Lighthouse:** $x = -1.00$ $ct = 2.00$

**Event 2:** Main Lighthouse blinks second time.

- **Ship space:** $x' = c \Delta$
- **Ship time:** $ct' = 2\Delta = 2.30$

For: $\beta = \frac{1}{2}$  $e^\rho = \sqrt{3}$

\[
cosh \rho = \frac{1}{\sqrt{1-\beta^2}} = \frac{2}{\sqrt{3}} = 1.155 = \Delta
\]

\[
\sinh \rho = \frac{\beta}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{3}} = 0.577
\]

\[\text{http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=104}\]
Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1st RelativIt animations).
2005 and 2016 animations of lighthouses and ships in \((x,y)\) scenarios and Minkowski \((x,ct)\) plots
Lighthouse \((x,y)\) frame: Dual concentric circular wavefronts serve as timing device
  Ship frame: time dilation \(\Delta=\cosh \rho = 1.15\) of Lighthouse blinks
  Simultaneous events in Lighthouse \((x,y)\) frame: Not so in Ship \((x',y')\) frame
Lighthouse-square \((x,ct)\) plots correlated with Ship-square \((x',ct')\) plots
Overlapped Lighthouse \((x,ct)\) and Ship \((x',ct')\) frame Minkowski plots correlate inconsistencies

\[\rightarrow\] Ship \((x',y')\) frame: Dual un-concentric circular wavefronts map space-time
  Pythagorean derivation of time-dilation factor \(\Delta=\cosh \rho\)
  Un-concentric derivation of stellar aberration \(k\)-angle \(\sigma\)

Per-spacetime 4-vector \((\omega_0,\omega_x,\omega_y,\omega_z) = (\omega,ck_x,ck_y,ck_z)\) transformation
  “Occam-sword” geometry: A pattern recognition aid
  Relating velocity parameter \(\beta=u/c\) to rapidity \(\rho\) to \(k\)-angle \(\sigma\) to \(u/c\)-angle \(\nu\)
  Circular arc-area \(\sigma\) vs. hyperbolic arc-area \(\rho\)
  Each circular trig function has a hyperbolic “country-cousin” function

Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration \(k\)-angle \(\sigma\)
Simple 2015 animations of lighthouses and ships in \((x,y)\) scenarios and Minkowski \((x,ct)\) plots

RelativIt Web Simulation
Relativistic Events in Main Lighthouse’s Frame

\[
\begin{pmatrix}
  x' \\
  ct'
\end{pmatrix} = \begin{pmatrix}
  \cosh \rho & \sinh \rho \\
  \sinh \rho & \cosh \rho
\end{pmatrix}\begin{pmatrix}
  x \\
  ct
\end{pmatrix} = \begin{pmatrix}
  \frac{1}{\sqrt{1-\beta^2}} & \frac{\beta}{\sqrt{1-\beta^2}} \\
  \frac{\beta}{\sqrt{1-\beta^2}} & \frac{1}{\sqrt{1-\beta^2}}
\end{pmatrix}\begin{pmatrix}
  x \\
  ct
\end{pmatrix} = \begin{pmatrix}
  1.155 & 0.577 \\
  0.577 & 1.155
\end{pmatrix}\begin{pmatrix}
  x \\
  ct
\end{pmatrix}
\]

Ship time \(t' = 1.006\)
Ship \(v/c\) (Rel. to Lhsc.) = -0.500
Ship \(v/c\) (Rel. to Obs.) = 0.000
Lhsc \(v/c\) (Rel. to Obs.) = 0.500

\[
\begin{pmatrix}
  x' \\
  ct'
\end{pmatrix} = \begin{pmatrix}
  \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\
  \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}}
\end{pmatrix}\begin{pmatrix}
  x \\
  ct
\end{pmatrix} = \begin{pmatrix}
  -\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}} \\
  -\frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}}
\end{pmatrix} = \begin{pmatrix}
  0 \\
  \sqrt{3}
\end{pmatrix}
\]

for: \(\beta = \frac{1}{2}\) or: \(e^\rho = \sqrt{3}\)
\(\cosh \rho = \frac{1}{\sqrt{1-\beta^2}} = \frac{2}{\sqrt{3}} = 1.155 = \Delta\)
\(\sinh \rho = \frac{\beta}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{3}} = 0.577\)

Event 0:
Ship passes Main Lighthouse.

<table>
<thead>
<tr>
<th>(Lighthouse space)</th>
<th>(x = 0)</th>
<th>(x = -1.00)</th>
<th>(x = 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Lighthouse time)</td>
<td>(ct = 0)</td>
<td>(ct = 2.00)</td>
<td>(ct = 2.00)</td>
</tr>
</tbody>
</table>

Event 1: Ship gets hit by first blink from Main Lighthouse.

<table>
<thead>
<tr>
<th>(Ship space)</th>
<th>(x' = 0)</th>
<th>(x' = 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Ship time)</td>
<td>(ct' = 0)</td>
<td>(ct' = 1.73)</td>
</tr>
</tbody>
</table>

Event 2: Main Lighthouse blinks second time.

<table>
<thead>
<tr>
<th>(Ship space)</th>
<th>(x' = c \Delta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Ship time)</td>
<td>(ct' = 2\Delta = 2.30)</td>
</tr>
</tbody>
</table>

Caution: May be confusing
http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=103

http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=104

Tuesday, April 19, 2016
Simple 2015 animations of lighthouses and ships in \((x,y)\) scenarios and Minkowski \((x,ct)\) plots

\[
\begin{align*}
(x') &= \left( \cosh \rho \sinh \rho \right) (x) \\
(\ct') &= \left( \frac{\beta - 1}{\sqrt{1 - \beta^2}} \right) (\ct)
\end{align*}
\]

RelativIt Web Simulation
Relativistic Events in Main Lighthouse’s Frame

For:
\[ \beta = \frac{1}{2} \]
\[ e^\rho = \sqrt{3} \]

\[
\begin{align*}
\cosh \rho &= \frac{1}{\sqrt{1 - \beta^2}} = 1.155 = \Delta \\
\sinh \rho &= \frac{\beta}{\sqrt{1 - \beta^2}} = 0.577
\end{align*}
\]

Ship Graph
Ref time \( t = 1.15 \) sec.
\( v/c = -0.50 \) litesec/sec.

Ship time \( t' = 1.153 \)
Ship \( v/c \) (Rel.to Lthse.) = -0.500
Ship \( v/c \) (Rel.to Obs.) = 0.000
Lthse \( v/c \) (Rel.to Obs.) = 0.500

\[
\begin{align*}
\left( \begin{array}{c} x' \\ \ct' \end{array} \right) &= \left( \frac{2}{\sqrt{3}} \frac{1}{\sqrt{3}} \right) \left( \begin{array}{c} x \\ \ct \end{array} \right) = \left( \begin{array}{c} 0 \\ \sqrt{3} \end{array} \right)
\end{align*}
\]

Event 0:
Ship passes Main Lighthouse.

Event 1: Ship gets hit by first blink from Main Lighthouse.

Event 2: Main Lighthouse blinks second time.

<table>
<thead>
<tr>
<th>Event 0: Ship passes Main Lighthouse.</th>
<th>Event 1: Ship gets hit by first blink from Main Lighthouse.</th>
<th>Event 2: Main Lighthouse blinks second time.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Lighthouse space) ( x = 0 )</td>
<td>( x = -1.00 )</td>
<td>( x = 0 )</td>
</tr>
<tr>
<td>(Lighthouse time) ( ct = 0 )</td>
<td>( ct = 2.00 )</td>
<td>( ct = 2.00 )</td>
</tr>
<tr>
<td>(Ship space) ( x' = 0 )</td>
<td>( x' = 0 )</td>
<td>( x' = c \Delta )</td>
</tr>
<tr>
<td>(Ship time) ( ct' = 0 )</td>
<td>( ct' = 1.73 )</td>
<td>( ct' = 2 \Delta = 2.30 )</td>
</tr>
</tbody>
</table>

Click & Drag at bottom to control animation speed

\[
\begin{align*}
\left( \begin{array}{c} x' \\ \ct' \end{array} \right) &= \left( \begin{array}{c} 2 \\ 4 \end{array} \right) \frac{1}{\sqrt{3}} = \left( \begin{array}{c} 1.15 \\ 2.30 \end{array} \right)
\end{align*}
\]
Simple 2015 animations of lighthouses and ships in \((x,y)\) scenarios and Minkowski \((x,ct)\) plots

**RelativIt Web Simulation**

**Relativistic Events in Main Lighthouse’s Frame**

\[
\left( x', ct' \right) = \left( \cosh \rho \sinh \rho, \sinh \rho \cosh \rho \right) \left( x, ct \right) = \left( \frac{1 - \beta^2}{\sqrt{1 - \beta^2} \sqrt{1 - \beta^2}}, \frac{\beta}{\sqrt{1 - \beta^2}} \right) \left( x, ct \right) = \left( \frac{2 \frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{3}}}, \frac{2 \frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{3}}} \right) \left( 1.155, 0.577 \right) \left( x, ct \right)
\]

**Ship Graph**

Ref time \( t = 1.51 \) sec.

\( v/c = -0.50 \) litesec/sec.

**Ship time** \( t' = 1.506 \)

Ship \( v/c \) (Rel.to Lthse.) \( = -0.500 \)

Ship \( v/c \) (Rel.to Obs.) \( = 0.000 \)

Lthse \( v/c \) (Rel.to Obs.) \( = 0.500 \)

**Event 0:**

Ship passes Main Lighthouse.

(Lighthouse space) \( x = 0 \)

(Lighthouse time) \( ct = 0 \)

**Event 1:** Ship gets hit by first blink from Main Lighthouse.

(Ship space) \( x' = 0 \)

(Ship time) \( ct' = 0 \)

\( x = 0 \)

(Ship space) \( x' = 0 \)

(Ship time) \( ct' = 1.73 \)

\( c \Delta = 1.155 \)

\[ \frac{2}{\sqrt{3}} = \frac{3}{2} \]

\( \frac{\beta}{\sqrt{1 - \beta^2}} = 0.577 \)

\( \cosh \rho \frac{1}{\sqrt{1 - \beta^2}} = 1.155 \)

\[ \frac{\beta}{\sqrt{1 - \beta^2}} = 0.577 \]

\( e^\rho = \sqrt{3} \)

\( \beta = \frac{1}{2} \)

(for: or:)

\[
\left( \frac{2 \frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{3}}}, \frac{2 \frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{3}}} \right) \left( \frac{0}{2} \right) = \left( \frac{2 \frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{3}}}, \frac{2 \frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{3}}} \right) = \left( \frac{2}{4} \right) \frac{1}{\sqrt{3}} = \left( 1.15 \right)
\]

**Event 2:** Main Lighthouse blinks second time.

(Ship space) \( x' = c \Delta \)

(Ship time) \( ct' = 2 \Delta = 2.30 \)

**Event 1:** Ship gets hit by first blink from Main Lighthouse.

(Lighthouse space) \( x = 0 \)

(Lighthouse time) \( ct = 2.00 \)

**Event 2:** Main Lighthouse blinks second time.

(Ship space) \( x' = 0 \)

(Ship time) \( ct' = 1.73 \)

\( x' = c \Delta \)

\( ct' = 2 \Delta = 2.30 \)
Simple 2015 animations of lighthouses and ships in \((x,y)\) scenarios and Minkowski \((x,ct)\) plots

RelativIt Web Simulation
Relativistic Events in
Main Lighthouse’s Frame

\[
\begin{align*}
\begin{pmatrix}
    x' \\
    ct'
\end{pmatrix} &=\begin{pmatrix}
    \cosh \rho & \sinh \rho \\
    \sinh \rho & \cosh \rho
\end{pmatrix}\begin{pmatrix}
    x \\
    ct
\end{pmatrix} = \begin{pmatrix}
    \frac{1}{\sqrt{1-\beta^2}} & \frac{\beta}{\sqrt{1-\beta^2}} \\
    \frac{\beta}{\sqrt{1-\beta^2}} & \frac{1}{\sqrt{1-\beta^2}}
\end{pmatrix}\begin{pmatrix}
    x \\
    ct
\end{pmatrix} = \begin{pmatrix}
    1.155 \\
    0.577
\end{pmatrix} = \begin{pmatrix}
    x \\
    ct
\end{pmatrix}
\end{align*}
\]

Ship time \(t' = 1.733\)
Ship v/c (Rel. to Lhsse.) = -0.500
Ship v/c (Rel. to Obs.) = 0.000
Lhsse v/c (Rel. to Obs.) = 0.500

Ship Graph
Ref time \(t = 1.733\) sec.
v/c = -0.50 litesec/sec.

\[
\begin{align*}
\begin{pmatrix}
    x' \\
    ct'
\end{pmatrix} &=\begin{pmatrix}
    \frac{2}{3} & \frac{1}{3} \\
    \frac{1}{3} & \frac{2}{3}
\end{pmatrix}\begin{pmatrix}
    0 \\
    2
\end{pmatrix} = \begin{pmatrix}
    \frac{2}{3} \\
    \frac{2}{3}
\end{pmatrix} = \begin{pmatrix}
    0.667 \\
    0.667
\end{pmatrix} = \begin{pmatrix}
    \frac{2}{3} \\
    \frac{2}{3}
\end{pmatrix} = \begin{pmatrix}
    1.15 \\
    2.30
\end{pmatrix}
\end{align*}
\]

**for:** \(\beta = \frac{1}{2}\) \(\ e^\rho = \sqrt{3}\)
\[
\begin{align*}
\cosh \rho &= \frac{1}{\sqrt{1-\beta^2}} = \frac{2}{\sqrt{3}} = 1.155 = \Delta \\
\sinh \rho &= \frac{\beta}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{3}} = 0.577
\end{align*}
\]

**Event 0:**
Ship passes Main Lighthouse.
(Lighthouse space) \(x = 0\)
(Lighthouse time) \(ct = 0\)
(Ship space) \(x' = 0\)
(Ship time) \(ct' = 0\)

**Event 1:** Ship gets hit by first blink from Main Lighthouse.
(Lighthouse space) \(x = -1.00\)
(Lighthouse time) \(ct = 2.00\)
(Ship space) \(x' = 0\)
(Ship time) \(ct' = 1.73\)

**Event 2:** Main Lighthouse blinks second time.
(Lighthouse space) \(x = 0\)
(Lighthouse time) \(ct = 2.00\)
(Space) \(x' = c \Delta\)
(Ship time) \(ct' = 2\Delta = 2.30\)

Click & Drag at bottom to control animation speed.
Simple 2015 animations of lighthouses and ships in \((x,y)\) scenarios and Minkowski \((x,ct)\) plots

\[
\begin{pmatrix}
  x' \\
  ct'
\end{pmatrix}
= \begin{pmatrix}
  \cosh \rho & \sinh \rho \\
  \sinh \rho & \cosh \rho
\end{pmatrix}
\begin{pmatrix}
  x \\
  ct
\end{pmatrix}
= \begin{pmatrix}
  \frac{1}{\sqrt{1-\beta^2}} & \frac{\beta}{\sqrt{1-\beta^2}} \\
  \frac{\beta}{\sqrt{1-\beta^2}} & \frac{1}{\sqrt{1-\beta^2}}
\end{pmatrix}
\begin{pmatrix}
  x \\
  ct
\end{pmatrix}
= \begin{pmatrix}
  1.155 & 0.577 \\
  0.577 & 1.155
\end{pmatrix}
\begin{pmatrix}
  x \\
  ct
\end{pmatrix}
\]

Relativistic Events in Main Lighthouse’s Frame

Ship time \(t' = 2.006\)

Ship \(v/c\) (Rel. to Lhse.) = 0.500
Ship \(v/c\) (Rel. to Obs.) = 0.000
Lhse \(v/c\) (Rel. to Obs.) = 0.500

Ship Graph
Ref time \(t = 2.01\) sec.
\(v/c = -0.50\) litesec/sec.

for:
\[\beta = \frac{1}{2}\]
\[e^\rho = \sqrt{3}\]

\[
cosh \rho = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{3}} = 1.155 = \Delta
\]

\[
sinh \rho = \frac{\beta}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{3}} = 0.577
\]

Event 0:
Ship passes Main Lighthouse.

Event 1:
Ship gets hit by first blink from Main Lighthouse.

Event 2:
Main Lighthouse blinks second time.

<table>
<thead>
<tr>
<th>Event</th>
<th>Description</th>
<th>(Lighthouse space)</th>
<th>(Lighthouse time)</th>
<th>(Ship space)</th>
<th>(Ship time)</th>
<th>(Ship space)</th>
<th>(Ship time)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Ship passes Main Lighthouse.</td>
<td>(x = 0)</td>
<td>(ct = 0)</td>
<td>(x' = 0)</td>
<td>(ct' = 0)</td>
<td>(x = 0)</td>
<td>(ct = 2.00)</td>
</tr>
<tr>
<td>1</td>
<td>Ship gets hit by first blink from Main Lighthouse.</td>
<td>(x = -1.00)</td>
<td>(ct = 2.00)</td>
<td>(x' = 0)</td>
<td>(ct' = 1.73)</td>
<td>(x' = c \Delta)</td>
<td>(ct' = 2\Delta = 2.30)</td>
</tr>
<tr>
<td>2</td>
<td>Main Lighthouse blinks second time.</td>
<td>(x = 0)</td>
<td>(ct = 2.00)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Simple 2015 animations of lighthouses and ships in \((x,y)\) scenarios and Minkowski \((x,ct)\) plots

\[
\begin{pmatrix}
  x' \\
  ct'
\end{pmatrix} = \begin{pmatrix}
  \cosh \rho & \sinh \rho \\
  \sinh \rho & \cosh \rho
\end{pmatrix}\begin{pmatrix}
  x \\
  ct
\end{pmatrix} = \begin{pmatrix}
  \frac{1}{\sqrt{1-\beta^2}} & \frac{\beta}{\sqrt{1-\beta^2}} \\
  \frac{\beta}{\sqrt{1-\beta^2}} & \frac{1}{\sqrt{1-\beta^2}}
\end{pmatrix}\begin{pmatrix}
  x \\
  ct
\end{pmatrix} = \begin{pmatrix}
  1.155 & 0.577 \\
  0.577 & 1.155
\end{pmatrix}\begin{pmatrix}
  x \\
  ct
\end{pmatrix}
\]

\(\text{Ship time } t' = 2.256\)

\((\text{Ship space})\) 
\[x' = 0\]
\((\text{Ship time})\) 
\[ct' = 0\]

\(\text{Ship v/c (Rel. to Lighthouse) } = -0.500\)
\(\text{Ship v/c (Rel. to Obs.) } = 0.000\)
\(\text{Lighthouse v/c (Rel. to Obs.) } = 0.500\)

\(\beta = \frac{1}{2}\) 
\(\rho = \sqrt{3}\)
\(\cosh \rho = \frac{1}{\sqrt{1-\beta^2}} = \frac{2}{\sqrt{3}} = 1.155 = \Delta\)
\(\sinh \rho = \frac{\rho}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{3}} = 0.577\)

\(\text{Event 0: Ship passes Main Lighthouse.}\)
\(\text{Event 1: Ship gets hit by first blink from Main Lighthouse.}\)
\(\text{Event 2: Main Lighthouse blinks second time.}\)

\[
\begin{pmatrix}
  \frac{-2}{\sqrt{3}} \\
  \frac{1}{\sqrt{3}}
\end{pmatrix}\begin{pmatrix}
  0 \\
  2
\end{pmatrix} = \begin{pmatrix}
  \frac{2}{\sqrt{3}} \\
  \frac{2}{\sqrt{3}}
\end{pmatrix} = \begin{pmatrix}
  2 \\
  2\Delta
\end{pmatrix} = \begin{pmatrix}
  1.15 \\
  2.30
\end{pmatrix}
\]

\[\text{Caution: May be confusing}\]

\[\text{http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=104}\]

\[\text{http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=103}\]
Simple 2015 animations of lighthouses and ships in \((x,y)\) scenarios and Minkowski \((x,ct)\) plots

**RelativIt Web Simulation**

**Relativistic Events in Main Lighthouse’s Frame**

\[
\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \cosh \rho & \sinh \rho \\ \sinh \rho & \cosh \rho \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1-\beta^2}} & \frac{\beta}{\sqrt{1-\beta^2}} \\ \frac{\beta}{\sqrt{1-\beta^2}} & \frac{1}{\sqrt{1-\beta^2}} \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} 1.155 & 0.577 \\ 0.577 & 1.155 \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix}
\]

Click & Drag at bottom to control animation speed

**Ship time** \(t' = 2.316\)

**Ship v/c (Rel.to Lhse.)** = -0.500  
**Ship v/c (Rel.to Obs.)** = 0.000  
**Lhse v/c (Rel.to Obs.)** = 0.500

**Ship Graph**

Ref time \(t = 2.32\) sec.  
\(v/c = -0.50\) litesec/sec.

\[
\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \frac{1}{\sqrt{3}} \\ 2 \frac{2}{\sqrt{3}} \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \frac{1}{\sqrt{3}} = \begin{pmatrix} 1.15 \\ 2.30 \end{pmatrix}
\]

**for:**  
\(\beta = \frac{1}{2}\)  
\(e^\rho = \sqrt{3}\)

\[
\cosh \rho = \frac{1}{\sqrt{1-\beta^2}} = \frac{2}{\sqrt{3}} = 1.155 = \Delta
\]

\[
\sinh \rho = \frac{\beta}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{3}} = 0.577
\]

**Event 0:**  
Ship passes Main Lighthouse.

**Event 1:**  
Ship gets hit by first blink from Main Lighthouse.

**Event 2:**  
Main Lighthouse blinks second time.

<table>
<thead>
<tr>
<th>Event 0: Ship passes Main Lighthouse.</th>
<th>Event 1: Ship gets hit by first blink from Main Lighthouse.</th>
<th>Event 2: Main Lighthouse blinks second time.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Lighthouse space) (x = 0)</td>
<td>(x = -1.00)</td>
<td>(x = 0)</td>
</tr>
<tr>
<td>(Lighthouse time) (ct = 0)</td>
<td>(ct = 2.00)</td>
<td>(ct = 2.00)</td>
</tr>
<tr>
<td>(Ship space) (x' = 0)</td>
<td>(x' = 0)</td>
<td>(x' = c \Delta)</td>
</tr>
<tr>
<td>(Ship time) (ct' = 0)</td>
<td>(ct' = 1.73)</td>
<td>(ct' = 2 \Delta = 2.30)</td>
</tr>
</tbody>
</table>

Caution: May be confusing

http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=104

http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=103

Tuesday, April 19, 2016
Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1st RelativIt animations).

2005 and 2016 animations of lighthouses and ships in \((x,y)\) scenarios and Minkowski \((x,ct)\) plots

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➤ Pythagorean derivation of time-dilation factor \(\Delta = \cosh \rho\)
Un-concentric derivation of stellar aberration \(k\)-angle \(\sigma\)

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Relating velocity parameter \(\beta = u/c\) to rapidity \(\rho\) to \(k\)-angle \(\sigma\) to \(u/c\)-angle \(\nu\)
Circular arc-area \(\sigma\) vs. hyperbolic arc-area \(\rho\)
Each circular trig function has a hyperbolic “country-cousin” function

Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration \(k\)-angle \(\sigma\)
RelativeIt Web Simulation
Relativistic Events in Main Lighthouse’s Frame

\[
\left(\begin{array}{c}
\Delta
\end{array}\right) = \left(\begin{array}{c}
\frac{1}{\sqrt{1 - \beta^2}} \\
\frac{\beta}{\sqrt{1 - \beta^2}}
\end{array}\right)
\left(\begin{array}{c}
\frac{1}{\sqrt{1 - \beta^2}} \\
\frac{\beta}{\sqrt{1 - \beta^2}}
\end{array}\right)
\left(\begin{array}{c}
0 \\
1
\end{array}\right) = \left(\begin{array}{c}
\frac{1}{\sqrt{3}} \\
\frac{2}{\sqrt{3}}
\end{array}\right)
\left(\begin{array}{c}
1.155 \\
0.577
\end{array}\right)
\]

Ship time \( t' = 1.153 \)

Ship Graph
Ref time \( t = 1.15 \) sec.
\( \text{v/c} = -0.50 \) litesec/sec.

For:
\( \beta = \frac{1}{2} \)
\( e^\rho = \sqrt{3} \)

\begin{align*}
\cosh \rho &= \frac{1}{\sqrt{1 - \beta^2}} = 1.155 = \Delta \\
\sinh \rho &= \frac{\beta}{\sqrt{1 - \beta^2}} = 0.577
\end{align*}

Lighthouse space \( x = 0 \)
Lighthouse time \( ct = 0 \)
Ship space \( x' = 0 \)
Ship time \( ct' = 0 \)

<table>
<thead>
<tr>
<th>Event 0: Ship passes Main Lighthouse.</th>
<th>Event 1: Ship gets hit by first blink from Main Lighthouse.</th>
<th>Event 2: Main Lighthouse blinks second time.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x = 0 )</td>
<td>( x = -1.00 )</td>
<td>( x = 0 )</td>
</tr>
<tr>
<td>( ct = 0 )</td>
<td>( ct = 2.00 )</td>
<td>( ct = 2.00 )</td>
</tr>
<tr>
<td>( x' = 0 )</td>
<td>( x' = 0 )</td>
<td>( x' = c \Delta )</td>
</tr>
<tr>
<td>( ct' = 0 )</td>
<td>( ct' = 1.73 )</td>
<td>( ct' = 2 \Delta = 2.30 )</td>
</tr>
</tbody>
</table>

Pythagorean derivation of time-dilation factor \( \Delta = \cosh \rho \)
Pythagorean derivation of time-dilation factor $\Delta = \cosh \rho$

RelativIt Web Simulation
Relativistic Events in
Main Lighthouse’s Frame

Ship registers $1^{st}$ Lighthouse Blink
at its

for: $\beta = \frac{1}{2}$
or:
$e^\rho = \sqrt{3}$

$c = \cosh \rho = \frac{1}{\sqrt{1-\beta^2}} = 1.155 = \Delta$

$\sinh \rho = \frac{\beta}{\sqrt{1-\beta^2}} = 0.577$

$Lighthouse space \quad x = 0$
$Lighthouse time \quad ct = 0$

(Ship space) $x' = 0$
(Ship time) $ct' = 0$

Event 0: Ship passes Main Lighthouse.

Event 1: Ship gets hit by first blink from Main Lighthouse.

Event 2: Main Lighthouse blinks second time.

(Lighthouse space) $x = 0$
(Lighthouse time) $ct = 0$

$x = -1.00$
$ct = 2.00$

$x' = 0$
$ct' = 1.73$

$x' = c \Delta$
$ct' = 2\Delta = 2.30$

Tuesday, April 19, 2016
RelativIt Web Simulation
Relativistic Events in Main Lighthouse’s Frame

\[
\begin{pmatrix}
  x' \\
  ct'
\end{pmatrix} = \begin{pmatrix}
  \cosh \rho & \sinh \rho \\
  \sinh \rho & \cosh \rho
\end{pmatrix} \begin{pmatrix}
  x \\
  ct
\end{pmatrix} = \begin{pmatrix}
  \frac{1}{\sqrt{1-\beta^2}} & \frac{\beta}{\sqrt{1-\beta^2}} \\
  \frac{\beta}{\sqrt{1-\beta^2}} & \frac{1}{\sqrt{1-\beta^2}}
\end{pmatrix} \begin{pmatrix}
  x \\
  ct
\end{pmatrix} = \begin{pmatrix}
  \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \\
  \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}}
\end{pmatrix} \begin{pmatrix}
  1.155 \\
  0.577
\end{pmatrix} = \begin{pmatrix}
  x \\
  ct
\end{pmatrix}
\]

Ship time \( t' = 1.153 \)

\[
\Delta = \frac{c^2}{c^2 - v^2} = \frac{1}{\sqrt{1-v^2/c^2}} \]

\[\Delta^2 = \frac{c^2}{c^2 - v^2} = \frac{1}{1-v^2/c^2} \]

\[
\Delta = c \Delta = \frac{c^2}{c^2 - v^2} = \frac{1}{\sqrt{1-v^2/c^2}}
\]

\[
\cosh \rho = \frac{1}{\sqrt{1-\beta^2}} = 1.155 \]

\[
\sinh \rho = \frac{\beta}{\sqrt{1-\beta^2}} = 0.577
\]

Event 0: Ship passes Main Lighthouse.

\[
\begin{align*}
\text{(Lighthouse space)} & \quad x = 0 \\
\text{(Lighthouse time)} & \quad ct = 0 \\
\text{(Ship space)} & \quad x' = 0 \\
\text{(Ship time)} & \quad ct' = 0
\end{align*}
\]

Event 1: Ship gets hit by first blink from Main Lighthouse.

\[
\begin{align*}
\text{(Lighthouse space)} & \quad x = -1.00 \\
\text{(Lighthouse time)} & \quad ct = 2.00 \\
\text{(Ship space)} & \quad x' = 0 \\
\text{(Ship time)} & \quad ct' = 1.73
\end{align*}
\]

Event 2: Main Lighthouse blinks second time.

\[
\begin{align*}
\text{(Lighthouse space)} & \quad x = 0 \\
\text{(Lighthouse time)} & \quad ct = 2.00 \\
\text{(Ship space)} & \quad x' = c \Delta \\
\text{(Ship time)} & \quad ct' = 2 \Delta = 2.30
\end{align*}
\]
Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1st RelativIt animations).  
2005 and 2016 animations of lighthouses and ships in (x,y) scenarios and Minkowski (x,ct) plots

Lighthouse (x,y) frame: Dual concentric circular wavefronts serve as timing device

Ship frame: time dilation $\Delta=\cosh \rho=1.15$ of Lighthouse blinks

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Lighthouse-square (x,ct) plots correlated with Ship-square (x',ct') plots

Overlapped Lighthouse (x,ct) and Ship (x',ct') frame Minkowski plots correlate inconsistencies

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Pythagorean derivation of time-dilation factor $\Delta=\cosh \rho$

Un-concentric derivation of stellar aberration k-angle $\sigma$

Per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) = (\omega,ck_x,ck_y,ck_z)$ transformation

“Occam-sword” geometry: A pattern recognition aid

Relating velocity parameter $\beta=u/c$ to rapidity $\rho$ to k-angle $\sigma$ to u/c-angle $\nu$

Circular arc-area $\sigma$ vs. hyperbolic arc-area $\rho$

Each circular trig function has a hyperbolic “country-cousin” function

Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration k-angle $\sigma$
Relativistic Events in Ship’s Space-Time Frame

\[
\begin{pmatrix}
    x' \\
    ct'
\end{pmatrix} = \begin{pmatrix}
    \cosh \rho & \sinh \rho \\
    \sinh \rho & \cosh \rho
\end{pmatrix} \begin{pmatrix}
    x \\
    ct
\end{pmatrix} = \begin{pmatrix}
    \frac{1}{\sqrt{1-\beta^2}} & \frac{\beta}{\sqrt{1-\beta^2}} \\
    \frac{\beta}{\sqrt{1-\beta^2}} & \frac{1}{\sqrt{1-\beta^2}}
\end{pmatrix} \begin{pmatrix}
    x \\
    ct
\end{pmatrix} = \begin{pmatrix}
    1.155 & 0.577 \\
    0.577 & 1.155
\end{pmatrix} \begin{pmatrix}
    x \\
    ct
\end{pmatrix}
\]

\[c' = \frac{c}{\sqrt{1-\beta^2}} = 1.577\]
\[\sinh \rho = \frac{\beta}{\sqrt{1-\beta^2}} = 0.577\]

\[\cosh \rho = \frac{1}{\sqrt{1-\beta^2}} = 1.155 = \Delta\]

**Event 0:**
Ship passes Main Lighthouse.

**Event 1:** Ship gets hit by first blink from Main Lighthouse.

**Event 2:** Main Lighthouse blinks second time.
RelativIt Web Simulation

Relativistic Events in Ship’s Space-Time Frame

Stellar angle $\sigma$

for: $\beta = \frac{1}{2}$

$e^\rho = \sqrt{3}$

$\cosh \rho = \frac{1}{\sqrt{1 - \beta^2}} = \frac{2}{\sqrt{3}} = 1.155$

$\sinh \rho = \frac{\beta}{\sqrt{1 - \beta^2}} = \frac{1}{\sqrt{3}} = 0.577$

$u c = \frac{1}{2} = \sin \sigma$

2015 animations of lighthouses and ships in $(x,y)$ scenarios and Minkowski $(x,ct)$ plots

$\sigma = 30^\circ$

$(x', ct') = \left( \frac{1}{\sqrt{1 - \beta^2}} \right) \left( \begin{array}{c} x \\ ct \end{array} \right) = \left( \begin{array}{c} \frac{2}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{array} \right) \left( \begin{array}{c} 0 \\ \sqrt{3} \end{array} \right) = \left( \frac{2}{\sqrt{3}} \right)$

Ship Graph

Ref time $t' = -0.012$ sec.

$v/c = -0.50$ litesec/sec.

Ship v/c (Rel to Lhse.) = 0.500

Ship v/c (Rel to Obs.) = 0.000

Lhse v/c (Rel to Obs.) = 0.500

(Lighthouse space) $x = 0$

(Lighthouse time) $ct = 0$

(Ship space) $x' = 0$

(Ship time) $ct' = 0$

for:

$\beta = \frac{1}{2}$

$e^\rho = \sqrt{3}$

$cosh \rho = \frac{1}{\sqrt{1 - \beta^2}} = \frac{2}{\sqrt{3}} = 1.155$

$\sinh \rho = \frac{\beta}{\sqrt{1 - \beta^2}} = \frac{1}{\sqrt{3}} = 0.577$

$u c = \frac{1}{2} = \sin \sigma$

http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=105
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Per-spacetime 4-vector \((\omega_0, \omega_x, \omega_y, \omega_z) = (\omega, ck_x, ck_y, ck_z)\) transformation

(a) Laser frame \(\omega_0\)

\(k(\uparrow)\) \hspace{1cm} \(k(\rightarrow)\) \hspace{1cm} \(k(\downarrow)\)

Suppose starlight in lighthouse frame is straight down x-axis: \(\left(\omega_\downarrow, ck_{x\downarrow}, ck_{y\downarrow}, ck_{z\downarrow}\right) = (\omega_0, -\omega_0, 0, 0)\)
Per-spacetime 4-vector \((\omega_0, \omega_x, \omega_y, \omega_z) = (\omega, ck_x, ck_y, ck_z)\) transformation

(a) Laser frame \(\omega_0\)

\[
\begin{align*}
\mathbf{k}(\uparrow) & \quad \omega_0 \\
\mathbf{k}(\leftarrow) & \quad \omega_0 \\
\mathbf{k}(\rightarrow) & \quad \omega_0 \\
\mathbf{k}(\downarrow) & \quad \omega_0
\end{align*}
\]

along \(-z\)-axis: \((\omega_{\leftarrow}, ck_{\leftarrow x}, ck_{\leftarrow y}, ck_{\leftarrow z}) = (\omega_0, 0, 0, -\omega_0)\)

Suppose starlight in lighthouse frame is straight down \(x\)-axis: \((\omega_{\downarrow}, ck_{\downarrow x}, ck_{\downarrow y}, ck_{\downarrow z}) = (\omega_0, -\omega_0, 0, 0)\)

"South"
Per-spacetime 4-vector \((\omega_0, \omega_x, \omega_y, \omega_z) = (\omega, ck_x, ck_y, ck_z)\) transformation

(a) Laser frame \(\omega_0\)

Suppose starlight in lighthouse frame is straight down x-axis: \(\omega_{\downarrow}, ck_{x\downarrow}, ck_{y\downarrow}, ck_{z\downarrow}\) = \(\omega_0, -\omega_0, 0, 0\)

up +x-axis: \(\omega_{\uparrow}, ck_{x\uparrow}, ck_{y\uparrow}, ck_{z\uparrow}\) = \(\omega_0, +\omega_0, 0, 0\)

along -z-axis: \(\omega_{\leftarrow}, ck_{x\leftarrow}, ck_{y\leftarrow}, ck_{z\leftarrow}\) = \(\omega_0, 0, 0, -\omega_0\)

"North"

"West"

"South"
Per-spacetime 4-vector \((\omega_0, \omega_x, \omega_y, \omega_z) = (\omega, ck_x, ck_y, ck_z)\) transformation

Suppose starlight in lighthouse frame is straight down x-axis: \((\omega_\downarrow, ck_x_\downarrow, ck_y_\downarrow, ck_z_\downarrow) = (\omega_0, -\omega_0, 0, 0)\)
Per-spacetime 4-vector \((\omega_0, \omega_x, \omega_y, \omega_z) = (\omega, ck_x, ck_y, ck_z)\) transformation

(a) Laser frame \(\omega_0\)

\[
\begin{align*}
\omega_0 & \quad k(\uparrow) \\
\omega_0 & \quad k(\leftarrow) \\
\omega_0 k(\rightarrow) & \quad CW \text{ Laser-pair wavevectors}
\end{align*}
\]

(b) z-(→Moving) ship

\[
\begin{align*}
\omega_0 e^\rho & \quad k'(\uparrow) \\
\omega_0 e^{-\rho} & \quad k'(\leftarrow) \\
\omega_0 e^{-\rho} & \quad k'(\rightarrow)
\end{align*}
\]

Suppose starlight in lighthouse frame is straight down x-axis: \(\left(\omega_\downarrow, ck_x\downarrow, ck_y\downarrow, ck_z\downarrow\right) = (\omega_0, -\omega_0, 0, 0)\)

\(+\rho_z\)-rapidity ship frame sees starlight Lorentz transformed to: \(\left(\omega'_\downarrow, ck'_x\downarrow, ck'_y\downarrow, ck'_z\downarrow\right) = (\omega_0 \cosh \rho_z, -\omega_0, 0, -\omega_0 \sinh \rho_z)\)

\[
\begin{pmatrix}
\omega'_\downarrow \\
ck'_x\downarrow \\
ck'_y\downarrow \\
ck'_z\downarrow
\end{pmatrix} =
\begin{pmatrix}
\cosh \rho_z & 0 & -\sinh \rho_z \\
0 & 1 & 0 \\
0 & 0 & 1 \\
-\sinh \rho_z & \cosh \rho_z & 0
\end{pmatrix}
\begin{pmatrix}
\omega_\downarrow \\
ck_x\downarrow \\
ck_y\downarrow \\
ck_z\downarrow
\end{pmatrix} =
\begin{pmatrix}
\cosh \rho_z & 0 & -\sinh \rho_z \\
0 & 1 & 0 \\
0 & 0 & 1 \\
-\sinh \rho_z & \cosh \rho_z & 0
\end{pmatrix}
\begin{pmatrix}
\omega_0 \\
0 \\
0 \\
-\omega_0 \sinh \rho_z
\end{pmatrix} =
\begin{pmatrix}
\omega_0 \cosh \rho_z \\
-\omega_0 \\
0 \\
-\omega_0 \sinh \rho_z
\end{pmatrix}
\]
Per-spacetime 4-vector \((\omega_0, \omega_x, \omega_y, \omega_z) = (\omega, ck_x, ck_y, ck_z)\) transformation

(a) Laser frame \(\omega_0\)  

(b) \(z\)-(→ Moving) ship

Suppose starlight in lighthouse frame is straight down x-axis:  
\[
\left(\omega_{\downarrow}, ck_{x\downarrow}, ck_{y\downarrow}, ck_{z\downarrow}\right) = \left(\omega_0, -\omega_0, 0, 0\right)
\]

+ \(\rho_z\) -rapidity ship frame sees starlight Lorentz transformed to:  
\[
\left(\omega_{\downarrow}', ck_{x\downarrow}', ck_{y\downarrow}', ck_{z\downarrow}'\right) = \left(\omega_0 \cosh \rho_z, -\omega_0, 0, -\omega_0 \sinh \rho_z\right)
\]

Alternative ordering to \((\omega, ck_x, ck_y, ck_z)\):  
\[
\left(\omega_{\downarrow}', ck_{x\downarrow}', ck_{y\downarrow}', ck_{z\downarrow}'\right) = \left(\omega_0 \cosh \rho_z, -\omega_0, 0, -\omega_0 \sinh \rho_z\right)
\]

You can simplify notation by using ordering of 4-vector.

(But, we won’t do that, now)
**Per-spacetime 4-vector** \( (\omega_0, \omega_x, \omega_y, \omega_z) = (\omega, ck_x, ck_y, ck_z) \) transformation

\[
\begin{align*}
(a) \text{ Laser frame } &\quad \omega_0 \quad \omega_0 \quad \omega_0 \quad \omega_0 \\
\text{CW Laser-pair wavevectors} &\quad k(\downarrow) \quad k(\leftarrow) \quad k(\uparrow) \quad k(\rightarrow) \\
\end{align*}
\]

(b) \( z-(\rightarrow \text{Moving}) \) ship

1. **Lighthouse frame**
   - Starlight in lighthouse frame is straight down x-axis: \( (\omega_\downarrow, ck_x\downarrow, ck_y\downarrow, ck_z\downarrow) = (\omega_0, -\omega_0, 0, 0) \)

2. **Ship frame**
   - Starlight Lorentz transformed to: \( (\omega'_\downarrow, ck'_x\downarrow, ck'_y\downarrow, ck'_z\downarrow) = (\omega_0 \cosh \rho_z, -\omega_0, 0, -\omega_0 \sinh \rho_z) \)

\[
\begin{bmatrix}
\omega'_\downarrow \\
ck'_x\downarrow \\
ck'_y\downarrow \\
ck'_z\downarrow
\end{bmatrix}
= \begin{bmatrix}
\cosh \rho_z & -\sinh \rho_z \\
1 & 0 \\
0 & 1 \\
-\sinh \rho_z & \cosh \rho_z
\end{bmatrix}
\begin{bmatrix}
\omega_\downarrow \\
ck_x\downarrow \\
ck_y\downarrow \\
ck_z\downarrow
\end{bmatrix}
= \begin{bmatrix}
\cosh \rho_z & -\sinh \rho_z \\
1 & 0 \\
0 & 1 \\
-\sinh \rho_z & \cosh \rho_z
\end{bmatrix}
\begin{bmatrix}
\omega_0 \\
-\omega_0 \\
0 \\
-\omega_0 \sinh \rho_z
\end{bmatrix}
= \begin{bmatrix}
\omega_0 \cosh \rho_z \\
-\omega_0 \\
0 \\
-\omega_0 \sinh \rho_z
\end{bmatrix}
\]

After the 4-vector transformation, \( \omega_0 = \omega_\downarrow \) is **transverse Doppler shifted** to \( \omega_0 \cosh \rho_z \), while \( ck_z = 0 \) becomes \( ck'_z = -\omega_0 \sinh \rho_z \).

(The \( x \)-component is unchanged: \( ck'_x = -\omega_0 = ck_x \) and so is \( y \)-component: \( ck'_y = 0 = ck_y \).)
Per-spacetime 4-vector \((\omega_0, \omega_x, \omega_y, \omega_z) = (\omega, ck_x, ck_y, ck_z)\) transformation

(a) Laser frame \(\omega_0\) — (Lighthouse frame)
(b) z-(→Moving) ship

Suppose starlight in lighthouse frame is straight down \(x\)-axis : \((\omega_\downarrow, ck_x_\downarrow, ck_y_\downarrow, ck_z_\downarrow) = (\omega_0,-\omega_0,0,0)\)

+ \(\rho_z\) -rapidity ship frame sees starlight Lorentz transformed to :
\[
\begin{pmatrix}
\omega'_\downarrow \\
ck'_{x\downarrow} \\
ck'_{y\downarrow} \\
ck'_{z\downarrow}
\end{pmatrix}

= \begin{pmatrix}
\cosh \rho_z & -\sinh \rho_z \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{pmatrix}
\begin{pmatrix}
\omega_\downarrow \\
c_{kx}_{\downarrow} \\
c_{ky}_{\downarrow} \\
c_{kz}_{\downarrow}
\end{pmatrix}
\]

\[
= \begin{pmatrix}
\cosh \rho_z & -\sinh \rho_z \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{pmatrix}
\begin{pmatrix}
\omega_0 \\
-\omega_0 \\
0 \\
0
\end{pmatrix}
\]

\[
= \begin{pmatrix}
\omega_0 \cosh \rho_z \\
-\omega_0 \\
0 \\
-\omega_0 \sinh \rho_z
\end{pmatrix}
\]

After the 4-vector transformation, \(\omega_0=\omega_\downarrow\) is **transverse Doppler shifted** to \(\omega_0 \cosh \rho_z\), while \(ck_z=0\) becomes \(ck'_z = -\omega_0 \sinh \rho_z\). (The \(x\)-component is unchanged: \(ck'_{x'} = -\omega_0 = ck_x\) and so is \(y\)-component: \(ck'_{y'} = 0 = ck_y\).)
Per-spacetime 4-vector \((\omega_0,\omega_x,\omega_y,\omega_z) = (\omega,ck_x,ck_y,ck_z)\) transformation

(a) Laser frame \(\omega_0\)

(b) z-(Moving) ship

Suppose starlight in lighthouse frame is straight down x-axis: \((\omega_\downarrow,ck_{x\downarrow},ck_{y\downarrow},ck_{z\downarrow}) = (\omega_0,-\omega_0,0,0)\)

\(+\rho_z\) -rapidity ship frame sees starlight Lorentz transformed to: \((\omega'_\downarrow,ck'_{x\downarrow},ck'_{y\downarrow},ck'_{z\downarrow}) = (\omega_0 \cosh \rho_z,-\omega_0,0,-\omega_0 \sinh \rho_z)\)

After the 4-vector transformation, \(\omega_0=\omega_\downarrow\) is transverse Doppler shifted to \(\omega_0 \cosh \rho_z\), while \(ck_z=0\) becomes \(ck'_z = -\omega_0 \sinh \rho_z\).

(The x-component is unchanged: \(ck'_x = -\omega_0 = ck_x\) and so is y-component: \(ck'_y = 0 = ck_y\).)

Recall hyperbolic invariant to Lorentz transform: \(\omega^2-c^2k^2 = \omega'^2-c^2k'^2\) (=0 for 1-CW light)

The 4-vector form of this is: \(\omega^2-c^2k\cdot k = \omega'^2-c^2k'\cdot k'\) (=0 

\[\begin{pmatrix}
\omega'_\downarrow \\
ck'_{x\downarrow} \\
ck'_{y\downarrow} \\
ck'_{z\downarrow}
\end{pmatrix} = \begin{pmatrix}
\cosh \rho_z & 0 & -\sinh \rho_z \\
0 & 1 & 0 \\
-\sinh \rho_z & 0 & \cosh \rho_z
\end{pmatrix} \begin{pmatrix}
\omega_\downarrow \\
ck_{x\downarrow} \\
ck_{y\downarrow} \\
ck_{z\downarrow}
\end{pmatrix} = \begin{pmatrix}
\cosh \rho_z & 0 & -\sinh \rho_z \\
0 & 1 & 0 \\
-\sinh \rho_z & 0 & \cosh \rho_z
\end{pmatrix} \begin{pmatrix}
\omega_0 \\
-\omega_0 \\
0 \\
0
\end{pmatrix} = \begin{pmatrix}
\omega_0 \cosh \rho_z \\
-\omega_0 \\
0 \\
-\omega_0 \sinh \rho_z
\end{pmatrix} = \begin{pmatrix}
\omega_0 \sec \rho_z \\
-\omega_0 \\
0 \\
-\omega_0 \tan \rho_z
\end{pmatrix} \]
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Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration \(k\)-angle \(\sigma\)
Pattern recognition: “Occam’s Sword”

Fig. 5.10 CW cosmic speedometer.
Geometry of Lorentz boost of counter-propagating waves.
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Phase angle $\nu$

$sinh \rho = tan \sigma$

$tanh \rho = sin \sigma = \frac{u}{c} = tan \nu$

$e^\rho = sinh \rho + cosh \rho$

$c = 1$

$\sigma$

$\nu$
Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1st RelativIt animations).

2005 and 2016 animations of lighthouses and ships in \((x,y)\) scenarios and Minkowski \((x,ct)\) plots

Lighthouse \((x,y)\) frame: Dual concentric circular wavefronts serve as timing device

Ship frame: time dilation \(\Delta = \cosh \rho = 1.15\) of Lighthouse blinks

Simultaneous events in Lighthouse \((x,y)\) frame: Not so in Ship \((x',y')\) frame

Lighthouse-square \((x,ct)\) plots correlated with Ship-square \((x',ct')\) plots

Overlapped Lighthouse \((x,ct)\) and Ship \((x',ct')\) frame Minkowski plots correlate inconsistencies

Ship \((x',y')\) frame: Dual un-concentric circular wavefronts map space-time

Pythagorean derivation of time-dilation factor \(\Delta = \cosh \rho\)

Un-concentric derivation of stellar aberration \(k\)-angle \(\sigma\)

Per-spacetime 4-vector \((\omega_0, \omega_x, \omega_y, \omega_z) = (\omega, ck_x, ck_y, ck_z)\) transformation

“Occam-sword” geometry: A pattern recognition aid

Relating velocity parameter \(\beta = u/c\) to rapidity \(\rho\) to \(k\)-angle \(\sigma\) to \(u/c\)-angle \(\nu\)

Circular arc-area \(\sigma\) vs. hyperbolic arc-area \(\rho\)

Each circular trig function has a hyperbolic “country-cousin” function

Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration \(k\)-angle \(\sigma\)
Relating **Longitudinal** relativity parameter: Rapidity \( \rho = \log_e(\text{Doppler Shift}) \)

**to Transverse** relativity parameter: Stellar aberration angle \( \sigma \)

(a) Circular Functions

\[
\begin{align*}
\sin(\sigma) &= 0.6000 \\
\tan(\sigma) &= 0.7500 \\
\sec(\sigma) &= 1.2500
\end{align*}
\]
Relating **Longitudinal** relativity parameter: Rapidity $\rho = \log_e(\text{Doppler Shift})$

to **Transverse** relativity parameter: Stellar aberration angle $\sigma$

$\sinh \rho = \tan \sigma$

$tanh \rho = \sin \sigma = \frac{u}{c} = \tan \nu$

![Diagram showing hyperbolic and circular functions with examples of $\sin(\sigma)$, $\tan(\sigma)$, $\sinh(\rho)$, $\cosh(\rho)$, and $\tanh(\rho)$. The diagram also illustrates the relation between longitudinal and transverse parameters through hyperbolic and trigonometric functions.]
The straight scoop on “angle” and “rapidity” (They’re area!)

\[ y/x = \tanh \theta = \frac{v}{c} \]

-1.0

\[ y = \sinh \rho \]

\[ x = \cosh \rho \]

The “Area” being calculated is the total Gray Area between hyperbola pairs, X axis, and sloping u-line

\[ \text{Area} = \frac{1}{2} \text{base} \cdot \text{altitude} - \text{area under curve} = \frac{1}{2} xy - \int y \, dx \]
The straight scoop on “angle” and “rapidity” (They’re area!)

The “Area” being calculated is the total Gray Area between hyperbola pairs, X axis, and sloping u-line

Useful hyperbolic identities

\[ \text{Area} = \frac{1}{2} \text{base} \uparrow \text{altitude} - \text{area under curve} = \frac{1}{2} xy - \int y \, dx \]

\[ \text{Area} = \frac{1}{2} \sinh \rho \cosh \rho - \int \sinh \rho \, d(\cosh \rho) \]

\[ \sinh^2 \rho = \left( \frac{e^\rho - e^{-\rho}}{2} \right)^2 = \frac{1}{4} (e^{2\rho} + e^{-2\rho} - 2) = \frac{\cosh 2\rho - 1}{2} \]

\[ \sinh \rho \cosh \rho = \left( \frac{e^\rho - e^{-\rho}}{2} \right) \left( \frac{e^\rho + e^{-\rho}}{2} \right) = \frac{1}{4} (e^{2\rho} - e^{-2\rho}) = \frac{1}{2} \sinh 2\rho \]
The straight scoop on “angle” and “rapidity” (They’re area!)

\[ y/x = \tanh \theta = v/c \]

\[ y = \sinh \rho \]
\[ x = \cosh \rho \]

The “Area” being calculated is the total Gray Area between hyperbola pairs, X axis, and sloping u-line

Useful hyperbolic identities

\[ \sinh^2 \rho = \left( \frac{e^\rho - e^{-\rho}}{2} \right)^2 = \frac{1}{4} \left( e^{2\rho} + e^{-2\rho} - 2 \right) = \frac{\cosh 2\rho - 1}{2} \]

\[ \sinh \theta \cosh \theta = \left( \frac{e^\theta - e^{-\theta}}{2} \right) \left( \frac{e^\theta + e^{-\theta}}{2} \right) = \frac{1}{4} \left( e^{2\theta} - e^{-2\theta} \right) = \frac{1}{2} \sinh 2\theta \]

\[ \int \cosh a\rho \, d\rho = \frac{1}{a} \sinh a\rho \]
The straight scoop on “angle” and “rapidity” (They’re area!)

\[
\begin{align*}
\frac{\text{Area}}{2} &= \frac{1}{2} \text{base} \cdot \text{altitude} - \text{area under curve} = \frac{1}{2} xy - \int y \, dx \\
\frac{\text{Area}}{2} &= \frac{1}{2} \sinh \rho \cosh \rho - \int \sinh \rho \, d(\cosh \rho) \\
\frac{\text{Area}}{2} &= \frac{1}{2} \sinh \rho \cosh \rho - \int \sinh^2 \rho \, d\rho = \frac{1}{4} \sinh 2\rho - \int \frac{\cosh 2\rho - 1}{2} \, d\rho \\
&= \frac{1}{4} \sinh 2\rho - \frac{1}{4} \sinh 2\rho + \int \frac{1}{2} \, d\rho \\
&= \frac{\rho}{2}
\end{align*}
\]

**Amazing result:** \( \text{Area} = \rho \) is rapidity

The “Area” being calculated is the **total** Gray Area between hyperbola pairs, \( X \) axis, and sloping u-line

Useful hyperbolic identities

\[
\begin{align*}
\sinh^2 \rho &= \left( \frac{e^\rho - e^{-\rho}}{2} \right)^2 = \frac{1}{4} (e^{2\rho} + e^{-2\rho} - 2) = \frac{\cosh 2\rho - 1}{2} \\
\sinh \rho \cosh \rho &= \left( \frac{e^\rho - e^{-\rho}}{2} \right) \left( \frac{e^\rho + e^{-\rho}}{2} \right) = \frac{1}{4} (e^{2\rho} - e^{-2\rho}) = \frac{1}{2} \sinh 2\rho \\
\int \cosh a\theta \, d\theta &= \frac{1}{a} \sinh a\theta
\end{align*}
\]
Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1st RelativIt animations).

2005 and 2016 animations of lighthouses and ships in \((x,y)\) scenarios and Minkowski \((x,ct)\) plots

Lighthouse \((x,y)\) frame: Dual concentric circular wavefronts serve as timing device

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Un-concentric derivation of stellar aberration \(k\)-angle \(\sigma\)

Per-spacetime 4-vector \((\omega_0,\omega_x,\omega_y,\omega_z) = (\omega,ck_x,ck_y,ck_z)\) transformation

“Occam-sword” geometry: A pattern recognition aid

Relating velocity parameter \(\beta = u/c\) to rapidity \(\rho\) to \(k\)-angle \(\sigma\) to \(u/c\)-angle \(\nu\)

Circular arc-area \(\sigma\) vs. hyperbolic arc-area \(\rho\)

Each circular trig function has a hyperbolic “country-cousin” function

Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration \(k\)-angle \(\sigma\)
Circular Functions

\[ \begin{align*}
\sin(\sigma) &= 0.6000 \\
\tan(\sigma) &= 0.7500 \\
\sec(\sigma) &= 1.2500 \\
\cos(\sigma) &= 0.8000 \\
\cot(\sigma) &= 1.3333 \\
csc(\sigma) &= 1.6667
\end{align*} \]

Hyperbolic Functions

\[ \begin{align*}
\tanh(\rho) &= 0.6000 \\
\sinh(\rho) &= 0.7500 \\
cosh(\rho) &= 1.2500 \\
\sech(\rho) &= 0.8000 \\
csch(\rho) &= 1.3333 \\
\coth(\rho) &= 1.6667 \\
\cot(\sigma) &= csch(\rho) \\
csc(\sigma) &= coth(\rho) \\
\cos(\sigma) &= sech(\rho) \\
tan(\sigma) &= sinh(\rho) = sinh(\rho) \\
sin(\sigma) &= tan(\sigma) = sinh(\rho) \\
sec(\sigma) &= cosh(\rho) \\
\end{align*} \]

RelaWavity Web Simulation
Relating Rapidity and Stellar Abberation
Summary of optical wave parameters for relativity and QM

and their geometry

\[ v' = \omega'/2\pi \]

axis

(Units of 300THz)

An aid to pattern recognition:

**Occam's Sword**

(u/c = 3/5)

RelaWavity Web Simulation

\{perSpace - perTime All\}

Tuesday, April 19, 2016
An aid to pattern recognition:

**Table of 12 wave parameters**

(includes inverses) for relativity

...and values for $u/c=3/5$

RelaWavity Web Simulation
Relativistic Terms (Dual plot w/expanded table)
Fig. 5.5
Relativistic wave mechanics geometry.
(a) Overview.
(b-d) Details of contacting tangents.

(c) Basic construction given \( u/c = 45/53 \)
(d) \( u/c = 3/5 \)
Spectral details of Lorentz boost of North-South-East-West plane-wave 4-vectors \((\omega_0, \omega_x, \omega_y, \omega_z)\)

\[\sigma = 30^\circ = 0.524\]

\[\rho = 0.549\]

\[e^\rho = \sqrt{3}\]

\[e^{-\rho} = 1/\sqrt{3}\]

\[u/c = \sin \sigma = 1/2\]

\[u/c = \tanh \rho = 1/2\]

\[\omega_0 \tan \sigma = \omega_0 \sinh \rho = 1/\sqrt{3}\]

*South* starlight in lighthouse frame is straight down x-axis: \((\omega_\downarrow, ck_\downarrow, ck_\downarrow, ck_\downarrow) = (\omega_0, -\omega_0, 0, 0)\)

+ \(\rho_z\)-rapidity ship frame sees starlight Lorentz transformed to: \((\omega'_\downarrow, ck'_\downarrow, ck'_\downarrow, ck'_\downarrow) = (\omega_0 \cosh \rho_z, -\omega_0, 0, -\omega_0 \sinh \rho_z)\)

\[
\begin{pmatrix}
\omega'_\downarrow \\
ck'_\downarrow \\
ck'_\downarrow \\
ck'_\downarrow
\end{pmatrix}
=
\begin{pmatrix}
\cosh \rho_z & -\sinh \rho_z \\
1 & 0 \\
1 & 0 \\
-\sinh \rho_z & \cosh \rho_z
\end{pmatrix}
\begin{pmatrix}
\omega_\downarrow \\
ck_\downarrow \\
ck_\downarrow \\
ck_\downarrow
\end{pmatrix}
=
\begin{pmatrix}
\cosh \rho_z & -\sinh \rho_z \\
1 & 0 \\
1 & 0 \\
-\sinh \rho_z & \cosh \rho_z
\end{pmatrix}
\begin{pmatrix}
\omega_0 \\
-\omega_0 \\
0 \\
-\omega_0 \sinh \rho_z
\end{pmatrix}
=
\begin{pmatrix}
\omega_0 \cosh \rho_z \\
-\omega_0 \\
0 \\
-\omega_0 \sinh \rho_z
\end{pmatrix}
\begin{pmatrix}
\omega_0 \sec \sigma \\
-\omega_0 \\
0 \\
-\omega_0 \tan \sigma
\end{pmatrix}
\]
Lecture 27 discusses Lorentz boost of North-South-East-West plane-wave 4-vectors \((\omega_0, \omega_x, \omega_y, \omega_z)\).

For ship going \(u = c\) tanh \(\rho\) along \(z\)-axis:

- West starlight \((\omega_0, 0, 0, -\omega_0)\) is blue shifted by \(e^\rho = \cosh \rho + \sinh \rho\):
  \[
  \begin{pmatrix}
  \omega' \\
  c k'_{x} \\
  c k'_{y} \\
  c k'_{z}
  \end{pmatrix} = \omega_0 \begin{pmatrix}
  \cosh \rho_z + \sinh \rho_z \\
  0 \\
  0 \\
  -\sinh \rho_z - \cosh \rho_z
  \end{pmatrix} = \omega_0 \begin{pmatrix}
  e^\rho \\
  0 \\
  0 \\
  -e^\rho
  \end{pmatrix}
  \]

- Blue shift factor is \(e^\rho = \cosh \rho + \sinh \rho = \sec \sigma + \tan \sigma\)

- East starlight \((\omega_0, 0, 0, +\omega_0)\) is red shifted by \(e^{-\rho} = \cosh \rho - \sinh \rho\):
  \[
  \begin{pmatrix}
  \omega' \\
  c k'_{x} \\
  c k'_{y} \\
  c k'_{z}
  \end{pmatrix} = \omega_0 \begin{pmatrix}
  \cosh \rho_z - \sinh \rho_z \\
  0 \\
  0 \\
  -\sinh \rho_z + \cosh \rho_z
  \end{pmatrix} = \omega_0 \begin{pmatrix}
  e^{-\rho} \\
  0 \\
  0 \\
  -e^{-\rho}
  \end{pmatrix}
  \]

- Red shift factor is \(e^{-\rho} = \cosh \rho - \sinh \rho = \sec \sigma - \tan \sigma\)

\[
\begin{align*}
\sigma &= 30^\circ = 0.524 \\
\rho &= 0.549 \\
e^\rho &= \sqrt{3} \\
e^{-\rho} &= 1/\sqrt{3}
\end{align*}
\]
Faster Lorentz boost of North-South-East-West plane-wave 4-vectors \((\omega_0, \omega_x, \omega_y, \omega_z)\)

Lorentz boost by \(\sigma=60^\circ\) or \(e^{+\rho}=2+\sqrt{3}\)

Lighthouse view \((\omega, c\mathbf{k})\) of wave-vectors

Ship-frame view \((\omega', c\mathbf{k}')\) of wave-vectors

\[
\sigma = 60^\circ = 1.047 \\
\rho = 1.317 \\
e^\rho = 2+\sqrt{3} \\
e^{-\rho} = 2-\sqrt{3}
\]

\omega_0 \sin \sigma = \omega_0 \tanh \rho = \omega_0 \sqrt{3/2}

\omega_0 \sec \sigma = \omega_0 \cosh \rho

\omega_0 e^{+\rho} = \omega_0 (2+\sqrt{3})

\omega_0 e^{-\rho} = \omega_0 (2-\sqrt{3})

\omega/c = \sin \sigma = \sqrt{3/2}

\omega/c = \tanh \rho = \sqrt{3/2}

Red shift

Blue shift
Faster Lorentz boost of North-South-East-West plane-wave 4-vectors ($\omega_0, \omega_x, \omega_y, \omega_z$)

\[ \sigma = 60^\circ \text{ or } e^\rho = 2 + \sqrt{3} \]

How does Lorentz boost affect vector of arbitrary $\theta$?

Lighthouse view ($\omega, c\mathbf{k}$) of wave-vectors

Ship-frame view ($\omega', c\mathbf{k}'$) of wave-vectors

\[ u/c = \sin \sigma = \sqrt{3}/2 \]

\[ u/c = \tanh \rho = \sqrt{3}/2 \]

\[ \sigma = 60^\circ = 1.047 \]

\[ \rho = 1.317 \]

\[ e^\rho = 2 + \sqrt{3} \]

\[ e^{-\rho} = 2 - \sqrt{3} \]
Faster Lorentz boost of North-South-East-West plane-wave 4-vectors \((\omega_0, \omega_x, \omega_y, \omega_z)\)

\[ \text{Lorentz boost by } \sigma = 60^\circ \text{ or } e^{i\rho} = 2 + \sqrt{3} \]

How does Lorentz boost affect vector of arbitrary \(\theta\)?

Lighthouse view \((\omega, c\mathbf{k})\) of wave-vectors

Ship-frame view \((\omega', c\mathbf{k}')\) of wave-vectors

Let lab starlight ray at polar angle \(\theta\) have \(\mathbf{k} \uparrow \theta = \omega_0 (1, \cos \theta, 0, -\sin \theta)\). Then ship going \(\mathbf{u}\) along \(z\)-axis sees:

\[
\begin{pmatrix}
\omega'_x \\
\omega'_y \\
\omega'_z
\end{pmatrix} =
\begin{pmatrix}
\cosh \rho_z & -\sinh \rho_z \\
-\sinh \rho_z & \cosh \rho_z \\
1 & 1
\end{pmatrix}
\begin{pmatrix}
\omega_0 \\
\omega_0 \cos \theta \\
0
\end{pmatrix} =
\begin{pmatrix}
\cosh \rho_z + \sinh \rho_z \sin \theta \\
\cos \theta \\
-\sin \rho_z - \cosh \rho_z \sin \theta
\end{pmatrix} =
\begin{pmatrix}
\sec \sigma + \tan \sigma \sin \theta \\
\cos \theta \\
-\tan \sigma - \sec \sigma \sin \theta
\end{pmatrix}
\]

\[ u/c = \sin \sigma = \sqrt{3}/2 \]

\[ u/c = \tanh \rho = \sqrt{3}/2 \]
Faster Lorentz boost of North-South-East-West plane-wave 4-vectors \((\omega_0, \omega_x, \omega_y, \omega_z)\)

\(\text{Lorentz boost by } \sigma = 60^\circ \text{ or } e^{i \rho} = 2 + \sqrt{3}\)

How does Lorentz boost affect vector of arbitrary \(\theta\)?

Lighthouse view \((\omega, c \vec{k})\) of wave-vectors

Ship-frame view \((\omega', c \vec{k}')\) of wave-vectors

Let lab starlight ray at polar angle \(\theta\) have \(\vec{k} \uparrow \theta = \omega_\theta (1, \cos \theta, 0, -\sin \theta)\). Then ship going \(u\) along \(z\)-axis sees:

\[
\begin{pmatrix}
\omega'_{\theta} \\
ck'_{x\uparrow \theta} \\
ck'_{y\uparrow \theta} \\
ck'_{z\uparrow \theta}
\end{pmatrix} =
\begin{pmatrix}
\cosh \rho_z & \cdot & -\sinh \rho_z \\
\cdot & 1 & \cdot \\
\cdot & 1 & \cdot \\
-\sinh \rho_z & \cdot & \cosh \rho_z
\end{pmatrix}
\begin{pmatrix}
\omega_0 \\
0 \\
0 \\
-\omega_0 \sin \theta
\end{pmatrix}
= \omega_0
\begin{pmatrix}
\cosh \rho_z + \sinh \rho_z \sin \theta \\
\omega_0 \cos \theta \\
0 \\
-\sinh \rho_z - \cosh \rho_z \sin \theta
\end{pmatrix}
= \omega_0
\begin{pmatrix}
\sec \sigma + \tan \sigma \sin \theta \\
\cos \theta \\
0 \\
-\tan \sigma - \sec \sigma \sin \theta
\end{pmatrix}
Faster Lorentz boost of North-South-East-West plane-wave 4-vectors ($\omega_0, \omega_x, \omega_y, \omega_z$)

Lorentz boost by $\sigma = 60^\circ$ or $e^{i\rho} = 2 + \sqrt{3}$

How does Lorentz boost affect vector of arbitrary $\theta$?

Let lab starlight ray at polar angle $\theta$ have $\mathbf{k}^\uparrow \theta = \omega_0 (1, \cos \theta, 0, -\sin \theta)$. Then ship going $\mathbf{u}$-along-$z$-axis sees:

$$
\begin{pmatrix}
\omega'_{\theta} \\
ck'_{x\theta} \\
ck'_{y\theta} \\
ck'_{z\theta}
\end{pmatrix}
= 
\begin{pmatrix}
\cosh \rho_z & \cdot & -\sinh \rho_z \\
\cdot & 1 & \cdot \\
-\sinh \rho_z & \cdot & \cosh \rho_z
\end{pmatrix}
\begin{pmatrix}
\omega_0 \\
\omega_0 \cos \theta \\
0
\end{pmatrix}
= 
\omega_0
\begin{pmatrix}
\cosh \rho_z + \sinh \rho_z \sin \theta \\
\omega_0 \cos \theta \\
-\omega_0 \sin \theta
\end{pmatrix}
= 
\omega_0
\begin{pmatrix}
\sec \sigma + \tan \sigma \sin \theta \\
\cos \theta \\
0
\end{pmatrix}
= 
\omega_0
\begin{pmatrix}
\sec \sigma + \tan \sigma \sin \theta \\
\cos \theta \\
0
\end{pmatrix}
\begin{pmatrix}
\cos \theta \\
0 \\
-\tan \sigma - \sec \sigma \sin \theta
\end{pmatrix}
$$

Lighthouse view ($\omega, c\mathbf{k}$) of wave-vectors

Ship-frame view ($\omega', c\mathbf{k}')$ of wave-vectors

Faster Lorentz boost of North-South-East-West plane-wave 4-vectors ($\omega_0, \omega_x, \omega_y, \omega_z$)
Faster Lorentz boost of North-South-East-West plane-wave 4-vectors \((\omega_0,\omega_x,\omega_y,\omega_z)\)

Lorentz boost by \(\sigma=60^\circ\) or \(e^{i\rho}=2+\sqrt{3}\)

How does Lorentz boost affect vector of arbitrary \(\theta\)?

Let lab starlight ray at polar angle \(\theta\) have \(\mathbf{k} \uparrow \theta = \omega_0 (1,\cos \theta,0,-\sin \theta)\). Then ship going \(\mathbf{u}\) along \(z\)-axis sees:

\[
\begin{align*}
\left(\begin{array}{c}
\omega'_x \theta \\
\omega'_y \theta \\
\omega'_z \theta \\
\end{array}\right) &= \left(\begin{array}{ccc}
cosh \rho_z & -\sinh \rho_z & \omega_0 \\
1 & 0 & \omega_0 \cos \theta \\
-\sinh \rho_z & \cosh \rho_z & -\omega_0 \sin \theta \\
\end{array}\right) \\
&= \omega_0 \left(\begin{array}{ccc}
cosh \rho_z + \sinh \rho_z \sin \theta & \cos \theta & 0 \\
\cos \theta & 0 & 0 \\
-\sinh \rho_z - \cosh \rho_z \sin \theta & 0 & -\tan \sigma - \sec \sigma \sin \theta \\
\end{array}\right)
\end{align*}
\]
Faster Lorentz boost of North-South-East-West plane-wave 4-vectors \((\omega_0,\omega_x,\omega_y,\omega_z)\)

\[ \omega_{\sec\sigma}\sin\theta = \omega_{\tan\sigma} \]

\[ \omega_{\sec\sigma} = \omega_0 \cosh \rho \]

\[ u/c = \sin \sigma = \sqrt{3}/2 \]

\[ u/c = \tanh \rho = \sqrt{3}/2 \]

\[ \rho = 1.317 \]

\[ e^{\rho} = 2 + \sqrt{3} \]

\[ e^{-\rho} = 2 - \sqrt{3} \]

\[ \omega_{\sec\sigma} = \omega_0 \cos \theta \]

\[ \cos \theta \]

\[ \omega_0 \tan \sigma \]

\[ 0 \]

\[ \omega_0 \sec \sigma \sin \theta \]

\[ \cos \theta \]

\[ 0 \]

\[ \omega_0 \sec \sigma + \tan \sigma \sin \theta \]

\[ \omega_0 \cosh \rho \]

\[ \omega_0 \cos \theta \]

\[ 0 \]

\[ \omega_0 \sec \sigma \sin \theta \]

\[ \cos \theta \]

\[ 0 \]

\[ \omega_0 \tan \sigma \]

\[ 0 \]

\[ -\tan \sigma - \sec \sigma \sin \theta \]

\[ \omega_0 \sec \sigma \sin \theta \]

\[ \cos \theta \]

\[ 0 \]

\[ -\tan \sigma - \sec \sigma \sin \theta \]

Let lab starlight ray at polar angle \(\theta\) have \(k^\uparrow\theta = \omega_0 (1, \cos \theta, 0, -\sin \theta)\). Then ship going \(u\) along \(z\)-axis sees:

\[
\begin{pmatrix}
\omega'_{\uparrow\theta} \\
ck'_{x\uparrow\theta} \\
ck'_{y\uparrow\theta} \\
ck'_{z\uparrow\theta}
\end{pmatrix} = \begin{pmatrix}
\cosh \rho_z & -\sinh \rho_z & 0 & 0 \\
0 & 1 & 0 & 0 \\
-\sinh \rho_z & \cosh \rho_z & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix} \begin{pmatrix}
\omega_0 \\
0 \\
-\omega_0 \sin \theta \\
0
\end{pmatrix} = \begin{pmatrix}
\cosh \rho_z + \sinh \rho_z \sin \theta \\
\omega_0 \cos \theta \\
-\sinh \rho_z - \cosh \rho_z \sin \theta \\
0
\end{pmatrix} = \begin{pmatrix}
\sec \sigma + \tan \sigma \sin \theta \\
\cos \theta \\
0 \\
-\tan \sigma - \sec \sigma \sin \theta
\end{pmatrix}
\]
Review of geometric construction, per-space-time \((\omega, ck)\) dispersion hyperbola \(\omega = B \cosh \rho\).

A quick flip to space-time \((ct, x)\) construction: Minkowski coordinate grid.

Lorentz transformations of Phase vector \(P'\) and Group vector \(G'\) in per-space-time.

Lorentz matrix transformation of \((x, ct)\) space-time coordinates.

Two Famous-Name Coefficients: Lorentz space contraction and Einsein time dilation.

Heighway Paradoxes: A relativistic “He said-She-said...” argument.

Phase invariance...derives Lorentz transformations...and vice-versa.

Another view of phasor-invariance.

Geometry of invariant hyperbolas.

Algebra of invariant hyperbolas.

Proper time \(\tau_0\) and proper frequency \(\omega_0\).

A politically incorrect analogy of rotation to Lorentz transformation.

Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration angle \(\sigma\).

Relating rapidity \(\rho\) to stellar aberration angle \(\sigma\) and circular or hyperbolic arc-area.

Each circular trig function has a hyperbolic “country-cousin” function.

Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1\(^{st}\) RelativIt animations).
Comparing Longitudinal relativity parameter: Rapidity $\rho = \log_e$ (Doppler Shift) to Transverse relativity parameter: Stellar aberration angle $\sigma$.


Observer fixed below star sees it directly overhead. Observer going $u$ sees star at angle $\sigma$ in $u$ direction.

Stellar aberration angle $\sigma$:

$$c \tanh \rho = u = c \sin \sigma$$

We used notion $\sigma$ for stellar-ab-angle, (a “flipped-out” $\rho$).

Epstein seemed uninterested in $\rho$ analysis or in relation of $\sigma$ and $\rho$.
Relating **Longitudinal** relativity parameter: \( \text{Rapidity } \rho = \log_e(\text{Doppler Shift}) \)

to **Transverse** relativity parameter: Stellar aberration angle \( \sigma \)


**Proper time** \( c\tau \) vs. coordinate space \( x \) - (L. C. Epstein’s “Cosmic Speedometer”)  
Particles \( P \) and \( P' \) have speed \( u \) in \( (x',ct') \) and speed \( c \) in \( (x, c\tau) \)

**Proper time** \( C\tau \)

\[
c\tau = \sqrt{(ct')^2 - (x')^2}
\]

**Coordinate** \( x' = (u/c)ct' = ut' \)

**Einstein time dilation:**

\[
ct' = c\tau \sec\sigma = c\tau \cosh\rho = c\tau/\sqrt{1-u^2/c^2}
\]

**Lorentz length contraction:**

\[
L' = L \sech\rho = L\cos\sigma = L \cdot \sqrt{1-u^2/c^2}
\]

**Proper Time simultaneity:**

\[
c \Delta\tau = L' \sinh\rho = L \cos\sigma \sinh\rho = L \cos\sigma \tan\sigma = L \sin\sigma = L/\sqrt{c^2/u^2-1} \sim L u/c
\]
Relating **Longitudinal** relativity parameter: Rapidity $\rho = \log_e(\text{Doppler Shift})$

to **Transverse** relativity parameter: Stellar aberration angle $\sigma$


**Proper time** $c\tau$ vs. coordinate space $x$ - (L. C. Epstein’s “Cosmic Speedometer”)

Particles $P$ and $P'$ have speed $u$ in $(x',ct')$ and speed $c$ in $(x,c\tau)$

Proper time $c\tau$

$ct = \sqrt{(ct')^2 -(x')^2}$

Coordinate $x' = (u/c)ct' = ut'$

Einstein time dilation:

$ct' = c\tau \sec \sigma = c\tau \cosh \rho = c\tau / \sqrt{1-u^2/c^2}$

Lorentz length contraction:

$L' = L \sech \rho = L \cos \sigma = L \cdot \sqrt{1-u^2/c^2}$

Proper Time asimultaneity:

$c \Delta \tau = L' \sinh \rho = L \cos \sigma \sinh \rho$

$= L \cos \sigma \tan \sigma$

$= L \sin \sigma = L / \sqrt{c^2/u^2-1} \sim L u/c$

Epstein’s trick is to turn a hyperbolic form $c\tau = \sqrt{(ct')^2 -(x')^2}$ into a circular form:

$\sqrt{(c\tau)^2 + (x')^2} = (ct')$

Then everything (and everybody) always goes speed $c$ through $(x',c\tau)$ space!
Geometry of invariant hyperbolas

**Euclid’s 3-means (300 BC)**
Geometric “heart” of wave mechanics

**Thales (580 BC) rectangle-in-circle**
Relates to wave interference by (Galilean) phasor angular velocity addition

---

**Figure 10a** Euclidian mean geometry for counter-moving waves of frequency 1 and 4. (300THz units).

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**Pirelli Site animations:**
http://www.uark.edu/ua/pirelli/php/means_1.php
http://www.uark.edu/ua/pirelli/php/half_sum_2.php
Geometry of invariant hyperbolas

**Euclidian wave geometry with time-reversal symmetry imply dispersion hyperbolas:** \( \omega = nB \cosh \rho \)

Time \( r=1/b \) symmetry shows geometry of 2-CW grid transformation that leaves hyperbolas invariant.

http://www.uark.edu/ua/modphys/markup/RelaWavityWeb.html?plotType=3l5&minkGridPosCells=2
Algebra of invariant hyperbolas: Proper time $\tau_0$ and proper frequency $\omega_0$

\[
\begin{pmatrix}
ck \\
\omega
\end{pmatrix} = \begin{pmatrix}
\cosh \rho & \sinh \rho \\
\sinh \rho & \cosh \rho
\end{pmatrix} \begin{pmatrix}
ck' \\
\omega'
\end{pmatrix}
\]

\[
\begin{pmatrix}
x \\
ct
\end{pmatrix} = \begin{pmatrix}
\cosh \rho & \sinh \rho \\
\sinh \rho & \cosh \rho
\end{pmatrix} \begin{pmatrix}
x' \\
ct'
\end{pmatrix}
\]

Hyperbolic invariants to Lorentz transformation

Per-space-time invariant:

\[
\omega_0^2 = \omega^2 - (ck)^2 = \omega'^2 - (ck')^2
\]

$\omega_0$ is called “proper frequency” or rate of “aging”

\[
\omega_0 = \omega \sqrt{1- \frac{k^2}{c^2}} = \omega' \sqrt{1- \frac{k'^2}{c^2}}
\]

\[
= \omega \sqrt{1- \frac{u^2}{c^2}} = \omega' \sqrt{1- \frac{u'^2}{c^2}}
\]

$\omega_0$ is called “proper frequency” or rate of “aging”

Space-time invariant:

\[
(c\tau_0)^2 = (ct)^2 - x^2 = (ct')^2 - (x')^2
\]

$\tau_0$ is called “proper time” or “age”:

\[
\tau_0 = t \sqrt{1- \frac{x^2}{(ct)^2}} = t' \sqrt{1- \frac{x'^2}{(ct')^2}}
\]

\[
= t \sqrt{1- \frac{u^2}{c^2}} = t' \sqrt{1- \frac{u'^2}{c^2}}
\]

The “grand-daddy-of ‘em all” invariant

Phase invariance:

\[
\Phi_0 = k \cdot x - \omega \cdot t = k' \cdot x' - \omega' \cdot t'
\]

Proof:

\[
ck \cdot x' - \omega' \cdot ct' = ck \cdot x - \omega \cdot ct
\]

\[
ck \cdot \cosh - \omega \cdot \sinh = \omega' \cdot \cosh - ct' \cdot \sinh
\]

\[
ck \cdot \cosh^2 - ck \cdot x \cdot \sinh = ck \cdot x
\]

\[
ck \cdot x \cdot \cosh^2 - \omega \cdot ct \cdot \sinh = -\omega \cdot ct
\]
A politically incorrect analogy of rotation to Lorentz transformation

Fig. 2.B.1 Town map according to a "tipsy" surveyor.

Fig. 2.B.2 Diagram and formulas for reconciliation of the two surveyor's data.

Reminder: Component-based derivation is clumsy!

Circular invariants $r^2 = x^2 + y^2$

You may apply (Jacobian) transform matrix:

$$
\begin{pmatrix}
\langle x | x' \rangle & \langle y | x' \rangle \\
\langle y | y' \rangle & \langle y | y' \rangle
\end{pmatrix} = \begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
$$

or the inverse (Kajobian) transformation:

$$
\begin{pmatrix}
\langle x' | x \rangle & \langle x' | y \rangle \\
\langle y' | x \rangle & \langle y' | y \rangle
\end{pmatrix} = \begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix}
$$

to any vector $\mathbf{V} = |V\rangle = |x\rangle\langle x|V\rangle + |y\rangle\langle y|V\rangle$

$$
=x\langle x'|V\rangle + y\langle y'|V\rangle
$$
(a) Rotation Transformation and Invariants

\[
x = 1.65 \\
y = -0.85 \\
x^2 + y^2 = 3.43 \\
x' = 1.00 \\
y' = -1.56 \\
x'^2 + y'^2 = 3.43
\]

\[
\gamma = 0 \\
\theta = 0 \\
\theta + \theta' = 0.5236
\]

\[
x' = x \cos \theta - y \sin \theta = \frac{x}{\sqrt{1 + \frac{b^2}{c^2}}} + \frac{- \left( \frac{b}{c} \right) y}{\sqrt{1 + \frac{b^2}{c^2}}}
\]

\[
y' = x \sin \theta + y \cos \theta = \frac{\left( \frac{b}{c} \right) x}{\sqrt{1 + \frac{b^2}{c^2}}} + \frac{y}{\sqrt{1 + \frac{b^2}{c^2}}}
\]

(b) Lorentz Transformation and Invariants

\[
x = 1.5453 \\
ct = 0.9819 \\
x^2 - (ct)^2 = 1.42 \\
x' = 2.3512 \\
ct' = 2.0260 \\
x'^2 - (ct')^2 = 1.42
\]

\[
\gamma = 0.5493 \\
\theta = 0 \\
\theta + \theta' = -0.5493
\]

\[
x' = \frac{x}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{\frac{v}{c} ct}{\sqrt{1 - \frac{v^2}{c^2}}} = x \cosh \rho + y \sinh \rho
\]

\[
ct' = \frac{\frac{v}{c} x}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{ct}{\sqrt{1 - \frac{v^2}{c^2}}} = x \sinh \rho + y \cosh \rho
\]
**Light-cone-sections are hyperbolas**

*Main Lighthouse on (x=0,y=0) time line*

*North Lighthouse-1 on (x=0,y=1) time line*

*Fig. 2.B.5 Space-Space-Time plot of world lines for Lighthouses. North Lighthouse blink waves trace light cones.*