Lecture 25 Relawavity Introduction 3
Tuesday 4.12.2016

Relawavity: Relativistic wave mechanics III. 2nd-order effects

(Unit 3 4.12.16)

➔ Review: rapidity $\rho=\rho_{AB}$, Doppler shifts $e^{\pm\rho}$, and SR velocity parameter $V_{group}/c=\beta_{AB}=u_{AB}/c=\tanh\rho_{AB}$

Geometric construction steps 1-4 : 1-octave Doppler ($e^{+\rho}=2$, $e^{-\rho}=½$), ($\beta_{AB}=u_{AB}/c=3/5$)

Reviewing wave coefficients we'll need to know (backwards and forwards)

Comparison of group and phase dynamics: *FAST(er)* ($\beta=u/c=3/5$) *vs* SLOW(er) ($\beta=u/c=1/5$)

Thales Mean Geometry (*Thales of Miletus 624-543 BCE*) and its role in Relawavity

Geometric construction steps 5,6,...: Per-space-time (ω,ck) dispersion hyperbola ω = B$\cosh\rho$...

A quick flip to space-time *(ct,x)* construction: Minkowski coordinate grid

Lorentz transformations of Phase vector **P′** and Group vector **G′** in per-space-time

Lorentz matrix transformation of *(x,ct)* space-time coordinates

Two Famous-Name Coefficients: Lorentz space contraction and Einsein time dilation

Heighway Paradoxes: A relativistic "*He said-She-said*" argument

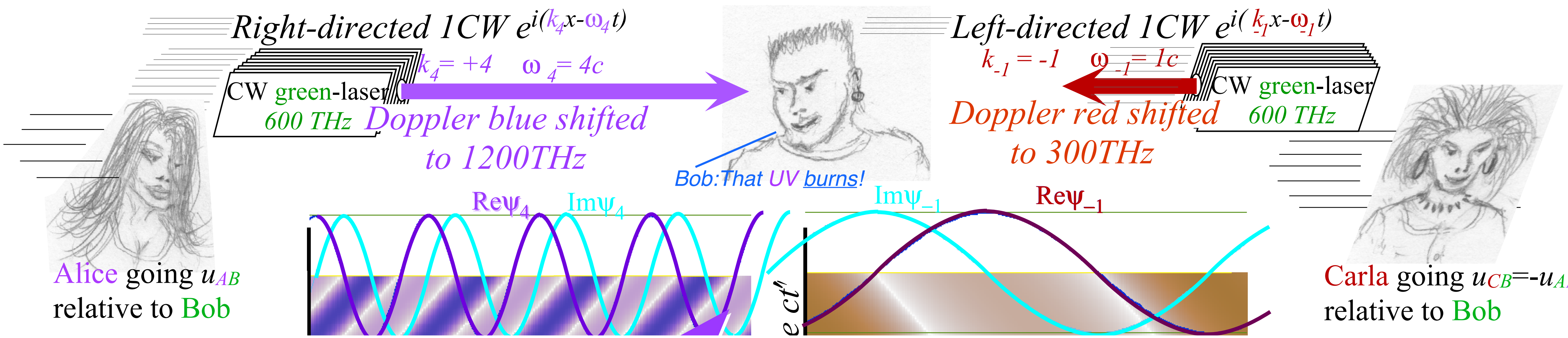
Phase invariance…derives Lorentz transformations

Another view: phas*or*-invariance and proper time

Yet another view: The Epstein space-proper-time approach to SR

Review: rapidity $\rho=\rho_{AB}$, Dopplers $e^{\pm\rho}$, and velocity $\beta_{AB}=u_{AB}/c=\tanh\rho$

Imagine Bob sees a pair of counter-propagating laser beams with wavevectors $k_R=+\omega_R/c$ and $k_L=-\omega_L/c$
$\omega_R=\omega_A$ going left-to-right (from Alice's$_{600\text{THz}}$laser) and $\omega_L=\omega_C$ going right-to-left (from Carla's$_{600\text{THz}}$laser).



We ask two questions:

(1.) To what velocity u_E must Bob accelerate so he sees beams with equal frequency ω_E ?

(2.) What is that frequency ω_E ?

Reply to Query (1.) has a *Jeopardy*-style answer-by-question:
What is the beam group velocity?

Given: $\omega_{group}=\dfrac{\omega_R-\omega_L}{2}$ and: $k_{group}=\dfrac{k_R-k_L}{2}$

$$u_E=V_{group}=\frac{\omega_{group}}{k_{group}}=\frac{\omega_R-\omega_L}{k_R-k_L}=c\,\frac{\omega_R-\omega_L}{\omega_R+\omega_L}=c\,\frac{1200-300}{1200+300}=\frac{3}{5}c$$

with $k_R=+\omega_R/c$ and $k_L=-\omega_L/c$

$$\frac{u_E}{c}=\frac{u_{AB}}{c}=\frac{e^{\rho_{AB}}-e^{-\rho_{AB}}}{e^{\rho_{AB}}+e^{-\rho_{AB}}}=\frac{\sinh\rho_{AB}}{\cosh\rho_{AB}}=\tanh\rho_{AB}=\frac{3}{5}$$

Using *Rapidity*: $\rho_{AB}=\log_e\langle A|B\rangle$

Given: $\omega_R=e^{\rho_{AB}}\omega_{600}$

and: $\omega_L=e^{\rho_{CB}}\omega_{600}=e^{-\rho_{AB}}\omega_{600}$

Reply to Query (2.) in similar style:
What ω_E is blue-shift $b\omega_L$ of ω_L <u>and</u> red-shift ω_R/b of ω_R ?

Blue-shift $b=e^{\rho_{AB}}$

Red-shift $r=b^{-1}=e^{-\rho_{AB}}$

$$\omega_E=b\,\omega_L=\omega_R/b \quad\Rightarrow\quad b=\sqrt{\omega_R/\omega_L} \quad\Rightarrow\quad \omega_E=\sqrt{\omega_R\cdot\omega_L}=\sqrt{1200\cdot 300}=600THz$$

(Geometric Mean)

Review: rapidity $\rho=\rho_{AB}$, Doppler shifts $e^{\pm\rho}$, and SR velocity parameter $V_{group}/c=\beta_{AB}=u_{AB}/c=\tanh\rho_{AB}$

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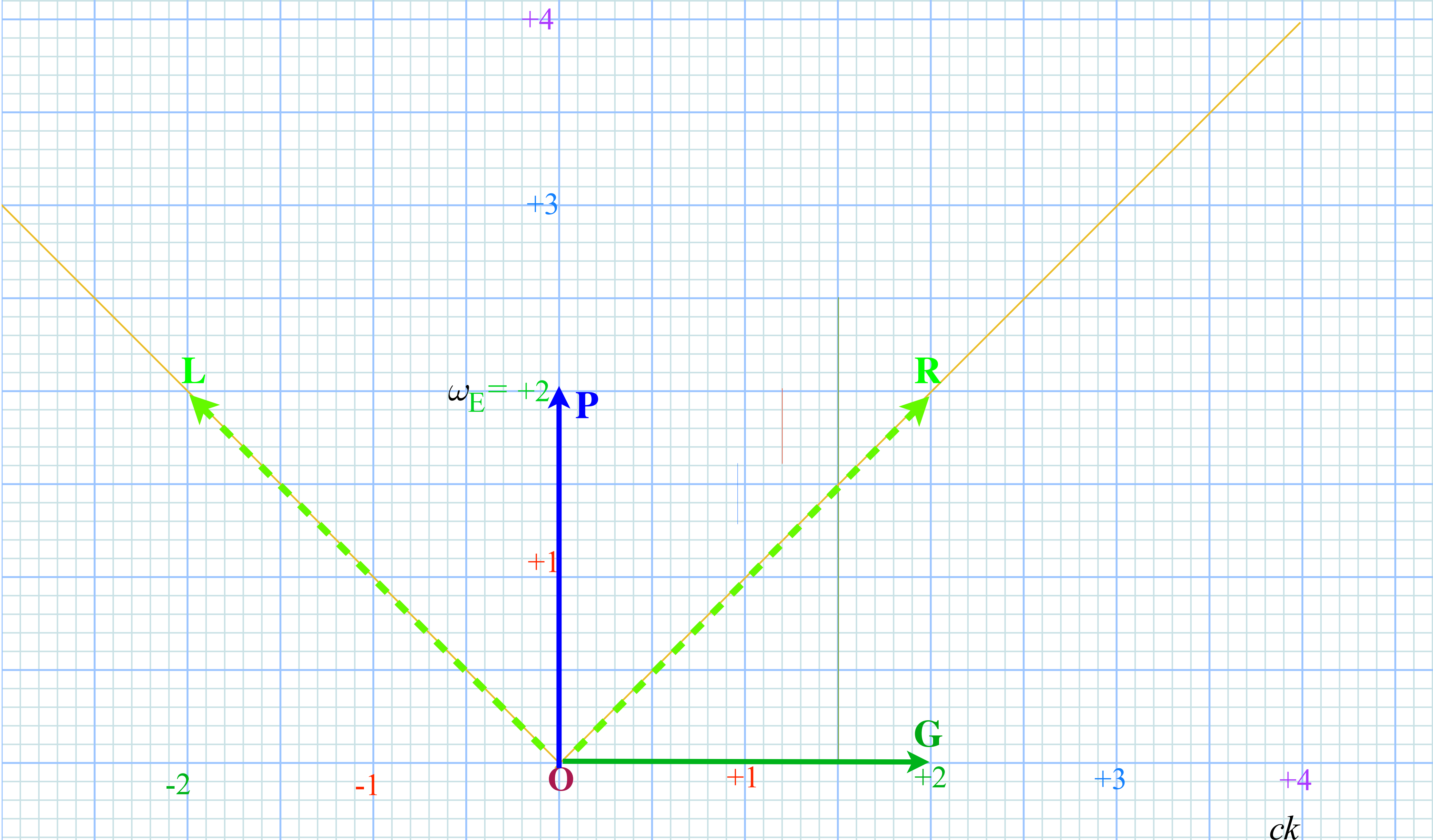

Angular 2π-factors

$$k_A=2\pi\kappa_A$$
$$\omega_A=2\pi\upsilon_A$$

$$k_{phase}=2\pi\kappa_{phase}$$
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$$k_{group}=2\pi\kappa_{group}$$
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$$\mathbf{P}=\tfrac{1}{2}(\mathbf{R}+\mathbf{L})$$

$$\mathbf{G}=\tfrac{1}{2}(\mathbf{R}-\mathbf{L})$$

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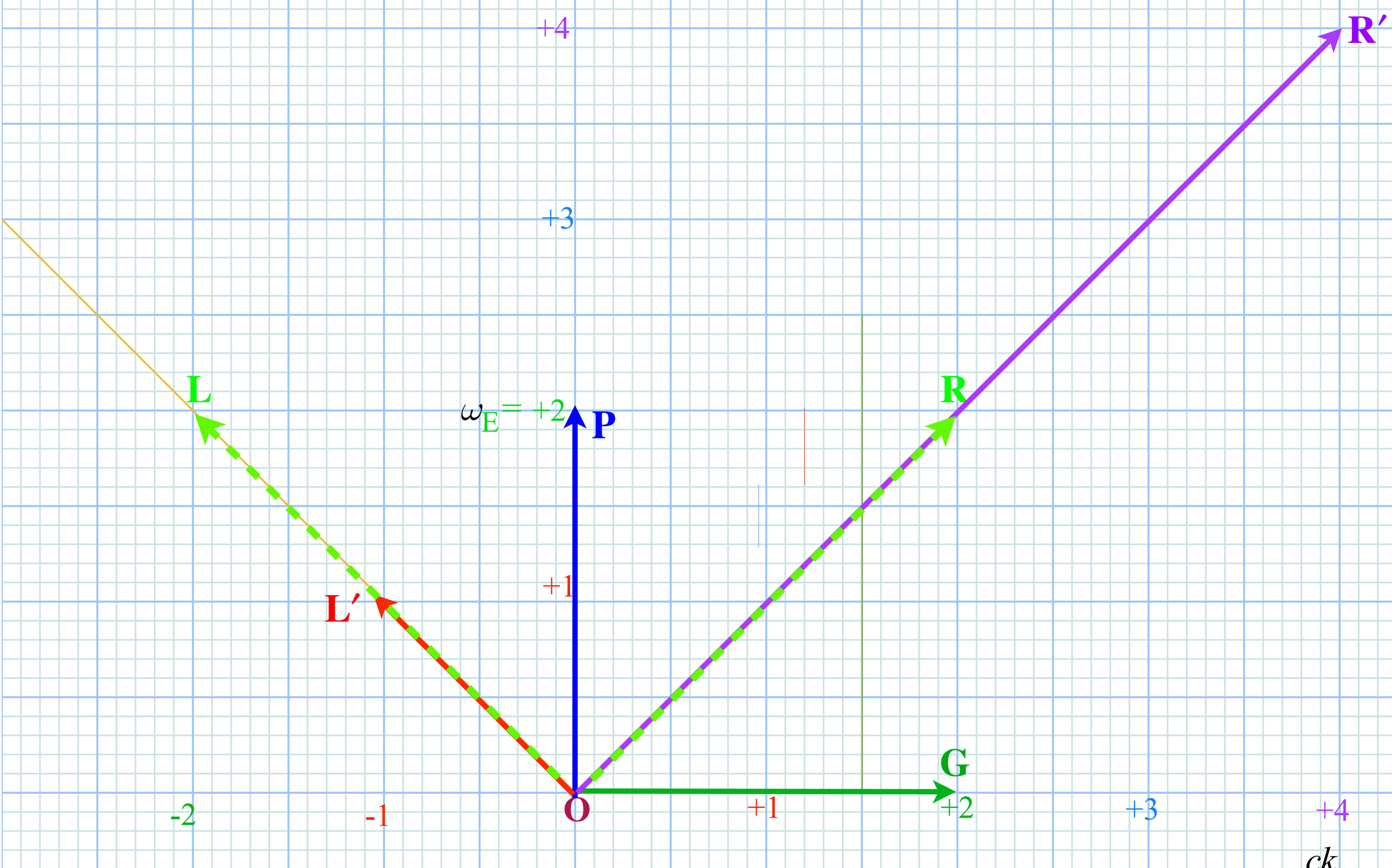

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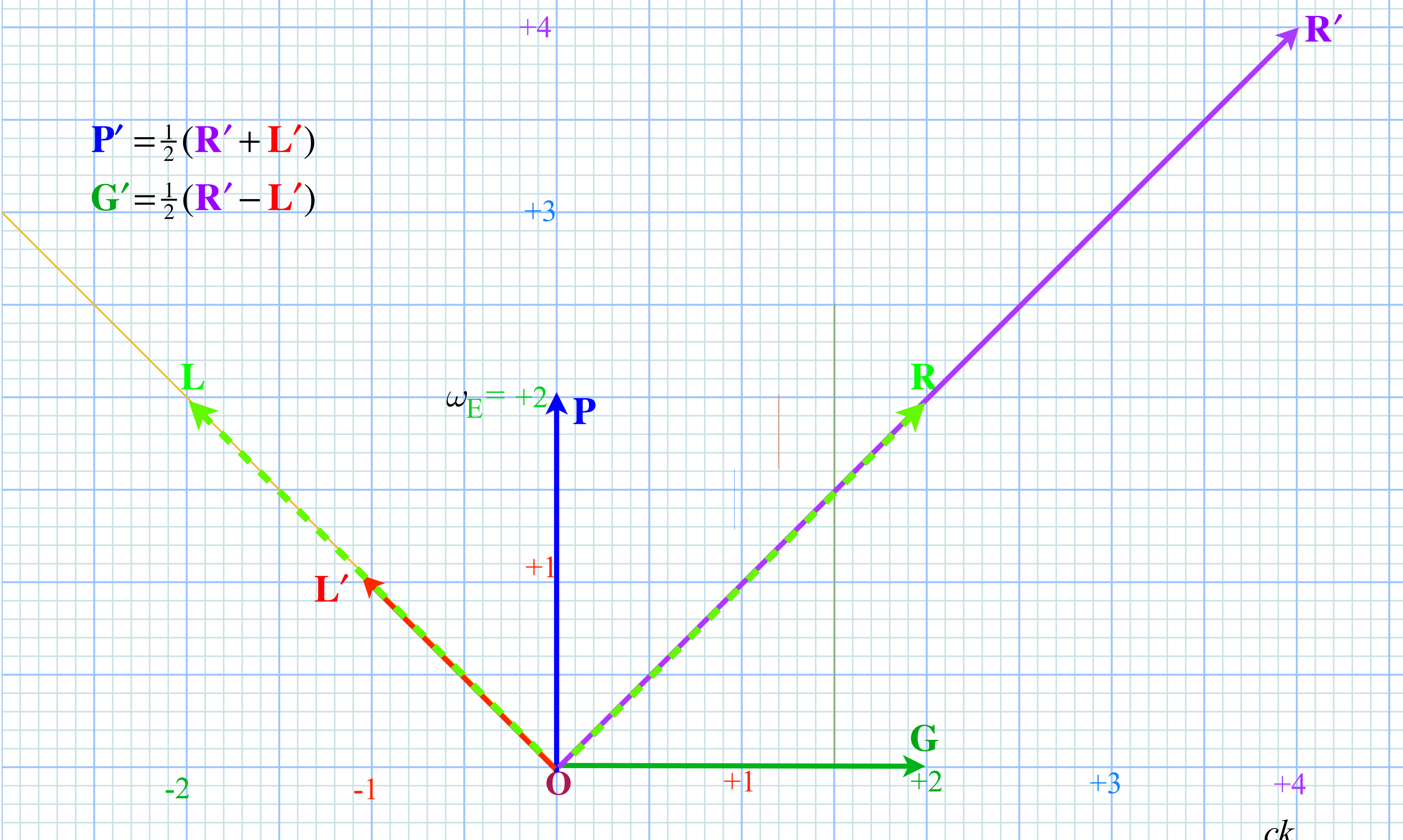

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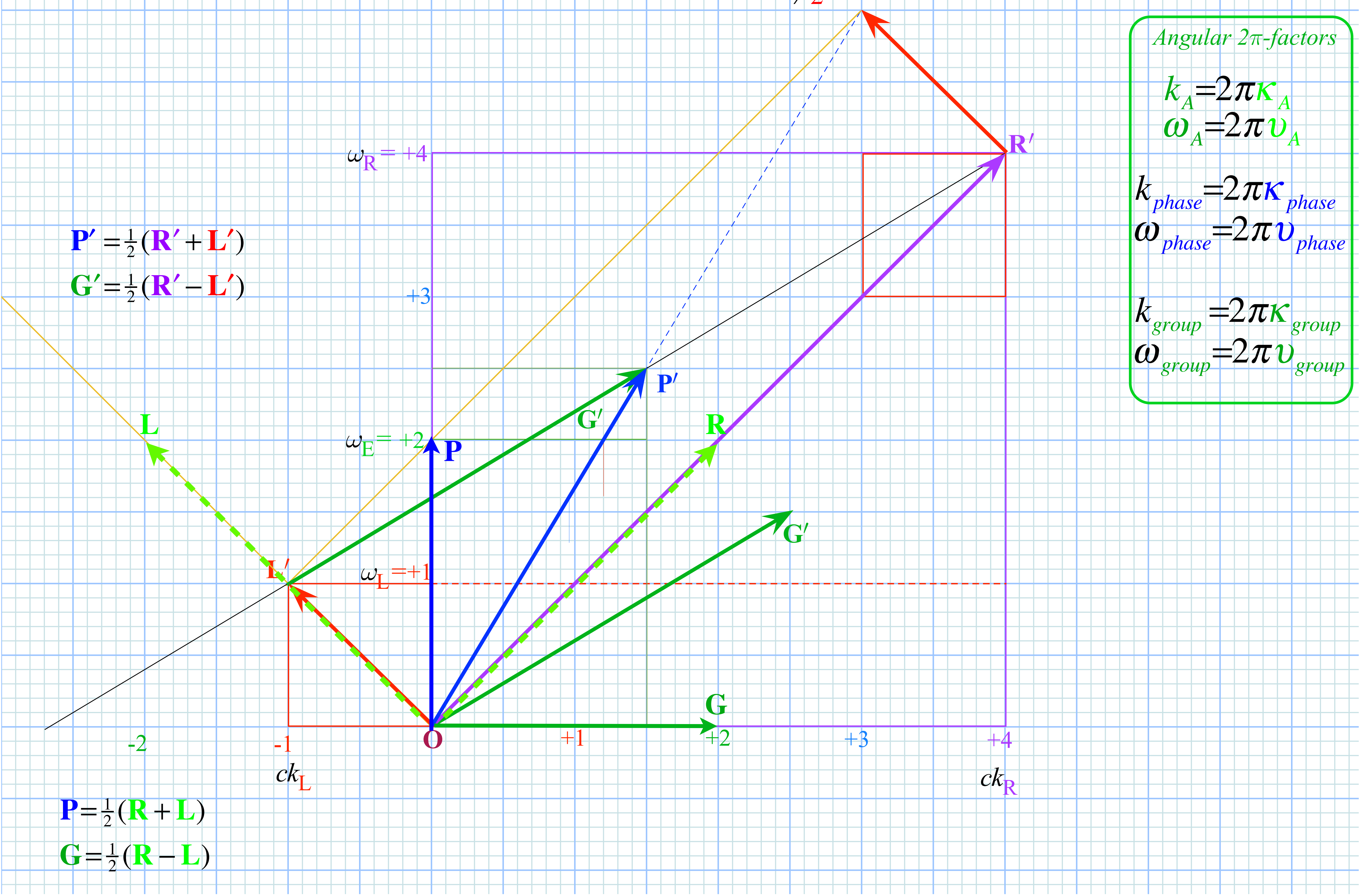

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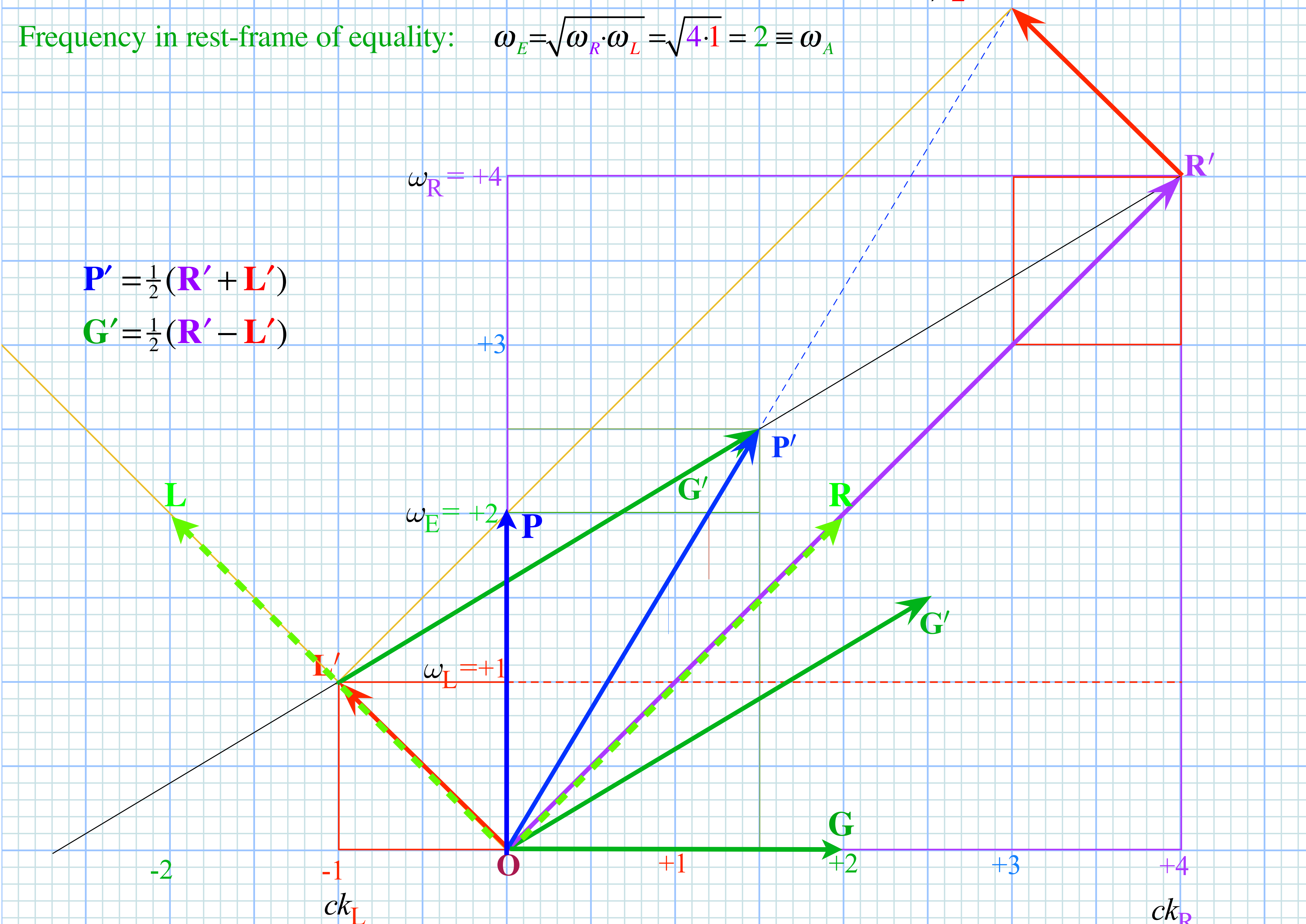
Frequency in rest-frame of equality: $\omega_E=\sqrt{\omega_R\cdot\omega_L}=\sqrt{4\cdot 1}=2\equiv\omega_A$



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2-CW Phase frequency $\omega_{phase}=\frac{1}{2}(\omega_R+\omega_L)=\frac{1}{2}(4+1)=2.5=\omega_E\cosh\rho$

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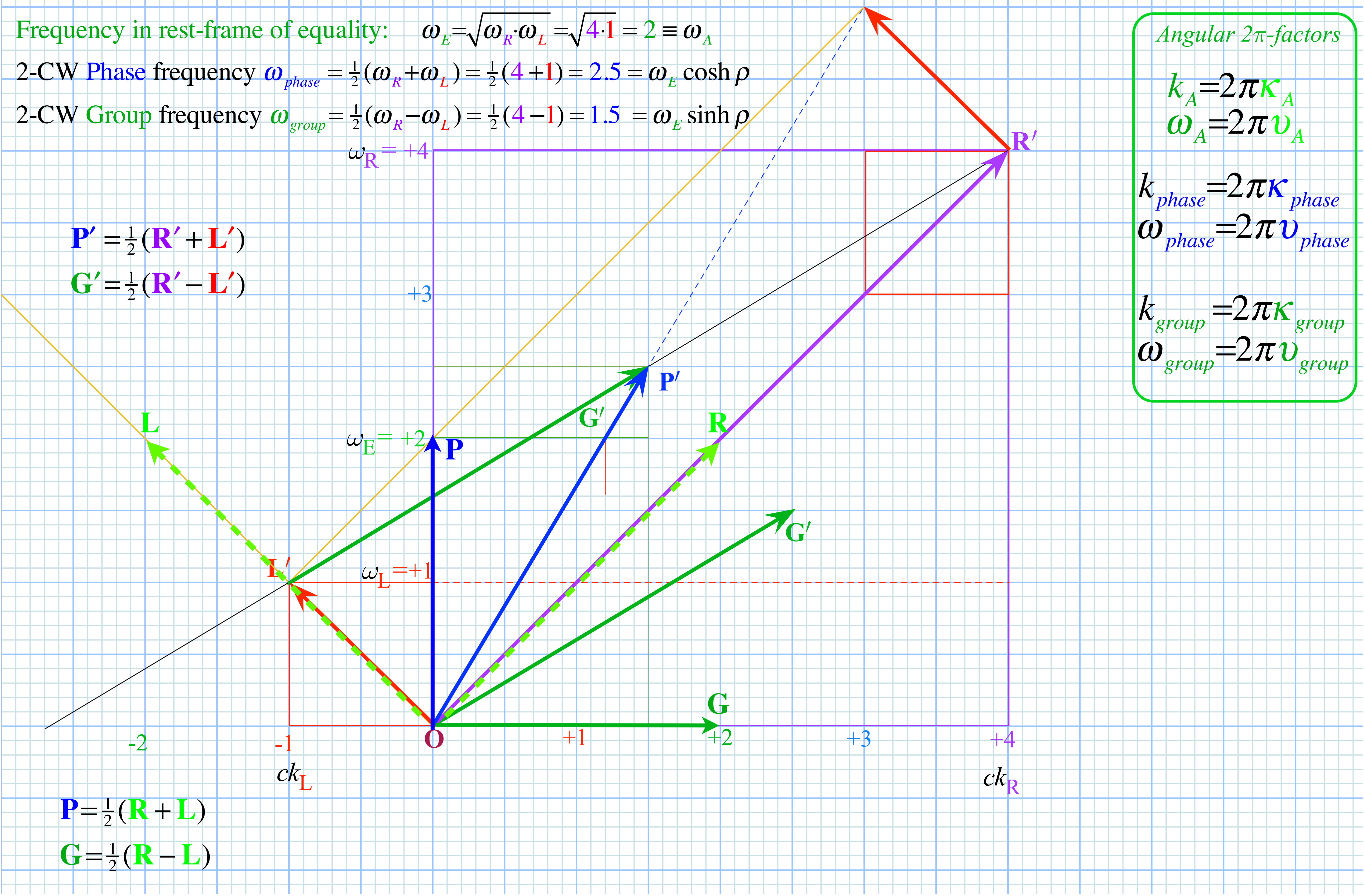

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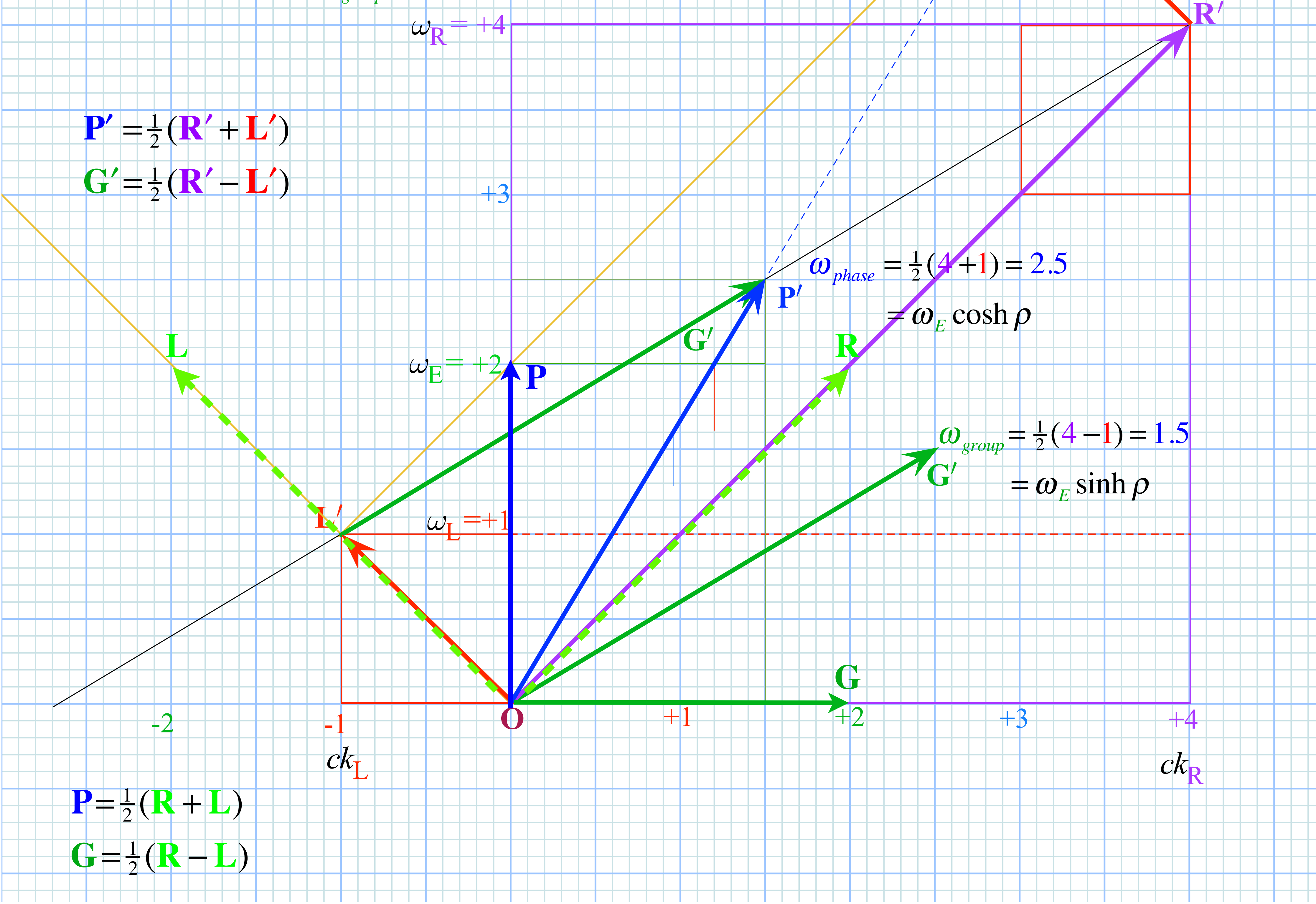

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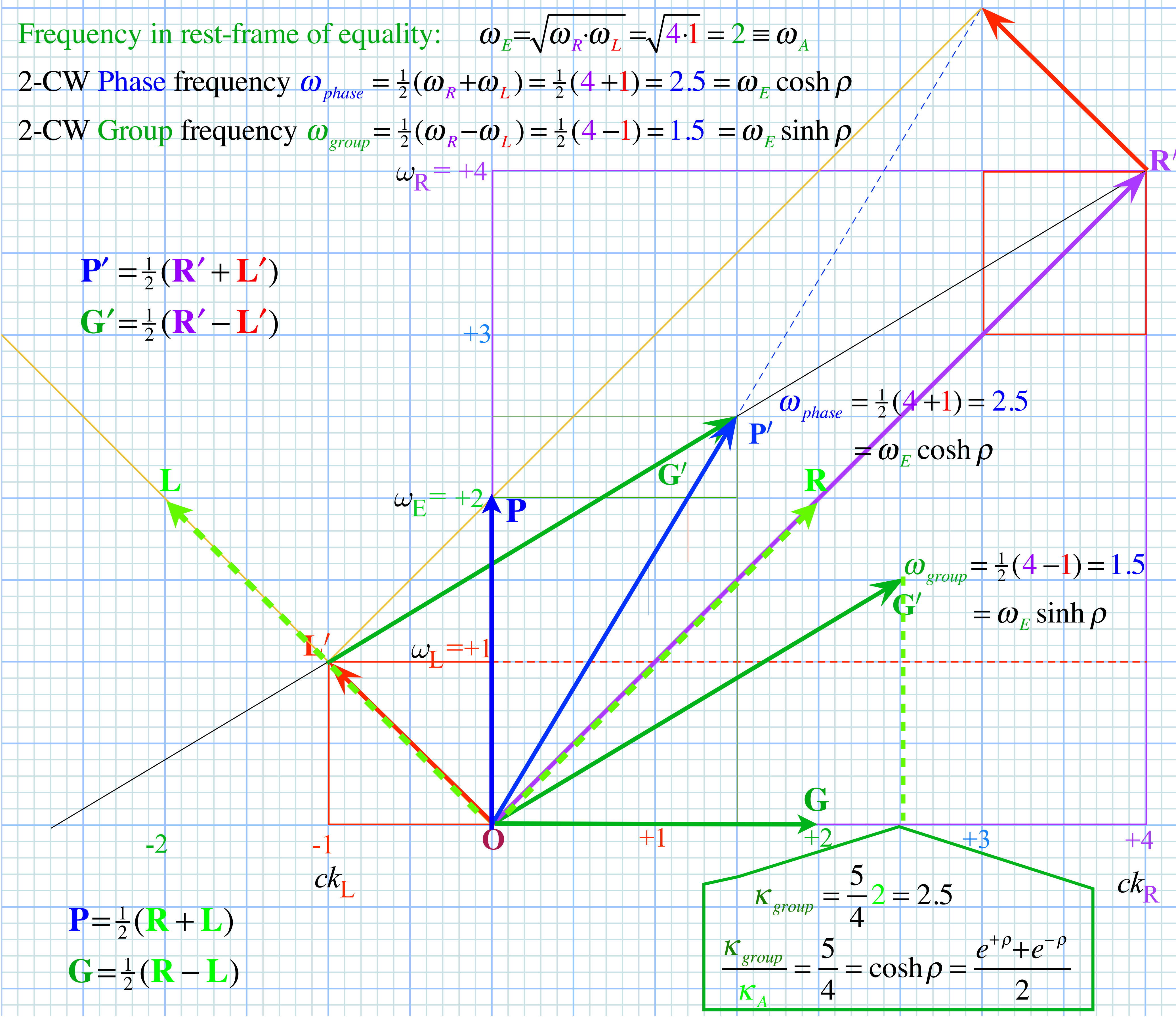

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$\kappa_{group}=\frac{5}{4}2=2.5$

$\frac{\kappa_{group}}{\kappa_A}=\frac{5}{4}=\cosh\rho=\frac{e^{+\rho}+e^{-\rho}}{2}$

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$\frac{5}{4}=\cosh\rho=\frac{1}{2}(e^{+\rho}+e^{-\rho})$

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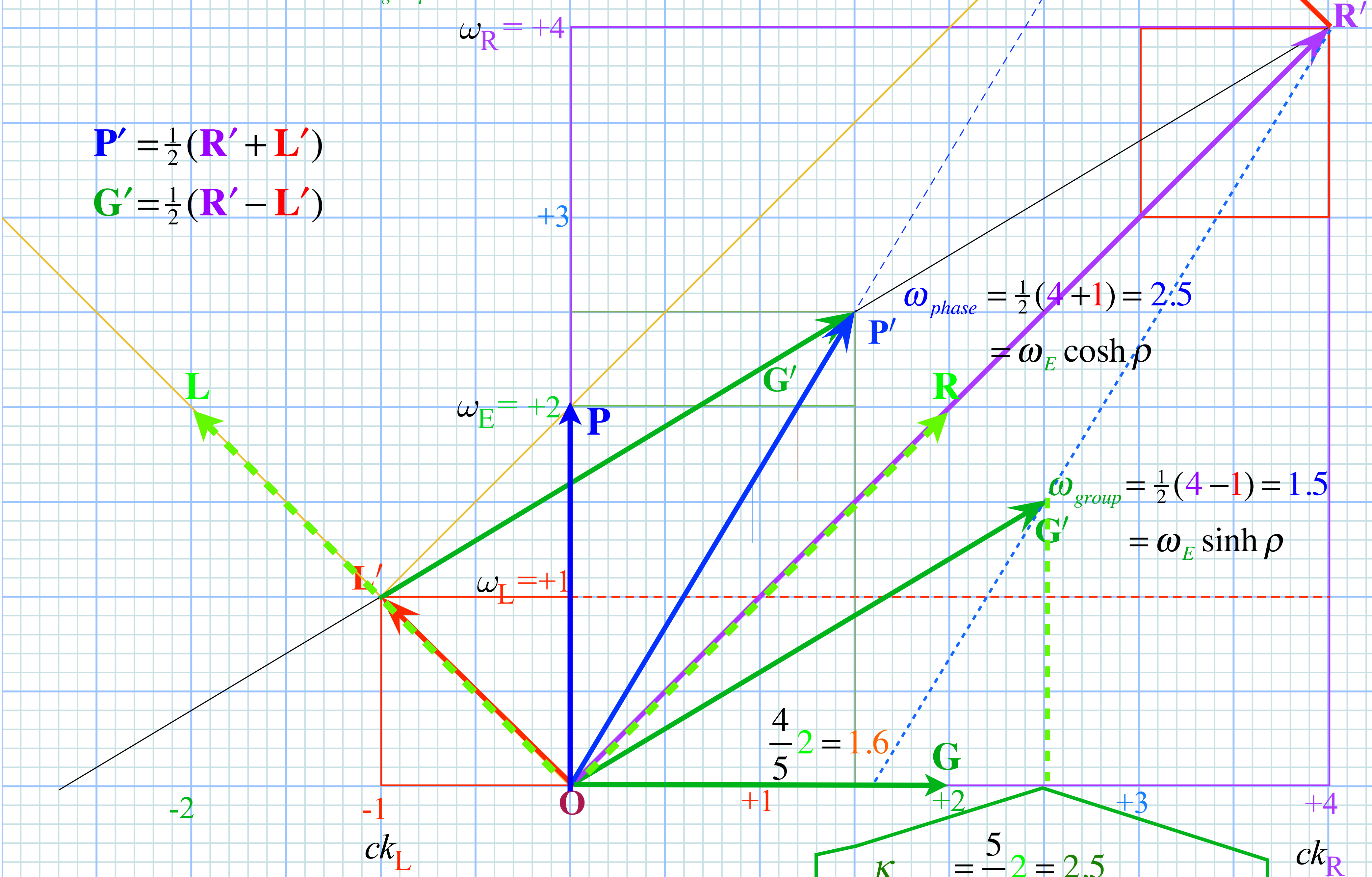

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$\frac{5}{4}=\cosh\rho=\frac{1}{2}(e^{+\rho}+e^{-\rho})$

$\frac{3}{4}=\sinh\rho=\frac{1}{2}(e^{+\rho}-e^{-\rho})$

$\kappa_{group}=\frac{5}{4}2=2.5$

$\frac{\kappa_{group}}{\kappa_A}=\frac{5}{4}=\cosh\rho=\frac{e^{+\rho}+e^{-\rho}}{2}$

Lorentz contraction of λ in spacetime

$\frac{\lambda_{group}}{\lambda_A}=\frac{4}{5}=\operatorname{sech}\rho=\frac{2}{e^{+\rho}+e^{-\rho}}$

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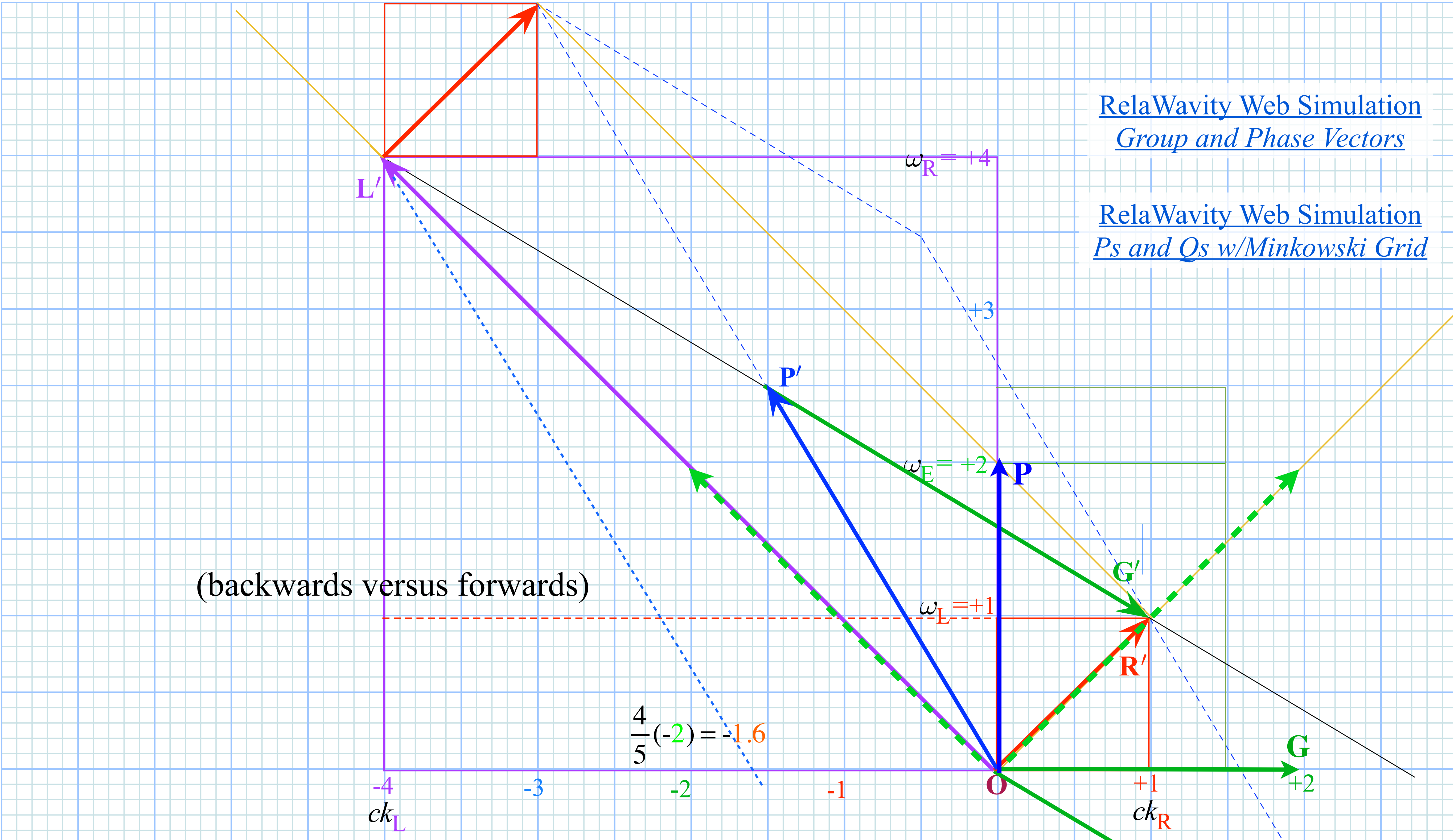
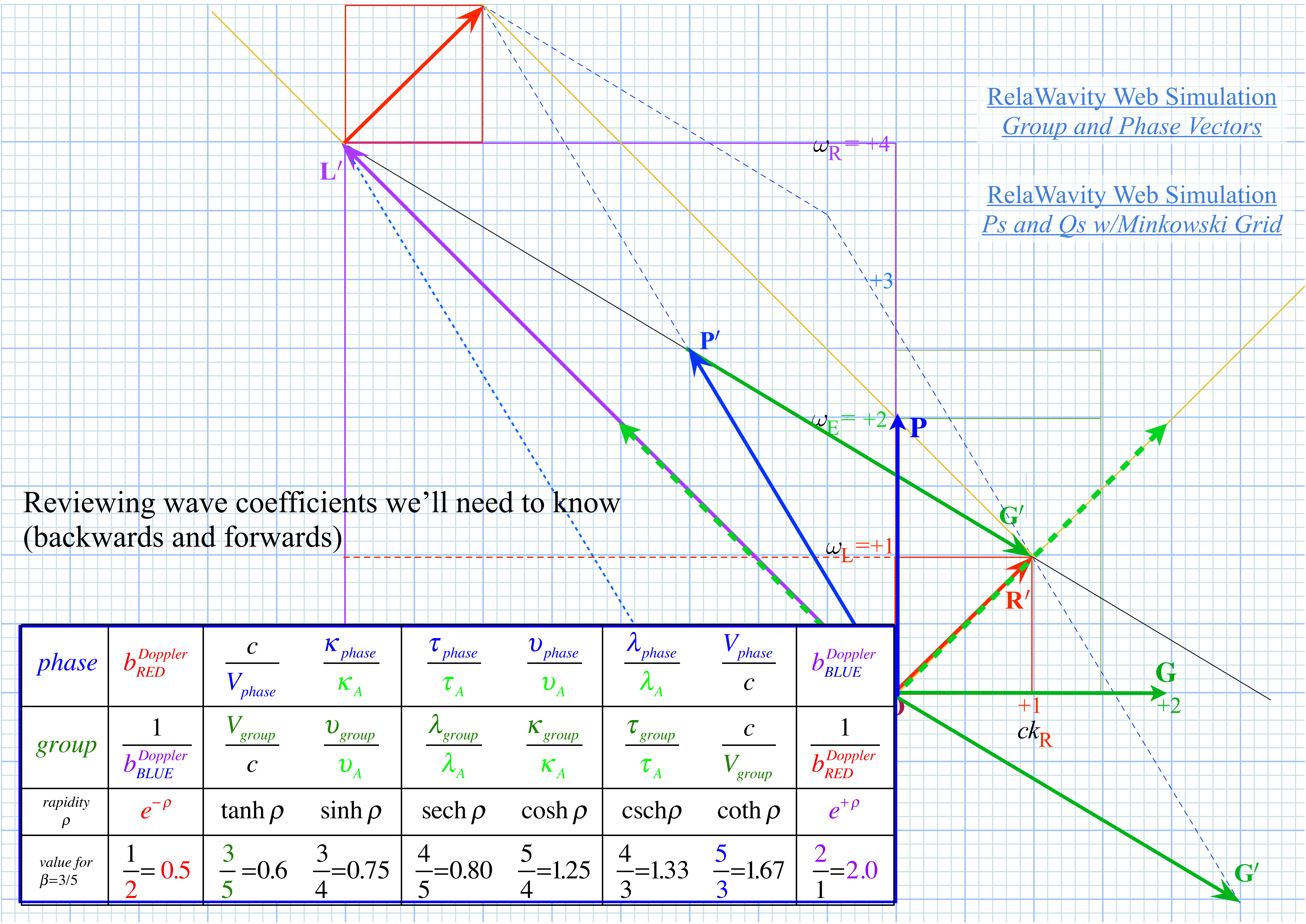


Caution: u_{AB} (linear velocity of Alice relative to Bob) equals $-u_{BA}$ (minus B relative to A)

Convention: u_{AB} is positive if Alice goes left-to-right according to Bob

while Bob goes right-to-left (negatively) according to Alice

Rapidity ρ_{AB} requires further discussion...



RelaWavity Web Simulation
Group and Phase Vectors

RelaWavity Web Simulation
Ps and Qs w/Minkowski Grid

Reviewing wave coefficients we'll need to know (backwards and forwards)

phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Phase wavenumber flips to *Phase wavelength* $\lambda = 1/\kappa$

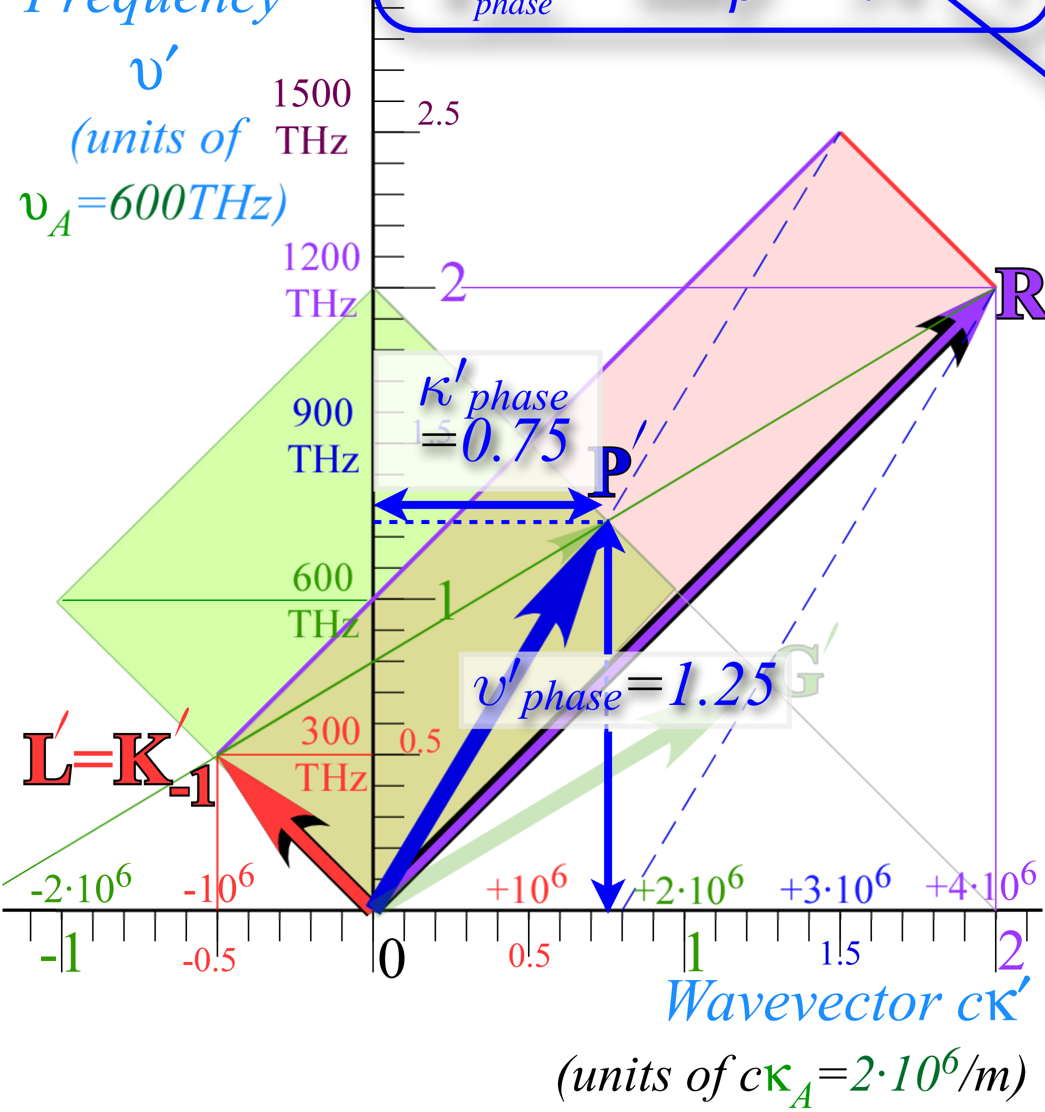
$\kappa'_{phase} = \kappa_A \sinh\rho = 3/4$ flips to $\lambda'_{phase} = \lambda_A \operatorname{csch}\rho = 4/3$

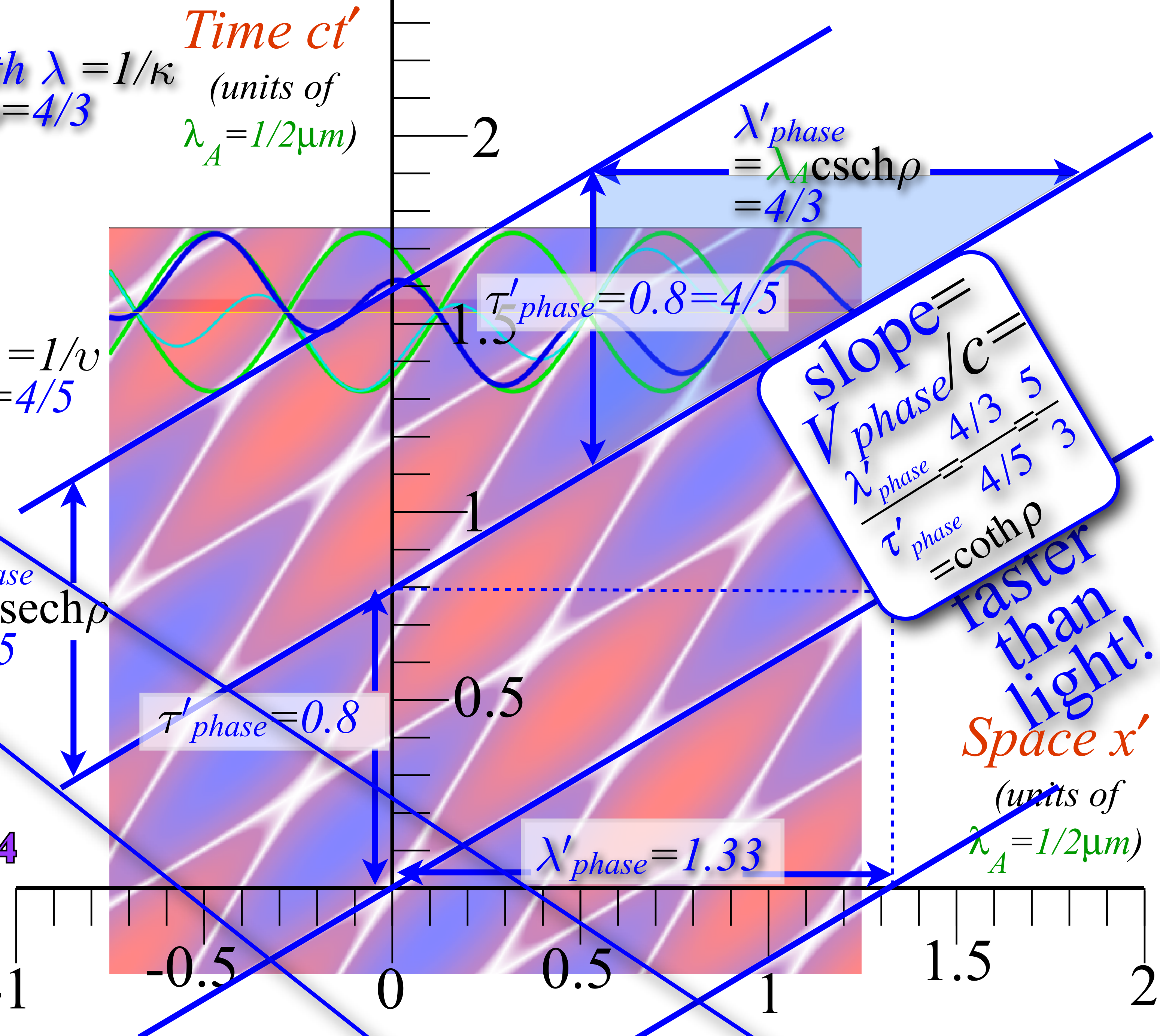
$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Phase frequency flips to *Phase period* $\tau = 1/\upsilon$

$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4$ flips to $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5$

$$\mathbf{P}\text{-slope} = V_{phase}/c = \frac{\upsilon'_{phase}}{\kappa'_{phase}} = \frac{\cosh\rho}{\sinh\rho} = \frac{5/4}{3/4} = \frac{5}{3}$$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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Group wavenumber
$\kappa'_{group} = \kappa_A \cosh\rho = 5/4 = 1.25$

Group wavelength $\lambda = 1/\kappa$
$\lambda'_{group} = \lambda_A \operatorname{sech}\rho = 4/5 = 0.8$

$$\mathbf{G}' = \begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

Group frequency
$\upsilon'_{group} = \upsilon_A \sinh\rho = 3/4 = 0.75$

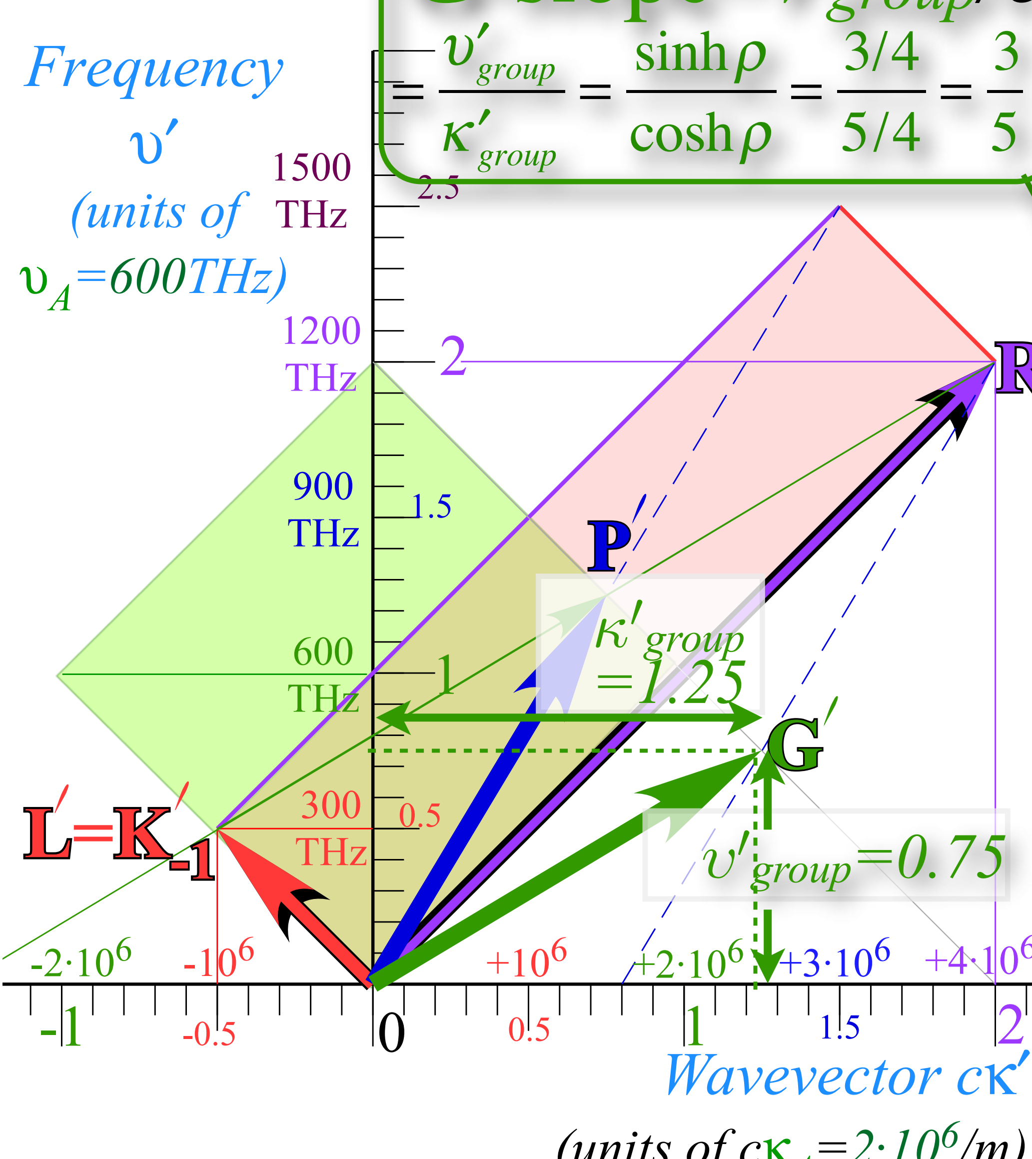
flips to

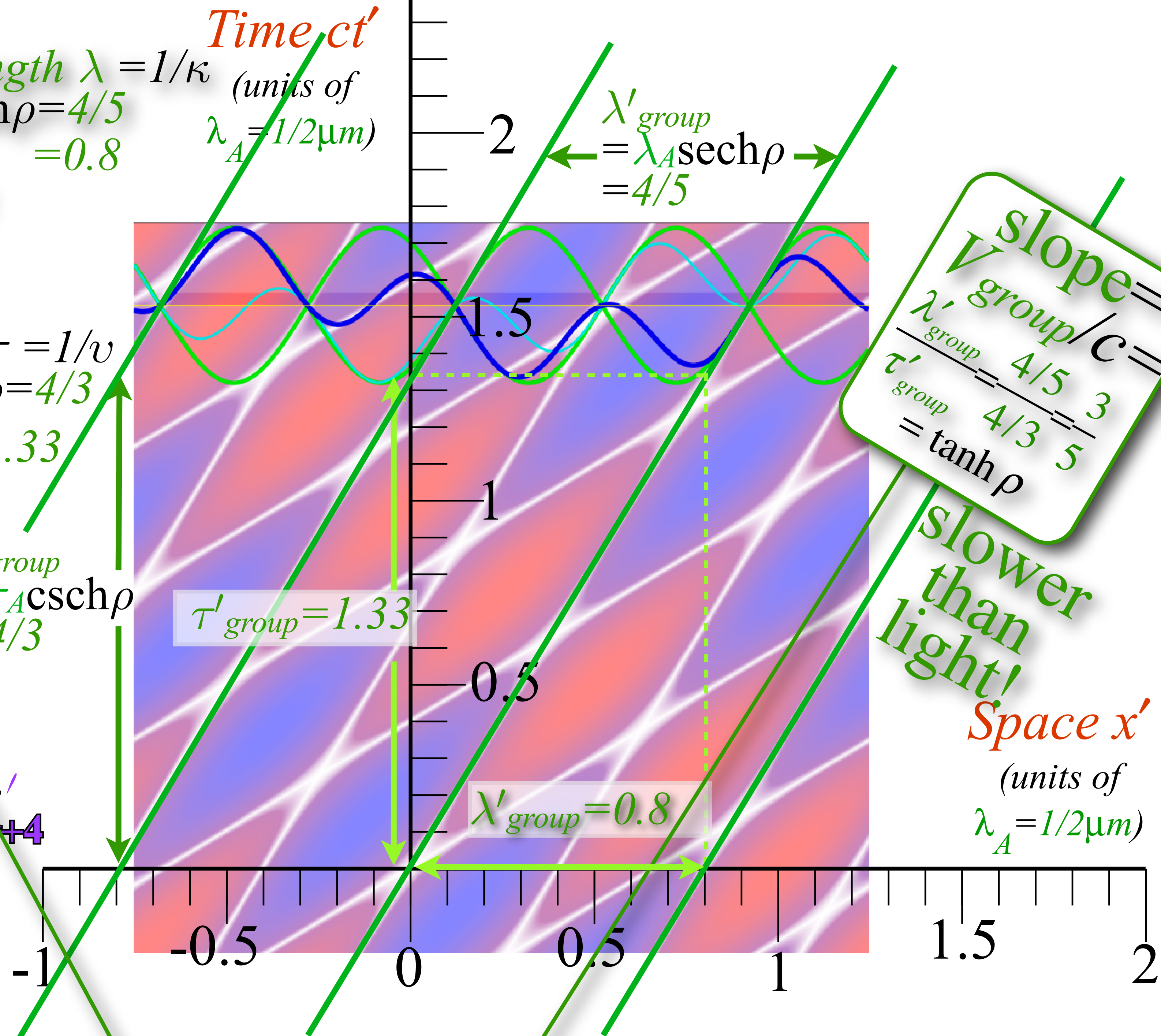
Group period $\tau = 1/\upsilon$
$\tau'_{group} = \tau_A \operatorname{csch}\rho = 4/3 = 1.33$

G-slope $= V_{group}/c$

$$= \frac{\upsilon'_{group}}{\kappa'_{group}} = \frac{\sinh\rho}{\cosh\rho} = \frac{3/4}{5/4} = \frac{3}{5}$$

$\tau'_{group} = \tau_A \operatorname{csch}\rho = 4/3$





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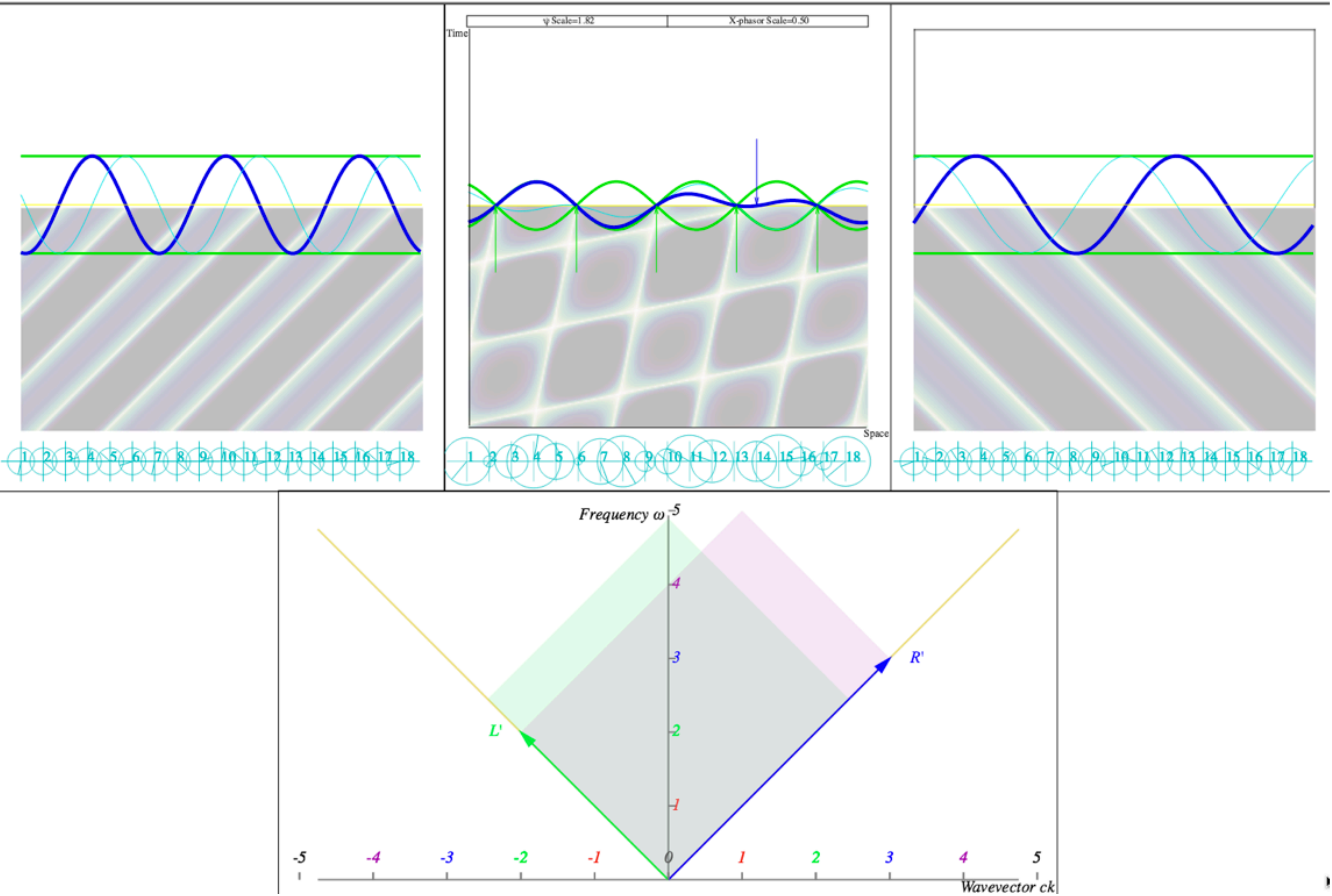
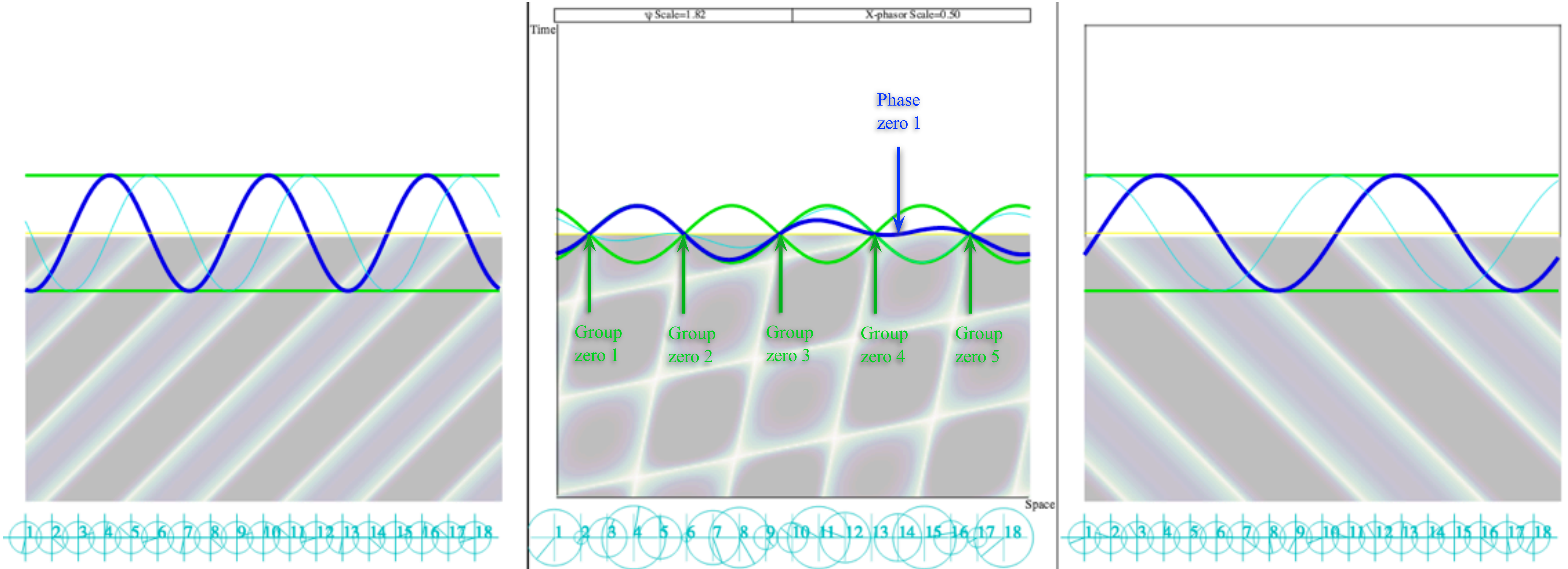
Comparison of group and phase dynamics: SLOW(er) (β=u/c=$1/5$)



BohrIt Web Simulation - *2 CW ct vs x Plot (ck = -2, 3) Multi-panel with Zero Tracers*

Comparison of group and phase dynamics: SLOW(er) ($\beta = u/c = 1/5$)

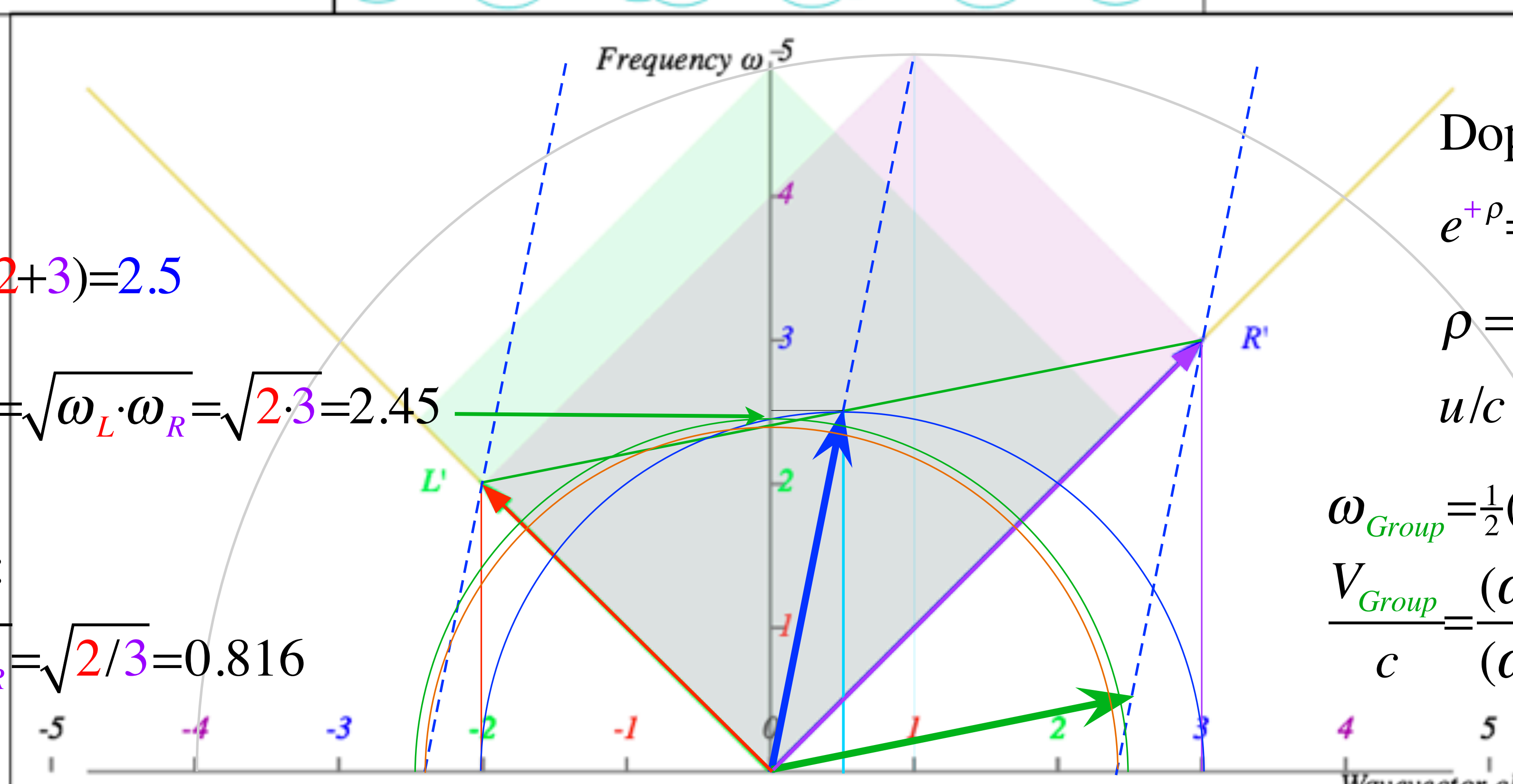


$$\omega_{Phase} = \tfrac{1}{2}(\omega_L + \omega_R) = \tfrac{1}{2}(2+3) = 2.5$$

Rest frequency $\omega_E = \sqrt{\omega_L \cdot \omega_R} = \sqrt{2 \cdot 3} = 2.45$

Doppler red:

$$e^{-\rho} = \sqrt{\omega_L / \omega_R} = \sqrt{2/3} = 0.816$$



Doppler blue:

$$e^{+\rho} = \sqrt{\omega_R / \omega_L} = \sqrt{3/2} = 1.225$$

$$\rho = \ln \sqrt{3/2} = 0.203$$

$$u/c = 0.2 = V_{group}/c$$

$$\omega_{Group} = \tfrac{1}{2}(\omega_R - \omega_L) = \tfrac{1}{2}(3-2) = 0.5$$

$$\frac{V_{Group}}{c} = \frac{(\omega_R - \omega_L)}{(\omega_R + \omega_L)} = \frac{(3-2)}{(3+2)} = \frac{1}{5}$$

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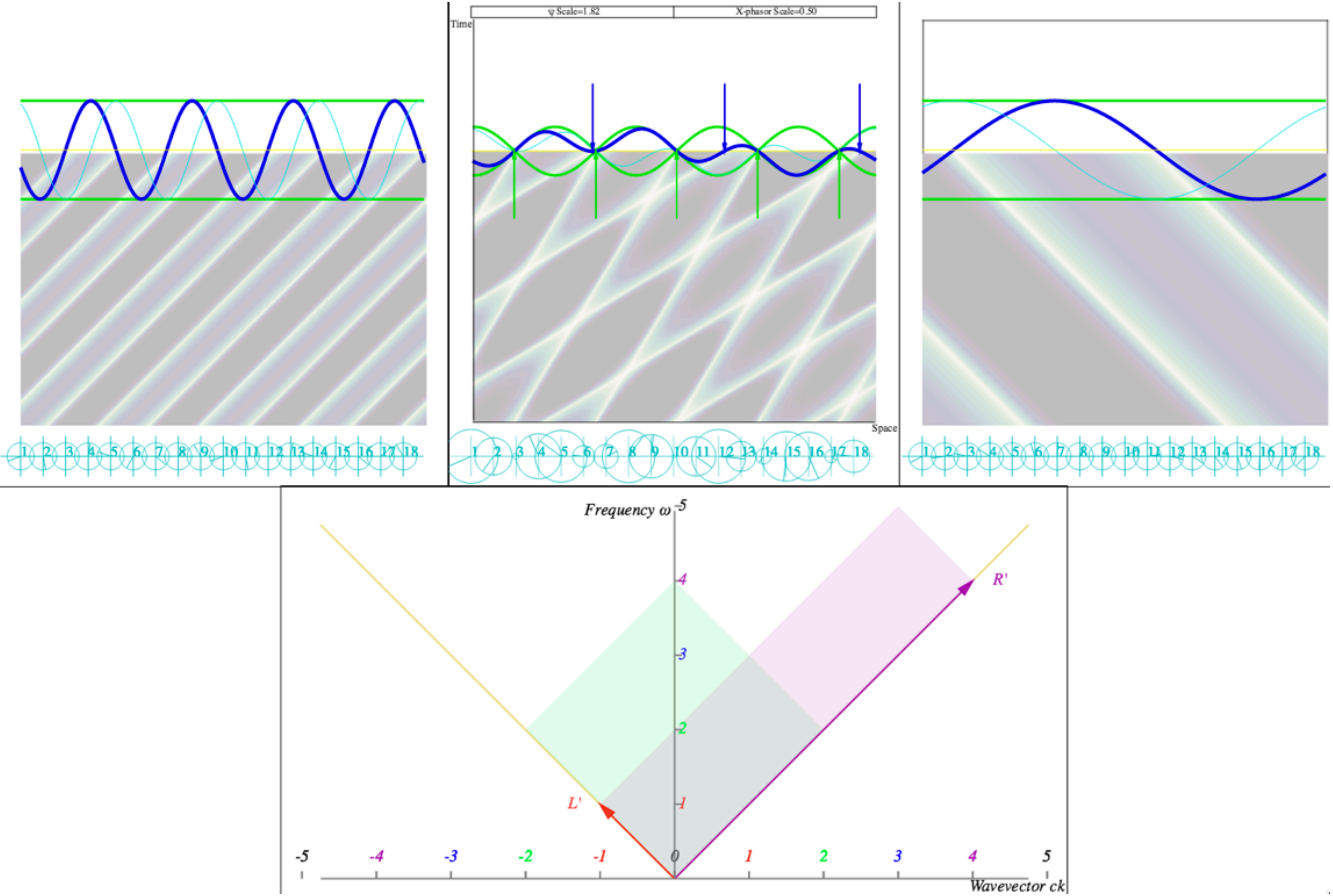
Comparison of group and phase dynamics: *FAST*$_{(er)}$ (β=u/c=3/5)



BohrIt Web Simulation - *2 CW ct vs x Plot (ck = -1, 4) Multi-panel with Zero Tracers*

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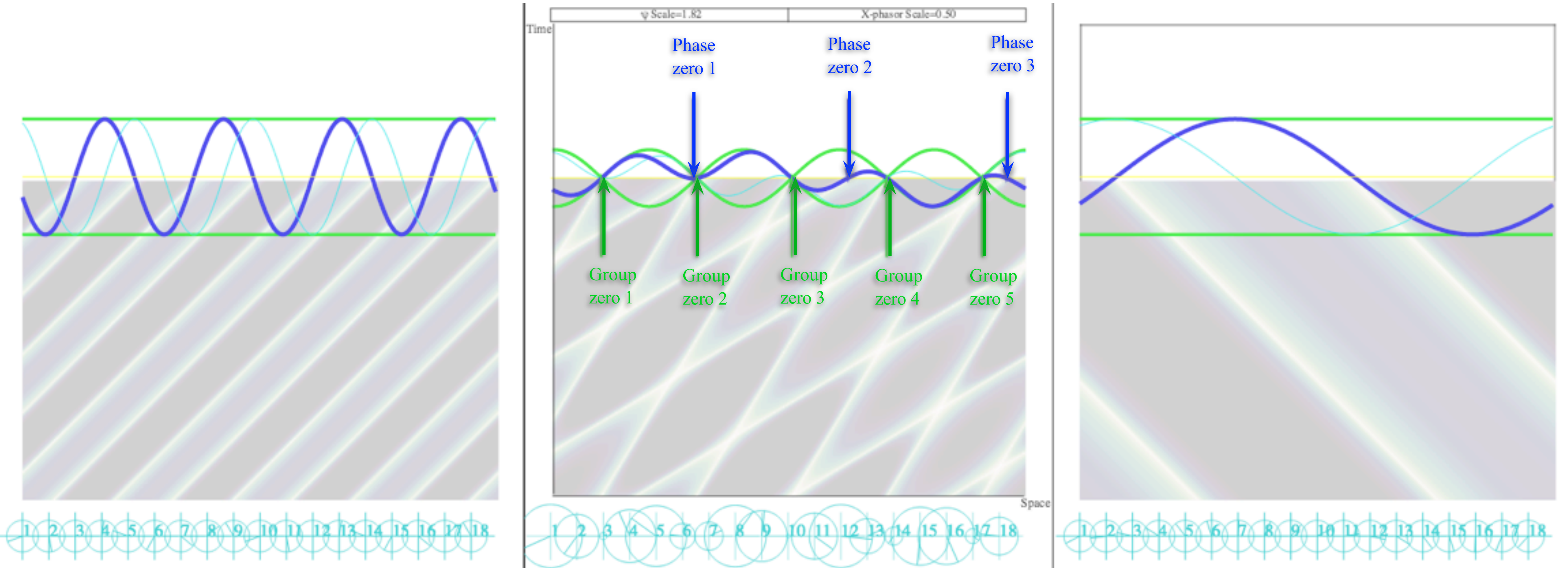

$$\omega_{Phase} = \tfrac{1}{2}(\omega_L + \omega_R) = \tfrac{1}{2}(1+4) = 2.5 = \omega_E \cosh\rho$$

Rest frequency $\omega_E = \sqrt{\omega_L \cdot \omega_R} = \sqrt{1 \cdot 4} = 2$

Doppler red:

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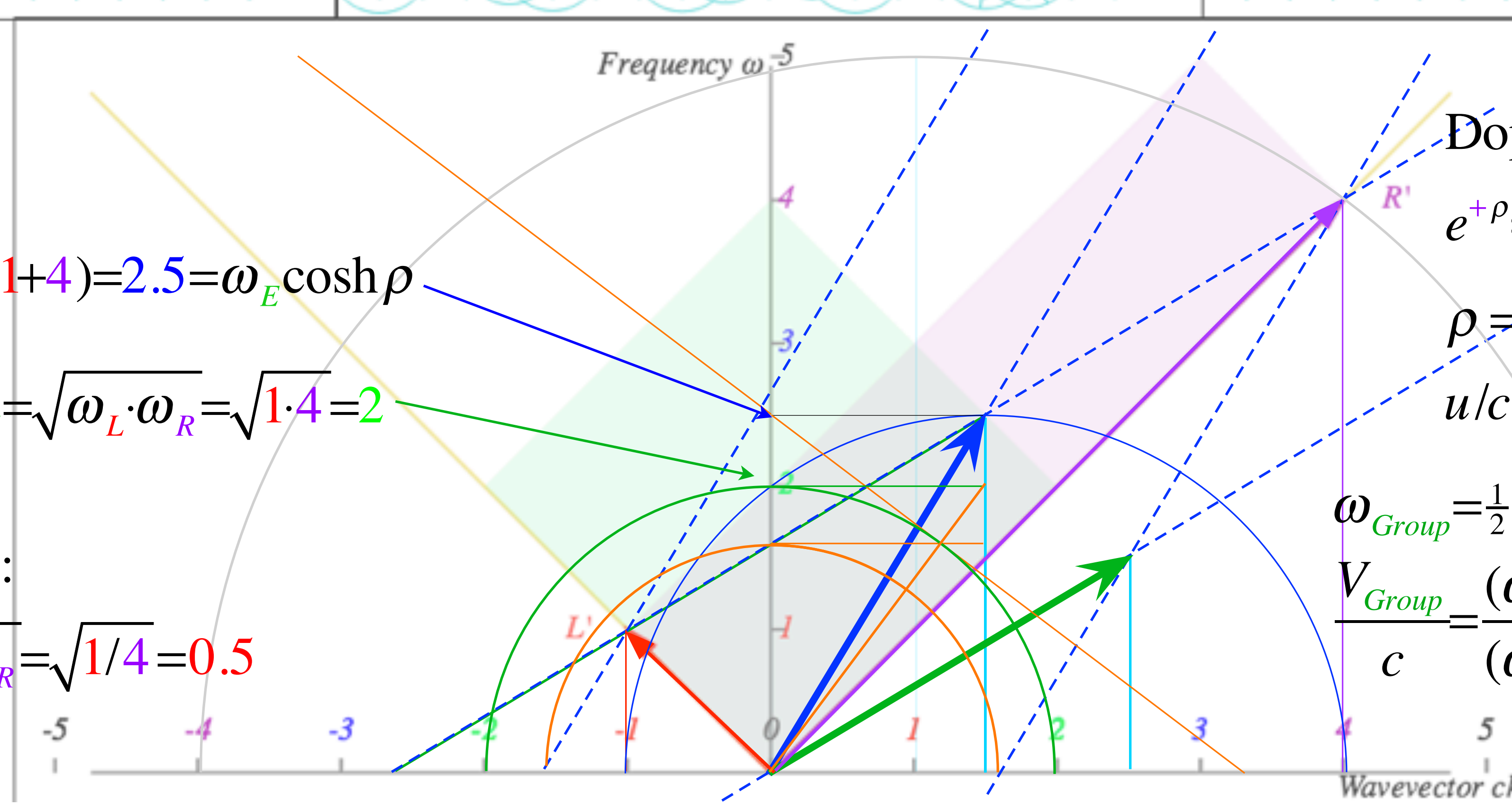

Doppler blue:

$$e^{+\rho} = \sqrt{\omega_R/\omega_L} = \sqrt{4/1} = 2$$

$$\rho = \ln\sqrt{4/1} = 0.693$$

$$u/c = 0.6 = V_{group}/c$$

$$\omega_{Group} = \tfrac{1}{2}(\omega_R - \omega_L) = \tfrac{1}{2}(4-1) = 1.5$$

$$\frac{V_{Group}}{c} = \frac{(\omega_R - \omega_L)}{(\omega_R + \omega_L)} = \frac{(4-1)}{(4+1)} = \frac{3}{5}$$

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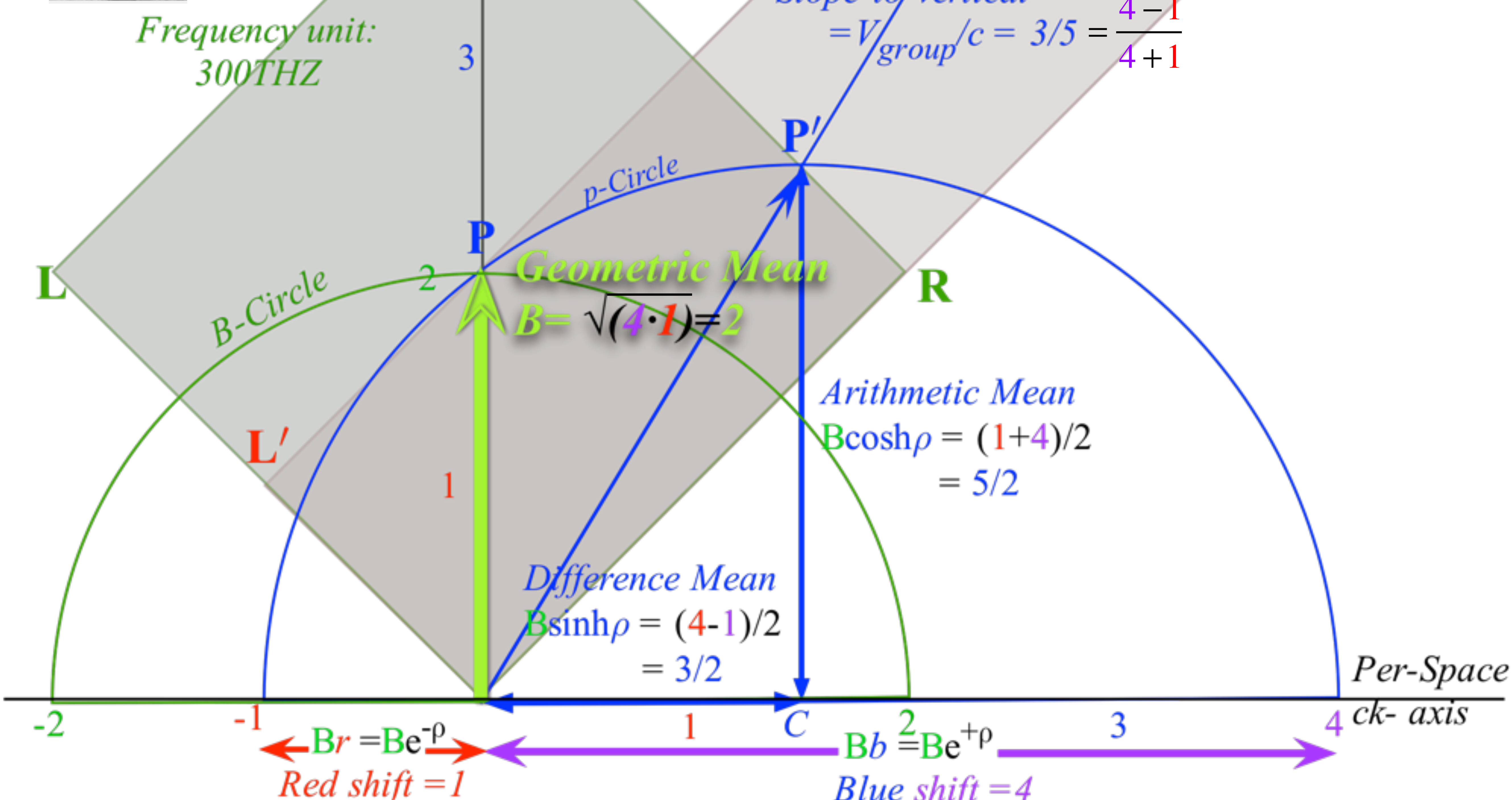
helps "Relawavity"



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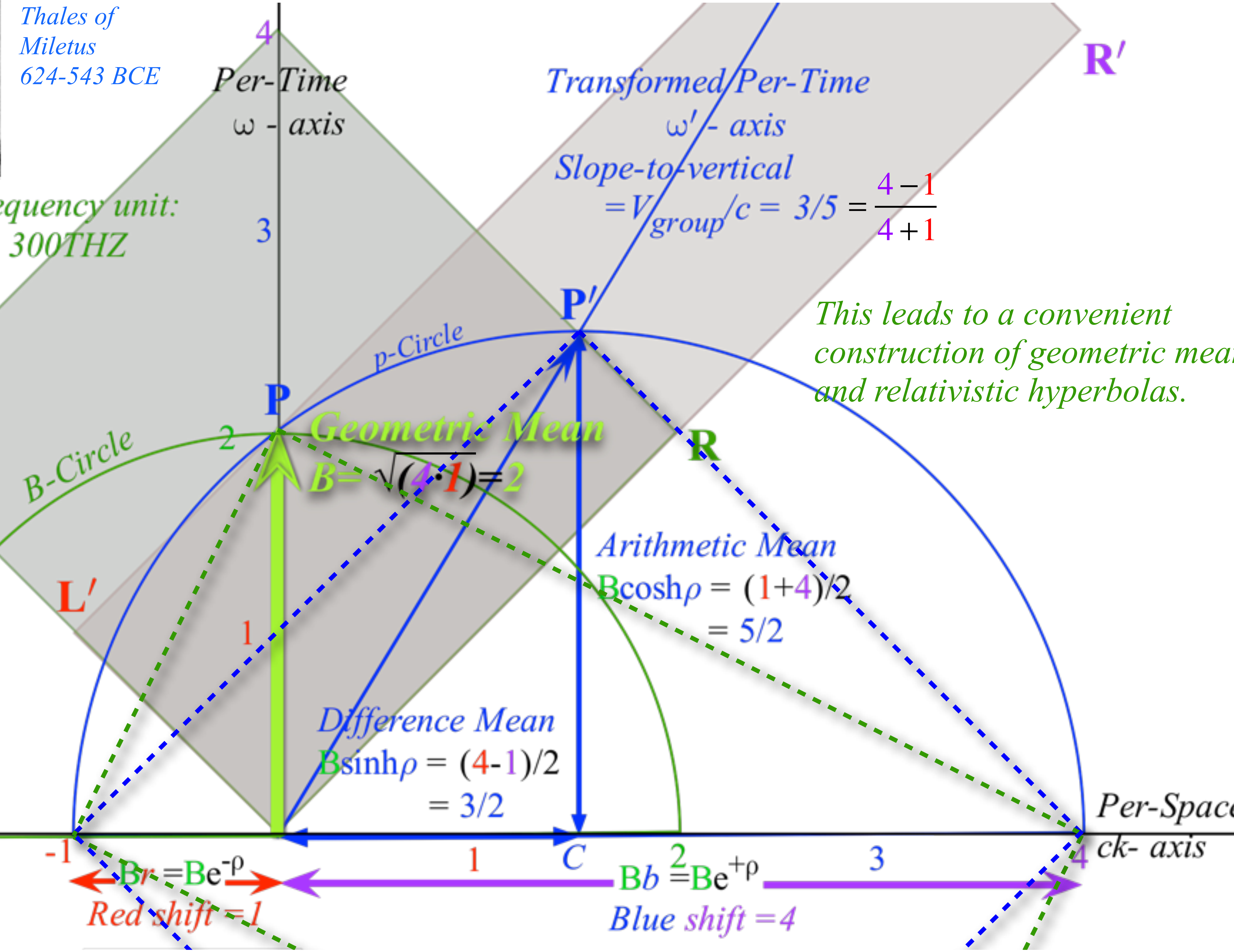

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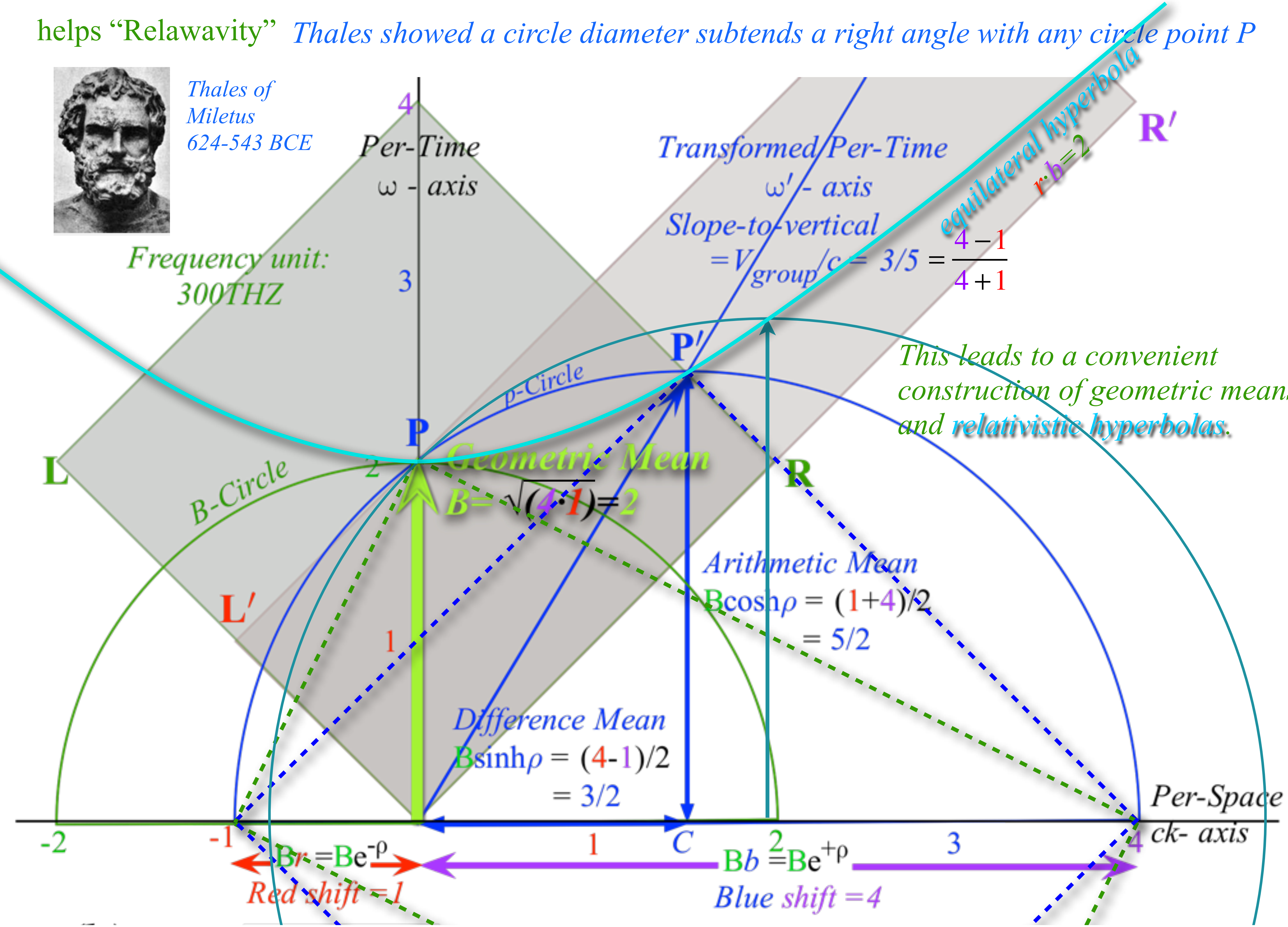

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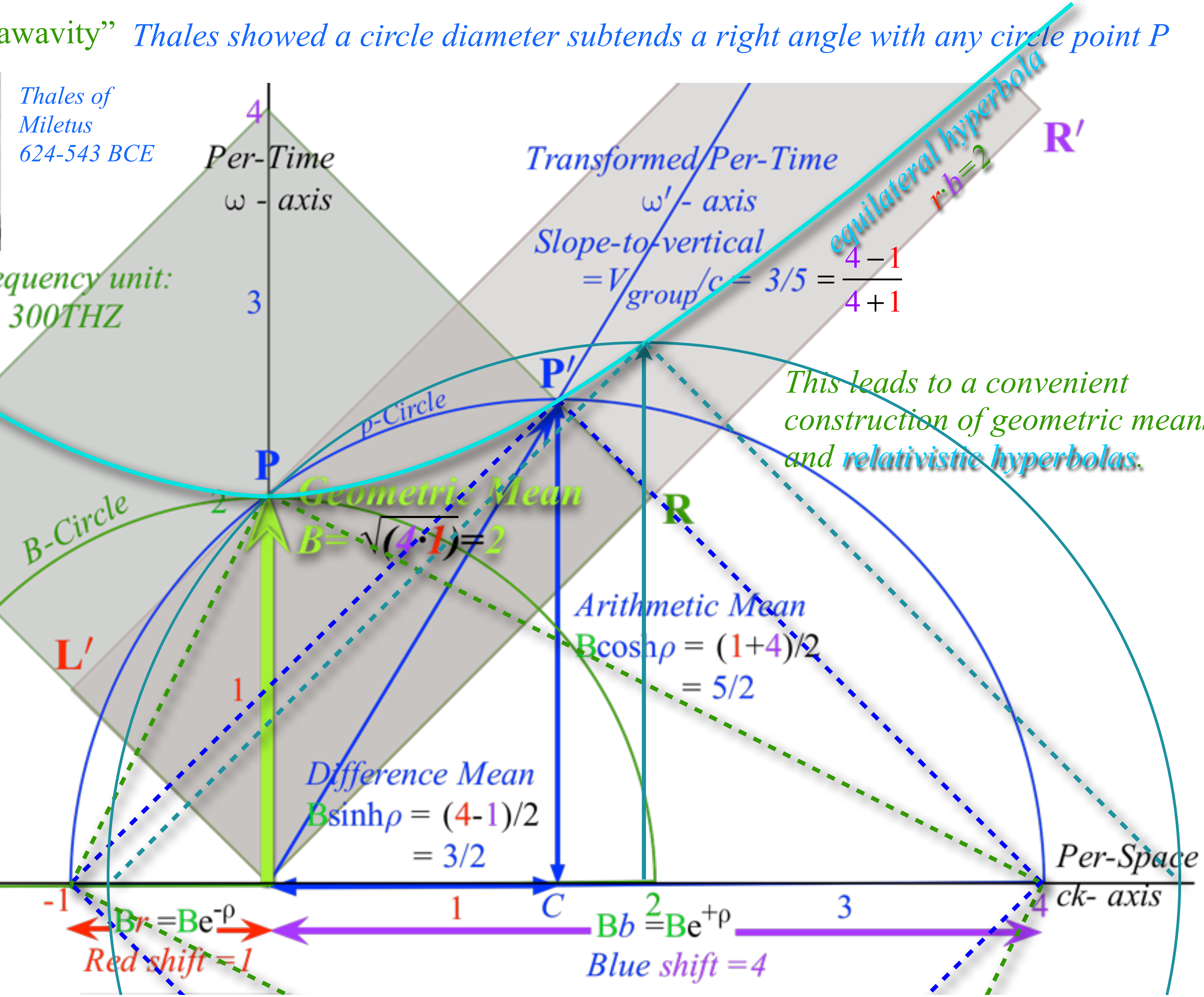

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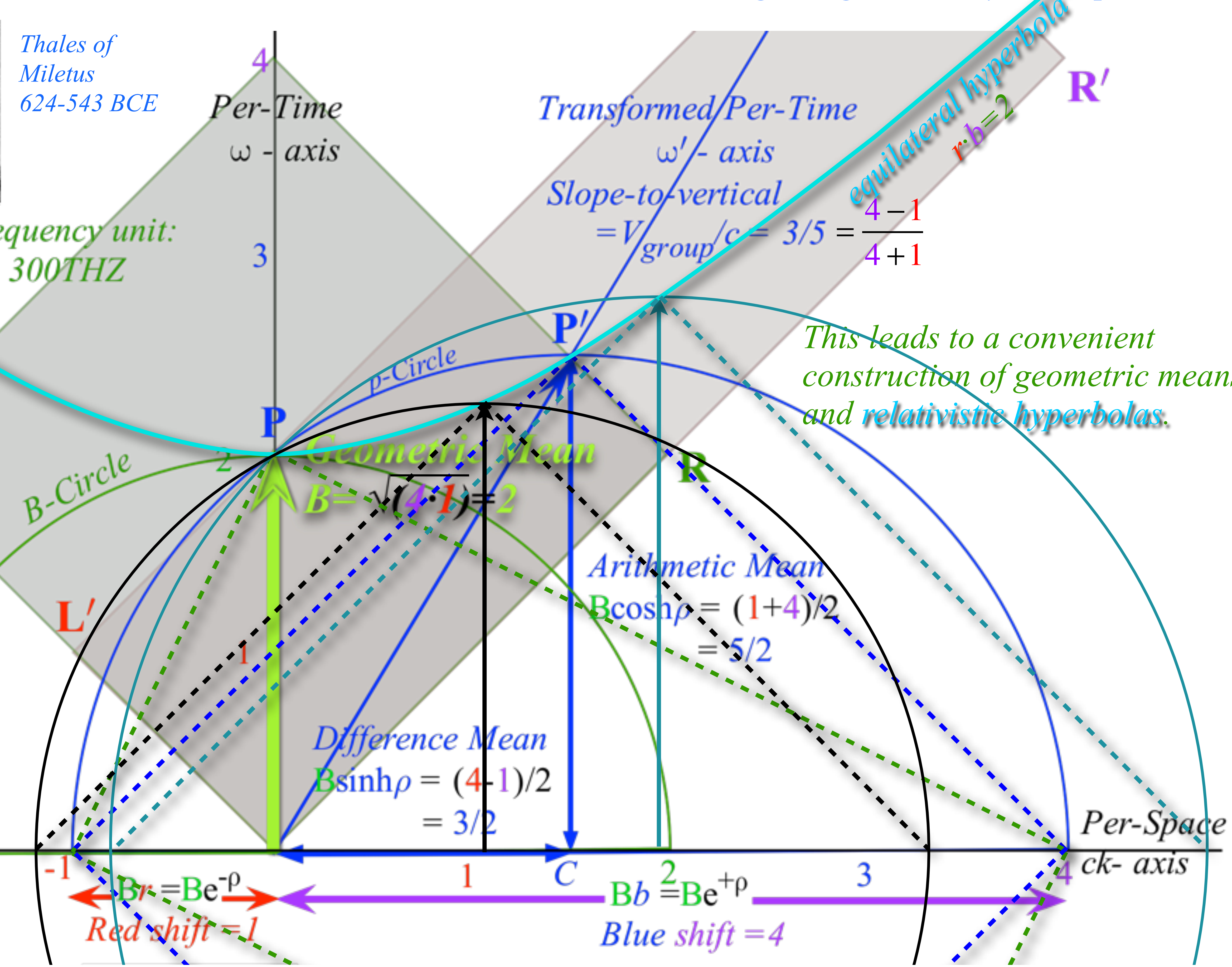

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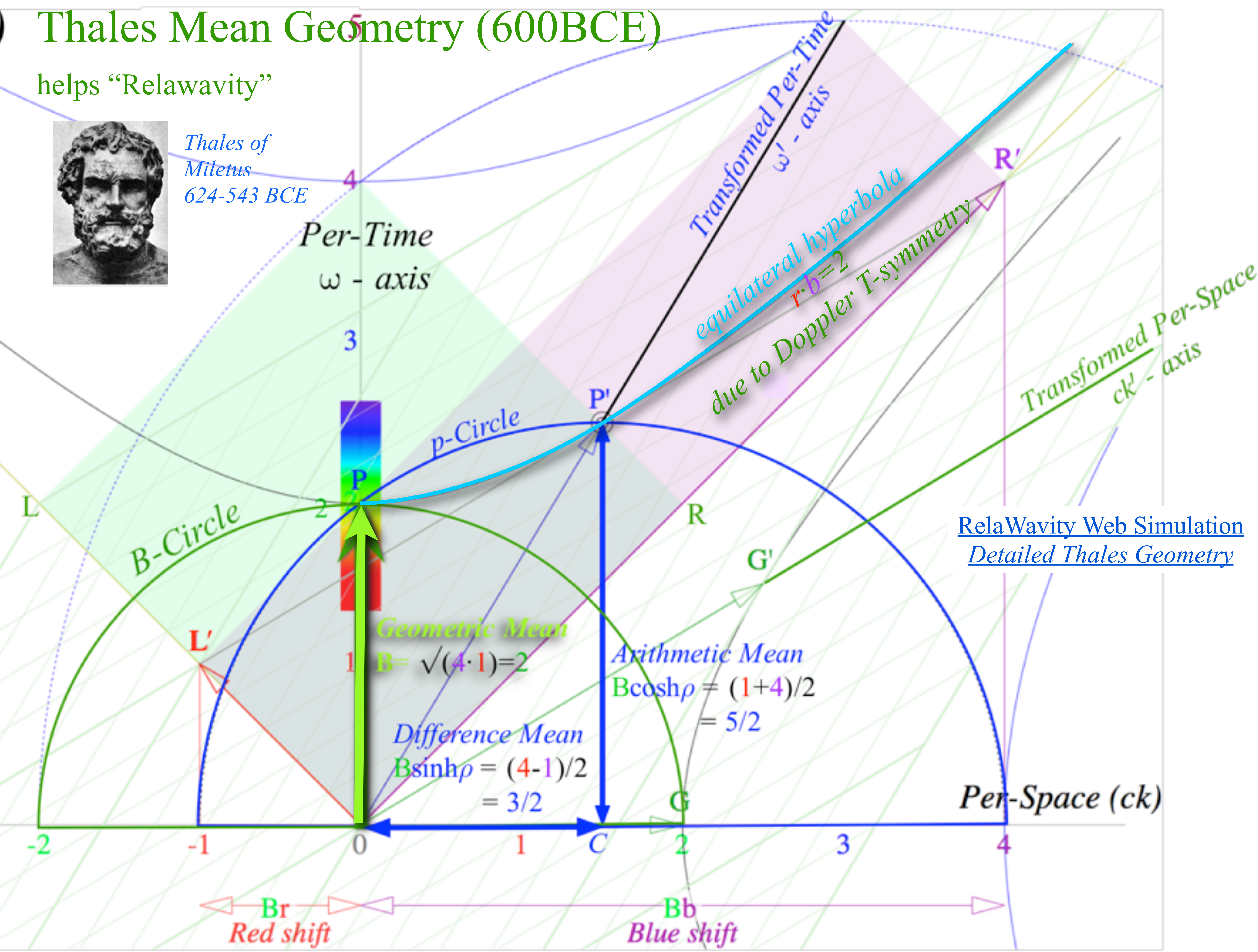

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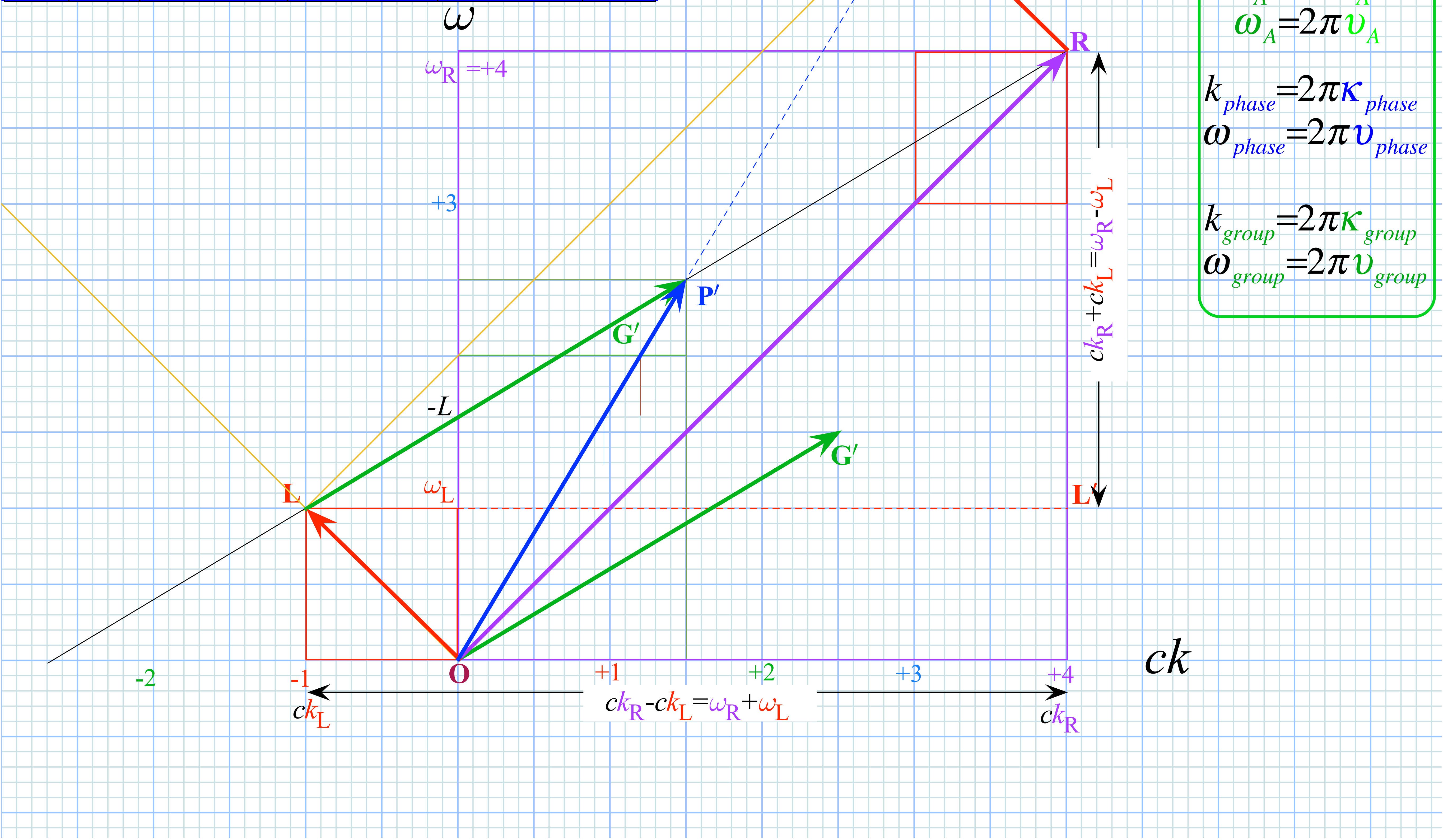
phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$ =0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0



phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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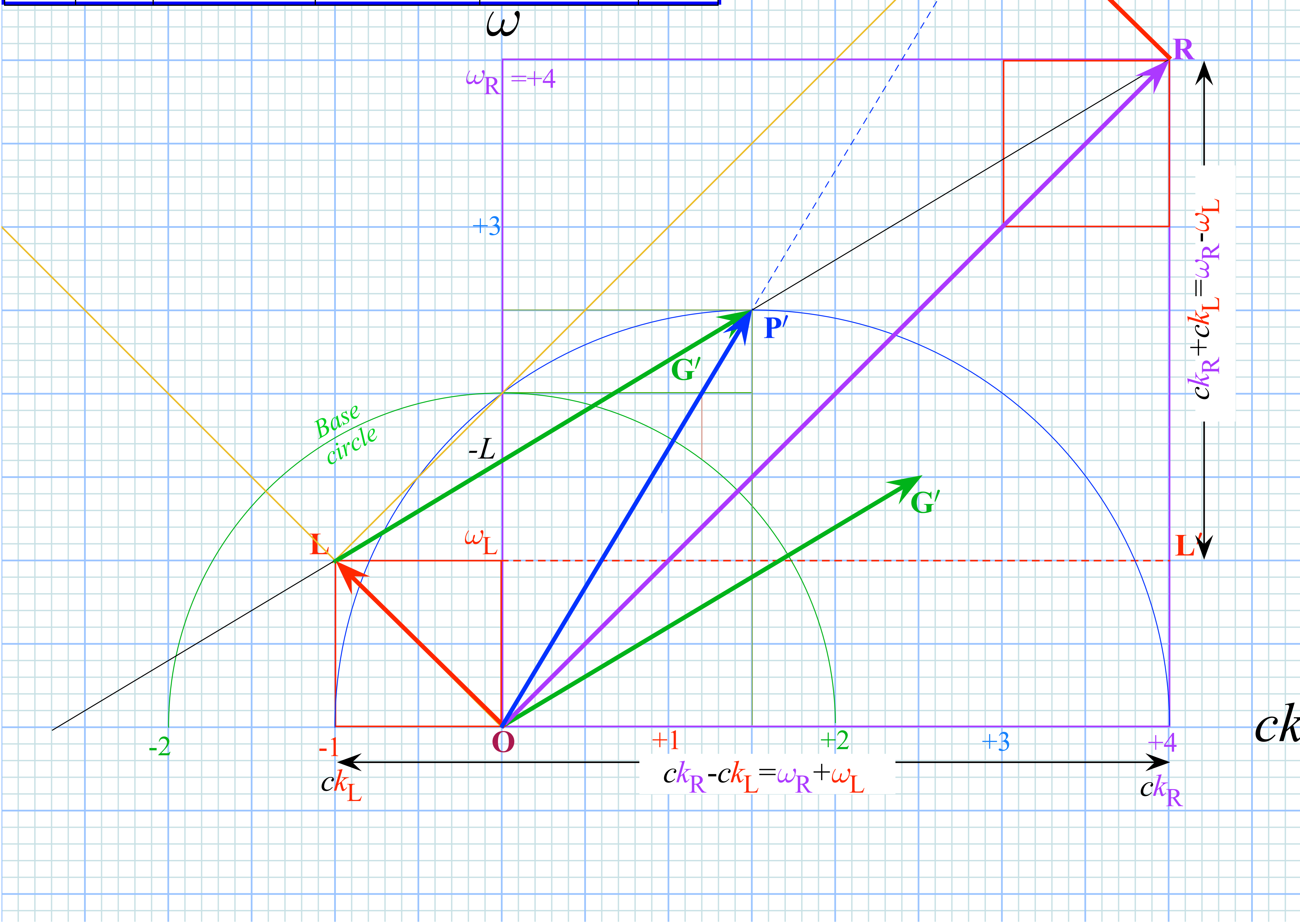

Angular 2π-factors

$k_A = 2\pi\kappa_A$
$\omega_A = 2\pi\upsilon_A$

$k_{phase} = 2\pi\kappa_{phase}$
$\omega_{phase} = 2\pi\upsilon_{phase}$

$k_{group} = 2\pi\kappa_{group}$
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phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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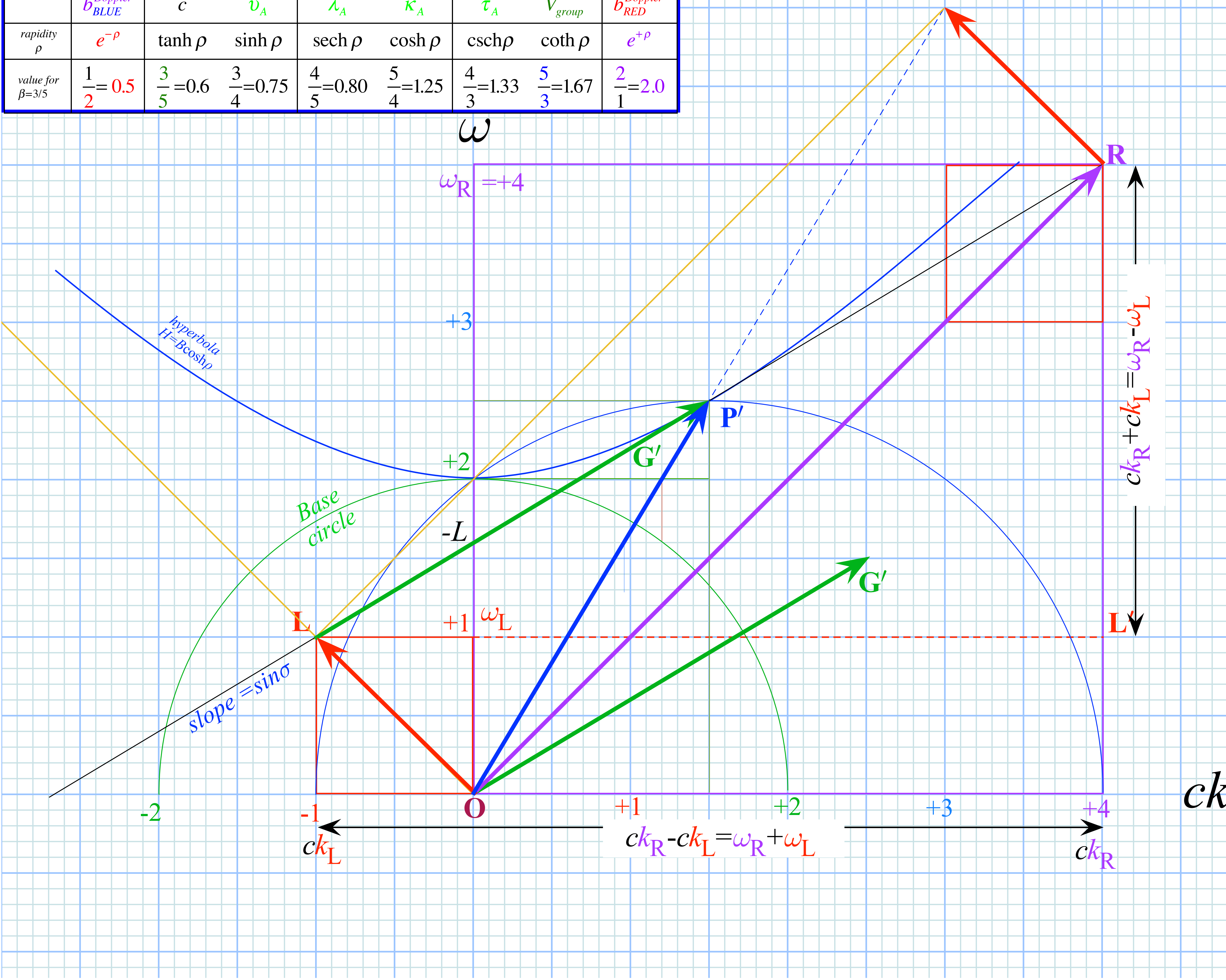

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group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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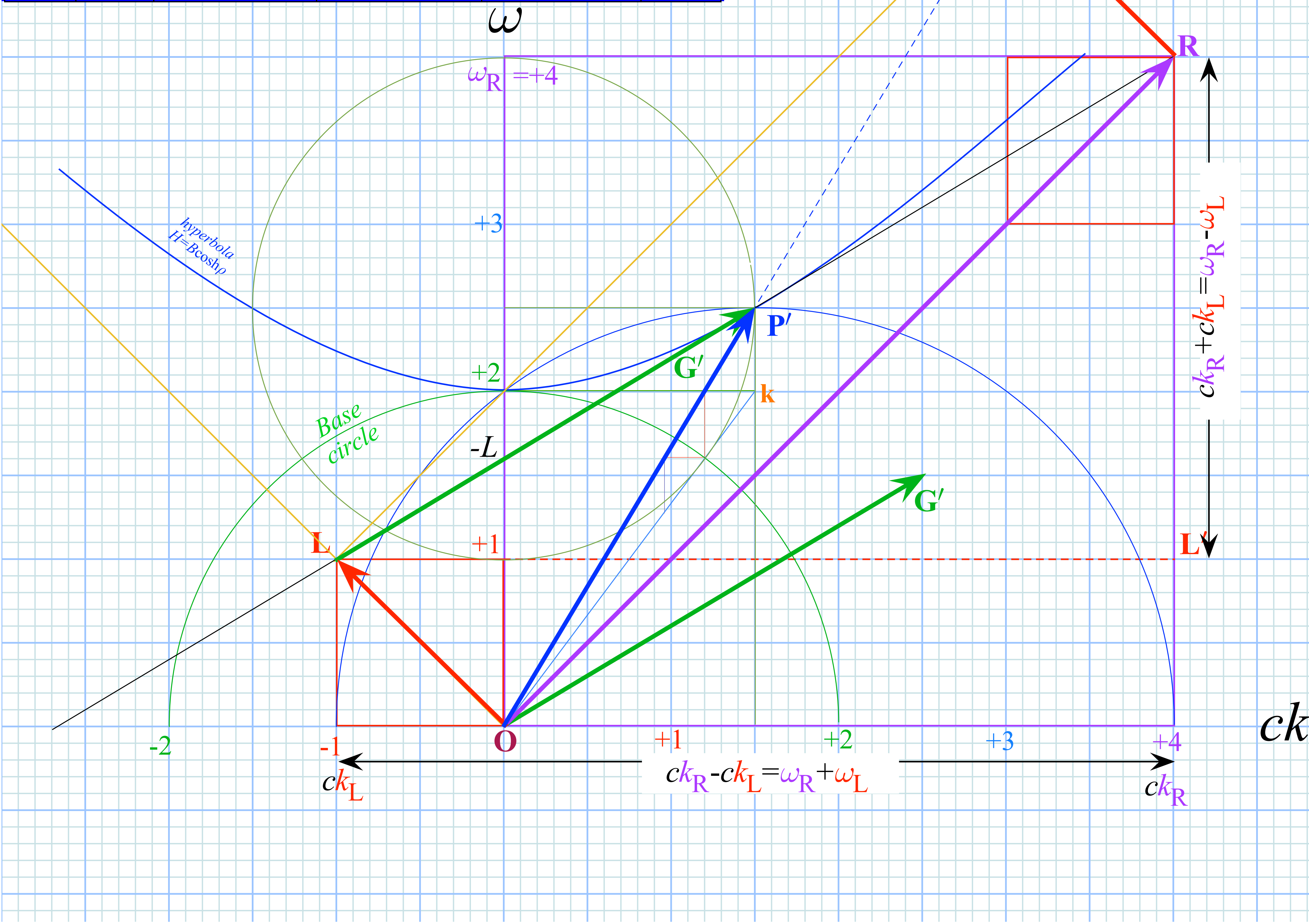

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phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
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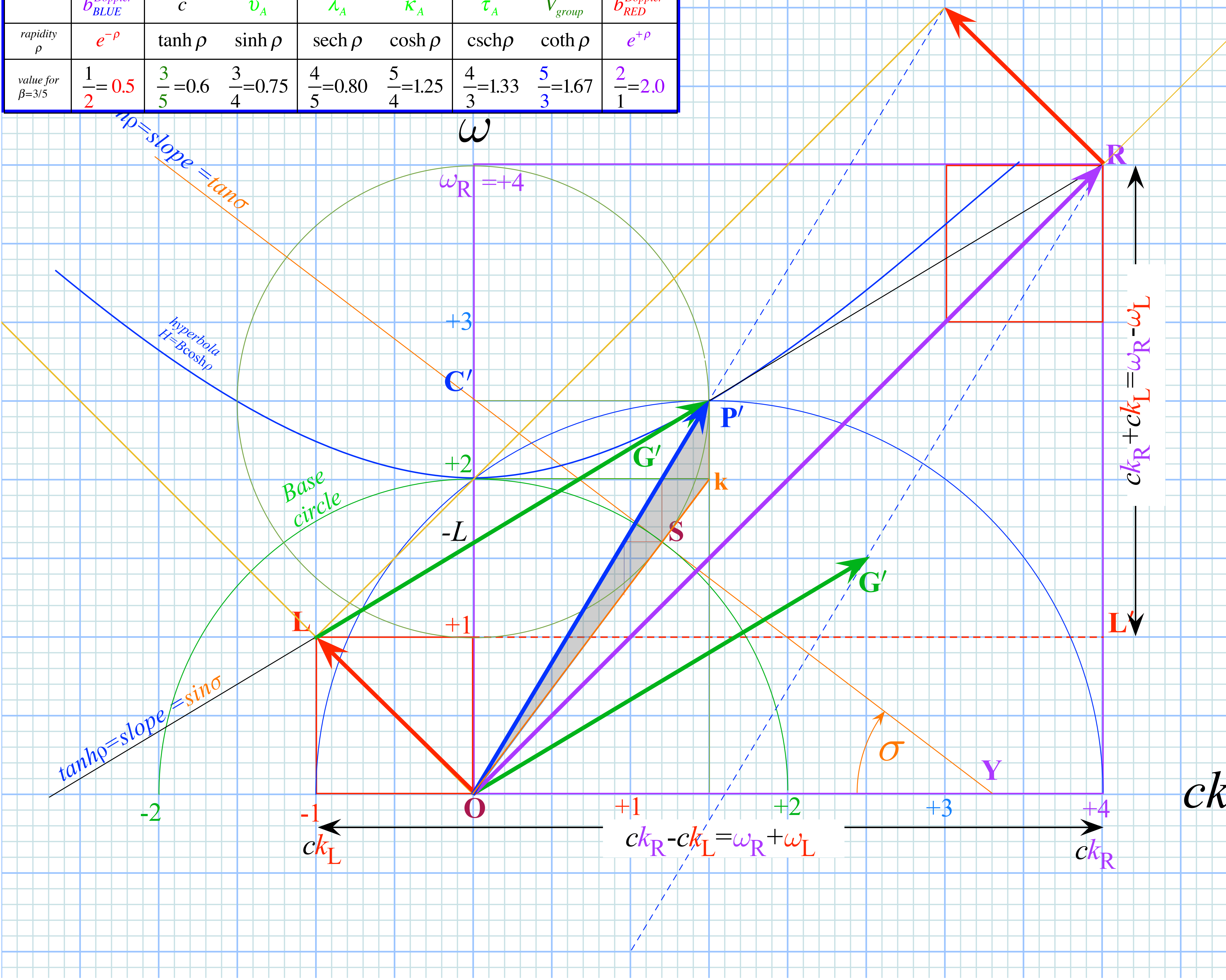

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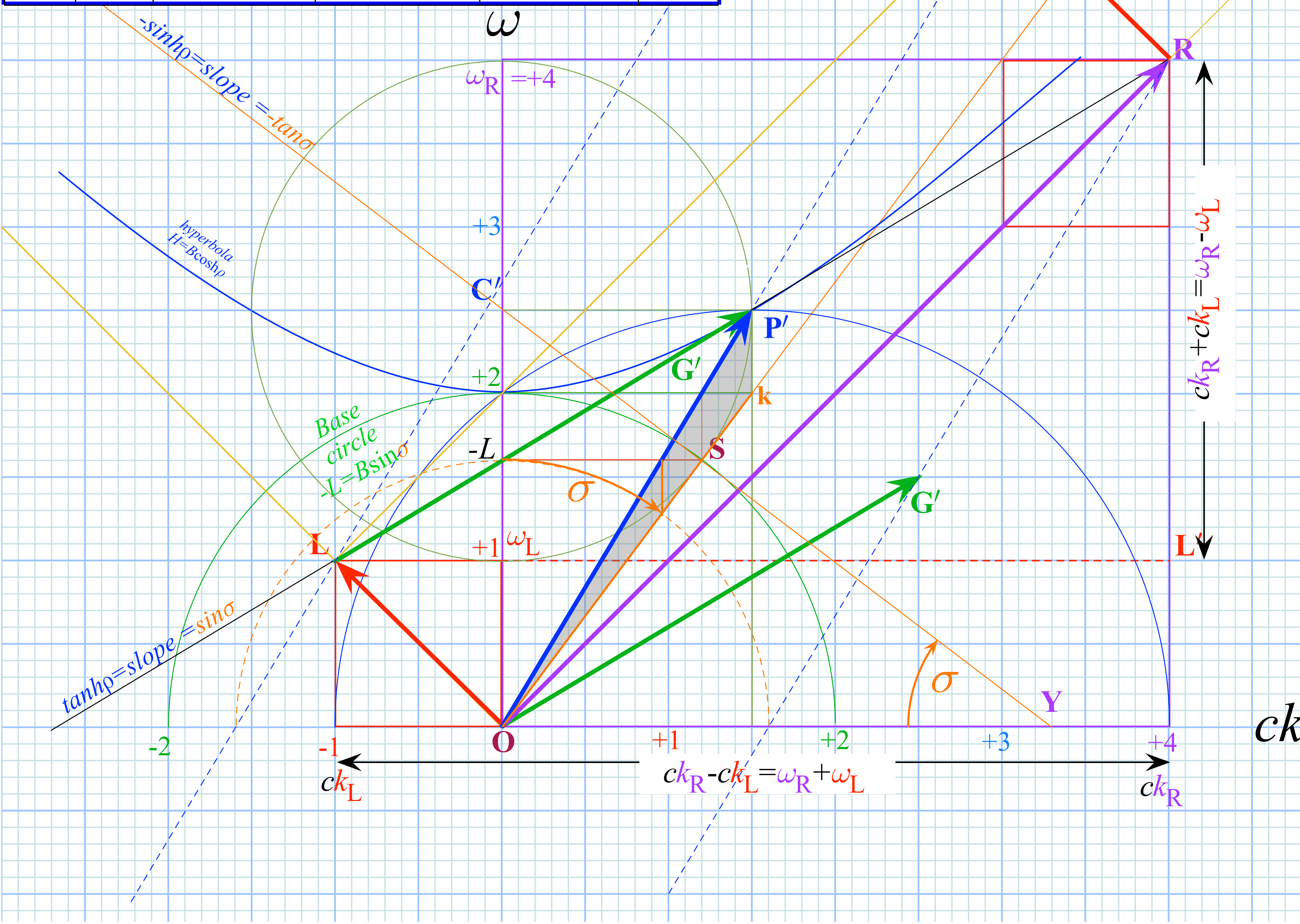

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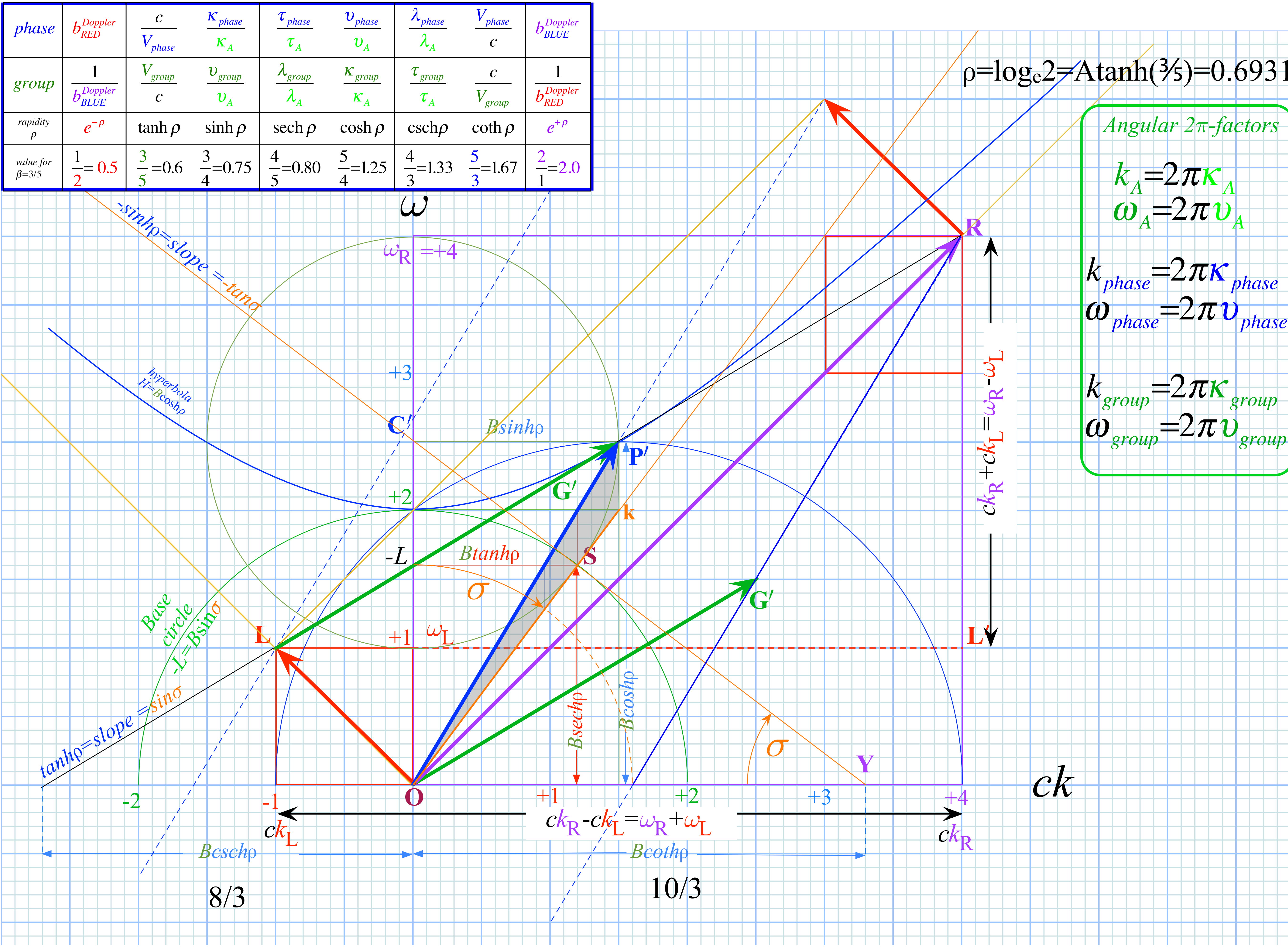

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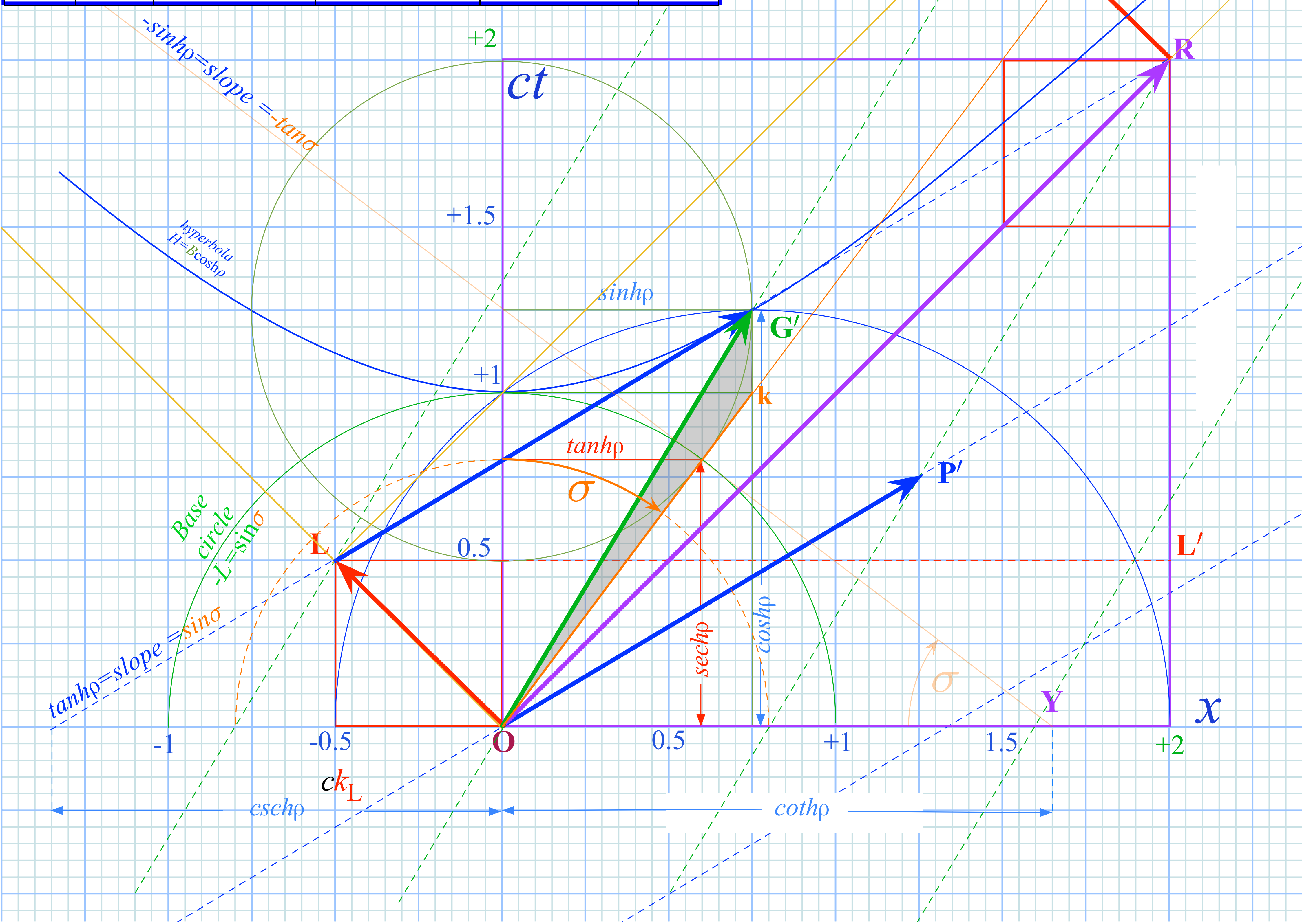

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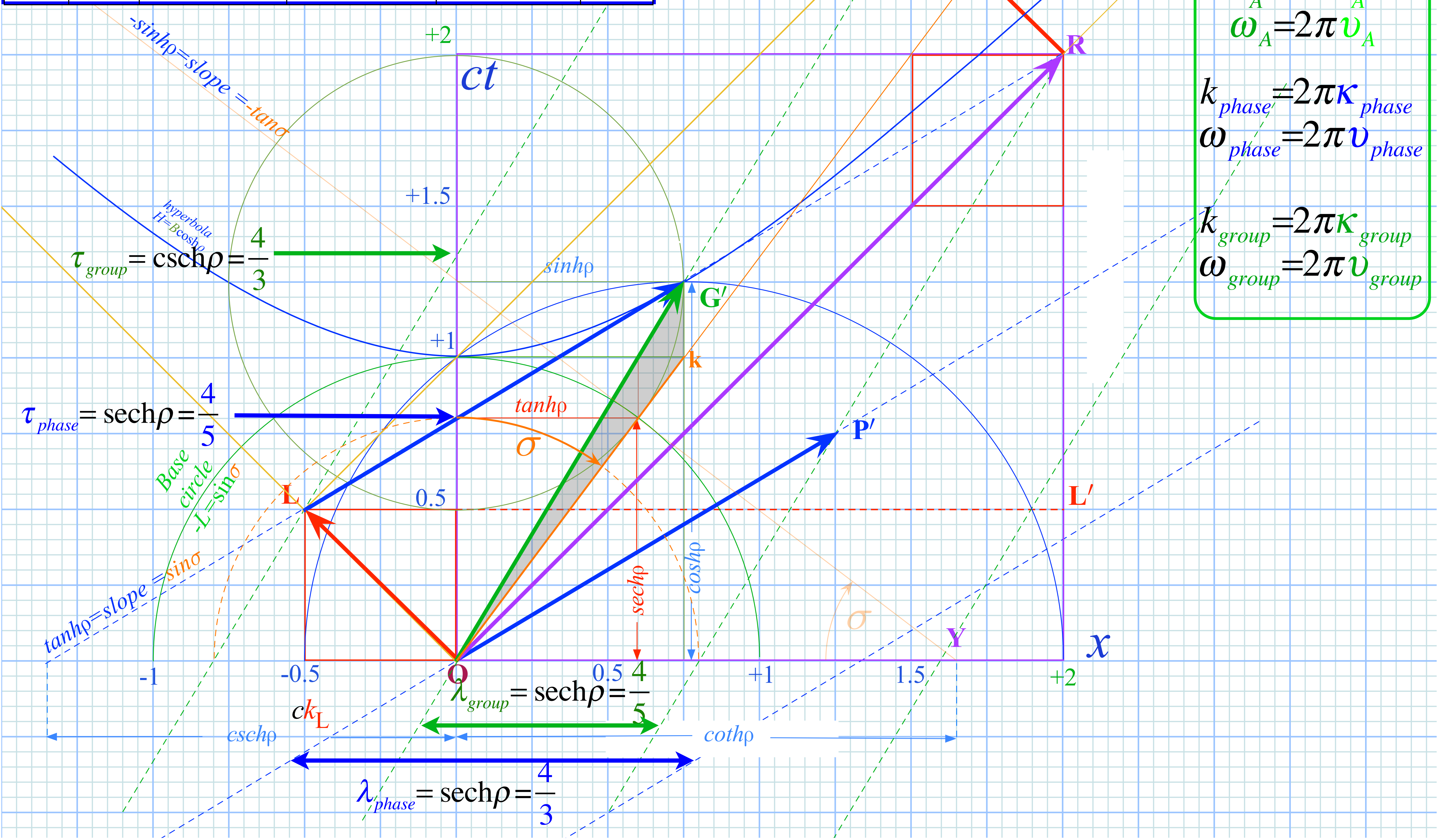
$$k_{group}=2\pi\kappa_{group}$$
$$\omega_{group}=2\pi\upsilon_{group}$$

phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$



Angular 2π-factors

$k_A = 2\pi\kappa_A$

$\omega_A = 2\pi\upsilon_A$

$k_{phase} = 2\pi\kappa_{phase}$

$\omega_{phase} = 2\pi\upsilon_{phase}$

$k_{group} = 2\pi\kappa_{group}$

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Lorentz transformations...

write **G′** *and* **P′** *in terms of* **G** *and* **P** *using* $\cosh\rho$ *and* $\sinh\rho$

$$\mathbf{G}' = \begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

$$= \upsilon_A \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cosh\rho + \upsilon_A \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sinh\rho$$

$$\mathbf{G}' = \mathbf{G}\cosh\rho + \mathbf{P}\sinh\rho$$

$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

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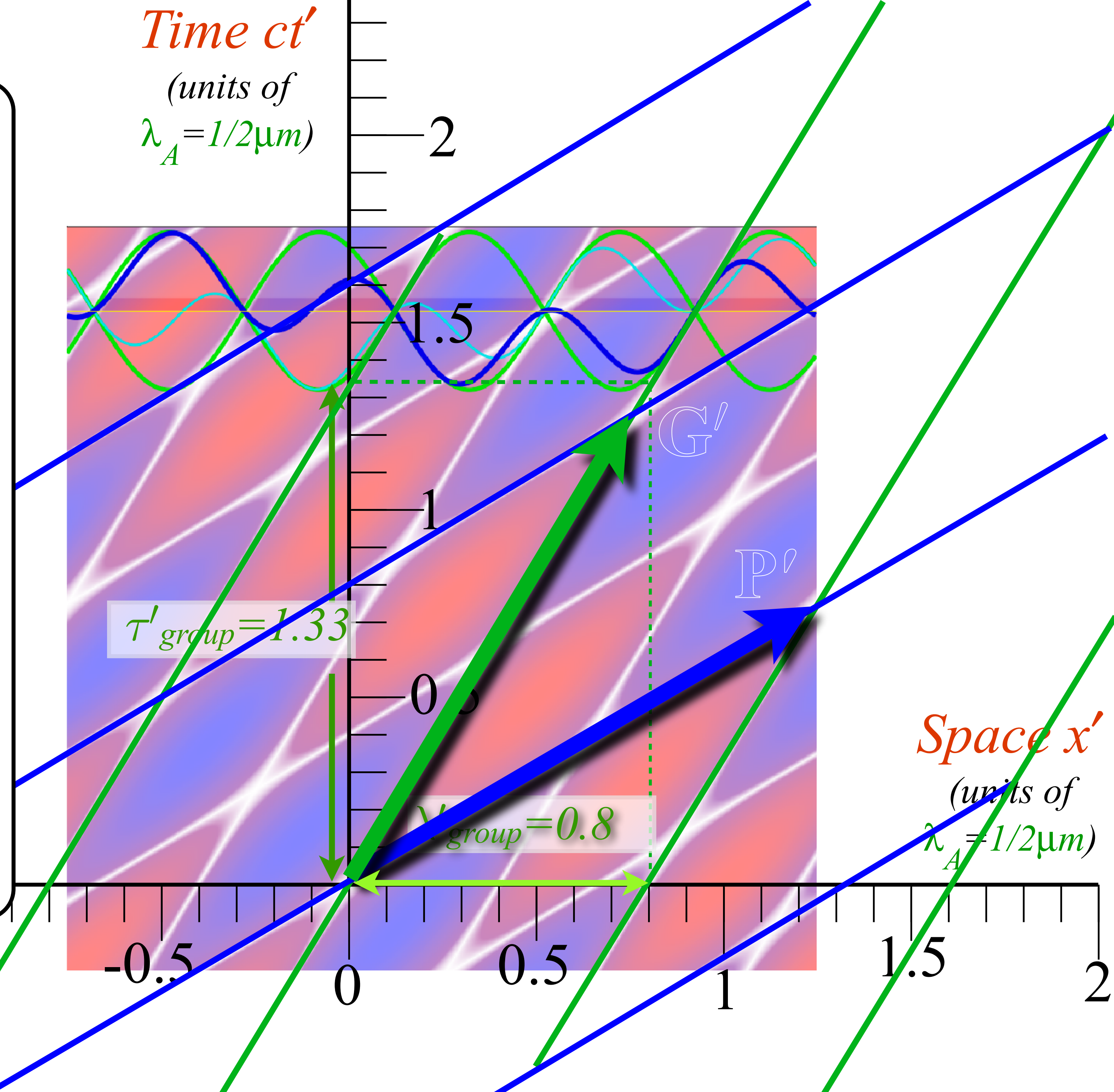

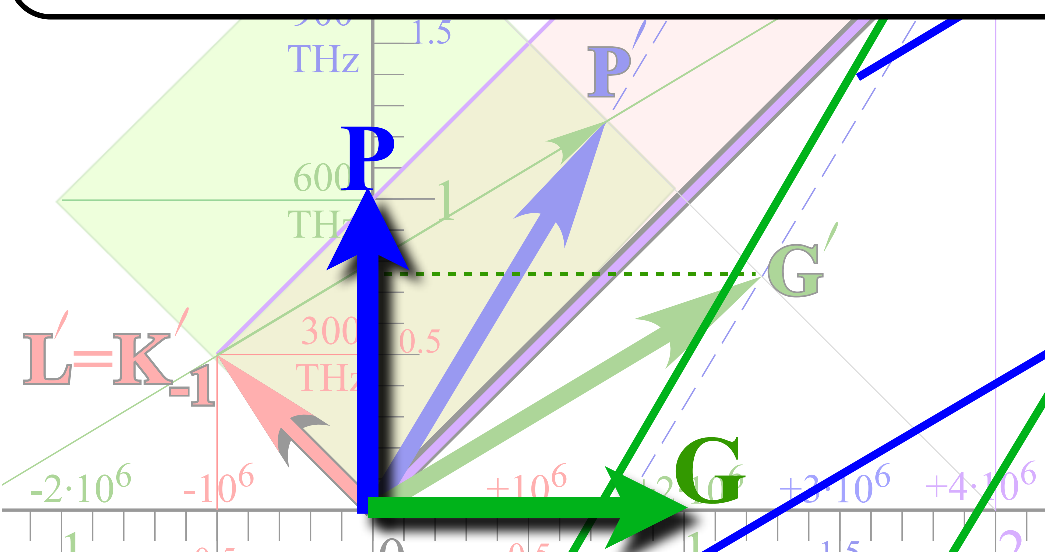

$$\begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix}$$

Lorentz transform matrix

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$\coth\rho$	$e^{+\rho}$
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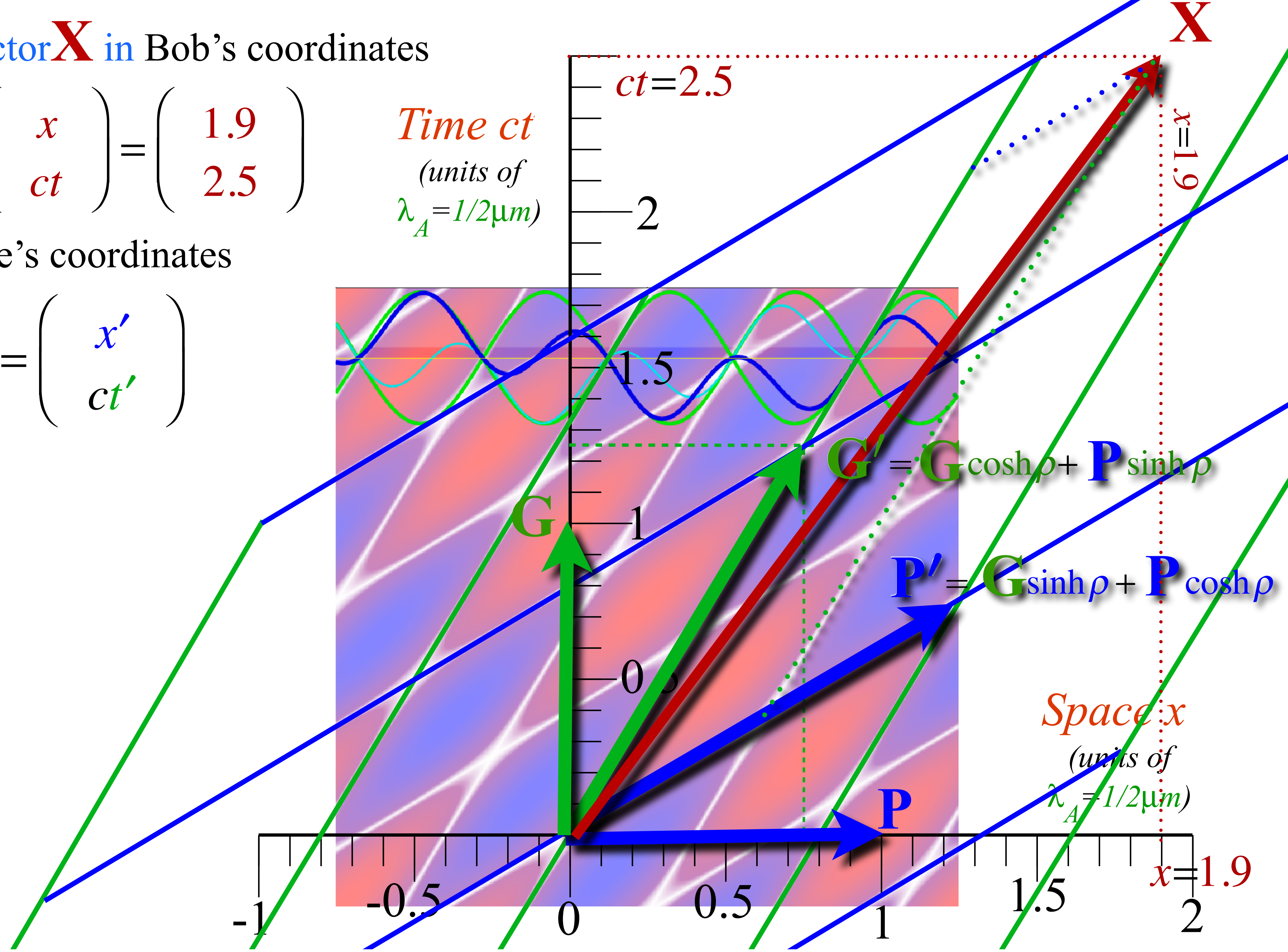
Lorentz transformations of coordinates

Space-time position vector **X** in Bob's coordinates

$$\mathbf{X} = x\mathbf{P} + ct\mathbf{G} = \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} 1.9 \\ 2.5 \end{pmatrix}$$

Same vector **X** in Alice's coordinates

$$\mathbf{X} = x'\mathbf{P}' + ct'\mathbf{G}' = \begin{pmatrix} x' \\ ct' \end{pmatrix}$$



Lorentz transform matrix for $u/c=3/5$

$$\begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} = \begin{pmatrix} \frac{5}{4} & \frac{3}{4} \\ \frac{3}{4} & \frac{5}{4} \end{pmatrix}$$

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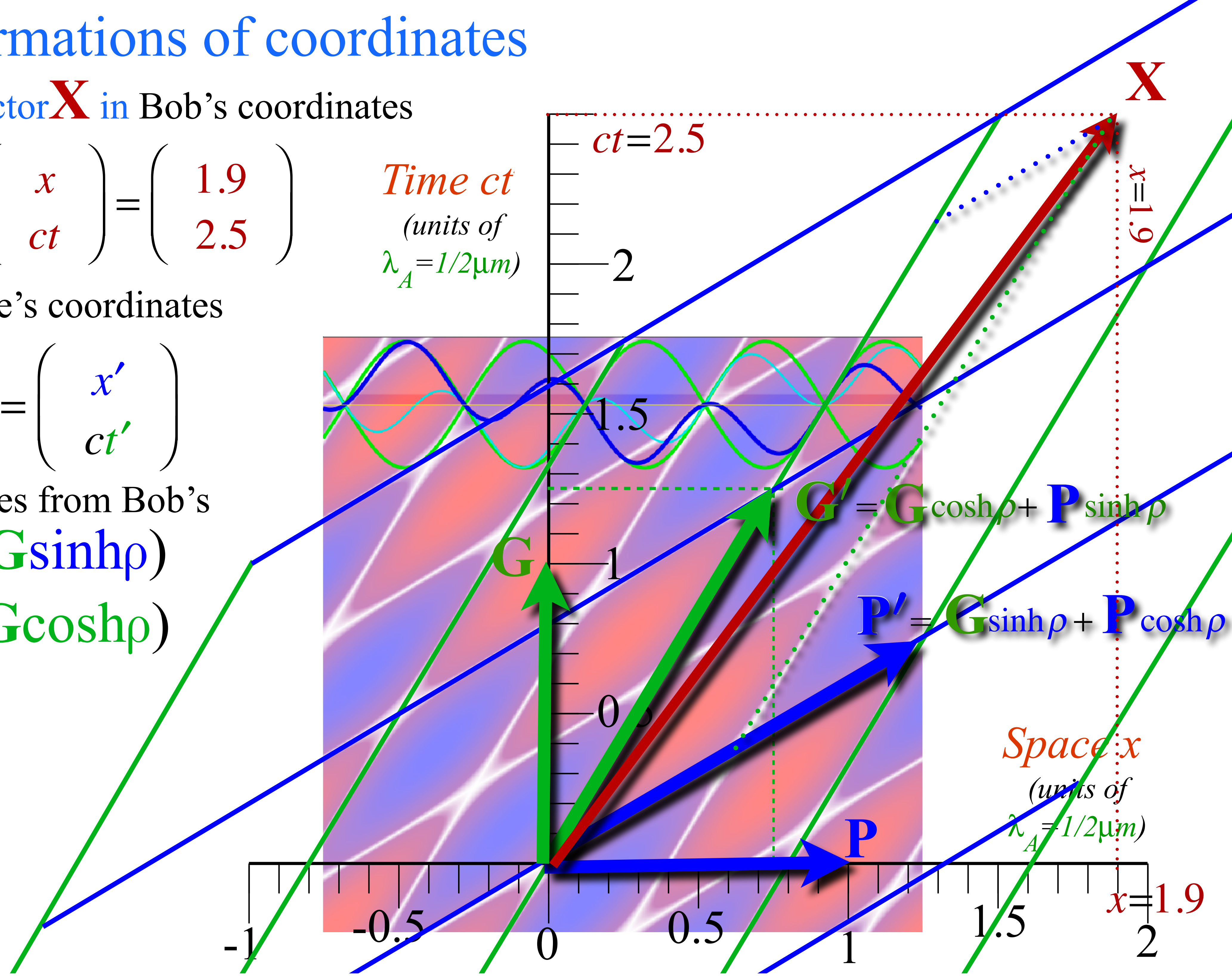
$$\mathbf{X} = x'(\mathbf{P}\cosh\rho + \mathbf{G}\sinh\rho) + ct'(\mathbf{P}\sinh\rho + \mathbf{G}\cosh\rho)$$



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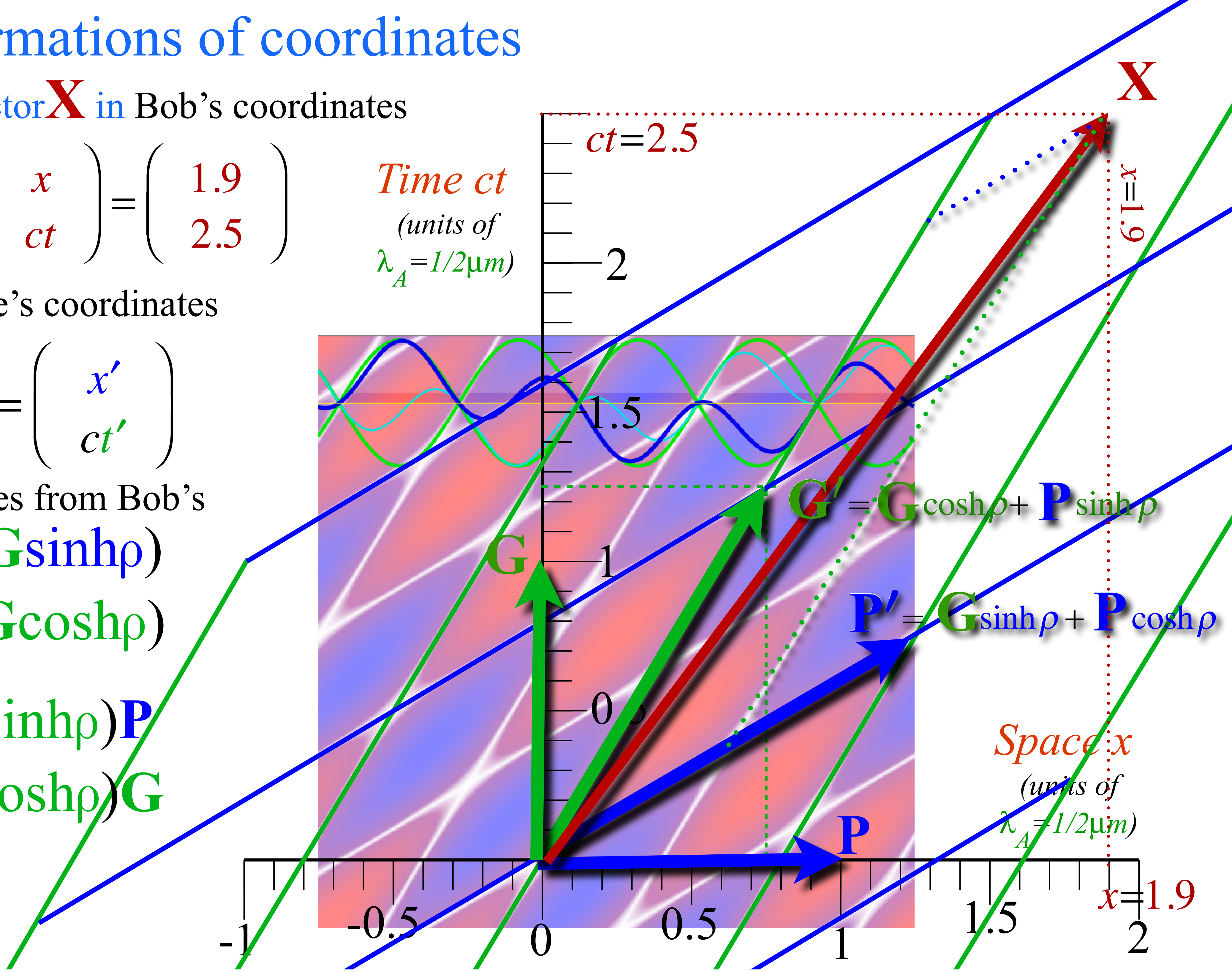
Rearrange:

$$\mathbf{X} = (x'\cosh\rho + ct'\sinh\rho)\mathbf{P} + (x'\sinh\rho + ct'\cosh\rho)\mathbf{G}$$



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Rearrange:

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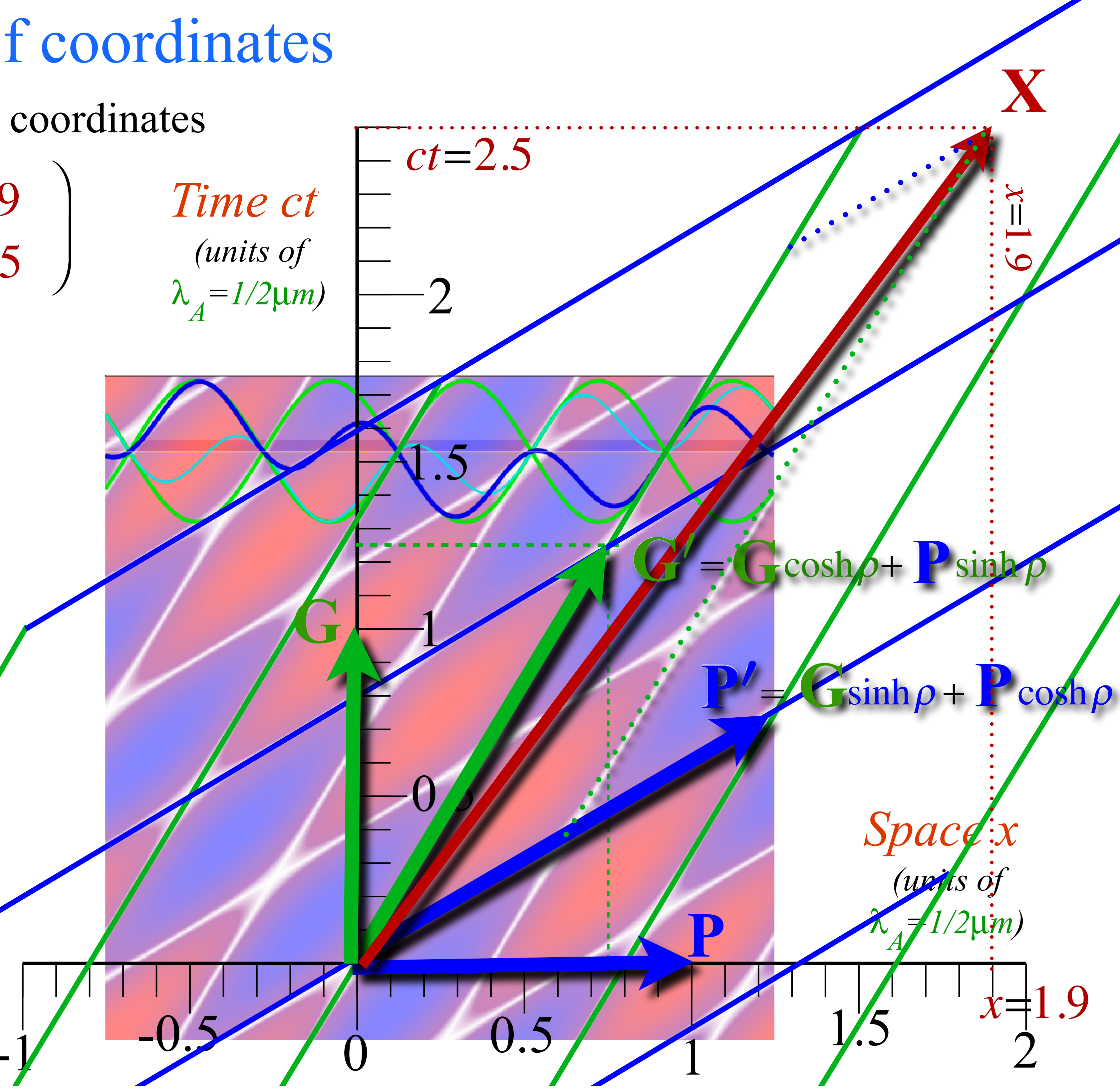
$$+\, (x'\sinh\rho + ct'\cosh\rho)\mathbf{G}$$
$$= x\mathbf{P}$$
$$+ ct\mathbf{G}$$

Put this in matrix form:



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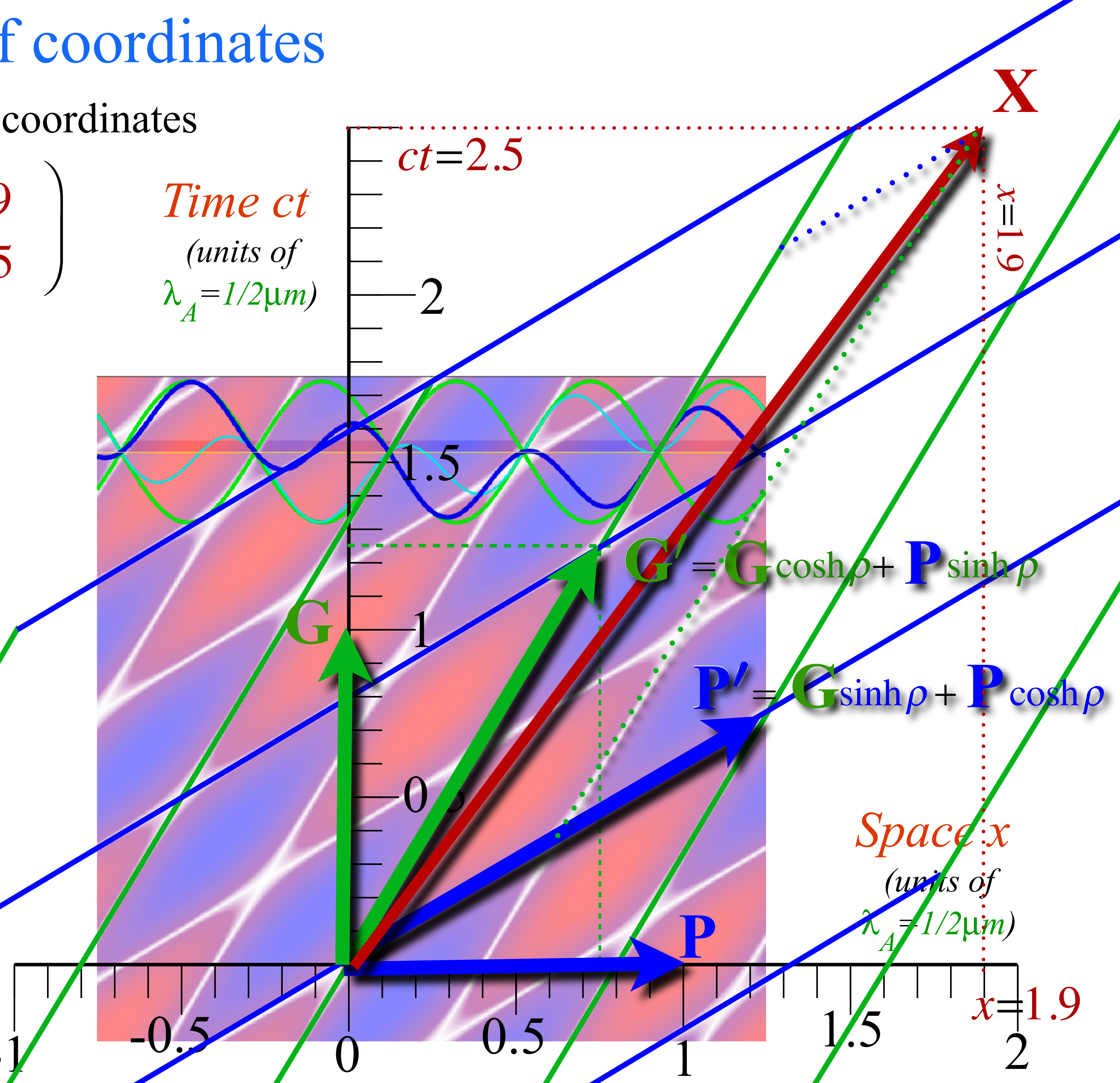

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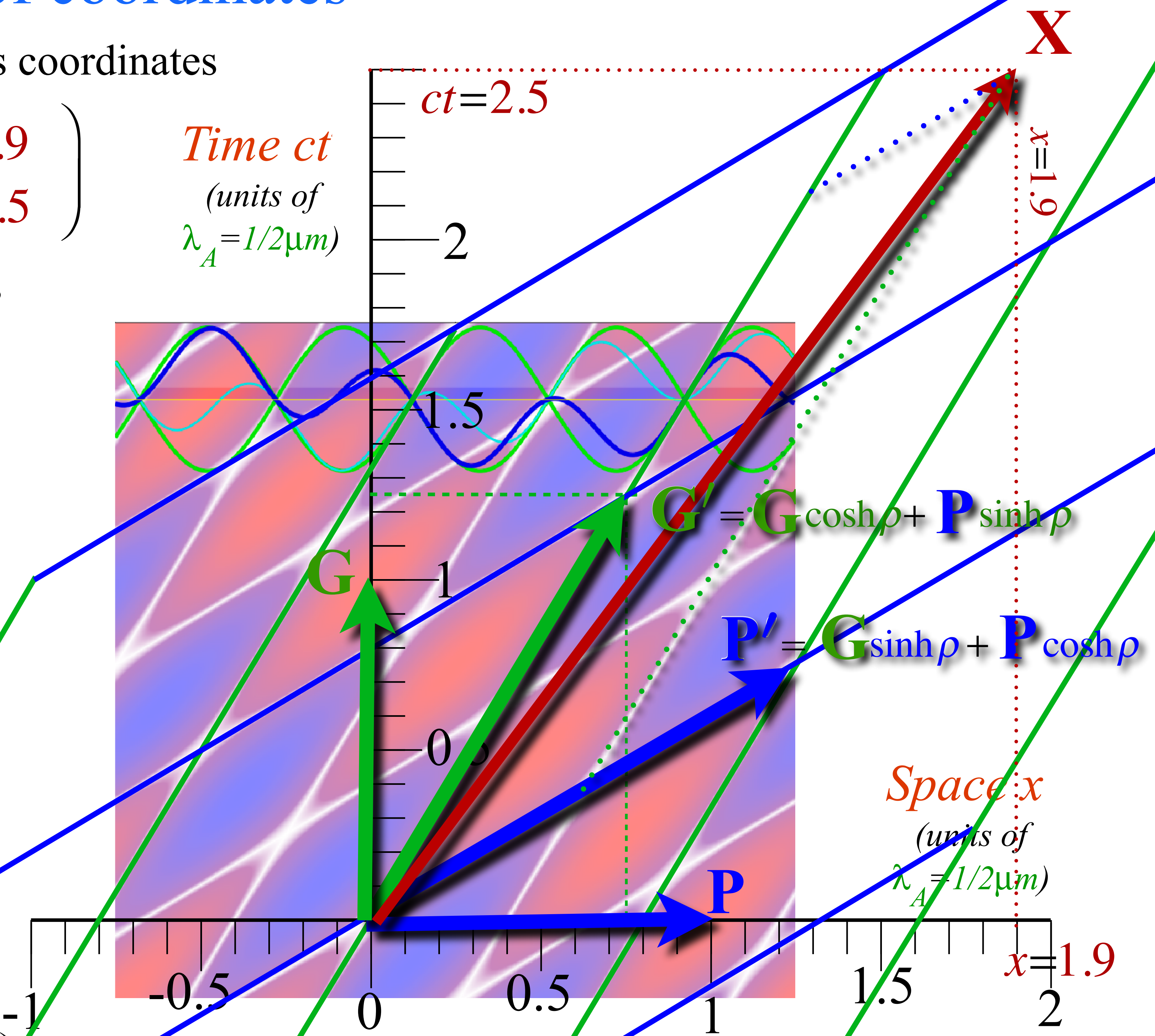

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Matrix inverse:

Set ρ to -ρ

$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \cosh\rho & -\sinh\rho \\ -\sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} 1.9 \\ 2.5 \end{pmatrix}$$

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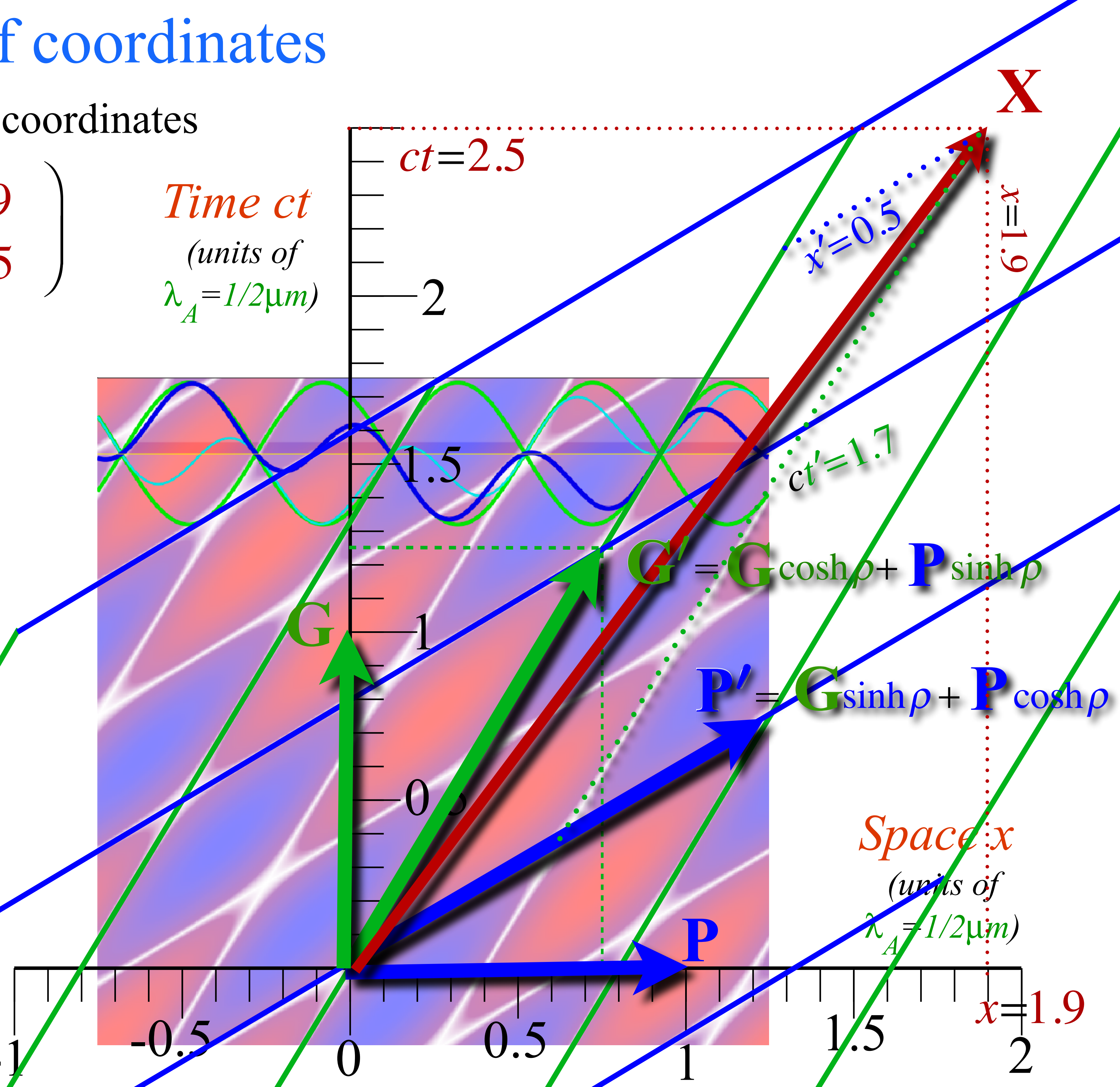

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Matrix inverse:

Set ρ to $-\rho$

$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \cosh\rho & -\sinh\rho \\ -\sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} 1.9 \\ 2.5 \end{pmatrix} = \begin{pmatrix} \frac{5}{4}1.9 - \frac{3}{4}2.5 \\ -\frac{3}{4}1.9 + \frac{5}{4}2.5 \end{pmatrix} = \begin{pmatrix} 0.5 \\ 1.7 \end{pmatrix}$$

Lorentz transform matrix for $u/c = 3/5$

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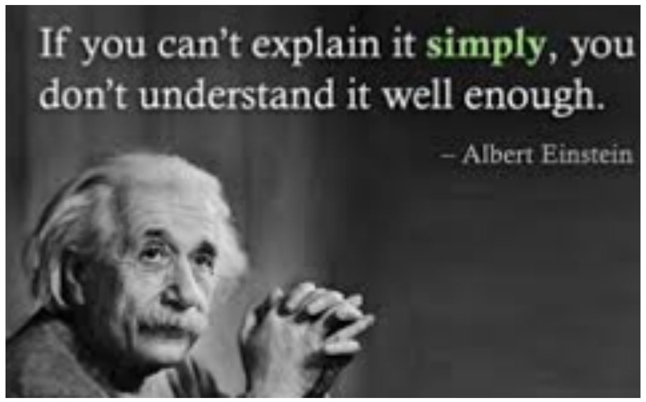
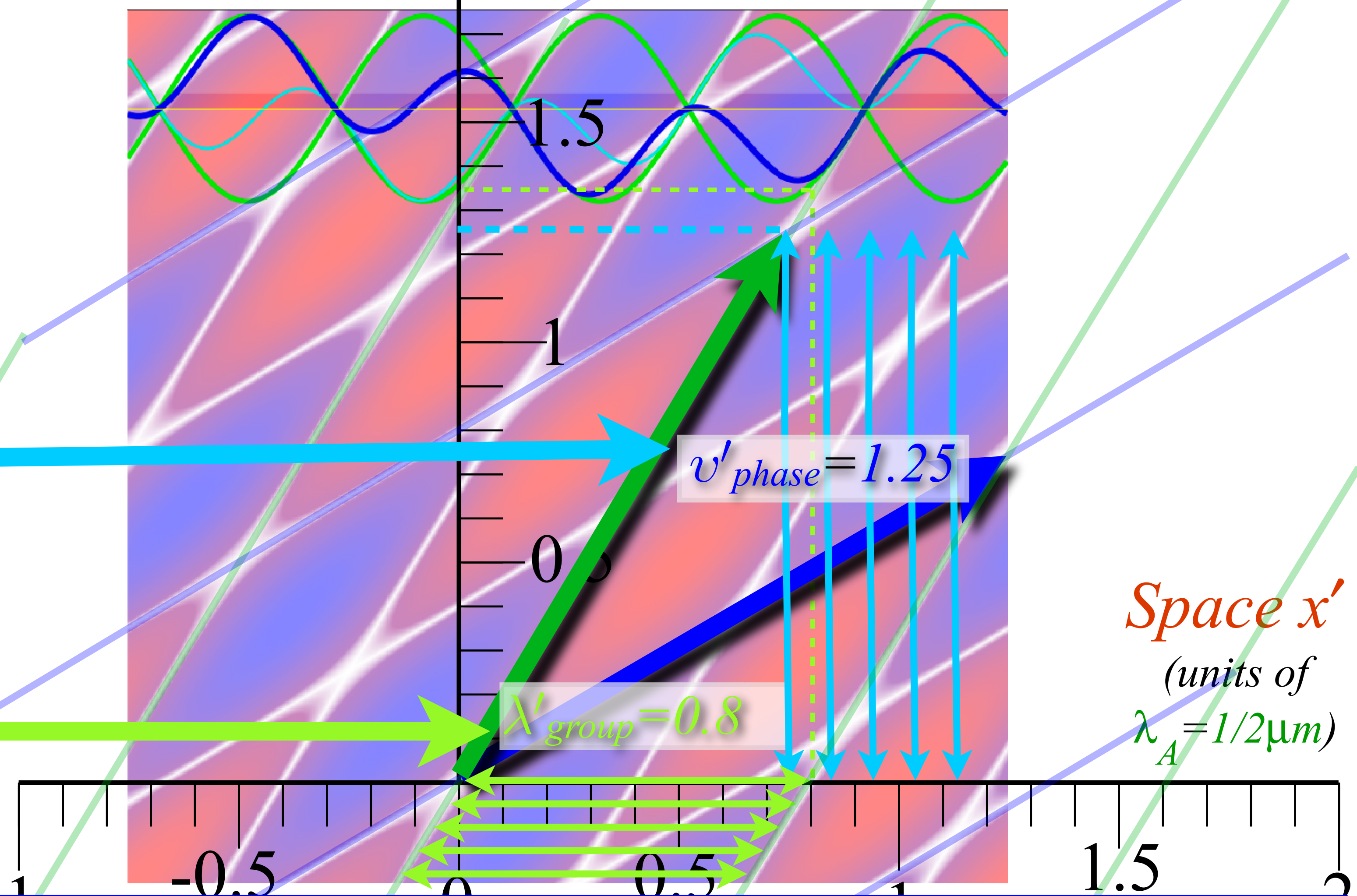


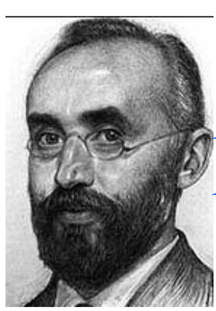
Albert Einstein 1859-1955

Herman Minkowski 1864-1909



This number is called an: ***Einstein time-dilation***
(dilated by 25% here)

This number is called a: ***Lorentz length-contraction***
(contracted by 20% here)



Hendrik A. Lorentz 1853-1928

Old-Fashioned Notation

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$ =0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

[RelaWavity Web Simulation](#)
[Relativistic *Terms (Dual plot w/expanded table)*](#)

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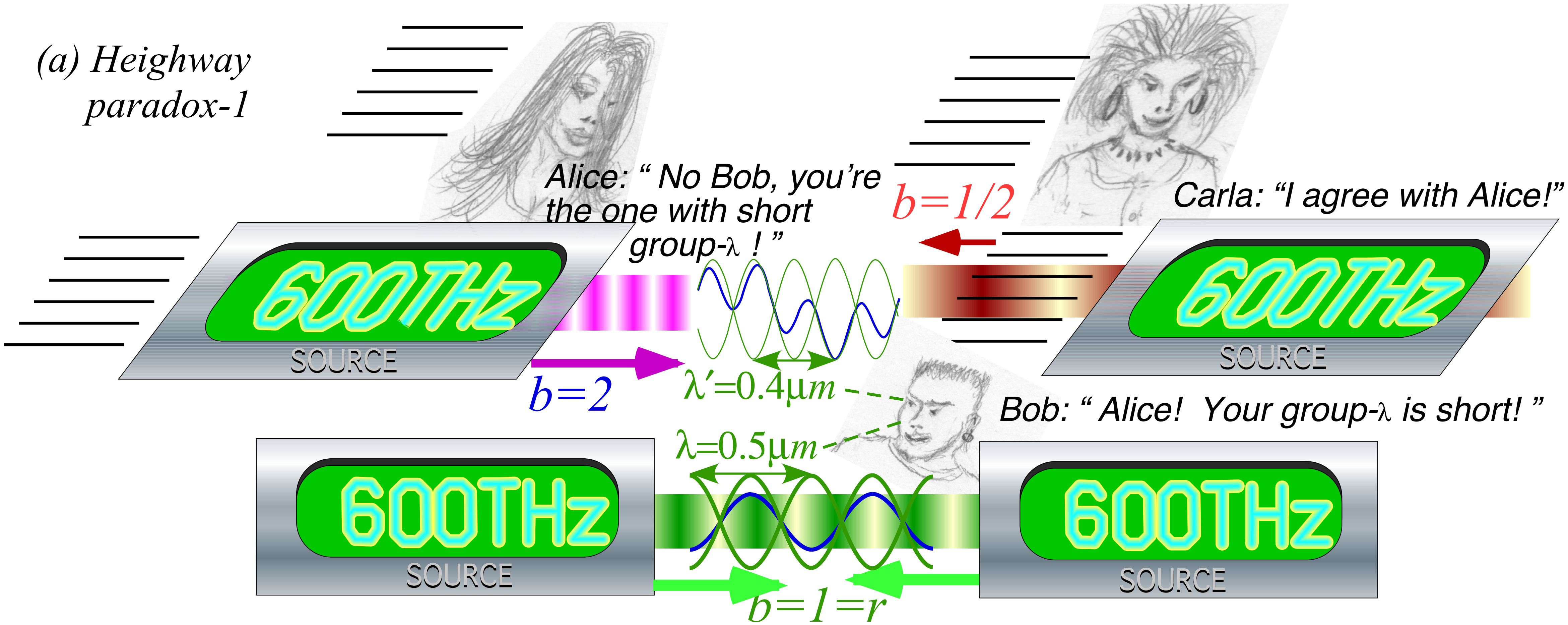

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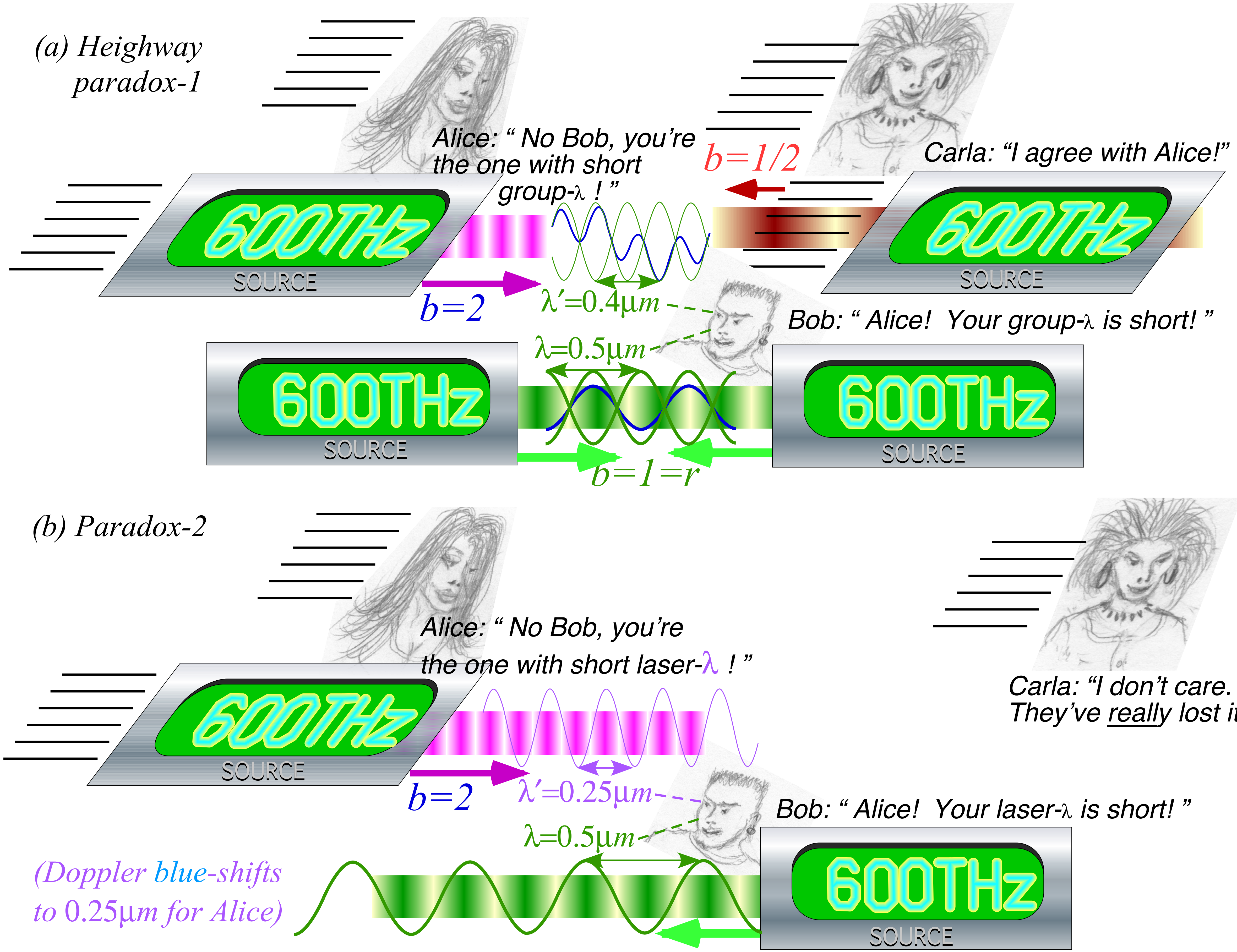

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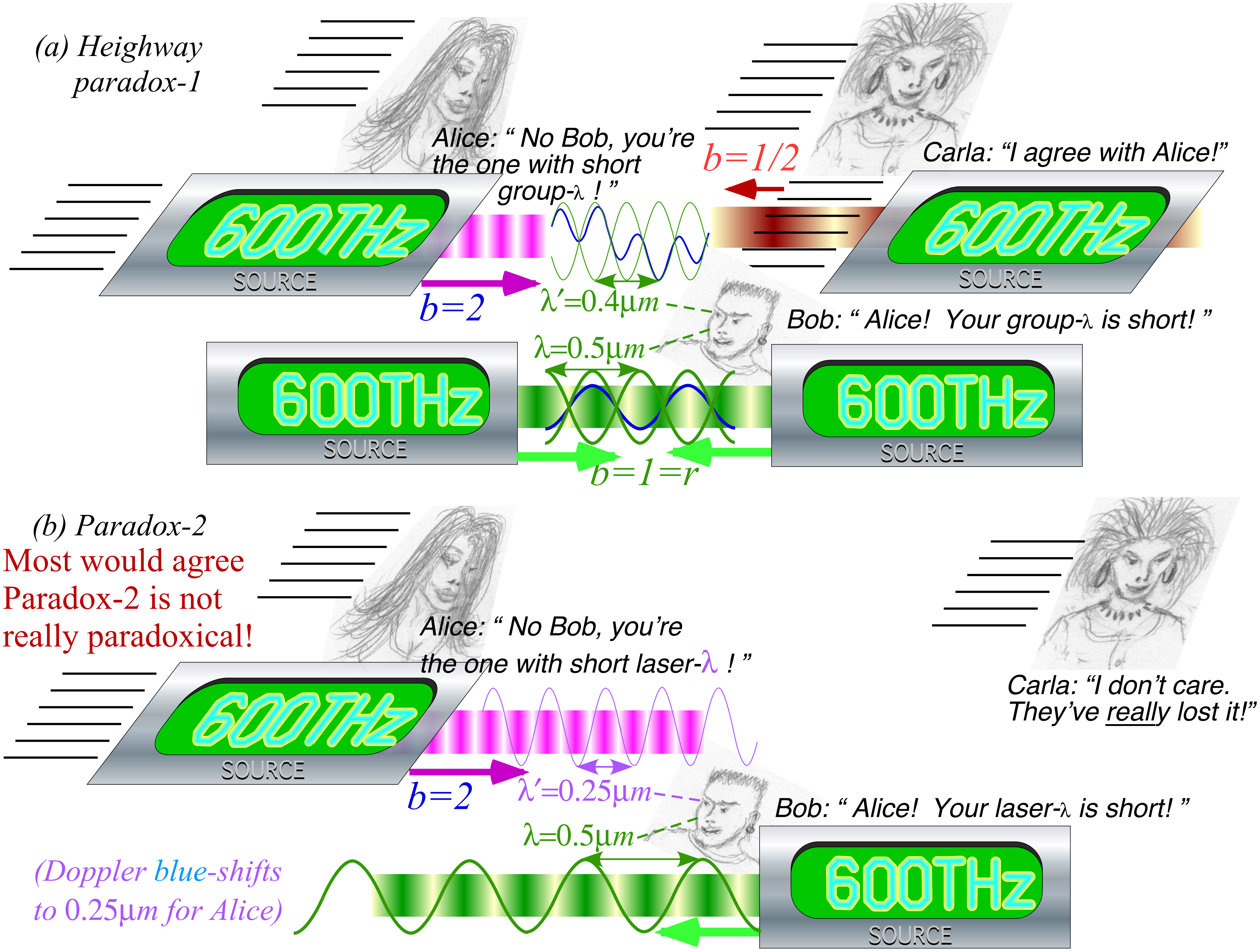

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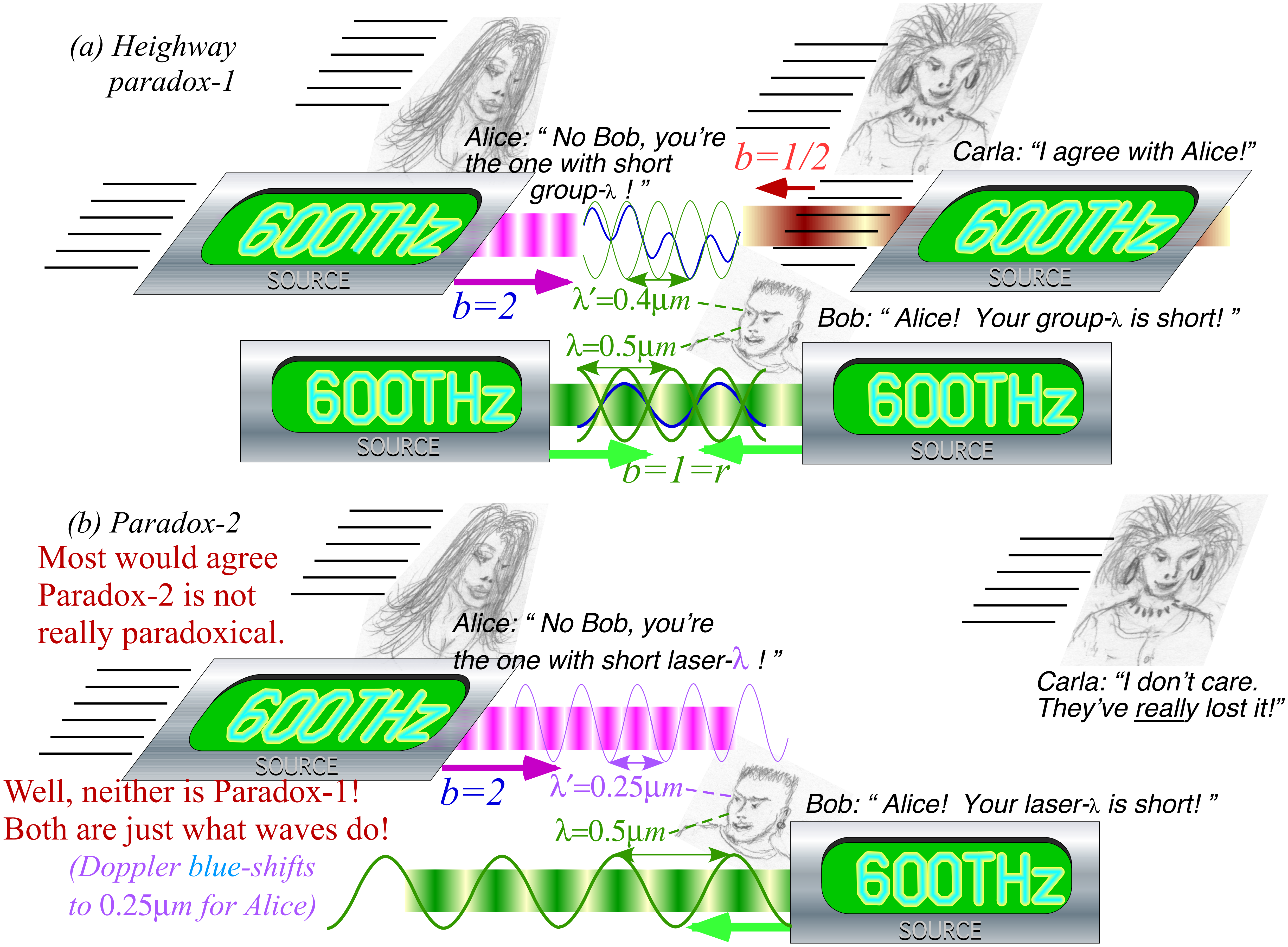

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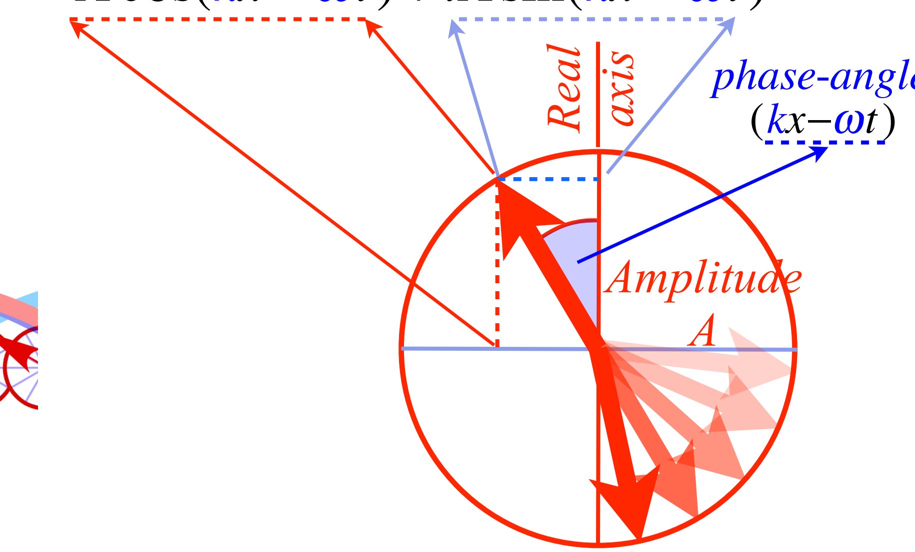

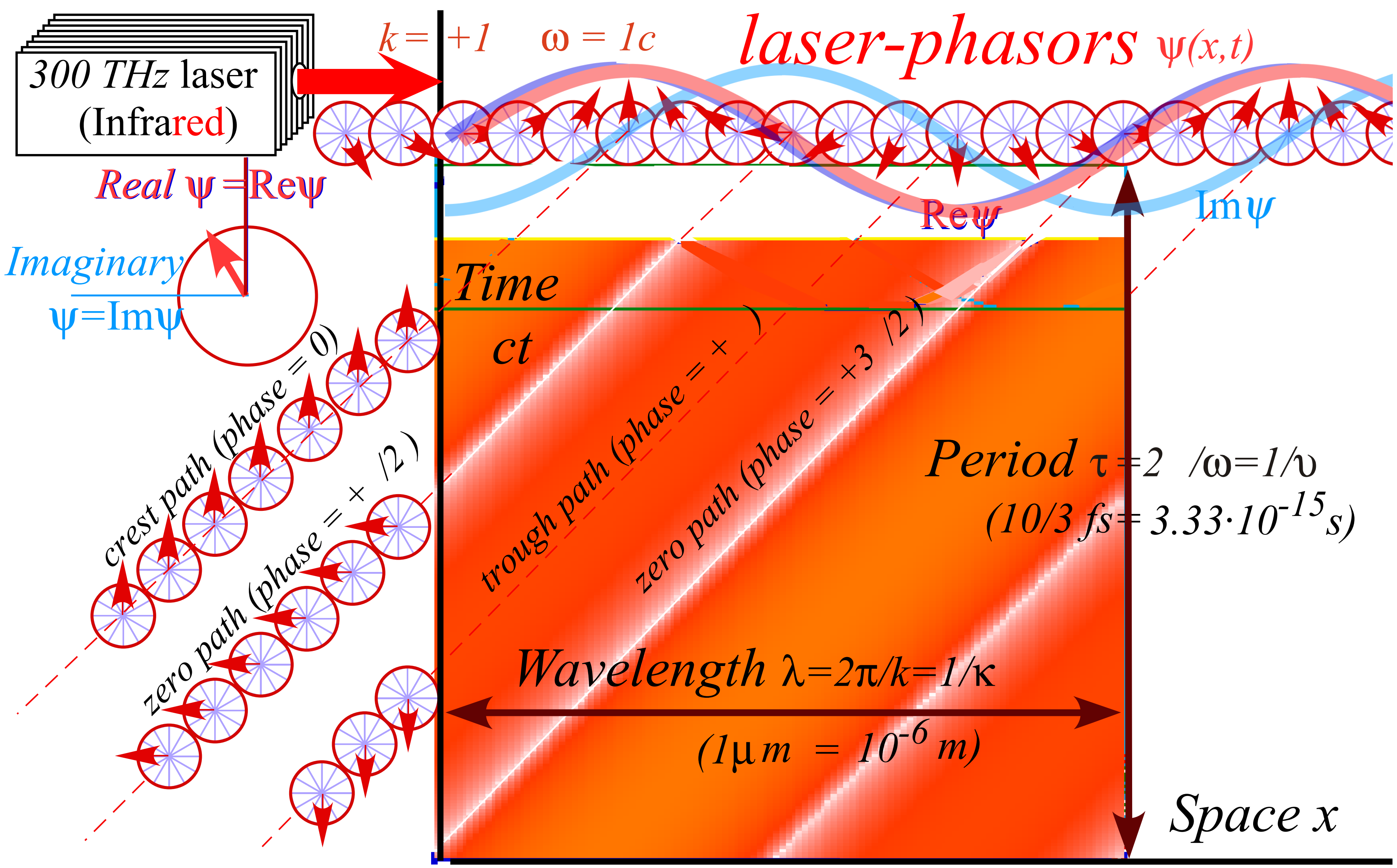

Fig. 4
Unit 3

BohrIt Web Simulation
1 CW *ct vs x* Plot
(ck = +1)

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...derives Lorentz transformations...

phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$

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Review: rapidity $\rho = \rho_{AB}$, Doppler shifts $e^{\pm\rho}$, and SR velocity parameter $V_{group}/c = \beta_{AB} = u_{AB}/c = \tanh\rho_{AB}$

Geometric construction steps 1-4 : 1-octave Doppler ($e^{+\rho} = 2$, $e^{-\rho} = \frac{1}{2}$), ($\beta_{AB} = u_{AB}/c = 3/5$)

Reviewing wave coefficients we'll need to know (backwards and forwards)

Comparison of group and phase dynamics: *FAST(er)* ($\beta = u/c = 3/5$) *vs* SLOW(er) ($\beta = u/c = 1/5$)

Thales Mean Geometry (*Thales of Miletus 624-543 BCE*) and its role in Relawavity

Geometric construction steps 5,6,...: Per-space-time (ω, ck) dispersion hyperbola $\omega = B\cosh\rho$...

A quick flip to space-time *(ct,x)* construction: Minkowski coordinate grid

Lorentz transformations of Phase vector **P′** and Group vector **G′** in per-space-time

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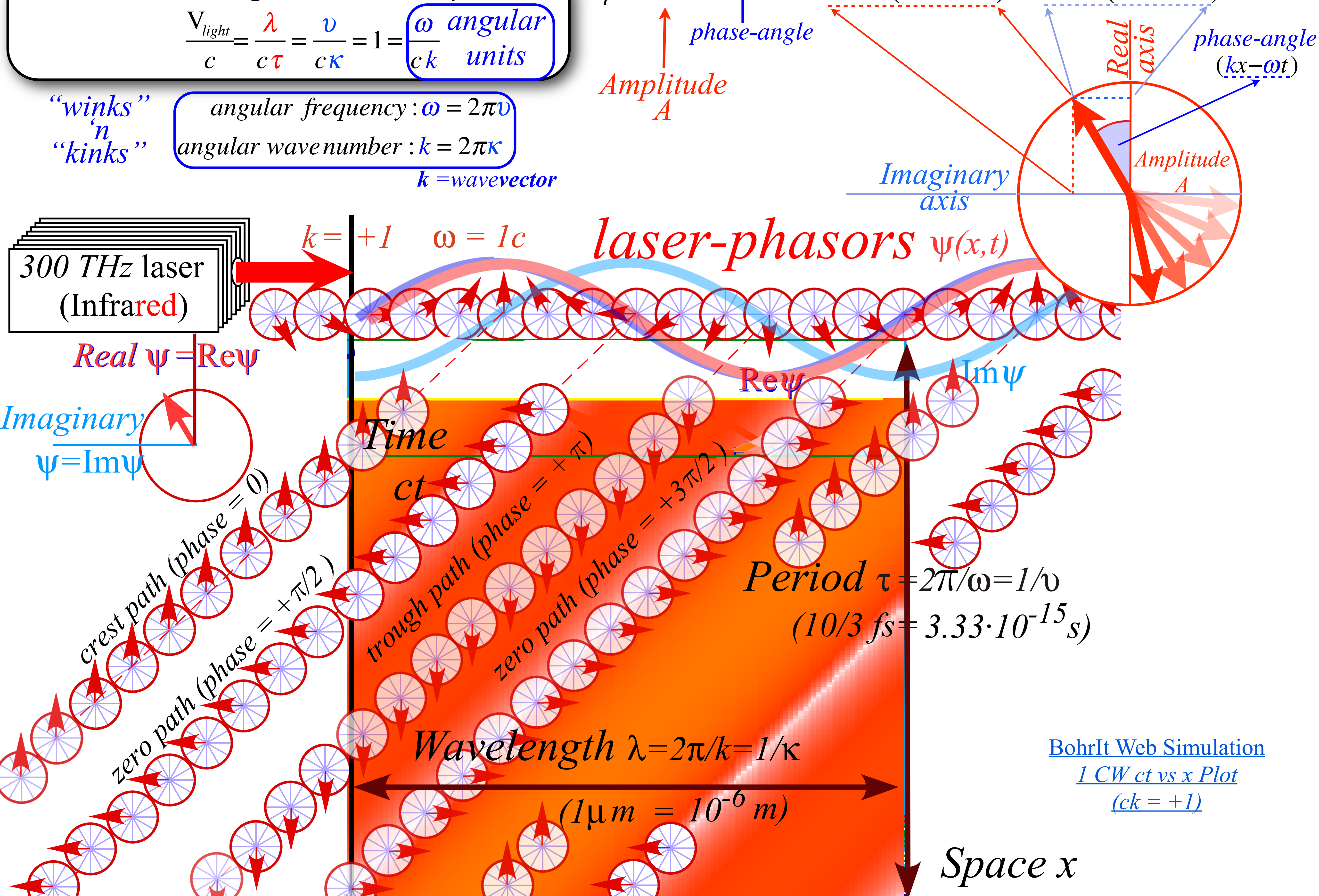
1CW Laser-phasor wave function

$$\psi = A\cdot e^{i(kx-\omega t)} = A\cdot\cos(kx-\omega t) + iA\cdot\sin(kx-\omega t)$$



1CW Laser-phasor wave function

Dimensionless Light wave-velocity c/c=1

$$\frac{V_{light}}{c} = \frac{\lambda}{c\tau} = \frac{\upsilon}{c\kappa} = 1 = \frac{\omega}{ck}\ \text{angular units}$$

"winks" 'n "kinks"

angular frequency: $\omega = 2\pi\upsilon$
angular wavenumber: $k = 2\pi\kappa$

$\boldsymbol{k}$ = wave**vector**

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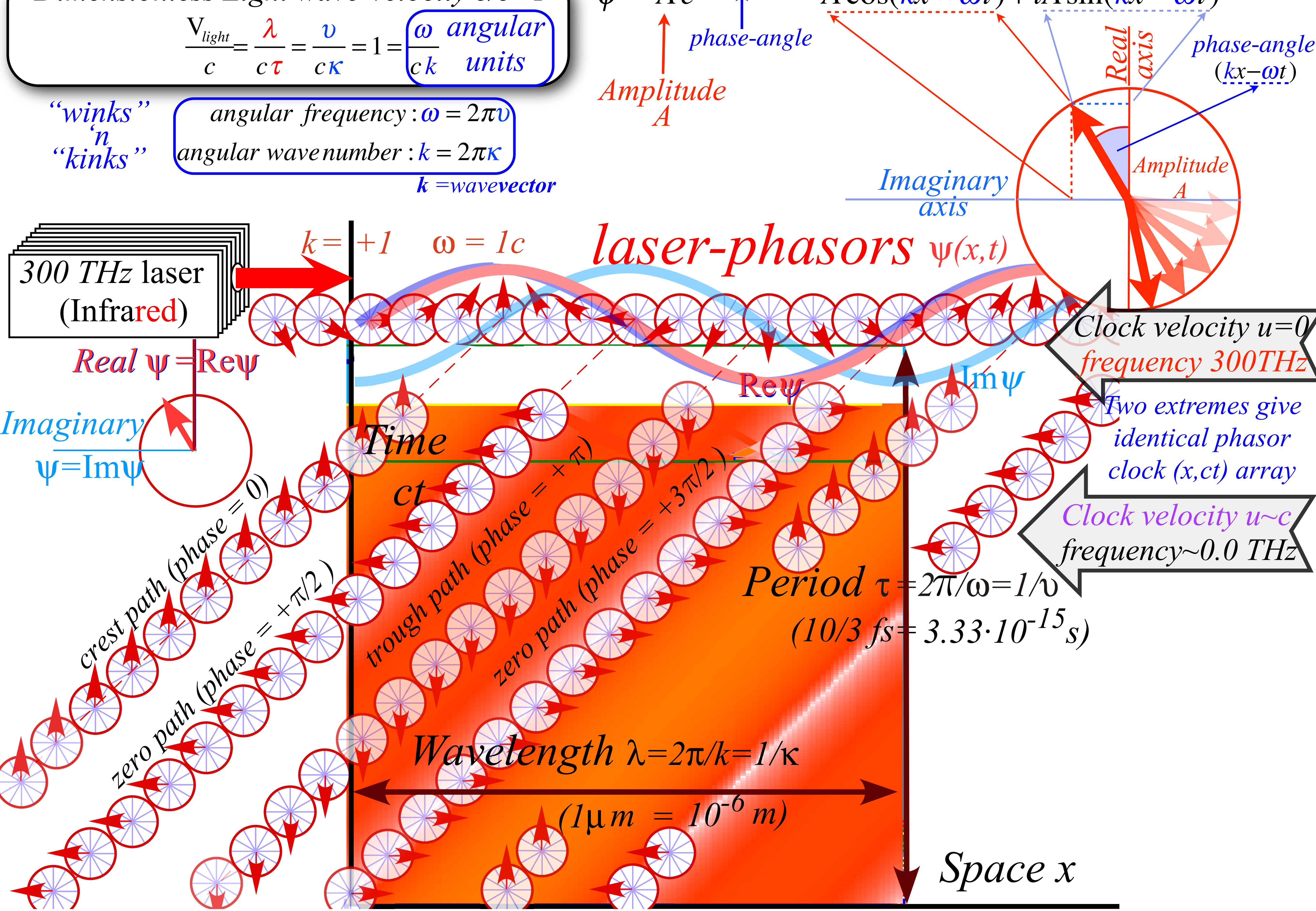

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Amplitude A — phase-angle — phase-angle $(kx-\omega t)$

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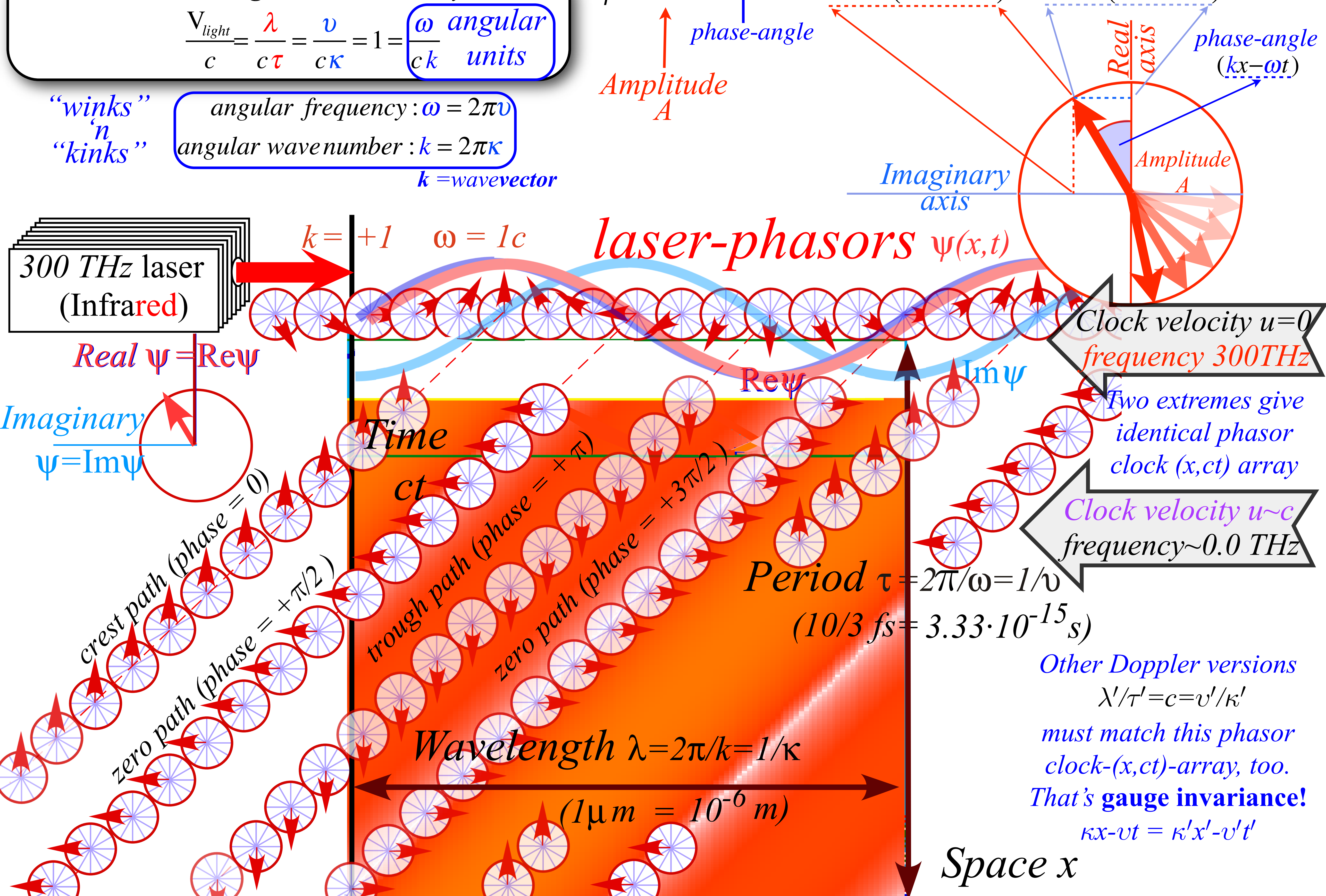

Other Doppler versions

$\lambda'/\tau' = c = \upsilon'/\kappa'$

must match this phasor clock-(x,ct)-array, too.

That's ***gauge invariance!***

$\kappa x - \upsilon t = \kappa' x' - \upsilon' t'$

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Review of Doppler-shift and Rapidity ρ_{AB} calculation: *Galileo's Revenge Part I Lect. 23 p.64-75*

Relating rapidity ρ_{AB} and relativity velocity parameter $\beta_{AB}=u_{AB}/c$

Review of ½-sum-½-difference Phase and Group factors giving relativistic space-axes and time-axes

Colliding-CW space-time (x,ct)-graph *vs* Colliding PW space-time (R,L)-baseball diamond

Review of ½-sum-½-difference of phasor angular velocity: *Galileo's Revenge Part II* (Pirelli site)

Elementary models: 2-comb Moire′ patterns and cosine-law constructions

Bob, Alice, and Carla combine Doppler shifted ½-sum-½-difference Phase and Group factors

Doppler shifted Phase vector $\mathbf{P}'$ and Group vector $\mathbf{G}'$ in per-space-time

Minkowski coordinate grid in space-time

Animations that compare Doppler shifted colliding CW with colliding PW

The 16 parameters of Doppler-shifted 2-CW Minkowski geometry

Doppler shifted Phase parameters

Doppler shifted Group parameters

Lorentz transformation matrix and Two Famous-Name Coefficients

Thales Mean Geometry (*Thales of Miletus 624-543 BCE*) and its role in Relawavity

Detailed geometric construction of relawavity plot for 1-octave Doppler ($\beta_{AB}=u_{AB}/c=3/5$)

➔ Stellar aberration and the Epstein approach to SR ➔

Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

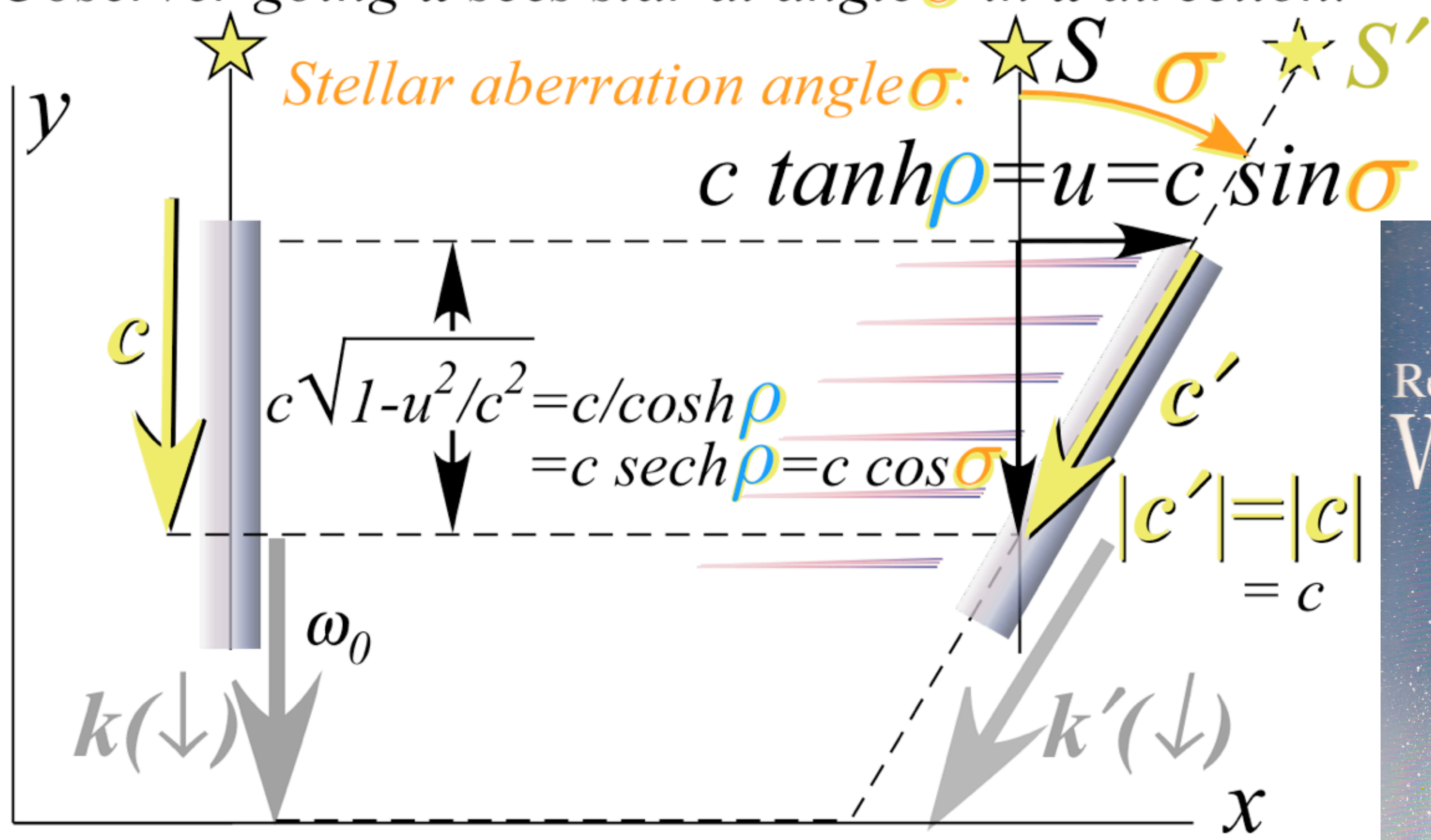
to a Transverse*relativity parameter: Stellar aberration angle σ

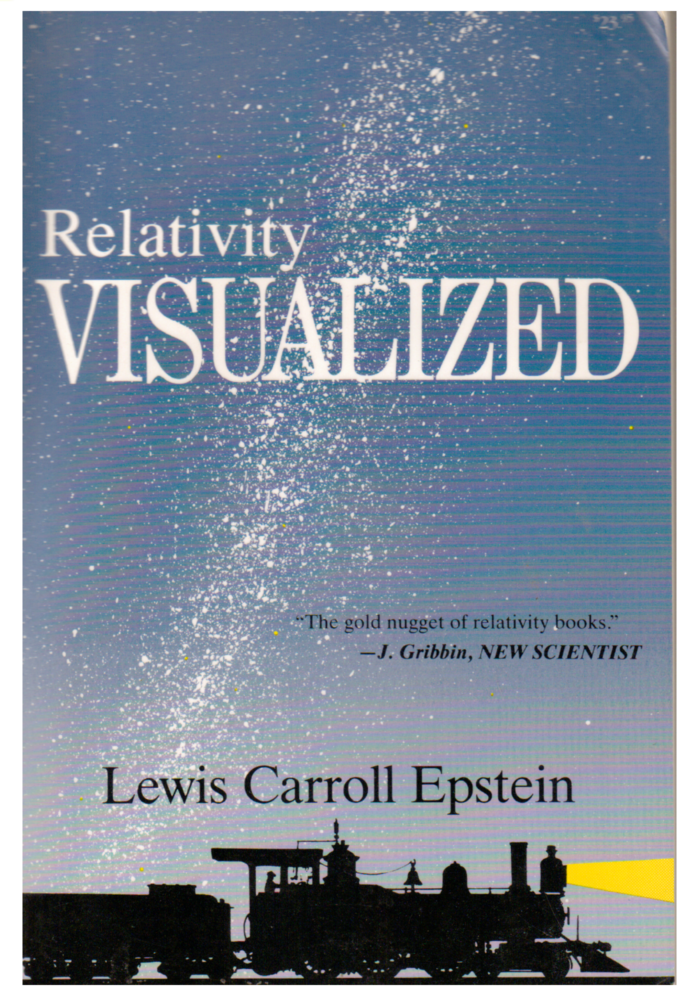
*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-

Observer fixed below star sees it directly overhead.

*Observer going **u** sees star at angle σ in **u** direction.*

We used notion σ for stellar-ab-angle, (a "flipped-out" ρ). Epstein not interested in ρ analysis or in relation of σ and ρ.





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Comparing <u>Longitudinal</u> relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

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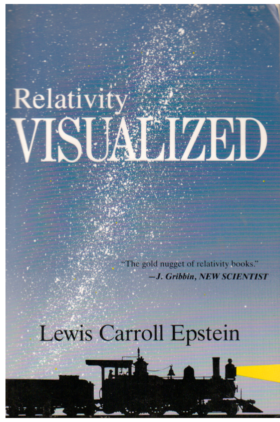
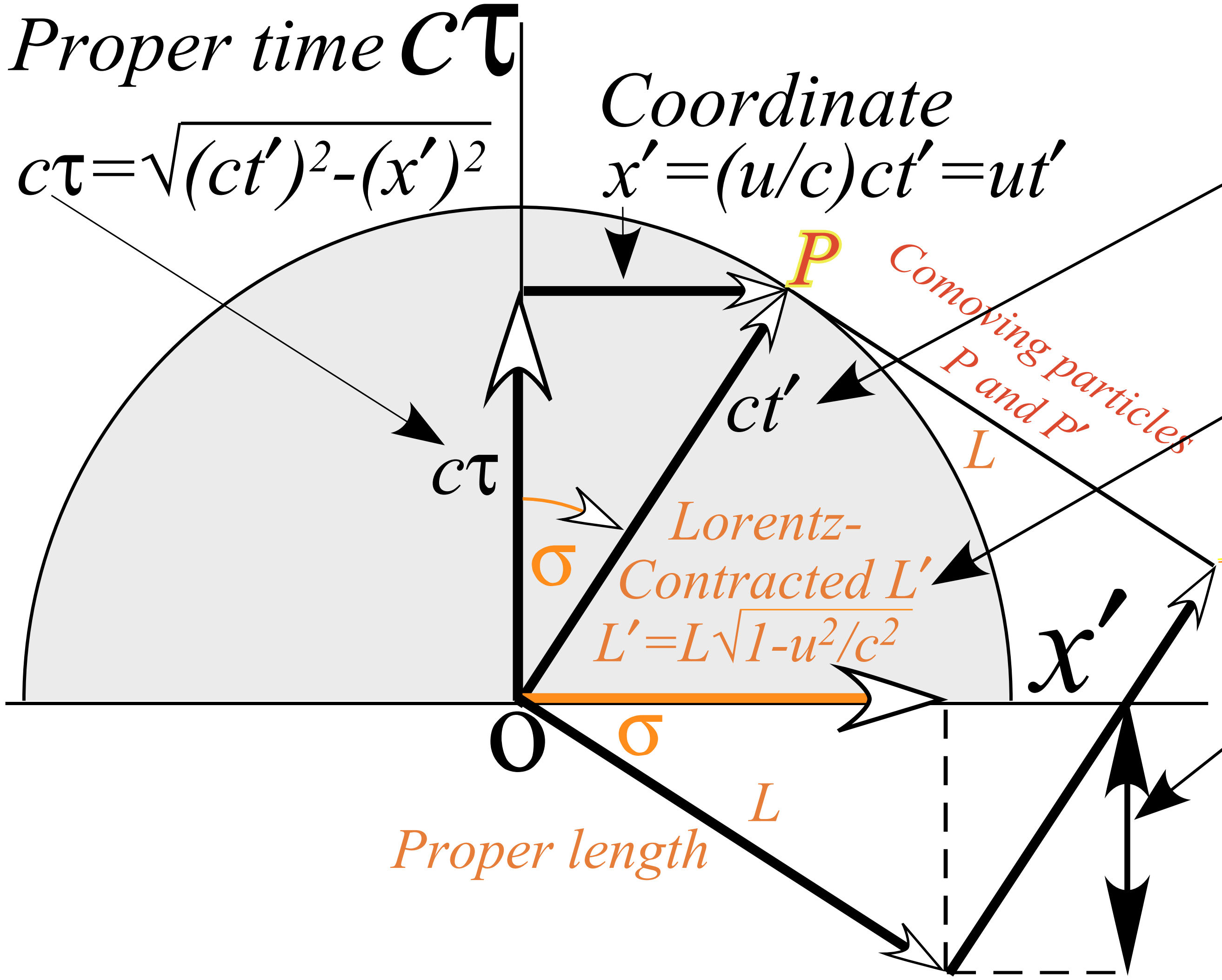


Proper time cτ vs. coordinate space x - (L. C. Epstein's "Cosmic Speedometer")

Particles P and P′ have speed u in (x′,ct′) and speed c in (x, cτ)



Einstein time dilation:

$$ct' = c\tau \sec\sigma = c\tau \cosh\rho = c\tau/\sqrt{1-u^2/c^2}$$

Lorentz length contraction:

$$L' = L\,\mathrm{sech}\rho = L\cos\sigma = L\cdot\sqrt{1-u^2/c^2}$$

Proper Time asimultaneity:

$$c\,\Delta\tau = L'\sinh\rho = L\cos\sigma\sinh\rho$$

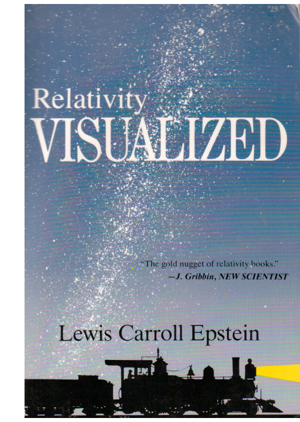
$$= L\cos\sigma\tan\sigma$$

$$= L\sin\sigma \quad = L/\sqrt{c^2/u^2-1} \sim L\,u/c$$

Comparing Longitudinal relativity parameter: Rapidity $\rho = \log_e$(Doppler Shift)

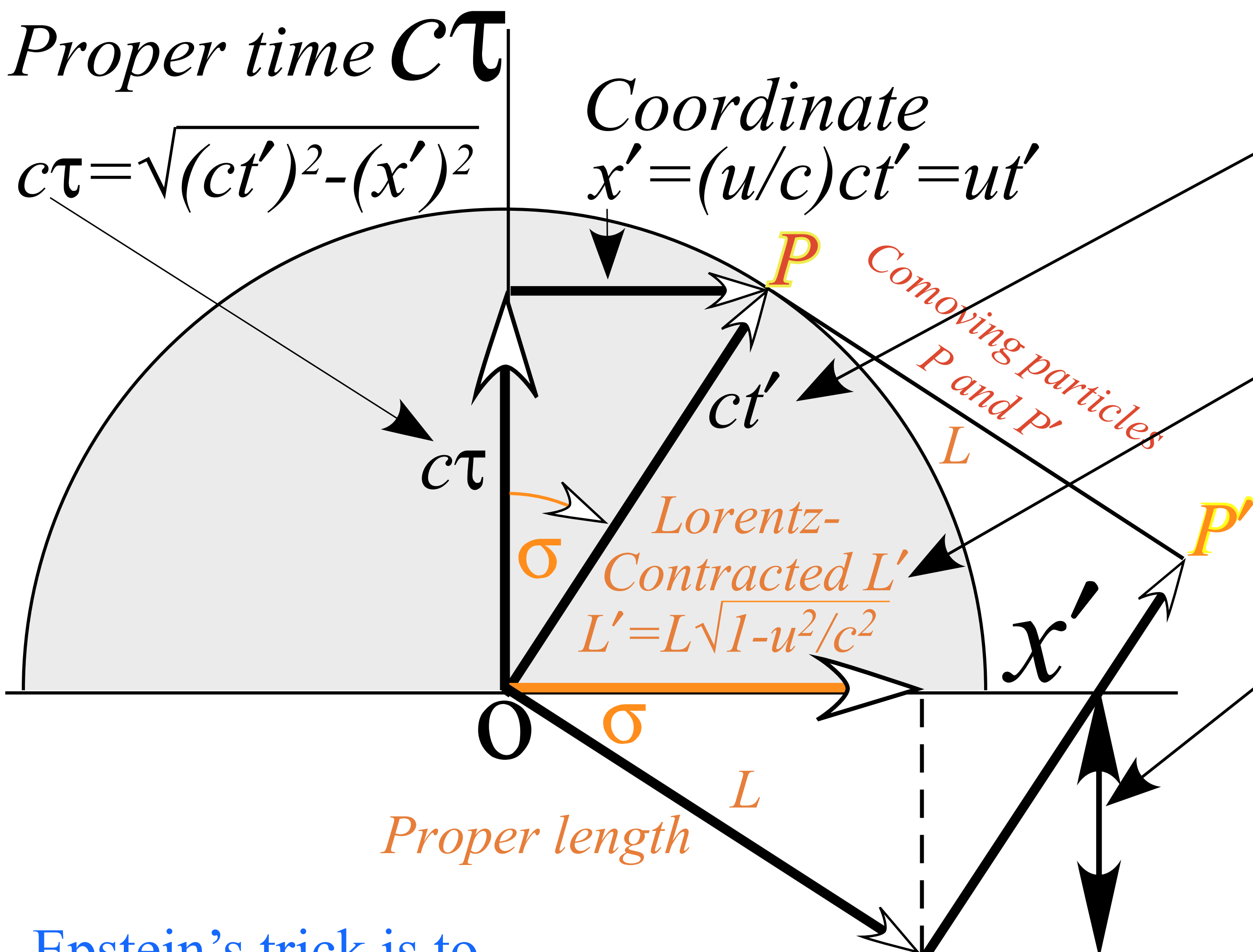
to a Transverse*relativity parameter: Stellar aberration angle σ

*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-



Proper time $c\tau$ vs. coordinate space x - (L. C. Epstein's "Cosmic Speedometer")

Particles P and P′ have speed u in (x',ct') and speed c in $(x, c\tau)$



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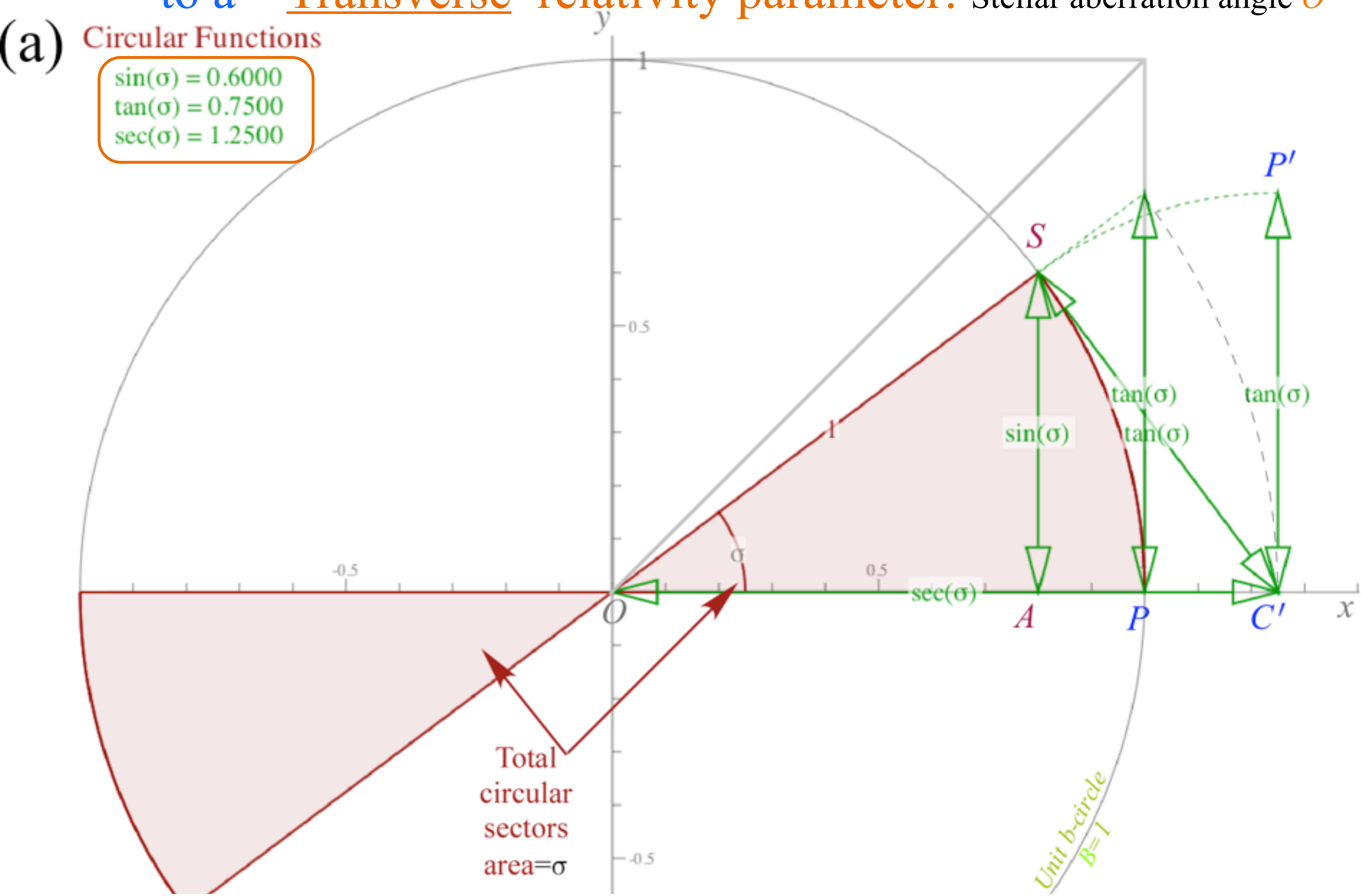
Epstein's trick is to turn a hyperbolic form $c\tau=\sqrt{(ct')^2-(x')^2}$ into a circular form:

$$\sqrt{(c\tau)^2+(x')^2}=(ct')$$

Then everything (and everybody) always goes speed c through $(x',c\tau)$ space!

Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

to a Transverse relativity parameter: Stellar aberration angle σ

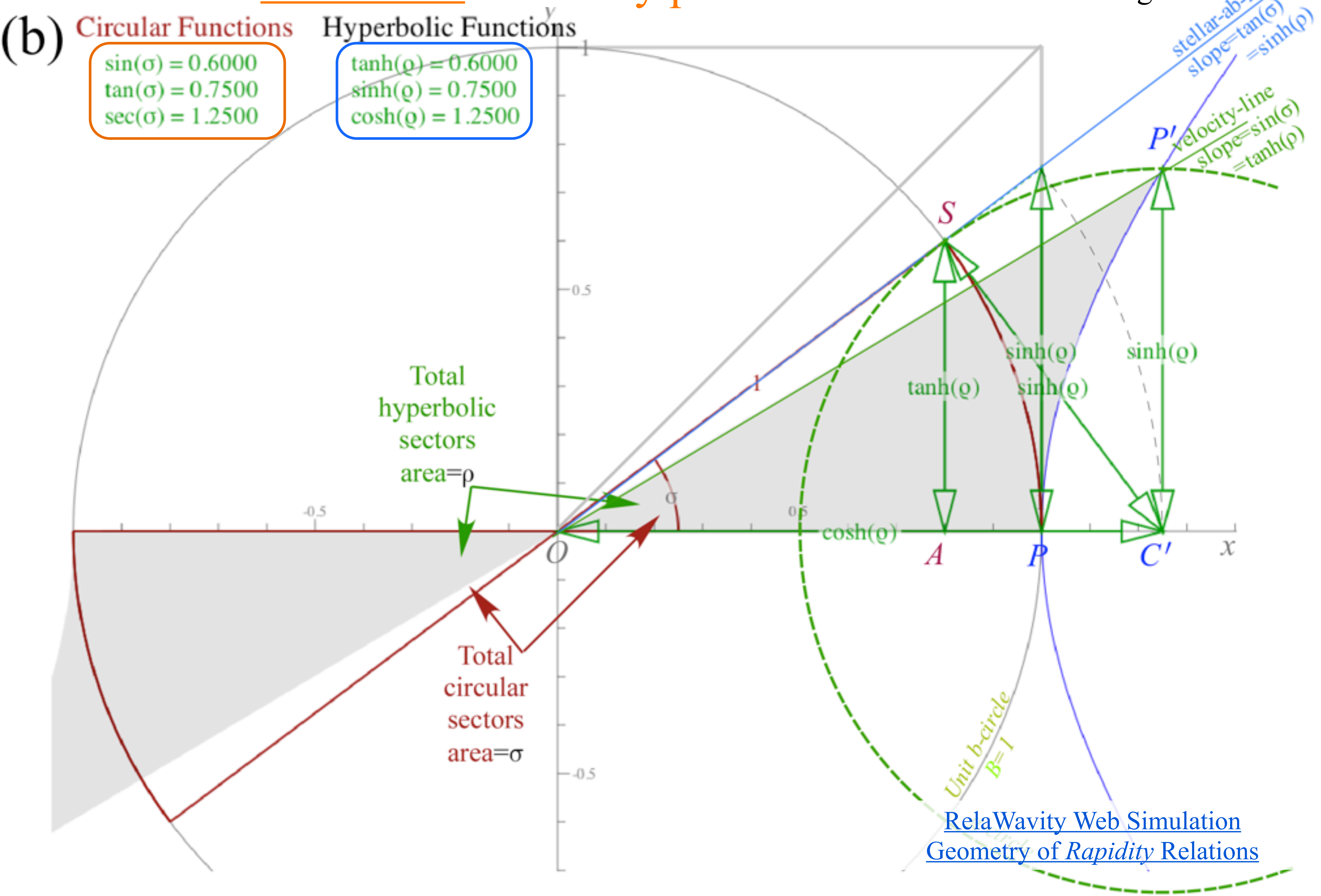


RelaWavity Web Simulation

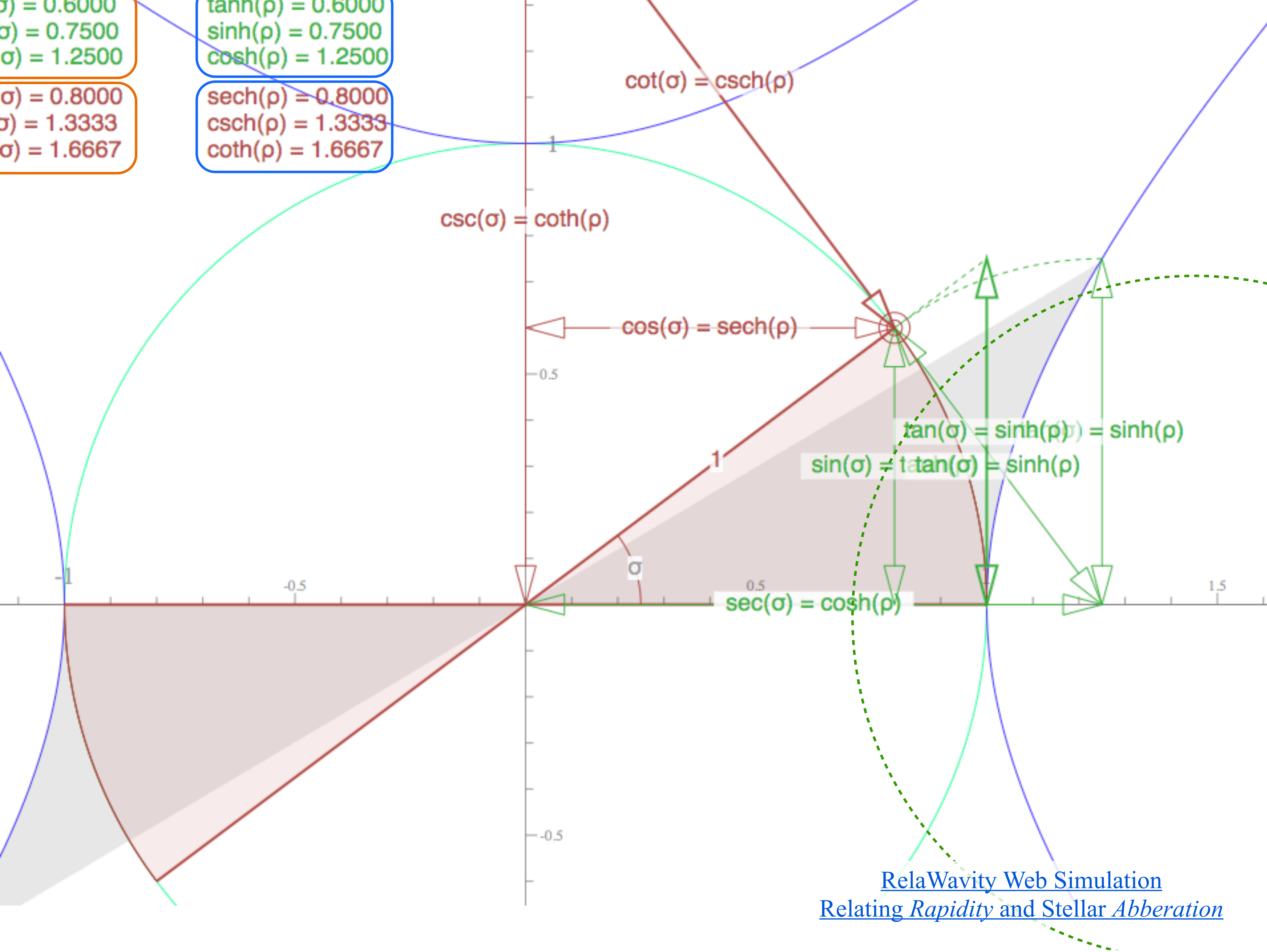
Geometry of Stellar Aberation Angle

Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

to a Transverse relativity parameter: Stellar aberration angle σ



RelaWavity Web Simulation
Geometry of *Rapidity* Relations

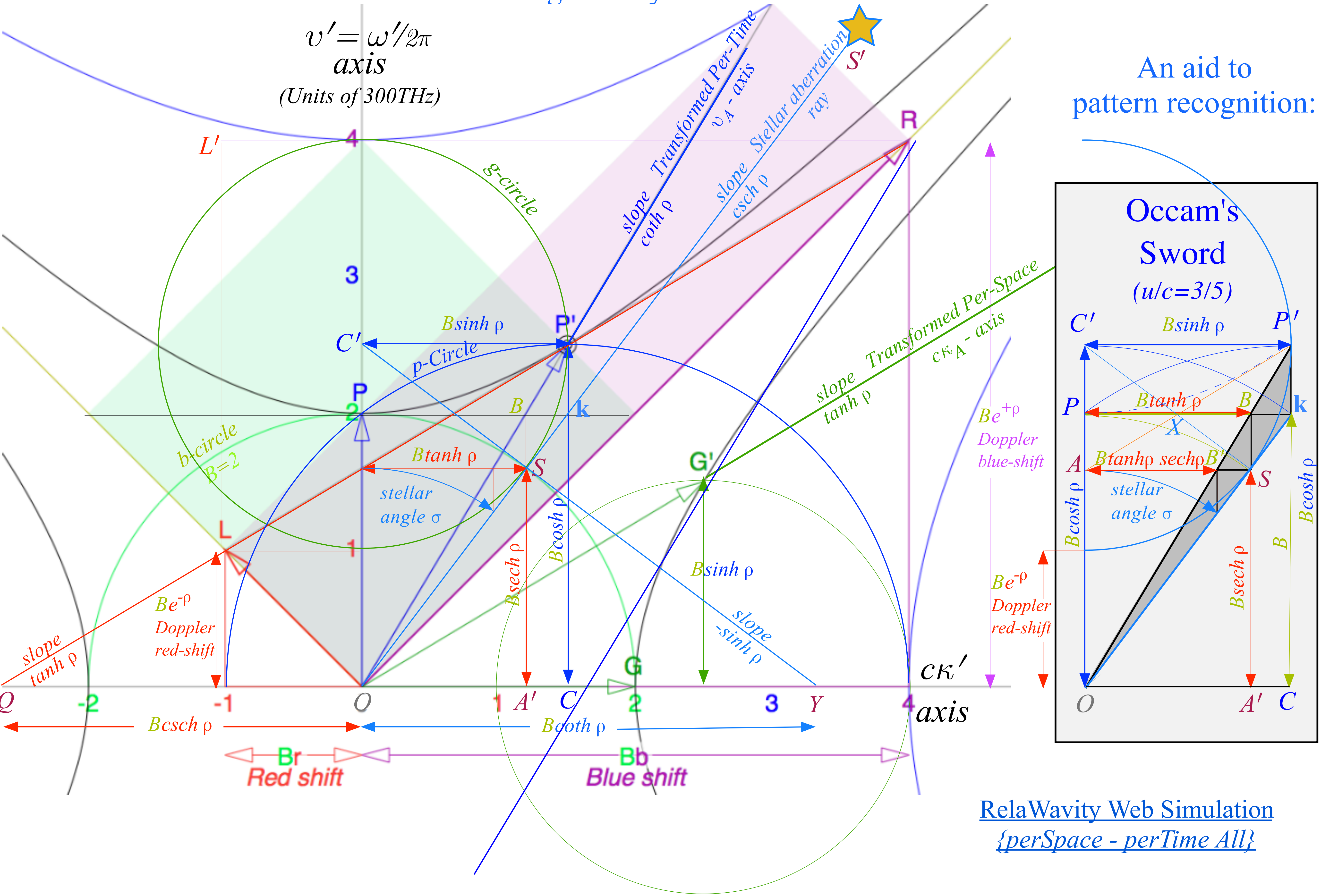


[RelaWavity Web Simulation](#)
[Relating *Rapidity* and Stellar *Abberation*](#)

Summary of optical wave parameters for relativity and QM

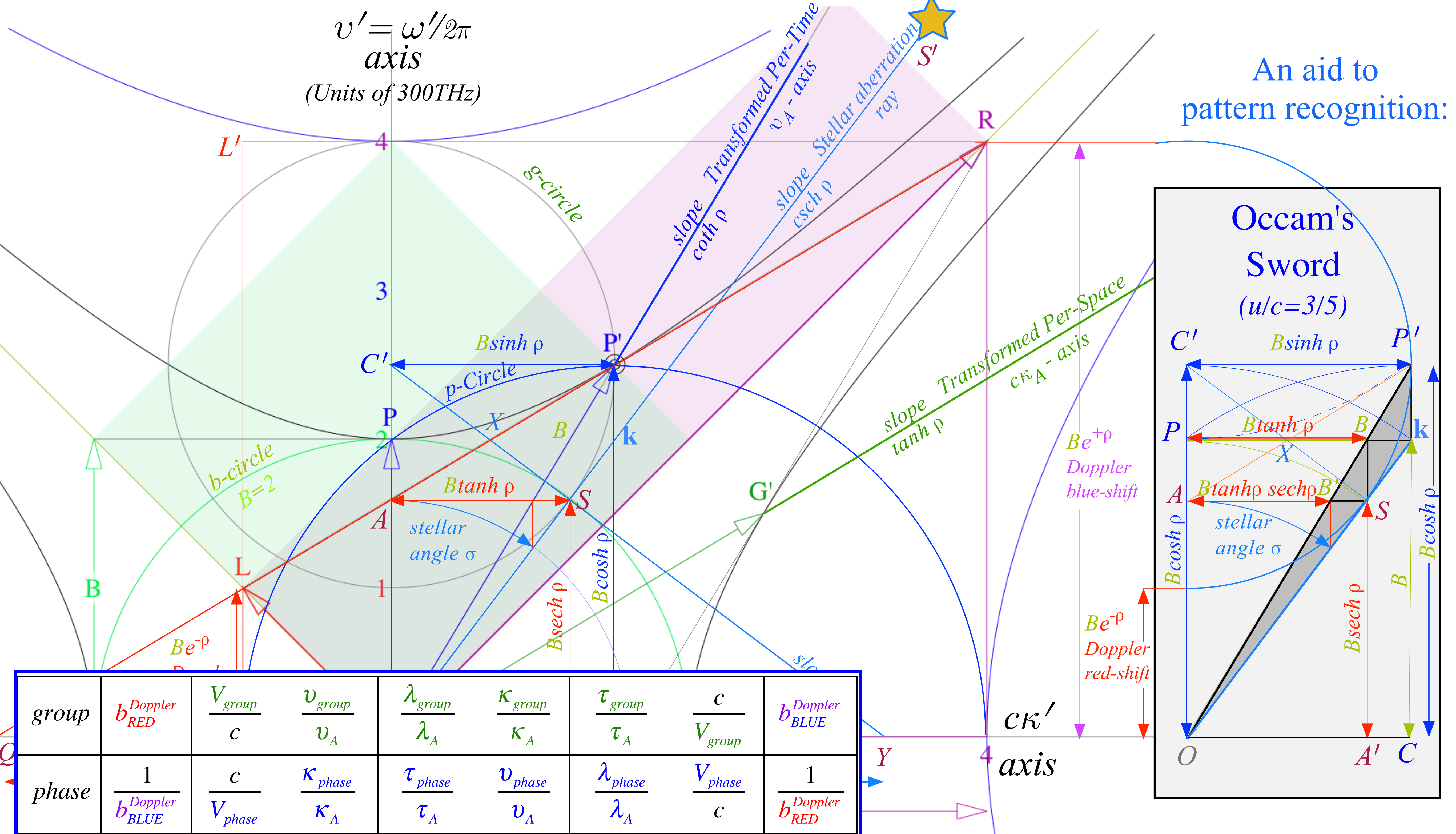
...and their geometry

An aid to pattern recognition:



RelaWavity Web Simulation {perSpace - perTime All}

An aid to pattern recognition:



group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β*=3/5*	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Table of 12 wave parameters (includes inverses) for relativity

...and values for *u/c=3/5*

RelaWavity Web Simulation

Relativistic *Terms (Dual plot w/expanded table)*


+10
+9
+8
+7
+6
+5
+4
+3
+2
+1
-3 -2 -1 O +1 +2 +3 +4 +5 +6 +7 +8 +9 +10