Relawavity: Relativistic wave mechanics II. 2\textsuperscript{nd}-order effects

(4.05.16)

Review of Doppler-shift and Rapidity $\rho_{AB}$ calculation: \textit{Galileo’s Revenge Part I Lect. 23 p.64-75}

Relating rapidity $\rho_{AB}$ and relativity velocity parameter $\beta_{AB}=u_{AB}/c$

Review of ½-sum-½-difference Phase and Group factors giving relativistic space-axes and time-axes
Colliding-CW space-time ($x,ct$)-graph vs Colliding PW space-time ($R,L$)-baseball diamond

Review of ½-sum-½-difference of phasor angular velocity: \textit{Galileo’s Revenge Part II} (Pirelli site)
Elementary models: 2-comb Moire' patterns and cosine-law constructions

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Thales Mean Geometry (\textit{Thales of Miletus 624-543 BCE}) and its role in Relawavity
Detailed geometric construction of relawavity plot for 1-octave Doppler ($\beta_{AB}=u_{AB}/c=3/5$)

Stellar aberration and the Epstein approach to SR
Review Doppler-shift and Rapidity calculation

Alice: Hey, Bob and Carla! Read off your Doppler shift ratios \( \langle B|A \rangle \) and \( \langle C|A \rangle \) to my 600THz beam.

Also, rapidity \( \rho_{BA} \) and \( \rho_{CA} \) relative to me.

Now, Carla, what’s your rapidity \( \rho_{CB} \) relative to Bob?

Bob-Alice Doppler ratio:
\[
\langle B|A \rangle = \frac{v_B}{v_A} = \frac{1200}{600} = 2
\]

Bob-Alice rapidity:
\[
\rho_{BA} = \log_e \langle B|A \rangle = \log_e \frac{2}{1} = 0.69 \quad \text{(so:} \rho_{AB} = -0.69)\]

Carla-Alice Doppler ratio:
\[
\langle C|A \rangle = \frac{v_C}{v_A} = \frac{400}{600} = \frac{2}{3}
\]

Carla-Alice rapidity:
\[
\rho_{CA} = \log_e \langle C|A \rangle = \log_e \frac{2}{3}
\]

Carla-Bob Doppler ratio:
\[
\langle C|B \rangle = \frac{v_C}{v_B} = \frac{v_C}{v_A} \cdot \frac{v_A}{v_B} = \langle C|A \rangle \langle A|B \rangle
\]

Carla-Bob rapidity:
\[
e^{\rho_{CB}} = e^{\rho_{CA}} e^{\rho_{AB}} \quad \text{implied:} \quad \rho_{CB} = \rho_{CA} + \rho_{AB}
\]

\[
e^{\rho_{CB}} = e^{\rho_{CA}} e^{\rho_{AB}} = e^{-0.41} e^{-0.69} = e^{-1.10}
\]

Galileo’s Revenge (part 1)
Rapidity adds just like Galilean velocity

\[\rho_{CB} = \rho_{CA} + \rho_{AB} = -0.41 - 0.69 = -1.10\]
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Imagine Bob sees a pair of counter-propagating laser beams with wavevectors $k_R=+\omega_R/c$ and $k_L=-\omega_L/c$ and $\omega_R=\omega_A$ going left-to-right (from Alice's $600\,\text{THz}$ laser) and $\omega_L=\omega_C$ going right-to-left (from Carla's $600\,\text{THz}$ laser).
Relating rapidity $\rho_{AB}$ and relativity velocity parameter $\beta_{AB} = u_{AB}/c$

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(1.) To what velocity $u_E$ must Bob accelerate so he sees beams with equal frequency $\omega_E$?
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What is the beam group velocity?
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$$u_E = V_{\text{group}} = \frac{\omega_{\text{group}}}{k_{\text{group}}} = \frac{\omega_R - \omega_L}{k_R - k_L}$$
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$u_E = V_{group} = \frac{\omega_{group}}{k_{group}} = \frac{\omega_R - \omega_L}{k_R - k_L} = c \frac{\omega_R - \omega_L}{\omega_R + \omega_L} = c \frac{1200 - 300}{1200 + 300} = \frac{3}{5}c$

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Reply to Query (2.) in similar style:

What $\omega_E$ is blue-shift $b\omega_L$ of $\omega_L$ and red-shift $\omega_R/b$ of $\omega_R$?

$$\omega_E = b \omega_L = \omega_R/b$$
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*(Geometric Mean)*
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With $k_R=+\omega_R/c$ and $k_L=-\omega_L/c$

Using Rapidity:

$\rho_{BA} = \log_e \langle B | A \rangle$

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**Using Rapidity:**

\[ \rho_{AB} = \log_e(A/B) \]

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Blue-shift $b = e^{\rho_{AB}}$  Red-shift $r = b^{-1} = e^{-\rho_{AB}}$

Thus $\omega_E = b\omega_L = \omega_R/b \quad \Rightarrow \quad b = \sqrt{\omega_R/\omega_L} \quad \Rightarrow \quad \omega_E = \sqrt{\omega_R \cdot \omega_L} = \sqrt{1200 \cdot 300} = 600 \text{ THz}$ (Geometric Mean)

Thursday, April 7, 2016
Relating rapidity $\rho_{AB}$ and relativity velocity parameter $\beta_{AB}=u_{AB}/c$

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\[
\frac{u_E}{c} = \frac{u_{AB}}{c} = \frac{e^{\rho_{AB}}-e^{-\rho_{AB}}}{e^{\rho_{AB}}+e^{-\rho_{AB}}} = \frac{\sinh \rho_{AB}}{\cosh \rho_{AB}} = \frac{3}{5} \quad \text{Using Rapidity:} \quad \rho_{AB} = \log_e \left( \frac{A}{B} \right)
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Colliding 2CW laser beams

Right-moving wave $e^{i(kx-\omega t)}$

$\begin{align*}
&k = +2 \\
&\omega = 2c
\end{align*}$

Alice’s laser

$\begin{align*}
&\text{CW Dye-laser} \\
&600 \text{ THz}
\end{align*}$

Left-moving wave $e^{i(-kx-\omega t)}$

$\begin{align*}
&k = -2 \\
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\end{align*}$

Carla’s laser

$\begin{align*}
&\text{CW Dye-laser} \\
&600 \text{ THz}
\end{align*}$

Wavelength $\lambda = 2\pi/k = 1/\kappa$

$(1/2\mu m = 0.5 \cdot 10^{-6} m)$

Period $\tau = 2\pi/\omega = 1/\nu$

$(5/3 fs = 1.67 \cdot 10^{-15} s)$

Alice: OK, Bob. We're gonna' hit you from both sides, now!

Carla: Look out, Bob!

Bob: Yikes!

Thursday, April 7, 2016
Right-moving CW $e^{ikx-\omega t}$

Left-moving CW $e^{i(-kx-\omega t)}$

Carla:
You get zeros of any wave-sum $e^{ia} + e^{ib}$ by factoring it into phase and group parts.

Bob:
Cool! You guys made me a space-time graph out of real zeros.

How’d it do that?

BohrIt Web Simulation
1 CW ct vs x Plot (ck = +1)
Single panel with Zero Tracers

BohrIt Web Simulation
2 CW ct vs x Plot (ck = ±2)
Multi-panel with Zero Tracers
Edge 0

**Right-moving CW** $e^{i(kx-\omega t)}$

- $k = +2$
- $\omega = 2c$
- CW Dye-laser 600 THz

**Left-moving CW** $e^{i(-kx-\omega t)}$

- $k = -2$
- $\omega = 2c$
- CW Dye-laser 600 THz

---

**Carla:**

Easy!

You get zeros of any wave-sum $e^{ia}+e^{ib}$ by factoring it into phase and group parts.

Remember your algebra? Exponents of products add.

So, half-sum $\frac{a+b}{2}$ plus half-diff $\frac{a-b}{2}$ gives $a$, and half-sum $\frac{a+b}{2}$ minus half-diff $\frac{a-b}{2}$ gives $b$.

**Presto!**

You factor $e^{ia}+e^{ib}$ into $e^\frac{a+b}{2}\left(e^{i\frac{a-b}{2}} + e^{-i\frac{a-b}{2}}\right)$. 

**Alice** 1CW phase: $a = kx - \omega t$

**Carla** 1CW phase: $b = -kx - \omega t$
Right-moving CW \( e^{i(kx - \omega t)} \)

Left-moving CW \( e^{i(-kx - \omega t)} \)

Wavelength \( \lambda = \frac{2\pi}{k} = \frac{1}{\nu} \) (1/2\( \mu m = 0.5 \cdot 10^{-6} m \))

Period \( \tau = \frac{2\pi}{\omega} = \frac{1}{\nu} \) (5/3fs = 1.67 \cdot 10^{-15}s)

\[
\begin{align*}
\text{Bob:} & \quad \text{Cool! You guys made me a space-time graph out of real zeros. How’d it do that?}
\end{align*}
\]

\[
\begin{align*}
\text{Carla:} & \quad \text{Easy! You get zeros of any wave-sum } e^{ia} + e^{ib} \text{ by factoring it into phase and group parts.}
\end{align*}
\]

\[
\begin{align*}
\text{Remember your algebra? Exponents of products add.}
\end{align*}
\]

\[
\begin{align*}
\text{So, half-sum } & \quad \frac{a+b}{2} \quad \text{plus half-diff } \frac{a-b}{2} \quad \text{gives } a,
\end{align*}
\]

\[
\begin{align*}
\text{and half-sum } & \quad \frac{a+b}{2} \quad \text{minus half-diff } \frac{a-b}{2} \quad \text{gives } b.
\end{align*}
\]

\[
\begin{align*}
\text{Presto! You factor } e^{ia} + e^{ib} \text{ into } e^{i\frac{a+b}{2}} \left( e^{i\frac{a-b}{2}} + e^{-i\frac{a-b}{2}} \right)
\end{align*}
\]

\[
\begin{align*}
\text{Alice 1CW phase: } a = kx - \omega t
\end{align*}
\]

\[
\begin{align*}
\text{Carla 1CW phase: } b = -kx - \omega t
\end{align*}
\]

\[
\begin{align*}
\text{Bob’s 2CW Group-phase: } +k = \frac{a-b}{2}
\end{align*}
\]

\[
\begin{align*}
\text{Group wave: } e^{-ikx} + e^{-ikx} = 2\cos kx \text{ is standing wave (does not vary with time } t)\]
\]
Let’s plot this in per-spacetime?!

Cool! You guys made me a space-time graph out of real zeros.

How’d it do that?

You get zeros of any wave-sum $e^{ia} + e^{ib}$ by factoring it into phase and group parts.

Remember your algebra? Exponents of products add.

So, half-sum $\frac{a+b}{2}$ plus half-diff $\frac{a-b}{2}$ gives $a$,

and half-sum $\frac{a+b}{2}$ minus half-diff $\frac{a-b}{2}$ gives $b$.

Presto!
You factor $e^{ia} + e^{ib}$ into $e^{\frac{i}{2}} \left( e^{\frac{i}{2} a-b} + e^{-\frac{i}{2} a-b} \right)$

Alice 1CW phase: $a = kx - \omega t$
Carla 1CW phase: $b = -kx - \omega t$

Bob’s 2CW Group-phase: $+k = \frac{a-b}{2}$

Group wave: $e^{-i\omega t} (e^{ikx} + e^{-ikx}) = 2\cos kx$

is standing wave (does not vary with time $t$)

Bob’s 2CW Phase-phase: $-\omega = \frac{a+b}{2}$

Phase wave real part: $\text{Re}(e^{-i\omega t}) = \cos(\omega t)$

is “instanton” wave (does not vary in space $x$)
Standing 2CW in per-space-time
Frequency
\( \omega = 2\pi \nu \)

\[ \Psi(x,t) = (e^{i\omega t})(2\cos kx) = e^{i(kx-\omega t)} + e^{i(-kx-\omega t)} \]

**Standing 2CW in space-time**

**Phase vector**
1/2-sum:
\[ \text{Phase vector} \]
\[ K_{\text{phase}} = \frac{R+L}{2} \]

**Group vector**
1/2-difference
\[ K_{\text{group}} = \frac{R-L}{2} \]

Bob: The \( P \) and \( G \) vectors are scale models of zero-grid lattice vectors (but \( P \) and \( G \) switch places).

\[ L = K_{-2} \]
\[ R = K_{+2} \]
\[ L = P - G \]
\[ R = P + G \]

Carla: OK, Bob! It looks like a baseball diamond with \( P \) at Pitcher’s mound and \( G \) at the Grandstand*. I’m on 1st base! (R)

*Thanks, Woody!
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The \((\Psi, \kappa)\) "Baseball Diamond"

The \((\nu, \kappa)\) \(\lambda=2\pi/k=1/\kappa\)

\((0.5\mu m=0.5 \cdot 10^{-6} m)\)

Period \(\tau=2\pi/\omega=1/\nu\)

\((1.67 fs=0.167 \cdot 10^{-15} s)\)

\(\Psi(x,t) = (e^{i\omega t}) (2\cos kx) = e^{i(kx-\omega t)} + e^{i(-kx-\omega t)}\)

Standing 2CW in per-space-time

Frequency \(\omega=2\pi \nu\)

Phase vector 1/2-sum:

\(\mathbf{K}_{\text{phase}} = \mathbf{P} = \frac{\mathbf{R} + \mathbf{L}}{2}\)

\(\mathbf{L} = \mathbf{K}_{-2}\)

\(\mathbf{P} = \mathbf{R} + \mathbf{G}\)

\(\mathbf{G} = \mathbf{P} - \mathbf{G}\)

Pitcher’s mound 1st base (Alice)

Grandstand 2nd base (Carla)

Wavevector \(ck=2\pi \kappa c\)

Group vector 1/2-difference

\(\mathbf{K}_{\text{group}} = \mathbf{G} = \frac{\mathbf{R} - \mathbf{L}}{2}\)

Bob: The \(\mathbf{P}\) and \(\mathbf{G}\) vectors are scale models of zero-grid lattice vectors (but \(\mathbf{P}\) and \(\mathbf{G}\) switch places)

It looks like a baseball diamond with \(\mathbf{P}\) at Pitcher’s mound and \(\mathbf{G}\) at the Grandstand*. Ok, I’m on 3rd base! L.

*Thanks, Woody!
Continuous Waves (CW) trace “Cartesian squares” in space-time

(a) CW squares
1 femtosecond
1.0 fs = 10^{-15}s
1 micron
1.0 µm = 10^{-6} meter

Pulse Waves (PW) trace “baseball diamonds” in space-time

(b) PW diamonds

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0 0.5 µm 1.0 µm
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BohrIt Web Simulation

2 CW ct vs x Plot
(ck = ±2)

RelAvativity Site

Phase and Group Vectors in per-Time vs per-Space

\[ R = P + G \]
\[ L = P - G \]
\[ P = \frac{R + L}{2} \]
\[ G = \frac{R - L}{2} \]
$R = P + G$

$L = P - G$

$P = \frac{R + L}{2}$

$G = \frac{R - L}{2}$

**BohrIt Multi-Panel Simulation**

*2 PW ct vs x Plot*  
$(\beta = u/c = 0)$

**BohrIt Simulation**

*2 PW ct vs x Plot*  
$(\beta = u/c = 0)$
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More at Pirelli Challenge page: *Un Grande Affaire’ - Light Meets Light*
You get zeros of any wave-sum $e^{ia} + e^{ib}$ by factoring it into phase and group parts.

Remember your algebra? Exponents of products add.

So, half-sum $\frac{a+b}{2}$ plus half-diff $\frac{a-b}{2}$ gives $a$, and half-sum $\frac{a+b}{2}$ minus half-diff $\frac{a-b}{2}$ gives $b$.

Presto!

You factor $e^{ia} + e^{ib}$ into $e^{i \frac{a+b}{2}} \left( e^{i \frac{a-b}{2}} + e^{-i \frac{a-b}{2}} \right)$

More at Pirelli Challenge page: 'Un Grande Affare’ - Light Meets Light
(a) Sum of Wave Phasor Array

Red phasor B: \(\Psi_A = e^{i\alpha}\) with \(\alpha\) and \(\beta\)
GREEN phasor A: \(\Psi_B = e^{i\beta}\) with \(\cos\beta\) and \(\sin\beta\)

\(r = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 0\)

(b) Typical Phasor Sum:

\(\Psi_{A+B} = \Psi_A + \Psi_B\)

(c) Phasor-relative views

\(\Psi_A - \Psi_B = (\alpha - \beta) / 2\)

Galileo’s Revenge (part 2)
Phasor angular velocity adds just like Galilean velocity

Happy now?

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More at Pirelli Challenge page: \textit{'Un Grande Affare’ - Light Meets Light}
Alice: Now our 600THz lasers move left-to-right. My 600THz laser is going so fast its beam blasts you with UV 1200THz.

Carla's 600THz laser is going away so you get a nice infrared 300THz.

Bob: That UV burns! I need to put on my sunglasses.
My UV 1200THz vector is fierce! You’ll need glasses to see P' and G' lines or coordinates.

Frequency $\nu'$ (units of $\nu_A = 600THz$)

Bob: Sunglasses help. Wow! Your 1st baseline $R'$ is Doppler blued up by $e^{\nu' p} = 2$.

Evenson axiom says, “Stay on your baseline!”

Carla: My IR 300THz L' baseline is a lot nicer!
**Evenson Axiom:**

Stay on your baseline!

**Doppler shift:**

UV 1200THz \( \mathbf{R}' \) vector is fierce! You'll need glasses to see \( \mathbf{P}' \) and \( \mathbf{G}' \) lines or coordinates.

**New Baseline:**

My UV 300THz \( \mathbf{L}' \) is a lot nicer! (and half as long.)

**Frequency:**

\( \nu' = e^{+\rho} \frac{1}{2} \nu_A = 1200 \text{THz} \)

**Wavevector:**

\( c\kappa' = e^{+\rho} c\kappa_A = 2 c\kappa_A \)

**Exson axiom says:**

Stay on your baseline!
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More at Pirelli Challenge page: *Un Grande Affare’ - Light Meets Light*
Alice: OK. My UV 1200THz vector is fierce! You’ll need glasses to see \( \mathbf{P}' \) and \( \mathbf{G}' \) lines or coordinates.

Carla: My UV 300THz is a lot nicer! (and half as long.)

Bob: Sunglasses help. Wow! Your 1st baseline is Doppler blued up by \( e^{+\Delta} = 2 \).

But, Carla’s 3rd baseline \( \mathbf{L}' \) is Doppler red shifted by \( e^{-\Delta} = 1/2 \):

New “Pitcher-mound” \( \mathbf{P}' \) (Phase pt.) is 1/2-sum \( \frac{\mathbf{R}'+\mathbf{L}'}{2} \):

\[
\begin{align*}
\mathbf{v}' & = \frac{c \mathbf{k}'}{2} = \mathbf{v}_A \left( \begin{array}{c} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{array} \right) + \mathbf{v}_A \left( \begin{array}{c} -1/2 \\ +1/2 \\ 2+1/2 \\ 2+1/2 \\ 2+1/2 \\ 2+1/2 \end{array} \right) \\
& = \mathbf{v}_A \left( \begin{array}{c} 3/4 \\ 5/4 \end{array} \right) \\
& = \mathbf{v}_A \left( \begin{array}{c} 3/4 \\ 5/4 \end{array} \right) \\
& = \mathbf{v}_A \left( \begin{array}{c} 3/4 \\ 5/4 \end{array} \right)
\end{align*}
\]

RelaWavity Simulation

Shifted \( b=2 \) Phase and Group Vectors in per-Time vs per-Space
Alice: OK. My UV 1200THz \( R' \) vector is fierce! You’ll need glasses to see \( P' \) and \( G' \) lines or coordinates.

Carla: My UV 300THz \( L' \) 3rd baseline is a lot nicer! (and half as long.)

Bob: Sunglasses help. Wow! Your 1st baseline is Doppler blued up by \( e^{+\rho} = 2 \).

But, Carla’s 3rd baseline \( L' \) is Doppler red shifted by \( e^{-\rho} = 1/2 \).

New “Pitcher-mound” \( P' \) (Phase pt.) is 1/2-sum \( (R' + L')/2 \):

\[
K'_{phase} = \frac{P'}{2} = \frac{R' + L'}{2}
\]

\[
\begin{align*}
\nu'_{phase} &= \frac{\nu_A}{2} \begin{pmatrix} e^{+\rho} \\ e^{+\rho} \end{pmatrix} + \nu_A \begin{pmatrix} -e^{-\rho} \\ +e^{-\rho} \end{pmatrix} = \nu_A \\
&= \nu_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \nu_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}
\end{align*}
\]

RelaWavity Simulation

**Shifted (b=2) Phase and Group Vectors in per-Time vs per-Space**
Alice: OK. My UV 1200THz vector is fierce! You’ll need glasses to see P' and G' lines or coordinates.

Carla: My UV 300THz is a lot nicer! (and half as long.)
Alice: OK.

My UV 1200THz $\mathbf{R}'$ vector is fierce!
You'll need glasses to see $\mathbf{P}'$ and $\mathbf{G}'$ lines or coordinates.

Carla: My UV 300THz $\mathbf{L}'$ vector is 3rd baseline is a lot nicer!
(and half as long.)

Bob: Sunglasses help.
Wow! Your 1st baseline $\mathbf{R}'$ is Doppler blued up by $e^{v_1}=2$;

But, Carla’s 3rd baseline $\mathbf{L}'$ is Doppler red shifted by $e^{v_1}=1/2; \quad \text{New "Pitcher-mound"} \quad \mathbf{P}' \quad \text{(Phase pt.)}$ is 1/2-sum $\left(\mathbf{R}'+\mathbf{L}'\right)/2$: $\quad \frac{e^{v_1}+e^{-v_1}}{2}$

New “Grandstand” $\mathbf{G}'$ (Group pt.) $\quad \frac{e^{v_1}-e^{-v_1}}{2}$

But, Carla’s 3rd baseline $\mathbf{L}'$ is Doppler red shifted by $e^{v_1}=1/2; \quad \text{New "Pitcher-mound"} \quad \mathbf{P}' \quad \text{(Phase pt.)}$ is 1/2-sum $\left(\mathbf{R}'+\mathbf{L}'\right)/2$: $\quad \frac{e^{v_1}+e^{-v_1}}{2}$

New “Grandstand” $\mathbf{G}'$ (Group pt.) $\quad \frac{e^{v_1}-e^{-v_1}}{2}$
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More at Pirelli Challenge page: *'Un Grande Affare’ - Light Meets Light*
Frequency
\( \nu' \)
(units of \( \nu_A = 600 \text{THz} \))

\[ \begin{align*}
\text{Wavelength} & = 2\pi/k = 1/\kappa \\
(1/4\mu m) & = 0.25 \times 10^{-6} \text{m} \\
(1\mu m) & = 10^{-6} \text{m}
\end{align*} \]

\[ \begin{align*}
\text{Wavelength} & = 2\pi/k = 1/\kappa \\
\text{Group vector} \quad G' = \frac{R' - L'}{2} \\
\text{Phase vector} \quad P' = R' + L' \quad (1/2-\text{sum vector})
\end{align*} \]

Bob: The spacetime wave-zeros replicate the same pattern.
Frequency

\( \nu' \)  
(units of \( \nu_A = 600 \text{THz} \))

**2CW per-Spacetime Plot**

2CW Minkowski-Spacetime Grid

Bob: The spacetime wave-zeros replicate
the same pattern.

(Except \( P' \)-phase and
\( G' \)-group indicators get switched again.)

**Let's measure these in careful detail!**

**2CW Minkowski-spacetime grid**

Wavevector \( ck' \)
(units of \( ck_A = 2 \cdot 10^6 / \text{m} \))

Phase vector \( P \)
1/2-sum vector \( K_{phase}' = \frac{P' + L'}{2} \)

Group vector \( G \)
1/2-diff vector \( K_{group}' = \frac{G' - L'}{2} \)
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More at Pirelli Challenge page: *'Un Grande Affare’ - Light Meets Light*
BohrIt Web Simulation
2 CW Minkowski Plot (ck = -1, +4)
BohrIt Web Simulation

2 PW ct vs x Plot (β = u/c = 3/5)
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More at Pirelli Challenge page: *'Un Grande Affaire’ - Light Meets Light*
The 16 parameters of 2CW interference

Start with the Dopplers

Space $x'\quad$ (units of $\lambda_A = 1/2\mu m$)

Time $ct'\quad$ (units of $\lambda_A = 1/2\mu m$)

RelaWavity Web Simulation - 16 Relativity Dimensions
The 16 parameters of 2CW interference

\[ P' = \begin{pmatrix} cK_{\text{phase}}' \\ v_{\text{phase}}' \end{pmatrix} = \begin{pmatrix} \sinh \rho \\ \cosh \rho \end{pmatrix} = \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix} \]

Phase frequency \( v_{\text{phase}}' = v_A \cosh \rho = 5/4 = 1.25 \)
Phase period \( \tau'_{\text{phase}} = \tau_A \text{sech} \rho = 4/5 \)

Frequency \( v' \)
(units of \( v_A = 600 \text{THz} \))

Wavevector \( ck' \)
(units of \( cK_A = 2 \cdot 10^6 \text{m} \))

RelaWavity Web Simulation - 16 Relativity Dimensions

Time \( ct' \)
(units of \( \lambda_A = 1/2 \mu m \))

Start with the Dopplers
...then do the phase waves

Space \( x' \)
(units of \( \lambda_A = 1/2 \mu m \))

<table>
<thead>
<tr>
<th>phase</th>
<th>( b_{\text{Doppler RED}} )</th>
<th>( c )</th>
<th>( K' )</th>
<th>( \kappa' )</th>
<th>( \tau'_{\text{phase}} )</th>
<th>( v'_{\text{phase}} )</th>
<th>( \lambda' )</th>
<th>( \kappa' )</th>
<th>( \tau'_{\text{group}} )</th>
<th>( v'_{\text{group}} )</th>
<th>( b_{\text{Doppler BLUE}} )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>( v_A )</td>
<td>( K_A )</td>
<td>( c_A )</td>
<td>( \kappa_A )</td>
<td>( \tau_A )</td>
<td>( v_A )</td>
<td>( \lambda_A )</td>
<td>( \kappa_A )</td>
<td>( \tau_A )</td>
<td>( v_A )</td>
</tr>
<tr>
<td>rapidity ( \rho )</td>
<td>( e^{-\rho} )</td>
<td>( \tanh \rho )</td>
<td>( \sinh \rho )</td>
<td>( \text{sech} \rho )</td>
<td>( \cosh \rho )</td>
<td>( \text{sech} \rho )</td>
<td>( \coth \rho )</td>
<td>( e^{+\rho} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>value for ( \beta = 3/5 )</td>
<td>( 1/2 = 0.5 )</td>
<td>( 3/5 = 0.6 )</td>
<td>( 3/4 = 0.75 )</td>
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<td>( 5/3 = 1.67 )</td>
<td>( 2/1 = 2.0 )</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
The 16 parameters of 2CW interference

\[
P' = \begin{pmatrix} c \kappa'_{phase} \\ \nu'_{phase} \end{pmatrix} = \nu_A \begin{pmatrix} \sinh \rho \\ \cosh \rho \end{pmatrix} = \nu_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}
\]

Phase frequency \( \nu'_{phase} = \nu_A \cosh \rho = 5/4 = 1.25 \)
Phase period \( \tau = \frac{1}{\nu} \)

Start with the Dopplers ...
then do the phase waves

\[
\tau'_{phase} = \tau_A \text{sech} \rho = 4/5
\]

\[
\tau'_{phase} = 0.8
\]

Frequency \( \nu' \)
(units of \( \nu_A = 600 \text{THz} \))

Wavevector \( \mathbf{c k'} \)
(units of \( \mathbf{c k_A} = 2 \cdot 10^6 / \text{m} \))

\[
\mathbf{L} = \mathbf{K}_1, \quad \mathbf{R} = \mathbf{K}_A
\]

\[
e^{-\rho} \nu_A = \frac{1}{2} \nu_A
\]

RelaWavity Web Simulation - 16 Relativity Dimensions
Phase wavenumber \( \kappa'_{\text{phase}} = \kappa_A \sinh \rho = 3/4 \) to Phase wavelength \( \lambda'_{\text{phase}} = \lambda_A \cosh \rho = 4/3 \)

\[
P' = \begin{pmatrix} c \kappa'_{\text{phase}} \\ \nu'_{\text{phase}} \end{pmatrix} = \nu_A \begin{pmatrix} \sinh \rho \\ \cosh \rho \end{pmatrix} = \nu_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}
\]

Phase frequency \( \nu'_{\text{phase}} = \nu_A \cosh \rho = 5/4 \) to Phase period \( \tau'_{\text{phase}} = \tau_A \sech \rho = 4/5 \)

Frequency \( \nu' \)
(units of \( \nu_A = 600 \text{THz} \))

1500 THz
1200 THz
900 THz
600 THz
300 THz
60 THz
90 THz
120 THz
1500 THz

Wavevector \( c \kappa' \)
(units of \( c \kappa_A = 2 \cdot 10^6 / \text{m} \))

RelaWavity Web Simulation - 16 Relativity Dimensions

Phase wavenumber \( \kappa_{\text{phase}} \) flips to Phase wavelength \( \lambda = 1/\kappa \).

\( \kappa'_{\text{phase}} = \kappa_A \sinh \rho = 3/4 \)

\( \lambda'_{\text{phase}} = \lambda_A \cosh \rho = 4/3 \)

\( \tau'_{\text{phase}} = \tau_A \sech \rho = 4/5 \)

Time \( ct' \)
(units of \( \lambda_A = 1/2 \mu m \))

2
1.5
1
0.5
0
-0.5
-1
0
0.5
1
1.5
2

Space \( x' \)
(units of \( \lambda_A = 1/2 \mu m \))

0
0.5
1
1.5
2

<table>
<thead>
<tr>
<th>group</th>
<th>( b_{\text{Doppler RED}} )</th>
<th>( V_{\text{blue}} )</th>
<th>( K_{\text{phase}} )</th>
<th>( \kappa_{\text{phase}} )</th>
<th>( \nu_{\text{phase}} )</th>
<th>( \lambda_{\text{phase}} )</th>
<th>( V_{\text{phase}} )</th>
<th>( b_{\text{Doppler BLUE}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>phase</td>
<td>( \frac{1}{b_{\text{Doppler RED}}} )</td>
<td>( V_{\text{red}} )</td>
<td>( \frac{\kappa_{\text{phase}}}{\kappa_{A}} )</td>
<td>( \frac{\nu_{\text{phase}}}{\nu_{A}} )</td>
<td>( \frac{\lambda_{\text{phase}}}{\lambda_{A}} )</td>
<td>( \frac{V_{\text{phase}}}{V_{A}} )</td>
<td>( \frac{1}{b_{\text{Doppler BLUE}}} )</td>
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<table>
<thead>
<tr>
<th>rapidity ( \rho )</th>
<th>( e^{-\rho} )</th>
<th>( \cosh \rho )</th>
<th>( \sinh \rho )</th>
<th>( \sech \rho )</th>
<th>( \cosh \rho )</th>
<th>( \sech \rho )</th>
<th>( \cosh \rho )</th>
<th>( e^\rho )</th>
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<td>value for ( \beta = 3/5 )</td>
<td>( \frac{1}{2} = 0.5 )</td>
<td>( \frac{3}{5} = 0.6 )</td>
<td>( \frac{3}{4} = 0.75 )</td>
<td>( \frac{4}{5} = 0.80 )</td>
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<td>( \frac{4}{3} = 1.33 )</td>
<td>( \frac{5}{3} = 1.67 )</td>
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Phase frequency flips to Phase period

Phase wavenumber flips to Phase wavelength

\[ \kappa'_{\text{phase}} = \kappa_A \sinh \rho = \frac{3}{4} \]
\[ \lambda'_{\text{phase}} = \lambda_A \cosh \rho = \frac{4}{3} \]

Time \( c't' \)

Phase wavenumber \( \kappa'_{\text{phase}} \) to Phase wavelength \( \lambda'_{\text{phase}} \)

\( \kappa'_{\text{phase}} = \kappa_A \sinh \rho = \frac{3}{4} \)
\( \lambda'_{\text{phase}} = \lambda_A \cosh \rho = \frac{4}{3} \)

Frequency

\( \nu' \)
(units of \( \nu_A = 600 \text{THz} \))

Wavevector \( c \kappa' \)
(units of \( c \kappa_A = 2 \cdot 10^6 / \text{m} \))

RelaWavity Web Simulation - 16 Relativity Dimensions
Phase wavenumber \( \kappa'_{\text{phase}} = \kappa_A \sinh \rho = 3/4 \) flips to Phase wavelength \( \lambda'_{\text{phase}} = \lambda_A \cosh \rho = 4/3 \)

\[
P' = \begin{pmatrix} c \kappa'_{\text{phase}} \\ \nu'_{\text{phase}} \end{pmatrix} = \nu_A \begin{pmatrix} \sinh \rho \\ \cosh \rho \end{pmatrix} = \nu_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}
\]

Frequency \( \nu' \) (units of \( \nu_A = 600 \text{THz} \))

Phase frequency \( \nu'_{\text{phase}} = \nu_A \cosh \rho = 5/4 \) flips to Phase period \( \tau' = 1/\nu'_{\text{phase}} = \tau_A \text{sech} \rho = 4/5 \)

Phase wavenumber \( \kappa'_{\text{phase}} = \kappa_A \sinh \rho = 0.75 \) flips to Phase wavelength \( \lambda'_{\text{phase}} = \lambda_A \cosh \rho = 4/3 \)

\[
R' = K'_{\text{phase}} = \begin{pmatrix} c \kappa'_{\text{phase}} \\ \nu'_{\text{phase}} \end{pmatrix} = \nu_A \begin{pmatrix} \sinh \rho \\ \cosh \rho \end{pmatrix} = \nu_A \begin{pmatrix} 0.75 \\ 4/3 \end{pmatrix}
\]

RelaWavity Web Simulation - 16 Relativity Dimensions
The diagram illustrates the relationship between phase wavenumber and phase wavelength, showing how they flip to their counterparts in a transformed coordinate system. It also highlights the velocity of light and its change in a reference frame.

- **Phase wavenumber:** $\kappa'_{\text{phase}} = \kappa_{A} \sinh \rho = 3/4$
- **Phase wavelength:** $\lambda'_{\text{phase}} = \lambda_{A} \text{csch} \rho = 4/3$

The diagram further explains the transformation of frequency and period, showing how velocities and wavelengths change in different reference frames.
Review of Doppler-shift and Rapidity $\rho_{AB}$ calculation: *Galileo’s Revenge Part I Lect. 23 p.64-75*
Relating rapidity $\rho_{AB}$ and relativity velocity parameter $\beta_{AB}=u_{AB}/c$

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More at Pirelli Challenge page: *'Un Grande Affare’ - Light Meets Light*
The 16 dimensions of 2CW interference

\[
G' = \begin{pmatrix}
\cosh \rho \\
\sinh \rho
\end{pmatrix} = \nu_A 
\begin{pmatrix}
2/3 \\
2/3
\end{pmatrix}
\]

Group frequency
\\
\nu'_\text{group} = \nu_A \sinh \rho = 3/4
\\
= 0.75
\\

Group period
\\
\tau'_\text{group} = \tau_A \csc h \rho = 4/3
\\
= 1.33
\\

Start with the Dopplers
\ldots then do the phase waves
\ldots then the group waves
\\

Frequency
\\
\nu'
\\
(\text{units of } \nu_A = 600 \text{THz})
\\
1500 THz
\\
1200 THz
\\
900 THz
\\
600 THz
\\
300 THz
\\
\nu'_\text{group} = 0.75
\\

Wavevector \( c \kappa'_A \)
\\
(\text{units of } c \kappa_A = 2 \times 10^6 / \text{m})
\\
RelaWavity Web Simulation - 16 Relativity Dimensions

\[ R = K_{-4} \]

\[ L = K_{-1} \]

\[ P = K_1 \]

\[ \text{Space } x' \]

(\text{units of } \lambda_A = 1/2 \mu m)

\[ 2.0 \]

\[ 1.5 \]

\[ 1.0 \]

\[ 0.5 \]

\[ -0.5 \]

\[ -1.0 \]

\[ -1.5 \]

\[ -2.0 \]

\[ \text{Time } ct' \]

(\text{units of } \lambda_A = 1/2 \mu m)

\[ 2 \]

\[ 1.5 \]

\[ 1.0 \]

\[ 0.5 \]

\[ -0.5 \]

\[ -1.0 \]

\[ -1.5 \]

\[ -2.0 \]
The 16 dimensions of 2CW interference

\[
G' = \left( \begin{array}{cc}
\frac{c\kappa_{\text{group}}'}{u_{\text{group}}'} & \cosh \rho \\
\frac{u_{\text{group}}'}{u_{\text{group}}'} & \sinh \rho
\end{array} \right) = \nu_A \left( \begin{array}{cc}
\frac{5}{4} \\
\frac{3}{4}
\end{array} \right)
\]

**Group frequency**

\[u_{\text{group}}' = \nu_A \sinh \rho = 3/4 = 0.75\]

**Group period**

\[\tau = \frac{1}{\nu} \]

\[\tau'_{\text{group}} = \tau_A \text{csch} \rho = 4/3 = 1.33\]

**Start with the Dopplers**

...then do the phase waves

...then the group waves

**Frequency**

\[\nu' (\text{units of } \nu_A = 600\text{THz})\]

- 1500 THz
- 1200 THz
- 900 THz
- 600 THz
- 300 THz
- 2 -1.10^6
- 1.10^6
- 2.10^6
- 3.10^6
- 4.10^6
- 0

**Wavevector**

\[\kappa'_{\text{group}} = 1.25\]

\[\kappa_{\text{group}}' = 0.75\]

\[\nu_{\text{group}}' = \nu_A \sinh \rho = 3/4 = 0.75\]

**Space**

\[x' (\text{units of } \lambda_A = 1/2\mu m)\]

**RelaWavity Web Simulation - 16 Relativity Dimensions**

<table>
<thead>
<tr>
<th>phase</th>
<th>b_{\text{Doppler \ RED}}</th>
<th>c</th>
<th>\kappa_{\text{phase}}/\kappa_A</th>
<th>\lambda_{\text{phase}}/\lambda_A</th>
<th>V_{\text{phase}}/c</th>
<th>b_{\text{Doppler \ BLUE}}</th>
</tr>
</thead>
<tbody>
<tr>
<td>group</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>rapidity</td>
<td>\rho</td>
<td>e^{-\rho}</td>
<td>\tanh \rho</td>
<td>\sinh \rho</td>
<td>\sech \rho</td>
<td>\cosh \rho</td>
</tr>
<tr>
<td>value for</td>
<td>$\beta = 3/5$</td>
<td>$1/2 = 0.5$</td>
<td>$3/5 = 0.6$</td>
<td>$3/4 = 0.75$</td>
<td>$4/5 = 0.80$</td>
<td>$5/4 = 1.25$</td>
</tr>
</tbody>
</table>
Start with the Dopplers...then do the phase waves...then the group waves.

\[ \rho = \frac{\kappa_A}{2} \]

\[ \kappa'_{\text{group}} = \kappa_A \cosh \rho = \frac{5}{4} \]

\[ \lambda'_{\text{group}} = \lambda_A \text{sech} \rho = \frac{4}{5} \]

\[ \tau'_{\text{group}} = \tau_A \csc \rho = \frac{4}{3} \]

\[ \tau'_{\text{group}} = 1.33 \]

\[ \rho = 0.75 \]

\[ \kappa'_{\text{group}} = 1.25 \]

\[ \lambda'_{\text{group}} = 0.8 \]

\[ \tau'_{\text{group}} = 1.25 \]

\[ \rho = 1/\kappa \]

\[ \kappa'_{\text{group}} = \kappa_A \cosh \rho = 1.25 \]

\[ \lambda'_{\text{group}} = \lambda_A \text{sech} \rho = 0.8 \]

\[ \tau'_{\text{group}} = \tau_A \csc \rho = 1.25 \]

\[ \rho = 2 \]

\[ \kappa'_{\text{group}} = 2 \]

\[ \lambda'_{\text{group}} = 0.8 \]

\[ \tau'_{\text{group}} = 1.25 \]

\[ \rho = \frac{\kappa_A}{2} \]

\[ \kappa'_{\text{group}} = 1.25 \]

\[ \lambda'_{\text{group}} = 0.8 \]

\[ \tau'_{\text{group}} = 1.25 \]

\[ \rho = 2 \]

\[ \kappa'_{\text{group}} = 2 \]

\[ \lambda'_{\text{group}} = 0.8 \]

\[ \tau'_{\text{group}} = 1.25 \]

\[ \rho = \frac{\kappa_A}{2} \]

\[ \kappa'_{\text{group}} = 1.25 \]

\[ \lambda'_{\text{group}} = 0.8 \]

\[ \tau'_{\text{group}} = 1.25 \]

\[ \rho = 2 \]

\[ \kappa'_{\text{group}} = 2 \]

\[ \lambda'_{\text{group}} = 0.8 \]

\[ \tau'_{\text{group}} = 1.25 \]

\[ \rho = \frac{\kappa_A}{2} \]

\[ \kappa'_{\text{group}} = 1.25 \]

\[ \lambda'_{\text{group}} = 0.8 \]

\[ \tau'_{\text{group}} = 1.25 \]

\[ \rho = 2 \]

\[ \kappa'_{\text{group}} = 2 \]

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\[ \tau'_{\text{group}} = 1.25 \]

\[ \rho = 2 \]

\[ \kappa'_{\text{group}} = 2 \]

\[ \lambda'_{\text{group}} = 0.8 \]

\[ \tau'_{\text{group}} = 1.25 \]

\[ \rho = \frac{\kappa_A}{2} \]

\[ \kappa'_{\text{group}} = 1.25 \]

\[ \lambda'_{\text{group}} = 0.8 \]

\[ \tau'_{\text{group}} = 1.25 \]

\[ \rho = 2 \]

\[ \kappa'_{\text{group}} = 2 \]

\[ \lambda'_{\text{group}} = 0.8 \]

\[ \tau'_{\text{group}} = 1.25 \]

\[ \rho = \frac{\kappa_A}{2} \]

\[ \kappa'_{\text{group}} = 1.25 \]

\[ \lambda'_{\text{group}} = 0.8 \]

\[ \tau'_{\text{group}} = 1.25 \]

\[ \rho = 2 \]

\[ \kappa'_{\text{group}} = 2 \]

\[ \lambda'_{\text{group}} = 0.8 \]

\[ \tau'_{\text{group}} = 1.25 \]
Group wavenumber $k'_{\text{group}} = \kappa_A \cosh \rho = \frac{5}{4}$

$= 1.25$

Group wavelength $\lambda'_{\text{group}} = \lambda_A \text{sech} \rho = \frac{4}{5}$

$= 0.8$

Time $ct'$

(units of $\lambda_A = 1/2 \mu m$)

Start with the Dopplers

...then do the phase waves

...then the group waves

Group frequency $\nu'_{\text{group}} = \nu_A \sinh \rho = \frac{3}{4}$

$= 0.75$

Group period $\tau'_{\text{group}} = \tau_A \text{csch} \rho = \frac{4}{3}$

$= 1.33$

Space $x'$

(units of $\lambda_A = 1/2 \mu m$)

Frequency $\nu'$

(units of $\nu_A = 600 \text{THz}$)

Wavevector $\mathbf{c}k'$

(units of $\mathbf{c}k_A = 2 \cdot 10^6 / m$)

RelaWavity Web Simulation - 16 Relativity Dimensions

<table>
<thead>
<tr>
<th>Phase</th>
<th>$\beta$</th>
<th>$\kappa_{\text{phase}}$</th>
<th>$\nu_{\text{phase}}$</th>
<th>$\nu_A$</th>
<th>$\lambda_{\text{phase}}$</th>
<th>$\lambda_A$</th>
<th>$V_{\text{phase}}$</th>
<th>$c$</th>
<th>$1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>group</td>
<td>$b_{\text{Doppler}}$</td>
<td>$V_{\text{phase}}$</td>
<td>$\nu_{\text{group}}$</td>
<td>$\lambda'_{\text{group}}$</td>
<td>$\kappa'_{\text{group}}$</td>
<td>$\tau'_{\text{group}}$</td>
<td>$\tau_A$</td>
<td>$V_{\text{group}}$</td>
<td>$c$</td>
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</table>
| $\rho$ | $e^{-\rho}$ | $\tanh \rho$ | $\sinh \rho$ | $\text{sech} \rho$ | $\cosh \rho$ | $\text{csch} \rho$ | $\coth \rho$ | $e^{+\rho}$ |}

| $\beta = 3/5$ | $\frac{1}{2} = 0.5$ | $\frac{3}{5} = 0.6$ | $\frac{3}{4} = 0.75$ | $\frac{4}{5} = 0.80$ | $\frac{5}{4} = 1.25$ | $\frac{4}{3} = 1.33$ | $\frac{5}{3} = 1.67$ | $\frac{2}{1} = 2.0$ |
Group wavenumber
\( \kappa'_{\text{group}} = \kappa_A \cosh \rho = \frac{5}{4} \)
= 1.25

Group frequency
\( v'_{\text{group}} = \frac{\sinh \rho}{\kappa'_{\text{group}}} = \frac{3}{4} \)
= 0.75

G-slope = \( \frac{V_{\text{group}}}{c} \)
\( \kappa'_{\text{group}} = \frac{\cosh \rho}{\kappa A} = \frac{5}{4} \)

Group wavelength
\( \lambda'_{\text{group}} = \lambda_A \text{sech} \rho = \frac{4}{5} \)
= 0.8

Time \( ct' \)

Group period
\( T'_{\text{group}} = T_A \text{csch} \rho = \frac{4}{3} \)
= 1.33

Space \( x' \)

(units of \( \lambda_A = 1/2 \mu m \))
Review of Doppler-shift and Rapidity \( \rho_{AB} \) calculation: *Galileo’s Revenge Part I* Lect. 23 p.64-75
Relating rapidity \( \rho_{AB} \) and relativity velocity parameter \( \beta_{AB} = u_{AB}/c \)

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Doppler shifted Group parameters

\[
\rightarrow \quad \text{Lorentz transformation matrix and Two Famous-Name Coefficients} \quad \leftarrow
\]

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Detailed geometric construction of relawavity plot for 1-octave Doppler \((\beta_{AB} = u_{AB}/c = 3/5)\)

Stellar aberration and the Epstein approach to SR

More at Pirelli Challenge page: *'Un Grande Affare' - Light Meets Light*
Lorentz transformations... write $G'$ and $P'$ in terms of $G$ and $P$ using $\cosh \rho$ and $\sinh \rho$

\[
G' = \begin{pmatrix}
c'_{\text{group}} \\
u'_{\text{group}}
\end{pmatrix} = \nu_A \begin{pmatrix}
\cosh \rho \\
\sinh \rho
\end{pmatrix} = \nu_A \begin{pmatrix}
5/4 \\
3/4
\end{pmatrix}
= \nu_A \begin{pmatrix}
1 \\
0
\end{pmatrix} \cosh \rho + \nu_A \begin{pmatrix}
0 \\
1
\end{pmatrix} \sinh \rho
\]

\[
P' = \begin{pmatrix}
c'_{\text{phase}} \\
u'_{\text{phase}}
\end{pmatrix} = \nu_A \begin{pmatrix}
\sinh \rho \\
\cosh \rho
\end{pmatrix} = \nu_A \begin{pmatrix}
3/4 \\
5/4
\end{pmatrix}
= \nu_A \begin{pmatrix}
1 \\
0
\end{pmatrix} \sinh \rho + \nu_A \begin{pmatrix}
0 \\
1
\end{pmatrix} \cosh \rho
\]

$P' = G \sinh \rho + P \cosh \rho$

\[
\begin{pmatrix}
cosh \rho & \sinh \rho \\
\sinh \rho & c\cosh \rho
\end{pmatrix}
\]
Lorentz transform matrix

\[
\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
\text{phase} & b_{\text{Doppler}}^{\text{RED}} & c & \kappa_{\text{phase}} & \frac{\kappa}{\kappa_A} & \tau_{\text{phase}} & \frac{\tau}{\tau_A} & \nu_{\text{phase}} & \frac{\nu}{\nu_A} & \lambda_{\text{phase}} & \frac{\lambda}{\lambda_A} & V_{\text{phase}} & \frac{1}{b_{\text{Doppler}}^{\text{BLUE}}}
\hline
\text{group} & \frac{1}{b_{\text{Doppler}}^{\text{RED}}} & \frac{V_{\text{group}}}{c} & \frac{\nu_{\text{group}}}{\nu_A} & \frac{\lambda_{\text{group}}}{\lambda_A} & \frac{\kappa_{\text{group}}}{\kappa_A} & \frac{\tau_{\text{group}}}{\tau_A} & \frac{V_{\text{phase}}}{c} & \frac{1}{b_{\text{Doppler}}^{\text{BLUE}}}
\hline
\text{rapidity} \ \ \ \ \rho & e^{-\rho} & \tanh \rho & \sinh \rho & \sech \rho & \cosh \rho & \csch \rho & \coth \rho & e^{+\rho}
\hline
\text{value for } \beta = 3/5 & 1/2 = 0.5 & 3/5 = 0.6 & 3/4 = 0.75 & 4/5 = 0.80 & 5/4 = 1.25 & 4/3 = 1.33 & 5/3 = 1.67 & 2/1 = 2.0
\hline
\end{array}
\]
### Two Famous-Name Coefficients

**Time $ct'$**
(units of $\lambda_A = 1/2\mu m$)

**Space $x'$**
(units of $\lambda_A = 1/2\mu m$)

This number is called an **Einstein time-dilation**
(dilated by 25% here)

This number is called a **Lorentz length-contraction**
(contracted by 20% here)

---

**Old-Fashioned Notation**

Relativistic Terms (Dual plot w/ expanded table)

<table>
<thead>
<tr>
<th>phase group</th>
<th>$b_D$</th>
<th>$c/V_{phase}$</th>
<th>$\kappa_{phase}$</th>
<th>$\tau_{phase}$</th>
<th>$\nu_{phase}$</th>
<th>$\lambda_{phase}$</th>
<th>$V_{phase}$</th>
<th>$b_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{group}$</td>
<td>$V_{group}$</td>
<td>$\nu_{group}$</td>
<td>$\lambda_{group}$</td>
<td>$\kappa_{group}$</td>
<td>$\tau_{group}$</td>
<td>$\nu_{group}$</td>
<td>$\lambda_{group}$</td>
<td>$\kappa_{group}$</td>
</tr>
<tr>
<td>rapidity $\rho$</td>
<td>$e^{-\rho}$</td>
<td>$\tanh \rho$</td>
<td>$\sinh \rho$</td>
<td>$\sech \rho$</td>
<td>$\cosh \rho$</td>
<td>$\csch \rho$</td>
<td>$\coth \rho$</td>
<td>$e^{+\rho}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\beta = \frac{u}{c}$</th>
<th>$\sqrt{1-\beta^2}$</th>
<th>$\frac{1}{\sqrt{1+\beta^2}}$</th>
<th>$\sqrt{1-\beta^2} \frac{1}{1}$</th>
<th>$\sqrt{1-\beta^2} \frac{1}{1}$</th>
<th>$\sqrt{1-\beta^2} \frac{1}{1}$</th>
<th>$\sqrt{1+\beta^2} \frac{1}{1}$</th>
<th>$\sqrt{1+\beta^2} \frac{1}{1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta^2 = 0.5$</td>
<td>$\frac{3}{5} = 0.6$</td>
<td>$\frac{3}{4} = 0.75$</td>
<td>$\frac{4}{5} = 0.80$</td>
<td>$\frac{5}{4} = 1.25$</td>
<td>$\frac{4}{3} = 1.33$</td>
<td>$\frac{5}{3} = 1.67$</td>
<td>$\frac{2}{1} = 2.0$</td>
</tr>
</tbody>
</table>

---

**Relativity Web Simulation**

**Hendrik A. Lorentz**
1853-1928

**Albert Einstein**
1859-1955

**Herman Minkowski**
1864-1909

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*If you can't explain it *simply*, you don't understand it well enough.*

— Albert Einstein

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Thursday, April 7, 2016
Review of Doppler-shift and Rapidity $\rho_{AB}$ calculation: *Galileo’s Revenge Part I* Lect. 23 p.64-75

Relating rapidity $\rho_{AB}$ and relativity velocity parameter $\beta_{AB}=u_{AB}/c$

Review of $\frac{1}{2}$-sum-$\frac{1}{2}$-difference Phase and Group factors giving relativistic space-axes and time-axes

Colliding-CW space-time ($x,ct$)-graph vs Colliding PW space-time ($R,L$)-baseball diamond

Review of $\frac{1}{2}$-sum-$\frac{1}{2}$-difference of phasor angular velocity: *Galileo’s Revenge Part II* (Pirelli site)

Elementary models: 2-comb Moire' patterns and cosine-law constructions

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Thales Mean Geometry (*Thales of Miletus 624-543 BCE*) and its role in Relawavity

Detailed geometric construction of relawavity plot for 1-octave Doppler ($\beta_{AB}=u_{AB}/c=3/5$)

Stellar aberration and the Epstein approach to SR

More at Pirelli Challenge page: *'Un Grande Affaire’ - Light Meets Light*
Thales Mean Geometry (600BCE)

helps “Relawavity”

Thales of Miletus
624-543 BCE

Frequency unit:
300THZ

Per-Time
\( \omega - \text{axis} \)

Transformed Per-Time
\( \omega' - \text{axis} \)

Slope-to-vertical
\( = \frac{V_{\text{group}}/c = 3/5}{4+1} \)

Geometric Mean
\( B = \sqrt{4 \cdot 1} = 2 \)

Arithmetic Mean
\( B_{\text{cosh} \rho} = \frac{(1+4)}{2} = \frac{5}{2} \)

Difference Mean
\( B_{\text{sinh} \rho} = \frac{(4-1)}{2} = \frac{3}{2} \)

Red shift = 1

Blue shift = 4
Thales Mean Geometry (600 BCE)

Thales showed a circle diameter subtends a right angle with any circle point \( P \). This leads to a convenient construction of geometric means and relativistic hyperbolas.

**Thales of Miletus**
624-543 BCE

Frequency unit: 300 THZ

**Geometric Mean**

\[ B = \sqrt{(4-1)} = 2 \]

**Arithmetic Mean**

\[ B_{\text{cosh}} \rho = \frac{(1+4)}{2} = \frac{5}{2} \]

**Difference Mean**

\[ B_{\text{sinh}} \rho = \frac{(4-1)}{2} = \frac{3}{2} \]

**Transformed Per-Time**

\[ \omega' - \text{axis} \]

\[ \text{Slope-to-vertical} = \frac{4-1}{4+1} = \frac{3}{5} \]

This leads to a convenient construction of geometric means and relativistic hyperbolas.
Thales Mean Geometry (600BCE)

helps “Relawavity”  Thales showed a circle diameter subtends a right angle with any circle point P

This leads to a convenient construction of geometric means and relativistic hyperbolas.
Thales Mean Geometry (600BCE)

helps “Relawavity”

Thales of Miletus
624-543 BCE

\[ r \cdot b = 2 \]
due to Doppler T-symmetry

Per-Time
\( \omega - \text{axis} \)

Per-Space
\( ck' - \text{axis} \)

Geometric Mean
\[ B = \sqrt{(4 \cdot 1)} = 2 \]

Arithmetic Mean
\[ B \cosh \rho = \frac{(1+4)}{2} = \frac{5}{2} \]

Difference Mean
\[ B \sinh \rho = \frac{(4-1)}{2} = \frac{3}{2} \]

Red shift

Blue shift

RelaWavity Web Simulation
Detailed Thales Geometry
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hyperbola $H=bcosh\theta$

slope = -tan $\theta$

slope = sin $\theta$

Base circle $L=bsin\theta$

$ck_R = \omega_R - \omega_L$

$ck_L = \omega_R + \omega_L$

$ck_R - ck_L = \omega_R + \omega_L$

$ck_L + ck_R = \omega_R - \omega_L$
\[ \rho = \log e 2 = \text{Arctanh}(\frac{3}{5}) = \tanh^{-1}(\frac{3}{5}) = 0.6931 \]
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Comparing **Longitudinal** relativity parameter: Rapidity $\rho = \log_e(\text{Doppler Shift})$

*to a* **Transverse** relativity parameter: Stellar aberration angle $\sigma$


Observer fixed below star sees it directly overhead.
Observer going $\mathbf{u}$ sees star at angle $\sigma$ in $\mathbf{u}$ direction.

Stellar aberration angle $\sigma$:

$$c \tanh \rho = u = c \sin \sigma$$

We used notion $\sigma$ for stellar-ab-angle, (a “flipped-out” $\rho$). Epstein not interested in $\rho$ analysis or in relation of $\sigma$ and $\rho$. 
Comparing **Longitudinal** relativity parameter: Rapidity $\rho = \log_e(\text{Doppler Shift})$

to a **Transverse** relativity parameter: Stellar aberration angle $\sigma$


**Proper time** $c\tau$ vs. **coordinate space** $x$ - (L. C. Epstein’s “Cosmic Speedometer”)

Particles $P$ and $P'$ have speed $u$ in $(x', ct')$ and speed $c$ in $(x, c\tau)$

**Proper time** $c\tau$

$c\tau = \sqrt{(ct')^2 - (x')^2}$

**Coordinate**

$x' = (u/c)ct' = ut'$

**Einstein time dilation:**

$ct' = c\tau \sec \sigma = c\tau \cosh \rho = c\tau/\sqrt{1-u^2/c^2}$

**Lorentz length contraction:**

$L' = L \sech \rho = L \cos \sigma = L \sqrt{1-u^2/c^2}$

**Proper Time asimultaneity:**

$c \Delta \tau = L' \sinh \rho = L \cos \sigma \sinh \rho$

$= L \cos \sigma \tan \sigma$

$= L \sin \sigma = L/\sqrt{c^2/u^2-1} \sim L u/c$
Comparing **Longitudinal** relativity parameter: Rapidity $\rho = \log_e(\text{Doppler Shift})$

to a **Transverse** relativity parameter: Stellar aberration angle $\sigma$


**Proper time** $c\tau$ vs. coordinate space $x$ - (L. C. Epstein’s “Cosmic Speedometer”)

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**Coordinate**

$$x' = (u/c)ct' = ut'$$

**Einstein time dilation:**

$$ct' = c\tau \sec \sigma = c\tau \cosh \rho = c\tau / \sqrt{1 - u^2/c^2}$$

**Lorentz length contraction:**

$$L' = L \sech \rho = L \cos \sigma = L \cdot \sqrt{1 - u^2/c^2}$$

**Proper Time as simultaneity:**

$$c \Delta \tau = L' \sinh \rho = L \cos \sigma \sinh \rho$$

$$= L \cos \sigma \tan \sigma$$

$$= L \sin \sigma = L / \sqrt{c^2/u^2 - 1} \sim L u/c$$

Epstein’s trick is to turn a hyperbolic form $c\tau = \sqrt{(ct')^2 - (x')^2}$ into a circular form:

$$\sqrt{(c\tau)^2 + (x')^2} = (ct')$$

Then everything (and everybody) always goes speed $c$ through $(x', c\tau)$ space!
Comparing **Longitudinal** relativity parameter: Rapidity $\rho = \log_e(\text{Doppler Shift})$

to a **Transverse** relativity parameter: Stellar aberration angle $\sigma$

(a) Circular Functions

- $\sin(\sigma) = 0.6000$
- $\tan(\sigma) = 0.7500$
- $\sec(\sigma) = 1.2500$

Relativity Web Simulation

*Geometry of Stellar Aberration Angle*
Comparing **Longitudinal** relativity parameter: \( \rho = \log_e(\text{Doppler Shift}) \)

to a **Transverse** relativity parameter: Stellar aberration angle \( \sigma \)

Circular Functions
- \( \sin(\sigma) = 0.6000 \)
- \( \tan(\sigma) = 0.7500 \)
- \( \sec(\sigma) = 1.2500 \)

Hyperbolic Functions
- \( \tanh(\rho) = 0.6000 \)
- \( \sinh(\rho) = 0.7500 \)
- \( \cosh(\rho) = 1.2500 \)

**RelaWavity Web Simulation**
*Geometry of Rapidity Relations*
Summary of optical wave parameters for relativity and QM
...and their geometry

\[ v' = \frac{\omega'}{2\pi} \]
axis (Units of 300THz)

An aid to pattern recognition:

RelaWavity Web Simulation{perSpace - perTime All}
\[ u' = \omega' / 2\pi \]

axis

(Units of 300THz)

\[ B \sinh \rho \]

\[ B \tanh \rho \]

\[ B \csc \rho \]

\[ B \sech \rho \]

\[ B \cosh \rho \]

**Table of 12 wave parameters** (includes inverses) for relativity

...and values for \( u/c = 3/5 \)

- \( \beta = \frac{u}{c} \)
  - \( \beta = \frac{1}{2} = 0.5 \)
  - \( \beta = 0.6 \)
  - \( \beta = 0.75 \)
  - \( \beta = 0.80 \)
  - \( \beta = 1.25 \)
  - \( \beta = 1.33 \)
  - \( \beta = 1.67 \)
  - \( \beta = 2.0 \)

\[ \beta = \frac{u}{c} \]

\( \frac{1}{2} \)

\( \frac{3}{5} = 0.6 \)

\( \frac{3}{4} = 0.75 \)

\( \frac{4}{5} = 0.80 \)

\( \frac{5}{4} = 1.25 \)

\( \frac{4}{3} = 1.33 \)

\( \frac{5}{3} = 1.67 \)

\( \frac{2}{1} = 2.0 \)

An aid to pattern recognition:

**Occam's Sword**

(\( u/c = 3/5 \))

Relativity Web Simulation

Relativistic Terms (Dual plot w/expanded table)