The following is to acquaint you with some more exponential properties and phasor views of 2D-HO

Fun with Exponents & more of the Story of e

1.10.1 Consider a sequence of functions , $f_1(z)=z^z, f_2(z)=z^{f_1(z)}=z^{z^z}, f_3(z)=z^{f_2(z)}=z^{z^{z^z}}$,.... The function $f_N(z)$ has a finite limit $f_\infty(z)$ (as N approaches infinity) if number z is small enough .
(Hints: $z=1$ works! But, so does $z=\sqrt{2}$. Try solving for z and looking for a max value.)

(a) Find $f_\infty(\sqrt{2})=$____?

(b) Find an analytic expression for the limiting real z_{max} that involves the Euler constant. $e=2.718281828$... and check its numerical value.

This is to introduce phasor views of 2D-HO Geometry of *Phasor-to-Cartesian and vice-versa relations*

1.10.2 *A day in the life of a neutron starlet*
Suppose neutron starlet orbits inside the "Sophomore-Physics-Earth" (SPE) of radius $R_{SPE}=10$ units. (One unit is 637*km*.) Let it start at position $\mathbf{r}(0)=(7.0,3.0)$ with velocity $\mathbf{v}(0)=(6.0,8.0)$. (Let time unit be determined by setting SPE angular frequency to one ($\omega_{SPE}=1=2\pi\,\upsilon_{SPE}$).

(a) Use position-velocity-phasor graph paper to set times (phases) and amplitudes of x and y phasors.

(b) Use the phasors to locate the starlet position vector $\mathbf{r}(t)$ and its velocity $\mathbf{v}(t)$ for a complete orbit.

(c) Label times of each 12 orbit points at equal time intervals defined as follows by period $\tau_{SPE}=1/\upsilon_{SPE}$: $t=0$ is 12:00PM, $t=\tau_{SPE}/12$ is 1:00PM, $t=2\tau_{SPE}/12$ is 2:00PM, $t=3\tau_{SPE}/12$ is 3:00PM, and so forth. Label velocity vector points, too.

(d) Does the starlet ever penetrate the Earth surface? If it does, how might that affect its orbit?

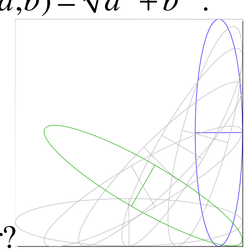
1.10.3 *12 days for a neutron starlet with an increasingly retarded x-phasor*
This project involves the "pincushion" graph paper for which the *x* and *y* phasors have the same unit-amplitude but their relative phase shifts so the x-phasor is retarded by ½ hour after each 24 hour orbit.

(a) Start by plotting an orbit starting with both phasors at 3:00 o'clock. (Should be a straight 45° line.) Then plot an orbit with the x-phasor starting at 2:30PM or 15° behind the y-phasor and maintaining that phase lag through the orbit. (Should be very narrow or eccentric ellipse.) Then plot an orbit with the x-phasor starting at 2:00PM or 30° behind the y-phasor and maintaining that phase lag through the orbit. (Should be a more rounded or less eccentric ellipse.) Continue plotting ellipses with phase lag 45°, 60°, 75°, 90°,..., 165°, and finally 180°, that last one being called "PI out of phase." Note what is special about the 90° case.

(b) Each of the ellipses drawn in part (a) has its own major radius *a* and minor radius *b* and may be circumscribed in its own *2a*-by-*2b* rectangle. With a ruler carefully sketch each of these rectangles.

(c) The hypotenuse of rectangular radii $r^{hypot}(a,b)=\sqrt{a^2+b^2}$ should be related. How?

(d) Derive the total energy of a 2D-IHO orbit and show it is proportional to $r^{hypot}(a,b)=\sqrt{a^2+b^2}$.



(e) Elliptical *5-by-1* surfboard rotates tangent to wall and floor. Where is its center?