

The following is to acquaint you with of some lesser known properties of exponentials and logarithms

1.8.1 Backsides of exponentials

- (a) Follow zig-zag scheme shown at the beginning of Lect. 11 to make plots of exponential $y=e^x$ at as many integer points $x= -2, -1, 0, 1, 2,..$ as is practical on full page graph paper provided online or in lab. Then add to the plot precise half-way points $x= -2.5, -1.5, -0.5, etc...$ as is practical. Show how a plot of $y=log_e x$ function is obtained from the graph
- (b) By algebra or geometry find tangent lines and their slope at integer points $x=-2, -1, 0, 1, 2,..$ (This is equivalent to solving the part (c) of this exercise.)
- (c) As a roller-coaster car moves down a track $y=e^x$ it shines one laser headlight beam along the track and another droplight beam vertically downward so both make spots on baseline $y=0$. Find the distance between spots as function of x .

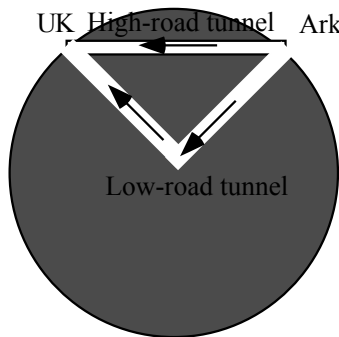
1.8.2 Sophomore-Physics-Earth

- (a) Follow the zig-zag scheme in Lect. 11 (or in Fig. 8.5 and 8.7 of text) to construct the potential and force curves of the Ideal Uniform Density Earth inside ($PE(x)=kx^2/2+PE(0)$) and outside ($PE(x)=-x^{-1}$).
- (b) On graph show focal point, latus-radius , and directrix of the inside PE parabola. Draw as accurately as possible the parabola's circle of curvature contacting it at $x=0$.
- (c) Draw a "kite" (see Fig. 8.4 in text) tangent to parabola at $x=l$ and another tangent at $x=1/2$.

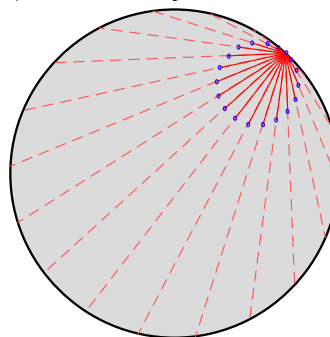
1.8.3 Tunnels to UK (5600 miles as an earthworm crawls) are shown below. One high-road is a direct route. A low-road turns at the Earth center. (Travel and turn-around are assumed frictionless and survivable.)

- (a) What is the time for each trip? Discuss using geometry or algebra arguments.

(a) Hi-road & low-road



(b) Lots of roads



- (b) Assume cars in subway tunnels depart Ark. at time $t=0$ as indicated above. Describe curve (thru dots shown) locating car positions at a later mid-trip time t before arrival and at arrival. (Thales geometry of circular chords may help. Recall superball figure 6.1 in text.)
- (c) What if the half-way turn-around point is above the Earth-center. Is trip quicker or slower?