Group Theory in Quantum Mechanics
Lecture 3 *(1.24.17)*
Analyzers, operators, and group axioms

(Quantum Theory for Computer Age - Ch. 1-2 of Unit 1)
(Principles of Symmetry, Dynamics, and Spectroscopy - Sec. 1-3 of Ch. 1)

Review: Axioms 1-4 and "Do-Nothing" vs " Do-Something" analyzers

Abstraction of Axiom-4 to define projection and unitary operators
- *Projection operators and resolution of identity*

Unitary operators and matrices that do something (or "nothing")
- *Diagonal unitary operators*
- *Non-diagonal unitary operators and †-conjugation relations*
- *Non-diagonal projection operators and Kronecker ⊗−products*
- *Axiom-4 similarity transformation*

Matrix representation of beam analyzers
- *Non-unitary "killer" devices: Sorter-counter, filter*
- *Unitary "non-killer" devices: 1/2-wave plate, 1/4-wave plate*

How analyzers "peek" and how that changes outcomes
- *Peeking polarizers and coherence loss*
  - *Classical Bayesian probability vs. Quantum probability*

Feynman $\langle j|k\rangle$-axioms compared to **Group axioms**

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Feynman amplitude axioms 1-4

Feynman-Dirac Interpretation of

$$\langle j | k' \rangle$$

=Amplitude of state-j after state-k′ is forced to choose from available m-type states

(1) The probability axiom

The first axiom deals with physical interpretation of amplitudes $\langle j | k' \rangle$.

Axiom 1: The absolute square $\left|\langle j | k' \rangle\right|^2 = \langle j | k' \rangle^* \langle j | k' \rangle$ *gives probability for occurrence in state-j of a system that started in state-k′=1′,2′,..,or n′ from one sorter and then was forced to choose between states j=1,2,...,n by another sorter.*

(2) The conjugation or inversion axiom *(time reversal symmetry)*

The second axiom concerns going backwards through a sorter or the reversal of amplitudes.

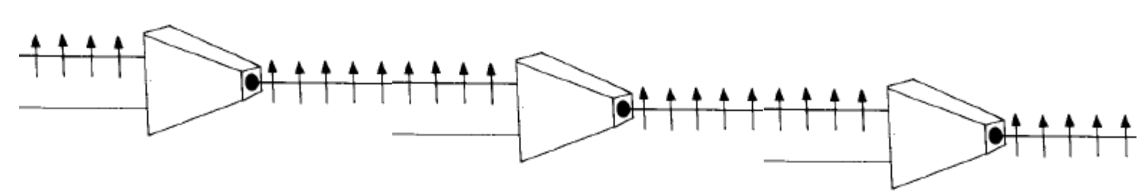
Axiom 2: The complex conjugate $\langle j | k' \rangle^*$ *of an amplitude* $\langle j | k' \rangle$ *equals its reverse:* $\langle j | k' \rangle^* = \langle k' | j \rangle$

(3) The orthonormality or identity axiom

The third axiom concerns the amplitude for "re measurement" by the same analyzer.

Axiom 3: If identical analyzers are used twice or more the amplitude for a passed state-k is one, and for all others it is zero:

$$\langle j | k \rangle = \delta_{jk} = \begin{Bmatrix} 1 \text{ if: } j=k \\ 0 \text{ if: } j \neq k \end{Bmatrix} = \langle j' | k' \rangle$$

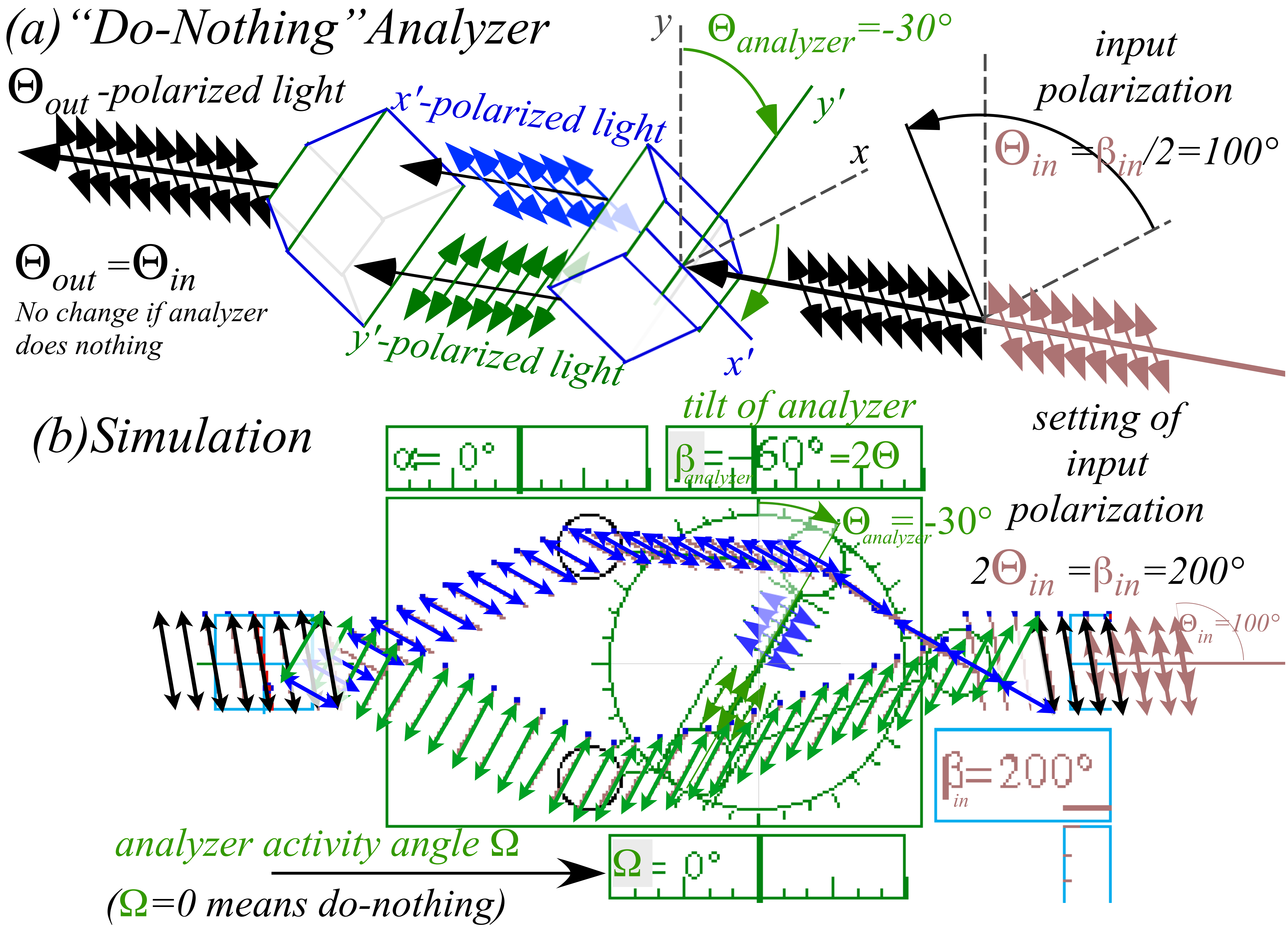


(4) The completeness or closure axiom

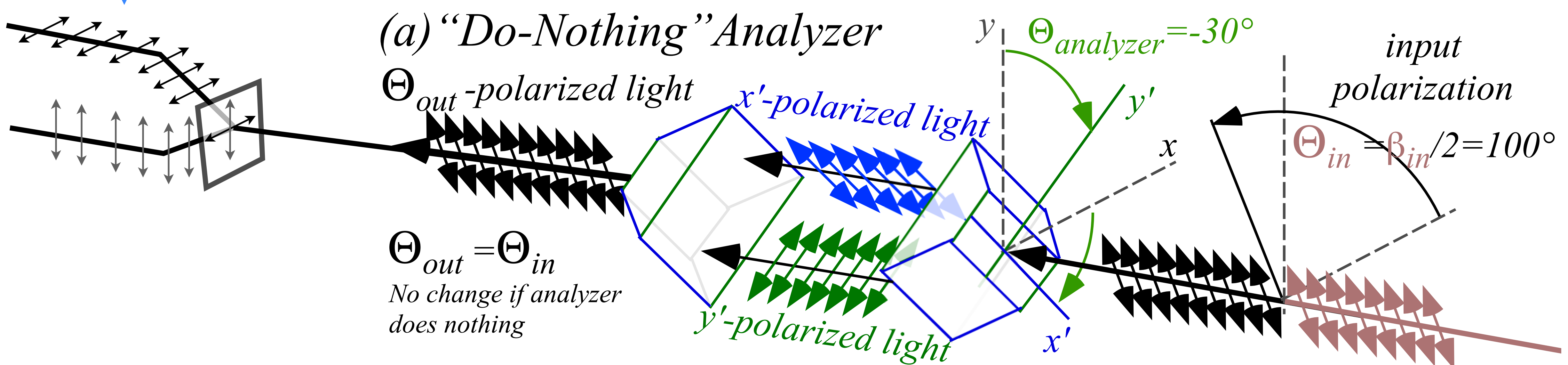
The fourth axiom concerns the "*Do-nothing*" property of an ideal analyzer, that is, a sorter followed by an "unsorter" or "put-back-togetherer" as sketched above.

Axiom 4. Ideal sorting followed by ideal recombination of amplitudes has no effect:

$$\langle j'' | m' \rangle = \sum_{k=1}^{n} \langle j'' | k \rangle \langle k | m' \rangle$$


(a) "Do-Nothing" Analyzer
Θ_{out} -polarized light
x'-polarized light
y'-polarized light
$\Theta_{out} = \Theta_{in}$
No change if analyzer does nothing
$\Theta_{analyzer} = -30°$
input polarization
$\Theta_{in} = \beta_{in}/2 = 100°$
x
y
x'
y'
(b) Simulation
tilt of analyzer
α= 0°
$\beta_{analyzer} = -60° = 2\Theta$
$\Theta_{analyzer} = -30°$
setting of input polarization
$2\Theta_{in} = \beta_{in} = 200°$
$\Theta_{in} = 100°$
$\beta_{in} = 200°$
analyzer activity angle Ω
Ω = 0°
(Ω=0 means do-nothing)

Imagine final xy-sorter analyzes output beam into x and y-components.



Amplitude in x or y-channel is sum over x' and y'-amplitudes

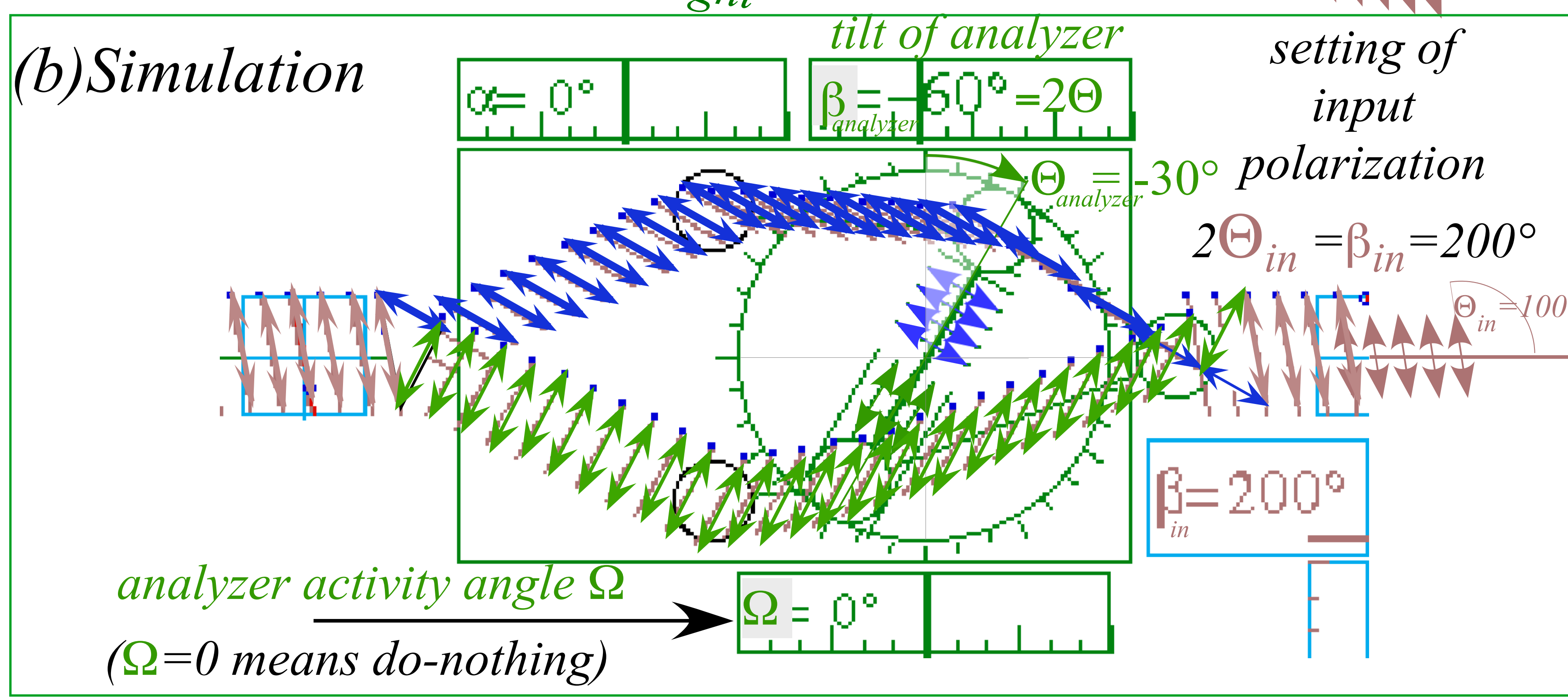
$\langle x'|\Theta_{in}\rangle = \cos(\Theta_{in} - \Theta)$

$\langle y'|\Theta_{in}\rangle = \sin(\Theta_{in} - \Theta)$

with relative angle $\Theta_{in} - \Theta$ of Θ_{in} to Θ-analyzer axes-(x', y') in products with final xy-sorter:

lab x-axis: $\langle x|x'\rangle = \cos\Theta = \langle y|y'\rangle$

y-axis: $\langle y|x'\rangle = \sin\Theta = -\langle x|y'\rangle$.



x-Output is: $\langle x|\Theta_{out}\rangle = \langle x|x'\rangle\langle x'|\Theta_{in}\rangle + \langle x|y'\rangle\langle y'|\Theta_{in}\rangle = \cos\Theta\cos(\Theta_{in}-\Theta) - \sin\Theta\sin(\Theta_{in}-\Theta) = \cos\Theta_{in}$

y-Output is: $\langle y|\Theta_{out}\rangle = \langle y|x'\rangle\langle x'|\Theta_{in}\rangle + \langle y|y'\rangle\langle y'|\Theta_{in}\rangle = \sin\Theta\cos(\Theta_{in}-\Theta) - \cos\Theta\sin(\Theta_{in}-\Theta) = \sin\Theta_{in}$.

(Recall $\cos(a+b) = \cos a \cos b - \sin a \sin b$ and $\sin(a+b) = \sin a \cos b + \cos a \sin b$)

Conclusion:

$\langle x|\Theta_{out}\rangle = \cos\Theta_{out} = \cos\Theta_{in}$ or: $\Theta_{out} = \Theta_{in}$ so "Do-Nothing" Analyzer in fact does nothing.

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Left abstraction gives ***bra-transform****:*

$$\langle j''| = \sum_{k=1}^{n} \langle j''|k\rangle\langle k|$$

Recall bra-ket
Transformation Matrix
$T_{m,n'} = \langle m|n'\rangle$

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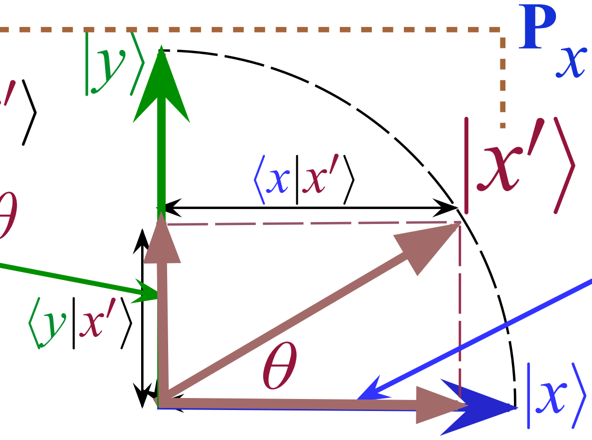

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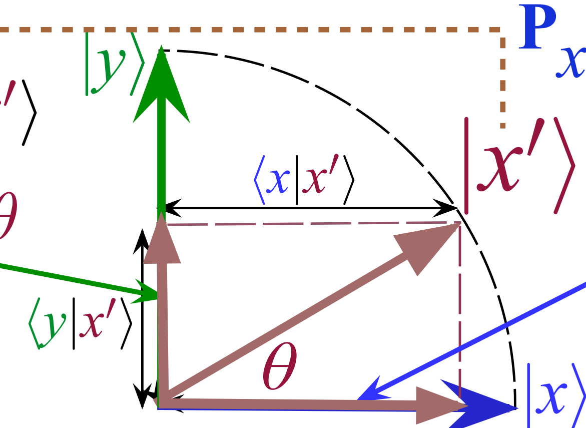

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Projections of general state $|\Psi\rangle$ ...

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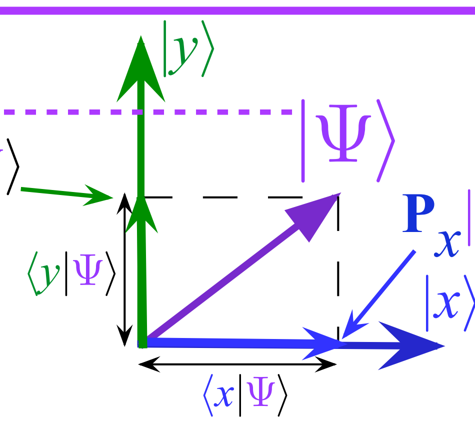

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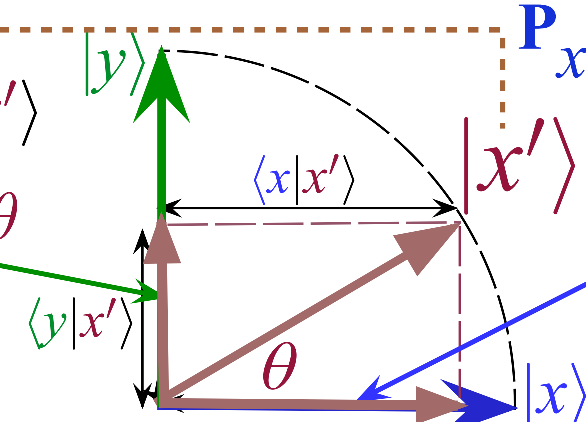

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Projections of general state $|\Psi\rangle$ ...

...must add up to $|\Psi\rangle$

$\mathbf{P}_x|\Psi\rangle + \mathbf{P}_y|\Psi\rangle = |\Psi\rangle$

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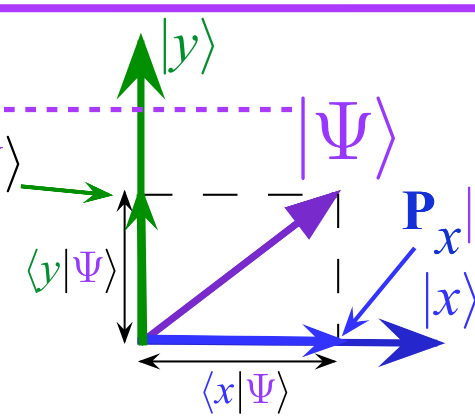

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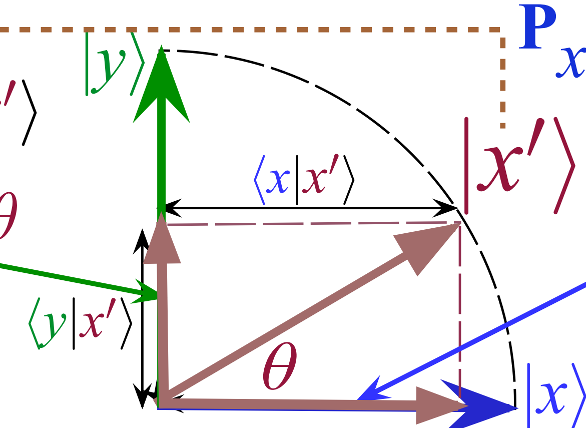

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Projections of general state $|\Psi\rangle$ *...*

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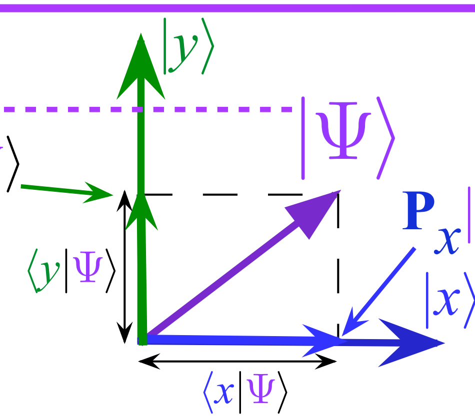
$\mathbf{P}_x|\Psi\rangle + \mathbf{P}_y|\Psi\rangle = |\Psi\rangle$

$(\mathbf{P}_x + \mathbf{P}_y)|\Psi\rangle = |\Psi\rangle$

$\mathbf{P}_y|\Psi\rangle = |y\rangle\langle y|\Psi\rangle$

$\mathbf{P}_x|\Psi\rangle = |x\rangle\langle x|\Psi\rangle$



...and so $\mathbf{P}_m$ *projectors must add up to identity operator...*

$$\mathbf{1} = \mathbf{P}_x + \mathbf{P}_y$$

Abstraction of Axiom 4 to define projection and unitary operators

Axiom 4: $\langle j''|m'\rangle = \sum_{k=1}^{n} \langle j''|k\rangle\langle k|m'\rangle$ *may be "abstracted" three different ways*

Left abstraction gives bra-transform:

$$\langle j''| = \sum_{k=1}^{n} \langle j''|k\rangle\langle k|$$

Right abstraction gives ket-transform:

$$|m'\rangle = \sum_{k=1}^{n} |k\rangle\langle k|m'\rangle$$

Center abstraction gives ket-bra identity operator:

$$\mathbf{1} = \sum_{k=1}^{n} |k\rangle\langle k| = \sum_{k=1}^{n} |k'\rangle\langle k'| = \sum_{k=1}^{n} |k''\rangle\langle k''| = \ldots$$

Resolution of Identity into Projectors $\{|1\rangle\langle 1|, |2\rangle\langle 2|..\}$ or $\{|1'\rangle\langle 1'|, |2'\rangle\langle 2'|..\}$ or $\{|1''\rangle\langle 1''|, |2''\rangle\langle 2''|..\}$

$\mathbf{P}_1 = |1\rangle\langle 1|$, $\mathbf{P}_2 = |2\rangle\langle 2|$,.. or $\mathbf{P}_{1'} = |1'\rangle\langle 1'|$, $\mathbf{P}_{2'} = |2'\rangle\langle 2'|$ *etc.*

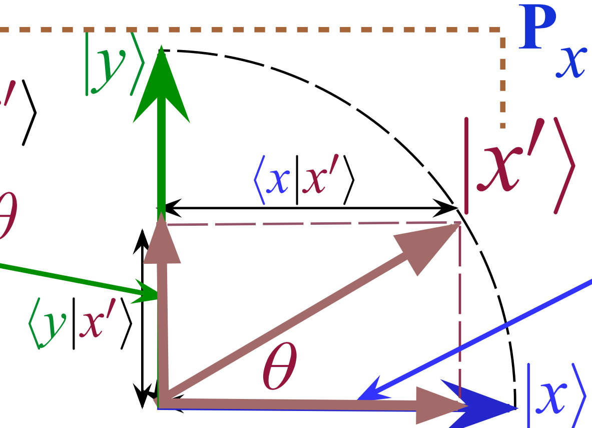
Recall bra-ket Transformation Matrix $T_{m,n'} = \langle m|n'\rangle$

$$\begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

Projections of unit vector $|x'\rangle$ *onto unit kets* $|x\rangle$ *and* $|y\rangle$

$\mathbf{P}_x|x'\rangle = |x\rangle\langle x|x'\rangle = |x\rangle\cos\theta$

$\mathbf{P}_y|x'\rangle = |y\rangle\langle y|x'\rangle = |y\rangle\sin\theta$



$$\begin{pmatrix} \langle x|\mathbf{P}_y|x\rangle & \langle x|\mathbf{P}_y|y\rangle \\ \langle y|\mathbf{P}_y|x\rangle & \langle y|\mathbf{P}_y|y\rangle \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \langle x|\mathbf{P}_x|x\rangle & \langle x|\mathbf{P}_x|y\rangle \\ \langle y|\mathbf{P}_x|x\rangle & \langle y|\mathbf{P}_x|y\rangle \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Projections of general state $|\Psi\rangle$ *...*

...must add up to $|\Psi\rangle$

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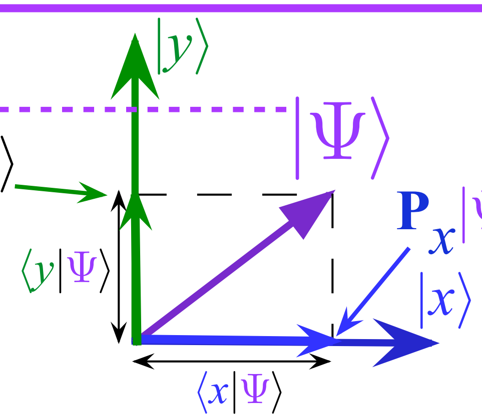

...and so $\mathbf{P}_m$ *projectors must add up to identity operator...*

$$\mathbf{1} = \mathbf{P}_x + \mathbf{P}_y$$

and identity matrix... $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

..as required by Axiom 4:

Review: Axioms 1-4 and"Do-Nothing"vs" Do-Something" analyzers

Abstraction of Axiom-4 to define projection and unitary operators

Projection operators and resolution of identity

→ *Unitary operators and matrices that do something (or "nothing")*

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Non-diagonal unitary operators and †-conjugation relations

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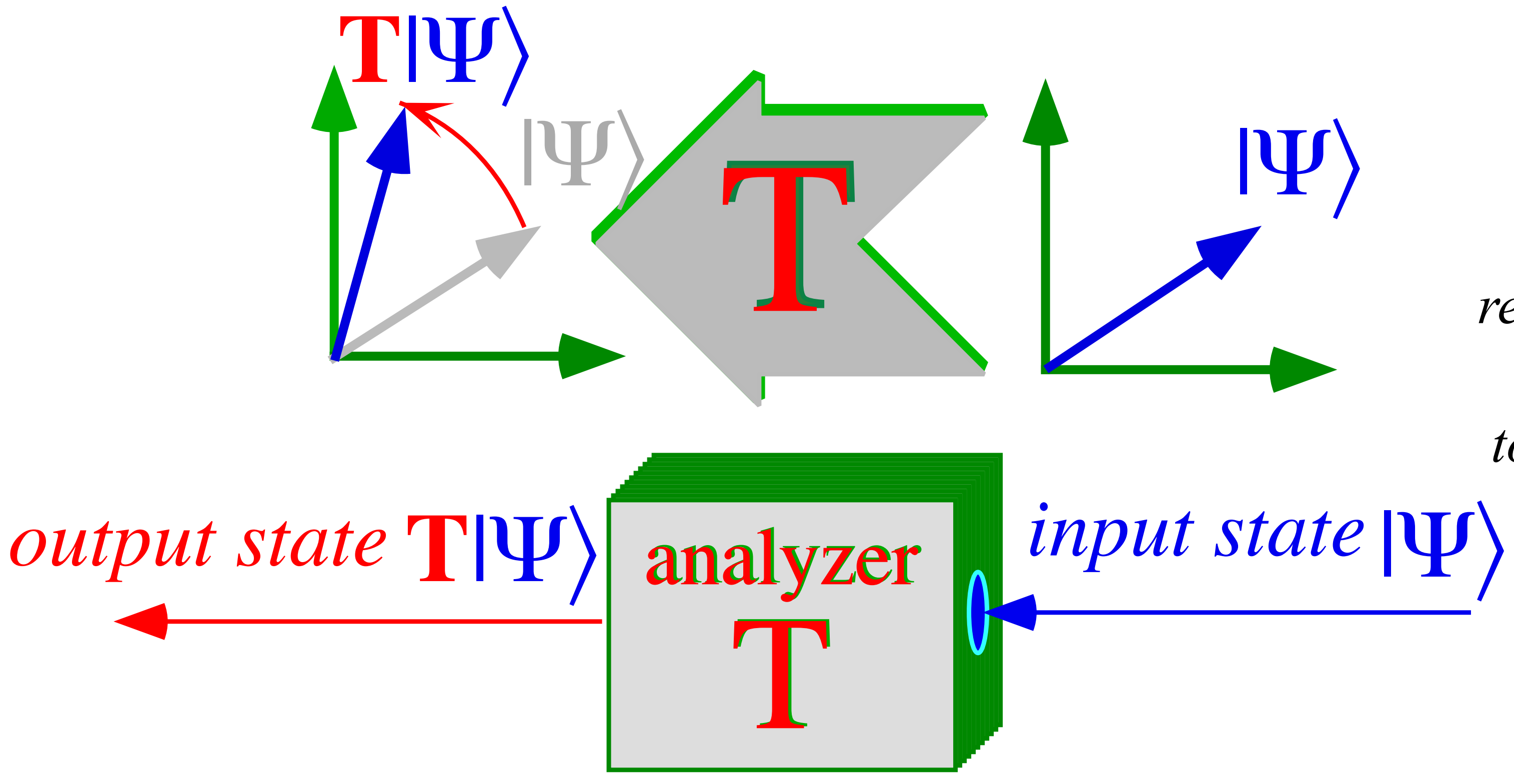

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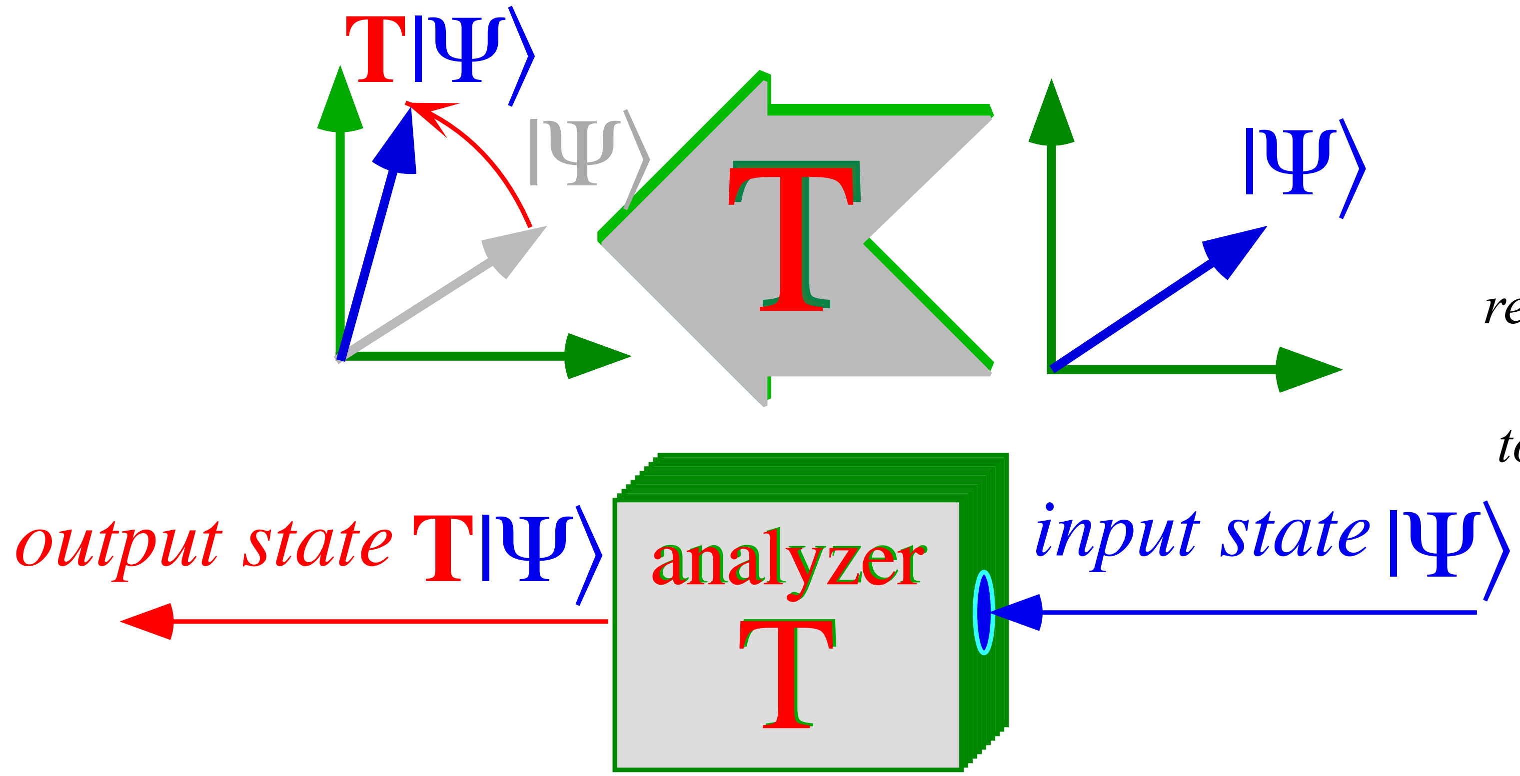

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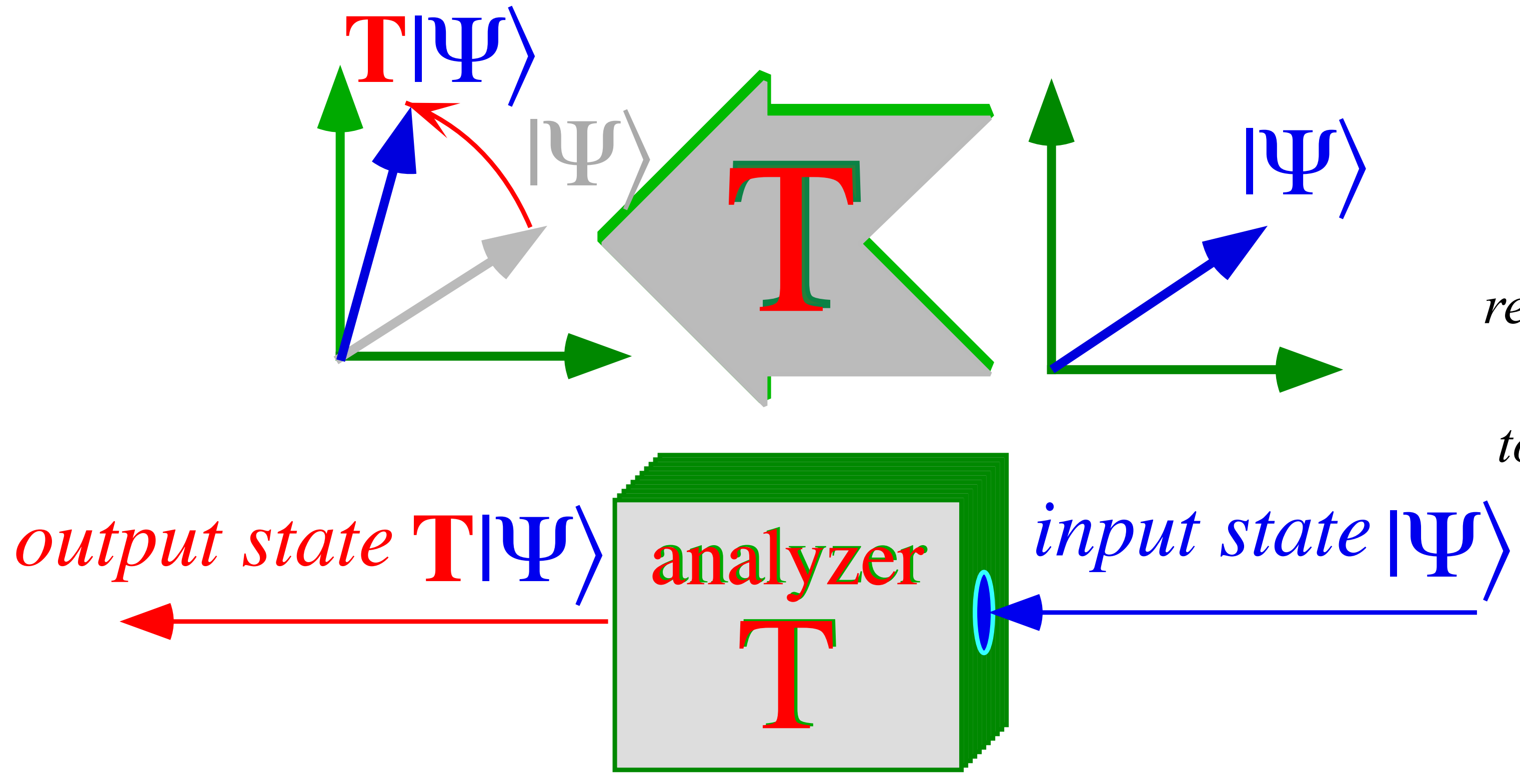

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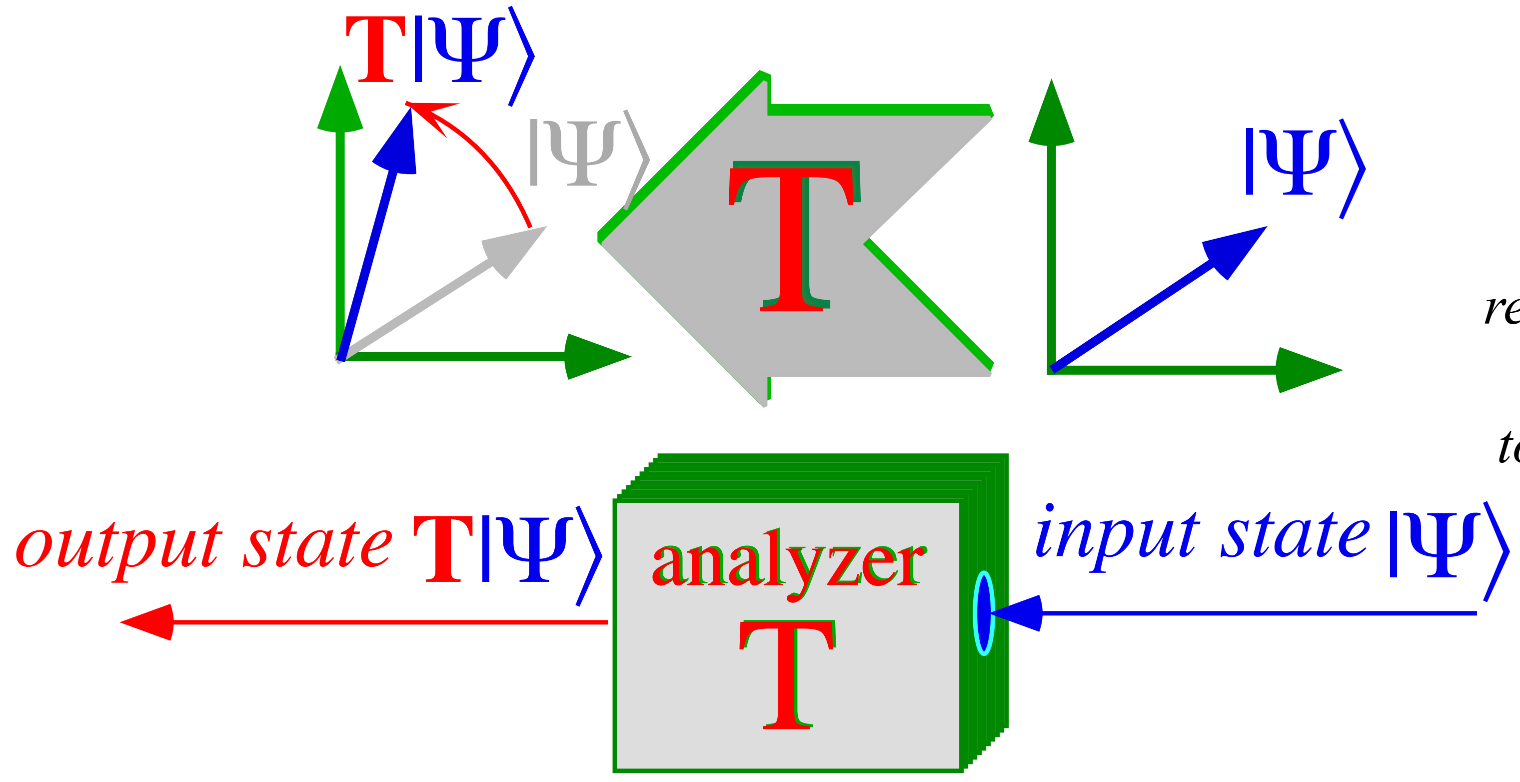

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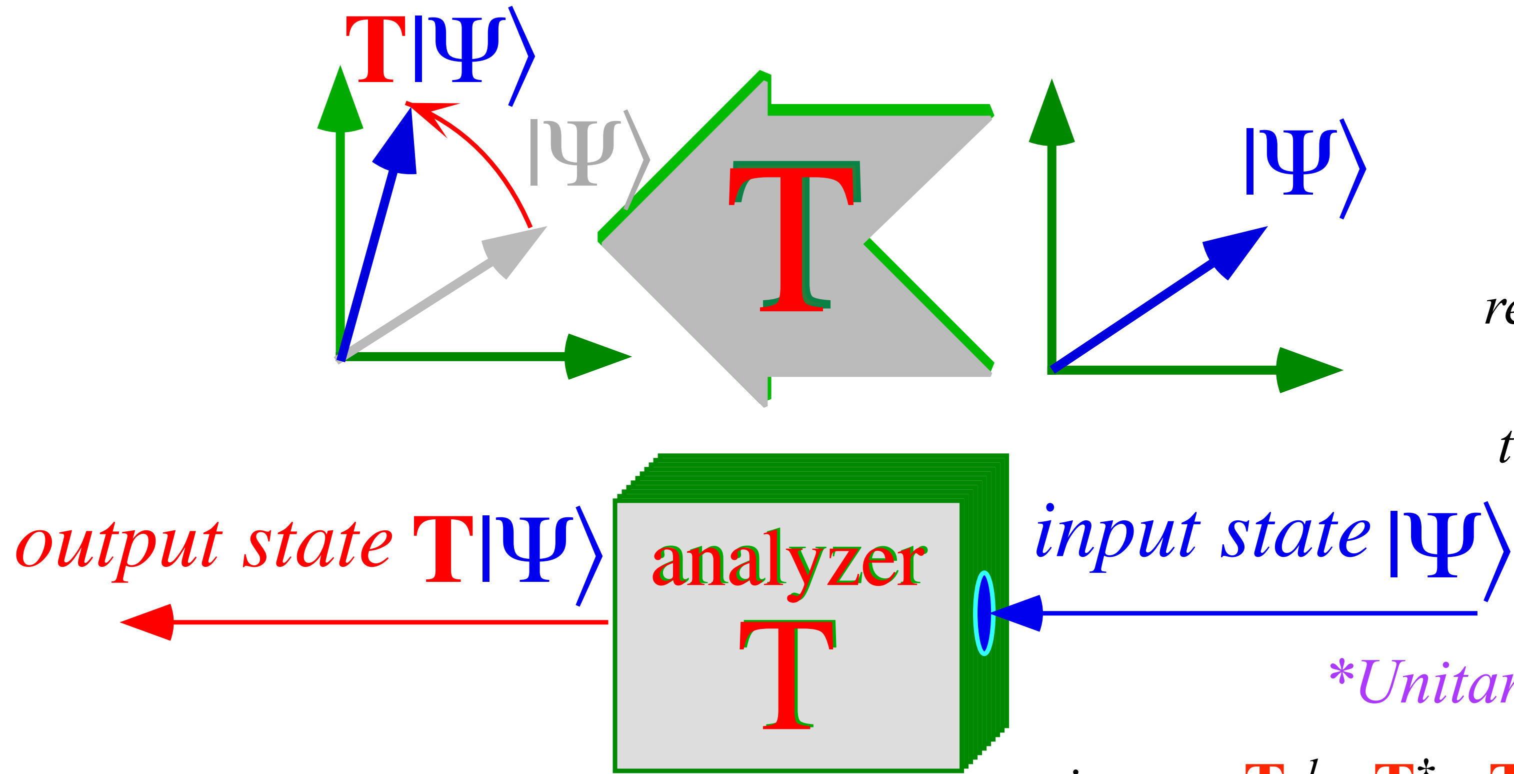

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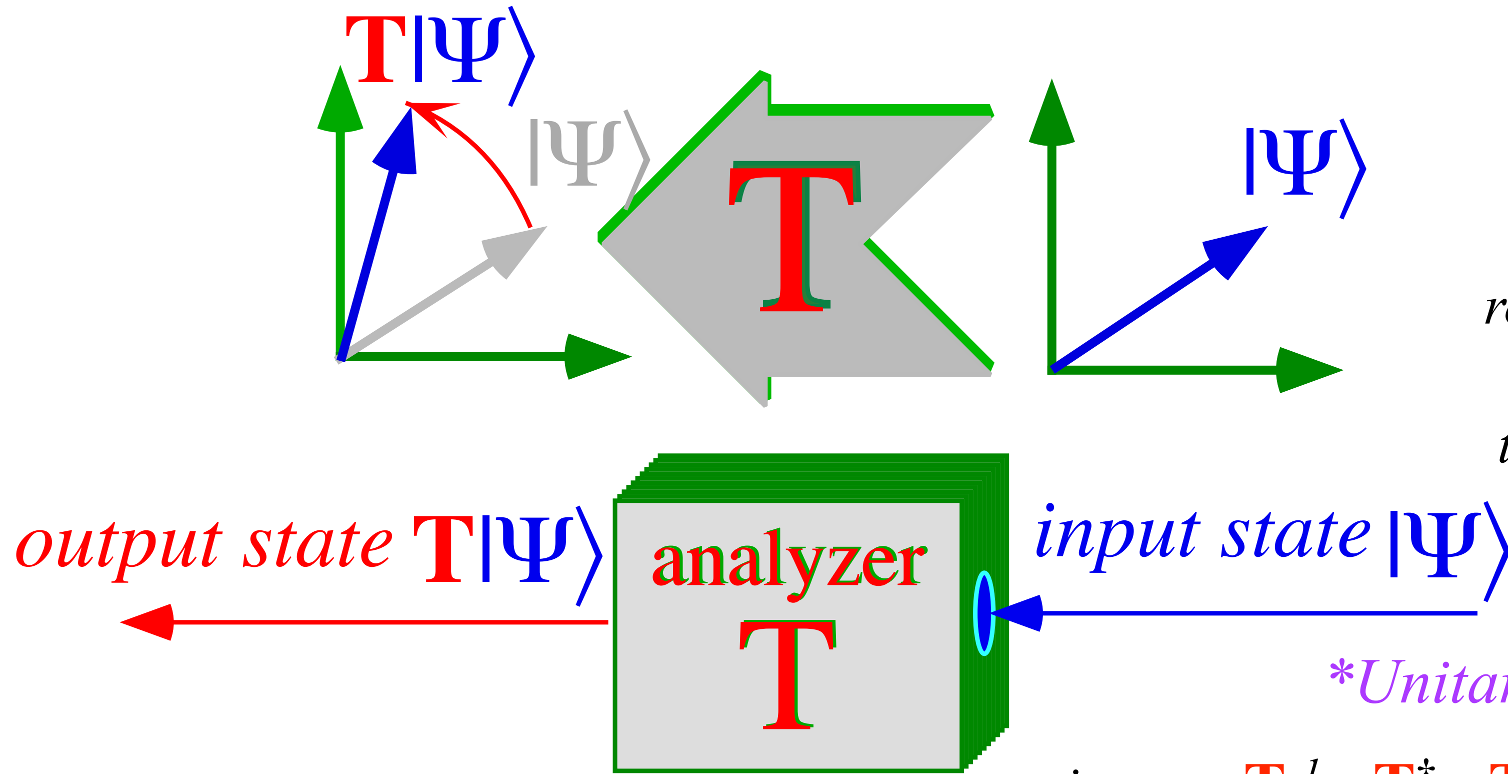

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Most "do-something" operators $\mathbf{T}'$ *are not diagonal, that is, not just* $|x\rangle\langle x|$ *and* $|y\rangle\langle y|$ *combinations.*

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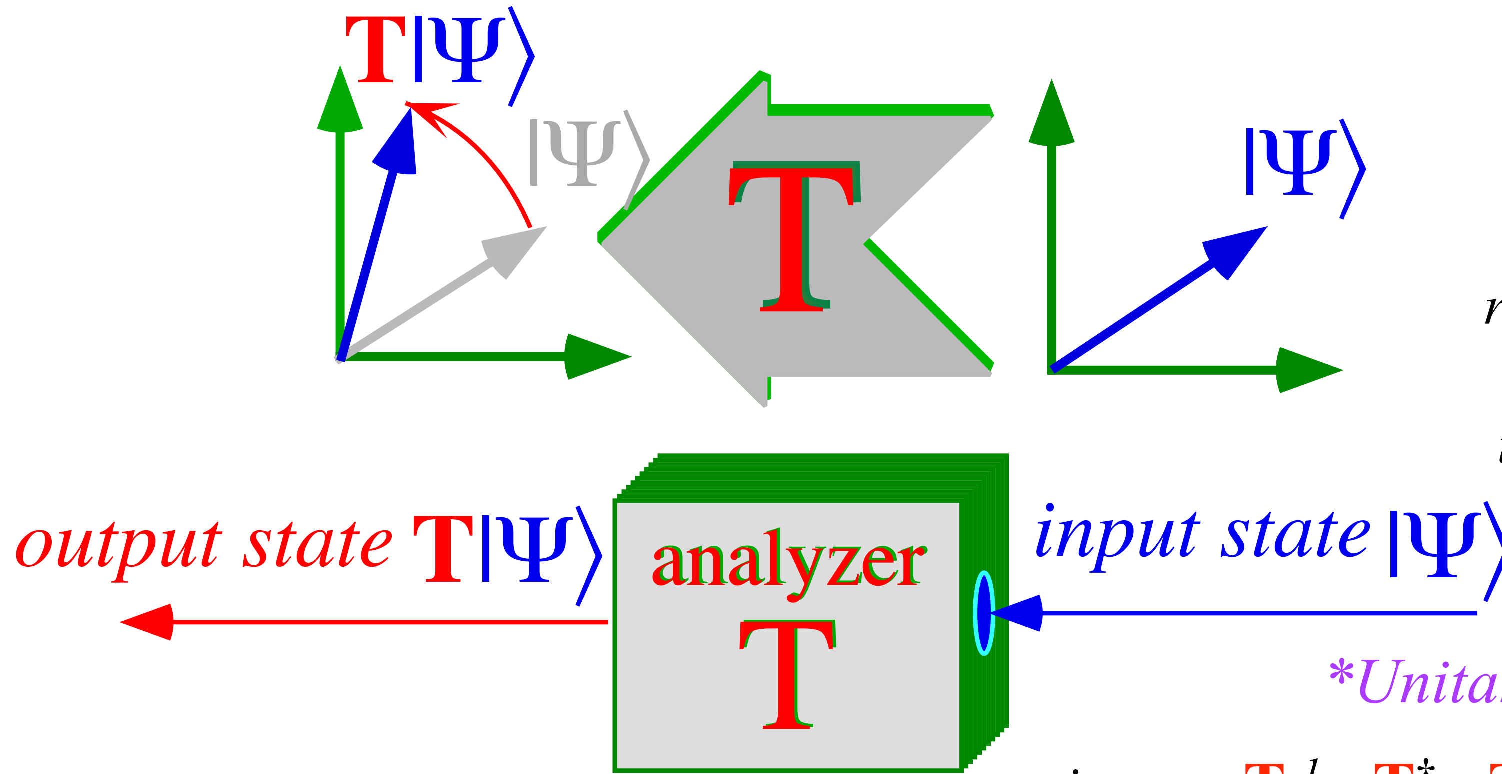

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(Matrix representation of $\mathbf{T}'$ *is a little more complicated. See following pages.)*

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so each transformed ket $|\Psi'\rangle = \mathbf{U}|\Psi\rangle$ has the same *probability* $\langle\Psi|\Psi\rangle = \langle\Psi'|\Psi'\rangle = \langle\Psi|\mathbf{U}^{\dagger}\mathbf{U}|\Psi\rangle$

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Example **U** *transfomation:*

$|x'\rangle = \mathbf{U}|x\rangle = \cos\phi\,|x\rangle + \sin\phi\,|y\rangle$

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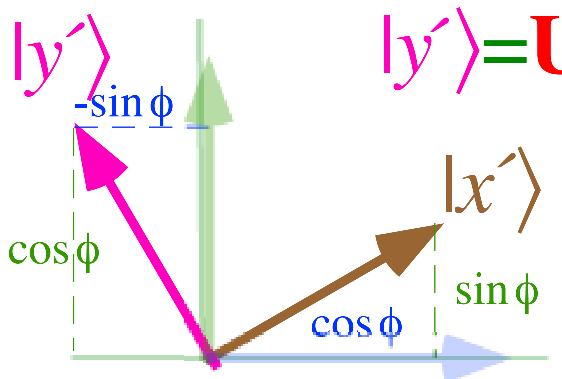

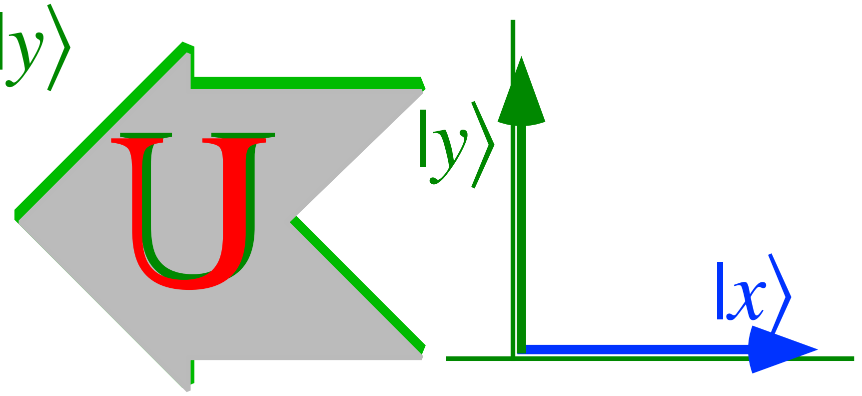

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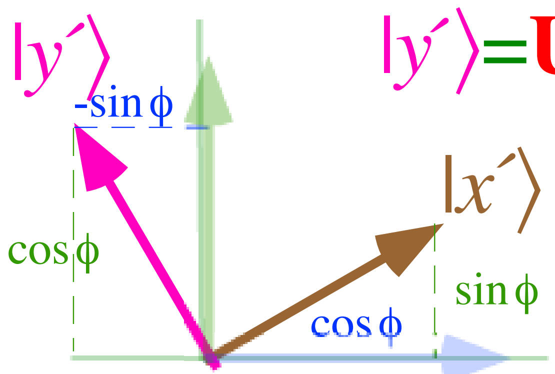

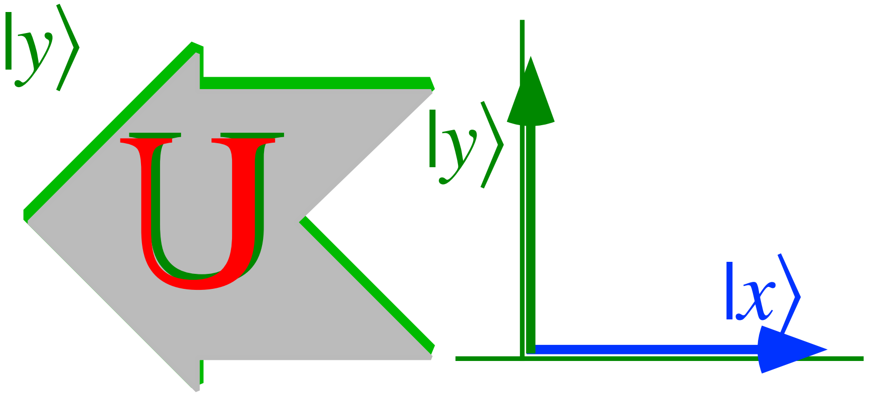

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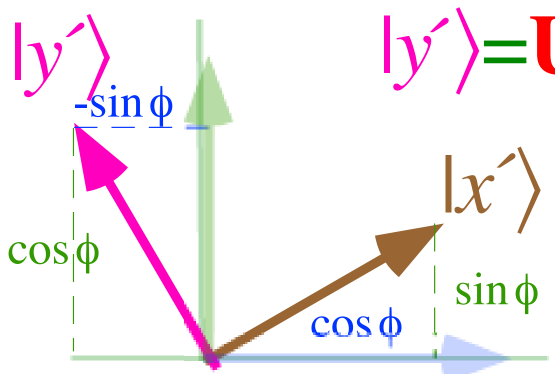

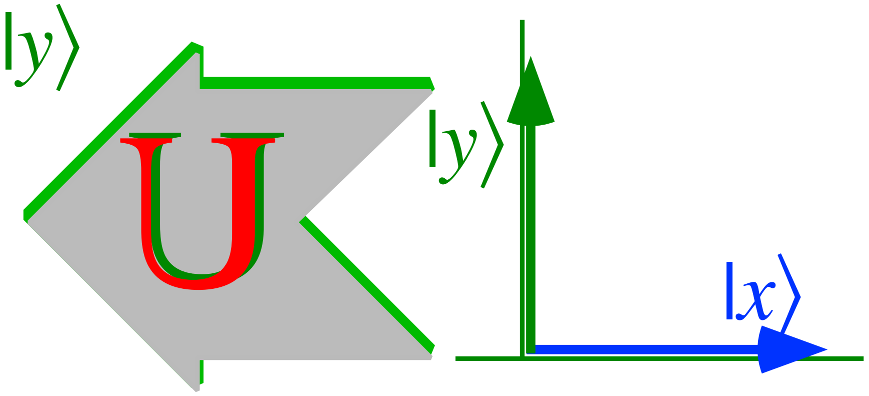

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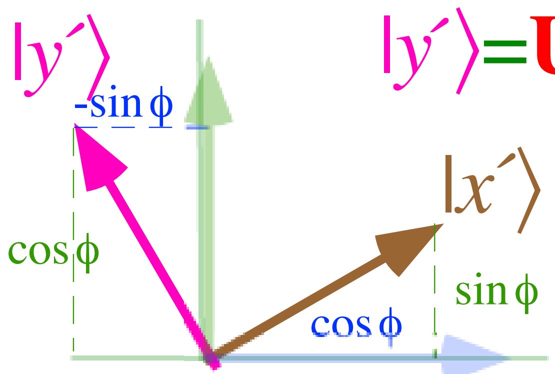

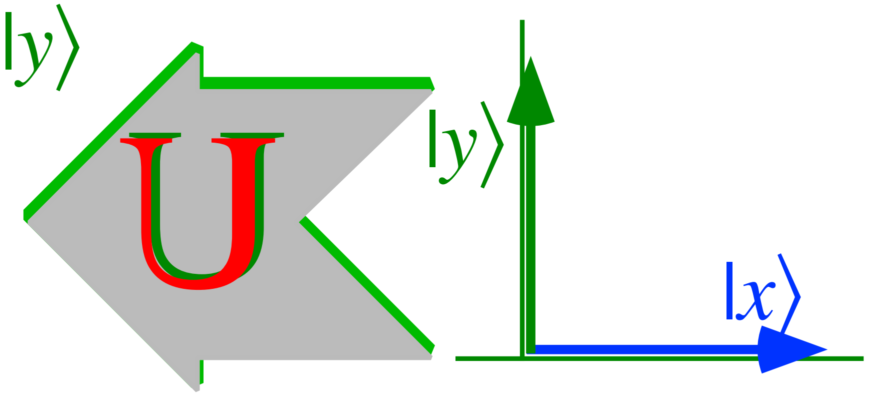

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...implies matrix representation of operator **U**

$$\begin{pmatrix} \langle x|\mathbf{U}|x\rangle & \langle x|\mathbf{U}|y\rangle \\ \langle y|\mathbf{U}|x\rangle & \langle y|\mathbf{U}|y\rangle \end{pmatrix} = \begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$$

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Example **U** *transfomation:* *(Rotation by ϕ=30°)*

$$|x'\rangle=\mathbf{U}|x\rangle=\cos\phi\,|x\rangle+\sin\phi\,|y\rangle$$

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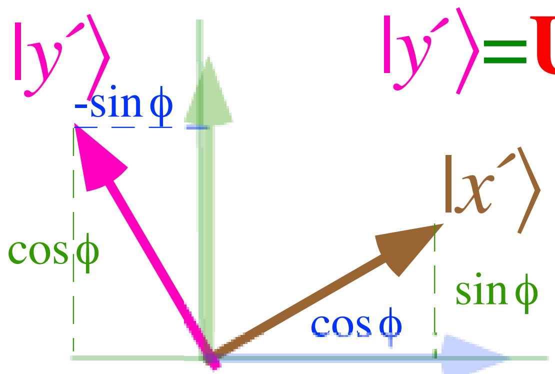

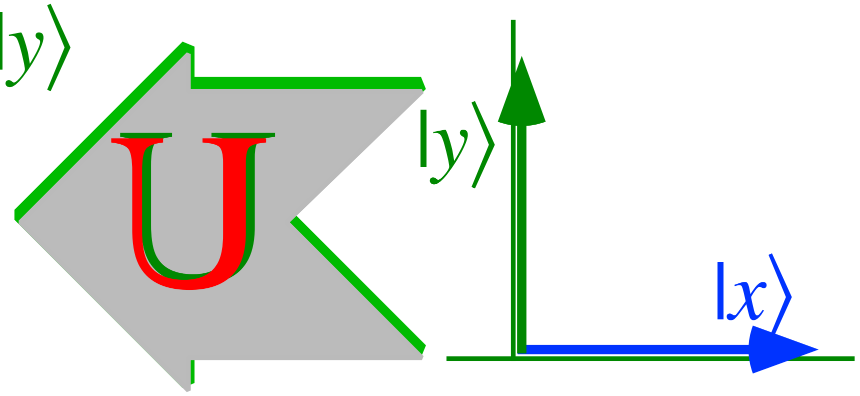

Ket definition: $|x'\rangle=\mathbf{U}|x\rangle$ implies: $\mathbf{U}^{\dagger}|x'\rangle=|x\rangle$ implies: $\langle x|=\langle x'|\mathbf{U}$ implies: $\langle x|\mathbf{U}^{\dagger}=\langle x'|$

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...implies matrix representation of operator **U** in either of the bases it connects is *exactly the same.*

$$\begin{pmatrix}\langle x|\mathbf{U}|x\rangle & \langle x|\mathbf{U}|y\rangle\\ \langle y|\mathbf{U}|x\rangle & \langle y|\mathbf{U}|y\rangle\end{pmatrix}=\begin{pmatrix}\langle x|x'\rangle & \langle x|y'\rangle\\ \langle y|x'\rangle & \langle y|y'\rangle\end{pmatrix}=\begin{pmatrix}\cos\phi & -\sin\phi\\ \sin\phi & \cos\phi\end{pmatrix}=\begin{pmatrix}\langle x'|\mathbf{U}|x'\rangle & \langle x'|\mathbf{U}|y'\rangle\\ \langle y'|\mathbf{U}|x'\rangle & \langle y'|\mathbf{U}|y'\rangle\end{pmatrix}$$

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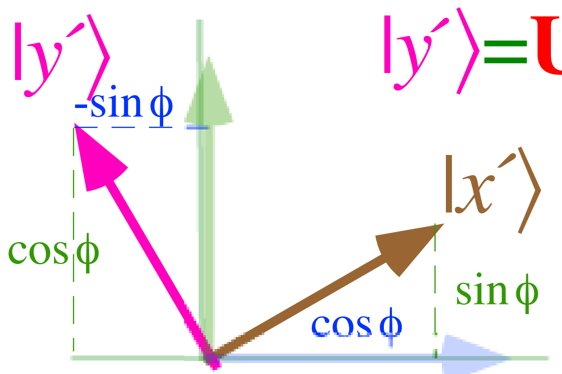

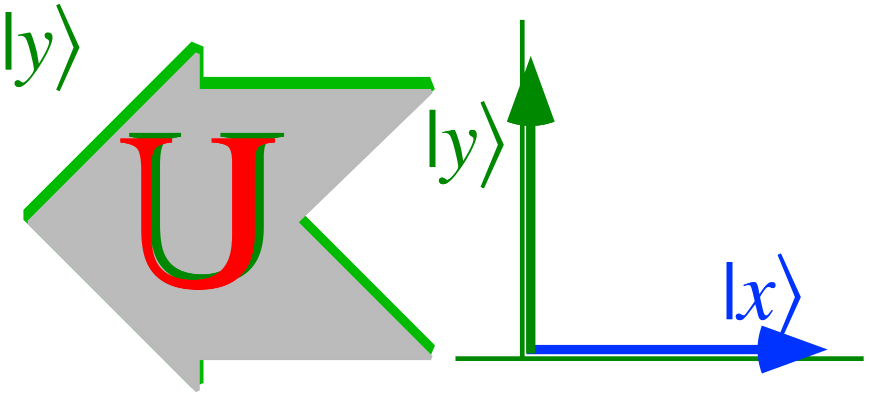

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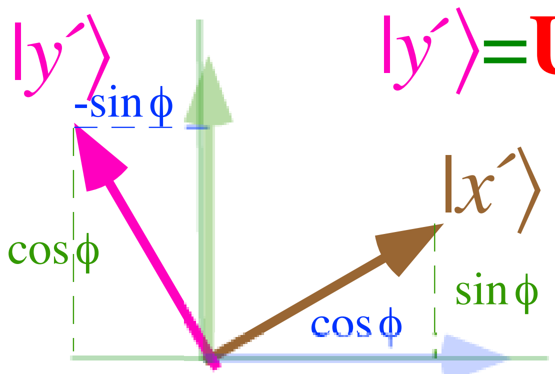

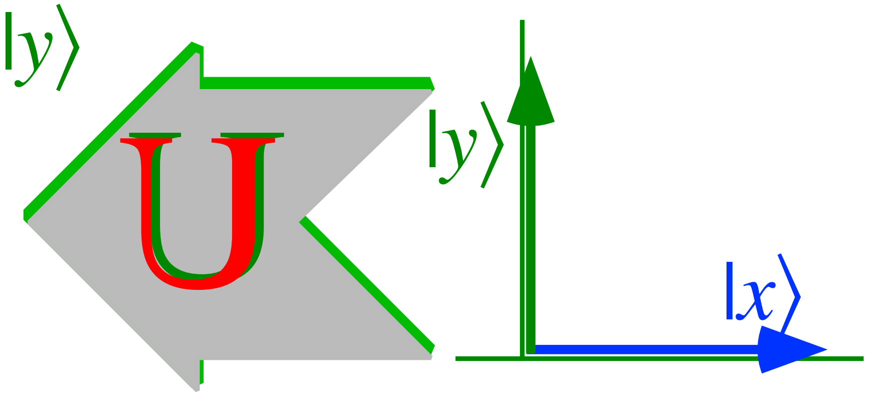

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$$= \begin{pmatrix} \langle x|x'\rangle^{*} & \langle y|x'\rangle^{*} \\ \langle x|y'\rangle^{*} & \langle y|y'\rangle^{*} \end{pmatrix}$$

Axiom-3 consistent with

inverse U *=tranpose-conjugate* $\mathbf{U}^{\dagger} = \mathbf{U}^{T*}$

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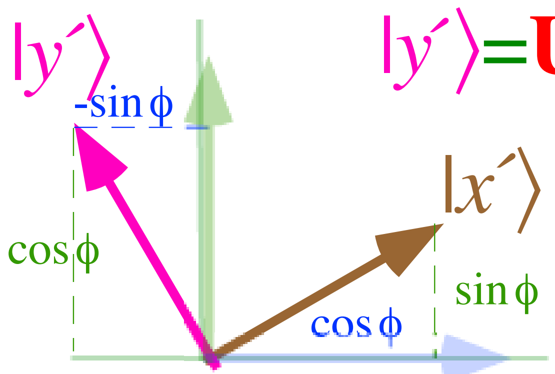

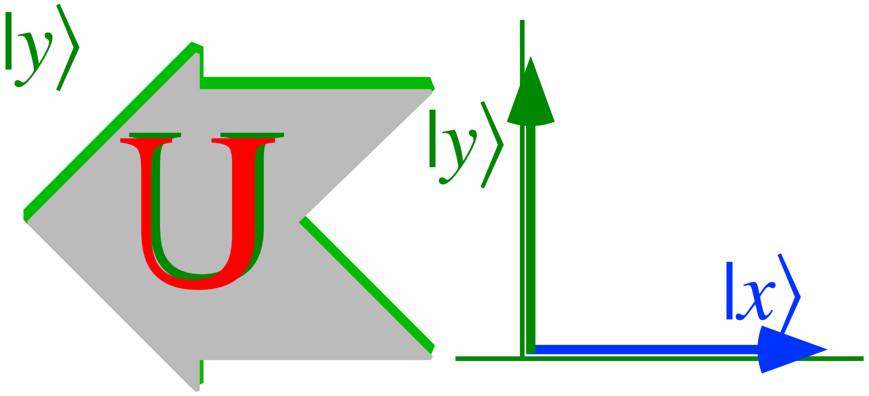

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...implies matrix representation of operator $\mathbf{U}$ in either of the bases it connects is *exactly the same.*

$$\begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} = \begin{pmatrix} \langle x|\mathbf{U}|x\rangle & \langle x|\mathbf{U}|y\rangle \\ \langle y|\mathbf{U}|x\rangle & \langle y|\mathbf{U}|y\rangle \end{pmatrix} = \begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} = \begin{pmatrix} \langle x'|\mathbf{U}|x'\rangle & \langle x'|\mathbf{U}|y'\rangle \\ \langle y'|\mathbf{U}|x'\rangle & \langle y'|\mathbf{U}|y'\rangle \end{pmatrix}$$

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Axiom-3 consistent with

inverse $\mathbf{U}$ *=tranpose-conjugate* $\mathbf{U}^{\dagger} = \mathbf{U}^{T*}$

Review: Axioms 1-4 and "Do-Nothing" vs " Do-Something" analyzers

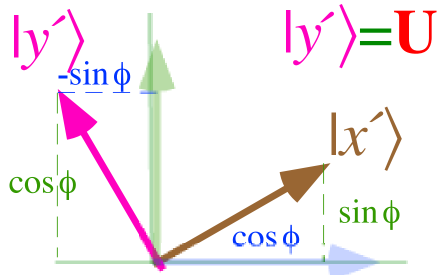
Abstraction of Axiom-4 to define projection and unitary operators
Projection operators and resolution of identity

Unitary operators and matrices that do something (or "nothing")
Diagonal unitary operators
Non-diagonal unitary operators and †-conjugation relations
→ *Non-diagonal projection operators and Kronecker ⊗−products*
Axiom-4 similarity transformation

Matrix representation of beam analyzers
Non-unitary "killer" devices: Sorter-counter, filter
Unitary "non-killer" devices: 1/2-wave plate, 1/4-wave plate

How analyzers "peek" and how that changes outcomes
Peeking polarizers and coherence loss
Classical Bayesian probability vs. Quantum probability

Feynman $\langle j|k\rangle$-axioms compared to **Group axioms**



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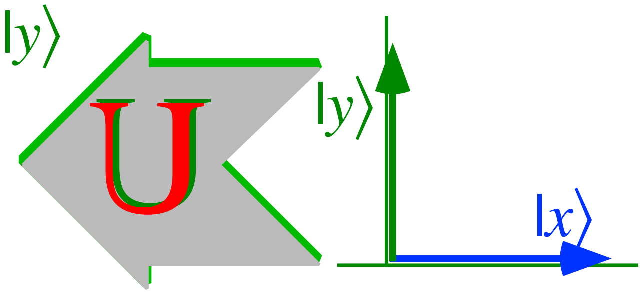

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Projector $\mathbf{P}_x = |x\rangle\langle x|$ in ϕ-tilted polarization bases $\{|x'\rangle, |y'\rangle\}$ is not diagonal.

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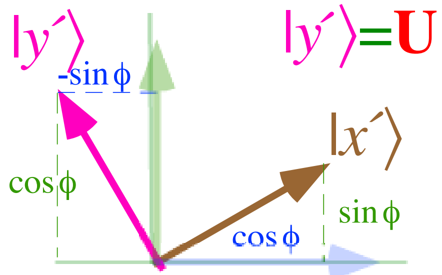

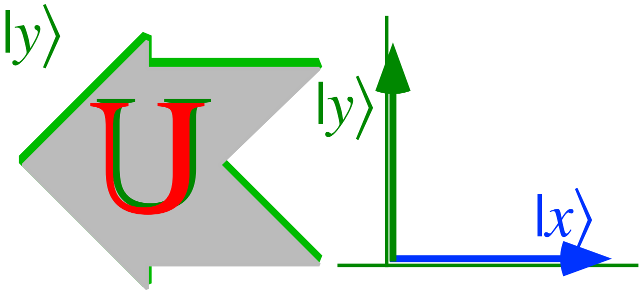

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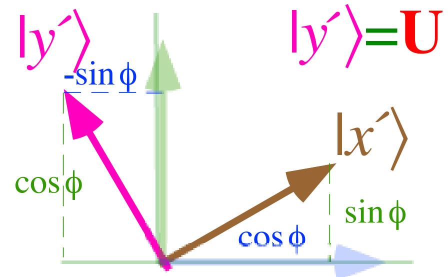

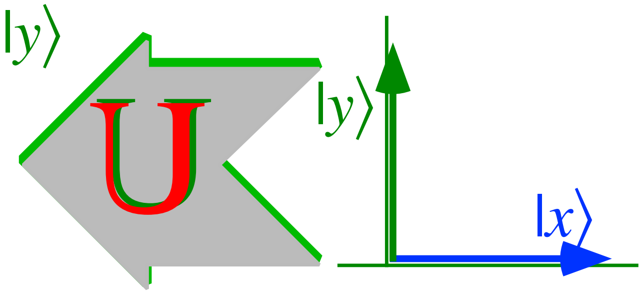

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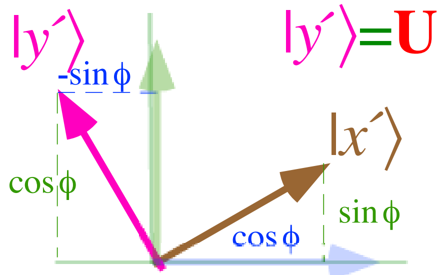

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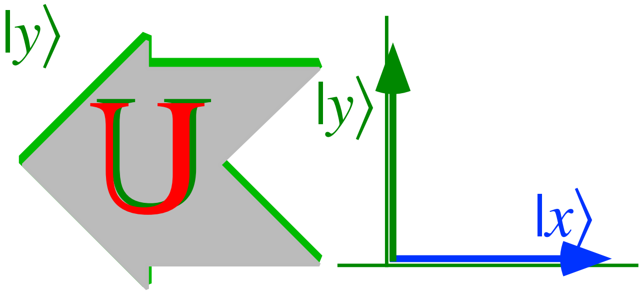

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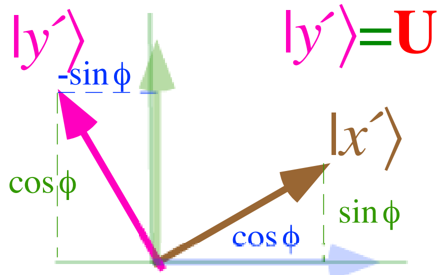

$|x'\rangle = \mathbf{U}|x\rangle = \cos\phi\,|x\rangle + \sin\phi\,|y\rangle$

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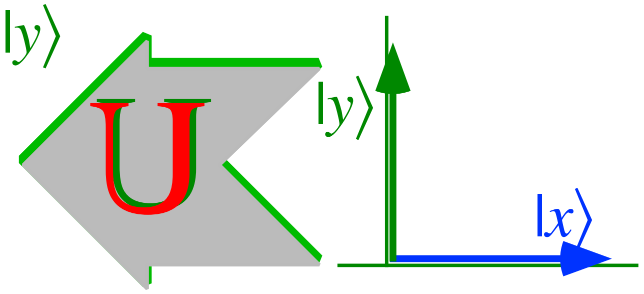

$$\begin{pmatrix} \langle x|\mathbf{U}|x\rangle & \langle x|\mathbf{U}|y\rangle \\ \langle y|\mathbf{U}|x\rangle & \langle y|\mathbf{U}|y\rangle \end{pmatrix} = \begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix}$$

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$$\begin{pmatrix} \langle x'|\mathbf{P}_x|x'\rangle & \langle x'|\mathbf{P}_x|y'\rangle \\ \langle y'|\mathbf{P}_x|x'\rangle & \langle y'|\mathbf{P}_x|y'\rangle \end{pmatrix} = \begin{pmatrix} \langle x'|x\rangle\langle x|x'\rangle & \langle x'|x\rangle\langle x|y'\rangle \\ \langle y'|x\rangle\langle x|x'\rangle & \langle y'|x\rangle\langle x|y'\rangle \end{pmatrix}$$

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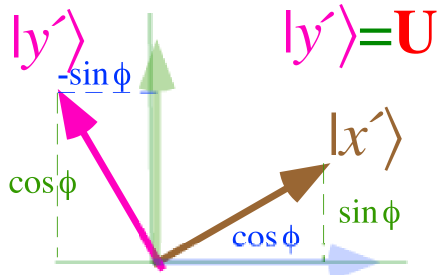

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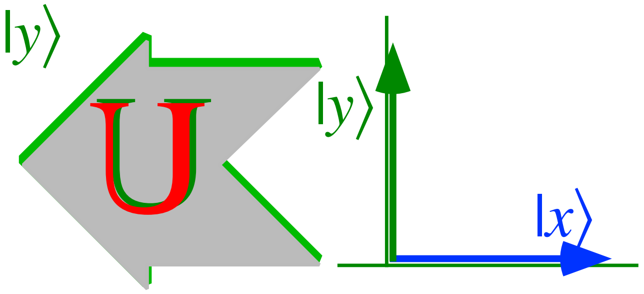

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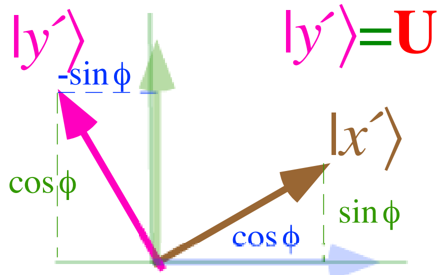
The $x'y'$-representation of $\mathbf{P}_X$:

$$\mathbf{P}_x = |x\rangle\langle x| \rightarrow \begin{pmatrix} \cos\phi \\ -\sin\phi \end{pmatrix} \otimes \begin{pmatrix} \cos\phi & -\sin\phi \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2\phi & -\sin\phi\cos\phi \\ -\sin\phi\cos\phi & \sin^2\phi \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}_{(\text{for } \phi=0)}$$



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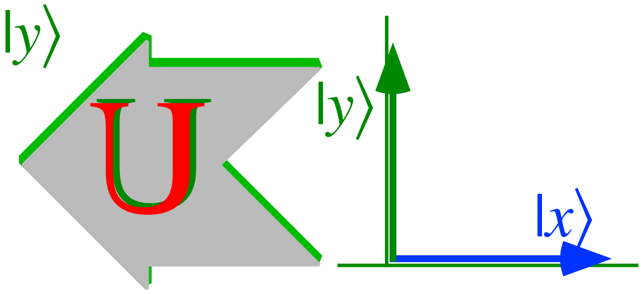

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The $x'y'$-representation of $\mathbf{P_y}$:

$$\mathbf{P}_y = |y\rangle\langle y| \rightarrow \begin{pmatrix} \sin\phi \\ \cos\phi \end{pmatrix} \otimes \begin{pmatrix} \sin\phi & \cos\phi \end{pmatrix}$$

$$= \begin{pmatrix} \sin^2\phi & \sin\phi\cos\phi \\ \sin\phi\cos\phi & \cos^2\phi \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}_{(\text{for } \phi=0)}$$

Review: Axioms 1-4 and"Do-Nothing"vs" Do-Something" analyzers

Abstraction of Axiom-4 to define projection and unitary operators
  Projection operators and resolution of identity

Unitary operators and matrices that do something (or "nothing")
  Diagonal unitary operators
  Non-diagonal unitary operators and †-conjugation relations
  Non-diagonal projection operators and Kronecker ⊗−products
  → *Axiom-4 similarity transformation*

Matrix representation of beam analyzers
  Non-unitary "killer" devices: Sorter-counter, filter
  Unitary "non-killer" devices: 1/2-wave plate, 1/4-wave plate

How analyzers "peek" and how that changes outcomes
  Peeking polarizers and coherence loss
    Classical Bayesian probability vs. Quantum probability

Feynman $\langle j|k\rangle$-axioms compared to **Group axioms**

Axiom-4 similarity transformations (Using: $\mathbf{1}=\sum|k\rangle\langle k|$ *)*

Axiom-4 is basically a matrix product as seen by comparing the following.

$$\langle j''|m'\rangle = \langle j''|\mathbf{1}|m'\rangle = \sum_{k=1}^{n} \langle j''|k\rangle\langle k|m'\rangle$$

$$\begin{pmatrix} \langle 1''|1'\rangle & \langle 1''|2'\rangle & \cdots & \langle 1''|n'\rangle \\ \langle 2''|1'\rangle & \langle 2''|2'\rangle & \cdots & \langle 2''|n'\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle n''|1'\rangle & \langle n''|2'\rangle & \cdots & \langle n''|n'\rangle \end{pmatrix} = \begin{pmatrix} \langle 1''|1\rangle & \langle 1''|2\rangle & \cdots & \langle 1''|n\rangle \\ \langle 2''|1\rangle & \langle 2''|2\rangle & \cdots & \langle 2''|n\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle n''|1\rangle & \langle n''|2\rangle & \cdots & \langle n''|n\rangle \end{pmatrix} \bullet \begin{pmatrix} \langle 1|1'\rangle & \langle 1|2'\rangle & \cdots & \langle 1|n'\rangle \\ \langle 2|1'\rangle & \langle 2|2'\rangle & \cdots & \langle 2|n'\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle n|1'\rangle & \langle n|2'\rangle & \cdots & \langle n|n'\rangle \end{pmatrix}$$

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$$T_{j''m'}\begin{pmatrix} prime \\ to \\ double-prime \end{pmatrix} = \sum_{k=1}^{n} T_{j''k}\begin{pmatrix} unprimed \\ to \\ double-prime \end{pmatrix} T_{k\,m'}\begin{pmatrix} prime \\ to \\ unprimed \end{pmatrix}$$

$$\mathbf{T}(b'' \leftarrow b') = \mathbf{T}(b'' \leftarrow b) \bullet \mathbf{T}(b \leftarrow b')$$

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(1) The closure axiom

Products ab = c are defined between any two group elements a and b, and the result c is contained in the group.

(2) The associativity axiom

Products (ab)c and a(bc) are equal for all elements a, b, and c in the group .

Transformation Group axioms

(3) The identity axiom

There is a unique element 1 (the identity) such that 1·a = a = a·1 for all elements a in the group ..

4) The inverse axiom

For all elements a in the group there is an inverse element a^{-1} such that $a^{-1}a = 1 = a \cdot a^{-1}$.

Axiom-4 is applied twice to transform operator matrix representation.

Example: *Find:*

$$\begin{pmatrix} \langle x'|\mathbf{P}_x|x'\rangle & \langle x'|\mathbf{P}_x|y'\rangle \\ \langle y'|\mathbf{P}_x|x'\rangle & \langle y'|\mathbf{P}_x|y'\rangle \end{pmatrix}$$

given:

$$\begin{pmatrix} \langle x|\mathbf{P}_x|x\rangle & \langle x|\mathbf{P}_x|y\rangle \\ \langle y|\mathbf{P}_x|x\rangle & \langle y|\mathbf{P}_x|y\rangle \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

and T-matrix:

$$\begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$$

The old "**P=1·P·1**-trick" where: $\mathbf{1} = \sum |k\rangle\langle k| = |x\rangle\langle x| + |y\rangle\langle y|$;

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$$\begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$$

The old "$\mathbf{P}=\mathbf{1}\cdot\mathbf{P}\cdot\mathbf{1}$-trick" where: $\mathbf{1}=\sum|k\rangle\langle k| = |x\rangle\langle x| + |y\rangle\langle y|$;

$$\langle x'|\mathbf{P}_x|y'\rangle = \langle x'|\mathbf{1}\cdot\mathbf{P}_x\cdot\mathbf{1}|y'\rangle = \langle x'|\left(|x\rangle\langle x| + |y\rangle\langle y|\right)\cdot\mathbf{P}_x\cdot\left(|x\rangle\langle x| + |y\rangle\langle y|\right)|y'\rangle = \left(\langle x'|x\rangle\langle x| + \langle x'|y\rangle\langle y|\right)\cdot\mathbf{P}_x\cdot\left(|x\rangle\langle x|y'\rangle + |y\rangle\langle y|y'\rangle\right)$$

$$= \langle x'|x\rangle\langle x|\mathbf{P}_x|x\rangle\langle x|y'\rangle + \langle x'|y\rangle\langle y|\mathbf{P}_x|x\rangle\langle x|y'\rangle + \langle x'|x\rangle\langle x|\mathbf{P}_x|y\rangle\langle y|y'\rangle + \dots$$

Axiom-4 is applied twice to transform operator matrix representation.

Example: *Find:* $\begin{pmatrix} \langle x'|\mathbf{P}_x|x'\rangle & \langle x'|\mathbf{P}_x|y'\rangle \\ \langle y'|\mathbf{P}_x|x'\rangle & \langle y'|\mathbf{P}_x|y'\rangle \end{pmatrix}$ *given:* $\begin{pmatrix} \langle x|\mathbf{P}_x|x\rangle & \langle x|\mathbf{P}_x|y\rangle \\ \langle y|\mathbf{P}_x|x\rangle & \langle y|\mathbf{P}_x|y\rangle \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ *and T-matrix:* $\begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$

The old "**P=1·P·1**-trick" where: $\mathbf{1} = \sum |k\rangle\langle k| = |x\rangle\langle x| + |y\rangle\langle y|$;

$$\langle x'|\mathbf{P}_x|y'\rangle = \langle x'|\mathbf{1}\cdot\mathbf{P}_x\cdot\mathbf{1}|y'\rangle = \langle x'|\big(|x\rangle\langle x| + |y\rangle\langle y|\big)\cdot\mathbf{P}_x\cdot\big(|x\rangle\langle x| + |y\rangle\langle y|\big)|y'\rangle = \big(\langle x'|x\rangle\langle x| + \langle x'|y\rangle\langle y|\big)\cdot\mathbf{P}_x\cdot\big(|x\rangle\langle x|y'\rangle + |y\rangle\langle y|y'\rangle\big)$$

$$= \langle x'|x\rangle\langle x|\mathbf{P}_x|x\rangle\langle x|y'\rangle + \langle x'|y\rangle\langle y|\mathbf{P}_x|x\rangle\langle x|y'\rangle + \langle x'|x\rangle\langle x|\mathbf{P}_x|y\rangle\langle y|y'\rangle + \langle x'|y\rangle\langle y|\mathbf{P}_x|y\rangle\langle y|y'\rangle$$

Axiom-4 is applied twice to transform operator matrix representation.

Example: *Find:*
$$\begin{pmatrix} \langle x'|\mathbf{P}_x|x'\rangle & \langle x'|\mathbf{P}_x|y'\rangle \\ \langle y'|\mathbf{P}_x|x'\rangle & \langle y'|\mathbf{P}_x|y'\rangle \end{pmatrix}$$
given:
$$\begin{pmatrix} \langle x|\mathbf{P}_x|x\rangle & \langle x|\mathbf{P}_x|y\rangle \\ \langle y|\mathbf{P}_x|x\rangle & \langle y|\mathbf{P}_x|y\rangle \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$
and T-matrix:
$$\begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$$

The old "**P=1·P·1**-trick" where: $\mathbf{1} = \sum |k\rangle\langle k| = |x\rangle\langle x| + |y\rangle\langle y|$;

$$\langle x'|\mathbf{P}_x|y'\rangle = \langle x'|\mathbf{1}\cdot\mathbf{P}_x\cdot\mathbf{1}|y'\rangle = \langle x'|\big(|x\rangle\langle x| + |y\rangle\langle y|\big)\cdot\mathbf{P}_x\cdot\big(|x\rangle\langle x| + |y\rangle\langle y|\big)|y'\rangle = \big(\langle x'|x\rangle\langle x| + \langle x'|y\rangle\langle y|\big)\cdot\mathbf{P}_x\cdot\big(|x\rangle\langle x|y'\rangle + |y\rangle\langle y|y'\rangle\big)$$
$$= \langle x'|x\rangle\langle x|\mathbf{P}_x|x\rangle\langle x|y'\rangle + \langle x'|y\rangle\langle y|\mathbf{P}_x|x\rangle\langle x|y'\rangle + \langle x'|x\rangle\langle x|\mathbf{P}_x|y\rangle\langle y|y'\rangle + \langle x'|y\rangle\langle y|\mathbf{P}_x|y\rangle\langle y|y'\rangle$$

More elegant matrix product:
$$\begin{pmatrix} \langle x'|\mathbf{P}_x|x'\rangle & \langle x'|\mathbf{P}_x|y'\rangle \\ \langle y'|\mathbf{P}_x|x'\rangle & \langle y'|\mathbf{P}_x|y'\rangle \end{pmatrix} = \begin{pmatrix} \langle x'|x\rangle & \langle x'|y\rangle \\ \langle y'|x\rangle & \langle y'|y\rangle \end{pmatrix} \begin{pmatrix} \langle x|\mathbf{P}_x|x\rangle & \langle x|\mathbf{P}_x|y\rangle \\ \langle y|\mathbf{P}_x|x\rangle & \langle y|\mathbf{P}_x|y\rangle \end{pmatrix} \begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix}$$

Axiom-4 is applied twice to transform operator matrix representation.

Example: *Find:* $\begin{pmatrix} \langle x'|\mathbf{P}_x|x'\rangle & \langle x'|\mathbf{P}_x|y'\rangle \\ \langle y'|\mathbf{P}_x|x'\rangle & \langle y'|\mathbf{P}_x|y'\rangle \end{pmatrix}$ *given:* $\begin{pmatrix} \langle x|\mathbf{P}_x|x\rangle & \langle x|\mathbf{P}_x|y\rangle \\ \langle y|\mathbf{P}_x|x\rangle & \langle y|\mathbf{P}_x|y\rangle \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ *and T-matrix:* $\begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$

The old "**P=1·P·1**-trick" where: $\mathbf{1} = \sum |k\rangle\langle k| = |x\rangle\langle x| + |y\rangle\langle y|$;

$$\langle x'|\mathbf{P}_x|y'\rangle = \langle x'|\mathbf{1}\cdot\mathbf{P}_x\cdot\mathbf{1}|y'\rangle = \langle x'|\big(|x\rangle\langle x| + |y\rangle\langle y|\big)\cdot\mathbf{P}_x\cdot\big(|x\rangle\langle x| + |y\rangle\langle y|\big)|y'\rangle = \big(\langle x'|x\rangle\langle x| + \langle x'|y\rangle\langle y|\big)\cdot\mathbf{P}_x\cdot\big(|x\rangle\langle x|y'\rangle + |y\rangle\langle y|y'\rangle\big)$$

$$= \langle x'|x\rangle\langle x|\mathbf{P}_x|x\rangle\langle x|y'\rangle + \langle x'|y\rangle\langle y|\mathbf{P}_x|x\rangle\langle x|y'\rangle + \langle x'|x\rangle\langle x|\mathbf{P}_x|y\rangle\langle y|y'\rangle + \langle x'|y\rangle\langle y|\mathbf{P}_x|y\rangle\langle y|y'\rangle$$

More elegant matrix product:

$$\begin{pmatrix} \langle x'|\mathbf{P}_x|x'\rangle & \langle x'|\mathbf{P}_x|y'\rangle \\ \langle y'|\mathbf{P}_x|x'\rangle & \langle y'|\mathbf{P}_x|y'\rangle \end{pmatrix} = \begin{pmatrix} \langle x'|x\rangle & \langle x'|y\rangle \\ \langle y'|x\rangle & \langle y'|y\rangle \end{pmatrix} \begin{pmatrix} \langle x|\mathbf{P}_x|x\rangle & \langle x|\mathbf{P}_x|y\rangle \\ \langle y|\mathbf{P}_x|x\rangle & \langle y|\mathbf{P}_x|y\rangle \end{pmatrix} \begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix}$$

$$= \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \langle x|\mathbf{P}_x|x\rangle & \langle x|\mathbf{P}_x|y\rangle \\ \langle y|\mathbf{P}_x|x\rangle & \langle y|\mathbf{P}_x|y\rangle \end{pmatrix} \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$$

$$= \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \quad \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$$

Axiom-4 is applied twice to transform operator matrix representation.

Example: *Find:* $\begin{pmatrix} \langle x'|\mathbf{P}_x|x'\rangle & \langle x'|\mathbf{P}_x|y'\rangle \\ \langle y'|\mathbf{P}_x|x'\rangle & \langle y'|\mathbf{P}_x|y'\rangle \end{pmatrix}$ *given:* $\begin{pmatrix} \langle x|\mathbf{P}_x|x\rangle & \langle x|\mathbf{P}_x|y\rangle \\ \langle y|\mathbf{P}_x|x\rangle & \langle y|\mathbf{P}_x|y\rangle \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ *and T-matrix:* $\begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$

The old "**P=1·P·1**-trick" where: $\mathbf{1} = \sum |k\rangle\langle k| = \quad |x\rangle\langle x| \ + \ |y\rangle\langle y|$;

$$\langle x'|\mathbf{P}_x|y'\rangle = \langle x'|\mathbf{1}\cdot\mathbf{P}_x\cdot\mathbf{1}|y'\rangle = \langle x'|\big(|x\rangle\langle x|+|y\rangle\langle y|\big)\cdot\mathbf{P}_x\cdot\big(|x\rangle\langle x|+|y\rangle\langle y|\big)|y'\rangle = \big(\langle x'|x\rangle\langle x|+\langle x'|y\rangle\langle y|\big)\cdot\mathbf{P}_x\cdot\big(|x\rangle\langle x|y'\rangle+|y\rangle\langle y|y'\rangle\big)$$

$$= \langle x'|x\rangle\langle x|\mathbf{P}_x|x\rangle\langle x|y'\rangle + \langle x'|y\rangle\langle y|\mathbf{P}_x|x\rangle\langle x|y'\rangle + \langle x'|x\rangle\langle x|\mathbf{P}_x|y\rangle\langle y|y'\rangle + \langle x'|y\rangle\langle y|\mathbf{P}_x|y\rangle\langle y|y'\rangle$$

More elegant matrix product:

$$\begin{pmatrix} \langle x'|\mathbf{P}_x|x'\rangle & \langle x'|\mathbf{P}_x|y'\rangle \\ \langle y'|\mathbf{P}_x|x'\rangle & \langle y'|\mathbf{P}_x|y'\rangle \end{pmatrix} = \begin{pmatrix} \langle x'|x\rangle & \langle x'|y\rangle \\ \langle y'|x\rangle & \langle y'|y\rangle \end{pmatrix} \begin{pmatrix} \langle x|\mathbf{P}_x|x\rangle & \langle x|\mathbf{P}_x|y\rangle \\ \langle y|\mathbf{P}_x|x\rangle & \langle y|\mathbf{P}_x|y\rangle \end{pmatrix} \begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix}$$

$$= \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \langle x|\mathbf{P}_x|x\rangle & \langle x|\mathbf{P}_x|y\rangle \\ \langle y|\mathbf{P}_x|x\rangle & \langle y|\mathbf{P}_x|y\rangle \end{pmatrix} \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$$

$$= \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$$

$$= \begin{pmatrix} \cos\phi & 0 \\ -\sin\phi & 0 \end{pmatrix} \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} = \begin{pmatrix} \cos^2\phi & -\cos\phi\sin\phi \\ -\sin\phi\cos\phi & \sin^2\phi \end{pmatrix}$$

Axiom-4 is applied twice to transform operator matrix representation.

Example: *Find:* $\begin{pmatrix} \langle x'|\mathbf{P}_x|x'\rangle & \langle x'|\mathbf{P}_x|y'\rangle \\ \langle y'|\mathbf{P}_x|x'\rangle & \langle y'|\mathbf{P}_x|y'\rangle \end{pmatrix}$ *given:* $\begin{pmatrix} \langle x|\mathbf{P}_x|x\rangle & \langle x|\mathbf{P}_x|y\rangle \\ \langle y|\mathbf{P}_x|x\rangle & \langle y|\mathbf{P}_x|y\rangle \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ *and T-matrix:* $\begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$

The old "**P=1·P·1**-trick" where: $\mathbf{1}=\sum|k\rangle\langle k| = |x\rangle\langle x| + |y\rangle\langle y|$;

$$\langle x'|\mathbf{P}_x|y'\rangle = \langle x'|\mathbf{1}\cdot\mathbf{P}_x\cdot\mathbf{1}|y'\rangle = \langle x'|\big(|x\rangle\langle x|+|y\rangle\langle y|\big)\cdot\mathbf{P}_x\cdot\big(|x\rangle\langle x|+|y\rangle\langle y|\big)|y'\rangle = \big(\langle x'|x\rangle\langle x|+\langle x'|y\rangle\langle y|\big)\cdot\mathbf{P}_x\cdot\big(|x\rangle\langle x|y'\rangle+|y\rangle\langle y|y'\rangle\big)$$

$$= \langle x'|x\rangle\langle x|\mathbf{P}_x|x\rangle\langle x|y'\rangle + \langle x'|y\rangle\langle y|\mathbf{P}_x|x\rangle\langle x|y'\rangle + \langle x'|x\rangle\langle x|\mathbf{P}_x|y\rangle\langle y|y'\rangle + \langle x'|y\rangle\langle y|\mathbf{P}_x|y\rangle\langle y|y'\rangle$$

More elegant matrix product:

$$\begin{pmatrix} \langle x'|\mathbf{P}_x|x'\rangle & \langle x'|\mathbf{P}_x|y'\rangle \\ \langle y'|\mathbf{P}_x|x'\rangle & \langle y'|\mathbf{P}_x|y'\rangle \end{pmatrix} = \begin{pmatrix} \langle x'|x\rangle & \langle x'|y\rangle \\ \langle y'|x\rangle & \langle y'|y\rangle \end{pmatrix} \begin{pmatrix} \langle x|\mathbf{P}_x|x\rangle & \langle x|\mathbf{P}_x|y\rangle \\ \langle y|\mathbf{P}_x|x\rangle & \langle y|\mathbf{P}_x|y\rangle \end{pmatrix} \begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix}$$

$$= \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \langle x|\mathbf{P}_x|x\rangle & \langle x|\mathbf{P}_x|y\rangle \\ \langle y|\mathbf{P}_x|x\rangle & \langle y|\mathbf{P}_x|y\rangle \end{pmatrix} \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$$

$$= \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$$

$$= \begin{pmatrix} \cos\phi & 0 \\ -\sin\phi & 0 \end{pmatrix} \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} = \begin{pmatrix} \cos^2\phi & -\cos\phi\sin\phi \\ -\sin\phi\cos\phi & \sin^2\phi \end{pmatrix}$$

This checks with the previous result 4-pages back: $\mathbf{P}_x = |x\rangle\langle x| \to \begin{pmatrix} \cos\phi \\ -\sin\phi \end{pmatrix} \otimes \begin{pmatrix} \cos\phi & -\sin\phi \end{pmatrix} = \begin{pmatrix} \cos^2\phi & -\sin\phi\cos\phi \\ -\sin\phi\cos\phi & \sin^2\phi \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}_{(\text{for } \phi=0)}$

Review: Axioms 1-4 and "Do-Nothing" vs " Do-Something" analyzers

Abstraction of Axiom-4 to define projection and unitary operators
Projection operators and resolution of identity

Unitary operators and matrices that do something (or "nothing")
Diagonal unitary operators
Non-diagonal unitary operators and †-conjugation relations
Non-diagonal projection operators and Kronecker ⊗−products
Axiom-4 similarity transformation

Matrix representation of beam analyzers
➡ *Non-unitary "killer" devices: Sorter-counter, filter*
Unitary "non-killer" devices: 1/2-wave plate, 1/4-wave plate

How analyzers "peek" and how that changes outcomes
Peeking polarizers and coherence loss
Classical Bayesian probability vs. Quantum probability

Feynman $\langle j|k\rangle$-axioms compared to **Group axioms**

(1) Optical analyzer in sorter-counter configuration

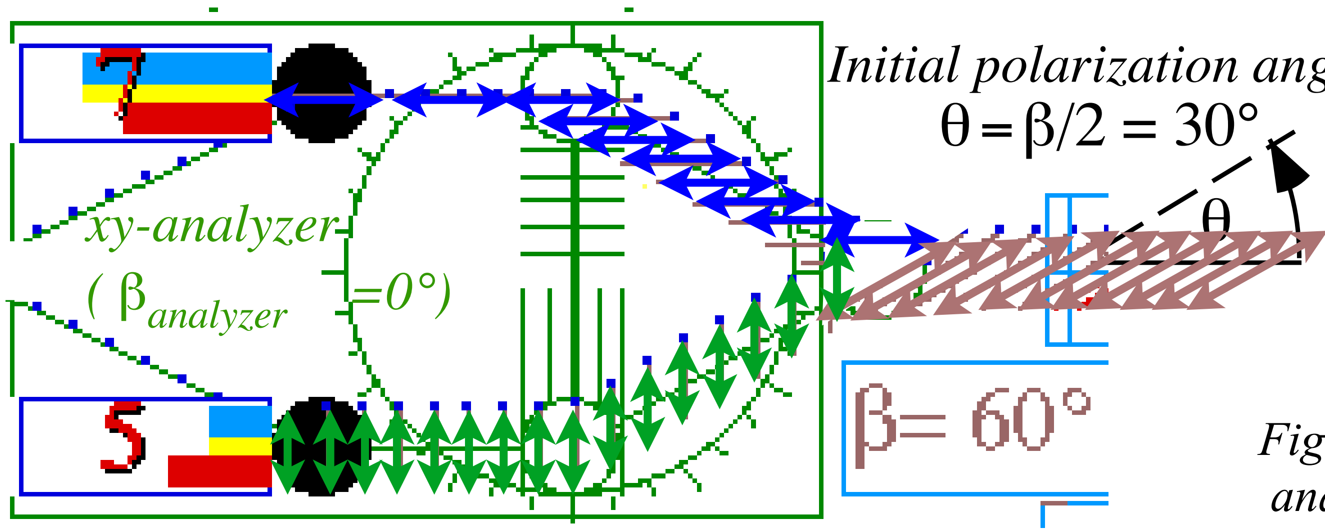
Analyzer reduced to a simple sorter-counter by blocking output of x-high-road and y-low-road with counters

$x\text{-counts} \sim \left| \langle x | x' \rangle \right|^2$

$= \cos^2\theta = 0.75$

$y\text{-counts} \sim \left| \langle y | x' \rangle \right|^2$

$= \sin^2\theta = 0.25$



Analyzer matrix:

$$\begin{pmatrix} \langle x|\mathbf{T}|x\rangle & \langle x|\mathbf{T}|y\rangle \\ \langle y|\mathbf{T}|x\rangle & \langle y|\mathbf{T}|y\rangle \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

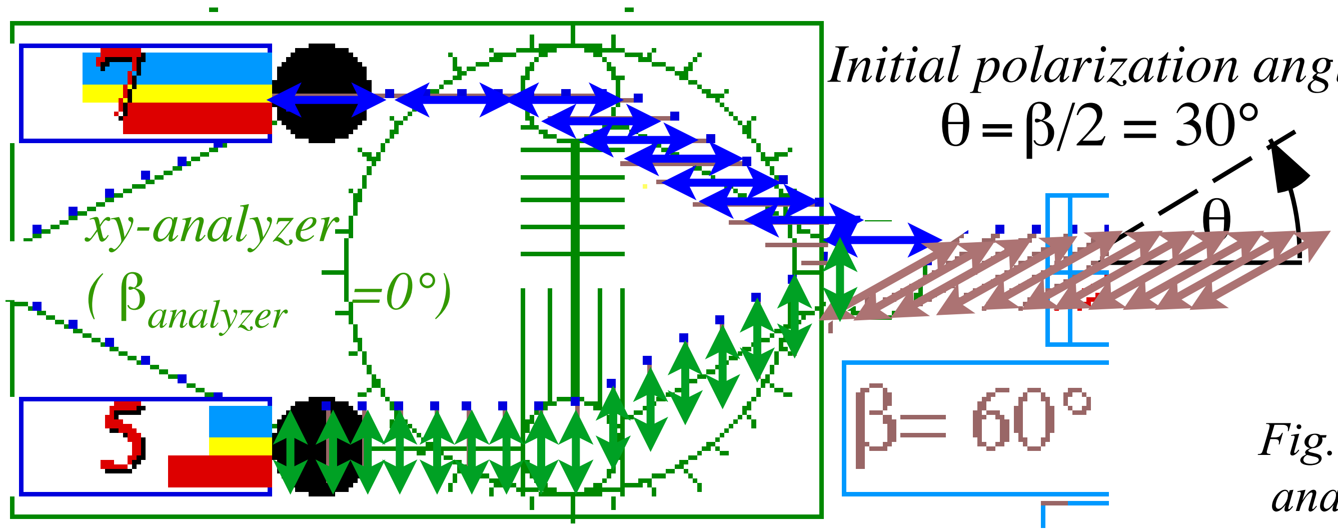
Fig. 1.3.3 Simulated polarization analyzer set up as a sorter-counter

(1) Optical analyzer in sorter-counter configuration

Analyzer reduced to a simple sorter-counter by blocking output of x-high-road and y-low-road with counters

$$x\text{-counts} \sim \left| \langle x | x' \rangle \right|^2 = \cos^2\theta = 0.75$$

$$y\text{-counts} \sim \left| \langle y | x' \rangle \right|^2 = \sin^2\theta = 0.25$$



Analyzer matrix:

$$\begin{pmatrix} \langle x|\mathbf{T}|x\rangle & \langle x|\mathbf{T}|y\rangle \\ \langle y|\mathbf{T}|x\rangle & \langle y|\mathbf{T}|y\rangle \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Fig. 1.3.3 Simulated polarization analyzer set up as a sorter-counter

(2) Optical analyzer in a filter configuration (Polaroid© sunglasses)

Analyzer blocks one path which may have photon counter without affecting function.

Analyzer matrix:

$$\begin{pmatrix} \langle x|\mathbf{P}_y|x\rangle & \langle x|\mathbf{P}_y|y\rangle \\ \langle y|\mathbf{P}_y|x\rangle & \langle y|\mathbf{P}_y|y\rangle \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$x\text{-counts} \sim \left| \langle y | x' \rangle \right|^2 = 0.75$$

(Blocked and filtered out)

$$y\text{-output} \sim \left| \langle y | x' \rangle \right|^2 = \sin^2\theta = 0.25$$

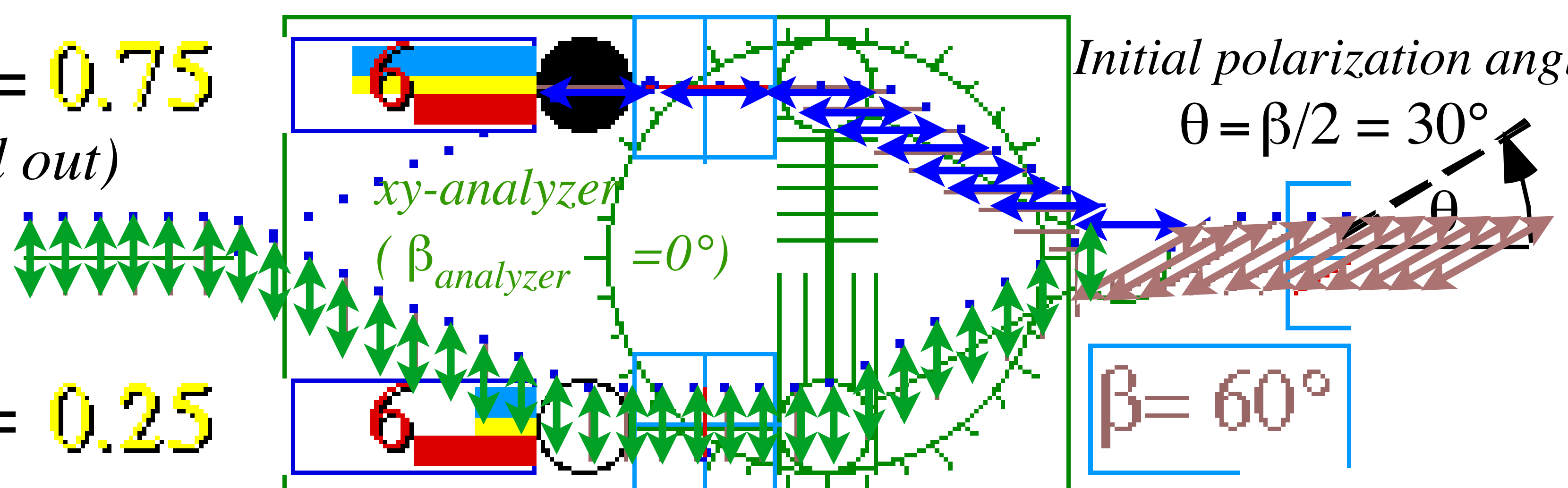


Fig. 1.3.4 Simulated polarization analyzer set up to filter out the x-polarized photons

Review: *Axioms 1-4 and "Do-Nothing" vs " Do-Something" analyzers*

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→ *Unitary "non-killer" devices: 1/2-wave plate, 1/4-wave plate*

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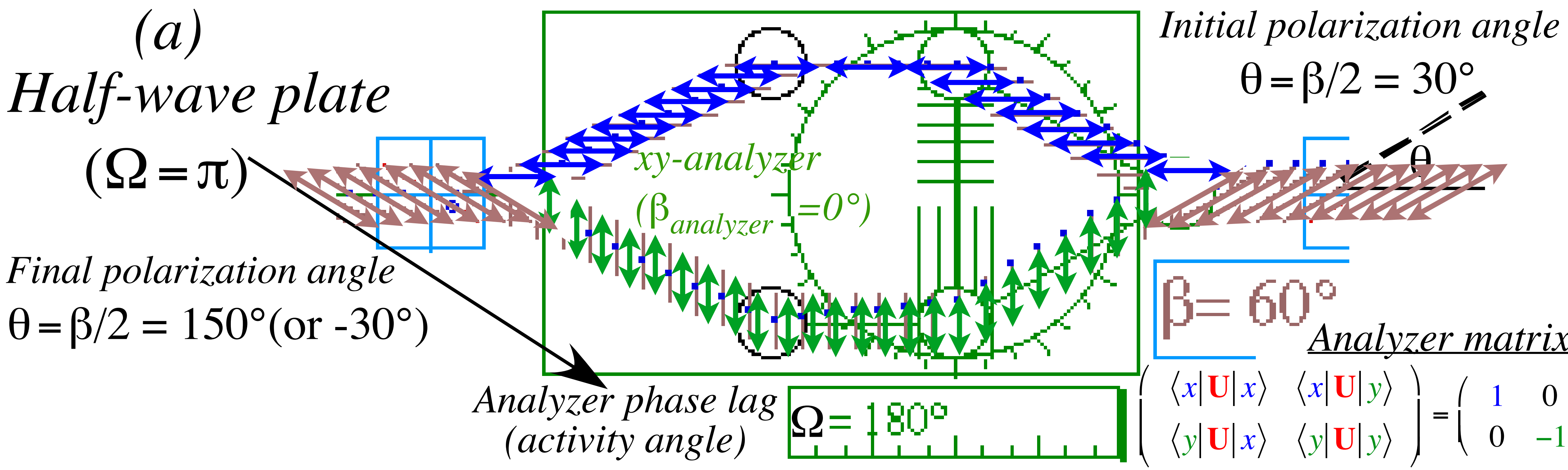
(3) Optical analyzers in the "control" configuration: Half or Quarter wave plates



$$\begin{pmatrix} \langle x|\mathbf{U}|x\rangle & \langle x|\mathbf{U}|y\rangle \\ \langle y|\mathbf{U}|x\rangle & \langle y|\mathbf{U}|y\rangle \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

(3) Optical analyzers in the "control" configuration: Half or Quarter wave plates

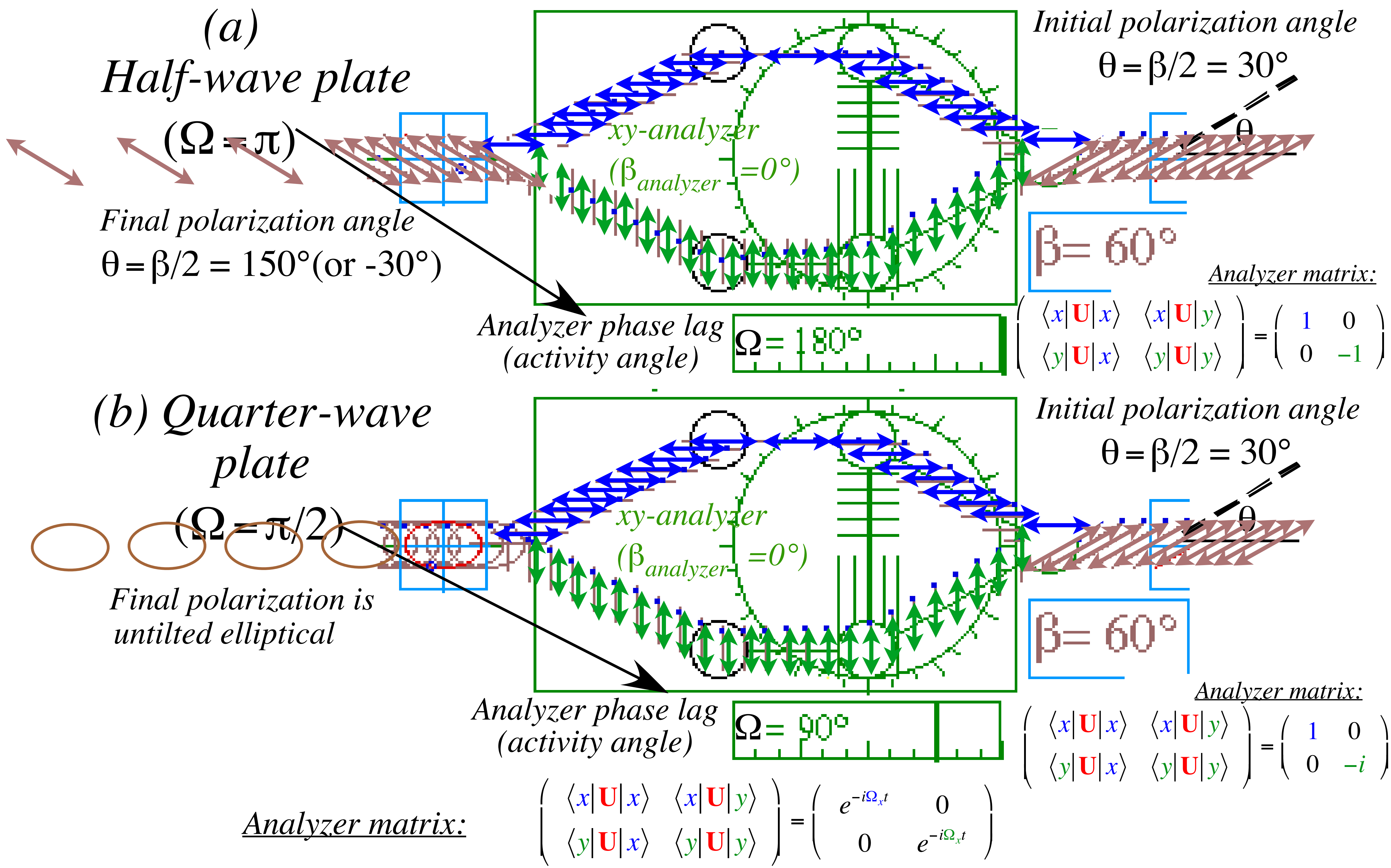
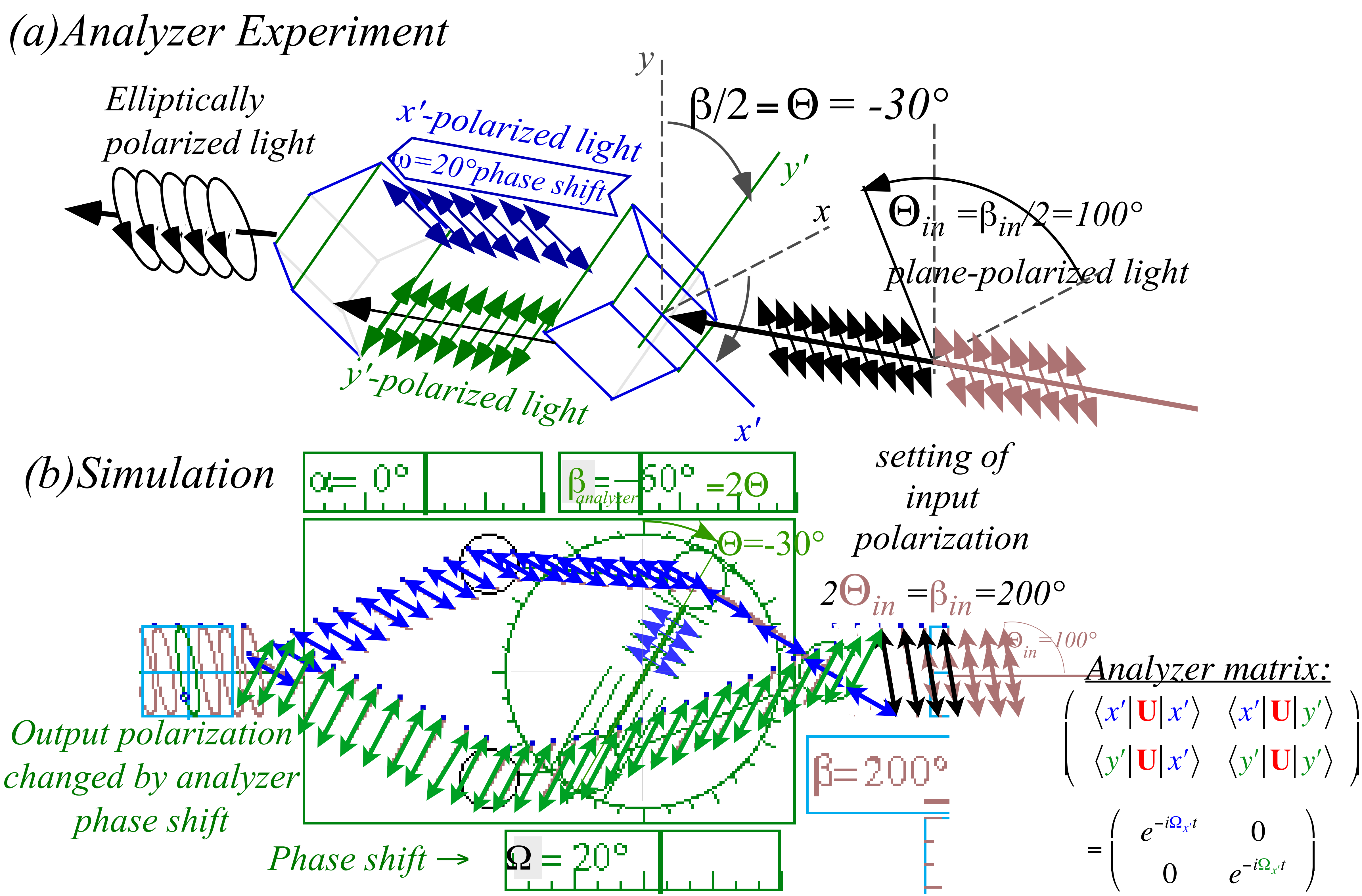


Fig. 1.3.5 Polarization control set to shift phase by (a) Half-wave (Ω = π) , (b) Quarter wave (Ω= π/2)



$$\begin{pmatrix} \langle x'|\mathbf{U}|x'\rangle & \langle x'|\mathbf{U}|y'\rangle \\ \langle y'|\mathbf{U}|x'\rangle & \langle y'|\mathbf{U}|y'\rangle \end{pmatrix} = \begin{pmatrix} e^{-i\Omega_{x'}t} & 0 \\ 0 & e^{-i\Omega_{x'}t} \end{pmatrix}$$

Similar to "do-nothing" analyzer but has extra phase factor $e^{-i\Omega_{x'}} = 0.94 - i\,0.34$ on the x'-path (top).

x-output: $\langle x|\Psi_{out}\rangle = \langle x|x'\rangle e^{-i\Omega_{x'}}\langle x'|\Psi_{in}\rangle + \langle x|y'\rangle\langle y'|\Psi_{in}\rangle = e^{-i\Omega_{x'}}\cos\Theta\cos(\Theta_{in}-\Theta) - \sin\Theta\sin(\Theta_{in}-\Theta)$

y-output: $\langle y|\Psi_{out}\rangle = \langle y|x'\rangle e^{-i\Omega_{x'}}\langle x'|\Psi_{in}\rangle + \langle x|y'\rangle\langle y'|\Psi_{in}\rangle = e^{-i\Omega_{x'}}\sin\Theta\cos(\Theta_{in}-\Theta) + \cos\Theta\sin(\Theta_{in}-\Theta)$

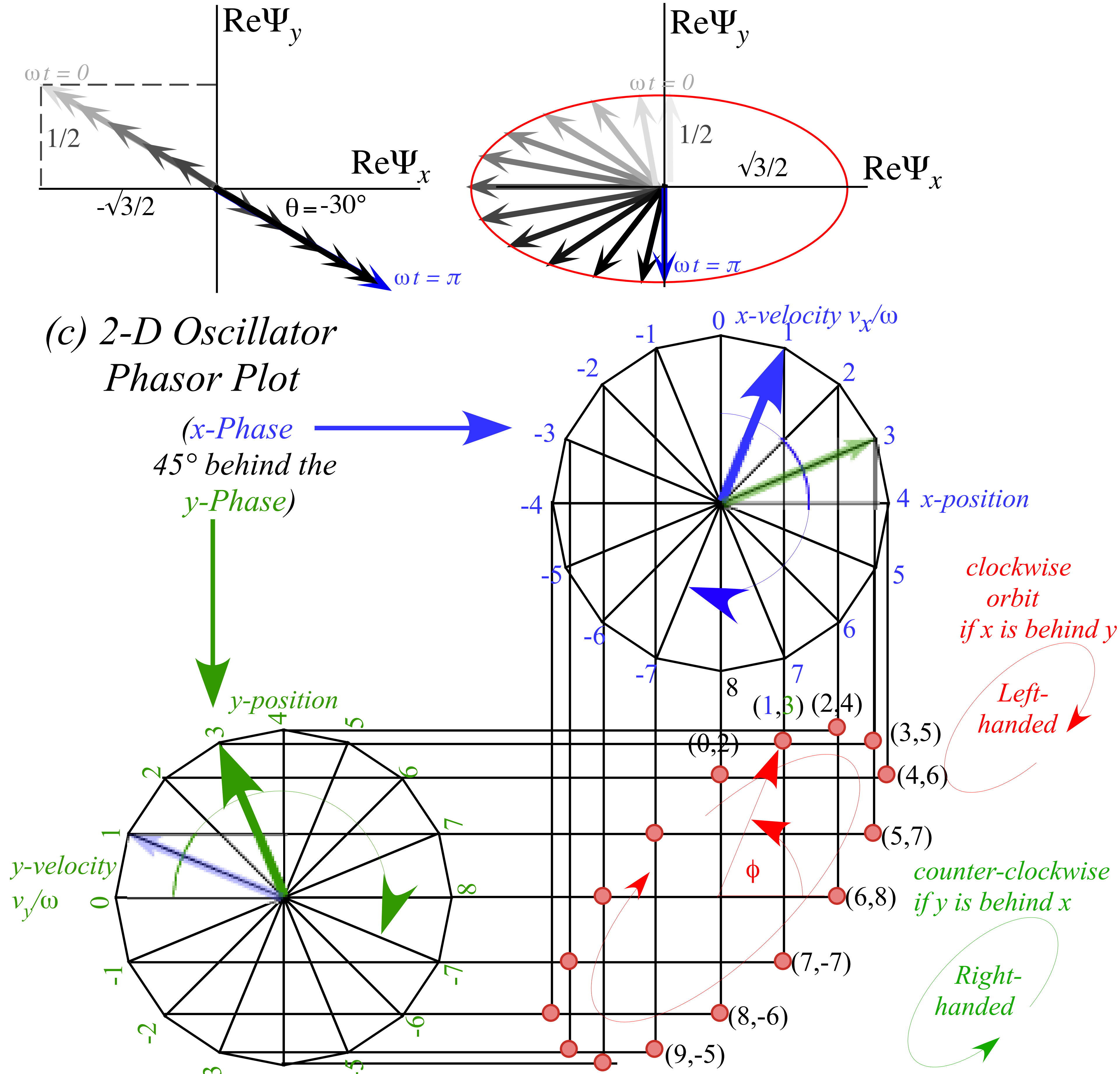


Fig. 1.3.6 Polarization states for (a) Half-wave (Ω = π) , (b) Quarter wave (Ω= π/2) (c) (Ω=−π/4)

[*BoxIt* web simulation - *With y-Phasor is on other side of xy plot*] [*RelaWavity* web simulation - *Contact ellipsometry*]

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Matrix representation of beam analyzers
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Unitary "non-killer" devices: 1/2-wave plate, 1/4-wave plate

→ *How analyzers "peek" and how that changes outcomes*
Peeking polarizers and coherence loss
Classical Bayesian probability vs. Quantum probability

Feynman $\langle j|k\rangle$-axioms *compared to* **Group axioms**

How analyzers may "peek" and how that changes outcomes

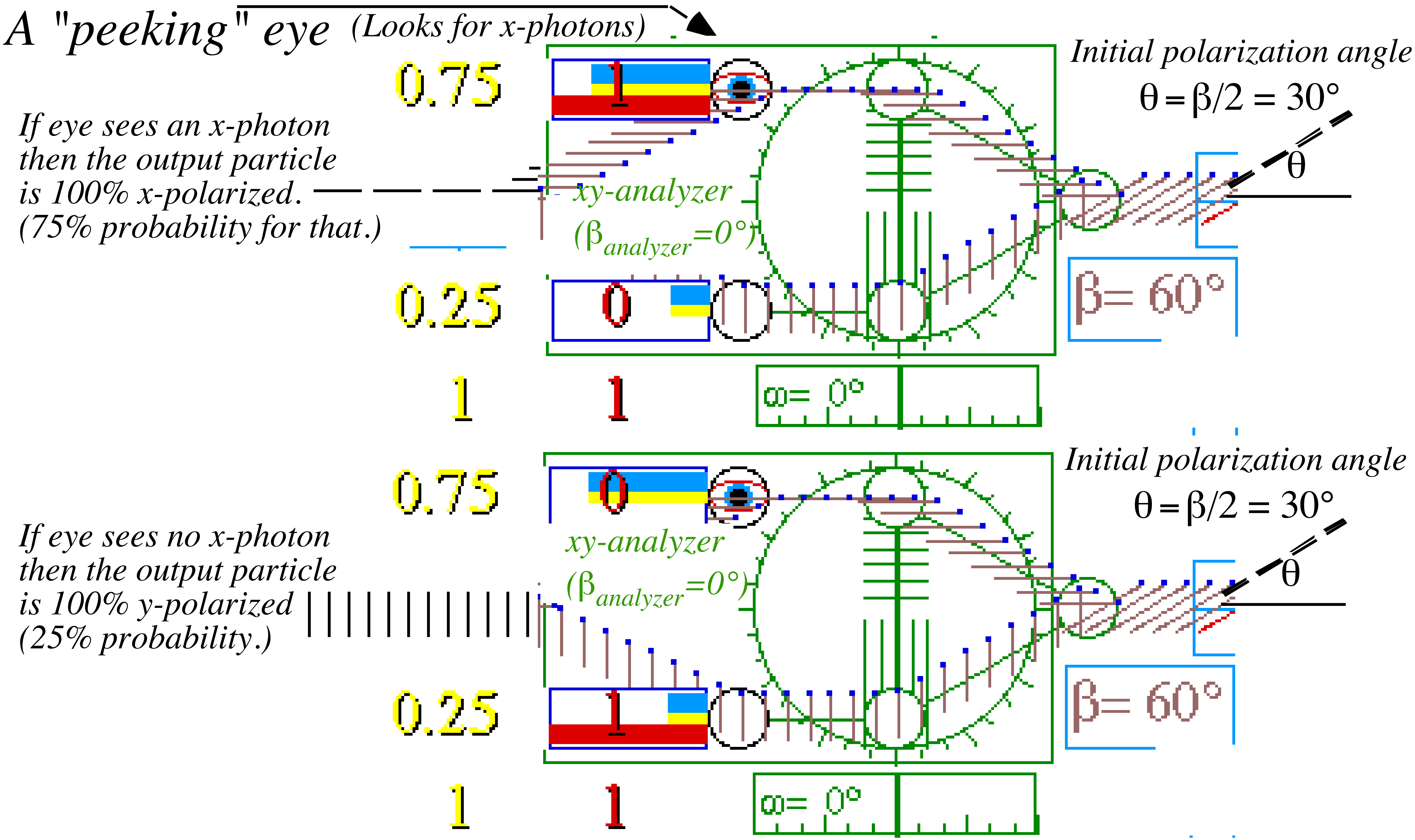


Fig. 1.3.7 Simulated polarization analyzer set up to "peek" if the photon is x-or y-polarized

How analyzers "peek" and how that changes outcomes

Simulations

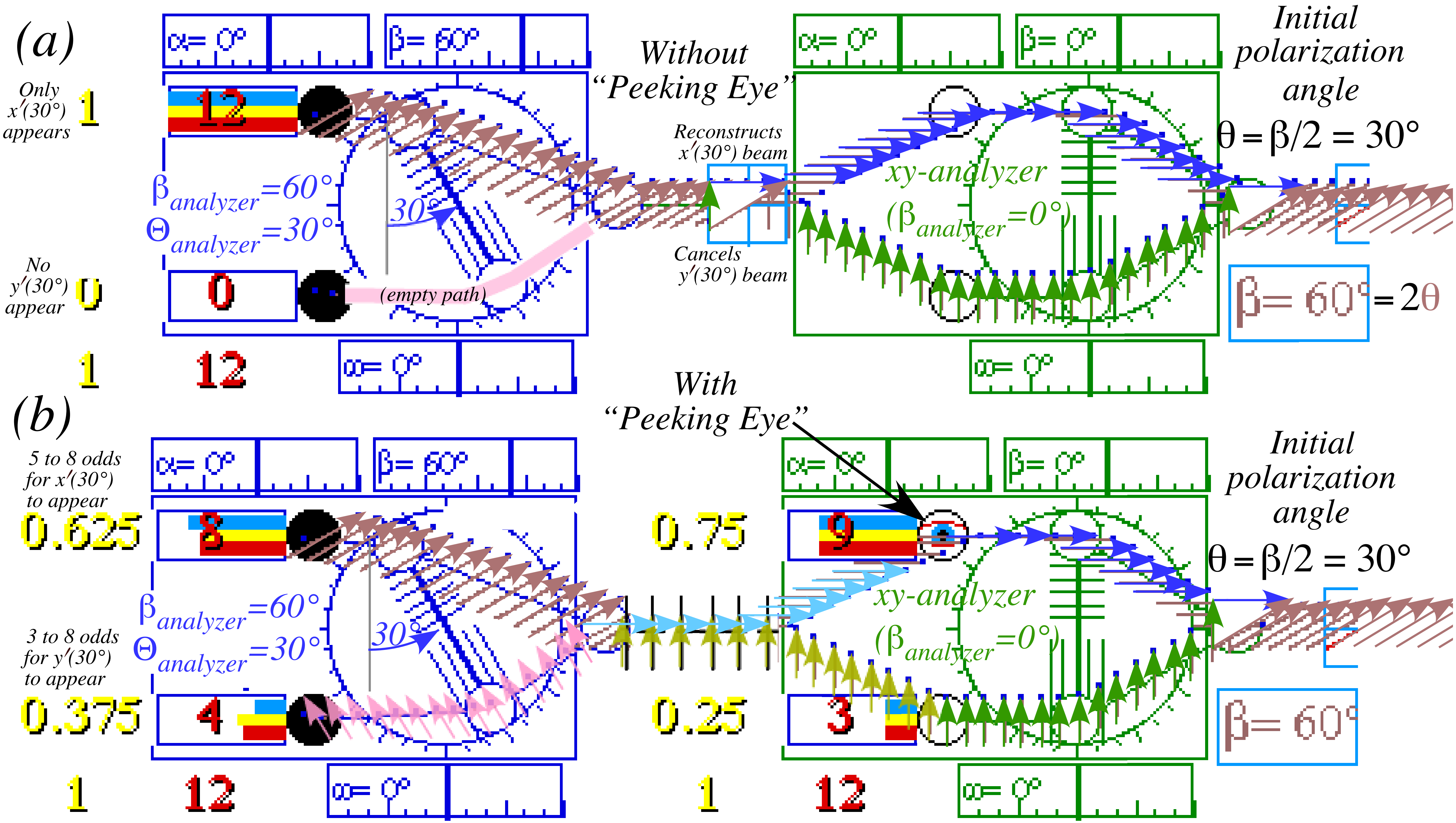
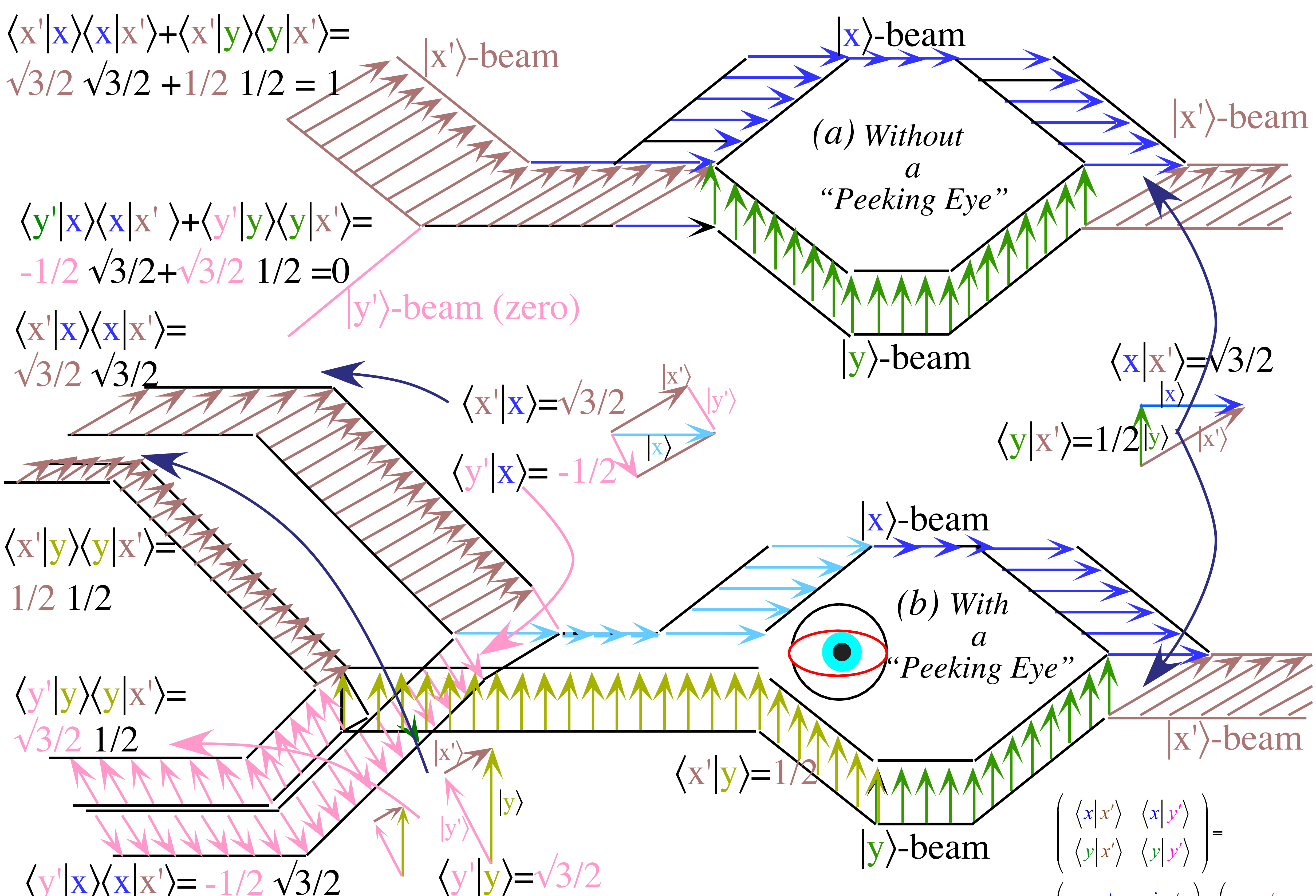


Fig. 1.3.8 Output with $\beta/2$=30° input to: (a) Coherent xy-"Do nothing" or
(b) Incoherent xy-"Peeking" devices

How analyzers "peek" and how that changes outcomes



$$\begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$$

Fig. 1.3.9 Beams-amplitudes of (a) xy-"Do nothing" and (b) xy-"Peeking" analyzer each with input $|x'\rangle$

How analyzers "peek" and how that changes outcomes

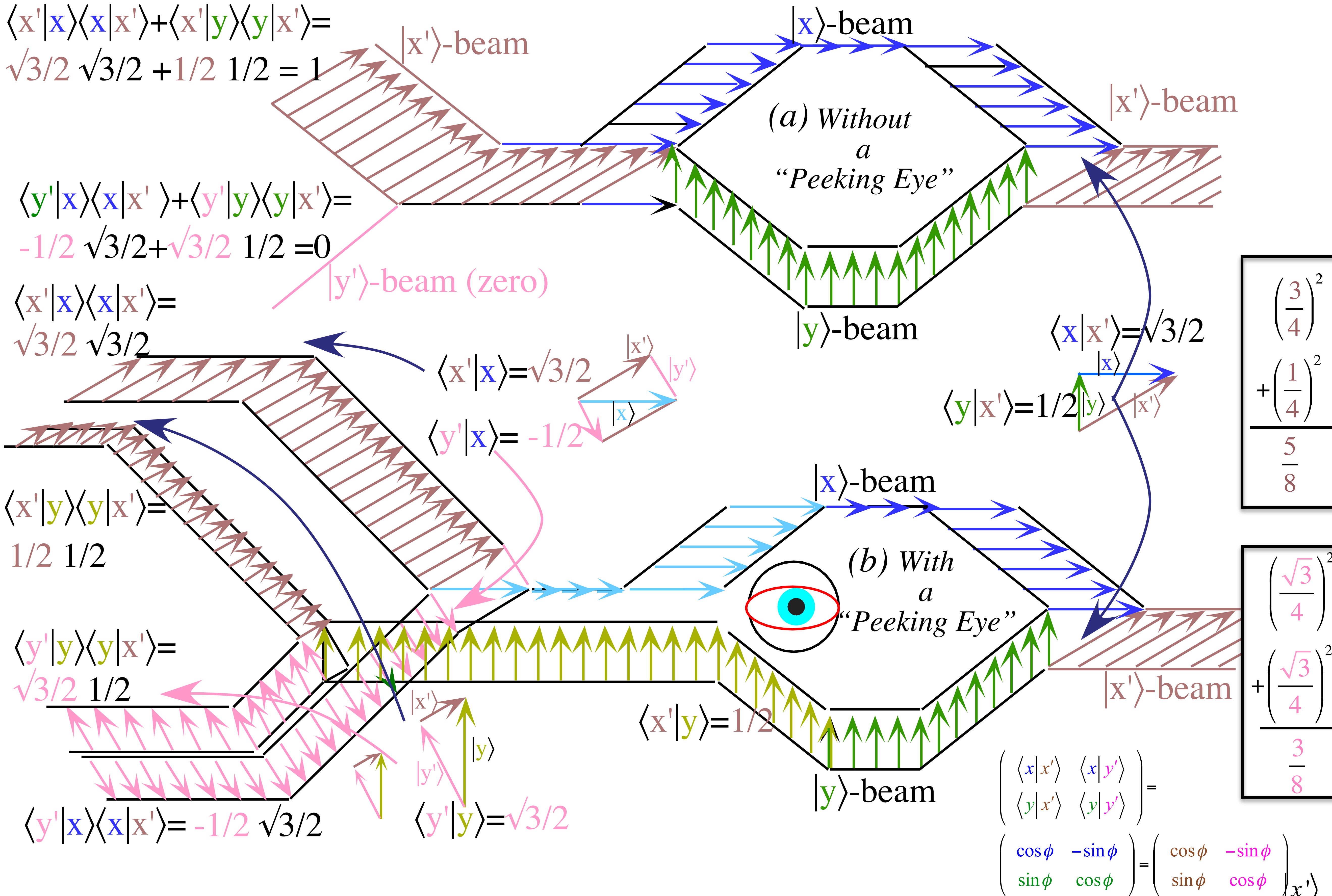


Fig. 1.3.9 Beams-amplitudes of (a) xy-"Do nothing" and (b) xy-"Peeking" analyzer each with input $|x'\rangle$

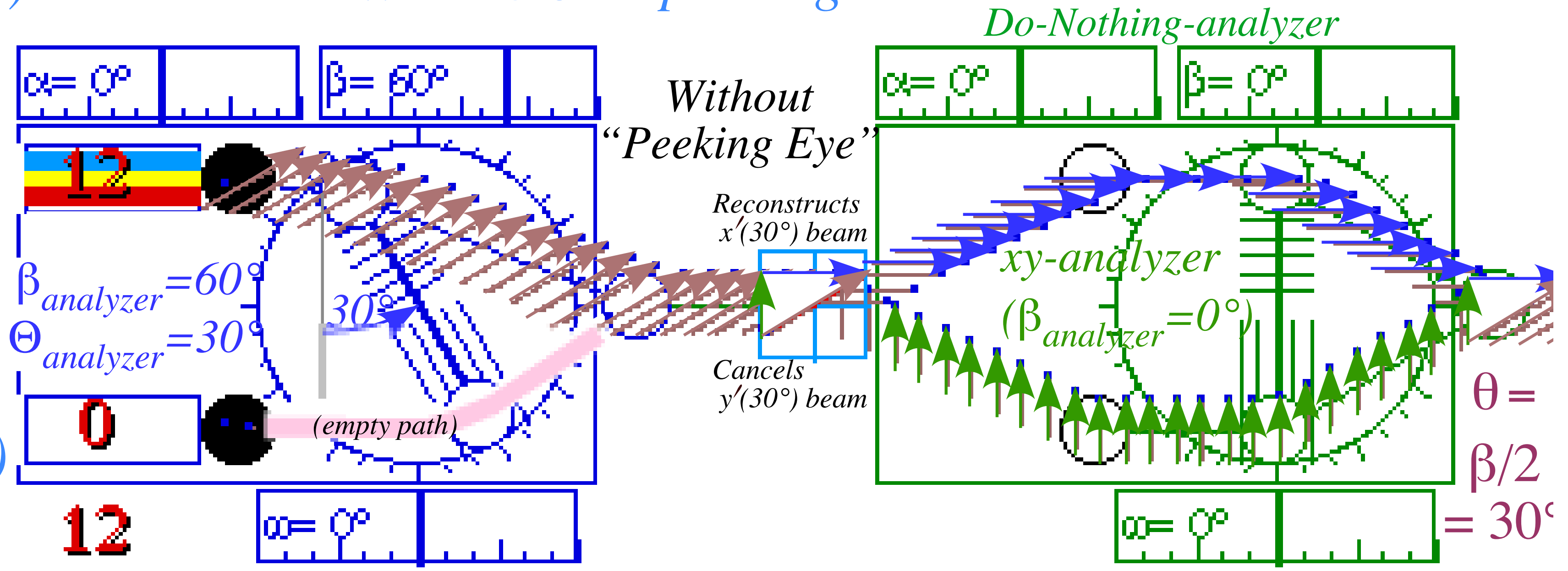
Amplitude $A(n')$ and Probability $P(n')$ at counter n' WITHOUT "peeking"

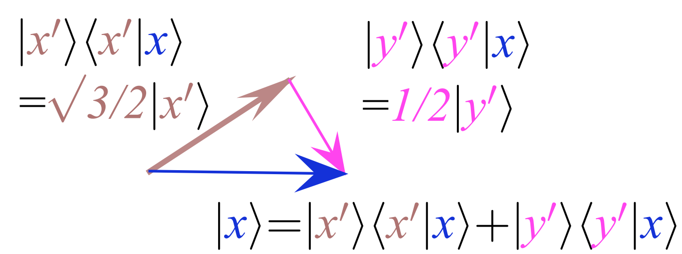
$$A(x') = \langle x'|x\rangle\langle x|x'\rangle + \langle x'|y\rangle\langle y|x'\rangle$$

$$= \frac{3}{4} + \frac{1}{4} = 1 = P(x')$$

$$A(y') = \langle y'|x\rangle\langle x|x'\rangle + \langle y'|y\rangle\langle y|x'\rangle$$

$$= \frac{-\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = 0 = P(y')$$





$$\begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$$

$$\begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix}$$

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$$A(x') = \langle x'|x\rangle(1)\langle x|x'\rangle + \langle x'|y\rangle\langle y|x'\rangle$$

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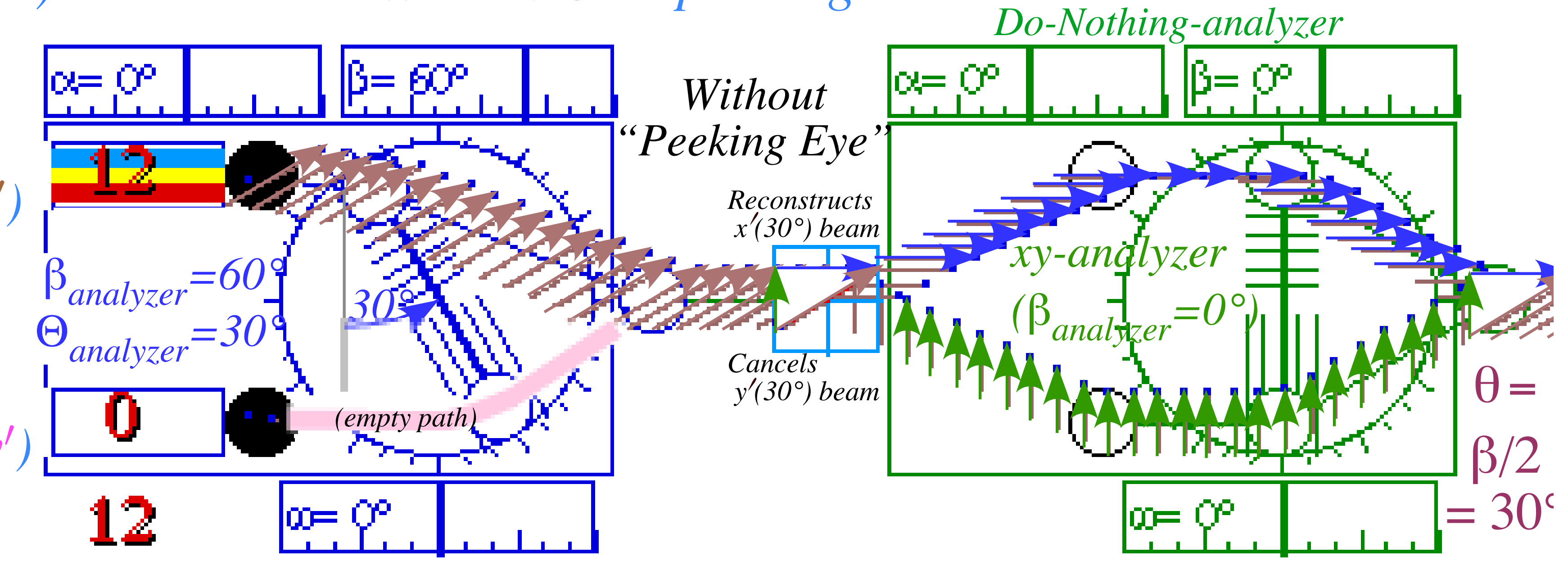


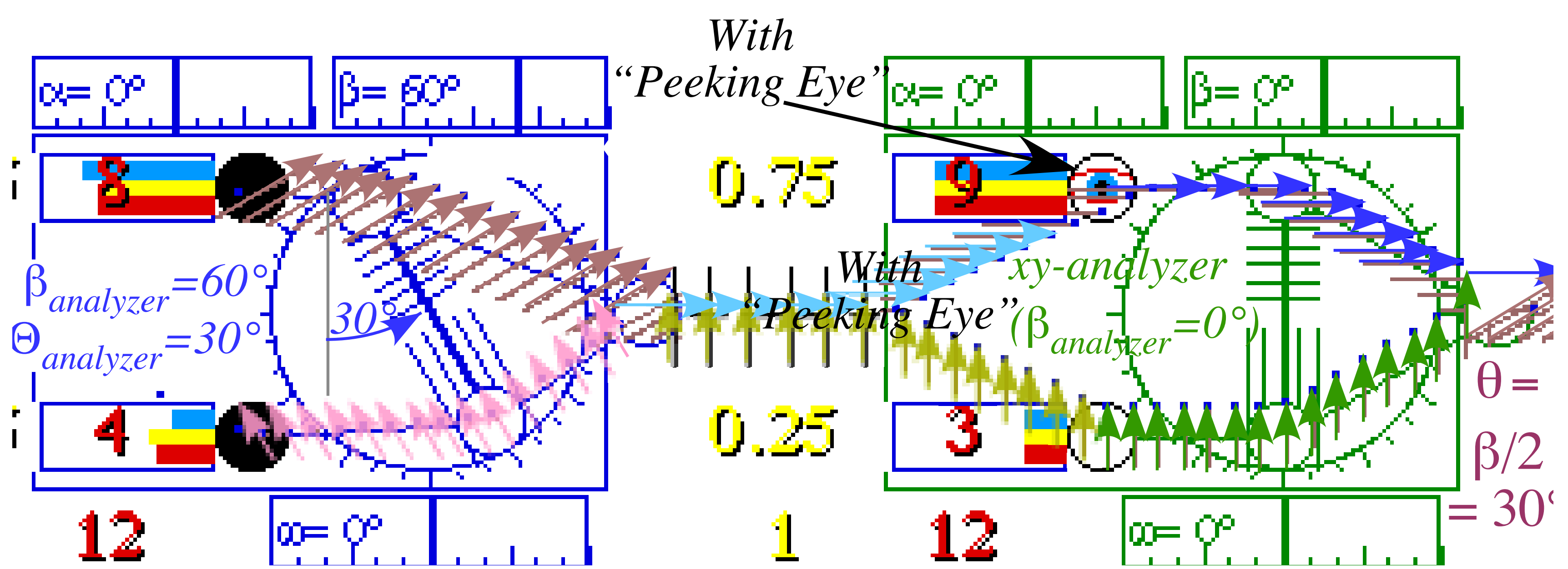
Amplitude $A(n')$ and Probability $P(n')$ at counter n' WITH "peeking"

Suppose "*x-eye*" puts phase $e^{i\phi}$ on each x-photon with random ϕ distributed over unit circle ($-\pi< \phi <\pi$).

So $e^{i\phi}$ averages to zero!

$$A(x') = \langle x'|x\rangle\left(e^{i\phi}\right)\langle x|x'\rangle + \langle x'|y\rangle\langle y|x'\rangle$$

$$= \frac{3}{4}\left(e^{i\phi}\right) + \frac{1}{4}$$



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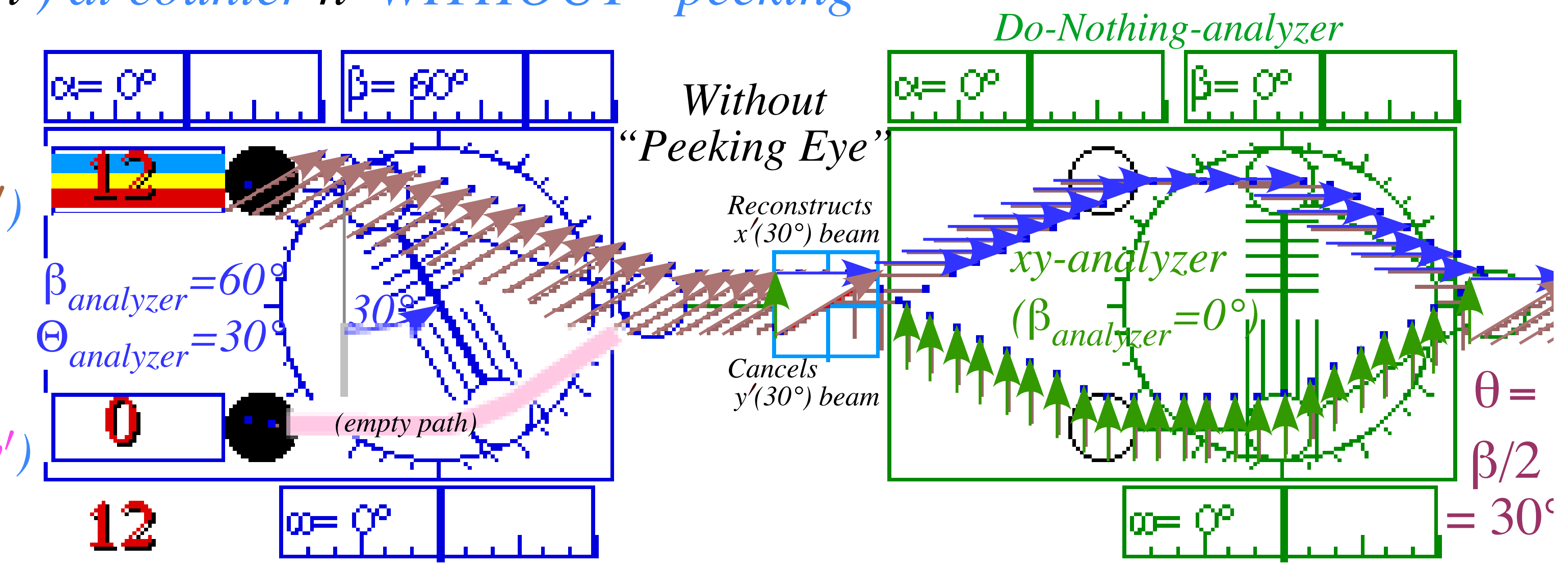

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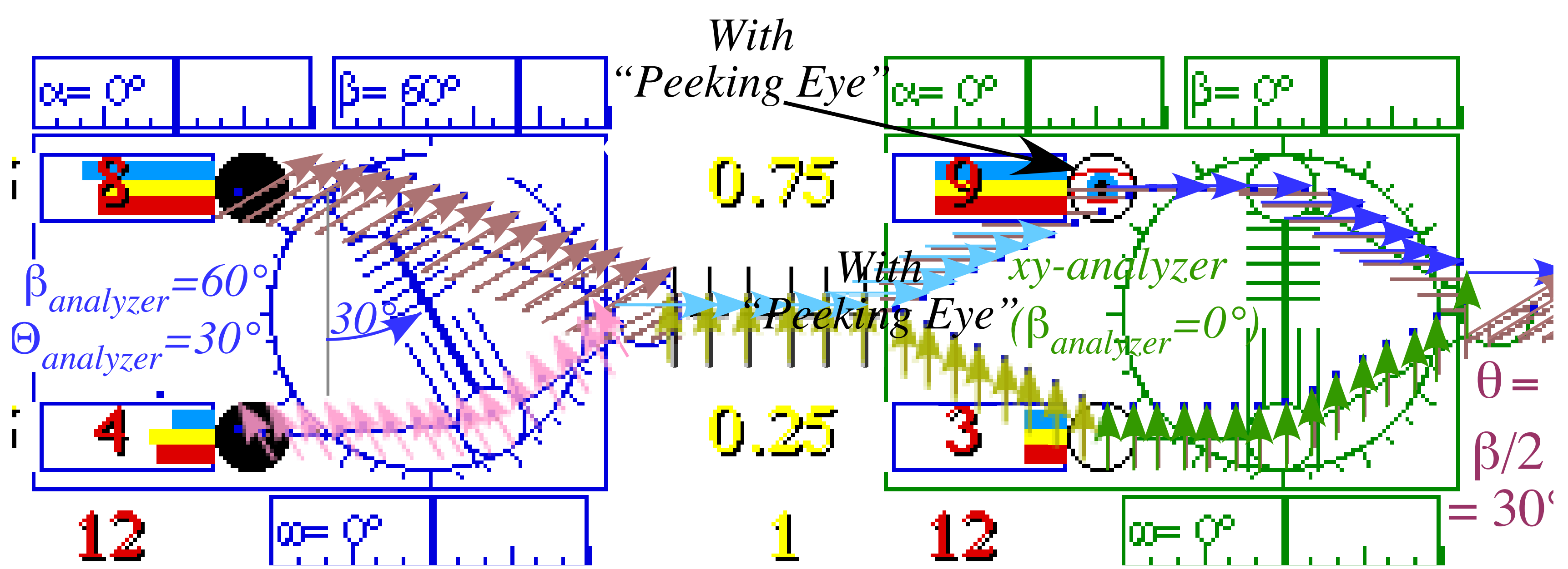
$$= \frac{5}{8} + \frac{3}{16}\left(e^{-i\phi} + e^{i\phi}\right) = \frac{5 + 3\cos\phi}{8}$$



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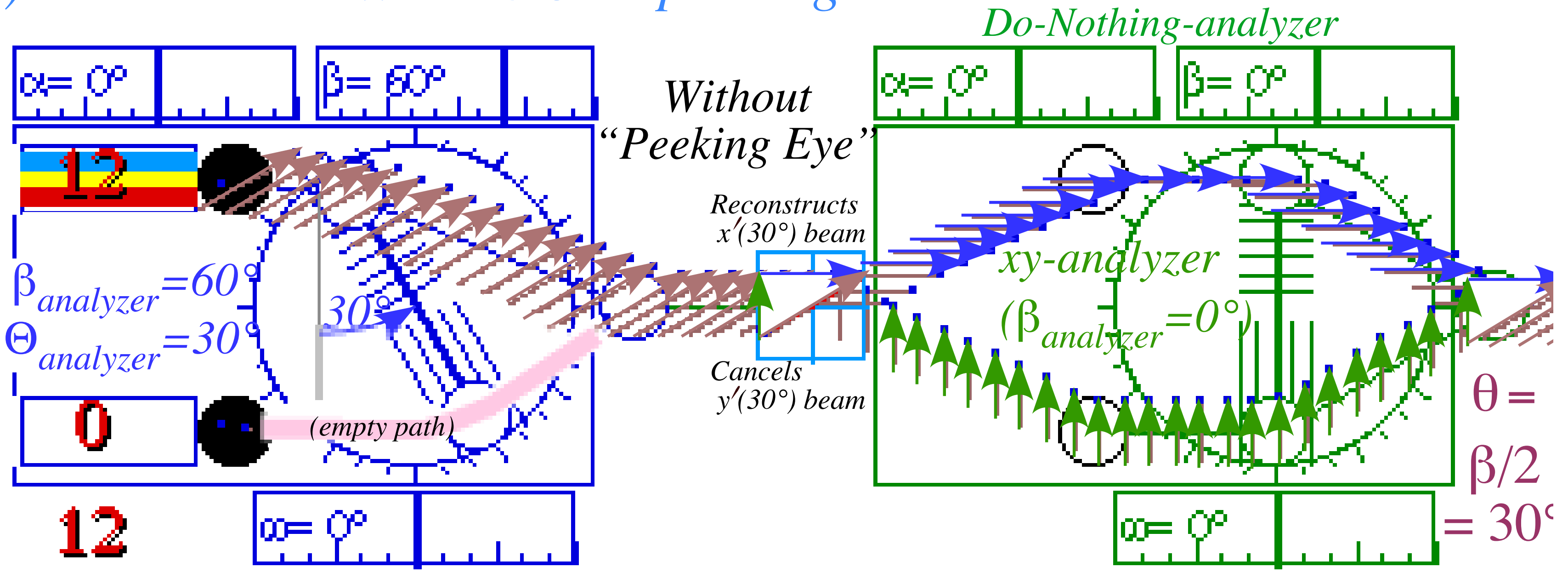

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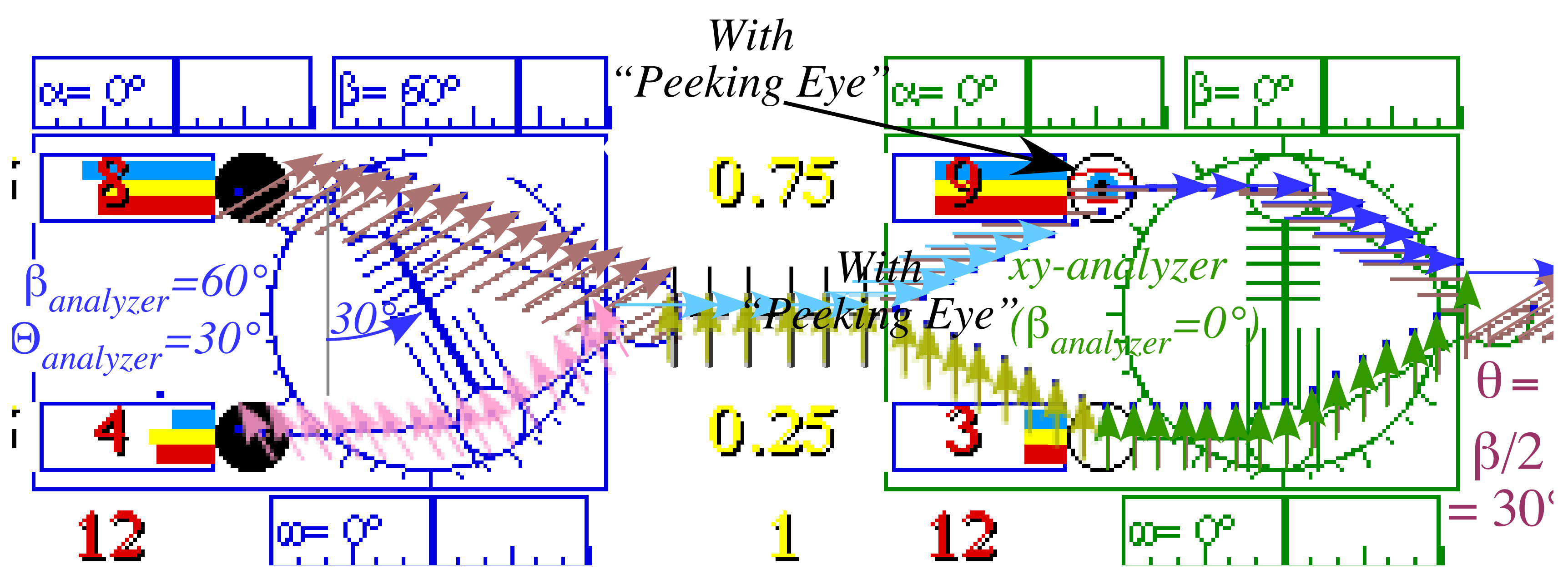
$$A(y') = \langle y'|x\rangle\left(e^{i\phi}\right)\langle x|x'\rangle + \langle y'|y\rangle\langle y|x'\rangle$$
$$= -\frac{\sqrt{3}}{4}\left(e^{i\phi}\right) + \frac{\sqrt{3}}{4}$$



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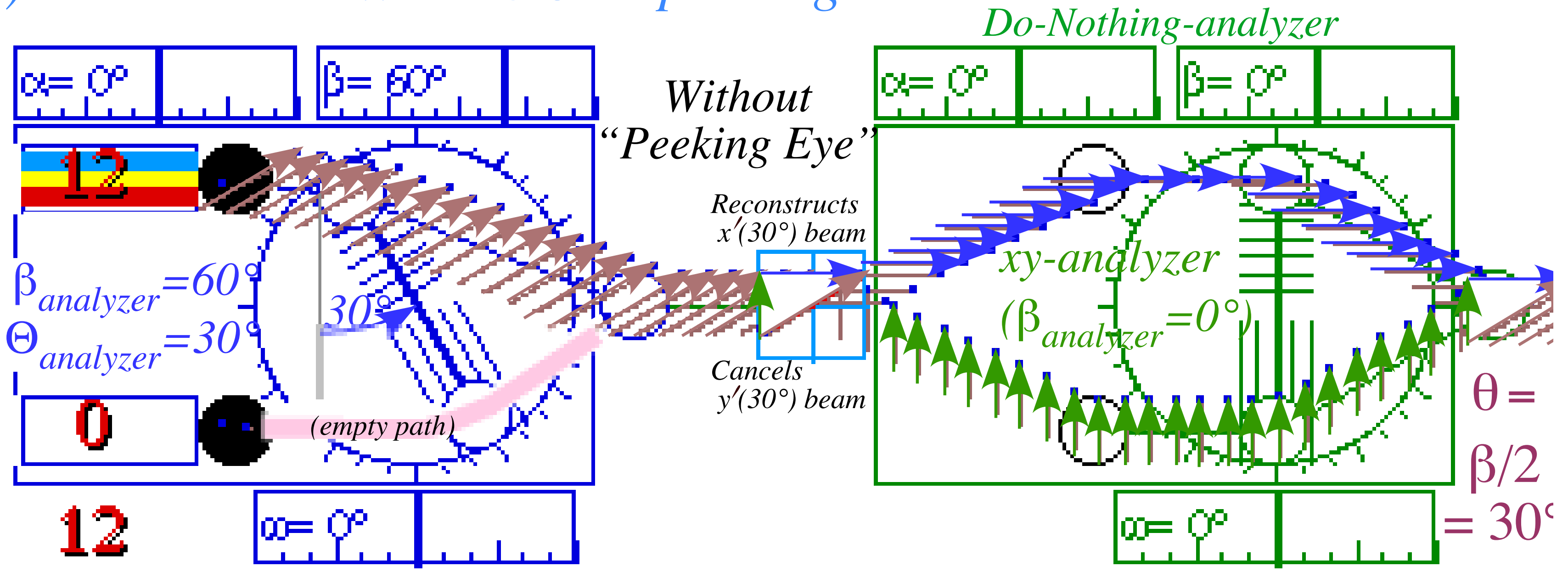

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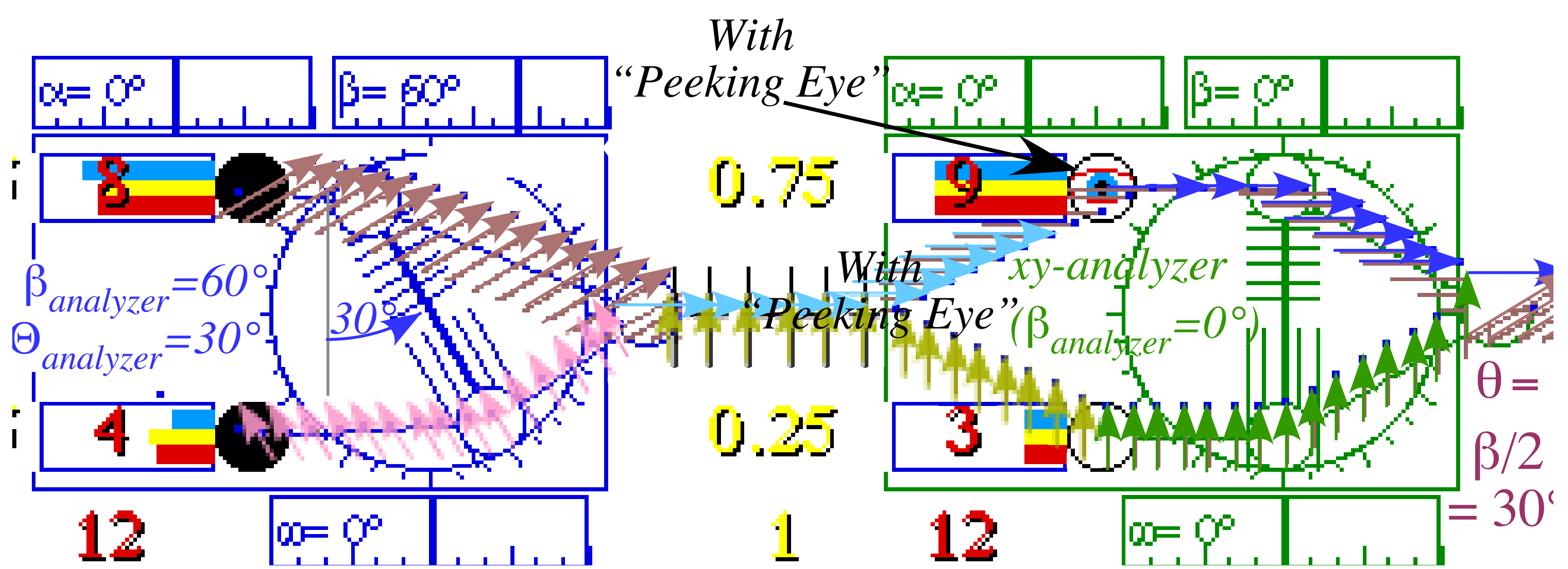
$$= \frac{5}{8} + \frac{3}{16}\left(e^{-i\phi} + e^{i\phi}\right) = \frac{5 + 3\cos\phi}{8}$$

$$A(y') = \langle y'|x\rangle\left(e^{i\phi}\right)\langle x|x'\rangle + \langle y'|y\rangle\langle y|x'\rangle$$
$$= -\frac{\sqrt{3}}{4}\left(e^{i\phi}\right) + \frac{\sqrt{3}}{4}$$
$$P(y') = \left(-\frac{\sqrt{3}}{4}\left(e^{i\phi}\right) + \frac{\sqrt{3}}{4}\right)^*\left(-\frac{\sqrt{3}}{4}\left(e^{i\phi}\right) + \frac{\sqrt{3}}{4}\right)$$
$$= \frac{3}{8} - \frac{3}{16}\left(e^{-i\phi} + e^{i\phi}\right) = \frac{3 - 3\cos\phi}{8}$$



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Abstraction of Axiom-4 to define projection and unitary operators
- *Projection operators and resolution of identity*

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- ➡ *Peeking polarizers and coherence loss*
  - *Classical Bayesian probability vs. Quantum probability*

Feynman $\langle j|k\rangle$-axioms compared to **Group axioms**

Classical Bayesian probability vs. Quantum probability

$$\begin{pmatrix} \text{Probability that} \\ \text{photon in } x'\text{-input} \\ \textit{becomes} \\ \text{photon in } x'\text{-counter} \end{pmatrix}_{classical} =$$

$$\begin{pmatrix} \text{probability that} \\ \text{photon in } x\text{-beam} \\ \textit{becomes} \\ \text{photon in } x'\text{-counter} \end{pmatrix} \cdot \begin{pmatrix} \text{probability that} \\ \text{photon in } x'\text{-input} \\ \textit{becomes} \\ \text{photon in } x\text{-beam} \end{pmatrix} + \begin{pmatrix} \text{probability that} \\ \text{photon in } y\text{-beam} \\ \textit{becomes} \\ \text{photon in } x'\text{-counter} \end{pmatrix} \cdot \begin{pmatrix} \text{probability that} \\ \text{photon in } x'\text{-input} \\ \textit{becomes} \\ \text{photon in } y\text{-beam} \end{pmatrix}$$

$$\begin{pmatrix} \text{Probability that} \\ \text{photon in } x'\text{-input} \\ \textit{becomes} \\ \text{photon in } x'\text{-counter} \end{pmatrix}_{classical} = \left(\left|\langle x'|x\rangle\right|^2\right) \cdot \left(\left|\langle x|x'\rangle\right|^2\right) + \left(\left|\langle x'|y\rangle\right|^2\right) \cdot \left(\left|\langle y|x'\rangle\right|^2\right)$$

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$$\begin{pmatrix} \text{Quantum probability} \\ \text{at } x'\text{-counter} \end{pmatrix} = \left|\langle x'|x\rangle\left(e^{i\phi}\right)\langle x|x'\rangle + \langle x'|y\rangle\langle y|x'\rangle\right|^2$$

Classical Bayesian probability vs. Quantum probability

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$$\begin{pmatrix} \text{Quantum probability} \\ \text{at } x'\text{-counter} \end{pmatrix} = \left|\langle x'|x\rangle\left(e^{i\phi}\right)\langle x|x'\rangle + \langle x'|y\rangle\langle y|x'\rangle\right|^2$$

$$= \left|\langle x'|x\rangle\langle x|x'\rangle\right|^2 + \left|\langle x'|y\rangle\langle y|x'\rangle\right|^2 + e^{-i\phi}\langle x'|x\rangle^*\langle x|x'\rangle^*\langle x'|y\rangle\langle y|x'\rangle + e^{i\phi}\langle x'|x\rangle\langle x|x'\rangle\langle x'|y\rangle^*\langle y|x'\rangle^* = 1$$

=(classical probability) + (Phase-sensitive or *quantum interference* terms)

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$$\begin{pmatrix}\text{probability that} \\ \text{photon in } x\text{-beam} \\ \textit{becomes} \\ \text{photon in } x'\text{-counter}\end{pmatrix} * \begin{pmatrix}\text{probability that} \\ \text{photon in } x'\text{-input} \\ \textit{becomes} \\ \text{photon in } x\text{-beam}\end{pmatrix} + \begin{pmatrix}\text{probability that} \\ \text{photon in } y\text{-beam} \\ \textit{becomes} \\ \text{photon in } x'\text{-counter}\end{pmatrix} * \begin{pmatrix}\text{probability that} \\ \text{photon in } x'\text{-input} \\ \textit{becomes} \\ \text{photon in } y\text{-beam}\end{pmatrix}$$

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$$\begin{pmatrix}\text{Quantum probability} \\ \text{at } x'\text{-counter}\end{pmatrix} = \left|\langle x'|x\rangle\left(e^{i\phi}\right)\langle x|x'\rangle + \langle x'|y\rangle\langle y|x'\rangle\right|^2$$

$$= \left|\langle x'|x\rangle\langle x|x'\rangle\right|^2 + \left|\langle x'|y\rangle\langle y|x'\rangle\right|^2 + e^{-i\phi}\langle x'|x\rangle^*\langle x|x'\rangle^*\langle x'|y\rangle\langle y|x'\rangle + e^{i\phi}\langle x'|x\rangle\langle x|x'\rangle\langle x'|y\rangle^*\langle y|x'\rangle^* = 1$$

=(classical probability) + (Phase-sensitive or *quantum interference* terms)

$$\begin{pmatrix}\text{Quantum probability} \\ \text{at } x'\text{-counter}\end{pmatrix} = \left|\langle x'|x\rangle\langle x|x'\rangle + \langle x'|y\rangle\langle y|x'\rangle\right|^2$$

Square of sum

Classical Bayesian probability vs. Quantum probability

$$\begin{pmatrix} \text{Probability that} \\ \text{photon in } x'\text{-input} \\ \textit{becomes} \\ \text{photon in } x'\text{-counter} \end{pmatrix}_{classical} = \qquad \begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix}$$

$$\begin{pmatrix} \text{probability that} \\ \text{photon in } x\text{-beam} \\ \textit{becomes} \\ \text{photon in } x'\text{-counter} \end{pmatrix} * \begin{pmatrix} \text{probability that} \\ \text{photon in } x'\text{-input} \\ \textit{becomes} \\ \text{photon in } x\text{-beam} \end{pmatrix} + \begin{pmatrix} \text{probability that} \\ \text{photon in } y\text{-beam} \\ \textit{becomes} \\ \text{photon in } x'\text{-counter} \end{pmatrix} * \begin{pmatrix} \text{probability that} \\ \text{photon in } x'\text{-input} \\ \textit{becomes} \\ \text{photon in } y\text{-beam} \end{pmatrix}$$

$$\begin{pmatrix} \text{Probability that} \\ \text{photon in } x'\text{-input} \\ \textit{becomes} \\ \text{photon in } x'\text{-counter} \end{pmatrix}_{classical} = \left(\left|\langle x'|x\rangle\right|^2\right)^* \left(\left|\langle x|x'\rangle\right|^2\right) + \left(\left|\langle x'|y\rangle\right|^2\right)^* \left(\left|\langle y|x'\rangle\right|^2\right) = \left(\left|\frac{\sqrt{3}}{2}\right|^2\right)^* \left(\left|\frac{\sqrt{3}}{2}\right|^2\right) + \left(\left|\frac{-1}{2}\right|^2\right)^* \left(\left|\frac{1}{2}\right|^2\right) = \frac{5}{8}$$

$$\begin{pmatrix} \text{Quantum probability} \\ \text{at } x'\text{-counter} \end{pmatrix} = \left|\langle x'|x\rangle\left(e^{i\phi}\right)\langle x|x'\rangle + \langle x'|y\rangle\langle y|x'\rangle\right|^2$$

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Square of sum

$$\begin{pmatrix} \text{Classical probability} \\ \text{at } x'\text{-counter} \end{pmatrix} = \left|\langle x'|x\rangle\langle x|x'\rangle\right|^2 + \left|\langle x'|y\rangle\langle y|x'\rangle\right|^2$$

Sum of squares

Review: Axioms 1-4 and "Do-Nothing" vs " Do-Something" analyzers

Abstraction of Axiom-4 to define projection and unitary operators
- *Projection operators and resolution of identity*

Unitary operators and matrices that do something (or "nothing")
- *Diagonal unitary operators*
- *Non-diagonal unitary operators and †-conjugation relations*
- *Non-diagonal projection operators and Kronecker ⊗−products*
- *Axiom-4 similarity transformation*

Matrix representation of beam analyzers
- *Non-unitary "killer" devices: Sorter-counter, filter*
- *Unitary "non-killer" devices: 1/2-wave plate, 1/4-wave plate*

How analyzers "peek" and how that changes outcomes
- *Peeking polarizers and coherence loss*
  - *Classical Bayesian probability vs. Quantum probability*

→ *Feynman $\langle j|k\rangle$-axioms compared to* **Group axioms**

Feynman $\langle j|k\rangle$-axioms compared to Group axioms

Axiom 1: $j'' \Leftrightarrow m'$ probability equals $|\langle j''|m'\rangle|^2 = |\langle m'|j''\rangle|^2$ (Probability)	Axiom 2: $\langle j''|m'\rangle^* = \langle m'|j''\rangle$ (T-reversal Conjugation)	Axiom 3: $\langle j|k\rangle = \delta_{jk} = \langle j'|k'\rangle = \langle j''|k''\rangle$ (Orthonormality)	Axiom 4: $\langle j''|m'\rangle = \sum_{k=1}^{n} \langle j''|k\rangle\langle k|m'\rangle$ (Completeness)

Group axioms

(1) The closure axiom

Products $ab = c$ are defined between any two group elements a and b, and the result c is contained in the group.

(2) The associativity axiom

Products $(ab)c$ and $a(bc)$ are equal for all elements a, b, and c in the group .

(3) The identity axiom

There is a unique element 1 (the identity) such that $1\cdot a = a = a\cdot 1$ for all elements a in the group ..

4) The inverse axiom

For all elements a in the group there is an inverse element a^{-1} such that $a^{-1}a = 1 = a\cdot a^{-1}$.

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*since analyzer-***A** *following analyzer-***B** *is analyzer-***AB** =**C**
*and analyzer-***B** *following analyzer-***A** *is analyzer-***BA** =**D**

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Feynman Axiom-4 consistent with **group axiom 2**
since analyzer matrix multiplication is associative

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Feynman Axiom-2 consistent with **group axiom 3**
*since "Do Nothing" analyzer =identity operator=***1**

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since inverse $\mathbf{U}$ *=tranpose-conjugate* $\mathbf{U}^\dagger = \mathbf{U}^{T*}$

Feynman $\langle j|k\rangle$-axioms *compared to* **Group axioms**

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(5) The commutative axiom (Abelian groups only)

All elements a in an Abelian group are mutually commuting: $a \cdot b = b \cdot a$.

Most analyzer sets (and most groups) are not Abelian (commutative)