Assignment 6 - **Group Theory and Quantum Physics 5093** All due Fri March 31 (3/31/17)

Main Reading: In text *QTCA* Unit 5 Ch. 15. Lectures 12-14

Try to finish Problem 15.1.1, 15.3.1a-c and 15.4.1 a-b by Fri March 17.

A Complete Completeness

15.1.1. The D-orthogonality relation (15.1.30) needs a completeness relation to go with it. Can you derive one? If so, do it, or else explain why not.

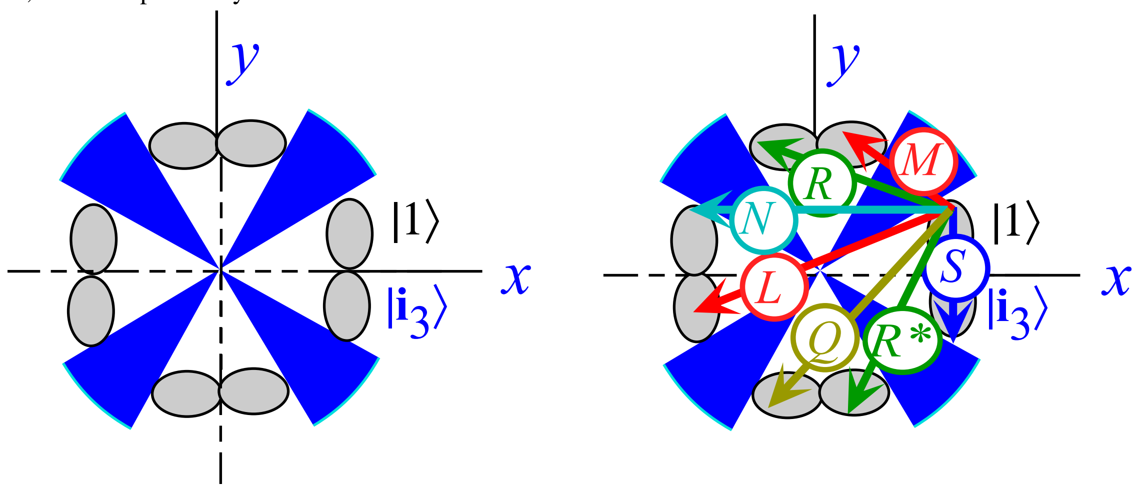

Fig. 15.4

The Square Deal

15.3.1. The analysis of D_3 needs to be extended to the group D_4 of a tetragonal 4-well ring.

(a) Derive an 8-by-8 D_4 group table like Fig. 15.1.2. (Construct an operator/state diagram.) Give a standing wave and moving wave irrep analogous to (15.1.8) and (15.1.10) and check it works for some products.

(b) Derive the D_4 class algebra analogous to (15.2.1) and reduce it so a complete D_4 character table is found. First, how many classes? (It should be more than four.)

(c) Determine the rank of D_4. Write out all the D_4 irrep projectors for the standing wave choice of basis that diagonalizes all elements of the D_2 subgroup from Fig. 15.1.1. Label your D_4 results using the standard labels $A_1, B_1, A_2, \ldots E_{n-1}$, for D_{2n} groups. (Let $A(B)$ parity be +(-) for $R_z(90°)$, and $_{1(2)}$ parity be +(-) for $R_x(180°)$.)

(d) Use the irrep projectors to produce a complete set of D_4 band states and sketch them in a way analogous to Fig. 15.3.2 or 3. (You may use actual solutions from previous problems.)

The Square Deal Continued

15.4.1. Apply analysis of the group D_4 of a tetragonal 4-well quantum ring as was done for D_3.

(a) Derive 8-by-8 D_4 dual regular representations like (15.1.15a) and (15.3.11d) for D_3.

(b) Derive the D_4 Hamiltonian analogous to (15.4.2b) based on the Fig. 15.4 above, and reduce to 2-by-2 blocks.

(c) (Extra Credit optional) Do a U(2) analysis of the residual 2-by-2 Hamiltonian matrix or matrices.

(d) Give eigensolutions if only S and M are non-zero. Consider S>>M and M>>S.

(e) Give eigensolutions if only S and R=R* are non-zero. Consider S>>R and R>>S.

A Super-Degenrate Square Deal

15.4.2. Let the Hamiltonian of the tetragonal 4-well quantum ring have symmetry $D_4^* D_4$.

(a) What form does its Hamiltonian matrix have in the original group basis?

(b) What form do the eigensolutions take? If possible, give answer in closed form.