Galloping Gertie
1. Galloping phase velocity of general 2-frequency plane \( A_1e^{i(k_1x-vt)}+A_2e^{i(k_2x-\omega t)} \) helps us understand relativistic QM. Derive this velocity \( V_{gallop}(t) \) as a function of Standing Wave Ratio \( SWR=(A_1-A_2)/(A_1+A_2) \), time \( t \), and \( \phi_{phase}=\omega t/k \). Then give a formula for maximum \( V_{gallop-\max} \) and minimum \( V_{gallop-\min} \) phase velocities achieved in terms of \( SWR \).

\( H_{2x2} = \begin{pmatrix} A & B \\ B & A \end{pmatrix} \) and high school algebra revisited
2. Being able to solve eigenvalue problems using symmetry projection implies that such projection can also solve certain types of algebraic polynomial equations, that is, secular equations. Indeed, the \( C_2(B) \) solution amounts to derivation of the quadratic formula for solutions to \( ax^2+bx^2+c=0 \). Show eigenvalues of \( H_{2x2}(\text{above}) \) given by \( C_2(B) \) algebra also re-derive quadratic equation. (Relate \( A \) and \( B \) to \( \omega/a \) and \( c/a \). Next we begin to generalize these results.)

\( H_{3x3} = \begin{pmatrix} 0 & A & B \\ B & 0 & A \\ A & B & 0 \end{pmatrix} \) and college algebra revisited
3. A \( C_3(\text{symmetric}) \) Hamiltonian matrix \( H_{3x3}(\text{above}) \) has a cubic secular equation. Use \( C_3 \)-solutions to derive a cubic formula for roots of \( ax^3+bx^2+cx+d=0 \). Hint: Simplify by dealing with secular equation \( x^3+0+cx+d=0 \) of traceless \( H_{3x3} \). Show that missing \( a \) and \( b \) parts are easy to tack on afterwards.

\( H_{4x4} = (?) \) and beyond (This is an extra credit problem)
4. A \( C_4(\text{symmetric}) \) Hamiltonian matrix \( H_{4x4}(\text{derive this}) \) has a quartic secular equation. Use \( C_4 \)-solutions to derive its eigensolutions. Discuss whether this gives roots for a general quartic \( ax^4+bx^3+cx^2+dx+e=0 \) or just for a special case.

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**Holey Pentagram Batman!**

5. Five identical quantum dots are originally connected by five identical evanescent tunneling “weak-links” as shown in Figure (a) above. Let a quantum dot pair linked by a single path of tunneling amplitude \( S \) (1st sketch) have a 1.0 GHz resonance as its lowest frequency and a 2.0 GHz resonance if linked by a two \( S \)-paths.) A link may have slightly weaker tunneling amplitude \( s<S \), as in Figure (b) or be removed entirely, as sketched in Figure (c).

(a) Derive and solve a model Hamiltonian matrix for the Figure (a) device. This should include a matrix representation and its eigenvectors and eigenvalues. These may be given in terms of the tunneling amplitude \( S \), but a numerical value of \( S \) in Hz should be used, too. Sketch \( \omega_m \) levels and \( \omega_{\text{proj}} \)-projecting polygon. Discuss its beats. Can it do perfect revivals?

(b) Approximate the Hamiltonian and energy levels perturbed by reducing lower link to \( s=0.9S \) in Figure (b). (Perturbation theory in Sec. 3.2(b) of QTforCA may be helpful.)

(c) Find Hamiltonian and its eigensolutions missing a link as in Figure (c). Give eigenvectors and values in \( n \)-dot linear chain models given in class. (See also p. 13-16 of U(2) Character Geometry or Sec. 14.1 of QTforCA Unit 5.)

6. List all distinct commutative (Abelian) groups in terms of distinct \( C_p \times C_q \times \ldots \) cross products.

(a) for order \( N=8 \). (b) \( N=9 \). (c) \( N=10 \). (d) \( N=11 \). (e) \( N=12 \). (f) \( N=16 \).

7. Consider character tables for Abelian groups using \((m)_p \times (p)_q \times \ldots \) notation where \((m)_n \) means \( m \)-waves-modulo-\( n \).

(a) List character tables of all distinct Abelian groups of order 8.

(b) List character tables of \( C_2 \times C_2 \) using \((m)_2 \times (p)_2 \) notation and \( C_5 \) using \((m)_5 \) notation.

(c) Show these two “six-groups” are not distinct Abelian groups, and relate \((m)_3 \times (p)_3 \) notation to \((m)_6 \) notation.

(d) Using notation \((p)_2 \times (m)_3 \) for \( C_2 \times C_3 \), show how that alters character table ordering.