Photonic Zeno

1. Imagine a series of $N$ polarization beam sorters like the ones in Fig. 1.2.1 or 1.2.3 are placed so the top $x$-output beam of each goes into the next sorter in line which is rotated clockwise by an angle $\phi$ relative to the one before. Suppose unit amplitude $x$-polarization ($\Psi_x = 1$, $\Psi_y = 0$) comes into the first sorter in the series.

(a) What angle $\phi$ makes the amplitude $1/2^N$ coming out of this series? (*Zeno attenuation*)
(b) What angle $\phi$ makes the intensity $1/2^N$ coming out of this series? (*Zeno depletion*)
(c) Suppose the objective is to have as much $y$-polarization as is practical come out of this series. How does the output amplitude and intensity vary with the number $N$?

How many ($N$) sorters are needed to give 99% photon conversion efficiency?

Electronic Zeno

2. Imagine a series of $N$ electron beam sorters like the ones in Fig. 1.1.6 or 1.2.4 are placed so the top $\uparrow$- (up) output beam of each goes into the next sorter in line which is rotated clockwise by an angle $\phi$ relative to the one before. Suppose unit amplitude $\uparrow$- spin ($\Psi_{\uparrow} = 1$, $\Psi_{\downarrow} = 0$) comes into the first sorter in the series.

(a) What angle $\phi$ makes the amplitude $1/2^N$ coming out of this series? (*Zeno attenuation*)
(b) What angle $\phi$ makes the intensity $1/2^N$ coming out of this series? (*Zeno depletion*)
(c) Suppose the objective is to maximize $\downarrow$-spin (down) output from this series. How does the output amplitude and intensity vary with the number $N$?

How many ($N$) sorters are needed to give 99% electron conversion efficiency?

(This is called *adiabatic reversal*.)

3. Effects of a 1/4-wave and a 1/2-wave plate are described in (1.3.1)- (1.3.3) and Fig. 1.3.6 for an input polarization angle of $\theta = 30^\circ$ relative to $x$-axis. Here consider $\theta = 45^\circ$.

(a) Describe effect of a "whole-wave" plate. ($\Omega = \_\_\_\_\_\_\_\?$ Give $\Psi$ and sketch $\text{Re}\Psi_x$ vs. $\text{Re}\Psi_y$ path.)
(b) Describe effect of a "1/3-wave" plate. ($\Omega = \_\_\_\_\_\_\_\_\_\_\?$ Give $\Psi$ and sketch $\text{Re}\Psi_x$ vs. $\text{Re}\Psi_y$ path.)
Polarizer exercise

1.2.1. A y-polarized light beam of unit amplitude (1 photon/sec.) enters the analyzer system as shown below. Fill in the blanks with numbers or symbols that tell as much as possible about what is present at each channel or branch.

\[
\langle x|x' \rangle \langle x|y' \rangle = \begin{array}{c}
\langle y|x' \rangle \\
\langle y'|y \rangle
\end{array}
\]

\[
\langle x'|x \rangle \langle x'|y \rangle = \begin{array}{c}
\langle y'|x \rangle \\
\langle y'|y \rangle
\end{array}
\]

State of output x channel
\[
\begin{array}{c}
\text{Amplitude=} \\
\text{Probability=}
\end{array}
\]

State of output y channel
\[
\begin{array}{c}
\text{Amplitude=} \\
\text{Probability=}
\end{array}
\]

State of x' channel
\[
\begin{array}{c}
\text{Amplitude=} \\
\text{Probability=}
\end{array}
\]

State of y' channel
\[
\begin{array}{c}
\text{Amplitude=} \\
\text{Probability=}
\end{array}
\]

State of input channel
\[
\begin{array}{c}
\text{Amplitude=} \\
\text{Probability=}
\end{array}
\]

A Dim View

1.2.2 (a) How far away from KUAF (10^5 Watts at 91.3 MHz) do you only get 1 photon/m^2s?
(b) How far away from a 10^5 Watt green light source do you only get 1 photon/m^2s? Assume (incorrectly) scalar isotropic coherent wave sources.
Give mks E-field amplitude in each case.
Assignment 1 - Solutions 1/17/13 Due Thur. Jan 24

Photonic Zeno Solutions

1. Imagine a series of \( N \) polarization beam sorters like the ones in Fig. 1.2.1 or 1.2.3

\( N = 1 \) amp. \( \langle x' | x \rangle = \cos \theta \) so \( N \)-transition amp: \( \langle x' | x \rangle^N = \cos^N \theta \)  

(a) What \( \theta \) makes the amp \( 1/2^N \) amp: \( \langle x' | x \rangle^N = \cos^N \theta \) \( \Rightarrow \cos^N \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3} = 60^\circ \) (Zeno attenuation)

(b) What \( \theta \) makes the intensity \( 1/2^N \)?  

\( \langle x' | x \rangle^N = \cos^N \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3} = 60^\circ \) (Zeno depletion)

(c) Supposing need \( y \)-polarization (angle \( \theta = \frac{\pi}{2} \) ) coming out of this series. .output amplitude  

\( \langle x' | x \rangle^N = \cos^N \theta = \cos^N \frac{\pi}{2N} \) and intensity \( I(N) = \langle x' | x \rangle^{2N} = \cos^{2N} \frac{\pi}{2N} \) varies with \( N \). \( I(2) = 0.25, I(4) = 0.53 \).  

How many \( (N) \) needed for 99% photon conversion? \( I(245) = 0.98997, I(246) = 0.99002 \).

Electronic Zeno

2. Imagine a series of \( N \) electron beam sorters like the ones in Fig. 1.1.6 or 1.2.4 are placed so the top \( \uparrow \)- (up) output beam

\( N = 1 \) amp. \( \langle \uparrow | \uparrow \rangle = \cos \frac{\phi}{2} \) so \( N \)-transition amp: \( \langle \uparrow | \uparrow \rangle^N = \cos^N \frac{\phi}{2} \)  

(a) What angle \( \phi \) has amp \( 1/2^N \)?  

amp: \( \langle \uparrow | \uparrow \rangle^N = \cos^N \frac{\phi}{2} = \frac{1}{2} \Rightarrow \frac{\phi}{2} = \frac{\pi}{3} = 60^\circ \) (Zeno attenuation)

(b) What angle \( \phi \) intensity \( 1/2^N \)?  

intensity: \( \langle \uparrow | \uparrow \rangle^N = \cos^N \frac{\phi}{2} = \frac{1}{2} \Rightarrow \frac{\phi}{2} = \frac{\pi}{4} = 45^\circ \) (Zeno depletion)

(c) Suppose need \( \downarrow \)-spin (down \( \phi = \pi \) ) output amplitude amp: \( \langle \uparrow | \uparrow \rangle^N = \cos^N \frac{\phi}{2} = \cos^N \frac{\pi}{2N} \) and  

intensity amp: \( \langle \uparrow | \uparrow \rangle^{2N} = \cos^{2N} \frac{\phi}{2} = \cos^{2N} \frac{\pi}{2N} \) varies with the number \( N \) in same way as Photonic case.

How many \( (N) \) sorters are needed to give 99% electron conversion efficiency?... same \( I(246) = 0.99 \) as Photonic case.  

(This is called adiabatic reversal.)

3. Effects of a 1/4-wave and a 1/2-wave plate are described in (1.3.1)-(1.3.3) and Fig. 1.3.6 for an input polarization angle of \( \theta = 30^\circ \) relative to \( x \)-axis. Here consider \( \theta = 45^\circ \).

(a) Describe effect of a "whole-wave" plate. (\( \Omega = 2\pi \)? Give \( \Psi \) and sketch \( \Re \Psi \) vs. \( \Re \Psi \) path.) "1/4-wave" is \( \Omega = 2\pi/4 \).

(b) Describe effect of a "1/3-wave" plate. \( \Omega = 2\pi/3 \) ? Give \( \Psi \) and sketch \( \Re \Psi \) vs. \( \Re \Psi \) path. "1/2-wave" is \( \Omega = 2\pi/2 \).
Solutions

Polarizer exercise

1.2.1. A y-polarized light beam of unit amplitude (1 photon/sec.) enters the analyzer system as shown below. Fill in the blanks with numbers or symbols that tell what is present at each channel or branch.

1.2.2 (a) How far away from a 10³ Watts at ν_{KUAF} = 91.3 MHz do you only get 1 photon/m²s?
(b) How far away from a 10⁶ Watt green light source do you only get 1 photon/m²s? Assume (incorrectly) scalar isotropic coherent wave sources. (c) Give mks E-field amplitude in each case.

(a) Total flux intensity \( S \) at radius \( r \) multiplied by area \( 4\pi r^2 \) of \( r \)-sphere around source is its total output \( O \) per second.

\[
4\pi r^2 (m^2) S (J/s/m^2) = 10^3 \text{Watt} = 10^3 J/s = 4\pi r^2 (n/s) h\nu J
\]

with: \( h = 6.63 \times 10^{-34} (J \cdot s) \) and: \( \nu_{KUAF} = 9.13 \times 10^8 \text{ (Hz or s}^{-1})\)

Let radius \( r_n \) have photon flux intensity of \( n \) photons per m² per sec. (n counts per unit area in each 1-sec. interval):

\[
r_n(m) = \sqrt{\frac{10^3 J/s}{4\pi (n/s/m^2) h\nu J}} = \sqrt{\frac{10^3 J m^2/s}{4\pi (n/s) 6.63 \times 10^{-34} (J \cdot s) h\nu J}} = \left( \frac{3.626 \times 10^{14} n^{-1/2}(m)}{ \nu = 9.13 \times 10^8 (s}^{-1}) \right)
\]

Total KUAF photon number output in 1 sec. is \( N = 10^3 J / h\nu = 1.65 \times 10^{30} / s \), the total photon population in sphere of radius \( c \).

Total \( N \) and total flux of \( 10^3 \) Watts are fixed. Flux intensity \( S(J \text{ per m}^2 \text{ per sec}) \) and photon counts \( n \) per m² per sec are also fixed in time but vary inversely with \( r^2 \) as does the square of electric E-field. \( S = Uc = \varepsilon \varepsilon_0 c = 10^3 / 4\pi r^2 = nh\nu \).

\[
|E| = \sqrt{\frac{nh\nu}{\varepsilon_0 c}} = \sqrt{\frac{n \cdot 6.63 \times 10^{-34} Js 91.3 \times 10^8 s} {8.842 \times 10^{-12} \varepsilon_0 \frac{C^2}{N m^2}}} = \sqrt{n} \frac{4.7 \times 10^{-12} N}{C}
\]

Setting \( n = 1 \) gives tiny E-fields. (~5 Volts per \( 10^{12} \) m.)

Average mks E-field amplitude is

\[
|E| = f_n = \frac{N\nu h}{\varepsilon_0 c} = \frac{10^3 \ J}{4\pi \ c^2 \ v_0} = \left( \frac{10^3 N \text{m/s}}{4\pi \ (3 \times 10^8 m)^2 \ 8.842 \times 10^{-12} \ v_0 ^2 \ C^2} \right) = 9.99 \times 10^8 \ \frac{N}{C} \sim 10^{-9} \ \text{Volt/m}
\]