

Group Theory in Quantum Mechanics

Lecture 3 (1.20.15)

Analyzers, operators, and group axioms

(Quantum Theory for Computer Age - Ch. 1-2 of Unit 1)

(Principles of Symmetry, Dynamics, and Spectroscopy - Sec. 1-3 of Ch. 1)

Review: Axioms 1-4 and “Do-Nothing” vs “ Do-Something” analyzers

Abstraction of Axiom-4 to define projection and unitary operators

Projection operators and resolution of identity

Unitary operators and matrices that do something (or “nothing”)

Diagonal unitary operators

Non-diagonal unitary operators and †-conjugation relations

Non-diagonal projection operators and Kronecker \otimes -products

Axiom-4 similarity transformation

Matrix representation of beam analyzers

Non-unitary “killer” devices: Sorter-counter, filter

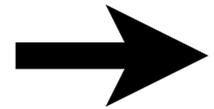
Unitary “non-killer” devices: 1/2-wave plate, 1/4-wave plate

How analyzers “peek” and how that changes outcomes

Peeking polarizers and coherence loss

Classical Bayesian probability vs. Quantum probability

*Feynman $\langle j|k\rangle$ -axioms compared to **Group axioms***



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Feynman amplitude axioms 1-4

Feynman-Dirac
Interpretation of

$$\langle j | k' \rangle$$

= Amplitude of state- j after
state- k' is forced to choose
from available m -type states

(1) The probability axiom

The first axiom deals with physical interpretation of amplitudes $\langle j | k' \rangle$.

Axiom 1: The absolute square $|\langle j | k' \rangle|^2 = \langle j | k' \rangle^ \langle j | k' \rangle$ gives probability for occurrence in state- j of a system that started in state- $k'=1',2',\dots,$ or n' from one sorter and then was forced to choose between states $j=1,2,\dots,n$ by another sorter.*

(2) The conjugation or inversion axiom (time reversal symmetry)

The second axiom concerns going backwards through a sorter or the reversal of amplitudes.

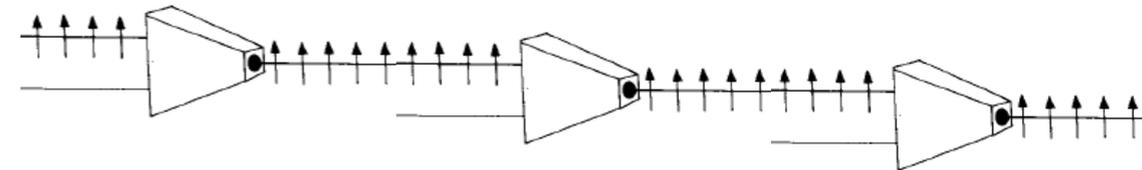
Axiom 2: The complex conjugate $\langle j | k' \rangle^$ of an amplitude $\langle j | k' \rangle$ equals its reverse: $\langle j | k' \rangle^* = \langle k' | j \rangle$*

(3) The orthonormality or identity axiom

The third axiom concerns the amplitude for "re measurement" by the same analyzer.

Axiom 3: If identical analyzers are used twice or more the amplitude for a passed state- k is one, and for all others it is zero:

$$\langle j | k \rangle = \delta_{jk} = \begin{cases} 1 \text{ if: } j=k \\ 0 \text{ if: } j \neq k \end{cases} = \langle j' | k' \rangle$$



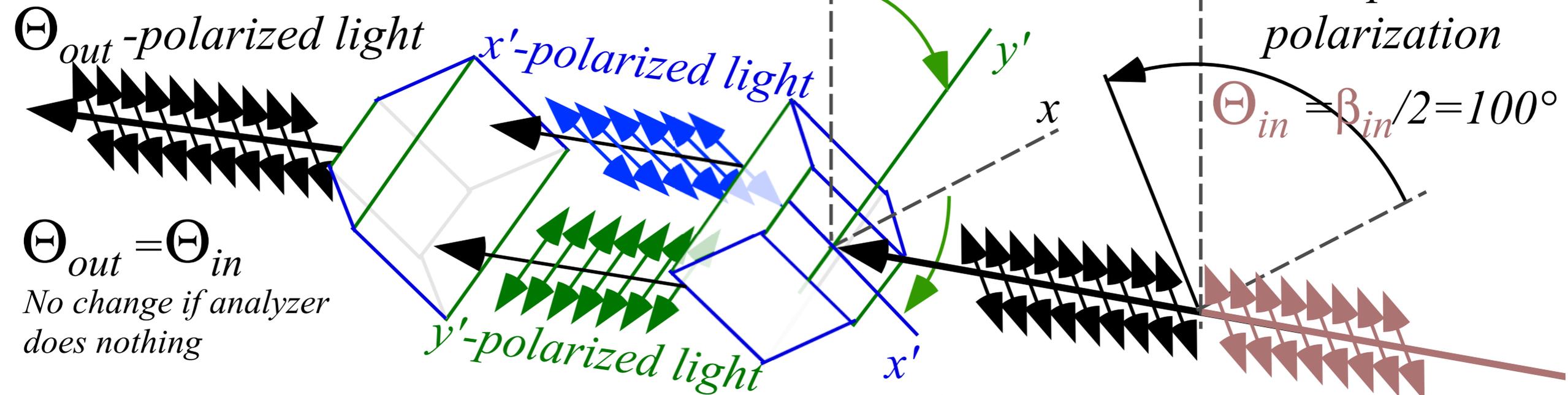
(4) The completeness or closure axiom

The fourth axiom concerns the "Do-nothing" property of an ideal analyzer, that is, a sorter followed by an "unsorter" or "put-back-togetherer" as sketched above.

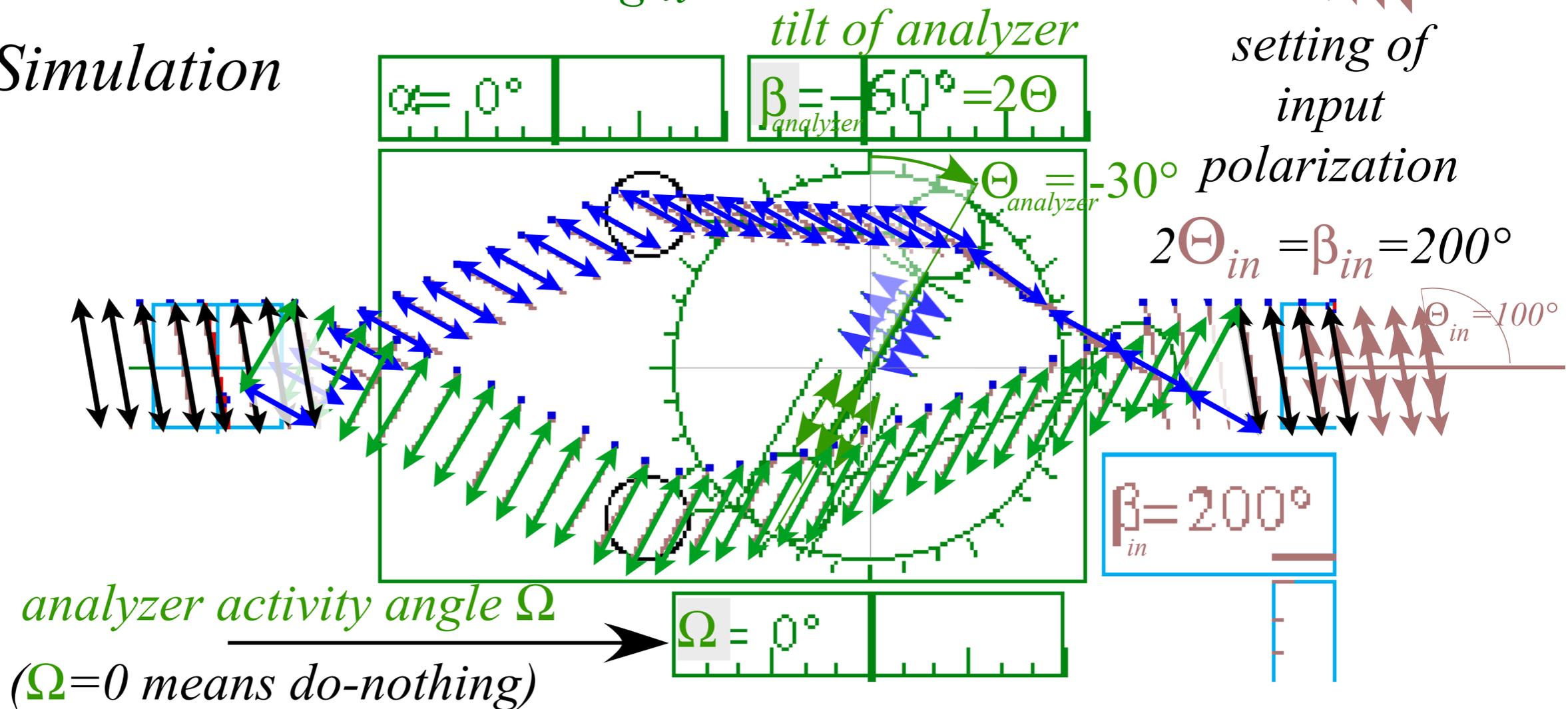
Axiom 4. Ideal sorting followed by ideal recombination of amplitudes has no effect:

$$\langle j'' | m' \rangle = \sum_{k=1}^n \langle j'' | k \rangle \langle k | m' \rangle$$

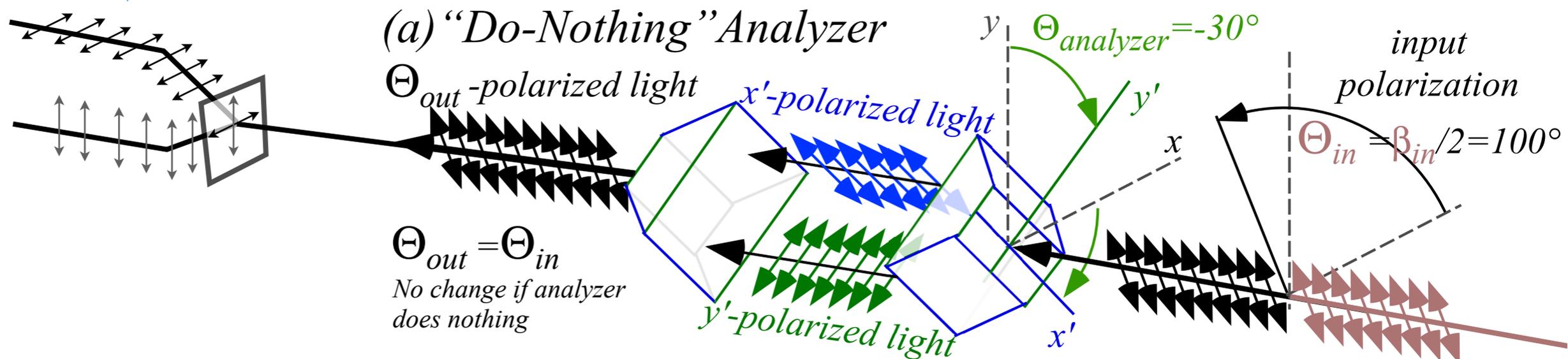
(a) "Do-Nothing" Analyzer



(b) Simulation



Imagine final xy -sorter analyzes output beam into x and y -components.



Amplitude in x or y -channel is sum over x' and y' -amplitudes

$$\langle x' | \Theta_{in} \rangle = \cos(\Theta_{in} - \Theta)$$

$$\langle y' | \Theta_{in} \rangle = \sin(\Theta_{in} - \Theta)$$

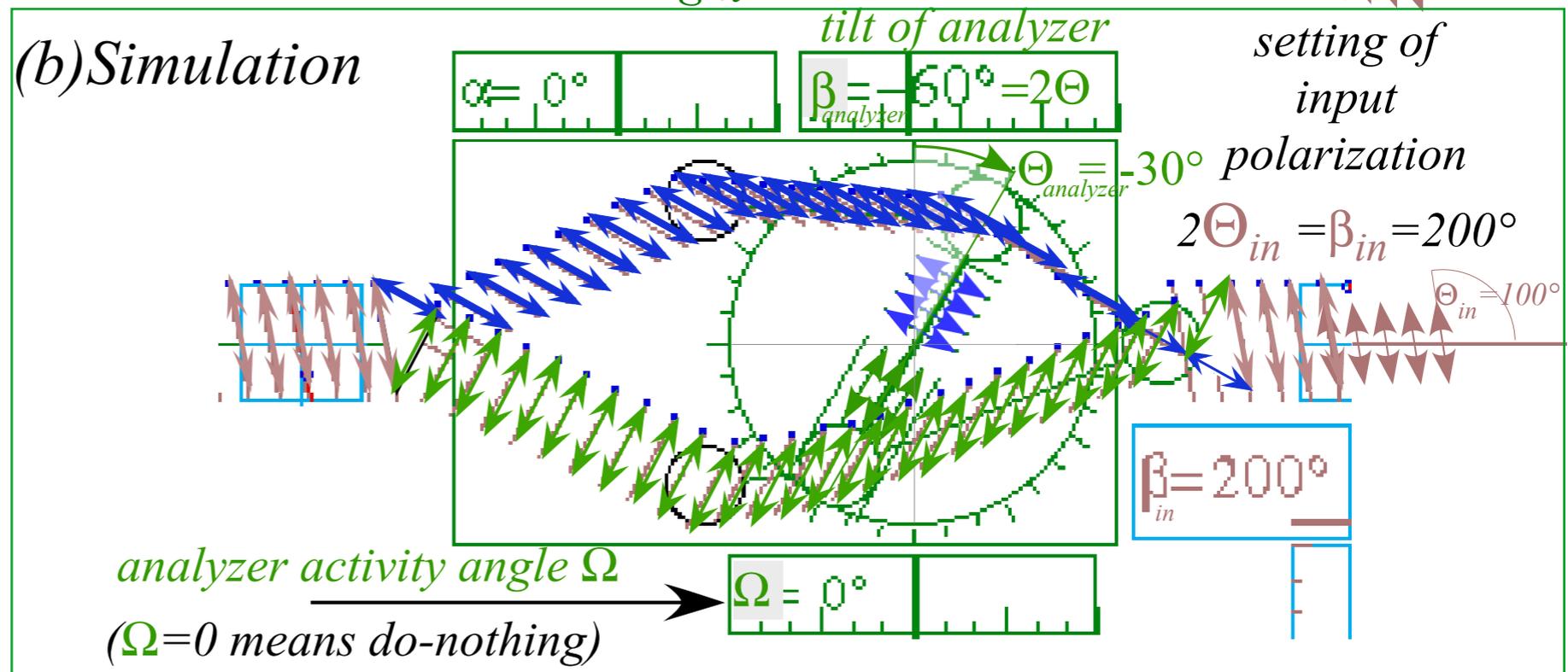
with relative angle $\Theta_{in} - \Theta$

of Θ_{in} to Θ -analyzer axes- (x', y')

in products with final xy -sorter:

$$\text{lab } x\text{-axis: } \langle x | x' \rangle = \cos \Theta = \langle y | y' \rangle$$

$$y\text{-axis: } \langle y | x' \rangle = \sin \Theta = -\langle x | y' \rangle.$$



$$x\text{-Output is: } \langle x | \Theta_{out} \rangle = \langle x | x' \rangle \langle x' | \Theta_{in} \rangle + \langle x | y' \rangle \langle y' | \Theta_{in} \rangle = \cos \Theta \cos(\Theta_{in} - \Theta) - \sin \Theta \sin(\Theta_{in} - \Theta) = \cos \Theta_{in}$$

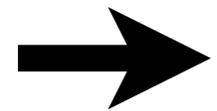
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(Recall $\cos(a+b) = \cos a \cos b - \sin a \sin b$ and $\sin(a+b) = \sin a \cos b + \cos a \sin b$)

Conclusion:

$$\langle x | \Theta_{out} \rangle = \cos \Theta_{out} = \cos \Theta_{in} \text{ or: } \Theta_{out} = \Theta_{in} \text{ so "Do-Nothing" Analyzer in fact does nothing.}$$

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Recall bra-ket
Transformation Matrix

$$T_{m,n'} = \langle m | n' \rangle$$
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Resolution of Identity into Projectors $\{|1\rangle\langle 1|, |2\rangle\langle 2|, \dots\}$ or $\{|1'\rangle\langle 1'|, |2'\rangle\langle 2'|, \dots\}$ or $\{|1''\rangle\langle 1''|, |2''\rangle\langle 2''|, \dots\}$

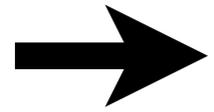
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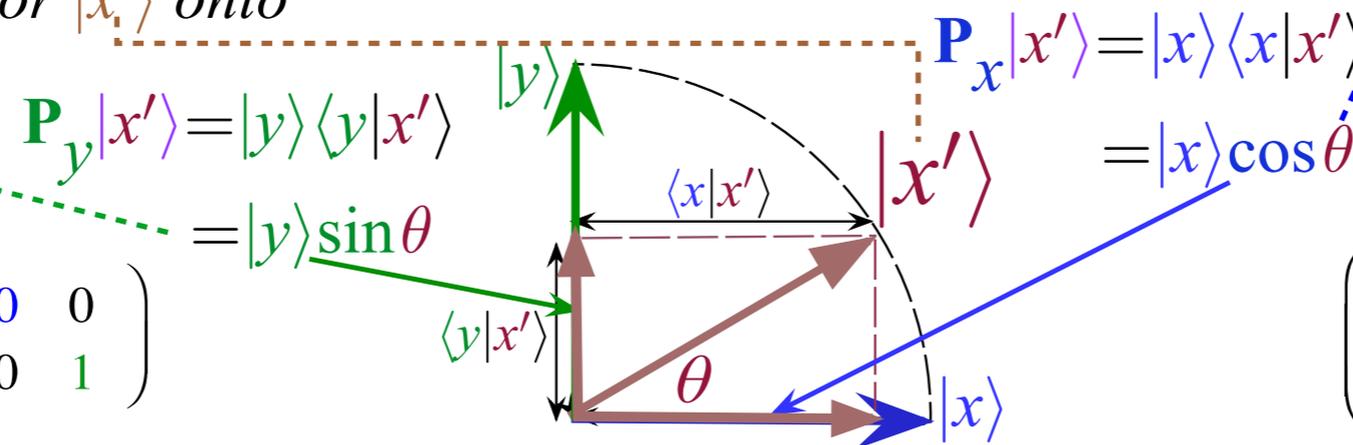
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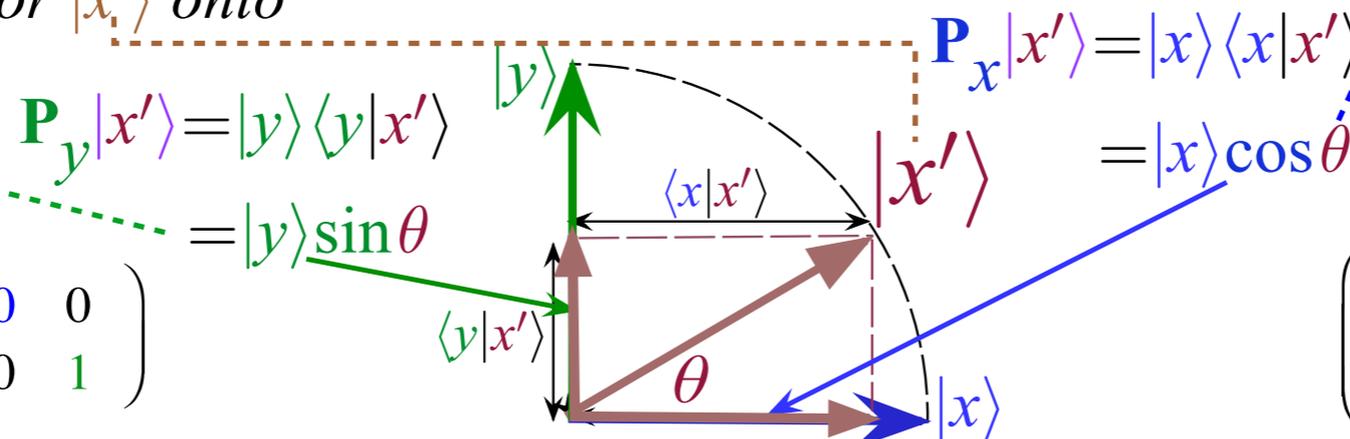
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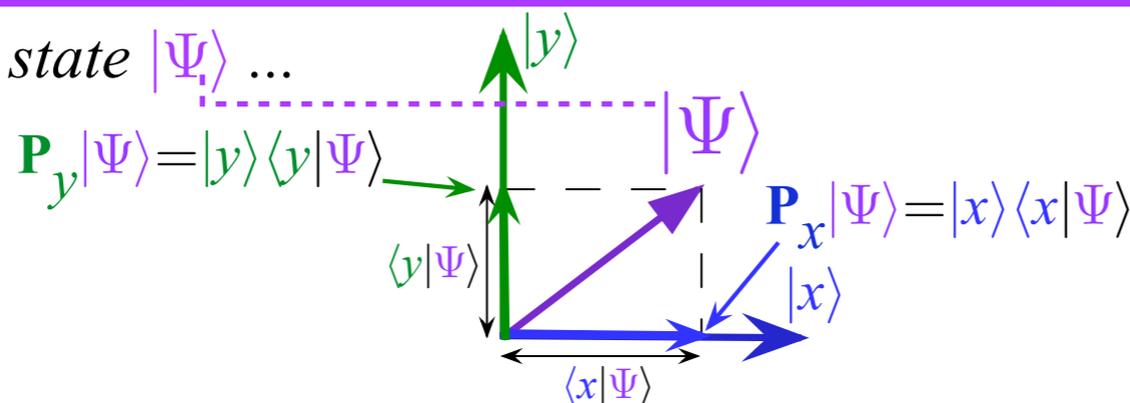
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Projections of general state $|\Psi\rangle$...



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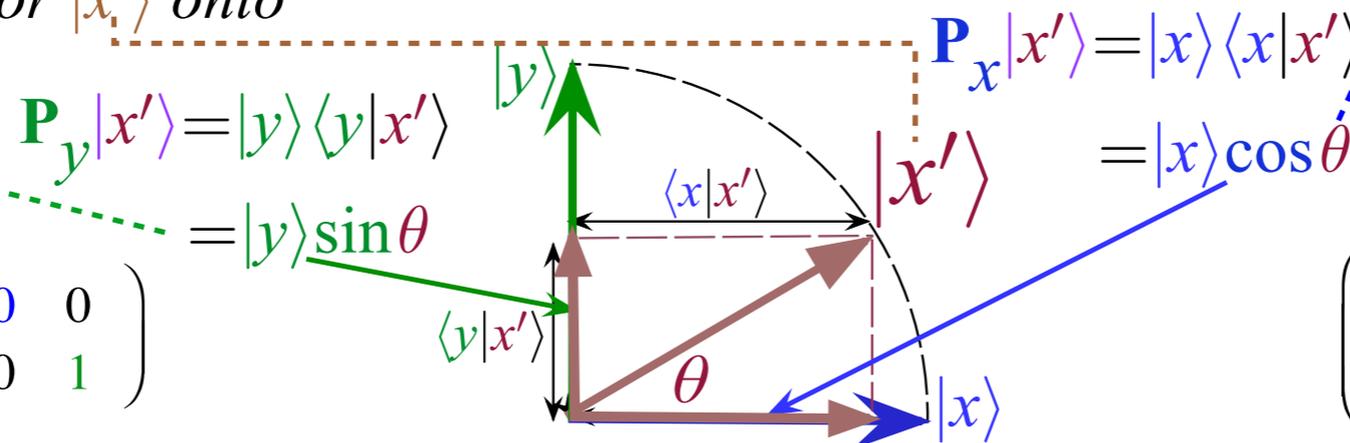
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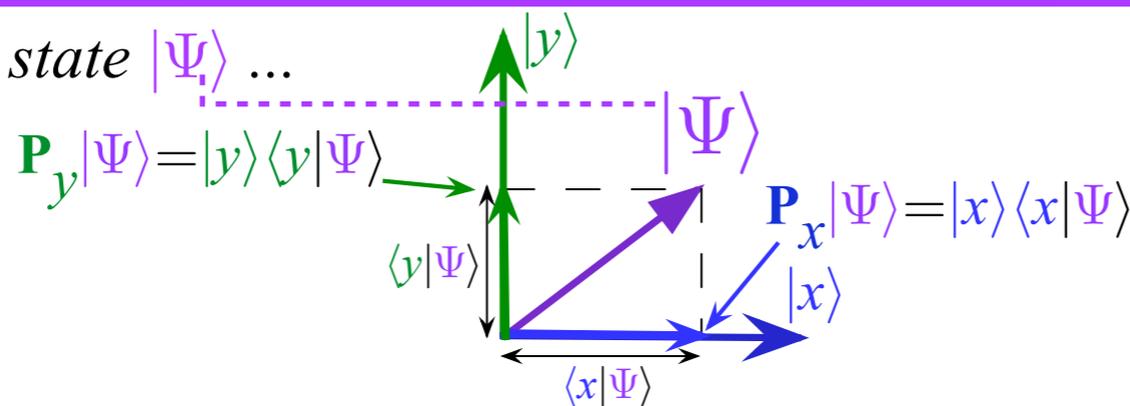
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...must add up to $|\Psi\rangle$

$$\mathbf{P}_x |\Psi\rangle + \mathbf{P}_y |\Psi\rangle = |\Psi\rangle$$

$$(\mathbf{P}_x + \mathbf{P}_y) |\Psi\rangle = |\Psi\rangle$$



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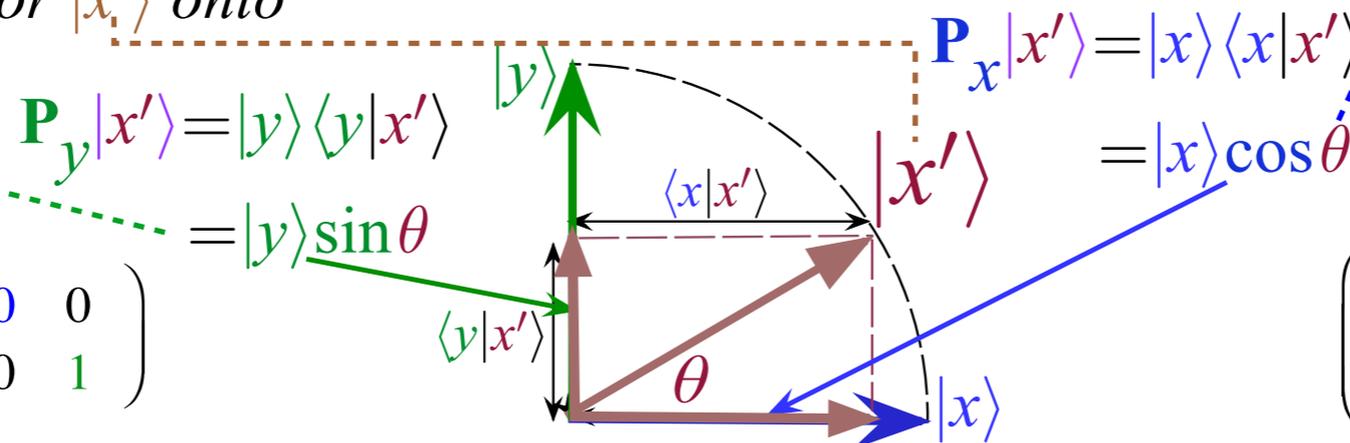
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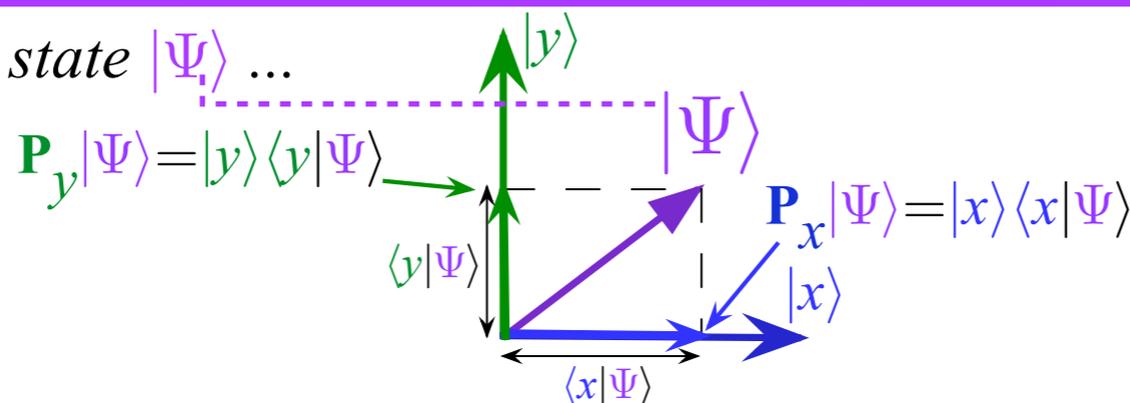
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...and so \mathbf{P}_m projectors must add up to identity operator...

$$\mathbf{1} = \mathbf{P}_x + \mathbf{P}_y$$

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Axiom 4: $\langle j'' | m' \rangle = \sum_{k=1}^n \langle j'' | k \rangle \langle k | m' \rangle$ may be "abstracted" three different ways

Left abstraction gives bra-transform:

$$\langle j'' | = \sum_{k=1}^n \langle j'' | k \rangle \langle k |$$

Right abstraction gives ket-transform:

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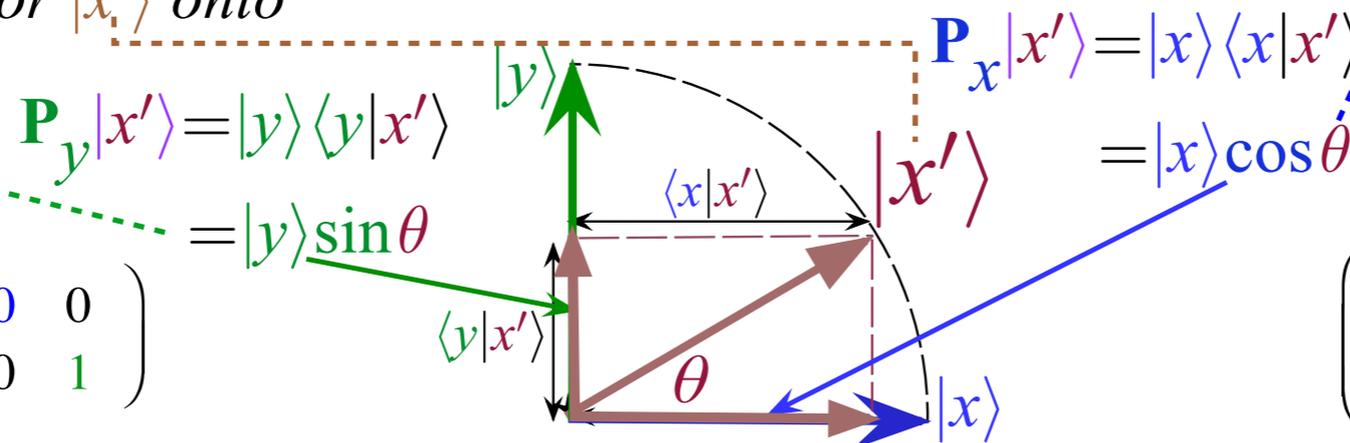
Center abstraction gives ket-bra identity operator:

$$\mathbf{1} = \sum_{k=1}^n | k \rangle \langle k | = \sum_{k=1}^n | k' \rangle \langle k' | = \sum_{k=1}^n | k'' \rangle \langle k'' | = \dots$$

Resolution of Identity into Projectors $\{|1\rangle\langle 1|, |2\rangle\langle 2|, \dots\}$ or $\{|1'\rangle\langle 1'|, |2'\rangle\langle 2'|, \dots\}$ or $\{|1''\rangle\langle 1''|, |2''\rangle\langle 2''|, \dots\}$

$\mathbf{P}_1 = |1\rangle\langle 1|, \mathbf{P}_2 = |2\rangle\langle 2|, \dots$ or $\mathbf{P}_{1'} = |1'\rangle\langle 1'|, \mathbf{P}_{2'} = |2'\rangle\langle 2'|$ etc. Recall bra-ket

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Transformation Matrix

$$T_{m,n'} = \langle m | n' \rangle$$

$$\begin{pmatrix} \langle x | x' \rangle & \langle x | y' \rangle \\ \langle y | x' \rangle & \langle y | y' \rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

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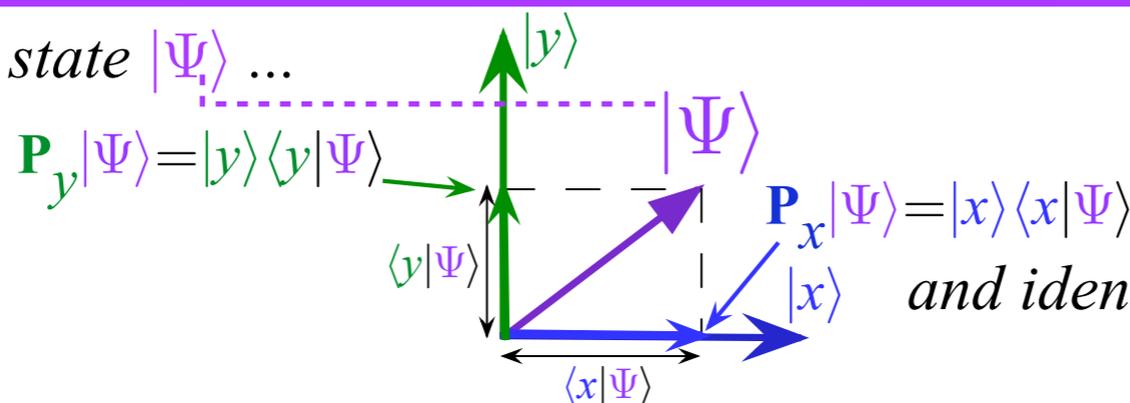
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Projections of general state $|\Psi\rangle$...

...must add up to $|\Psi\rangle$

$$\mathbf{P}_x |\Psi\rangle + \mathbf{P}_y |\Psi\rangle = |\Psi\rangle$$

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...and so \mathbf{P}_m projectors

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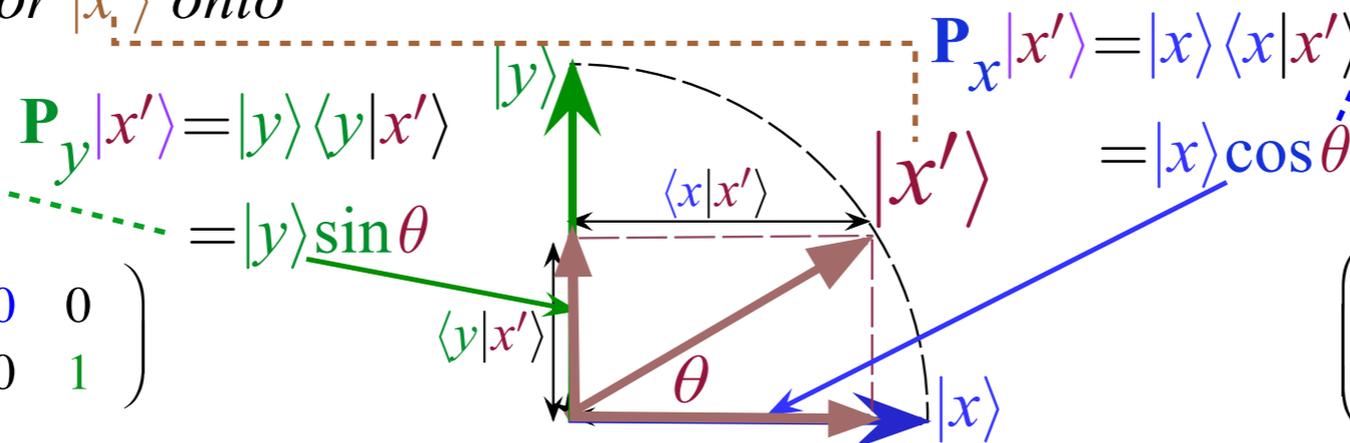
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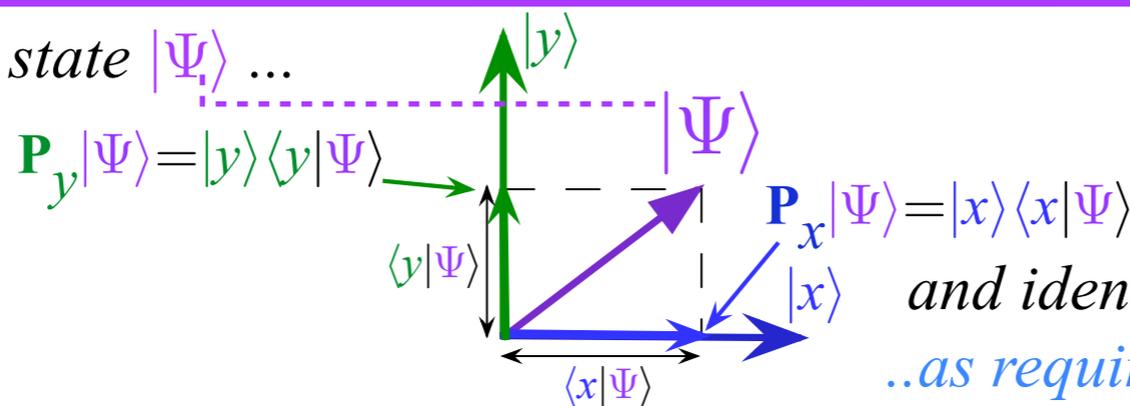
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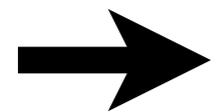
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and identity matrix... ..as required by Axiom 4:

Review: Axioms 1-4 and “Do-Nothing” vs “Do-Something” analyzers

*Abstraction of Axiom-4 to define projection and unitary operators
Projection operators and resolution of identity*



*Unitary operators and matrices that do something (or “nothing”)
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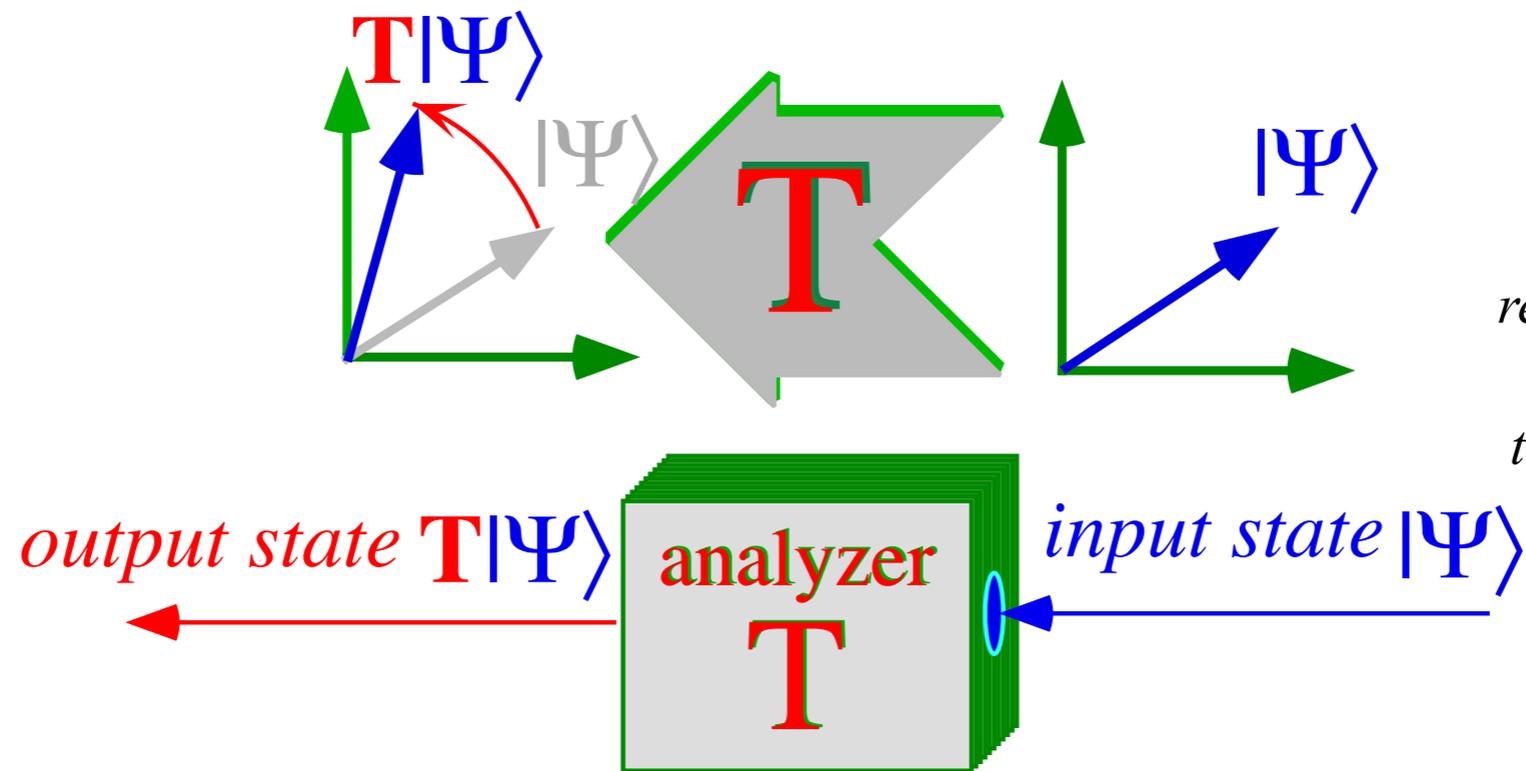


Fig. 3.1.1 Effect of analyzer represented by ket vector transformation of $|\Psi\rangle$ to new ket vector $T|\Psi\rangle$.

Unitary operators and matrices that do something (or “nothing”)

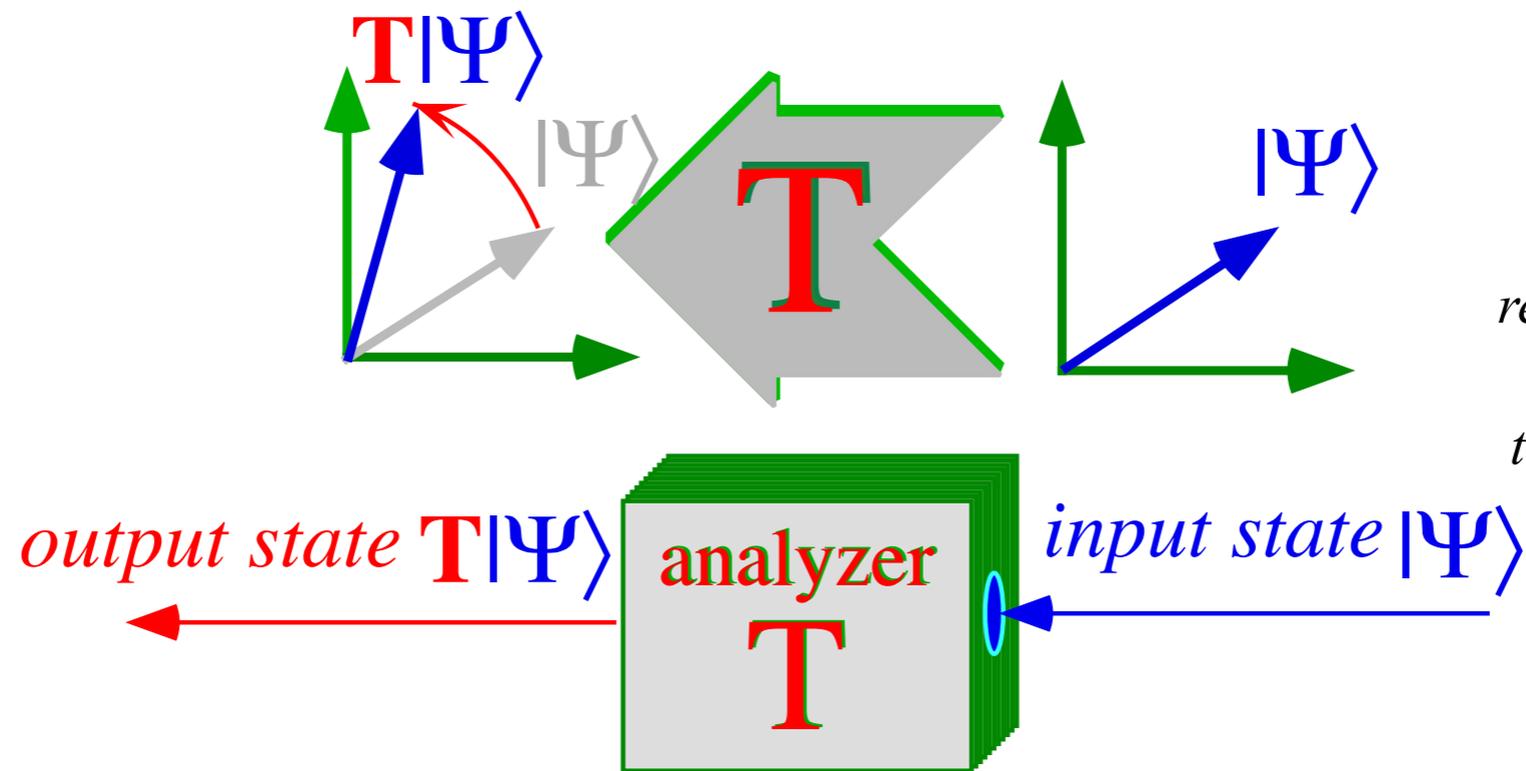


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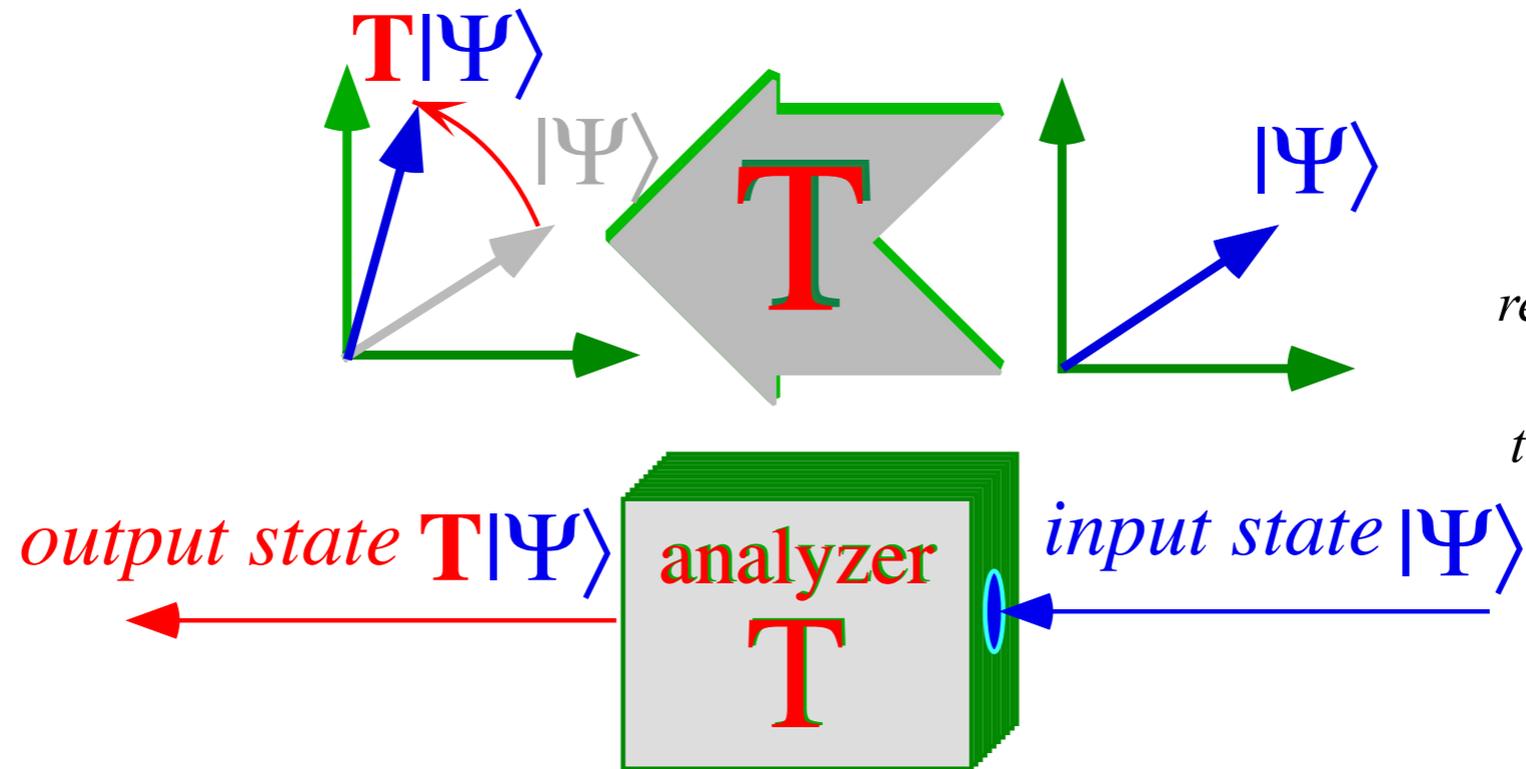


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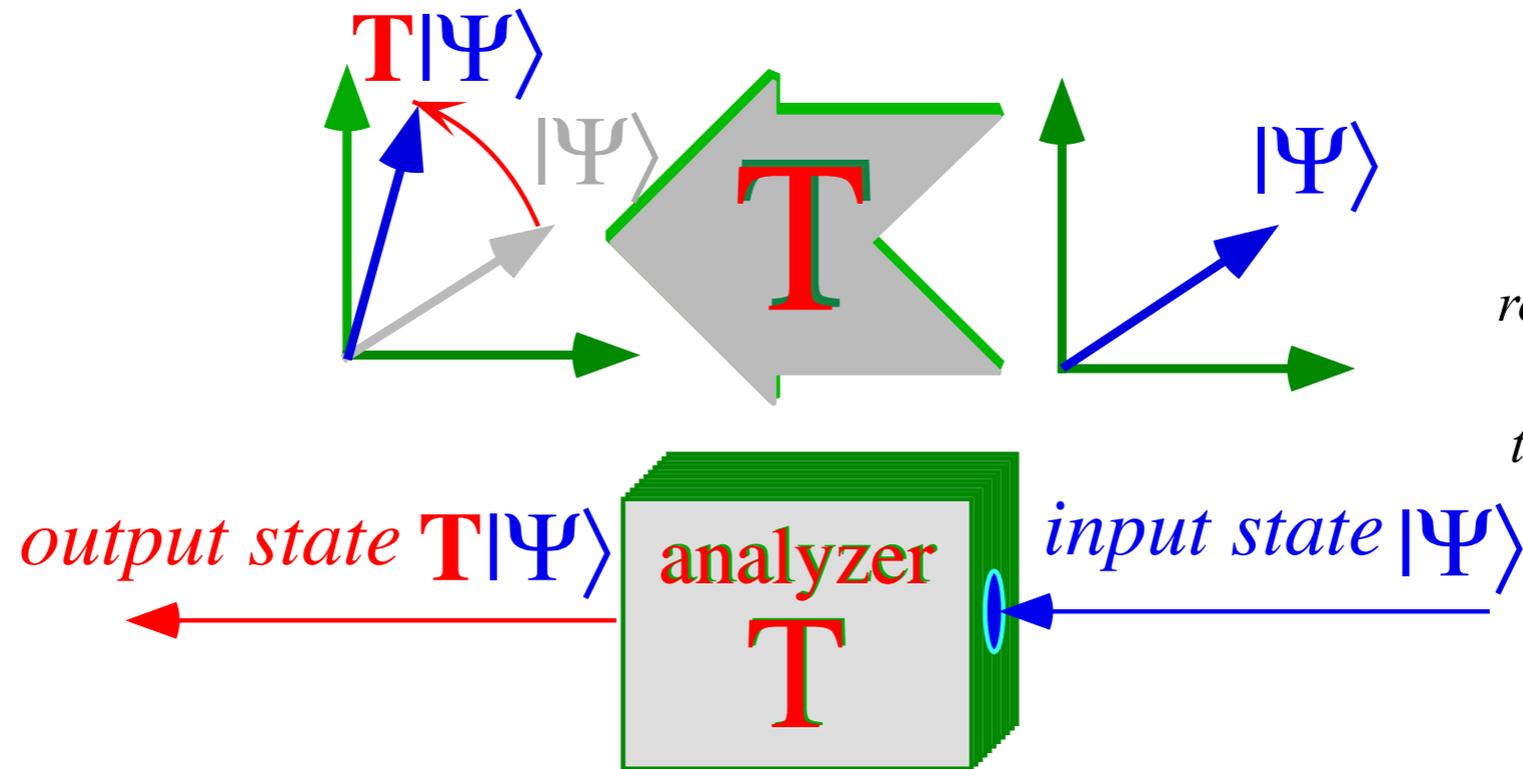


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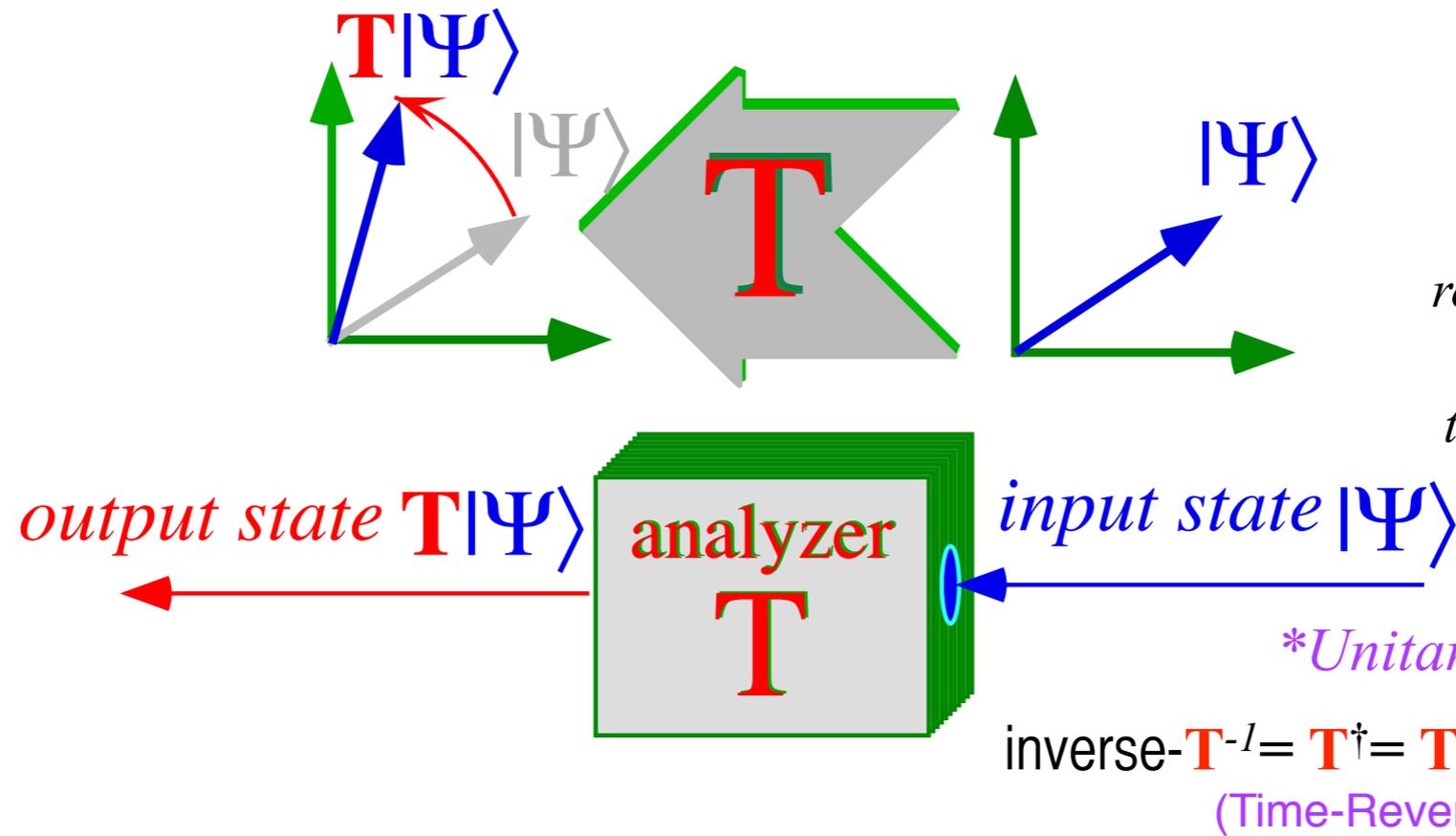


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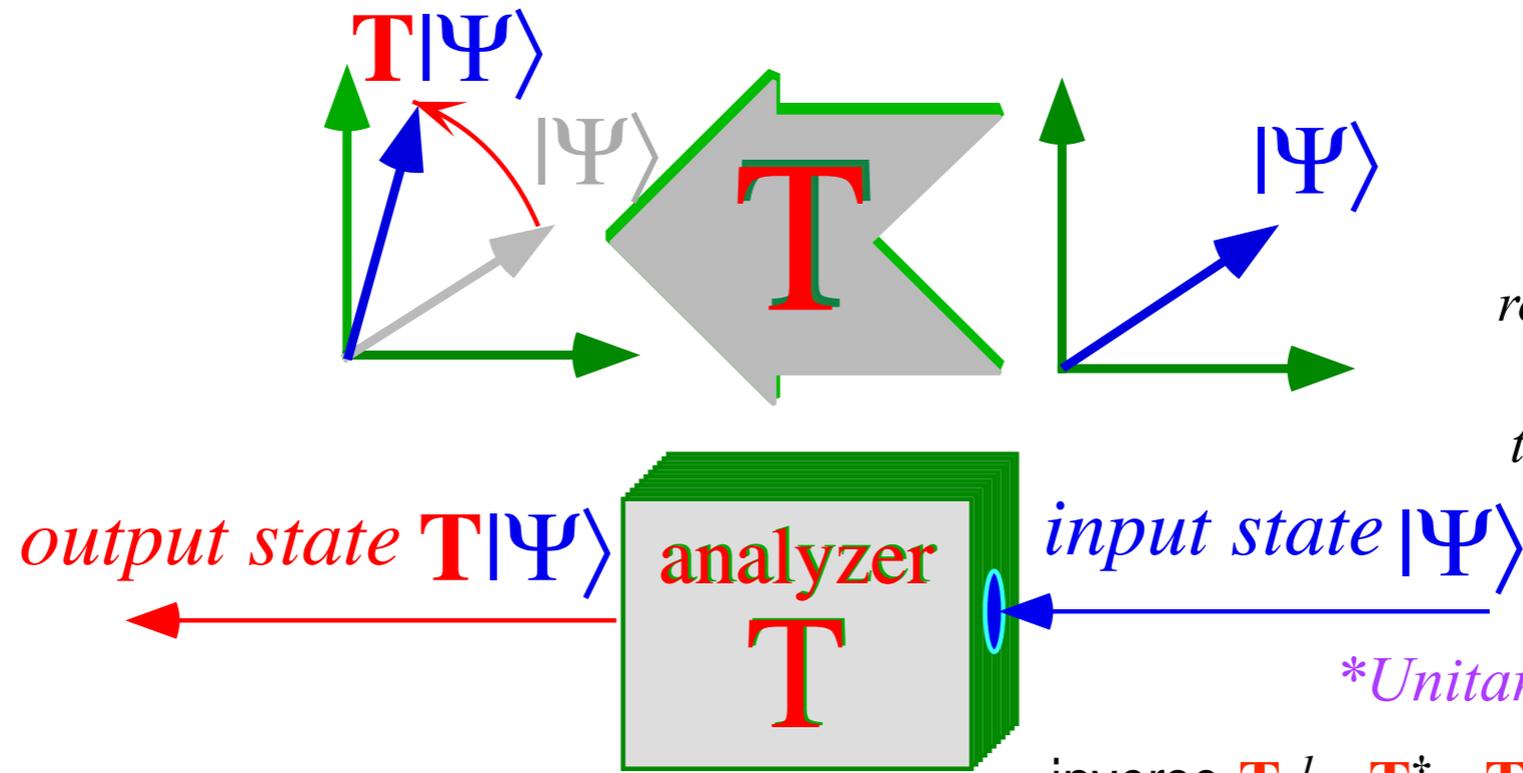


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Most “do-something” operators \mathbf{T}' are not diagonal, that is, not just $|x\rangle\langle x|$ and $|y\rangle\langle y|$ combinations.

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Unitary operators and matrices that do something (or “nothing”)

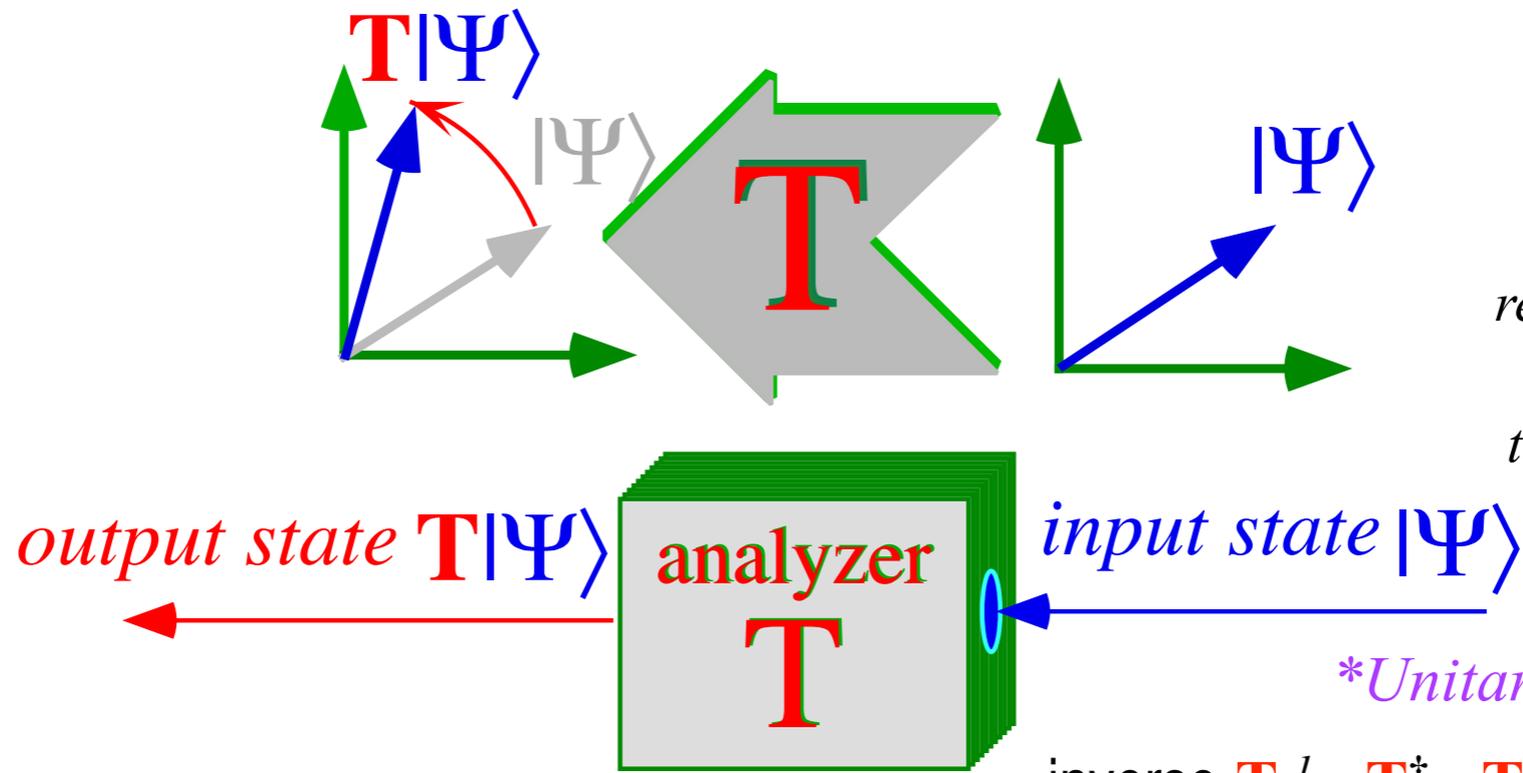


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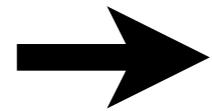
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(Matrix representation of \mathbf{T}' is a little more complicated. See following pages.)

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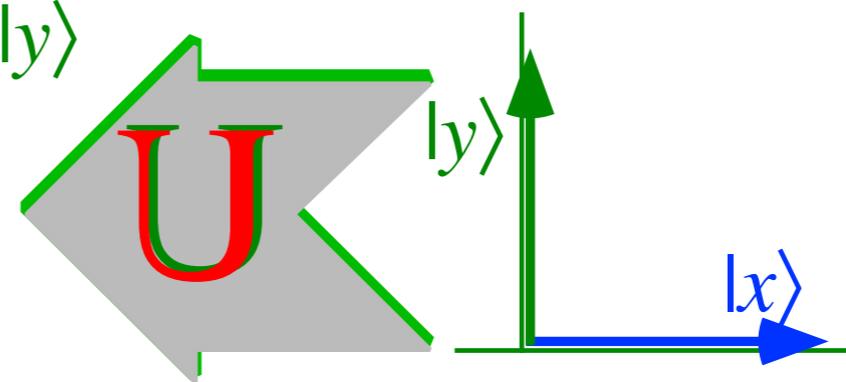
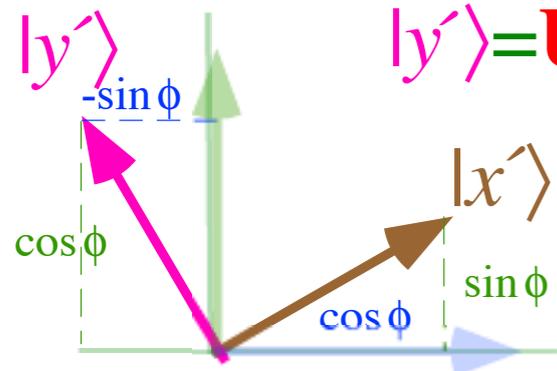
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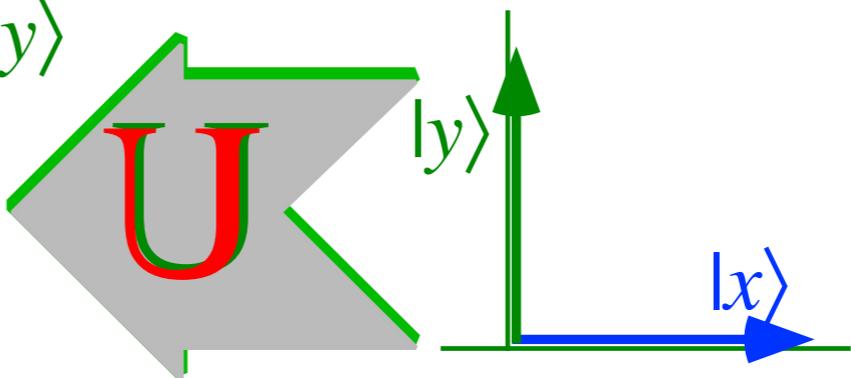
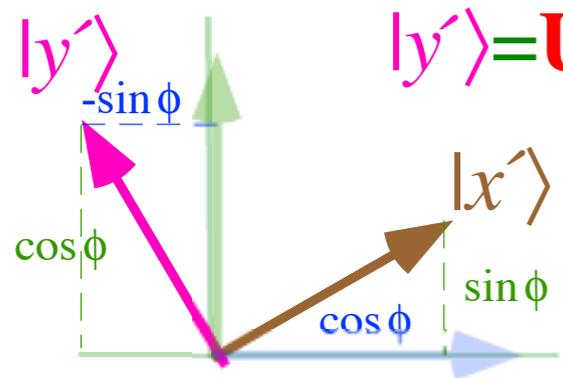
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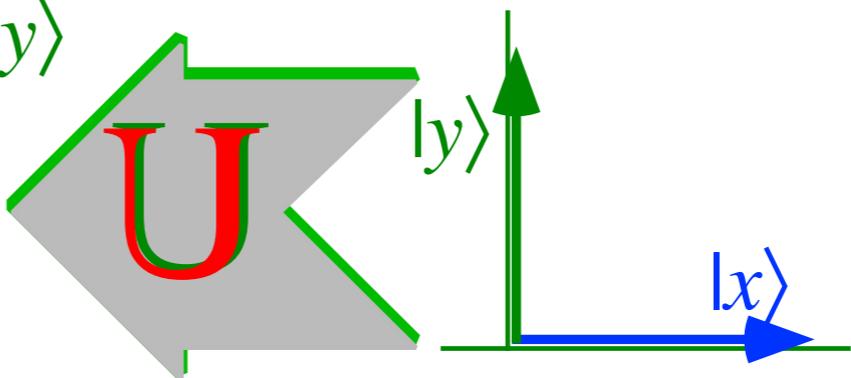
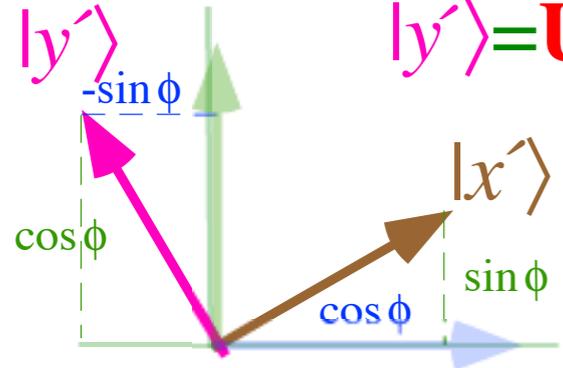
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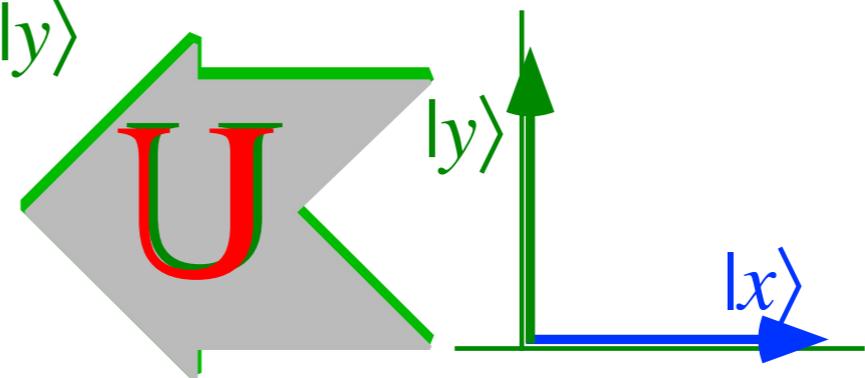
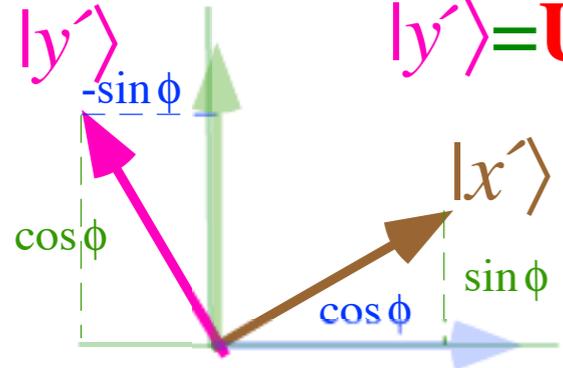
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Example \mathbf{U} transformation:

$$|x'\rangle = \mathbf{U}|x\rangle = \cos\phi |x\rangle + \sin\phi |y\rangle$$

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Ket definition: $|x'\rangle = \mathbf{U}|x\rangle$ implies: $\mathbf{U}^\dagger|x'\rangle = |x\rangle$ implies: $\langle x| = \langle x'|\mathbf{U}$ implies: $\langle x|\mathbf{U}^\dagger = \langle x'|$

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...implies matrix representation of operator \mathbf{U}

$$\begin{pmatrix} \langle x|\mathbf{U}|x\rangle & \langle x|\mathbf{U}|y\rangle \\ \langle y|\mathbf{U}|x\rangle & \langle y|\mathbf{U}|y\rangle \end{pmatrix} = \begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$$

Unitary operators U satisfy “easy inversion” relations: $U^{-1} = U^\dagger = U^{T*}$

They are “designed” to conserve *probability* and *overlap*

so each transformed ket $|\Psi'\rangle = U|\Psi\rangle$ has the same *probability* $\langle\Psi|\Psi\rangle = \langle\Psi'|\Psi'\rangle = \langle\Psi|U^\dagger U|\Psi\rangle$

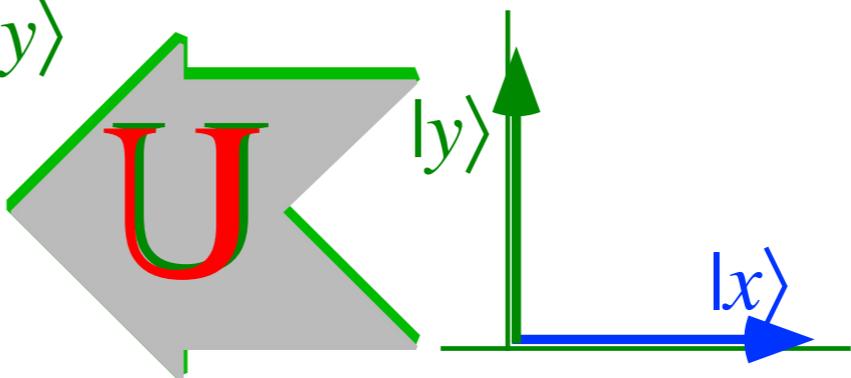
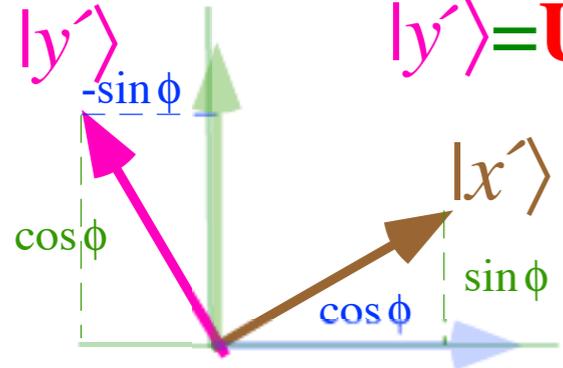
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Example U transformation: (Rotation by $\phi = 30^\circ$)

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Ket definition: $|x'\rangle = U|x\rangle$ implies: $U^\dagger|x'\rangle = |x\rangle$ implies: $\langle x| = \langle x'|U$ implies: $\langle x|U^\dagger = \langle x'|$

Ket definition: $|y'\rangle = U|y\rangle$ implies: $U^\dagger|y'\rangle = |y\rangle$ implies: $\langle y| = \langle y'|U$ implies: $\langle y|U^\dagger = \langle y'|$

...implies matrix representation of operator U in either of the bases it connects is *exactly the same*.

$$\begin{pmatrix} \langle x|U|x\rangle & \langle x|U|y\rangle \\ \langle y|U|x\rangle & \langle y|U|y\rangle \end{pmatrix} = \begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} = \begin{pmatrix} \langle x'|U|x'\rangle & \langle x'|U|y'\rangle \\ \langle y'|U|x'\rangle & \langle y'|U|y'\rangle \end{pmatrix}$$

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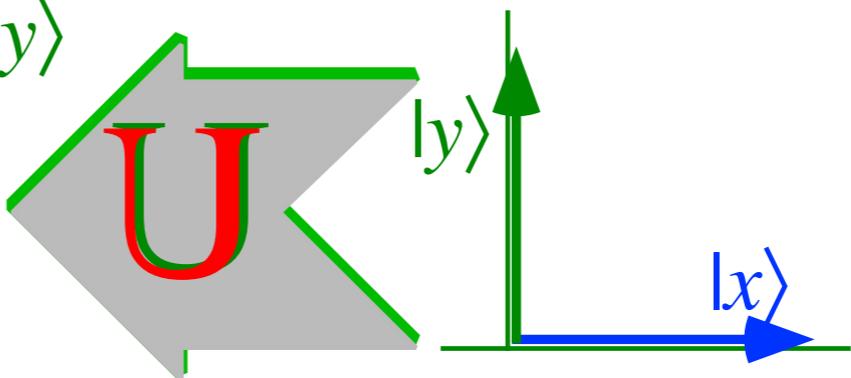
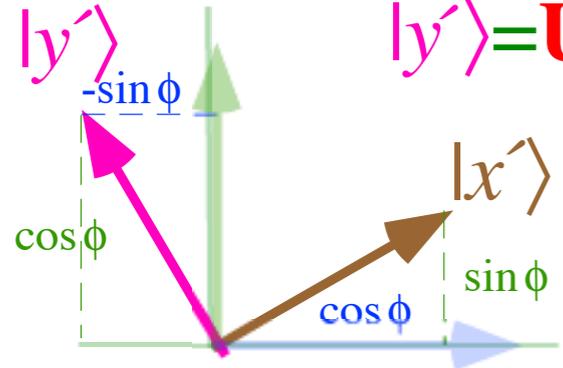
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So also is the inverse \mathbf{U}^\dagger

$$\begin{pmatrix} \langle x|\mathbf{U}|x\rangle & \langle x|\mathbf{U}|y\rangle \\ \langle y|\mathbf{U}|x\rangle & \langle y|\mathbf{U}|y\rangle \end{pmatrix} = \begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} = \begin{pmatrix} \langle x'|\mathbf{U}|x'\rangle & \langle x'|\mathbf{U}|y'\rangle \\ \langle y'|\mathbf{U}|x'\rangle & \langle y'|\mathbf{U}|y'\rangle \end{pmatrix}$$

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Unitary operators \mathbf{U} satisfy “easy inversion” relations: $\mathbf{U}^{-1} = \mathbf{U}^\dagger = \mathbf{U}^{T*}$

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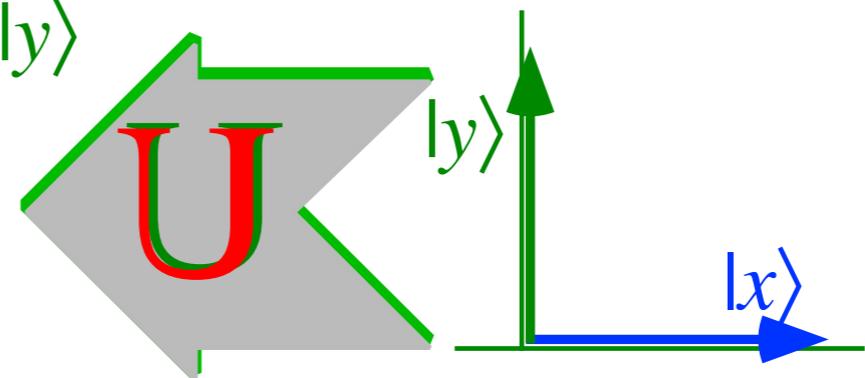
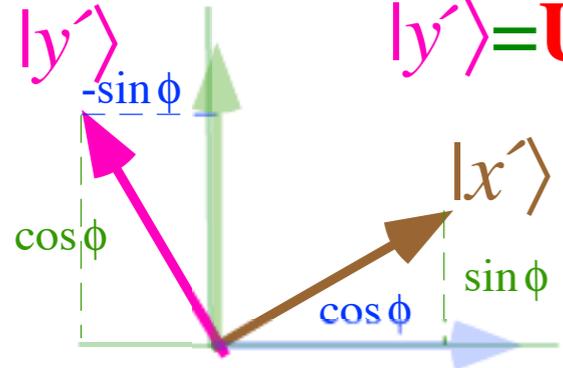
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$$\begin{aligned} \begin{pmatrix} \langle x|\mathbf{U}|x\rangle & \langle x|\mathbf{U}|y\rangle \\ \langle y|\mathbf{U}|x\rangle & \langle y|\mathbf{U}|y\rangle \end{pmatrix} &= \begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} = \begin{pmatrix} \langle x'|\mathbf{U}|x'\rangle & \langle x'|\mathbf{U}|y'\rangle \\ \langle y'|\mathbf{U}|x'\rangle & \langle y'|\mathbf{U}|y'\rangle \end{pmatrix} \\ \begin{pmatrix} \langle x|\mathbf{U}^\dagger|x\rangle & \langle x|\mathbf{U}^\dagger|y\rangle \\ \langle y|\mathbf{U}^\dagger|x\rangle & \langle y|\mathbf{U}^\dagger|y\rangle \end{pmatrix} &= \begin{pmatrix} \langle x'|x\rangle & \langle x'|y\rangle \\ \langle y'|x\rangle & \langle y'|y\rangle \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} = \begin{pmatrix} \langle x'|\mathbf{U}^\dagger|x'\rangle & \langle x'|\mathbf{U}^\dagger|y'\rangle \\ \langle y'|\mathbf{U}^\dagger|x'\rangle & \langle y'|\mathbf{U}^\dagger|y'\rangle \end{pmatrix} \\ &= \begin{pmatrix} \langle x|x'\rangle^* & \langle y|x'\rangle^* \\ \langle x|y'\rangle^* & \langle y|y'\rangle^* \end{pmatrix} \end{aligned}$$

Axiom-3 consistent with inverse $\mathbf{U} = \text{transpose-conjugate } \mathbf{U}^\dagger = \mathbf{U}^{T}$*

Unitary operators \mathbf{U} satisfy “easy inversion” relations: $\mathbf{U}^{-1} = \mathbf{U}^\dagger = \mathbf{U}^{T*}$

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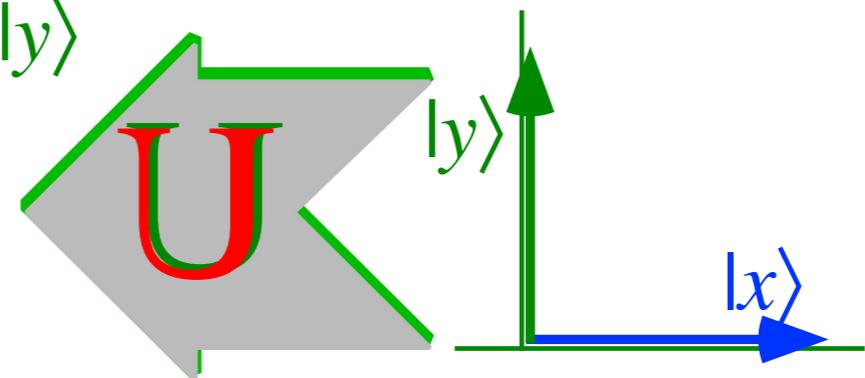
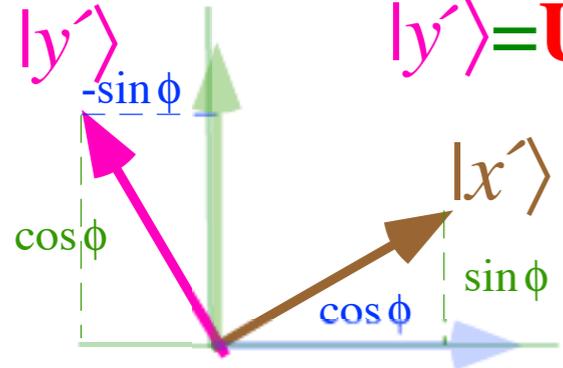
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So also is the inverse \mathbf{U}^\dagger

$$\begin{pmatrix} \langle x|\mathbf{U}^\dagger|x\rangle & \langle x|\mathbf{U}^\dagger|y\rangle \\ \langle y|\mathbf{U}^\dagger|x\rangle & \langle y|\mathbf{U}^\dagger|y\rangle \end{pmatrix} = \begin{pmatrix} \langle x'|x\rangle & \langle x'|y\rangle \\ \langle y'|x\rangle & \langle y'|y\rangle \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} = \begin{pmatrix} \langle x'|\mathbf{U}^\dagger|x'\rangle & \langle x'|\mathbf{U}^\dagger|y'\rangle \\ \langle y'|\mathbf{U}^\dagger|x'\rangle & \langle y'|\mathbf{U}^\dagger|y'\rangle \end{pmatrix}$$

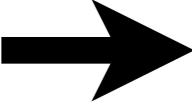
$$= \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} = \begin{pmatrix} \langle x|x'\rangle^* & \langle y|x'\rangle^* \\ \langle x|y'\rangle^* & \langle y|y'\rangle^* \end{pmatrix} \quad \text{Axiom-3 consistent with} \\ \text{inverse } \mathbf{U} = \text{transpose-conjugate } \mathbf{U}^\dagger = \mathbf{U}^{T*}$$

Review: Axioms 1-4 and “Do-Nothing” vs “Do-Something” analyzers

*Abstraction of Axiom-4 to define projection and unitary operators
Projection operators and resolution of identity*

*Unitary operators and matrices that do something (or “nothing”)
Diagonal unitary operators*

Non-diagonal unitary operators and †-conjugation relations

 *Non-diagonal projection operators and Kronecker \otimes -products
Axiom-4 similarity transformation*

Matrix representation of beam analyzers

Non-unitary “killer” devices: Sorter-counter, filter

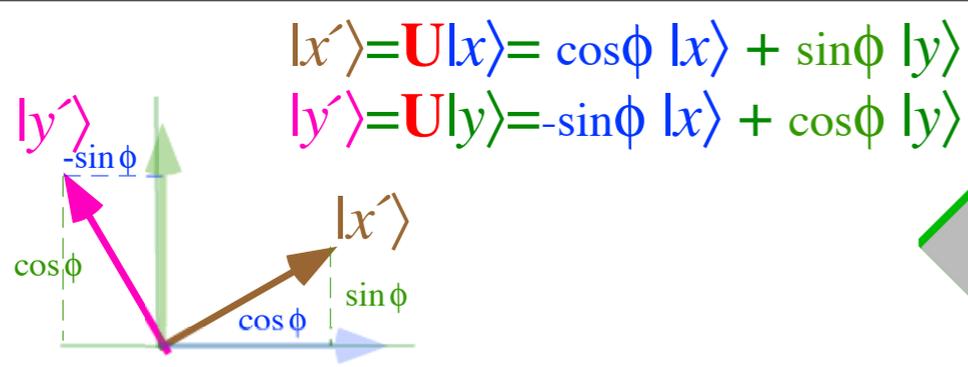
Unitary “non-killer” devices: 1/2-wave plate, 1/4-wave plate

How analyzers “peek” and how that changes outcomes

Peeking polarizers and coherence loss

Classical Bayesian probability vs. Quantum probability

*Feynman $\langle j|k \rangle$ -axioms compared to **Group axioms***

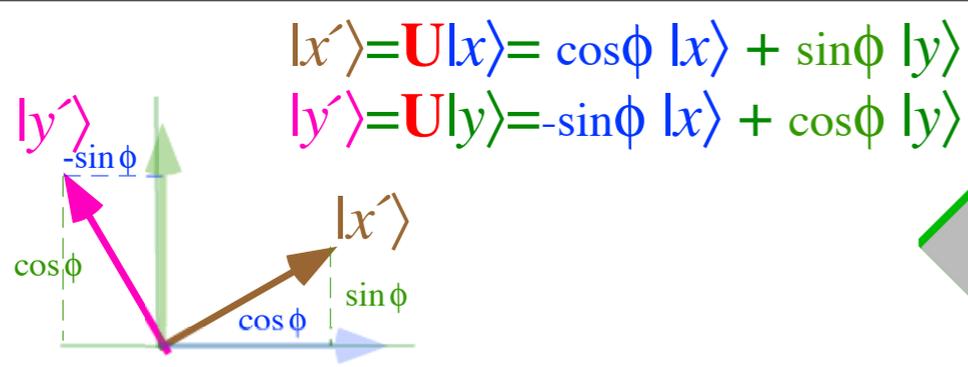


$$\begin{pmatrix} \langle x|\mathbf{U}|x\rangle & \langle x|\mathbf{U}|y\rangle \\ \langle y|\mathbf{U}|x\rangle & \langle y|\mathbf{U}|y\rangle \end{pmatrix} = \begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} \\
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Projector $\mathbf{P}_x = |x\rangle\langle x|$ in ϕ -tilted polarization bases $\{|x'\rangle, |y'\rangle\}$ is not diagonal.

$$\begin{pmatrix} \langle x'|\mathbf{P}_x|x'\rangle & \langle x'|\mathbf{P}_x|y'\rangle \\ \langle y'|\mathbf{P}_x|x'\rangle & \langle y'|\mathbf{P}_x|y'\rangle \end{pmatrix} = \begin{pmatrix} \langle x'|x\rangle\langle x|x'\rangle & \langle x'|x\rangle\langle x|y'\rangle \\ \langle y'|x\rangle\langle x|x'\rangle & \langle y'|x\rangle\langle x|y'\rangle \end{pmatrix}$$

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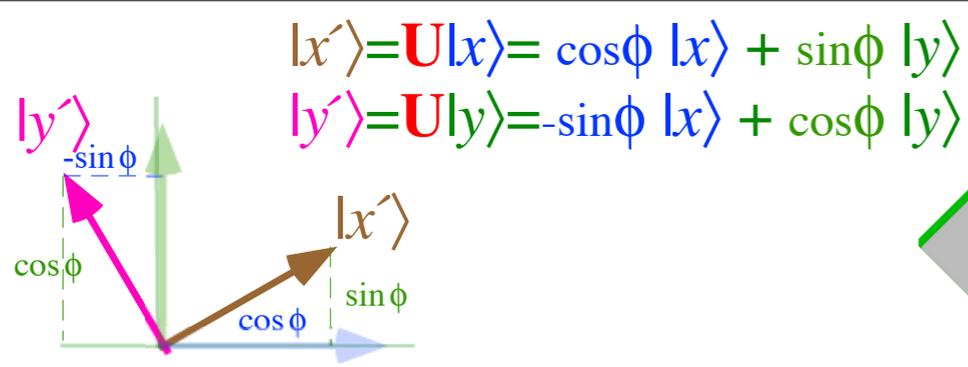
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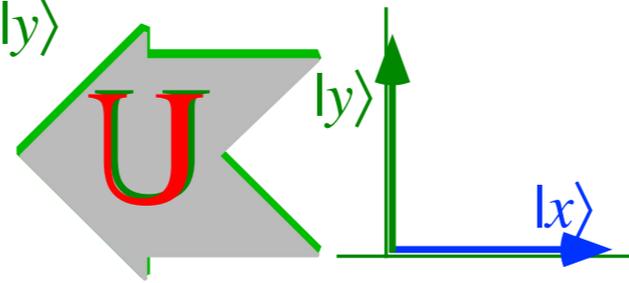
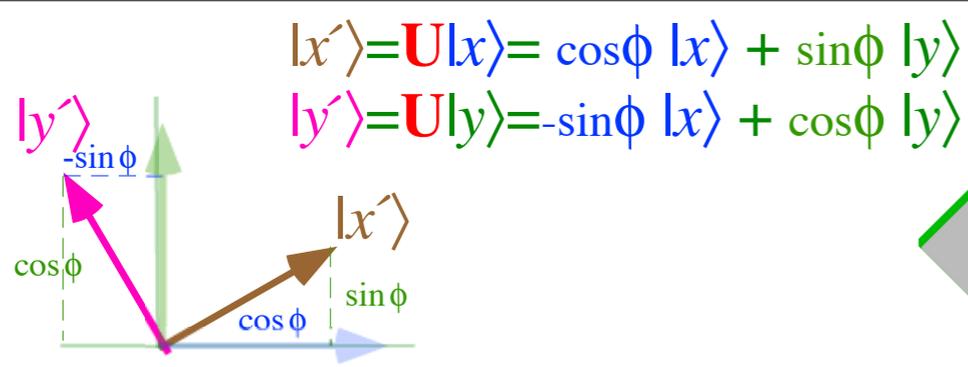
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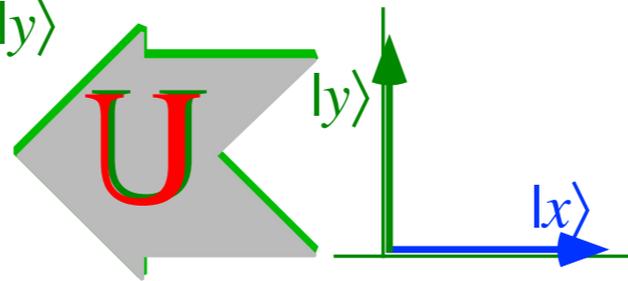
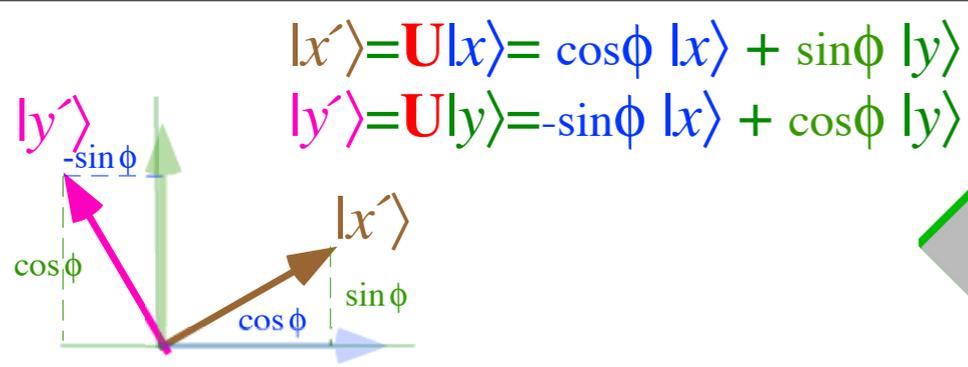
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$$\begin{pmatrix} \langle x|U|x\rangle & \langle x|U|y\rangle \\ \langle y|U|x\rangle & \langle y|U|y\rangle \end{pmatrix} = \begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} \\
 \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$$

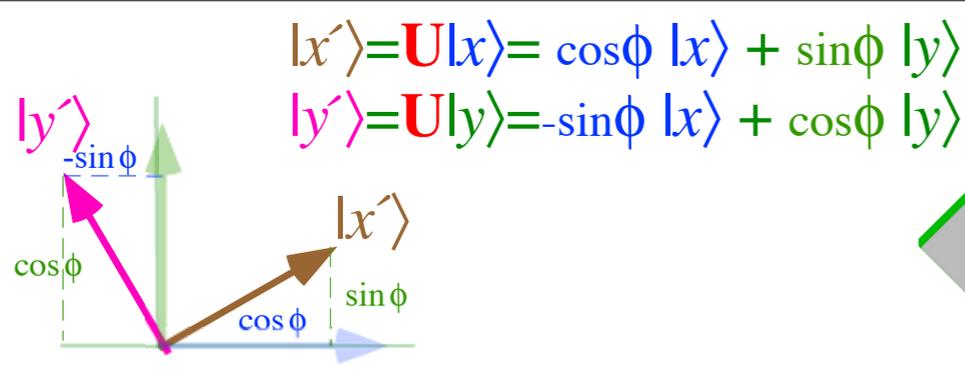
Projector $\mathbf{P}_x = |x\rangle\langle x|$ in ϕ -tilted polarization bases $\{|x'\rangle, |y'\rangle\}$ is not diagonal.

$$\begin{pmatrix} \langle x'|\mathbf{P}_x|x'\rangle & \langle x'|\mathbf{P}_x|y'\rangle \\ \langle y'|\mathbf{P}_x|x'\rangle & \langle y'|\mathbf{P}_x|y'\rangle \end{pmatrix} = \begin{pmatrix} \langle x'|x\rangle\langle x|x'\rangle & \langle x'|x\rangle\langle x|y'\rangle \\ \langle y'|x\rangle\langle x|x'\rangle & \langle y'|x\rangle\langle x|y'\rangle \end{pmatrix}$$

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Projector $\mathbf{P}_x = |x\rangle\langle x|$ is what is called an *outer* or *Kronecker tensor* (\otimes) *product* of ket $|x\rangle$ and bra $\langle x|$.

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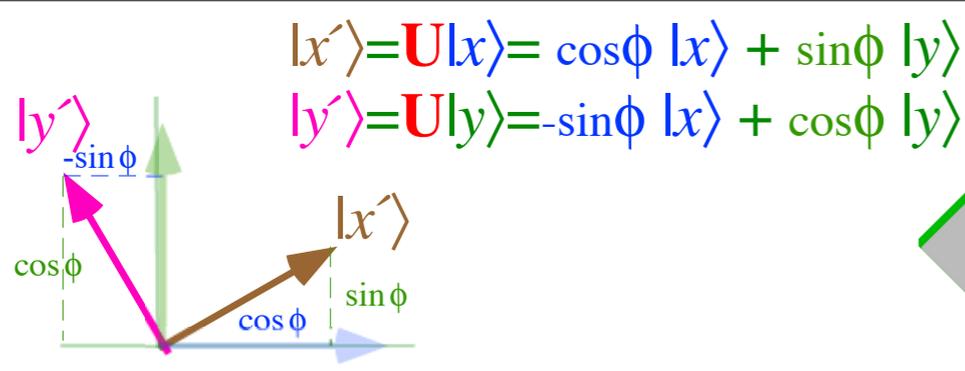
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The $x'y'$ -representation of \mathbf{P}_x :

$$\mathbf{P}_x = |x\rangle\langle x| \rightarrow \begin{pmatrix} \cos\phi \\ -\sin\phi \end{pmatrix} \otimes \begin{pmatrix} \cos\phi & -\sin\phi \end{pmatrix} \\
 = \begin{pmatrix} \cos^2\phi & -\sin\phi\cos\phi \\ -\sin\phi\cos\phi & \sin^2\phi \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}_{(\text{for } \phi=0)}$$



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$$= \begin{pmatrix} \langle x'|U|x'\rangle & \langle x'|U|y'\rangle \\ \langle y'|U|x'\rangle & \langle y'|U|y'\rangle \end{pmatrix}$$

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The $x'y'$ -representation of \mathbf{P}_y :

$$\mathbf{P}_y = |y\rangle\langle y| \rightarrow \begin{pmatrix} \sin\phi \\ \cos\phi \end{pmatrix} \otimes \begin{pmatrix} \sin\phi & \cos\phi \end{pmatrix} \\
 = \begin{pmatrix} \sin^2\phi & \sin\phi\cos\phi \\ \sin\phi\cos\phi & \cos^2\phi \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}_{(\text{for } \phi=0)}$$

Review: Axioms 1-4 and “Do-Nothing” vs “Do-Something” analyzers

*Abstraction of Axiom-4 to define projection and unitary operators
Projection operators and resolution of identity*

*Unitary operators and matrices that do something (or “nothing”)
Diagonal unitary operators*

Non-diagonal unitary operators and †-conjugation relations

Non-diagonal projection operators and Kronecker \otimes -products

 *Axiom-4 similarity transformation*

Matrix representation of beam analyzers

Non-unitary “killer” devices: Sorter-counter, filter

Unitary “non-killer” devices: 1/2-wave plate, 1/4-wave plate

How analyzers “peek” and how that changes outcomes

Peeking polarizers and coherence loss

Classical Bayesian probability vs. Quantum probability

*Feynman $\langle j|k \rangle$ -axioms compared to **Group axioms***

Axiom-4 similarity transformations (Using: $\mathbf{1} = \sum |k\rangle\langle k|$)

Axiom-4 is basically a matrix product as seen by comparing the following.

$$\langle j'' | m' \rangle = \langle j'' | \mathbf{1} | m' \rangle = \sum_{k=1}^n \langle j'' | k \rangle \langle k | m' \rangle$$

$$\begin{pmatrix} \langle 1'' | 1' \rangle & \langle 1'' | 2' \rangle & \cdots & \langle 1'' | n' \rangle \\ \langle 2'' | 1' \rangle & \langle 2'' | 2' \rangle & \cdots & \langle 2'' | n' \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle n'' | 1' \rangle & \langle n'' | 2' \rangle & \cdots & \langle n'' | n' \rangle \end{pmatrix} = \begin{pmatrix} \langle 1'' | 1 \rangle & \langle 1'' | 2 \rangle & \cdots & \langle 1'' | n \rangle \\ \langle 2'' | 1 \rangle & \langle 2'' | 2 \rangle & \cdots & \langle 2'' | n \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle n'' | 1 \rangle & \langle n'' | 2 \rangle & \cdots & \langle n'' | n \rangle \end{pmatrix} \bullet \begin{pmatrix} \langle 1 | 1' \rangle & \langle 1 | 2' \rangle & \cdots & \langle 1 | n' \rangle \\ \langle 2 | 1' \rangle & \langle 2 | 2' \rangle & \cdots & \langle 2 | n' \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle n | 1' \rangle & \langle n | 2' \rangle & \cdots & \langle n | n' \rangle \end{pmatrix}$$

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$$T_{j'' m'} \begin{pmatrix} \text{prime} \\ \text{to} \\ \text{double - prime} \end{pmatrix} = \sum_{k=1}^n T_{j'' k} \begin{pmatrix} \text{unprimed} \\ \text{to} \\ \text{double - prime} \end{pmatrix} T_{k m'} \begin{pmatrix} \text{prime} \\ \text{to} \\ \text{unprimed} \end{pmatrix}$$

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$$\mathbf{T}(b'' \leftarrow b') = \mathbf{T}(b'' \leftarrow b) \bullet \mathbf{T}(b \leftarrow b')$$

(1) The closure axiom

Products $ab = c$ are defined between any two group elements a and b , and the result c is contained in the group.

(2) The associativity axiom

Products $(ab)c$ and $a(bc)$ are equal for all elements a , b , and c in the group.

(3) The identity axiom

There is a unique element 1 (the identity) such that $1 \cdot a = a = a \cdot 1$ for all elements a in the group ..

4) The inverse axiom

For all elements a in the group there is an inverse element a^{-1} such that $a^{-1}a = 1 = a \cdot a^{-1}$.

Transformation Group axioms

Axiom-4 is applied twice to transform operator matrix representation.

Example: *Find:*

$$\left(\begin{array}{cc} \langle x' | \mathbf{P}_x | x' \rangle & \langle x' | \mathbf{P}_x | y' \rangle \\ \langle y' | \mathbf{P}_x | x' \rangle & \langle y' | \mathbf{P}_x | y' \rangle \end{array} \right) \quad \text{given:} \quad \left(\begin{array}{cc} \langle x | \mathbf{P}_x | x \rangle & \langle x | \mathbf{P}_x | y \rangle \\ \langle y | \mathbf{P}_x | x \rangle & \langle y | \mathbf{P}_x | y \rangle \end{array} \right) = \left(\begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array} \right) \quad \text{and } T\text{-matrix:}$$

$$\left(\begin{array}{cc} \langle x | x' \rangle & \langle x | y' \rangle \\ \langle y | x' \rangle & \langle y | y' \rangle \end{array} \right) = \left(\begin{array}{cc} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{array} \right)$$

The old “ $\mathbf{P}=\mathbf{1}\cdot\mathbf{P}\cdot\mathbf{1}$ -trick” where: $\mathbf{1}=\sum|k\rangle\langle k| = |x\rangle\langle x| + |y\rangle\langle y|;$

Axiom-4 is applied twice to transform operator matrix representation.

Example: *Find:*

$$\begin{pmatrix} \langle x' | \mathbf{P}_x | x' \rangle & \langle x' | \mathbf{P}_x | y' \rangle \\ \langle y' | \mathbf{P}_x | x' \rangle & \langle y' | \mathbf{P}_x | y' \rangle \end{pmatrix} \quad \text{given:} \quad \begin{pmatrix} \langle x | \mathbf{P}_x | x \rangle & \langle x | \mathbf{P}_x | y \rangle \\ \langle y | \mathbf{P}_x | x \rangle & \langle y | \mathbf{P}_x | y \rangle \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \text{and } T\text{-matrix:}$$

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$$\langle x' | \mathbf{P}_x | y' \rangle = \langle x' | \mathbf{1} \cdot \mathbf{P}_x \cdot \mathbf{1} | y' \rangle = \langle x' | (|x\rangle\langle x| + |y\rangle\langle y|) \cdot \mathbf{P}_x \cdot (|x\rangle\langle x| + |y\rangle\langle y|) | y' \rangle$$

Axiom-4 is applied twice to transform operator matrix representation.

Example: *Find:*

given:

and T-matrix:

$$\begin{pmatrix} \langle x' | \mathbf{P}_x | x' \rangle & \langle x' | \mathbf{P}_x | y' \rangle \\ \langle y' | \mathbf{P}_x | x' \rangle & \langle y' | \mathbf{P}_x | y' \rangle \end{pmatrix} \begin{pmatrix} \langle x | \mathbf{P}_x | x \rangle & \langle x | \mathbf{P}_x | y \rangle \\ \langle y | \mathbf{P}_x | x \rangle & \langle y | \mathbf{P}_x | y \rangle \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \langle x | x' \rangle & \langle x | y' \rangle \\ \langle y | x' \rangle & \langle y | y' \rangle \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix}$$

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Axiom-4 is applied twice to transform operator matrix representation.

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given:

and T-matrix:

$$\begin{pmatrix} \langle x' | \mathbf{P}_x | x' \rangle & \langle x' | \mathbf{P}_x | y' \rangle \\ \langle y' | \mathbf{P}_x | x' \rangle & \langle y' | \mathbf{P}_x | y' \rangle \end{pmatrix} \begin{pmatrix} \langle x | \mathbf{P}_x | x \rangle & \langle x | \mathbf{P}_x | y \rangle \\ \langle y | \mathbf{P}_x | x \rangle & \langle y | \mathbf{P}_x | y \rangle \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \langle x | x' \rangle & \langle x | y' \rangle \\ \langle y | x' \rangle & \langle y | y' \rangle \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix}$$

The old “ $\mathbf{P}=\mathbf{1}\cdot\mathbf{P}\cdot\mathbf{1}$ -trick” where: $\mathbf{1}=\sum|k\rangle\langle k| = |x\rangle\langle x| + |y\rangle\langle y|$;

$$\begin{aligned} \langle x' | \mathbf{P}_x | y' \rangle &= \langle x' | \mathbf{1} \cdot \mathbf{P}_x \cdot \mathbf{1} | y' \rangle = \langle x' | (|x\rangle\langle x| + |y\rangle\langle y|) \cdot \mathbf{P}_x \cdot (|x\rangle\langle x| + |y\rangle\langle y|) | y' \rangle = (\langle x' | x \rangle \langle x| + \langle x' | y \rangle \langle y|) \cdot \mathbf{P}_x \cdot (|x\rangle\langle x| + |y\rangle\langle y|) \\ &= \langle x' | x \rangle \langle x | \mathbf{P}_x | x \rangle \langle x | y' \rangle + \langle x' | y \rangle \langle y | \mathbf{P}_x | x \rangle \langle x | y' \rangle + \langle x' | x \rangle \langle x | \mathbf{P}_x | y \rangle \langle y | y' \rangle + \langle x' | y \rangle \langle y | \mathbf{P}_x | y \rangle \langle y | y' \rangle \end{aligned}$$

Axiom-4 is applied twice to transform operator matrix representation.

Example: Find:

given:

and T-matrix:

$$\begin{pmatrix} \langle x' | \mathbf{P}_x | x' \rangle & \langle x' | \mathbf{P}_x | y' \rangle \\ \langle y' | \mathbf{P}_x | x' \rangle & \langle y' | \mathbf{P}_x | y' \rangle \end{pmatrix} \begin{pmatrix} \langle x | \mathbf{P}_x | x \rangle & \langle x | \mathbf{P}_x | y \rangle \\ \langle y | \mathbf{P}_x | x \rangle & \langle y | \mathbf{P}_x | y \rangle \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \langle x | x' \rangle & \langle x | y' \rangle \\ \langle y | x' \rangle & \langle y | y' \rangle \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix}$$

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More elegant matrix product:

$$\begin{pmatrix} \langle x' | \mathbf{P}_x | x' \rangle & \langle x' | \mathbf{P}_x | y' \rangle \\ \langle y' | \mathbf{P}_x | x' \rangle & \langle y' | \mathbf{P}_x | y' \rangle \end{pmatrix} = \begin{pmatrix} \langle x' | x \rangle & \langle x' | y \rangle \\ \langle y' | x \rangle & \langle y' | y \rangle \end{pmatrix} \begin{pmatrix} \langle x | \mathbf{P}_x | x \rangle & \langle x | \mathbf{P}_x | y \rangle \\ \langle y | \mathbf{P}_x | x \rangle & \langle y | \mathbf{P}_x | y \rangle \end{pmatrix} \begin{pmatrix} \langle x | x' \rangle & \langle x | y' \rangle \\ \langle y | x' \rangle & \langle y | y' \rangle \end{pmatrix}$$

Axiom-4 is applied twice to transform operator matrix representation.

Example: Find:

given:

and T-matrix:

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More elegant matrix product:

$$\begin{aligned} \begin{pmatrix} \langle x' | \mathbf{P}_x | x' \rangle & \langle x' | \mathbf{P}_x | y' \rangle \\ \langle y' | \mathbf{P}_x | x' \rangle & \langle y' | \mathbf{P}_x | y' \rangle \end{pmatrix} &= \begin{pmatrix} \langle x' | x \rangle & \langle x' | y \rangle \\ \langle y' | x \rangle & \langle y' | y \rangle \end{pmatrix} \begin{pmatrix} \langle x | \mathbf{P}_x | x \rangle & \langle x | \mathbf{P}_x | y \rangle \\ \langle y | \mathbf{P}_x | x \rangle & \langle y | \mathbf{P}_x | y \rangle \end{pmatrix} \begin{pmatrix} \langle x | x' \rangle & \langle x | y' \rangle \\ \langle y | x' \rangle & \langle y | y' \rangle \end{pmatrix} \\ &= \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \langle x | \mathbf{P}_x | x \rangle & \langle x | \mathbf{P}_x | y \rangle \\ \langle y | \mathbf{P}_x | x \rangle & \langle y | \mathbf{P}_x | y \rangle \end{pmatrix} \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \\ &= \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \end{aligned}$$

Axiom-4 is applied twice to transform operator matrix representation.

Example: *Find:*

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More elegant matrix product:

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Axiom-4 is applied twice to transform operator matrix representation.

Example: Find:

given:

and T-matrix:

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More elegant matrix product:

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This checks with the previous result 4-pages back: $\mathbf{P}_x = |x\rangle\langle x| \rightarrow \begin{pmatrix} \cos \phi \\ -\sin \phi \end{pmatrix} \otimes \begin{pmatrix} \cos \phi & -\sin \phi \end{pmatrix} = \begin{pmatrix} \cos^2 \phi & -\sin \phi \cos \phi \\ -\sin \phi \cos \phi & \sin^2 \phi \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}_{(\text{for } \phi=0)}$

Review: Axioms 1-4 and “Do-Nothing” vs “Do-Something” analyzers

*Abstraction of Axiom-4 to define projection and unitary operators
Projection operators and resolution of identity*

*Unitary operators and matrices that do something (or “nothing”)
Diagonal unitary operators*

Non-diagonal unitary operators and †-conjugation relations

Non-diagonal projection operators and Kronecker \otimes -products

Axiom-4 similarity transformation

Matrix representation of beam analyzers



Non-unitary “killer” devices: Sorter-counter, filter

Unitary “non-killer” devices: 1/2-wave plate, 1/4-wave plate

How analyzers “peek” and how that changes outcomes

Peeking polarizers and coherence loss

Classical Bayesian probability vs. Quantum probability

*Feynman $\langle j|k \rangle$ -axioms compared to **Group axioms***

(1) Optical analyzer in sorter-counter configuration

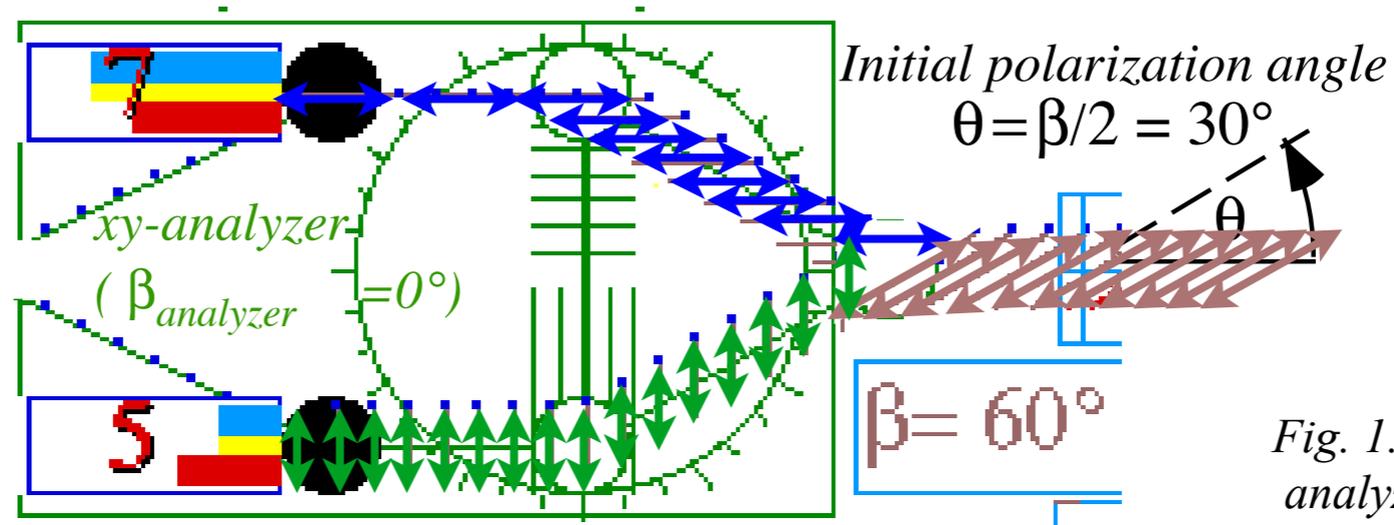
Analyzer reduced to a simple sorter-counter by blocking output of x -high-road and y -low-road with counters

$$x\text{-counts} \sim |\langle x|x' \rangle|^2$$

$$= \cos^2 \theta = 0.75$$

$$y\text{-counts} \sim |\langle y|x' \rangle|^2$$

$$= \sin^2 \theta = 0.25$$



Analyzer matrix:

$$\begin{pmatrix} \langle x|\mathbf{T}|x \rangle & \langle x|\mathbf{T}|y \rangle \\ \langle y|\mathbf{T}|x \rangle & \langle y|\mathbf{T}|y \rangle \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

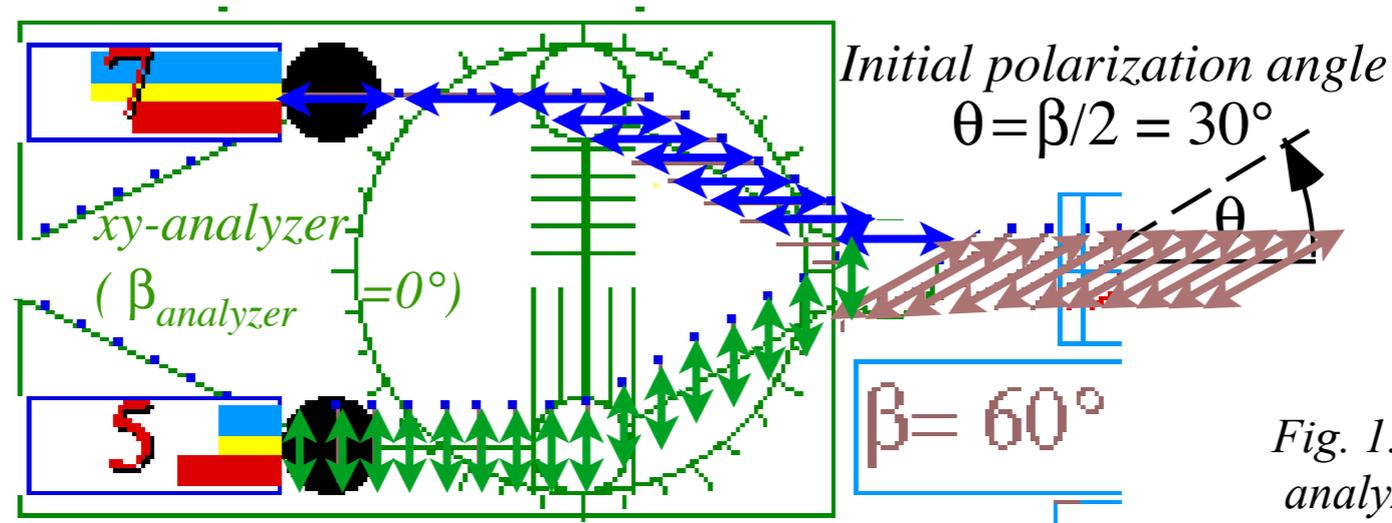
Fig. 1.3.3 Simulated polarization analyzer set up as a sorter-counter

(1) Optical analyzer in sorter-counter configuration

Analyzer reduced to a simple sorter-counter by blocking output of x -high-road and y -low-road with counters

$$x\text{-counts} \sim |\langle x|x' \rangle|^2 = \cos^2 \theta = 0.75$$

$$y\text{-counts} \sim |\langle y|x' \rangle|^2 = \sin^2 \theta = 0.25$$



Analyzer matrix:

$$\begin{pmatrix} \langle x|\mathbf{T}|x \rangle & \langle x|\mathbf{T}|y \rangle \\ \langle y|\mathbf{T}|x \rangle & \langle y|\mathbf{T}|y \rangle \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Fig. 1.3.3 Simulated polarization analyzer set up as a sorter-counter

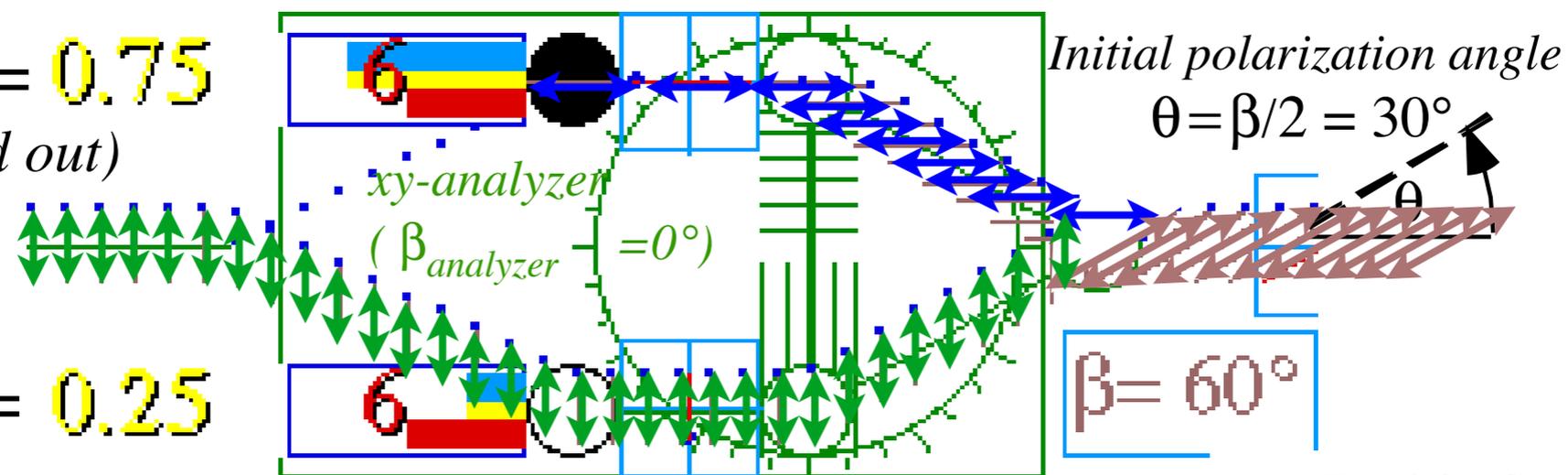
(2) Optical analyzer in a filter configuration (Polaroid© sunglasses)

Analyzer blocks one path which may have photon counter without affecting function.

$$x\text{-counts} \sim |\langle y|x' \rangle|^2 = 0.75$$

(Blocked and filtered out)

$$y\text{-output} \sim |\langle y|x' \rangle|^2 = \sin^2 \theta = 0.25$$



Analyzer matrix:

$$\begin{pmatrix} \langle x|\mathbf{P}_y|x \rangle & \langle x|\mathbf{P}_y|y \rangle \\ \langle y|\mathbf{P}_y|x \rangle & \langle y|\mathbf{P}_y|y \rangle \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Fig. 1.3.4 Simulated polarization analyzer set up to filter out the x -polarized photons

Review: Axioms 1-4 and “Do-Nothing” vs “Do-Something” analyzers

*Abstraction of Axiom-4 to define projection and unitary operators
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Matrix representation of beam analyzers

Non-unitary “killer” devices: Sorter-counter, filter

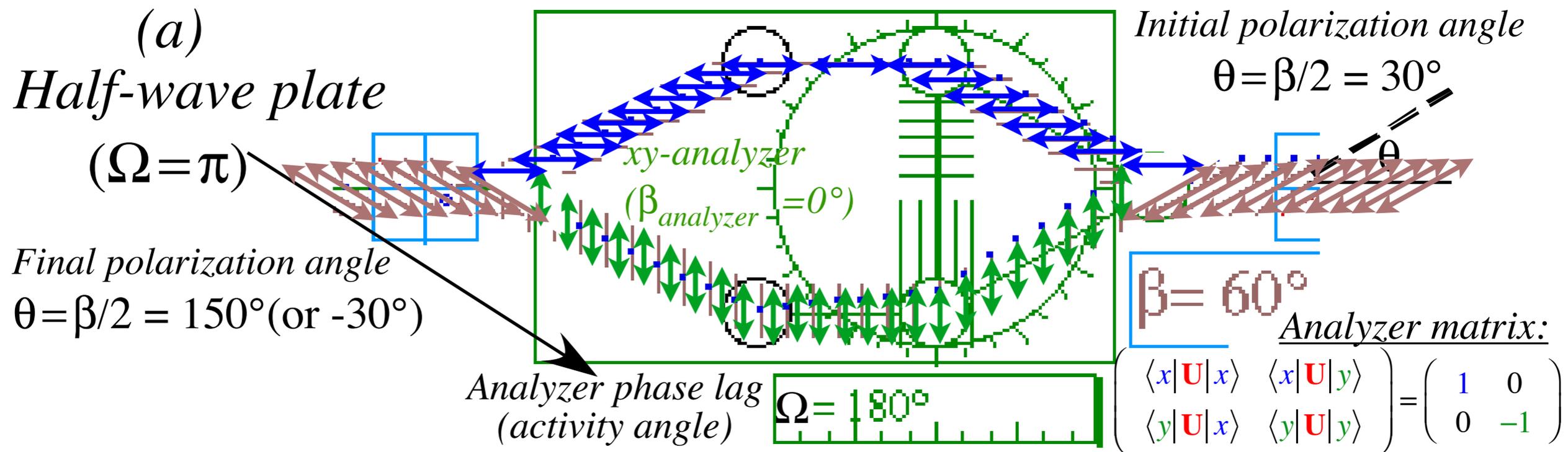
 *Unitary “non-killer” devices: 1/2-wave plate, 1/4-wave plate*

How analyzers “peek” and how that changes outcomes

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(3) Optical analyzers in the "control" configuration: Half or Quarter wave plates



(3) Optical analyzers in the "control" configuration: Half or Quarter wave plates

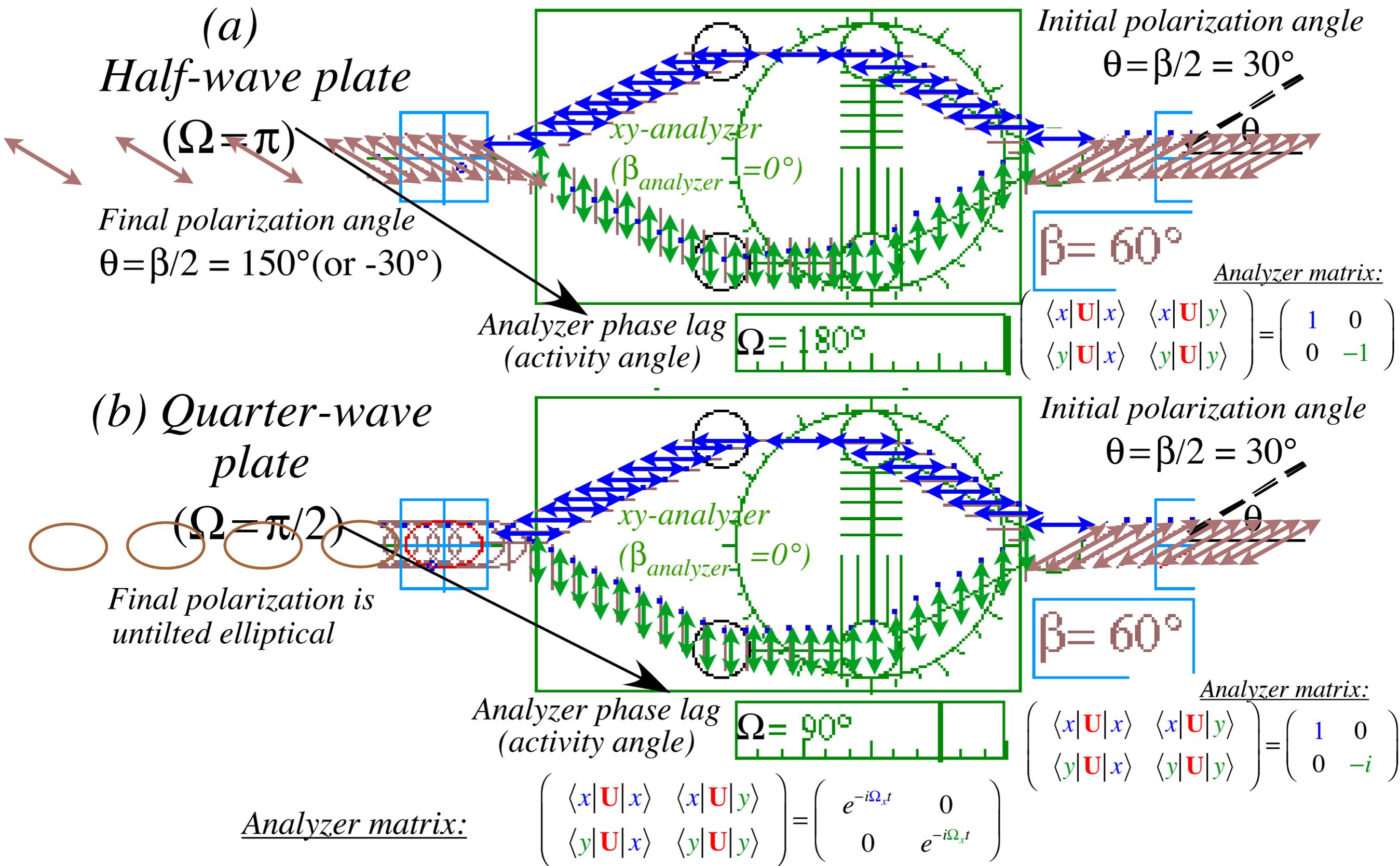
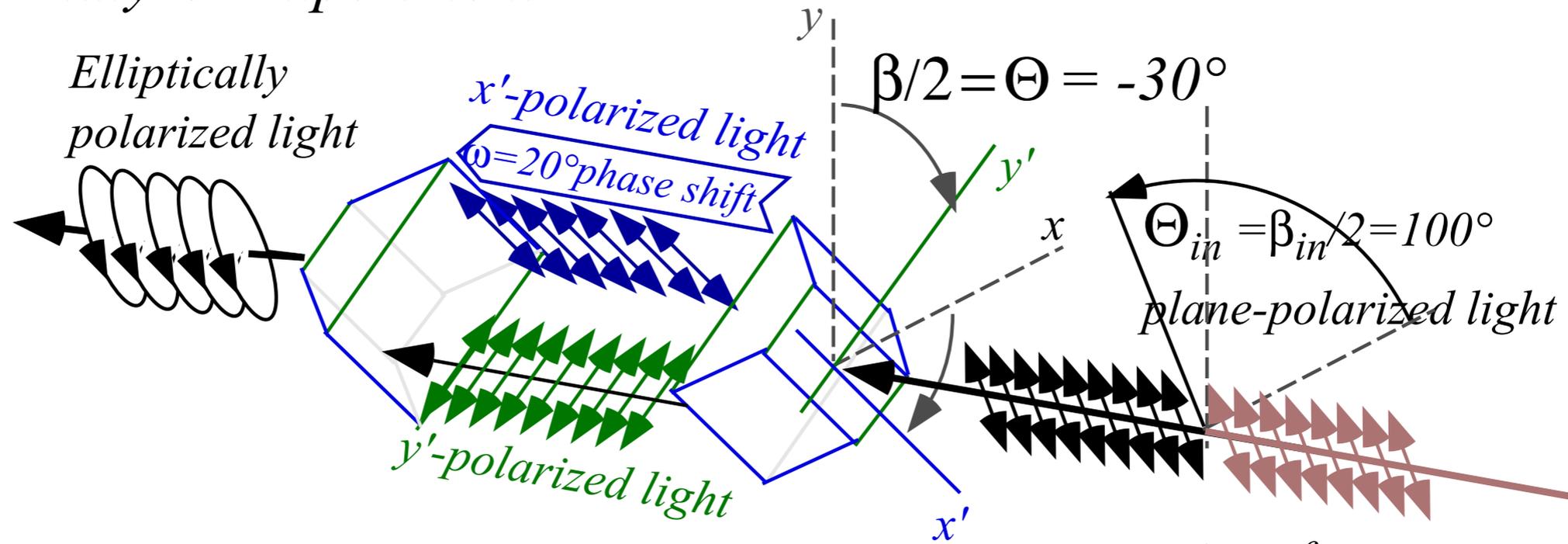
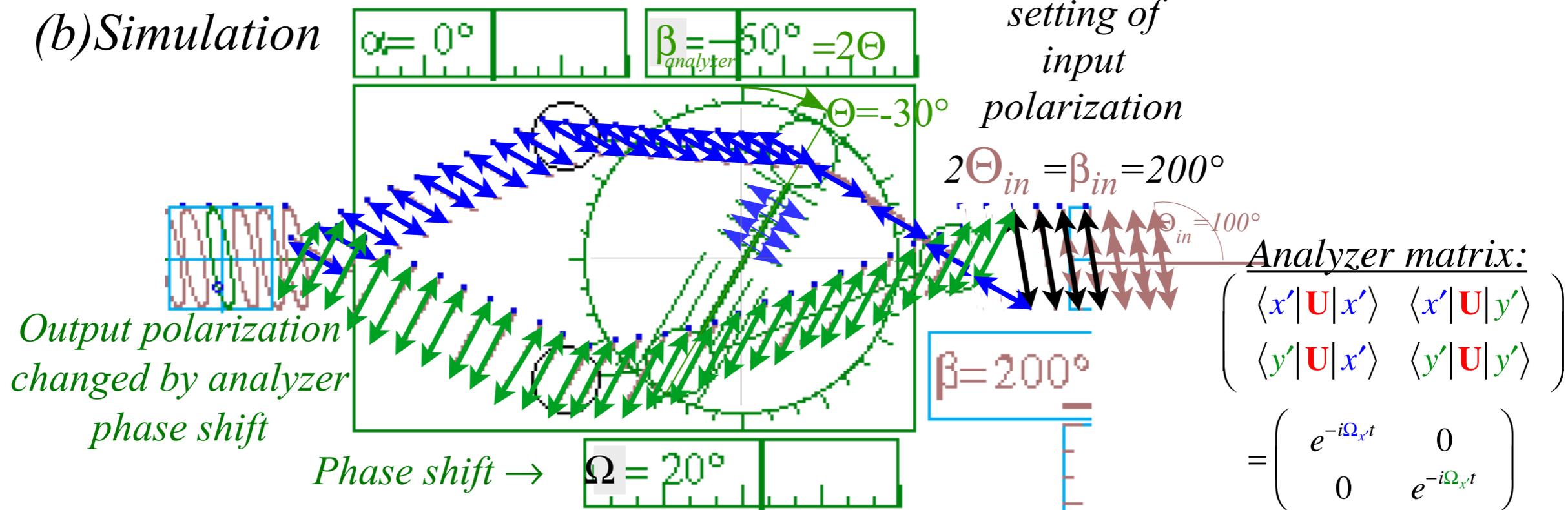


Fig. 1.3.5 Polarization control set to shift phase by (a) Half-wave ($\Omega = \pi$) , (b) Quarter wave ($\Omega = \pi/2$)

(a) Analyzer Experiment



(b) Simulation



Similar to "do-nothing" analyzer but has extra phase factor $e^{-i\Omega_{x'}} = 0.94 - i 0.34$ on the x' -path (top).

$$x\text{-output: } \langle x | \Psi_{out} \rangle = \langle x | x' \rangle e^{-i\Omega_{x'}} \langle x' | \Psi_{in} \rangle + \langle x | y' \rangle \langle y' | \Psi_{in} \rangle = e^{-i\Omega_{x'}} \cos \Theta \cos(\Theta_{in} - \Theta) - \sin \Theta \sin(\Theta_{in} - \Theta)$$

$$y\text{-output: } \langle y | \Psi_{out} \rangle = \langle y | x' \rangle e^{-i\Omega_{x'}} \langle x' | \Psi_{in} \rangle + \langle y | y' \rangle \langle y' | \Psi_{in} \rangle = e^{-i\Omega_{x'}} \sin \Theta \cos(\Theta_{in} - \Theta) + \cos \Theta \sin(\Theta_{in} - \Theta)$$

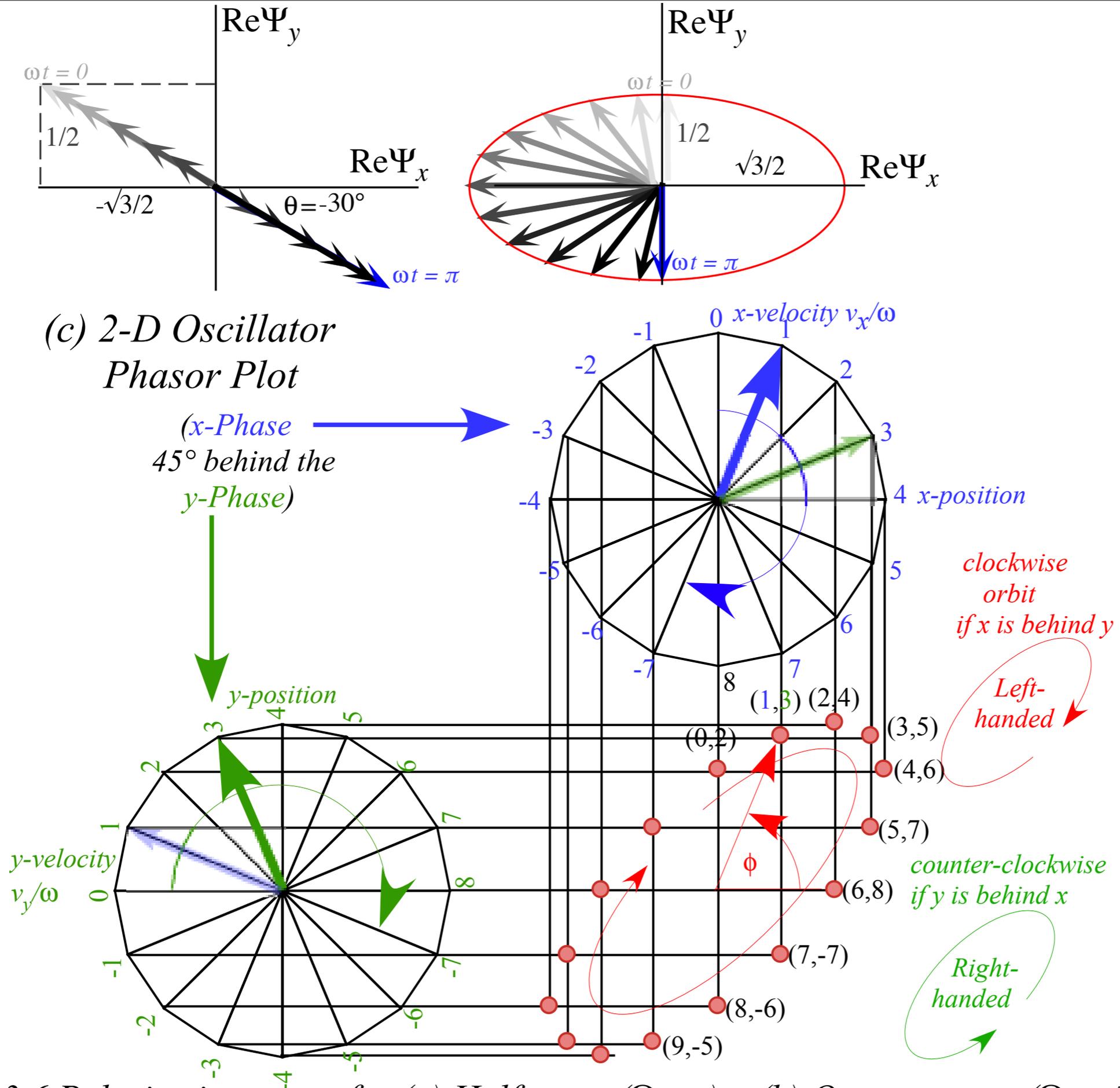


Fig. 1.3.6 Polarization states for (a) Half-wave ($\Omega=\pi$) , (b) Quarter wave ($\Omega=\pi/2$) (c) ($\Omega=-\pi/4$)

Review: Axioms 1-4 and “Do-Nothing” vs “Do-Something” analyzers

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Unitary “non-killer” devices: 1/2-wave plate, 1/4-wave plate



How analyzers “peek” and how that changes outcomes

Peeking polarizers and coherence loss

Classical Bayesian probability vs. Quantum probability

*Feynman $\langle j|k \rangle$ -axioms compared to **Group axioms***

How analyzers may "peek" and how that changes outcomes

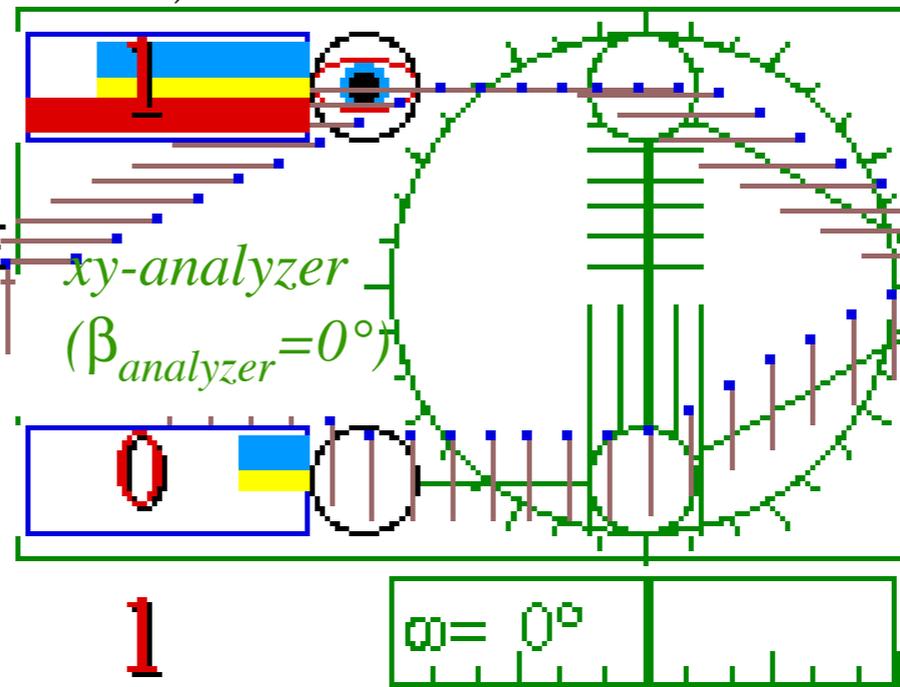
A "peeking" eye (Looks for x-photons)

If eye sees an x-photon
then the output particle
is 100% x-polarized.
(75% probability for that.)

0.75

0.25

1



Initial polarization angle

$$\theta = \beta/2 = 30^\circ$$

$$\beta = 60^\circ$$

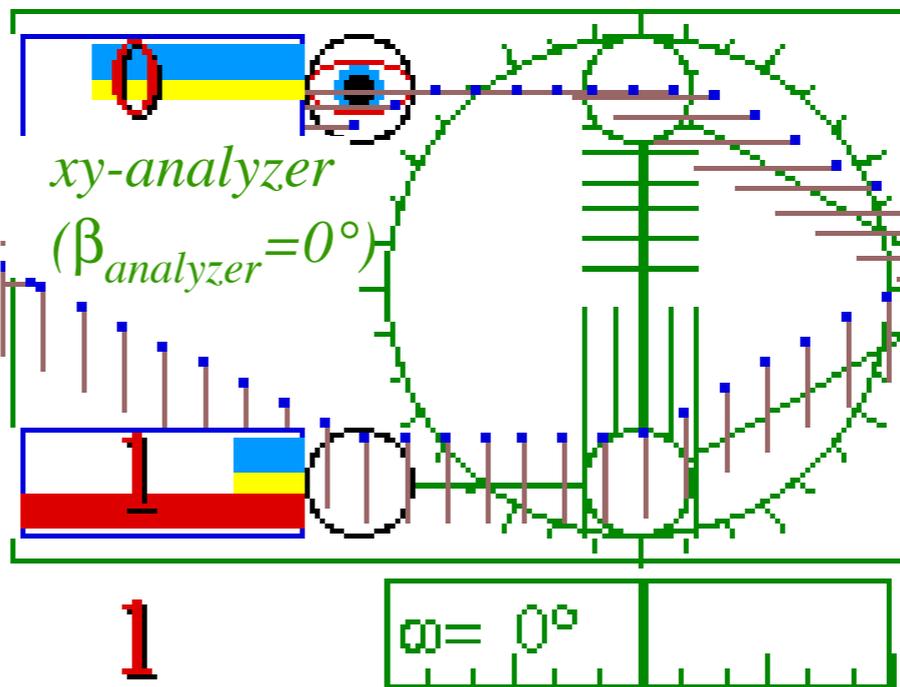
$$\omega = 0^\circ$$

If eye sees no x-photon
then the output particle
is 100% y-polarized
(25% probability.)

0.75

0.25

1



Initial polarization angle

$$\theta = \beta/2 = 30^\circ$$

$$\beta = 60^\circ$$

$$\omega = 0^\circ$$

Fig. 1.3.7 Simulated polarization analyzer set up to "peek" if the photon is x-or y-polarized

How analyzers "peek" and how that changes outcomes

Simulations

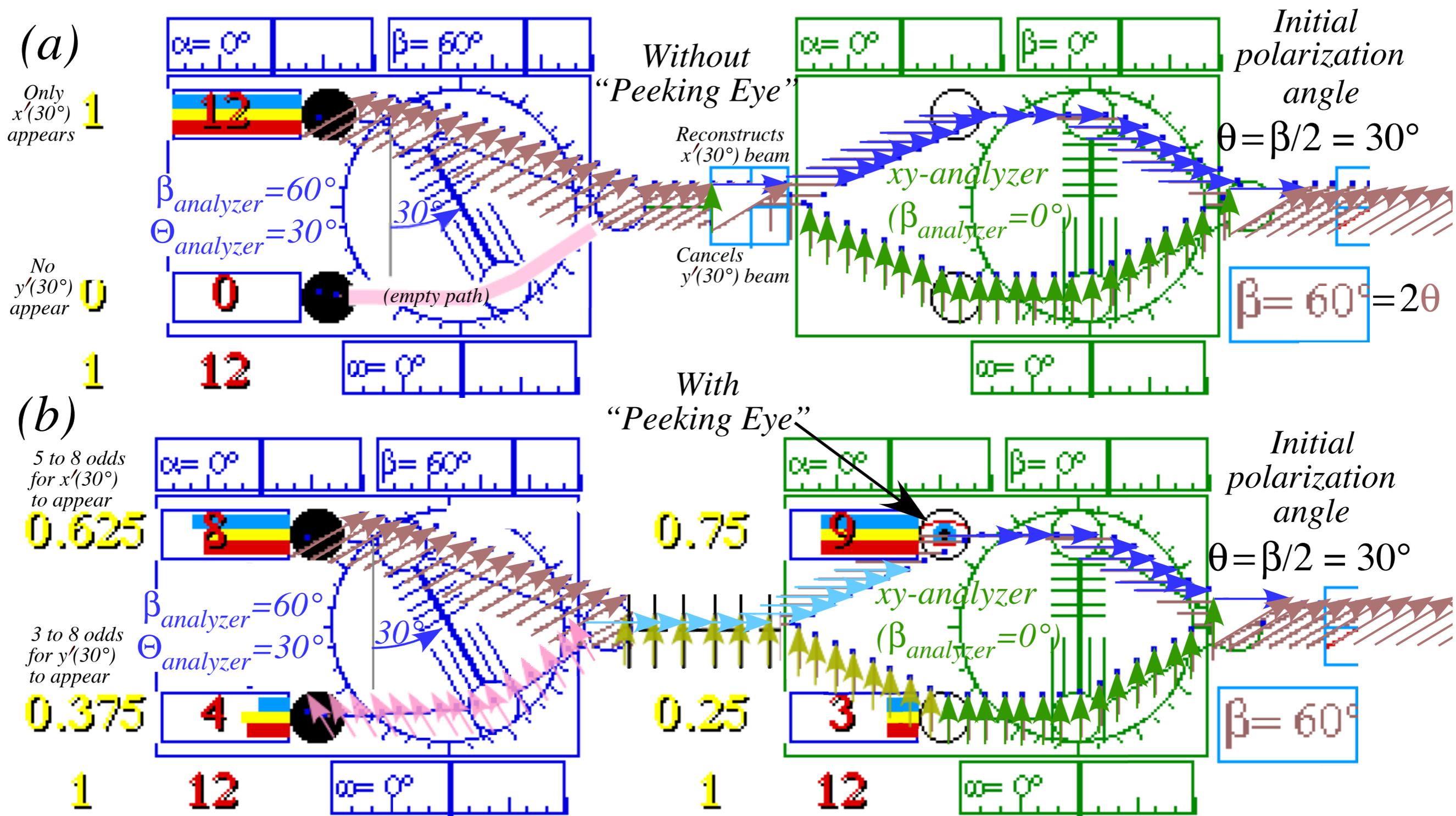


Fig. 1.3.8 Output with $\beta/2=30^\circ$ input to: (a) Coherent xy-"Do nothing" or (b) Incoherent xy-"Peeking" devices

How analyzers "peek" and how that changes outcomes

$$\langle x'|x \rangle \langle x|x' \rangle + \langle x'|y \rangle \langle y|x' \rangle = \sqrt{3}/2 \sqrt{3}/2 + 1/2 \cdot 1/2 = 1$$

$$\langle y'|x \rangle \langle x|x' \rangle + \langle y'|y \rangle \langle y|x' \rangle = -1/2 \sqrt{3}/2 + \sqrt{3}/2 \cdot 1/2 = 0$$

$$\langle x'|x \rangle \langle x|x' \rangle = \sqrt{3}/2 \sqrt{3}/2$$

$$\langle x'|y \rangle \langle y|x' \rangle = 1/2 \cdot 1/2$$

$$\langle y'|y \rangle \langle y|x' \rangle = \sqrt{3}/2 \cdot 1/2$$

$$\langle y'|x \rangle \langle x|x' \rangle = -1/2 \sqrt{3}/2$$

$$\langle y'|y \rangle = \sqrt{3}/2$$

$$\begin{pmatrix} \langle x|x' \rangle & \langle x|y' \rangle \\ \langle y|x' \rangle & \langle y|y' \rangle \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \Big|_{x'}$$

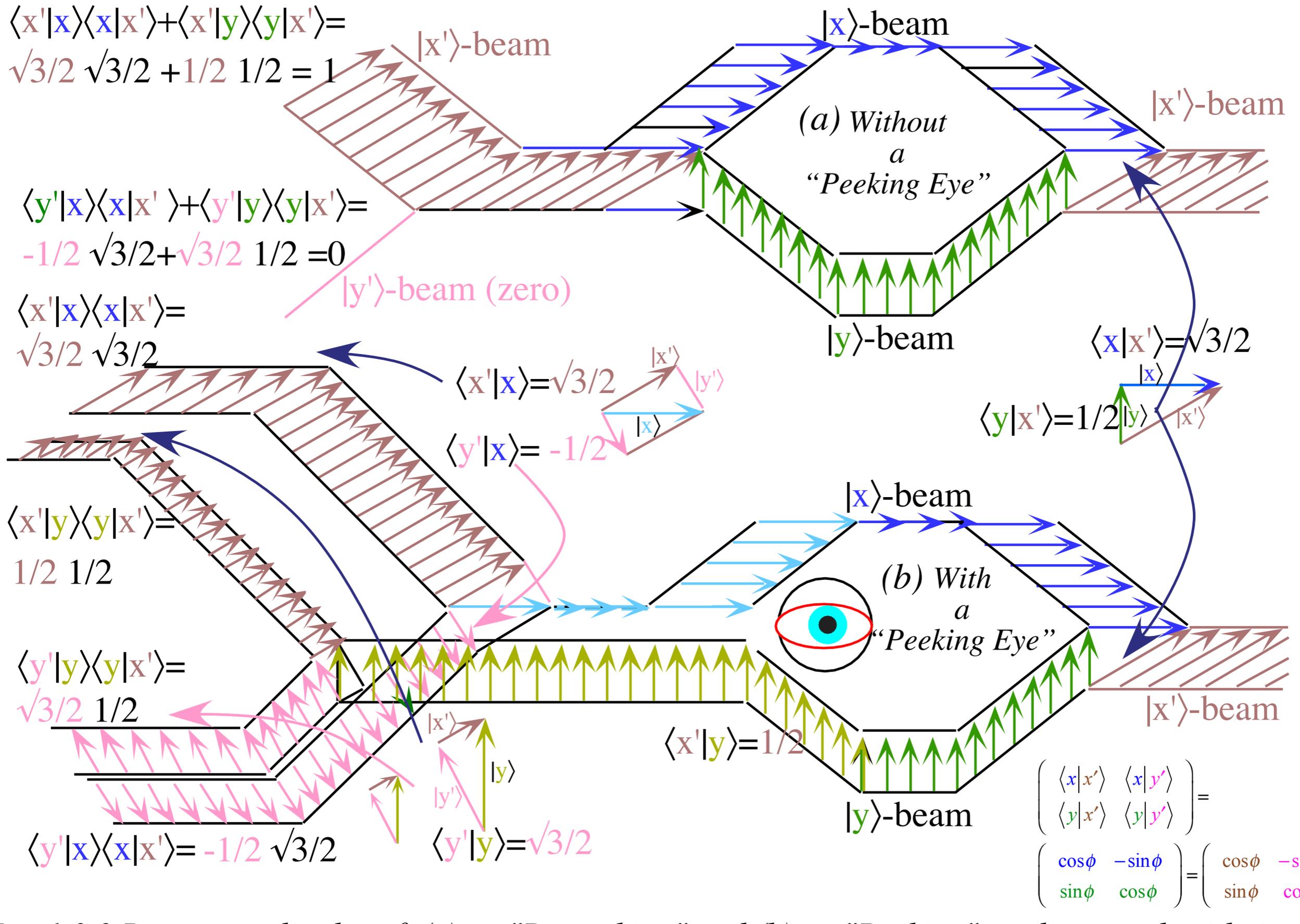


Fig. 1.3.9 Beams-amplitudes of (a) xy-"Do nothing" and (b) xy-"Peeking" analyzer each with input

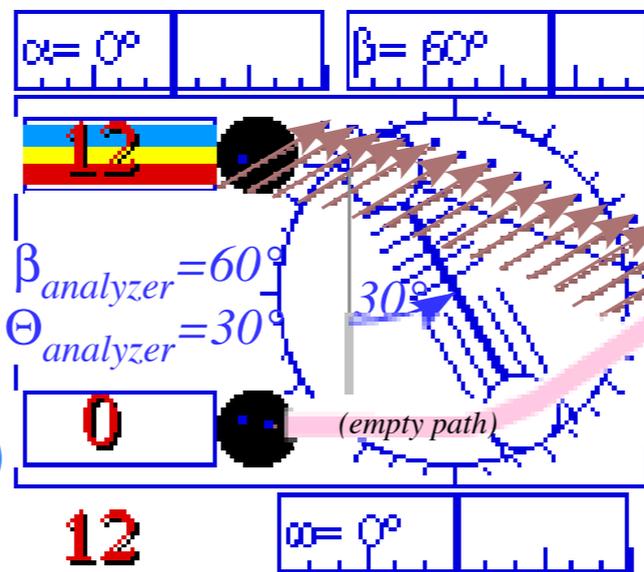
Amplitude $A(n')$ and Probability $P(n')$ at counter n' WITHOUT "peeking"

$$A(x') = \langle x' | x \rangle \langle x | x' \rangle + \langle x' | y \rangle \langle y | x' \rangle$$

$$= \frac{3}{4} + \frac{1}{4} = 1 = P(x')$$

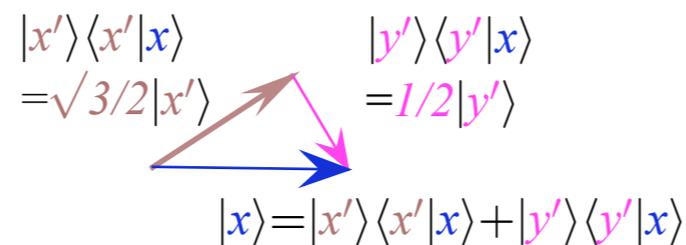
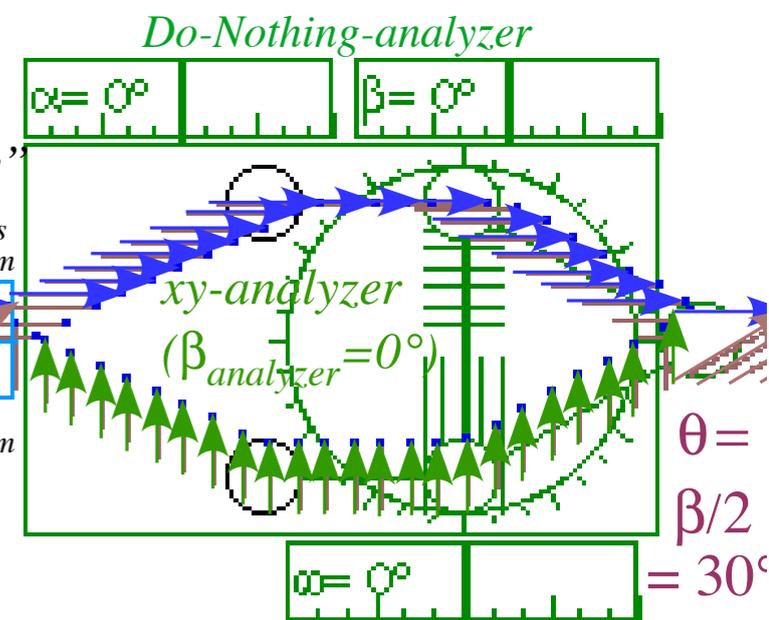
$$A(y') = \langle y' | x \rangle \langle x | x' \rangle + \langle y' | y \rangle \langle y | x' \rangle$$

$$= -\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = 0 = P(y')$$



Without "Peeking Eye"

Reconstructs $x'(30^\circ)$ beam
Cancels $y'(30^\circ)$ beam



$$\begin{pmatrix} \langle x | x' \rangle & \langle x | y' \rangle \\ \langle y | x' \rangle & \langle y | y' \rangle \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix}$$

$$\begin{pmatrix} \langle x | x' \rangle & \langle x | y' \rangle \\ \langle y | x' \rangle & \langle y | y' \rangle \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix}$$

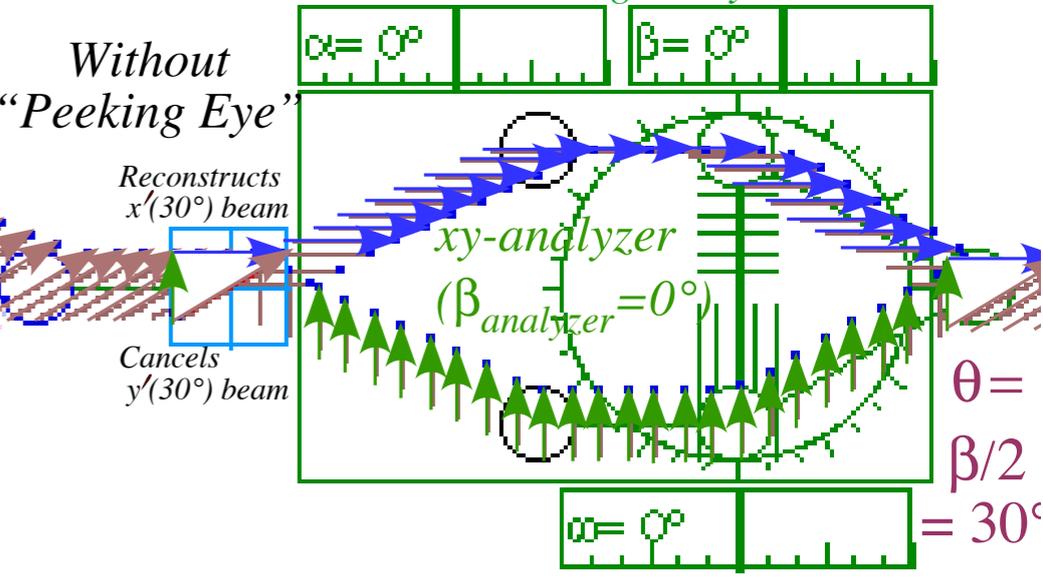
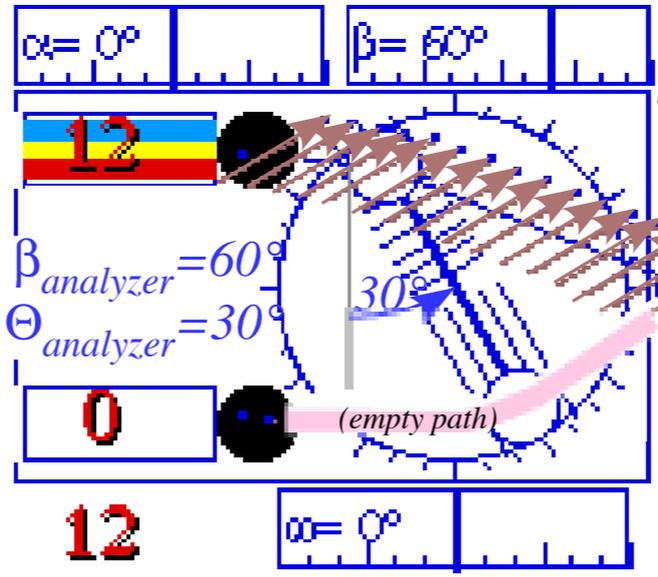
Amplitude $A(n')$ and Probability $P(n')$ at counter n' WITHOUT "peeking"

$$A(x') = \langle x'|x \rangle (1) \langle x|x' \rangle + \langle x'|y \rangle \langle y|x' \rangle$$

$$= \frac{3}{4}(1) + \frac{1}{4} = 1 = P(x')$$

$$A(y') = \langle y'|x \rangle (1) \langle x|x' \rangle + \langle y'|y \rangle \langle y|x' \rangle$$

$$= -\frac{\sqrt{3}}{4}(1) + \frac{\sqrt{3}}{4} = 0 = P(y')$$



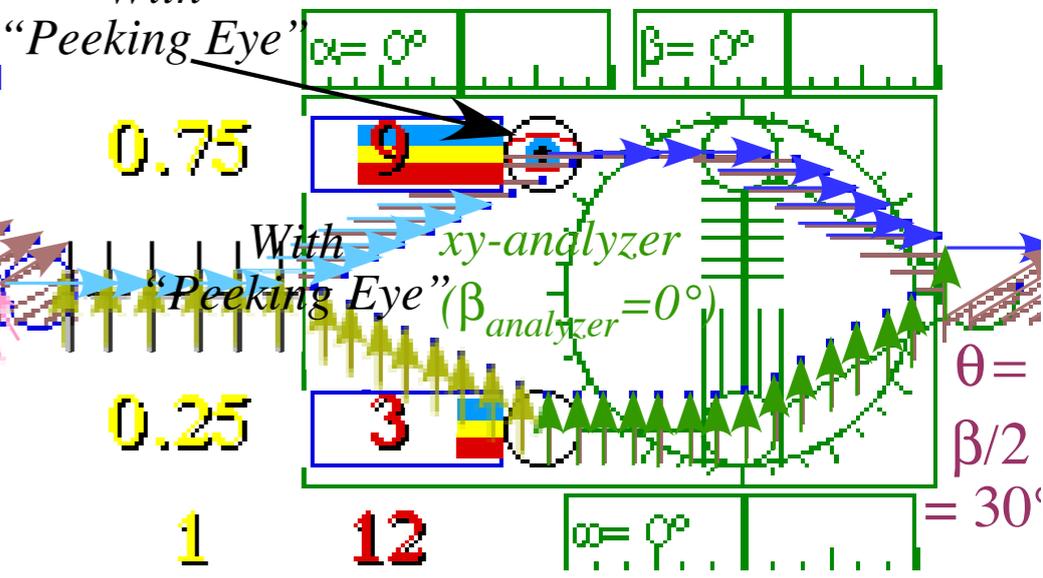
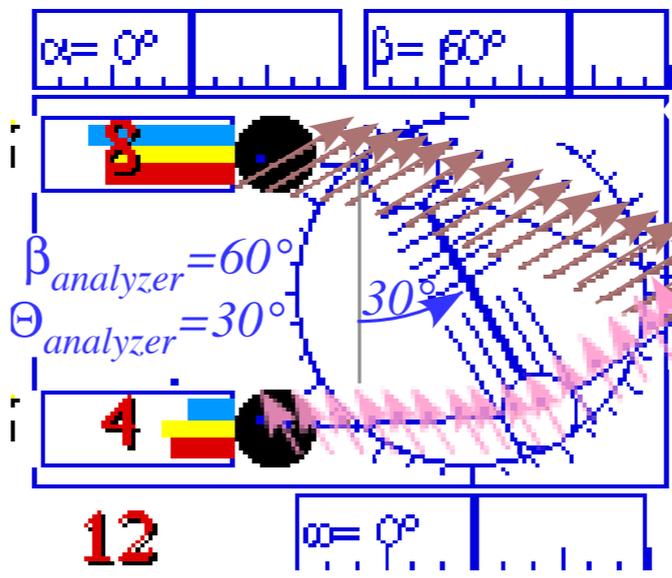
Amplitude $A(n')$ and Probability $P(n')$ at counter n' WITH "peeking"

Suppose "x-eye" puts phase $e^{i\phi}$ on each x-photon with random ϕ distributed over unit circle ($-\pi < \phi < \pi$).

So $e^{i\phi}$ averages to zero!

$$A(x') = \langle x'|x \rangle (e^{i\phi}) \langle x|x' \rangle + \langle x'|y \rangle \langle y|x' \rangle$$

$$= \frac{3}{4}(e^{i\phi}) + \frac{1}{4}$$



$$\begin{pmatrix} \langle x|x' \rangle & \langle x|y' \rangle \\ \langle y|x' \rangle & \langle y|y' \rangle \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix}$$

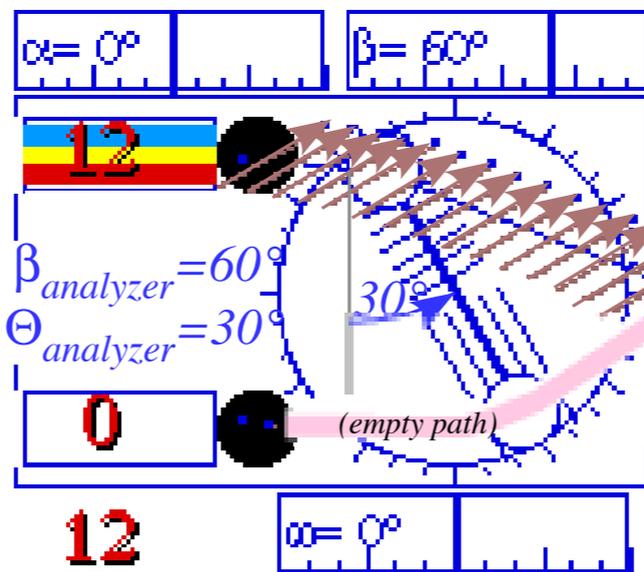
Amplitude $A(n')$ and Probability $P(n')$ at counter n' WITHOUT "peeking"

$$A(x') = \langle x'|x \rangle (1) \langle x|x' \rangle + \langle x'|y \rangle \langle y|x' \rangle$$

$$= \frac{3}{4}(1) + \frac{1}{4} = 1 = P(x')$$

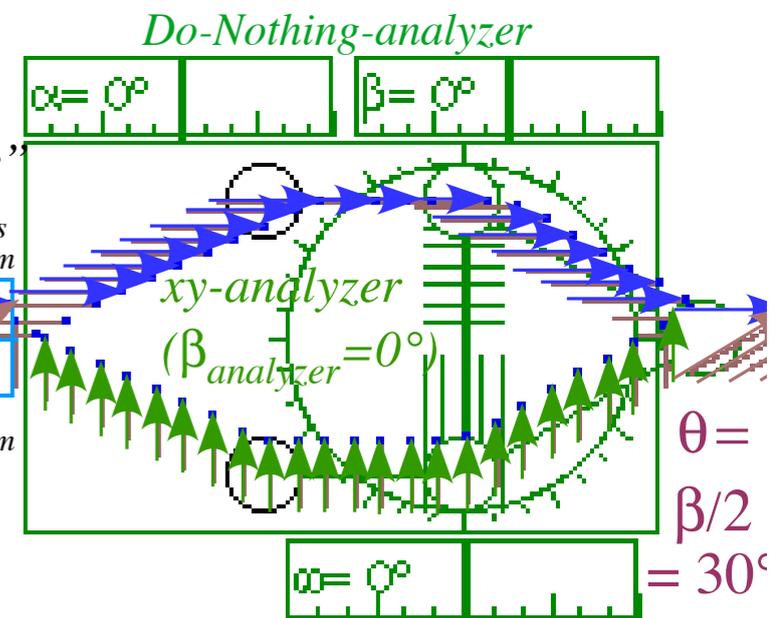
$$A(y') = \langle y'|x \rangle (1) \langle x|x' \rangle + \langle y'|y \rangle \langle y|x' \rangle$$

$$= -\frac{\sqrt{3}}{4}(1) + \frac{\sqrt{3}}{4} = 0 = P(y')$$



Without "Peeking Eye"

Reconstructs $x'(30^\circ)$ beam
Cancels $y'(30^\circ)$ beam



$\theta = \beta/2 = 30^\circ$

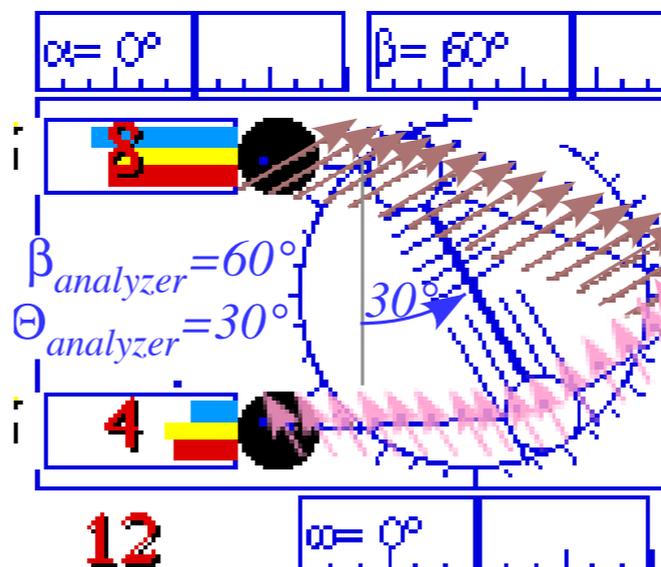
Amplitude $A(n')$ and Probability $P(n')$ at counter n' WITH "peeking"

Suppose "x-eye" puts phase $e^{i\phi}$ on each x -photon with random ϕ distributed over unit circle ($-\pi < \phi < \pi$).

So $e^{i\phi}$ averages to zero!

$$A(x') = \langle x'|x \rangle (e^{i\phi}) \langle x|x' \rangle + \langle x'|y \rangle \langle y|x' \rangle$$

$$= \frac{3}{4}(e^{i\phi}) + \frac{1}{4}$$

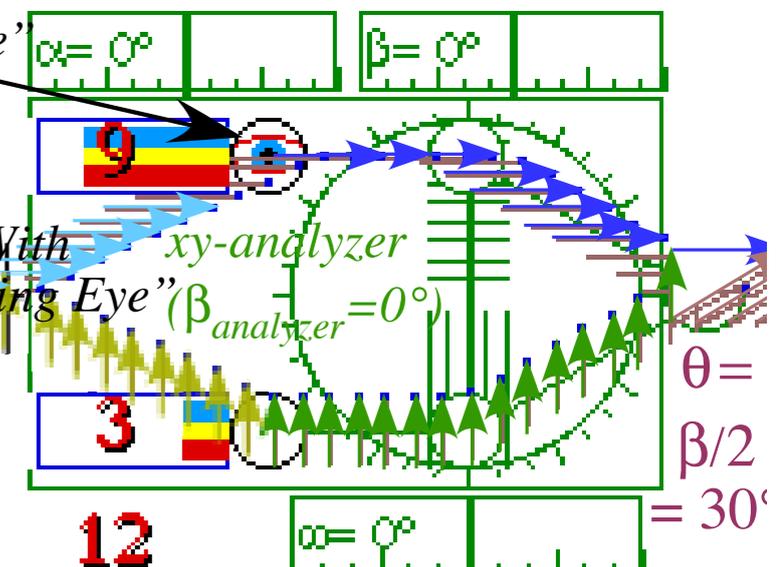


With "Peeking Eye"

0.75

0.25

1



$\theta = \beta/2 = 30^\circ$

$$\begin{pmatrix} \langle x|x' \rangle & \langle x|y' \rangle \\ \langle y|x' \rangle & \langle y|y' \rangle \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix}$$

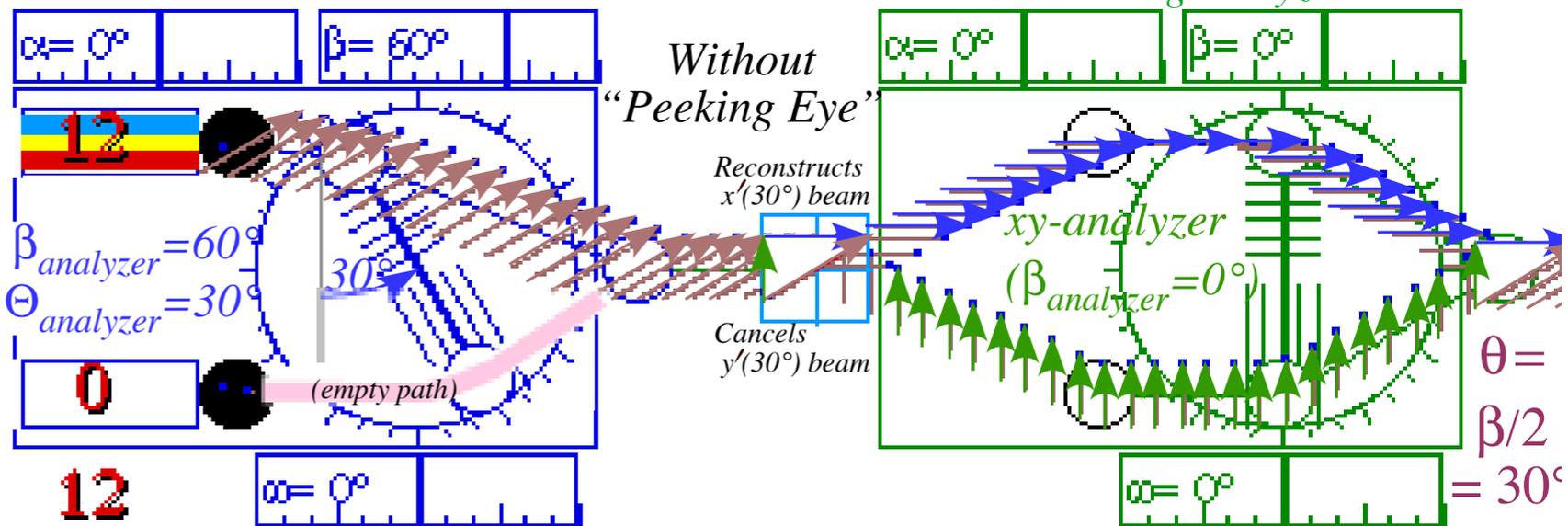
Amplitude $A(n')$ and Probability $P(n')$ at counter n' WITHOUT "peeking"

$$A(x') = \langle x'|x \rangle (1) \langle x|x' \rangle + \langle x'|y \rangle \langle y|x' \rangle$$

$$= \frac{3}{4}(1) + \frac{1}{4} = 1 = P(x')$$

$$A(y') = \langle y'|x \rangle (1) \langle x|x' \rangle + \langle y'|y \rangle \langle y|x' \rangle$$

$$= -\frac{\sqrt{3}}{4}(1) + \frac{\sqrt{3}}{4} = 0 = P(y')$$



Amplitude $A(n')$ and Probability $P(n')$ at counter n' WITH "peeking"

Suppose "x-eye" puts phase $e^{i\phi}$ on each x -photon with random ϕ distributed over unit circle ($-\pi < \phi < \pi$).

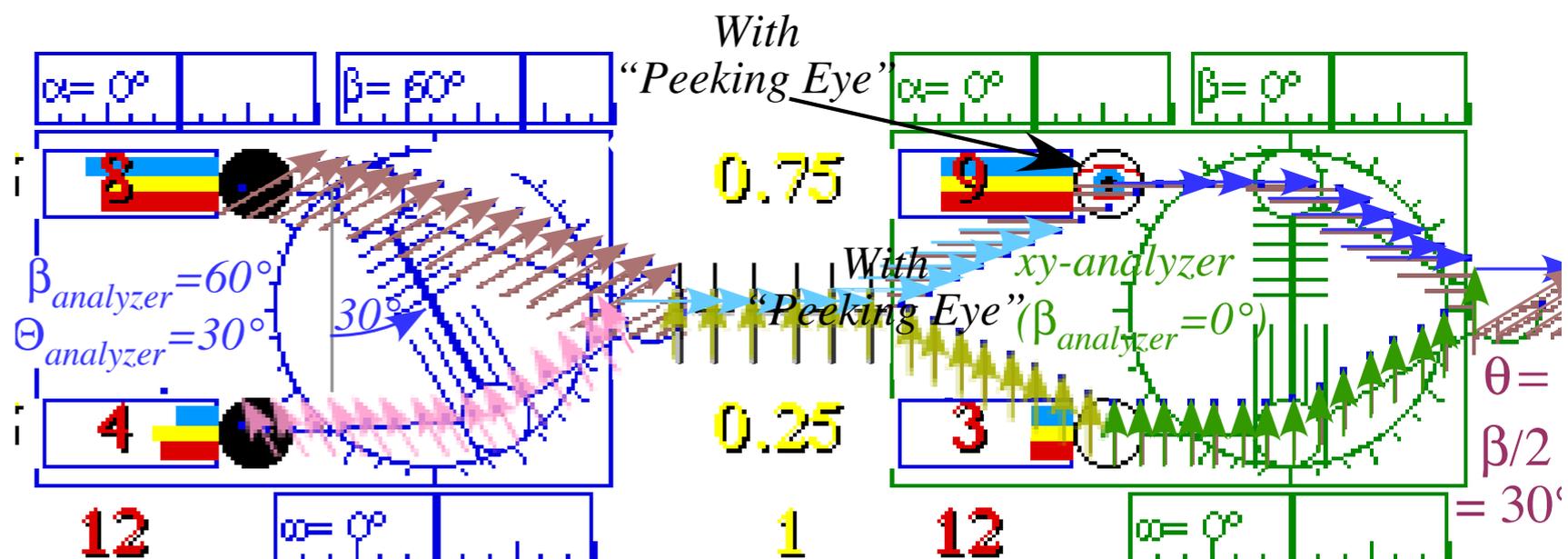
So $e^{i\phi}$ averages to zero!

$$A(x') = \langle x'|x \rangle (e^{i\phi}) \langle x|x' \rangle + \langle x'|y \rangle \langle y|x' \rangle$$

$$= \frac{3}{4}(e^{i\phi}) + \frac{1}{4}$$

$$P(x') = \left(\frac{3}{4}(e^{i\phi}) + \frac{1}{4} \right) \left(\frac{3}{4}(e^{-i\phi}) + \frac{1}{4} \right)$$

$$= \frac{5}{8} + \frac{3}{16}(e^{-i\phi} + e^{i\phi}) = \frac{5 + 3\cos\phi}{8}$$



$$\begin{pmatrix} \langle x|x' \rangle & \langle x|y' \rangle \\ \langle y|x' \rangle & \langle y|y' \rangle \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix}$$

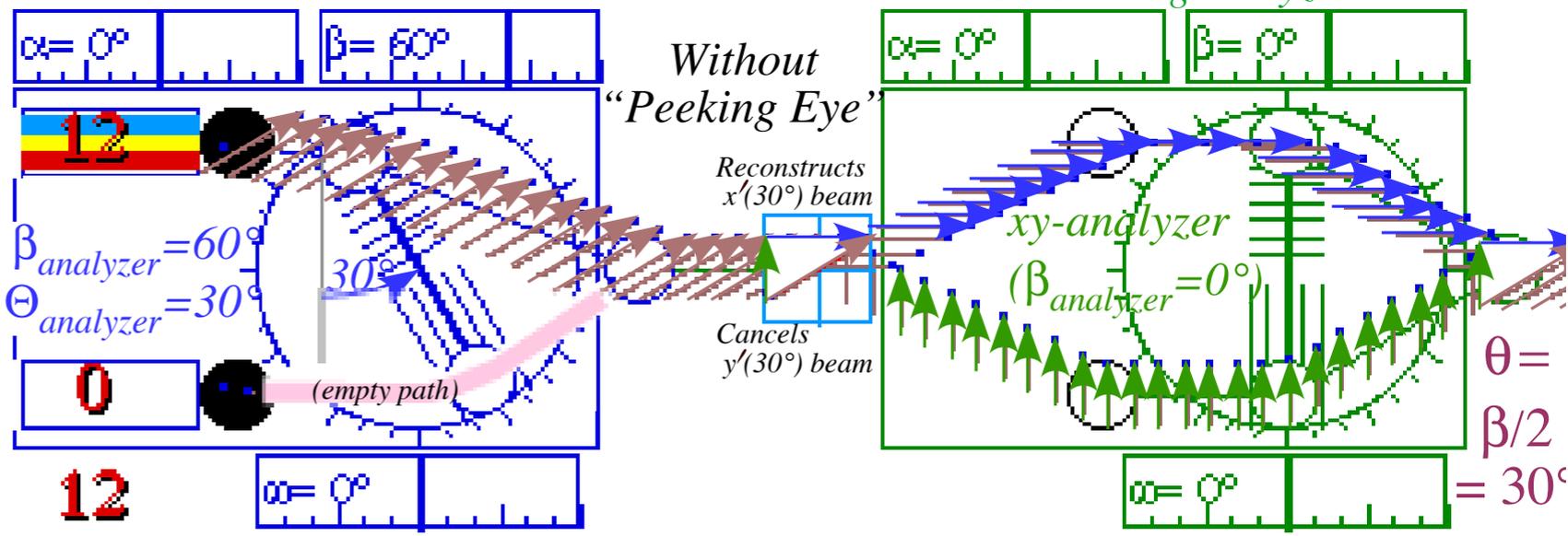
Amplitude $A(n')$ and Probability $P(n')$ at counter n' WITHOUT "peeking"

$$A(x') = \langle x'|x \rangle (1) \langle x|x' \rangle + \langle x'|y \rangle \langle y|x' \rangle$$

$$= \frac{3}{4}(1) + \frac{1}{4} = 1 = P(x')$$

$$A(y') = \langle y'|x \rangle (1) \langle x|x' \rangle + \langle y'|y \rangle \langle y|x' \rangle$$

$$= -\frac{\sqrt{3}}{4}(1) + \frac{\sqrt{3}}{4} = 0 = P(y')$$



Amplitude $A(n')$ and Probability $P(n')$ at counter n' WITH "peeking"

Suppose "x-eye" puts phase $e^{i\phi}$ on each x-photon with random ϕ distributed over unit circle ($-\pi < \phi < \pi$).

So $e^{i\phi}$ averages to zero!

$$A(x') = \langle x'|x \rangle (e^{i\phi}) \langle x|x' \rangle + \langle x'|y \rangle \langle y|x' \rangle$$

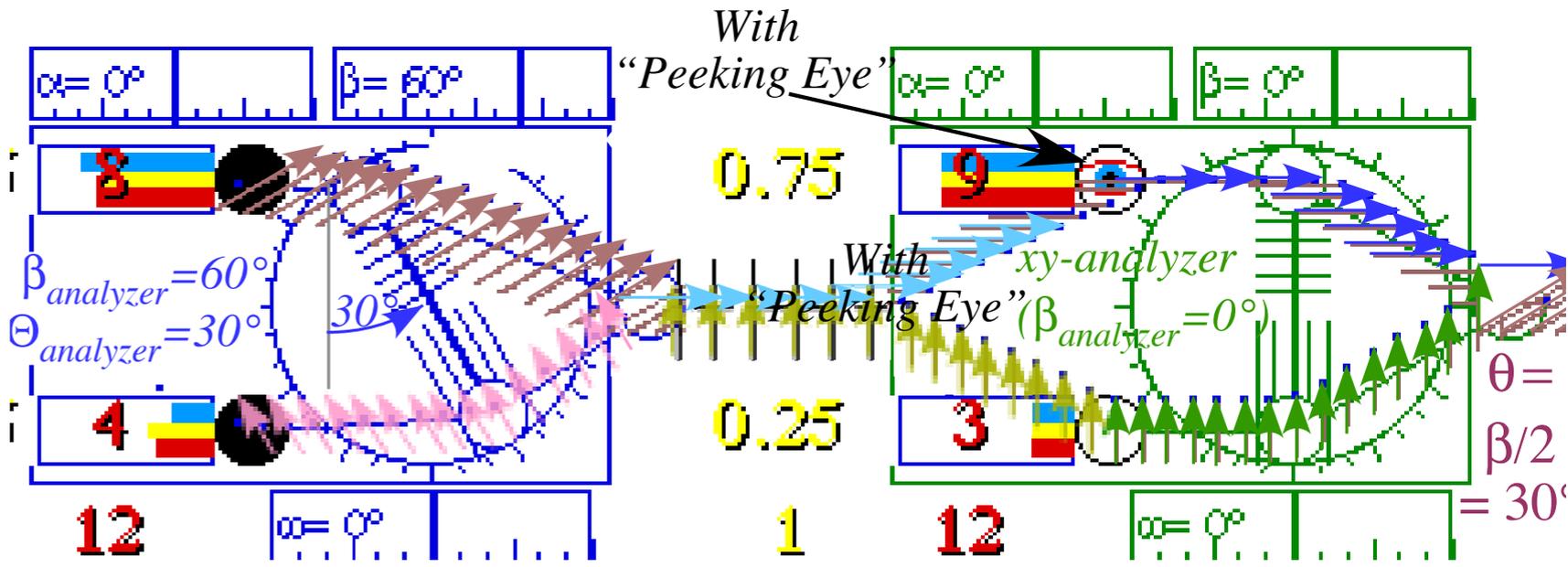
$$= \frac{3}{4}(e^{i\phi}) + \frac{1}{4}$$

$$P(x') = \left(\frac{3}{4}(e^{i\phi}) + \frac{1}{4} \right) \left(\frac{3}{4}(e^{-i\phi}) + \frac{1}{4} \right)$$

$$= \frac{5}{8} + \frac{3}{16}(e^{-i\phi} + e^{i\phi}) = \frac{5 + 3\cos\phi}{8}$$

$$A(y') = \langle y'|x \rangle (e^{i\phi}) \langle x|x' \rangle + \langle y'|y \rangle \langle y|x' \rangle$$

$$= -\frac{\sqrt{3}}{4}(e^{i\phi}) + \frac{\sqrt{3}}{4}$$



$$\begin{pmatrix} \langle x|x' \rangle & \langle x|y' \rangle \\ \langle y|x' \rangle & \langle y|y' \rangle \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix}$$

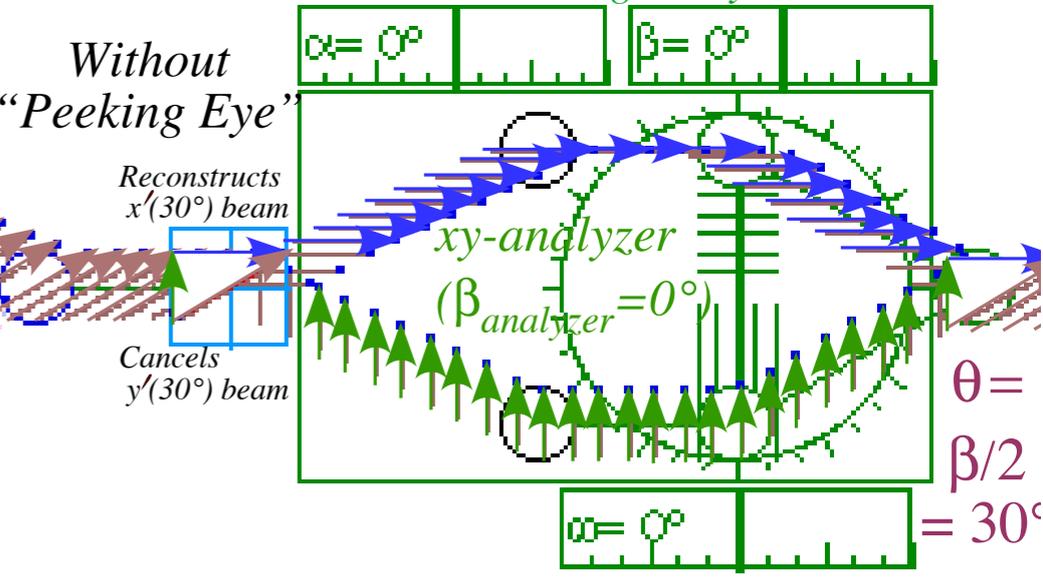
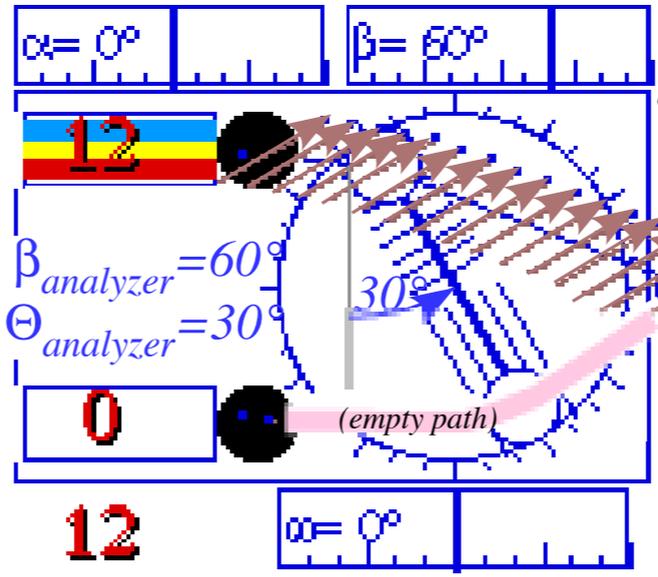
Amplitude $A(n')$ and Probability $P(n')$ at counter n' WITHOUT "peeking"

$$A(x') = \langle x'|x \rangle (1) \langle x|x' \rangle + \langle x'|y \rangle \langle y|x' \rangle$$

$$= \frac{3}{4}(1) + \frac{1}{4} = 1 = P(x')$$

$$A(y') = \langle y'|x \rangle (1) \langle x|x' \rangle + \langle y'|y \rangle \langle y|x' \rangle$$

$$= -\frac{\sqrt{3}}{4}(1) + \frac{\sqrt{3}}{4} = 0 = P(y')$$



Amplitude $A(n')$ and Probability $P(n')$ at counter n' WITH "peeking"

Suppose "x-eye" puts phase $e^{i\phi}$ on each x-photon with random ϕ distributed over unit circle ($-\pi < \phi < \pi$).

So $e^{i\phi}$ averages to zero!

$$A(x') = \langle x'|x \rangle (e^{i\phi}) \langle x|x' \rangle + \langle x'|y \rangle \langle y|x' \rangle$$

$$= \frac{3}{4}(e^{i\phi}) + \frac{1}{4}$$

$$P(x') = \left(\frac{3}{4}(e^{i\phi}) + \frac{1}{4} \right) \left(\frac{3}{4}(e^{-i\phi}) + \frac{1}{4} \right)$$

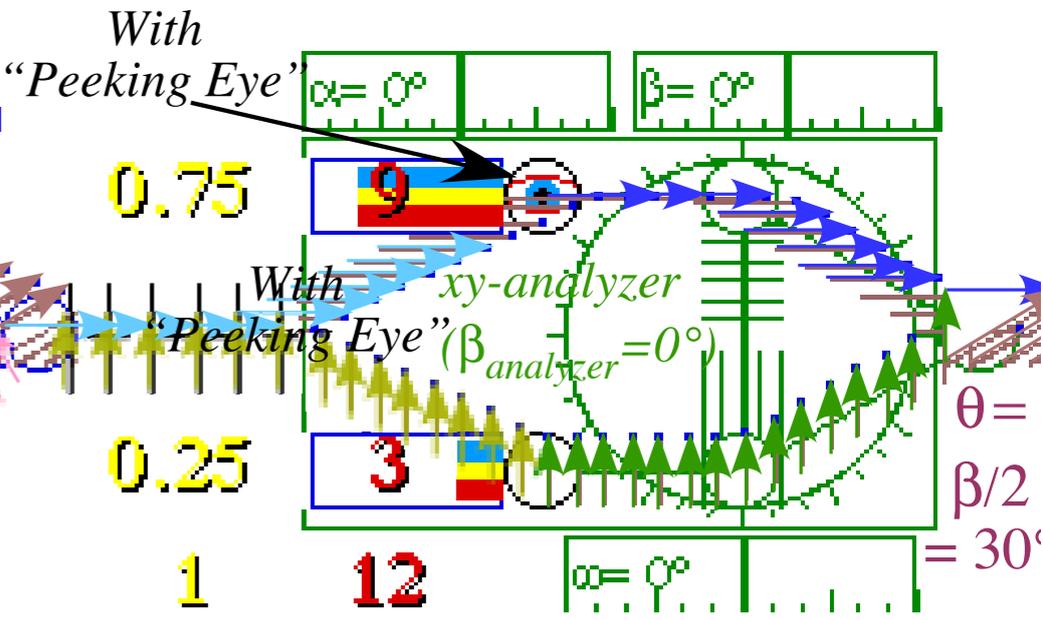
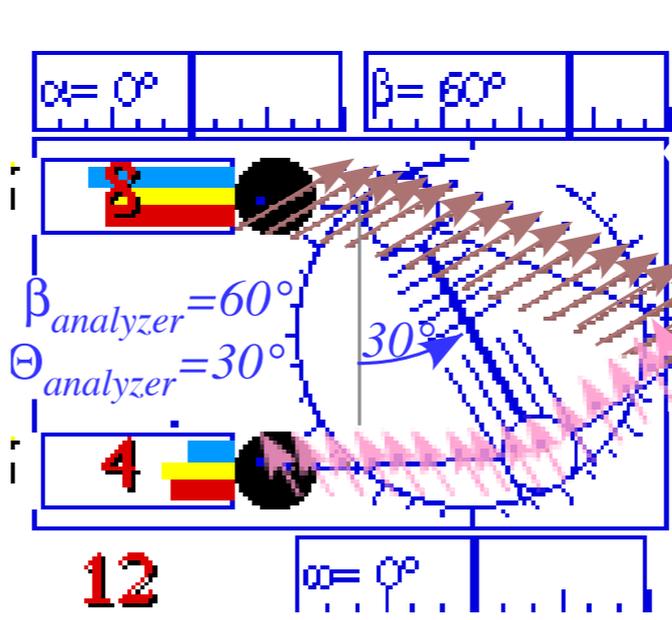
$$= \frac{5}{8} + \frac{3}{16}(e^{-i\phi} + e^{i\phi}) = \frac{5 + 3\cos\phi}{8}$$

$$A(y') = \langle y'|x \rangle (e^{i\phi}) \langle x|x' \rangle + \langle y'|y \rangle \langle y|x' \rangle$$

$$= -\frac{\sqrt{3}}{4}(e^{i\phi}) + \frac{\sqrt{3}}{4}$$

$$P(y') = \left(-\frac{\sqrt{3}}{4}(e^{i\phi}) + \frac{\sqrt{3}}{4} \right) \left(-\frac{\sqrt{3}}{4}(e^{-i\phi}) + \frac{\sqrt{3}}{4} \right)$$

$$= \frac{3}{8} - \frac{3}{16}(e^{-i\phi} + e^{i\phi}) = \frac{3 - 3\cos\phi}{8}$$



$$\begin{pmatrix} \langle x|x' \rangle & \langle x|y' \rangle \\ \langle y|x' \rangle & \langle y|y' \rangle \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix}$$

Review: Axioms 1-4 and “Do-Nothing” vs “Do-Something” analyzers

*Abstraction of Axiom-4 to define projection and unitary operators
Projection operators and resolution of identity*

*Unitary operators and matrices that do something (or “nothing”)
Diagonal unitary operators
Non-diagonal unitary operators and †-conjugation relations
Non-diagonal projection operators and Kronecker \otimes -products
Axiom-4 similarity transformation*

Matrix representation of beam analyzers

Non-unitary “killer” devices: Sorter-counter, filter

Unitary “non-killer” devices: 1/2-wave plate, 1/4-wave plate

How analyzers “peek” and how that changes outcomes



Peeking polarizers and coherence loss

Classical Bayesian probability vs. Quantum probability

*Feynman $\langle j|k \rangle$ -axioms compared to **Group axioms***

Classical Bayesian probability vs. Quantum probability

$$\begin{aligned}
 & \left(\begin{array}{l} \text{Probability that} \\ \text{photon in } x'\text{-input} \\ \text{becomes} \\ \text{photon in } x'\text{-counter} \end{array} \right)_{\text{classical}} = \\
 & \left(\begin{array}{l} \text{probability that} \\ \text{photon in } x\text{-beam} \\ \text{becomes} \\ \text{photon in } x'\text{-counter} \end{array} \right) \cdot \left(\begin{array}{l} \text{probability that} \\ \text{photon in } x'\text{-input} \\ \text{becomes} \\ \text{photon in } x\text{-beam} \end{array} \right) + \left(\begin{array}{l} \text{probability that} \\ \text{photon in } y\text{-beam} \\ \text{becomes} \\ \text{photon in } x'\text{-counter} \end{array} \right) \cdot \left(\begin{array}{l} \text{probability that} \\ \text{photon in } x'\text{-input} \\ \text{becomes} \\ \text{photon in } y\text{-beam} \end{array} \right) \\
 & \left(\begin{array}{l} \text{Probability that} \\ \text{photon in } x'\text{-input} \\ \text{becomes} \\ \text{photon in } x'\text{-counter} \end{array} \right)_{\text{classical}} = \left(|\langle x'|x \rangle|^2 \right) \cdot \left(|\langle x|x' \rangle|^2 \right) + \left(|\langle x'|y \rangle|^2 \right) \cdot \left(|\langle y|x' \rangle|^2 \right)
 \end{aligned}$$

Classical Bayesian probability vs. Quantum probability

$$\left(\begin{array}{l} \text{Probability that} \\ \text{photon in } x'\text{-input} \\ \text{becomes} \\ \text{photon in } x'\text{-counter} \end{array} \right)_{\text{classical}} = \left(\begin{array}{cc} \langle x|x' \rangle & \langle x|y' \rangle \\ \langle y|x' \rangle & \langle y|y' \rangle \end{array} \right) = \left(\begin{array}{cc} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{array} \right) = \left(\begin{array}{cc} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{array} \right)$$

$$\left(\begin{array}{l} \text{probability that} \\ \text{photon in } x\text{-beam} \\ \text{becomes} \\ \text{photon in } x'\text{-counter} \end{array} \right) \cdot \left(\begin{array}{l} \text{probability that} \\ \text{photon in } x'\text{-input} \\ \text{becomes} \\ \text{photon in } x\text{-beam} \end{array} \right) + \left(\begin{array}{l} \text{probability that} \\ \text{photon in } y\text{-beam} \\ \text{becomes} \\ \text{photon in } x'\text{-counter} \end{array} \right) \cdot \left(\begin{array}{l} \text{probability that} \\ \text{photon in } x'\text{-input} \\ \text{becomes} \\ \text{photon in } y\text{-beam} \end{array} \right)$$

$$\left(\begin{array}{l} \text{Probability that} \\ \text{photon in } x'\text{-input} \\ \text{becomes} \\ \text{photon in } x'\text{-counter} \end{array} \right)_{\text{classical}} = \left(|\langle x'|x \rangle|^2 \right) \cdot \left(|\langle x|x' \rangle|^2 \right) + \left(|\langle x'|y \rangle|^2 \right) \cdot \left(|\langle y|x' \rangle|^2 \right) = \left(\left| \frac{\sqrt{3}}{2} \right|^2 \right) \cdot \left(\left| \frac{\sqrt{3}}{2} \right|^2 \right) + \left(\left| \frac{-1}{2} \right|^2 \right) \cdot \left(\left| \frac{1}{2} \right|^2 \right) = \frac{5}{8}$$

Classical Bayesian probability vs. Quantum probability

$$\left(\begin{array}{l} \text{Probability that} \\ \text{photon in } x'\text{-input} \\ \text{becomes} \\ \text{photon in } x'\text{-counter} \end{array} \right)_{\text{classical}} = \left(\begin{array}{cc} \langle x|x' \rangle & \langle x|y' \rangle \\ \langle y|x' \rangle & \langle y|y' \rangle \end{array} \right) = \left(\begin{array}{cc} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{array} \right) = \left(\begin{array}{cc} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{array} \right)$$

$$\left(\begin{array}{l} \text{probability that} \\ \text{photon in } x\text{-beam} \\ \text{becomes} \\ \text{photon in } x'\text{-counter} \end{array} \right) \cdot \left(\begin{array}{l} \text{probability that} \\ \text{photon in } x'\text{-input} \\ \text{becomes} \\ \text{photon in } x\text{-beam} \end{array} \right) + \left(\begin{array}{l} \text{probability that} \\ \text{photon in } y\text{-beam} \\ \text{becomes} \\ \text{photon in } x'\text{-counter} \end{array} \right) \cdot \left(\begin{array}{l} \text{probability that} \\ \text{photon in } x'\text{-input} \\ \text{becomes} \\ \text{photon in } y\text{-beam} \end{array} \right)$$

$$\left(\begin{array}{l} \text{Probability that} \\ \text{photon in } x'\text{-input} \\ \text{becomes} \\ \text{photon in } x'\text{-counter} \end{array} \right)_{\text{classical}} = \left(|\langle x'|x \rangle|^2 \right) \cdot \left(|\langle x|x' \rangle|^2 \right) + \left(|\langle x'|y \rangle|^2 \right) \cdot \left(|\langle y|x' \rangle|^2 \right) = \left(\left| \frac{\sqrt{3}}{2} \right|^2 \right) \cdot \left(\left| \frac{\sqrt{3}}{2} \right|^2 \right) + \left(\left| \frac{-1}{2} \right|^2 \right) \cdot \left(\left| \frac{1}{2} \right|^2 \right) = \frac{5}{8}$$

$$\left(\begin{array}{l} \text{Quantum probability} \\ \text{at } x'\text{-counter} \end{array} \right) = \left| \langle x'|x \rangle (e^{i\phi}) \langle x|x' \rangle + \langle x'|y \rangle \langle y|x' \rangle \right|^2$$

Classical Bayesian probability vs. Quantum probability

$$\left(\begin{array}{l} \text{Probability that} \\ \text{photon in } x'\text{-input} \\ \text{becomes} \\ \text{photon in } x'\text{-counter} \end{array} \right)_{\text{classical}} = \begin{pmatrix} \langle x|x' \rangle & \langle x|y' \rangle \\ \langle y|x' \rangle & \langle y|y' \rangle \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix}$$

$$\left(\begin{array}{l} \text{probability that} \\ \text{photon in } x\text{-beam} \\ \text{becomes} \\ \text{photon in } x'\text{-counter} \end{array} \right) \cdot \left(\begin{array}{l} \text{probability that} \\ \text{photon in } x'\text{-input} \\ \text{becomes} \\ \text{photon in } x\text{-beam} \end{array} \right) + \left(\begin{array}{l} \text{probability that} \\ \text{photon in } y\text{-beam} \\ \text{becomes} \\ \text{photon in } x'\text{-counter} \end{array} \right) \cdot \left(\begin{array}{l} \text{probability that} \\ \text{photon in } x'\text{-input} \\ \text{becomes} \\ \text{photon in } y\text{-beam} \end{array} \right)$$

$$\left(\begin{array}{l} \text{Probability that} \\ \text{photon in } x'\text{-input} \\ \text{becomes} \\ \text{photon in } x'\text{-counter} \end{array} \right)_{\text{classical}} = \left(|\langle x'|x \rangle|^2 \right) \cdot \left(|\langle x|x' \rangle|^2 \right) + \left(|\langle x'|y \rangle|^2 \right) \cdot \left(|\langle y|x' \rangle|^2 \right) = \left(\left| \frac{\sqrt{3}}{2} \right|^2 \right) \cdot \left(\left| \frac{\sqrt{3}}{2} \right|^2 \right) + \left(\left| \frac{-1}{2} \right|^2 \right) \cdot \left(\left| \frac{1}{2} \right|^2 \right) = \frac{5}{8}$$

$$\left(\begin{array}{l} \text{Quantum probability} \\ \text{at } x'\text{-counter} \end{array} \right) = \left| \langle x'|x \rangle (e^{i\phi}) \langle x|x' \rangle + \langle x'|y \rangle \langle y|x' \rangle \right|^2$$

$$= \left| \langle x'|x \rangle \langle x|x' \rangle \right|^2 + \left| \langle x'|y \rangle \langle y|x' \rangle \right|^2 + e^{-i\phi} \langle x'|x \rangle^* \langle x|x' \rangle^* \langle x'|y \rangle \langle y|x' \rangle + e^{i\phi} \langle x'|x \rangle \langle x|x' \rangle \langle x'|y \rangle^* \langle y|x' \rangle^* = 1$$

$$= \left(\text{classical probability} \right) + \left(\text{Phase-sensitive or } \textit{quantum interference} \text{ terms} \right)$$

Review: Axioms 1-4 and “Do-Nothing” vs “Do-Something” analyzers

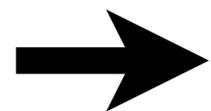
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Classical Bayesian probability vs. Quantum probability

*Feynman $\langle j|k \rangle$ -axioms compared to **Group axioms***

Classical Bayesian probability vs. Quantum probability

$$\left(\begin{array}{l} \text{Probability that} \\ \text{photon in } x'\text{-input} \\ \text{becomes} \\ \text{photon in } x'\text{-counter} \end{array} \right)_{\text{classical}} = \begin{pmatrix} \langle x|x' \rangle & \langle x|y' \rangle \\ \langle y|x' \rangle & \langle y|y' \rangle \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix}$$

$$\left(\begin{array}{l} \text{probability that} \\ \text{photon in } x\text{-beam} \\ \text{becomes} \\ \text{photon in } x'\text{-counter} \end{array} \right) * \left(\begin{array}{l} \text{probability that} \\ \text{photon in } x'\text{-input} \\ \text{becomes} \\ \text{photon in } x\text{-beam} \end{array} \right) + \left(\begin{array}{l} \text{probability that} \\ \text{photon in } y\text{-beam} \\ \text{becomes} \\ \text{photon in } x'\text{-counter} \end{array} \right) * \left(\begin{array}{l} \text{probability that} \\ \text{photon in } x'\text{-input} \\ \text{becomes} \\ \text{photon in } y\text{-beam} \end{array} \right)$$

$$\left(\begin{array}{l} \text{Probability that} \\ \text{photon in } x'\text{-input} \\ \text{becomes} \\ \text{photon in } x'\text{-counter} \end{array} \right)_{\text{classical}} = \left(|\langle x'|x \rangle|^2 \right) * \left(|\langle x|x' \rangle|^2 \right) + \left(|\langle x'|y \rangle|^2 \right) * \left(|\langle y|x' \rangle|^2 \right) = \left(\left| \frac{\sqrt{3}}{2} \right|^2 \right) * \left(\left| \frac{\sqrt{3}}{2} \right|^2 \right) + \left(\left| \frac{-1}{2} \right|^2 \right) * \left(\left| \frac{1}{2} \right|^2 \right) = \frac{5}{8}$$

$$\left(\begin{array}{l} \text{Quantum probability} \\ \text{at } x'\text{-counter} \end{array} \right) = \left| \langle x'|x \rangle (e^{i\phi}) \langle x|x' \rangle + \langle x'|y \rangle \langle y|x' \rangle \right|^2$$

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Square of sum

Classical Bayesian probability vs. Quantum probability

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$$\left(\begin{array}{l} \text{Probability that} \\ \text{photon in } x'\text{-input} \\ \text{becomes} \\ \text{photon in } x'\text{-counter} \end{array} \right)_{\text{classical}} = \left(|\langle x'|x \rangle|^2 \right) * \left(|\langle x|x' \rangle|^2 \right) + \left(|\langle x'|y \rangle|^2 \right) * \left(|\langle y|x' \rangle|^2 \right) = \left(\left| \frac{\sqrt{3}}{2} \right|^2 \right) * \left(\left| \frac{\sqrt{3}}{2} \right|^2 \right) + \left(\left| \frac{-1}{2} \right|^2 \right) * \left(\left| \frac{1}{2} \right|^2 \right) = \frac{5}{8}$$

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$$= \left(\text{classical probability} \right) + \left(\text{Phase-sensitive or } \textit{quantum interference} \text{ terms} \right)$$

$\left(\begin{array}{l} \text{Quantum probability} \\ \text{at } x'\text{-counter} \end{array} \right) = \left \langle x' x \rangle \langle x x' \rangle + \langle x' y \rangle \langle y x' \rangle \right ^2$ <p style="text-align: center; color: blue;"><i>Square of sum</i></p>	$\left(\begin{array}{l} \text{Classical probability} \\ \text{at } x'\text{-counter} \end{array} \right) = \left \langle x' x \rangle \langle x x' \rangle \right ^2 + \left \langle x' y \rangle \langle y x' \rangle \right ^2$ <p style="text-align: center; color: blue;"><i>Sum of squares</i></p>
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Review: Axioms 1-4 and “Do-Nothing” vs “Do-Something” analyzers

*Abstraction of Axiom-4 to define projection and unitary operators
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<p><i>Axiom 1: $j'' \Leftrightarrow m'$ probability equals $\langle j'' m'\rangle ^2 = \langle m' j''\rangle ^2$</i> (Probability)</p>	<p><i>Axiom 2: $\langle j'' m'\rangle^* = \langle m' j''\rangle$</i> (T-reversal Conjugation)</p>	<p><i>Axiom 3: $\langle j k\rangle = \delta_{jk} = \langle j' k'\rangle = \langle j'' k''\rangle$</i> (Orthonormality)</p>	<p><i>Axiom 4: $\langle j'' m'\rangle = \sum_{k=1}^n \langle j'' k\rangle \langle k m'\rangle$</i> (Completeness)</p>
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Group axioms

(1) The closure axiom

Products $ab = c$ are defined between any two group elements a and b , and the result c is contained in the group.

(2) The associativity axiom

Products $(ab)c$ and $a(bc)$ are equal for all elements a , b , and c in the group .

(3) The identity axiom

There is a unique element 1 (the identity) such that $1 \cdot a = a = a \cdot 1$ for all elements a in the group ..

4) The inverse axiom

For all elements a in the group there is an inverse element a^{-1} such that $a^{-1}a = 1 = a \cdot a^{-1}$.

<p><i>Axiom 1: $j'' \Leftrightarrow m'$ probability equals $\langle j'' m' \rangle ^2 = \langle m' j'' \rangle ^2$</i> (Probability)</p>	<p><i>Axiom 2: $\langle j'' m' \rangle^* = \langle m' j'' \rangle$</i> (T-reversal Conjugation)</p>	<p><i>Axiom 3: $\langle j k \rangle = \delta_{jk} = \langle j' k' \rangle = \langle j'' k'' \rangle$</i> (Orthonormality)</p>	<p><i>Axiom 4: $\langle j'' m' \rangle = \sum_{k=1}^n \langle j'' k \rangle \langle k m' \rangle$</i> (Completeness)</p>
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Group axioms

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Products $ab = c$ are defined between any two group elements a and b , and the result c is contained in the group.

*Feynman Axiom-4 consistent with group axiom 1 since analyzer-**A** following analyzer-**B** is analyzer-**AB** = **C** and analyzer-**B** following analyzer-**A** is analyzer-**BA** = **D***

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*Feynman Axiom-3 consistent with **group axiom 4** since inverse **U** =transpose-conjugate **U[†] = U^{T*}***

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Feynman Axiom-3 consistent with group axiom 4 since inverse U =transpose-conjugate $U^\dagger = U^{T}$*

(5) The commutative axiom (Abelian groups only)

All elements a in an Abelian group are mutually commuting: $a \cdot b = b \cdot a$.

Most analyzer sets (and most groups) are not Abelian (commutative)