Group Theory in Quantum Mechanics

Lecture 2 *(1.15.15)*

Quantum amplitudes, analyzers, and axioms

(Quantum Theory for Computer Age - Ch. 1 of Unit 1)
(Principles of Symmetry, Dynamics, and Spectroscopy - Sec. 1-2 of Ch. 1)

Review: *"Abstraction" of bra and ket vectors from a Transformation Matrix*
- *Introducing scalar and matrix products*

Planck's energy and N-quanta (Cavity/Beam wave mode)
- *Did Max Planck Goof? What's 1-photon worth?*
- *Feynman amplitude axiom 1*

What comes out of a beam sorter channel or branch-b?
- *Sample calculations*
- *Feynman amplitude axioms 2-3*

Beam analyzers: Sorter-unsorters
- *The "Do-Nothing" analyzer*
- *Feynman amplitude axiom 4*
- *Some "Do-Something" analyzers*
  - *Sorter-counter, Filter, 1/2-wave plate, 1/4-wave plate*

http://www.uark.edu/ua/modphys/pdfs/QTCA_Pdfs/QTCA_Text_2013/QTCA_Unit_1_Ch._1_2013.pdf

http://www.uark.edu/ua/modphys/pdfs/QTCA_Pdfs/QTCA_Text_2013/QTCA_Unit_7_Ch._22_2005.pdf

See also 2005 site:

http://www.uark.edu/ua/pirelli/php/light_energy_flux_1.php
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"Abstraction" of bra and ket vectors from a Transformation Matrix

Ket or column vectors

Bra or row vectors

Given Transformation Matrix $T_{m,n'}$: $\begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

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$\Downarrow$

$$|x'\rangle = \begin{pmatrix} \langle x|x'\rangle \\ \langle y|x'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

Abstracting ket $|\,n'\rangle$ state vectors

from

Transformation Matrix

$T_{m,n'} = \langle m|\, n'\rangle$

“Abstraction” of bra and ket vectors from a Transformation Matrix

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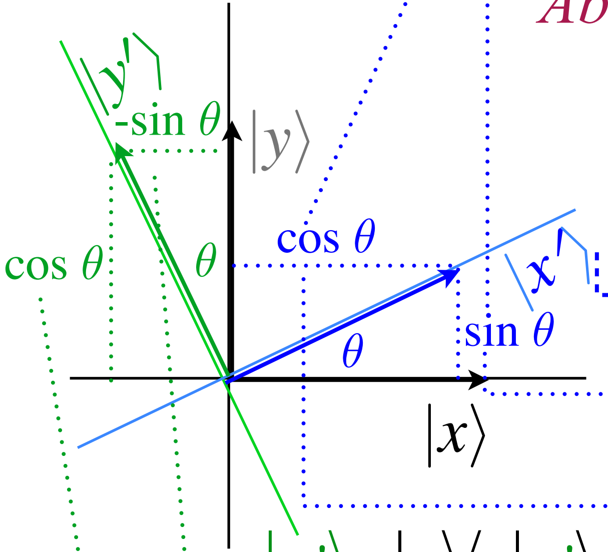
$$|x'\rangle=\begin{pmatrix}\langle x|x'\rangle\\ \langle y|x'\rangle\end{pmatrix}=\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix},\ |y'\rangle=\begin{pmatrix}\langle x|y'\rangle\\ \langle y|y'\rangle\end{pmatrix}=\begin{pmatrix}-\sin\theta\\ \cos\theta\end{pmatrix}$$

Abstracting ket $|n'\rangle$ state vectors
from
Transformation Matrix
$T_{m,n'}=\langle m|n'\rangle$



$$|x'\rangle=|x\rangle\langle x|x'\rangle+|y\rangle\langle y|x'\rangle$$
$$=|x\rangle(\cos\theta)+|y\rangle(\sin\theta)$$

$$|y'\rangle=|x\rangle\langle x|y'\rangle+|y\rangle\langle y|y'\rangle$$
$$=|x\rangle(-\sin\theta)+|y\rangle(\cos\theta)$$

(θ=+30°)-Rotated kets $\{|\mathbf{x}'\rangle,|\mathbf{y}'\rangle\}$ or $\{\mathbf{x}',\mathbf{y}'\}$ represented in page-aligned $\{|\mathbf{x}\rangle,|\mathbf{y}\rangle\}$ basis.

Ket vector algebra has the order of $T_{m,n'}$ transposed

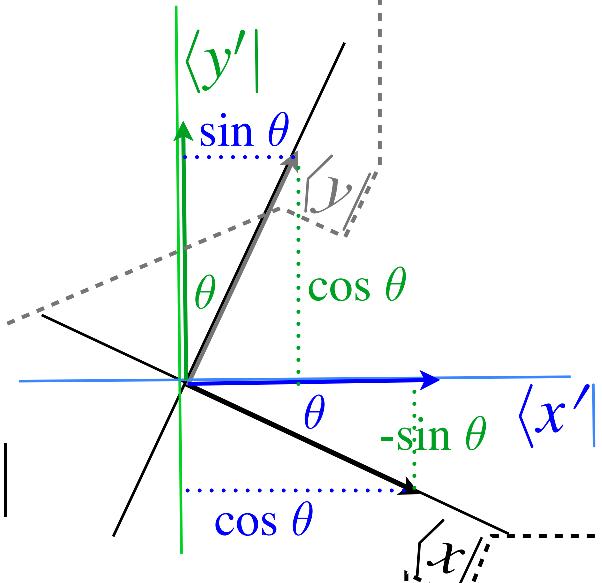
$$|x'\rangle=|x\rangle\langle x|x'\rangle+|y\rangle\langle y|x'\rangle=|x\rangle(\cos\theta)+|y\rangle(\sin\theta)$$
$$|y'\rangle=|x\rangle\langle x|y'\rangle+|y\rangle\langle y|y'\rangle=|x\rangle(-\sin\theta)+|y\rangle(\cos\theta)$$

Bra or row vectors

$$\langle x|=\begin{pmatrix}\langle x|x'\rangle & \langle x|y'\rangle\end{pmatrix}=\begin{pmatrix}\cos\theta & -\sin\theta\end{pmatrix}$$
$$\langle y|=\begin{pmatrix}\langle y|x'\rangle & \langle y|y'\rangle\end{pmatrix}=\begin{pmatrix}\sin\theta & \cos\theta\end{pmatrix}$$

Abstracting bra $\langle m|$ state vectors
from
Transformation Matrix
$T_{m,n'}=\langle m|n'\rangle$

$$\langle y|=\langle y|x'\rangle\langle x'|+\langle y|y'\rangle\langle y'|$$
$$=(\sin\theta)\langle x'|+(\cos\theta)\langle y'|$$



$$\langle x|=\langle x|x'\rangle\langle x'|+\langle x|y'\rangle\langle y'|$$
$$=(\cos\theta)\langle x'|+(-\sin\theta)\langle y'|$$

(θ=-30°)-Rotated bras $\{\langle\mathbf{x}|,\langle\mathbf{y}|\}$ or $\{\mathbf{x},\mathbf{y}\}$ represented in page-aligned $\{|\mathbf{x}'\rangle,|\mathbf{y}'\rangle\}$ basis.

Bra vector algebra has the same order as $T_{m,n'}$

$$\langle x|=\langle x|x'\rangle\langle x'|+\langle x|y'\rangle\langle y'|=(\cos\theta)\langle x'|+(-\sin\theta)\langle y'|$$
$$\langle y|=\langle y|x'\rangle\langle x'|+\langle y|y'\rangle\langle y'|=(\sin\theta)\langle x'|+(\cos\theta)\langle y'|$$

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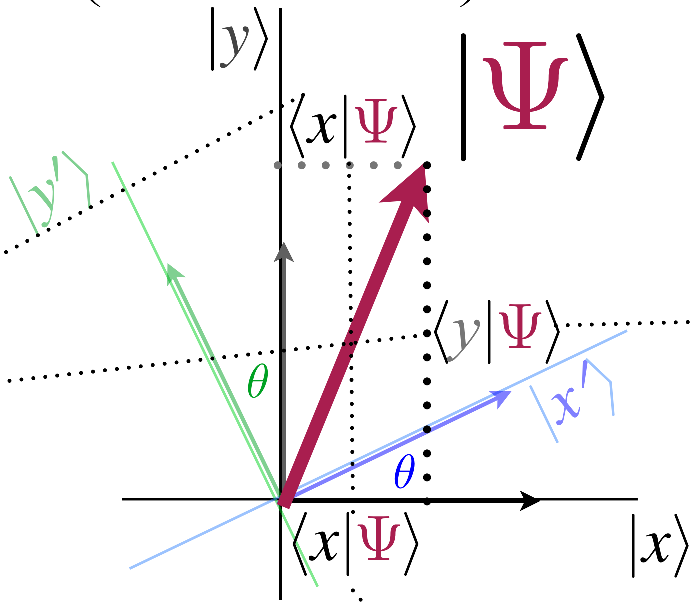
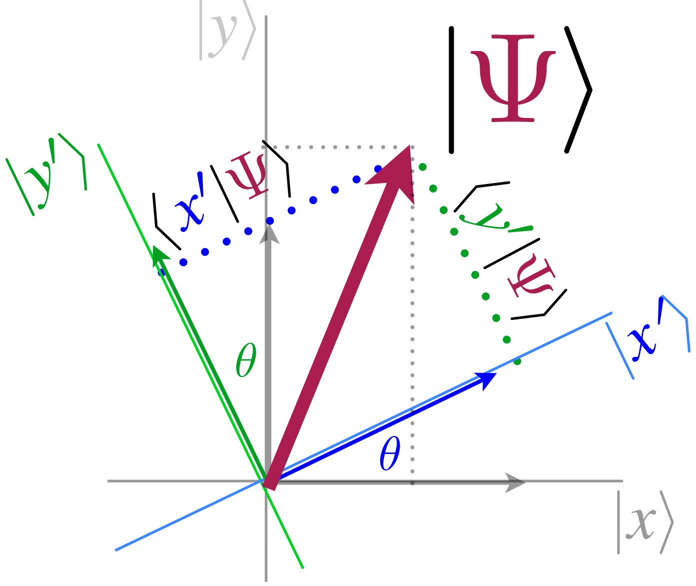
Transformation matrix $T_{m,n'}=\langle m|\,n'\rangle$ is array of dot or *scalar products* (dot products) of unit vectors or *direction cosines*.

$$\begin{pmatrix}\langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle\end{pmatrix}=\begin{pmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{pmatrix}=\begin{pmatrix}(\mathbf{x}\bullet\mathbf{x'}) & (\mathbf{x}\bullet\mathbf{y'}) \\ (\mathbf{y}\bullet\mathbf{x'}) & (\mathbf{y}\bullet\mathbf{y'})\end{pmatrix}$$

$\{\langle x|,\langle y|\}$
components
of $|\Psi\rangle$:
$\langle x|\Psi\rangle=\Psi_x$
$\langle y|\Psi\rangle=\Psi_y$





$\{\langle x'|,\langle y'|\}$
components
of $|\Psi\rangle$:
$\langle x'|\Psi\rangle=\Psi_{x'}$
$\langle y'|\Psi\rangle=\Psi_{y'}$

Any state $|\Psi\rangle$ can be expanded in any basis $\{\langle x|,\langle y|\}$, or $\{\langle x'|,\langle y'|\}$, ...*etc*.

$$|\Psi\rangle=|x\rangle\langle x|\Psi\rangle\ +\ |y\rangle\langle y|\Psi\rangle\ =\ |x'\rangle\langle x'|\Psi\rangle\ +\ |y'\rangle\langle y'|\Psi\rangle$$

Transformation matrix $T_{m,n'}$ relates $\{\langle x|\Psi\rangle,\langle y|\Psi\rangle\}$ amplitudes to $\{\langle x'|\Psi\rangle,\langle y'|\Psi\rangle\}$.

$$\begin{pmatrix}\langle x|\Psi\rangle \\ \langle y|\Psi\rangle\end{pmatrix}=\begin{pmatrix}\langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle\end{pmatrix}\begin{pmatrix}\langle x'|\Psi\rangle \\ \langle y'|\Psi\rangle\end{pmatrix}\quad \text{or:}\quad \begin{pmatrix}\Psi_x \\ \Psi_y\end{pmatrix}=\begin{pmatrix}\langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle\end{pmatrix}\begin{pmatrix}\Psi_{x'} \\ \Psi_{y'}\end{pmatrix}$$

Hybrid
Gibbs-Dirac
notation
(Ug-ly!)

Proof: $\langle x|=\langle x|x'\rangle\langle x'|+\langle x|y'\rangle\langle y'|$ implies: $\langle x|\Psi\rangle=\langle x|x'\rangle\langle x'|\Psi\rangle+\langle x|y'\rangle\langle y'|\Psi\rangle$

$\langle y|=\langle y|x'\rangle\langle x'|+\langle y|y'\rangle\langle y'|$ implies: $\langle y|\Psi\rangle=\langle y|x'\rangle\langle x'|\Psi\rangle+\langle y|y'\rangle\langle y'|\Psi\rangle$

Transformation matrix $T_{m,n'}=\langle m|\,n'\rangle$ is array of dot or *scalar products* (dot products) of unit vectors or *direction cosines*.

$$\begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} (\mathbf{x}\bullet\mathbf{x'}) & (\mathbf{x}\bullet\mathbf{y'}) \\ (\mathbf{y}\bullet\mathbf{x'}) & (\mathbf{y}\bullet\mathbf{y'}) \end{pmatrix}$$

$\{\langle x|,\langle y|\}$
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$\langle x|\Psi\rangle=\Psi_x$
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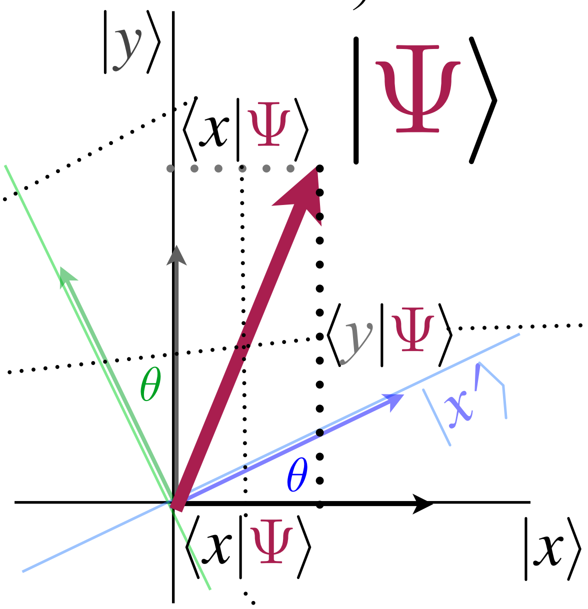

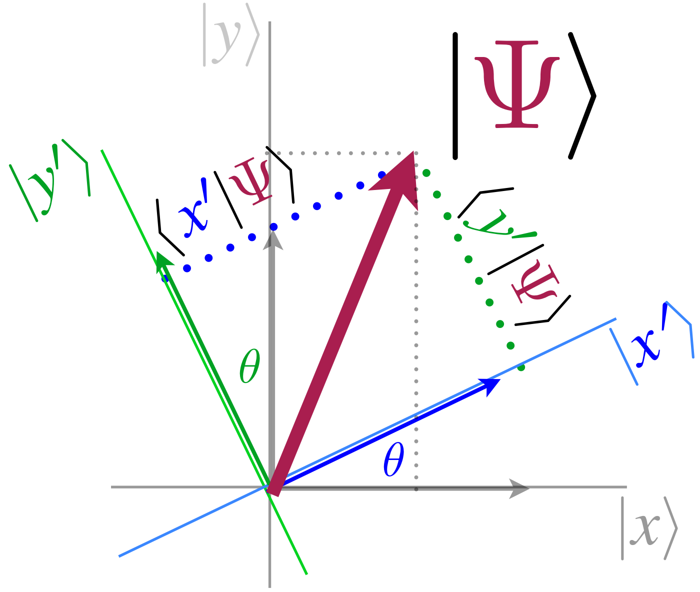

$\{\langle x'|,\langle y'|\}$
components
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$\langle x'|\Psi\rangle=\Psi_{x'}$
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Any state $|\Psi\rangle$ can be expanded in any basis $\{\langle x|,\langle y|\}$, or $\{\langle x'|,\langle y'|\}$, ...*etc*.

$$|\Psi\rangle=|x\rangle\langle x|\Psi\rangle + |y\rangle\langle y|\Psi\rangle = |x'\rangle\langle x'|\Psi\rangle + |y'\rangle\langle y'|\Psi\rangle$$

Transformation matrix $T_{m,n'}$ relates $\{\langle x|\Psi\rangle,\langle y|\Psi\rangle\}$ amplitudes to $\{\langle x'|\Psi\rangle,\langle y'|\Psi\rangle\}$.

$$\begin{pmatrix} \langle x|\Psi\rangle \\ \langle y|\Psi\rangle \end{pmatrix} = \begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} \begin{pmatrix} \langle x'|\Psi\rangle \\ \langle y'|\Psi\rangle \end{pmatrix} \quad \text{or:} \quad \begin{pmatrix} \Psi_x \\ \Psi_y \end{pmatrix} = \begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} \begin{pmatrix} \Psi_{x'} \\ \Psi_{y'} \end{pmatrix}$$

Hybrid
Gibbs-Dirac
notation
(Ug-ly!)

Inverse $(\dagger=\top{}^*=-1)$ matrix $T_{n',m}$ relates $\{\langle x'|\Psi\rangle,\langle y'|\Psi\rangle\}$ amplitudes to $\{\langle x|\Psi\rangle,\langle y|\Psi\rangle\}$.

$$\begin{pmatrix} \langle x'|\Psi\rangle \\ \langle y'|\Psi\rangle \end{pmatrix} = \begin{pmatrix} \langle x'|x\rangle & \langle x'|y\rangle \\ \langle y'|x\rangle & \langle y'|y\rangle \end{pmatrix} \begin{pmatrix} \langle x|\Psi\rangle \\ \langle y|\Psi\rangle \end{pmatrix} \quad \text{or:} \quad \begin{pmatrix} \Psi_{x'} \\ \Psi_{y'} \end{pmatrix} = \begin{pmatrix} \langle x'|x\rangle & \langle x'|y\rangle \\ \langle y'|x\rangle & \langle y'|y\rangle \end{pmatrix} \begin{pmatrix} \Psi_x \\ \Psi_y \end{pmatrix}$$

Hybrid
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(Still Ug-ly!)

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Planck's energy and N-quanta *(Cavity/Beam of volume V with wave mode of frequency $\omega=2\pi\nu$)*

Planck axiom: $\boldsymbol{E}$-field energy density U in cavity/beam mode-ω is: $U=N\hbar\omega/V=Nh\nu/V$ *(N "photons")*

$h=2\pi\hbar=6.6310^{-34}Js$ *Planck constant*

http://www.uark.edu/ua/modphys/pdfs/QTCA_Pdfs/QTCA_Text_2013/QTCA_Unit_1_Ch._1_2013.pdf

http://www.uark.edu/ua/modphys/pdfs/QTCA_Pdfs/QTCA_Text_2013/QTCA_Unit_7_Ch._22_2005.pdf

Planck's energy and ***N*-quanta** *(Cavity/Beam of volume V with wave mode of frequency $\omega=2\pi\nu$)*

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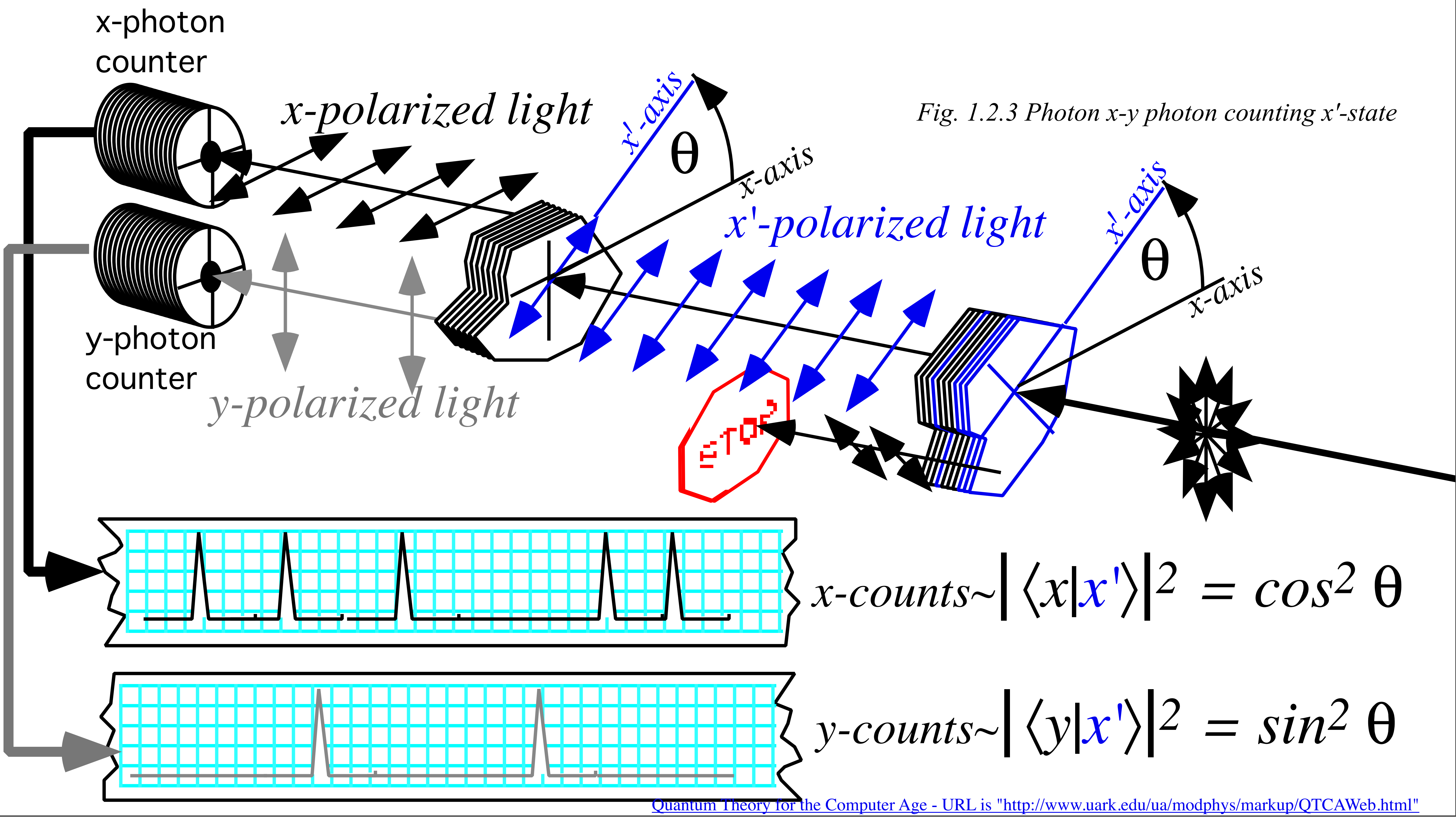

Fig. 1.2.3 Photon x-y photon counting x'-state

$$x\text{-counts}\sim\left|\langle x|x'\rangle\right|^2=\cos^2\theta$$

$$y\text{-counts}\sim\left|\langle y|x'\rangle\right|^2=\sin^2\theta$$

Quantum Theory for the Computer Age - URL is "http://www.uark.edu/ua/modphys/markup/QTCAWeb.html"

Planck's energy and N-quanta *(Cavity/Beam of volume V with wave mode of frequency $\omega=2\pi\nu$)*

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E-field vector $(E_x, E_y) = f\cdot(\Psi_x, \Psi_y)$ where *quantum field proportionality constant is f*.

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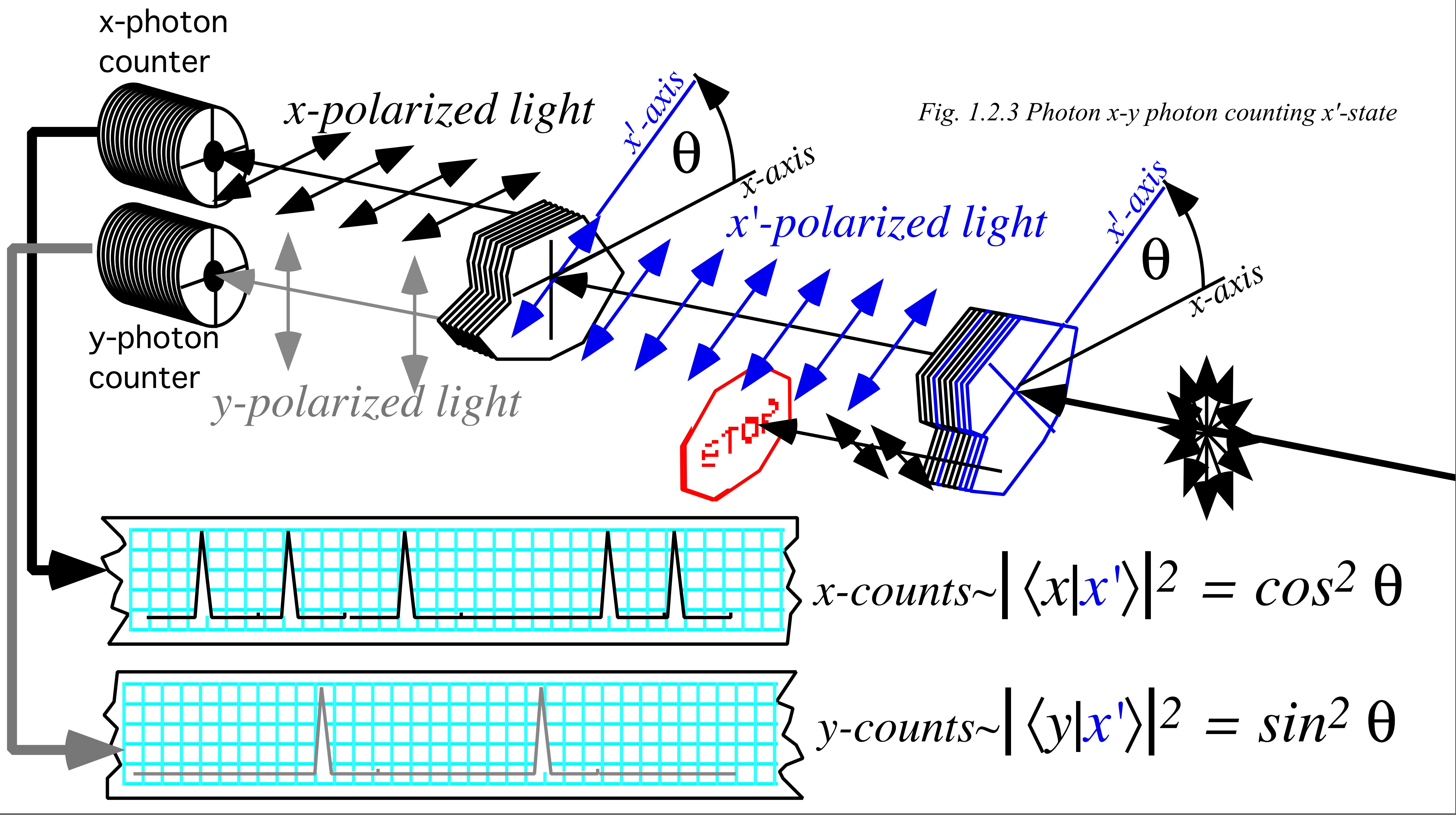

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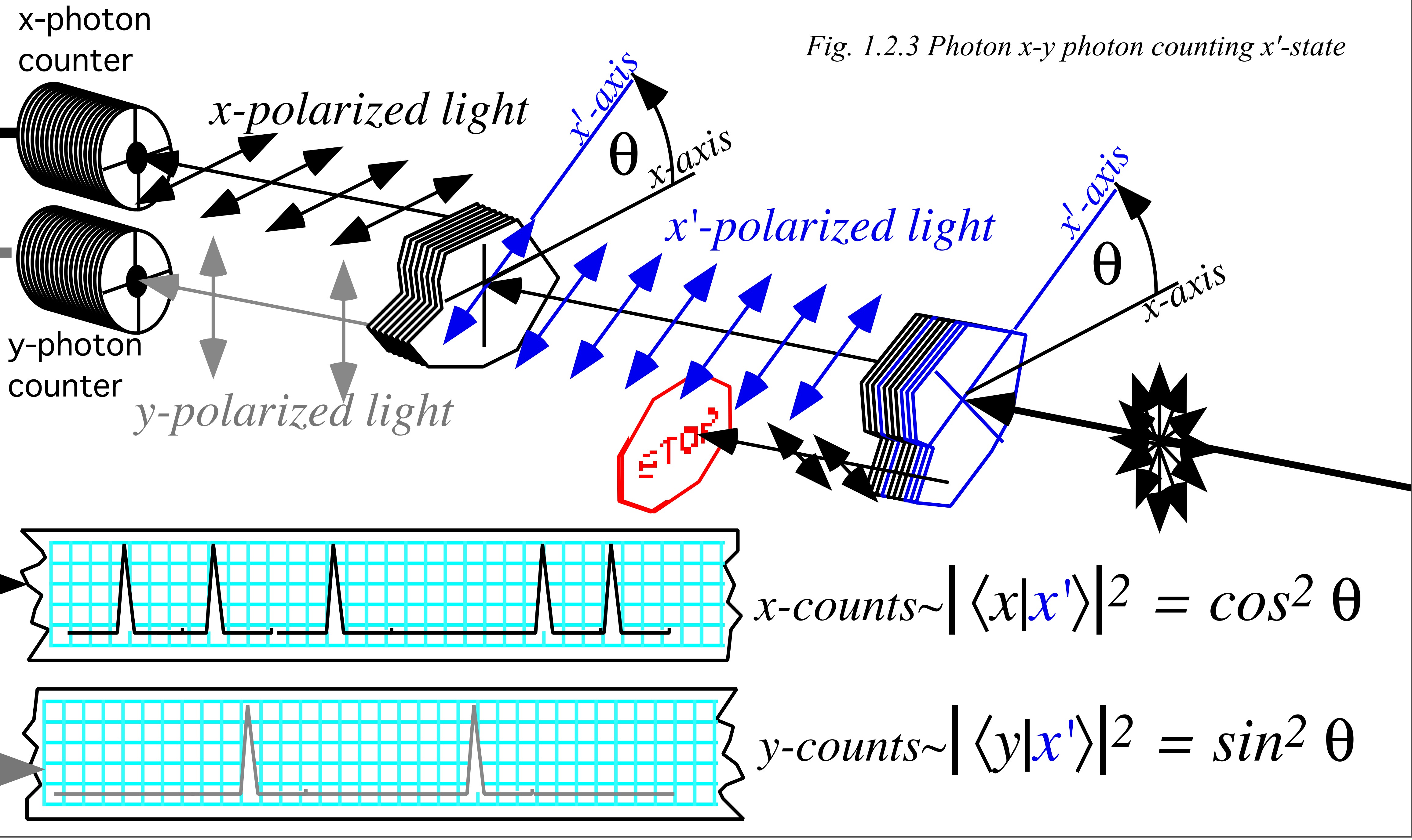
$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} E_x(0)e^{ikz-i\omega t} \\ E_y(0)e^{ikz-i\omega t} \end{pmatrix} \cong \begin{pmatrix} E_x(0)e^{-i\omega t} \\ E_y(0)e^{-i\omega t} \end{pmatrix} = f\begin{pmatrix} \Psi_x \\ \Psi_y \end{pmatrix}$$

$h=2\pi\hbar=6.63\cdot10^{-34}Js$ Planck constant



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$h=2\pi\hbar=6.626\ 075\cdot 10^{-34}Js$	*Planck constant*
$c=2.997\ 924\ 58\cdot 10^{8}\ ms^{-1}$	*Light speed*
$\varepsilon_0=8.854\cdot 10^{-12}C^2N^{-1}m^{-2}$	*Electrostatic constant*

Coulomb constant: $k=1/4\pi\varepsilon_0$

$= 9\cdot 10^{9}\ J/C$

Ψ-amplitude-squares sum to *exciton-number N*. (...or *photon-number N*)

$$N = \left|\Psi_x\right|^2 + \left|\Psi_y\right|^2 = \langle x|\Psi\rangle^* \langle x|\Psi\rangle + \langle y|\Psi\rangle^* \langle y|\Psi\rangle$$

Quantum em wave theory

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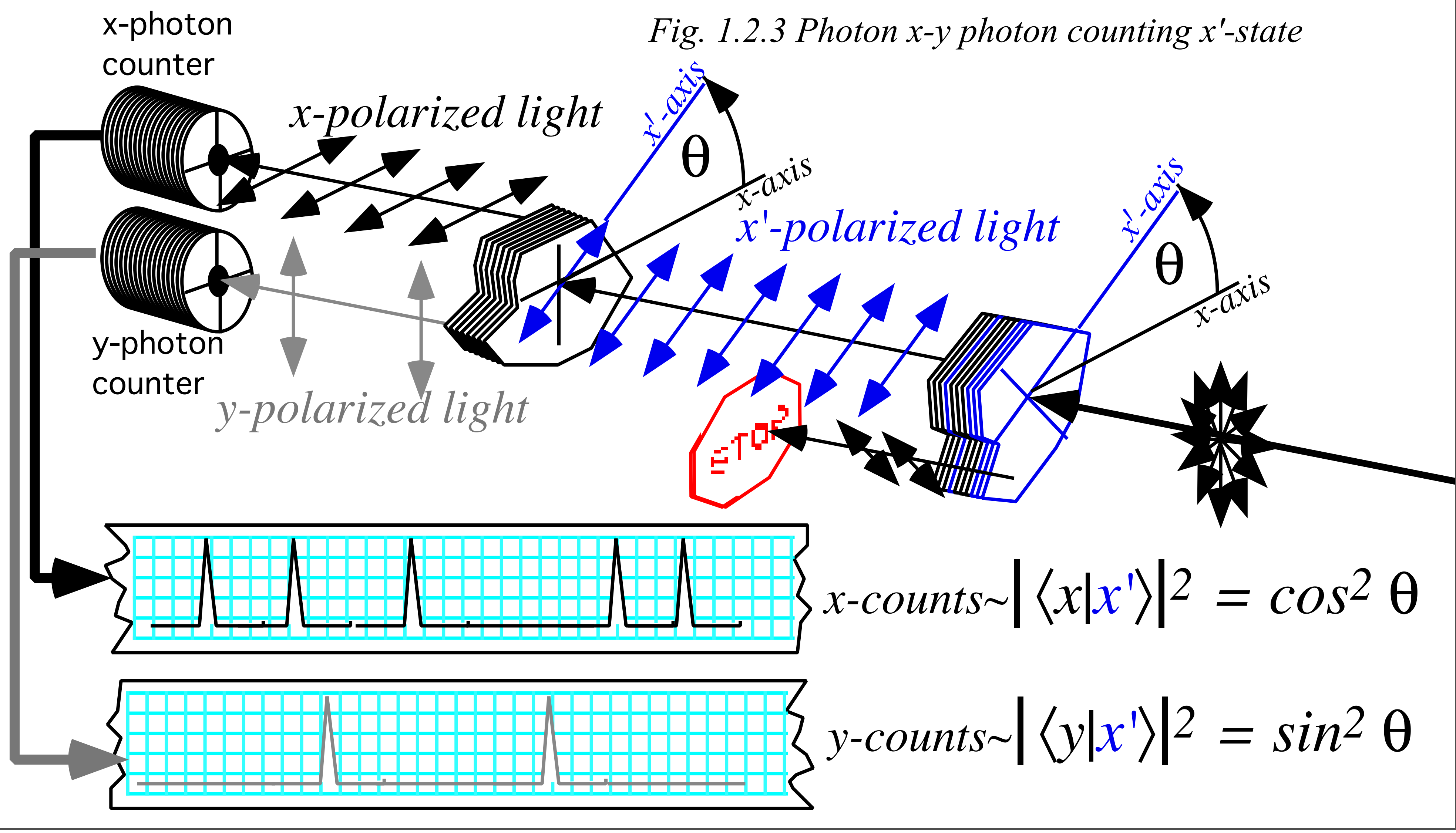

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$h=2\pi\hbar=6.626\,075\cdot10^{-34}Js$	Planck constant
$c=2.997\,924\,58\cdot10^{8}\,ms^{-1}$	Light speed
$\varepsilon_0=8.854\cdot10^{-12}C^2N^{-1}m^{-2}$	Electrostatic constant

Coulomb constant: $k=1/4\pi\varepsilon_0$
$=9\cdot10^{9}\,J/C$

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$$N=|\Psi_x|^2+|\Psi_y|^2=\langle x|\Psi\rangle^*\langle x|\Psi\rangle+\langle y|\Psi\rangle^*\langle y|\Psi\rangle$$

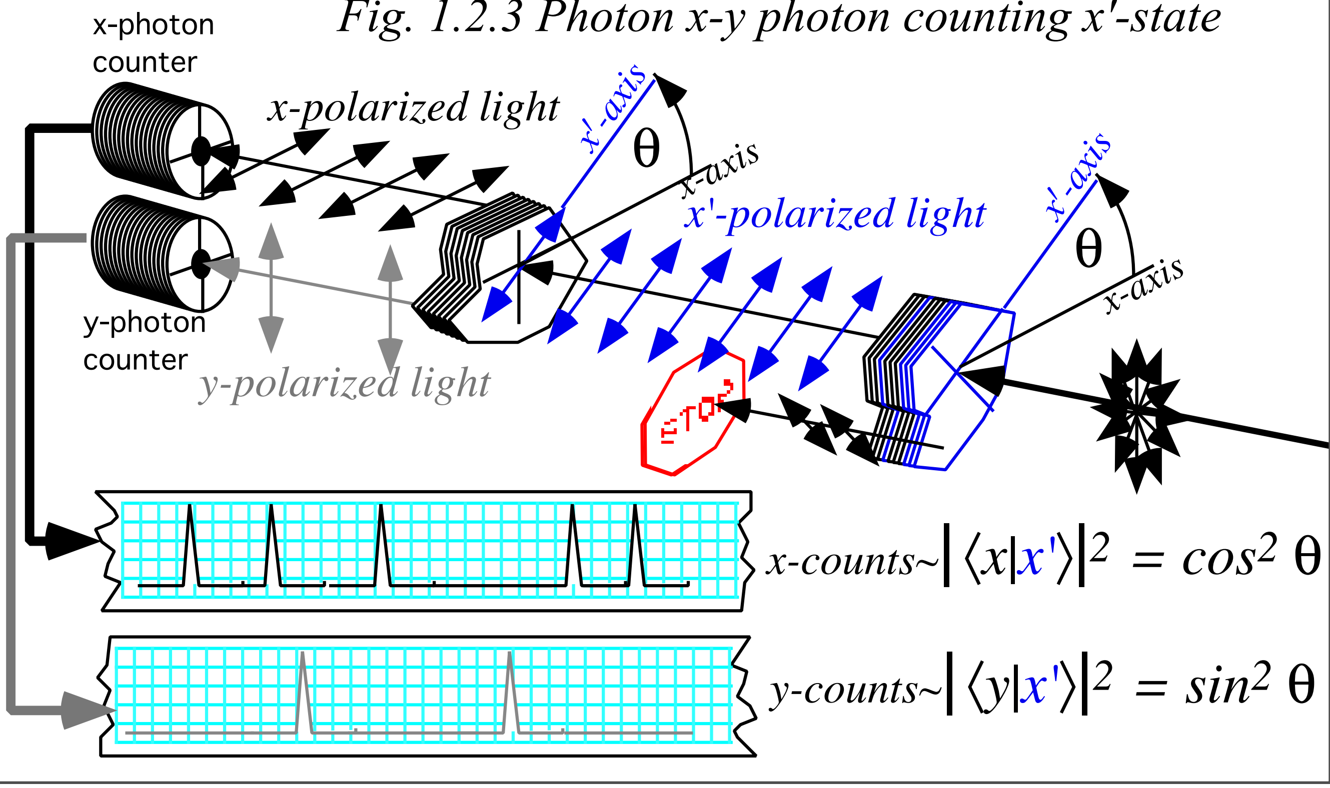
Quantum em wave theory

Poynting energy flux $S(J/m^2s)$ or *energy density $U(J/m^3)$* of light beam.

$$S=cU,\quad\text{where: }U=\varepsilon_0\left(|E_x|^2+|E_y|^2\right)=\varepsilon_0\left(E_x^*E_x+E_y^*E_y\right)=\varepsilon_0\left(E_x(0)^2+E_y(0)^2\right)$$

Classical em wave theory

Fig. 1.2.3 Photon x-y photon counting x'-state



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Quantum em wave theory

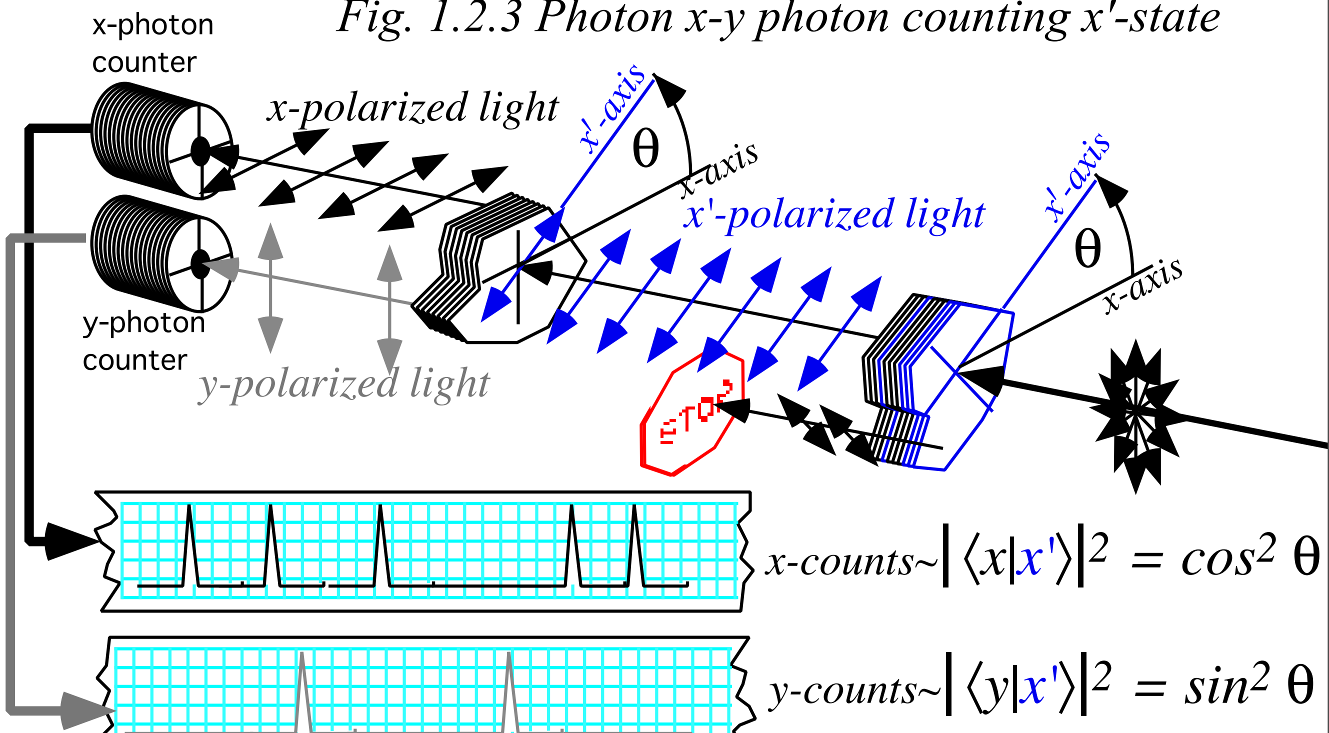
Poynting energy flux $S(J/m^2s)$ or *energy density $U(J/m^3)$* of light beam.

$$S=cU,\quad\text{where: } U=\varepsilon_0\left(\left|E_x\right|^2+\left|E_y\right|^2\right)=\varepsilon_0\left(E_x^*E_x+E_y^*E_y\right)=\varepsilon_0\left(E_x(0)^2+E_y(0)^2\right)$$

Classical em wave theory

Equate U to Planck's N-*"photon"* quantum energy density $N\hbar\omega/V$

Fig. 1.2.3 Photon x-y photon counting x'-state



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Poynting energy flux S(J/m²s) or *energy density U(J/m³)* of light beam.

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Equate U to Planck's N-*"photon"* quantum energy density $N\hbar\omega/V$

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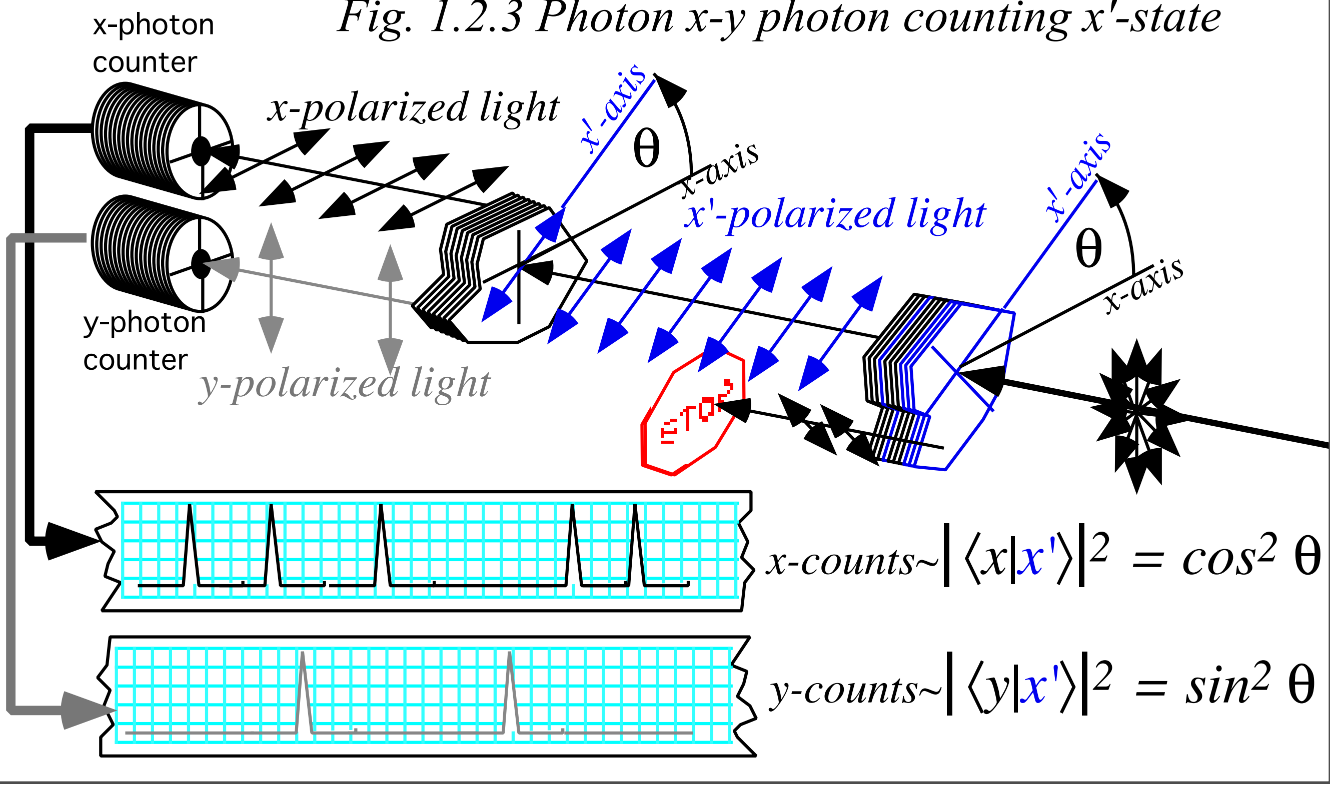
Fig. 1.2.3 Photon x-y photon counting x'-state



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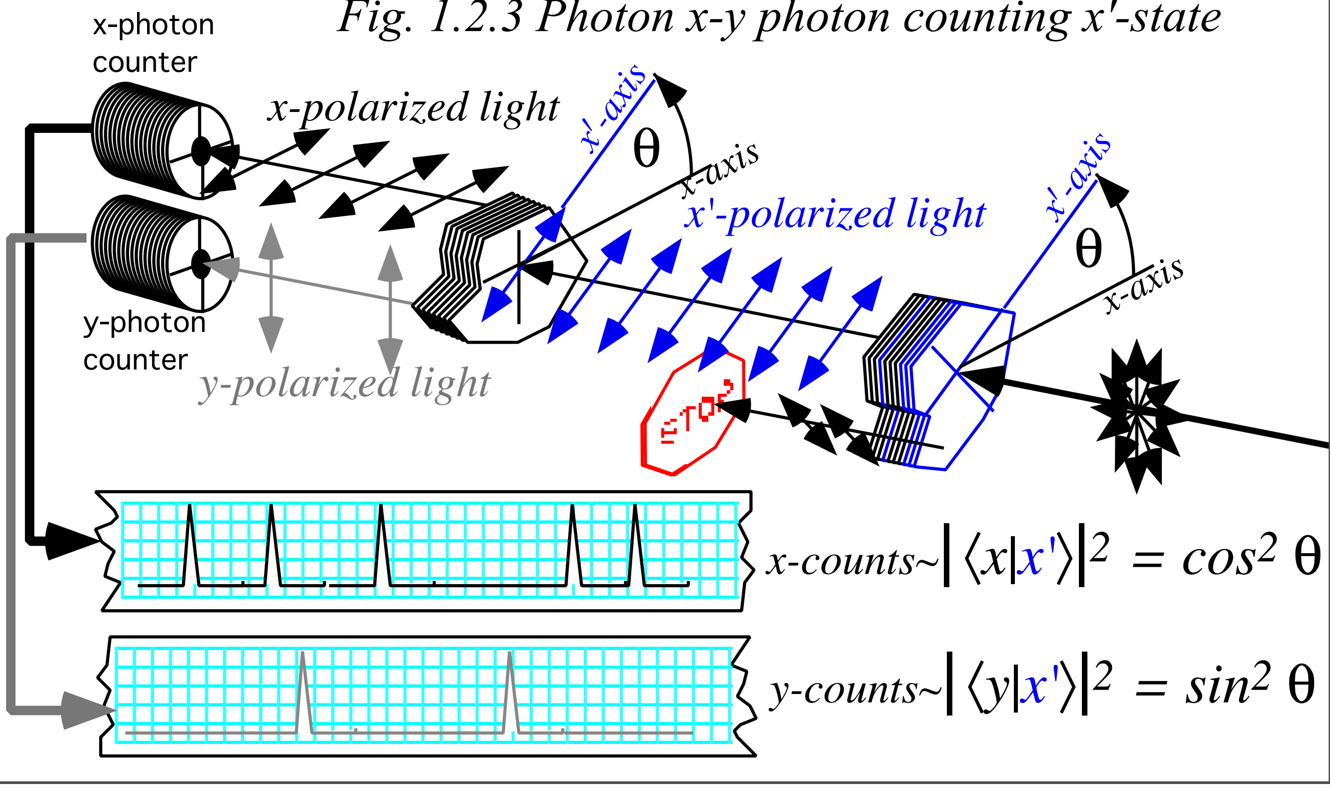

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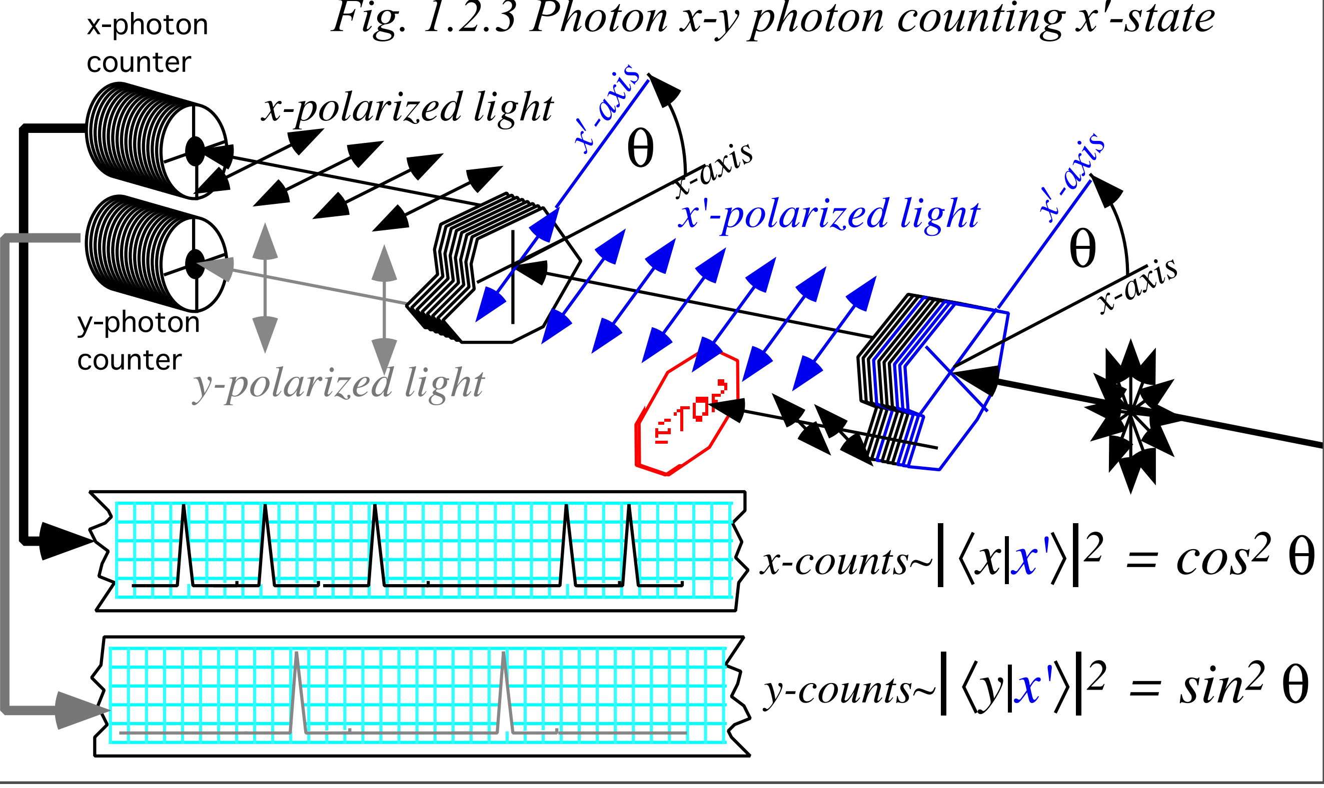

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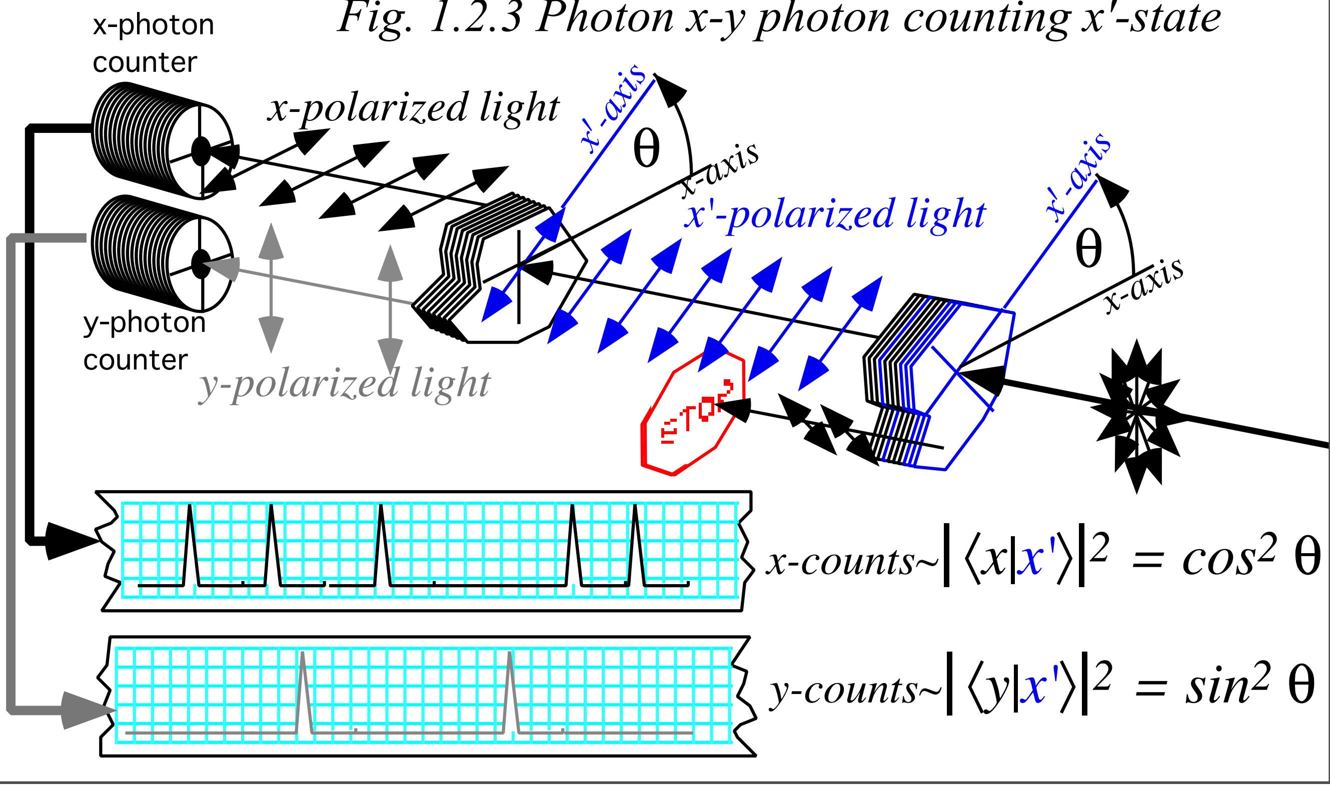

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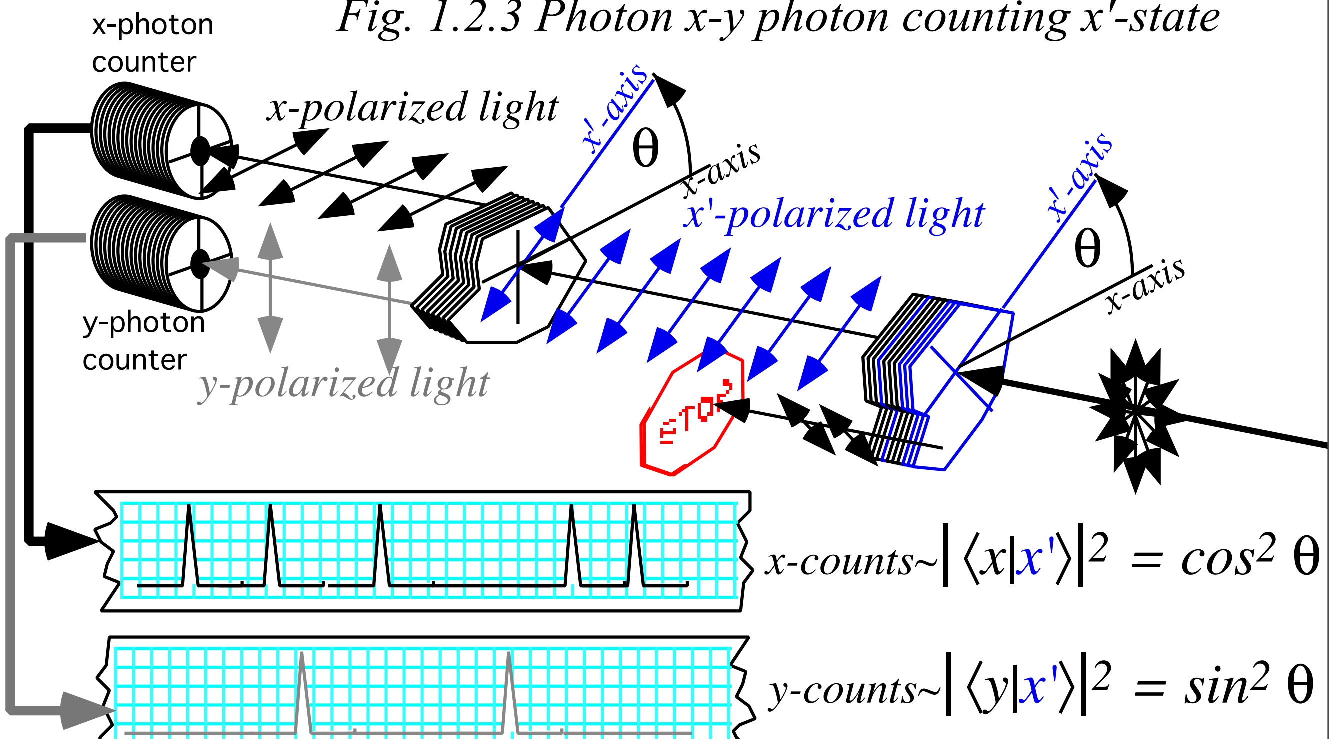

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Introducing scalar and matrix products

Planck's energy and N-quanta (Cavity/Beam wave mode)
➡ *Did Max Planck Goof? What's 1-photon worth?*
Feynman amplitude axiom 1

What comes out of a beam sorter channel or branch-b?
Sample calculations
Feynman amplitude axioms 2-3

Beam analyzers: Sorter-unsorters
The "Do-Nothing" analyzer
Feynman amplitude axiom 4
Some "Do-Something" analyzers
Sorter-counter, Filter, 1/2-wave plate, 1/4-wave plate

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http://www.uark.edu/ua/modphys/pdfs/QTCA_Pdfs/QTCA_Text_2013/QTCA_Unit_7_Ch._22_2005.pdf

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Was Planck's linear-in-frequency-υ energy axiom a Goof ?

Not at all. It's really not linear-in-frequency after all.

The $\varepsilon_0\,|\mathbf{E}|^2 = h\,N\cdot\upsilon$ *axiom IS in fact a product of TWO frequencies!*

The 2nd "frequency" is COUNT RATE N.

As shown later: Both frequencies transform by Relativistic Doppler factor $e^{\pm\rho}$*:*

Coherent frequency is light quality: $\upsilon' = e^{\pm\rho}\,\upsilon$. *Incoherent frequency is light quantity:* $N' = e^{\pm\rho}\,N$.

This would imply that the $|\mathbf{E}|$*-field also transforms like a frequency:* $|\mathbf{E}|' = = e^{\pm\rho}\,|\mathbf{E}|$.

Indeed, E-field amplitude is a frequency, too!

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Feynman amplitude axiom 1

Feynman-Dirac Interpretation of

$$\langle j|k'\rangle$$

=Amplitude of state-j after state-k′ is forced to choose from available m-type states

(1) The probability axiom

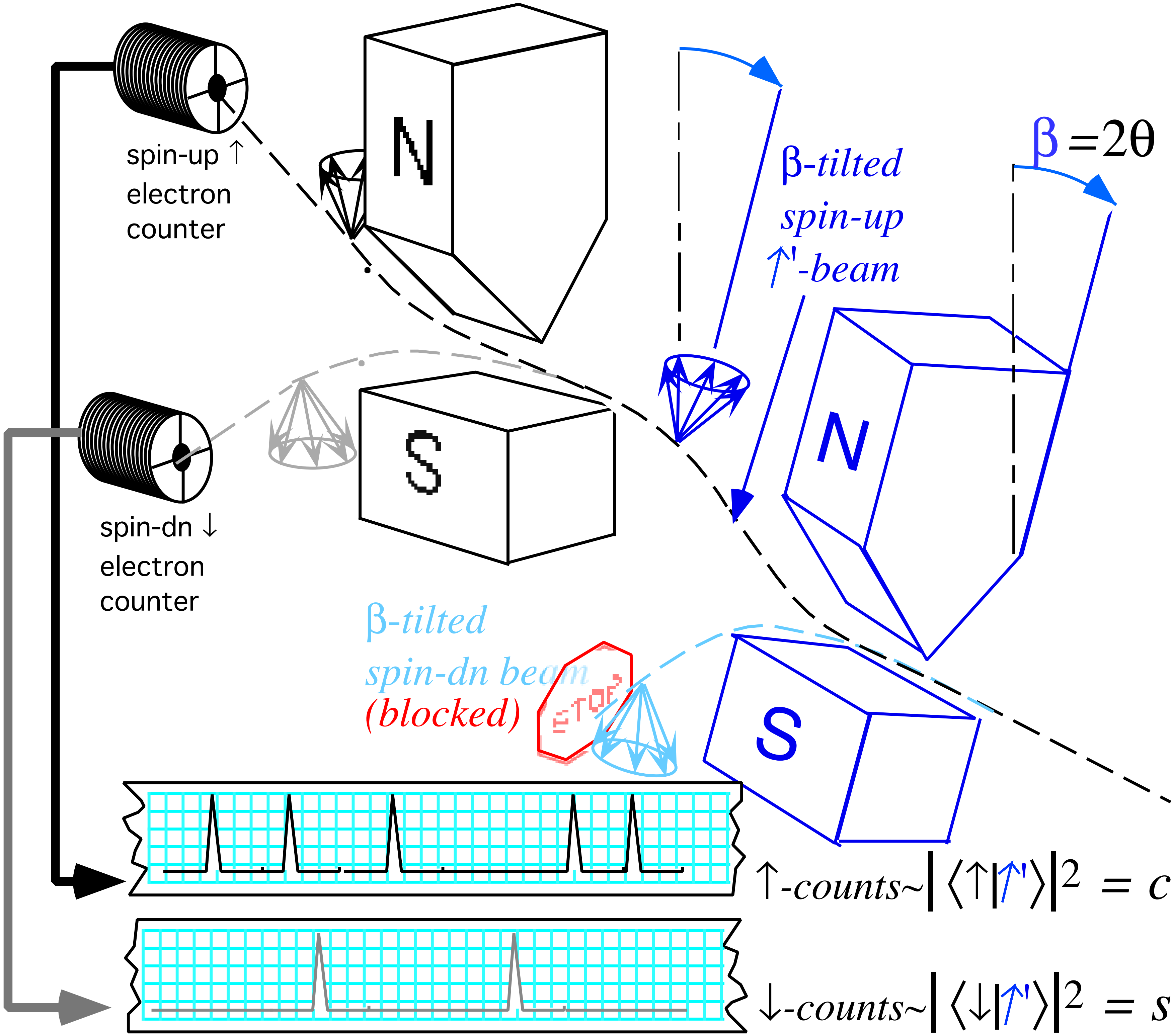
The first axiom deals with physical interpretation of amplitudes $\langle j|k'\rangle$.

Axiom 1: The absolute square $|\langle j|k'\rangle|^2 = \langle j|k'\rangle^* \langle j|k'\rangle$ *gives probability for occurrence in state-j of a system that started in state-k'=1',2',..,or n' from one sorter and then was forced to choose between states j=1,2,...,n by another sorter.*

probability

Amplitude axioms apply to all intensity-conserving systems

This includes, first of all, spin-1/2 electron, proton, ..., ^{13}C, ... particles (Fermions)



↑-counts~$\left|\langle\uparrow|\uparrow'\rangle\right|^2 = \cos^2\beta/2 = \cos^2\theta$

↓-counts~$\left|\langle\downarrow|\uparrow'\rangle\right|^2 = \sin^2\beta/2 = \sin^2\theta$

Fig. 1.2.4 Electron up-dn-spin counting of a tilted spin-up (↑')-state

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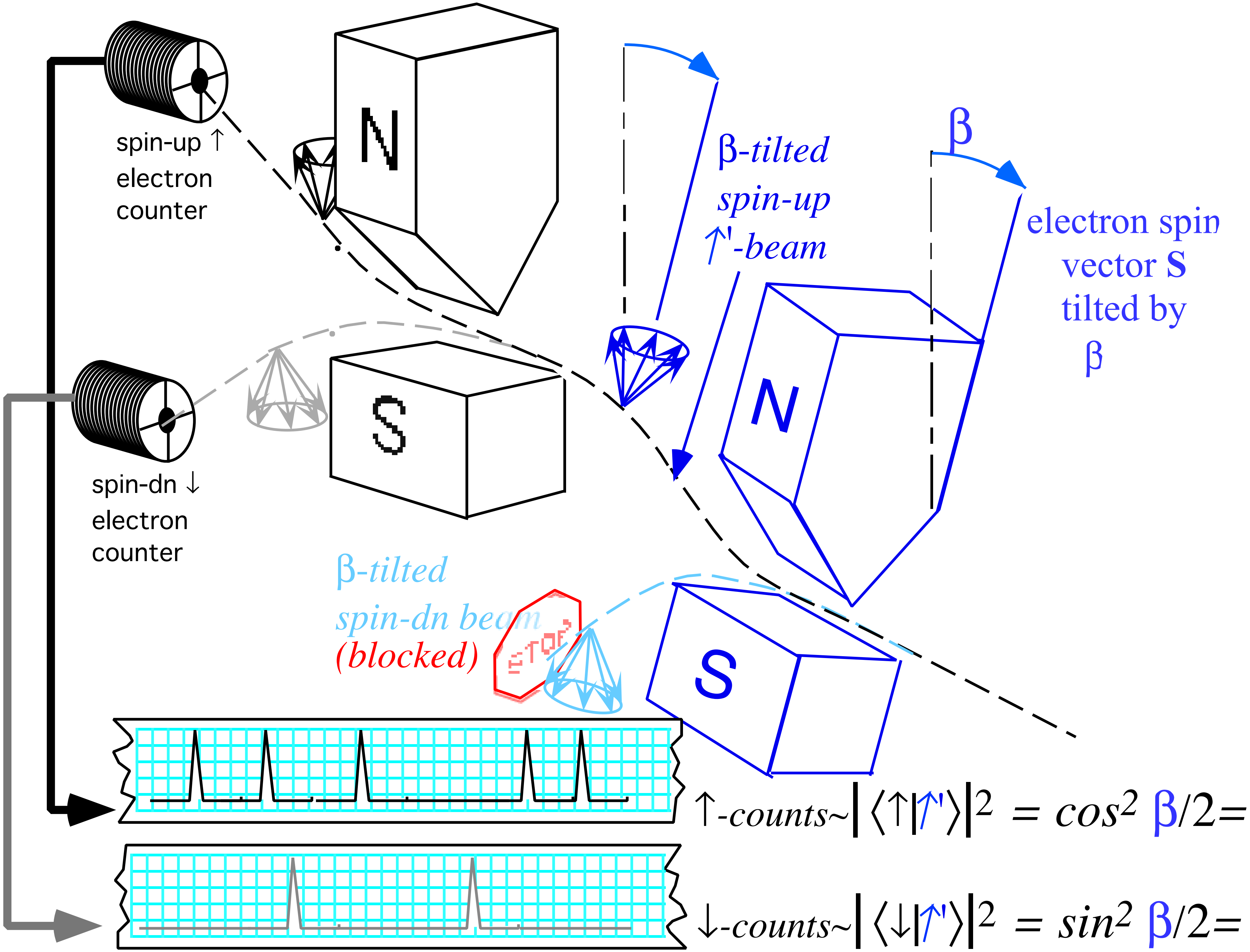

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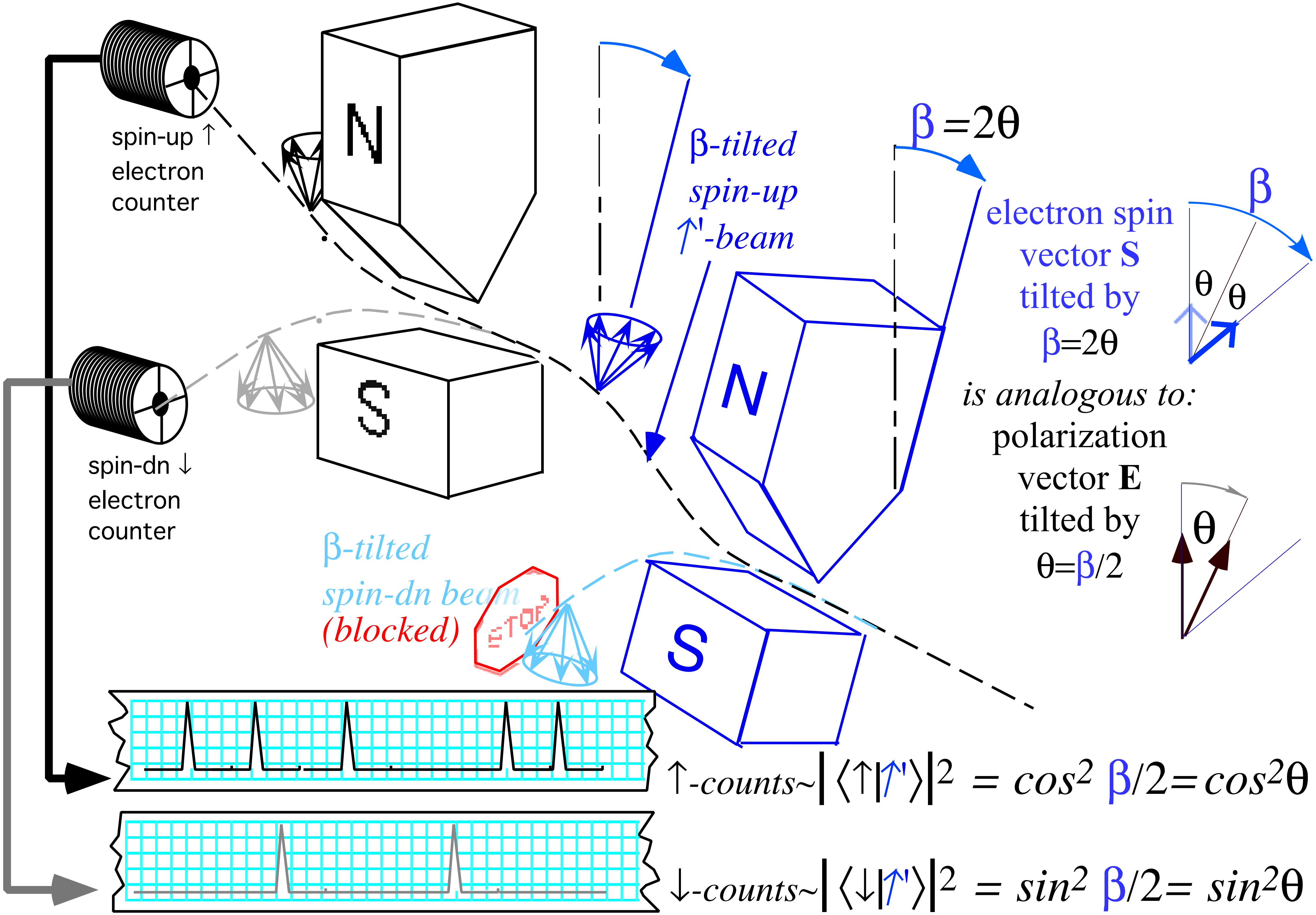

electron spin vector **S** tilted by β=2θ

is analogous to:

polarization vector **E** tilted by θ=β/2

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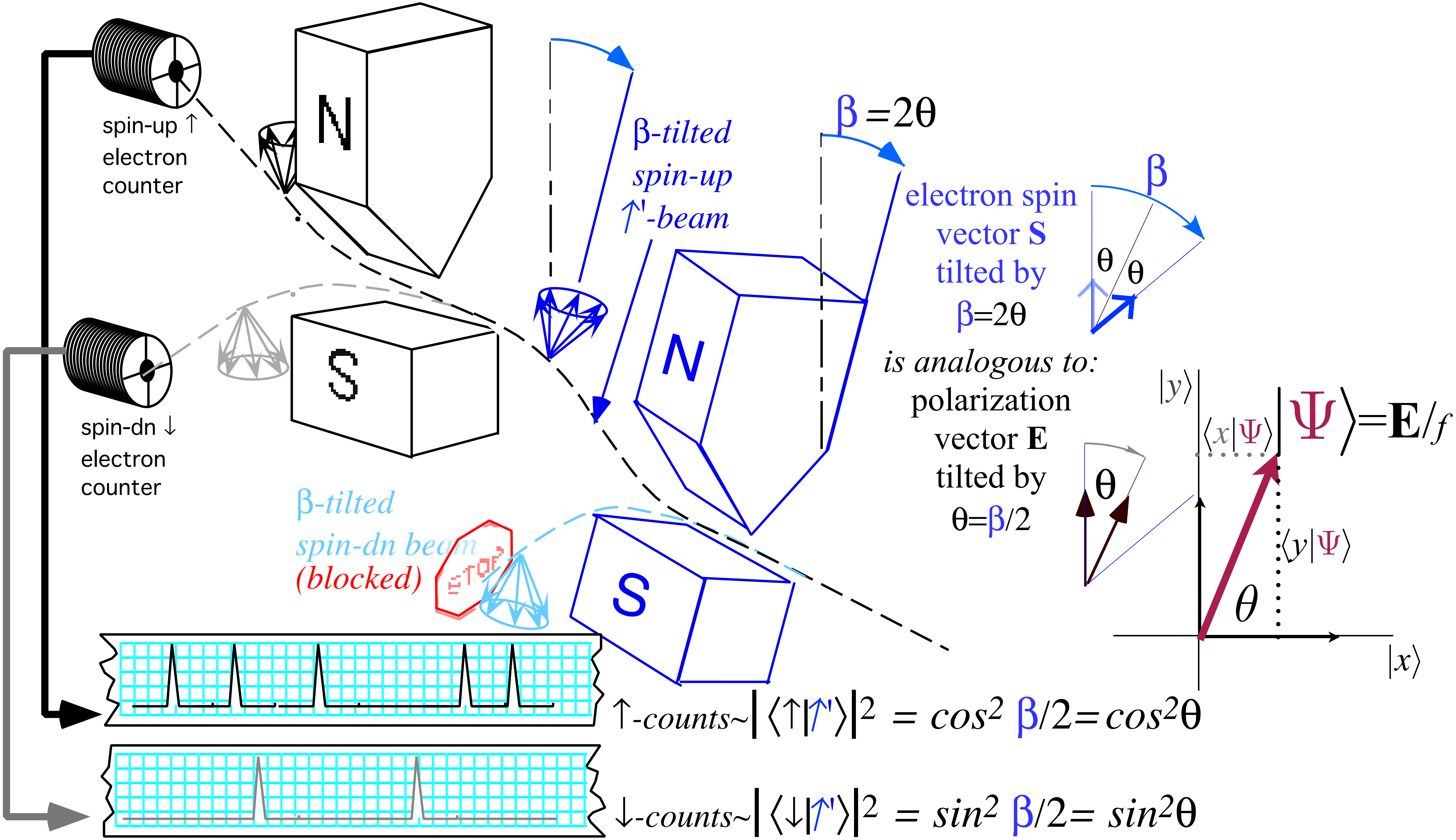

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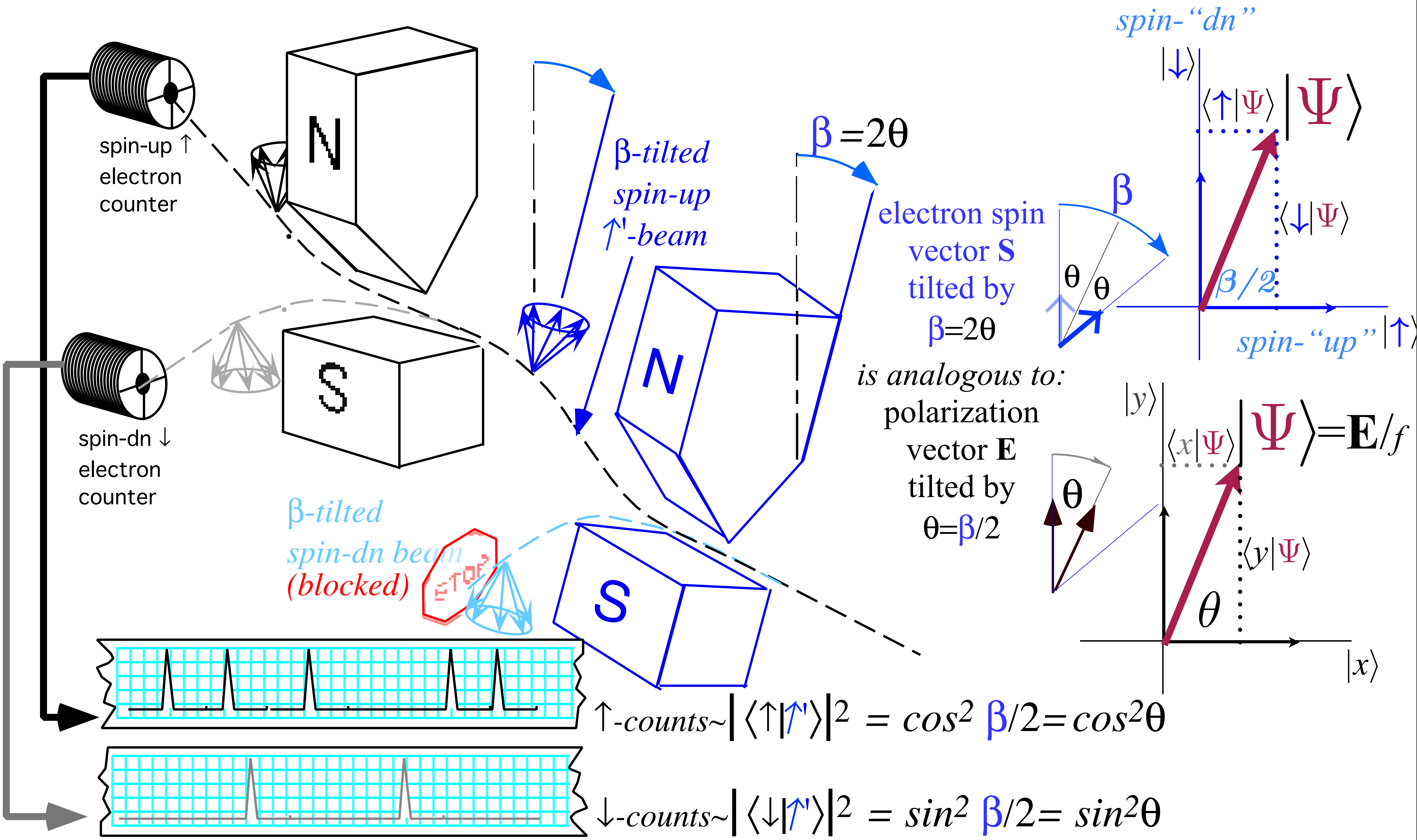

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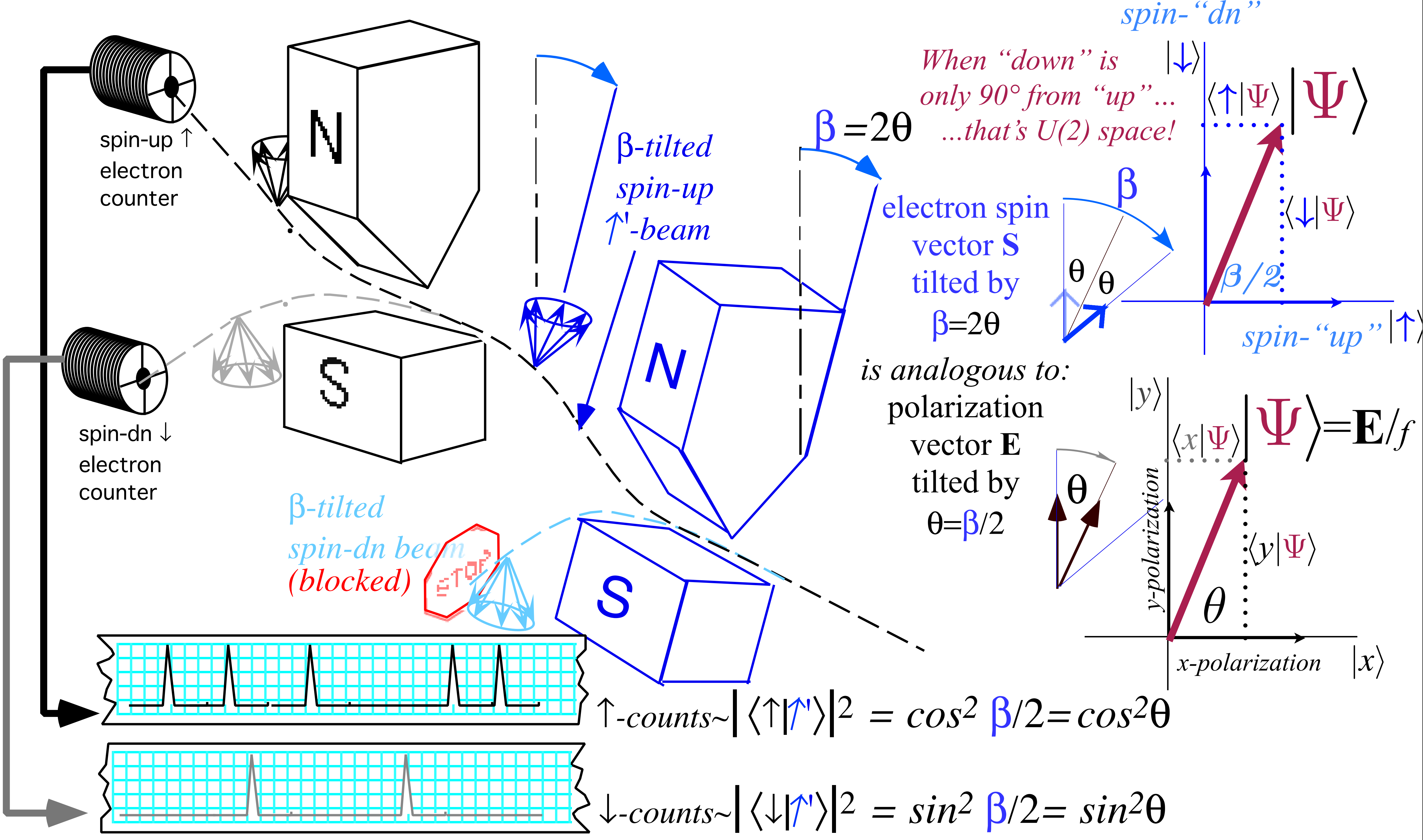

When "down" is only 90° from "up"... ...that's U(2) space!

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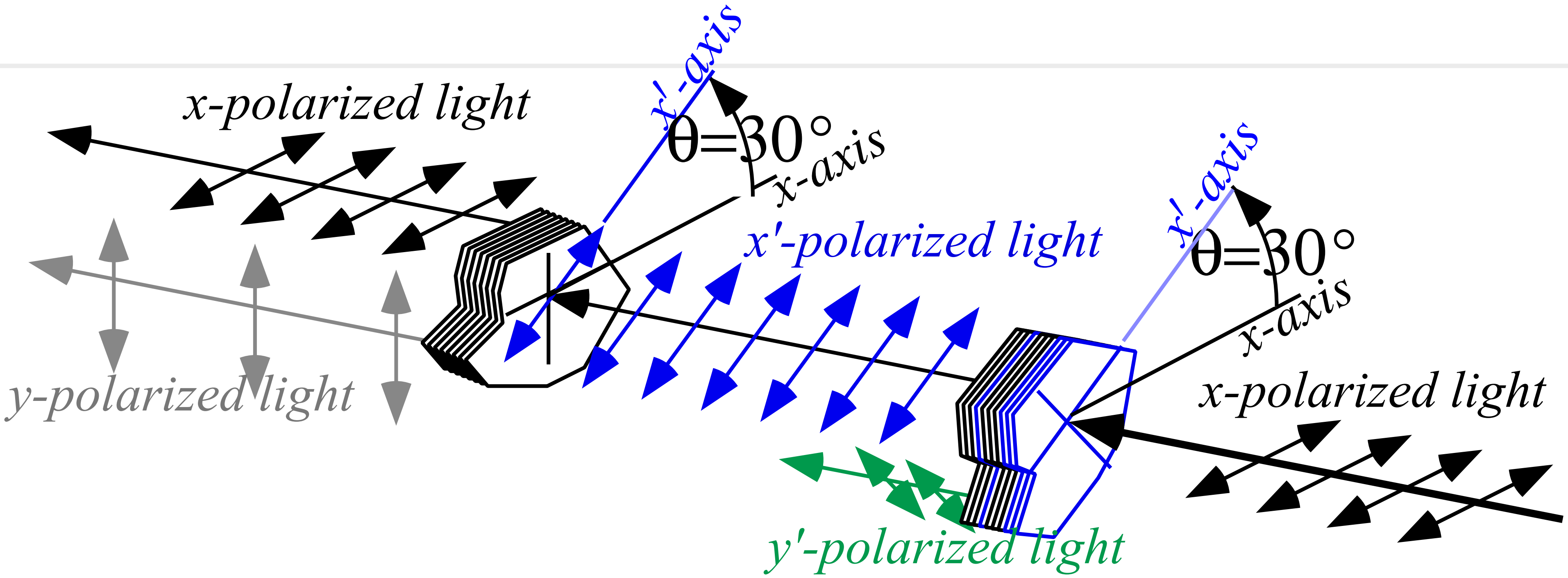

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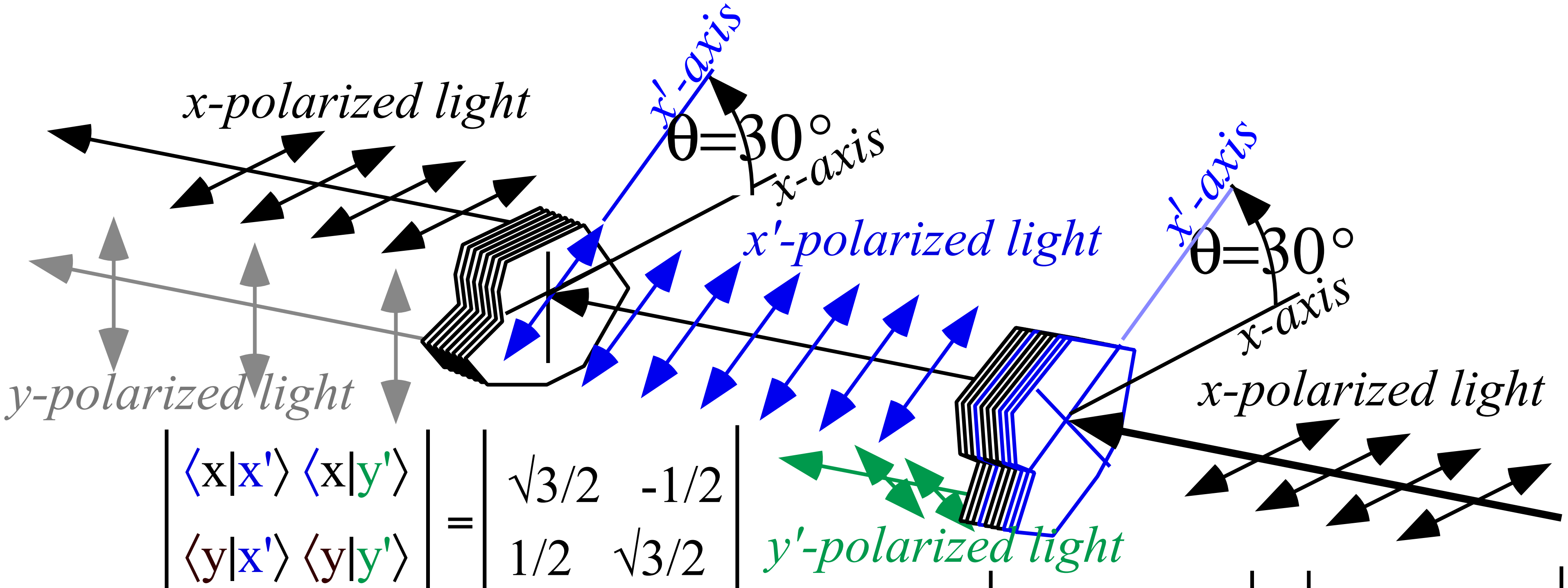

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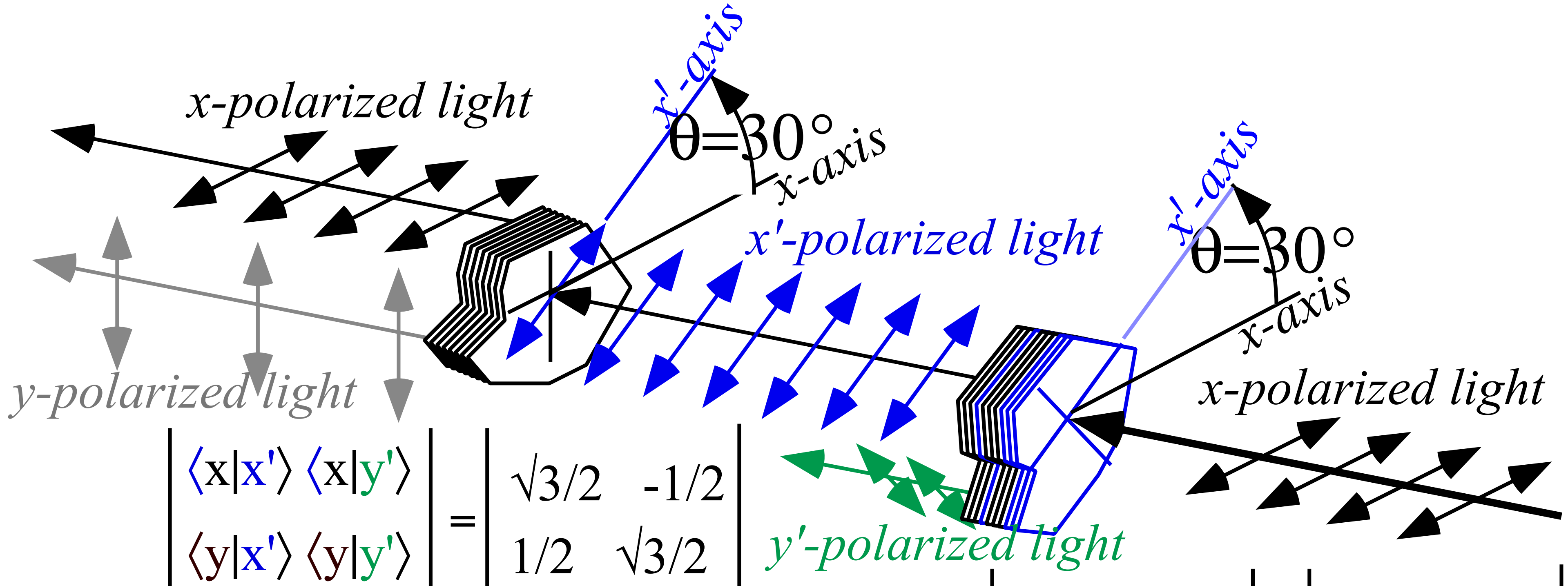

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State
$|x\rangle$

Amplitude=
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Probability=
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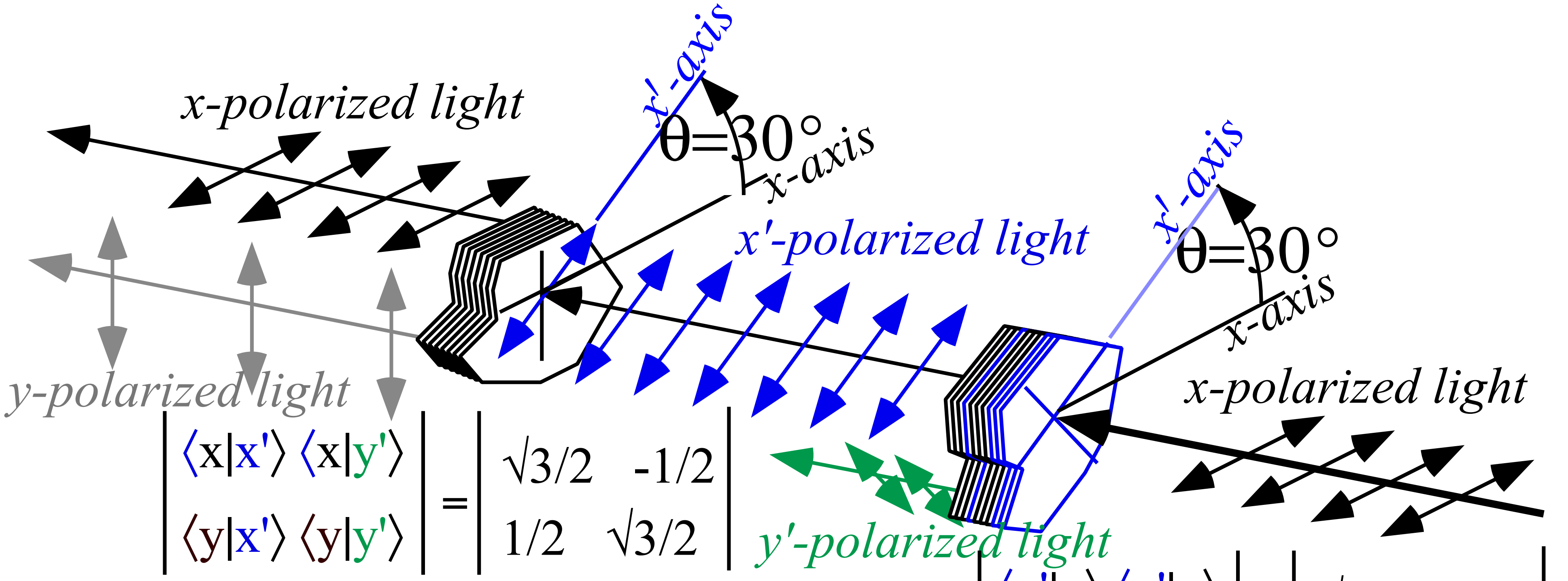

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Base State

$|x'\rangle\langle x'|x\rangle$

Amplitude=
$\sqrt{3}/2$=0.867
Probability=
0.75

Base State

$|x\rangle$

Amplitude=
1.0
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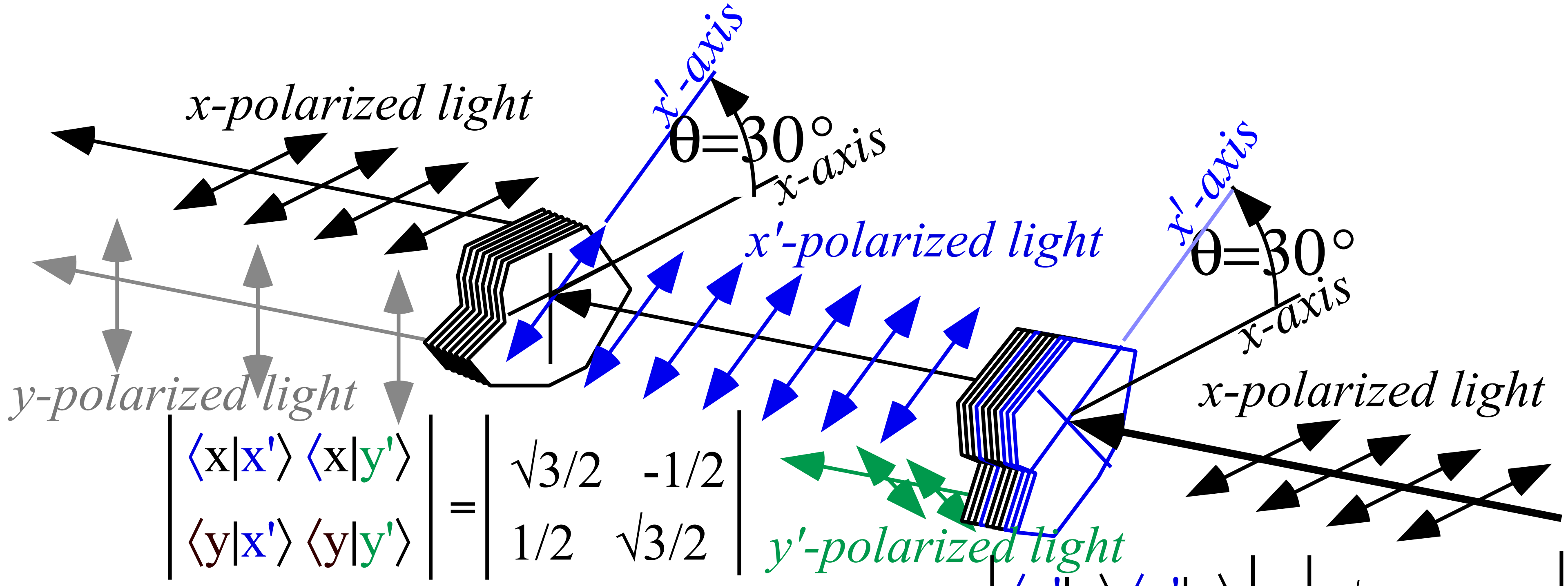
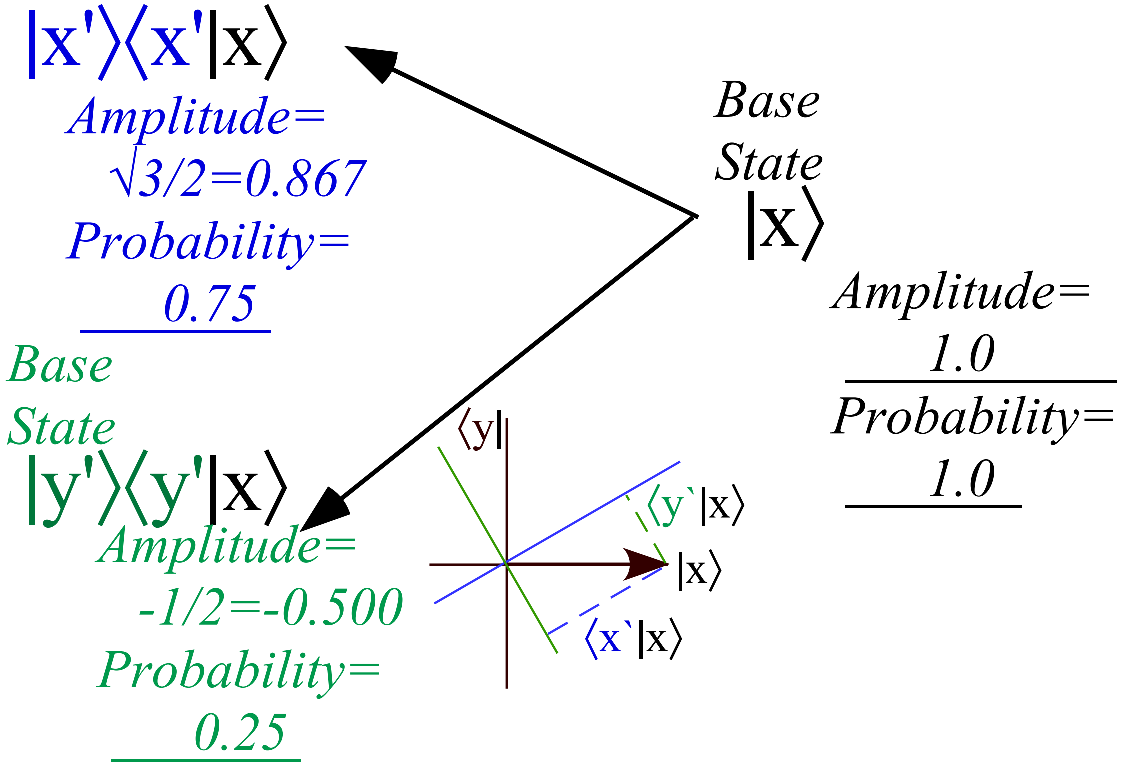


$$\begin{vmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{vmatrix} = \begin{vmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{vmatrix}$$

$$\begin{vmatrix} \langle x'|x\rangle & \langle x'|y\rangle \\ \langle y'|x\rangle & \langle y'|y\rangle \end{vmatrix} = \begin{vmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{vmatrix}$$



Base State
$|x'\rangle\langle x'|x\rangle$
Amplitude=
√3/2=0.867
Probability=
0.75

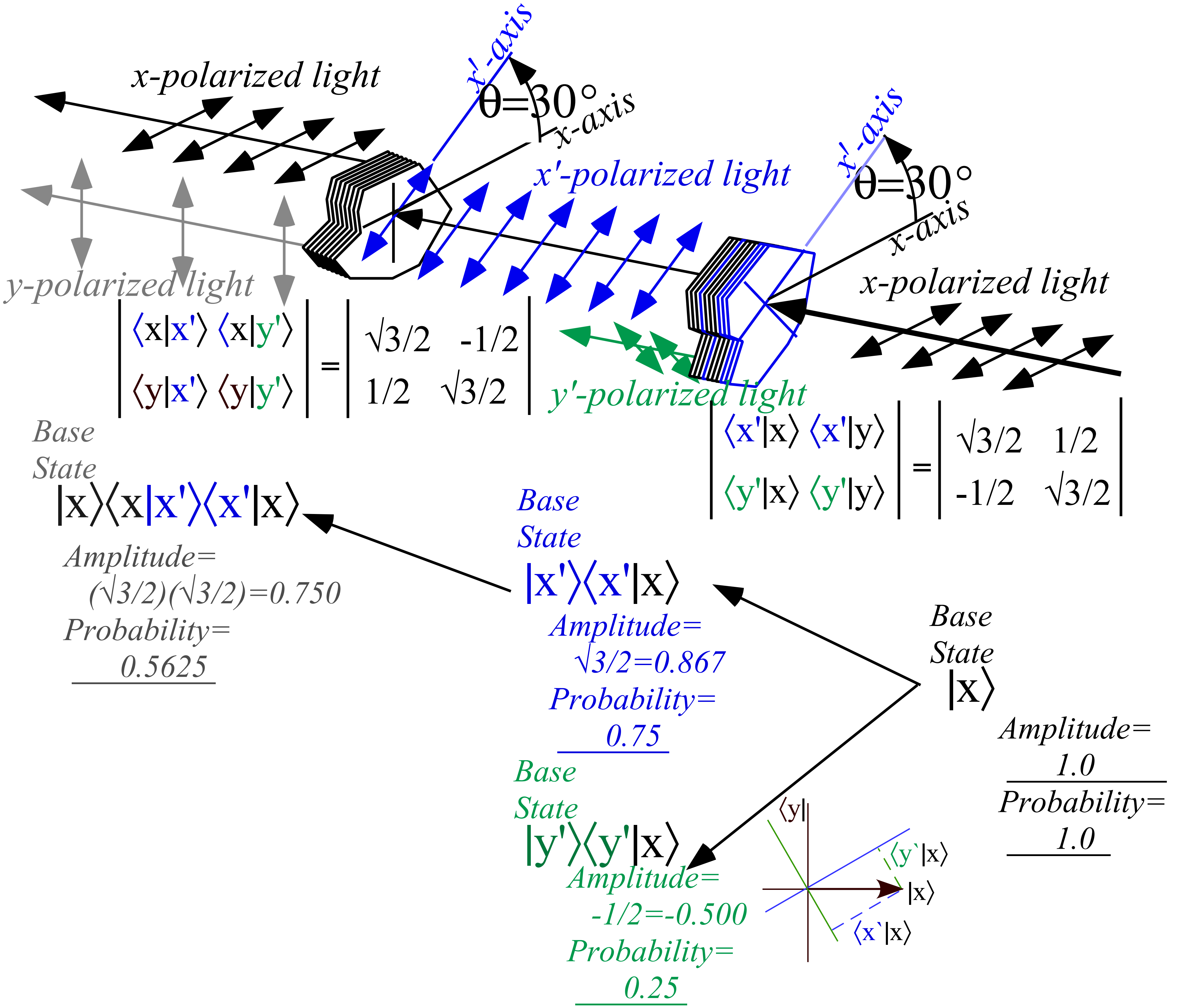
Base State
$|y'\rangle\langle y'|x\rangle$
Amplitude=
-1/2=-0.500
Probability=
0.25

Base State
$|x\rangle$
Amplitude=
1.0
Probability=
1.0

Q:What comes out of an analyzer channel or branch-b?

A: Want to determine or calculate:

its *base state* $|b\rangle$, its *amplitude* $\langle b|\Psi\rangle$, and its *probability* $|\langle b|\Psi\rangle|^2$ using *T*-matrices



$$\begin{vmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{vmatrix} = \begin{vmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{vmatrix}$$

$$\begin{vmatrix} \langle x'|x\rangle & \langle x'|y\rangle \\ \langle y'|x\rangle & \langle y'|y\rangle \end{vmatrix} = \begin{vmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{vmatrix}$$

Base State $|x\rangle$

Amplitude= 1.0

Probability= 1.0

Base State $|x'\rangle\langle x'|x\rangle$

Amplitude= √3/2=0.867

Probability= 0.75

Base State $|y'\rangle\langle y'|x\rangle$

Amplitude= -1/2=-0.500

Probability= 0.25

Base State $|x\rangle\langle x|x'\rangle\langle x'|x\rangle$

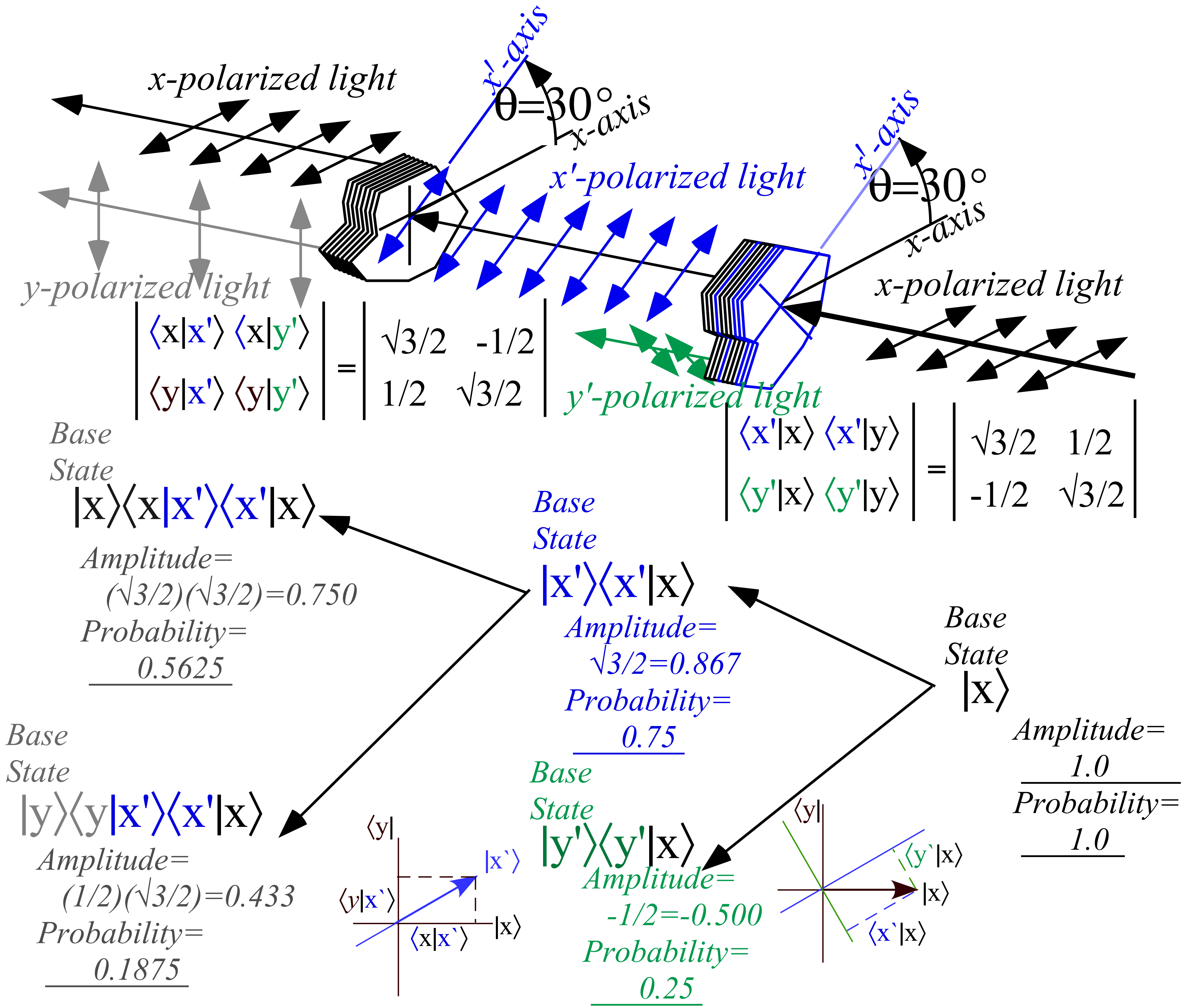
Amplitude= (√3/2)(√3/2)=0.750

Probability= 0.5625

Q:What comes out of an analyzer channel or branch-b?

A: Want to determine or calculate:

its *base state* $|b\rangle$, its *amplitude* $\langle b|\Psi\rangle$, and its *probability* $|\langle b|\Psi\rangle|^2$ using *T*-matrices



$$\left|\begin{matrix}\langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle\end{matrix}\right| = \left|\begin{matrix}\sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2\end{matrix}\right|$$

$$\left|\begin{matrix}\langle x'|x\rangle & \langle x'|y\rangle \\ \langle y'|x\rangle & \langle y'|y\rangle\end{matrix}\right| = \left|\begin{matrix}\sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2\end{matrix}\right|$$

Base State $|x\rangle$

Amplitude= *1.0*

Probability= *1.0*

Base State $|x'\rangle\langle x'|x\rangle$

Amplitude= $\sqrt{3}/2=0.867$

Probability= *0.75*

Base State $|y'\rangle\langle y'|x\rangle$

Amplitude= *-1/2=-0.500*

Probability= *0.25*

Base State $|x\rangle\langle x|x'\rangle\langle x'|x\rangle$

Amplitude= $(\sqrt{3}/2)(\sqrt{3}/2)=0.750$

Probability= *0.5625*

Base State $|y\rangle\langle y|x'\rangle\langle x'|x\rangle$

Amplitude= $(1/2)(\sqrt{3}/2)=0.433$

Probability= *0.1875*

$$\left\langle e_p \middle| D_o C_n B_m \middle| \Psi \right\rangle = \left\langle e_p \middle| d_o \right\rangle \left\langle d_o \middle| c_n \right\rangle \left\langle c_n \middle| b_m \right\rangle \left\langle b_m \middle| \Psi \right\rangle$$

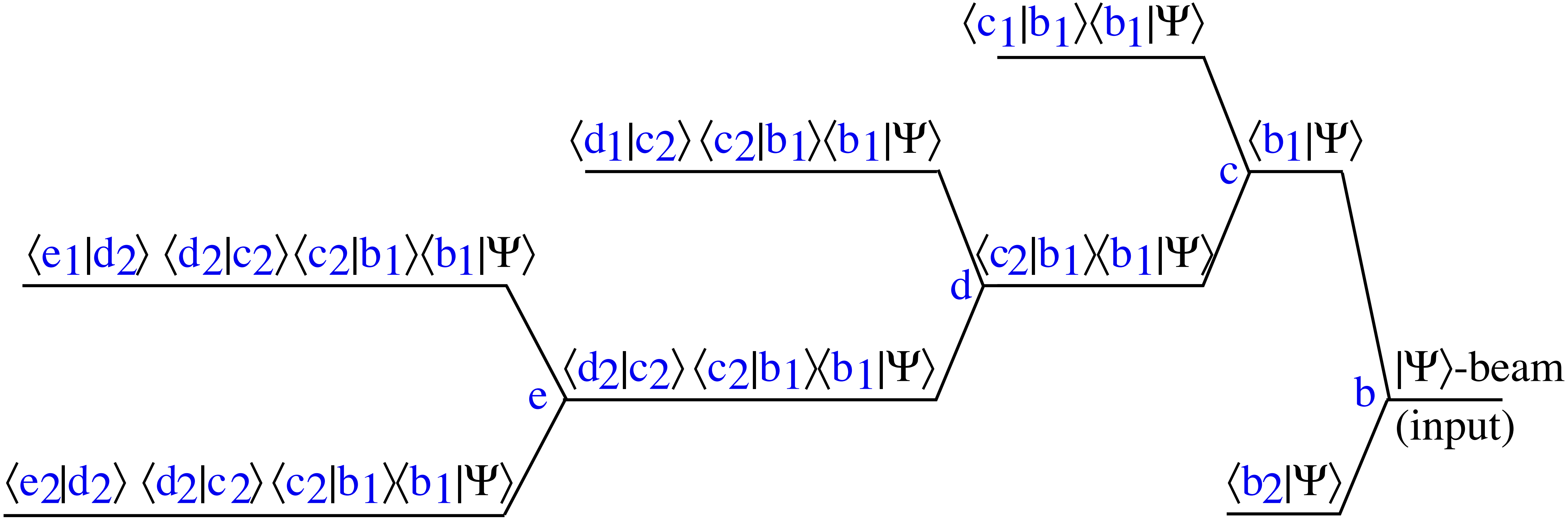


Fig. 1.3.10 Beams-amplitude products for successive beam sorting

Review: "Abstraction" of bra and ket vectors from a Transformation Matrix
  Introducing scalar and matrix products

Planck's energy and N-quanta (Cavity/Beam wave mode)
  Did Max Planck Goof? What's 1-photon worth?
  Feynman amplitude axiom 1

What comes out of a beam sorter channel or branch-b?
  Sample calculations
→ Feynman amplitude axioms 2-3

Beam analyzers: Sorter-unsorters
  The "Do-Nothing" analyzer
  Feynman amplitude axiom 4
  Some "Do-Something" analyzers
    Sorter-counter, Filter, 1/2-wave plate, 1/4-wave plate

Feynman amplitude axiom 1 *(Given above p.35)*

Feynman-Dirac Interpretation of

$$\langle j | k' \rangle$$

*=Amplitude of **state-j** after state-k′ is forced to choose from available **m-type** states*

(1) The probability axiom

The first axiom deals with physical interpretation of amplitudes $\langle j | k' \rangle$.

Axiom 1: The absolute square $\left|\langle j | k' \rangle\right|^2 = \langle j | k' \rangle^* \langle j | k' \rangle$ *gives probability for occurrence in **state-j** of a system that started in state-k′=1′,2′,..,or n′ from one sorter and then was forced to choose between states **j=1,2,...,n** by another sorter.*

Feynman amplitude axioms 1-2

Feynman-Dirac Interpretation of

$$\langle j | k' \rangle$$

*=Amplitude of state-j after state-k′ is forced to choose from available **m-type** states*

(1) The probability axiom

The first axiom deals with physical interpretation of amplitudes $\langle j | k' \rangle$.

Axiom 1: The absolute square $|\langle j | k' \rangle|^2 = \langle j | k' \rangle^* \langle j | k' \rangle$ *gives probability for occurrence in state-j of a system that started in state-k'=1',2',..,or n' from one sorter and then was forced to choose between states j=1,2,...,n by another sorter.*

(2) The conjugation or inversion axiom *(time reversal symmetry)*

The second axiom concerns going backwards through a sorter or the reversal of amplitudes.

Axiom 2: The complex conjugate $\langle j | k' \rangle^*$ *of an amplitude* $\langle j | k' \rangle$ *equals its reverse:* $\langle j | k' \rangle^* = \langle k' | j \rangle$

Feynman amplitude axioms 1-3

Feynman-Dirac Interpretation of

$$\langle j | k' \rangle$$

=Amplitude of state-j after state-k′ is forced to choose from available **m-type** states

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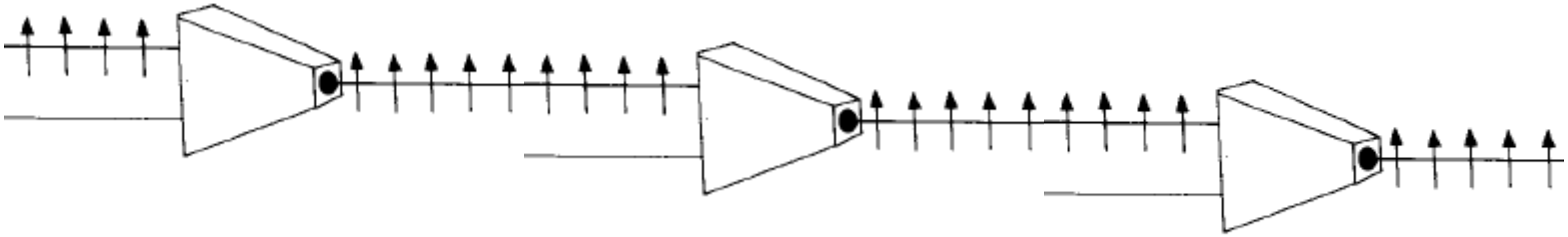
(3) The orthonormality or identity axiom

The third axiom concerns the amplitude for "re measurement" by the same analyzer.

Axiom 3: If identical analyzers are used twice or more the amplitude for a passed state-k is one, and for all others it is zero:

$$\langle j | k \rangle = \delta_{jk} = \begin{Bmatrix} 1 \text{ if: } j{=}k \\ 0 \text{ if: } j \neq k \end{Bmatrix} = \langle j' | k' \rangle$$



Review: "Abstraction" of bra and ket vectors from a Transformation Matrix
- *Introducing scalar and matrix products*

Planck's energy and N-quanta (Cavity/Beam wave mode)
- *Did Max Planck Goof? What's 1-photon worth?*
- *Feynman amplitude axiom 1*

What comes out of a beam sorter channel or branch-b?
- *Sample calculations*
- *Feynman amplitude axioms 2-3*

➡ *Beam analyzers: Sorter-unsorters*
- *The "Do-Nothing" analyzer*
- *Feynman amplitude axiom 4*
- *Some "Do-Something" analyzers*
  - *Sorter-counter, Filter, 1/2-wave plate, 1/4-wave plate*

Beam Analyzers: Fundamental Quantum Processes (Sorter-Unsorter pairs)

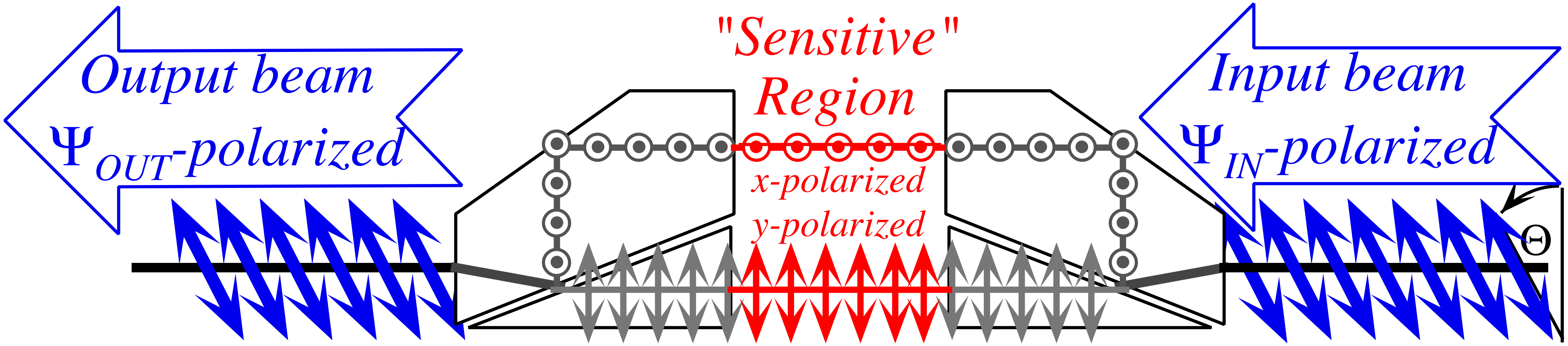


Fig. 1.3.1 Anatomy of ideal optical polarization analyzer in its "Do nothing" mode

Beam Analyzers: Fundamental Quantum Processes (Sorter-Unsorter pairs)

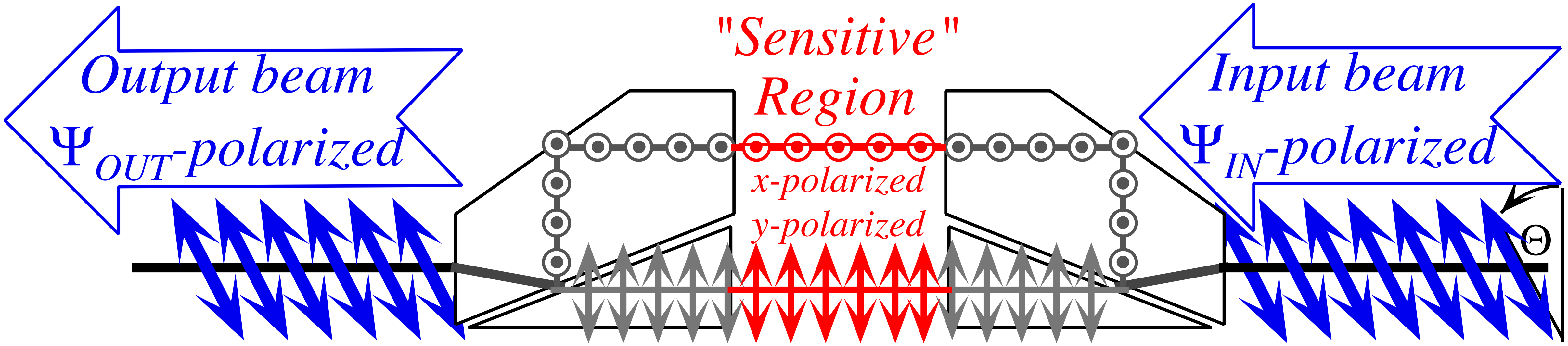


Fig. 1.3.1 Anatomy of ideal optical polarization analyzer in its "Do nothing" mode

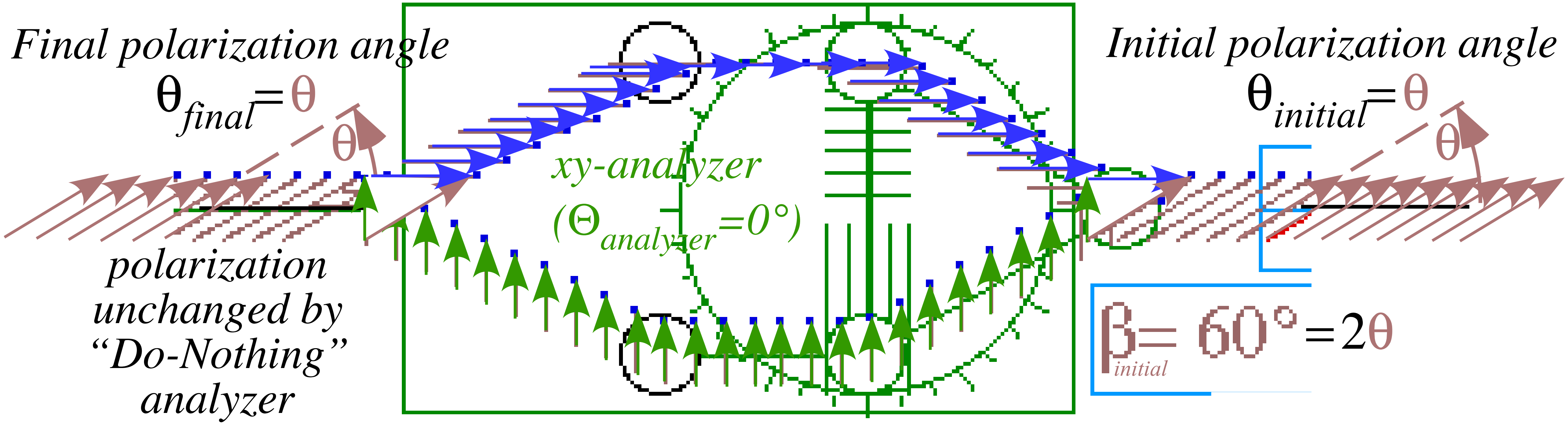


Fig. 1.3.2 Computer sketch of simulated polarization analyzer in "do-nothing" mode

Review: "Abstraction" of bra and ket vectors from a Transformation Matrix
  - *Introducing scalar and matrix products*

Planck's energy and N-quanta (Cavity/Beam wave mode)
  - *Did Max Planck Goof? What's 1-photon worth?*
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What comes out of a beam sorter channel or branch-b?
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Beam analyzers: Sorter-unsorters
  - *The "Do-Nothing" analyzer*
  - *Feynman amplitude axiom 4*
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Beam Analyzers: Fundamental Quantum Processes (Sorter-Unsorter pairs)

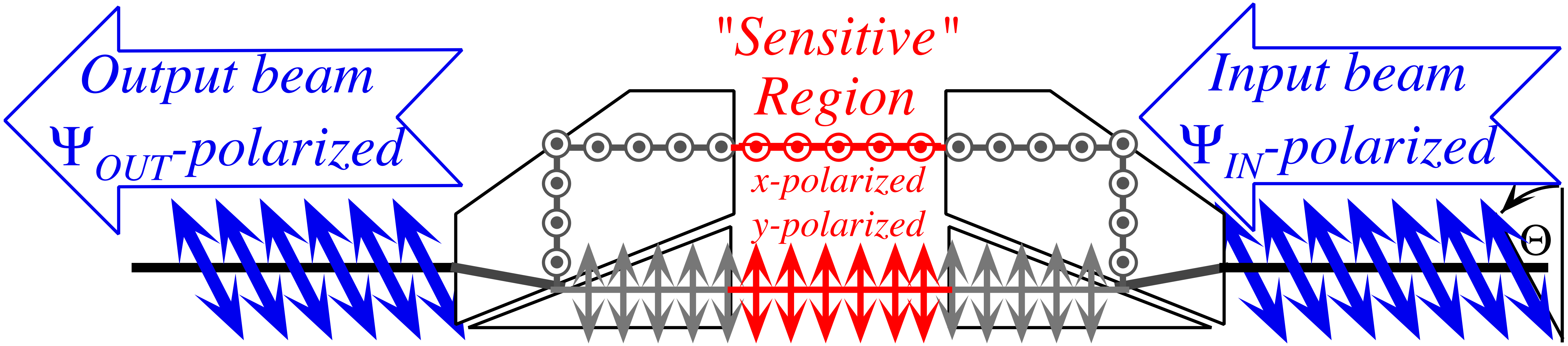


Fig. 1.3.1 Anatomy of ideal optical polarization analyzer in its "Do nothing" mode

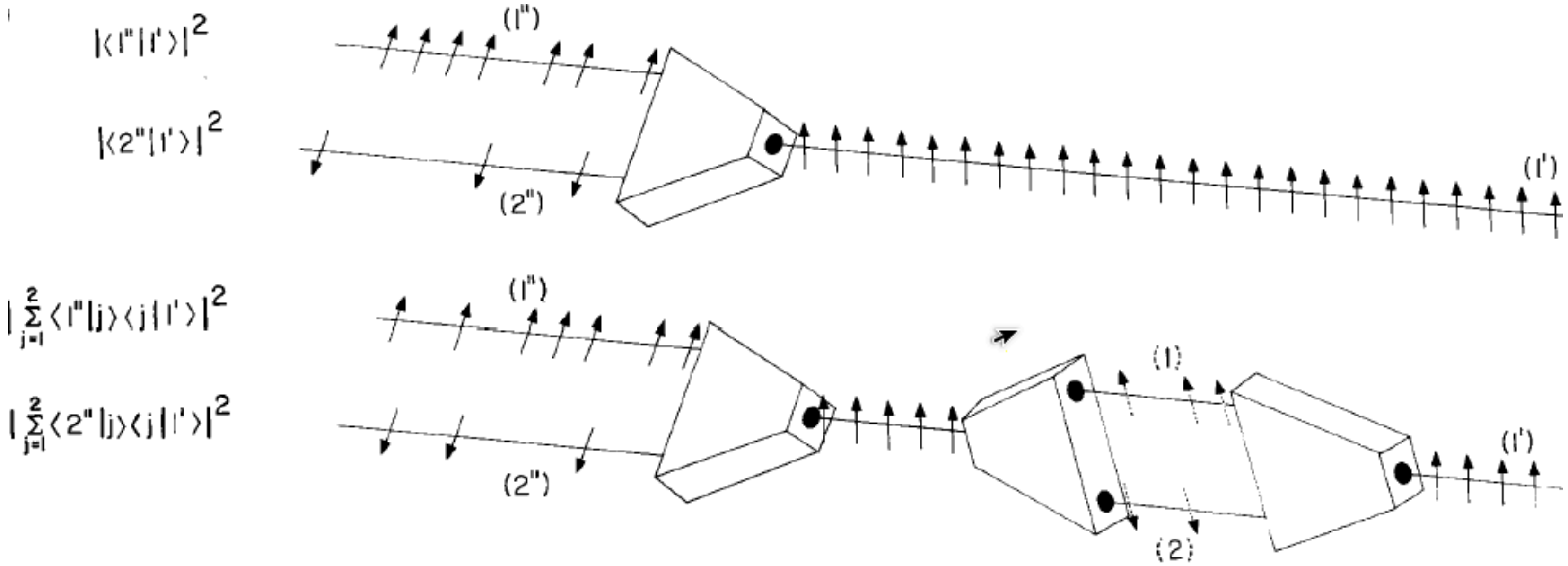
Feynman amplitude axiom 4

(4) The completeness or closure axiom

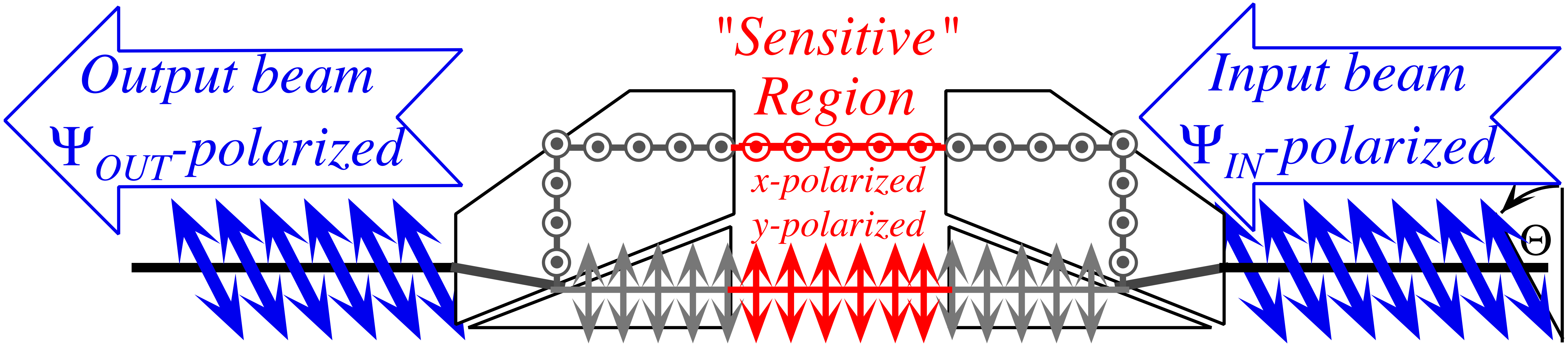
The fourth axiom concerns the "*Do-nothing*" property of an ideal analyzer, that is, a sorter followed by an "unsorter" or "put-back-togetherer" as sketched above.

Axiom 4. Ideal sorting followed by ideal recombination of amplitudes has no effect:

$$\langle j''|m'\rangle = \sum_{k=1}^{n} \langle j''|k\rangle\langle k|m'\rangle$$



Beam Analyzers: Fundamental Quantum Processes (Sorter-Unsorter pairs)



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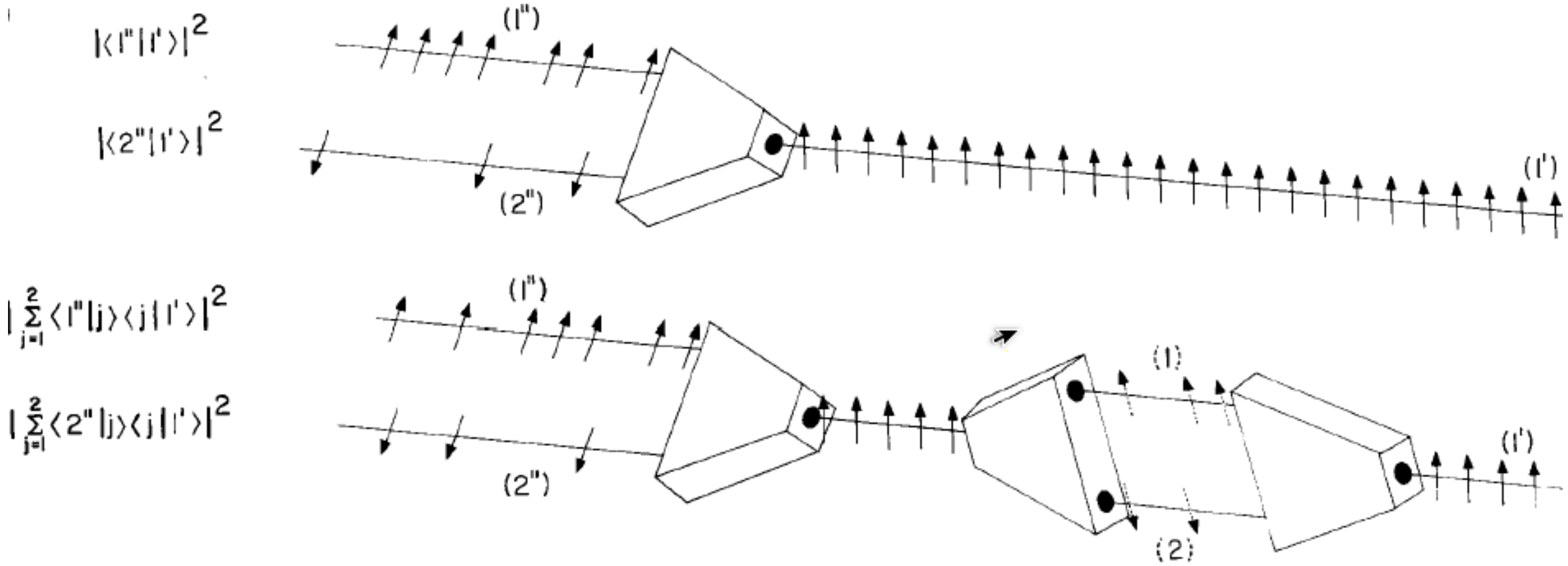

May use axioms 1-3 to prove special case:

$$1 = \langle m'|m'\rangle \quad \text{by 3}$$

$$\Updownarrow_{\text{by 1}}$$

$$1 = \sum_{k=1}^{n} |\langle k|m'\rangle|^2$$

$$\Updownarrow_{\text{by 1}}$$

$$1 = \sum_{k=1}^{n} \langle k|m'\rangle^* \langle k|m'\rangle$$

$$\Updownarrow_{\text{by 2}}$$

$$\sum_{k=1}^{n} \langle m'|k\rangle\langle k|m'\rangle$$

$$= \langle m'|m'\rangle$$

Feynman amplitude axioms 1-4

Feynman-Dirac Interpretation of

$$\langle j|k'\rangle$$

=Amplitude of ***state-j*** *after state-k′ is forced to choose from available* ***m-type*** *states*

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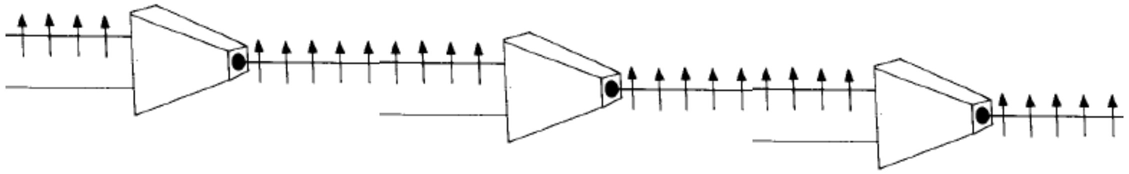

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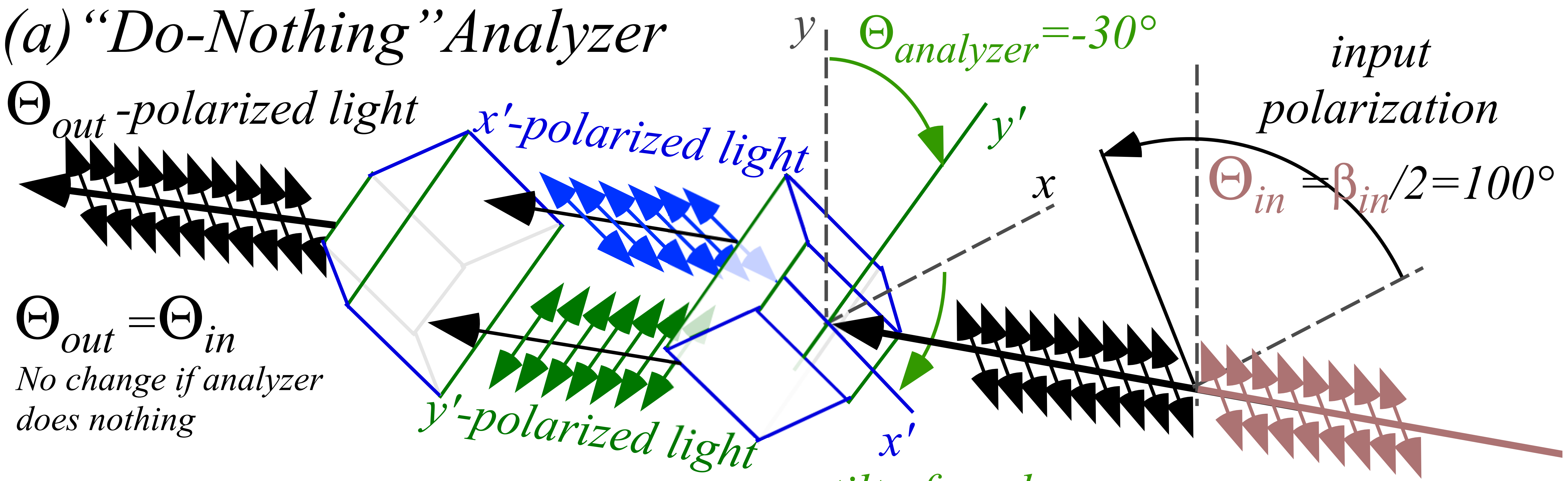
$$\langle j''|m'\rangle = \sum_{k=1}^{n} \langle j''|k\rangle\langle k|m'\rangle$$

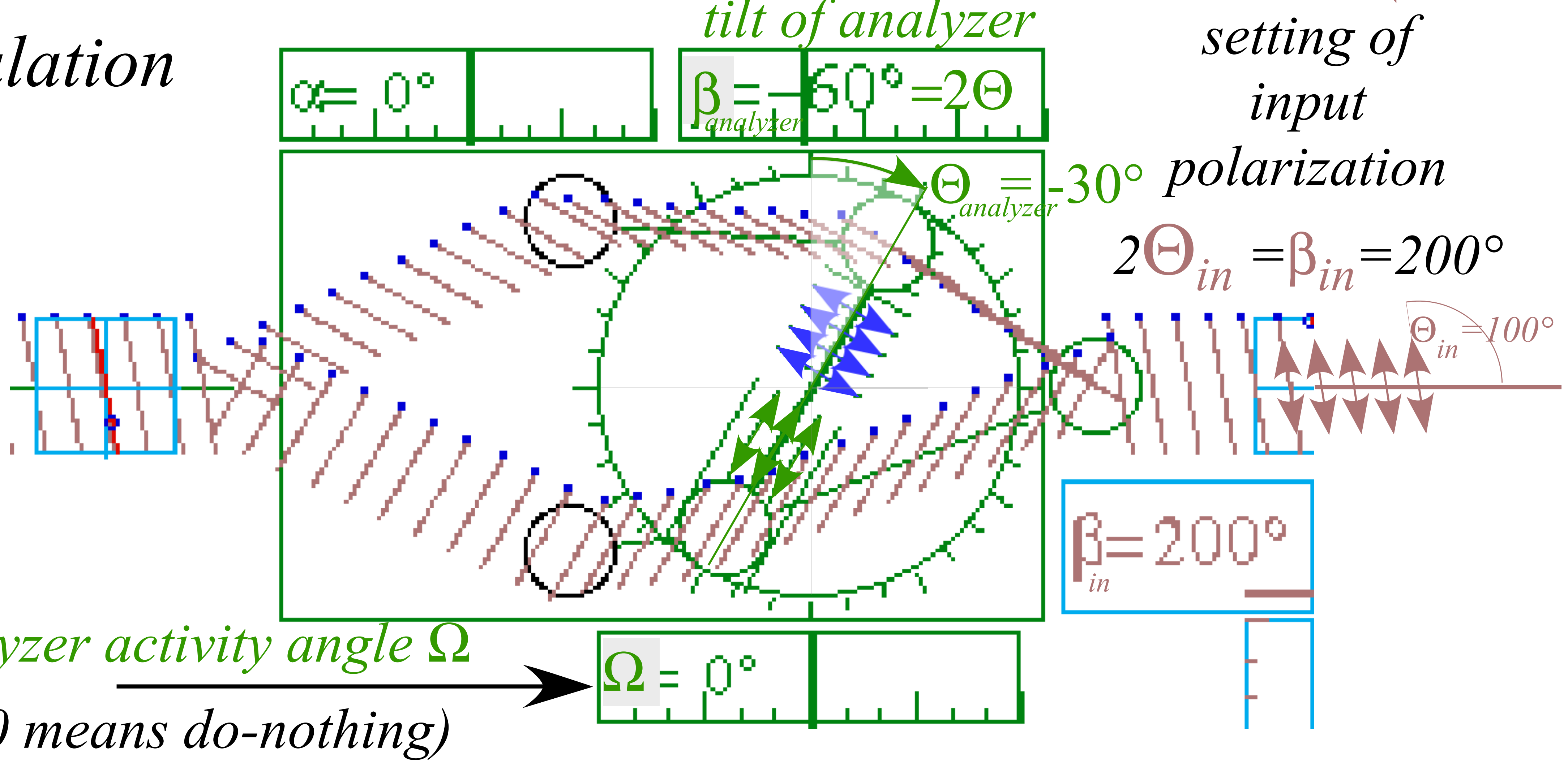
(a) “Do-Nothing” Analyzer



Θ_{out} =Θ_{in}

No change if analyzer does nothing

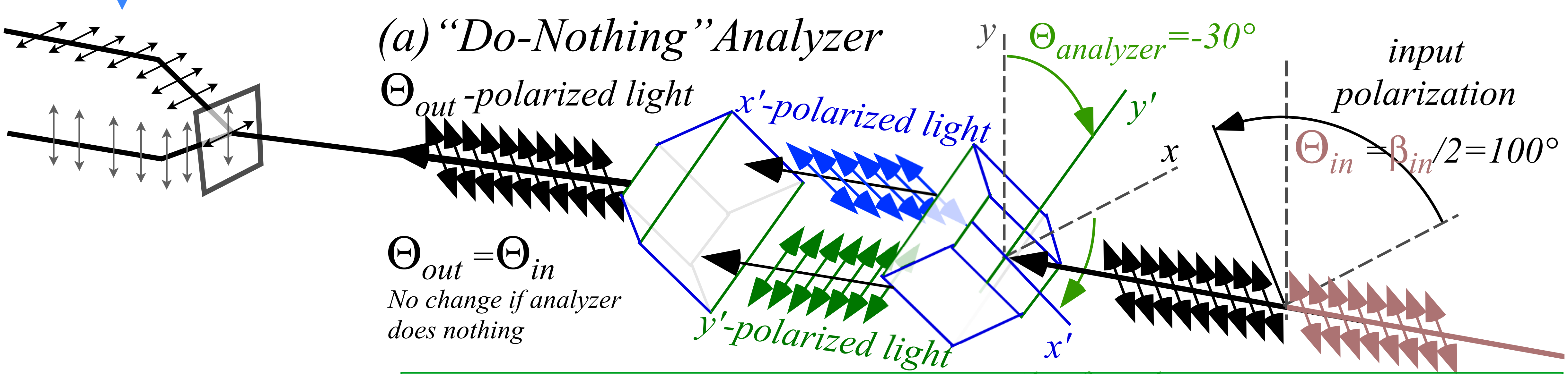
(b) Simulation



analyzer activity angle Ω

(Ω=0 means do-nothing)

Imagine final *xy*-sorter analyzes output beam into *x* and *y*-components.



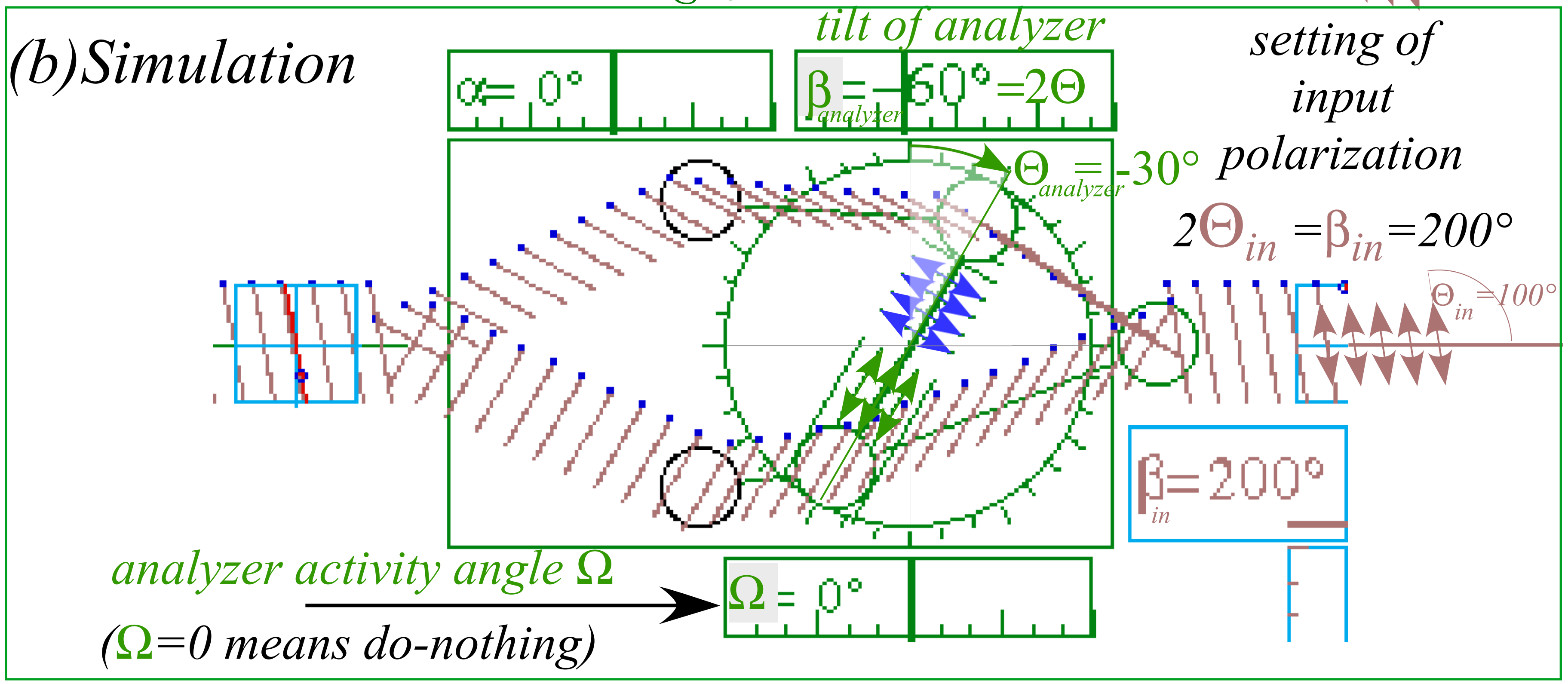
Amplitude in *x* or *y*-channel is sum over *x′* and *y′*-amplitudes

$\langle x'|\Theta_{in}\rangle = \cos(\Theta_{in} - \Theta)$

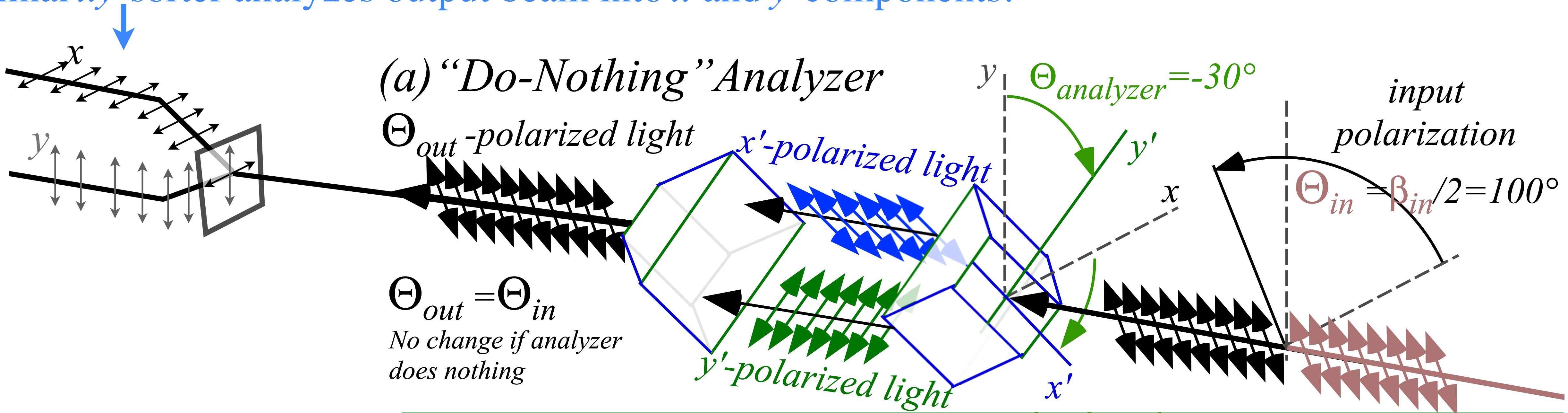
$\langle y'|\Theta_{in}\rangle = \sin(\Theta_{in} - \Theta)$

with relative angle $\Theta_{in} - \Theta$

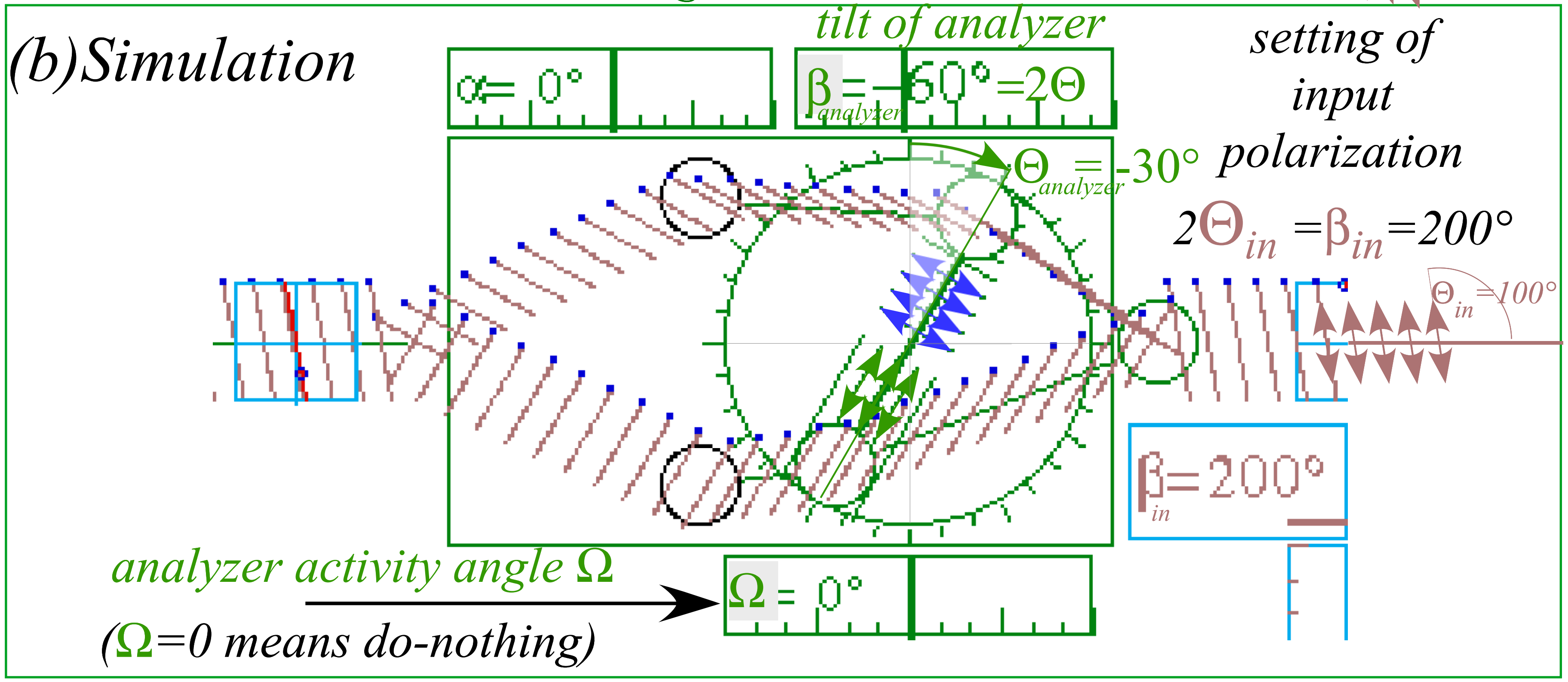
of Θ_{in} to Θ-analyzer axes-(x',y')



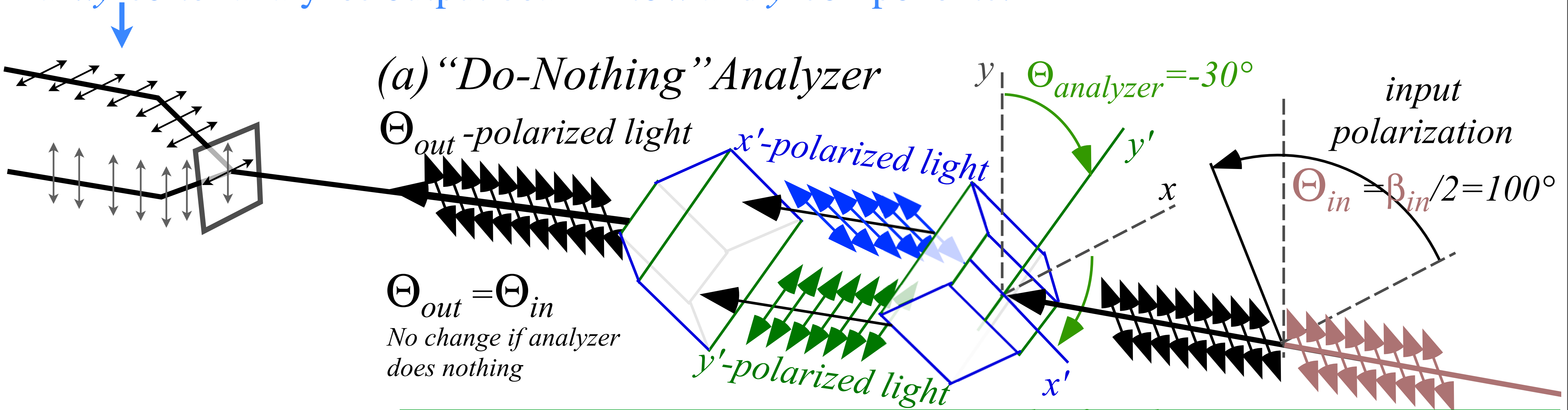
Imagine final *xy*-sorter analyzes output beam into *x* and *y*-components.



Amplitude in *x* or *y*-channel is
sum over *x′* and *y′*-amplitudes
$\langle x'|\Theta_{in}\rangle=\cos(\Theta_{in}-\Theta)$
$\langle y'|\Theta_{in}\rangle=\sin(\Theta_{in}-\Theta)$
with relative angle $\Theta_{in}-\Theta$
of Θ_{in} to Θ-analyzer axes-(x',y')
in products with final *xy*-sorter:
lab x-axis: $\langle x|x'\rangle=\cos\Theta=\langle y|y'\rangle$
y-axis: $\langle y|x'\rangle=\sin\Theta=-\langle x|y'\rangle$.



Imagine final *xy*-sorter analyzes output beam into *x* and *y*-components.



Amplitude in *x* or *y*-channel is sum over *x′* and *y′*-amplitudes

$\langle x'|\Theta_{in}\rangle = \cos(\Theta_{in} - \Theta)$

$\langle y'|\Theta_{in}\rangle = \sin(\Theta_{in} - \Theta)$

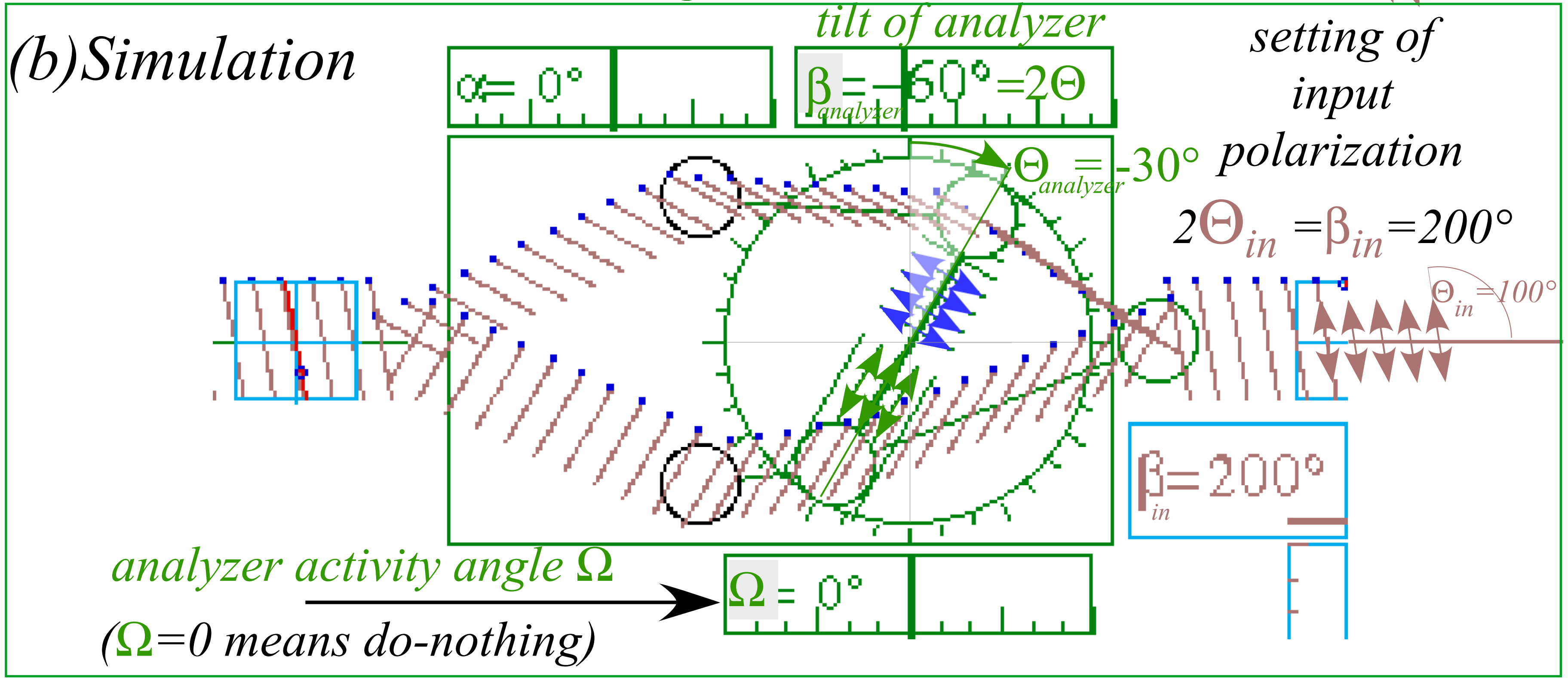
with relative angle $\Theta_{in} - \Theta$

of Θ_{in} to Θ-analyzer axes-(*x′*,*y′*)

in products with final *xy*-sorter:

lab x-axis: $\langle x|x'\rangle = \cos\Theta = \langle y|y'\rangle$

y-axis: $\langle y|x'\rangle = \sin\Theta = -\langle x|y'\rangle$.

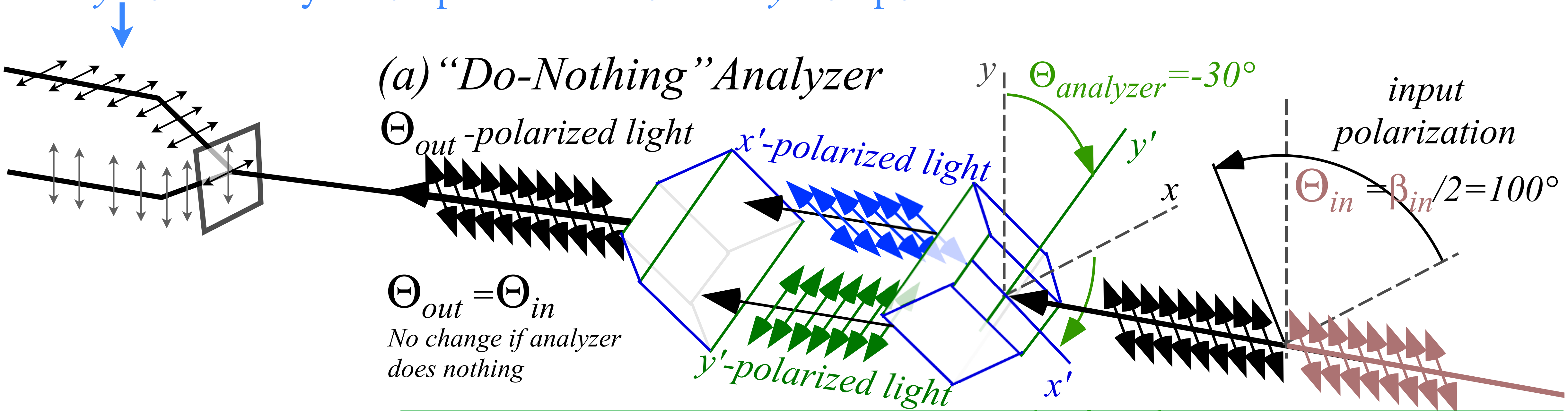


x-Output is: $\langle x|\Theta_{out}\rangle = \langle x|x'\rangle\langle x'|\Theta_{in}\rangle + \langle x|y'\rangle\langle y'|\Theta_{in}\rangle = \cos\Theta\cos(\Theta_{in}-\Theta) - \sin\Theta\sin(\Theta_{in}-\Theta) = \cos\Theta_{in}$

y-Output is: $\langle y|\Theta_{out}\rangle = \langle y|x'\rangle\langle x'|\Theta_{in}\rangle + \langle y|y'\rangle\langle y'|\Theta_{in}\rangle = \sin\Theta\cos(\Theta_{in}-\Theta) - \cos\Theta\sin(\Theta_{in}-\Theta) = \sin\Theta_{in}$.

(Recall $\cos(a+b) = \cos a\cos b - \sin a\sin b$ and $\sin(a+b) = \sin a\cos b + \cos a\sin b$)

Imagine final xy-sorter analyzes output beam into x and y-components.

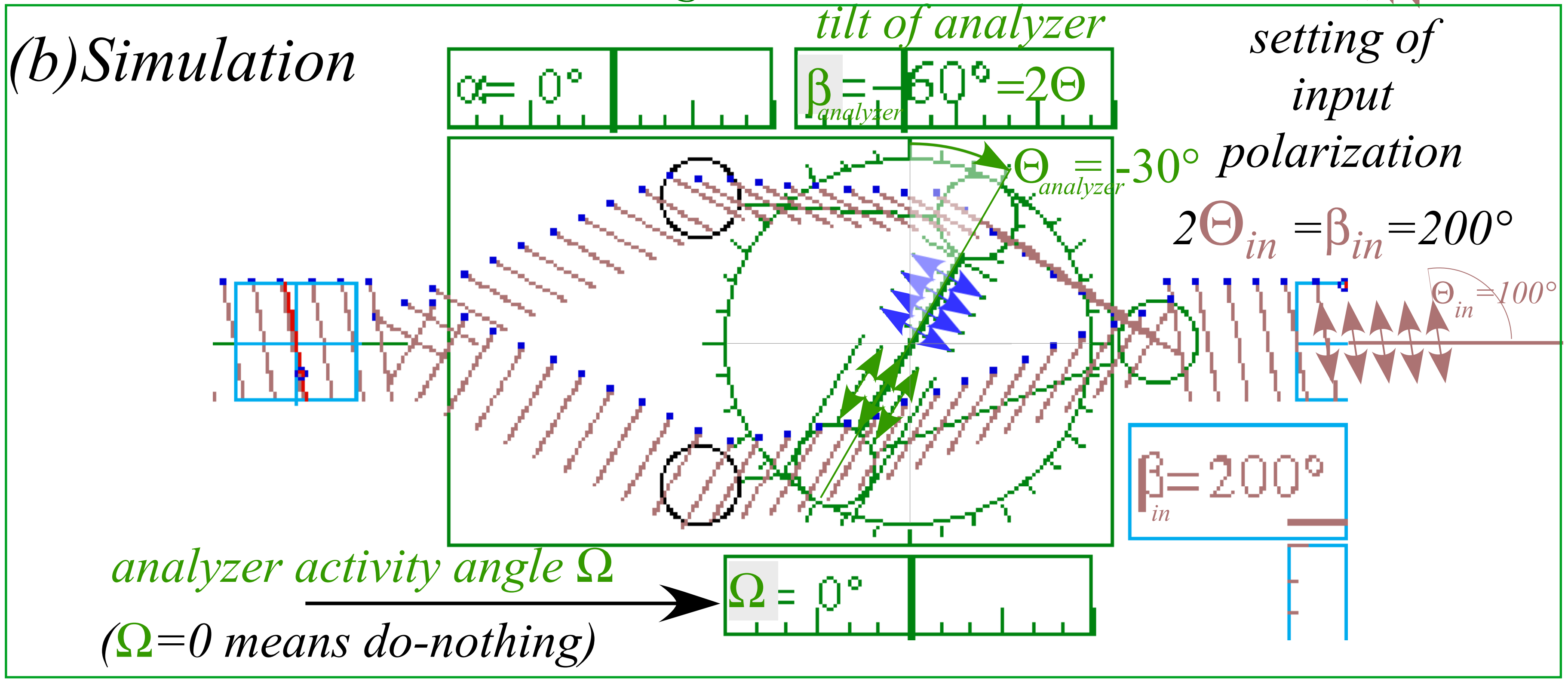


(a) "Do-Nothing" Analyzer

Θ_{out} -polarized light

$\Theta_{out} = \Theta_{in}$
No change if analyzer does nothing

Amplitude in x or y-channel is
sum over x' and y'-amplitudes
$\langle x'|\Theta_{in}\rangle=\cos(\Theta_{in}-\Theta)$
$\langle y'|\Theta_{in}\rangle=\sin(\Theta_{in}-\Theta)$
with relative angle $\Theta_{in}-\Theta$
of Θ_{in} to Θ-analyzer axes-(x',y')
in products with final xy-sorter:
lab x-axis: $\langle x|x'\rangle = \cos\Theta = \langle y|y'\rangle$
y-axis: $\langle y|x'\rangle = \sin\Theta = -\langle x|y'\rangle$.



(b) Simulation

x-Output is: $\langle x|\Theta_{out}\rangle = \langle x|x'\rangle\langle x'|\Theta_{in}\rangle + \langle x|y'\rangle\langle y'|\Theta_{in}\rangle = \cos\Theta\cos(\Theta_{in}-\Theta) - \sin\Theta\sin(\Theta_{in}-\Theta) = \cos\Theta_{in}$

y-Output is: $\langle y|\Theta_{out}\rangle = \langle y|x'\rangle\langle x'|\Theta_{in}\rangle + \langle y|y'\rangle\langle y'|\Theta_{in}\rangle = \sin\Theta\cos(\Theta_{in}-\Theta) - \cos\Theta\sin(\Theta_{in}-\Theta) = \sin\Theta_{in}$.

(Recall $\cos(a+b)=\cos a \cos b - \sin a \sin b$ and $\sin(a+b)=\sin a \cos b + \cos a \sin b$)

Conclusion:

$\langle x|\Theta_{out}\rangle = \cos\Theta_{out} = \cos\Theta_{in}$ or: $\Theta_{out} = \Theta_{in}$ so "Do-Nothing" Analyzer in fact does nothing.

Review: "Abstraction" of bra and ket vectors from a Transformation Matrix
  - *Introducing scalar and matrix products*

Planck's energy and N-quanta (Cavity/Beam wave mode)
  - *Did Max Planck Goof? What's 1-photon worth?*
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What comes out of a beam sorter channel or branch-b?
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Beam analyzers: Sorter-unsorters
  - *The "Do-Nothing" analyzer*
  - *Feynman amplitude axiom 4*
  - *Some "Do-Something" analyzers*
    - ➡ *Sorter-counter, Filter, 1/2-wave plate, 1/4-wave plate*

(1) Optical analyzer in sorter-counter configuration

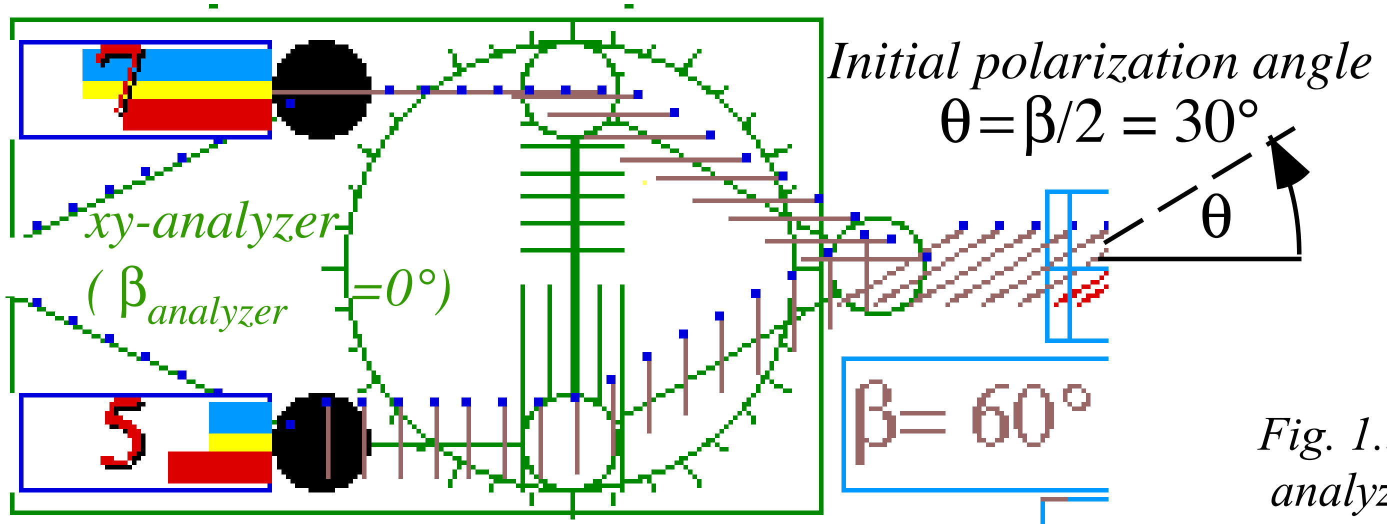
Analyzer reduced to a simple sorter-counter by blocking output of *x*-high-road and *y*-low-road with counters

$$x\text{-counts}\sim\left|\langle x|x'\rangle\right|^2$$

$$=\cos^2\theta=0.75$$

$$y\text{-counts}\sim\left|\langle y|x'\rangle\right|^2$$

$$=\sin^2\theta=0.25$$



Initial polarization angle
$\theta=\beta/2 = 30°$

Fig. 1.3.3 Simulated polarization analyzer set up as a sorter-counter

(1) Optical analyzer in sorter-counter configuration

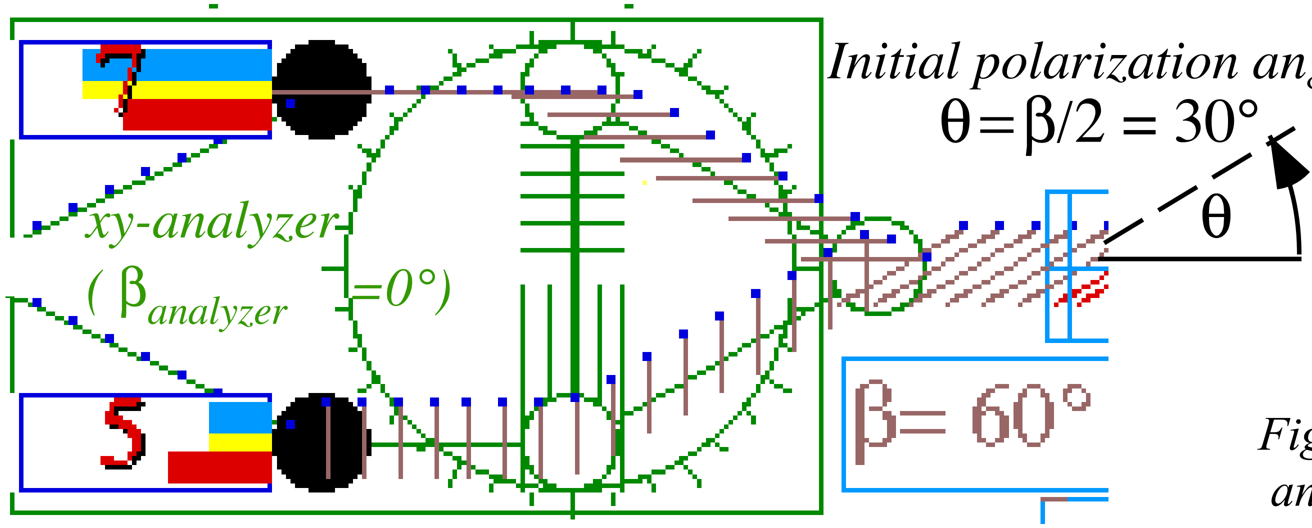
Analyzer reduced to a simple sorter-counter by blocking output of *x*-high-road and *y*-low-road with counters

$$x\text{-counts}\sim\left|\langle x|x'\rangle\right|^2$$

$$=\cos^2\theta=0.75$$

$$y\text{-counts}\sim\left|\langle y|x'\rangle\right|^2$$

$$=\sin^2\theta=0.25$$



Initial polarization angle

$\theta=\beta/2=30°$

Fig. 1.3.3 Simulated polarization analyzer set up as a sorter-counter

(2) Optical analyzer in a filter configuration (Polaroid© sunglasses)

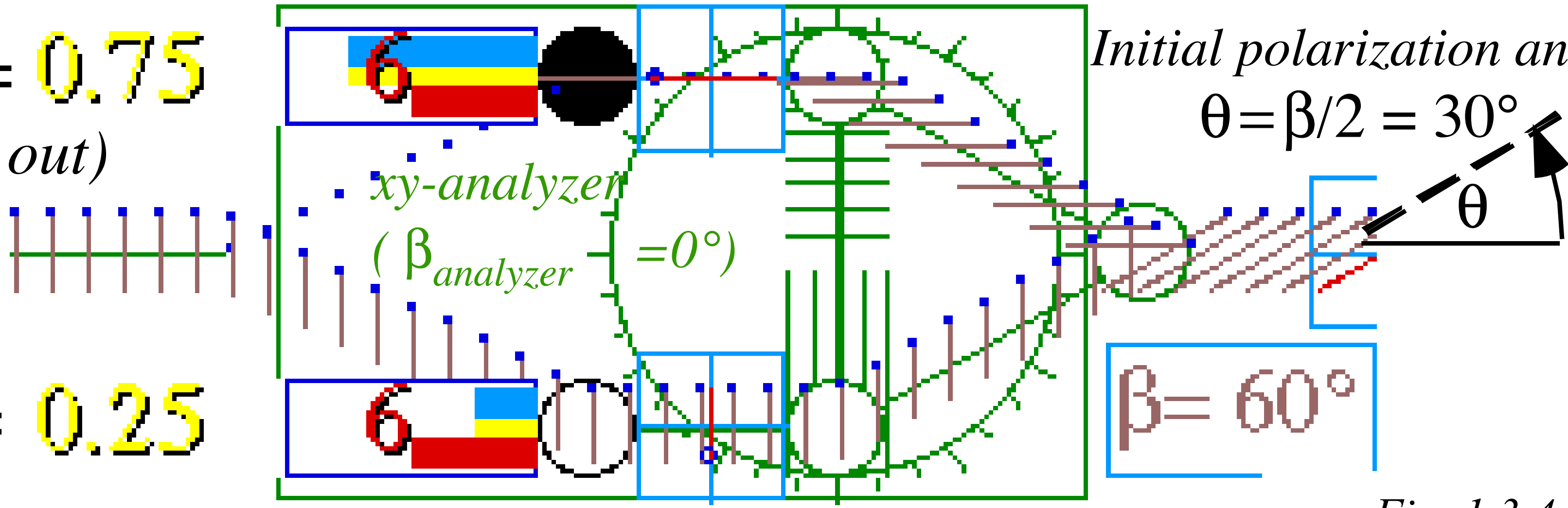
Analyzer blocks one path which may have photon counter without affecting function.

$$x\text{-counts}\sim\left|\langle y|x'\rangle\right|^2=0.75$$

(Blocked and filtered out)

$$y\text{-output}\sim\left|\langle y|x'\rangle\right|^2$$

$$=\sin^2\theta=0.25$$



Initial polarization angle

$\theta=\beta/2=30°$

Fig. 1.3.4 Simulated polarization analyzer set up to filter out the x-polarized photons

- *Review: "Abstraction" of bra and ket vectors from a Transformation Matrix*
  - *Introducing scalar and matrix products*
- *Planck's energy and N-quanta (Cavity/Beam wave mode)*
  - *Did Max Planck Goof? What's 1-photon worth?*
  - *Feynman amplitude axiom 1*
- *What comes out of a beam sorter channel or branch-b?*
  - *Sample calculations*
  - *Feynman amplitude axioms 2-3*
- *Beam analyzers: Sorter-unsorters*
  - *The "Do-Nothing" analyzer*
  - *Feynman amplitude axiom 4*
  - *Some "Do-Something" analyzers*
    - *Sorter-counter, Filter, 1/2-wave plate, 1/4-wave plate*

(3) Optical analyzers in the "control" configuration: Half or Quarter wave plates

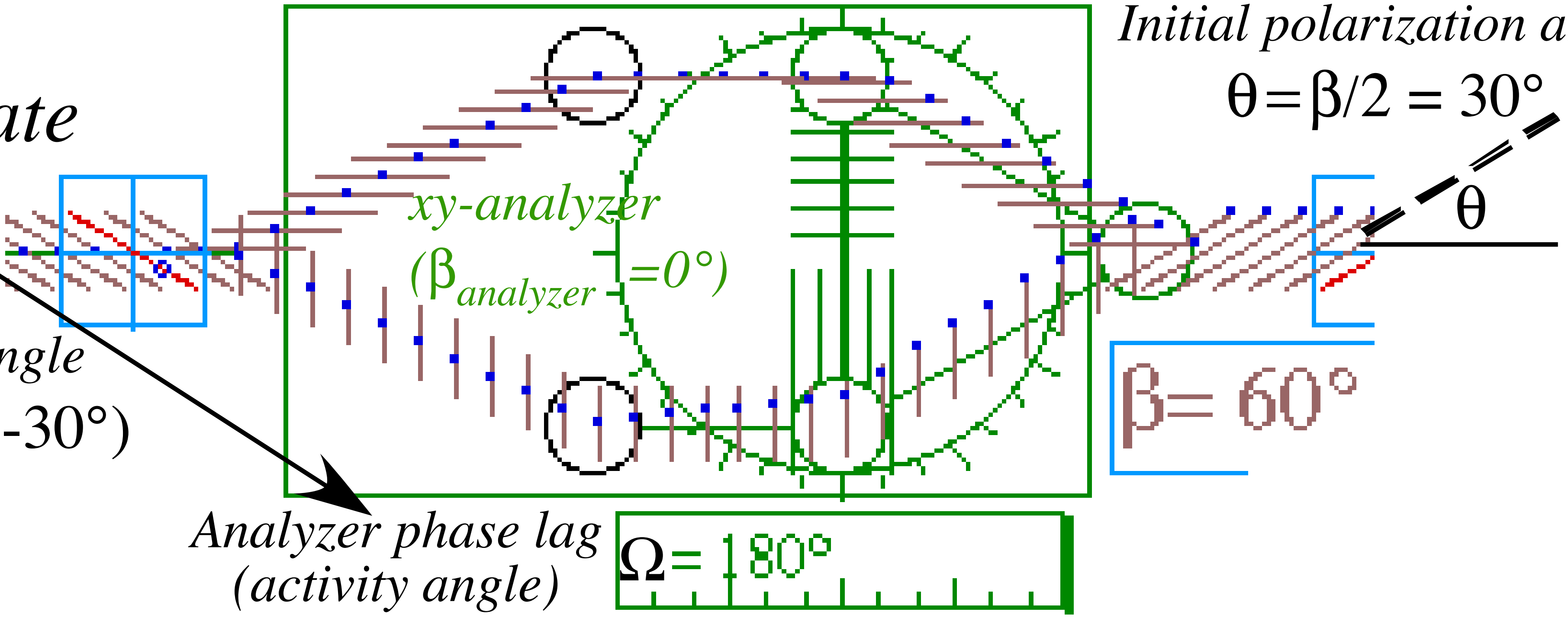
(a)

Half-wave plate

$(\Omega=\pi)$

Final polarization angle

$\theta=\beta/2$ = 150°(or -30°)



Initial polarization angle

$\theta=\beta/2$ = 30°

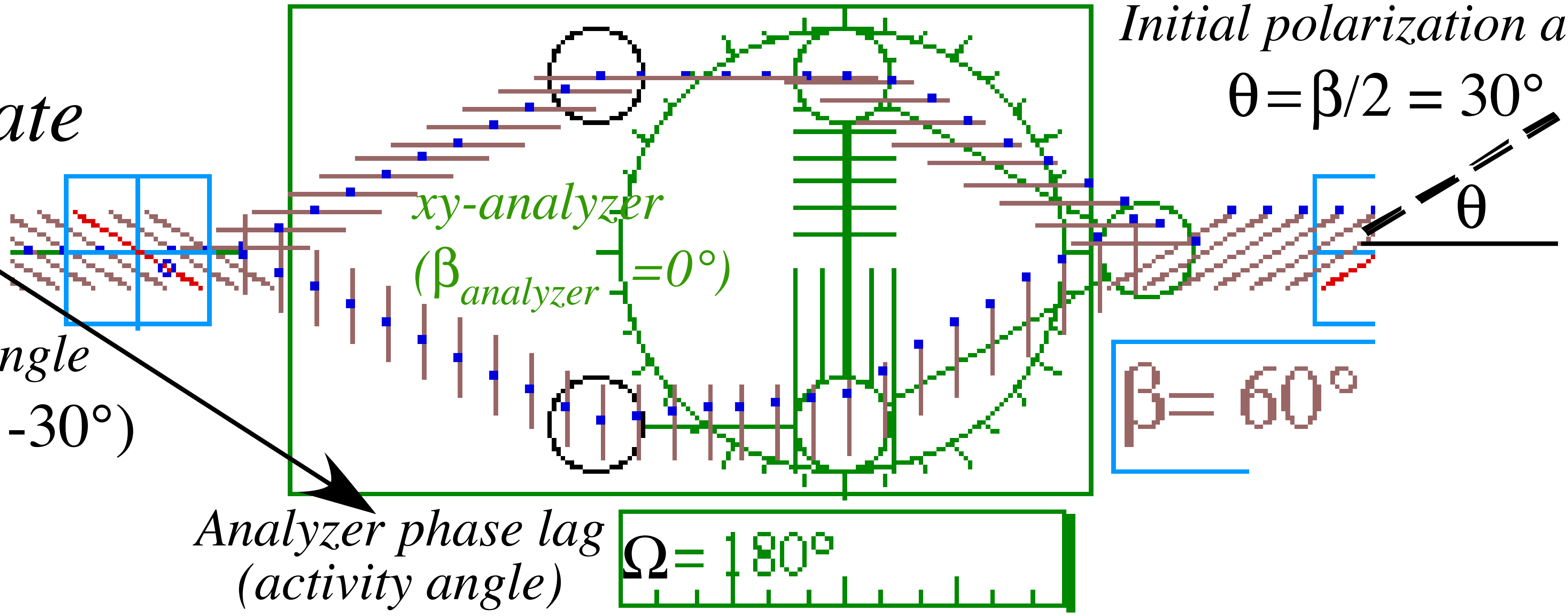
Analyzer phase lag
(activity angle)

(3) Optical analyzers in the "control" configuration: Half or Quarter wave plates

(a)

Half-wave plate

($\Omega=\pi$)



Initial polarization angle

$\theta=\beta/2 = 30°$

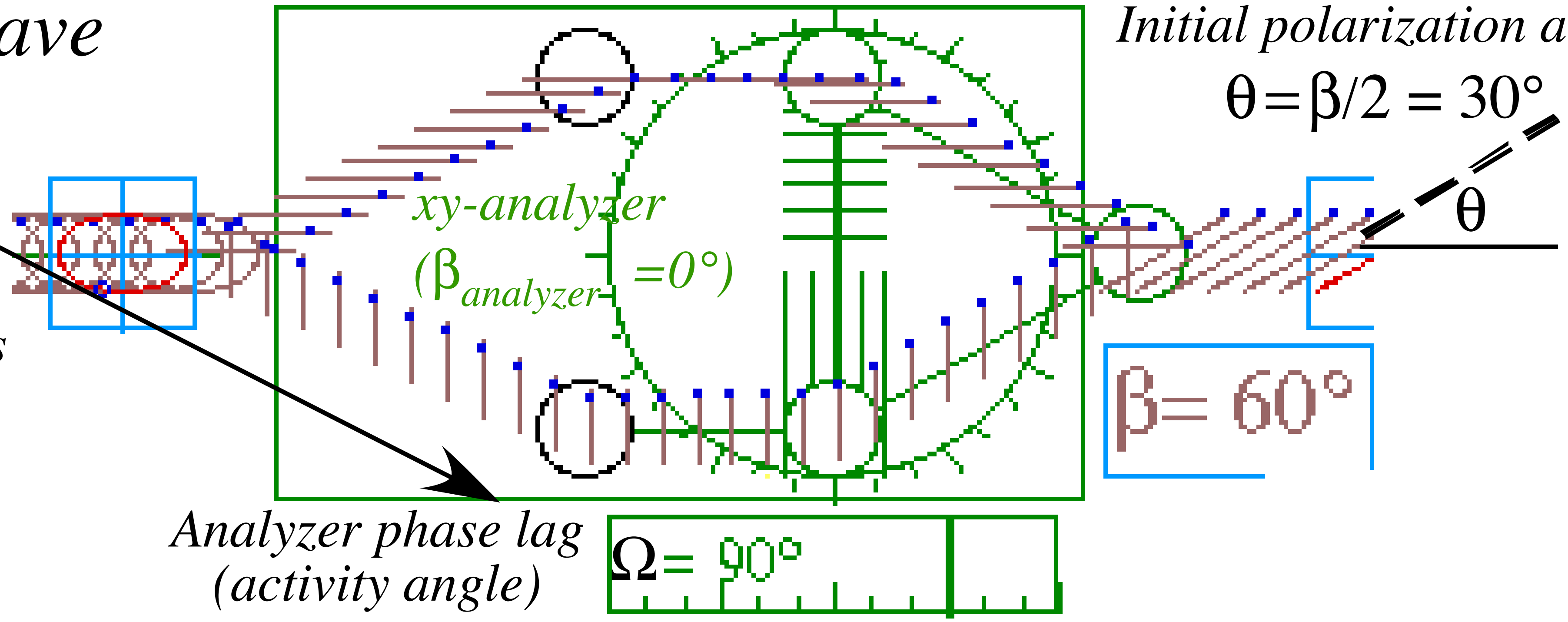
Final polarization angle

$\theta=\beta/2 = 150°$(or -30°)

Analyzer phase lag
(activity angle)

(b) Quarter-wave plate

($\Omega=\pi/2$)



Initial polarization angle

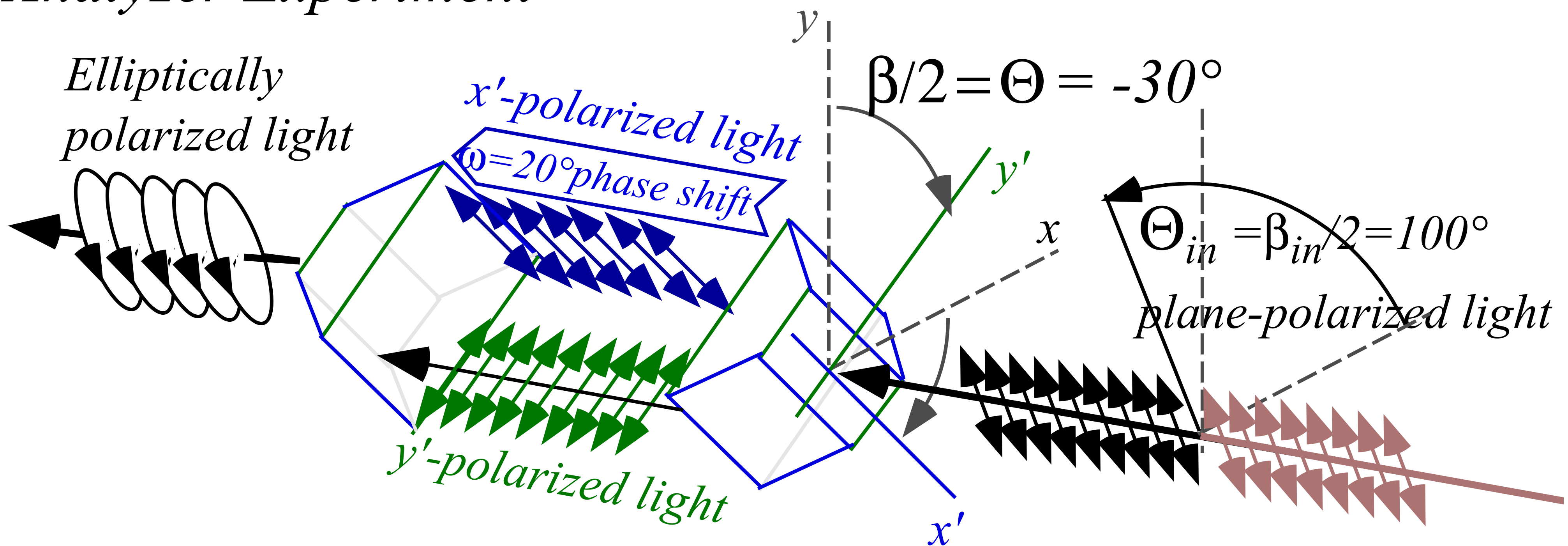
$\theta=\beta/2 = 30°$

Final polarization is
untilted elliptical

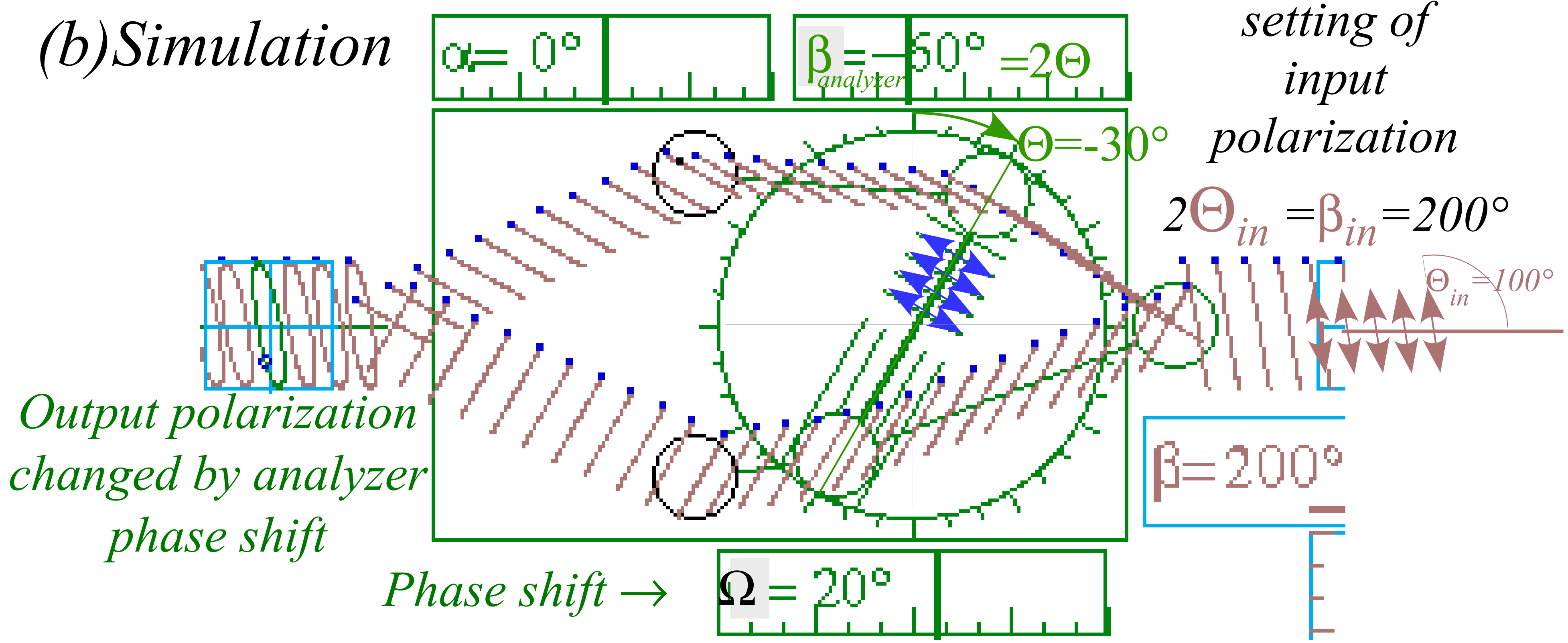
Analyzer phase lag
(activity angle)

Fig. 1.3.5 Polarization control set to shift phase by (a) Half-wave ($\Omega=\pi$) , (b) Quarter wave ($\Omega=\pi/2$)

(a)Analyzer Experiment



(b)Simulation



Similar to "do-nothing" analyzer but has extra phase factor $e^{-i\Omega} = 0.94\text{-}i\ 0.34$ on the x'-path .

x-output:

$$\langle x|\Psi_{out}\rangle = \langle x|x'\rangle e^{-i\Omega}\langle x'|\Psi_{in}\rangle + \langle x|y'\rangle\langle y'|\Psi_{in}\rangle = e^{-i\Omega}\cos\Theta\cos\left(\Theta_{in}-\Theta\right) - \sin\Theta\sin\left(\Theta_{in}-\Theta\right)$$

y-output:

$$\langle y|\Psi_{out}\rangle = \langle y|x'\rangle e^{-i\Omega}\langle x'|\Psi_{in}\rangle + \langle y|y'\rangle\langle y'|\Psi_{in}\rangle = e^{-i\Omega}\sin\Theta\cos\left(\Theta_{in}-\Theta\right) + \cos\Theta\sin\left(\Theta_{in}-\Theta\right)$$

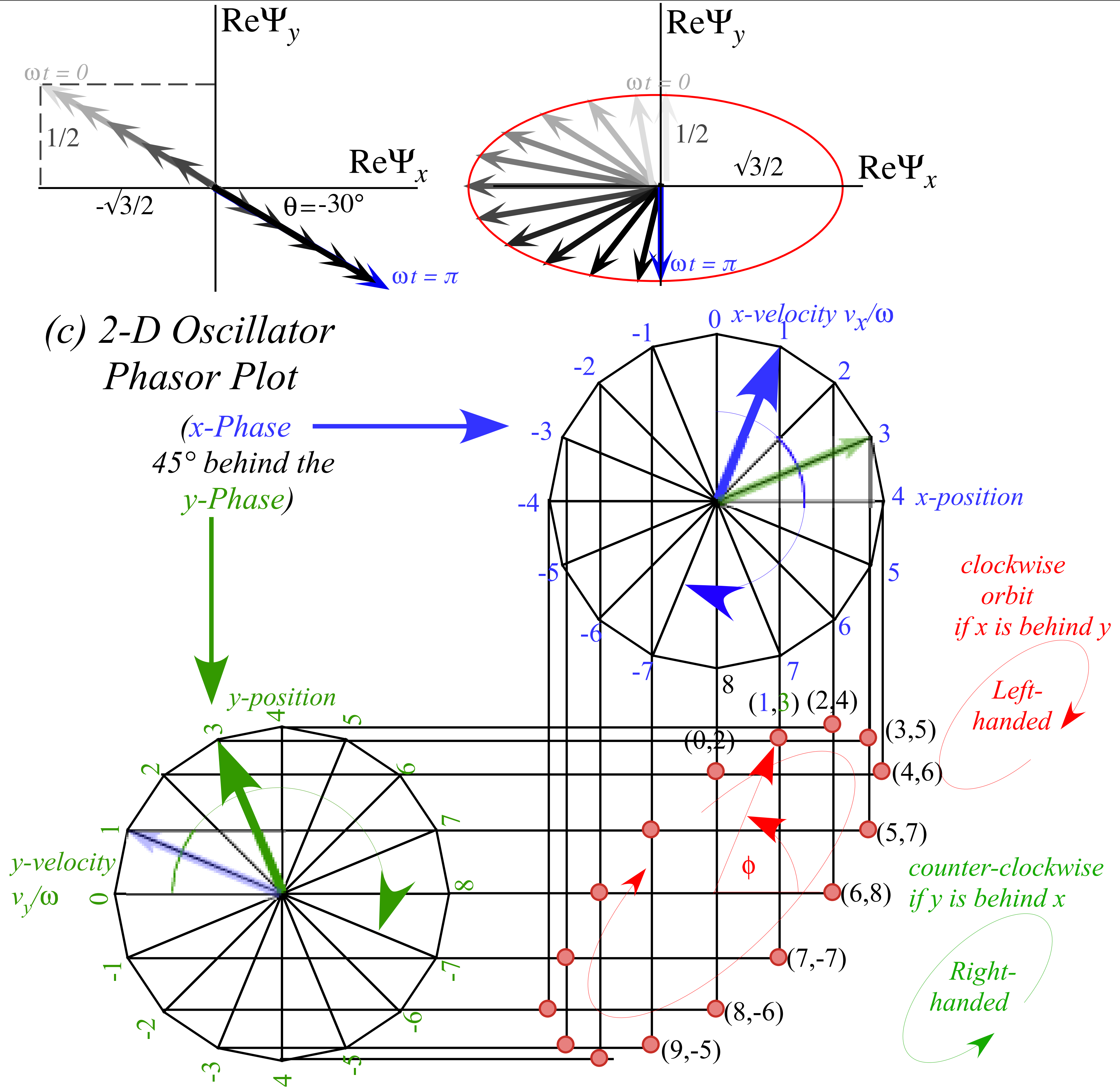
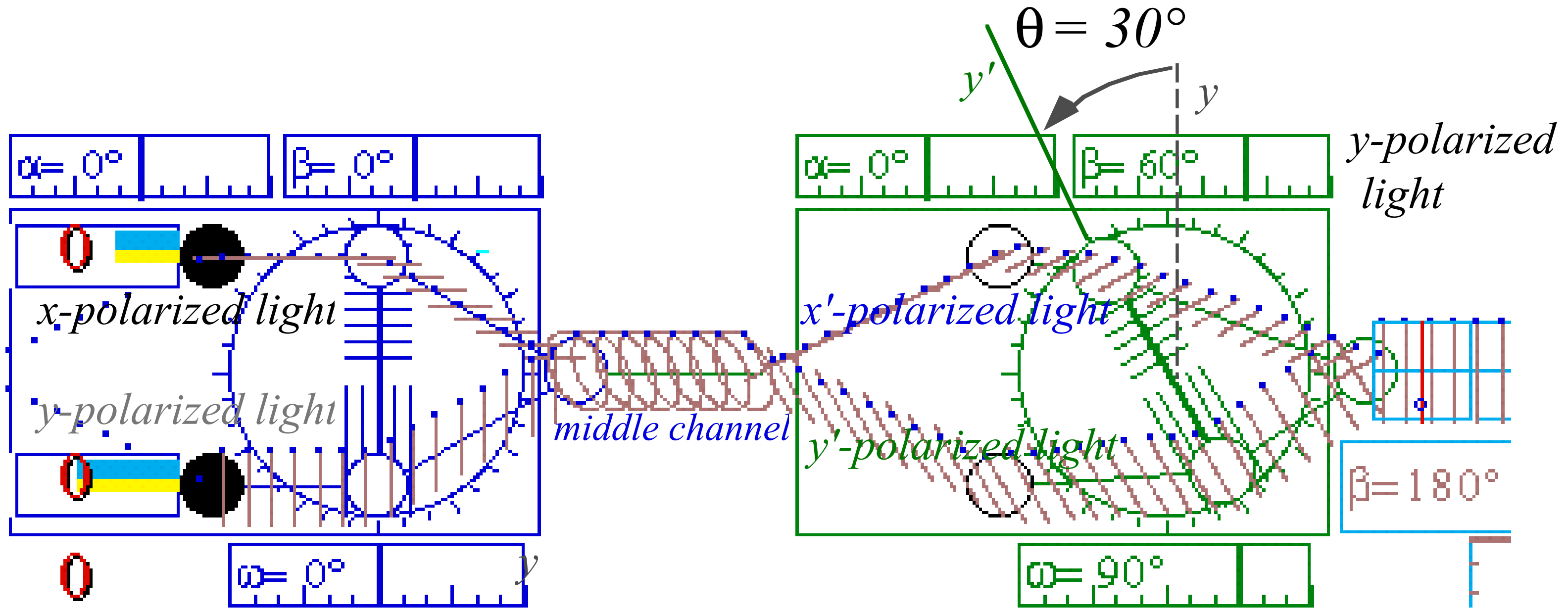


Fig. 1.3.6 Polarization states for (a) Half-wave (Ω=π) , (b) Quarter wave (Ω=π/2) (c) (Ω=–π/4)

Ellipsis in Middle

1.3.1 A y-polarized light beam of unit amplitude (1 photon/sec.) enters an active analyzer that is tilted by 30° as shown below. The active analyzer puts a ω = 90° phase factor e-iω in the x' beam.

Fill in the blanks with numbers or symbols that tell as much as possible about what is present at each channel or branch.



$$\begin{vmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{vmatrix} = \begin{vmatrix} \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} \end{vmatrix} \qquad \begin{vmatrix} \langle x'|x\rangle & \langle x'|y\rangle \\ \langle y'|x\rangle & \langle y'|y\rangle \end{vmatrix} = \begin{vmatrix} \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} \end{vmatrix}$$

State of output x channel

|__⟩ ____________

Amplitude=

____

Probability=

____

State of x' channel

|__⟩ ______

Amplitude=

____

Probability=

____

State of input channel

|y⟩1.0

Amplitude=

1.0

Probability=

1.0

State of middle channel

____

Probability=

____

State of output y channel

|__⟩ ____________

Amplitude=

____

Probability=

____

State of y' channel

|__⟩ ______

Amplitude=

____

Probability=

____

Note: The x' and y' quantities will not be graded. Use for work.