Group Theory in Quantum Mechanics

Lecture 21 *(4.16.13)*

Octahedral $O_h \supset O \supset D_4 \supset C_4$ eiegensolution in coset spaces

(Int.J.Mol.Sci, 14, 714(2013) p.755-774 , QTCA Unit 5 Ch. 15)
(PSDS - Ch. 4)

Review Octahedral $O \supset D_4 \supset C_4$ subgroup chain and coset bases

Coset factored splitting of $O \supset D_4 \supset C_4$ projectors and levels

- *Coset spaces based on $m_4(C_4)\uparrow O$*
- *Splitting class projectors into C_4 cosets and $m_4(C_4)\uparrow O$ bases*

General development of irep projectors $\mathbf{P}^{\mu}_{m_4 m_4}$

- *Calculating $\mathbf{P}^{E}_{0_4 0_4}$*
- *Calculating $\mathbf{P}^{E}_{2_4 2_4}$*
- *Calculating $\mathbf{P}^{T_1}_{0_4 0_4}$*
- *Calculating $\mathbf{P}^{T_1}_{1_4 1_4}$*
- *Calculating $\mathbf{P}^{T_2}_{2_4 2_4}$*

Structure and applications of various subgroup chain ireps

- $O_h \supset D_{4h} \supset C_{4v}$
- $O_h \supset D_{3h} \supset C_{3v}$
- $O_h \supset C_{2v}$

Review Octahedral $O \supset D_4 \supset C_4$ *subgroup chain and coset bases*

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Coset spaces based on $m_4(C_4)\uparrow O$

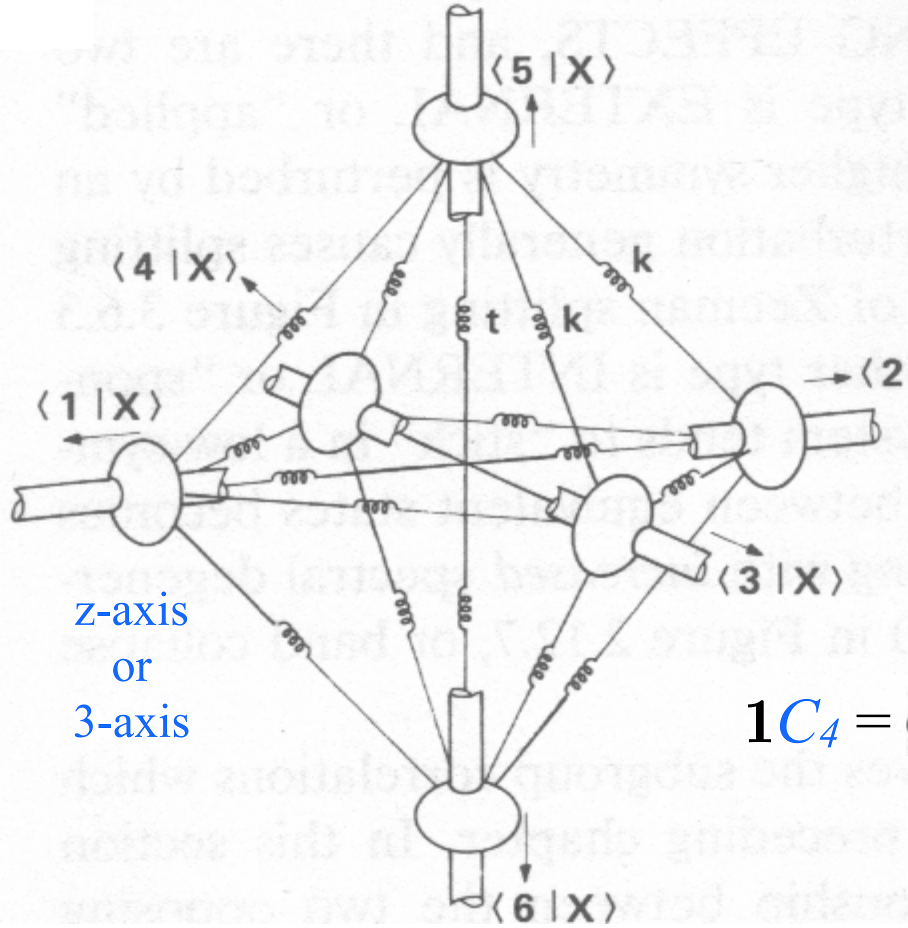
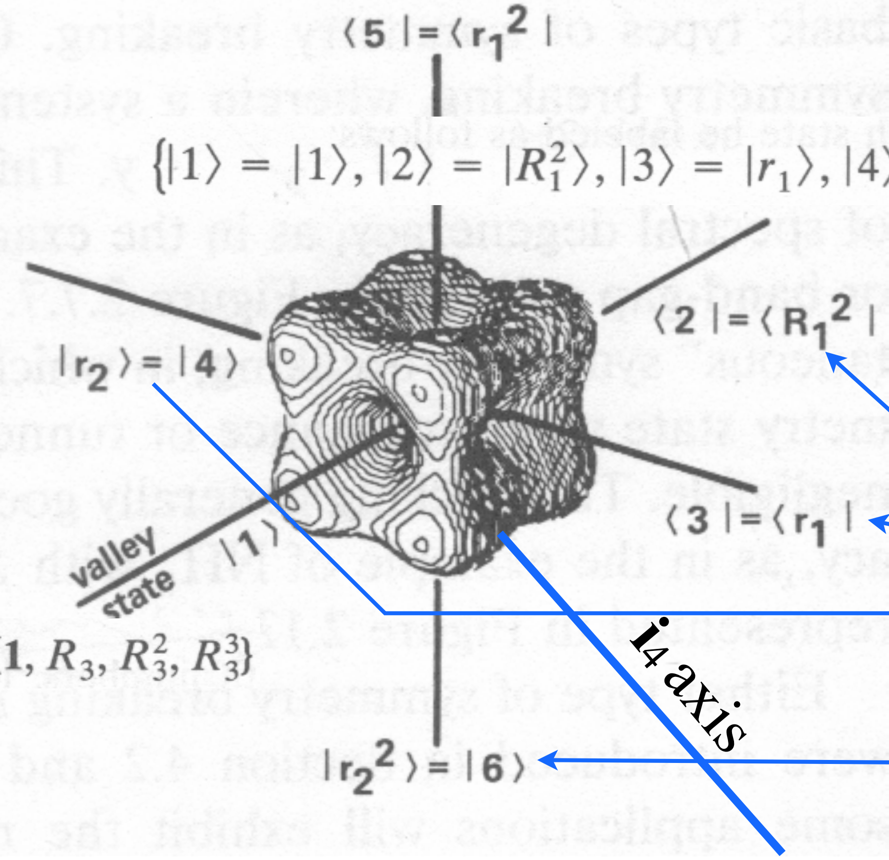


Fig. 4.3.1 PSDS



$$\{|1\rangle = |1\rangle, |2\rangle = |R_1^2\rangle, |3\rangle = |r_1\rangle, |4\rangle = |r_2\rangle, |5\rangle = |r_1^2\rangle, |6\rangle = |r_2^2\rangle$$

$$\mathbf{1}C_4 = C_4 = \{\mathbf{1}, R_3, R_3^2, R_3^3\}$$

Thus we label states by left *cosets* $\mathbf{r}_\ell C_4$ *of Local* C_4

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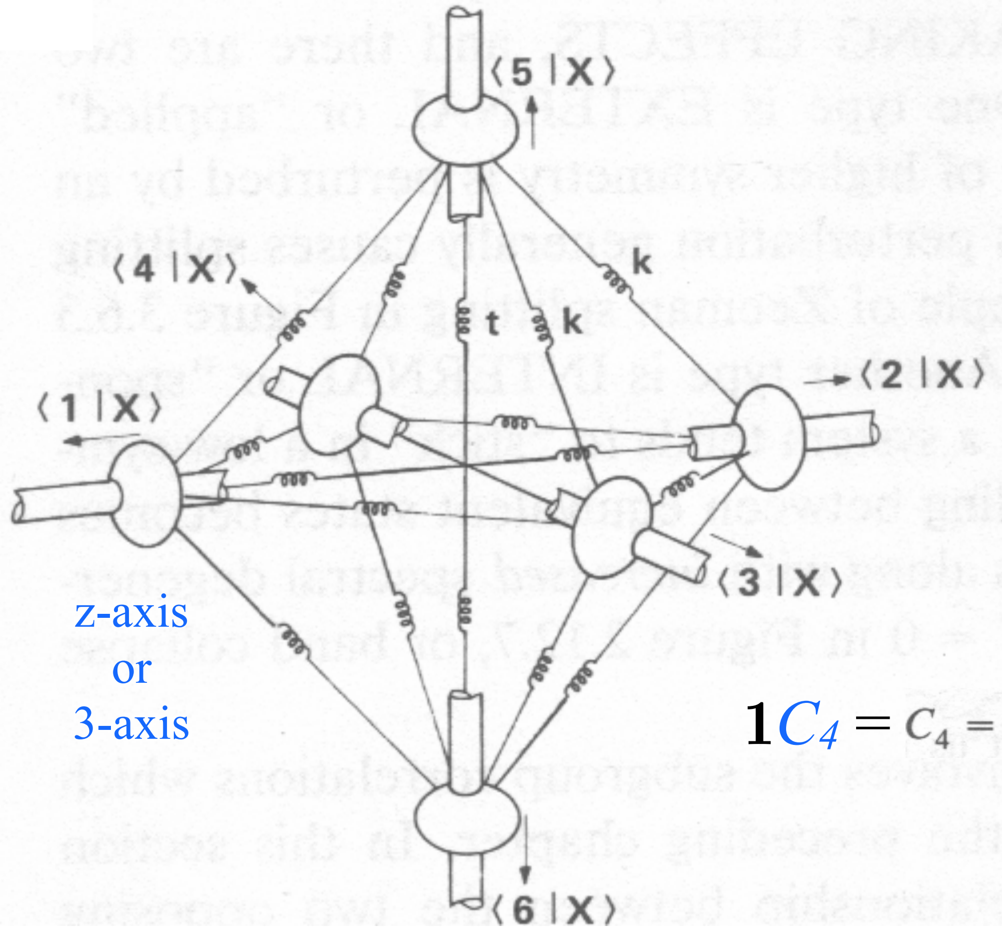
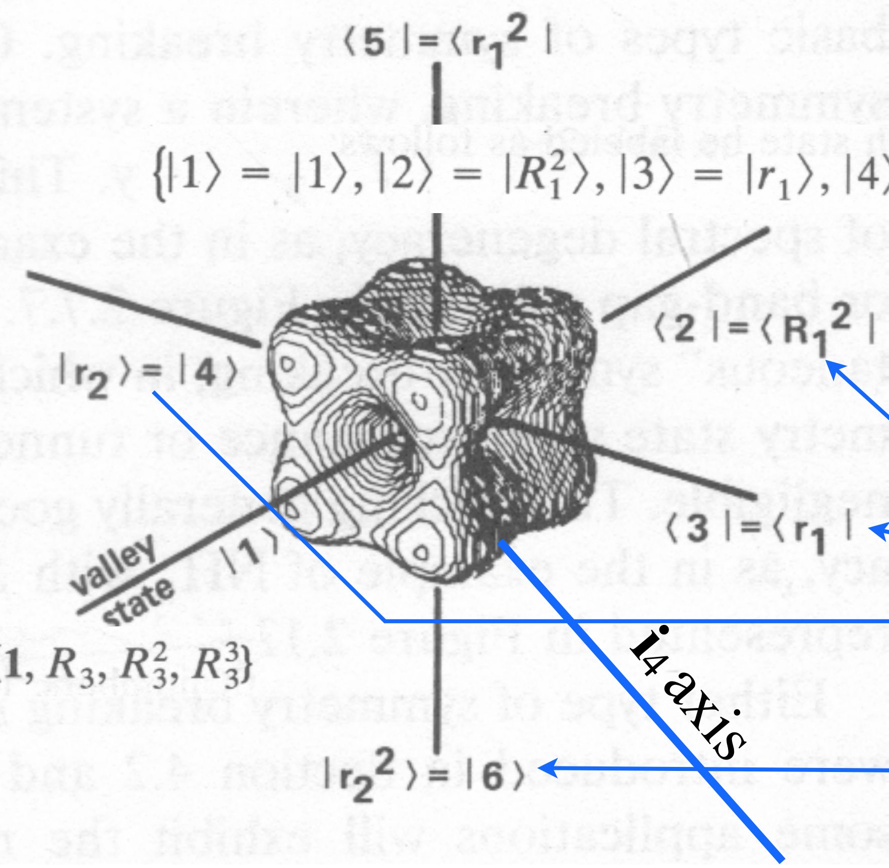


z-axis
or
3-axis



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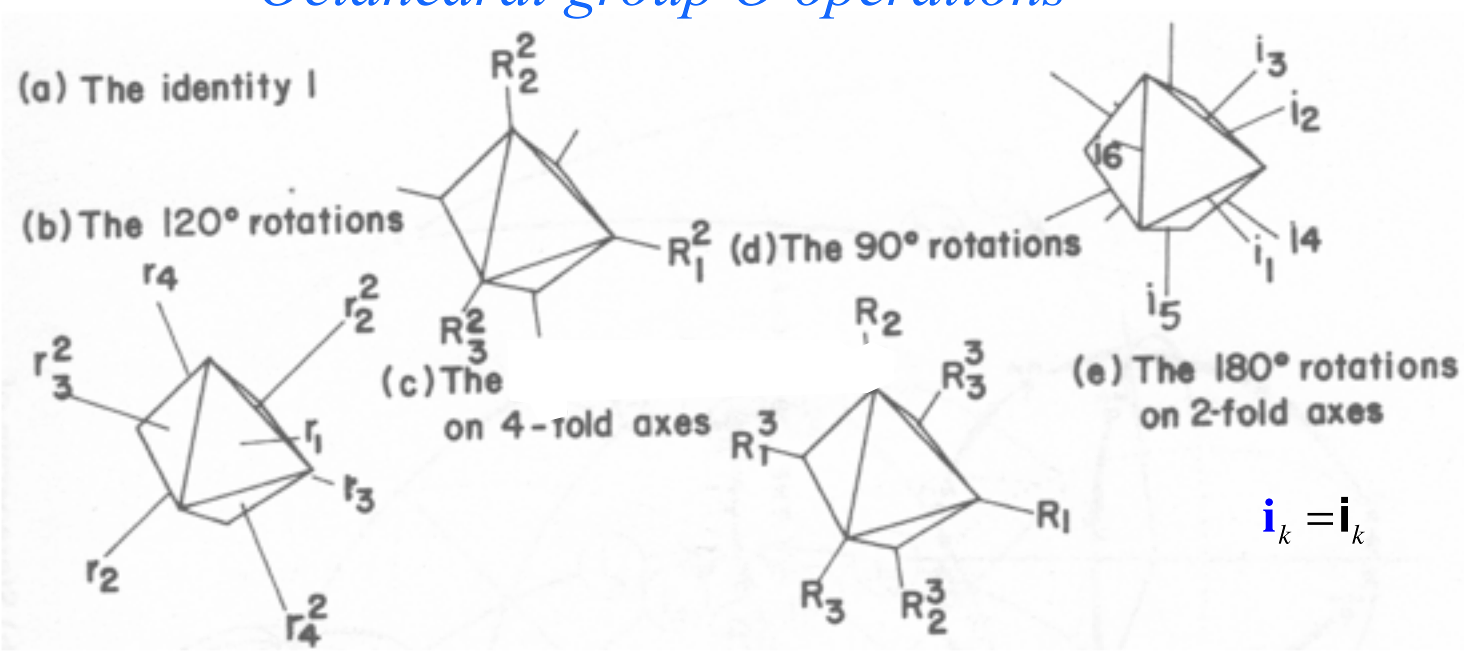
Octahedral group O operations

Class of 1: **1**



$\mathbf{i}_k = \mathbf{i}_k$

Coset spaces based on $m_4(C_4)\uparrow O$

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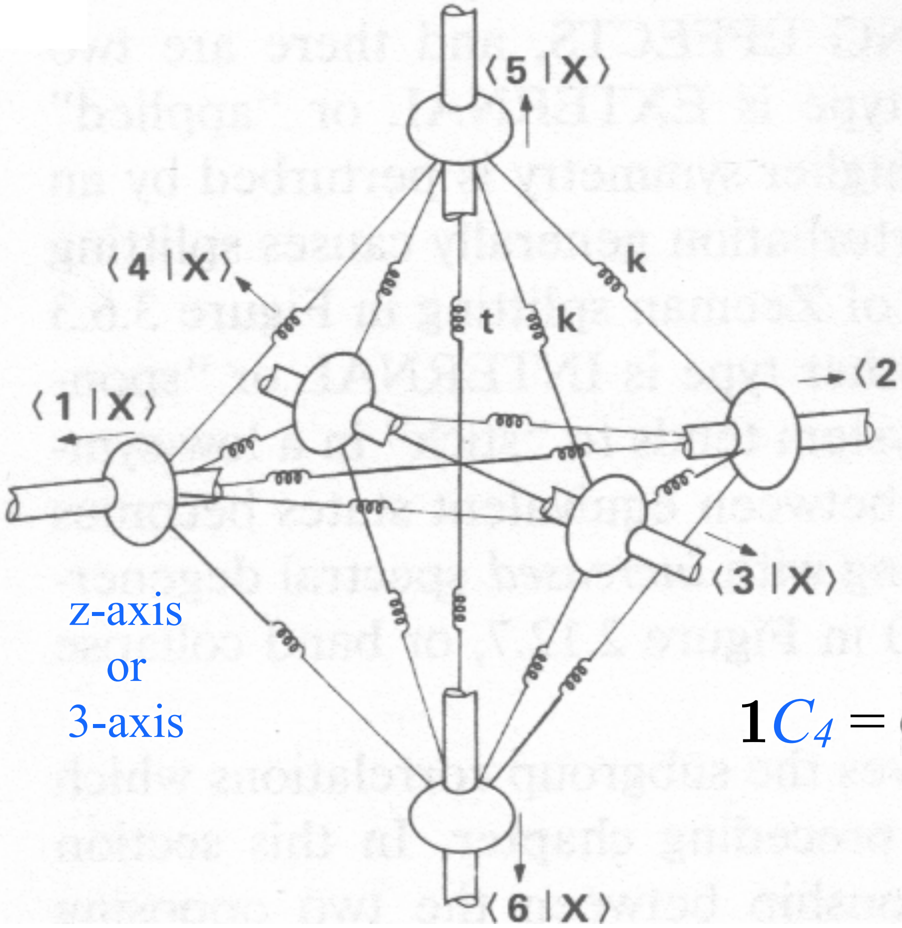

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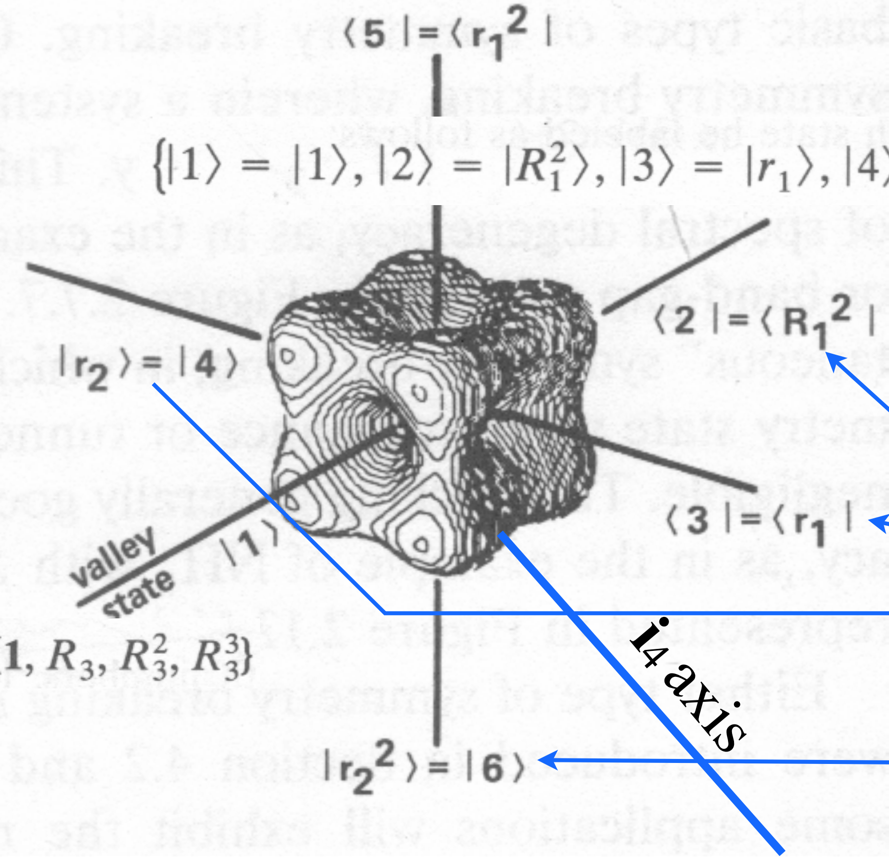

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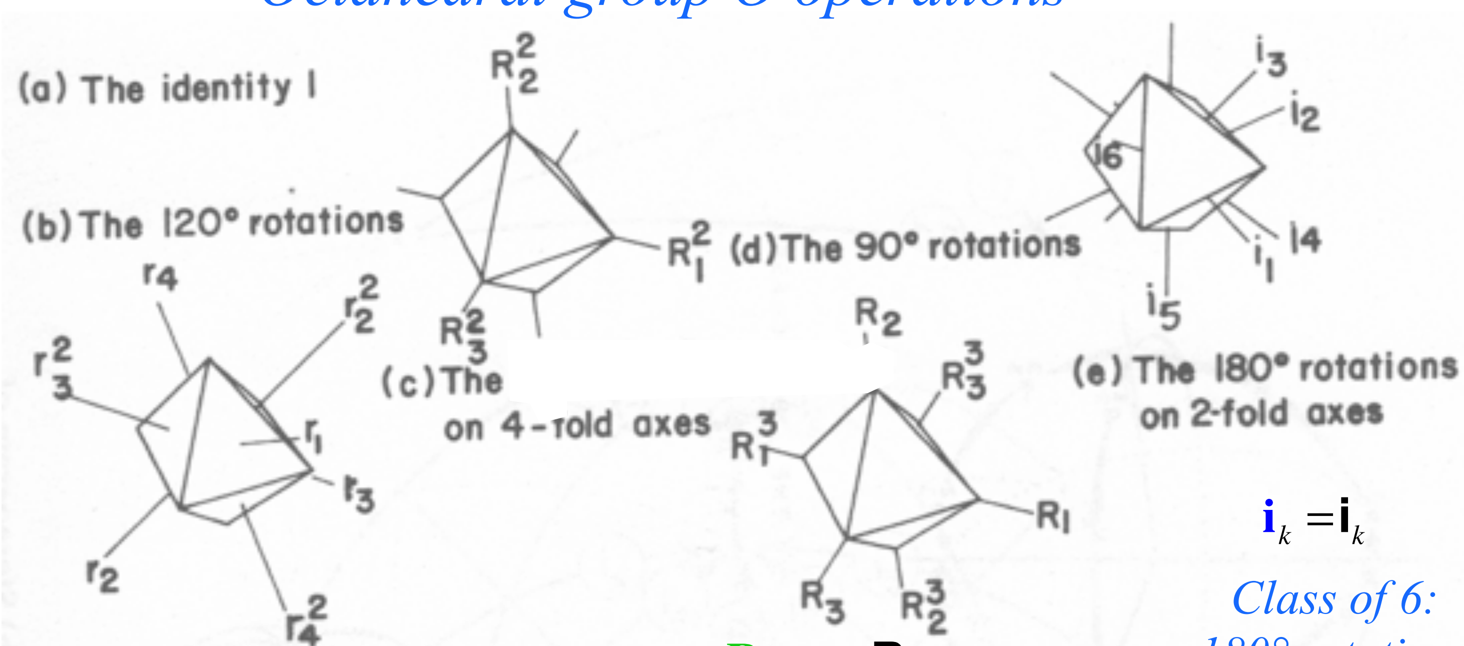

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Class of 6: 180° rotations on [110] diagonals

$\mathbf{R}_{x,y,z} = \mathbf{R}_{1,2,3}$

$\mathbf{r}_k = \mathbf{r}_k$ $\quad \tilde{\mathbf{r}}_k = \mathbf{r}_k^2 = \mathbf{r}_k^{-1}$

Class of 8: ±120° rotations on [111] axes

$\rho_{x,y,z} = \mathbf{R}_{1,2,3}^2$

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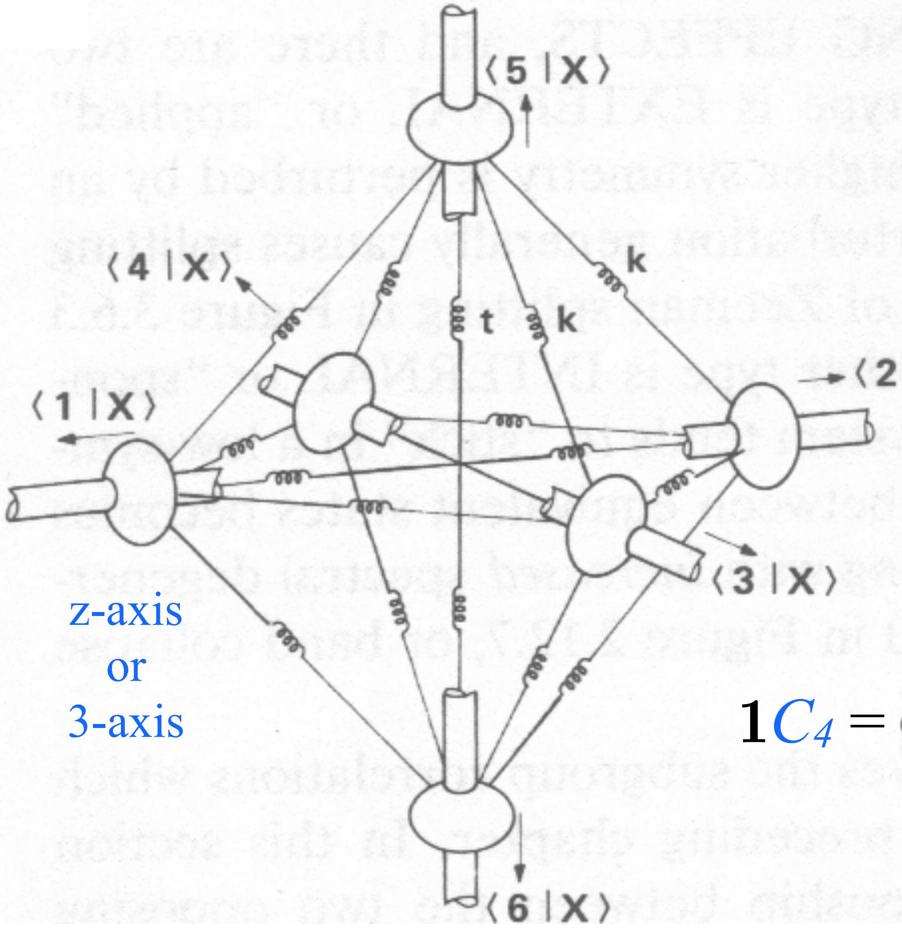
Coset spaces based on $m_4(C_4)\uparrow O$



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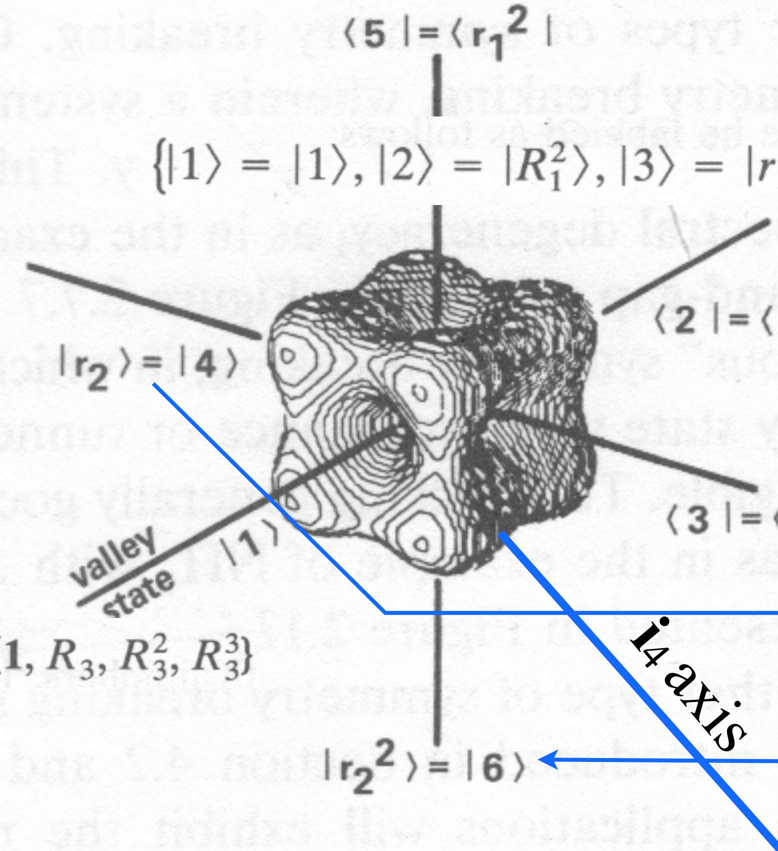

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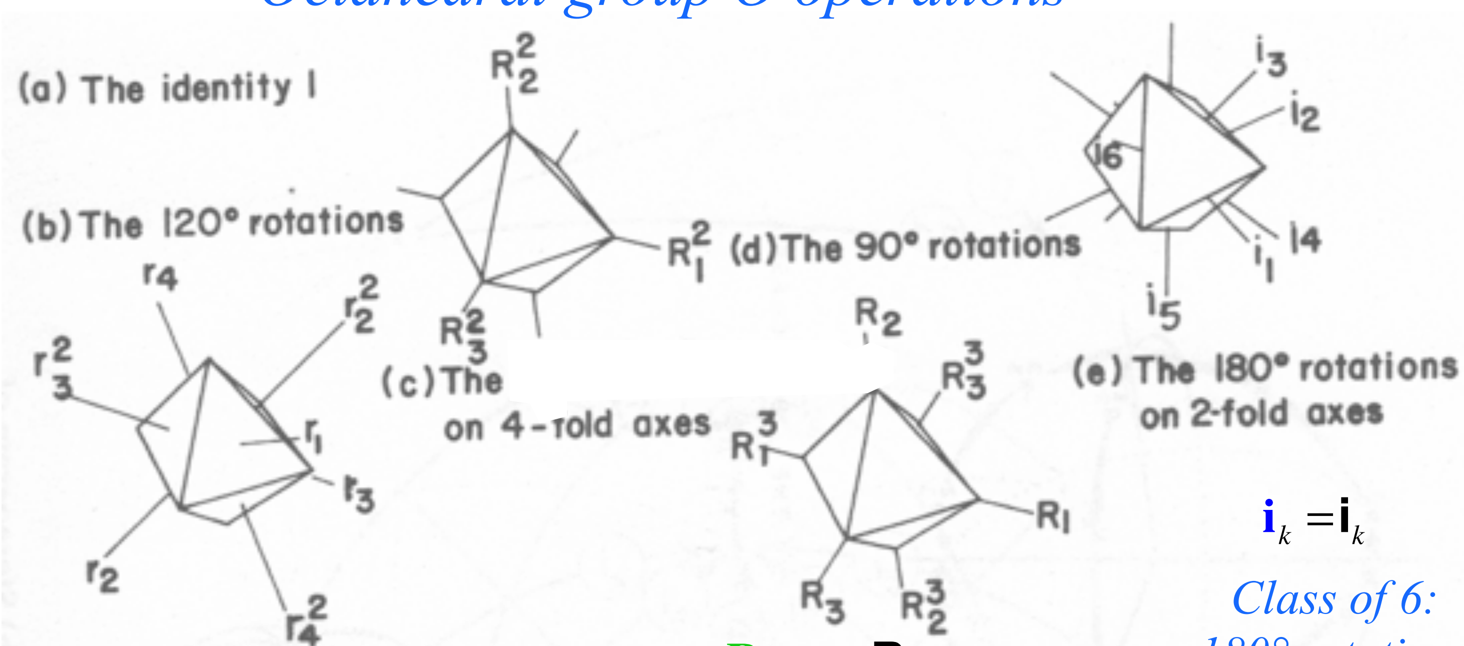

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With slightly new notation

$$\mathbf{1}C_4 = \mathbf{1}\{\mathbf{1}, \boldsymbol{\rho}_z, \mathbf{R}_z, \tilde{\mathbf{R}}_z\}$$
$$\boldsymbol{\rho}_x C_4 = \{\boldsymbol{\rho}_x, \boldsymbol{\rho}_y, \mathbf{i}_4, \mathbf{i}_3\}$$
$$\mathbf{r}_1 C_4 = \{\mathbf{r}_1, \mathbf{r}_4, \mathbf{i}_1, \mathbf{R}_y\}$$
$$\mathbf{r}_2 C_4 = \{\mathbf{r}_2, \mathbf{r}_3, \mathbf{i}_2, \tilde{\mathbf{R}}_y\}$$
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Splitting class projectors into C_4 cosets and $m_4(C_4)\uparrow O$ bases

$O \supset C_4$ *Correlation table shows splitting pathways and induced $m_4(C_4)\uparrow O$ reps*

$O\supset C_4$	0_4	1_4	2_4	3_4
$A_1\downarrow C_4$	1	·	·	·
$A_2\downarrow C_4$	·	·	1	·
$E\downarrow C_4$	1	·	1	·
$T_1\downarrow C_4$	1	1	·	1
$T_2\downarrow C_4$	·	1	1	1

$O: \chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}=\mathbf{1}$	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	ρ_{xyz}	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$C_4: \chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}=\mathbf{1}$	$\mathbf{R}_z$	ρ_z	$\tilde{\mathbf{R}}_z$
$\mu=0_4$	1	1	1	1
1_4	1	$-i$	-1	i
2_4	1	-1	1	-1
3_4	1	$-i$	-1	$-i$

Splitting class projectors into C_4 cosets and $m_4(C_4)\uparrow O$ bases

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$A_2 \downarrow C_4$	·	·	1	·
$E \downarrow C_4$	1	·	1	·
$T_1 \downarrow C_4$	1	1	·	1
$T_2 \downarrow C_4$	·	1	1	1

$\mathbf{1}\cdot\mathbf{P}^{\mu} =$	$(\mathbf{p}_{0_4}$	$+\mathbf{p}_{1_4}$	$+\mathbf{p}_{2_4}$	$+\mathbf{p}_{3_4})\cdot\mathbf{P}^{\mu}$
$\mathbf{1}\cdot\mathbf{P}^{A_1} =$	$\mathbf{P}^{A_1}_{0_4 0_4}$	$+0$	$+0$	$+0$
$\mathbf{1}\cdot\mathbf{P}^{A_2} =$	0	$+0$	$+\mathbf{P}^{A_2}_{2_4 2_4}$	$+0$
$\mathbf{1}\cdot\mathbf{P}^{E} =$	$\mathbf{P}^{E}_{0_4 0_4}$	$+0$	$+\mathbf{P}^{E}_{2_4 2_4}$	$+0$
$\mathbf{1}\cdot\mathbf{P}^{T_1} =$	$\mathbf{P}^{T_1}_{0_4 0_4}$	$+\mathbf{P}^{T_1}_{1_4 1_4}$	$+0$	$+\mathbf{P}^{T_1}_{3_4 3_4}$
$\mathbf{1}\cdot\mathbf{P}^{T_2} =$	0	$+\mathbf{P}^{T_2}_{1_4 1_4}$	$+\mathbf{P}^{T_2}_{2_4 2_4}$	$+\mathbf{P}^{T_2}_{3_4 3_4}$

$O: \chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}=\mathbf{1}$	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	ρ_{xyz}	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
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Splitting class projectors into C_4 cosets and $m_4(C_4)\uparrow O$ bases

$O\supset C_4$ Correlation table shows splitting pathways and induced $m_4(C_4)\uparrow O$ reps

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$\mathbf{1}\cdot\mathbf{P}^{\mu}=$	$(\mathbf{p}_{0_4}$	$+\mathbf{p}_{1_4}$	$+\mathbf{p}_{2_4}$	$+\mathbf{p}_{3_4})\cdot\mathbf{P}^{\mu}$
$\mathbf{1}\cdot\mathbf{P}^{A_1}=$	$\mathbf{P}^{A_1}_{0_40_4}$	+0	+0	+0
$\mathbf{1}\cdot\mathbf{P}^{A_2}=$	0	+0	$+\mathbf{P}^{A_2}_{2_42_4}$	+0
$\mathbf{1}\cdot\mathbf{P}^{E}=$	$\mathbf{P}^{E}_{0_40_4}$	+0	$+\mathbf{P}^{E}_{2_42_4}$	+0
$\mathbf{1}\cdot\mathbf{P}^{T_1}=$	$\mathbf{P}^{T_1}_{0_40_4}$	$+\mathbf{P}^{T_1}_{1_41_4}$	+0	$+\mathbf{P}^{T_1}_{3_43_4}$
$\mathbf{1}\cdot\mathbf{P}^{T_2}=$	0	$+\mathbf{P}^{T_2}_{1_41_4}$	$+\mathbf{P}^{T_2}_{2_42_4}$	$+\mathbf{P}^{T_2}_{3_43_4}$

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$O\supset C_4$ splitting done by C_4 projectors applied to O class projectors

$$\mathbf{P}^{E}=\frac{2}{8}\mathbf{1}-\frac{1}{8}\mathbf{c}_r+\frac{2}{8}\mathbf{c}_\rho+\frac{0}{8}\mathbf{c}_R-\frac{0}{8}\mathbf{c}_i$$

$$\mathbf{P}^{T_1}=\frac{3}{8}\mathbf{1}+\frac{0}{8}\mathbf{c}_r-\frac{1}{8}\mathbf{c}_\rho+\frac{1}{8}\mathbf{c}_R-\frac{1}{8}\mathbf{c}_i$$

$$\mathbf{P}^{T_2}=\frac{3}{8}\mathbf{1}+\frac{0}{8}\mathbf{c}_r-\frac{1}{8}\mathbf{c}_\rho-\frac{1}{8}\mathbf{c}_R+\frac{1}{8}\mathbf{c}_i$$

Splitting class projectors into C_4 cosets and $m_4(C_4)\uparrow O$ bases

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$\mathbf{1}\cdot\mathbf{P}^{\mu}=$	$(\mathbf{p}_{0_4}$	$+\mathbf{p}_{1_4}$	$+\mathbf{p}_{2_4}$	$+\mathbf{p}_{3_4})\cdot\mathbf{P}^{\mu}$
$\mathbf{1}\cdot\mathbf{P}^{A_1}=$	$\mathbf{P}^{A_1}_{0_40_4}$	$+0$	$+0$	$+0$
$\mathbf{1}\cdot\mathbf{P}^{A_2}=$	0	$+0$	$+\mathbf{P}^{A_2}_{2_42_4}$	$+0$
$\mathbf{1}\cdot\mathbf{P}^{E}=$	$\mathbf{P}^{E}_{0_40_4}$	$+0$	$+\mathbf{P}^{E}_{2_42_4}$	$+0$
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$\mathbf{1}\cdot\mathbf{P}^{T_2}=$	0	$+\mathbf{P}^{T_2}_{1_41_4}$	$+\mathbf{P}^{T_2}_{2_42_4}$	$+\mathbf{P}^{T_2}_{3_43_4}$

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$O\supset C_4$ *splitting done by C_4 projectors applied to O class projectors*

$$\mathbf{p}_{m_4}=\sum_{p=0}^{3}\frac{e^{2\pi i m\cdot p/4}}{4}\mathbf{R}_z^p=\begin{cases}\mathbf{p}_{0_4}=(\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{1_4}=(\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{2_4}=(\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{3_4}=(\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4\end{cases}$$

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2_4	1	-1	1	-1
3_4	1	$-i$	-1	$-i$

$$\mathbf{P}^{E}=\frac{2}{8}\mathbf{1}-\frac{1}{8}\mathbf{c}_r+\frac{2}{8}\mathbf{c}_\rho+\frac{0}{8}\mathbf{c}_R-\frac{0}{8}\mathbf{c}_i$$

$$\mathbf{P}^{T_1}=\frac{3}{8}\mathbf{1}+\frac{0}{8}\mathbf{c}_r-\frac{1}{8}\mathbf{c}_\rho+\frac{1}{8}\mathbf{c}_R-\frac{1}{8}\mathbf{c}_i$$

$$\mathbf{P}^{T_2}=\frac{3}{8}\mathbf{1}+\frac{0}{8}\mathbf{c}_r-\frac{1}{8}\mathbf{c}_\rho-\frac{1}{8}\mathbf{c}_R+\frac{1}{8}\mathbf{c}_i$$

Splitting class projectors into C_4 cosets and $m_4(C_4)\uparrow O$ bases

$O \supset C_4$ Correlation table shows splitting pathways and induced $m_4(C_4)\uparrow O$ reps

$O\supset C_4$	0_4	1_4	2_4	3_4
$A_1\downarrow C_4$	1	·	·	·
$A_2\downarrow C_4$	·	·	1	·
$E\downarrow C_4$	1	·	1	·
$T_1\downarrow C_4$	1	1	·	1
$T_2\downarrow C_4$	·	1	1	1

$1\cdot\mathbf{P}^\mu =$	$(\mathbf{p}_{0_4}$	$+\mathbf{p}_{1_4}$	$+\mathbf{p}_{2_4}$	$+\mathbf{p}_{3_4})\cdot\mathbf{P}^\mu$
$1\cdot\mathbf{P}^{A_1} =$	$\mathbf{P}^{A_1}_{0_40_4}$	$+0$	$+0$	$+0$
$1\cdot\mathbf{P}^{A_2} =$	0	$+0$	$+\mathbf{P}^{A_2}_{2_42_4}$	$+0$
$1\cdot\mathbf{P}^{E} =$	$\mathbf{P}^{E}_{0_40_4}$	$+0$	$+\mathbf{P}^{E}_{2_42_4}$	$+0$
$1\cdot\mathbf{P}^{T_1} =$	$\mathbf{P}^{T_1}_{0_40_4}$	$+\mathbf{P}^{T_1}_{1_41_4}$	$+0$	$+\mathbf{P}^{T_1}_{3_43_4}$
$1\cdot\mathbf{P}^{T_2} =$	0	$+\mathbf{P}^{T_2}_{1_41_4}$	$+\mathbf{P}^{T_2}_{2_42_4}$	$+\mathbf{P}^{T_2}_{3_43_4}$

$O:\chi^\mu_{\mathbf{g}}$	$\mathbf{g}=1$	$\mathbf{r}_{1-4}$, $\tilde{\mathbf{r}}_{1-4}$	ρ_{xyz}	$\mathbf{R}_{xyz}$, $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$O\supset C_4$ splitting done by C_4 projectors applied to O class projectors

$$\mathbf{p}_{m_4}=\sum_{p=0}^{3}\frac{e^{2\pi i m\cdot p/4}}{4}\mathbf{R}_z^p=\begin{cases}\mathbf{p}_{0_4}=(\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{1_4}=(\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{2_4}=(\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{3_4}=(\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4\end{cases}$$

$C_4:\chi^\mu_{\mathbf{g}}$	$\mathbf{g}=1$	$\mathbf{R}_z$	ρ_z	$\tilde{\mathbf{R}}_z$
$\mu=0_4$	1	1	1	1
1_4	1	$-i$	-1	i
2_4	1	-1	1	-1
3_4	1	$-i$	-1	$-i$

$$\mathbf{P}^E=\frac{2}{8}\mathbf{1}-\frac{1}{8}\mathbf{c}_r+\frac{2}{8}\mathbf{c}_\rho+\frac{0}{8}\mathbf{c}_R-\frac{0}{8}\mathbf{c}_i$$

$$\mathbf{P}^{T_1}=\frac{3}{8}\mathbf{1}+\frac{0}{8}\mathbf{c}_r-\frac{1}{8}\mathbf{c}_\rho+\frac{1}{8}\mathbf{c}_R-\frac{1}{8}\mathbf{c}_i$$

$$\mathbf{P}^{T_2}=\frac{3}{8}\mathbf{1}+\frac{0}{8}\mathbf{c}_r-\frac{1}{8}\mathbf{c}_\rho-\frac{1}{8}\mathbf{c}_R+\frac{1}{8}\mathbf{c}_i$$

General development of irep projectors follows

$$\mathbf{P}^\mu_{m_4m_4}\equiv\mathbf{p}_{m_4}\mathbf{P}^\mu=\mathbf{P}^\mu\mathbf{p}_{m_4}$$

Splitting class projectors into C_4 cosets and $m_4(C_4)\uparrow O$ bases

$O\supset C_4$ Correlation table shows splitting pathways and induced $m_4(C_4)\uparrow O$ reps

$O\supset C_4$	0_4	1_4	2_4	3_4
$A_1\downarrow C_4$	1	·	·	·
$A_2\downarrow C_4$	·	·	1	·
$E\downarrow C_4$	1	·	1	·
$T_1\downarrow C_4$	1	1	·	1
$T_2\downarrow C_4$	·	1	1	1

$1\cdot\mathbf{P}^{\mu}=$	$(\mathbf{p}_{0_4}$	$+\mathbf{p}_{1_4}$	$+\mathbf{p}_{2_4}$	$+\mathbf{p}_{3_4})\cdot\mathbf{P}^{\mu}$
$1\cdot\mathbf{P}^{A_1}=$	$\mathbf{P}^{A_1}_{0_40_4}$	$+0$	$+0$	$+0$
$1\cdot\mathbf{P}^{A_2}=$	0	$+0$	$+\mathbf{P}^{A_2}_{2_42_4}$	$+0$
$1\cdot\mathbf{P}^{E}=$	$\mathbf{P}^{E}_{0_40_4}$	$+0$	$+\mathbf{P}^{E}_{2_42_4}$	$+0$
$1\cdot\mathbf{P}^{T_1}=$	$\mathbf{P}^{T_1}_{0_40_4}$	$+\mathbf{P}^{T_1}_{1_41_4}$	$+0$	$+\mathbf{P}^{T_1}_{3_43_4}$
$1\cdot\mathbf{P}^{T_2}=$	0	$+\mathbf{P}^{T_2}_{1_41_4}$	$+\mathbf{P}^{T_2}_{2_42_4}$	$+\mathbf{P}^{T_2}_{3_43_4}$

$O:\chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}$=1	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	ρ_{xyz}	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$$\mathbf{p}_{m_4}=\sum_{p=0}^{3}\frac{e^{2\pi i m\cdot p/4}}{4}\mathbf{R}_z^p=\begin{cases}\mathbf{p}_{0_4}=(\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{1_4}=(\mathbf{1}+i\mathbf{R}_z\text{-}\rho_z\text{-}i\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{2_4}=(\mathbf{1}\text{-}\mathbf{R}_z+\rho_z\text{-}\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{3_4}=(\mathbf{1}\text{-}i\mathbf{R}_z\text{-}\rho_z+i\tilde{\mathbf{R}}_z)/4\end{cases}$$

$C_4:\chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}$=1	$\mathbf{R}_z$	ρ_z	$\tilde{\mathbf{R}}_z$
$\mu=0_4$	1	1	1	1
1_4	1	$-i$	-1	i
2_4	1	-1	1	-1
3_4	1	$-i$	-1	$-i$

$O\supset C_4$ splitting done by C_4 projectors applied to O class projectors

$$\mathbf{P}^{E}=\frac{2}{8}\mathbf{1}-\frac{1}{8}\mathbf{c}_r+\frac{2}{8}\mathbf{c}_\rho+\frac{0}{8}\mathbf{c}_R-\frac{0}{8}\mathbf{c}_i$$

$$\mathbf{P}^{T_1}=\frac{3}{8}\mathbf{1}+\frac{0}{8}\mathbf{c}_r-\frac{1}{8}\mathbf{c}_\rho+\frac{1}{8}\mathbf{c}_R-\frac{1}{8}\mathbf{c}_i$$

$$\mathbf{P}^{T_2}=\frac{3}{8}\mathbf{1}+\frac{0}{8}\mathbf{c}_r-\frac{1}{8}\mathbf{c}_\rho-\frac{1}{8}\mathbf{c}_R+\frac{1}{8}\mathbf{c}_i$$

General development of irep projectors follows

$$\mathbf{P}^{\mu}_{m_4m_4}\equiv\mathbf{p}_{m_4}\mathbf{P}^{\mu}=\mathbf{P}^{\mu}\mathbf{p}_{m_4}$$

...followed by examples:

$$\mathbf{P}^{T_1}_{0_40_4}\equiv\mathbf{p}_{0_4}\mathbf{P}^{T_1}=\mathbf{P}^{T_1}\mathbf{p}_{0_4}$$

$$\mathbf{P}^{T_1}_{1_41_4}\equiv\mathbf{p}_{1_4}\mathbf{P}^{T_1}=\mathbf{P}^{T_1}\mathbf{p}_{1_4}$$

etc.

Splitting class projectors into C_4 cosets and $m_4(C_4)\uparrow O$ bases

$O\supset C_4$ *Correlation table shows splitting pathways and induced $m_4(C_4)\uparrow O$ reps*

$O\supset C_4$	0_4	1_4	2_4	3_4
$A_1\downarrow C_4$	1	·	·	·
$A_2\downarrow C_4$	·	·	1	·
$E\downarrow C_4$	1	·	1	·
$T_1\downarrow C_4$	1	1	·	1
$T_2\downarrow C_4$	·	1	1	1

$\mathbf{1}\cdot\mathbf{P}^{\mu}=$	$(\mathbf{p}_{0_4}$	$+\mathbf{p}_{1_4}$	$+\mathbf{p}_{2_4}$	$+\mathbf{p}_{3_4})\cdot\mathbf{P}^{\mu}$
$\mathbf{1}\cdot\mathbf{P}^{A_1}=$	$\mathbf{P}^{A_1}_{0_40_4}$	$+0$	$+0$	$+0$
$\mathbf{1}\cdot\mathbf{P}^{A_2}=$	0	$+0$	$+\mathbf{P}^{A_2}_{2_42_4}$	$+0$
$\mathbf{1}\cdot\mathbf{P}^{E}=$	$\mathbf{P}^{E}_{0_40_4}$	$+0$	$+\mathbf{P}^{E}_{2_42_4}$	$+0$
$\mathbf{1}\cdot\mathbf{P}^{T_1}=$	$\mathbf{P}^{T_1}_{0_40_4}$	$+\mathbf{P}^{T_1}_{1_41_4}$	$+0$	$+\mathbf{P}^{T_1}_{3_43_4}$
$\mathbf{1}\cdot\mathbf{P}^{T_2}=$	0	$+\mathbf{P}^{T_2}_{1_41_4}$	$+\mathbf{P}^{T_2}_{2_42_4}$	$+\mathbf{P}^{T_2}_{3_43_4}$

$O:\chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}=\mathbf{1}$	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	ρ_{xyz}	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$$\mathbf{p}_{m_4}=\sum_{p=0}^{3}\frac{e^{2\pi i m\cdot p/4}}{4}\mathbf{R}_z^p=\begin{cases}\mathbf{p}_{0_4}=(\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{1_4}=(\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{2_4}=(\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{3_4}=(\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4\end{cases}$$

$C_4:\chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}=\mathbf{1}$	$\mathbf{R}_z$	ρ_z	$\tilde{\mathbf{R}}_z$
$\mu=0_4$	1	1	1	1
1_4	1	$-i$	-1	i
2_4	1	-1	1	-1
3_4	1	$-i$	-1	$-i$

$O\supset C_4$ *splitting done by C_4 projectors applied to O class projectors*

$$\mathbf{P}^{E}=\frac{2}{8}\mathbf{1}-\frac{1}{8}\mathbf{c}_r+\frac{2}{8}\mathbf{c}_\rho+\frac{0}{8}\mathbf{c}_R-\frac{0}{8}\mathbf{c}_i$$

$$\mathbf{P}^{T_1}=\frac{3}{8}\mathbf{1}+\frac{0}{8}\mathbf{c}_r-\frac{1}{8}\mathbf{c}_\rho+\frac{1}{8}\mathbf{c}_R-\frac{1}{8}\mathbf{c}_i$$

$$\mathbf{P}^{T_2}=\frac{3}{8}\mathbf{1}+\frac{0}{8}\mathbf{c}_r-\frac{1}{8}\mathbf{c}_\rho-\frac{1}{8}\mathbf{c}_R+\frac{1}{8}\mathbf{c}_i$$

This leads to combinations of cosets and irep "factoring"

$$\mathbf{1}C_4=\mathbf{1}\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\}$$
$$\rho_x C_4=\{\rho_x,\rho_y,\mathbf{i}_4,\mathbf{i}_3\}$$
$$\mathbf{r}_1 C_4=\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\}$$
$$\mathbf{r}_2 C_4=\{\mathbf{r}_2,\mathbf{r}_3,\mathbf{i}_2,\tilde{\mathbf{R}}_y\}$$
$$\tilde{\mathbf{r}}_1 C_4=\{\tilde{\mathbf{r}}_1,\tilde{\mathbf{r}}_3,\tilde{\mathbf{R}}_x,\mathbf{i}_6\}$$
$$\tilde{\mathbf{r}}_2 C_4=\{\tilde{\mathbf{r}}_2,\tilde{\mathbf{r}}_4,\mathbf{R}_x,\mathbf{i}_5\}$$

General development of irep projectors follows

$$\mathbf{P}^{\mu}_{m_4m_4}\equiv\mathbf{p}_{m_4}\mathbf{P}^{\mu}=\mathbf{P}^{\mu}\mathbf{p}_{m_4}$$

...followed by examples:

$$\mathbf{P}^{T_1}_{0_40_4}\equiv\mathbf{p}_{0_4}\mathbf{P}^{T_1}=\mathbf{P}^{T_1}\mathbf{p}_{0_4}$$

$$\mathbf{P}^{T_1}_{1_41_4}\equiv\mathbf{p}_{1_4}\mathbf{P}^{T_1}=\mathbf{P}^{T_1}\mathbf{p}_{1_4}$$

etc.

Review Octahedral $O \supset D_4 \supset C_4$ *subgroup chain and coset bases*

Coset factored splitting of $O \supset D_4 \supset C_4$ *projectors and levels*

Coset spaces based on $m_4(C_4)\uparrow O$

Splitting class projectors into C_4 *cosets and* $m_4(C_4)\uparrow O$ *bases*

General development of irep projectors $\mathbf{P}^{\mu}_{m4m4}$

Calculating $\mathbf{P}^{E}_{0_4 0_4}$

Calculating $\mathbf{P}^{E}_{2_4 2_4}$

Calculating $\mathbf{P}^{T_1}_{0_4 0_4}$

Calculating $\mathbf{P}^{T_1}_{1_4 1_4}$

Calculating $\mathbf{P}^{T_2}_{2_4 2_4}$

Structure and applications of various subgroup chain ireps

$O_h \supset D_{4h} \supset C_{4v}$

$O_h \supset D_{3h} \supset C_{3v}$

$O_h \supset C_{2v}$

General development of irep projectors $\mathbf{P}^{\mu}_{m4m4}$

$$\mathbf{P}^{\mu}_{m_4 m_4} = \mathbf{p}^{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}^{m_4}$$

$$= \sum_{g}^{\circ O} \frac{\ell^{\mu}}{\circ O}(\chi_g^{\mu *}) \cdot \mathbf{g}\left(\mathbf{p}^{m_4}\right)$$

$$= \left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\mathbf{1}}^{\mu *}\right)\cdot \mathbf{1}\left(\mathbf{1} + d_{\rho_z}^{m4}\boldsymbol{\rho}_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4}$$

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$$\boldsymbol{\rho}_x C_4 = \boldsymbol{\rho}_x\left\{\mathbf{1},\boldsymbol{\rho}_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\boldsymbol{\rho}_x,\boldsymbol{\rho}_y,\mathbf{i}_4,\mathbf{i}_3\right\} Coset$$

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$$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1},\boldsymbol{\rho}_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\right\} Coset$$

General development of irep projectors $\mathbf{P}^{\mu}_{m4m4}$

$$\mathbf{P}^{\mu}_{m_4 m_4} = \mathbf{p}^{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}^{m_4}$$

$$= \sum_{g}^{\circ O} \frac{\ell^{\mu}}{\circ O}(\chi_g^{\mu *}) \cdot \mathbf{g}\left(\mathbf{p}^{m_4}\right)$$

$$= \left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\mathbf{1}}^{\mu *}\right) \cdot \mathbf{1}\left(\mathbf{1} + d_{\rho_z}^{m4}\boldsymbol{\rho}_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} \quad = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1 \cdot \mathbf{1} + d_{\rho_z}^{m4}\boldsymbol{\rho}_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

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$$\boldsymbol{\rho}_x C_4 = \boldsymbol{\rho}_x\left\{\mathbf{1}, \boldsymbol{\rho}_z, \mathbf{R}_z, \tilde{\mathbf{R}}_z\right\} = \left\{\boldsymbol{\rho}_x, \boldsymbol{\rho}_y, \mathbf{i}_4, \mathbf{i}_3\right\} Coset$$

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$$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1}, \boldsymbol{\rho}_z, \mathbf{R}_z, \tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1, \mathbf{r}_4, \mathbf{i}_1, \mathbf{R}_y\right\} Coset$$

General development of irep projectors $\mathbf{P}^{\mu}_{m4m4}$

$$\mathbf{P}^{\mu}_{m_4 m_4} = \mathbf{p}^{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}^{m_4}$$

$$= \sum_{g}^{\circ O} \frac{\ell^{\mu}}{\circ O}(\chi_g^{\mu *}) \cdot \mathbf{g}\left(\mathbf{p}^{m_4}\right)$$

$$= \left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\mathbf{1}}^{\mu *}\right) \cdot \mathbf{1}\left(\mathbf{1} + d_{\rho_z}^{m4}\boldsymbol{\rho}_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} \quad = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1 \cdot \mathbf{1} + d_{\rho_z}^{m4}\boldsymbol{\rho}_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\rho_z}^{\mu *}\right) \cdot \boldsymbol{\rho}_z\left(\mathbf{1} + d_{\rho_z}^{m4}\boldsymbol{\rho}_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4}$$

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$$\boldsymbol{\rho}_x C_4 = \boldsymbol{\rho}_x\left\{\mathbf{1}, \boldsymbol{\rho}_z, \mathbf{R}_z, \tilde{\mathbf{R}}_z\right\} = \left\{\boldsymbol{\rho}_x, \boldsymbol{\rho}_y, \mathbf{i}_4, \mathbf{i}_3\right\} Coset$$

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$$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1}, \boldsymbol{\rho}_z, \mathbf{R}_z, \tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1, \mathbf{r}_4, \mathbf{i}_1, \mathbf{R}_y\right\} Coset$$

General development of irep projectors $\mathbf{P}^{\mu}_{m4m4}$

$$\mathbf{P}^{\mu}_{m_4 m_4} = \mathbf{p}^{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}^{m_4}$$

$$= \sum_{g}^{\circ O} \frac{\ell^{\mu}}{\circ O} (\chi_g^{\mu *}) \cdot \mathbf{g}\left(\mathbf{p}^{m_4}\right)$$

$$= \left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\mathbf{1}}^{\mu *}\right)\cdot \mathbf{1}\left(\mathbf{1} + d_{\rho_z}^{m4}\boldsymbol{\rho}_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} \quad = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d_{\rho_z}^{m4}\boldsymbol{\rho}_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\rho_z}^{\mu *}\right)\cdot \boldsymbol{\rho}_z\left(\mathbf{1} + d_{\rho_z}^{m4}\boldsymbol{\rho}_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(1\cdot\boldsymbol{\rho}_z + d_{\rho_z}^{m4}\mathbf{1} + d_{R_z}^{m4}\tilde{\mathbf{R}}_z + d_{\tilde{R}_z}^{m4}\mathbf{R}_z\right)$$

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$$\boldsymbol{\rho}_x C_4 = \boldsymbol{\rho}_x\left\{\mathbf{1},\boldsymbol{\rho}_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\boldsymbol{\rho}_x,\boldsymbol{\rho}_y,\mathbf{i}_4,\mathbf{i}_3\right\} Coset$$

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$$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1},\boldsymbol{\rho}_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\right\} Coset$$

General development of irep projectors $\mathbf{P}^{\mu}_{m4m4}$

$$\mathbf{P}^{\mu}_{m_4 m_4} = \mathbf{p}^{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}^{m_4}$$

$$= \sum_{g}^{°O} \frac{\ell^{\mu}}{°O}(\chi_g^{\mu *}) \cdot \mathbf{g}\left(\mathbf{p}^{m_4}\right)$$

$$= \left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\mathbf{1}}^{\mu *}\right)\cdot \mathbf{1}\left(\mathbf{1} + d_{\rho_z}^{m_4}\boldsymbol{\rho}_z + d_{R_z}^{m_4}\mathbf{R}_z + d_{\tilde{R}_z}^{m_4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} \quad = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d_{\rho_z}^{m_4}\boldsymbol{\rho}_z + d_{R_z}^{m_4}\mathbf{R}_z + d_{\tilde{R}_z}^{m_4}\tilde{\mathbf{R}}_z\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(\ 1\cdot\mathbf{1} + d_{\rho_z}^{m_4}\boldsymbol{\rho}_z + d_{R_z}^{m_4}\mathbf{R}_z + d_{\tilde{R}_z}^{m_4}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\rho_z}^{\mu *}\right)\cdot \boldsymbol{\rho}_z\left(\mathbf{1} + d_{\rho_z}^{m_4}\boldsymbol{\rho}_z + d_{R_z}^{m_4}\mathbf{R}_z + d_{\tilde{R}_z}^{m_4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(1\cdot\boldsymbol{\rho}_z + d_{\rho_z}^{m_4}\mathbf{1} + d_{R_z}^{m_4}\tilde{\mathbf{R}}_z + d_{\tilde{R}_z}^{m_4}\mathbf{R}_z\right) = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(\ d_{\rho_z}^{m_4}\mathbf{1} + 1\cdot\boldsymbol{\rho}_z + d_{\tilde{R}_z}^{m_4}\mathbf{R}_z + d_{R_z}^{m_4}\tilde{\mathbf{R}}_z\right)$$

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$$\boldsymbol{\rho}_x C_4 = \boldsymbol{\rho}_x\left\{\mathbf{1}, \boldsymbol{\rho}_z, \mathbf{R}_z, \tilde{\mathbf{R}}_z\right\} = \left\{\boldsymbol{\rho}_x, \boldsymbol{\rho}_y, \mathbf{i}_4, \mathbf{i}_3\right\} Coset$$

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$$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1}, \boldsymbol{\rho}_z, \mathbf{R}_z, \tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1, \mathbf{r}_4, \mathbf{i}_1, \mathbf{R}_y\right\} Coset$$

General development of irep projectors $\mathbf{P}^{\mu}{}_{m4m4}$

$$\mathbf{P}^{\mu}_{m_4 m_4} = \mathbf{p}^{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}^{m_4}$$

$$= \sum_{g}^{\circ O} \frac{\ell^{\mu}}{\circ O}(\chi_g^{\mu *}) \cdot \mathbf{g}\left(\mathbf{p}^{m_4}\right)$$

$$= \left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\mathbf{1}}^{\mu *}\right)\cdot \mathbf{1}\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\rho_z}^{\mu *}\right)\cdot \rho_z\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(1\cdot\rho_z + d_{\rho_z}^{m4}\mathbf{1} + d_{R_z}^{m4}\tilde{\mathbf{R}}_z + d_{\tilde{R}_z}^{m4}\mathbf{R}_z\right) = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(d_{\rho_z}^{m4}\mathbf{1} + 1\cdot\rho_z + d_{\tilde{R}_z}^{m4}\mathbf{R}_z + d_{R_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\mathbf{R}_z}^{\mu *}\right)\cdot \mathbf{R}_z\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4}$$

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$$\rho_x C_4 = \rho_x\left\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\rho_x,\rho_y,\mathbf{i}_4,\mathbf{i}_3\right\} Coset$$

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$$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\right\} Coset$$

General development of irep projectors $\mathbf{P}^{\mu}_{m4m4}$

$$\mathbf{P}^{\mu}_{m_4 m_4} = \mathbf{p}^{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}^{m_4}$$

$$= \sum_{g}^{\circ O} \frac{\ell^{\mu}}{\circ O}(\chi_g^{\mu *}) \cdot \mathbf{g}\left(\mathbf{p}^{m_4}\right)$$

$$= \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\mathbf{1}}^{\mu *}) \cdot \mathbf{1}\left(\mathbf{1} + d_{\rho_z}^{m4}\boldsymbol{\rho}_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1 \cdot \mathbf{1} + d_{\rho_z}^{m4}\boldsymbol{\rho}_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1 \cdot \mathbf{1} + d_{\rho_z}^{m4}\boldsymbol{\rho}_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\rho_z}^{\mu *}) \cdot \boldsymbol{\rho}_z\left(\mathbf{1} + d_{\rho_z}^{m4}\boldsymbol{\rho}_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(1 \cdot \boldsymbol{\rho}_z + d_{\rho_z}^{m4}\mathbf{1} + d_{R_z}^{m4}\tilde{\mathbf{R}}_z + d_{\tilde{R}_z}^{m4}\mathbf{R}_z\right) = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(d_{\rho_z}^{m4}\mathbf{1} + 1 \cdot \boldsymbol{\rho}_z + d_{\tilde{R}_z}^{m4}\mathbf{R}_z + d_{R_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\mathbf{R}_z}^{\mu *}) \cdot \mathbf{R}_z\left(\mathbf{1} + d_{\rho_z}^{m4}\boldsymbol{\rho}_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(1 \cdot \mathbf{R}_z + d_{\rho_z}^{m4}\tilde{\mathbf{R}}_z + d_{R_z}^{m4}\boldsymbol{\rho}_z + d_{\tilde{R}_z}^{m4}\mathbf{1}\right) =$$

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$$\boldsymbol{\rho}_x C_4 = \boldsymbol{\rho}_x\left\{\mathbf{1}, \boldsymbol{\rho}_z, \mathbf{R}_z, \tilde{\mathbf{R}}_z\right\} = \left\{\boldsymbol{\rho}_x, \boldsymbol{\rho}_y, \mathbf{i}_4, \mathbf{i}_3\right\} \textit{Coset}$$

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$$+ \left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\mathbf{R}_z}^{\mu *}\right) \cdot \mathbf{R}_z\left(\mathbf{1} + d_{\rho_z}^{m4}\boldsymbol{\rho}_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(1 \cdot \mathbf{R}_z + d_{\rho_z}^{m4}\tilde{\mathbf{R}}_z + d_{R_z}^{m4}\boldsymbol{\rho}_z + d_{\tilde{R}_z}^{m4}\mathbf{1}\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(d_{\tilde{R}_z}^{m4}\mathbf{1} + d_{R_z}^{m4}\boldsymbol{\rho}_z + 1 \cdot \mathbf{R}_z + d_{\rho_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

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$$\boldsymbol{\rho}_x C_4 = \boldsymbol{\rho}_x\left\{\mathbf{1}, \boldsymbol{\rho}_z, \mathbf{R}_z, \tilde{\mathbf{R}}_z\right\} = \left\{\boldsymbol{\rho}_x, \boldsymbol{\rho}_y, \mathbf{i}_4, \mathbf{i}_3\right\} Coset$$

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$$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1}, \boldsymbol{\rho}_z, \mathbf{R}_z, \tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1, \mathbf{r}_4, \mathbf{i}_1, \mathbf{R}_y\right\} Coset$$

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$$+\left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\rho_z}^{\mu *}\right)\cdot \rho_z\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(1\cdot\rho_z + d_{\rho_z}^{m4}\mathbf{1} + d_{R_z}^{m4}\tilde{\mathbf{R}}_z + d_{\tilde{R}_z}^{m4}\mathbf{R}_z\right) = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(d_{\rho_z}^{m4}\mathbf{1} + 1\cdot\rho_z + d_{\tilde{R}_z}^{m4}\mathbf{R}_z + d_{R_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+\left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\mathbf{R}_z}^{\mu *}\right)\cdot \mathbf{R}_z\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(1\cdot\mathbf{R}_z + d_{\rho_z}^{m4}\tilde{\mathbf{R}}_z + d_{R_z}^{m4}\rho_z + d_{\tilde{R}_z}^{m4}\mathbf{1}\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(d_{\tilde{R}_z}^{m4}\mathbf{1} + d_{R_z}^{m4}\rho_z + 1\cdot\mathbf{R}_z + d_{\rho_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+\left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\tilde{\mathbf{R}}_z}^{\mu *}\right)\cdot \tilde{\mathbf{R}}_z\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(1\cdot\tilde{\mathbf{R}}_z + d_{\rho_z}^{m4}\mathbf{R}_z + d_{R_z}^{m4}\mathbf{1} + d_{\tilde{R}_z}^{m4}\rho_z\right) = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(d_{R_z}^{m4}\mathbf{1} + d_{\tilde{R}_z}^{m4}\rho_z + d_{\rho_z}^{m4}\mathbf{R}_z + 1\cdot\tilde{\mathbf{R}}_z\right)$$

$$\rho_x C_4 = \rho_x\left\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\rho_x,\rho_y,\mathbf{i}_4,\mathbf{i}_3\right\} Coset$$

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$$+ \left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\rho_z}^{\mu *}\right)\cdot \rho_z\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(1\cdot\rho_z + d_{\rho_z}^{m4}\mathbf{1} + d_{R_z}^{m4}\tilde{\mathbf{R}}_z + d_{\tilde{R}_z}^{m4}\mathbf{R}_z\right) = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(d_{\rho_z}^{m4}\mathbf{1} + 1\cdot\rho_z + d_{\tilde{R}_z}^{m4}\mathbf{R}_z + d_{R_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\mathbf{R}_z}^{\mu *}\right)\cdot \mathbf{R}_z\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(1\cdot\mathbf{R}_z + d_{\rho_z}^{m4}\tilde{\mathbf{R}}_z + d_{R_z}^{m4}\rho_z + d_{\tilde{R}_z}^{m4}\mathbf{1}\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(d_{\tilde{R}_z}^{m4}\mathbf{1} + d_{R_z}^{m4}\rho_z + 1\cdot\mathbf{R}_z + d_{\rho_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\tilde{\mathbf{R}}_z}^{\mu *}\right)\cdot \tilde{\mathbf{R}}_z\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(1\cdot\tilde{\mathbf{R}}_z + d_{\rho_z}^{m4}\mathbf{R}_z + d_{R_z}^{m4}\mathbf{1} + d_{\tilde{R}_z}^{m4}\rho_z\right) = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(d_{R_z}^{m4}\mathbf{1} + d_{\tilde{R}_z}^{m4}\rho_z + d_{\rho_z}^{m4}\mathbf{R}_z + 1\cdot\tilde{\mathbf{R}}_z\right)$$

$$\rho_x C_4 = \rho_x\left\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\rho_x,\rho_y,\mathbf{i}_4,\mathbf{i}_3\right\} Coset$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\rho_x}^{\mu *}\right)\cdot \rho_x\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} =$$

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$$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\right\} Coset$$

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$$+\left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\tilde{\mathbf{R}}_z}^{\mu *}\right)\cdot \tilde{\mathbf{R}}_z\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(1\cdot\tilde{\mathbf{R}}_z + d_{\rho_z}^{m4}\mathbf{R}_z + d_{R_z}^{m4}\mathbf{1} + d_{\tilde{R}_z}^{m4}\rho_z\right) = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(d_{R_z}^{m4}\mathbf{1} + d_{\tilde{R}_z}^{m4}\rho_z + d_{\rho_z}^{m4}\mathbf{R}_z + 1\cdot\tilde{\mathbf{R}}_z\right)$$

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$$+\left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\rho_x}^{\mu *}\right)\cdot \rho_x\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_x}^{\mu *}}{96}\right)\left(1\cdot\rho_x + d_{\rho_z}^{m4}\rho_y + d_{R_z}^{m4}\mathbf{i}_4 + d_{\tilde{R}_z}^{m4}\mathbf{i}_3\right) =$$

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$$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\right\} \mathit{Coset}$$

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$$= \left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\mathbf{1}}^{\mu *}\right)\cdot \mathbf{1}\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+\left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\rho_z}^{\mu *}\right)\cdot \rho_z\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(1\cdot\rho_z + d_{\rho_z}^{m4}\mathbf{1} + d_{R_z}^{m4}\tilde{\mathbf{R}}_z + d_{\tilde{R}_z}^{m4}\mathbf{R}_z\right) = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(d_{\rho_z}^{m4}\mathbf{1} + 1\cdot\rho_z + d_{\tilde{R}_z}^{m4}\mathbf{R}_z + d_{R_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+\left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\mathbf{R}_z}^{\mu *}\right)\cdot \mathbf{R}_z\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(1\cdot\mathbf{R}_z + d_{\rho_z}^{m4}\tilde{\mathbf{R}}_z + d_{R_z}^{m4}\rho_z + d_{\tilde{R}_z}^{m4}\mathbf{1}\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(d_{\tilde{R}_z}^{m4}\mathbf{1} + d_{R_z}^{m4}\rho_z + 1\cdot\mathbf{R}_z + d_{\rho_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+\left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\tilde{\mathbf{R}}_z}^{\mu *}\right)\cdot \tilde{\mathbf{R}}_z\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(1\cdot\tilde{\mathbf{R}}_z + d_{\rho_z}^{m4}\mathbf{R}_z + d_{R_z}^{m4}\mathbf{1} + d_{\tilde{R}_z}^{m4}\rho_z\right) = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(d_{R_z}^{m4}\mathbf{1} + d_{\tilde{R}_z}^{m4}\rho_z + d_{\rho_z}^{m4}\mathbf{R}_z + 1\cdot\tilde{\mathbf{R}}_z\right)$$

$$\rho_x C_4 = \rho_x\left\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\rho_x,\rho_y,\mathbf{i}_4,\mathbf{i}_3\right\} Coset$$

$$+\left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\rho_x}^{\mu *}\right)\cdot \rho_x\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_x}^{\mu *}}{96}\right)\left(1\cdot\rho_x + d_{\rho_z}^{m4}\rho_y + d_{R_z}^{m4}\mathbf{i}_4 + d_{\tilde{R}_z}^{m4}\mathbf{i}_3\right) = \left(\frac{\ell^{\mu}\chi_{\rho_x}^{\mu *}}{96}\right)\left(1\cdot\rho_x + d_{\rho_z}^{m4}\rho_y + d_{R_z}^{m4}\mathbf{i}_4 + d_{\tilde{R}_z}^{m4}\mathbf{i}_3\right)$$

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$$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\right\} Coset$$

General development of irep projectors $\mathbf{P}^{\mu}{}_{m4m4}$

$$\mathbf{P}^{\mu}_{m_4 m_4} = \mathbf{p}^{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}^{m_4}$$

$$= \sum_{g}^{\circ O} \frac{\ell^{\mu}}{{}^{\circ}O}(\chi_g^{\mu *}) \cdot \mathbf{g}\left(\mathbf{p}^{m_4}\right)$$

$$= \left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\mathbf{1}}^{\mu *}\right)\cdot \mathbf{1}\left(\mathbf{1} + d_{\rho_z}^{m4}\boldsymbol{\rho}_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d_{\rho_z}^{m4}\boldsymbol{\rho}_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d_{\rho_z}^{m4}\boldsymbol{\rho}_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+\left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\rho_z}^{\mu *}\right)\cdot \boldsymbol{\rho}_z\left(\mathbf{1} + d_{\rho_z}^{m4}\boldsymbol{\rho}_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(1\cdot\boldsymbol{\rho}_z + d_{\rho_z}^{m4}\mathbf{1} + d_{R_z}^{m4}\tilde{\mathbf{R}}_z + d_{\tilde{R}_z}^{m4}\mathbf{R}_z\right) = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(d_{\rho_z}^{m4}\mathbf{1} + 1\cdot\boldsymbol{\rho}_z + d_{\tilde{R}_z}^{m4}\mathbf{R}_z + d_{R_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+\left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\mathbf{R}_z}^{\mu *}\right)\cdot \mathbf{R}_z\left(\mathbf{1} + d_{\rho_z}^{m4}\boldsymbol{\rho}_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(1\cdot\mathbf{R}_z + d_{\rho_z}^{m4}\tilde{\mathbf{R}}_z + d_{R_z}^{m4}\boldsymbol{\rho}_z + d_{\tilde{R}_z}^{m4}\mathbf{1}\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(d_{\tilde{R}_z}^{m4}\mathbf{1} + d_{R_z}^{m4}\boldsymbol{\rho}_z + 1\cdot\mathbf{R}_z + d_{\rho_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+\left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\tilde{\mathbf{R}}_z}^{\mu *}\right)\cdot \tilde{\mathbf{R}}_z\left(\mathbf{1} + d_{\rho_z}^{m4}\boldsymbol{\rho}_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(1\cdot\tilde{\mathbf{R}}_z + d_{\rho_z}^{m4}\mathbf{R}_z + d_{R_z}^{m4}\mathbf{1} + d_{\tilde{R}_z}^{m4}\boldsymbol{\rho}_z\right) = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(d_{R_z}^{m4}\mathbf{1} + d_{\tilde{R}_z}^{m4}\boldsymbol{\rho}_z + d_{\rho_z}^{m4}\mathbf{R}_z + 1\cdot\tilde{\mathbf{R}}_z\right)$$

$$\boldsymbol{\rho}_x C_4 = \boldsymbol{\rho}_x\left\{\mathbf{1},\boldsymbol{\rho}_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\boldsymbol{\rho}_x,\boldsymbol{\rho}_y,\mathbf{i}_4,\mathbf{i}_3\right\} Coset$$

$$+\left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\rho_x}^{\mu *}\right)\cdot \boldsymbol{\rho}_x\left(\mathbf{1} + d_{\rho_z}^{m4}\boldsymbol{\rho}_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_x}^{\mu *}}{96}\right)\left(1\cdot\boldsymbol{\rho}_x + d_{\rho_z}^{m4}\boldsymbol{\rho}_y + d_{R_z}^{m4}\mathbf{i}_4 + d_{\tilde{R}_z}^{m4}\mathbf{i}_3\right) = \left(\frac{\ell^{\mu}\chi_{\rho_x}^{\mu *}}{96}\right)\left(1\cdot\boldsymbol{\rho}_x + d_{\rho_z}^{m4}\boldsymbol{\rho}_y + d_{R_z}^{m4}\mathbf{i}_4 + d_{\tilde{R}_z}^{m4}\mathbf{i}_3\right)$$

$$+\left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\rho_y}^{\mu *}\right)\cdot \boldsymbol{\rho}_y\left(\mathbf{1} + d_{\rho_z}^{m4}\boldsymbol{\rho}_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4}$$

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$$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1},\boldsymbol{\rho}_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\right\} Coset$$

General development of irep projectors $\mathbf{P}^{\mu}_{m4m4}$

$$\mathbf{P}^{\mu}_{m_4 m_4} = \mathbf{p}^{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}^{m_4}$$

$$= \sum_g^{\circ O} \frac{\ell^{\mu}}{{}^{\circ}O}(\chi_g^{\mu *})\cdot \mathbf{g}\left(\mathbf{p}^{m_4}\right)$$

$$= \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\mathbf{1}}^{\mu *})\cdot \mathbf{1}\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\rho_z}^{\mu *})\cdot \rho_z\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(1\cdot\rho_z + d_{\rho_z}^{m4}\mathbf{1} + d_{R_z}^{m4}\tilde{\mathbf{R}}_z + d_{\tilde{R}_z}^{m4}\mathbf{R}_z\right) = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(d_{\rho_z}^{m4}\mathbf{1} + 1\cdot\rho_z + d_{\tilde{R}_z}^{m4}\mathbf{R}_z + d_{R_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\mathbf{R}_z}^{\mu *})\cdot \mathbf{R}_z\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(1\cdot\mathbf{R}_z + d_{\rho_z}^{m4}\tilde{\mathbf{R}}_z + d_{R_z}^{m4}\rho_z + d_{\tilde{R}_z}^{m4}\mathbf{1}\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(d_{\tilde{R}_z}^{m4}\mathbf{1} + d_{R_z}^{m4}\rho_z + 1\cdot\mathbf{R}_z + d_{\rho_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\tilde{\mathbf{R}}_z}^{\mu *})\cdot \tilde{\mathbf{R}}_z\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(1\cdot\tilde{\mathbf{R}}_z + d_{\rho_z}^{m4}\mathbf{R}_z + d_{R_z}^{m4}\mathbf{1} + d_{\tilde{R}_z}^{m4}\rho_z\right) = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(d_{R_z}^{m4}\mathbf{1} + d_{\tilde{R}_z}^{m4}\rho_z + d_{\rho_z}^{m4}\mathbf{R}_z + 1\cdot\tilde{\mathbf{R}}_z\right)$$

$$\rho_x C_4 = \rho_x\left\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\rho_x,\rho_y,\mathbf{i}_4,\mathbf{i}_3\right\} Coset$$

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$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\rho_y}^{\mu *})\cdot \rho_y\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_y}^{\mu *}}{96}\right)\left(1\cdot\rho_y + d_{\rho_z}^{m4}\rho_x + d_{R_z}^{m4}\mathbf{i}_3 + d_{\tilde{R}_z}^{m4}\mathbf{i}_4\right) =$$

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$$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\right\} Coset$$

General development of irep projectors $\mathbf{P}^{\mu}_{m4m4}$

$$\mathbf{P}^{\mu}_{m_4 m_4} = \mathbf{p}^{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}^{m_4}$$

$$= \sum_{g}^{\circ O} \frac{\ell^{\mu}}{\circ O}(\chi_g^{\mu *}) \cdot \mathbf{g}\left(\mathbf{p}^{m_4}\right)$$

$$= \left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\mathbf{1}}^{\mu *}\right)\cdot \mathbf{1}\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+\left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\rho_z}^{\mu *}\right)\cdot \rho_z\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(1\cdot\rho_z + d_{\rho_z}^{m4}\mathbf{1} + d_{R_z}^{m4}\tilde{\mathbf{R}}_z + d_{\tilde{R}_z}^{m4}\mathbf{R}_z\right) = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(d_{\rho_z}^{m4}\mathbf{1} + 1\cdot\rho_z + d_{\tilde{R}_z}^{m4}\mathbf{R}_z + d_{R_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+\left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\mathbf{R}_z}^{\mu *}\right)\cdot \mathbf{R}_z\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(1\cdot\mathbf{R}_z + d_{\rho_z}^{m4}\tilde{\mathbf{R}}_z + d_{R_z}^{m4}\rho_z + d_{\tilde{R}_z}^{m4}\mathbf{1}\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(d_{\tilde{R}_z}^{m4}\mathbf{1} + d_{R_z}^{m4}\rho_z + 1\cdot\mathbf{R}_z + d_{\rho_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+\left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\tilde{\mathbf{R}}_z}^{\mu *}\right)\cdot \tilde{\mathbf{R}}_z\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(1\cdot\tilde{\mathbf{R}}_z + d_{\rho_z}^{m4}\mathbf{R}_z + d_{R_z}^{m4}\mathbf{1} + d_{\tilde{R}_z}^{m4}\rho_z\right) = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(d_{R_z}^{m4}\mathbf{1} + d_{\tilde{R}_z}^{m4}\rho_z + d_{\rho_z}^{m4}\mathbf{R}_z + 1\cdot\tilde{\mathbf{R}}_z\right)$$

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$$+\left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\rho_x}^{\mu *}\right)\cdot \rho_x\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_x}^{\mu *}}{96}\right)\left(1\cdot\rho_x + d_{\rho_z}^{m4}\rho_y + d_{R_z}^{m4}\mathbf{i}_4 + d_{\tilde{R}_z}^{m4}\mathbf{i}_3\right) = \left(\frac{\ell^{\mu}\chi_{\rho_x}^{\mu *}}{96}\right)\left(1\cdot\rho_x + d_{\rho_z}^{m4}\rho_y + d_{R_z}^{m4}\mathbf{i}_4 + d_{\tilde{R}_z}^{m4}\mathbf{i}_3\right)$$

$$+\left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\rho_y}^{\mu *}\right)\cdot \rho_y\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_y}^{\mu *}}{96}\right)\left(1\cdot\rho_y + d_{\rho_z}^{m4}\rho_x + d_{R_z}^{m4}\mathbf{i}_3 + d_{\tilde{R}_z}^{m4}\mathbf{i}_4\right) = \left(\frac{\ell^{\mu}\chi_{\rho_y}^{\mu *}}{96}\right)\left(d_{\rho_z}^{m4}\rho_x + 1\cdot\rho_y + d_{\tilde{R}_z}^{m4}\mathbf{i}_4 + d_{R_z}^{m4}\mathbf{i}_3\right)$$

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$$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\right\} Coset$$

General development of irep projectors $\mathbf{P}^{\mu}_{m4m4}$

$$\mathbf{P}^{\mu}_{m_4m_4} = \mathbf{p}^{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}^{m_4}$$

$$= \sum_g^{\circ O} \frac{\ell^{\mu}}{{}^{\circ}O} (\chi_g^{\mu *}) \cdot \mathbf{g}\left(\mathbf{p}^{m_4}\right)$$

$$= \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\mathbf{1}}^{\mu *}) \cdot \mathbf{1}\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\rho_z}^{\mu *}) \cdot \rho_z\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(1\cdot\rho_z + d_{\rho_z}^{m4}\mathbf{1} + d_{R_z}^{m4}\tilde{\mathbf{R}}_z + d_{\tilde{R}_z}^{m4}\mathbf{R}_z\right) = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(d_{\rho_z}^{m4}\mathbf{1} + 1\cdot\rho_z + d_{\tilde{R}_z}^{m4}\mathbf{R}_z + d_{R_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\mathbf{R}_z}^{\mu *}) \cdot \mathbf{R}_z\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(1\cdot\mathbf{R}_z + d_{\rho_z}^{m4}\tilde{\mathbf{R}}_z + d_{R_z}^{m4}\rho_z + d_{\tilde{R}_z}^{m4}\mathbf{1}\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(d_{\tilde{R}_z}^{m4}\mathbf{1} + d_{R_z}^{m4}\rho_z + 1\cdot\mathbf{R}_z + d_{\rho_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\tilde{\mathbf{R}}_z}^{\mu *}) \cdot \tilde{\mathbf{R}}_z\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(1\cdot\tilde{\mathbf{R}}_z + d_{\rho_z}^{m4}\mathbf{R}_z + d_{R_z}^{m4}\mathbf{1} + d_{\tilde{R}_z}^{m4}\rho_z\right) = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(d_{R_z}^{m4}\mathbf{1} + d_{\tilde{R}_z}^{m4}\rho_z + d_{\rho_z}^{m4}\mathbf{R}_z + 1\cdot\tilde{\mathbf{R}}_z\right)$$

$$\rho_x C_4 = \rho_x\left\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\rho_x,\rho_y,\mathbf{i}_4,\mathbf{i}_3\right\} Coset$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\rho_x}^{\mu *}) \cdot \rho_x\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_x}^{\mu *}}{96}\right)\left(1\cdot\rho_x + d_{\rho_z}^{m4}\rho_y + d_{R_z}^{m4}\mathbf{i}_4 + d_{\tilde{R}_z}^{m4}\mathbf{i}_3\right) = \left(\frac{\ell^{\mu}\chi_{\rho_x}^{\mu *}}{96}\right)\left(1\cdot\rho_x + d_{\rho_z}^{m4}\rho_y + d_{R_z}^{m4}\mathbf{i}_4 + d_{\tilde{R}_z}^{m4}\mathbf{i}_3\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\rho_y}^{\mu *}) \cdot \rho_y\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_y}^{\mu *}}{96}\right)\left(1\cdot\rho_y + d_{\rho_z}^{m4}\rho_x + d_{R_z}^{m4}\mathbf{i}_3 + d_{\tilde{R}_z}^{m4}\mathbf{i}_4\right) = \left(\frac{\ell^{\mu}\chi_{\rho_y}^{\mu *}}{96}\right)\left(d_{\rho_z}^{m4}\rho_x + 1\cdot\rho_y + d_{\tilde{R}_z}^{m4}\mathbf{i}_4 + d_{R_z}^{m4}\mathbf{i}_3\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\mathbf{i}_4}^{\mu *}) \cdot \mathbf{i}_4\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{i}_4}^{\mu *}}{96}\right)\left(1\cdot\mathbf{i}_4 + d_{\rho_z}^{m4}\mathbf{i}_3 + d_{R_z}^{m4}\rho_y + d_{\tilde{R}_z}^{m4}\rho_x\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{i}_4}^{\mu *}}{96}\right)\left(d_{\tilde{R}_z}^{m4}\rho_x + d_{R_z}^{m4}\rho_y + 1\cdot\mathbf{i}_4 + d_{\rho_z}^{m4}\mathbf{i}_3\right)$$

$+$

$$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\right\} Coset$$

General development of irep projectors $\mathbf{P}^{\mu}_{m4m4}$

$$\mathbf{P}^{\mu}_{m_4m_4} = \mathbf{p}^{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}^{m_4}$$

$$= \sum_g^{\circ O} \frac{\ell^{\mu}}{\circ O}(\chi_g^{\mu *}) \cdot \mathbf{g}\left(\mathbf{p}^{m_4}\right)$$

$$= \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\mathbf{1}}^{\mu *}) \cdot \mathbf{1}\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\rho_z}^{\mu *}) \cdot \rho_z\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(1\cdot\rho_z + d_{\rho_z}^{m4}\mathbf{1} + d_{R_z}^{m4}\tilde{\mathbf{R}}_z + d_{\tilde{R}_z}^{m4}\mathbf{R}_z\right) = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(d_{\rho_z}^{m4}\mathbf{1} + 1\cdot\rho_z + d_{\tilde{R}_z}^{m4}\mathbf{R}_z + d_{R_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\mathbf{R}_z}^{\mu *}) \cdot \mathbf{R}_z\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(1\cdot\mathbf{R}_z + d_{\rho_z}^{m4}\tilde{\mathbf{R}}_z + d_{R_z}^{m4}\rho_z + d_{\tilde{R}_z}^{m4}\mathbf{1}\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(d_{\tilde{R}_z}^{m4}\mathbf{1} + d_{R_z}^{m4}\rho_z + 1\cdot\mathbf{R}_z + d_{\rho_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\tilde{\mathbf{R}}_z}^{\mu *}) \cdot \tilde{\mathbf{R}}_z\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(1\cdot\tilde{\mathbf{R}}_z + d_{\rho_z}^{m4}\mathbf{R}_z + d_{R_z}^{m4}\mathbf{1} + d_{\tilde{R}_z}^{m4}\rho_z\right) = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(d_{R_z}^{m4}\mathbf{1} + d_{\tilde{R}_z}^{m4}\rho_z + d_{\rho_z}^{m4}\mathbf{R}_z + 1\cdot\tilde{\mathbf{R}}_z\right)$$

$$\rho_x C_4 = \rho_x\left\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\rho_x,\rho_y,\mathbf{i}_4,\mathbf{i}_3\right\} Coset$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\rho_x}^{\mu *}) \cdot \rho_x\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_x}^{\mu *}}{96}\right)\left(1\cdot\rho_x + d_{\rho_z}^{m4}\rho_y + d_{R_z}^{m4}\mathbf{i}_4 + d_{\tilde{R}_z}^{m4}\mathbf{i}_3\right) = \left(\frac{\ell^{\mu}\chi_{\rho_x}^{\mu *}}{96}\right)\left(1\cdot\rho_x + d_{\rho_z}^{m4}\rho_y + d_{R_z}^{m4}\mathbf{i}_4 + d_{\tilde{R}_z}^{m4}\mathbf{i}_3\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\rho_y}^{\mu *}) \cdot \rho_y\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_y}^{\mu *}}{96}\right)\left(1\cdot\rho_y + d_{\rho_z}^{m4}\rho_x + d_{R_z}^{m4}\mathbf{i}_3 + d_{\tilde{R}_z}^{m4}\mathbf{i}_4\right) = \left(\frac{\ell^{\mu}\chi_{\rho_y}^{\mu *}}{96}\right)\left(d_{\rho_z}^{m4}\rho_x + 1\cdot\rho_y + d_{\tilde{R}_z}^{m4}\mathbf{i}_4 + d_{R_z}^{m4}\mathbf{i}_3\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\mathbf{i}_4}^{\mu *}) \cdot \mathbf{i}_4\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{i}_4}^{\mu *}}{96}\right)\left(1\cdot\mathbf{i}_4 + d_{\rho_z}^{m4}\mathbf{i}_3 + d_{R_z}^{m4}\rho_y + d_{\tilde{R}_z}^{m4}\rho_x\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{i}_4}^{\mu *}}{96}\right)\left(d_{\tilde{R}_z}^{m4}\rho_x + d_{R_z}^{m4}\rho_y + 1\cdot\mathbf{i}_4 + d_{\rho_z}^{m4}\mathbf{i}_3\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\mathbf{i}_3}^{\mu *}) \cdot \mathbf{i}_3\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{i}_3}^{\mu *}}{96}\right)\left(1\cdot\mathbf{i}_3 + d_{\rho_z}^{m4}\mathbf{i}_4 + d_{R_z}^{m4}\mathbf{1} + d_{\tilde{R}_z}^{m4}\rho_y\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{i}_3}^{\mu *}}{96}\right)\left(d_{R_z}^{m4}\rho_x + d_{\tilde{R}_z}^{m4}\rho_y + d_{\rho_z}^{m4}\mathbf{i}_4 + 1\cdot\mathbf{i}_3\right)$$

$$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\right\} Coset$$

etc. etc.

General development of irep projectors $\mathbf{P}^{\mu}_{m4m4}$

$$\mathbf{P}^{\mu}_{m_4 m_4} = \mathbf{p}_{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}_{m_4}$$

$$= \sum_{g}^{°O} \frac{\ell^{\mu}}{°O}(\chi_g^{\mu *})\cdot \mathbf{g}\cdot\left(\mathbf{p}_{m_4}\right) = \sum_{g}^{°O} \frac{\ell^{\mu}}{4°O}(\chi_g^{\mu *})\cdot \mathbf{g}\cdot\left(d_1^{m_4}\mathbf{1} + d_{\boldsymbol{\rho}_z}^{m_4}\boldsymbol{\rho}_z + d_{\mathbf{R}_z}^{m_4}\mathbf{R}_z + d_{\tilde{\mathbf{R}}_z}^{m_4}\tilde{\mathbf{R}}_z\right)$$

$$\mathbf{1}C_4 = \mathbf{1}\left\{\mathbf{1},\boldsymbol{\rho}_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} \quad \boldsymbol{\rho}_x C_4 = \left\{\boldsymbol{\rho}_x,\boldsymbol{\rho}_y,\mathbf{i}_4,\mathbf{i}_3\right\} \quad \mathbf{r}_1 C_4 = \left\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\right\} \quad \mathbf{r}_2 C_4 = \left\{\mathbf{r}_2,\mathbf{r}_3,\mathbf{i}_2,\tilde{\mathbf{R}}_y\right\} \quad \tilde{\mathbf{r}}_1 C_4 = \left\{\tilde{\mathbf{r}}_1,\tilde{\mathbf{r}}_3,\tilde{\mathbf{R}}_x,\mathbf{i}_6\right\} \quad \tilde{\mathbf{r}}_2 C_4 = \left\{\tilde{\mathbf{r}}_2,\tilde{\mathbf{r}}_4,\mathbf{R}_x,\mathbf{i}_5\right.$$

$$=\tfrac{\ell^\mu}{96}\chi_{\mathbf{1}}^{\mu *}(1,d_{\rho_z}^{m4},d_{R_z}^{m4},d_{\tilde{R}_z}^{m4})+\tfrac{\ell^\mu}{96}\chi_{\boldsymbol{\rho}_x}^{\mu *}(1,d_{\rho_z}^{m4},d_{R_z}^{m4},d_{\tilde{R}_z}^{m4})+\tfrac{\ell^\mu}{96}\chi_{\mathbf{r}_1}^{\mu *}(1,d_{\rho_z}^{m4},d_{R_z}^{m4},d_{\tilde{R}_z}^{m4})+\tfrac{\ell^\mu}{96}\chi_{\mathbf{r}_2}^{\mu *}(1,d_{\rho_z}^{m4},d_{R_z}^{m4},d_{\tilde{R}_z}^{m4})+\tfrac{\ell^\mu}{96}\chi_{\tilde{\mathbf{r}}_1}^{\mu *}(1,d_{\rho_z}^{m4},d_{R_z}^{m4},d_{\tilde{R}_z}^{m4})+\tfrac{\ell^\mu}{96}\chi_{\tilde{\mathbf{r}}_2}^{\mu *}(1,d_{\rho_z}^{m4},d_{R_z}^{m4},d_{\tilde{R}}$$

$$+\tfrac{\ell^\mu}{96}\chi_{\rho_z}^{\mu *}(d_{\rho_z}^{m4},1,d_{\tilde{R}_z}^{m4},d_{R_z}^{m4})+\tfrac{\ell^\mu}{96}\chi_{\boldsymbol{\rho}_y}^{\mu *}(d_{\rho_z}^{m4},1,d_{\tilde{R}_z}^{m4},d_{R_z}^{m4})+\tfrac{\ell^\mu}{96}\chi_{\mathbf{r}_4}^{\mu *}(d_{\rho_z}^{m4},1,d_{\tilde{R}_z}^{m4},d_{R_z}^{m4})+\tfrac{\ell^\mu}{96}\chi_{\mathbf{r}_3}^{\mu *}(d_{\rho_z}^{m4},1,d_{\tilde{R}_z}^{m4},d_{R_z}^{m4})+\tfrac{\ell^\mu}{96}\chi_{\tilde{\mathbf{r}}_3}^{\mu *}(d_{\rho_z}^{m4},1,d_{\tilde{R}_z}^{m4},d_{R_z}^{m4})+\tfrac{\ell^\mu}{96}\chi_{\tilde{\mathbf{r}}_4}^{\mu *}(d_{\rho_z}^{m4},1,d_{\tilde{R}_z}^{m4},d$$

$$+\tfrac{\ell^\mu}{96}\chi_{\mathbf{R}_z}^{\mu *}(d_{\tilde{R}_z}^{m4},d_{R_z}^{m4},1,d_{\rho_z}^{m4})+\tfrac{\ell^\mu}{96}\chi_{\mathbf{i}_4}^{\mu *}(d_{\tilde{R}_z}^{m4},d_{R_z}^{m4},1,d_{\rho_z}^{m4})+\tfrac{\ell^\mu}{96}\chi_{\mathbf{i}_1}^{\mu *}(d_{\tilde{R}_z}^{m4},d_{R_z}^{m4},1,d_{\rho_z}^{m4})+\tfrac{\ell^\mu}{96}\chi_{\mathbf{i}_2}^{\mu *}(d_{\tilde{R}_z}^{m4},d_{R_z}^{m4},1,d_{\rho_z}^{m4})+\tfrac{\ell^\mu}{96}\chi_{\tilde{\mathbf{R}}_x}^{\mu *}(d_{\tilde{R}_z}^{m4},d_{R_z}^{m4},1,d_{\rho_z}^{m4})+\tfrac{\ell^\mu}{96}\chi_{\mathbf{R}_x}^{\mu *}(d_{\tilde{R}_z}^{m4},d_{R_z}^{m4},1,d$$

$$+\tfrac{\ell^\mu}{96}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}(d_{R_z}^{m4},d_{\tilde{R}_z}^{m4},d_{\rho_z}^{m4},1)+\tfrac{\ell^\mu}{96}\chi_{\mathbf{i}_3}^{\mu *}(d_{R_z}^{m4},d_{\tilde{R}_z}^{m4},d_{\rho_z}^{m4},1)+\tfrac{\ell^\mu}{96}\chi_{\mathbf{R}_y}^{\mu *}(d_{R_z}^{m4},d_{\tilde{R}_z}^{m4},d_{\rho_z}^{m4},1)+\tfrac{\ell^\mu}{96}\chi_{\tilde{\mathbf{R}}_y}^{\mu *}(d_{R_z}^{m4},d_{\tilde{R}_z}^{m4},d_{\rho_z}^{m4},1)+\tfrac{\ell^\mu}{96}\chi_{\mathbf{i}_6}^{\mu *}(d_{R_z}^{m4},d_{\tilde{R}_z}^{m4},d_{\rho_z}^{m4},1)+\tfrac{\ell^\mu}{96}\chi_{\mathbf{i}_5}^{\mu *}(d_{R_z}^{m4},d_{\tilde{R}_z}^{m4},d_{\rho_z}^{m4},1$$

$$\tfrac{1}{96}(\_\mathbf{1}+\_\boldsymbol{\rho}_z+\_\mathbf{R}_z+\_\tilde{\mathbf{R}}_z \;+\_\boldsymbol{\rho}_x+\_\boldsymbol{\rho}_y+\_\mathbf{i}_4+\_\tilde{\mathbf{R}}_z \;+\_\mathbf{r}_1+\_\mathbf{r}_4+\_\mathbf{i}_1+\_\mathbf{R}_y \;+\_\mathbf{r}_2+\_\mathbf{r}_3+\_\mathbf{i}_2+\_\tilde{\mathbf{R}}_y \;+\_\tilde{\mathbf{r}}_1+\_\tilde{\mathbf{r}}_3+\_\tilde{\mathbf{R}}_x+\_\mathbf{i}_6 \;+\_\tilde{\mathbf{r}}_2+\_\tilde{\mathbf{r}}_4+\_\mathbf{R}_x+\_\mathbf{i}_5$$

$$\left\{\mathbf{1},\boldsymbol{\rho}_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} \quad \left\{\boldsymbol{\rho}_x,\boldsymbol{\rho}_y,\mathbf{i}_3,\mathbf{i}_4\right\} \quad \left\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\right\} \quad \left\{\mathbf{r}_2,\mathbf{r}_3,\mathbf{i}_2,\tilde{\mathbf{R}}_y\right\} \quad \left\{\tilde{\mathbf{r}}_1,\tilde{\mathbf{r}}_3,\tilde{\mathbf{R}}_x,\mathbf{i}_6\right\} \quad \left\{\tilde{\mathbf{r}}_2,\tilde{\mathbf{r}}_4,\mathbf{R}_x,\mathbf{i}_5\right\}$$

Review Octahedral $O \supset D_4 \supset C_4$ subgroup chain and coset bases

Coset factored splitting of $O \supset D_4 \supset C_4$ projectors and levels

Coset spaces based on $m_4(C_4)\uparrow O$

Splitting class projectors into C_4 cosets and $m_4(C_4)\uparrow O$ bases

General development of irep projectors $\mathbf{P}^{\mu}_{m4m4}$

Calculating $\mathbf{P}^{E}_{0_4 0_4}$

Calculating $\mathbf{P}^{E}_{2_4 2_4}$

Calculating $\mathbf{P}^{T_1}_{0_4 0_4}$

Calculating $\mathbf{P}^{T_1}_{1_4 1_4}$

Calculating $\mathbf{P}^{T_2}_{2_4 2_4}$

Structure and applications of various subgroup chain ireps

$O_h \supset D_{4h} \supset C_{4v}$

$O_h \supset D_{3h} \supset C_{3v}$

$O_h \supset C_{2v}$

Calculating $\mathbf{P}^{E}_{0_4 0_4}$

$O\supset C_4$	0_4	1_4	2_4	3_4
$A_1\downarrow C_4$	1	·	·	·
$A_2\downarrow C_4$	·	·	1	·
$E\downarrow C_4$	1	·	1	·
$T_1\downarrow C_4$	1	1	·	1
$T_2\downarrow C_4$	·	1	1	1

$O: \chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}=\mathbf{1}$	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	$\boldsymbol{\rho}_{xyz}$	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$$\mathbf{p}_{m_4}=\sum_{p=0}^{3}\frac{e^{2\pi i m\cdot p/4}}{4}\mathbf{R}_z^p=\begin{cases}\mathbf{p}_{0_4}=(\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{1_4}=(\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{2_4}=(\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{3_4}=(\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4\end{cases}$$

$$\mathbf{P}^{E}_{0_4 0_4}=\mathbf{p}_{0_4}\mathbf{P}^{E}=\mathbf{P}^{E}\mathbf{p}_{0_4}$$

$$=\sum_{g}^{\circ O}\frac{\ell^E}{\circ O}(\chi^E_g)\cdot\mathbf{g}\cdot\left(\mathbf{p}_{0_4}\right)=\sum_{g}^{\circ O}\frac{2}{96}(\chi^E_g)\cdot\mathbf{g}\cdot\left(1\cdot\mathbf{1}+1\cdot\boldsymbol{\rho}_z+1\cdot\mathbf{R}_z+1\cdot\tilde{\mathbf{R}}_z\right)$$

Calculating $\mathbf{P}^{E}_{0_4 0_4}$

$O \supset C_4$	0_4	1_4	2_4	3_4
$A_1 \downarrow C_4$	1	·	·	·
$A_2 \downarrow C_4$	·	·	1	·
$E \downarrow C_4$	1	·	1	·
$T_1 \downarrow C_4$	1	1	·	1
$T_2 \downarrow C_4$	·	1	1	1

$O: \chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}=\mathbf{1}$	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	ρ_{xyz}	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$$\mathbf{p}_{m_4} = \sum_{p=0}^{3} \frac{e^{2\pi i m\cdot p/4}}{4} \mathbf{R}_z^p = \begin{cases} \mathbf{p}_{0_4} = (\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{1_4} = (\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{2_4} = (\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{3_4} = (\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4 \end{cases}$$

$$\mathbf{P}^{E}_{0_4 0_4} = \mathbf{p}_{0_4}\mathbf{P}^{E} = \mathbf{P}^{E}\mathbf{p}_{0_4}$$

$$= \sum_{g}^{\circ O} \frac{\ell^E}{{}^\circ O} (\chi^E_g) \cdot \mathbf{g} \cdot (\mathbf{p}_{0_4}) = \sum_{g}^{\circ O} \frac{2}{96} (\chi^E_g) \cdot \mathbf{g} \cdot (1 \cdot \mathbf{1} + 1 \cdot \rho_z + 1 \cdot \mathbf{R}_z + 1 \cdot \tilde{\mathbf{R}}_z)$$

$$\mathbf{1}C_4 = \mathbf{1}\{\mathbf{1}, \rho_z, \mathbf{R}_z, \tilde{\mathbf{R}}_z\} \quad \rho_x C_4 = \{\rho_x, \rho_y, \mathbf{i}_4, \mathbf{i}_3\} \quad \mathbf{r}_1 C_4 = \{\mathbf{r}_1, \mathbf{r}_4, \mathbf{i}_1, \mathbf{R}_y\} \quad \mathbf{r}_2 C_4 = \{\mathbf{r}_2, \mathbf{r}_3, \mathbf{i}_2, \tilde{\mathbf{R}}_y\} \quad \tilde{\mathbf{r}}_1 C_4 = \{\tilde{\mathbf{r}}_1, \tilde{\mathbf{r}}_3, \tilde{\mathbf{R}}_x, \mathbf{i}_6\} \quad \tilde{\mathbf{r}}_2 C_4 = \{\tilde{\mathbf{r}}_2, \tilde{\mathbf{r}}_4, \mathbf{R}_x, \mathbf{i}_5\}$$

$$= \tfrac{1}{48}\chi^E_{\mathbf{1}}(1, d^{14}_{\rho_z}, d^{14}_{R_z}, d^{14}_{\tilde{R}_z}) + \tfrac{1}{48}\chi^E_{\rho_x}(1, d^{14}_{\rho_z}, d^{14}_{R_z}, d^{14}_{\tilde{R}_z}) + \tfrac{1}{48}\chi^E_{\mathbf{r}_1}(1, d^{14}_{\rho_z}, d^{14}_{R_z}, d^{14}_{\tilde{R}_z}) + \tfrac{1}{48}\chi^E_{\mathbf{r}_2}(1, d^{14}_{\rho_z}, d^{14}_{R_z}, d^{14}_{\tilde{R}_z}) + \tfrac{1}{48}\chi^E_{\tilde{\mathbf{r}}_1}(1, d^{14}_{\rho_z}, d^{14}_{R_z}, d^{14}_{\tilde{R}_z}) + \tfrac{1}{48}\chi^E_{\tilde{\mathbf{r}}_2}(1, d^{14}_{\rho_z}, d^{14}_{R_z}, d^{14}_{\tilde{R}_z})$$

$$+ \tfrac{1}{48}\chi^E_{\rho_z}(d^{14}_{\rho_z}, 1, d^{14}_{\tilde{R}_z}, d^{14}_{R_z}) + \tfrac{1}{48}\chi^E_{\rho_y}(d^{14}_{\rho_z}, 1, d^{14}_{\tilde{R}_z}, d^{14}_{R_z}) + \tfrac{1}{48}\chi^E_{\mathbf{r}_4}(d^{14}_{\rho_z}, 1, d^{14}_{\tilde{R}_z}, d^{14}_{R_z}) + \tfrac{1}{48}\chi^E_{\mathbf{r}_3}(d^{14}_{\rho_z}, 1, d^{14}_{\tilde{R}_z}, d^{14}_{R_z}) + \tfrac{1}{48}\chi^E_{\tilde{\mathbf{r}}_3}(d^{14}_{\rho_z}, 1, d^{14}_{\tilde{R}_z}, d^{14}_{R_z}) + \tfrac{1}{48}\chi^E_{\tilde{\mathbf{r}}_4}(d^{14}_{\rho_z}, 1, d^{14}_{\tilde{R}_z}, d^{14}_{R_z})$$

$$+ \tfrac{1}{48}\chi^E_{\mathbf{R}_z}(d^{14}_{\tilde{R}_z}, d^{14}_{R_z}, 1, d^{14}_{\rho_z}) + \tfrac{1}{48}\chi^E_{\mathbf{i}_4}(d^{14}_{\tilde{R}_z}, d^{14}_{R_z}, 1, d^{14}_{\rho_z}) + \tfrac{1}{48}\chi^E_{\mathbf{i}_1}(d^{14}_{\tilde{R}_z}, d^{14}_{R_z}, 1, d^{14}_{\rho_z}) + \tfrac{1}{48}\chi^E_{\mathbf{i}_2}(d^{14}_{\tilde{R}_z}, d^{14}_{R_z}, 1, d^{14}_{\rho_z}) + \tfrac{1}{48}\chi^E_{\tilde{\mathbf{R}}_x}(d^{14}_{\tilde{R}_z}, d^{14}_{R_z}, 1, d^{14}_{\rho_z}) + \tfrac{1}{48}\chi^E_{\mathbf{R}_x}(d^{14}_{\tilde{R}_z}, d^{14}_{R_z}, 1, d^{14}_{\rho_z})$$

$$+ \tfrac{1}{48}\chi^E_{\tilde{\mathbf{R}}_z}(d^{14}_{R_z}, d^{14}_{\tilde{R}_z}, d^{14}_{\rho_z}, 1) + \tfrac{1}{48}\chi^E_{\mathbf{i}_3}(d^{14}_{R_z}, d^{14}_{\tilde{R}_z}, d^{14}_{\rho_z}, 1) + \tfrac{1}{48}\chi^E_{\mathbf{R}_y}(d^{14}_{R_z}, d^{14}_{\tilde{R}_z}, d^{14}_{\rho_z}, 1) + \tfrac{1}{48}\chi^E_{\tilde{\mathbf{R}}_y}(d^{14}_{R_z}, d^{14}_{\tilde{R}_z}, d^{14}_{\rho_z}, 1) + \tfrac{1}{48}\chi^E_{\mathbf{i}_6}(d^{14}_{R_z}, d^{14}_{\tilde{R}_z}, d^{14}_{\rho_z}, 1) + \tfrac{1}{48}\chi^E_{\mathbf{i}_5}(d^{14}_{R_z}, d^{14}_{\tilde{R}_z}, d^{14}_{\rho_z}, 1)$$

Calculating $\mathbf{P}^{E}_{0_4 0_4}$

$O\supset C_4$	0_4	1_4	2_4	3_4
$A_1\downarrow C_4$	1	·	·	·
$A_2\downarrow C_4$	·	·	1	·
$E\downarrow C_4$	1	·	1	·
$T_1\downarrow C_4$	1	1	·	1
$T_2\downarrow C_4$	·	1	1	1

$O:\chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}=\mathbf{1}$	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	ρ_{xyz}	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$$\mathbf{p}_{m_4}=\sum_{p=0}^{3}\frac{e^{2\pi i m\cdot p/4}}{4}\mathbf{R}_z^p=\begin{cases}\mathbf{p}_{0_4}=(\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{1_4}=(\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{2_4}=(\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{3_4}=(\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4\end{cases}$$

$$\mathbf{P}^{E}_{0_4 0_4}=\mathbf{p}_{0_4}\mathbf{P}^{E}=\mathbf{P}^{E}\mathbf{p}_{0_4}$$

$$=\sum_{g}^{\circ O}\frac{\ell^E}{\circ O}(\chi^E_g)\cdot\mathbf{g}\cdot\left(\mathbf{p}_{0_4}\right)=\sum_{g}^{\circ O}\frac{2}{96}(\chi^E_g)\cdot\mathbf{g}\cdot\left(1\cdot\mathbf{1}+1\cdot\rho_z+1\cdot\mathbf{R}_z+1\cdot\tilde{\mathbf{R}}_z\right)$$

$$\mathbf{1}C_4=\mathbf{1}\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\}\quad \rho_x C_4=\{\rho_x,\rho_y,\mathbf{i}_4,\mathbf{i}_3\}\quad \mathbf{r}_1C_4=\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\}\quad \mathbf{r}_2C_4=\{\mathbf{r}_2,\mathbf{r}_3,\mathbf{i}_2,\tilde{\mathbf{R}}_y\}\quad \tilde{\mathbf{r}}_1C_4=\{\tilde{\mathbf{r}}_1,\tilde{\mathbf{r}}_3,\tilde{\mathbf{R}}_x,\mathbf{i}_6\}\quad \tilde{\mathbf{r}}_2C_4=\{\tilde{\mathbf{r}}_2,\tilde{\mathbf{r}}_4,\mathbf{R}_x,\mathbf{i}_5\}$$

$$=\tfrac{1}{48}\chi^E_{\mathbf{1}}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z})+\tfrac{1}{48}\chi^E_{\rho_x}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z})+\tfrac{1}{48}\chi^E_{\mathbf{r}_1}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z})+\tfrac{1}{48}\chi^E_{\mathbf{r}_2}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z})+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{r}}_1}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z})+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{r}}_2}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z})$$

$$+\tfrac{1}{48}\chi^E_{\rho_z}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z})+\tfrac{1}{48}\chi^E_{\rho_y}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z})+\tfrac{1}{48}\chi^E_{\mathbf{r}_4}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z})+\tfrac{1}{48}\chi^E_{\mathbf{r}_3}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z})+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{r}}_3}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z})+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{r}}_4}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z})$$

$$+\tfrac{1}{48}\chi^E_{\mathbf{R}_z}(d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z})+\tfrac{1}{48}\chi^E_{\mathbf{i}_4}(d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z})+\tfrac{1}{48}\chi^E_{\mathbf{i}_1}(d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z})+\tfrac{1}{48}\chi^E_{\mathbf{i}_2}(d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z})+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{R}}_x}(d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z})+\tfrac{1}{48}\chi^E_{\mathbf{R}_x}(d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z})$$

$$+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{R}}_z}(d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z},d^{1_4}_{\rho_z},1)+\tfrac{1}{48}\chi^E_{\mathbf{i}_3}(d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z},d^{1_4}_{\rho_z},1)+\tfrac{1}{48}\chi^E_{\mathbf{R}_y}(d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z},d^{1_4}_{\rho_z},1)+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{R}}_y}(d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z},d^{1_4}_{\rho_z},1)+\tfrac{1}{48}\chi^E_{\mathbf{i}_6}(d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z},d^{1_4}_{\rho_z},1)+\tfrac{1}{48}\chi^E_{\mathbf{i}_5}(d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z},d^{1_4}_{\rho_z},1)$$

$$=\tfrac{1}{48}(+2)(1,+1,+1,+1)+\tfrac{1}{48}(+2)(1,+1,+1,+1)+\tfrac{1}{48}(-1)(1,+1,+1,+1)+\tfrac{1}{48}(-1)(1,+1,+1,+1)+\tfrac{1}{48}(-1)(1,+1,+1,+1)+\tfrac{1}{48}(-1)(1,+1,+1,+1)$$

$$+\tfrac{1}{48}(+2)(+1,1,+1,+1)+\tfrac{1}{48}(+2)(+1,1,+1,+1)+\tfrac{1}{48}(-1)(+1,1,+1,+1)+\tfrac{1}{48}(-1)(+1,1,+1,+1)+\tfrac{1}{48}(-1)(+1,1,+1,+1)+\tfrac{1}{48}(-1)(+1,1,+1,+1)$$

$$+\tfrac{1}{48}(0)(+1,+1,1,+1)+\tfrac{1}{48}(0)(+1,+1,1,+1)+\tfrac{1}{48}(0)(+1,+1,1,+1)+\tfrac{1}{48}(0)(+1,+1,1,+1)+\tfrac{1}{48}(0)(+1,+1,1,+1)+\tfrac{1}{48}(0)(+1,+1,1,+1)$$

$$+\tfrac{1}{48}(0)(+1,+1,+1,1)+\tfrac{1}{48}(0)(+1,+1,+1,1)+\tfrac{1}{48}(0)(+1,+1,+1,1)+\tfrac{1}{48}(0)(+1,+1,+1,1)+\tfrac{1}{48}(0)(+1,+1,+1,1)+\tfrac{1}{48}(0)(+1,+1,+1,1)$$

4, 4, 4, 4, *4, 4, 4, 4,* *-2,-2,-2,-2,* *-2,-2,-2,-2,* *-2,-2,-2,-2,* *-2,-2,-2,-2,*

Calculating $\mathbf{P}^{E}_{0_4 0_4}$

$O\supset C_4$	0_4	1_4	2_4	3_4
$A_1\downarrow C_4$	1	·	·	·
$A_2\downarrow C_4$	·	·	1	·
$E\downarrow C_4$	1	·	1	·
$T_1\downarrow C_4$	1	1	·	1
$T_2\downarrow C_4$	·	1	1	1

$O: \chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}=1$	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	ρ_{xyz}	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$$\mathbf{p}_{m_4}=\sum_{p=0}^{3}\frac{e^{2\pi i m\cdot p/4}}{4}\mathbf{R}_z^p=\begin{cases}\mathbf{p}_{0_4}=(\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{1_4}=(\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{2_4}=(\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{3_4}=(\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4\end{cases}$$

$$\mathbf{P}^{E}_{0_4 0_4}=\mathbf{p}_{0_4}\mathbf{P}^{E}=\mathbf{P}^{E}\mathbf{p}_{0_4}$$

$$=\sum_{g}^{\circ O}\frac{\ell^E}{\circ O}(\chi^E_g)\cdot\mathbf{g}\cdot(\mathbf{p}_{0_4})=\sum_{g}^{\circ O}\frac{2}{96}(\chi^E_g)\cdot\mathbf{g}\cdot(1\cdot\mathbf{1}+1\cdot\rho_z+1\cdot\mathbf{R}_z+1\cdot\tilde{\mathbf{R}}_z)$$

$$\mathbf{1}C_4=\mathbf{1}\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\}\quad \rho_x C_4=\{\rho_x,\rho_y,\mathbf{i}_4,\mathbf{i}_3\}\quad \mathbf{r}_1C_4=\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\}\quad \mathbf{r}_2C_4=\{\mathbf{r}_2,\mathbf{r}_3,\mathbf{i}_2,\tilde{\mathbf{R}}_y\}\quad \tilde{\mathbf{r}}_1C_4=\{\tilde{\mathbf{r}}_1,\tilde{\mathbf{r}}_3,\tilde{\mathbf{R}}_x,\mathbf{i}_6\}\quad \tilde{\mathbf{r}}_2C_4=\{\tilde{\mathbf{r}}_2,\tilde{\mathbf{r}}_4,\mathbf{R}_x,\mathbf{i}_5\}$$

$$=\tfrac{1}{48}\chi^E_{\mathbf{1}}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z})+\tfrac{1}{48}\chi^E_{\rho_x}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z})+\tfrac{1}{48}\chi^E_{\mathbf{r}_1}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z})+\tfrac{1}{48}\chi^E_{\mathbf{r}_2}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z})+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{r}}_1}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z})+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{r}}_2}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z})$$

$$+\tfrac{1}{48}\chi^E_{\rho_z}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z})+\tfrac{1}{48}\chi^E_{\rho_y}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z})+\tfrac{1}{48}\chi^E_{\mathbf{r}_4}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z})+\tfrac{1}{48}\chi^E_{\mathbf{r}_3}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z})+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{r}}_3}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z})+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{r}}_4}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z})$$

$$+\tfrac{1}{48}\chi^E_{\mathbf{R}_z}(d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z})+\tfrac{1}{48}\chi^E_{\mathbf{i}_4}(d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z})+\tfrac{1}{48}\chi^E_{\mathbf{i}_1}(d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z})+\tfrac{1}{48}\chi^E_{\mathbf{i}_2}(d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z})+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{R}}_x}(d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z})+\tfrac{1}{48}\chi^E_{\mathbf{R}_x}(d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z})$$

$$+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{R}}_z}(d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z},d^{1_4}_{\rho_z},1)+\tfrac{1}{48}\chi^E_{\mathbf{i}_3}(d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z},d^{1_4}_{\rho_z},1)+\tfrac{1}{48}\chi^E_{\mathbf{R}_y}(d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z},d^{1_4}_{\rho_z},1)+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{R}}_y}(d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z},d^{1_4}_{\rho_z},1)+\tfrac{1}{48}\chi^E_{\mathbf{i}_6}(d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z},d^{1_4}_{\rho_z},1)+\tfrac{1}{48}\chi^E_{\mathbf{i}_5}(d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z},d^{1_4}_{\rho_z},1)$$

$$=\tfrac{1}{48}(+2)(1,+1,+1,+1)+\tfrac{1}{48}(+2)(1,+1,+1,+1)+\tfrac{1}{48}(-1)(1,+1,+1,+1)+\tfrac{1}{48}(-1)(1,+1,+1,+1)+\tfrac{1}{48}(-1)(1,+1,+1,+1)+\tfrac{1}{48}(-1)(1,+1,+1,+1)$$

$$+\tfrac{1}{48}(+2)(+1,1,+1,+1)+\tfrac{1}{48}(+2)(+1,1,+1,+1)+\tfrac{1}{48}(-1)(+1,1,+1,+1)+\tfrac{1}{48}(-1)(+1,1,+1,+1)+\tfrac{1}{48}(-1)(+1,1,+1,+1)+\tfrac{1}{48}(-1)(+1,1,+1,+1)$$

$$+\tfrac{1}{48}(0)(+1,+1,1,+1)+\tfrac{1}{48}(0)(+1,+1,1,+1)+\tfrac{1}{48}(0)(+1,+1,1,+1)+\tfrac{1}{48}(0)(+1,+1,1,+1)+\tfrac{1}{48}(0)(+1,+1,1,+1)+\tfrac{1}{48}(0)(+1,+1,1,+1)$$

$$+\tfrac{1}{48}(0)(+1,+1,+1,1)+\tfrac{1}{48}(0)(+1,+1,+1,1)+\tfrac{1}{48}(0)(+1,+1,+1,1)+\tfrac{1}{48}(0)(+1,+1,+1,1)+\tfrac{1}{48}(0)(+1,+1,+1,1)+\tfrac{1}{48}(0)(+1,+1,+1,1)$$

4, 4, 4, 4, *4, 4, 4, 4,* *-2,-2,-2,-2,* *-2,-2,-2,-2,* *-2,-2,-2,-2,* *-2,-2,-2,-2,*

$$\tfrac{1}{12}(\underline{1}\mathbf{1}+\underline{1}\rho_z+\underline{1}\mathbf{R}_z+\underline{1}\tilde{\mathbf{R}}_z+\underline{1}\rho_x+\underline{1}\rho_y+\underline{1}\mathbf{i}_4+\underline{1}\tilde{\mathbf{R}}_z\quad \underline{-\tfrac{1}{2}}\mathbf{r}_1\,\underline{-\tfrac{1}{2}}\mathbf{r}_4\,\underline{-\tfrac{1}{2}}\mathbf{i}_1\,\underline{-\tfrac{1}{2}}\mathbf{R}_y\quad \underline{-\tfrac{1}{2}}\mathbf{r}_2\,\underline{-\tfrac{1}{2}}\mathbf{r}_3\,\underline{-\tfrac{1}{2}}\mathbf{i}_2\,\underline{-\tfrac{1}{2}}\tilde{\mathbf{R}}_y\quad \underline{-\tfrac{1}{2}}\tilde{\mathbf{r}}_1\,\underline{-\tfrac{1}{2}}\tilde{\mathbf{r}}_3\,\underline{-\tfrac{1}{2}}\tilde{\mathbf{R}}_x\,\underline{-\tfrac{1}{2}}\mathbf{i}_6\quad \underline{-\tfrac{1}{2}}\tilde{\mathbf{r}}_2\,\underline{-\tfrac{1}{2}}\tilde{\mathbf{r}}_4\,\underline{-\tfrac{1}{2}}\mathbf{R}_x\,\underline{-\tfrac{1}{2}}\mathbf{i}_5)$$

Review Octahedral $O \supset D_4 \supset C_4$ *subgroup chain and coset bases*

Coset factored splitting of $O \supset D_4 \supset C_4$ *projectors and levels*

Coset spaces based on $m_4(C_4)\uparrow O$

Splitting class projectors into C_4 *cosets and* $m_4(C_4)\uparrow O$ *bases*

General development of irep projectors $\mathbf{P}^{\mu}_{m4m4}$

Calculating $\mathbf{P}^{E}_{0_4 0_4}$

Calculating $\mathbf{P}^{E}_{2_4 2_4}$

Calculating $\mathbf{P}^{T_1}_{0_4 0_4}$

Calculating $\mathbf{P}^{T_1}_{1_4 1_4}$

Calculating $\mathbf{P}^{T_2}_{2_4 2_4}$

Structure and applications of various subgroup chain ireps

$O_h \supset D_{4h} \supset C_{4v}$

$O_h \supset D_{3h} \supset C_{3v}$

$O_h \supset C_{2v}$

Calculating $\mathbf{P}^{E}_{2_4 2_4}$

$O\supset C_4$	0_4	1_4	2_4	3_4
$A_1\downarrow C_4$	1	·	·	·
$A_2\downarrow C_4$	·	·	1	·
$E\downarrow C_4$	1	·	1	·
$T_1\downarrow C_4$	1	1	·	1
$T_2\downarrow C_4$	·	1	1	1

$O:\chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}=\mathbf{1}$	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	$\boldsymbol{\rho}_{xyz}$	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$$\mathbf{p}_{m_4}=\sum_{p=0}^{3}\frac{e^{2\pi i m\cdot p/4}}{4}\mathbf{R}_z^p=\begin{cases}\mathbf{p}_{0_4}=(\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{1_4}=(\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{2_4}=(\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{3_4}=(\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4\end{cases}$$

$$\mathbf{P}^{E}_{2_4 2_4}=\mathbf{p}_{2_4}\mathbf{P}^{E}=\mathbf{P}^{E}\mathbf{p}_{2_4}$$

$$=\sum_{g}^{\circ O}\frac{\ell^E}{\circ O}(\chi_g^E)\cdot\mathbf{g}\cdot(\mathbf{p}_{2_4})=\sum_{g}^{\circ O}\frac{2}{96}(\chi_g^E)\cdot\mathbf{g}\cdot(1\cdot\mathbf{1}+1\cdot\boldsymbol{\rho}_z-1\cdot\mathbf{R}_z-1\cdot\tilde{\mathbf{R}}_z)$$

$$\mathbf{1}C_4=\mathbf{1}\{\mathbf{1},\boldsymbol{\rho}_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\}\quad \boldsymbol{\rho}_xC_4=\{\boldsymbol{\rho}_x,\boldsymbol{\rho}_y,\mathbf{i}_4,\mathbf{i}_3\}\quad \mathbf{r}_1C_4=\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\}\quad \mathbf{r}_2C_4=\{\mathbf{r}_2,\mathbf{r}_3,\mathbf{i}_2,\tilde{\mathbf{R}}_y\}\quad \tilde{\mathbf{r}}_1C_4=\{\tilde{\mathbf{r}}_1,\tilde{\mathbf{r}}_3,\tilde{\mathbf{R}}_x,\mathbf{i}_6\}\quad \tilde{\mathbf{r}}_2C_4=\{\tilde{\mathbf{r}}_2,\tilde{\mathbf{r}}_4,\mathbf{R}_x,\mathbf{i}_5\}$$

$$=\tfrac{1}{48}\chi^E_{\mathbf{1}}(1,d^{2_4}_{\rho_z},d^{2_4}_{R_z},d^{2_4}_{\tilde{R}_z})+\tfrac{1}{48}\chi^E_{\boldsymbol{\rho}_x}(1,d^{2_4}_{\rho_z},d^{2_4}_{R_z},d^{2_4}_{\tilde{R}_z})+\tfrac{1}{48}\chi^E_{\mathbf{r}_1}(1,d^{2_4}_{\rho_z},d^{2_4}_{R_z},d^{2_4}_{\tilde{R}_z})+\tfrac{1}{48}\chi^E_{\mathbf{r}_2}(1,d^{2_4}_{\rho_z},d^{2_4}_{R_z},d^{2_4}_{\tilde{R}_z})+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{r}}_1}(1,d^{2_4}_{\rho_z},d^{2_4}_{R_z},d^{2_4}_{\tilde{R}_z})+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{r}}_2}(1,d^{2_4}_{\rho_z},d^{2_4}_{R_z},d^{2_4}_{\tilde{R}_z})$$

$$+\tfrac{1}{48}\chi^E_{\rho_z}(d^{2_4}_{\rho_z},1,d^{2_4}_{\tilde{R}_z},d^{2_4}_{R_z})+\tfrac{1}{48}\chi^E_{\boldsymbol{\rho}_y}(d^{2_4}_{\rho_z},1,d^{2_4}_{\tilde{R}_z},d^{2_4}_{R_z})+\tfrac{1}{48}\chi^E_{\mathbf{r}_4}(d^{2_4}_{\rho_z},1,d^{2_4}_{\tilde{R}_z},d^{2_4}_{R_z})+\tfrac{1}{48}\chi^E_{\mathbf{r}_3}(d^{2_4}_{\rho_z},1,d^{2_4}_{\tilde{R}_z},d^{2_4}_{R_z})+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{r}}_3}(d^{2_4}_{\rho_z},1,d^{2_4}_{\tilde{R}_z},d^{2_4}_{R_z})+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{r}}_4}(d^{2_4}_{\rho_z},1,d^{2_4}_{\tilde{R}_z},d^{2_4}_{R_z})$$

$$+\tfrac{1}{48}\chi^E_{\mathbf{R}_z}(d^{2_4}_{\tilde{R}_z},d^{2_4}_{R_z},1,d^{2_4}_{\rho_z})+\tfrac{1}{48}\chi^E_{\mathbf{i}_4}(d^{2_4}_{\tilde{R}_z},d^{2_4}_{R_z},1,d^{2_4}_{\rho_z})+\tfrac{1}{48}\chi^E_{\mathbf{i}_1}(d^{2_4}_{\tilde{R}_z},d^{2_4}_{R_z},1,d^{2_4}_{\rho_z})+\tfrac{1}{48}\chi^E_{\mathbf{i}_2}(d^{2_4}_{\tilde{R}_z},d^{2_4}_{R_z},1,d^{2_4}_{\rho_z})+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{R}}_x}(d^{2_4}_{\tilde{R}_z},d^{2_4}_{R_z},1,d^{2_4}_{\rho_z})+\tfrac{1}{48}\chi^E_{\mathbf{R}_x}(d^{2_4}_{\tilde{R}_z},d^{2_4}_{R_z},1,d^{2_4}_{\rho_z})$$

$$+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{R}}_z}(d^{2_4}_{R_z},d^{2_4}_{\tilde{R}_z},d^{2_4}_{\rho_z},1)+\tfrac{1}{48}\chi^E_{\mathbf{i}_3}(d^{2_4}_{R_z},d^{2_4}_{\tilde{R}_z},d^{2_4}_{\rho_z},1)+\tfrac{1}{48}\chi^E_{\mathbf{R}_y}(d^{2_4}_{R_z},d^{2_4}_{\tilde{R}_z},d^{2_4}_{\rho_z},1)+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{R}}_y}(d^{2_4}_{R_z},d^{2_4}_{\tilde{R}_z},d^{2_4}_{\rho_z},1)+\tfrac{1}{48}\chi^E_{\mathbf{i}_6}(d^{2_4}_{R_z},d^{2_4}_{\tilde{R}_z},d^{2_4}_{\rho_z},1)+\tfrac{1}{48}\chi^E_{\mathbf{i}_5}(d^{2_4}_{R_z},d^{2_4}_{\tilde{R}_z},d^{2_4}_{\rho_z},1)$$

$$=\tfrac{1}{48}(+2)(1,+1,-1,-1)\;=\tfrac{1}{48}(+2)(1,+1,-1,-1)\;+\tfrac{1}{48}(-1)(1,+1,-1,-1)\;+\tfrac{1}{48}(-1)(1,+1,-1,-1)\;+\tfrac{1}{48}(-1)(1,+1,-1,-1)\;+\tfrac{1}{48}(-1)(1,+1,-1,-1)$$

$$+\tfrac{1}{48}(+2)(+1,1,-1,-1)\;+\tfrac{1}{48}(+2)(+1,1,-1,-1)\;+\tfrac{1}{48}(-1)(+1,1,-1,-1)\;+\tfrac{1}{48}(-1)(+1,1,-1,-1)\;+\tfrac{1}{48}(-1)(+1,1,-1,-1)\;+\tfrac{1}{48}(-1)(+1,1,-1,-1)$$

$$+\tfrac{1}{48}(0)(-1,-1,1,+1)\;+\tfrac{1}{48}(0)(-1,-1,1,+1)\;+\tfrac{1}{48}(0)(-1,-1,1,+1)\;+\tfrac{1}{48}(0)(-1,-1,1,+1)\;+\tfrac{1}{48}(0)(-1,-1,1,+1)\;+\tfrac{1}{48}(0)(-1,-1,1,+1)$$

$$+\tfrac{1}{48}(0)(-1,-1,+1,1)\;+\tfrac{1}{48}(0)(-1,-1,+1,1)\;+\tfrac{1}{48}(0)(-1,-1,+1,1)\;+\tfrac{1}{48}(0)(-1,-1,+1,1)\;+\tfrac{1}{48}(0)(-1,-1,+1,1)\;+\tfrac{1}{48}(0)(-1,-1,+1,1)$$

4, 4,-4,-4, *4, 4,-4,-4,* *-2,-2, 2, 2,* *-2,-2, 2, 2,* *-2,-2, 2, 2,* *-2,-2, 2, 2,*

$$\tfrac{1}{12}(\underline{1}\mathbf{1}+\underline{1}\boldsymbol{\rho}_z-\underline{1}\mathbf{R}_z-\underline{1}\tilde{\mathbf{R}}_z+\underline{1}\boldsymbol{\rho}_x+\underline{1}\boldsymbol{\rho}_y-\underline{1}\mathbf{i}_4-\underline{1}\mathbf{i}_3\;\underline{-\tfrac{1}{2}}\mathbf{r}_1\underline{-\tfrac{1}{2}}\mathbf{r}_4\underline{+\tfrac{1}{2}}\mathbf{i}_1\underline{+\tfrac{1}{2}}\mathbf{R}_y\;\underline{-\tfrac{1}{2}}\mathbf{r}_2\underline{-\tfrac{1}{2}}\mathbf{r}_3\underline{+\tfrac{1}{2}}\mathbf{i}_2\underline{+\tfrac{1}{2}}\tilde{\mathbf{R}}_y\;\underline{-\tfrac{1}{2}}\tilde{\mathbf{r}}_1\underline{-\tfrac{1}{2}}\tilde{\mathbf{r}}_3\underline{+\tfrac{1}{2}}\tilde{\mathbf{R}}_x\underline{+\tfrac{1}{2}}\mathbf{i}_6\;\underline{-\tfrac{1}{2}}\tilde{\mathbf{r}}_2\underline{-\tfrac{1}{2}}\tilde{\mathbf{r}}_4\underline{+\tfrac{1}{2}}\mathbf{R}_x\underline{+\tfrac{1}{2}}\mathbf{i}_5)$$

Calculating $\mathbf{P}^{T_1}{}_{0_4 0_4}$

$O\supset C_4$	0_4	1_4	2_4	3_4
$A_1\downarrow C_4$	1	·	·	·
$A_2\downarrow C_4$	·	·	1	·
$E\downarrow C_4$	1	·	1	·
$T_1\downarrow C_4$	1	1	·	1
$T_2\downarrow C_4$	·	1	1	1

$O:\chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}=\mathbf{1}$	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	ρ_{xyz}	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$$\mathbf{p}_{m_4}=\sum_{p=0}^{3}\frac{e^{2\pi i m\cdot p/4}}{4}\mathbf{R}_z^p=\begin{cases}\mathbf{p}_{0_4}=(\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{1_4}=(\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{2_4}=(\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{3_4}=(\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4\end{cases}$$

$$\mathbf{P}^{T_1}_{0_4 0_4}=\mathbf{p}_{0_4}\mathbf{P}^{T_1}=\mathbf{P}^{T_1}\mathbf{p}_{0_4}$$

$$=\sum_{g}^{\circ O}\frac{\ell^{T_1}}{\circ O}(\chi_g^{T_1})\cdot\mathbf{g}\cdot(\mathbf{p}_{0_4})=\sum_{g}^{\circ O}\frac{3}{96}(\chi_g^{T_1})\cdot\mathbf{g}\cdot\left(1\cdot\mathbf{1}+1\cdot\rho_z+1\cdot\mathbf{R}_z+1\cdot\tilde{\mathbf{R}}_z\right)$$

$$\mathbf{1}C_4=\mathbf{1}\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\}\quad \rho_x C_4=\{\rho_x,\rho_y,\mathbf{i}_4,\mathbf{i}_3\}\quad \mathbf{r}_1C_4=\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\}\quad \mathbf{r}_2C_4=\{\mathbf{r}_2,\mathbf{r}_3,\mathbf{i}_2,\tilde{\mathbf{R}}_y\}\quad \tilde{\mathbf{r}}_1C_4=\{\tilde{\mathbf{r}}_1,\tilde{\mathbf{r}}_3,\tilde{\mathbf{R}}_x,\mathbf{i}_6\}\quad \tilde{\mathbf{r}}_2C_4=\{\tilde{\mathbf{r}}_2,\tilde{\mathbf{r}}_4,\mathbf{R}_x,\mathbf{i}_5\}$$

$$=\tfrac{1}{32}\chi^{T_1}_{\mathbf{1}}(1,d^{04}_{\rho_z},d^{04}_{R_z},d^{04}_{\tilde R_z})+\tfrac{1}{32}\chi^{T_1}_{\rho_x}(1,d^{04}_{\rho_z},d^{04}_{R_z},d^{04}_{\tilde R_z})+\tfrac{1}{32}\chi^{T_1}_{\mathbf{r}_1}(1,d^{04}_{\rho_z},d^{04}_{R_z},d^{04}_{\tilde R_z})+\tfrac{1}{32}\chi^{T_1}_{\mathbf{r}_2}(1,d^{04}_{\rho_z},d^{04}_{R_z},d^{04}_{\tilde R_z})+\tfrac{1}{32}\chi^{T_1}_{\tilde{\mathbf{r}}_1}(1,d^{04}_{\rho_z},d^{04}_{R_z},d^{04}_{\tilde R_z})+\tfrac{1}{32}\chi^{T_1}_{\tilde{\mathbf{r}}_2}(1,d^{04}_{\rho_z},d^{04}_{R_z},d^{04}_{\tilde R_z})$$

$$+\tfrac{1}{32}\chi^{T_1}_{\rho_z}(d^{04}_{\rho_z},1,d^{04}_{\tilde R_z},d^{04}_{R_z})+\tfrac{1}{32}\chi^{T_1}_{\rho_y}(d^{04}_{\rho_z},1,d^{04}_{\tilde R_z},d^{04}_{R_z})+\tfrac{1}{32}\chi^{T_1}_{\mathbf{r}_4}(d^{04}_{\rho_z},1,d^{04}_{\tilde R_z},d^{04}_{R_z})+\tfrac{1}{32}\chi^{T_1}_{\mathbf{r}_3}(d^{04}_{\rho_z},1,d^{04}_{\tilde R_z},d^{04}_{R_z})+\tfrac{1}{32}\chi^{T_1}_{\tilde{\mathbf{r}}_3}(d^{04}_{\rho_z},1,d^{04}_{\tilde R_z},d^{04}_{R_z})+\tfrac{1}{32}\chi^{T_1}_{\tilde{\mathbf{r}}_4}(d^{04}_{\rho_z},1,d^{04}_{\tilde R_z},d^{04}_{R_z})$$

$$+\tfrac{1}{32}\chi^{T_1}_{\mathbf{R}_z}(d^{04}_{\tilde R_z},d^{04}_{R_z},1,d^{04}_{\rho_z})+\tfrac{1}{32}\chi^{T_1}_{\mathbf{i}_4}(d^{04}_{\tilde R_z},d^{04}_{R_z},1,d^{04}_{\rho_z})+\tfrac{1}{32}\chi^{T_1}_{\mathbf{i}_1}(d^{04}_{\tilde R_z},d^{04}_{R_z},1,d^{04}_{\rho_z})+\tfrac{1}{32}\chi^{T_1}_{\mathbf{i}_2}(d^{04}_{\tilde R_z},d^{04}_{R_z},1,d^{04}_{\rho_z})+\tfrac{1}{32}\chi^{T_1}_{\tilde{\mathbf{R}}_x}(d^{04}_{\tilde R_z},d^{04}_{R_z},1,d^{04}_{\rho_z})+\tfrac{1}{32}\chi^{T_1}_{\mathbf{R}_x}(d^{04}_{\tilde R_z},d^{04}_{R_z},1,d^{04}_{\rho_z})$$

$$+\tfrac{1}{32}\chi^{T_1}_{\tilde{\mathbf{R}}_z}(d^{04}_{R_z},d^{04}_{\tilde R_z},d^{04}_{\rho_z},1)+\tfrac{1}{32}\chi^{T_1}_{\mathbf{i}_3}(d^{04}_{R_z},d^{04}_{\tilde R_z},d^{04}_{\rho_z},1)+\tfrac{1}{32}\chi^{T_1}_{\mathbf{R}_y}(d^{04}_{R_z},d^{04}_{\tilde R_z},d^{04}_{\rho_z},1)+\tfrac{1}{32}\chi^{T_1}_{\tilde{\mathbf{R}}_y}(d^{04}_{R_z},d^{04}_{\tilde R_z},d^{04}_{\rho_z},1)+\tfrac{1}{32}\chi^{T_1}_{\mathbf{i}_6}(d^{04}_{R_z},d^{04}_{\tilde R_z},d^{04}_{\rho_z},1)+\tfrac{1}{32}\chi^{T_1}_{\mathbf{i}_5}(d^{04}_{R_z},d^{04}_{\tilde R_z},d^{04}_{\rho_z},1)$$

$$=\tfrac{1}{32}(+3)(1,+1,+1,+1)+\tfrac{1}{32}(-1)(1,+1,+1,+1)+\tfrac{1}{32}(0)(1,+1,+1,+1)+\tfrac{1}{32}(0)(1,+1,+1,+1)+\tfrac{1}{32}(0)(1,+1,+1,+1)+\tfrac{1}{32}(0)(1,+1,+1,+1)$$

$$+\tfrac{1}{32}(-1)(+1,1,+1,+1)+\tfrac{1}{32}(-1)(+1,1,+1,+1)+\tfrac{1}{32}(0)(+1,1,+1,+1)+\tfrac{1}{32}(0)(+1,1,+1,+1)+\tfrac{1}{32}(0)(+1,1,+1,+1)+\tfrac{1}{32}(0)(+1,1,+1,+1)$$

$$+\tfrac{1}{32}(+1)(+1,+1,1,+1)+\tfrac{1}{32}(-1)(+1,+1,1,+1)+\tfrac{1}{32}(-1)(+1,+1,1,+1)+\tfrac{1}{32}(-1)(+1,+1,1,+1)+\tfrac{1}{32}(+1)(+1,+1,1,+1)+\tfrac{1}{32}(+1)(+1,+1,1,+1)$$

$$+\tfrac{1}{32}(+1)(+1,+1,+1,1)+\tfrac{1}{32}(-1)(+1,+1,+1,1)+\tfrac{1}{32}(+1)(+1,+1,+1,1)+\tfrac{1}{32}(+1)(+1,+1,+1,1)+\tfrac{1}{32}(-1)(+1,+1,+1,1)+\tfrac{1}{32}(-1)(+1,+1,+1,1)$$

4, 4, 0, 0, *-4,-4,-4,-4,* *0, 0, 0, 0,* *0, 0, 0, 0,* *0, 0, 0, 0,* *0, 0, 0, 0,*

$$\tfrac{1}{8}(\underline{1}\mathbf{1}+\underline{1}\rho_z+\underline{1}\mathbf{R}_z+\underline{1}\tilde{\mathbf{R}}_z\ \underline{-1}\rho_x\ \underline{-1}\rho_y\ \underline{-1}\mathbf{i}_4\ \underline{-1}\mathbf{i}_3\quad +\underline{0}\mathbf{r}_1+\underline{0}\mathbf{r}_4+\underline{0}\mathbf{i}_1+\underline{0}\mathbf{R}_y\ +\underline{0}\mathbf{r}_2+\underline{0}\mathbf{r}_3+\underline{0}\mathbf{i}_2+\underline{0}\tilde{\mathbf{R}}_y\ +\underline{0}\tilde{\mathbf{r}}_1+\underline{0}\tilde{\mathbf{r}}_3+\underline{0}\tilde{\mathbf{R}}_x+\underline{0}\mathbf{i}_6\ +\underline{0}\tilde{\mathbf{r}}_2+\underline{0}\tilde{\mathbf{r}}_4+\underline{0}\mathbf{R}_x+\underline{0}\mathbf{i}_5)$$

Calculating $\mathbf{P}^{T_1}{}_{1_4 1_4}$

$O\supset C_4$	0_4	1_4	2_4	3_4
$A_1\downarrow C_4$	1	·	·	·
$A_2\downarrow C_4$	·	·	1	·
$E\downarrow C_4$	1	·	1	·
$T_1\downarrow C_4$	1	1	·	1
$T_2\downarrow C_4$	·	1	1	1

$O:\chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}=\mathbf{1}$	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	ρ_{xyz}	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$$\mathbf{p}_{m_4}=\sum_{p=0}^{3}\frac{e^{2\pi i m\cdot p/4}}{4}\mathbf{R}_z^p=\begin{cases}\mathbf{p}_{0_4}=(\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{1_4}=(\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{2_4}=(\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{3_4}=(\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4\end{cases}$$

$$\mathbf{P}^{T_1}_{1_4 1_4}=\mathbf{p}_{1_4}\mathbf{P}^{T_1}=\mathbf{P}^{T_1}\mathbf{p}_{1_4}$$

$$=\sum_{g}^{\circ O}\frac{\ell^{T_1}}{\circ O}(\chi^{T_1}_g)\cdot\mathbf{g}\cdot(\mathbf{p}_{1_4})=\sum_{g}^{\circ O}\frac{3}{96}(\chi^{T_1}_g)\cdot\mathbf{g}\cdot(1\cdot\mathbf{1}-1\cdot\rho_z+i\cdot\mathbf{R}_z-i\cdot\tilde{\mathbf{R}}_z)$$

$$\mathbf{1}C_4=\mathbf{1}\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\}\quad \rho_x C_4=\{\rho_x,\rho_y,\mathbf{i}_4,\mathbf{i}_3\}\quad \mathbf{r}_1C_4=\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\}\quad \mathbf{r}_2C_4=\{\mathbf{r}_2,\mathbf{r}_3,\mathbf{i}_2,\tilde{\mathbf{R}}_y\}\quad \tilde{\mathbf{r}}_1C_4=\{\tilde{\mathbf{r}}_1,\tilde{\mathbf{r}}_3,\tilde{\mathbf{R}}_x,\mathbf{i}_6\}\quad \tilde{\mathbf{r}}_2C_4=\{\tilde{\mathbf{r}}_2,\tilde{\mathbf{r}}_4,\mathbf{R}_x,\mathbf{i}_5\}$$

$$=\tfrac{1}{32}\chi^{T_1}_{\mathbf{1}}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z})+\tfrac{1}{32}\chi^{T_1}_{\rho_x}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z})+\tfrac{1}{32}\chi^{T_1}_{\mathbf{r}_1}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z})+\tfrac{1}{32}\chi^{T_1}_{\mathbf{r}_2}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z})+\tfrac{1}{32}\chi^{T_1}_{\tilde{\mathbf{r}}_1}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z})+\tfrac{1}{32}\chi^{T_1}_{\tilde{\mathbf{r}}_2}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z})$$

$$+\tfrac{1}{32}\chi^{T_1}_{\rho_z}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z})+\tfrac{1}{32}\chi^{T_1}_{\rho_y}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z})+\tfrac{1}{32}\chi^{T_1}_{\mathbf{r}_4}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z})+\tfrac{1}{32}\chi^{T_1}_{\mathbf{r}_3}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z})+\tfrac{1}{32}\chi^{T_1}_{\tilde{\mathbf{r}}_3}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z})+\tfrac{1}{32}\chi^{T_1}_{\tilde{\mathbf{r}}_4}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z})$$

$$+\tfrac{1}{32}\chi^{T_1}_{\mathbf{R}_z}(d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z})+\tfrac{1}{32}\chi^{T_1}_{\mathbf{i}_4}(d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z})+\tfrac{1}{32}\chi^{T_1}_{\mathbf{i}_1}(d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z})+\tfrac{1}{32}\chi^{T_1}_{\mathbf{i}_2}(d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z})+\tfrac{1}{32}\chi^{T_1}_{\tilde{\mathbf{R}}_x}(d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z})+\tfrac{1}{32}\chi^{T_1}_{\mathbf{R}_x}(d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z})$$

$$+\tfrac{1}{32}\chi^{T_1}_{\tilde{\mathbf{R}}_z}(d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z},d^{1_4}_{\rho_z},1)+\tfrac{1}{32}\chi^{T_1}_{\mathbf{i}_3}(d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z},d^{1_4}_{\rho_z},1)+\tfrac{1}{32}\chi^{T_1}_{\mathbf{R}_y}(d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z},d^{1_4}_{\rho_z},1)+\tfrac{1}{32}\chi^{T_1}_{\tilde{\mathbf{R}}_y}(d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z},d^{1_4}_{\rho_z},1)+\tfrac{1}{32}\chi^{T_1}_{\mathbf{i}_6}(d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z},d^{1_4}_{\rho_z},1)+\tfrac{1}{32}\chi^{T_1}_{\mathbf{i}_5}(d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z},d^{1_4}_{\rho_z},1)$$

$$=\tfrac{1}{32}(+3)(1,-1,+i,-i)+\tfrac{1}{32}(-1)(1,-1,+i,-i)+\tfrac{1}{32}(0)(1,-1,+i,-i)+\tfrac{1}{32}(0)(1,-1,+i,-i)+\tfrac{1}{32}(0)(1,-1,+i,-i)+\tfrac{1}{32}(0)(1,-1,+i,-i)$$

$$+\tfrac{1}{32}(-1)(-1,1,-i,+i)+\tfrac{1}{32}(-1)(-1,1,-i,+i)+\tfrac{1}{32}(0)(-1,1,-i,+i)+\tfrac{1}{32}(0)(-1,1,-i,+i)+\tfrac{1}{32}(0)(-1,1,-i,+i)+\tfrac{1}{32}(0)(-1,1,-i,+i)$$

$$+\tfrac{1}{32}(+1)(-i,+i,1,-1)+\tfrac{1}{32}(-1)(-i,+i,1,-1)+\tfrac{1}{32}(-1)(-i,+i,1,-1)+\tfrac{1}{32}(-1)(-i,+i,1,-1)+\tfrac{1}{32}(+1)(-i,+i,1,-1)+\tfrac{1}{32}(+1)(-i,+i,1,-1)$$

$$+\tfrac{1}{32}(+1)(+i,-i,-1,1)+\tfrac{1}{32}(-1)(+i,-i,-1,1)+\tfrac{1}{32}(+1)(+i,-i,-1,1)+\tfrac{1}{32}(+1)(+i,-i,-1,1)+\tfrac{1}{32}(-1)(+i,-i,-1,1)+\tfrac{1}{32}(-1)(+i,-i,-1,1)$$

$+4,-4,\ 4i,-4i,\quad 0,\ 0,\ 0,\ 0,\quad +2i,\ -2i,-2,\ +2,\quad +2i,\ -2i,-2,\ +2,\quad -2i,+2i,+2,\ -2\quad -2i,+2i,+2,\ -2.$

$$\tfrac{1}{8}\big(1\mathbf{1}-1\rho_z+i\mathbf{R}_z-i\tilde{\mathbf{R}}_z+0\rho_x+0\rho_y+0\mathbf{i}_4+0\mathbf{i}_3+\tfrac{i}{2}\mathbf{r}_1-\tfrac{i}{2}\mathbf{r}_4-\tfrac{1}{2}\mathbf{i}_1+\tfrac{1}{2}\mathbf{R}_y+\tfrac{i}{2}\mathbf{r}_2-\tfrac{i}{2}\mathbf{r}_3-\tfrac{1}{2}\mathbf{i}_2+\tfrac{1}{2}\tilde{\mathbf{R}}_y-\tfrac{i}{2}\tilde{\mathbf{r}}_1+\tfrac{i}{2}\tilde{\mathbf{r}}_3+\tfrac{1}{2}\tilde{\mathbf{R}}_x-\tfrac{1}{2}\mathbf{i}_6-\tfrac{i}{2}\tilde{\mathbf{r}}_2+\tfrac{i}{2}\tilde{\mathbf{r}}_4+\tfrac{1}{2}\mathbf{R}_x-\tfrac{1}{2}\mathbf{i}_5\big)$$

Calculating $\mathbf{P}^{T_2}{}_{2_4 2_4}$

$O \supset C_4$	0_4	1_4	2_4	3_4
$A_1 \downarrow C_4$	1	·	·	·
$A_2 \downarrow C_4$	·	·	1	·
$E \downarrow C_4$	1	·	1	·
$T_1 \downarrow C_4$	1	1	·	1
$T_2 \downarrow C_4$	·	1	1	1

$O: \chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}=1$	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	ρ_{xyz}	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$$\mathbf{p}_{m_4} = \sum_{p=0}^{3} \frac{e^{2\pi i m \cdot p/4}}{4} \mathbf{R}_z^p = \begin{cases} \mathbf{p}_{0_4} = (\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{1_4} = (\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{2_4} = (\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{3_4} = (\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4 \end{cases}$$

$$\mathbf{P}^{T_2}_{2_4 2_4} = \mathbf{p}_{2_4}\mathbf{P}^{T_2} = \mathbf{P}^{T_2}\mathbf{p}_{2_4}$$

$$= \sum_{g}^{°O} \frac{\ell^{T_2}}{°O} (\chi_g^{T_2}) \cdot \mathbf{g} \cdot (\mathbf{p}_{2_4}) = \sum_{g}^{°O} \frac{3}{96} (\chi_g^{T_2}) \cdot \mathbf{g} \cdot \left(1 \cdot \mathbf{1} + 1 \cdot \rho_z - 1 \cdot \mathbf{R}_z - 1 \cdot \tilde{\mathbf{R}}_z \right)$$

$$\mathbf{1}C_4 = \mathbf{1}\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\} \quad \rho_x C_4 = \{\rho_x,\rho_y,\mathbf{i}_4,\mathbf{i}_3\} \quad \mathbf{r}_1 C_4 = \{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\} \quad \mathbf{r}_2 C_4 = \{\mathbf{r}_2,\mathbf{r}_3,\mathbf{i}_2,\tilde{\mathbf{R}}_y\} \quad \tilde{\mathbf{r}}_1 C_4 = \{\tilde{\mathbf{r}}_1,\tilde{\mathbf{r}}_3,\tilde{\mathbf{R}}_x,\mathbf{i}_6\} \quad \tilde{\mathbf{r}}_2 C_4 = \{\tilde{\mathbf{r}}_2,\tilde{\mathbf{r}}_4,\mathbf{R}_x,\mathbf{i}_5\}$$

$$= \tfrac{1}{32}\chi^{T_2}_{\mathbf{1}}(1,d^{24}_{\rho_z},d^{24}_{R_z},d^{24}_{\tilde{R}_z}) + \tfrac{1}{32}\chi^{T_2}_{\rho_x}(1,d^{24}_{\rho_z},d^{24}_{R_z},d^{24}_{\tilde{R}_z}) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{r}_1}(1,d^{24}_{\rho_z},d^{24}_{R_z},d^{24}_{\tilde{R}_z}) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{r}_2}(1,d^{24}_{\rho_z},d^{24}_{R_z},d^{24}_{\tilde{R}_z}) + \tfrac{1}{32}\chi^{T_2}_{\tilde{\mathbf{r}}_1}(1,d^{24}_{\rho_z},d^{24}_{R_z},d^{24}_{\tilde{R}_z}) + \tfrac{1}{32}\chi^{T_2}_{\tilde{\mathbf{r}}_2}(1,d^{24}_{\rho_z},d^{24}_{R_z},d^{24}_{\tilde{R}_z})$$

$$+ \tfrac{1}{32}\chi^{T_2}_{\rho_z}(d^{24}_{\rho_z},1,d^{24}_{\tilde{R}_z},d^{24}_{R_z}) + \tfrac{1}{32}\chi^{T_2}_{\rho_y}(d^{24}_{\rho_z},1,d^{24}_{\tilde{R}_z},d^{24}_{R_z}) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{r}_4}(d^{24}_{\rho_z},1,d^{24}_{\tilde{R}_z},d^{24}_{R_z}) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{r}_3}(d^{24}_{\rho_z},1,d^{24}_{\tilde{R}_z},d^{24}_{R_z}) + \tfrac{1}{32}\chi^{T_2}_{\tilde{\mathbf{r}}_3}(d^{24}_{\rho_z},1,d^{24}_{\tilde{R}_z},d^{24}_{R_z}) + \tfrac{1}{32}\chi^{T_2}_{\tilde{\mathbf{r}}_4}(d^{24}_{\rho_z},1,d^{24}_{\tilde{R}_z},d^{24}_{R_z})$$

$$+ \tfrac{1}{32}\chi^{T_2}_{\mathbf{R}_z}(d^{24}_{\tilde{R}_z},d^{24}_{R_z},1,d^{24}_{\rho_z}) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{i}_4}(d^{24}_{\tilde{R}_z},d^{24}_{R_z},1,d^{24}_{\rho_z}) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{i}_1}(d^{24}_{\tilde{R}_z},d^{24}_{R_z},1,d^{24}_{\rho_z}) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{i}_2}(d^{24}_{\tilde{R}_z},d^{24}_{R_z},1,d^{24}_{\rho_z}) + \tfrac{1}{32}\chi^{T_2}_{\tilde{\mathbf{R}}_x}(d^{24}_{\tilde{R}_z},d^{24}_{R_z},1,d^{24}_{\rho_z}) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{R}_x}(d^{24}_{\tilde{R}_z},d^{24}_{R_z},1,d^{24}_{\rho_z})$$

$$+ \tfrac{1}{32}\chi^{T_2}_{\tilde{\mathbf{R}}_z}(d^{24}_{R_z},d^{24}_{\tilde{R}_z},d^{24}_{\rho_z},1) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{i}_3}(d^{24}_{R_z},d^{24}_{\tilde{R}_z},d^{24}_{\rho_z},1) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{R}_y}(d^{24}_{R_z},d^{24}_{\tilde{R}_z},d^{24}_{\rho_z},1) + \tfrac{1}{32}\chi^{T_2}_{\tilde{\mathbf{R}}_y}(d^{24}_{R_z},d^{24}_{\tilde{R}_z},d^{24}_{\rho_z},1) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{i}_6}(d^{24}_{R_z},d^{24}_{\tilde{R}_z},d^{24}_{\rho_z},1)) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{i}_5}(d^{24}_{R_z},d^{24}_{\tilde{R}_z},d^{24}_{\rho_z},1)$$

$$= \tfrac{1}{32}(+3)(1,+1,-1,-1) + \tfrac{1}{32}(-1)(1,+1,-1,-1) + \tfrac{1}{32}(0)(1,+1,-1,-1) + \tfrac{1}{32}(0)(1,+1,-1,-1) + \tfrac{1}{32}(0)(1,+1,-1,-1) + \tfrac{1}{32}(0)(1,+1,-1,-1)$$

$$+ \tfrac{1}{32}(-1)(+1,1,-1,-1) + \tfrac{1}{32}(-1)(+1,1,-1,-1) + \tfrac{1}{32}(0)(+1,1,-1,-1) + \tfrac{1}{32}(0)(+1,1,-1,-1) + \tfrac{1}{32}(0)(+1,1,-1,-1) + \tfrac{1}{32}(0)(+1,1,-1,-1)$$

$$+ \tfrac{1}{32}(-1)(-1,-1,1,+1) + \tfrac{1}{32}(+1)(-1,-1,1,+1) + \tfrac{1}{32}(+1)(-1,-1,1,+1) + \tfrac{1}{32}(+1)(-1,-1,1,+1) + \tfrac{1}{32}(-1)(-1,-1,1,+1) + \tfrac{1}{32}(-1)(-1,-1,1,+1)$$

$$+ \tfrac{1}{32}(-1)(-1,-1,+1,1) + \tfrac{1}{32}(+1)(-1,-1,+1,1) + \tfrac{1}{32}(-1)(-1,-1,+1,1) + \tfrac{1}{32}(-1)(-1,-1,+1,1) + \tfrac{1}{32}(+1)(-1,-1,+1,1) + \tfrac{1}{32}(+1)(-1,-1,+1,1)$$

4, 4, -4, -4, *-4, -4, 4, 4,* *0, 0, 0, 0,* *0, 0, 0, 0,* *0, 0, 0, 0,* *0, 0, 0, 0,*

$$\tfrac{1}{8}(\underline{1}\mathbf{1}+\underline{1}\rho_z-\underline{1}\mathbf{R}_z-\underline{1}\tilde{\mathbf{R}}_z \;\underline{-1}\rho_x\underline{-1}\rho_y+\underline{1}\mathbf{i}_4+\underline{1}\mathbf{i}_3 \;+\underline{0}\mathbf{r}_1+\underline{0}\mathbf{r}_4+\underline{0}\mathbf{i}_1+\underline{0}\mathbf{R}_y \;+\underline{0}\mathbf{r}_2+\underline{0}\mathbf{r}_3+\underline{0}\mathbf{i}_2+\underline{0}\tilde{\mathbf{R}}_y \;+\underline{0}\tilde{\mathbf{r}}_1+\underline{0}\tilde{\mathbf{r}}_3+\underline{0}\tilde{\mathbf{R}}_x+\underline{0}\mathbf{i}_6 \;+\underline{0}\tilde{\mathbf{r}}_2+\underline{0}\tilde{\mathbf{r}}_4+\underline{0}\mathbf{R}_x+\underline{0}\mathbf{i}_5)$$

Calculating $\mathbf{P}^{T_2}{}_{1_4 1_4}$

$O \supset C_4$	0_4	1_4	2_4	3_4
$A_1 \downarrow C_4$	1	·	·	·
$A_2 \downarrow C_4$	·	·	1	·
$E \downarrow C_4$	1	·	1	·
$T_1 \downarrow C_4$	1	1	·	1
$T_2 \downarrow C_4$	·	1	1	1

$O: \chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}=\mathbf{1}$	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	ρ_{xyz}	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu = A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$$\mathbf{p}_{m_4} = \sum_{p=0}^{3} \frac{e^{2\pi i m \cdot p/4}}{4} \mathbf{R}_z^p = \begin{cases} \mathbf{p}_{0_4} = (\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{1_4} = (\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{2_4} = (\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{3_4} = (\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4 \end{cases}$$

$$\mathbf{P}^{T_2}_{1_4 1_4} = \mathbf{p}_{1_4}\mathbf{P}^{T_2} = \mathbf{P}^{T_2}\mathbf{p}_{1_4}$$

$$= \sum_{g}^{\circ O} \frac{\ell^{T_2}}{\circ O} (\chi_g^{T_2}) \cdot \mathbf{g} \cdot (\mathbf{p}_{1_4}) = \sum_{g}^{\circ O} \frac{3}{96} (\chi_g^{T_2}) \cdot \mathbf{g} \cdot (1 \cdot \mathbf{1} - 1 \cdot \rho_z + i \cdot \mathbf{R}_z - i \cdot \tilde{\mathbf{R}}_z)$$

$$\mathbf{1}C_4 = \mathbf{1}\{\mathbf{1}, \rho_z, \mathbf{R}_z, \tilde{\mathbf{R}}_z\} \quad \rho_x C_4 = \{\rho_x, \rho_y, \mathbf{i}_4, \mathbf{i}_3\} \quad \mathbf{r}_1 C_4 = \{\mathbf{r}_1, \mathbf{r}_4, \mathbf{i}_1, \mathbf{R}_y\} \quad \mathbf{r}_2 C_4 = \{\mathbf{r}_2, \mathbf{r}_3, \mathbf{i}_2, \tilde{\mathbf{R}}_y\} \quad \tilde{\mathbf{r}}_1 C_4 = \{\tilde{\mathbf{r}}_1, \tilde{\mathbf{r}}_3, \tilde{\mathbf{R}}_x, \mathbf{i}_6\} \quad \tilde{\mathbf{r}}_2 C_4 = \{\tilde{\mathbf{r}}_2, \tilde{\mathbf{r}}_4, \mathbf{R}_x, \mathbf{i}_5\}$$

$$= \tfrac{1}{32}\chi^{T_2}_{\mathbf{1}}(1, d^{14}_{\rho_z}, d^{14}_{R_z}, d^{14}_{\tilde{R}_z}) + \tfrac{1}{32}\chi^{T_2}_{\rho_x}(1, d^{14}_{\rho_z}, d^{14}_{R_z}, d^{14}_{\tilde{R}_z}) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{r}_1}(1, d^{14}_{\rho_z}, d^{14}_{R_z}, d^{14}_{\tilde{R}_z}) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{r}_2}(1, d^{14}_{\rho_z}, d^{14}_{R_z}, d^{14}_{\tilde{R}_z}) + \tfrac{1}{32}\chi^{T_2}_{\tilde{\mathbf{r}}_1}(1, d^{14}_{\rho_z}, d^{14}_{R_z}, d^{14}_{\tilde{R}_z}) + \tfrac{1}{32}\chi^{T_2}_{\tilde{\mathbf{r}}_2}(1, d^{14}_{\rho_z}, d^{14}_{R_z}, d^{14}_{\tilde{R}_z})$$

$$+ \tfrac{1}{32}\chi^{T_2}_{\rho_z}(d^{14}_{\rho_z}, 1, d^{14}_{\tilde{R}_z}, d^{14}_{R_z}) + \tfrac{1}{32}\chi^{T_2}_{\rho_y}(d^{14}_{\rho_z}, 1, d^{14}_{\tilde{R}_z}, d^{14}_{R_z}) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{r}_4}(d^{14}_{\rho_z}, 1, d^{14}_{\tilde{R}_z}, d^{14}_{R_z}) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{r}_3}(d^{14}_{\rho_z}, 1, d^{14}_{\tilde{R}_z}, d^{14}_{R_z}) + \tfrac{1}{32}\chi^{T_2}_{\tilde{\mathbf{r}}_3}(d^{14}_{\rho_z}, 1, d^{14}_{\tilde{R}_z}, d^{14}_{R_z}) + \tfrac{1}{32}\chi^{T_2}_{\tilde{\mathbf{r}}_4}(d^{14}_{\rho_z}, 1, d^{14}_{\tilde{R}_z}, d^{14}_{R_z})$$

$$+ \tfrac{1}{32}\chi^{T_2}_{\mathbf{R}_z}(d^{14}_{\tilde{R}_z}, d^{14}_{R_z}, 1, d^{14}_{\rho_z}) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{i}_4}(d^{14}_{\tilde{R}_z}, d^{14}_{R_z}, 1, d^{14}_{\rho_z}) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{i}_1}(d^{14}_{\tilde{R}_z}, d^{14}_{R_z}, 1, d^{14}_{\rho_z}) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{i}_2}(d^{14}_{\tilde{R}_z}, d^{14}_{R_z}, 1, d^{14}_{\rho_z}) + \tfrac{1}{32}\chi^{T_2}_{\tilde{\mathbf{R}}_x}(d^{14}_{\tilde{R}_z}, d^{14}_{R_z}, 1, d^{14}_{\rho_z}) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{R}_x}(d^{14}_{\tilde{R}_z}, d^{14}_{R_z}, 1, d^{14}_{\rho_z})$$

$$+ \tfrac{1}{32}\chi^{T_2}_{\tilde{\mathbf{R}}_z}(d^{14}_{R_z}, d^{14}_{\tilde{R}_z}, d^{14}_{\rho_z}, 1) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{i}_3}(d^{14}_{R_z}, d^{14}_{\tilde{R}_z}, d^{14}_{\rho_z}, 1) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{R}_y}(d^{14}_{R_z}, d^{14}_{\tilde{R}_z}, d^{14}_{\rho_z}, 1) + \tfrac{1}{32}\chi^{T_2}_{\tilde{\mathbf{R}}_y}(d^{14}_{R_z}, d^{14}_{\tilde{R}_z}, d^{14}_{\rho_z}, 1) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{i}_6}(d^{14}_{R_z}, d^{14}_{\tilde{R}_z}, d^{14}_{\rho_z}, 1) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{i}_5}(d^{14}_{R_z}, d^{14}_{\tilde{R}_z}, d^{14}_{\rho_z}, 1)$$

$$= \tfrac{1}{32}(+3)(1, -1, +i, -i) + \tfrac{1}{32}(-1)(1, -1, +i, -i) + \tfrac{1}{32}(0)(1, -1, +i, -i) + \tfrac{1}{32}(0)(1, -1, +i, -i) + \tfrac{1}{32}(0)(1, -1, +i, -i) + \tfrac{1}{32}(0)(1, -1, +i, -i)$$

$$+ \tfrac{1}{32}(-1)(-1, 1, -i, +i) + \tfrac{1}{32}(-1)(-1, 1, -i, +i) + \tfrac{1}{32}(0)(-1, 1, -i, +i) + \tfrac{1}{32}(0)(-1, 1, -i, +i) + \tfrac{1}{32}(0)(-1, 1, -i, +i) + \tfrac{1}{32}(0)(-1, 1, -i, +i)$$

$$+ \tfrac{1}{32}(-1)(-i, +i, 1, -1) + \tfrac{1}{32}(+1)(-i, +i, 1, -1) + \tfrac{1}{32}(+1)(-i, +i, 1, -1) + \tfrac{1}{32}(+1)(-i, +i, 1, -1) + \tfrac{1}{32}(-1)(-i, +i, 1, -1) + \tfrac{1}{32}(-1)(-i, +i, 1, -1)$$

$$+ \tfrac{1}{32}(-1)(+i, -i, -1, 1) + \tfrac{1}{32}(+1)(+i, -i, -1, 1) + \tfrac{1}{32}(-1)(+i, -i, -1, 1) + \tfrac{1}{32}(-1)(+i, -i, -1, 1) + \tfrac{1}{32}(+1)(+i, -i, -1, 1) + \tfrac{1}{32}(+1)(+i, -i, -1, 1)$$

+4,-4, 4i,-4i,   *0,0,0,0,*   *-2i,2i,2,-2,*   *-2i,2i,2,-2,*   *2i,-2i,-2,2,*   *2i,-2i,-2,2.*

$$\tfrac{1}{8}(1\mathbf{1} - 1\rho_z + i\mathbf{R}_z - i\tilde{\mathbf{R}}_z + 0\rho_x + 0\rho_y + 0\mathbf{i}_4 + 0\mathbf{i}_3 - \tfrac{i}{2}\mathbf{r}_1 + \tfrac{i}{2}\mathbf{r}_4 + \tfrac{1}{2}\mathbf{i}_1 - \tfrac{1}{2}\mathbf{R}_y - \tfrac{i}{2}\mathbf{r}_2 + \tfrac{i}{2}\mathbf{r}_3 + \tfrac{1}{2}\mathbf{i}_2 - \tfrac{1}{2}\tilde{\mathbf{R}}_y + \tfrac{i}{2}\tilde{\mathbf{r}}_1 - \tfrac{i}{2}\tilde{\mathbf{r}}_3 - \tfrac{1}{2}\tilde{\mathbf{R}}_x + \tfrac{1}{2}\mathbf{i}_6 + \tfrac{i}{2}\tilde{\mathbf{r}}_2 - \tfrac{i}{2}\tilde{\mathbf{r}}_4 - \tfrac{1}{2}\mathbf{R}_x + \tfrac{1}{2}\mathbf{i}_5$$

Review Octahedral $O \supset D_4 \supset C_4$ *subgroup chain and coset bases*

Coset factored splitting of $O \supset D_4 \supset C_4$ *projectors and levels*

Coset spaces based on $m_4(C_4)\uparrow O$

Splitting class projectors into C_4 *cosets and* $m_4(C_4)\uparrow O$ *bases*

General development of irep projectors $\mathbf{P}^{\mu}_{m4m4}$

Calculating $\mathbf{P}^{E}_{0_4 0_4}$

Calculating $\mathbf{P}^{E}_{2_4 2_4}$

Calculating $\mathbf{P}^{T_1}_{0_4 0_4}$

Calculating $\mathbf{P}^{T_1}_{1_4 1_4}$

Calculating $\mathbf{P}^{T_2}_{2_4 2_4}$

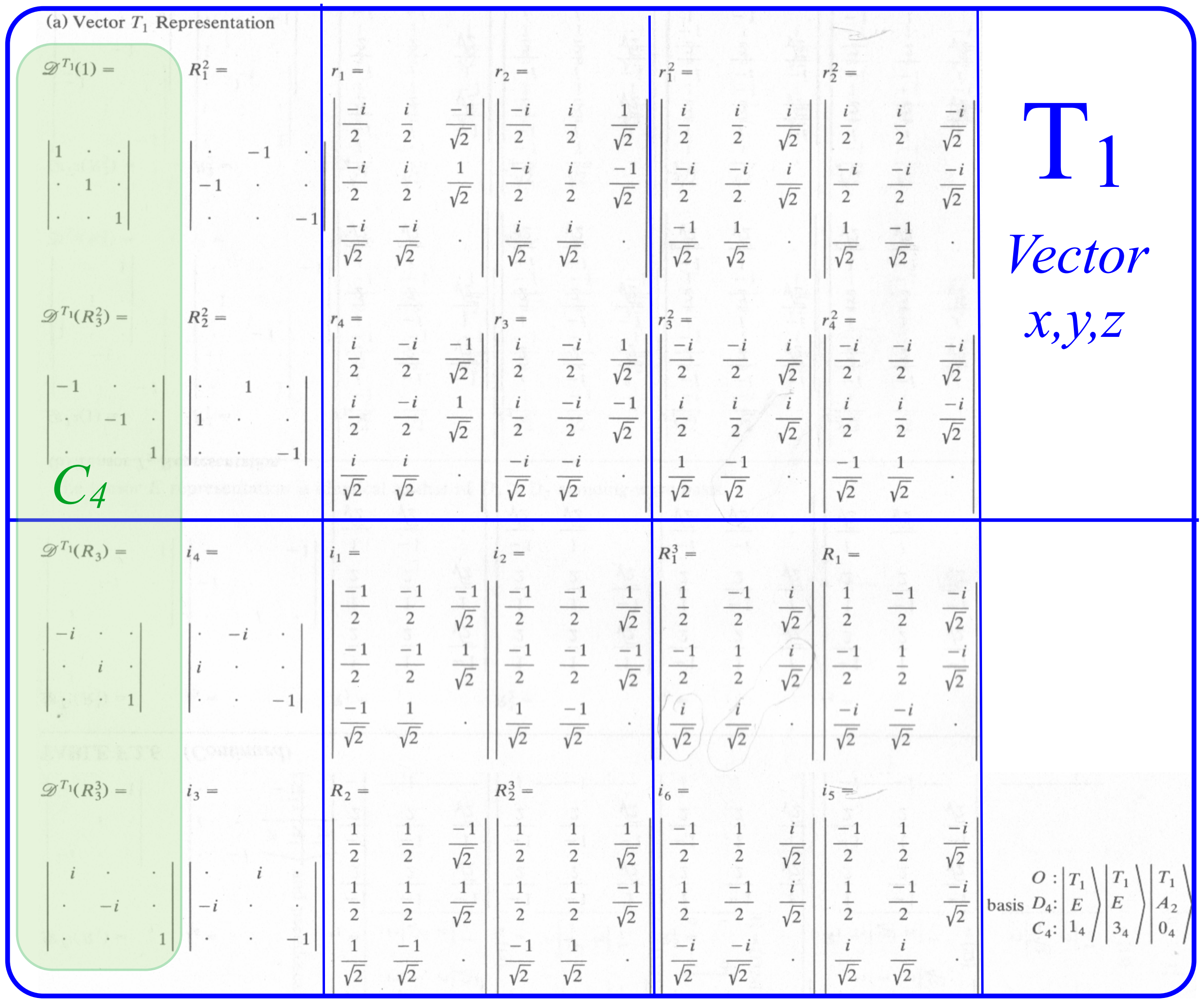
Structure and applications of various subgroup chain ireps

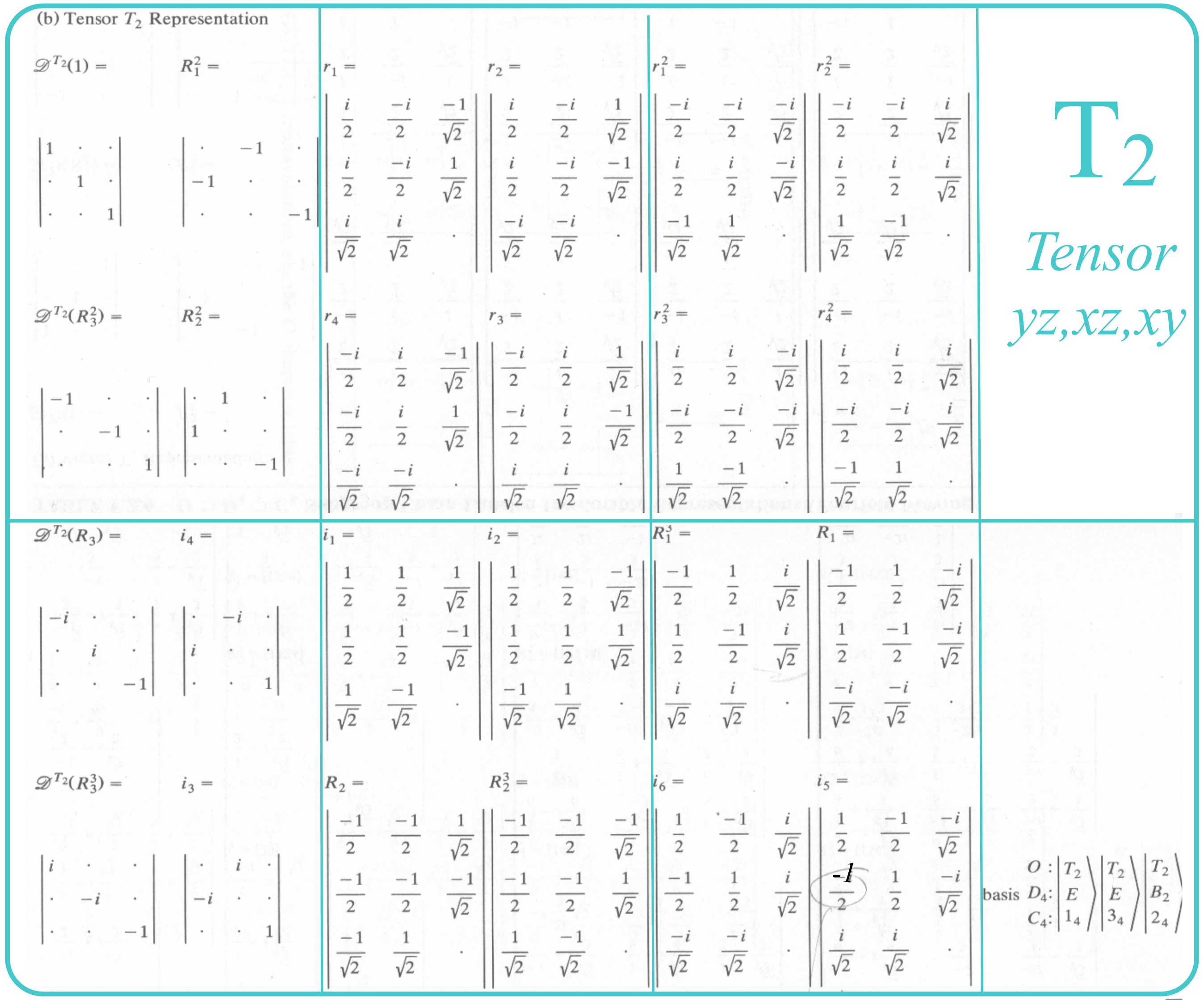
$O_h \supset D_{4h} \supset C_{4v}$

$O_h \supset D_{3h} \supset C_{3v}$

$O_h \supset C_{2v}$

Ireps for O⊃D_4⊃C_4 subgroup chain





E

Tensor

$x^2+y^2-2z^2$

$(x^2-y^2)\sqrt{3}$

$1=[1][2][3][4]$: $\begin{pmatrix}1&0\\0&1\end{pmatrix}$ $R_1^2=[13][24]$: $\begin{pmatrix}1&0\\0&1\end{pmatrix}$ $r_1=[132]$: $\begin{pmatrix}-\frac{1}{2}&-\frac{\sqrt{3}}{2}\\+\frac{\sqrt{3}}{2}&-\frac{1}{2}\end{pmatrix}$ $r_2=[124]$: $\begin{pmatrix}-\frac{1}{2}&-\frac{\sqrt{3}}{2}\\+\frac{\sqrt{3}}{2}&-\frac{1}{2}\end{pmatrix}$ $r_1^2=[123]$: $\begin{pmatrix}-\frac{1}{2}&+\frac{\sqrt{3}}{2}\\-\frac{\sqrt{3}}{2}&-\frac{1}{2}\end{pmatrix}$ $r_2^2=[142]$: $\begin{pmatrix}-\frac{1}{2}&+\frac{\sqrt{3}}{2}\\-\frac{\sqrt{3}}{2}&-\frac{1}{2}\end{pmatrix}$

$R_3^2=[12][34]$: $\begin{pmatrix}1&0\\0&1\end{pmatrix}$ $R_2^2=[14][23]$: $\begin{pmatrix}1&0\\0&1\end{pmatrix}$ $r_4=[234]$: $\begin{pmatrix}-\frac{1}{2}&-\frac{\sqrt{3}}{2}\\+\frac{\sqrt{3}}{2}&-\frac{1}{2}\end{pmatrix}$ $r_3=[124]$: $\begin{pmatrix}-\frac{1}{2}&-\frac{\sqrt{3}}{2}\\+\frac{\sqrt{3}}{2}&-\frac{1}{2}\end{pmatrix}$ $r_3^2=[134]$: $\begin{pmatrix}-\frac{1}{2}&+\frac{\sqrt{3}}{2}\\-\frac{\sqrt{3}}{2}&-\frac{1}{2}\end{pmatrix}$ $r_4^2=[243]$: $\begin{pmatrix}-\frac{1}{2}&+\frac{\sqrt{3}}{2}\\-\frac{\sqrt{3}}{2}&-\frac{1}{2}\end{pmatrix}$

$R_3=[1423]$: $\begin{pmatrix}1&0\\0&-1\end{pmatrix}$ $i_4=[12]$: $\begin{pmatrix}1&0\\0&-1\end{pmatrix}$ $i_1=[14]$: $\begin{pmatrix}-\frac{1}{2}&+\frac{\sqrt{3}}{2}\\+\frac{\sqrt{3}}{2}&+\frac{1}{2}\end{pmatrix}$ $i_2=[23]$: $\begin{pmatrix}-\frac{1}{2}&+\frac{\sqrt{3}}{2}\\+\frac{\sqrt{3}}{2}&+\frac{1}{2}\end{pmatrix}$ $R_1^3=[1432]$: $\begin{pmatrix}-\frac{1}{2}&-\frac{\sqrt{3}}{2}\\-\frac{\sqrt{3}}{2}&+\frac{1}{2}\end{pmatrix}$ $R_1=[1234]$: $\begin{pmatrix}-\frac{1}{2}&-\frac{\sqrt{3}}{2}\\-\frac{\sqrt{3}}{2}&+\frac{1}{2}\end{pmatrix}$

$R_3^3=[1324]$: $\begin{pmatrix}1&0\\0&-1\end{pmatrix}$ $i_3=[34]$: $\begin{pmatrix}1&0\\0&-1\end{pmatrix}$ $R_2=[1243]$: $\begin{pmatrix}-\frac{1}{2}&+\frac{\sqrt{3}}{2}\\+\frac{\sqrt{3}}{2}&+\frac{1}{2}\end{pmatrix}$ $R_2^3=[1342]$: $\begin{pmatrix}-\frac{1}{2}&+\frac{\sqrt{3}}{2}\\+\frac{\sqrt{3}}{2}&+\frac{1}{2}\end{pmatrix}$ $i_6=[24]$: $\begin{pmatrix}-\frac{1}{2}&-\frac{\sqrt{3}}{2}\\-\frac{\sqrt{3}}{2}&+\frac{1}{2}\end{pmatrix}$ $i_5=[13]$: $\begin{pmatrix}-\frac{1}{2}&-\frac{\sqrt{3}}{2}\\-\frac{\sqrt{3}}{2}&+\frac{1}{2}\end{pmatrix}$

basis: O: $|E\rangle$, $|E\rangle$; D_4: $|A_1\rangle$, $|B_1\rangle$; C_4: $|0_4\rangle$, $|2_4\rangle$

$O:\chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}$=1	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	ρ_{xyz}	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

Ireps for $O \supset D_4 \supset D_2$ *subgroup chain*

T_1 — *Vector* x,y,z

D_2:

$\mathscr{D}^{T_2}(1) = \begin{vmatrix} 1 & \cdot & \cdot \\ \cdot & 1 & \cdot \\ \cdot & \cdot & 1 \end{vmatrix}$ $\quad R_1^2 = \begin{vmatrix} 1 & \cdot & \cdot \\ \cdot & -1 & \cdot \\ \cdot & \cdot & -1 \end{vmatrix}$

$\mathscr{D}^{T_2}(R_3^2) = \begin{vmatrix} -1 & \cdot & \cdot \\ \cdot & -1 & \cdot \\ \cdot & \cdot & 1 \end{vmatrix}$ $\quad R_2^2 = \begin{vmatrix} -1 & \cdot & \cdot \\ \cdot & 1 & \cdot \\ \cdot & \cdot & -1 \end{vmatrix}$

$r_1 = \begin{vmatrix} \cdot & \cdot & 1 \\ -1 & \cdot & \cdot \\ \cdot & -1 & \cdot \end{vmatrix}$ $\quad r_2 = \begin{vmatrix} \cdot & \cdot & -1 \\ -1 & \cdot & \cdot \\ \cdot & 1 & \cdot \end{vmatrix}$ $\quad r_1^2 = \begin{vmatrix} \cdot & -1 & \cdot \\ \cdot & \cdot & -1 \\ 1 & \cdot & \cdot \end{vmatrix}$ $\quad r_2^2 = \begin{vmatrix} \cdot & -1 & \cdot \\ \cdot & \cdot & 1 \\ -1 & \cdot & \cdot \end{vmatrix}$

$r_4 = \begin{vmatrix} \cdot & \cdot & 1 \\ 1 & \cdot & \cdot \\ \cdot & 1 & \cdot \end{vmatrix}$ $\quad r_3 = \begin{vmatrix} \cdot & \cdot & -1 \\ 1 & \cdot & \cdot \\ \cdot & -1 & \cdot \end{vmatrix}$ $\quad r_3^2 = \begin{vmatrix} \cdot & 1 & \cdot \\ \cdot & \cdot & -1 \\ -1 & \cdot & \cdot \end{vmatrix}$ $\quad r_4^2 = \begin{vmatrix} \cdot & 1 & \cdot \\ \cdot & \cdot & 1 \\ 1 & \cdot & \cdot \end{vmatrix}$

$\mathscr{D}^{T_2}(R_3) = \begin{vmatrix} \cdot & -1 & \cdot \\ 1 & \cdot & \cdot \\ \cdot & \cdot & -1 \end{vmatrix}$ $\quad i_4 = \begin{vmatrix} \cdot & -1 & \cdot \\ -1 & \cdot & \cdot \\ \cdot & \cdot & 1 \end{vmatrix}$ $\quad i_1 = \begin{vmatrix} \cdot & \cdot & -1 \\ \cdot & 1 & \cdot \\ -1 & \cdot & \cdot \end{vmatrix}$ $\quad i_2 = \begin{vmatrix} \cdot & \cdot & 1 \\ \cdot & 1 & \cdot \\ 1 & \cdot & \cdot \end{vmatrix}$ $\quad R_1^3 = \begin{vmatrix} -1 & \cdot & \cdot \\ \cdot & \cdot & 1 \\ \cdot & -1 & \cdot \end{vmatrix}$ $\quad R_1 = \begin{vmatrix} -1 & \cdot & \cdot \\ \cdot & \cdot & -1 \\ \cdot & 1 & \cdot \end{vmatrix}$

$\mathscr{D}^{T_2}(R_3^3) = \begin{vmatrix} \cdot & 1 & \cdot \\ -1 & \cdot & \cdot \\ \cdot & \cdot & -1 \end{vmatrix}$ $\quad i_3 = \begin{vmatrix} \cdot & 1 & \cdot \\ 1 & \cdot & \cdot \\ \cdot & \cdot & 1 \end{vmatrix}$ $\quad R_2 = \begin{vmatrix} \cdot & \cdot & -1 \\ \cdot & -1 & \cdot \\ 1 & \cdot & \cdot \end{vmatrix}$ $\quad R_2^3 = \begin{vmatrix} \cdot & \cdot & 1 \\ \cdot & -1 & \cdot \\ -1 & \cdot & \cdot \end{vmatrix}$ $\quad i_6 = \begin{vmatrix} 1 & \cdot & \cdot \\ \cdot & \cdot & 1 \\ \cdot & 1 & \cdot \end{vmatrix}$ $\quad i_5 = \begin{vmatrix} 1 & \cdot & \cdot \\ \cdot & \cdot & -1 \\ \cdot & -1 & \cdot \end{vmatrix}$

basis:

O	T_1	T_1	T_1
D_4	E	E	A_2
D_2	B_1 $\rangle$	B_2 $\rangle$	A_2 $\rangle$

T_2 — *Tensor* yz,xz,xy

$\mathscr{D}^{T_2}(1) = \begin{vmatrix} 1 & \cdot & \cdot \\ \cdot & 1 & \cdot \\ \cdot & \cdot & 1 \end{vmatrix}$ $\quad R_1^2 = \begin{vmatrix} 1 & \cdot & \cdot \\ \cdot & -1 & \cdot \\ \cdot & \cdot & -1 \end{vmatrix}$

$\mathscr{D}^{T_2}(R_3^2) = \begin{vmatrix} -1 & \cdot & \cdot \\ \cdot & -1 & \cdot \\ \cdot & \cdot & 1 \end{vmatrix}$ $\quad R_2^2 = \begin{vmatrix} -1 & \cdot & \cdot \\ \cdot & 1 & \cdot \\ \cdot & \cdot & -1 \end{vmatrix}$

$r_1 = \begin{vmatrix} \cdot & \cdot & 1 \\ -1 & \cdot & \cdot \\ \cdot & -1 & \cdot \end{vmatrix}$ $\quad r_2 = \begin{vmatrix} \cdot & \cdot & -1 \\ -1 & \cdot & \cdot \\ \cdot & 1 & \cdot \end{vmatrix}$ $\quad r_1^2 = \begin{vmatrix} \cdot & -1 & \cdot \\ \cdot & \cdot & -1 \\ 1 & \cdot & \cdot \end{vmatrix}$ $\quad r_2^2 = \begin{vmatrix} \cdot & -1 & \cdot \\ \cdot & \cdot & 1 \\ -1 & \cdot & \cdot \end{vmatrix}$

$r_4 = \begin{vmatrix} \cdot & \cdot & 1 \\ 1 & \cdot & \cdot \\ \cdot & 1 & \cdot \end{vmatrix}$ $\quad r_3 = \begin{vmatrix} \cdot & \cdot & -1 \\ 1 & \cdot & \cdot \\ \cdot & -1 & \cdot \end{vmatrix}$ $\quad r_3^2 = \begin{vmatrix} \cdot & 1 & \cdot \\ \cdot & \cdot & -1 \\ -1 & \cdot & \cdot \end{vmatrix}$ $\quad r_4^2 = \begin{vmatrix} \cdot & 1 & \cdot \\ \cdot & \cdot & 1 \\ 1 & \cdot & \cdot \end{vmatrix}$

$\mathscr{D}^{T_2}(R_3) = \begin{vmatrix} \cdot & -1 & \cdot \\ 1 & \cdot & \cdot \\ \cdot & \cdot & -1 \end{vmatrix}$ $\quad i_4 = \begin{vmatrix} \cdot & -1 & \cdot \\ -1 & \cdot & \cdot \\ \cdot & \cdot & 1 \end{vmatrix}$ $\quad i_1 = \begin{vmatrix} \cdot & \cdot & -1 \\ \cdot & 1 & \cdot \\ -1 & \cdot & \cdot \end{vmatrix}$ $\quad i_2 = \begin{vmatrix} \cdot & \cdot & 1 \\ \cdot & 1 & \cdot \\ 1 & \cdot & \cdot \end{vmatrix}$ $\quad R_1^3 = \begin{vmatrix} -1 & \cdot & \cdot \\ \cdot & \cdot & 1 \\ \cdot & -1 & \cdot \end{vmatrix}$ $\quad R_1 = \begin{vmatrix} -1 & \cdot & \cdot \\ \cdot & \cdot & -1 \\ \cdot & 1 & \cdot \end{vmatrix}$

$\mathscr{D}^{T_2}(R_3^3) = \begin{vmatrix} \cdot & 1 & \cdot \\ -1 & \cdot & \cdot \\ \cdot & \cdot & -1 \end{vmatrix}$ $\quad i_3 = \begin{vmatrix} \cdot & 1 & \cdot \\ 1 & \cdot & \cdot \\ \cdot & \cdot & 1 \end{vmatrix}$ $\quad R_2 = \begin{vmatrix} \cdot & \cdot & -1 \\ \cdot & -1 & \cdot \\ 1 & \cdot & \cdot \end{vmatrix}$ $\quad R_2^3 = \begin{vmatrix} \cdot & \cdot & 1 \\ \cdot & -1 & \cdot \\ -1 & \cdot & \cdot \end{vmatrix}$ $\quad i_6 = \begin{vmatrix} 1 & \cdot & \cdot \\ \cdot & \cdot & 1 \\ \cdot & 1 & \cdot \end{vmatrix}$ $\quad i_5 = \begin{vmatrix} 1 & \cdot & \cdot \\ \cdot & \cdot & -1 \\ \cdot & -1 & \cdot \end{vmatrix}$

basis:

O	T_2	T_2	T_2
D_4	E	E	B_2
D_2	B_1 $\rangle$	B_2 $\rangle$	A_2 $\rangle$

E — *Tensor* $x^2+y^2-2z^2$, $(x^2-y^2)\sqrt{3}$

$\mathscr{D}^{E}(1) = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ $\quad R_1^2 = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ $\quad \mathscr{D}^{E}(R_3^2) = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ $\quad R_2^2 = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

$r_1 = r_2 = r_4 = r_3 = \begin{vmatrix} \frac{-1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{-1}{2} \end{vmatrix}$

$r_1^2 = r_2^2 = r_3^2 = r_4^2 = \begin{vmatrix} \frac{-1}{2} & \frac{\sqrt{3}}{2} \\ \frac{-\sqrt{3}}{2} & \frac{-1}{2} \end{vmatrix}$

$\mathscr{D}^{E}(R_3) = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$ $\quad i_4 = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$ $\quad \mathscr{D}^{E}(R_3^3) = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$ $\quad i_3 = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$

$i_1 = i_2 = R_2 = R_2^3 = \begin{vmatrix} \frac{-1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{vmatrix}$

$R_1^3 = R_1 = i_6 = i_5 = \begin{vmatrix} \frac{-1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{-\sqrt{3}}{2} & \frac{1}{2} \end{vmatrix}$

basis:

O	E	E
D_4	A_1	B_1
D_2	A_1 $\rangle$	A_1 $\rangle$

O: $\chi_{\mathbf{g}}^{\mu}$	$\mathbf{g}=\mathbf{1}$	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	$\boldsymbol{\rho}_{xyz}$	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

Ireps for $O \supset D_3 \supset C_2$ subgroup chain

T_1 — Vector u, v, w

C_2:

$\mathscr{D}^{T_1}(1) = \begin{vmatrix} 1 & \cdot & \cdot \\ \cdot & 1 & \cdot \\ \cdot & \cdot & 1 \end{vmatrix}$ $\quad i_4 = [12]$: $\begin{vmatrix} 1 & \cdot & \cdot \\ \cdot & -1 & \cdot \\ \cdot & \cdot & -1 \end{vmatrix}$

$R_1^2 = [13][24]$: $\begin{vmatrix} \cdot & \frac{\sqrt3}{3} & \frac{\sqrt6}{3} \\ \frac{\sqrt3}{3} & \frac{-2}{3} & \frac{\sqrt2}{3} \\ \frac{\sqrt6}{3} & \frac{\sqrt2}{3} & \frac{-1}{3} \end{vmatrix}$ $\quad R_3 = [1423]$: $\begin{vmatrix} \cdot & -\frac{\sqrt3}{3} & -\frac{\sqrt6}{3} \\ \frac{\sqrt3}{3} & \frac{2}{3} & -\frac{\sqrt2}{3} \\ \frac{\sqrt6}{3} & -\frac{\sqrt2}{3} & \frac{1}{3} \end{vmatrix}$

$r_1 = [132]$: $\begin{vmatrix} \frac{-1}{2} & -\frac{\sqrt3}{2} & \cdot \\ \frac{\sqrt3}{2} & \frac{-1}{2} & \cdot \\ \cdot & \cdot & 1 \end{vmatrix}$ $\quad i_5 = [13]$: $\begin{vmatrix} \frac{-1}{2} & -\frac{\sqrt3}{2} & \cdot \\ -\frac{\sqrt3}{2} & \frac{1}{2} & \cdot \\ \cdot & \cdot & -1 \end{vmatrix}$

$r_4 = [234]$: $\begin{vmatrix} \frac{1}{2} & -\frac{\sqrt3}{6} & \frac{\sqrt6}{3} \\ -\frac{\sqrt3}{2} & \frac{-1}{6} & \frac{\sqrt2}{3} \\ \cdot & -\frac{\sqrt8}{3} & \frac{-1}{3} \end{vmatrix}$ $\quad i_6 = [24]$: $\begin{vmatrix} \frac{-1}{2} & \frac{\sqrt3}{6} & -\frac{\sqrt6}{3} \\ \frac{\sqrt3}{6} & \frac{-5}{6} & -\frac{\sqrt2}{3} \\ -\frac{\sqrt6}{3} & -\frac{\sqrt2}{3} & \frac{1}{3} \end{vmatrix}$

$r_1^2 = [123]$: $\begin{vmatrix} \frac{-1}{2} & \frac{\sqrt3}{2} & \cdot \\ -\frac{\sqrt3}{2} & \frac{-1}{2} & \cdot \\ \cdot & \cdot & 1 \end{vmatrix}$ $\quad i_2 = [23]$: $\begin{vmatrix} \frac{-1}{2} & \frac{\sqrt3}{2} & \cdot \\ \frac{\sqrt3}{2} & \frac{1}{2} & \cdot \\ \cdot & \cdot & -1 \end{vmatrix}$

$r_2^2 = [142]$: $\begin{vmatrix} \frac{-1}{2} & -\frac{\sqrt3}{6} & \frac{\sqrt6}{3} \\ \frac{\sqrt3}{6} & \frac{5}{6} & \frac{\sqrt2}{3} \\ -\frac{\sqrt6}{3} & \frac{\sqrt2}{3} & \frac{-1}{3} \end{vmatrix}$ $\quad R_2^3 = [1342]$: $\begin{vmatrix} \frac{1}{2} & \frac{\sqrt3}{6} & -\frac{\sqrt6}{3} \\ -\frac{\sqrt3}{2} & \frac{1}{6} & -\frac{\sqrt2}{3} \\ \cdot & \frac{\sqrt8}{3} & \frac{1}{3} \end{vmatrix}$

$R_2^2 = [14][23]$: $\begin{vmatrix} \cdot & -\frac{\sqrt3}{3} & -\frac{\sqrt6}{3} \\ -\frac{\sqrt3}{3} & \frac{-2}{3} & \frac{\sqrt2}{3} \\ -\frac{\sqrt6}{3} & \frac{\sqrt2}{3} & \frac{-1}{3} \end{vmatrix}$ $\quad R_3^3 = [1324]$: $\begin{vmatrix} \cdot & \frac{\sqrt3}{3} & \frac{\sqrt6}{3} \\ -\frac{\sqrt3}{3} & \frac{2}{3} & -\frac{\sqrt2}{3} \\ -\frac{\sqrt6}{3} & -\frac{\sqrt2}{3} & \frac{1}{3} \end{vmatrix}$

$R_3^2 = [12][34]$: $\begin{vmatrix} -1 & \cdot & \cdot \\ \cdot & \frac{1}{3} & -\frac{\sqrt8}{3} \\ \cdot & -\frac{\sqrt8}{3} & \frac{-1}{3} \end{vmatrix}$ $\quad i_3 = [34]$: $\begin{vmatrix} -1 & \cdot & \cdot \\ \cdot & \frac{-1}{3} & \frac{\sqrt8}{3} \\ \cdot & \frac{\sqrt8}{3} & \frac{1}{3} \end{vmatrix}$

$r_2 = [124]$: $\begin{vmatrix} \frac{-1}{2} & \frac{\sqrt3}{6} & -\frac{\sqrt6}{3} \\ -\frac{\sqrt3}{6} & \frac{5}{6} & \frac{\sqrt2}{3} \\ \frac{\sqrt6}{3} & \frac{\sqrt2}{3} & \frac{-1}{3} \end{vmatrix}$ $\quad R_1 = [1234]$: $\begin{vmatrix} \frac{1}{2} & -\frac{\sqrt3}{6} & \frac{\sqrt6}{3} \\ \frac{\sqrt3}{2} & \frac{1}{6} & -\frac{\sqrt2}{3} \\ \cdot & \frac{\sqrt8}{3} & \frac{1}{3} \end{vmatrix}$

$r_3 = [143]$: $\begin{vmatrix} \frac{1}{2} & \frac{\sqrt3}{2} & \cdot \\ \frac{\sqrt3}{6} & \frac{-1}{6} & -\frac{\sqrt8}{3} \\ -\frac{\sqrt6}{3} & \frac{\sqrt2}{3} & \frac{-1}{3} \end{vmatrix}$ $\quad R_1^3 = [1432]$: $\begin{vmatrix} \frac{1}{2} & \frac{\sqrt3}{2} & \cdot \\ -\frac{\sqrt3}{6} & \frac{1}{6} & \frac{\sqrt8}{3} \\ \frac{\sqrt6}{3} & -\frac{\sqrt2}{3} & \frac{1}{3} \end{vmatrix}$

$r_3^2 = [134]$: $\begin{vmatrix} \frac{1}{2} & \frac{\sqrt3}{6} & -\frac{\sqrt6}{3} \\ \frac{\sqrt3}{2} & \frac{-1}{6} & \frac{\sqrt2}{3} \\ \cdot & -\frac{\sqrt8}{3} & \frac{-1}{3} \end{vmatrix}$ $\quad i_1 = [14]$: $\begin{vmatrix} \frac{-1}{2} & -\frac{\sqrt3}{6} & \frac{\sqrt6}{3} \\ -\frac{\sqrt3}{6} & \frac{-5}{6} & -\frac{\sqrt2}{3} \\ \frac{\sqrt6}{3} & -\frac{\sqrt2}{3} & \frac{1}{3} \end{vmatrix}$

$r_4^2 = [243]$: $\begin{vmatrix} \frac{1}{2} & -\frac{\sqrt3}{2} & \cdot \\ -\frac{\sqrt3}{6} & \frac{-1}{6} & -\frac{\sqrt8}{3} \\ \frac{\sqrt6}{3} & \frac{\sqrt2}{3} & \frac{-1}{3} \end{vmatrix}$ $\quad R_2 = [1243]$: $\begin{vmatrix} \frac{1}{2} & -\frac{\sqrt3}{2} & \cdot \\ \frac{\sqrt3}{6} & \frac{1}{6} & \frac{\sqrt8}{3} \\ -\frac{\sqrt6}{3} & -\frac{\sqrt2}{3} & \frac{1}{3} \end{vmatrix}$

basis: $\begin{matrix} O \\ D_3 \\ C_2 \end{matrix} \left| \begin{matrix} T_1 \\ E \\ 0_2 \end{matrix} \right\rangle \left| \begin{matrix} T_1 \\ E \\ 1_2 \end{matrix} \right\rangle \left| \begin{matrix} T_1 \\ A_2 \\ 1_2 \end{matrix} \right\rangle$

T_2 — Tensor vw, uw, uv

$\mathscr{D}^{T_2}(1) = \begin{vmatrix} 1 & \cdot & \cdot \\ \cdot & 1 & \cdot \\ \cdot & \cdot & 1 \end{vmatrix}$ $\quad i_4 = [12]$: $\begin{vmatrix} 1 & \cdot & \cdot \\ \cdot & 1 & \cdot \\ \cdot & \cdot & -1 \end{vmatrix}$

$R_1^2 = [13][24]$: $\begin{vmatrix} \frac{-1}{3} & -\frac{\sqrt2}{3} & \frac{\sqrt6}{3} \\ -\frac{\sqrt2}{3} & \frac{-2}{3} & -\frac{\sqrt3}{3} \\ \frac{\sqrt6}{3} & -\frac{\sqrt3}{3} & \cdot \end{vmatrix}$ $\quad R_3 = [1423]$: $\begin{vmatrix} \frac{-1}{3} & -\frac{\sqrt2}{3} & -\frac{\sqrt6}{3} \\ -\frac{\sqrt2}{3} & \frac{-2}{3} & \frac{\sqrt3}{3} \\ \frac{\sqrt6}{3} & -\frac{\sqrt3}{3} & \cdot \end{vmatrix}$

$r_1 = [132]$: $\begin{vmatrix} 1 & \cdot & \cdot \\ \cdot & \frac{-1}{2} & -\frac{\sqrt3}{2} \\ \cdot & \frac{\sqrt3}{2} & \frac{-1}{2} \end{vmatrix}$ $\quad i_5 = [13]$: $\begin{vmatrix} 1 & \cdot & \cdot \\ \cdot & \frac{-1}{2} & -\frac{\sqrt3}{2} \\ \cdot & -\frac{\sqrt3}{2} & \frac{1}{2} \end{vmatrix}$

$r_4 = [234]$: $\begin{vmatrix} \frac{-1}{3} & \frac{\sqrt8}{3} & \cdot \\ -\frac{\sqrt2}{3} & \frac{-1}{6} & \frac{\sqrt3}{2} \\ \frac{\sqrt6}{3} & \frac{\sqrt3}{6} & \frac{1}{2} \end{vmatrix}$ $\quad i_6 = [24]$: $\begin{vmatrix} \frac{-1}{3} & -\frac{\sqrt2}{3} & \frac{\sqrt6}{3} \\ -\frac{\sqrt2}{3} & \frac{5}{6} & \frac{\sqrt3}{6} \\ \frac{\sqrt6}{3} & \frac{\sqrt3}{6} & \frac{1}{2} \end{vmatrix}$

$r_1^2 = [123]$: $\begin{vmatrix} 1 & \cdot & \cdot \\ \cdot & \frac{-1}{2} & \frac{\sqrt3}{2} \\ \cdot & -\frac{\sqrt3}{2} & \frac{-1}{2} \end{vmatrix}$ $\quad i_2 = [23]$: $\begin{vmatrix} 1 & \cdot & \cdot \\ \cdot & \frac{-1}{2} & \frac{\sqrt3}{2} \\ \cdot & \frac{\sqrt3}{2} & \frac{1}{2} \end{vmatrix}$

$r_2^2 = [142]$: $\begin{vmatrix} \frac{-1}{3} & -\frac{\sqrt2}{3} & -\frac{\sqrt6}{3} \\ -\frac{\sqrt2}{3} & \frac{5}{6} & -\frac{\sqrt3}{6} \\ \frac{\sqrt6}{3} & \frac{\sqrt3}{6} & \frac{-1}{2} \end{vmatrix}$ $\quad R_2^3 = [1342]$: $\begin{vmatrix} \frac{-1}{3} & \frac{\sqrt8}{3} & \cdot \\ -\frac{\sqrt2}{3} & \frac{-1}{6} & -\frac{\sqrt3}{2} \\ \frac{\sqrt6}{3} & \frac{\sqrt3}{6} & \frac{-1}{2} \end{vmatrix}$

$R_2^2 = [14][23]$: $\begin{vmatrix} \frac{-1}{3} & -\frac{\sqrt2}{3} & -\frac{\sqrt6}{3} \\ -\frac{\sqrt2}{3} & \frac{-2}{3} & \frac{\sqrt3}{3} \\ -\frac{\sqrt6}{3} & \frac{\sqrt3}{3} & \cdot \end{vmatrix}$ $\quad R_3^3 = [1324]$: $\begin{vmatrix} \frac{-1}{3} & -\frac{\sqrt2}{3} & \frac{\sqrt6}{3} \\ -\frac{\sqrt2}{3} & \frac{-2}{3} & \frac{\sqrt3}{3} \\ -\frac{\sqrt6}{3} & \frac{\sqrt3}{3} & \cdot \end{vmatrix}$

$R_3^2 = [12][34]$: $\begin{vmatrix} \frac{-1}{3} & \frac{\sqrt8}{3} & \cdot \\ \frac{\sqrt8}{3} & \frac{1}{3} & \cdot \\ \cdot & \cdot & -1 \end{vmatrix}$ $\quad i_3 = [34]$: $\begin{vmatrix} \frac{-1}{3} & \frac{\sqrt8}{3} & \cdot \\ \frac{\sqrt8}{3} & \frac{1}{3} & \cdot \\ \cdot & \cdot & 1 \end{vmatrix}$

$r_2 = [124]$: $\begin{vmatrix} \frac{-1}{3} & -\frac{\sqrt2}{3} & \frac{\sqrt6}{3} \\ -\frac{\sqrt2}{3} & \frac{5}{6} & \frac{\sqrt3}{6} \\ -\frac{\sqrt6}{3} & -\frac{\sqrt3}{6} & \frac{-1}{2} \end{vmatrix}$ $\quad R_1 = [1234]$: $\begin{vmatrix} \frac{-1}{3} & \frac{\sqrt8}{3} & \cdot \\ -\frac{\sqrt2}{3} & \frac{-1}{6} & \frac{\sqrt3}{2} \\ -\frac{\sqrt6}{3} & -\frac{\sqrt3}{6} & \frac{-1}{2} \end{vmatrix}$

$r_3 = [143]$: $\begin{vmatrix} \frac{-1}{3} & -\frac{\sqrt2}{3} & -\frac{\sqrt6}{3} \\ \frac{\sqrt8}{3} & \frac{-1}{6} & -\frac{\sqrt3}{6} \\ \cdot & -\frac{\sqrt3}{2} & \frac{1}{2} \end{vmatrix}$ $\quad R_1^3 = [1432]$: $\begin{vmatrix} \frac{-1}{3} & -\frac{\sqrt2}{3} & -\frac{\sqrt6}{3} \\ \frac{\sqrt8}{3} & \frac{-1}{6} & -\frac{\sqrt3}{6} \\ \cdot & \frac{\sqrt3}{2} & \frac{-1}{2} \end{vmatrix}$

$r_3^2 = [134]$: $\begin{vmatrix} \frac{-1}{3} & \frac{\sqrt8}{3} & \cdot \\ -\frac{\sqrt2}{3} & \frac{-1}{6} & -\frac{\sqrt3}{2} \\ -\frac{\sqrt6}{3} & -\frac{\sqrt3}{6} & \frac{1}{2} \end{vmatrix}$ $\quad i_1 = [14]$: $\begin{vmatrix} \frac{-1}{3} & -\frac{\sqrt2}{3} & -\frac{\sqrt6}{3} \\ -\frac{\sqrt2}{3} & \frac{5}{6} & -\frac{\sqrt3}{6} \\ -\frac{\sqrt6}{3} & -\frac{\sqrt3}{6} & \frac{1}{2} \end{vmatrix}$

$r_4^2 = [243]$: $\begin{vmatrix} \frac{-1}{3} & -\frac{\sqrt2}{3} & \frac{\sqrt6}{3} \\ \frac{\sqrt8}{3} & \frac{-1}{6} & \frac{\sqrt3}{6} \\ \cdot & \frac{\sqrt3}{2} & \frac{1}{2} \end{vmatrix}$ $\quad R_2 = [1243]$: $\begin{vmatrix} \frac{-1}{3} & -\frac{\sqrt2}{3} & \frac{\sqrt6}{3} \\ \frac{\sqrt8}{3} & \frac{-1}{6} & \frac{\sqrt3}{6} \\ \cdot & -\frac{\sqrt3}{2} & \frac{-1}{2} \end{vmatrix}$

basis: $\begin{matrix} O \\ D_3 \\ C_2 \end{matrix} \left| \begin{matrix} T_2 \\ B_2 \\ 0_2 \end{matrix} \right\rangle \left| \begin{matrix} T_2 \\ E \\ 0_2 \end{matrix} \right\rangle \left| \begin{matrix} T_2 \\ E \\ 1_2 \end{matrix} \right\rangle$

$\mathscr{D}^E(1) = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ $\quad$ C_2 $\quad$ $i_4 = [12]$ $\begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$

$R_1^2 = [13][24]$ $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ $\quad$ $R_3 = [1423]$ $\begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$

$r_1 = [132]$ $\begin{vmatrix} \frac{-1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{vmatrix}$ $\quad$ $i_5 = [13]$ $\begin{vmatrix} \frac{-1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{-\sqrt{3}}{2} & \frac{1}{2} \end{vmatrix}$

$r_4 = [234]$ $\begin{vmatrix} \frac{-1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{vmatrix}$ $\quad$ $i_6 = [24]$ $\begin{vmatrix} \frac{-1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{-\sqrt{3}}{2} & \frac{1}{2} \end{vmatrix}$

$r_1^2 = [123]$ $\begin{vmatrix} \frac{-1}{2} & \frac{\sqrt{3}}{2} \\ \frac{-\sqrt{3}}{2} & \frac{-1}{2} \end{vmatrix}$ $\quad$ $i_2 = [23]$ $\begin{vmatrix} \frac{-1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{vmatrix}$

$r_2^2 = [142]$ $\begin{vmatrix} \frac{-1}{2} & \frac{\sqrt{3}}{2} \\ \frac{-\sqrt{3}}{2} & \frac{-1}{2} \end{vmatrix}$ $\quad$ $R_2^3 = [1342]$ $\begin{vmatrix} \frac{-1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{vmatrix}$

$R_2^2 = [14][23]$ $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ $\quad$ $R_3^3 = [1324]$ $\begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$

$R_3^2 = [12][34]$ $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ $\quad$ $i_3 = [34]$ $\begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$

$r_2 = [124]$ $\begin{vmatrix} \frac{-1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{-1}{2} \end{vmatrix}$ $\quad$ $R_1 = [1234]$ $\begin{vmatrix} \frac{-1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{-\sqrt{3}}{2} & \frac{1}{2} \end{vmatrix}$

$r_3 = [143]$ $\begin{vmatrix} \frac{-1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{-1}{2} \end{vmatrix}$ $\quad$ $R_1^3 = [1432]$ $\begin{vmatrix} \frac{-1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{-\sqrt{3}}{2} & \frac{1}{2} \end{vmatrix}$

$r_3^2 = [134]$ $\begin{vmatrix} \frac{-1}{2} & \frac{\sqrt{3}}{2} \\ \frac{-\sqrt{3}}{2} & \frac{-1}{2} \end{vmatrix}$ $\quad$ $i_1 = [14]$ $\begin{vmatrix} \frac{-1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{vmatrix}$

$r_4^2 = [243]$ $\begin{vmatrix} \frac{-1}{2} & \frac{\sqrt{3}}{2} \\ \frac{-\sqrt{3}}{2} & \frac{-1}{2} \end{vmatrix}$ $\quad$ $R_2 = [1243]$ $\begin{vmatrix} \frac{-1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{vmatrix}$

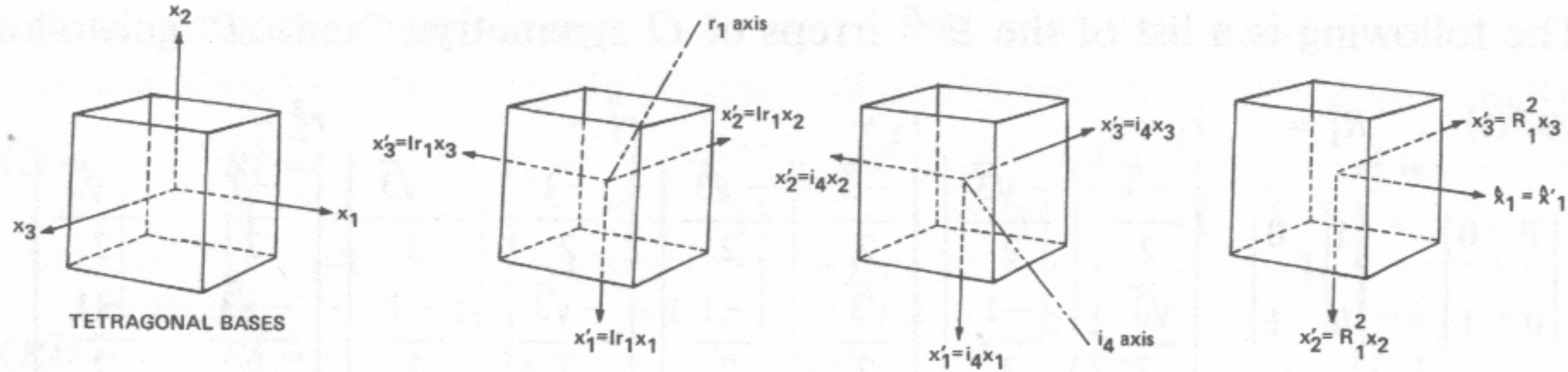
E *Tensor*

$u^2+v^2-2w^2$

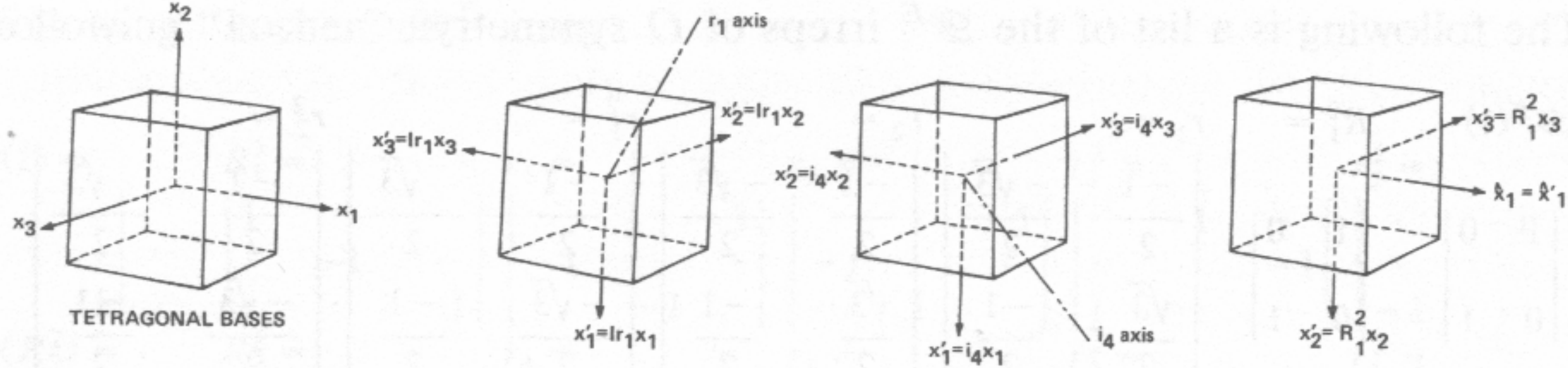
$(u^2-v^2)\sqrt{3}$

basis: $\begin{vmatrix} O \\ D_3 \\ C_2 \end{vmatrix} \begin{matrix} E \\ E \\ 0_2 \end{matrix} \Big\rangle \Big| \begin{matrix} E \\ E \\ 1_2 \end{matrix} \Big\rangle$

$O: \chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}$=1	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	$\boldsymbol{\rho}_{xyz}$	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1



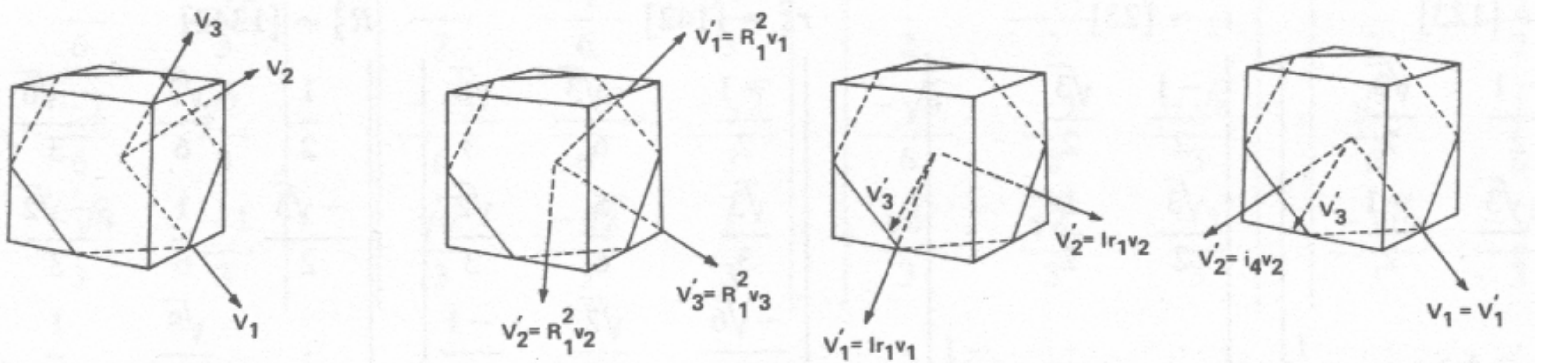
$$D^{T1u}(Ir_1) = \begin{pmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} \qquad D^{T1u}(i_4) = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad D^{T1u}(R_1^2) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$



$$D^{T1u}(Ir_1) = \begin{pmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} \qquad D^{T1u}(i_4) = \begin{pmatrix} \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & -1 \end{pmatrix} \qquad D^{T1u}(R_1^2) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

TRIGONAL BASES

$$D^{T1u}(R_1^2) = \begin{pmatrix} 0 & \sqrt{3}/3 & \sqrt{6}/3 \\ \sqrt{3}/3 & -2/3 & \sqrt{2}/3 \\ \sqrt{6}/3 & \sqrt{2}/3 & -1/3 \end{pmatrix} \qquad D^{T1u}(Ir_1) = \begin{pmatrix} \begin{pmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{pmatrix} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & -1 \end{pmatrix} \qquad D^{T1u}(i_4) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

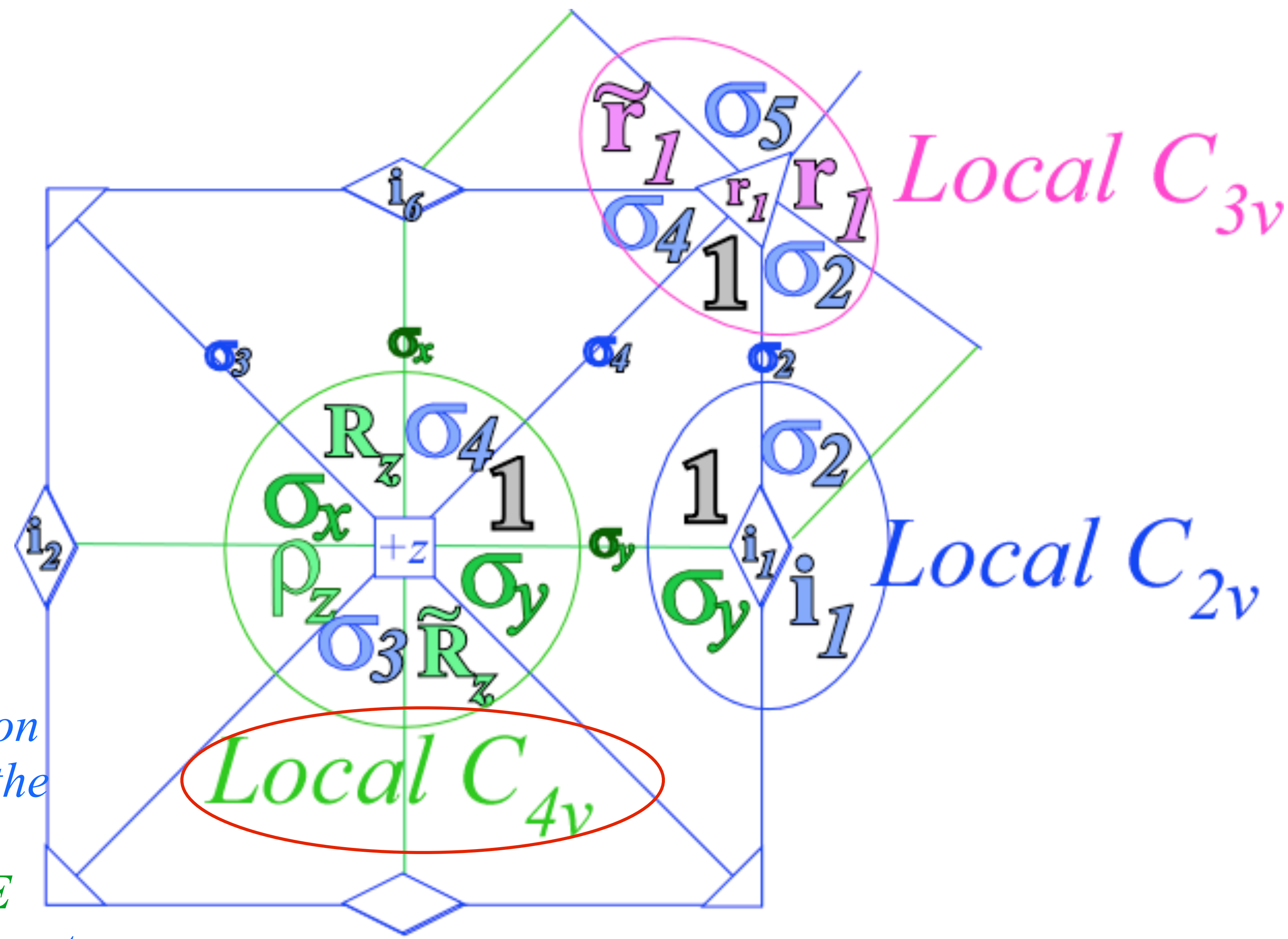


O⊃C_4	0_4	1_4	2_4	3_4
$A_1 \downarrow C_4$	1	·	·	·
$A_2 \downarrow C_4$	·	·	1	·
$E \downarrow C_4$	1	·	1	·
$T_1 \downarrow C_4$	1	1	·	1
$T_2 \downarrow C_4$	·	1	1	1

$O_h \supset C_{4v}$	A'	B'	A''	B''	E
$A_{1g} \downarrow C_{4v}$	1	·	·	·	·
$A_{2g} \downarrow C_{4v}$	·	1	·	·	·
$E_g \downarrow C_{4v}$	1	1	·	·	·
$T_{1g} \downarrow C_{4v}$	·	·	1	·	1
$T_{2g} \downarrow C_{4v}$	·	·	·	1	1
$A_{1g} \downarrow C_{4v}$	·	·	1	·	·
$A_{2u} \downarrow C_{4v}$	·	·	·	1	·
$E_u \downarrow C_{4v}$	·	·	1	1	·
$T_{1u} \downarrow C_{4v}$	1	·	·	·	1
$T_{2u} \downarrow C_{4v}$	·	1	·	·	1

,

$O_h \supset C_{4v}$ correlation predicts the parity of the A_1T_1E cluster is not uniformly even (g) or odd (u): $A_{1g}T_{1u}E_g$



$0_4\uparrow O$ cluster
$Symmetry_{parity}$
$A_{1g}T_{1u}E_g$

$$E^{A_1}=H+T+4S$$
$$E^{T_1}=H-T$$
$$E^{E}=H+T-2S$$

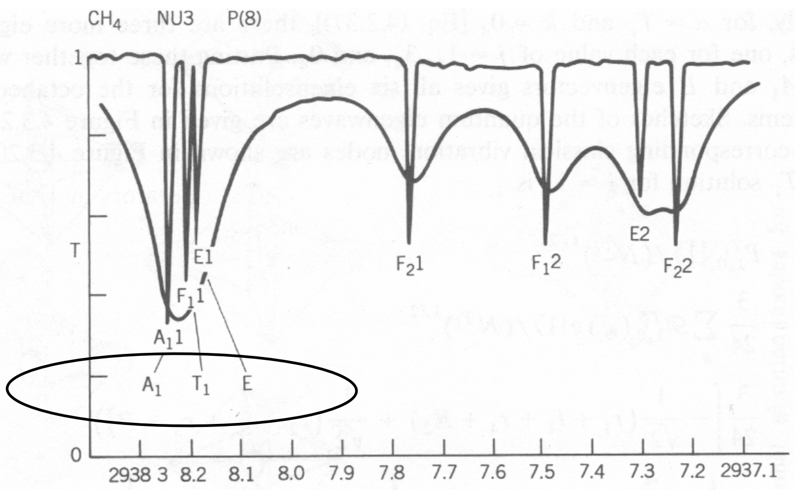
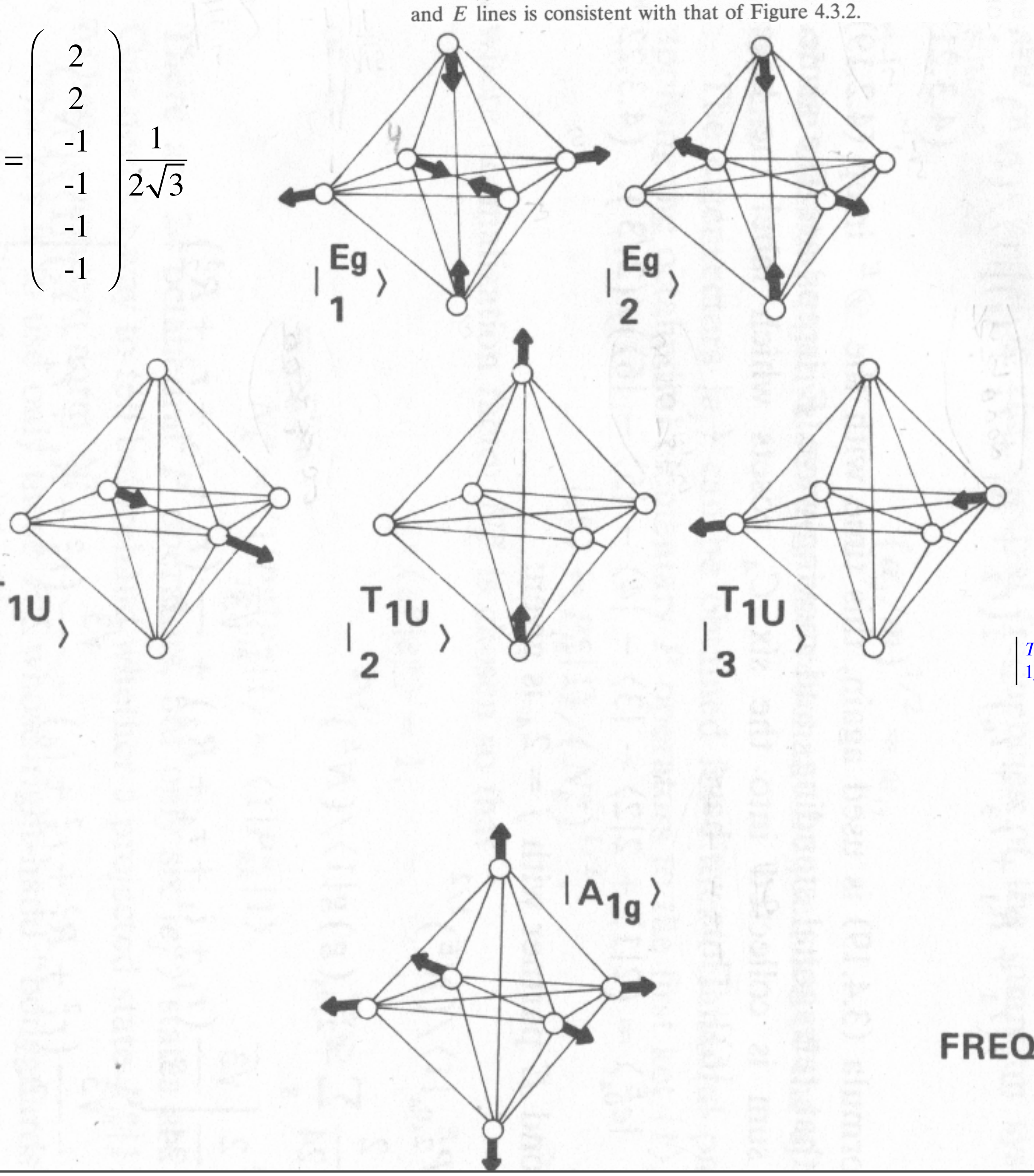
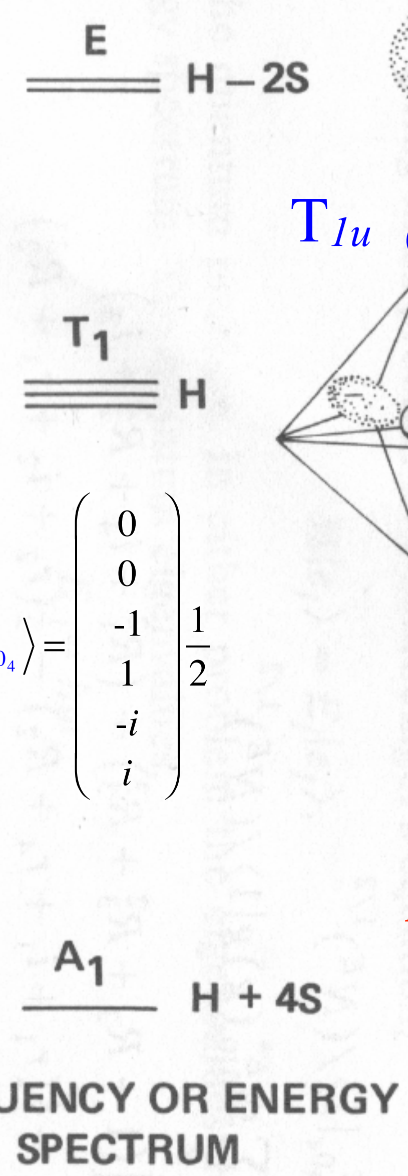


Figure 4.3.3 Evidence of an (A_1T_1E) spectral cluster in methane laser spectra. (Courtesy of Dr. Allan Pine, MIT Lincoln Laboratories, from *Journal of Optical Society of America* **66**, 97 (1976)). The ordering and approximate spacing of the A_1T_1 and E lines is consistent with that of Figure 4.3.2.

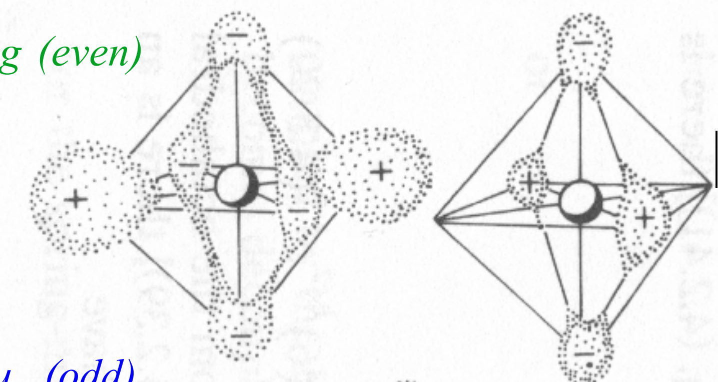
$$\begin{pmatrix} \langle 1|\mathbf{H}|1\rangle & \langle 1|\mathbf{H}|2\rangle & \cdots & \langle 1|\mathbf{H}|6\rangle \\ \langle 2|\mathbf{H}|1\rangle & \langle 2|\mathbf{H}|2\rangle & \cdots & \langle 2|\mathbf{H}|6\rangle \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \langle 6|\mathbf{H}|1\rangle & \langle 6|\mathbf{H}|2\rangle & \cdots & \langle 6|\mathbf{H}|6\rangle \end{pmatrix} = \begin{pmatrix} H & T & S & S & S & S \\ T & H & S & S & S & S \\ S & S & H & T & S & S \\ S & S & T & H & S & S \\ S & S & S & S & H & T \\ S & S & S & S & T & H \end{pmatrix}.$$

$$\left|{}^{E}_{0_4 0_4}\right\rangle = \begin{pmatrix} 2 \\ 2 \\ -1 \\ -1 \\ -1 \\ -1 \end{pmatrix}\frac{1}{2\sqrt{3}}$$



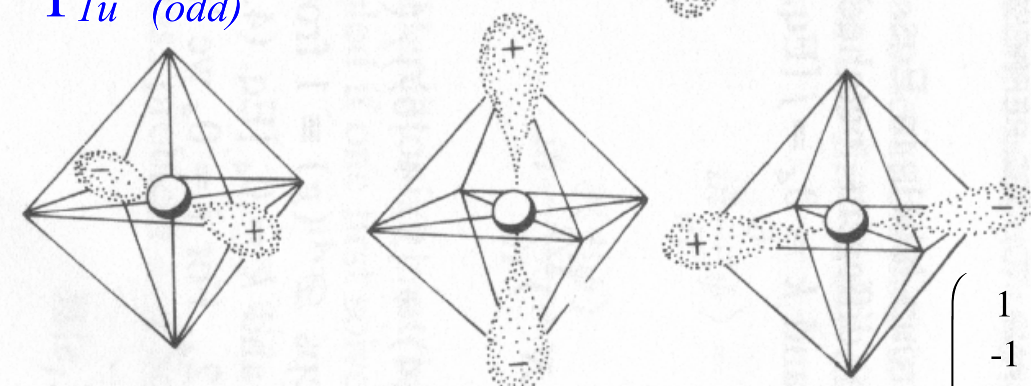


E_g *(even)*



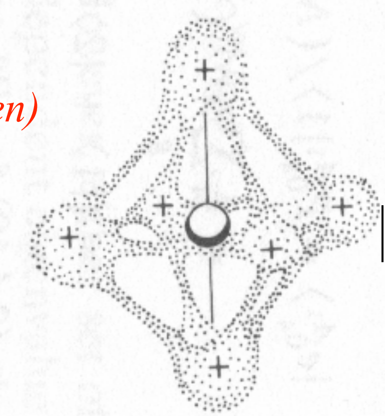
$$\left|{}^{E}_{2_4 0_4}\right\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ -1 \\ -1 \end{pmatrix}\frac{1}{2}$$

T_{1u} *(odd)*



$$\left|{}^{T_1}_{1_4 0_4}\right\rangle = \begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \\ -i \\ i \end{pmatrix}\frac{1}{2} \qquad \left|{}^{T_1}_{0_4 0_4}\right\rangle = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}\frac{1}{\sqrt{2}}$$

A_{1g} *(even)*



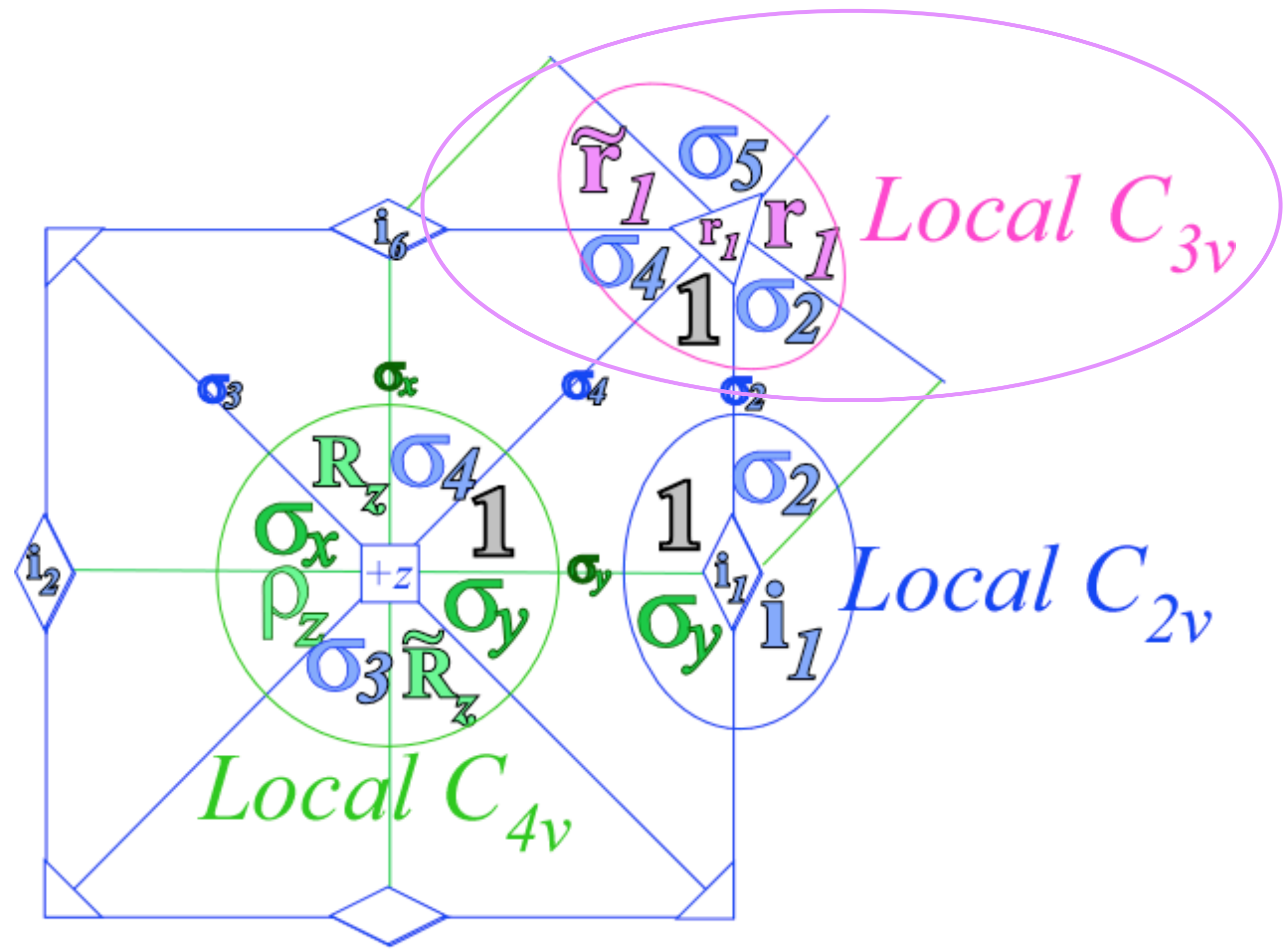
$$\left|{}^{A_1}_{0_4 0_4}\right\rangle = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}\frac{1}{\sqrt{6}}$$

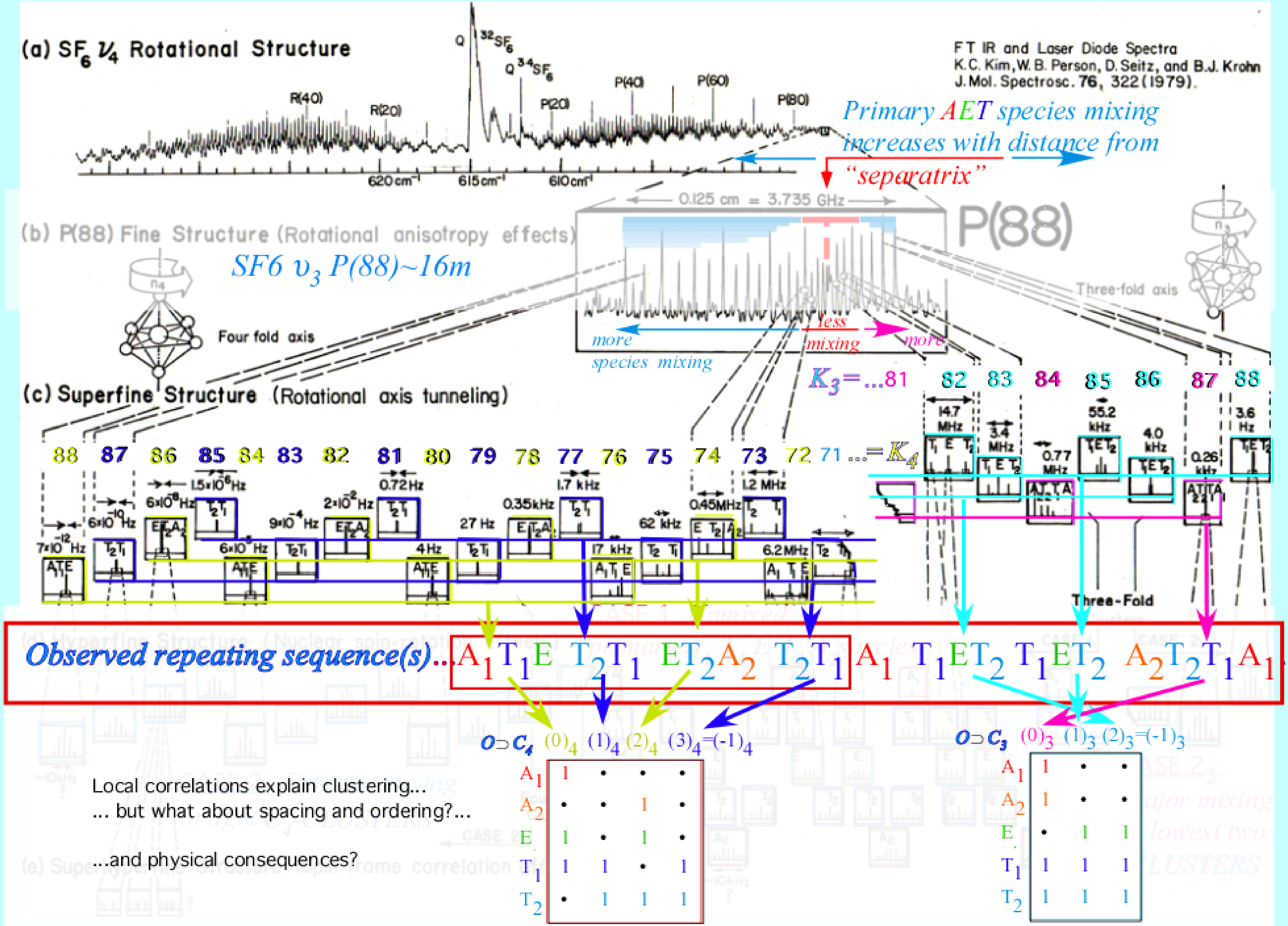
$O \supset C_3$	0_3	1_3	2_3
$A_1 \downarrow C_3$	1	·	·
$A_2 \downarrow C_3$	1	·	·
$E \downarrow C_3$	·	1	1
$T_1 \downarrow C_3$	1	1	1
$T_2 \downarrow C_3$	1	1	1

,

$O_h \supset C_{3v}$	A'	A''	E
$A_{1g} \downarrow C_{3v}$	1	·	·
$A_{2g} \downarrow C_{3v}$	·	1	·
$E_g \downarrow C_{3v}$	·	·	1
$T_{1g} \downarrow C_{3v}$	·	1	1
$T_{2g} \downarrow C_{3v}$	1	·	1
$A_{1g} \downarrow C_{3v}$	·	1	·
$A_{2u} \downarrow C_{3v}$	1	·	·
$E_u \downarrow C_{3v}$	·	·	1
$T_{1u} \downarrow C_{3v}$	1	·	1
$T_{2u} \downarrow C_{3v}$	·	1	1

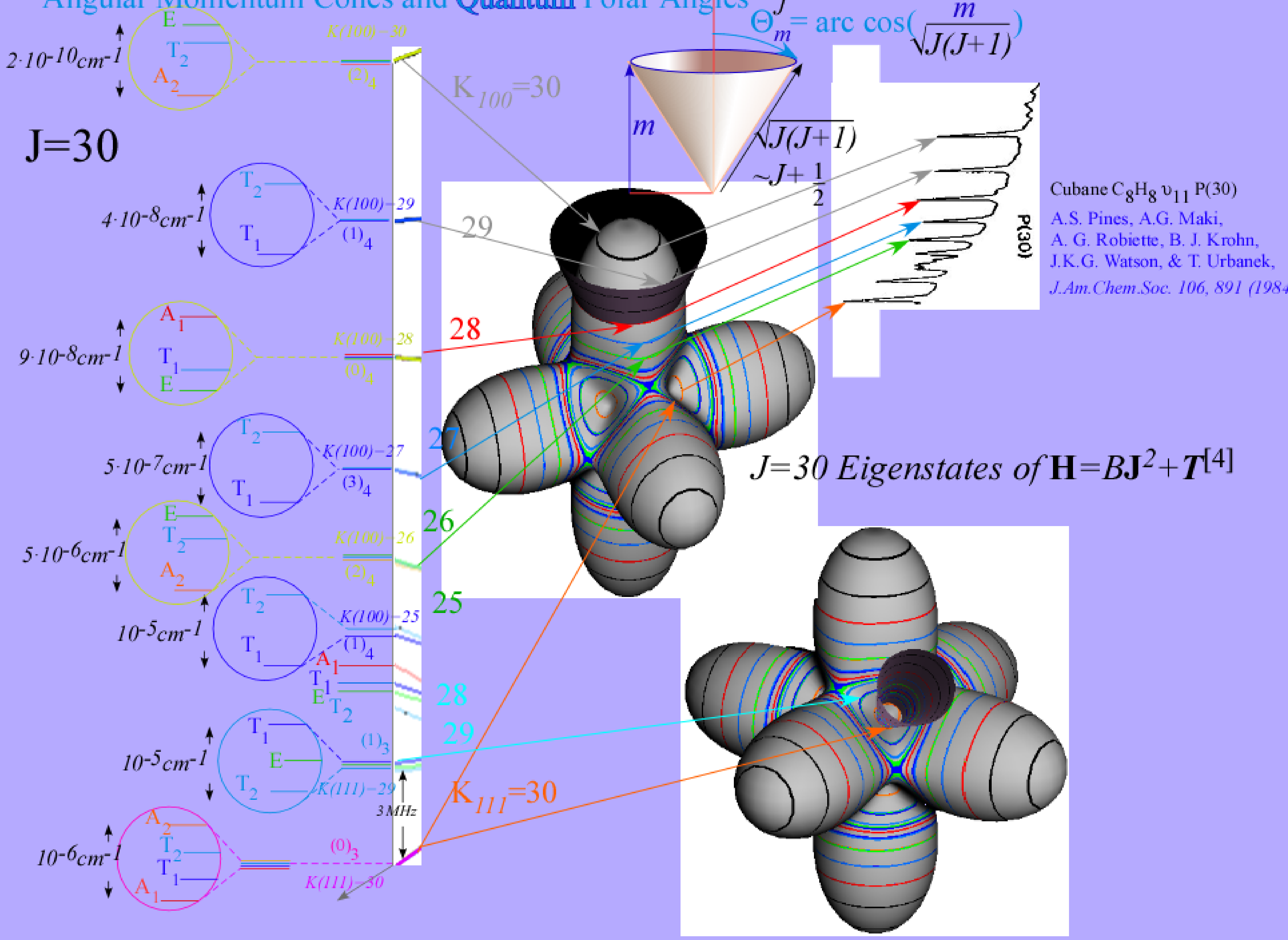
,




(a) SF6 ν4 Rotational Structure
FT IR and Laser Diode Spectra
K.C. Kim, W.B. Person, D. Seitz, and B.J. Krohn
J. Mol. Spectrosc. 76, 322 (1979).
Q 32SF6
Q 34SF6
R(40)
R(20)
P(20)
P(40)
P(60)
P(80)
620 cm-1
615 cm-1
610 cm-1
Primary AET species mixing
increases with distance from
"separatrix"
0.125 cm = 3.735 GHz
P(88)
(b) P(88) Fine Structure (Rotational anisotropy effects)
SF6 υ3 P(88)~16m
Four fold axis
Three-fold axis
more
species mixing
less
mixing
more
K3=...81 82 83 84 85 86 87 88
(c) Superfine Structure (Rotational axis tunneling)
88 87 86 85 84 83 82 81 80 79 78 77 76 75 74 73 72 71 ...=K4
Three-Fold
Observed repeating sequence(s)...A1 T1 E T2 T1 E T2 A2 T2 T1 A1 T1 E T2 T1 E T2 A2 T2 T1 A1...
O⊃C4 (0)4 (1)4 (2)4 (3)4=(-1)4
O⊃C3 (0)3 (1)3 (2)3=(-1)3
Local correlations explain clustering...
... but what about spacing and ordering?...
...and physical consequences?

Angular Momentum Cones and Quantum Polar Angles

$$\Theta_m^J = \arccos\left(\frac{m}{\sqrt{J(J+1)}}\right)$$



Cubane C_8H_8 ν_{11} P(30)
A.S. Pines, A.G. Maki, A. G. Robiette, B. J. Krohn, J.K.G. Watson, & T. Urbanek, *J.Am.Chem.Soc. 106, 891 (1984)*

$O\supset C_4$	0_4	1_4	2_4	3_4
$A_1\downarrow C_4$	1	·	·	·
$A_2\downarrow C_4$	·	·	1	·
$E\downarrow C_4$	1	·	1	·
$T_1\downarrow C_4$	1	1	·	1
$T_2\downarrow C_4$	·	1	1	1

$O\supset C_3$	0_3	1_3	2_3
$A_1\downarrow C_3$	1	·	·
$A_2\downarrow C_3$	1	·	·
$E\downarrow C_3$	·	1	1
$T_1\downarrow C_3$	1	1	1
$T_2\downarrow C_3$	1	1	1

$O\supset C_2(\mathbf{i}_1)$	0_2	1_2
$A_1\downarrow C_2$	1	·
$A_2\downarrow C_2$	·	1
$E\downarrow C_2$	1	1
$T_1\downarrow C_2$	1	2
$T_2\downarrow C_2$	2	1

$O\supset C_2(\rho_z)$	0_2	1_2
$A_1\downarrow C_2$	1	·
$A_2\downarrow C_2$	1	·
$E\downarrow C_2$	2	·
$T_1\downarrow C_2$	1	2
$T_2\downarrow C_2$	1	2

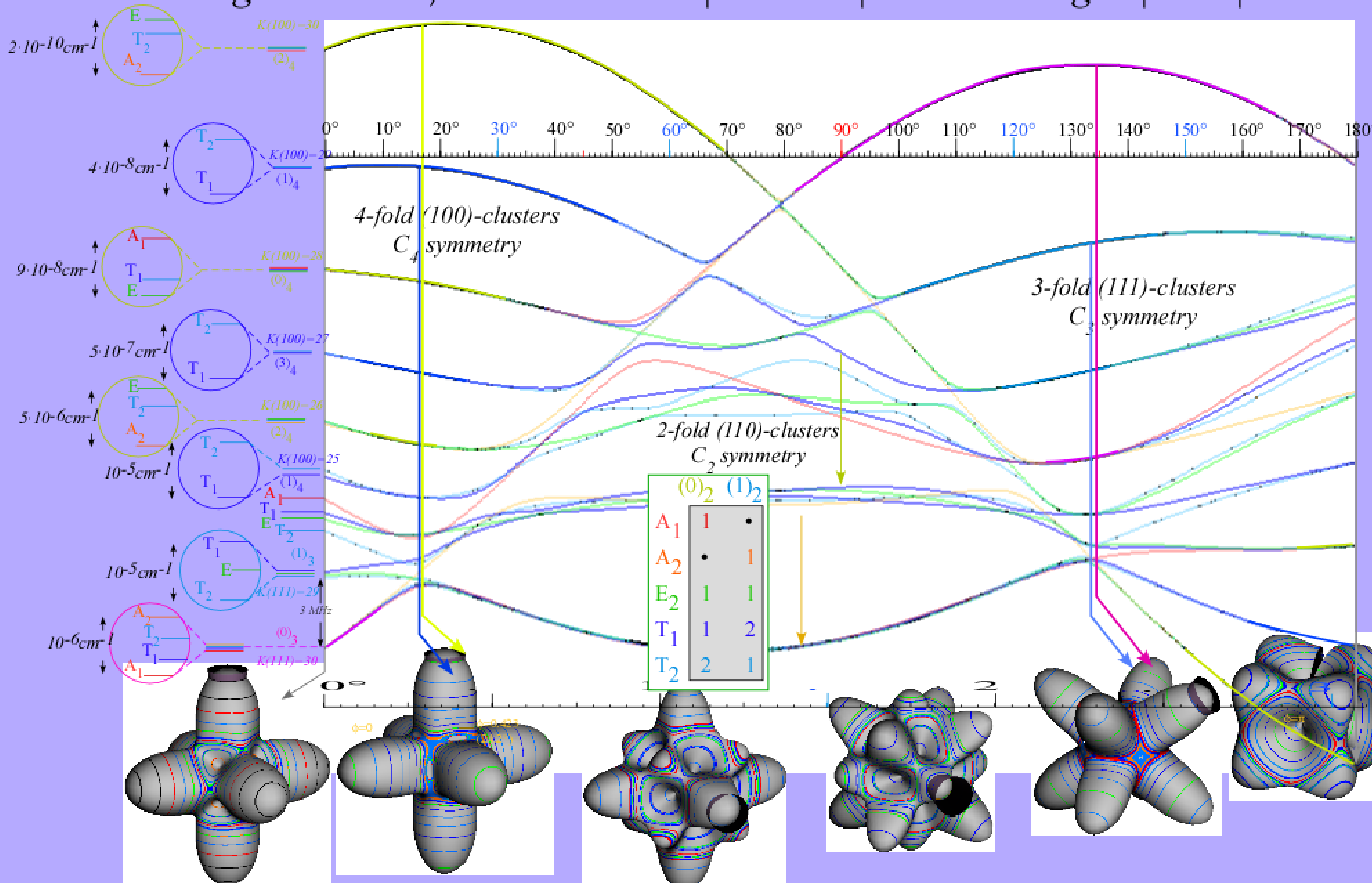
$O_h\supset C_{4v}$	A'	B'	A''	B''	E
$A_{1g}\downarrow C_{4v}$	1	·	·	·	·
$A_{2g}\downarrow C_{4v}$	·	1	·	·	·
$E_g\downarrow C_{4v}$	1	1	·	·	·
$T_{1g}\downarrow C_{4v}$	·	·	1	·	1
$T_{2g}\downarrow C_{4v}$	·	·	·	1	1
$A_{1g}\downarrow C_{4v}$	·	·	1	·	·
$A_{2u}\downarrow C_{4v}$	·	·	·	1	·
$E_u\downarrow C_{4v}$	·	·	1	1	·
$T_{1u}\downarrow C_{4v}$	1	·	·	·	1
$T_{2u}\downarrow C_{4v}$	·	1	·	·	1

$O_h\supset C_{3v}$	A'	A''	E
$A_{1g}\downarrow C_{3v}$	1	·	·
$A_{2g}\downarrow C_{3v}$	·	1	·
$E_g\downarrow C_{3v}$	·	·	1
$T_{1g}\downarrow C_{3v}$	·	1	1
$T_{2g}\downarrow C_{3v}$	1	·	1
$A_{1g}\downarrow C_{3v}$	·	1	·
$A_{2u}\downarrow C_{3v}$	1	·	·
$E_u\downarrow C_{3v}$	·	·	1
$T_{1u}\downarrow C_{3v}$	1	·	1
$T_{2u}\downarrow C_{3v}$	·	1	1

$O_h\supset C_{2v}^i$	A'	B'	A''	B''
$A_{1g}\downarrow C_{2v}^i$	1	·	·	·
$A_{2g}\downarrow C_{2v}^i$	·	1	·	·
$E_g\downarrow C_{2v}^i$	1	1	·	·
$T_{1g}\downarrow C_{2v}^i$	·	1	1	1
$T_{2g}\downarrow C_{2v}^i$	1	·	1	1
$A_{1g}\downarrow C_{2v}^i$	·	·	1	·
$A_{2u}\downarrow C_{2v}^i$	·	·	·	1
$E_u\downarrow C_{2v}^i$	·	·	1	1
$T_{1u}\downarrow C_{2v}^i$	1	1	·	1
$T_{2u}\downarrow C_{2v}^i$	1	1	1	·

$O_h\supset C_{2v}^z$	A'	B'	A''	B''
$A_{1g}\downarrow C_{2v}^z$	1	·	·	·
$A_{2g}\downarrow C_{2v}^z$	1	·	·	·
$E_g\downarrow C_{2v}^z$	2	·	·	·
$T_{1g}\downarrow C_{2v}^z$	·	1	1	1
$T_{2g}\downarrow C_{2v}^z$	·	1	1	1
$A_{1g}\downarrow C_{2v}^z$	·	·	1	·
$A_{2u}\downarrow C_{2v}^z$	·	·	1	·
$E_u\downarrow C_{2v}^z$	·	·	2	·
$T_{1u}\downarrow C_{2v}^z$	1	1	·	1
$T_{2u}\downarrow C_{2v}^z$	1	1	·	1

Eigenvalues of $\mathbf{H}=B\mathbf{J}^2+\cos\phi\,\mathbf{T}^{[4]}+\sin\phi\,\mathbf{T}^{[6]}$ *vs. mix angle* ϕ*:* $0<\phi<\pi$



	$(0)_2$	$(1)_2$
A_1	1	•
A_2	•	1
E_2	1	1
T_1	1	2
T_2	2	1

Ireps for $O \supset D_4 \supset C_4$ *subgroup chain*

T_1

Vector

x,y,z

T_2

Tensor

yz,xz,xy

E

Tensor

$x^2+y^2-2z^2$

$(x^2-y^2)\sqrt{3}$

C_4 subgroup correlation to O

$O\supset C_4$	$(0)_4$	$(1)_4$	$(2)_4$	$(3)_4=(-1)_4$
A_1	1	•	•	•
A_2	•	•	1	•
E	1	•	1	•
T_1	1	1	•	1
T_2	•	1	1	1

C_4 Projectors to split octahedral P^α

$$\mathbf{p}_{m_4} = \sum_{p=0}^{3} \frac{e^{2\pi i m\cdot p/4}}{4}\mathbf{R}_z^p = \begin{cases} \mathbf{p}_{0_4} = (\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{1_4} = (\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{2_4} = (\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{3_4} = (\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4 \end{cases}$$

$\mathbf{1}\cdot\mathbf{P}^\alpha =$	$(\mathbf{p}_{0_4}$	$+\mathbf{p}_{1_4}$	$+\mathbf{p}_{2_4}$	$+\mathbf{p}_{3_4})\cdot\mathbf{P}^\alpha$
$\mathbf{1}\cdot\mathbf{P}^{A_1} =$	$\mathbf{P}^{A_1}_{0_40_4}$	$+0$	$+0$	$+0$
$\mathbf{1}\cdot\mathbf{P}^{A_2} =$	0	$+0$	$+\mathbf{P}^{A_2}_{2_42_4}$	$+0$
$\mathbf{1}\cdot\mathbf{P}^{E} =$	$\mathbf{P}^{E}_{0_40_4}$	$+0$	$+\mathbf{P}^{E}_{2_42_4}$	$+0$
$\mathbf{1}\cdot\mathbf{P}^{T_1} =$	$\mathbf{P}^{T_1}_{0_40_4}$	$+\mathbf{P}^{T_1}_{1_41_4}$	$+0$	$+\mathbf{P}^{T_1}_{3_43_4}$
$\mathbf{1}\cdot\mathbf{P}^{T_2} =$	0	$+\mathbf{P}^{T_2}_{1_41_4}$	$+\mathbf{P}^{T_2}_{2_42_4}$	$+\mathbf{P}^{T_2}_{3_43_4}$

10 split $O\supset C_4$ octahedral P^α related to 10 split sub-classes

$\mathbf{P}^{(\alpha)}_{n_4n_4}(O\supset C_4)$	$\mathbf{1}$	$r_1r_2\tilde{r}_3\tilde{r}_4$	$\tilde{r}_1\tilde{r}_2r_3r_4$	$\rho_x\rho_y$	ρ_z	$R_x\tilde{R}_xR_y\tilde{R}_y$	R_z	$\tilde{R}_z$	$i_1i_2i_5i_6$	i_3i_4
$24\cdot\mathbf{P}^{A_1}_{0_40_4}$	1	1	1	1	1	1	1	1	1	1
$24\cdot\mathbf{P}^{A_2}_{2_42_4}$	1	1	1	1	1	-1	-1	-1	-1	-1
$12\cdot\mathbf{P}^{E}_{0_40_4}$	1	$-\frac{1}{2}$	$-\frac{1}{2}$	1	1	$-\frac{1}{2}$	1	1	$-\frac{1}{2}$	1
$12\cdot\mathbf{P}^{E}_{2_42_4}$	1	$-\frac{1}{2}$	$-\frac{1}{2}$	1	1	$+\frac{1}{2}$	-1	-1	$+\frac{1}{2}$	-1
$8\cdot\mathbf{P}^{T_1}_{1_41_4}$	1	$-\frac{i}{2}$	$+\frac{i}{2}$	0	-1	$+\frac{1}{2}$	$-i$	$+i$	$-\frac{1}{2}$	0
$8\cdot\mathbf{P}^{T_1}_{3_43_4}$	1	$+\frac{i}{2}$	$-\frac{i}{2}$	0	-1	$+\frac{1}{2}$	$+i$	$-i$	$-\frac{1}{2}$	0
$8\cdot\mathbf{P}^{T_1}_{0_40_4}$	1	0	0	-1	1	0	1	1	0	-1
$8\cdot\mathbf{P}^{T_2}_{1_41_4}$	1	$+\frac{i}{2}$	$-\frac{i}{2}$	0	-1	$-\frac{1}{2}$	$-i$	$+i$	$+\frac{1}{2}$	0
$8\cdot\mathbf{P}^{T_2}_{3_43_4}$	1	$-\frac{i}{2}$	$+\frac{i}{2}$	0	-1	$-\frac{1}{2}$	$+i$	$-i$	$+\frac{1}{2}$	0
$8\cdot\mathbf{P}^{T_2}_{2_42_4}$	1	0	0	-1	1	0	-1	-1	0	1

$\ell^{A_1} = 1$
$\ell^{A_2} = 1$
$\ell^{E} = 2$
$\ell^{T_1} = 3$
$\ell^{T_2} = 3$

Example: $G=O$

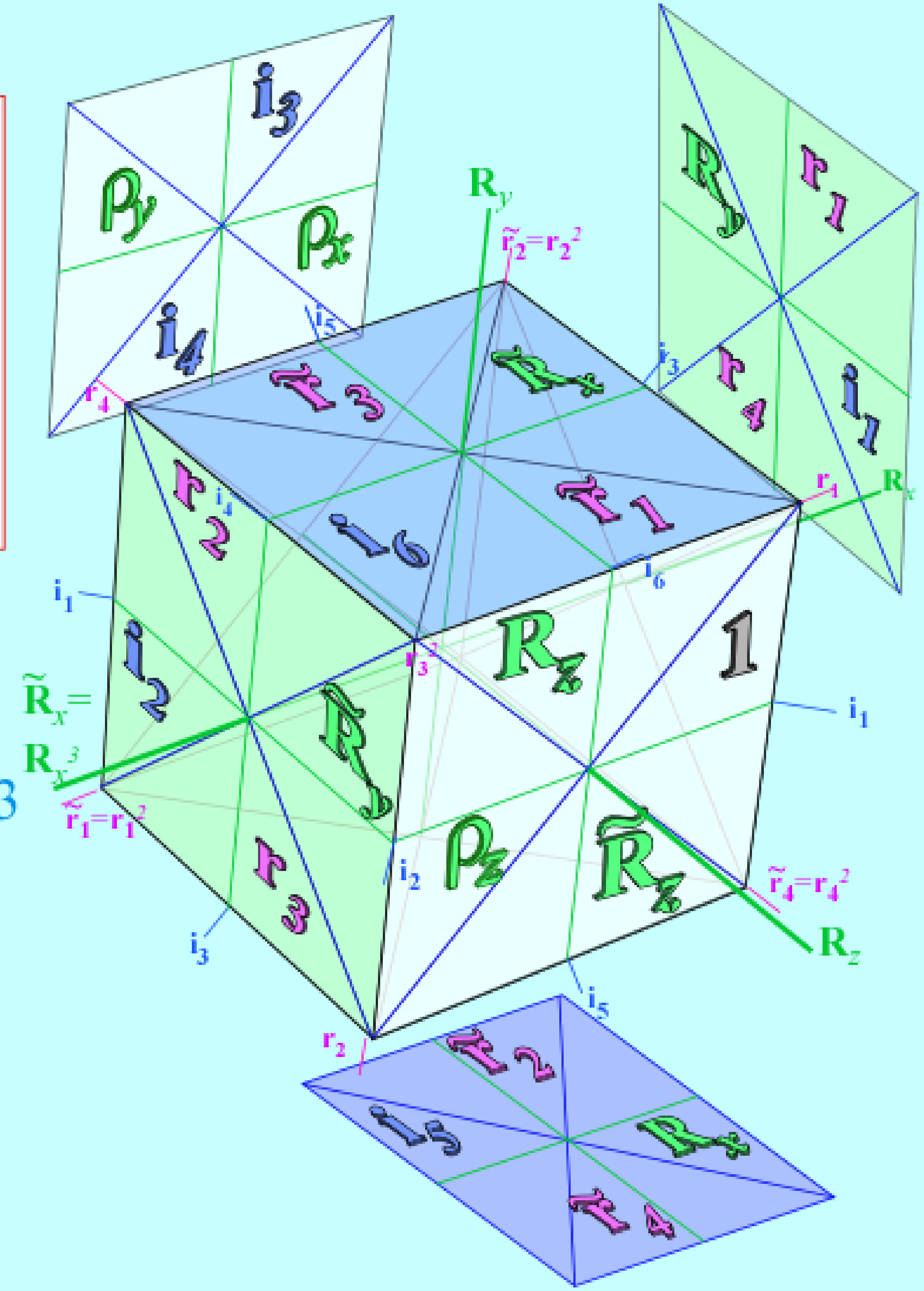
Cubic-Octahedral Group O

Centrum: $\kappa(O)=\Sigma_{(\alpha)} (\ell^{\alpha})^0 = 1^0+1^0+2^0+3^0+3^0=5$

Rank: $\rho(O)=\Sigma_{(\alpha)} (\ell^{\alpha})^1 = 1^1+1^1+2^1+3^1+3^1=10$

Order: $^o(O)=\Sigma_{(\alpha)} (\ell^{\alpha})^0 = 1^2+1^2+2^2+3^2+3^2=24$

	O group $\chi^{\alpha}_{\kappa_g}$	$g=1$	r_{1-4} $\tilde{r}_{1-4}$	ρ_{xyz}	R_{xyz} $\tilde{R}_{xyz}$	i_{1-6}
s-orbital r^2	$\alpha = A_1$	1	1	1	1	1
	A_2	1	1	1	−1	−1
d-orbitals $\{x^2+y^2-2z^2, x^2-y^2\}$	E	2	−1	2	0	0
p-orbitals $\{x, y, z\}$	T_1	3	0	−1	1	−1
$\{xz, yz, xy\}$ d-orbitals	T_2	3	0	−1	−1	1



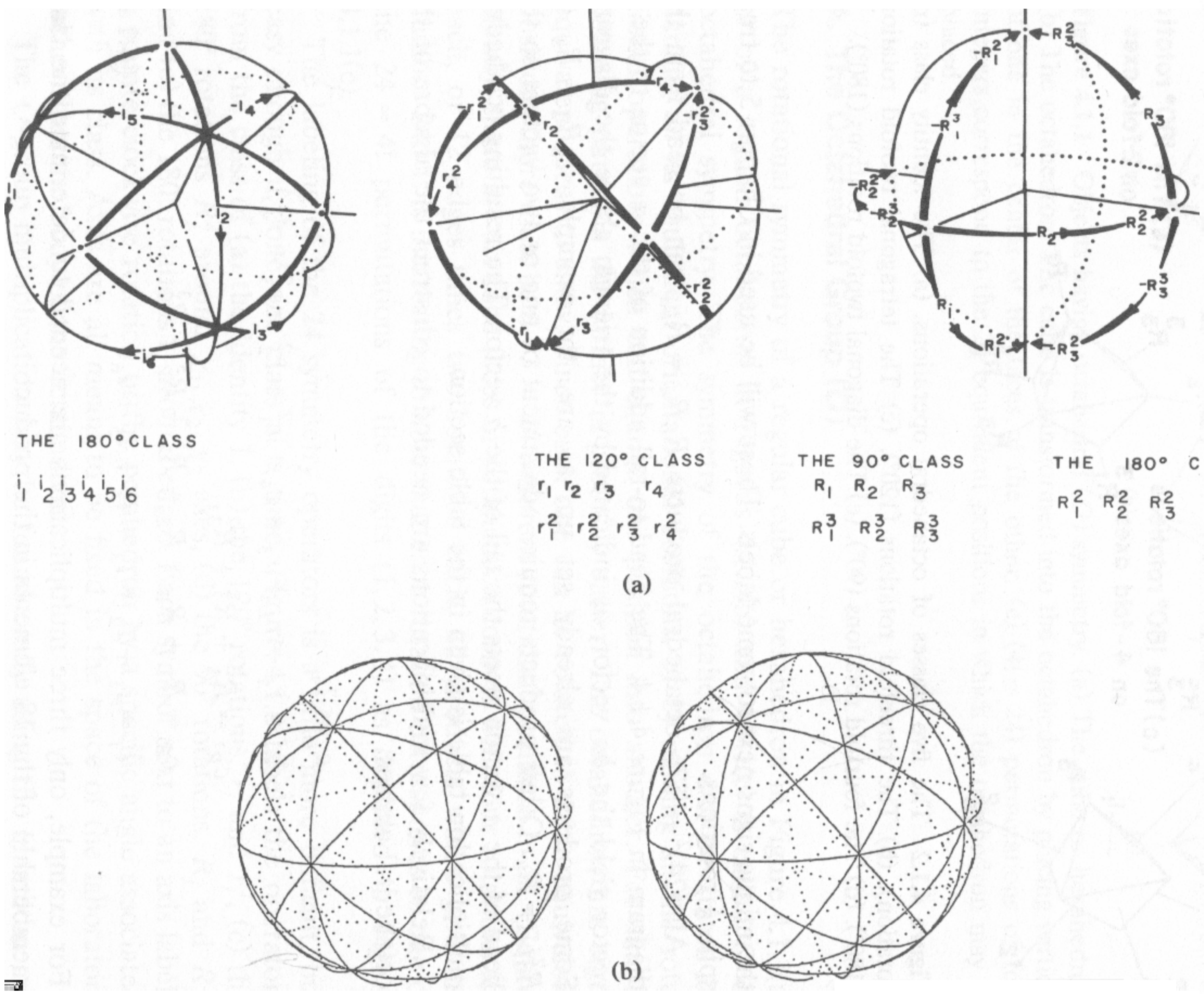
$O \supset C_4$

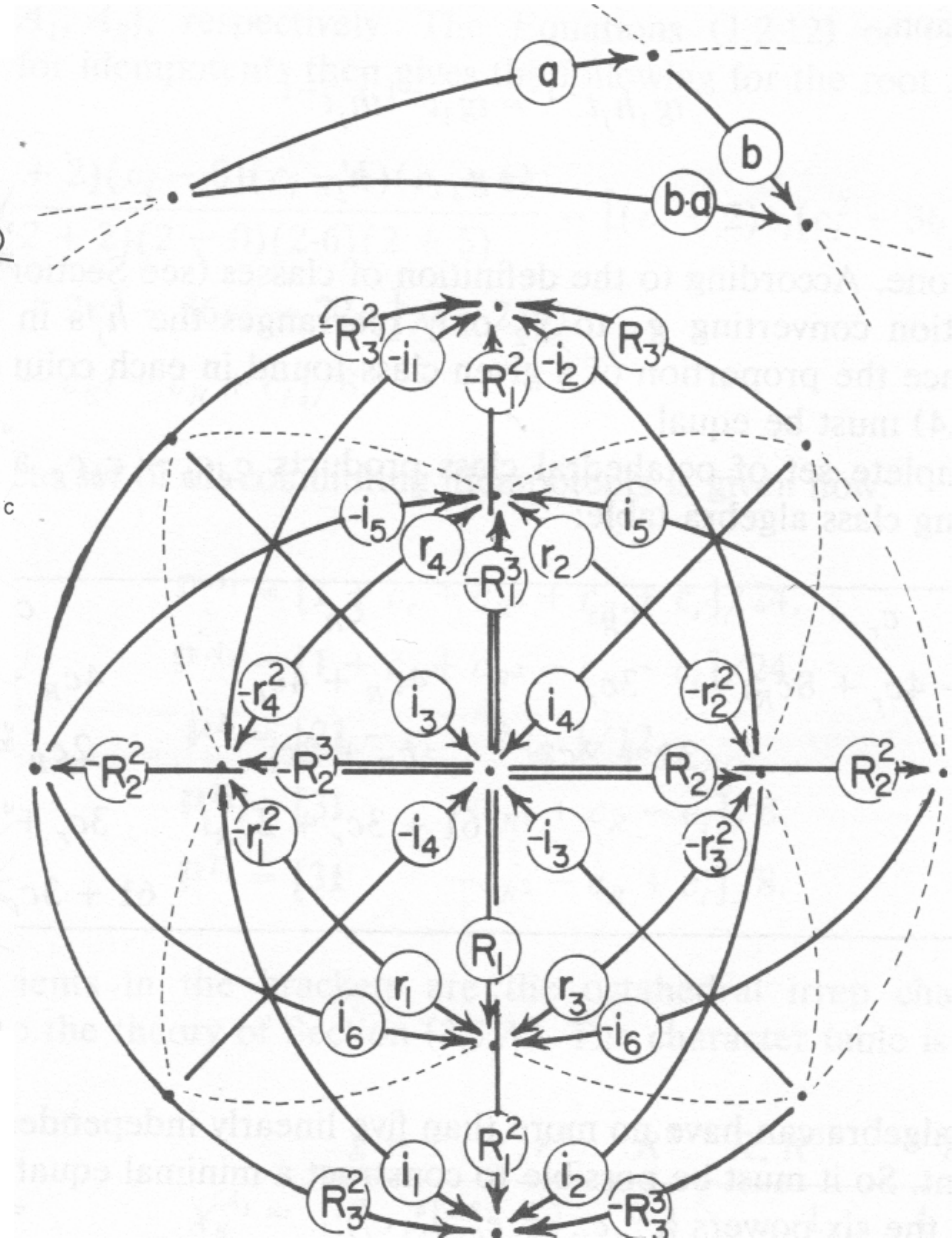
	$(0)_4$	$(1)_4$	$(2)_4$	$(3)_4=(-1)_4$
A_1	1	•	•	•
A_2	•	•	1	•
E	1	•	1	•
T_1	1	1	•	1
T_2	•	1	1	1

$O \supset C_3$

	$(0)_3$	$(1)_3$	$(2)_3=(-1)_3$
A_1	1	•	•
A_2	1	•	•
E	•	1	1
T_1	1	1	1
T_2	1	1	1

Octahedral O and spin-O⊂U(2) rotation nomogram from Fig. 4.1.3-4 *Principles of Symmetry, Dynamics and Spectroscopy*





Octahedral O⊃D_4⊃C_4 subgroup correlations

$\chi^\mu_g(O)$	$\mathbf{g}=\mathbf{1}$	$\mathbf{r}_{1...4}$	180° $\boldsymbol{\rho}_{xyz}$	90° $\mathbf{R}_{xyz}$	180° $\mathbf{i}_{1...6}$
A_1	1	1	1	1	1
A_2	1	1	1	−1	−1
E	2	−1	2	0	0
T_1	3	0	−1	1	−1
T_2	3	0	−1	−1	1

O↓D_4 subduction

D_4: **1**, $\boldsymbol{\rho}_{z180°}$, $\mathbf{R}_{z\pm90°}$, $\boldsymbol{\rho}_{z180°}$, $\mathbf{i}_{3,4}$

$A_1(O)\downarrow D_4$ = 1, 1, 1, 1, 1 .

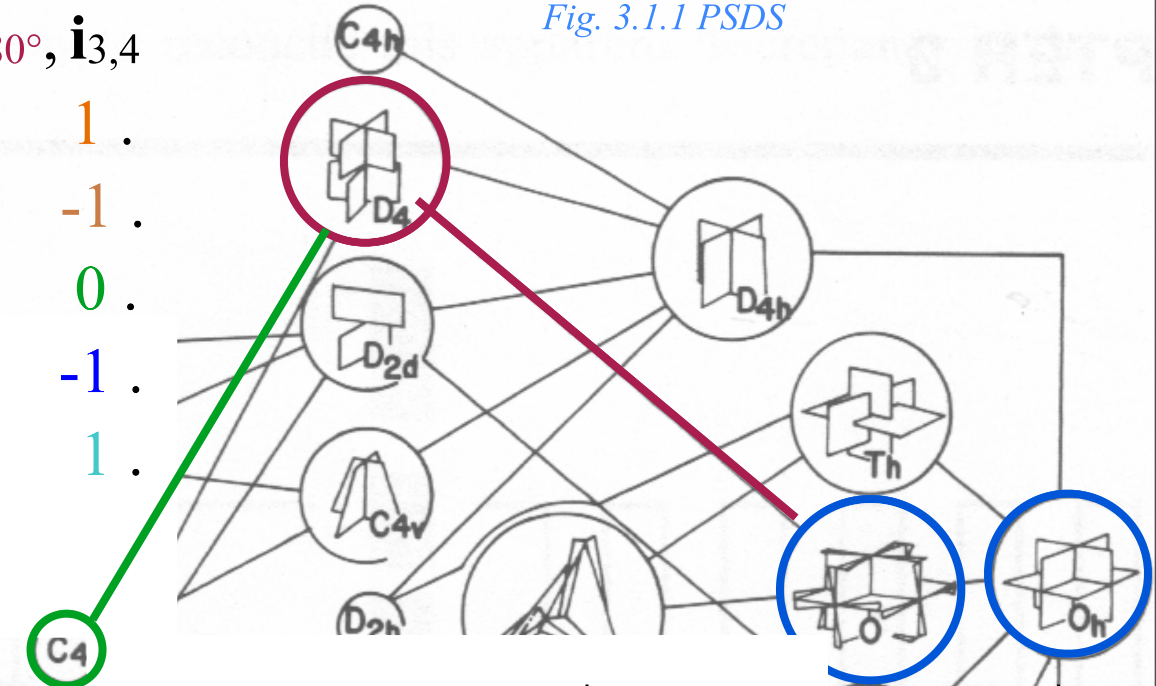
$A_2(O)\downarrow D_4$ = 1, 1, -1, 1, -1 .

$E(O)\downarrow D_4$ = 2, 2, 0, 2, 0 .

$T_2(O)\downarrow D_4$ = 3, -1, 1, -1, -1 .

$T_2(O)\downarrow D_4$ = 3, -1, -1, -1, 1 .

Fig. 3.1.1 PSDS



$\chi^\mu_g(D_4)$	$\mathbf{g}=\mathbf{1}$	$\boldsymbol{\rho}_{z180°}$	$\mathbf{R}_{z\pm90°}$	$\boldsymbol{\rho}_{x,y180°}$	$\mathbf{i}_{3,4}$
A_1	1	1	1	1	1
B_1	1	1	−1	1	−1
A_2	1	1	1	−1	−1
B_2	1	1	−1	−1	1
E	2	−2	0	0	0

D_4↓C_4 subduction

1, $\mathbf{R}_{z+90°}$, $\boldsymbol{\rho}_{z180°}$, $\mathbf{R}_{z-90°}$

$A_1(D_4)\downarrow C_4$ = 1, 1, 1, 1. $=(0)_4$

$B_1(D_4)\downarrow C_4$ = 1, -1, 1, -1. $=(2)_4$

$A_2(D_4)\downarrow C_4$ = 1, 1, 1, 1. $=(0)_4$

$B_2(D_4)\downarrow C_4$ = 1, -1, 1, -1. $=(2)_4$

$E(D_4)\downarrow C_4$ = 2, 0, -2, 0 . $=(1)_4\oplus(3)_4$

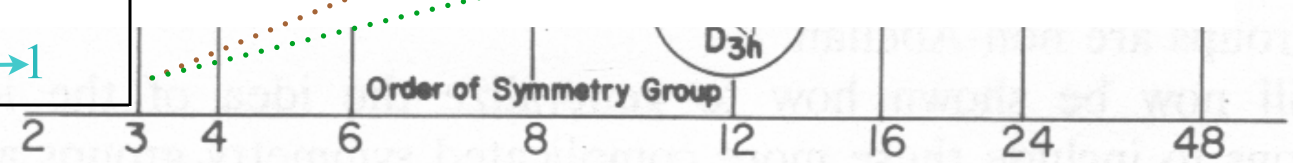
O↓D_4	A_1	B_1	A_2	B_2	E
A_1	1	·	·	·	·
A_2	·	1	·	·	·
E	1	1	·	·	·
T_1	·	·	1	·	1
T_2	·	·	·	1	1

$\chi^\mu_g(C_4)$	$\mathbf{g}=\mathbf{1}$	$\mathbf{R}_{z+90°}$	$\mathbf{R}_{z+180°}$	$\mathbf{R}_{z-90°}$
$(0)_4$	1	1	1	1
$(1)_4$	1	i	−1	$-i$
$(2)_4$	1	−1	1	−1
$(3)_4$	1	$-i$	−1	i

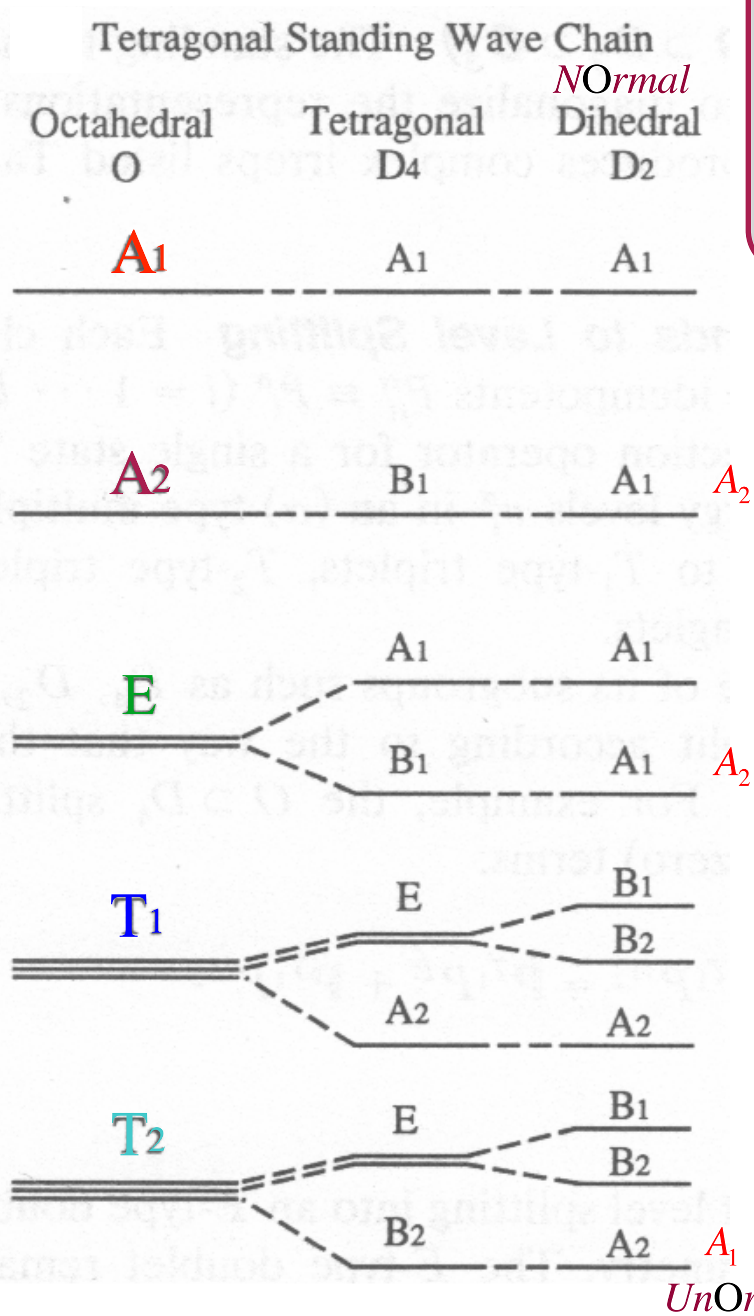
O↓C_4 subduction

O↓C_4	0_4	1_4	2_4	$3_4=\bar{1}_4$
A_1	1	·	·	·
A_2	·	·	1	·
E	1	·	1	·
T_1	1	1	·	1
T_2	·	1	1	1

D_4↓C_4	0_4	1_4	2_4	$3_4=\bar{1}_4$
A_1	1	·	·	·
B_1	·	·	1	·
A_2	1	·	·	·
B_2	·	·	1	·
E	·	1	·	1



$O_h \supset O \supset D_4 \supset C_4$ subgroup splitting



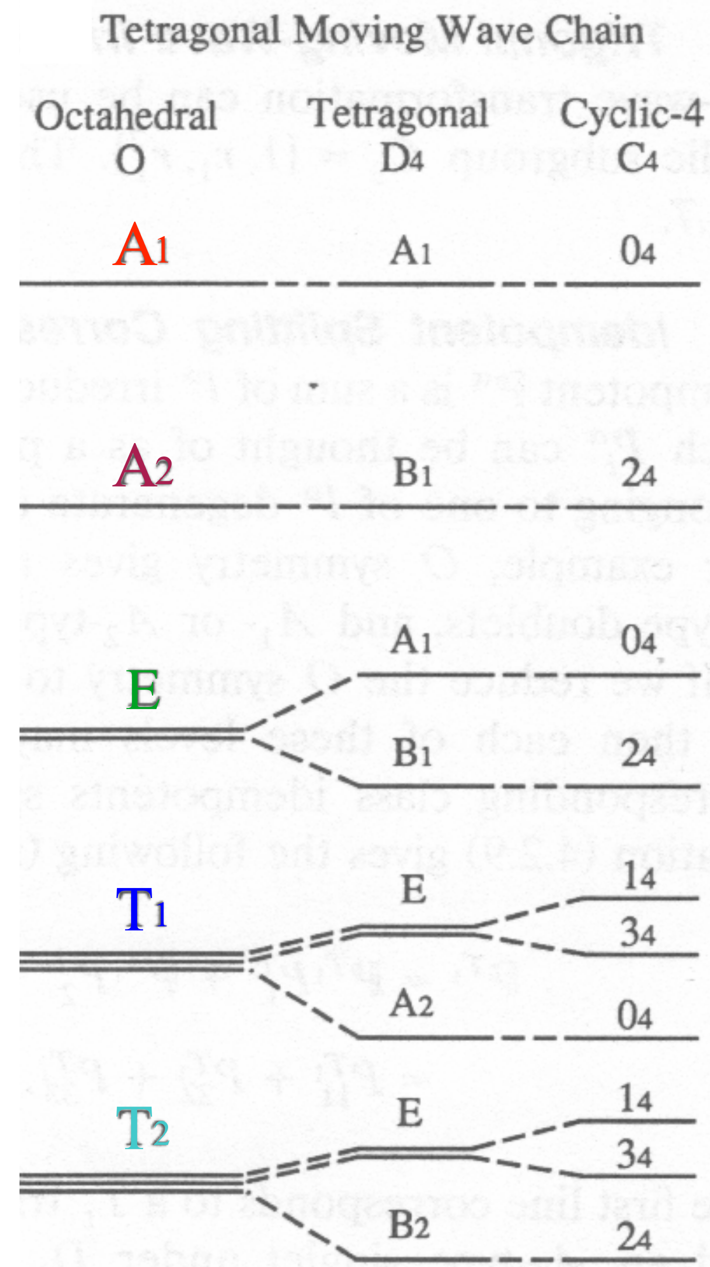
D_4	$\mathbf{1}$	ρ_z	$\mathbf{R}_z$	$\rho_{x,y}$	$\mathbf{i}_{3,4}$
A_1	1	1	1	1	1
B_1	1	1	-1	1	-1
A_2	1	1	1	-1	-1
B_2	1	1	-1	-1	1
E	2	-2	0	0	0

NOrmal $D_2 = \{1, \mathbf{R}_3^2, \mathbf{R}_1^2, \mathbf{R}_2^2\}$

$D_4 \downarrow D_2$	A_1	B_1	A_2	B_2
A_1	1	·	·	·
B_1	1	·	·	·
A_2	·	·	1	·
B_2	·	·	1	·
E	·	1	·	1

*Un*Ormal $D_2 = \{1, \mathbf{R}_3^2, \mathbf{i}_3, \mathbf{i}_4\}$

$D_4 \downarrow D_2$	A_1	B_1	A_2	B_2
A_1	1	·	·	·
B_1	·	·	1	·
A_2	·	·	1	·
B_2	1	·	·	·
E	·	1	·	1



$D_2^{Nm}\{\ \mathbf{1},\ \mathbf{R}_z^2,\ \mathbf{R}_x^2,\ \mathbf{R}_y^2\}$

$D_2^{Un}\{\ \mathbf{1},\ \mathbf{R}_z^2,\ \mathbf{i}_3,\ \mathbf{i}_4\}$

A_1	1	1	1	1
B_1	1	-1	1	-1
A_2	1	1	-1	-1
B_2	1	-1	-1	1

$-1_4 =$

$D_4 \downarrow C_4$	0_4	1_4	2_4	3_4
A_1	1	·	·	·
B_1	·	·	1	·
A_2	1	·	·	·
B_2	·	·	1	·
E	·	1	·	1

$\mathbf{r}, \tilde{\mathbf{r}}_i \quad \rho_{xyz} \quad \mathbf{R}, \tilde{\mathbf{R}}_{xyz}$

O	$\mathbf{1}$	$\mathbf{r}$	$\mathbf{R}^2$	$\mathbf{R}^3$	$\mathbf{i}_k$
A_1	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

NOrmal $D_2 = \{1, \mathbf{R}_3^2, \mathbf{R}_1^2, \mathbf{R}_2^2\}$

$O \downarrow D_2$	A_1	B_1	A_2	B_2
A_1	1	·	·	·
A_2	1	·	·	·
E	2	·	·	·
T_1	·	1	1	1
T_2	·	1	1	1

*Un*Ormal $D_2 = \{1, \mathbf{R}_3^2, \mathbf{i}_3, \mathbf{i}_4\}$

$O \downarrow D_2$	A_1	B_1	A_2	B_2
A_1	1	·	·	·
A_2	·	·	1	·
E	1	·	1	·
T_1	·	1	1	1
T_2	1	1	·	1

$O \downarrow D_4$	A_1	B_1	A_2	B_2	E
A_1	1	·	·	·	·
A_2	·	1	·	·	·
E	1	1	·	·	·
T_1	·	·	1	·	1
T_2	·	·	·	1	1

$-1_4 =$

$O \downarrow C_4$	0_4	1_4	2_4	3_4
A_1	1	·	·	·
A_2	·	·	1	·
E	1	·	1	·
T_1	1	1	·	1
T_2	·	1	1	1

$O_h \supset O \supset D_4 \supset C_{4v} \supset C_{2v}$ *subgroup splitting*

$\downarrow C_{4v}$	A'	B'	A''	B''	E
$\mathcal{D}^{A_{1g}}$	1	·	·	·	·
$\mathcal{D}^{A_{2g}}$	·	1	·	·	·
$\mathcal{D}^{E_g}$	1	1	·	·	·
$\mathcal{D}^{T_{1g}}$	·	·	1	·	1
$\mathcal{D}^{T_{2g}}$	·	·	·	1	1
$\mathcal{D}^{A_{1u}}$	·	·	1	·	·
$\mathcal{D}^{A_{2u}}$	·	·	·	1	·
$\mathcal{D}^{E_u}$	·	·	1	1	·
$\mathcal{D}^{T_{1u}}$	1	·	·	·	1
$\mathcal{D}^{T_{2u}}$	·	1	·	·	1

$\downarrow C_{2v}$	A'	B'	A''	B''
$\mathcal{D}^{A_{1g}}$	1	·	·	·
$\mathcal{D}^{A_{2g}}$	·	1	·	·
$\mathcal{D}^{E_g}$	1	1	·	·
$\mathcal{D}^{T_{1g}}$	·	1	1	1
$\mathcal{D}^{T_{2g}}$	1	·	1	1
$\mathcal{D}^{A_{1u}}$	·	·	1	·
$\mathcal{D}^{A_{2u}}$	·	·	·	1
$\mathcal{D}^{E_u}$	·	·	1	1
$\mathcal{D}^{T_{1u}}$	1	1	·	1
$\mathcal{D}^{T_{2u}}$	1	1	1	·

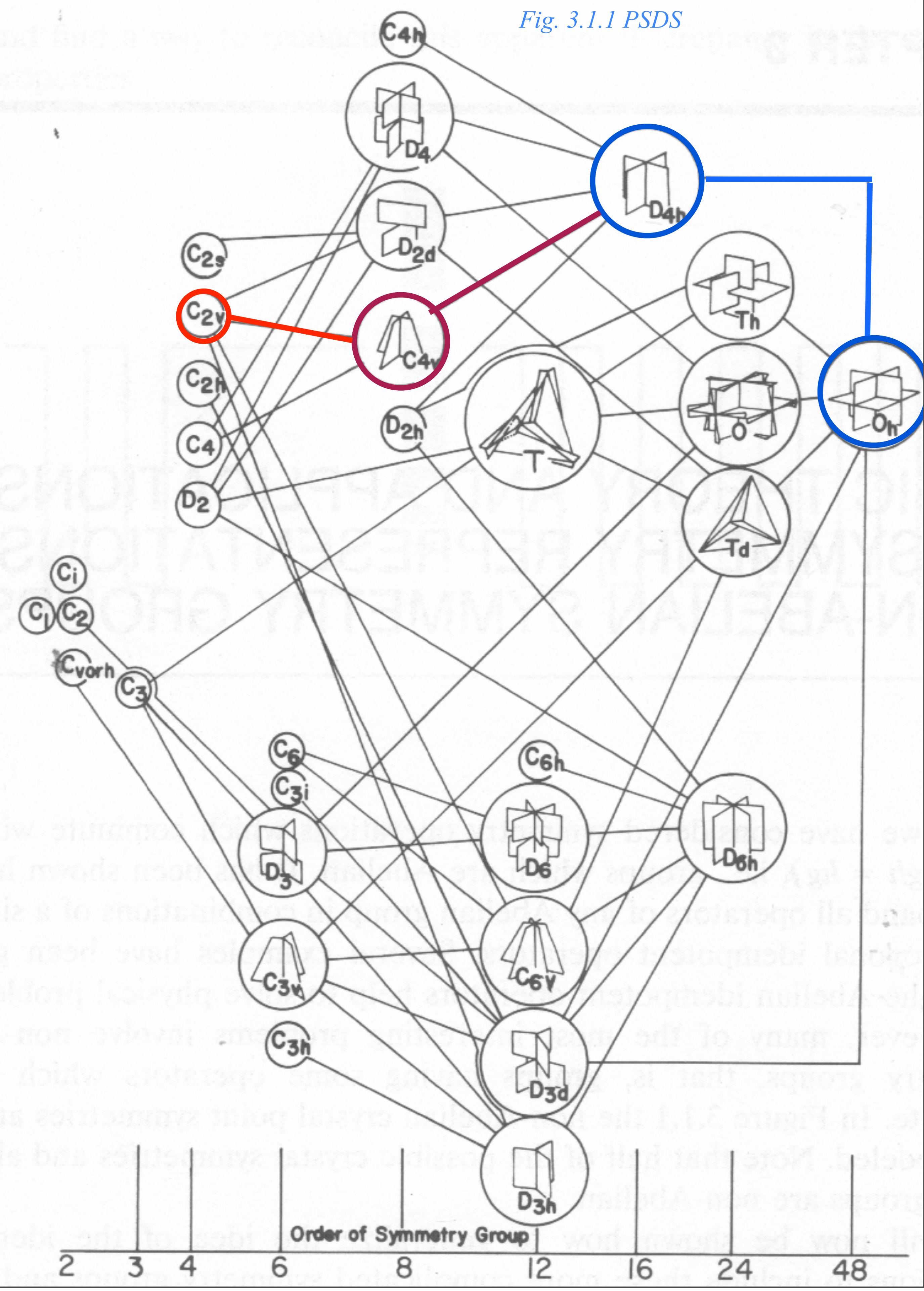


Fig. 3.1.1 PSDS

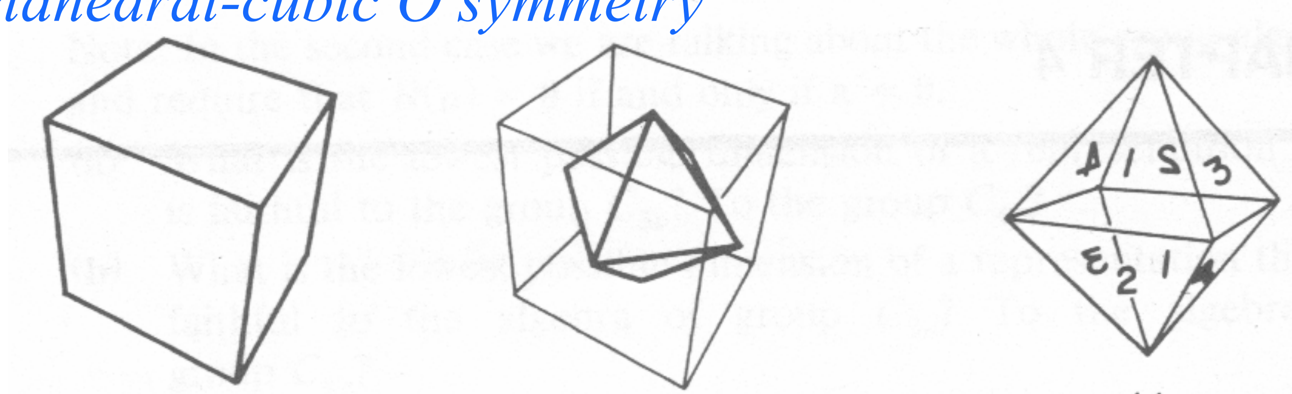
$\mathbf{1}$	$\mathbf{r}_1$	$\mathbf{r}_2$	$\mathbf{r}_3$	$\mathbf{r}_4$	$\tilde{\mathbf{r}}_1$	$\tilde{\mathbf{r}}_2$	$\tilde{\mathbf{r}}_3$	$\tilde{\mathbf{r}}_4$	ρ_x	ρ_y	ρ_z	$\mathbf{R}_x$	$\mathbf{R}_y$	$\mathbf{R}_z$	$\tilde{\mathbf{R}}_x$	$\tilde{\mathbf{R}}_y$	$\tilde{\mathbf{R}}_z$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{i}_4$	$\mathbf{i}_5$	$\mathbf{i}_6$
ρ_z	$\mathbf{r}_3$	$\mathbf{r}_4$	$\mathbf{r}_1$	$\mathbf{r}_2$	$\tilde{\mathbf{r}}_4$	$\tilde{\mathbf{r}}_3$	$\tilde{\mathbf{r}}_2$	$\tilde{\mathbf{r}}_1$	ρ_y	ρ_x	$\mathbf{1}$	$\mathbf{i}_6$	$\mathbf{i}_2$	$\tilde{\mathbf{R}}_z$	$\mathbf{i}_5$	$\mathbf{i}_1$	$\mathbf{R}_z$	$\tilde{\mathbf{R}}_y$	$\mathbf{R}_y$	$\mathbf{i}_4$	$\mathbf{i}_3$	$\tilde{\mathbf{R}}_x$	$\mathbf{R}_x$
$\mathbf{R}_z$	$\mathbf{i}_6$	$\mathbf{i}_5$	$\mathbf{R}_x$	$\tilde{\mathbf{R}}_x$	$\tilde{\mathbf{R}}_y$	$\mathbf{R}_y$	$\mathbf{i}_2$	$\mathbf{i}_1$	$\mathbf{i}_3$	$\mathbf{i}_4$	$\tilde{\mathbf{R}}_z$	$\mathbf{r}_1$	$\tilde{\mathbf{r}}_3$	ρ_z	$\mathbf{r}_2$	$\tilde{\mathbf{r}}_4$	$\mathbf{1}$	$\tilde{\mathbf{r}}_1$	$\tilde{\mathbf{r}}_2$	ρ_y	ρ_x	$\mathbf{r}_4$	$\mathbf{r}_3$
$\tilde{\mathbf{R}}_z$	$\mathbf{R}_x$	$\tilde{\mathbf{R}}_x$	$\mathbf{i}_6$	$\mathbf{i}_5$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{R}_y$	$\tilde{\mathbf{R}}_y$	$\mathbf{i}_4$	$\mathbf{i}_3$	$\mathbf{R}_z$	$\mathbf{r}_3$	$\tilde{\mathbf{r}}_2$	$\mathbf{1}$	$\mathbf{r}_4$	$\tilde{\mathbf{r}}_1$	ρ_z	$\tilde{\mathbf{r}}_4$	$\tilde{\mathbf{r}}_3$	ρ_x	ρ_y	$\mathbf{r}_2$	$\mathbf{r}_1$

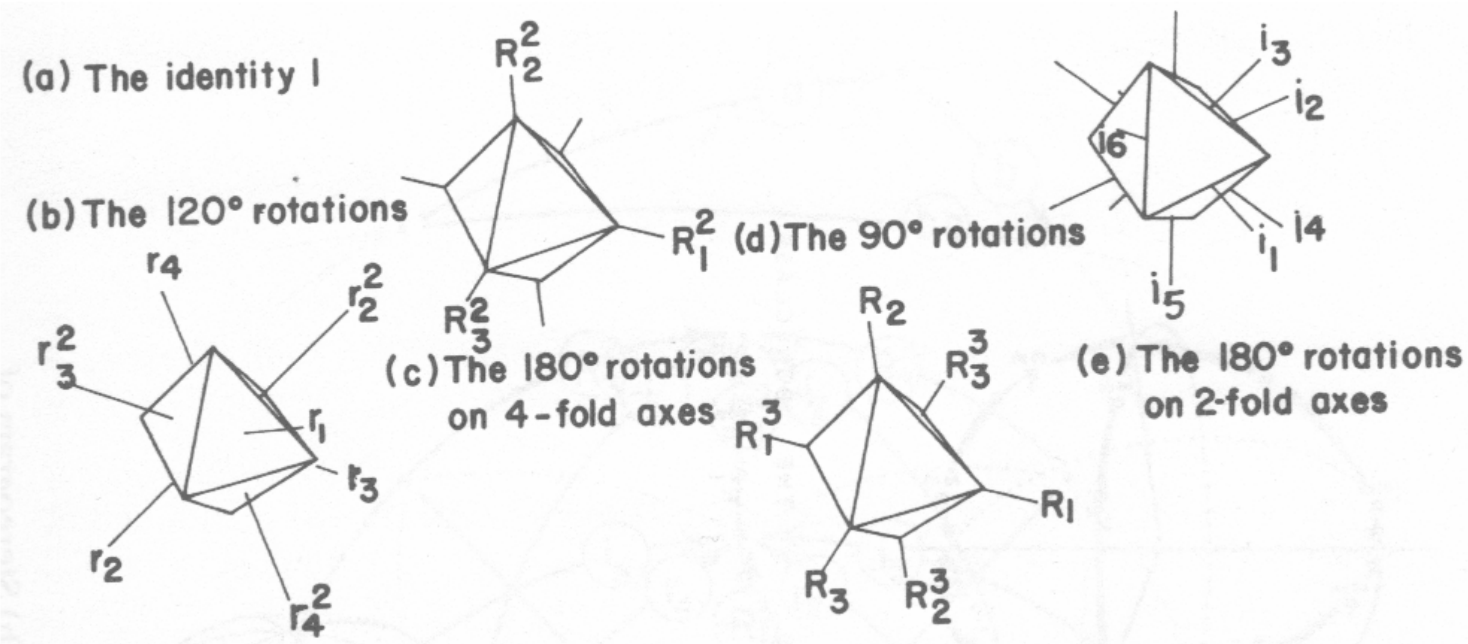
$\mathbf{1}$	$\mathbf{r}_1$	$\mathbf{r}_2$	$\mathbf{r}_3$	$\mathbf{r}_4$	$\tilde{\mathbf{r}}_1$	$\tilde{\mathbf{r}}_2$	$\tilde{\mathbf{r}}_3$	$\tilde{\mathbf{r}}_4$	ρ_x	ρ_y	ρ_z	$\mathbf{R}_x$	$\mathbf{R}_y$	$\mathbf{R}_z$	$\tilde{\mathbf{R}}_x$	$\tilde{\mathbf{R}}_y$	$\tilde{\mathbf{R}}_z$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{i}_4$	$\mathbf{i}_5$	$\mathbf{i}_6$
ρ_z	$\mathbf{r}_3$	$\mathbf{r}_4$	$\mathbf{r}_1$	$\mathbf{r}_2$	$\tilde{\mathbf{r}}_4$	$\tilde{\mathbf{r}}_3$	$\tilde{\mathbf{r}}_2$	$\tilde{\mathbf{r}}_1$	ρ_y	ρ_x	$\mathbf{1}$	$\mathbf{i}_6$	$\mathbf{i}_2$	$\tilde{\mathbf{R}}_z$	$\mathbf{i}_5$	$\mathbf{i}_1$	$\mathbf{R}_z$	$\tilde{\mathbf{R}}_y$	$\mathbf{R}_y$	$\mathbf{i}_4$	$\mathbf{i}_3$	$\tilde{\mathbf{R}}_x$	$\mathbf{R}_x$
$\mathbf{R}_z$	$\mathbf{i}_6$	$\mathbf{i}_5$	$\mathbf{R}_x$	$\tilde{\mathbf{R}}_x$	$\tilde{\mathbf{R}}_y$	$\mathbf{R}_y$	$\mathbf{i}_2$	$\mathbf{i}_1$	$\mathbf{i}_3$	$\mathbf{i}_4$	$\tilde{\mathbf{R}}_z$	$\mathbf{r}_1$	$\tilde{\mathbf{r}}_3$	ρ_z	$\mathbf{r}_2$	$\tilde{\mathbf{r}}_4$	$\mathbf{1}$	$\tilde{\mathbf{r}}_1$	$\tilde{\mathbf{r}}_2$	ρ_y	ρ_x	$\mathbf{r}_4$	$\mathbf{r}_3$
$\tilde{\mathbf{R}}_z$	$\mathbf{R}_x$	$\tilde{\mathbf{R}}_x$	$\mathbf{i}_6$	$\mathbf{i}_5$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{R}_y$	$\tilde{\mathbf{R}}_y$	$\mathbf{i}_4$	$\mathbf{i}_3$	$\mathbf{R}_z$	$\mathbf{r}_3$	$\tilde{\mathbf{r}}_2$	$\mathbf{1}$	$\mathbf{r}_4$	$\tilde{\mathbf{r}}_1$	ρ_z	$\tilde{\mathbf{r}}_4$	$\tilde{\mathbf{r}}_3$	ρ_x	ρ_y	$\mathbf{r}_2$	$\mathbf{r}_1$

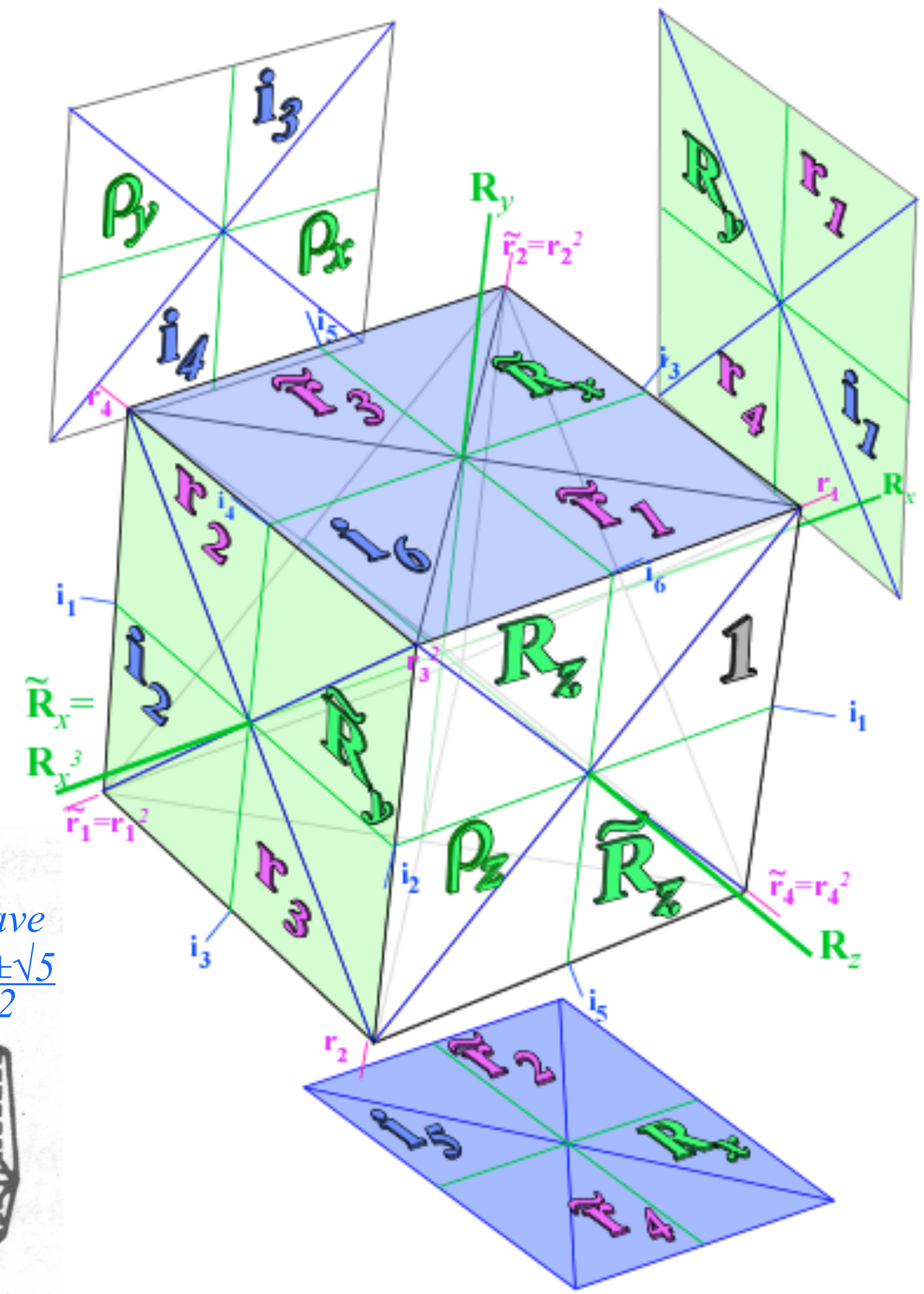
Introduction to octahedral/ tetrahedral symmetry $O_h \supset O \sim T_d \supset T$

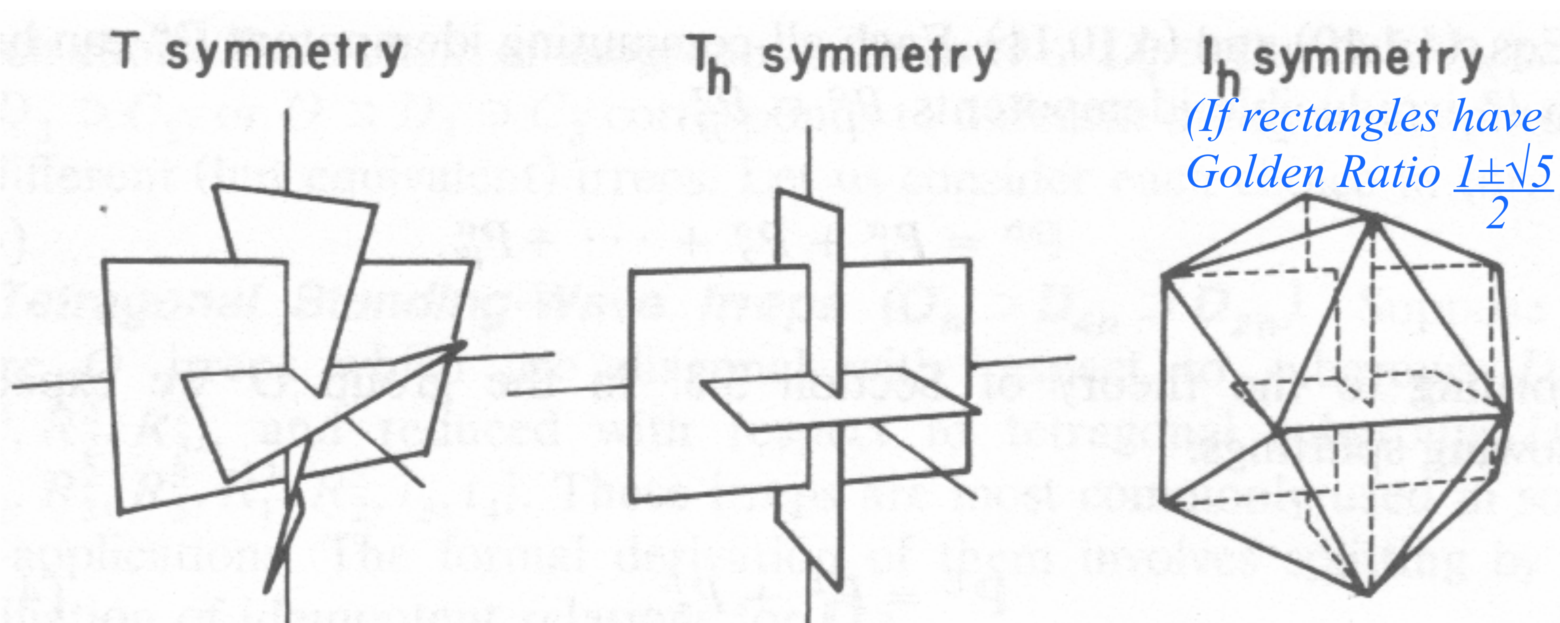
Octahedral-cubic O symmetry

Order °O=6 hexahedron squares · 4 pts =24
=8 octahedron triangles · 3 pts =24
=12 lines · 2 pts =24 positions









Introduction to octahedral tetrahedral symmetry $O_h \supset O \sim T_d \supset T$

Octahedral groups $O_h \supset O \sim T_d$ and $O_h \supset T_h \supset T$

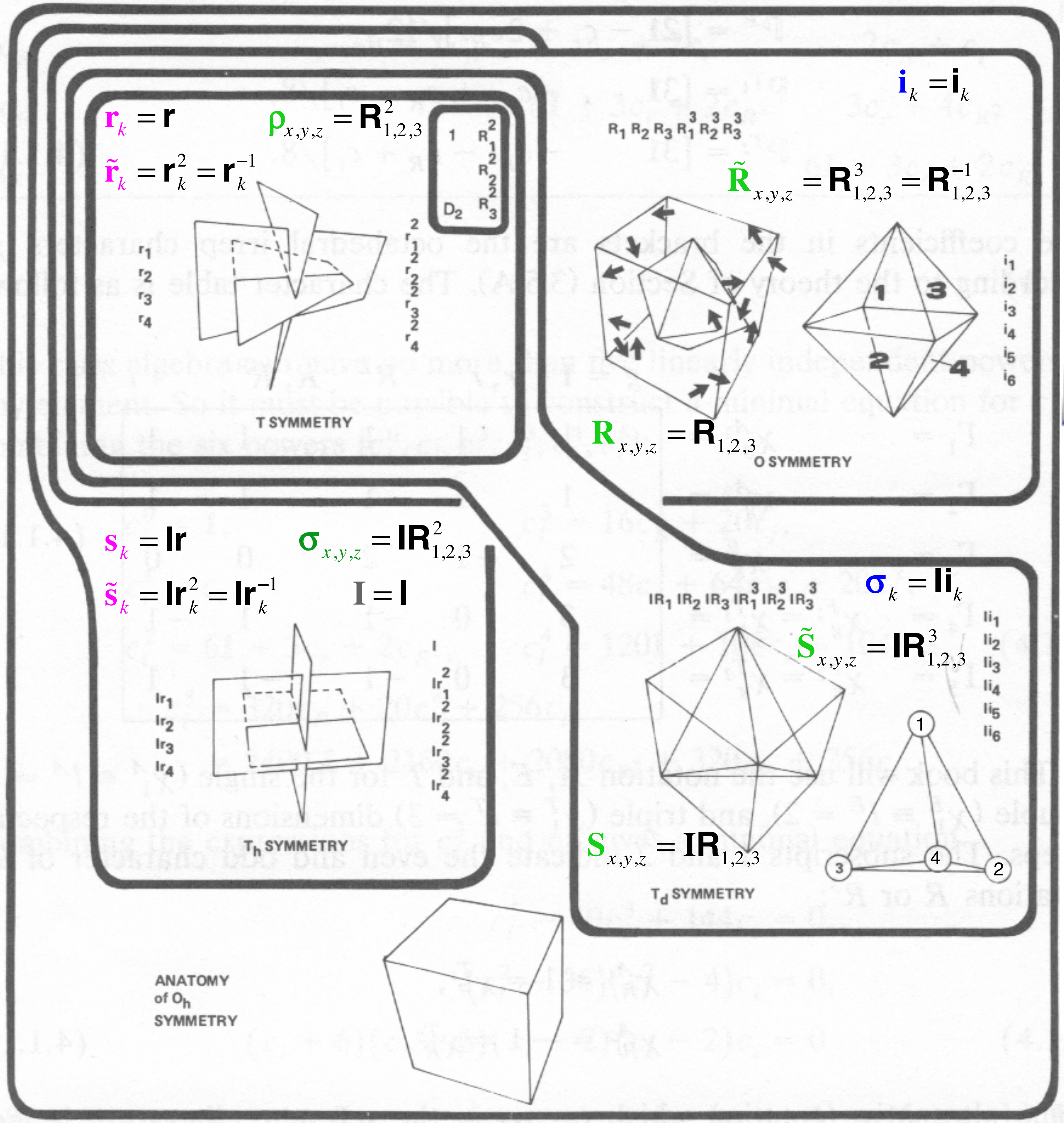


Figure 4.1.5 The full octahedral group (O_h) and four non-Abelian subgroups T, T_h, T_d, and O. The Abelian D_2 subgroup of T is indicated also.

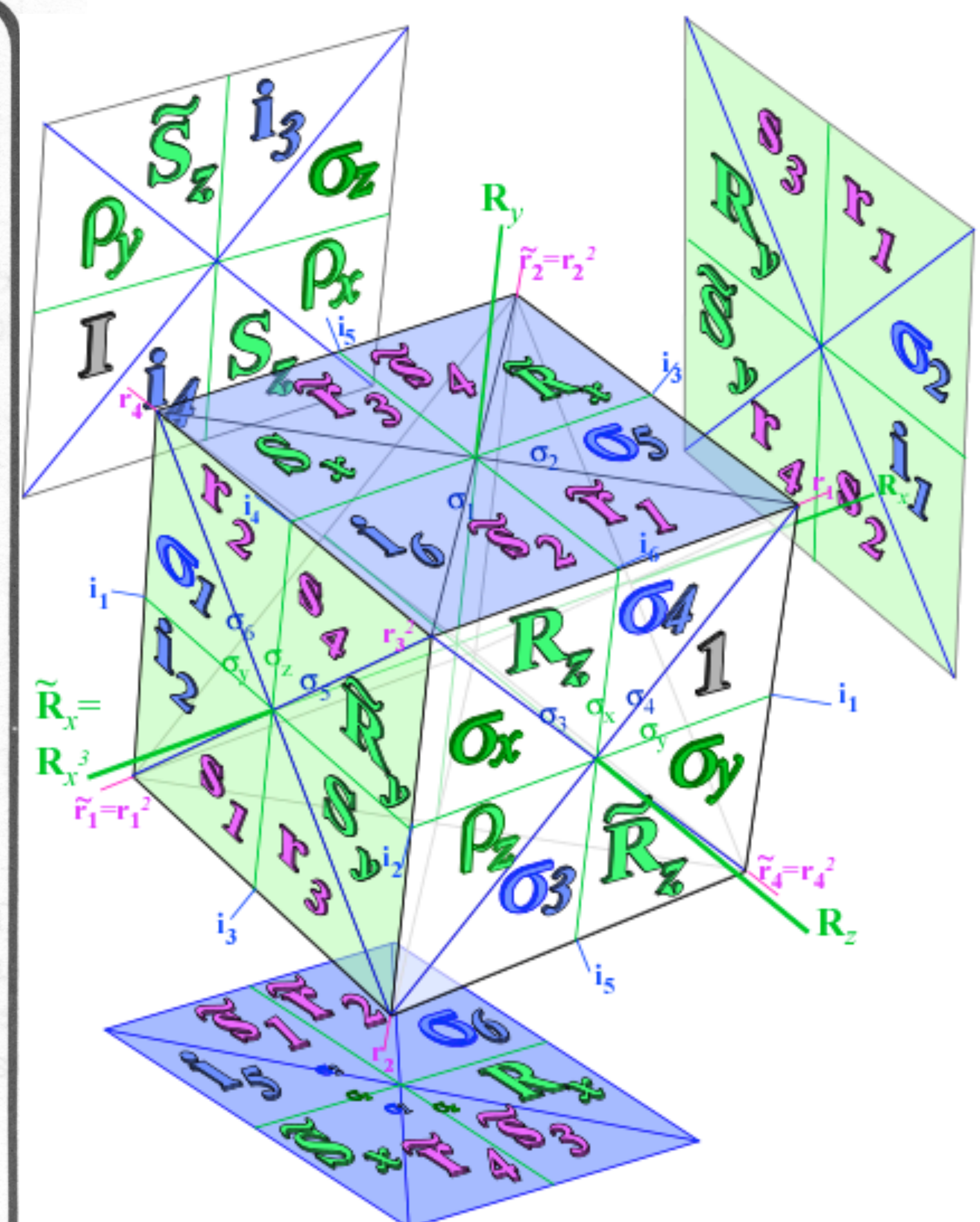
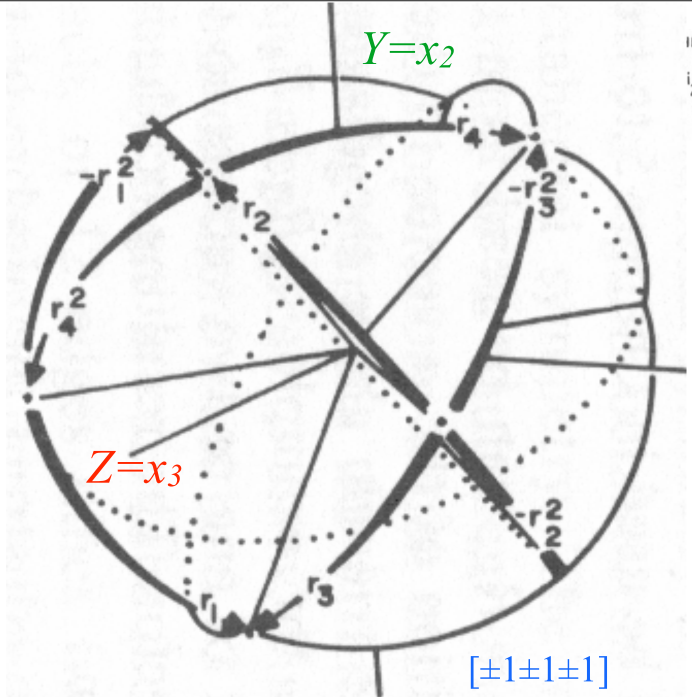
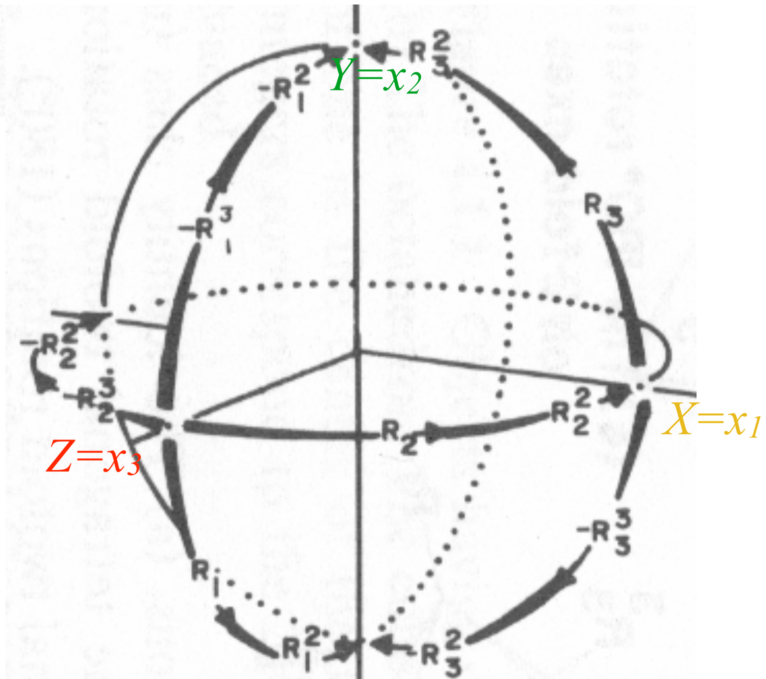
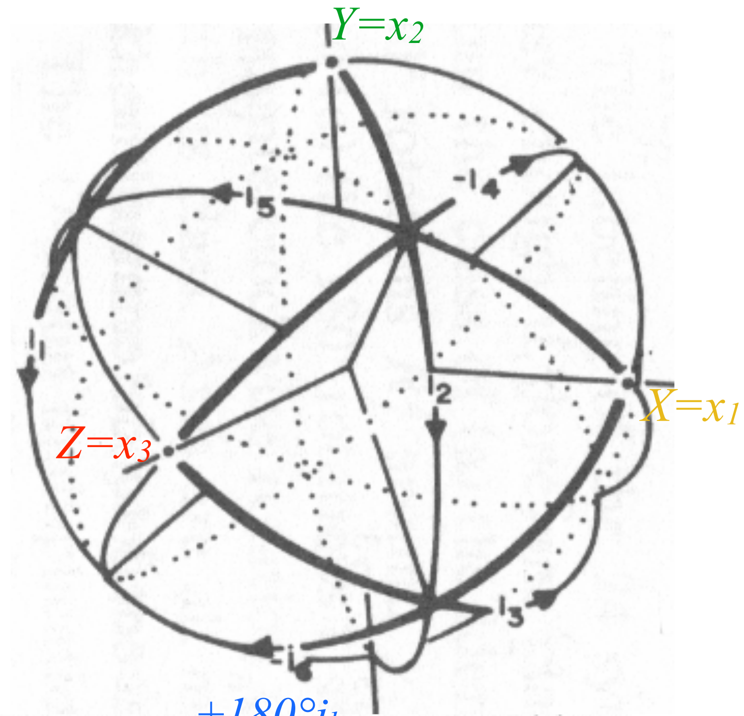


Fig. 4.1.5 from Principles of Symmetry, Dynamics and Spectroscopy







+120°: $[1\,1\,1]$ $[\bar{1}\,\bar{1}\,1]$ $[1\,\bar{1}\,\bar{1}]$ $[\bar{1}\,1\,\bar{1}]$ -120°: $[\bar{1}\,\bar{1}\,\bar{1}]$ $[1\,1\,\bar{1}]$ $[\bar{1}\,1\,1]$ $[1\,\bar{1}\,1]$ ±180° XYZ: $[1\,0\,0]$ $[0\,1\,0]$ $[0\,0\,1]$ +90° XYZ: $[1\,0\,0]$ $[0\,1\,0]$ $[0\,0\,1]$ -90° XYZ: $[\bar{1}\,0\,0]$ $[0\,\bar{1}\,0]$ $[0\,0\,\bar{1}]$ ±180°i_k: $[1\,0\,1]$ $[1\,0\,\bar{1}]$ $[1\,1\,0]$ $[1\,\bar{1}\,0]$ $[0\,1\,\bar{1}]$ $[0\,1\,1]$

1	r_1	r_2	r_3	r_4	r_1^2	r_2^2	r_3^2	r_4^2	R_1^2	R_2^2	R_3^2	R_1	R_2	R_3	R_1^3	R_2^3	R_3^3	i_1	i_2	i_3	i_4	i_5	i_6
r_1	r_1^2	$-r_4^2$	$-r_2^2$	$-r_3^2$	-1	$-R_2^2$	$-R_3^2$	$-R_1^2$	$-r_2$	$-r_3$	$-r_4$	i_3	i_6	i_1	$-R_3$	$-R_1$	$-R_2$	R_1^3	i_5	R_2^3	i_2	$-i_4$	R_3^3
r_2	$-r_3^2$	r_2^2	$-r_4^2$	$-r_1^2$	R_2^2	-1	R_1^2	$-R_3^2$	r_1	r_4	$-r_3$	R_3	$-R_1^3$	i_2	i_3	$-i_5$	R_2^3	i_6	$-R_1$	R_2	$-i_1$	R_3^3	i_4
r_3	$-r_4^2$	$-r_1^2$	r_3^2	$-r_2^2$	R_3^2	$-R_1^2$	-1	R_2^2	$-r_4$	r_1	r_2	$-i_4$	R_1	$-R_2^3$	R_3^3	i_6	i_2	i_5	$-R_1^3$	i_1	R_2	$-i_3$	R_3
r_4	$-r_2^2$	$-r_3^2$	$-r_1^2$	r_4^2	R_1^2	R_3^2	$-R_2^2$	-1	r_3	$-r_2$	r_1	$-R_3^3$	$-i_5$	R_2	$-i_4$	R_1^3	i_1	R_1	i_6	$-i_2$	R_2^3	R_3	i_3
r_1^2	-1	R_1^2	R_2^2	R_3^2	$-r_1$	r_3	r_4	r_2	r_4^2	r_2^2	r_3^2	R_2^3	R_3^3	R_1^3	$-i_1$	$-i_3$	$-i_6$	$-R_3$	$-i_4$	$-R_1$	i_5	$-i_2$	$-R_2$
r_2^2	$-R_1^2$	-1	R_3^2	$-R_2^2$	r_4	$-r_2$	r_1	r_3	$-r_3^2$	$-r_1^2$	r_4^2	i_2	$-i_3$	$-R_1$	R_2	$-R_3^3$	$-i_5$	i_4	$-R_3$	$-R_1^3$	$-i_6$	R_2^3	$-i_1$
r_3^2	$-R_2^2$	$-R_3^2$	-1	R_1^2	r_2	r_4	$-r_3$	r_1	r_2^2	$-r_4^2$	$-r_1^2$	$-R_2$	$-i_4$	$-i_6$	i_2	R_3	$-R_1^3$	$-i_3$	$-R_3^3$	i_5	R_1	$-i_1$	$-R_2^3$
r_4^2	$-R_3^2$	R_2^2	$-R_1^2$	-1	r_3	r_1	r_2	$-r_4$	$-r_1^2$	r_3^2	$-r_2^2$	$-i_1$	$-R_3$	$-i_5$	$-R_2^3$	$-i_4$	R_1	$-R_3^3$	i_3	$-i_6$	R_1^3	R_2	$-i_2$
R_1^2	$-r_4$	r_3	$-r_2$	r_1	r_2^2	$-r_1^2$	r_4^2	$-r_3^2$	-1	R_3^2	$-R_2^2$	R_1^3	i_1	$-i_4$	$-R_1$	i_2	$-i_3$	$-R_2$	$-R_2^3$	R_3^3	R_3	$-i_6$	i_5
R_2^2	$-r_2$	r_1	r_4	$-r_3$	r_3^2	$-r_4^2$	$-r_1^2$	r_2^2	$-R_3^2$	-1	R_1^2	$-i_5$	R_2^3	i_3	$-i_6$	$-R_2$	$-i_4$	$-i_2$	i_1	$-R_3$	R_3^3	R_1	R_1^3
R_3^2	$-r_3$	$-r_4$	r_1	r_2	r_4^2	r_3^2	$-r_2^2$	$-r_1^2$	R_2^2	$-R_1^2$	-1	i_6	i_2	R_3^3	$-i_5$	$-i_1$	$-R_3$	R_2^3	$-R_2$	i_4	$-i_3$	R_1^3	$-R_1$
R_1	i_1	$-R_2^3$	$-i_2$	R_2	R_3^3	$-i_3$	$-R_3$	i_4	R_1^3	i_6	i_5	R_1^2	r_1	$-r_4^2$	-1	$-r_3$	r_2^2	$-r_4$	r_2	r_1^2	$-r_3^2$	$-R_2^2$	R_3^2
R_2	i_3	R_3	$-R_3^3$	i_4	R_1^3	i_5	$-i_6$	$-R_1$	$-i_2$	R_2^3	i_1	$-r_2^2$	R_2^2	r_1	r_3^2	-1	$-r_4$	R_1^2	R_3^2	$-r_2$	$-r_3$	$-r_4^2$	r_1^2
R_3	i_6	i_5	R_1	$-R_1^3$	R_2^3	$-R_2$	$-i_2$	$-i_1$	i_3	i_4	R_3^3	r_1	$-r_3^2$	R_3^2	$-r_2$	r_4^2	-1	r_1^2	r_2^2	R_2^2	$-R_1^2$	$-r_4$	$-r_3$
R_1^3	$-R_2$	$-i_2$	R_2^3	i_1	$-i_3$	$-R_3^3$	i_4	R_3	$-R_1$	i_5	$-i_6$	-1	$-r_4$	r_3^2	$-R_1^2$	r_2	$-r_1^2$	$-r_1$	r_3	r_2^2	$-r_4^2$	$-R_3^2$	$-R_2^2$
R_2^3	$-R_3$	i_3	i_4	R_3^3	$-i_6$	R_1	$-R_1^3$	i_5	$-i_1$	$-R_2$	$-i_2$	r_4^2	-1	$-r_2$	$-r_1^2$	$-R_2^2$	r_3	$-R_3^2$	R_1^2	$-r_1$	$-r_4$	$-r_2^2$	r_3^2
R_3^3	$-R_1$	R_1^3	i_6	i_5	$-i_1$	$-i_2$	R_2	$-R_2^3$	i_4	$-i_3$	$-R_3$	$-r_3$	r_2^2	-1	r_4	$-r_1^2$	$-R_3^2$	r_4^2	r_3^2	$-R_1^2$	$-R_2^2$	$-r_2$	$-r_1$
i_1	R_3^3	$-i_4$	i_3	R_3	$-R_1$	$-i_6$	$-i_5$	$-R_1^3$	R_2^3	i_2	$-R_2$	r_1^2	R_3^2	$-r_4$	r_4^2	$-R_1^2$	$-r_1$	-1	$-R_2^2$	$-r_3$	r_2	r_3^2	r_2^2
i_2	i_4	R_3^3	R_3	$-i_3$	$-i_5$	R_1^3	R_1	$-i_6$	R_2	$-i_1$	R_2^3	$-r_3^2$	$-R_1^2$	$-r_3$	$-r_2^2$	$-R_3^2$	$-r_2$	R_2^2	-1	r_4	$-r_1$	r_1^2	r_4^2
i_3	R_1^3	R_1	$-i_5$	i_6	$-R_2$	$-R_2^3$	$-i_1$	i_2	$-R_3$	R_3^3	$-i_4$	$-r_2$	r_1^2	R_1^2	$-r_1$	r_2^2	$-R_2^2$	r_3^2	$-r_4^2$	-1	R_3^2	r_3	$-r_4$
i_4	$-i_5$	i_6	$-R_1^3$	$-R_1$	$-i_2$	i_1	$-R_2^3$	$-R_2$	$-R_3^3$	$-R_3$	i_3	r_4	r_4^2	R_2^2	r_3	r_3^2	R_1^2	$-r_2^2$	r_1^2	$-R_3^3$	-1	r_1	$-r_2$
i_5	i_2	$-R_2$	i_1	$-R_2^3$	i_4	$-R_3$	i_3	$-R_3^3$	i_6	$-R_1^3$	$-R_1$	R_3^2	r_2	r_2^2	R_2^2	r_4	r_4^2	$-r_3$	$-r_1$	$-r_3^2$	$-r_1^2$	-1	$-R_1^2$
i_6	R_2^3	i_1	R_2	i_2	$-R_3$	$-i_4$	$-R_3^3$	$-i_3$	$-i_5$	$-R_1$	R_1^3	R_2^2	$-r_3$	r_1^2	$-R_3^2$	$-r_1$	r_3^2	$-r_2$	$-r_4$	r_4^2	r_2^2	R_1^2	-1

Octahedral O and spin-O⊂U(2) rotation product Table F.2.1 from Principles of Symmetry, Dynamics and Spectroscopy