Lecture 8
Mon. 9.23.2019

Quadratic form geometry and development of mechanics of Lagrange and Hamilton

(Ch. 12 of Unit 1 and Ch. 4-5 of Unit 7)

Review of partial differential calculus
Chain rule and order $\partial^2\Psi/\partial x\partial y = \partial^2\Psi/\partial y\partial x$ symmetry

Scaling transformation between Lagrangian and Hamiltonian views of KE
Introducing 0^{th} Lagrange and 0^{th} Hamilton differential equations of mechanics
Introducing 1^{st} Lagrange and 1^{st} Hamilton differential equations of mechanics

Introducing the Poincare´ and Legendre contact transformations
Geometry of Legendre contact transformation (Preview of Unit 8 relativistic quantum mechanics)
Example from thermodynamics
Legendre transform: special case of General Contact Transformation (lights,camera, ACTION!)

An elementary contact transformation from sophomore physics
Algebra-calculus development of "The Volcanoes of Io" and "The Atoms of NIST"
Intuitive-geometric development of " " " and " " "

Link ⇒ CoulIt - Simulation of the Volcanoes of Io

Link ⇒ RelaWavity - Physical Terms H(p) & L(u)

This Lecture's Reference Link Listing

Web Resources - front page
UAF Physics UTube channel

Quantum Theory for the Computer Age
Principles of Symmetry, Dynamics, and Spectroscopy
Classical Mechanics with a Bang!
Modern Physics and its Classical Foundations

2017 Group Theory for QM
2018 Adv CM
2018 AMOP
2019 Advanced Mechanics

Lecture #8

"RelaWavity" Web Simulations:
2-CW laser wave, Lagrangian vs Hamiltonian,
Physical Terms Lagrangian L(u) vs Hamiltonian H(p)
CoulIt Web Simulation of the Volcanoes of Io
BohrIt Multi-Panel Plot:
Relativistically shifted Time-Space plots of 2 CW light waves

NASA Astronomy Picture of the Day -
Io: The Prometheus Plume *(Just Image)*
NASA Galileo - *Io's Alien Volcanoes*
New Horizons - *Volcanic Eruption Plume on Jupiter's moon IO*
NASA Galileo - A Hawaiian-Style Volcano on Io

AMOP Ch 0 Space-Time Symmetry - 2019
Seminar at Rochester Institute of Optics, Aux. slides-2018

Select, exciting, and related Research & Articles of Interest:

These *are* hot off the presses. Out in MISC for quick reference.
Burning a hole in reality—design for a new laser may be powerful enough to pierce space-time - Sumner-Daily KOS-2019
Trampoline mirror may push laser pulse through fabric of the Universe - Lee-ArsTechnica-2019
Achieving_Extreme_Light_Intensities_using_Optically_Curved_Relativistic_Plasma_Mirrors_-_Vincenti-prl-2019
A_Soft_Matter_Computer_for_Soft_Robots_-_Garrad-sr-2019
Correlated_Insulator_Behaviour_at_Half-Filling_in_Magic-Angle_Graphene_Superlattices_-_cao-n-2018

Sorting_ultracold_atoms_in_a_three-dimensional_optical_lattice_in_a_realization_of_Maxwell's_Demon - Kumar-n-2018
Synthetic_three-dimensional_atomic_structures_assembled_atom_by_atom - Barredo-n-2018

Older ones:
Wave-particle_duality_of_C60_molecules - Arndt-ltn-1999
Optical_Vortex_Knots_-_One_Photon__At_A_Time - Tempone-Wiltshire-Sr-2018
Baryon_Deceleration_by_Strong_Chromofields_in_Ultrarelativistic_, Nuclear_Collisions - Mishustin-PhysRevC-2007, *APS Link & Abstract*
Hadronic Molecules - Guo-x-2017
Hidden-charm_pentaquark_and_tetraquark_states - Chen-pr-2016

Running Reference Link Listing

Lectures #7 through #7

In reverse order

BoxIt Web Simulations:

- *Generic/Default*
- *Most Basic A-Type*
- *Basic A-Type w/reference lines*
- *Basic A-Type A-Type with Potential energy*
- *A-Type with Potential energy and Stokes Plot*
- *A-Type w/3 time rates of change*
- *A-Type w/3 time rates of change with Stokes Plot*
- *B-Type (A=1.0, B=-0.05, C=0.0, D=1.0)*

RelaWavity Web Elliptical Motion Simulations:

- *Orbits with b/a=0.125*
- *Orbits with b/a=0.5*
- *Orbits with b/a=0.7*
- *Exegesis with b/a=0.125*
- *Exegesis with b/a=0.5*
- *Exegesis with b/a=0.7*
- *Contact Ellipsometry*

Pirelli Site: Phasors animimation

CMwithBang Lecture #6, page=70 (9.10.18)

Running Reference Link Listing

Lectures #6 through #1

In reverse order

RelaWavity Web Simulation: Contact Ellipsometry
BoxIt Web Simulation: Elliptical Motion (A-Type)
CMwBang Course: Site Title Page
Pirelli Relativity Challenge: Describing Wave Motion With Complex Phasors
UAF Physics UTube channel

Velocity Amplification in Collision Experiments Involving Superballs - Harter, 1971
MIT OpenCourseWare: High School/Physics/Impulse and Momentum
Hubble Site: Supernova - SN 1987A

BounceIt Web Animation - Scenarios:
Generic Scenario: 2-Balls dropped no Gravity (7:1) - V vs V Plot (Power=4)
1-Ball dropped w/Gravity=0.5 w/Potential Plot: Power=1, Power=4
7:1 - V vs V Plot: Power=1
3-Ball Stack (10:3:1) w/Newton plot (y vs t) - Power=4
3-Ball Stack (10:3:1) w/Newton plot (y vs t) - Power=1
3-Ball Stack (10:3:1) w/Newton plot (y vs t) - Power=1 w/Gaps
4-Ball Stack (27:9:3:1) w/Newton plot (y vs t) - Power=4
4-Newton's Balls (1:1:1:1) w/Newtonian plot (y vs t) - Power=4 w/Gaps
6-Ball Totally Inelastic (1:1:1:1:1:1) w/Gaps: Newtonian plot (t vs x), V6 vs V5 plot
5-Ball Totally Inelastic Pile-up w/ 5-Stationary-Balls - Minkowski plot (t vs x1) w/Gaps
1-Ball Totally Inelastic Pile-up w/ 5-Stationary-Balls - Vx2 vs Vx1 plot w/Gaps

BounceItIt Web Animation - Scenarios:
49:1 y vs t, 49:1 V2 vs V1, 1:500:1 - 1D Gas Model w/ faux restorative force (Cool),
1:500:1 - 1D Gas (Warm), 1:500:1 - 1D Gas Model (Cool, Zoomed in),
Farey Sequence - Wolfram
Fractions - Ford-AMM-1938
Monstermash BounceItIt Animations:
1000:1 - V2 vs V1, 1000:1 with t vs x - Minkowski Plot
Quantum Revivals of Morse Oscillators and Farey-Ford Geometry - Li-Harter-2013
Quantum_Revivals_of_Morse_Oscillators_and_Farey-Ford_Geometry - Li-Harter-cpl-2015
Quant. Revivals of Morse Oscillators and Farey-Ford Geom. - Harter-Li-CPL-2015 (Publ.)
Velocity_Amplification_in_Collision_Experiments_Involving_Superballs-Harter-1971
WaveIt Web Animation - Scenarios:
Quantum_Carpet, Quantum_Carpet_wMBars,
Quantum_Carpet_BCar, Quantum_Carpet_BCar_wMBars
Wave Node Dynamics and Revival Symmetry in Quantum Rotors - Harter-JMS-2001
Wave Node Dynamics and Revival Symmetry in Quantum Rotors - Harter-jms-2001 (Publ.)

BounceIt Dual plots
m_1:m_2 = 3:1
v_2 vs v_1 and V_2 vs V_1, $(v_1, v_2)=(1, 0.1)$, $(v_1, v_2)=(1, 0)$
y_2 vs y_1 plots: $(v_1, v_2)=(1, 0.1)$, $(v_1, v_2)=(1, 0)$, $(v_1, v_2)=(1, -1)$
Estrangian plot V_2 vs V_1: $(v_1, v_2)=(0, 1)$, $(v_1, v_2)=(1, -1)$
m_1:m_2 = 4:1
v2 vs v1, y2 vs y1
m_1:m_2 = 100:1, $(v_1, v_2)=(1, 0)$: V2 vs V1 Estrangian plot, y2 vs y1 plot

With g=0 and 70:10 mass ratio
With non zero g, velocity dependent damping and mass ratio of 70:35
M_1=49, M_2=1 with Newtonian time plot
M_1=49, M_2=1 with V_2 vs V_1 plot
Example with friction
Low force constant with drag displaying a Pass-thru, Fall-Thru, Bounce-Off
m1:m2= 3:1 and (v1, v2) = (1, 0) Comparison with Estrangian

AJP article on superball dynamics
AAPT Summer Reading List
Scitation.org - AIP publications
HarterSoft Youtube Channel

X2 paper: Velocity Amplification in Collision Experiments Involving Superballs - Harter, et. al. 1971 (pdf)
Car Collision Web Simulator: https://modphys.hosted.uark.edu/markup/CMMotionWeb.html
Superball Collision Web Simulator: https://modphys.hosted.uark.edu/markup/BounceItWeb.html; with Scenarios: 1007
BounceIt web simulation with g=0 and 70:10 mass ratio
With non zero g, velocity dependent damping and mass ratio of 70:35
Elastic Collision Dual Panel Space vs Space: Space vs Time (Newton) , Time vs. Space(Minkowski)
Inelastic Collision Dual Panel Space vs Space: Space vs Time (Newton), Time vs. Space(Minkowski)
Matrix Collision Simulator: M_1=49, M_2=1 V_2 vs V_1 plot <<Under Construction>>

More Advanced QM and classical references at the end of this Lecture

Review of partial differential calculus

Chain rule and order $\partial^2\Psi/\partial x\partial y = \partial^2\Psi/\partial y\partial x$ symmetry

Scaling transformation between Lagrangian and Hamiltonian views of KE

Introducing 0^{th} Lagrange and 0^{th} Hamilton differential equations of mechanics

Introducing 1^{st} Lagrange and 1^{st} Hamilton differential equations of mechanics

Introducing the Poincare´ and Legendre contact transformations

Geometry of Legendre contact transformation (Preview of Unit 8 relativistic quantum mechanics)

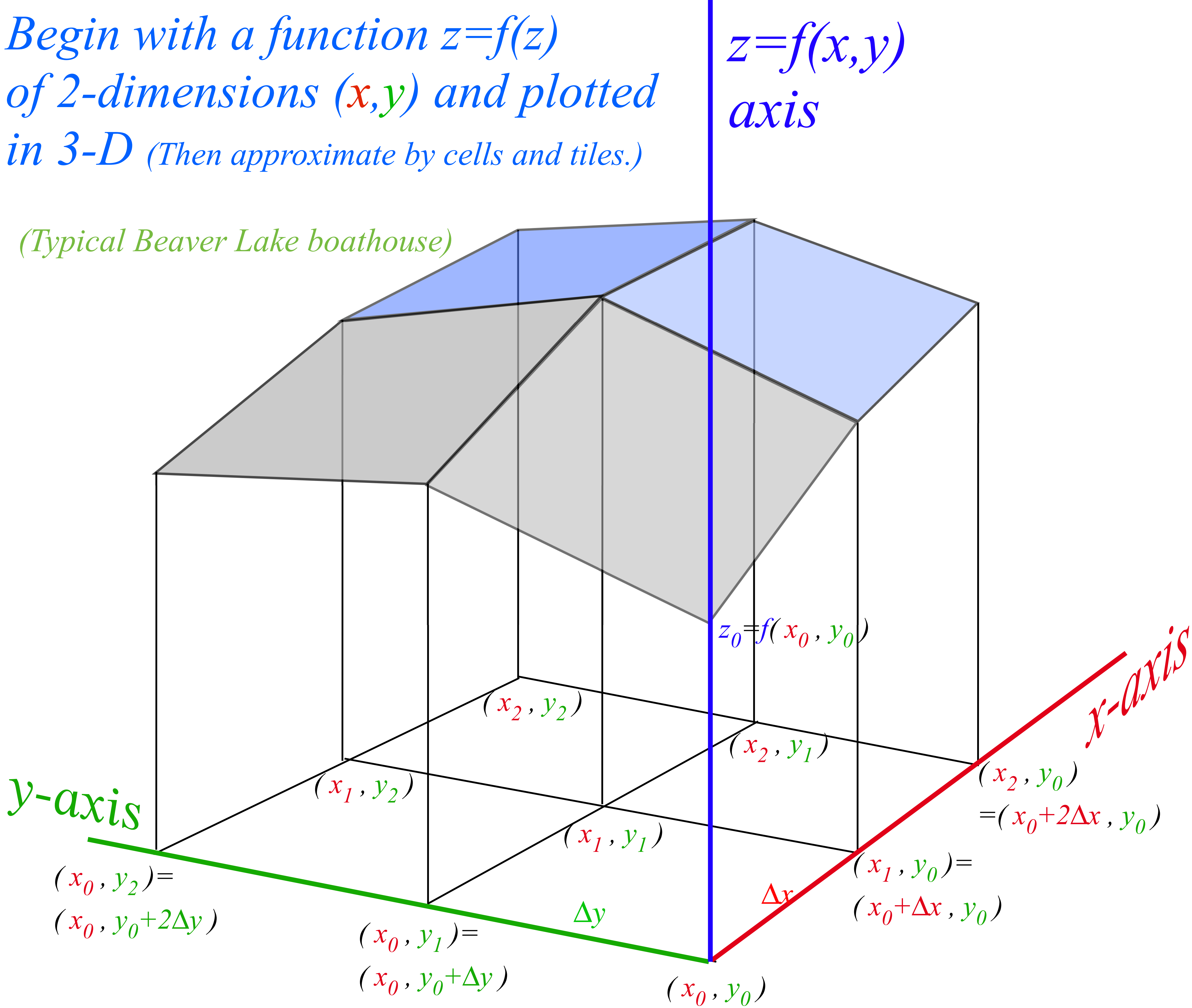
Example from thermodynamics

Legendre transform: special case of General Contact Transformation (lights,camera, ACTION!)

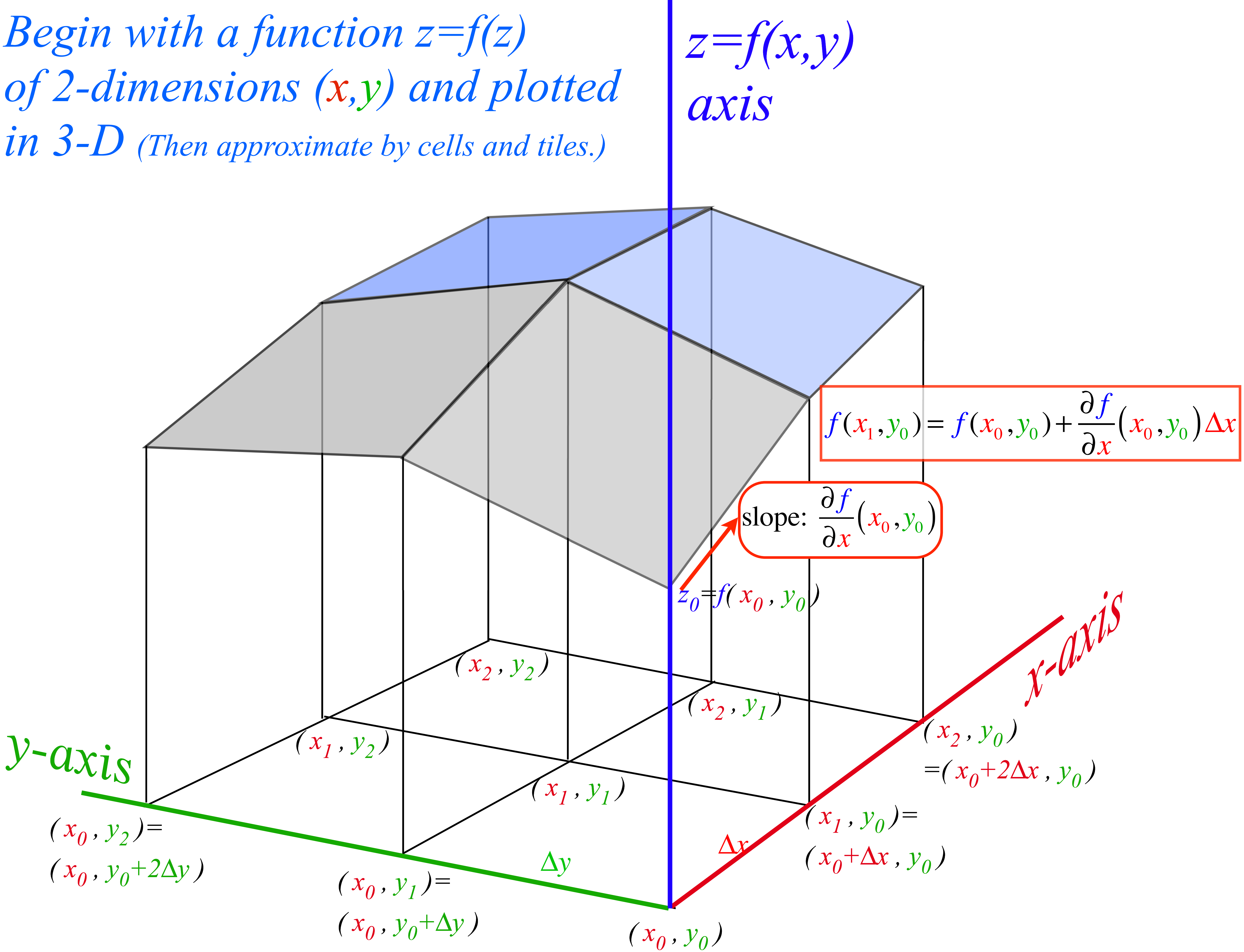
An elementary contact transformation from sophomore physics

Algebra-calculus development of "The Volcanoes of Io" and "The Atoms of NIST"

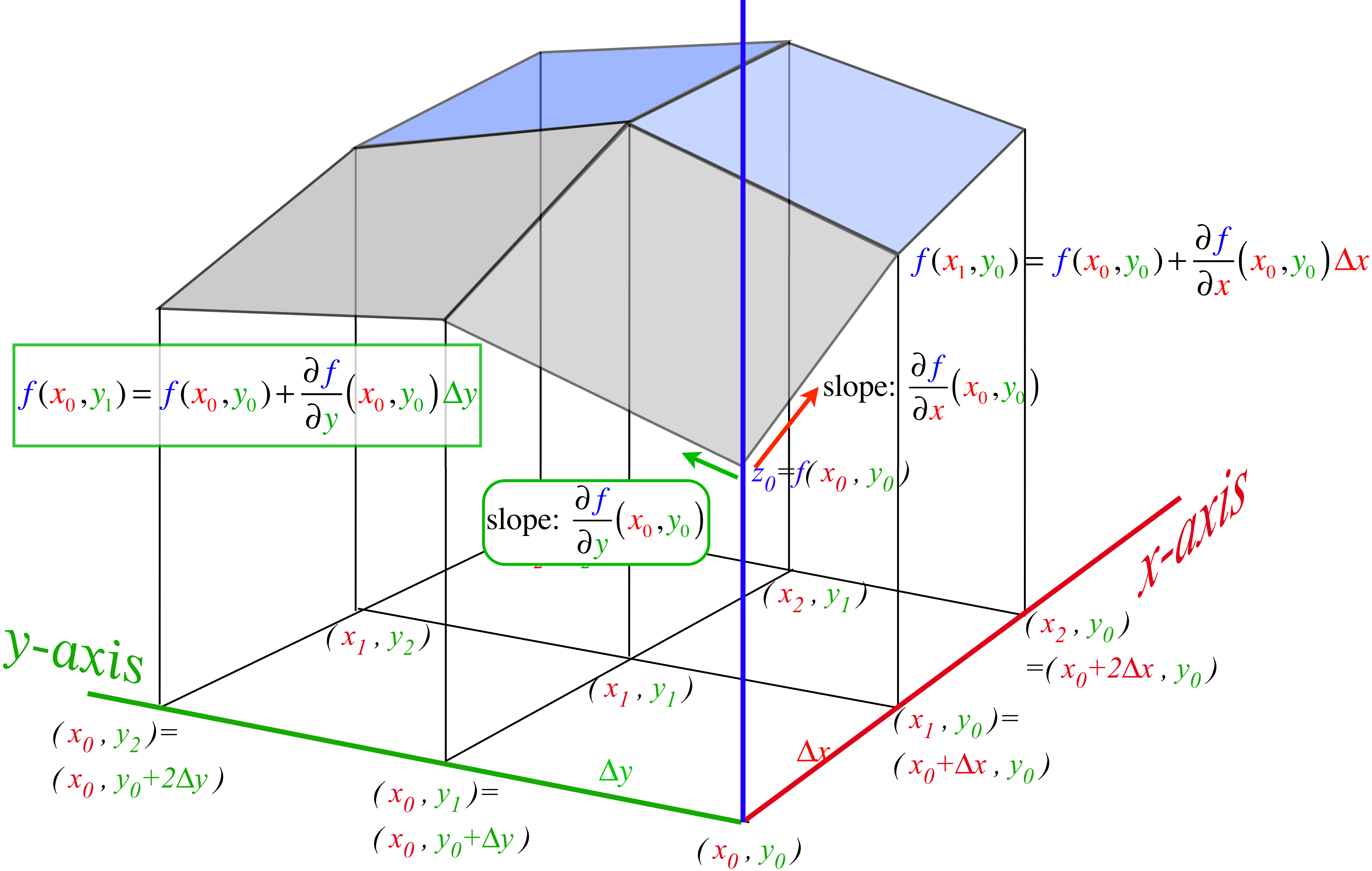
Intuitive-geometric development of " " " and " " "


Begin with a function z=f(z)
of 2-dimensions (x,y) and plotted
in 3-D (Then approximate by cells and tiles.)
z=f(x,y)
axis
(Typical Beaver Lake boathouse)
z0=f(x0 , y0)
(x2 , y2)
(x2 , y1)
(x1 , y2)
(x2 , y0)
=(x0+2Δx , y0)
x-axis
y-axis
(x1 , y1)
(x1 , y0)=
(x0+Δx , y0)
Δx
Δy
(x0 , y2)=
(x0 , y0+2Δy)
(x0 , y1)=
(x0 , y0+Δy)
(x0 , y0)

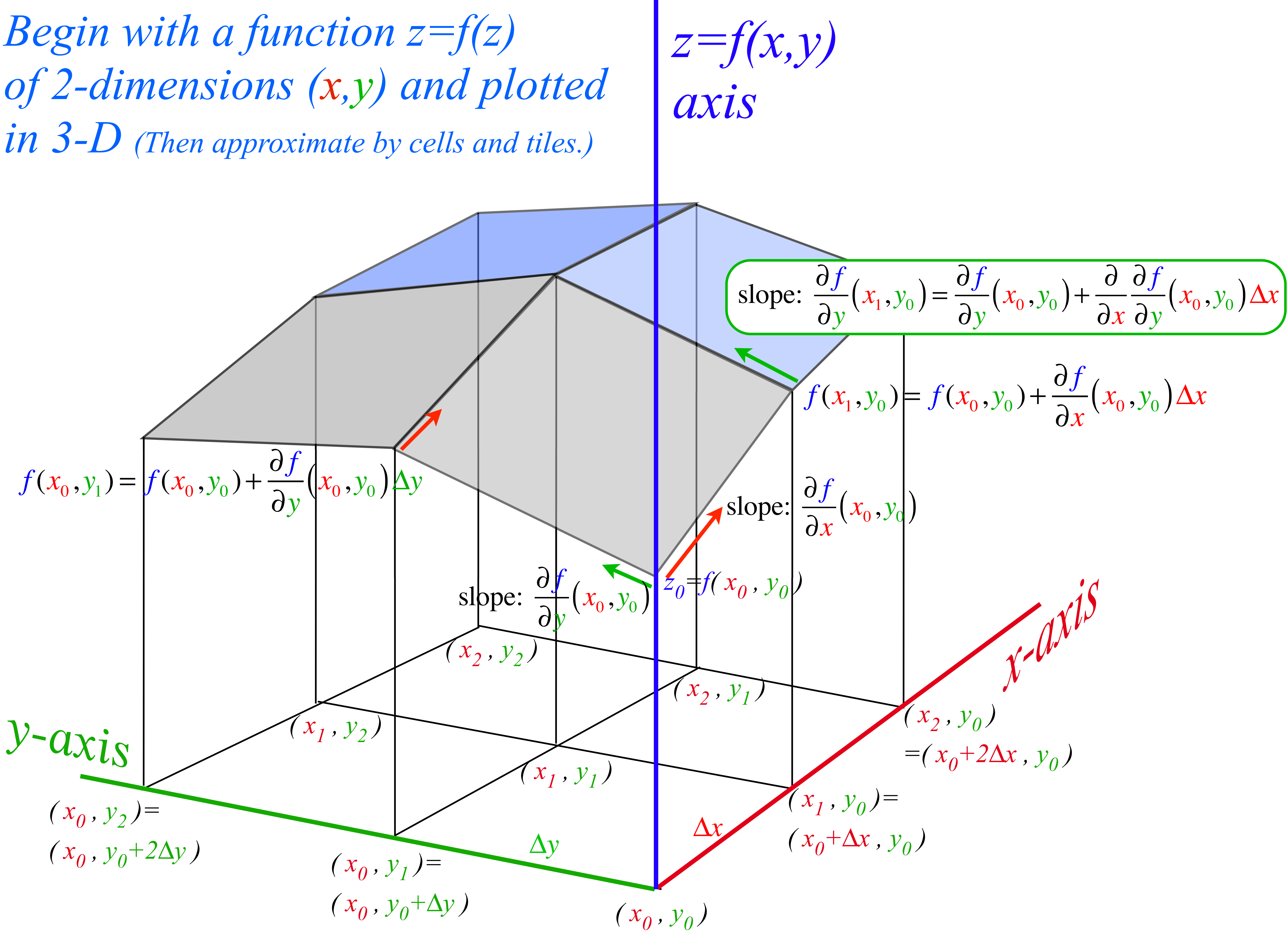
Begin with a function z=f(z)
of 2-dimensions (x,y) and plotted
in 3-D *(Then approximate by cells and tiles.)*



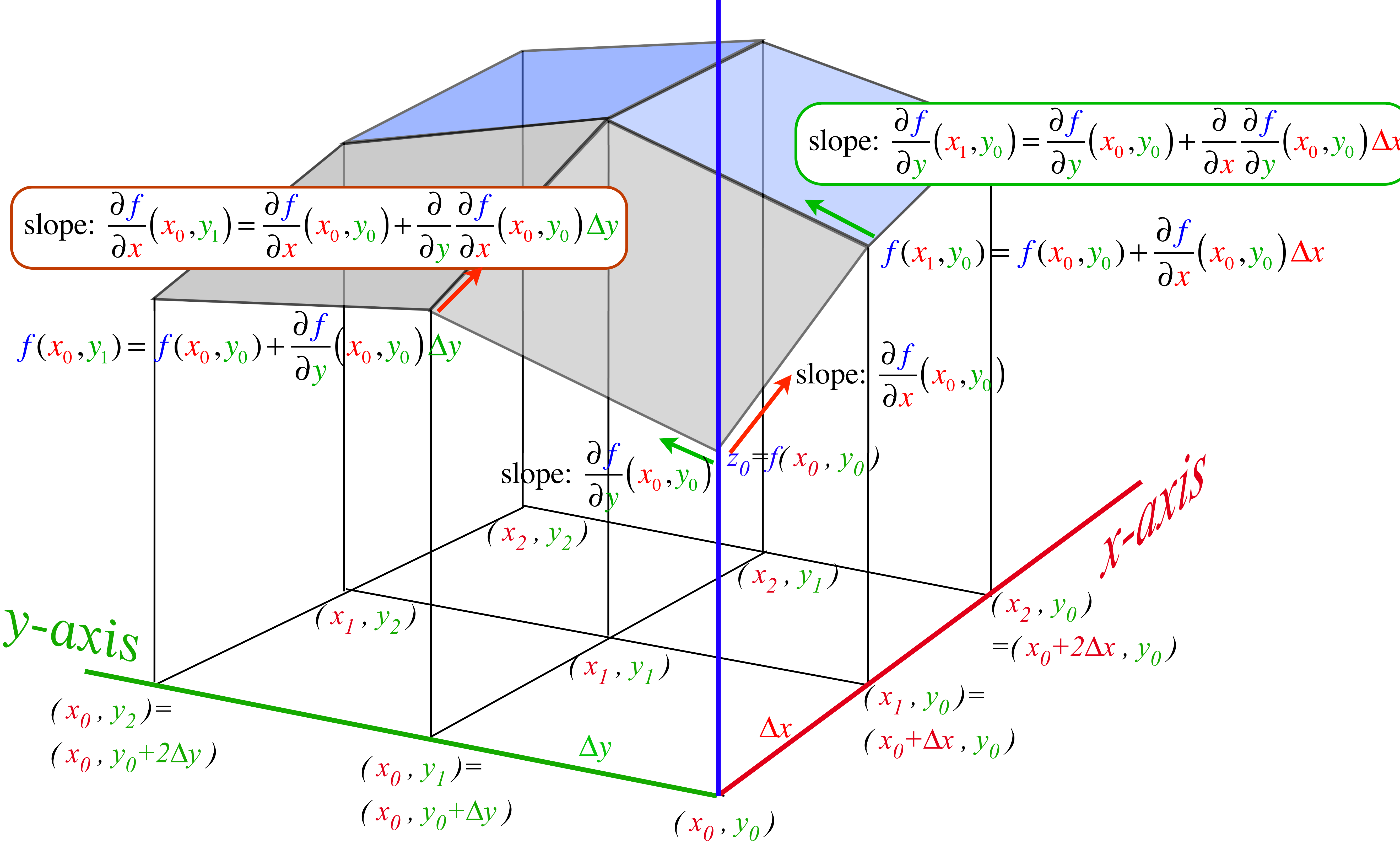
Begin with a function z=f(z) of 2-dimensions (x,y) and plotted in 3-D *(Then approximate by cells and tiles.)*

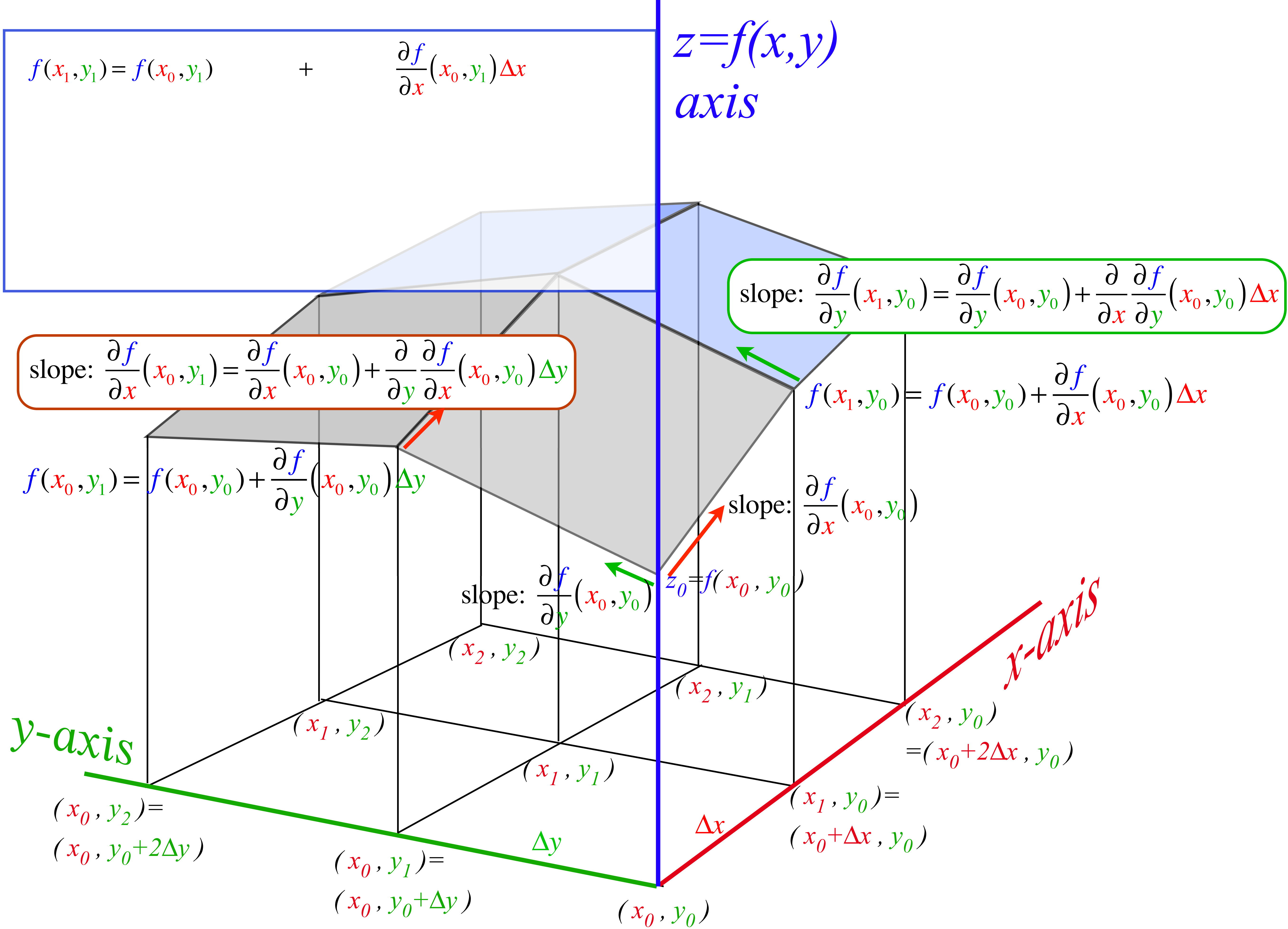


Begin with a function z=f(z)
of 2-dimensions (x,y) and plotted
in 3-D *(Then approximate by cells and tiles.)*

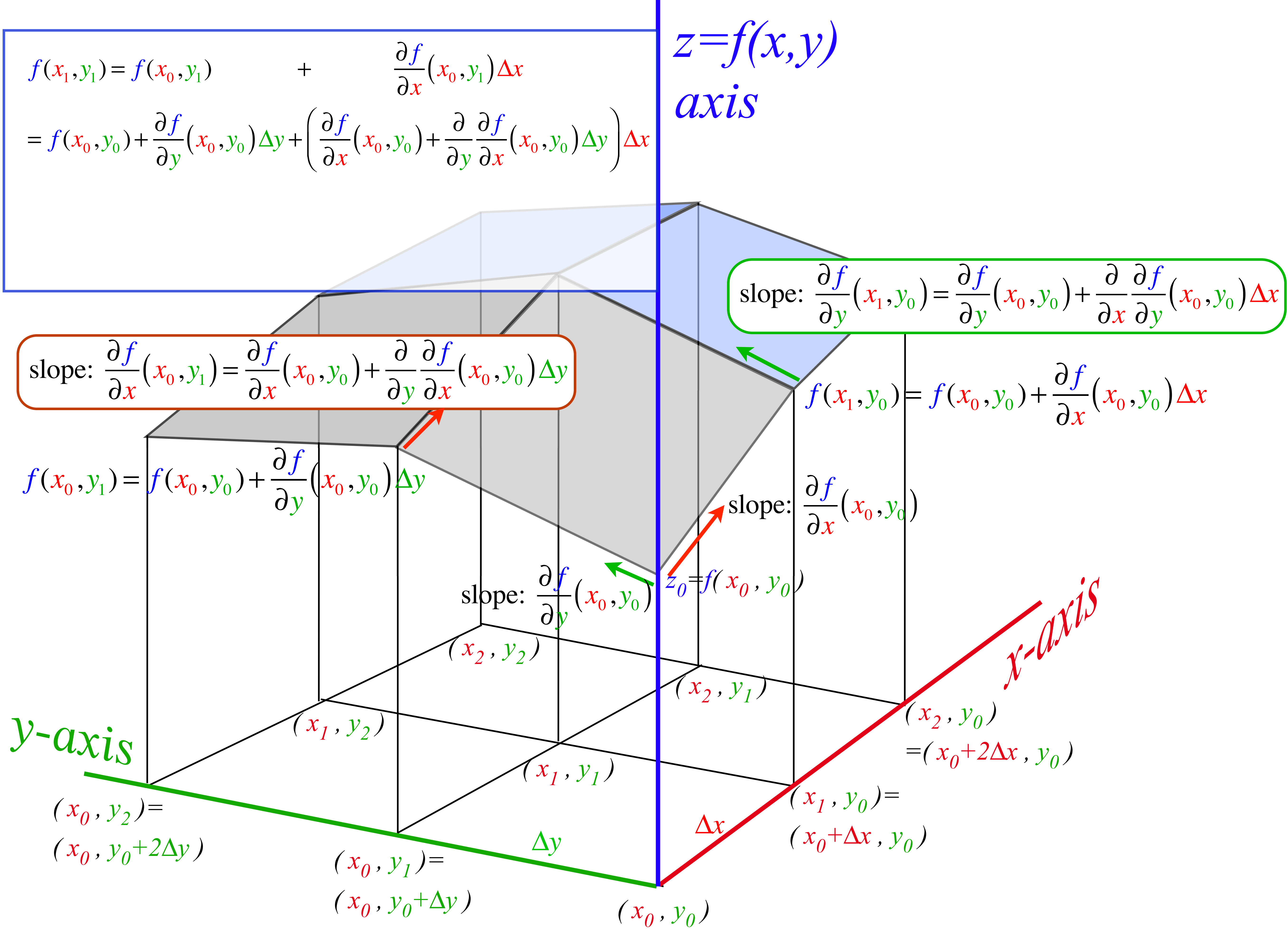


Begin with a function z=f(z) of 2-dimensions (x,y) and plotted in 3-D *(Then approximate by cells and tiles.)*

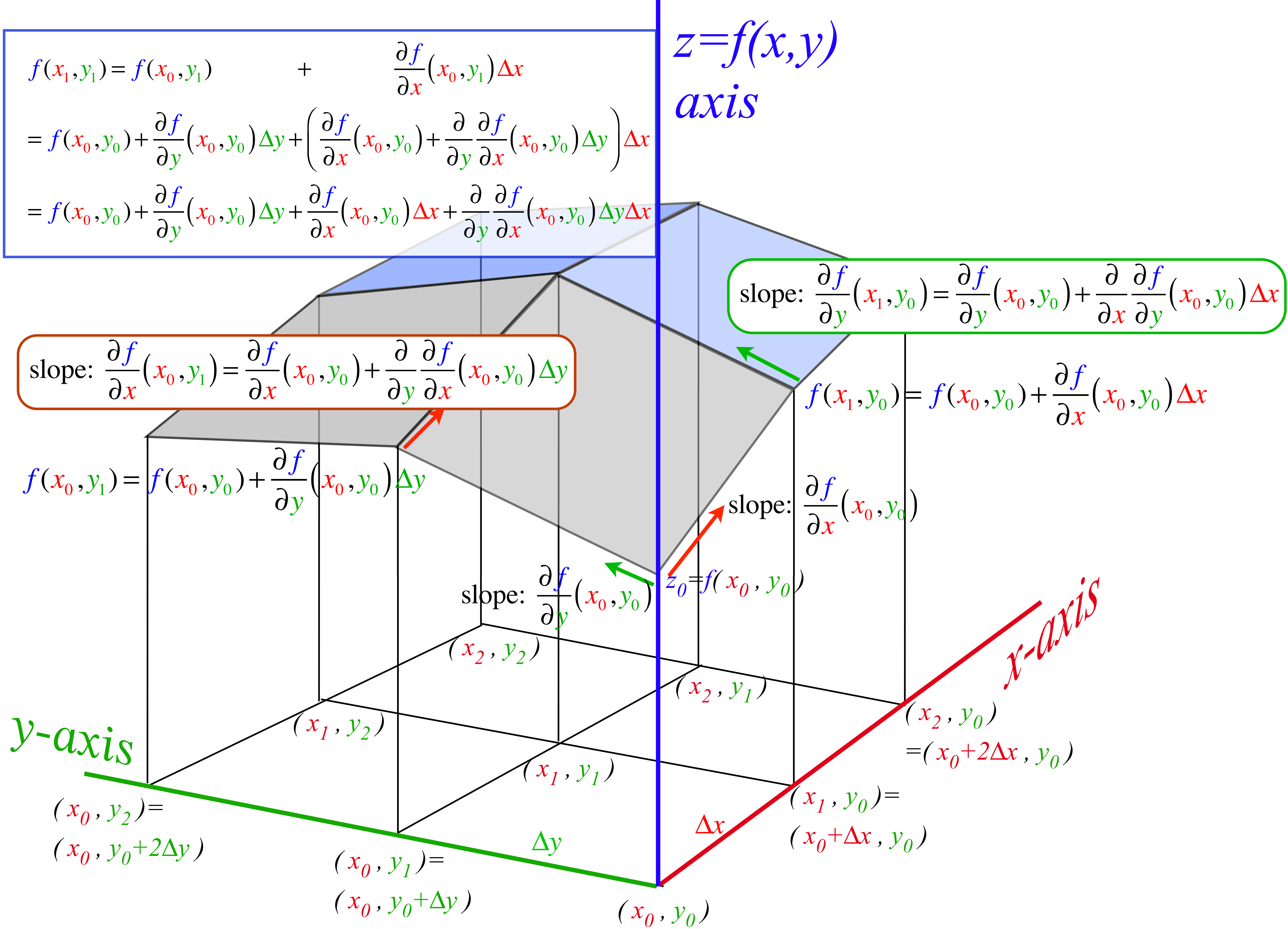




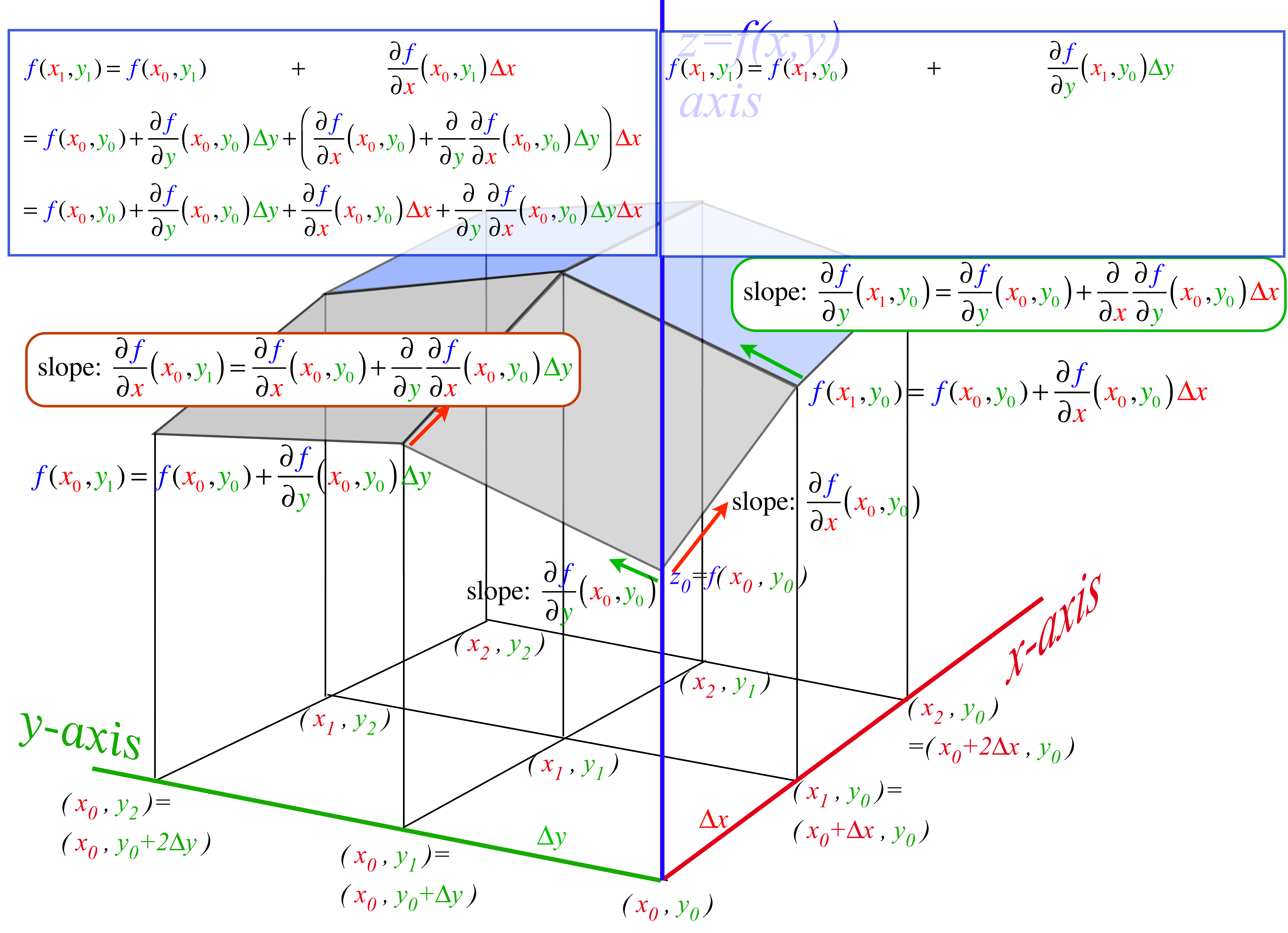
$f(x_1,y_1)=f(x_0,y_1) + \frac{\partial f}{\partial x}(x_0,y_1)\Delta x$
$z=f(x,y)$ axis
slope: $\frac{\partial f}{\partial y}(x_1,y_0)=\frac{\partial f}{\partial y}(x_0,y_0)+\frac{\partial}{\partial x}\frac{\partial f}{\partial y}(x_0,y_0)\Delta x$
slope: $\frac{\partial f}{\partial x}(x_0,y_1)=\frac{\partial f}{\partial x}(x_0,y_0)+\frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta y$
$f(x_1,y_0)=f(x_0,y_0)+\frac{\partial f}{\partial x}(x_0,y_0)\Delta x$
$f(x_0,y_1)=f(x_0,y_0)+\frac{\partial f}{\partial y}(x_0,y_0)\Delta y$
slope: $\frac{\partial f}{\partial x}(x_0,y_0)$
slope: $\frac{\partial f}{\partial y}(x_0,y_0)$
$z_0=f(x_0,y_0)$
(x_2,y_2)
(x_2,y_1)
(x_1,y_2)
(x_1,y_1)
(x_2,y_0)
$=(x_0+2\Delta x,y_0)$
$(x_1,y_0)=$
$(x_0+\Delta x,y_0)$
x-axis
y-axis
$(x_0,y_2)=$
$(x_0,y_0+2\Delta y)$
$(x_0,y_1)=$
$(x_0,y_0+\Delta y)$
Δy
Δx
(x_0,y_0)



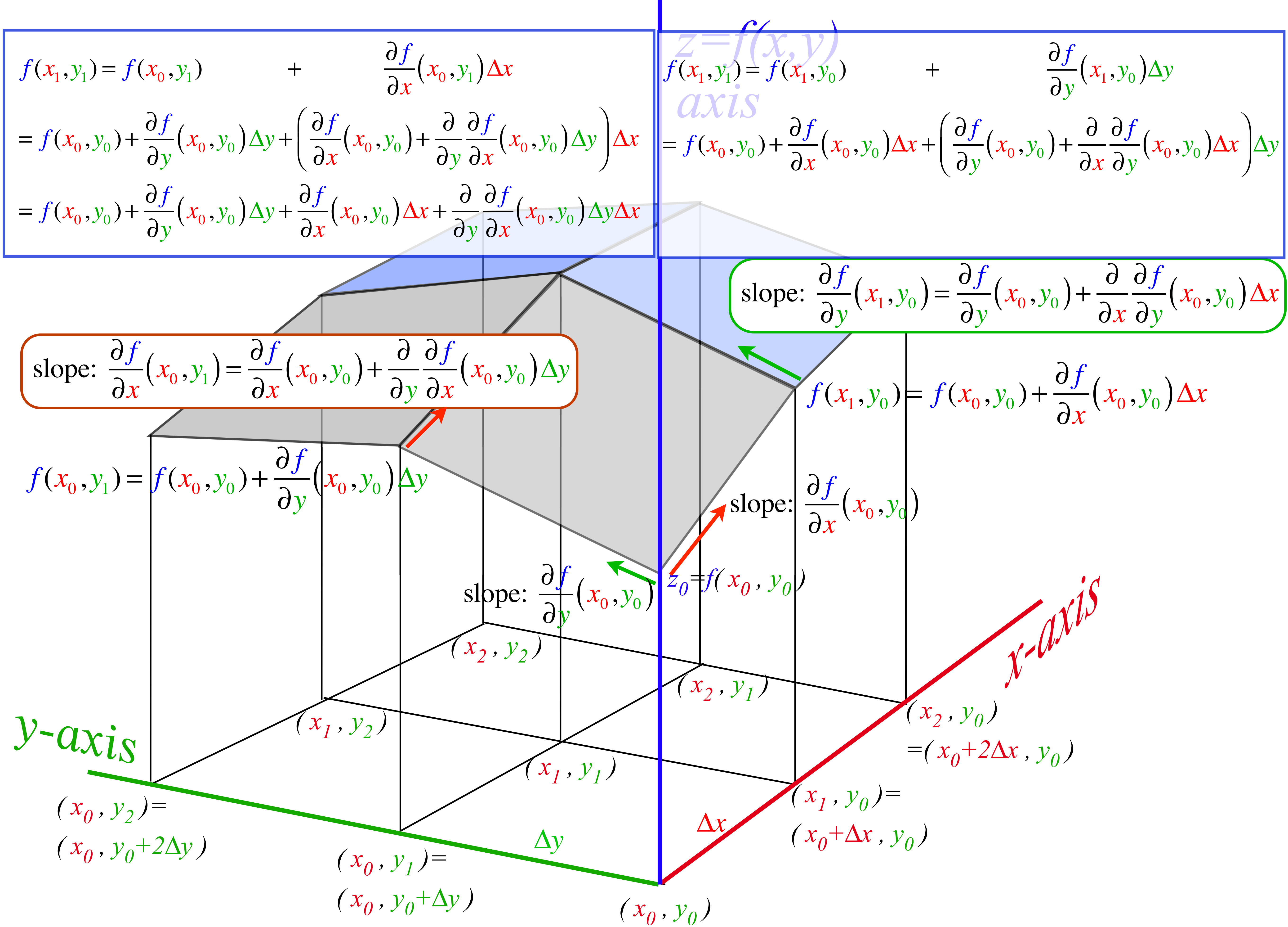
$f(x_1,y_1)=f(x_0,y_1) + \frac{\partial f}{\partial x}(x_0,y_1)\Delta x$
$= f(x_0,y_0)+\frac{\partial f}{\partial y}(x_0,y_0)\Delta y+\left(\frac{\partial f}{\partial x}(x_0,y_0)+\frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta y\right)\Delta x$
z=f(x,y) axis
slope: $\frac{\partial f}{\partial y}(x_1,y_0)=\frac{\partial f}{\partial y}(x_0,y_0)+\frac{\partial}{\partial x}\frac{\partial f}{\partial y}(x_0,y_0)\Delta x$
slope: $\frac{\partial f}{\partial x}(x_0,y_1)=\frac{\partial f}{\partial x}(x_0,y_0)+\frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta y$
$f(x_1,y_0)=f(x_0,y_0)+\frac{\partial f}{\partial x}(x_0,y_0)\Delta x$
$f(x_0,y_1)=f(x_0,y_0)+\frac{\partial f}{\partial y}(x_0,y_0)\Delta y$
slope: $\frac{\partial f}{\partial x}(x_0,y_0)$
slope: $\frac{\partial f}{\partial y}(x_0,y_0)$
$z_0=f(x_0,y_0)$
(x_2,y_2)
(x_2,y_1)
(x_1,y_2)
(x_2,y_0)
$=(x_0+2\Delta x,y_0)$
x-axis
y-axis
(x_1,y_1)
$(x_1,y_0)=$
$(x_0+\Delta x,y_0)$
Δx
Δy
$(x_0,y_2)=$
$(x_0,y_0+2\Delta y)$
$(x_0,y_1)=$
$(x_0,y_0+\Delta y)$
(x_0,y_0)



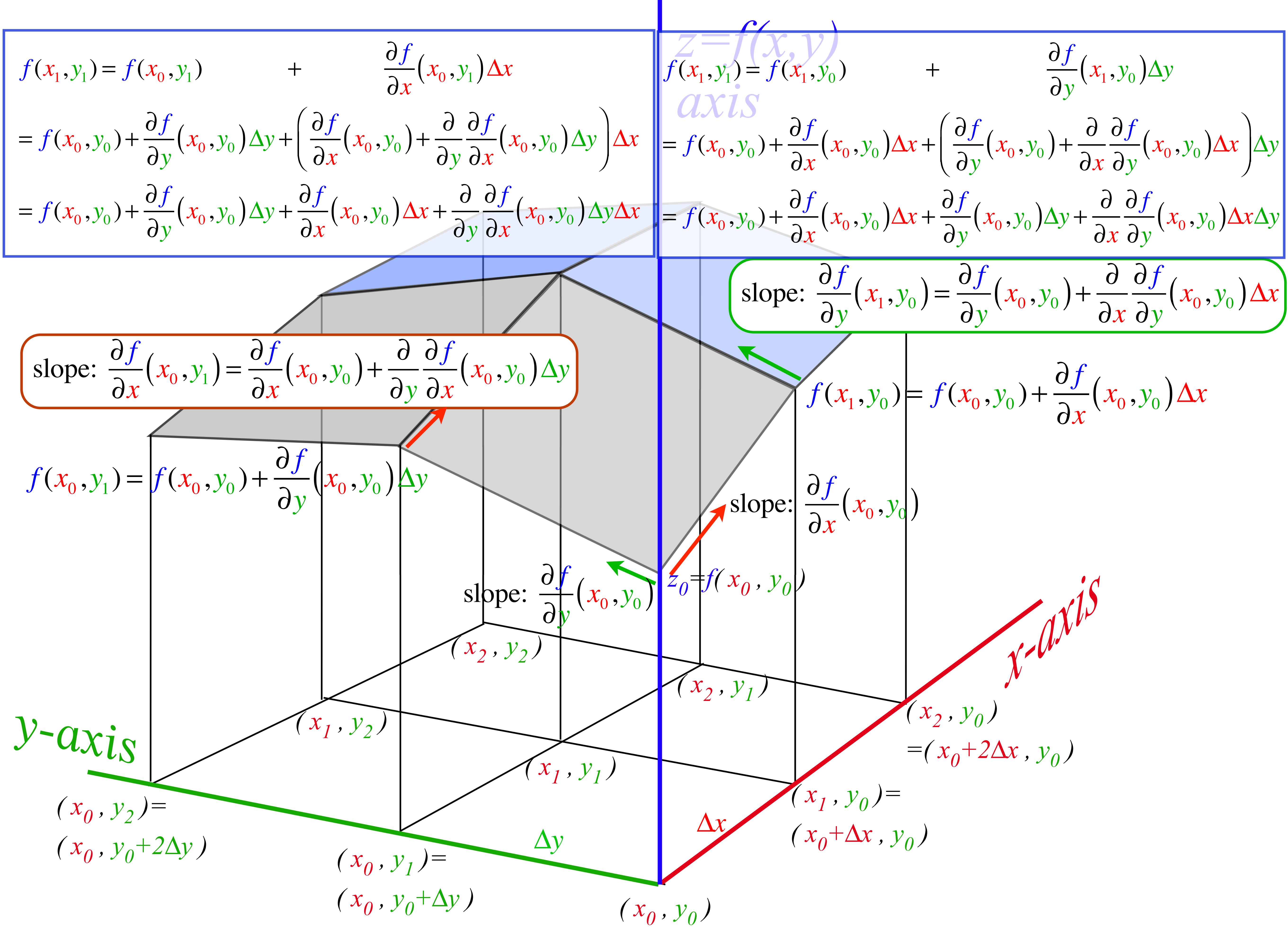
$f(x_1,y_1) = f(x_0,y_1) + \frac{\partial f}{\partial x}(x_0,y_1)\Delta x$
$= f(x_0,y_0) + \frac{\partial f}{\partial y}(x_0,y_0)\Delta y + \left(\frac{\partial f}{\partial x}(x_0,y_0) + \frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta y\right)\Delta x$
$= f(x_0,y_0) + \frac{\partial f}{\partial y}(x_0,y_0)\Delta y + \frac{\partial f}{\partial x}(x_0,y_0)\Delta x + \frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta y\Delta x$
z=f(x,y) axis
slope: $\frac{\partial f}{\partial y}(x_1,y_0) = \frac{\partial f}{\partial y}(x_0,y_0) + \frac{\partial}{\partial x}\frac{\partial f}{\partial y}(x_0,y_0)\Delta x$
slope: $\frac{\partial f}{\partial x}(x_0,y_1) = \frac{\partial f}{\partial x}(x_0,y_0) + \frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta y$
$f(x_1,y_0) = f(x_0,y_0) + \frac{\partial f}{\partial x}(x_0,y_0)\Delta x$
$f(x_0,y_1) = f(x_0,y_0) + \frac{\partial f}{\partial y}(x_0,y_0)\Delta y$
slope: $\frac{\partial f}{\partial x}(x_0,y_0)$
slope: $\frac{\partial f}{\partial y}(x_0,y_0)$
$z_0=f(x_0,y_0)$
(x_2,y_2)
(x_2,y_1)
(x_1,y_2)
(x_1,y_1)
(x_2,y_0) $=(x_0+2\Delta x, y_0)$
$(x_1,y_0)=$ $(x_0+\Delta x, y_0)$
x-axis
y-axis
$(x_0,y_2)=$ $(x_0, y_0+2\Delta y)$
$(x_0,y_1)=$ $(x_0, y_0+\Delta y)$
Δy
Δx
(x_0,y_0)



$f(x_1,y_1)=f(x_0,y_1)+\frac{\partial f}{\partial x}(x_0,y_1)\Delta x$
$=f(x_0,y_0)+\frac{\partial f}{\partial y}(x_0,y_0)\Delta y+\left(\frac{\partial f}{\partial x}(x_0,y_0)+\frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta y\right)\Delta x$
$=f(x_0,y_0)+\frac{\partial f}{\partial y}(x_0,y_0)\Delta y+\frac{\partial f}{\partial x}(x_0,y_0)\Delta x+\frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta y\Delta x$
$f(x_1,y_1)=f(x_1,y_0)+\frac{\partial f}{\partial y}(x_1,y_0)\Delta y$
z=f(x,y) axis
slope: $\frac{\partial f}{\partial y}(x_1,y_0)=\frac{\partial f}{\partial y}(x_0,y_0)+\frac{\partial}{\partial x}\frac{\partial f}{\partial y}(x_0,y_0)\Delta x$
slope: $\frac{\partial f}{\partial x}(x_0,y_1)=\frac{\partial f}{\partial x}(x_0,y_0)+\frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta y$
$f(x_1,y_0)=f(x_0,y_0)+\frac{\partial f}{\partial x}(x_0,y_0)\Delta x$
$f(x_0,y_1)=f(x_0,y_0)+\frac{\partial f}{\partial y}(x_0,y_0)\Delta y$
slope: $\frac{\partial f}{\partial x}(x_0,y_0)$
slope: $\frac{\partial f}{\partial y}(x_0,y_0)$
$z_0=f(x_0,y_0)$
x-axis
y-axis
(x_2,y_2)
(x_2,y_1)
(x_1,y_2)
(x_1,y_1)
(x_2,y_0) $=(x_0+2\Delta x,y_0)$
$(x_1,y_0)=$ $(x_0+\Delta x,y_0)$
$(x_0,y_2)=$ $(x_0,y_0+2\Delta y)$
$(x_0,y_1)=$ $(x_0,y_0+\Delta y)$
(x_0,y_0)
Δx
Δy



$f(x_1,y_1)=f(x_0,y_1)+\frac{\partial f}{\partial x}(x_0,y_1)\Delta x$
$=f(x_0,y_0)+\frac{\partial f}{\partial y}(x_0,y_0)\Delta y+\left(\frac{\partial f}{\partial x}(x_0,y_0)+\frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta y\right)\Delta x$
$=f(x_0,y_0)+\frac{\partial f}{\partial y}(x_0,y_0)\Delta y+\frac{\partial f}{\partial x}(x_0,y_0)\Delta x+\frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta y\Delta x$
z=f(x,y) axis
$f(x_1,y_1)=f(x_1,y_0)+\frac{\partial f}{\partial y}(x_1,y_0)\Delta y$
$=f(x_0,y_0)+\frac{\partial f}{\partial x}(x_0,y_0)\Delta x+\left(\frac{\partial f}{\partial y}(x_0,y_0)+\frac{\partial}{\partial x}\frac{\partial f}{\partial y}(x_0,y_0)\Delta x\right)\Delta y$
slope: $\frac{\partial f}{\partial y}(x_1,y_0)=\frac{\partial f}{\partial y}(x_0,y_0)+\frac{\partial}{\partial x}\frac{\partial f}{\partial y}(x_0,y_0)\Delta x$
slope: $\frac{\partial f}{\partial x}(x_0,y_1)=\frac{\partial f}{\partial x}(x_0,y_0)+\frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta y$
$f(x_1,y_0)=f(x_0,y_0)+\frac{\partial f}{\partial x}(x_0,y_0)\Delta x$
$f(x_0,y_1)=f(x_0,y_0)+\frac{\partial f}{\partial y}(x_0,y_0)\Delta y$
slope: $\frac{\partial f}{\partial x}(x_0,y_0)$
slope: $\frac{\partial f}{\partial y}(x_0,y_0)$
$z_0=f(x_0,y_0)$
x-axis
y-axis
(x_2,y_2)
(x_2,y_1)
(x_1,y_2)
(x_2,y_0)
$=(x_0+2\Delta x,y_0)$
(x_1,y_1)
$(x_1,y_0)=$
$(x_0+\Delta x,y_0)$
Δx
Δy
$(x_0,y_2)=$
$(x_0,y_0+2\Delta y)$
$(x_0,y_1)=$
$(x_0,y_0+\Delta y)$
(x_0,y_0)



$f(x_1,y_1)=f(x_0,y_1)+\frac{\partial f}{\partial x}(x_0,y_1)\Delta x$
$=f(x_0,y_0)+\frac{\partial f}{\partial y}(x_0,y_0)\Delta y+\left(\frac{\partial f}{\partial x}(x_0,y_0)+\frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta y\right)\Delta x$
$=f(x_0,y_0)+\frac{\partial f}{\partial y}(x_0,y_0)\Delta y+\frac{\partial f}{\partial x}(x_0,y_0)\Delta x+\frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta y\Delta x$
z=f(x,y) axis
$f(x_1,y_1)=f(x_1,y_0)+\frac{\partial f}{\partial y}(x_1,y_0)\Delta y$
$=f(x_0,y_0)+\frac{\partial f}{\partial x}(x_0,y_0)\Delta x+\left(\frac{\partial f}{\partial y}(x_0,y_0)+\frac{\partial}{\partial x}\frac{\partial f}{\partial y}(x_0,y_0)\Delta x\right)\Delta y$
$=f(x_0,y_0)+\frac{\partial f}{\partial x}(x_0,y_0)\Delta x+\frac{\partial f}{\partial y}(x_0,y_0)\Delta y+\frac{\partial}{\partial x}\frac{\partial f}{\partial y}(x_0,y_0)\Delta x\Delta y$
slope: $\frac{\partial f}{\partial y}(x_1,y_0)=\frac{\partial f}{\partial y}(x_0,y_0)+\frac{\partial}{\partial x}\frac{\partial f}{\partial y}(x_0,y_0)\Delta x$
slope: $\frac{\partial f}{\partial x}(x_0,y_1)=\frac{\partial f}{\partial x}(x_0,y_0)+\frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta y$
$f(x_1,y_0)=f(x_0,y_0)+\frac{\partial f}{\partial x}(x_0,y_0)\Delta x$
$f(x_0,y_1)=f(x_0,y_0)+\frac{\partial f}{\partial y}(x_0,y_0)\Delta y$
slope: $\frac{\partial f}{\partial x}(x_0,y_0)$
slope: $\frac{\partial f}{\partial y}(x_0,y_0)$
$z_0=f(x_0,y_0)$
(x_2,y_2)
(x_2,y_1)
(x_1,y_2)
(x_1,y_1)
(x_2,y_0)
$=(x_0+2\Delta x,y_0)$
x-axis
y-axis
$(x_0,y_2)=$
$(x_0,y_0+2\Delta y)$
$(x_1,y_0)=$
$(x_0+\Delta x,y_0)$
Δx
Δy
$(x_0,y_1)=$
$(x_0,y_0+\Delta y)$
(x_0,y_0)

Review of partial differential calculus

→ *Chain rule and order $\partial^2\Psi/\partial x\partial y = \partial^2\Psi/\partial y\partial x$ symmetry*

Scaling transformation between Lagrangian and Hamiltonian views of KE

Introducing 0^{th} Lagrange and 0^{th} Hamilton differential equations of mechanics

Introducing 1^{st} Lagrange and 1^{st} Hamilton differential equations of mechanics

Introducing the Poincare´ and Legendre contact transformations

Geometry of Legendre contact transformation (Preview of Unit 8 relativistic quantum mechanics)

Example from thermodynamics

Legendre transform: special case of General Contact Transformation (lights,camera, ACTION!)

An elementary contact transformation from sophomore physics

Algebra-calculus development of "The Volcanoes of Io" and "The Atoms of NIST"

Intuitive-geometric development of " " " and " " "

What the geometry indicates....(Two important results)

$$f(x_1, y_1) = f(x_0, y_0) + \frac{\partial f}{\partial x}(x_0, y_0)\Delta x + \frac{\partial f}{\partial y}(x_0, y_0)\Delta y + \frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0, y_0)\Delta x \Delta y$$

$$= f(x_0, y_0) + \frac{\partial f}{\partial y}(x_0, y_0)\Delta y + \frac{\partial f}{\partial x}(x_0, y_0)\Delta x + \frac{\partial}{\partial x}\frac{\partial f}{\partial y}(x_0, y_0)\Delta y \Delta x$$

What the geometry indicates....(Two important results)

$$f(x_1,y_1) = f(x_0,y_0) + \frac{\partial f}{\partial x}(x_0,y_0)\Delta x + \frac{\partial f}{\partial y}(x_0,y_0)\Delta y + \frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta x\Delta y$$

$$= f(x_0,y_0) + \frac{\partial f}{\partial y}(x_0,y_0)\Delta y + \frac{\partial f}{\partial x}(x_0,y_0)\Delta x + \frac{\partial}{\partial x}\frac{\partial f}{\partial y}(x_0,y_0)\Delta y\Delta x$$

If $f(x,y)$ is contiuous around (x_0,y_0) and (x_1,y_1) then $\frac{\partial}{\partial y}\frac{\partial f}{\partial x}$ equals $\frac{\partial}{\partial x}\frac{\partial f}{\partial y}$

What the geometry indicates....(Two important results)

$$f(x_1,y_1)=f(x_0,y_0)+\frac{\partial f}{\partial x}(x_0,y_0)\Delta x+\frac{\partial f}{\partial y}(x_0,y_0)\Delta y+\frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta x\Delta y$$

$$=f(x_0,y_0)+\frac{\partial f}{\partial y}(x_0,y_0)\Delta y+\frac{\partial f}{\partial x}(x_0,y_0)\Delta x+\frac{\partial}{\partial x}\frac{\partial f}{\partial y}(x_0,y_0)\Delta y\Delta x$$

If $f(x,y)$ is contiuous around (x_0,y_0) and (x_1,y_1) then $\frac{\partial}{\partial y}\frac{\partial f}{\partial x}$ equals $\frac{\partial}{\partial x}\frac{\partial f}{\partial y}$

1. Chain rules

$$\left[f(x_1,y_1)-f(x_0,y_0)\right]=df=\frac{\partial f}{\partial x}(x_0,y_0)dx+\frac{\partial f}{\partial y}(x_0,y_0)dy\ldots_{(keep\ 1^{st}-order\ terms\ only!)}$$

$$\frac{df}{dt}=\frac{\partial f}{\partial x}(x_0,y_0)\frac{dx}{dt}+\frac{\partial f}{\partial y}(x_0,y_0)\frac{dy}{dt}$$

$$\dot{f}=\frac{\partial f}{\partial x}\dot{x}+\frac{\partial f}{\partial y}\dot{y}\qquad (shorthand\ notation)$$

What the geometry indicates....(Two important results)

$$f(x_1,y_1)=f(x_0,y_0)+\frac{\partial f}{\partial x}(x_0,y_0)\Delta x+\frac{\partial f}{\partial y}(x_0,y_0)\Delta y+\frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta x\Delta y$$

$$=f(x_0,y_0)+\frac{\partial f}{\partial y}(x_0,y_0)\Delta y+\frac{\partial f}{\partial x}(x_0,y_0)\Delta x+\frac{\partial}{\partial x}\frac{\partial f}{\partial y}(x_0,y_0)\Delta y\Delta x$$

If $f(x,y)$ is contiuous around (x_0,y_0) and (x_1,y_1) then $\frac{\partial}{\partial y}\frac{\partial f}{\partial x}$ equals $\frac{\partial}{\partial x}\frac{\partial f}{\partial y}$

1. Chain rules

$$[f(x_1,y_1)-f(x_0,y_0)]=df=\frac{\partial f}{\partial x}(x_0,y_0)dx+\frac{\partial f}{\partial y}(x_0,y_0)dy\ldots_{(keep\ 1^{st}-order\ terms\ only!)}$$

$$\frac{df}{dt}=\frac{\partial f}{\partial x}(x_0,y_0)\frac{dx}{dt}+\frac{\partial f}{\partial y}(x_0,y_0)\frac{dy}{dt}$$

$$\dot{f}=\frac{\partial f}{\partial x}\dot{x}+\frac{\partial f}{\partial y}\dot{y}\quad (shorthand\ notation)\quad =\partial_x f\,\dot{x}+\partial_y f\,\dot{y}$$

2. Symmetry of partial deriv. ordering

(pay attention to the 2^{nd} – order terms, too!)

$$\frac{\partial}{\partial y}\frac{\partial f}{\partial x}=\frac{\partial}{\partial x}\frac{\partial f}{\partial y}\quad \text{or:}\quad \frac{\partial^2 f}{\partial y\partial x}=\frac{\partial^2 f}{\partial x\partial y}\quad \text{or:}\quad \partial_y\partial_x f=\partial_x\partial_y f$$

(shorthand notation)

What the geometry indicates....(Two important results)

$$f(x_1,y_1) = f(x_0,y_0) + \frac{\partial f}{\partial x}(x_0,y_0)\Delta x + \frac{\partial f}{\partial y}(x_0,y_0)\Delta y + \frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta x\Delta y$$

$$= f(x_0,y_0) + \frac{\partial f}{\partial y}(x_0,y_0)\Delta y + \frac{\partial f}{\partial x}(x_0,y_0)\Delta x + \frac{\partial}{\partial x}\frac{\partial f}{\partial y}(x_0,y_0)\Delta y\Delta x$$

If $f(x,y)$ is contiuous around (x_0,y_0) and (x_1,y_1) then $\frac{\partial}{\partial y}\frac{\partial f}{\partial x}$ equals $\frac{\partial}{\partial x}\frac{\partial f}{\partial y}$

1. Chain rules

$$[f(x_1,y_1) - f(x_0,y_0)] = df = \frac{\partial f}{\partial x}(x_0,y_0)dx + \frac{\partial f}{\partial y}(x_0,y_0)dy \ldots_{(keep\ 1^{st}-order\ terms\ only!)}$$

$$\frac{df}{dt} = \frac{\partial f}{\partial x}(x_0,y_0)\frac{dx}{dt} + \frac{\partial f}{\partial y}(x_0,y_0)\frac{dy}{dt}$$

$$\dot{f} = \frac{\partial f}{\partial x}\dot{x} + \frac{\partial f}{\partial y}\dot{y} \qquad (shorthand\ notation) \qquad = \partial_x f\,\dot{x} + \partial_y f\,\dot{y}$$

2. Symmetry of partial deriv. ordering

(pay attention to the 2^{nd}-order terms, too!)

$$\frac{\partial}{\partial y}\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\frac{\partial f}{\partial y} \quad \text{or:} \quad \frac{\partial^2 f}{\partial y\,\partial x} = \frac{\partial^2 f}{\partial x\,\partial y} \quad \text{or:} \quad \partial_y\partial_x f = \partial_x\partial_y f$$

(shorthand notation)

$$Let: \vec{\nabla} = \begin{pmatrix} \partial_x & \partial_y \end{pmatrix} \quad so: \vec{\nabla}f \bullet \mathbf{dr} = \begin{pmatrix} \partial_x f & \partial_y f \end{pmatrix} \bullet \begin{pmatrix} dx \\ dy \end{pmatrix} = \partial_x f\, dx + \partial_y f\, dy = df$$

Review of partial differential calculus

Chain rule and order $\partial^2\Psi/\partial x\partial y = \partial^2\Psi/\partial y\partial x$ symmetry

→ *Scaling transformation between Lagrangian and Hamiltonian views of KE*

Introducing 0th Lagrange and 0th Hamilton differential equations of mechanics

Introducing 1st Lagrange and 1st Hamilton differential equations of mechanics

Introducing the Poincare´ and Legendre contact transformations

Geometry of Legendre contact transformation (Preview of Unit 8 relativistic quantum mechanics)

Example from thermodynamics

Legendre transform: special case of General Contact Transformation (lights,camera, ACTION!)

An elementary contact transformation from sophomore physics

Algebra-calculus development of "The Volcanoes of Io" and "The Atoms of NIST"

Intuitive-geometric development of " " " and " " "

Three ways to express energy: Consider kinetic energy (KE) first

1. **Lagrangian** *is explicit function of* **velocity**: $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

$$L(v_k \ldots) = \tfrac{1}{2}(m_1 v_1^2 + m_2 v_2^2 + \ldots) = L(\mathbf{v}\ldots) = \tfrac{1}{2}\mathbf{v}\bullet\mathbf{M}\bullet\mathbf{v} + \ldots = \tfrac{1}{2}\begin{pmatrix} v_1 & v_2 \end{pmatrix}\begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} + \ldots$$

2. **"Estrangian"** *is explicit function of* **R**-*rescaled velocity:*

(or l'Estrangian

*or: "**speedinum**" V* $\mathbf{V} = \mathbf{R}\bullet\mathbf{v}$ or: $\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} \sqrt{m_1} & 0 \\ 0 & \sqrt{m_2} \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

$$E(V_k \ldots) = \tfrac{1}{2}(V_1^2 + V_2^2 + \ldots) = E(\mathbf{V}\ldots) = \tfrac{1}{2}\mathbf{V}\bullet\mathbf{1}\bullet\mathbf{V} + \ldots = \tfrac{1}{2}\begin{pmatrix} V_1 & V_2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} + \ldots$$

3. **Hamiltonian** *is explicit function of* **M=R**^{2}-*rescaled velocity:*

or: ***momentum p*** $\mathbf{p} = \mathbf{M}\bullet\mathbf{v}$ or: $\begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} m_1 v_1 \\ m_2 v_2 \end{pmatrix}$

$$H(p_k \ldots) = \tfrac{1}{2}\left(\frac{p_1^2}{m_1} + \frac{p_2^2}{m_2} + \ldots\right) = H(\mathbf{p}\ldots) = \tfrac{1}{2}\mathbf{p}\bullet\mathbf{M}^{-1}\bullet\mathbf{p} + \ldots = \tfrac{1}{2}\begin{pmatrix} p_1 & p_2 \end{pmatrix}\begin{pmatrix} 1/m_1 & 0 \\ 0 & 1/m_2 \end{pmatrix}\begin{pmatrix} p_1 \\ p_2 \end{pmatrix} + \ldots$$

Review of partial differential calculus

Chain rule and order $\partial^2\Psi/\partial x\partial y = \partial^2\Psi/\partial y\partial x$ symmetry

Scaling transformation between Lagrangian and Hamiltonian views of KE

→ *Introducing 0^{th} Lagrange and 0^{th} Hamilton differential equations of mechanics*

Introducing 1^{st} Lagrange and 1^{st} Hamilton differential equations of mechanics

Introducing the Poincare´ and Legendre contact transformations

Geometry of Legendre contact transformation (Preview of Unit 8 relativistic quantum mechanics)

Example from thermodynamics

Legendre transform: special case of General Contact Transformation (lights,camera, ACTION!)

An elementary contact transformation from sophomore physics

Algebra-calculus development of "The Volcanoes of Io" and "The Atoms of NIST"

Intuitive-geometric development of " " " and " " "

Introducing the (partial $\frac{\partial ?}{\partial ?}$) differential equations of mechanics

Starts out with simple demands for explicit-dependence, "loyalty" or "fealty to the colors"

Lagrangian *and* ***Estrangian***
have *<u>no</u>* *explicit dependence*
on ***momentum*** **p**=**M**•**v**

$$\frac{\partial L}{\partial p_k} \equiv 0 \equiv \frac{\partial E}{\partial p_k}$$

Hamiltonian *and* ***Estrangian***
have *<u>no</u>* *explicit dependence*
on ***velocity*** **v**=$\mathbf{M}^{-1}$•**p**

$$\frac{\partial H}{\partial v_k} \equiv 0 \equiv \frac{\partial E}{\partial v_k}$$

Lagrangian *and* ***Hamiltonian***
have *<u>no</u>* *explicit dependence*
on ***speedinum*** **V**=$\mathbf{M}^{1/2}$•**v**

$$\frac{\partial L}{\partial V_k} \equiv 0 \equiv \frac{\partial H}{\partial V_k}$$

Review of partial differential calculus

Chain rule and order $\partial^2\Psi/\partial x\partial y = \partial^2\Psi/\partial y\partial x$ symmetry

Scaling transformation between Lagrangian and Hamiltonian views of KE

Introducing 0^{th} Lagrange and 0^{th} Hamilton differential equations of mechanics

→ *Introducing 1^{st} Lagrange and 1^{st} Hamilton differential equations of mechanics*

Introducing the Poincare´ and Legendre contact transformations

Geometry of Legendre contact transformation (Preview of Unit 8 relativistic quantum mechanics)

Example from thermodynamics

Legendre transform: special case of General Contact Transformation (lights,camera, ACTION!)

An elementary contact transformation from sophomore physics

Algebra-calculus development of "The Volcanoes of Io" and "The Atoms of NIST"

Intuitive-geometric development of " " " and " " "

Introducing the (partial$\frac{\partial ?}{\partial ?}$) differential equations of mechanics

Starts out with simple demands for explicit-dependence, "loyalty" or "fealty to the colors"

Lagrangian *and* ***Estrangian***
have *no* *explicit dependence*
on ***momentum*** $\mathbf{p}=\mathbf{M}\bullet\mathbf{v}$

$$\frac{\partial L}{\partial p_k} \equiv 0 \equiv \frac{\partial E}{\partial p_k}$$

Hamiltonian *and* ***Estrangian***
have *no* *explicit dependence*
on ***velocity*** $\mathbf{v}=\mathbf{M}^{-1}\bullet\mathbf{p}$

$$\frac{\partial H}{\partial v_k} \equiv 0 \equiv \frac{\partial E}{\partial v_k}$$

Lagrangian *and* ***Hamiltonian***
have *no* *explicit dependence*
on ***speedinum*** $\mathbf{V}=\mathbf{M}^{1/2}\bullet\mathbf{v}$

$$\frac{\partial L}{\partial V_k} \equiv 0 \equiv \frac{\partial H}{\partial V_k}$$

Such non-dependencies hold in spite of "under-the-table" matrix and partial-differential connections†

$$\nabla_v L = \frac{\partial L}{\partial \mathbf{v}} = \frac{\partial}{\partial \mathbf{v}} \frac{\mathbf{v}\bullet\mathbf{M}\bullet\mathbf{v}}{2}$$
$$= \mathbf{M}\bullet\mathbf{v} = \mathbf{p}$$

$$\begin{pmatrix} \frac{\partial L}{\partial v_1} \\ \frac{\partial L}{\partial v_2} \end{pmatrix} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$$

Lagrange's 1st equation(s)

$$\frac{\partial L}{\partial v_k} = p_k \quad \text{or:} \quad \frac{\partial L}{\partial \mathbf{v}} = \mathbf{p}$$

$$\nabla_p H = \mathbf{v} = \frac{\partial H}{\partial \mathbf{p}} = \frac{\partial}{\partial \mathbf{p}} \frac{\mathbf{p}\bullet\mathbf{M}^{-1}\bullet\mathbf{p}}{2}$$
$$= \mathbf{M}^{-1}\bullet\mathbf{p} = \mathbf{v}$$

$$\begin{pmatrix} \frac{\partial H}{\partial p_1} \\ \frac{\partial H}{\partial p_2} \end{pmatrix} = \begin{pmatrix} m_1^{-1} & 0 \\ 0 & m_2^{-1} \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

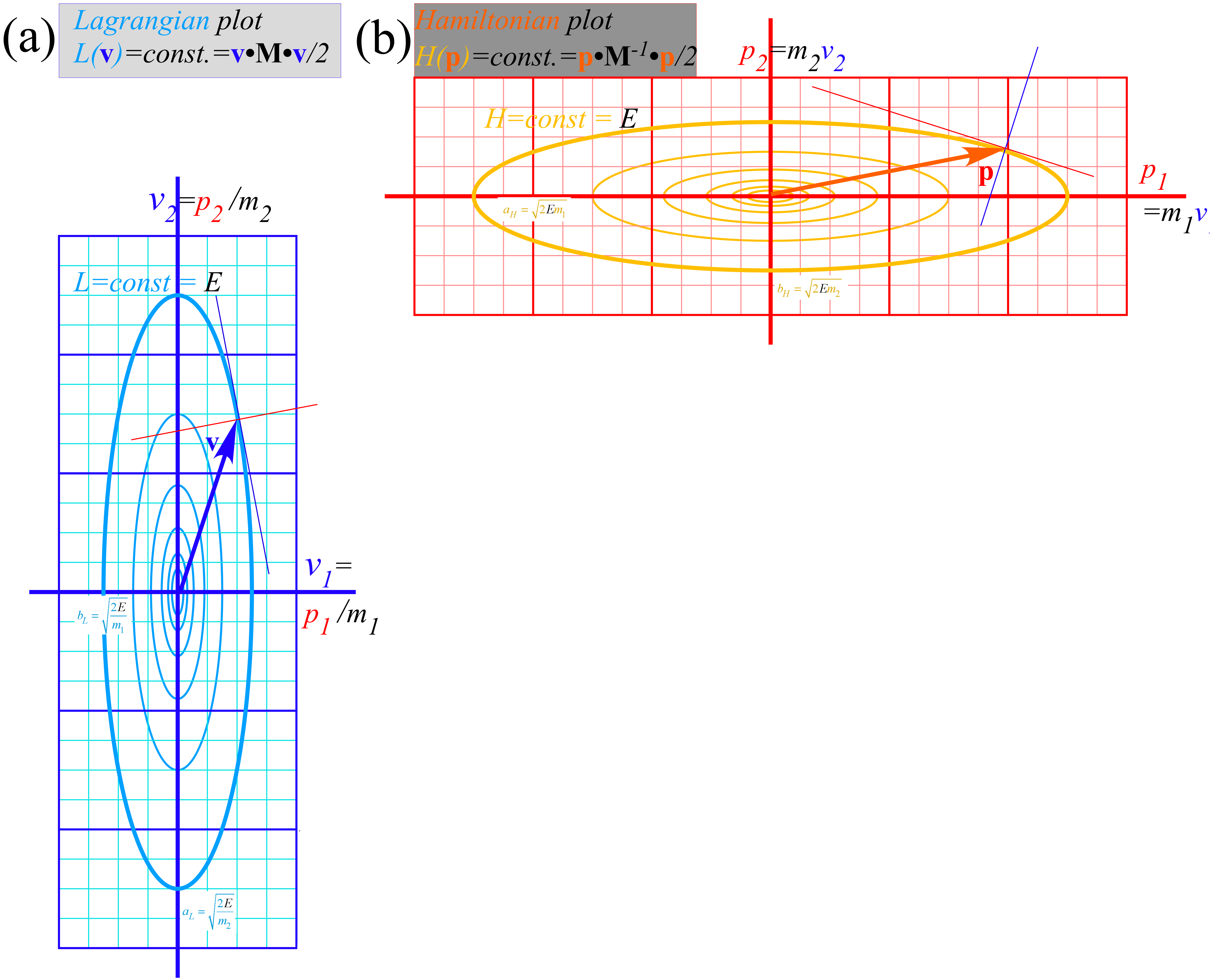
Hamilton's 1st equation(s)

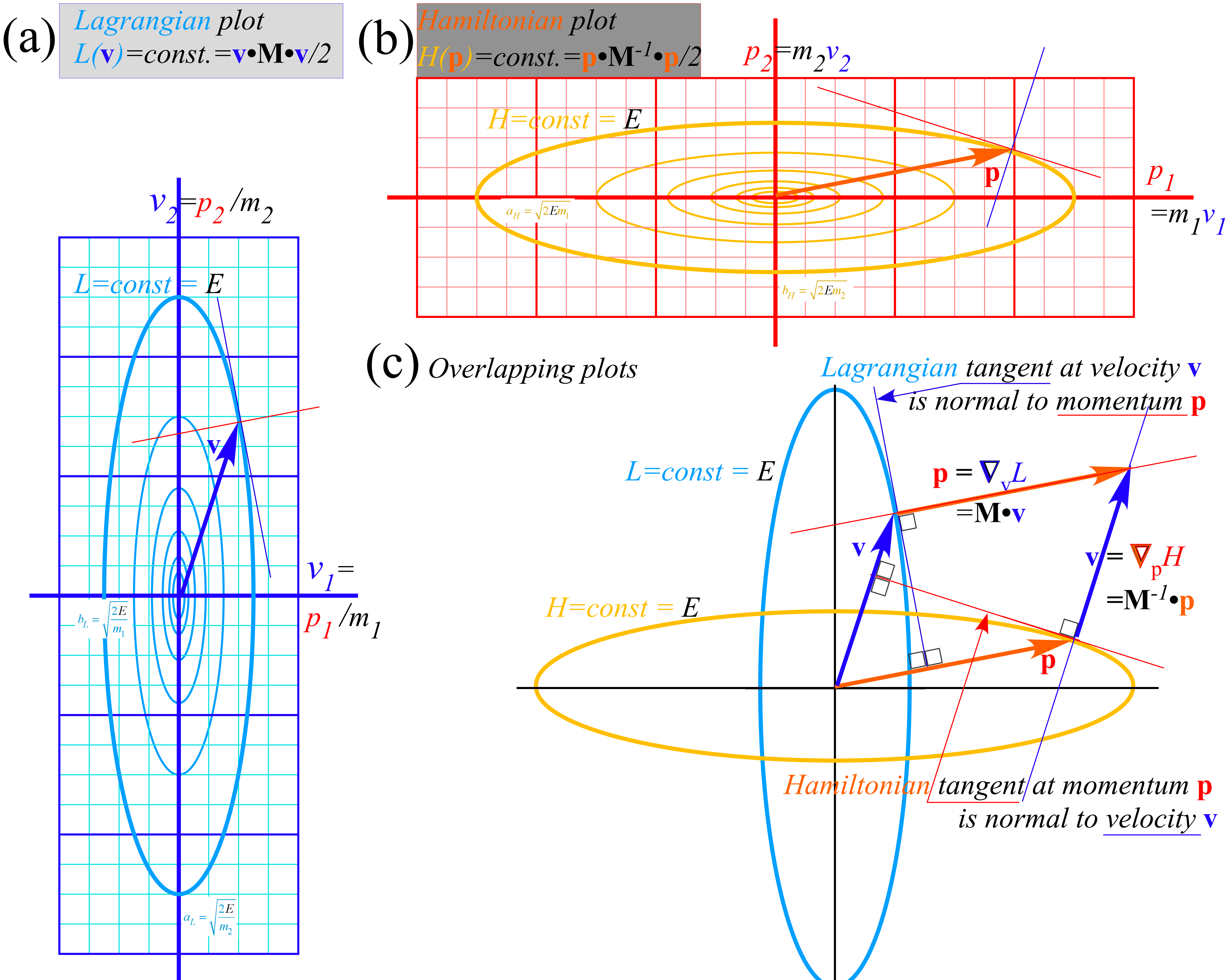
$$\frac{\partial H}{\partial p_k} = v_k \quad \text{or:} \quad \frac{\partial H}{\partial \mathbf{p}} = \mathbf{v}$$

Estrangian is neglected for now.
(It is related to dual ellipse geometry
in Lecture 7 p. 71-79 and 80-85)

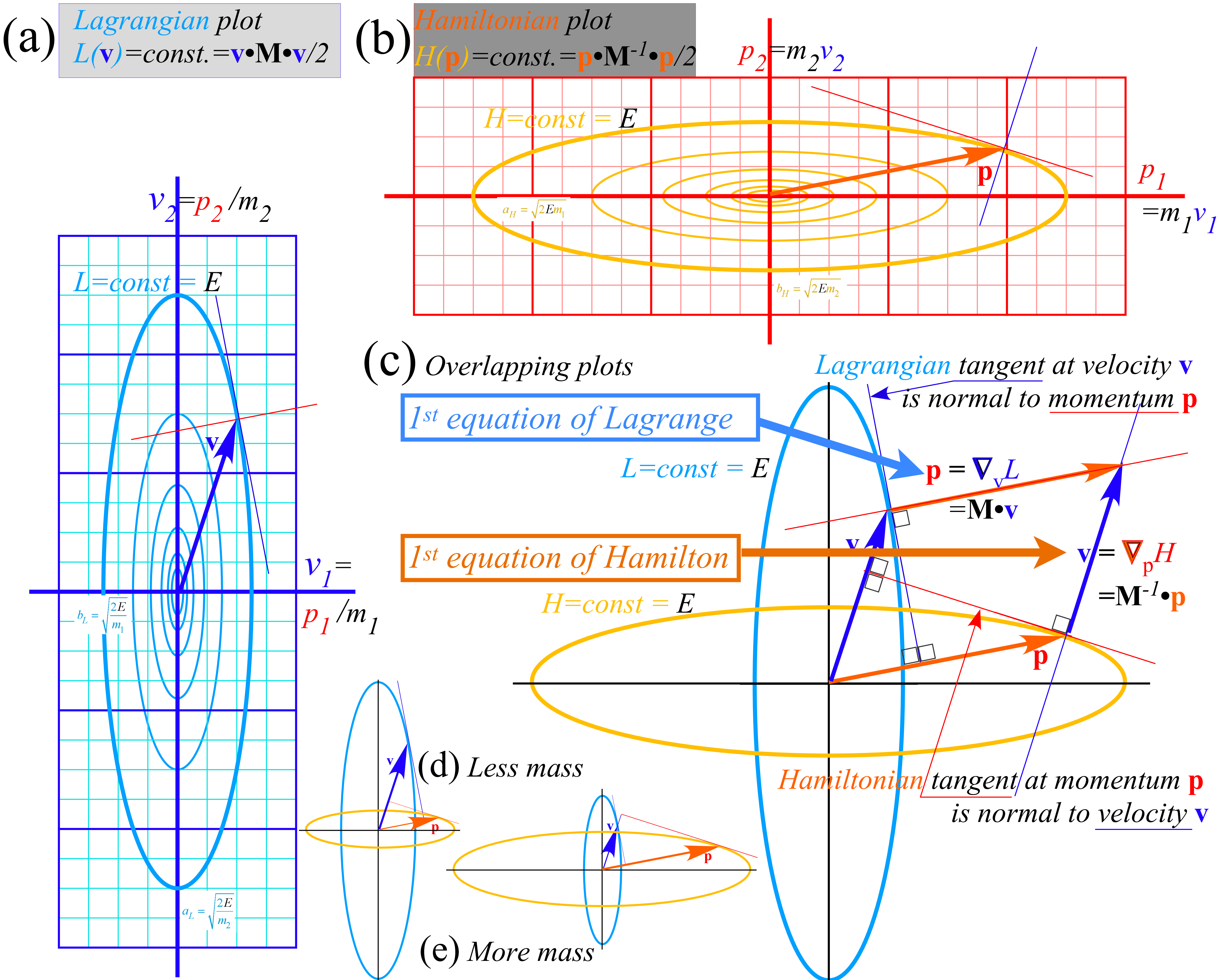
†non-dependency due to
stationary-value effects
as shown on p.39 to p.48

Unit 1
Fig. 12.2




(a) Lagrangian plot
L(v)=const.=v•M•v/2
(b) Hamiltonian plot
H(p)=const.=p•M^-1•p/2
p2=m2v2
H=const = E
p
p1
=m1v1
v2=p2/m2
L=const = E
v
v1=
p1/m1
(c) Overlapping plots
Lagrangian tangent at velocity v
is normal to momentum p
L=const = E
p = ∇vL
=M•v
v
v = ∇pH
=M^-1•p
H=const = E
p
Hamiltonian tangent at momentum p
is normal to velocity v

Unit 1
Fig. 12.2



Review of partial differential calculus

Chain rule and order $\partial^2\Psi/\partial x\partial y = \partial^2\Psi/\partial y\partial x$ symmetry

Scaling transformation between Lagrangian and Hamiltonian views of KE

Introducing 0^{th} Lagrange and 0^{th} Hamilton differential equations of mechanics

Introducing 1^{st} Lagrange and 1^{st} Hamilton differential equations of mechanics

→ *Introducing the Poincare´ and Legendre contact transformations*

Geometry of Legendre contact transformation (Preview of Unit 8 relativistic quantum mechanics)

Example from thermodynamics

Legendre transform: special case of General Contact Transformation (lights,camera, ACTION!)

An elementary contact transformation from sophomore physics

Algebra-calculus development of "The Volcanoes of Io" and "The Atoms of NIST"

Intuitive-geometric development of " " " and " " "

Introducing the Poincare′ and Legendre contact transformations

Given matrix relation: $\mathbf{p}=\mathbf{M}\cdot\mathbf{v}$ *or its inverse:* $\mathbf{v}=\mathbf{M}^{-1}\cdot\mathbf{p}$ *you might be tempted to rewrite*

Q-forms $L(\mathbf{v}..)=(1/2)\mathbf{v}\cdot\mathbf{M}\cdot\mathbf{v}$ *or* $H(\mathbf{p}..)=(1/2)\mathbf{p}\cdot\mathbf{M}^{-1}\cdot\mathbf{p}$ *to be* $H=(1/2)\mathbf{p}\cdot\mathbf{v}$ *or equivalently* $L=(1/2)\mathbf{v}\cdot\mathbf{p}$.

Introducing the Poincare′ and Legendre contact transformations

Given matrix relation: $\mathbf{p}=\mathbf{M}\cdot\mathbf{v}$ *or its inverse:* $\mathbf{v}=\mathbf{M}^{-1}\cdot\mathbf{p}$ *you might be tempted to rewrite*

Q-forms $L(\mathbf{v}..)=(1/2)\mathbf{v}\cdot\mathbf{M}\cdot\mathbf{v}$ *or* $H(\mathbf{p}..)=(1/2)\mathbf{p}\cdot\mathbf{M}^{-1}\cdot\mathbf{p}$ *to be* $H=(1/2)\mathbf{p}\cdot\mathbf{v}$ *or equivalently* $L=(1/2)\mathbf{v}\cdot\mathbf{p}$.

Numerically-CORRECT, but Differentially-WRONG!

Introducing the Poincare′ and Legendre contact transformations

Given matrix relation: $\mathbf{p}=\mathbf{M}\bullet\mathbf{v}$ *or its inverse:* $\mathbf{v}=\mathbf{M}^{-1}\bullet\mathbf{p}$ *you might be tempted to rewrite*

Q-forms $L(\mathbf{v}..)=(1/2)\mathbf{v}\bullet\mathbf{M}\bullet\mathbf{v}$ *or* $H(\mathbf{p}..)=(1/2)\mathbf{p}\bullet\mathbf{M}^{-1}\bullet\mathbf{p}$ *to be* $H=(1/2)\mathbf{p}\bullet\mathbf{v}$ *or equivalently* $L=(1/2)\mathbf{v}\bullet\mathbf{p}$.

Numerically-CORRECT, but Differentially-WRONG! *(In classical physics* $\mathbf{p}\bullet\mathbf{v}$ *and* $\mathbf{v}\bullet\mathbf{p}$ *are identical)*

Instead try: $H(\mathbf{p}..)=\mathbf{p}\bullet\mathbf{v}-(1/2)\mathbf{v}\bullet\mathbf{p}=\mathbf{p}\bullet\mathbf{v}-L(\mathbf{v}..)$ *or else:* $L(\mathbf{v}..)=\mathbf{p}\bullet\mathbf{v}-H(\mathbf{p}..)$

Introducing the Poincare′ and Legendre contact transformations

Given matrix relation: $\mathbf{p}=\mathbf{M}\cdot\mathbf{v}$ *or its inverse:* $\mathbf{v}=\mathbf{M}^{-1}\cdot\mathbf{p}$ *you might be tempted to rewrite*

Q-forms $L(\mathbf{v}..)=(1/2)\mathbf{v}\cdot\mathbf{M}\cdot\mathbf{v}$ *or* $H(\mathbf{p}..)=(1/2)\mathbf{p}\cdot\mathbf{M}^{-1}\cdot\mathbf{p}$ *to be* $H=(1/2)\mathbf{p}\cdot\mathbf{v}$ *or equivalently* $L=(1/2)\mathbf{v}\cdot\mathbf{p}$.

Numerically-CORRECT, but Differentially-WRONG!

Instead try: $H(\mathbf{p}..)=\mathbf{p}\cdot\mathbf{v}-(1/2)\mathbf{v}\cdot\mathbf{p}=\mathbf{p}\cdot\mathbf{v}-L(\mathbf{v}..)$ *or else:* $L(\mathbf{v}..)=\mathbf{p}\cdot\mathbf{v}-H(\mathbf{p}..)$

That is ... the ***Legendre contact transformation***

$$L(\mathbf{v})=\mathbf{p}\bullet\mathbf{v}-H(\mathbf{p}) \qquad or: \qquad H(\mathbf{p})=\mathbf{p}\bullet\mathbf{v}-L(\mathbf{v})$$

Introducing the Poincare′ and Legendre contact transformations

Given matrix relation: $\mathbf{p}=\mathbf{M}\bullet\mathbf{v}$ *or its inverse:* $\mathbf{v}=\mathbf{M}^{-1}\bullet\mathbf{p}$ *you might be tempted to rewrite*

Q-forms $L(\mathbf{v}..)=(1/2)\mathbf{v}\bullet\mathbf{M}\bullet\mathbf{v}$ *or* $H(\mathbf{p}..)=(1/2)\mathbf{p}\bullet\mathbf{M}^{-1}\bullet\mathbf{p}$ *to be* $H=(1/2)\mathbf{p}\bullet\mathbf{v}$ *or equivalently* $L=(1/2)\mathbf{v}\bullet\mathbf{p}$.

Numerically-CORRECT, but Differentially-WRONG!

Instead try: $H(\mathbf{p}..)=\mathbf{p}\bullet\mathbf{v}-(1/2)\mathbf{v}\bullet\mathbf{p}=\mathbf{p}\bullet\mathbf{v}-L(\mathbf{v}..)$ *or else:* $L(\mathbf{v}..)=\mathbf{p}\bullet\mathbf{v}-H(\mathbf{p}..)$

That is ... the Legendre contact transformation

$$L(\mathbf{v})=\mathbf{p}\bullet\mathbf{v}-H(\mathbf{p}) \qquad or: \qquad H(\mathbf{p})=\mathbf{p}\bullet\mathbf{v}-L(\mathbf{v})$$

Now explicit dependency (non)-relations give the right derivatives

$$\frac{\partial L(\mathbf{v})}{\partial \mathbf{p}}=\frac{\partial}{\partial \mathbf{p}}\mathbf{p}\bullet\mathbf{v}-\frac{\partial H(\mathbf{p})}{\partial \mathbf{p}}$$

$$0=\mathbf{v}-\frac{\partial H(\mathbf{p})}{\partial \mathbf{p}}$$

$$\frac{\partial H(\mathbf{p})}{\partial \mathbf{v}}=\frac{\partial}{\partial \mathbf{v}}\mathbf{p}\bullet\mathbf{v}-\frac{\partial L(\mathbf{v})}{\partial \mathbf{v}}$$

$$0=\mathbf{p}-\frac{\partial L(\mathbf{v})}{\partial \mathbf{v}}$$

Introducing the Poincare′ and Legendre contact transformations

Given matrix relation: $\mathbf{p}=\mathbf{M}\bullet\mathbf{v}$ *or its inverse:* $\mathbf{v}=\mathbf{M}^{-1}\bullet\mathbf{p}$ *you might be tempted to rewrite*

Q-forms $L(\mathbf{v}..)=(1/2)\mathbf{v}\bullet\mathbf{M}\bullet\mathbf{v}$ *or* $H(\mathbf{p}..)=(1/2)\mathbf{p}\bullet\mathbf{M}^{-1}\bullet\mathbf{p}$ *to be* $H=(1/2)\mathbf{p}\bullet\mathbf{v}$ *or equivalently* $L=(1/2)\mathbf{v}\bullet\mathbf{p}$.

Numerically-CORRECT, but Differentially-WRONG!

Instead try: $H(\mathbf{p}..)=\mathbf{p}\bullet\mathbf{v}-(1/2)\mathbf{v}\bullet\mathbf{p}=\mathbf{p}\bullet\mathbf{v}-L(\mathbf{v}..)$ *or else:* $L(\mathbf{v}..)=\mathbf{p}\bullet\mathbf{v}-H(\mathbf{p}..)$

That is ... the Legendre contact transformation

$$L(\mathbf{v})=\mathbf{p}\bullet\mathbf{v}-H(\mathbf{p}) \qquad \text{or:} \qquad H(\mathbf{p})=\mathbf{p}\bullet\mathbf{v}-L(\mathbf{v})$$

Now explicit dependency (non)-relations give the right derivatives

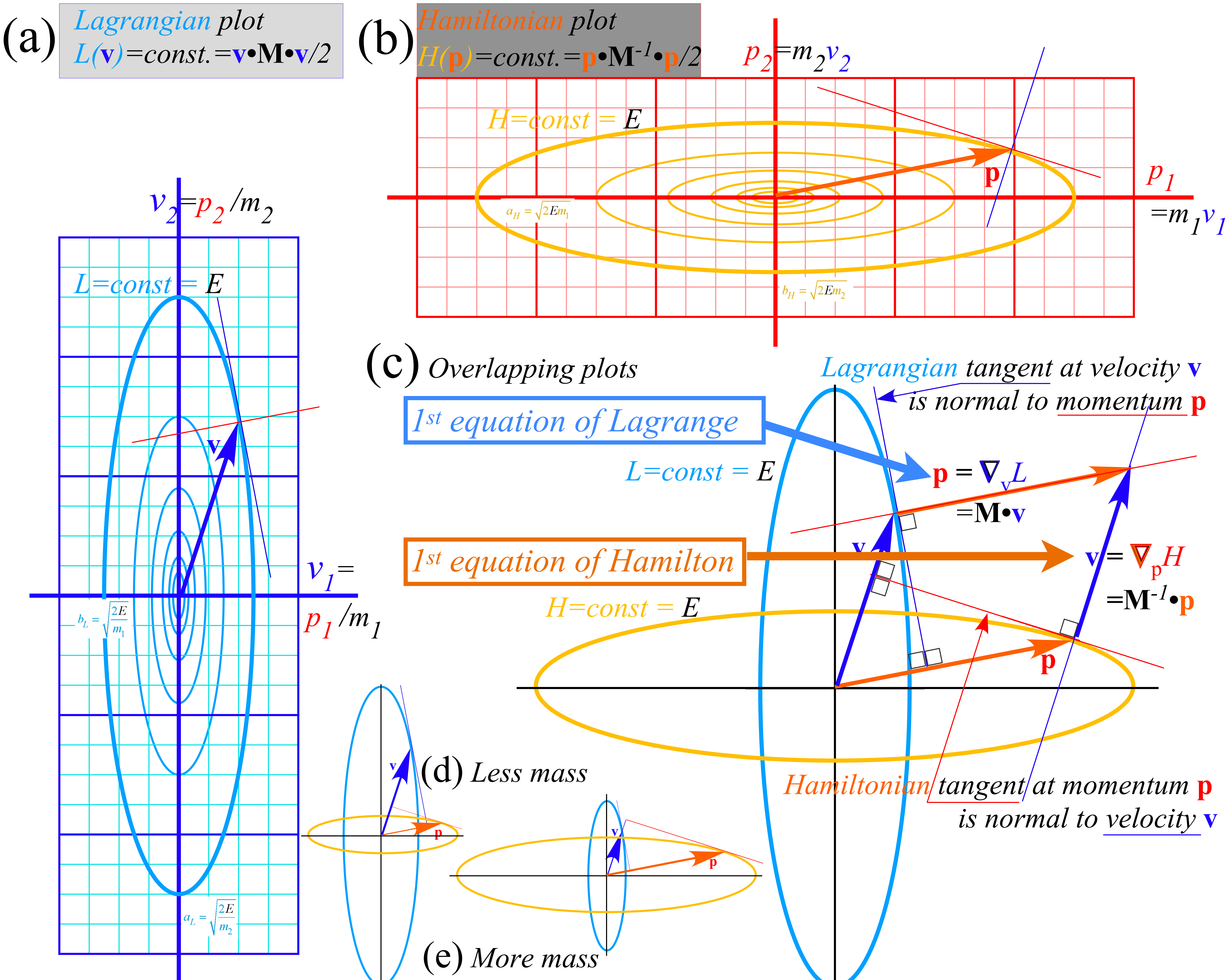
$$\frac{\partial L(\mathbf{v})}{\partial \mathbf{p}}=\frac{\partial}{\partial \mathbf{p}}\mathbf{p}\bullet\mathbf{v}-\frac{\partial H(\mathbf{p})}{\partial \mathbf{p}} \qquad \frac{\partial H(\mathbf{p})}{\partial \mathbf{v}}=\frac{\partial}{\partial \mathbf{v}}\mathbf{p}\bullet\mathbf{v}-\frac{\partial L(\mathbf{v})}{\partial \mathbf{v}}$$

$$0=\mathbf{v}-\frac{\partial H(\mathbf{p})}{\partial \mathbf{p}} \qquad 0=\mathbf{p}-\frac{\partial L(\mathbf{v})}{\partial \mathbf{v}}$$

That is *Hamilton's 1st equation(s)* *and* *Lagrange's 1st equation(s)*

$$\mathbf{v}=\frac{\partial H(\mathbf{p})}{\partial \mathbf{p}} \qquad \mathbf{p}=\frac{\partial L(\mathbf{v})}{\partial \mathbf{v}}$$

Unit 1
Fig. 12.2



Review of partial differential calculus

Chain rule and order $\partial^2\Psi/\partial x\partial y = \partial^2\Psi/\partial y\partial x$ symmetry

Scaling transformation between Lagrangian and Hamiltonian views of KE

Introducing 0th Lagrange and 0th Hamilton differential equations of mechanics

Introducing 1st Lagrange and 1st Hamilton differential equations of mechanics

Introducing the Poincare´ and Legendre contact transformations

Geometry of Legendre contact transformation (Preview of Unit 8 relativistic quantum mechanics)

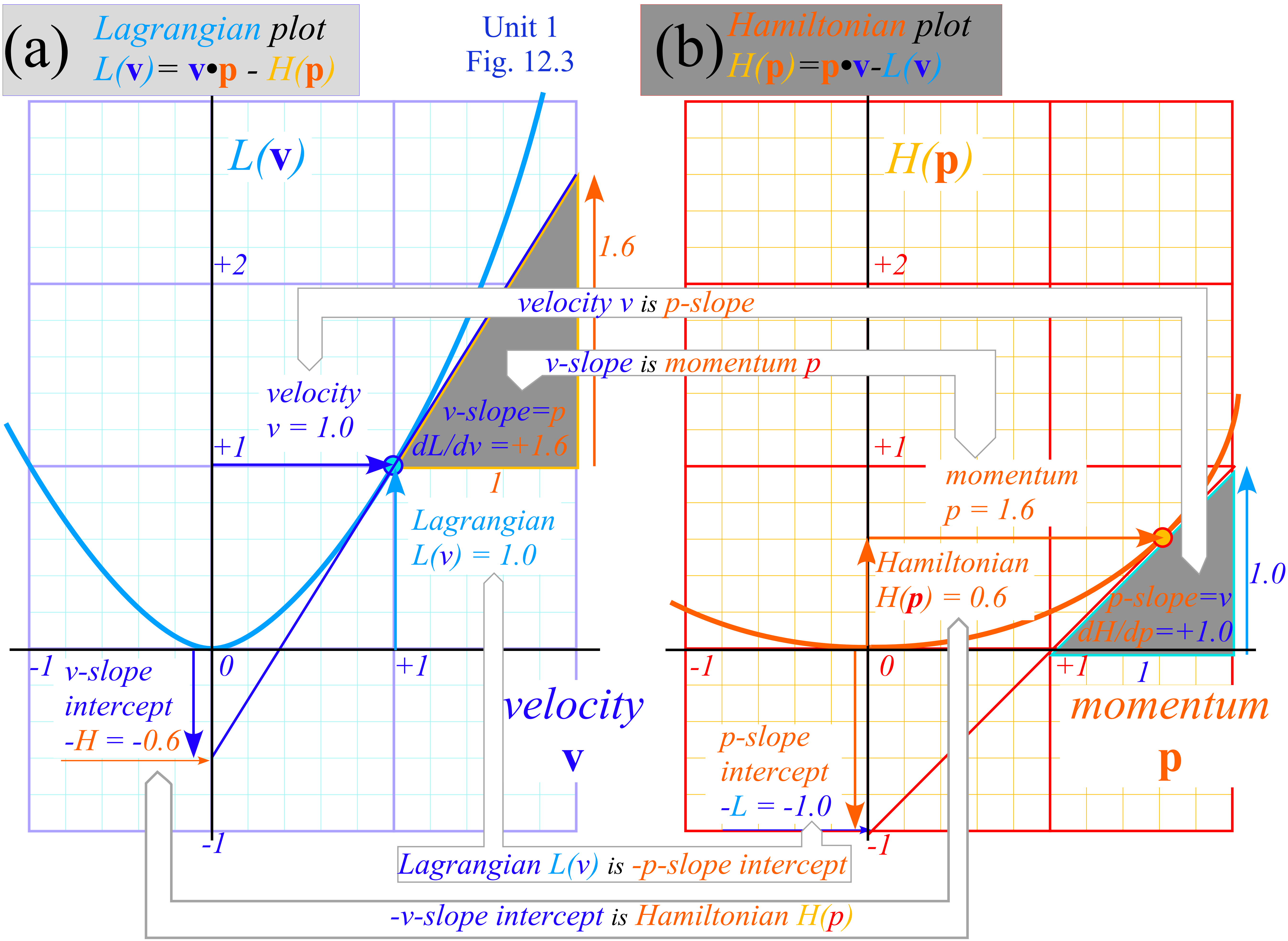
Example from thermodynamics

Legendre transform: special case of General Contact Transformation (lights,camera, ACTION!)

An elementary contact transformation from sophomore physics

Algebra-calculus development of "The Volcanoes of Io" and "The Atoms of NIST"

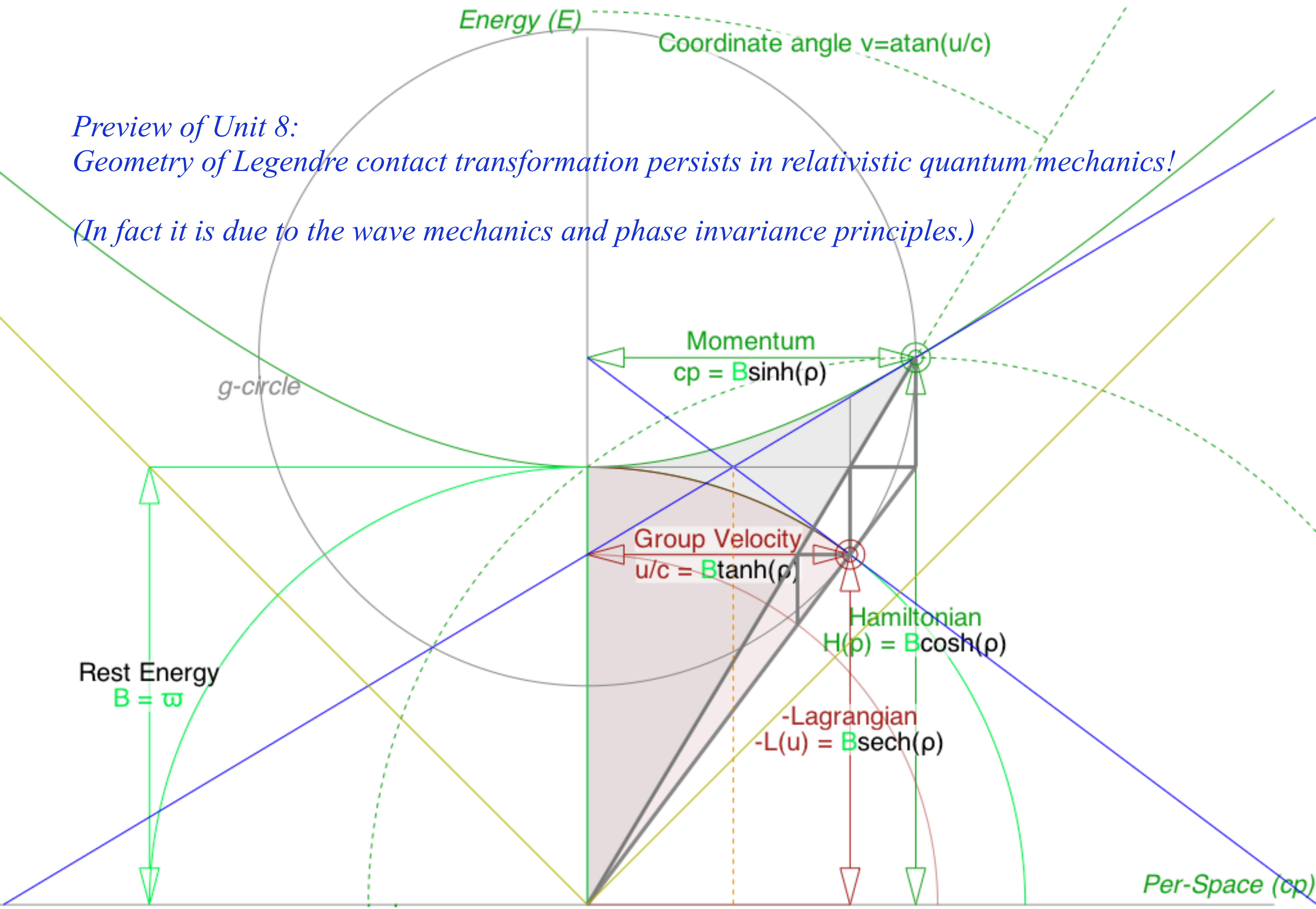
Intuitive-geometric development of " " " and " " "



(a) Lagrangian plot
L(v)= v•p - H(p)
Unit 1
Fig. 12.3
(b) Hamiltonian plot
H(p)=p•v-L(v)
L(v)
+2
+1
0
-1
+1
1.6
1
velocity v is p-slope
v-slope is momentum p
velocity
v = 1.0
v-slope=p
dL/dv =+1.6
Lagrangian
L(v) = 1.0
v-slope
intercept
-H = -0.6
velocity
v
H(p)
+2
+1
-1
0
momentum
p = 1.6
Hamiltonian
H(p) = 0.6
p-slope=v
dH/dp=+1.0
1.0
1
momentum
p
p-slope
intercept
-L = -1.0
Lagrangian L(v) is -p-slope intercept
-v-slope intercept is Hamiltonian H(p)

Preview of Unit 8:
Geometry of Legendre contact transformation persists in relativistic quantum mechanics!

(In fact it is due to the wave mechanics and phase invariance principles.)



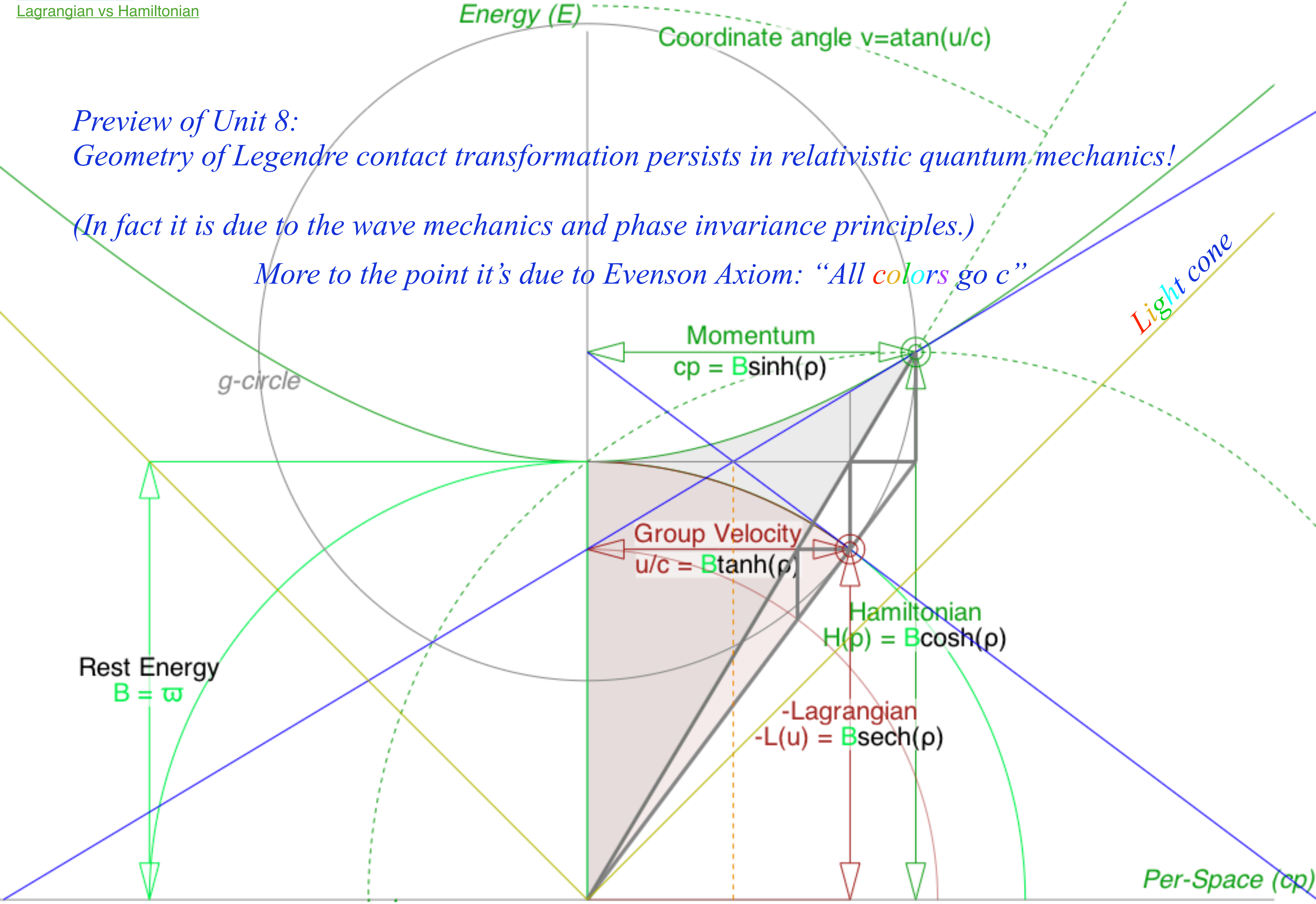
Link ⇒ RelaWavity - Physical Terms H(p) & L(u)

“Relawavity” and quantum basis of Lagrangian & Hamiltonian mechanics:
2-CW laser wave
Lagrangian vs Hamiltonian

Preview of Unit 8:
Geometry of Legendre contact transformation persists in relativistic quantum mechanics!

(In fact it is due to the wave mechanics and phase invariance principles.)

More to the point it’s due to Evenson Axiom: “All colors go c”



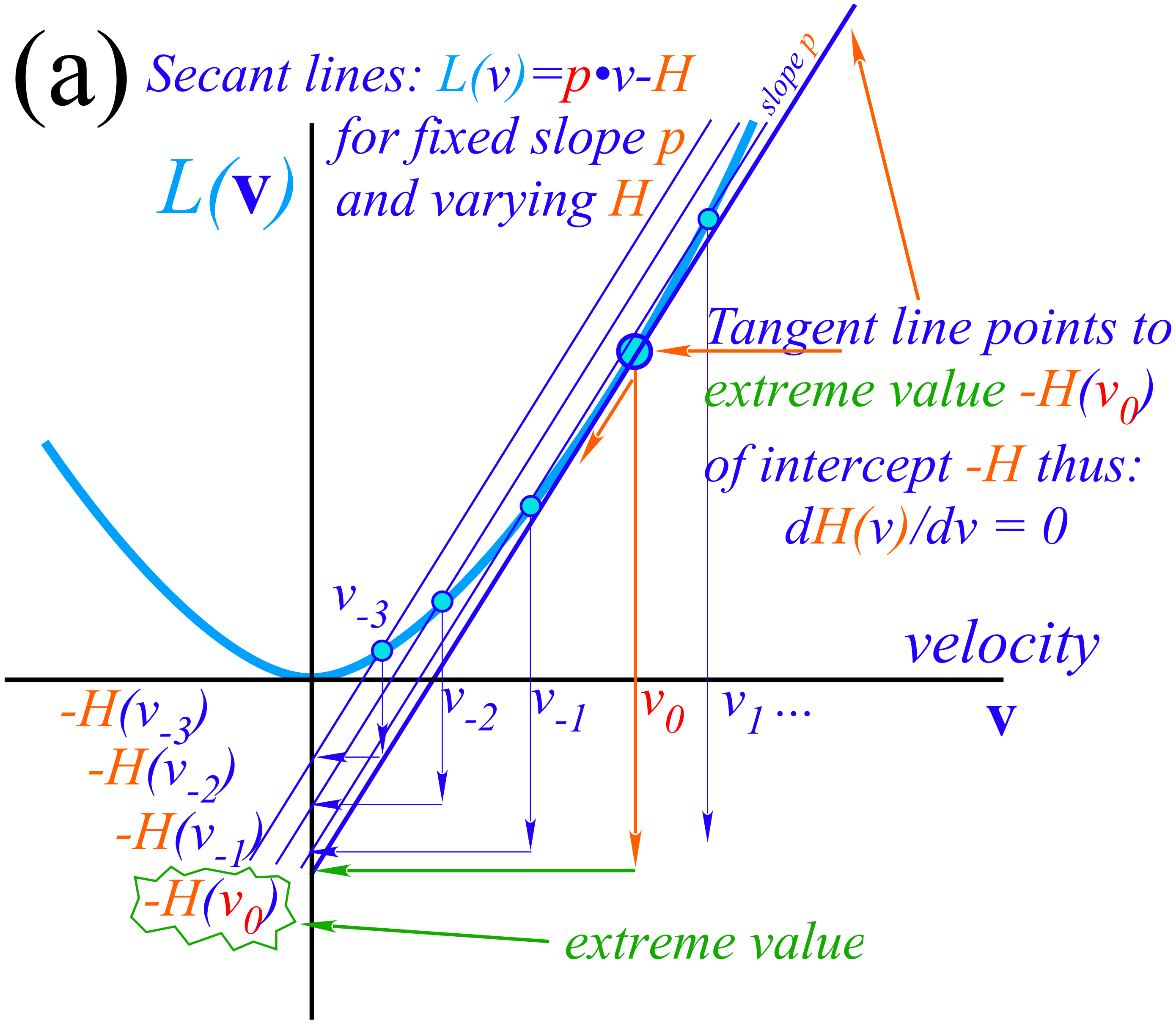
Link ⇒ RelaWavity - Physical Terms H(p) & L(u)

How Legendre contact transformations work...(to make $\frac{\partial H}{\partial v}=0$ or $\frac{\partial L}{\partial p}=0$)

Secant lines $L(\mathbf{v}) = p \bullet v - H$ *of fixed slope* $p=\frac{\partial L}{\partial v}$
and decreasing intercept $-H(v_{-2}) > -H(v_{-1}) > \ldots$
for increasing velocity $v_{-2} > v_{-1} > \ldots > v_0$
lead to unique tangent to $L(\mathbf{v})$*-curve at the tangent contact point* $v=v_0$ *that has* *max* $H(p\ v_0)$
Thus $\frac{\partial H}{\partial v}=0$

Unit 1
Fig. 12.4

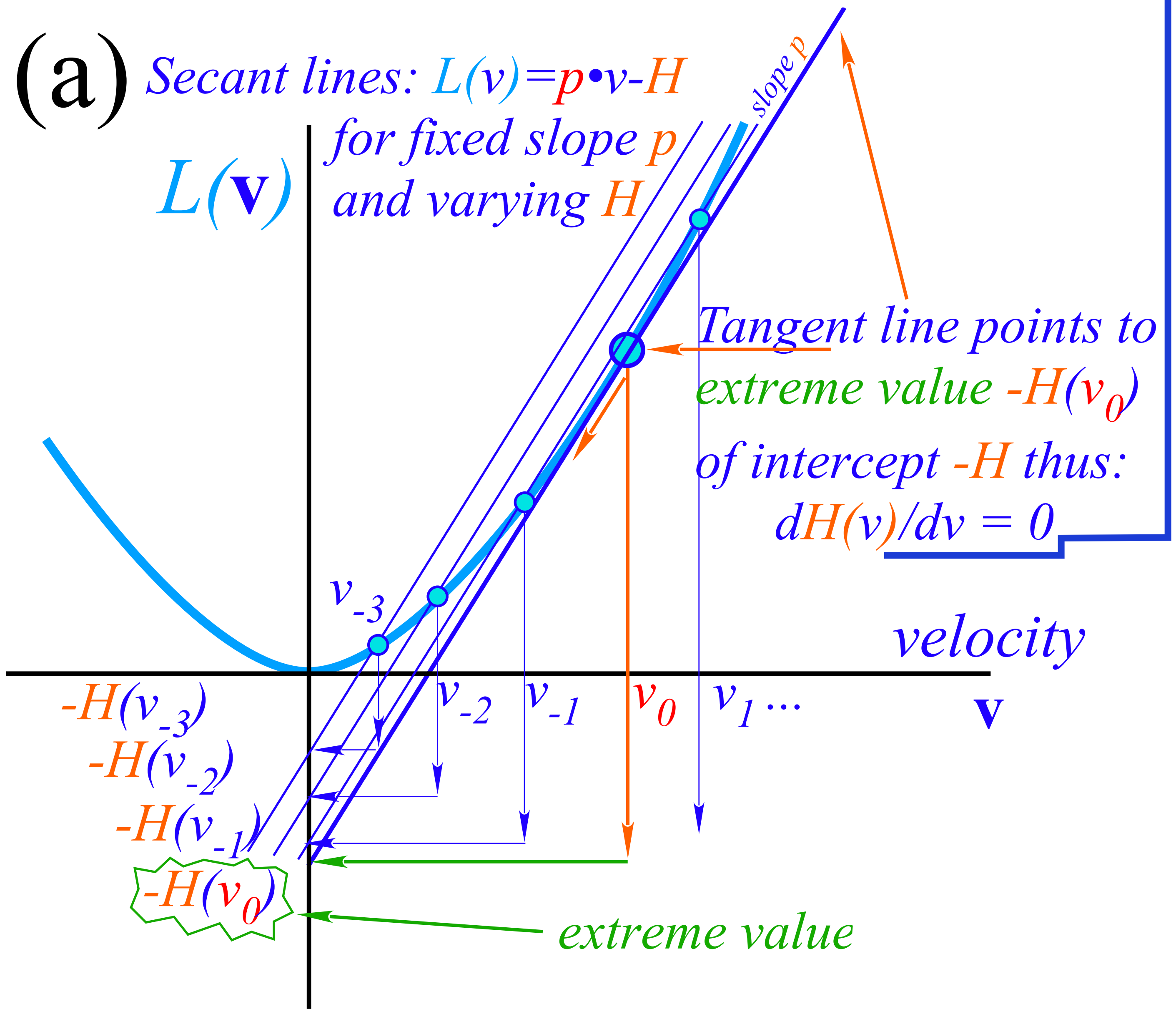
(a) *Secant lines:* $L(v)=p \bullet v-H$



How Legendre contact transformations work... (to make $\frac{\partial H}{\partial v}=0$ or $\frac{\partial L}{\partial p}=0$)

Secant lines $L(\mathbf{v}) = p \bullet v - H$ *of fixed slope* $p=\frac{\partial L}{\partial v}$
and decreasing intercept $-H(v_{-2}) > -H(v_{-1}) > \ldots$
for increasing velocity $v_{-2} > v_{-1} > \ldots > v_0$
lead to unique tangent to $L(\mathbf{v})$*-curve at the tangent contact point* $v=v_0$ *that has* <u>*max*</u> $H(p\ v_0)$
Thus $\frac{\partial H}{\partial v}=0$

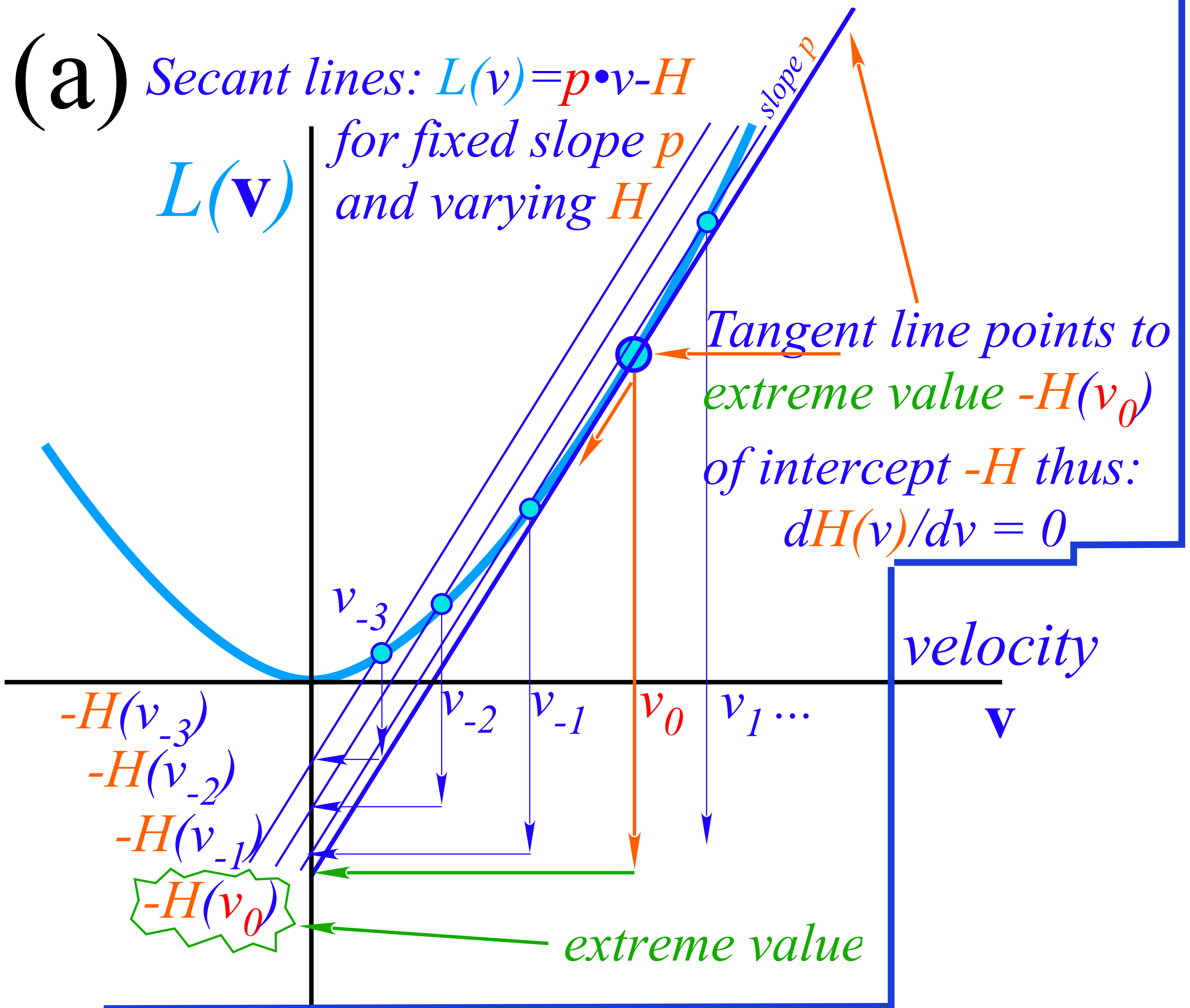
Unit 1
Fig. 12.4



How Legendre contact transformations work... (to make $\frac{\partial H}{\partial v}=0$ or $\frac{\partial L}{\partial p}=0$)

Secant lines $L(\mathbf{v}) = p \bullet v - H$ *of fixed slope* $p=\frac{\partial L}{\partial v}$
and decreasing intercept $-H(v_{-2}) > -H(v_{-1}) > \ldots$
for increasing velocity $v_{-2} > v_{-1} > \ldots > v_0$
lead to unique tangent to $L(\mathbf{v})$*-curve at the tangent contact point* $v=v_0$ *that has* <u>*max*</u> $H(p\ v_0)$
Thus $\frac{\partial H}{\partial v}=0$

Unit 1
Fig. 12.4



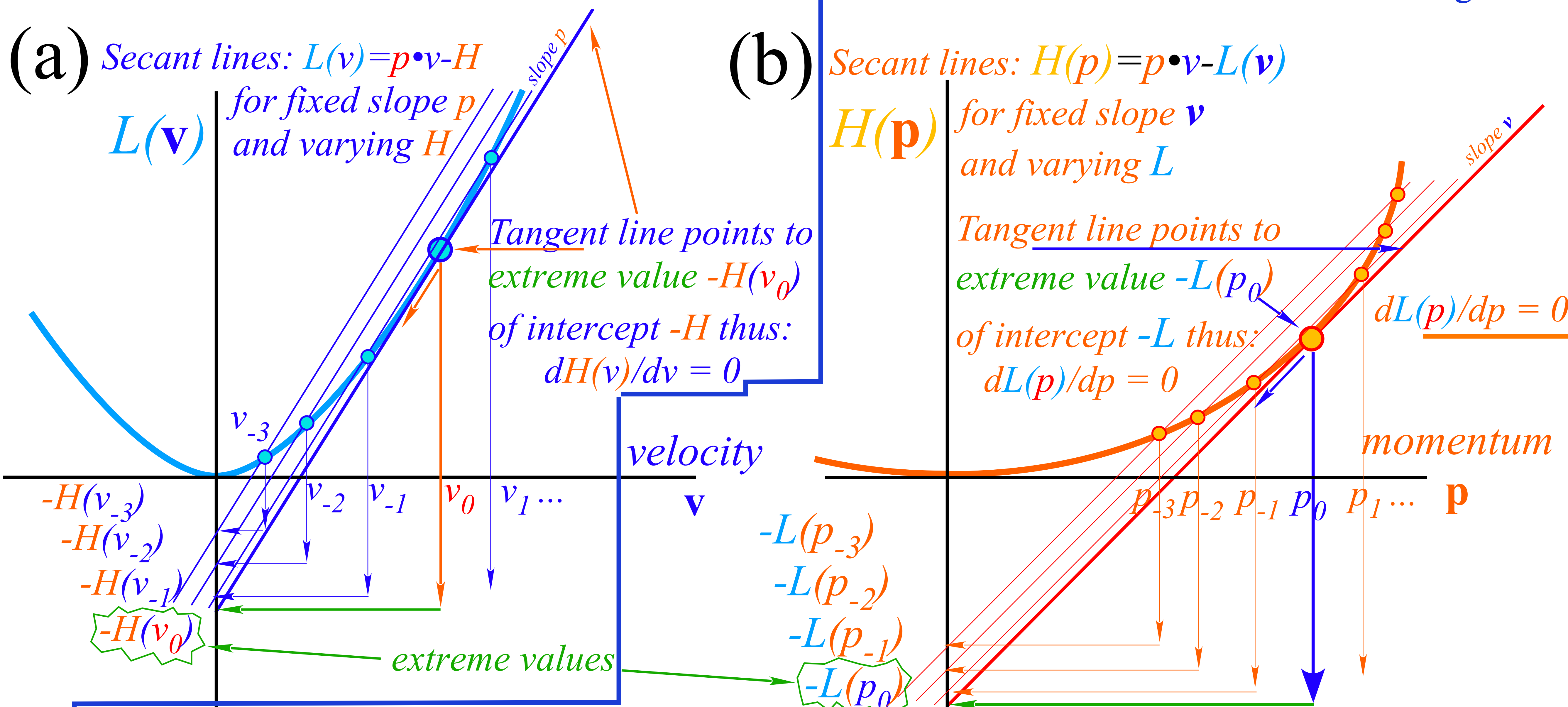
$\frac{\partial H}{\partial v}=0$ at each point $v=\frac{\partial H}{\partial p}$ of $L(v)$ with slope $p=\frac{\partial L}{\partial v}$

How Legendre contact transformations work...(to make $\frac{\partial H}{\partial v}=0$ or $\frac{\partial L}{\partial p}=0$)

Secant lines $L(\mathbf{v}) = p \bullet v - H$ *of fixed slope* $p=\frac{\partial L}{\partial v}$
and decreasing intercept $-H(v_{-2}) > -H(v_{-1}) > \ldots$
for increasing velocity $v_{-2} > v_{-1} > \ldots > v_0$
*lead to unique tangent to L(***v***)-curve at the tangent contact point v=v₀ that has* *max* *H(p v₀)*
Thus $\frac{\partial H}{\partial v}=0$

(Similarly...)

Unit 1
Fig. 12.4



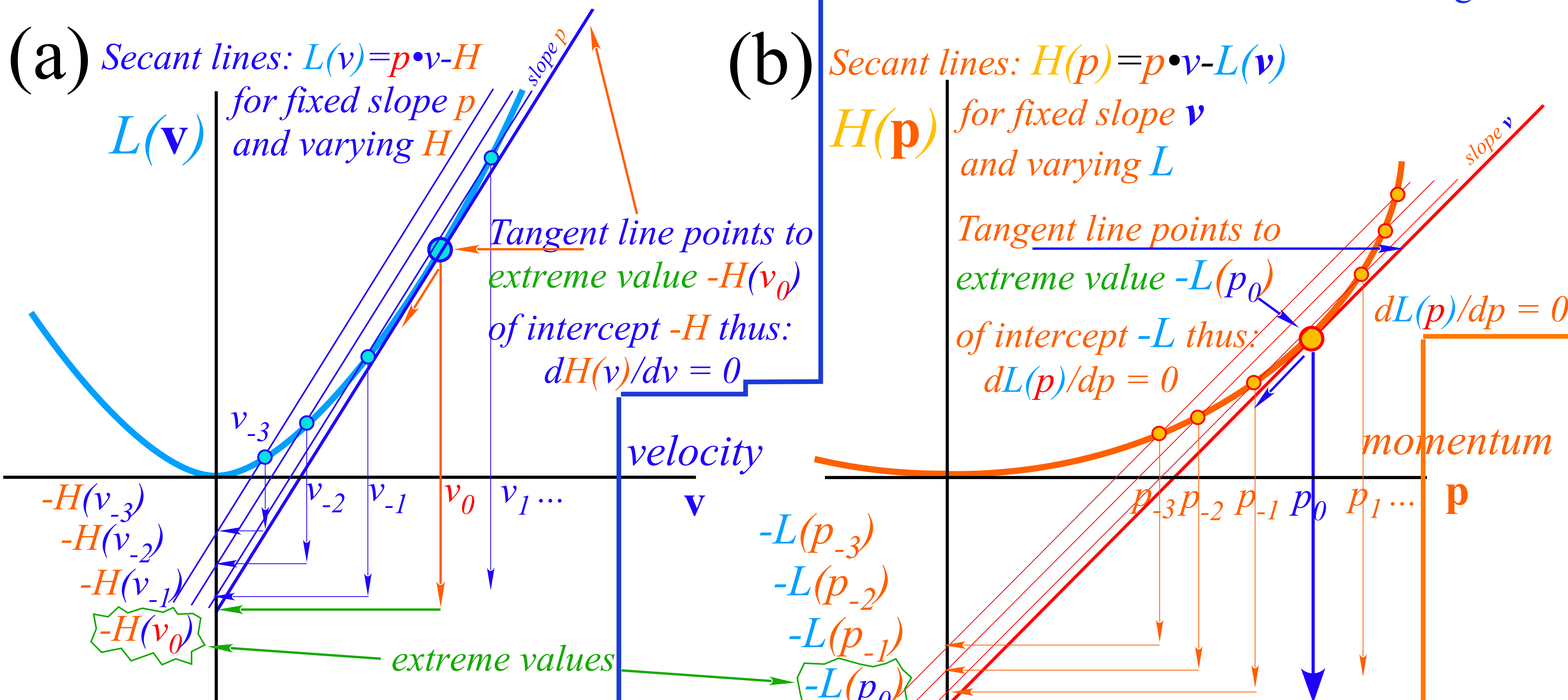
$\frac{\partial H}{\partial v}=0$ at each point $v=\frac{\partial H}{\partial p}$ of $L(v)$ with slope $p=\frac{\partial L}{\partial v}$

How Legendre contact transformations work...(to make $\frac{\partial H}{\partial v}=0$ or $\frac{\partial L}{\partial p}=0$)

Secant lines $L(\mathbf{v}) = p \bullet v - H$ *of fixed slope* $p=\frac{\partial L}{\partial v}$
and decreasing intercept $-H(v_{-2}) > -H(v_{-1}) > \ldots$
for increasing velocity $v_{-2} > v_{-1} > \ldots > v_0$
lead to unique tangent to $L(\mathbf{v})$*-curve at the tangent contact point* $v=v_0$ *that has* <u>*max*</u> $H(p\ v_0)$
Thus $\frac{\partial H}{\partial v}=0$

(Similarly...)
(Recall "Zeroth" equations on <u>*p.28*</u> *)*

Unit 1
Fig. 12.4



$\frac{\partial H}{\partial v}=0$ at each point $v=\frac{\partial H}{\partial p}$ of $L(v)$ with slope $p=\frac{\partial L}{\partial v}$

$\frac{\partial L}{\partial p}=0$ at each point $p=\frac{\partial L}{\partial v}$ of $H(p)$ with slope $v=\frac{\partial H}{\partial p}$

Review of partial differential calculus
Chain rule and order $\partial^2\Psi/\partial x\partial y = \partial^2\Psi/\partial y\partial x$ symmetry

Scaling transformation between Lagrangian and Hamiltonian views of KE
Introducing 0th Lagrange and 0th Hamilton differential equations of mechanics
Introducing 1st Lagrange and 1st Hamilton differential equations of mechanics

Introducing the Poincare′ and Legendre contact transformations
Geometry of Legendre contact transformation (Preview of Unit 8 relativistic quantum mechanics)
→ *Example from thermodynamics*
Legendre transform: special case of General Contact Transformation (lights,camera, ACTION!)

An elementary contact transformation from sophomore physics
Algebra-calculus development of "The Volcanoes of Io" and "The Atoms of NIST"
Intuitive-geometric development of " " " and " " "

Example of Legendre contact transformation in thermodynamics

Internal energy $U(S,V)$ is defined as a function of entropy S and volume V.

A new function *enthalpy* $H(S,P)$ depends on entropy and *pressure* P.

It is a Legendre transform $H(S,P)=P\cdot V+U$ of energy $U(S,V)$ to new variable $P=-(\frac{\partial U}{\partial V})_S$.

Example of Legendre contact transformation in thermodynamics

Lagrangian $L(r,v)$ position r velocity v

Internal energy $U(S,V)$ is defined as a function of entropy S and volume V.

Hamiltonian $H(r,p)$ position r *momentum* p

A new function *enthalpy* $H(S,P)$ depends on entropy and *pressure* P.

$H(r,p) = p \cdot v - L$ Lagrangian $L(r,v)$ $p = \left(\frac{\partial L}{\partial v}\right)_r$

It is a Legendre transform $H(S,P)=P \cdot V+U$ of energy $U(S,V)$ to new variable $P = -\left(\frac{\partial U}{\partial V}\right)_S$.

Example of Legendre contact transformation in thermodynamics

Lagrangian $L(r,v)$ position r velocity v

Internal energy U(S,V) is defined as a function of entropy S and volume V.

Hamiltonian $H(r,p)$ position r *momentum* p

A new function *enthalpy H(S,P)* depends on entropy and *pressure P*.

$H(r,p)= p \cdot v - L$ Lagrangian $L(r,v)$ $p=(\frac{\partial L}{\partial v})_r$

It is a Legendre transform $H(S,P)=P \cdot V+U$ of energy $U(S,V)$ to new variable $P=-(\frac{\partial U}{\partial V})_S$.

Except for ±signs, it's our Hamiltonian $H(p)=p \cdot v-L(v)$ going from Lagrangian $L(v)$

to use new variable momentum $p=(\frac{\partial L}{\partial v})_x$.

Review of partial differential calculus
Chain rule and order $\partial^2\Psi/\partial x\partial y = \partial^2\Psi/\partial y\partial x$ symmetry

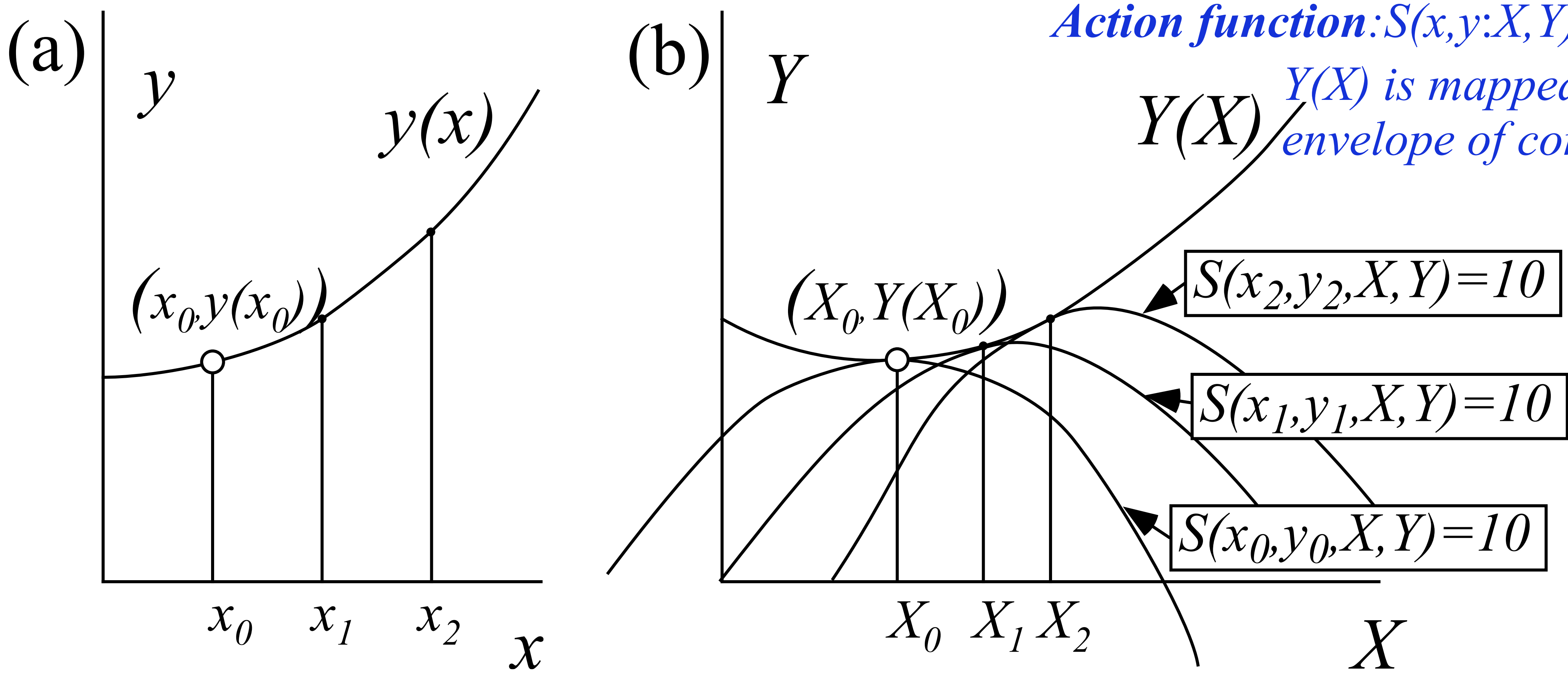
Scaling transformation between Lagrangian and Hamiltonian views of KE
Introducing 0th Lagrange and 0th Hamilton differential equations of mechanics
Introducing 1st Lagrange and 1st Hamilton differential equations of mechanics

Introducing the Poincare′ and Legendre contact transformations
Geometry of Legendre contact transformation (Preview of Unit 8 relativistic quantum mechanics)
Example from thermodynamics
→ *Legendre transform: special case of General Contact Transformation (lights,camera, ACTION!)*

An elementary contact transformation from sophomore physics
Algebra-calculus development of "The Volcanoes of Io" and "The Atoms of NIST"
Intuitive-geometric development of " " " and " " "

Legendre transform: special case of General Contact Transformation

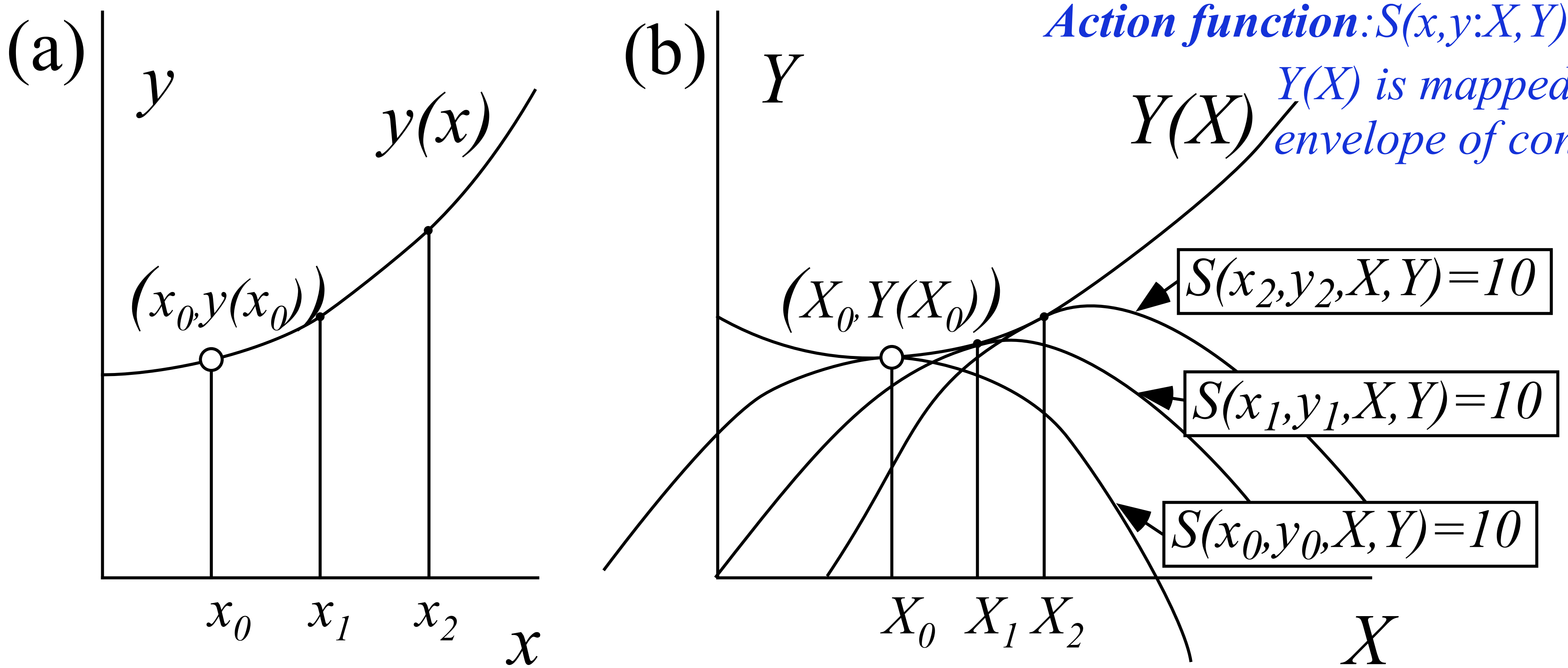
Active-Contact-Transformation Generator or
Action function: $S(x,y{:}X,Y)$=const. *does mapping.*
$Y(X)$ is mapped from $y(x)$ as an envelope of contacting S=const. curves.



Unit 1
Fig. 12.7

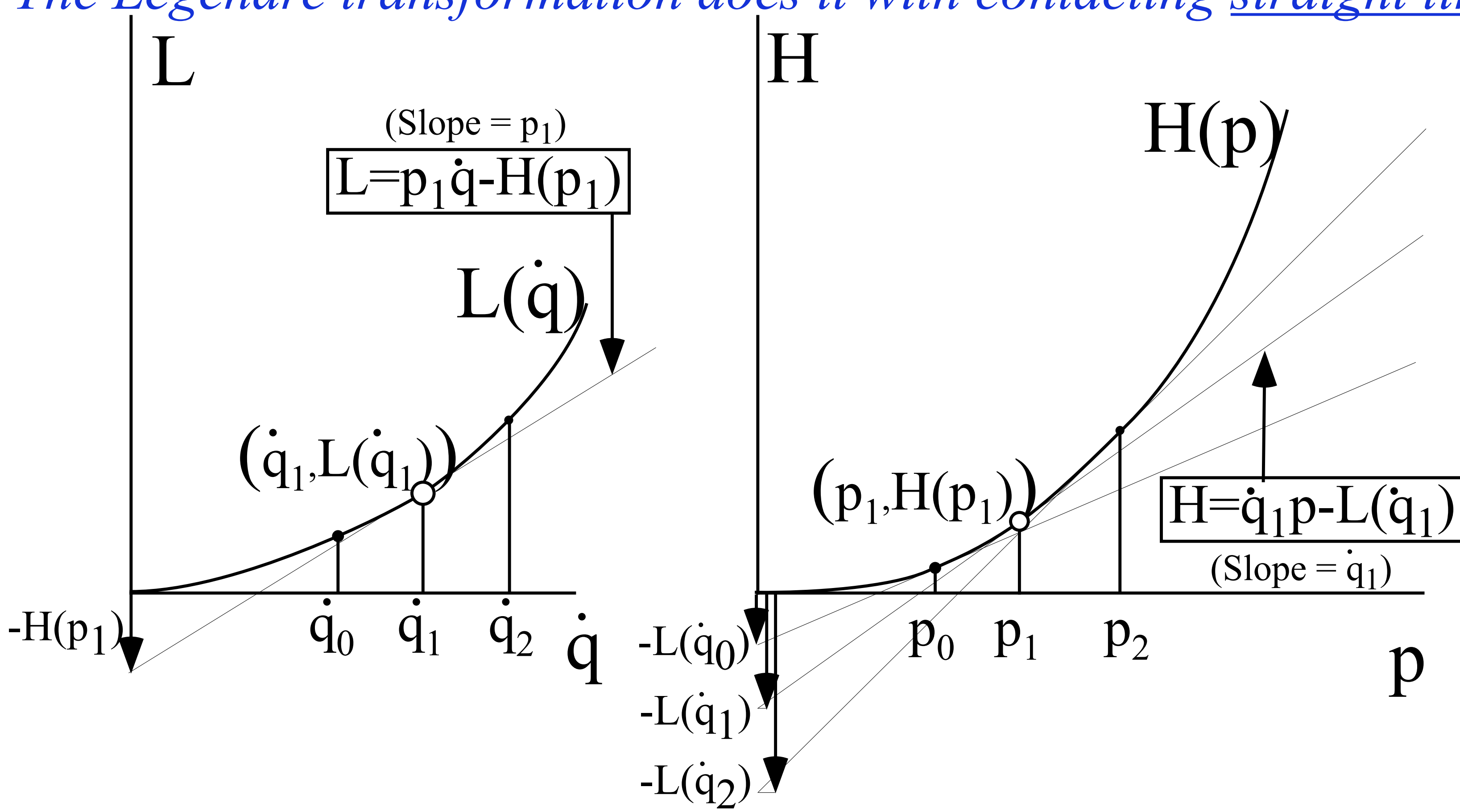
Legendre transform: special case of General Contact Transformation

Active-Contact-Transformation Generator or ***Action function****: S(x,y:X,Y)=const. does mapping.*
Y(X) is mapped from y(x) as an envelope of contacting S=const. curves.



Unit 1
Fig. 12.7

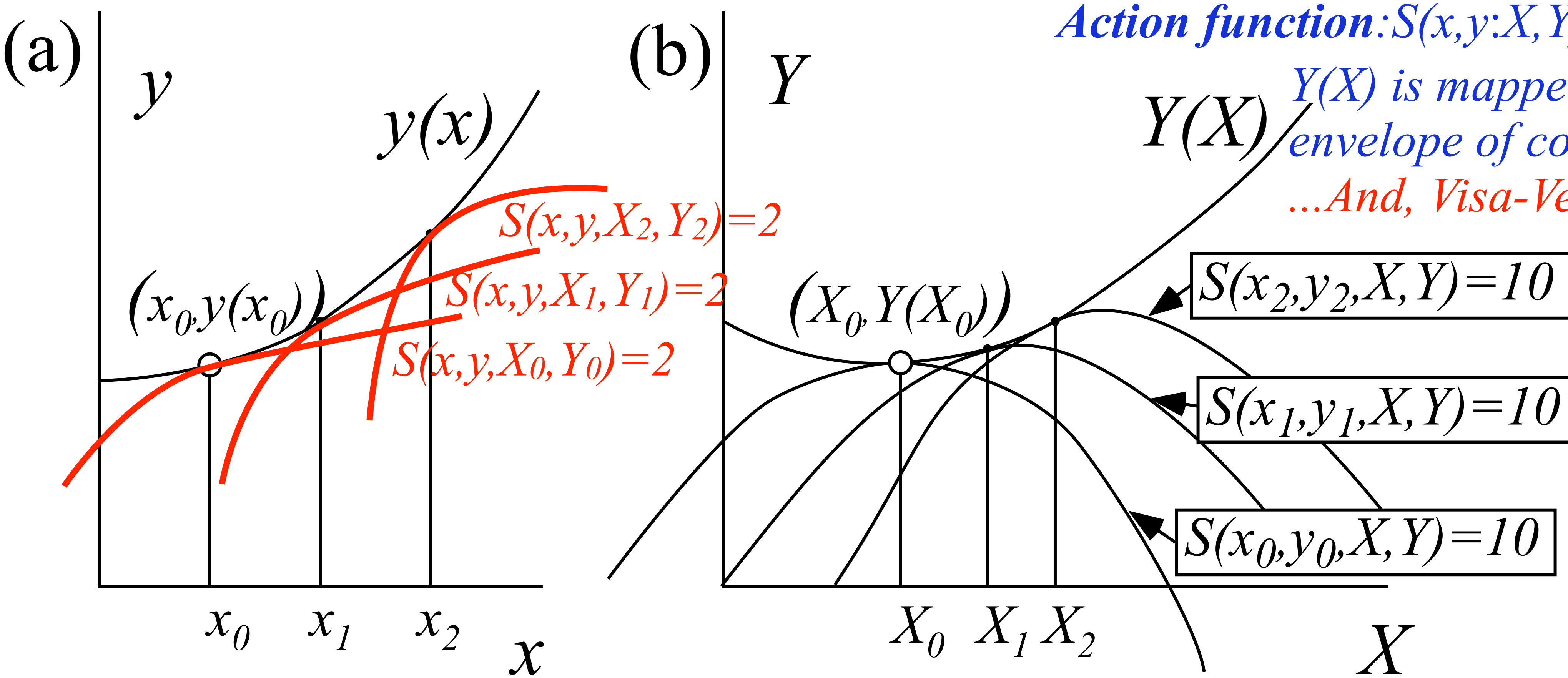
The Legendre transformation does it with contacting *<u>straight line</u>* *tangents.*



Unit 1
Fig. 12.9

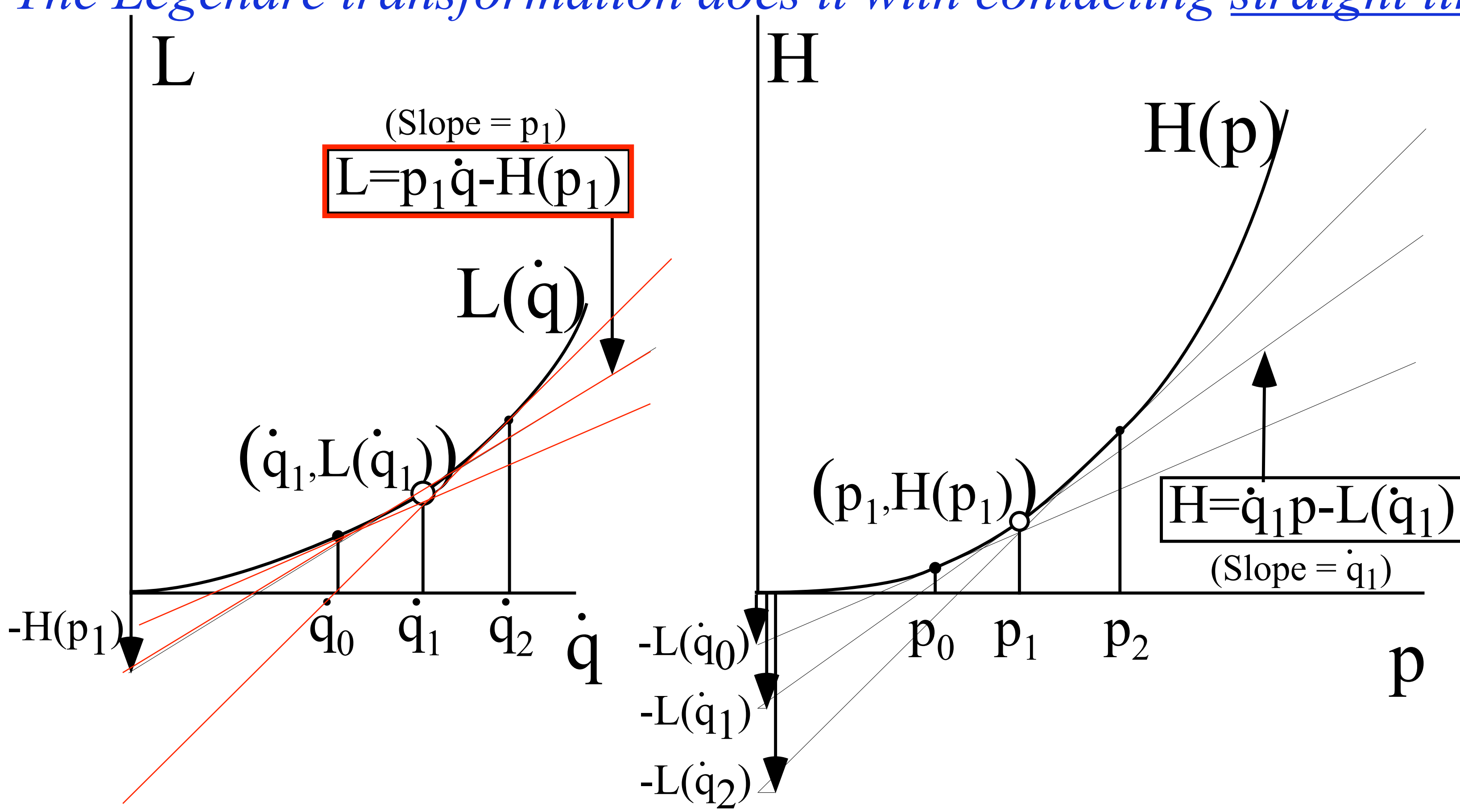
Legendre transform: special case of General Contact Transformation

Active-Contact-Transformation Generator or
Action function:*S(x,y:X,Y)=const. does mapping.*
Y(X) is mapped from y(x) as an envelope of contacting S=const. curves.
...And, Visa-Versa !...



Unit 1
Fig. 12.7

The Legendre transformation does it with contacting <u>straight line</u> tangents.



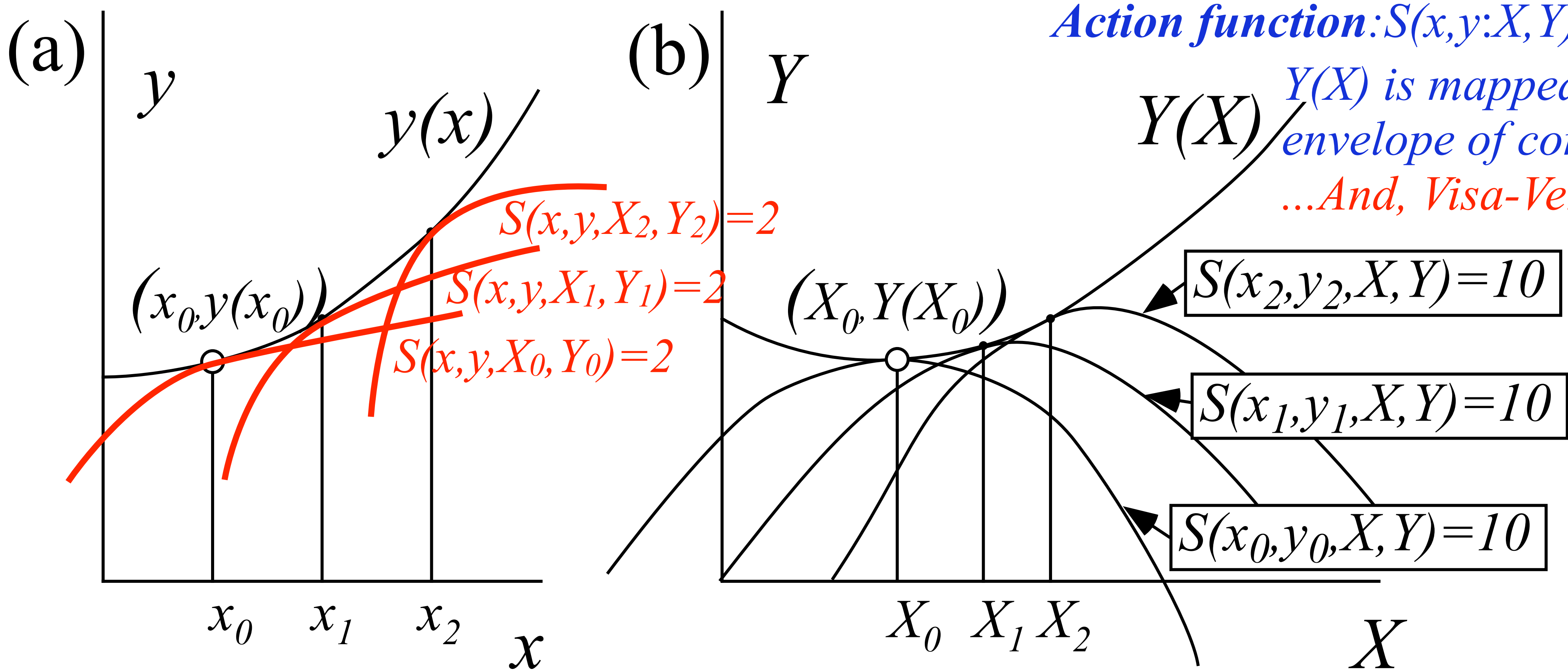
Unit 1
Fig. 12.9

Legendre transform: special case of General Contact Transformation

Active-Contact-Transformation Generator or
Action function: *S(x,y:X,Y)=const. does mapping.*

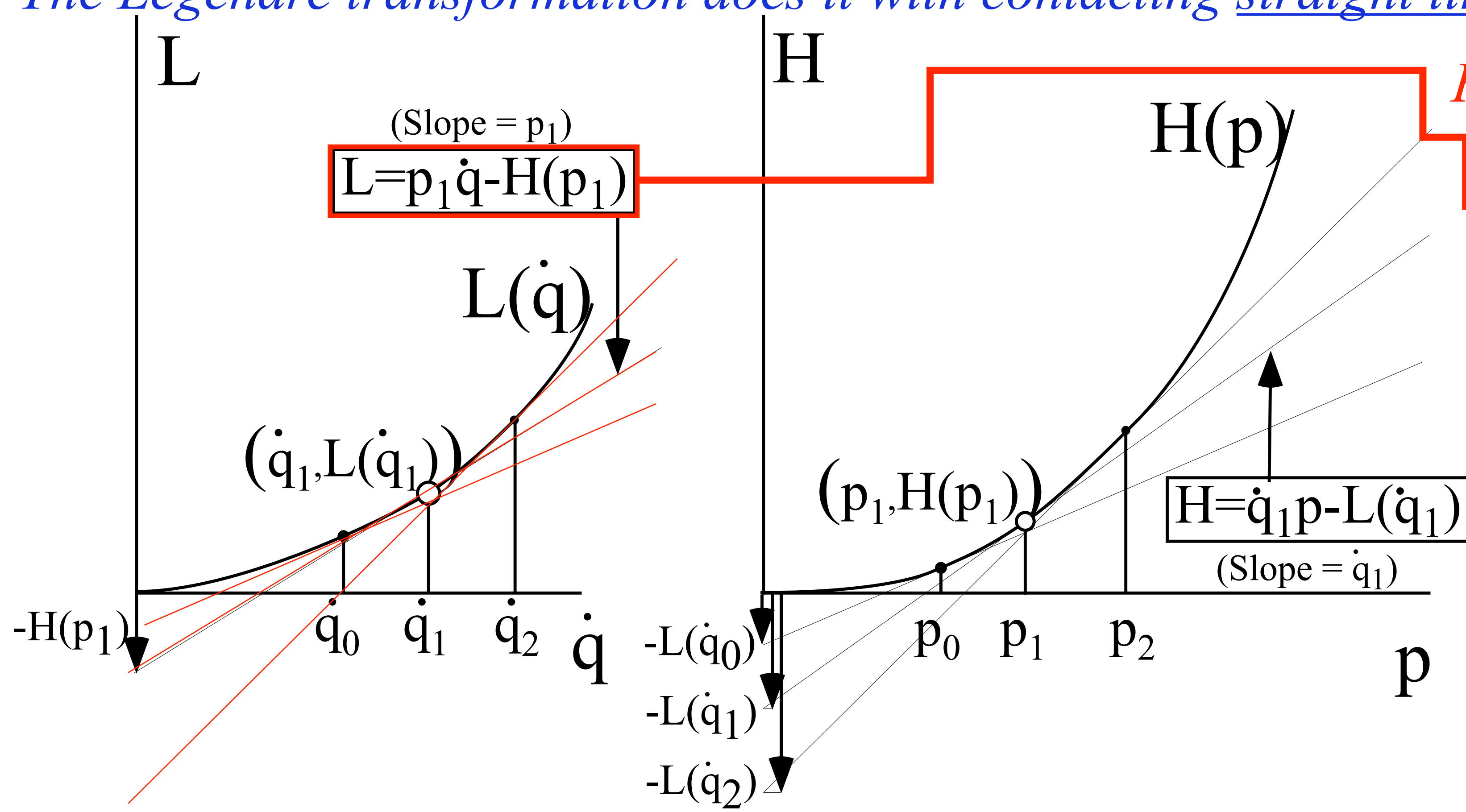
Y(X) is mapped from y(x) as an envelope of contacting S=const. curves.

...And, Visa-Versa !...



Unit 1
Fig. 12.7

The Legendre transformation does it with contacting straight line tangents.



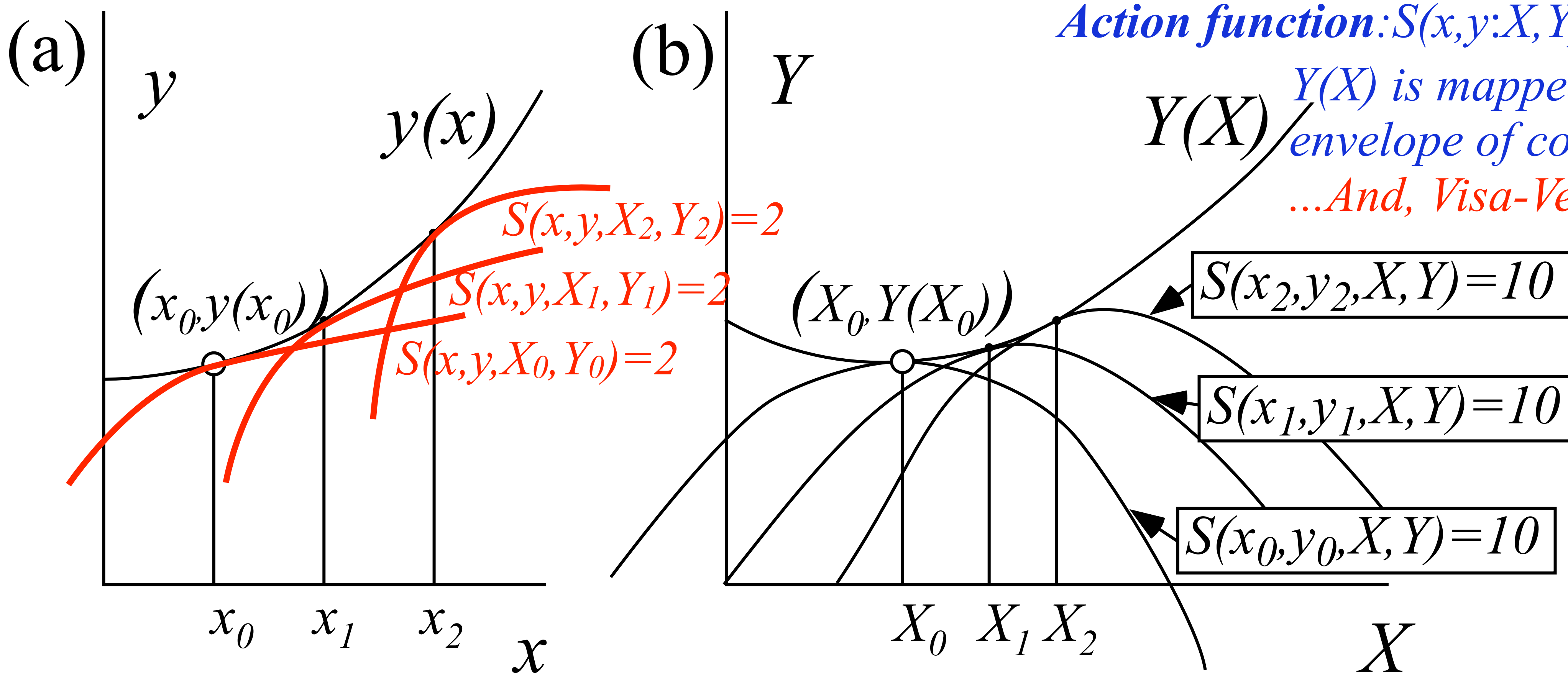
Poincare's differential action

$$dS = L\,dt = p\cdot\dot{q}\,dt - H\cdot dt$$
$$= p\cdot dq - H\cdot dt$$

Unit 1
Fig. 12.9

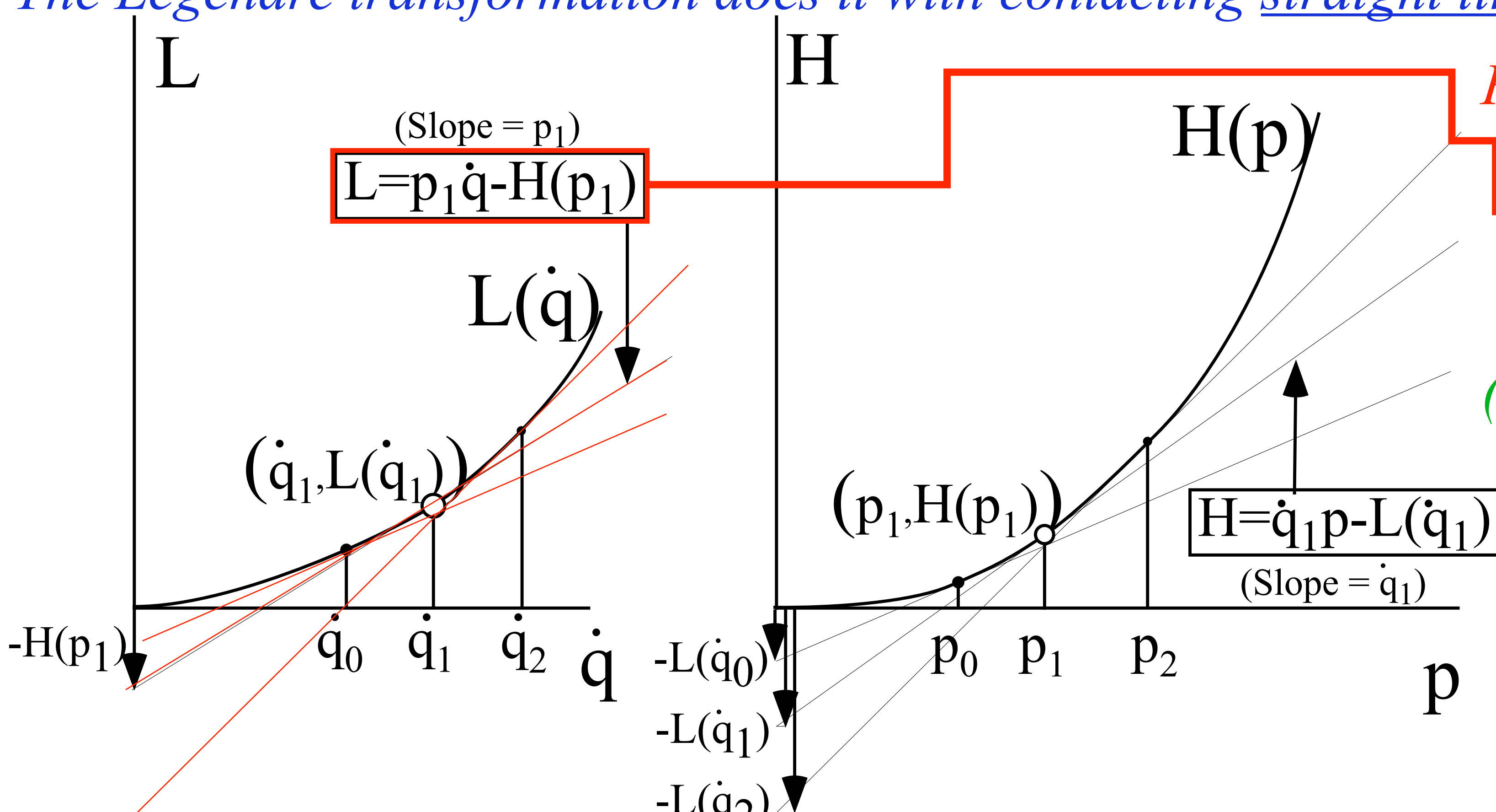
Legendre transform: special case of General Contact Transformation

Active-Contact-Transformation Generator or
Action function:*S(x,y:X,Y)=const. does mapping.*
Y(X) is mapped from y(x) as an envelope of contacting S=const. curves.
...And, Visa-Versa !...



Unit 1
Fig. 12.7

The Legendre transformation does it with contacting straight line tangents.



Poincare's differential action

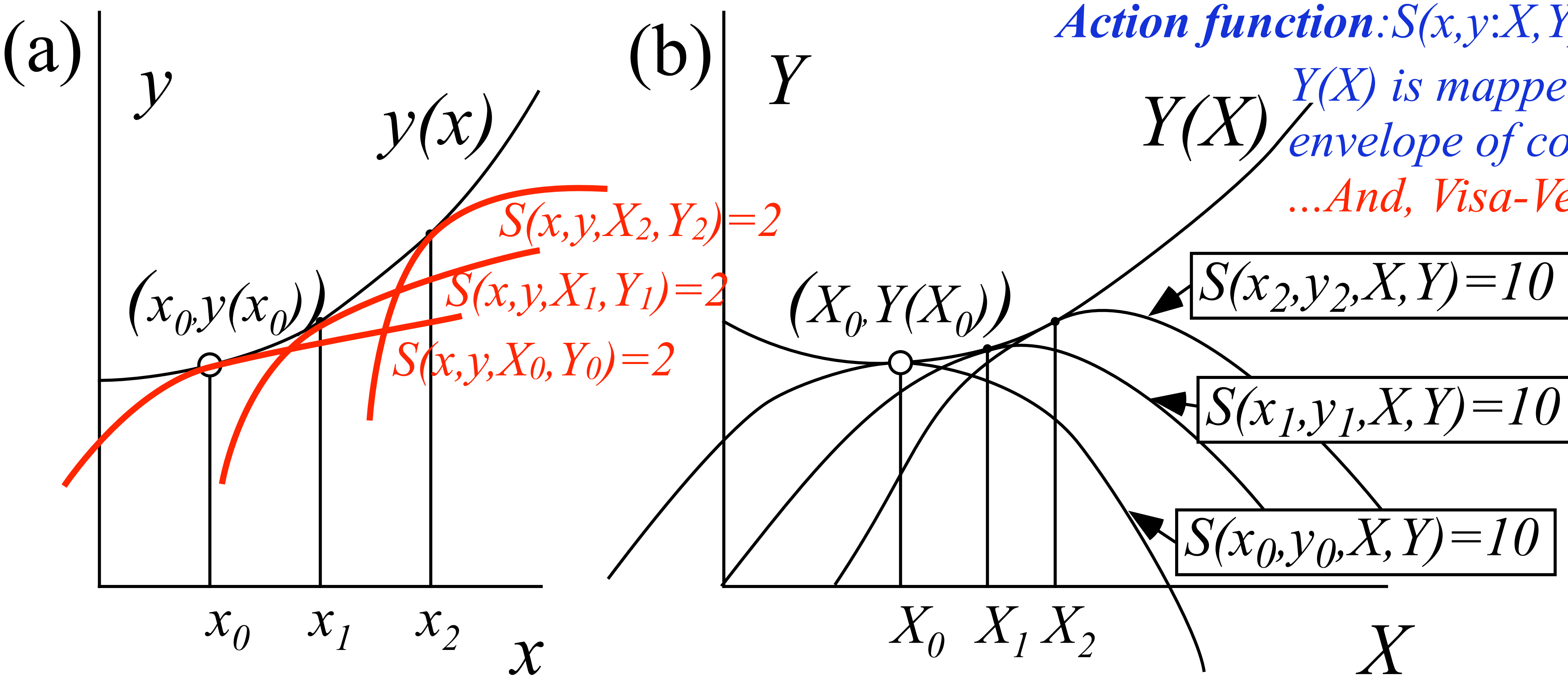
$$dS = L\,dt = p \cdot \dot{q}\,dt - H \cdot dt$$
$$= p \cdot dq - H \cdot dt$$

$$dS = L\,dt = \hbar k \cdot dr - \hbar\omega \cdot dt$$
(Quantum phase differential)

Unit 1
Fig. 12.9

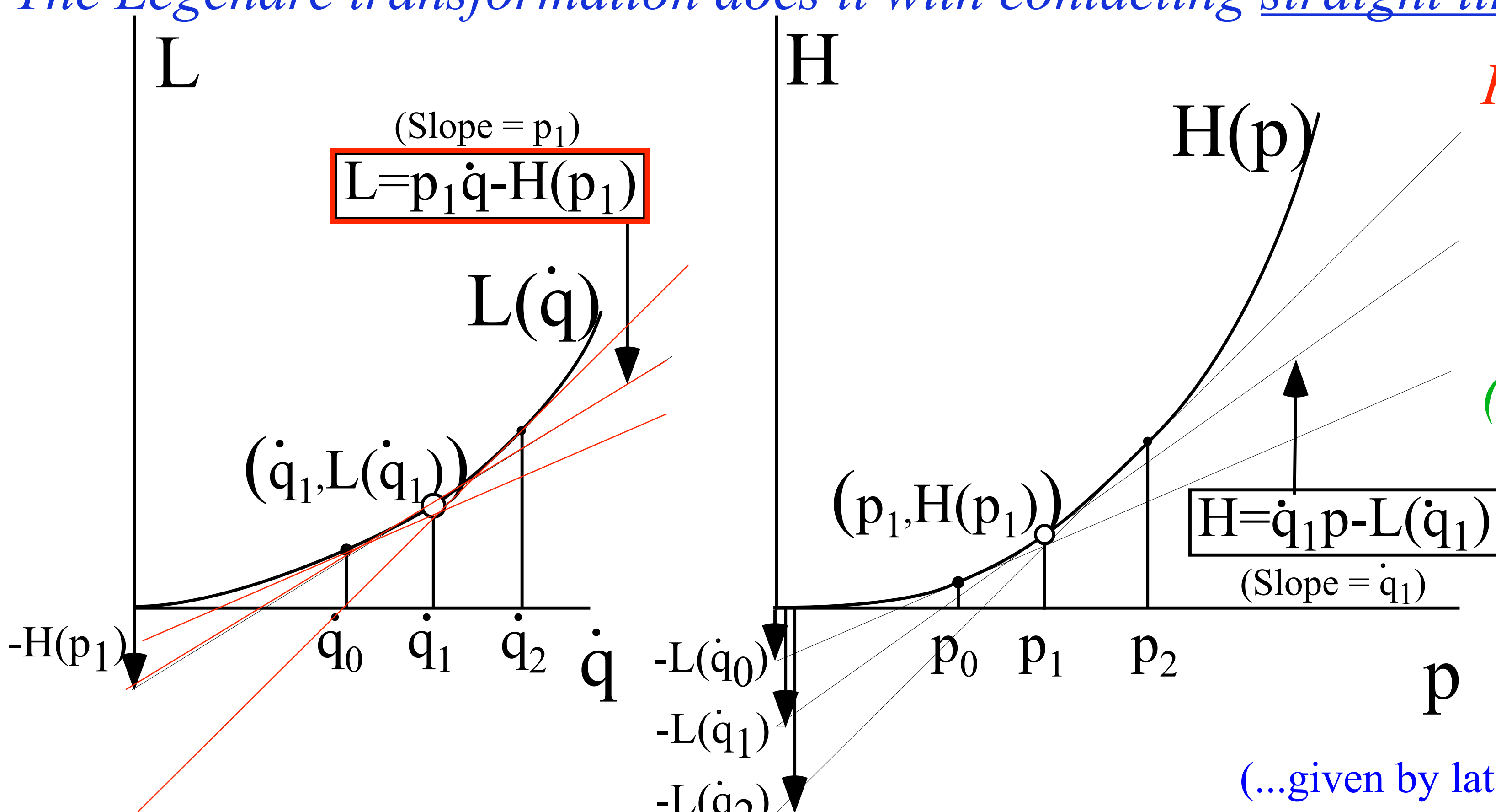
Legendre transform: special case of General Contact Transformation

Active-Contact-Transformation Generator or ***Action function****: S(x,y:X,Y)=const. does mapping.*
Y(X) is mapped from y(x) as an envelope of contacting S=const. curves.
...And, Visa-Versa !...



Unit 1
Fig. 12.7

The Legendre transformation does it with contacting straight line tangents.



Poincare's differential action

$$dS = L\,dt = p\cdot\dot{q}\,dt - H\cdot dt$$

$$= p\cdot dq - H\cdot dt$$

$$dS = L\,dt = \hbar k\cdot dr - \hbar\omega\cdot dt$$

(Quantum phase differential)

Unit 1
Fig. 12.9

This extraordinary claim needs extraordinary proof!

(...given by later lectures for Ch. 12 Unit 1 and Unit 8.)

Review of partial differential calculus

Chain rule and order $\partial^2\Psi/\partial x\partial y = \partial^2\Psi/\partial y\partial x$ symmetry

Scaling transformation between Lagrangian and Hamiltonian views of KE

Introducing 0^{th} Lagrange and 0^{th} Hamilton differential equations of mechanics

Introducing 1^{st} Lagrange and 1^{st} Hamilton differential equations of mechanics

Introducing the Poincare´ and Legendre contact transformations

Geometry of Legendre contact transformation (Preview of Unit 8 relativistic quantum mechanics)

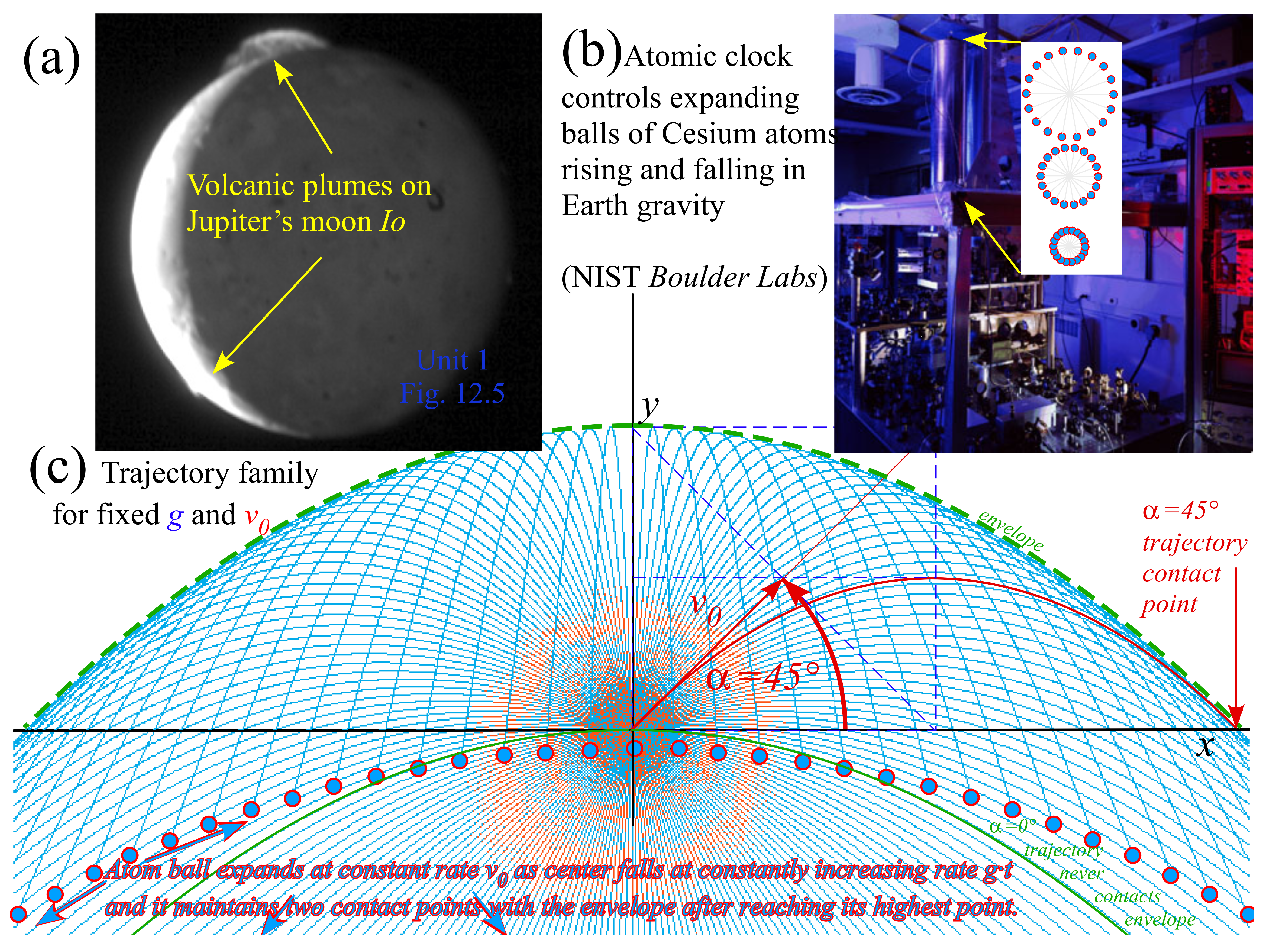
Example from thermodynamics

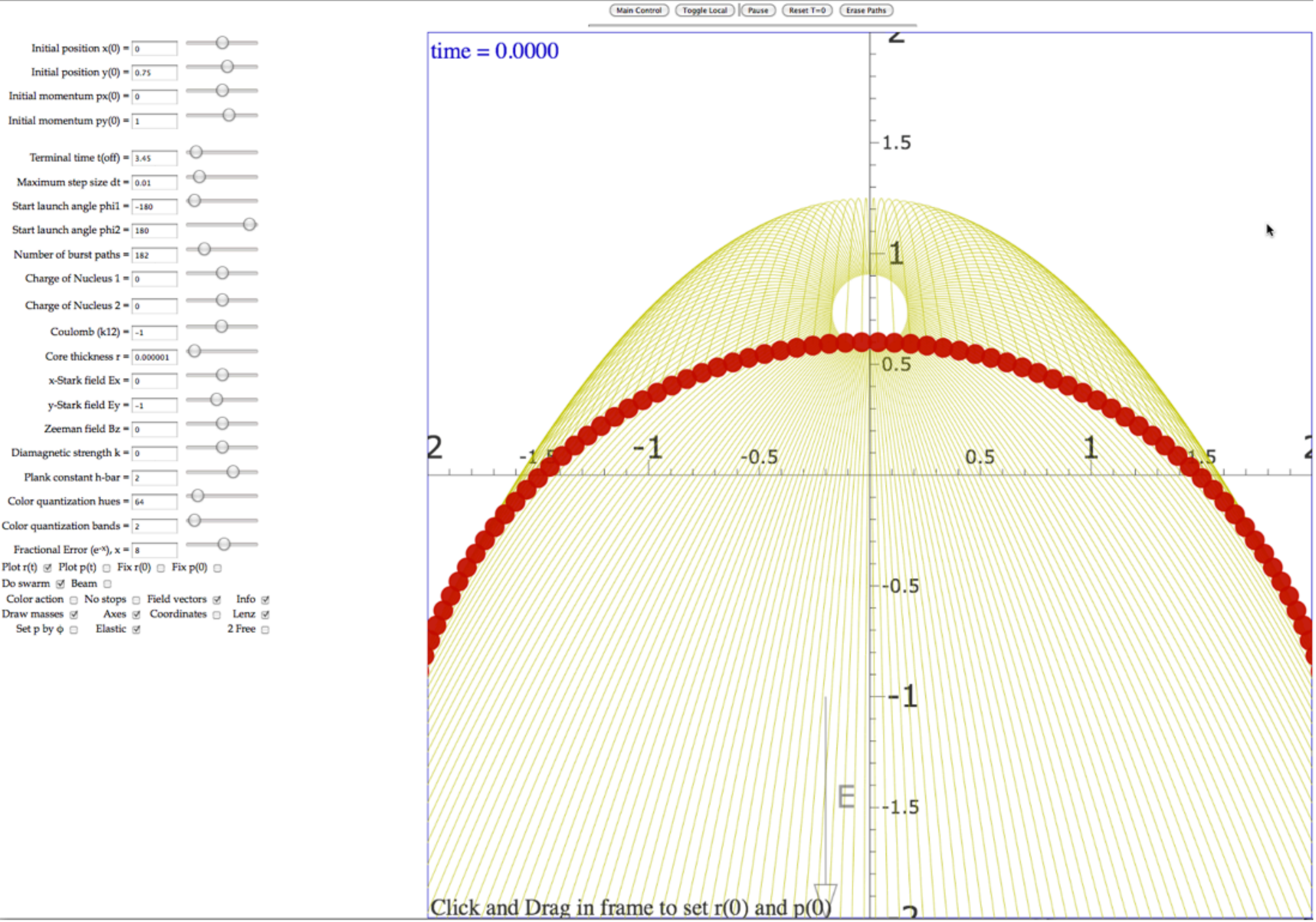
Legendre transform: special case of General Contact Transformation (lights,camera, ACTION!)

→ *An elementary contact transformation from sophomore physics*

Algebra-calculus development of "The Volcanoes of Io" and "The Atoms of NIST"

Intuitive-geometric development of " " " and " " "


(a)
Volcanic plumes on Jupiter's moon *Io*
Unit 1
Fig. 12.5
(b) Atomic clock controls expanding balls of Cesium atoms rising and falling in Earth gravity
(NIST *Boulder Labs*)
(c) Trajectory family for fixed g and v_0
y
x
envelope
α =45° trajectory contact point
v_0
α =45°
α =0° trajectory never contacts envelope
Atom ball expands at constant rate v_0 as center falls at constantly increasing rate g·t
and it maintains two contact points with the envelope after reaching its highest point.



[Link ⇒ CoulIt - Simulation of the *Volcanoes of Io*]

Review of partial differential calculus

Chain rule and order $\partial^2\Psi/\partial x\partial y = \partial^2\Psi/\partial y\partial x$ symmetry

Scaling transformation between Lagrangian and Hamiltonian views of KE

Introducing 0^{th} Lagrange and 0^{th} Hamilton differential equations of mechanics

Introducing 1^{st} Lagrange and 1^{st} Hamilton differential equations of mechanics

Introducing the Poincare´ and Legendre contact transformations

Geometry of Legendre contact transformation (Preview of Unit 8 relativistic quantum mechanics)

Example from thermodynamics

Legendre transform: special case of General Contact Transformation (lights,camera, ACTION!)

An elementary contact transformation from sophomore physics

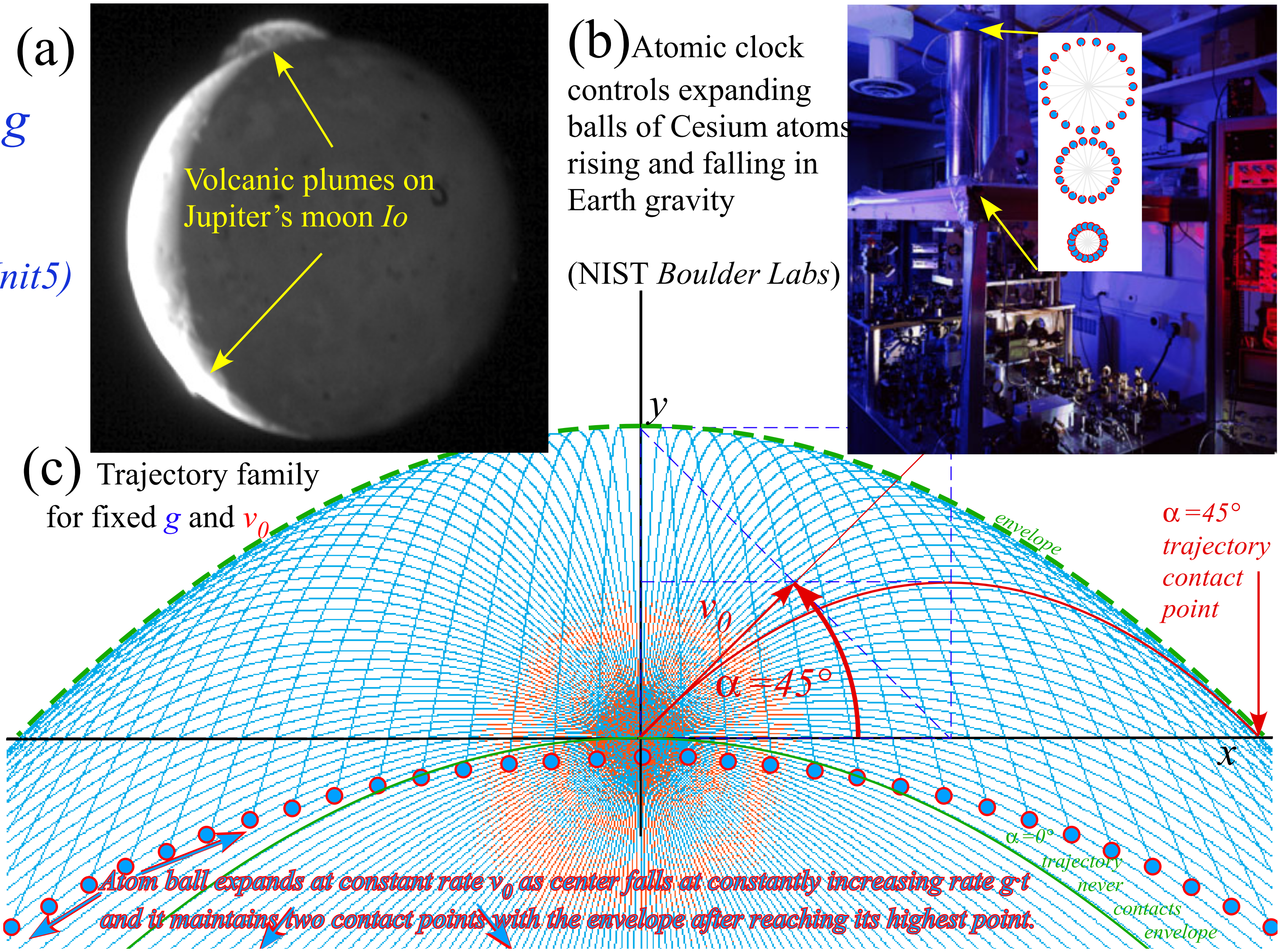
→ *Algebra-calculus development of "The Volcanoes of Io" and "The Atoms of NIST"*

Intuitive-geometric development of " " " and " " "

Constant gravity g assumed here...

Excellent for NIST

OK for Io (fixed in Unit5)



Unit 1
Fig. 12.5

UP-1 formulas for trajectories in constant gravity g

$$x(t) = (v_0 \cos\alpha)t \qquad y(t) = (v_0 \sin\alpha)t - \frac{1}{2}gt^2$$

$$\dot{x}(0) = v_x(0) = v_0 \cos\alpha \qquad \dot{y}(0) = v_y(0) = v_0 \sin\alpha$$

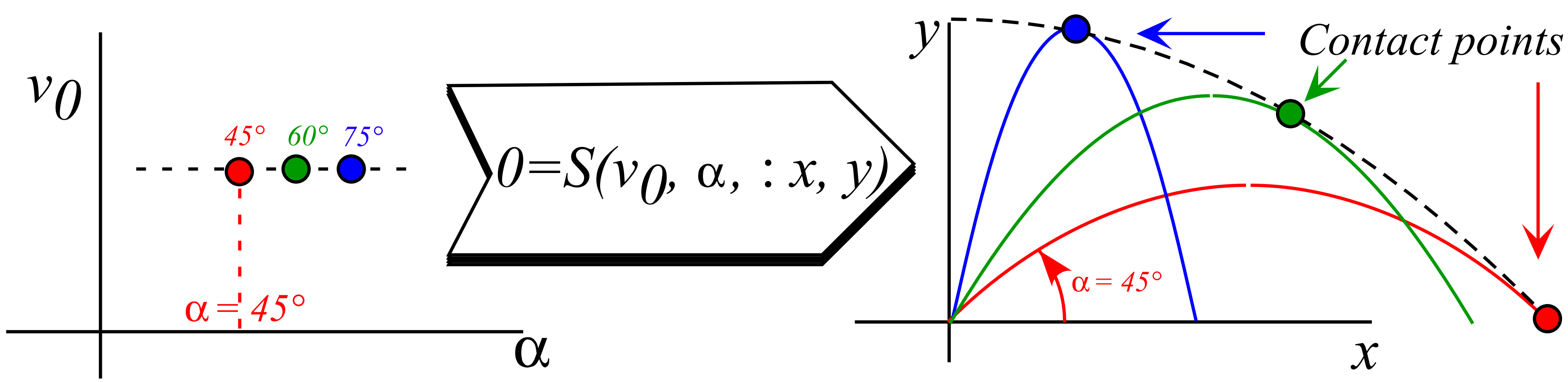
Substitute time $t = x/(v_0 \cos\alpha)$ into $y(t)$

$$y(x) = \frac{v_0 \sin\alpha}{v_0 \cos\alpha} x - \frac{gx^2}{2v_0^2 \cos^2\alpha}$$

$$y(x) = x\tan\alpha - \frac{gx^2}{2v_0^2 \cos^2\alpha}$$

Convert y(x) solution into Active Contact Transformation Generator $S(v_0, \alpha : x, y)$

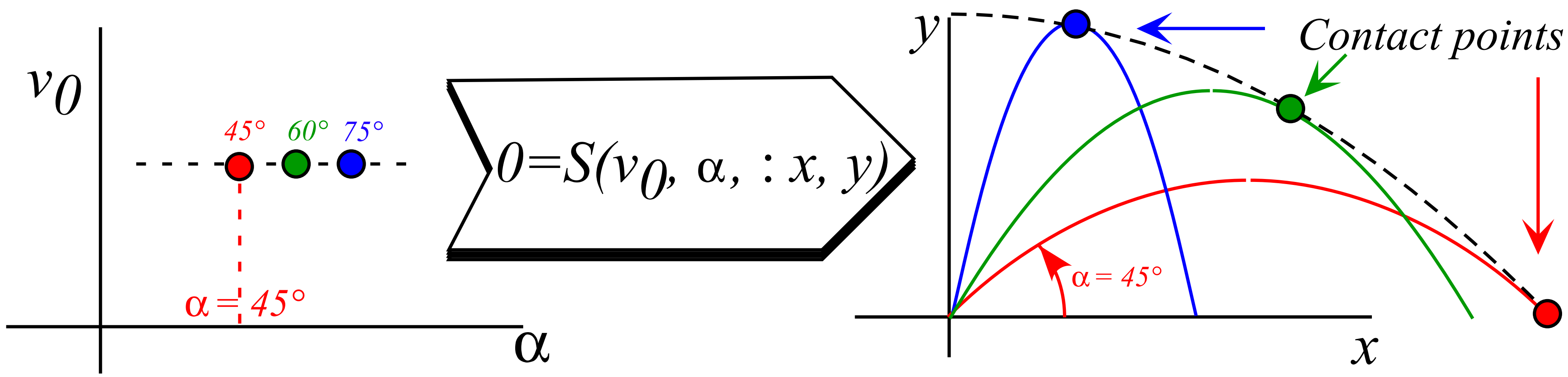
$$y(x) = x\tan\alpha - \frac{gx^2}{2v_0^2\cos^2\alpha}$$ *becomes:* $$S(v_0,\alpha : x,y) = -y + x\tan\alpha - \frac{gx^2}{2{v_0}^2\cos^2\alpha} = 0$$



Unit 1
Fig. 12.6

Convert y(x) solution into Active Contact Transformation Generator $S(v_0, \alpha : x, y)$

$$y(x) = x\tan\alpha - \frac{gx^2}{2v_0^2\cos^2\alpha} \qquad \textit{becomes:} \qquad S(v_0,\alpha : x,y) = -y + x\tan\alpha - \frac{gx^2}{2{v_0}^2\cos^2\alpha} = 0$$

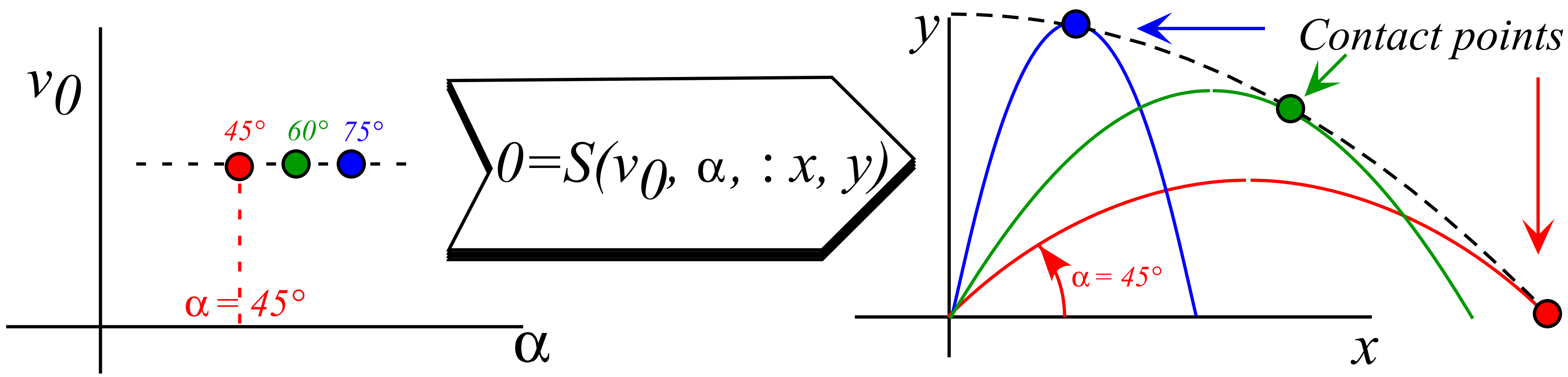


Unit 1
Fig. 12.6

Envelopes of the v_0-trajectory region contain extremal *contact points* with each trajectory where: $\frac{\partial S(v_0,\alpha : x,y)}{\partial\alpha} = 0$

Convert y(x) solution into Active Contact Transformation Generator $S(v_0, \alpha : x, y)$

$$y(x) = x\tan\alpha - \frac{gx^2}{2v_0^2\cos^2\alpha}$$ *becomes:* $$S(v_0, \alpha : x, y) = -y + x\tan\alpha - \frac{gx^2}{2{v_0}^2\cos^2\alpha} = 0$$



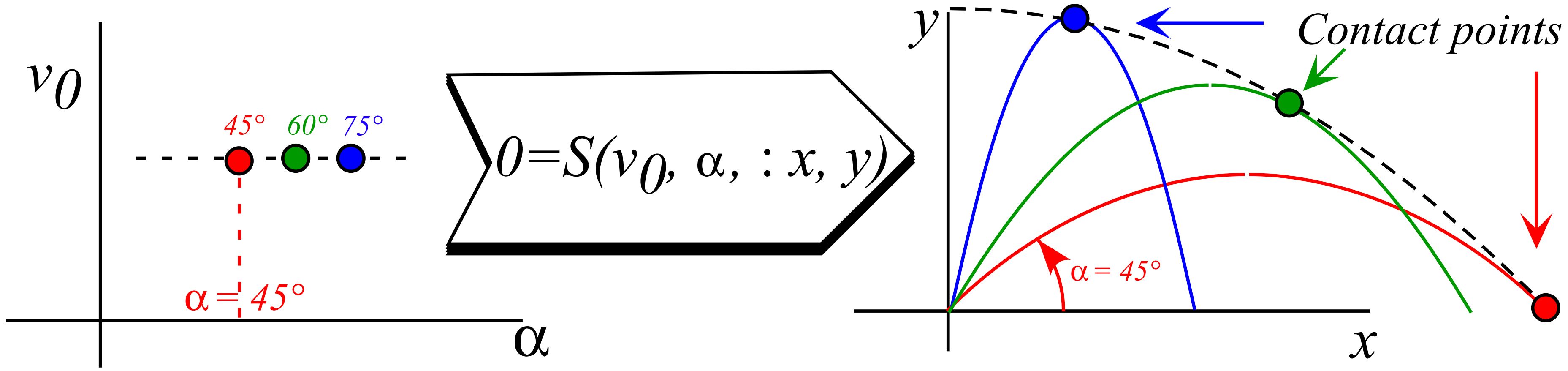
Unit 1
Fig. 12.6

Envelopes of the v_0-trajectory region contain extremal *contact points* with each trajectory where: $$\frac{\partial S(v_0, \alpha : x, y)}{\partial \alpha} = 0$$

$$x\frac{\partial \tan\alpha}{\partial \alpha} - \frac{gx^2}{2{v_0}^2}\frac{\partial \cos^{-2}\alpha}{\partial \alpha} = 0 = \frac{x}{\cos^2\alpha} - \frac{gx^2}{2{v_0}^2}\frac{2\sin\alpha}{\cos^3\alpha}$$

Convert y(x) solution into Active Contact Transformation Generator $S(v_0, \alpha : x, y)$

$$y(x) = x\tan\alpha - \frac{gx^2}{2v_0^2\cos^2\alpha} \qquad \textit{becomes:} \qquad S(v_0,\alpha : x,y) = -y + x\tan\alpha - \frac{gx^2}{2{v_0}^2\cos^2\alpha} = 0$$



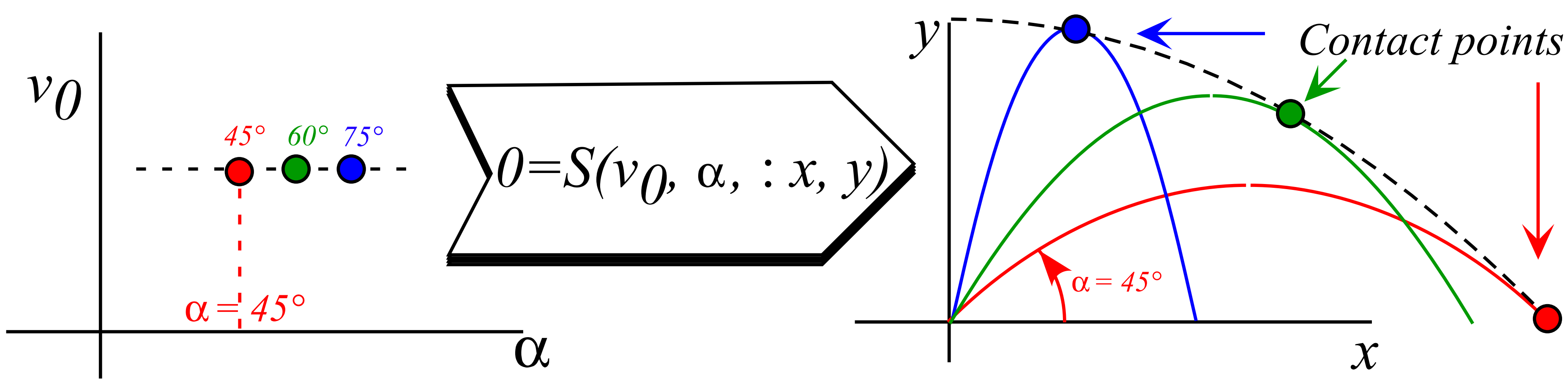
Unit 1
Fig. 12.6

Envelopes of the v_0-trajectory region contain extremal *contact points* with each trajectory where: $\frac{\partial S(v_0,\alpha : x,y)}{\partial\alpha} = 0$

$$x\frac{\partial\tan\alpha}{\partial\alpha} - \frac{gx^2}{2{v_0}^2}\frac{\partial\cos^{-2}\alpha}{\partial\alpha} = 0 = \frac{x}{\cos^2\alpha} - \frac{gx^2}{2{v_0}^2}\frac{2\sin\alpha}{\cos^3\alpha} \qquad \textit{gives:} \qquad \tan\alpha = \frac{v_0^2}{gx} \quad \text{or:} \quad x = \frac{v_0^2}{g\tan\alpha}\ .$$

Convert y(x) solution into Active Contact Transformation Generator $S(v_0, \alpha : x, y)$

$$y(x) = x\tan\alpha - \frac{gx^2}{2v_0^2\cos^2\alpha}$$ *becomes:* $$S(v_0, \alpha : x, y) = -y + x\tan\alpha - \frac{gx^2}{2{v_0}^2\cos^2\alpha} = 0$$



Unit 1
Fig. 12.6

Envelopes of the v_0-trajectory region contain extremal *contact points* with each trajectory where: $$\frac{\partial S(v_0, \alpha : x, y)}{\partial \alpha} = 0$$

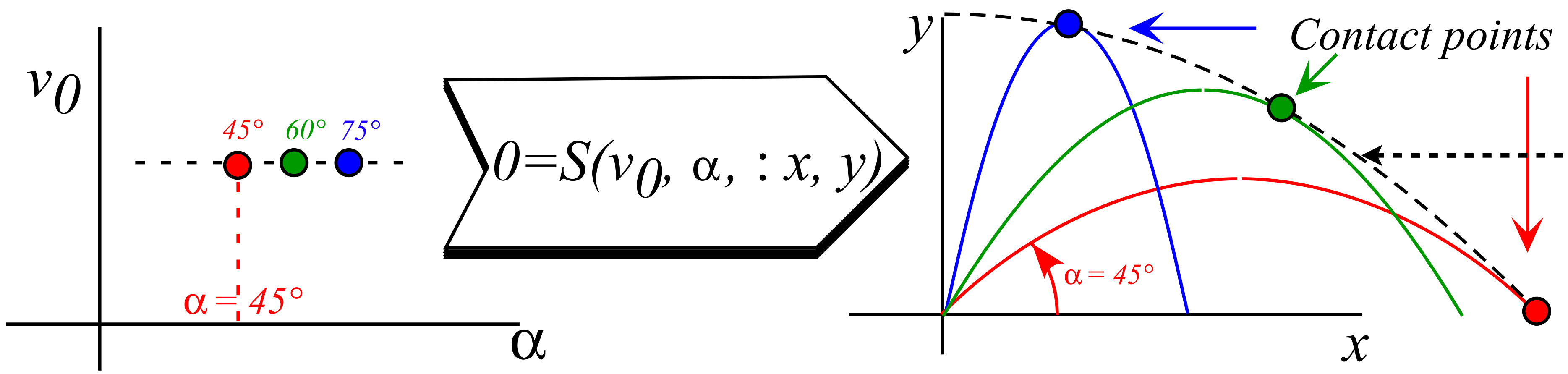
$$x\frac{\partial \tan\alpha}{\partial\alpha} - \frac{gx^2}{2{v_0}^2}\frac{\partial \cos^{-2}\alpha}{\partial\alpha} = 0 = \frac{x}{\cos^2\alpha} - \frac{gx^2}{2{v_0}^2}\frac{2\sin\alpha}{\cos^3\alpha}$$

$$\tan\alpha = \frac{v_0^2}{gx} \quad \text{or:} \quad x = \frac{v_0^2}{g\tan\alpha}.$$

$$y_{env}(x) = x\tan\alpha - \frac{gx^2}{2v_0^2}\left(1 + \tan^2\alpha\right) \Rightarrow y_{env}(x) = x\frac{v_0^2}{gx} - \frac{gx^2}{2v_0^2}\left(1 + \frac{v_0^4}{g^2x^2}\right)$$

Convert y(x) solution into Active Contact Transformation Generator $S(v_0, \alpha : x, y)$

$$y(x) = x\tan\alpha - \frac{gx^2}{2v_0^2\cos^2\alpha} \qquad \textit{becomes:} \qquad S(v_0,\alpha : x,y) = -y + x\tan\alpha - \frac{gx^2}{2{v_0}^2\cos^2\alpha} = 0$$



Envelopes of the v_0-trajectory region contain extremal *contact points* with each trajectory where: $\frac{\partial S(v_0,\alpha : x,y)}{\partial\alpha} = 0$

$$x\frac{\partial\tan\alpha}{\partial\alpha} - \frac{gx^2}{2{v_0}^2}\frac{\partial\cos^{-2}\alpha}{\partial\alpha} = 0 = \frac{x}{\cos^2\alpha} - \frac{gx^2}{2{v_0}^2}\frac{2\sin\alpha}{\cos^3\alpha}$$

$$\tan\alpha = \frac{v_0^2}{gx} \quad \text{or:} \quad x = \frac{v_0^2}{g\tan\alpha}\ .$$

$$y_{env}(x) = x\tan\alpha - \frac{gx^2}{2v_0^2}\left(1+\tan^2\alpha\right) \Rightarrow y_{env}(x) = x\frac{v_0^2}{gx} - \frac{gx^2}{2v_0^2}\left(1+\frac{v_0^4}{g^2x^2}\right)$$

$$y_{env}(x) = \frac{v_0^2}{g} - \frac{gx^2}{2v_0^2} - \frac{gx^2}{2v_0^2}\frac{v_0^4}{g^2x^2} = \frac{v_0^2}{2g} - \frac{gx^2}{2v_0^2}$$

Envelope function

Review of partial differential calculus

Chain rule and order $\partial^2\Psi/\partial x\partial y = \partial^2\Psi/\partial y\partial x$ symmetry

Scaling transformation between Lagrangian and Hamiltonian views of KE

Introducing 0^{th} Lagrange and 0^{th} Hamilton differential equations of mechanics

Introducing 1^{st} Lagrange and 1^{st} Hamilton differential equations of mechanics

Introducing the Poincare´ and Legendre contact transformations

Geometry of Legendre contact transformation (Preview of Unit 8 relativistic quantum mechanics)

Example from thermodynamics

Legendre transform: special case of General Contact Transformation (lights,camera, ACTION!)

An elementary contact transformation from sophomore physics

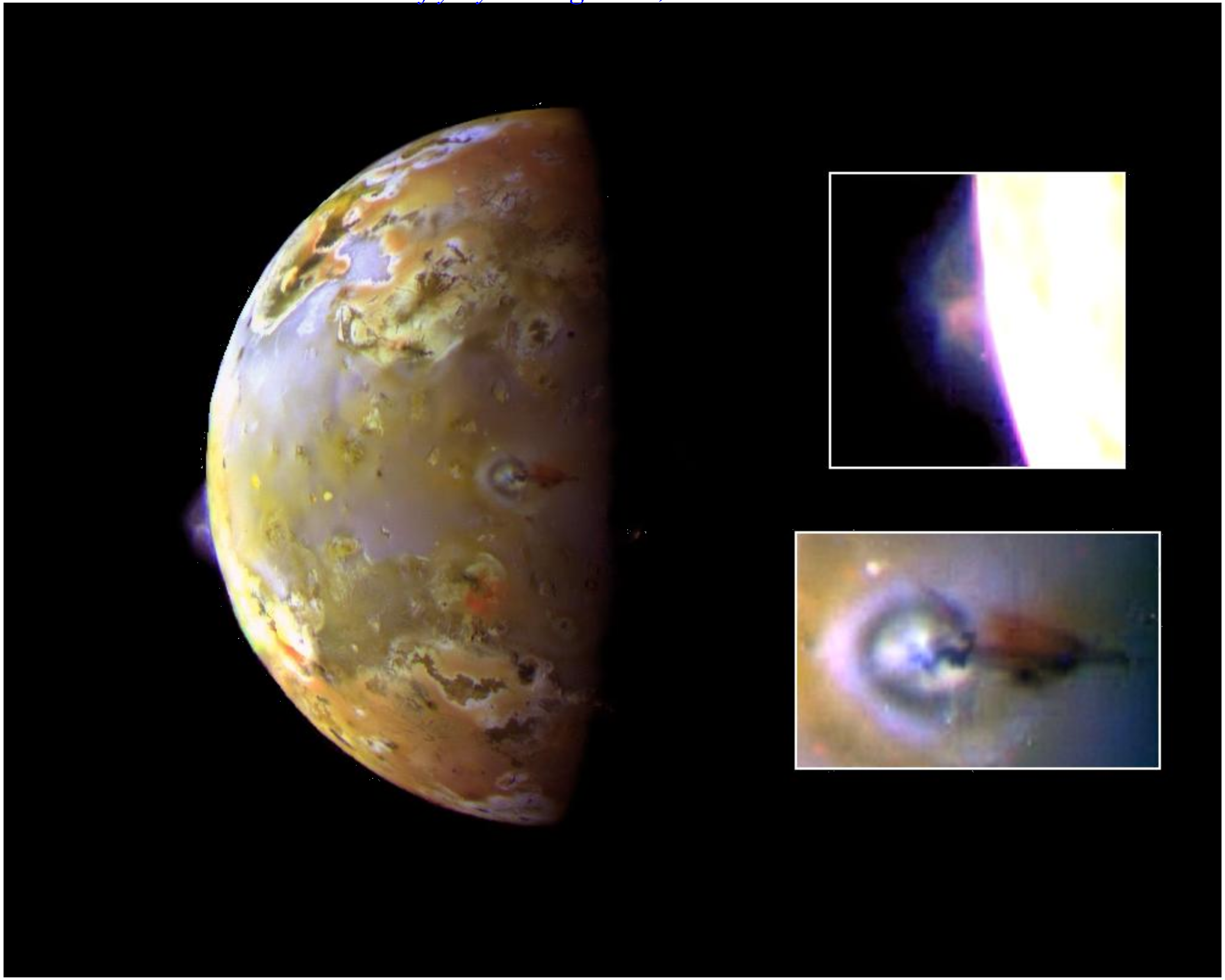
Algebra-calculus development of "The Volcanoes of Io" and "The Atoms of NIST"

→ *Intuitive-geometric development of " " " and " " "*

The Plumes of Prometheus

NASA-Galileo Project

Io fly-by on August 18, 1997



NASA Astronomy Picture of the Day - *Io: The Prometheus Plume* *(Just Image)*

New Horizons - *Volcanic Eruption Plume on Jupiter's moon IO*

NASA Galileo - *Io's Alien Volcanoes*

NASA Galileo - A Hawaiian-Style Volcano on Io

Io's Alien Volcanoes

Science @NASA
Inform Inspire Involve
science.nasa.gov

Space Science News home

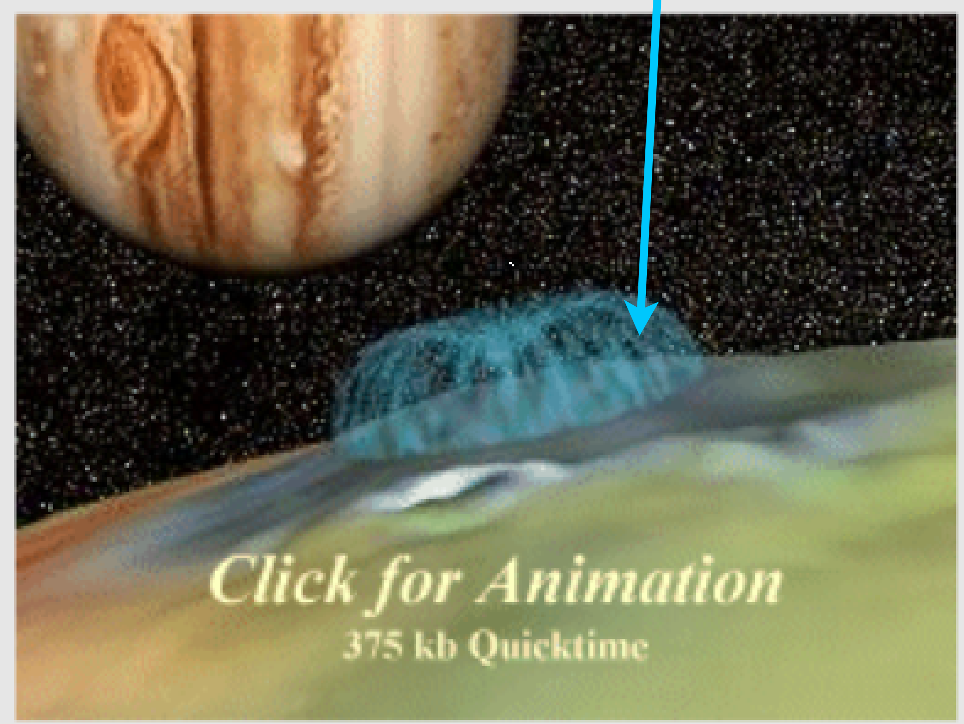
Pretty bad sketch of plumes
(LasVegas model of planetary ejecta?)

Io's Alien Volcanoes

Do these guys need a geometry lesson?

Need to fly parabola kite geometry...

Scientists are eager for a closer look at the solar system's strangest and most active volcanoes when Galileo flies by Io on October 11.



October 4, 1999: Thirty years ago, before the Voyager probes visited Jupiter, if you had described Io to a literary critic it would have been declared overwrought science fiction. Jupiter's strange moon is literally bursting with volcanoes. Dozens of active vents pepper the landscape which also includes gigantic frosty plains, towering mountains and volcanic rings the size of California. The volcanoes themselves are the hottest spots in the solar system with temperatures exceeding 1800 K (1527 C). The plumes which rise 300 km into space are so large they can be seen from Earth by the Hubble Space Telescope. Confounding common sense, these high-rising ejecta seem to be made up of, not blisteringly hot lava, but frozen sulfur dioxide. And to top it all off, Io bears a striking resemblance to a pepperoni pizza. Simply unbelievable.

Right: Digital Radiance simulation of Pillan Patera just before the Galileo flyby. click for animation → .

NASA Astronomy Picture of the Day - *Io: The Prometheus Plume* *(Just Image)*

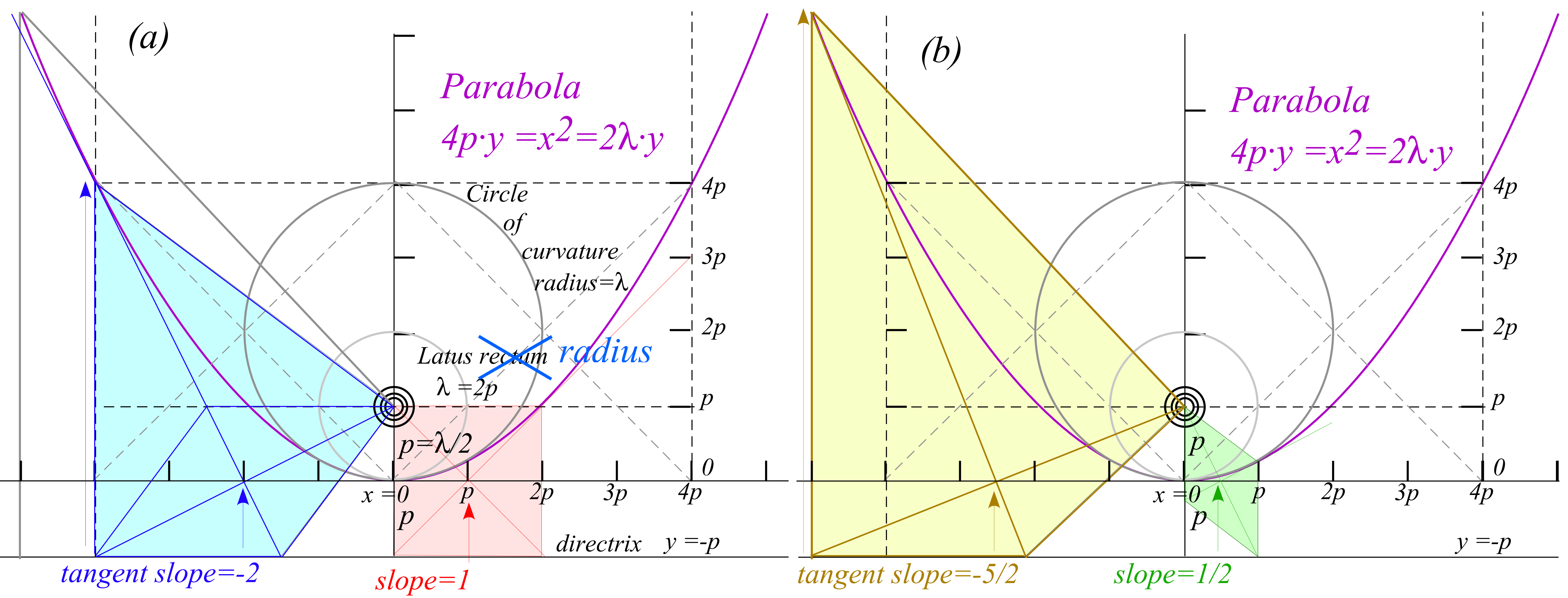
NASA Galileo - *Io's Alien Volcanoes*

New Horizons - *Volcanic Eruption Plume on Jupiter's moon IO*

NASA Galileo - A Hawaiian-Style Volcano on Io

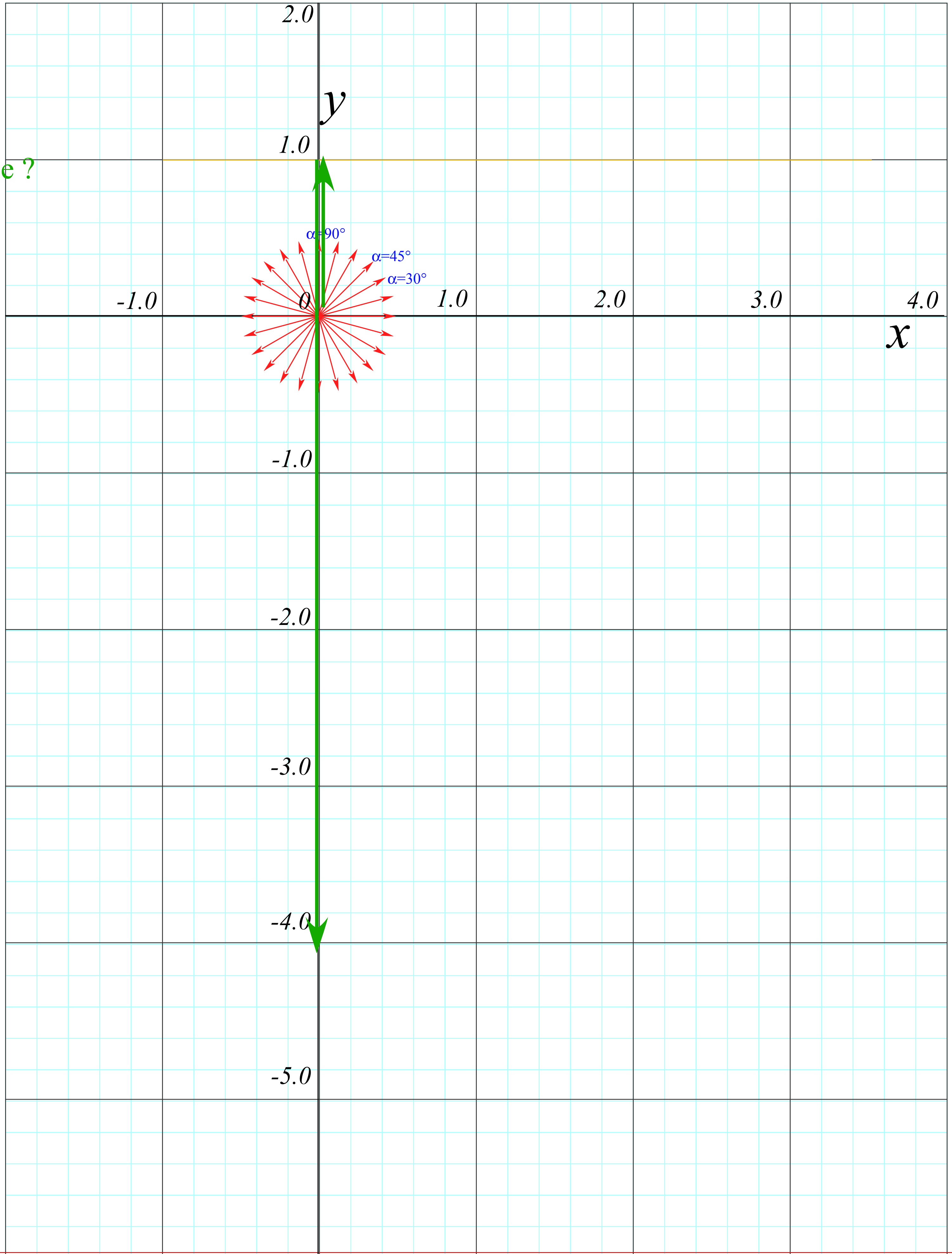
...conventional parabolic geometry...carried to extremes...

Recall Lecture 6 p.26 and p. 48-49 for kite geometry and application



Unit 1
Fig. 9.4

Say $\alpha=90°$ path rises to *1.0*
then drops. When at $y=1.0$...
Q1. ...where is its focus?
Q2. ...where is the blast wave?
Q3. ...how high can $\alpha=45°$ path path rise ?

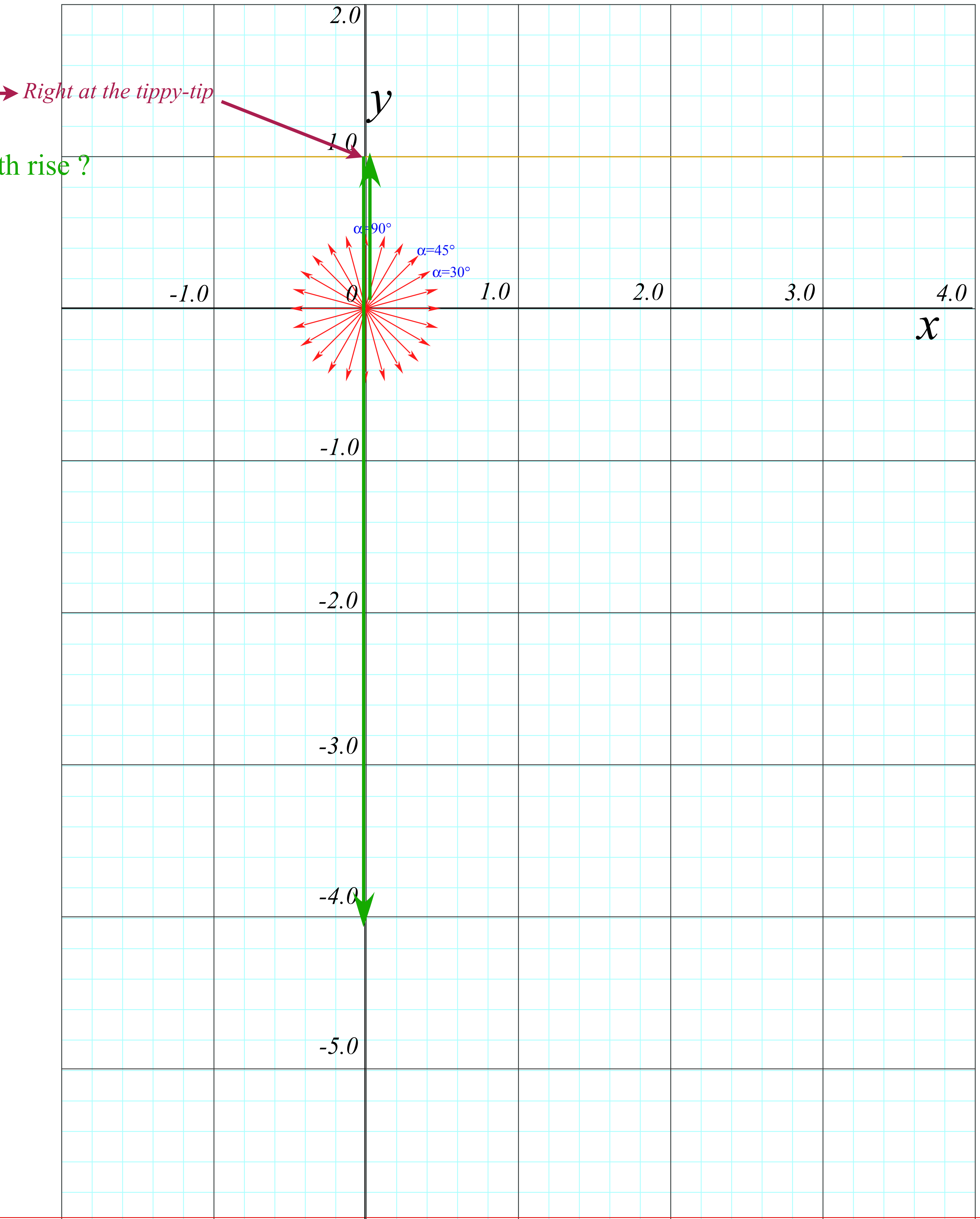


Say $\alpha=90°$ path rises to *1.0* then drops. When at $y=1.0$...

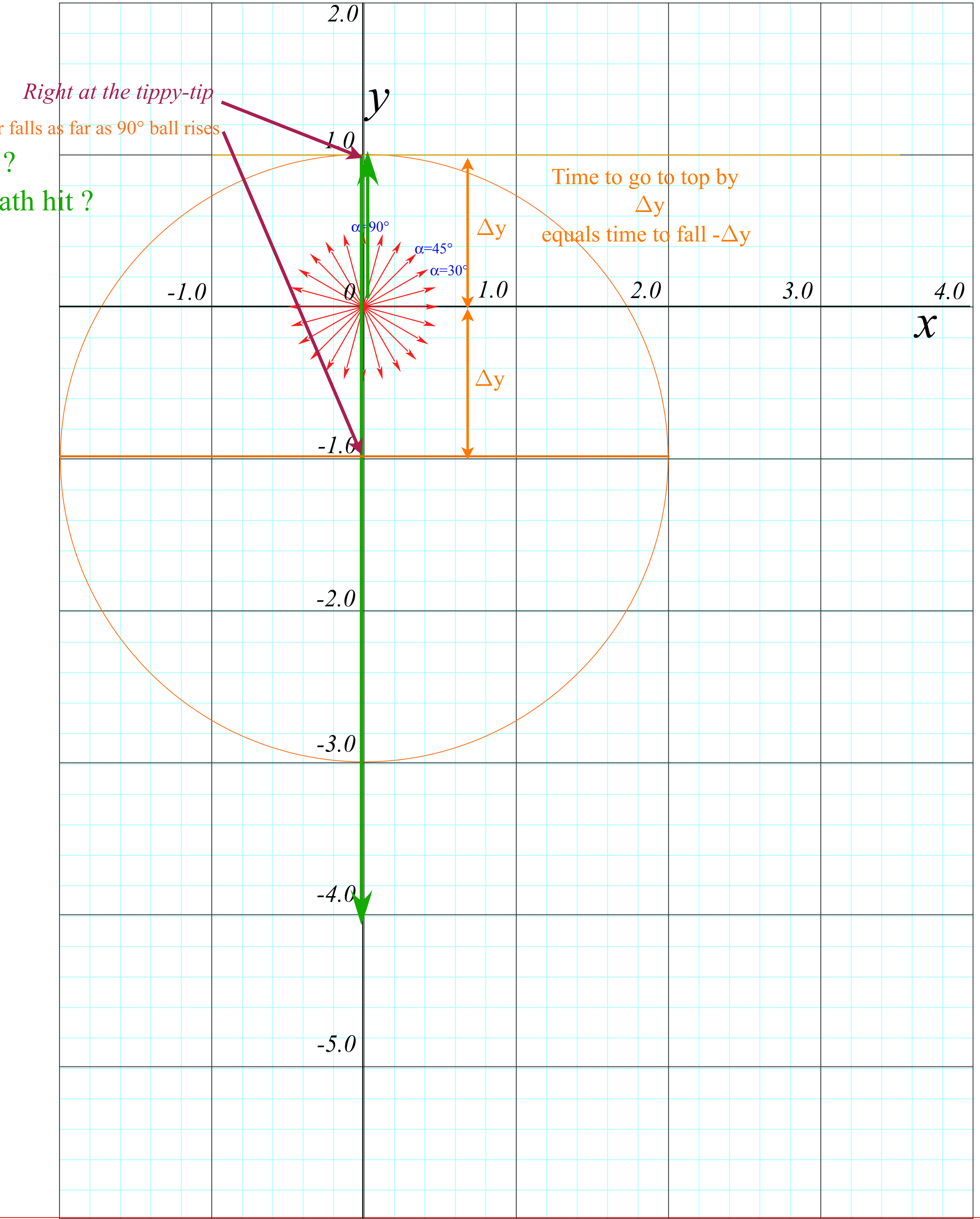
Q1. ...where is its focus?

Q2. ...where is the blast wave?

Q3. ...how high can $\alpha=45°$ path path rise ?



Say α=90° path rises to *1.0*
then drops. When at *y=1.0*...
Q1. ...where is its focus?
Q2. ...where is the blast wave?
Q3. How high can α=45° path rise ?
Q4. Where on *x-axis* does α=45° path hit ?



Say $\alpha=90°$ path rises to *1.0*

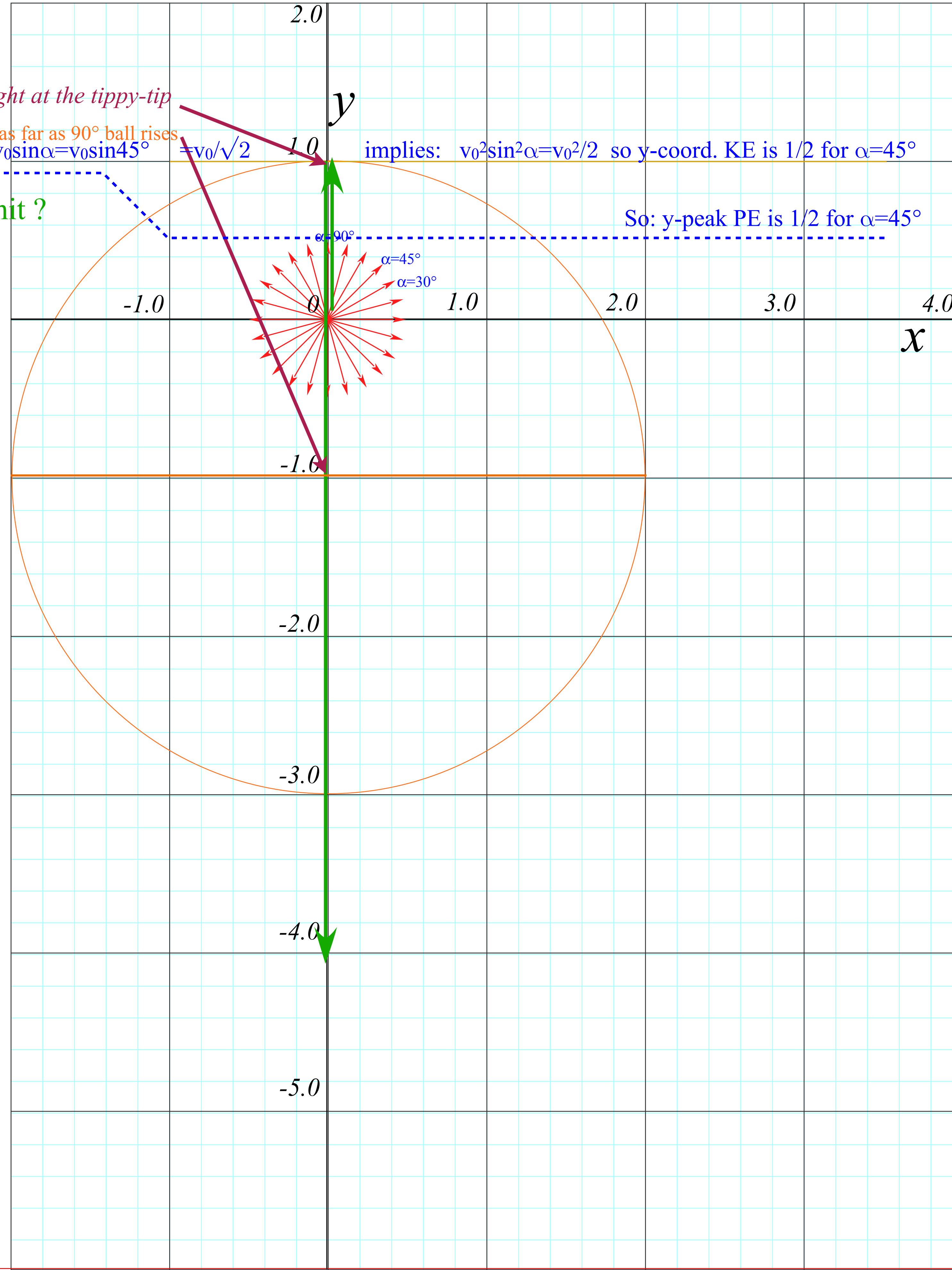
then drops. When at *y=1.0*...

Q1. ...where is its focus?

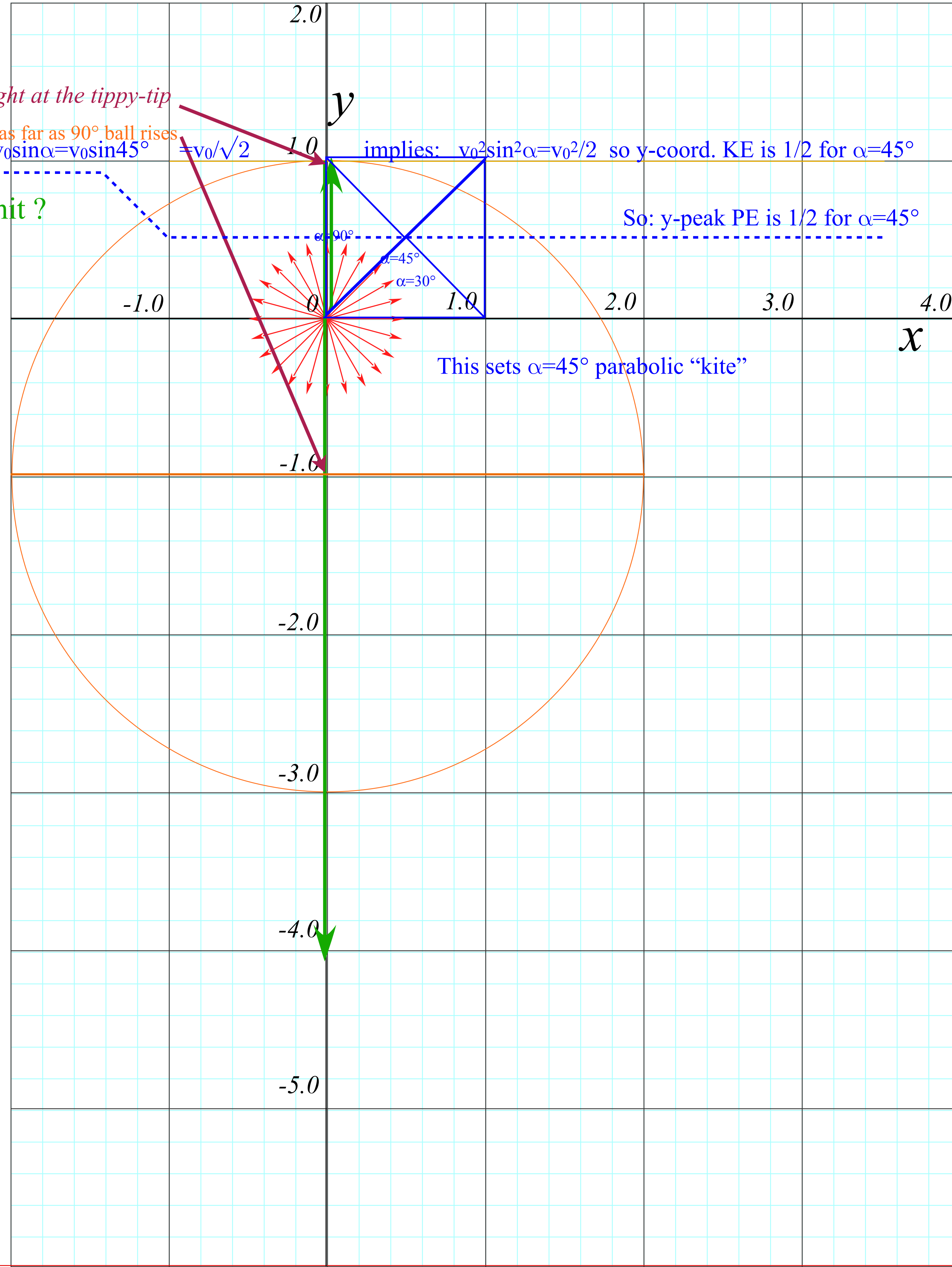
Q2. ...where is the blast wave?

Q3. How high can $\alpha=45°$ path rise ?

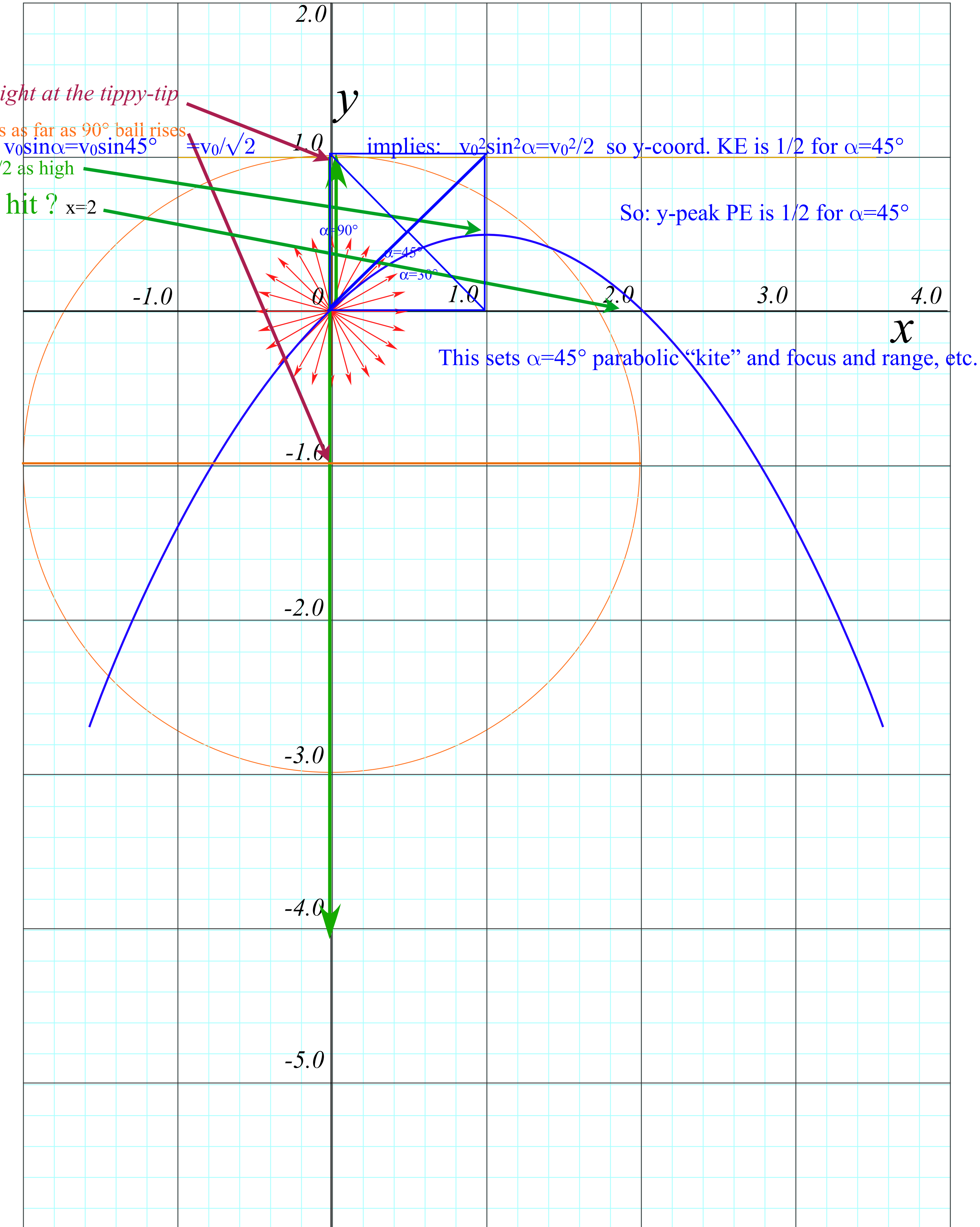
Q4. Where on *x-axis* does $\alpha=45°$ path hit ?



Say α=90° path rises to *1.0*
then drops. When at *y=1.0*...
Q1. ...where is its focus?
Q2. ...where is the blast wave? center falls as far as 90° ball rises
Q3. How high can α=45° path rise ?
Q4. Where on *x-axis* does α=45° path hit ?



Say $\alpha=90°$ path rises to *1.0*
then drops. When at *y=1.0*...
Q1. ...where is its focus?
Q2. ...where is the blast wave? center falls as far as 90° ball rises
Q3. How high can $\alpha=45°$ path rise ? 1/2 as high
Q4. Where on *x-axis* does $\alpha=45°$ path hit ? x=2
Q5. Where is blast wave then?
Q6 Where is $\alpha=45°$ path focus?
Q7 Guess for all-path envelope?
and its focus? directrix?



Say α=90° path rises to *1.0* then drops. When at *y=1.0*...

Q1. ...where is its focus? *Right at the tippy-tip*

Q2. ...where is the blast wave? center falls as far as 90° ball rises

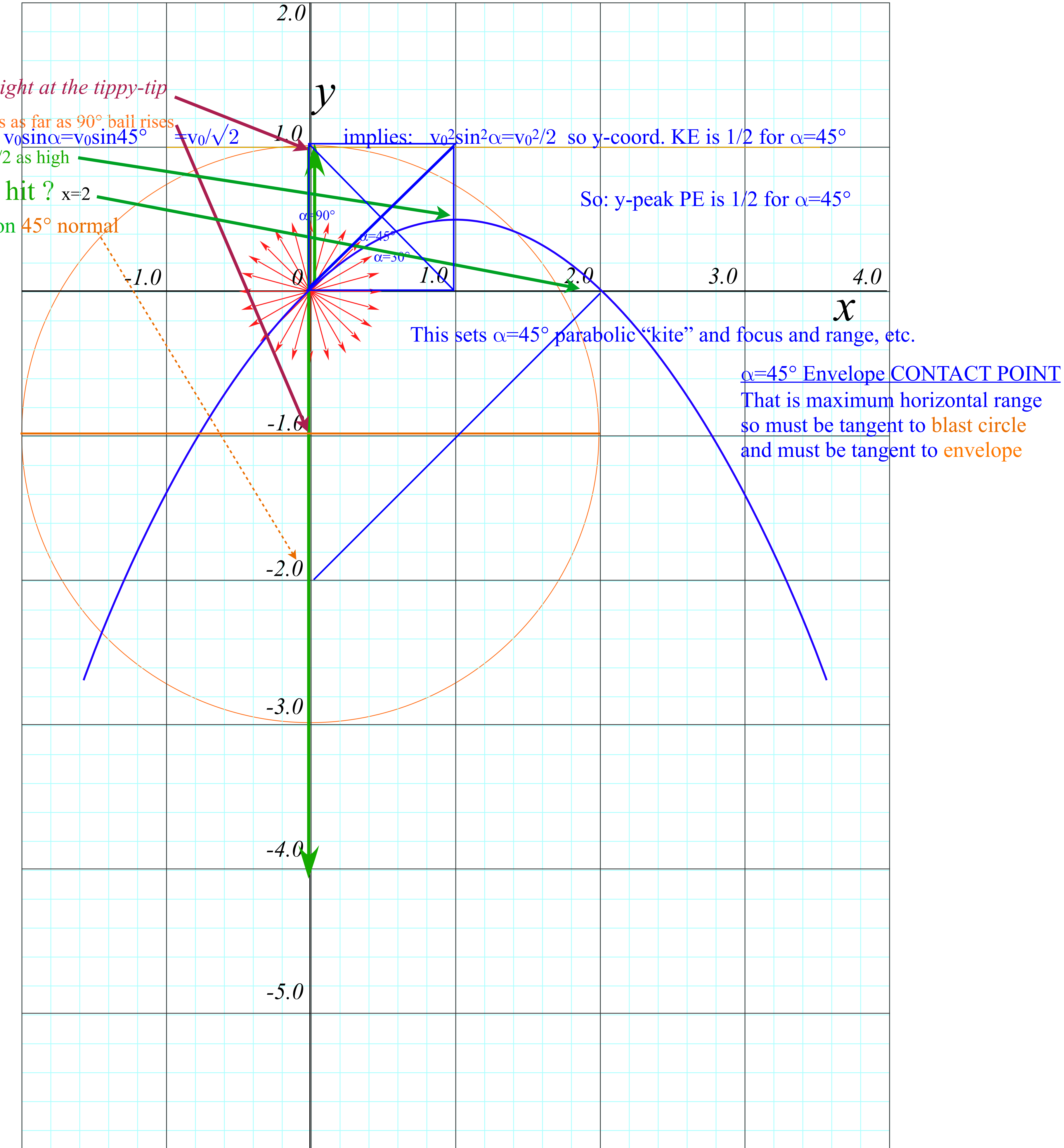
Q3. How high can α=45° path rise ? 1/2 as high

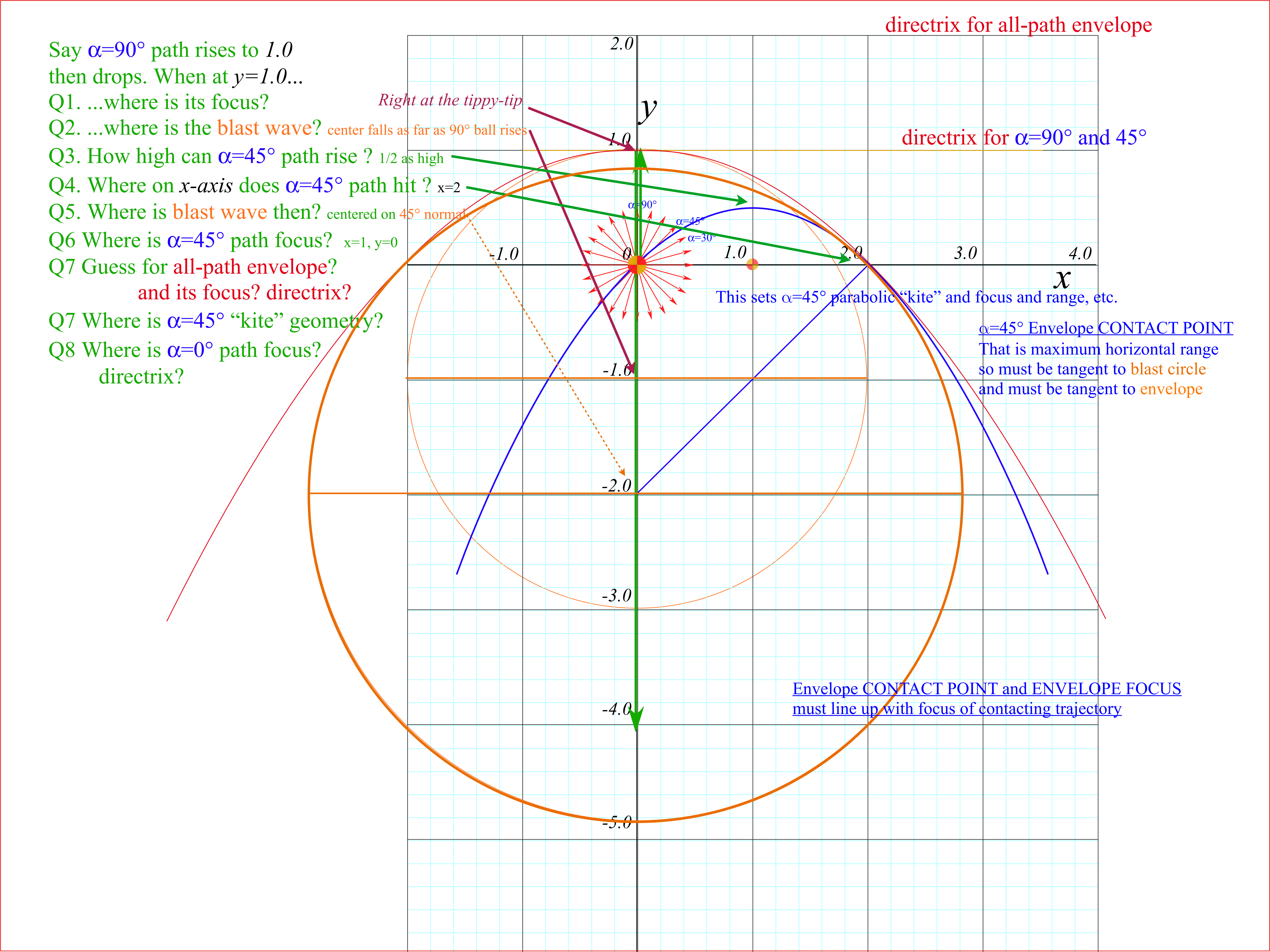
Q4. Where on *x-axis* does α=45° path hit ? x=2

Q5. Where is blast wave then? centered on 45° normal

Q6 Where is α=45° path focus?

Q7 Guess for all-path envelope?
and its focus? directrix?

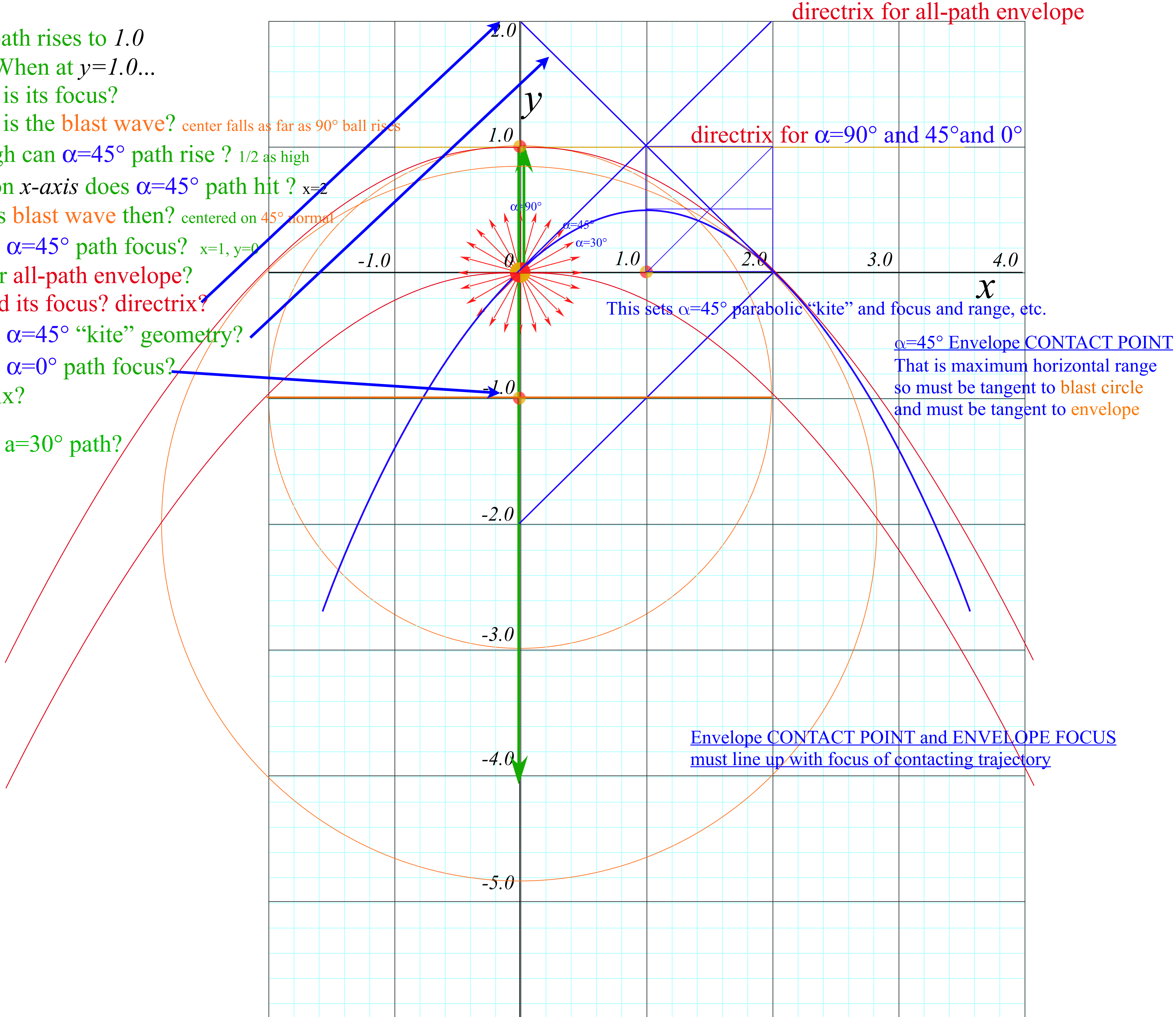


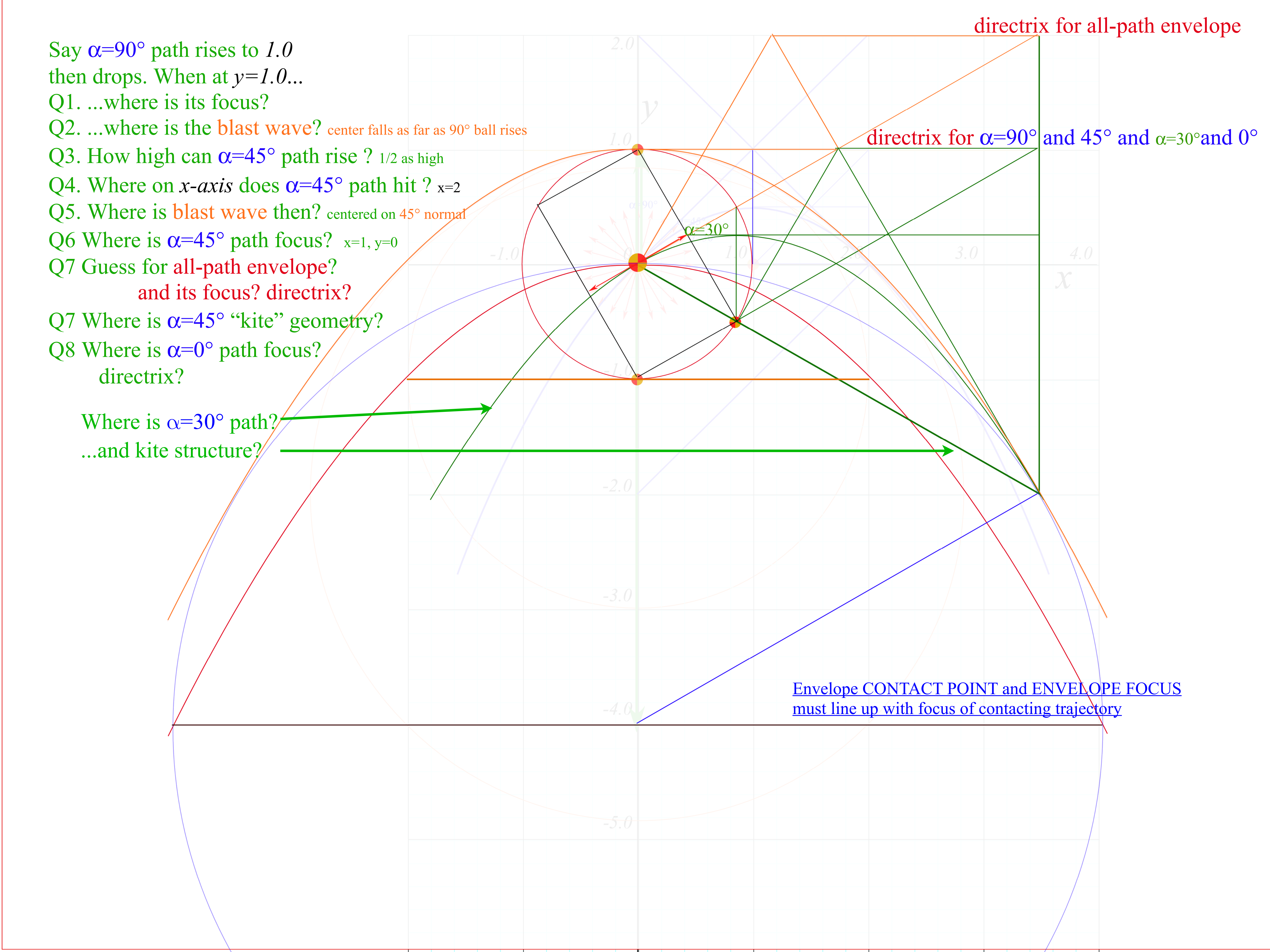


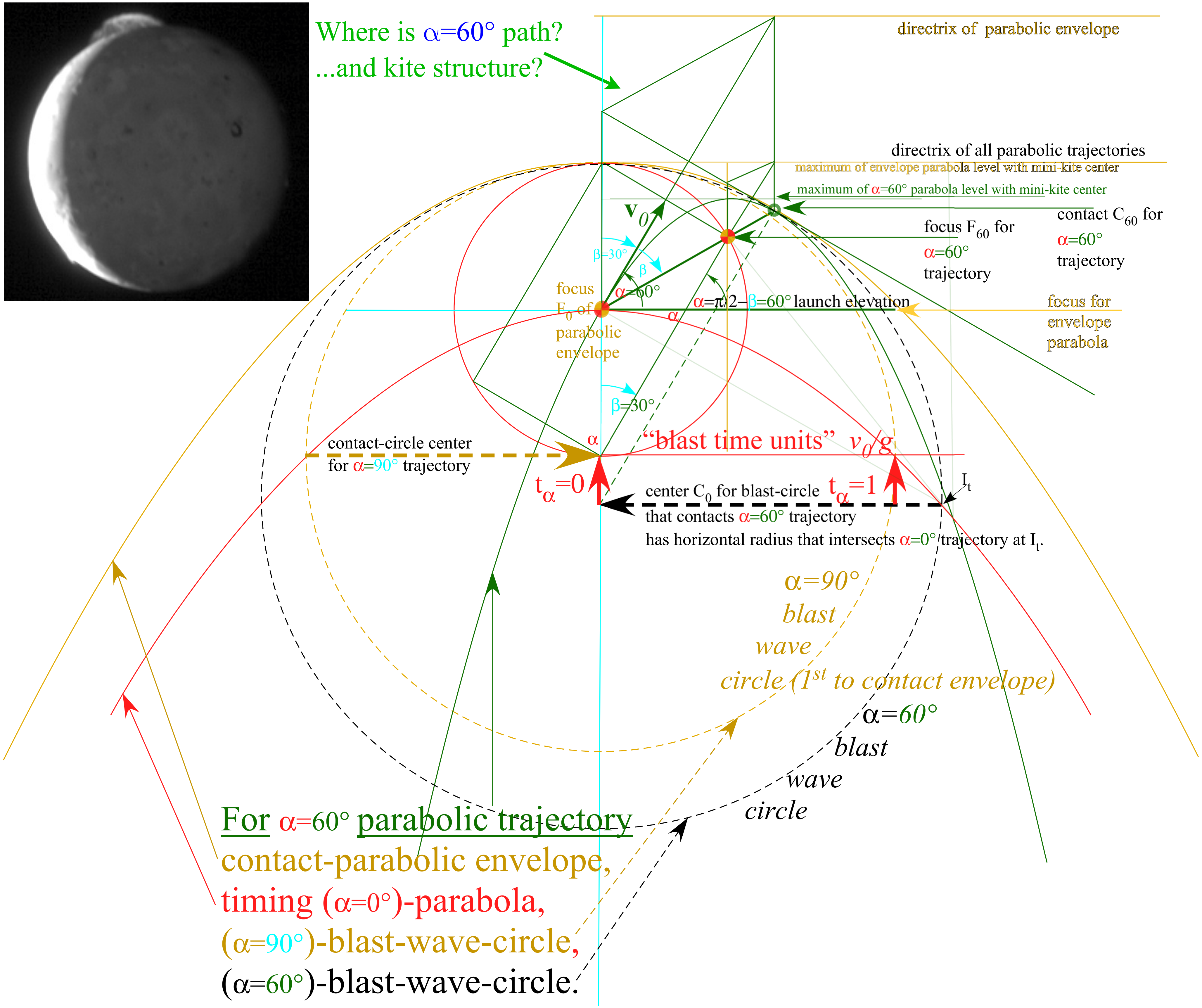
Say α=90° path rises to 1.0
then drops. When at y=1.0...
Q1. ...where is its focus?
Q2. ...where is the blast wave? center falls as far as 90° ball rises
Q3. How high can α=45° path rise ? 1/2 as high
Q4. Where on x-axis does α=45° path hit ? x=2
Q5. Where is blast wave then? centered on 45° normal
Q6 Where is α=45° path focus? x=1, y=0
Q7 Guess for all-path envelope?
and its focus? directrix?
Q7 Where is α=45° "kite" geometry?
Q8 Where is α=0° path focus?
directrix?
Right at the tippy-tip
directrix for all-path envelope
directrix for α=90° and 45°
α=90°
α=45°
α=30°
This sets α=45° parabolic "kite" and focus and range, etc.
α=45° Envelope CONTACT POINT
That is maximum horizontal range
so must be tangent to blast circle
and must be tangent to envelope
Envelope CONTACT POINT and ENVELOPE FOCUS
must line up with focus of contacting trajectory
2.0
1.0
-1.0
-2.0
-3.0
-4.0
-5.0
0
1.0
2.0
3.0
4.0
x
y

Say α=90° path rises to *1.0*
then drops. When at *y=1.0*...
Q1. ...where is its focus?
Q2. ...where is the blast wave? center falls as far as 90° ball rises
Q3. How high can α=45° path rise ? 1/2 as high
Q4. Where on *x-axis* does α=45° path hit ? x=2
Q5. Where is blast wave then? centered on 45° normal
Q6 Where is α=45° path focus? x=1, y=0
Q7 Guess for all-path envelope?
and its focus? directrix?
Q7 Where is α=45° "kite" geometry?
Q8 Where is α=0° path focus?
directrix?

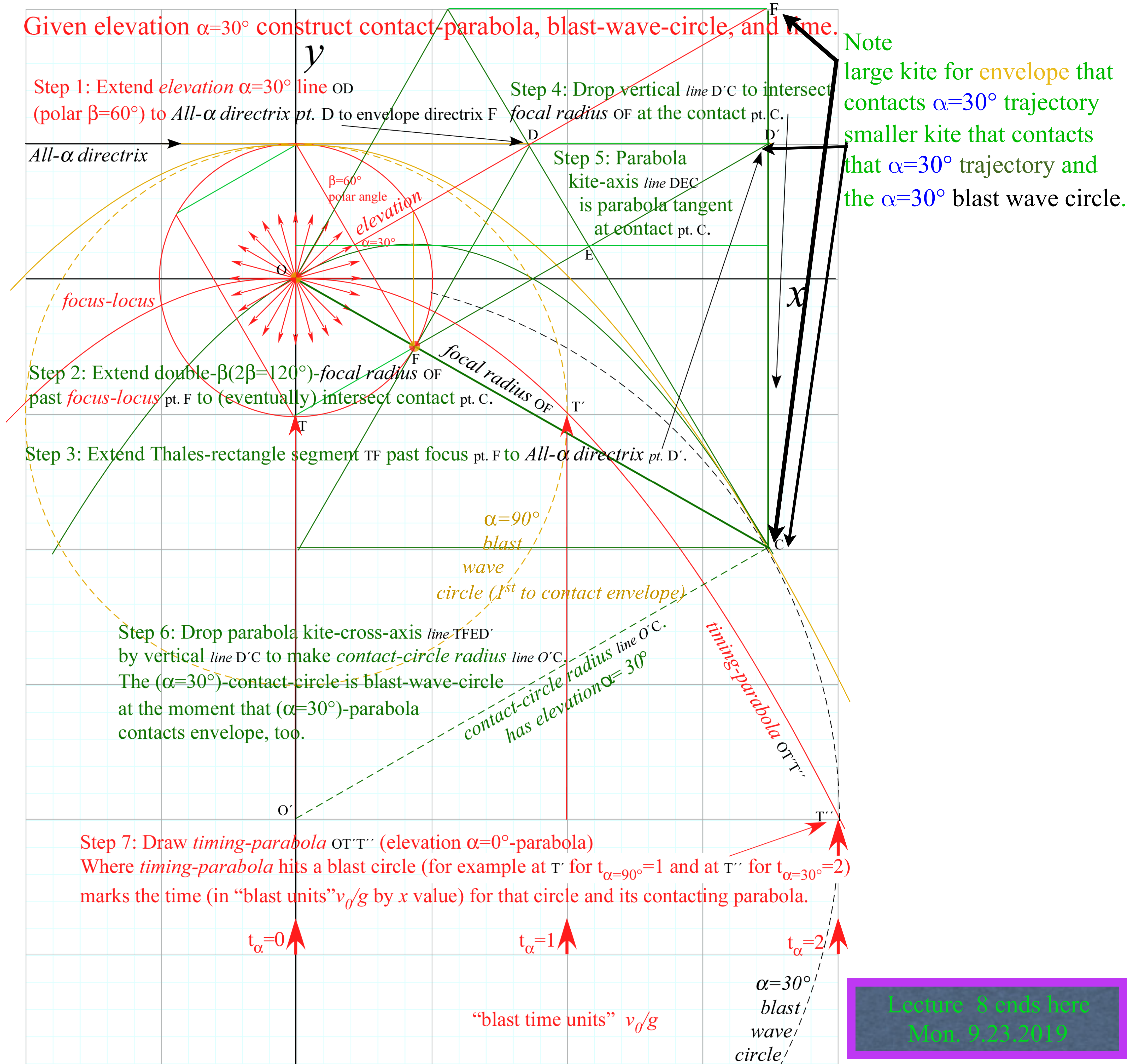
Where is a=30° path?




Say α=90° path rises to 1.0
then drops. When at y=1.0...
Q1. ...where is its focus?
Q2. ...where is the blast wave? center falls as far as 90° ball rises
Q3. How high can α=45° path rise ? 1/2 as high
Q4. Where on x-axis does α=45° path hit ? x=2
Q5. Where is blast wave then? centered on 45° normal
Q6 Where is α=45° path focus? x=1, y=0
Q7 Guess for all-path envelope?
and its focus? directrix?
Q7 Where is α=45° "kite" geometry?
Q8 Where is α=0° path focus?
directrix?
Where is α=30° path?
...and kite structure?
directrix for all-path envelope
directrix for α=90° and 45° and α=30° and 0°
α=30°
y
x
2.0
1.0
-1.0
1.0
3.0
4.0
-2.0
-3.0
-4.0
-5.0
Envelope CONTACT POINT and ENVELOPE FOCUS
must line up with focus of contacting trajectory



Where is α=60° path?
...and kite structure?
directrix of parabolic envelope
directrix of all parabolic trajectories
maximum of envelope parabola level with mini-kite center
maximum of α=60° parabola level with mini-kite center
contact C60 for
α=60°
trajectory
focus F60 for
α=60°
trajectory
v0
β=30°
β
α=60°
focus
F0 of
parabolic
envelope
α=π/2−β=60° launch elevation
α
focus for
envelope
parabola
β=30°
α
"blast time units" v0/g
contact-circle center
for α=90° trajectory
tα=0
tα=1
center C0 for blast-circle
that contacts α=60° trajectory
has horizontal radius that intersects α=0° trajectory at It.
It
α=90°
blast
wave
circle (1st to contact envelope)
α=60°
blast
wave
circle
For α=60° parabolic trajectory
contact-parabolic envelope,
timing (α=0°)-parabola,
(α=90°)-blast-wave-circle,
(α=60°)-blast-wave-circle.


Given elevation α=30° construct contact-parabola, blast-wave-circle, and time.
y
x
Step 1: Extend elevation α=30° line OD
(polar β=60°) to All-α directrix pt. D to envelope directrix F
All-α directrix
Step 4: Drop vertical line D′C to intersect
focal radius OF at the contact pt. C.
Step 5: Parabola
kite-axis line DEC
is parabola tangent
at contact pt. C.
β=60°
polar angle
elevation
α=30°
O
D
D′
E
F
C
T
T′
T′′
O′
focus-locus
focal radius OF
Step 2: Extend double-β(2β=120°)-focal radius OF
past focus-locus pt. F to (eventually) intersect contact pt. C.
Step 3: Extend Thales-rectangle segment TF past focus pt. F to All-α directrix pt. D′.
α=90°
blast
wave
circle (1st to contact envelope)
Step 6: Drop parabola kite-cross-axis line TFED′
by vertical line D′C to make contact-circle radius line O′C.
The (α=30°)-contact-circle is blast-wave-circle
at the moment that (α=30°)-parabola
contacts envelope, too.
contact-circle radius line O′C.
has elevation α= 30°
timing-parabola OT′T′′
Step 7: Draw timing-parabola OT′T′′ (elevation α=0°-parabola)
Where timing-parabola hits a blast circle (for example at T′ for t_α=90°=1 and at T′′ for t_α=30°=2)
marks the time (in "blast units" v_0/g by x value) for that circle and its contacting parabola.
t_α=0
t_α=1
t_α=2
α=30°
blast
wave
circle
"blast time units" v_0/g
Note
large kite for envelope that
contacts α=30° trajectory
smaller kite that contacts
that α=30° trajectory and
the α=30° blast wave circle.
Lecture 8 ends here
Mon. 9.23.2019

AMOP reference links *(Updated list given on **2nd** and 3rd pages of each class presentation)*

Web Resources - front page | Quantum Theory for the Computer Age | 2014 AMOP
UAF Physics UTube channel | Principles of Symmetry, Dynamics, and Spectroscopy | 2017 Group Theory for QM
Classical Mechanics with a Bang! | 2018 AMOP
Modern Physics and its Classical Foundations

Representaions Of Multidimensional Symmetries In Networks - harter-jmp-1973

Alternative Basis for the Theory of Complex Spectra

- Alternative_Basis_for_the_Theory_of_Complex_Spectra_I_-_harter-pra-1973
- Alternative_Basis_for_the_Theory_of_Complex_Spectra_II_-_harter-patterson-pra-1976
- Alternative_Basis_for_the_Theory_of_Complex_Spectra_III_-_patterson-harter-pra-1977

Frame Transformation Relations And Multipole Transitions In Symmetric Polyatomic Molecules - RMP-1978

Asymptotic eigensolutions of fourth and sixth rank octahedral tensor operators - Harter-Patterson-JMP-1979

Rotational energy surfaces and high- J eigenvalue structure of polyatomic molecules - Harter - Patterson - 1984

Galloping waves and their relativistic properties - ajp-1985-Harter

Rovibrational Spectral Fine Structure Of Icosahedral Molecules - Cpl 1986 (Alt Scan)

Theory of hyperfine and superfine levels in symmetric polyatomic molecules.

- I) Trigonal and tetrahedral molecules: Elementary spin-1/2 cases in vibronic ground states - PRA-1979-Harter-Patterson (Alt scan)
- II) Elementary cases in octahedral hexafluoride molecules - Harter-PRA-1981 (Alt scan)

Rotation–vibration spectra of icosahedral molecules.

- I) Icosahedral symmetry analysis and fine structure - harter-weeks-jcp-1989 (Alt scan)
- II) Icosahedral symmetry, vibrational eigenfrequencies, and normal modes of buckminsterfullerene - weeks-harter-jcp-1989 (Alt scan)
- III) Half-integral angular momentum - harter-reimer-jcp-1991

Rotation-vibration scalar coupling zeta coefficients and spectroscopic band shapes of buckminsterfullerene - Weeks-Harter-CPL-1991 (Alt scan)

Nuclear spin weights and gas phase spectral structure of 12C60 and 13C60 buckminsterfullerene -Harter-Reimer-Cpl-1992 - (Alt1, Alt2 Erratum)

Gas Phase Level Structure of C60 Buckyball and Derivatives Exhibiting Broken Icosahedral Symmetry - reimer-diss-1996

Fullerene symmetry reduction and rotational level fine structure/ the Buckyball isotopomer 12C 13C59 - jcp-Reimer-Harter-1997 (HiRez)

Wave Node Dynamics and Revival Symmetry in Quantum Rotors - harter - jms - 2001

Molecular Symmetry and Dynamics - Ch32-Springer Handbooks of Atomic, Molecular, and Optical Physics - Harter-2006

Resonance and Revivals

- I) QUANTUM ROTOR AND INFINITE-WELL DYNAMICS - ISMSLi2012 (Talk) OSU knowledge Bank
- II) Comparing Half-integer Spin and Integer Spin - Alva-ISMS-Ohio2013-R777 (Talks)
- III) Quantum Resonant Beats and Revivals in the Morse Oscillators and Rotors - (2013-Li-Diss)

Resonance and Revivals in Quantum Rotors - Comparing Half-integer Spin and Integer Spin - Alva-ISMS-Ohio2013-R777 (Talk)

Molecular Eigensolution Symmetry Analysis and Fine Structure - IJMS-harter-mitchell-2013

Quantum Revivals of Morse Oscillators and Farey-Ford Geometry - Li-Harter-cpl-2013

QTCA Unit 10 Ch 30 - 2013

AMOP Ch 0 Space-Time Symmetry - 2019

**Index/Search is disabled - a web based A.M.O.P. oriented reference page, with thumbnail/previews, greater control over the information display,*
https://modphys.hosted.uark.edu/markup/AMOP_References.html
and eventually full on Apache-SOLR *Index and search for* nuanced, whole-site content/metadata level *searching.*

AMOP reference links *(Updated list given on 2nd and 3rd pages of each class presentation)*

(Int.J.Mol.Sci, 14, 714(2013) p.755-774 , QTCA Unit 7 Ch. 23-26), (PSDS - Ch. 5, 7)

Int.J.Mol.Sci, 14, 714(2013), QTCA Unit 8 Ch. 23-25, QTCA Unit 9 Ch. 26, PSDS Ch. 5, PSDS Ch. 7

Intro spin ½ coupling
Unit 8 Ch. 24 p3

H atom hyperfine-B-level crossing
Unit 8 Ch. 24 p15

Hyperf. theory Ch. 24 p48.

Hyperf. theory Ch. 24 p48.
Deeper theory ends p53

Intro 2p3p coupling
Unit 8 Ch. 24 p17.

Intro LS-jj coupling
Unit 8 Ch. 24 p22.

CG coupling derived (start)
Unit 8 Ch. 24 p39.

CG coupling derived (formula)
Unit 8 Ch. 24 p44.

Lande' g-factor
Unit 8 Ch. 24 p26.

Irrep Tensor building
Unit 8 Ch. 25 p5.

Irrep Tensor Tables
Unit 8 Ch. 25 p12.

Wigner-Eckart tensor Theorem.
Unit 8 Ch. 25 p17.

Tensors Applied to d,f-levels.
Unit 8 Ch. 25 p21.

Tensors Applied to high J levels.
Unit 8 Ch. 25 p63.

Intro 3-particle coupling.
Unit 8 Ch. 25 p28.

Intro 3,4-particle Young Tableaus
GrpThLect29 p42.

Young Tableau Magic Formulae
GrpThLect29 p46-48.

**Index/Search is disabled - a web based A.M.O.P. oriented reference page, with thumbnail/previews, greater control over the information display,*
https://modphys.hosted.uark.edu/markup/AMOP_References.html
and eventually full on Apache-SOLR *Index and search for* nuanced, whole-site content/metadata level *searching.*

AMOP reference links *(Updated list given on 2nd and **3rd** and **4th** pages of each class presentation)*

Predrag Cvitanovic's: Birdtrack Notation, Calculations, and Simplification

Chaos_Classical_and_Quantum_-_2018-Cvitanovic-ChaosBook
Group Theory - PUP_Lucy_Day_-_Diagrammatic_notation_-_Ch4
Simplification_Rules_for_Birdtrack_Operators_-_Alcock-Zeilinger-Weigert-zeilinger-jmp-2017
Group Theory - Birdtracks_Lies_and_Exceptional_Groups_-_Cvitanovic-2011
Simplification_rules_for_birdtrack_operators-_jmp-alcock-zeilinger-2017
Birdtracks for SU(N) - 2017-Keppeler

Frank Rioux's: UMA method of vibrational induction

Quantum_Mechanics_Group_Theory_and_C60_-_Frank_Rioux_-_Department_of_Chemistry_Saint_Johns_U
Symmetry_Analysis_for_H20-_H20GrpTheory-_Rioux
Quantum_Mechanics-Group_Theory_and_C60_-_JChemEd-Rioux-1994
Group_Theory_Problems-_Rioux-_SymmetryProblemsX
Comment_on_the_Vibrational_Analysis_for_C60_and_Other_Fullerenes_Rioux-RSP

Supplemental AMOP Techniques & Experiment

Many Correlation Tables are Molien Sequences - Klee (Draft 2016)
High-resolution_spectroscopy_and_global_analysis_of_CF4_rovibrational_bands_to_model_its_atmospheric_absorption-_carlos-Boudon-jqsrt-2017
Symmetry and Chirality - Continuous_Measures_-_Avnir
*

Special Topics & Colloquial References

r-process_nucleosynthesis_from_matter_ejected_in_binary_neutron_star_mergers-PhysRevD-Bovard-2017

**Index/Search is disabled - a web based A.M.O.P. oriented reference page, with thumbnail/previews, greater control over the information display,*
https://modphys.hosted.uark.edu/markup/AMOP_References.html
and eventually full on Apache-SOLR *Index and search for* nuanced, whole-site content/metadata level *searching.*