2019 CMwBang! site

Class YouTube Channel

Lecture 5
Wed. 9.11.2019

Dynamics of Potentials and Force Fields

(Ch. 7 and part of Ch. 8 of Unit 1)

Potential energy dynamics of Superballs and related things

Thales geometry and "Sagittal approximation" to superball force law

Geometry and dynamics of single ball bounce

(a) Constant force F=-k (linear potential V=kx)

Some physics of dare-devil diving 80 ft. into kidee pool

(b) Linear force F=-kx (quadratic potential $V=\frac{1}{2}kx^2$ (like balloon))

(c) Non-linear force (like superball-floor or ball-bearing-anvil)

Geometry and potential dynamics of 2-ball bounce

A parable of RumpCo. vs CrapCorp. (introducing 3-mass potential-driven dynamics)

A story of USC pre-meds visiting Whammo Manufacturing Co.

Geometry and dynamics of n-ball bounces

Analogy with shockwave and acoustical horn amplifier

Advantages of a geometric $m_1, m_2, m_3, \ldots$ series

A story of Stirling Colgate (Palmolive) and core-collapse supernovae

Many-body 1D collisions

Elastic examples: Western buckboard

Bouncing columns and Newton's cradle

Inelastic examples: "Zig-zag geometry" of freeway crashes

Super-elastic examples: This really **is** *"Rocket-Science"*

This Lecture's Reference Link Listing

Web Resources - front page
UAF Physics UTube channel

AJP article on superball dynamics
AAPT Summer Reading List
Scitation.org - AIP publications
HarterSoft Youtube Channel

Quantum Theory for the Computer Age
Principles of Symmetry, Dynamics, and Spectroscopy
Classical Mechanics with a Bang!
Modern Physics and its Classical Foundations

2017 Group Theory for QM
2018 Adv CM
2018 AMOP
2019 Advanced Mechanics

Lecture #5

X2 paper: *Velocity Amplification in Collision Experiments Involving Superballs - Harter, et. al. 1971 (pdf)*
MIT OpenCourseWare: High School/Physics/Impulse and Momentum
Hubble Site: Supernova - SN 1987A

BounceItIt Web Animation - Scenarios:

Generic Scenario: 2-Balls dropped no Gravity (7:1) - V vs V Plot (Power=4)
1-Ball dropped w/Gravity=0.5 w/Potential Plot: Power=1, Power=4
7:1 - V vs V Plot: Power=1
3-Ball Stack (10:3:1) w/Newton plot (y vs t) - Power=4
3-Ball Stack (10:3:1) w/Newton plot (y vs t) - Power=1
3-Ball Stack (10:3:1) w/Newton plot (y vs t) - Power=1 w/Gaps
4-Ball Stack (27:9:3:1) w/Newton plot (y vs t) - Power=4
4-Newton's Balls (1:1:1:1) w/Newtonian plot (y vs t) - Power=4 w/Gaps
6-Ball Totally Inelastic Collision (1:1:1:1:1:1) w/Gaps: Newtonian plot (t vs x), V6 vs V5 plot
5-Ball Totally Inelastic Pile-up w/ 5-Stationary-Balls - Minkowski plot (t vs x1) w/Gaps
1-Ball Totally Inelastic Pile-up w/ 5-Stationary-Balls - Vx2 vs Vx1 plot w/Gaps

Running Reference Link Listing

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Prior to Lecture #5

BounceItIt Web Animation - Scenarios:
49:1 y vs t, *49:1 V2 vs V1*, *1:500:1 - 1D Gas Model w/ faux restorative force (Cool)*,
1:500:1 - 1D Gas (Warm), *1:500:1 - 1D Gas Model (Cool, Zoomed in)*,
Farey Sequence - Wolfram
Fractions - Ford-AMM-1938
Monstermash BounceItIt Animations:
1000:1 - V2 vs V1, *1000:1 with t vs x - Minkowski Plot*
Quantum Revivals of Morse Oscillators and Farey-Ford Geometry - Li-Harter-2013
Quantum_Revivals_of_Morse_Oscillators_and_Farey-Ford_Geometry - Li-Harter-cpl-2015
Quant. Revivals of Morse Oscillators and Farey-Ford Geom. - Harter-Li-CPL-2015 (Publ.)
Velocity_Amplification_in_Collision_Experiments_Involving_Superballs-Harter-1971
WaveIt Web Animation - Scenarios:
Quantum_Carpet, *Quantum_Carpet_wMBars*,
Quantum_Carpet_BCar, *Quantum_Carpet_BCar_wMBars*
Wave Node Dynamics and Revival Symmetry in Quantum Rotors - Harter-JMS-2001
Wave Node Dynamics and Revival Symmetry in Quantum Rotors - Harter-jms-2001 (Publ.)

BounceIt Dual plots
m_1:m_2 = 3:1
v_2 vs v_1 and V_2 vs V_1, $(v_1, v_2)=(1, 0.1)$, $(v_1, v_2)=(1, 0)$
y_2 vs y_1 plots: $(v_1, v_2)=(1, 0.1)$, $(v_1, v_2)=(1, 0)$, $(v_1, v_2)=(1, -1)$
Estrangian plot *V_2 vs V_1: $(v_1, v_2)=(0, 1)$, $(v_1, v_2)=(1, -1)$*
m_1:m_2 = 4:1
v2 vs v1, *y2 vs y1*
m_1:m_2 = 100:1, *$(v_1, v_2)=(1, 0)$: V2 vs V1 Estrangian plot, y2 vs y1 plot*
With g=0 and 70:10 mass ratio
With non zero g, velocity dependent damping and mass ratio of 70:35
M_1=49, M_2=1 with Newtonian time plot
M_1=49, M_2=1 with V_2 vs V_1 plot
Example with friction
Low force constant with drag displaying a Pass-thru, *Fall-Thru*, *Bounce-Off*
m1:m2= 3:1 and (v1, v2) = (1, 0) Comparison with Estrangian

X2 paper: *Velocity Amplification in Collision Experiments Involving Superballs - Harter, et. al. 1971 (pdf)*
Car Collision Web Simulator: https://modphys.hosted.uark.edu/markup/CMMotionWeb.html
Superball Collision Web Simulator: https://modphys.hosted.uark.edu/markup/BounceItWeb.html; with Scenarios: 1007
BounceIt web simulation with g=0 and 70:10 mass ratio
With non zero g, velocity dependent damping and mass ratio of 70:35
Elastic Collision Dual Panel Space vs Space: Space vs Time (Newton) , Time vs. Space(Minkowski)
Inelastic Collision Dual Panel Space vs Space: Space vs Time (Newton), Time vs. Space(Minkowski)
Matrix Collision Simulator:M_1=49, M_2=1 V_2 vs V_1 plot <<Under Construction>>

More Advanced QM and classical references at the end of this Lecture

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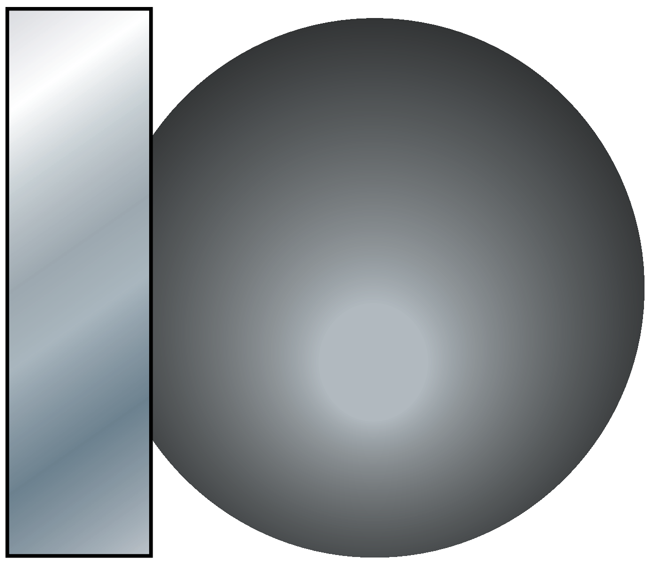
Super-elastic examples: This really **is** *"Rocket-Science"*

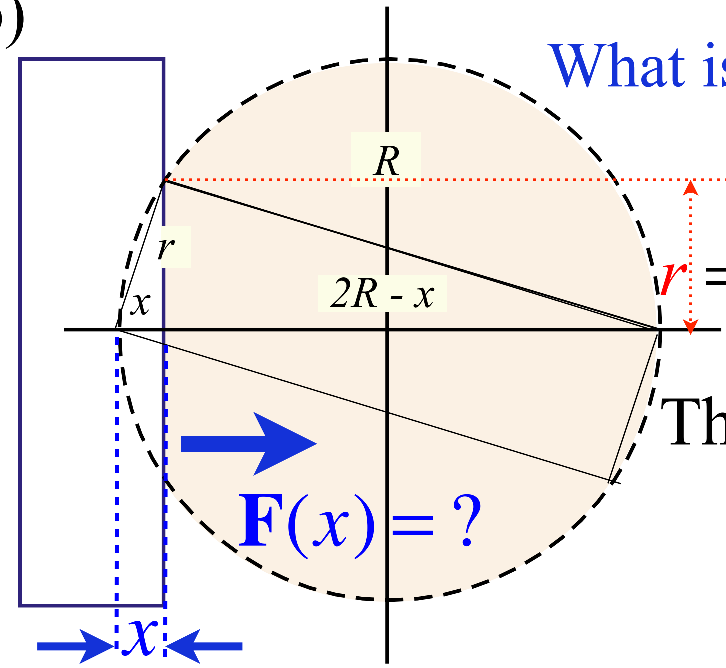
Potential Energy Geometry of Superballs and Related things

(a)



Unit 1
Fig. 7.1
(modified)

(b)



What is superball bounce force law $\mathbf{F}(x)$?

$$r = \sqrt{x(2R - x)} \quad \left(\approx \sqrt{2Rx} \quad for: \ x << R\right)$$

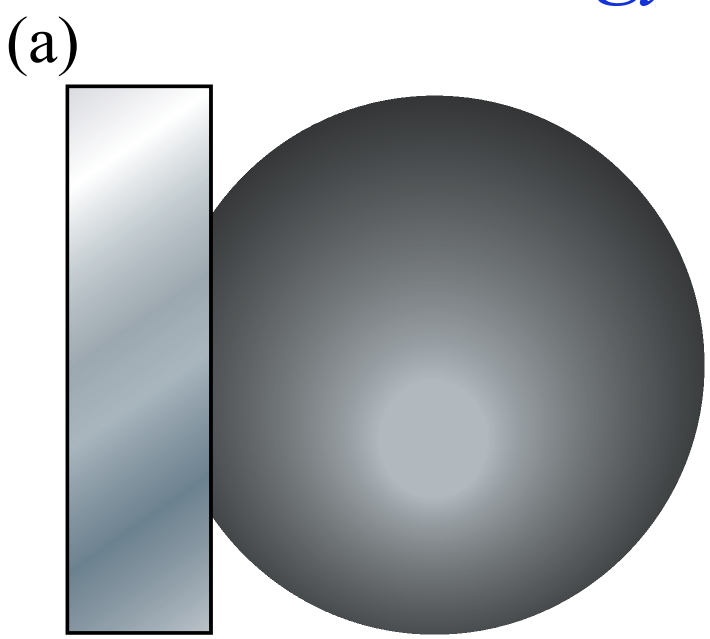
Thales' geometry and "Sagittal†" approx.

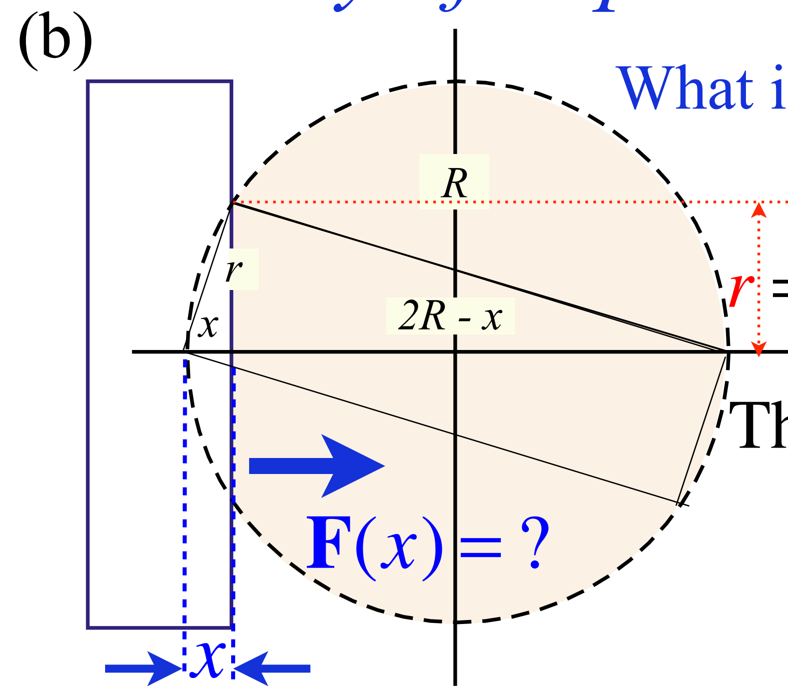
$$\frac{x}{r} = \frac{r}{2R - x}$$

† "bow"

Used for "thin-lense" optics

Potential Energy Geometry of Superballs and Related things





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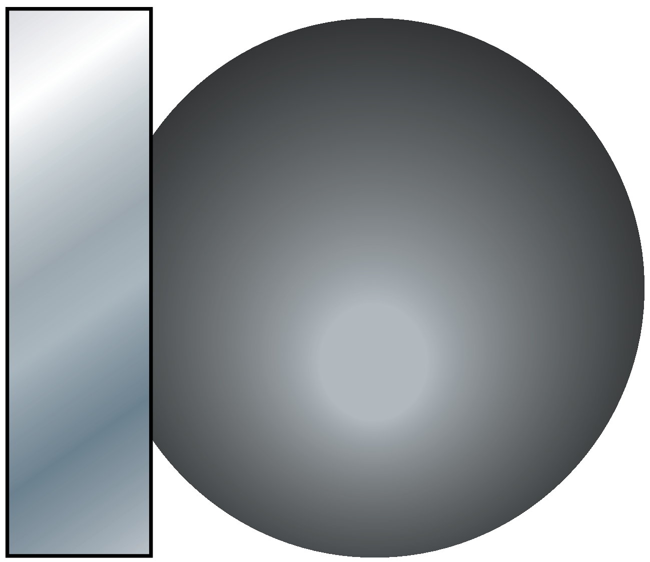
If superball was a balloon its bounce force law would be linear $F=-k\cdot x$ (Hooke Law)

$$F_{balloon}(x) = \underset{(Pressure)\cdot(Area)}{P\cdot A} = P\cdot\pi r^2$$

$$\approx P\cdot\pi 2Rx$$

Potential Energy Geometry of Superballs and Related things

(a)



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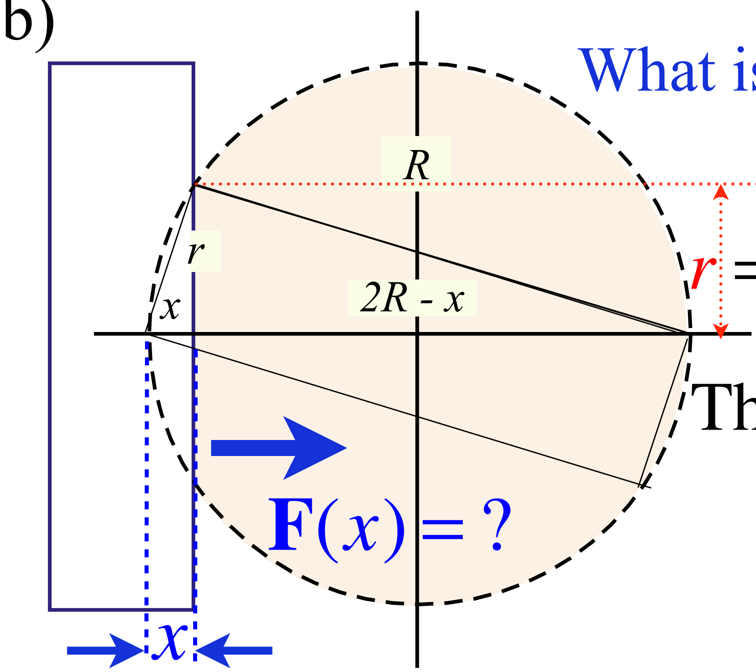

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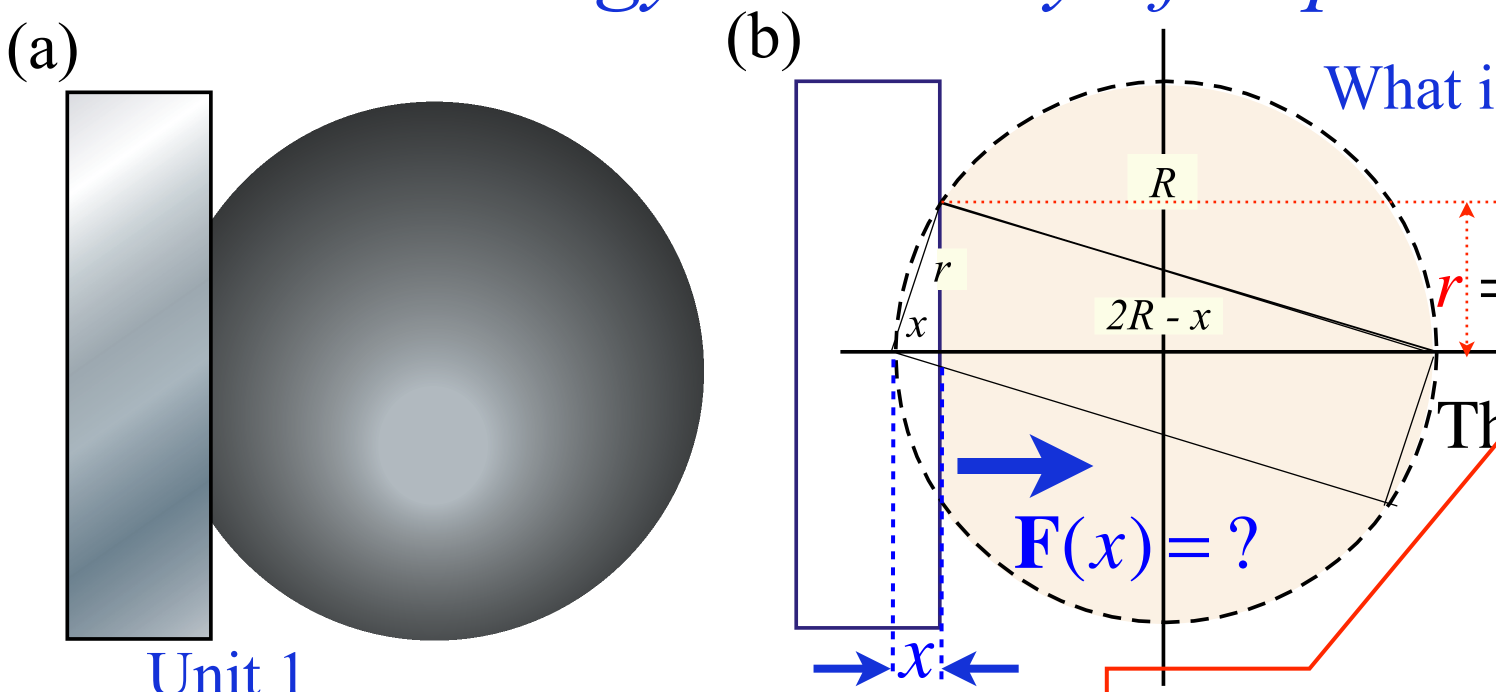
$$= kx$$

Potential Energy Geometry of Superballs and Related things



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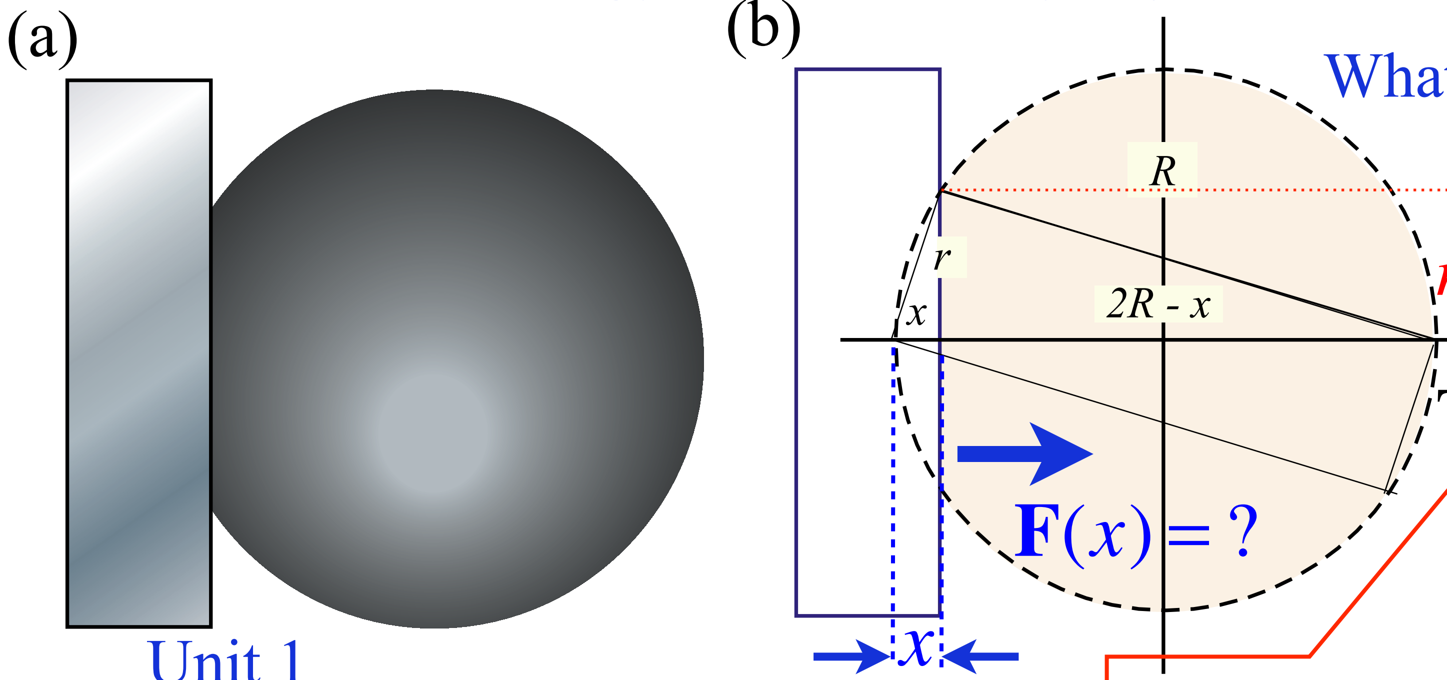
Instead superball force law depends on bulk *volume* modulus and is non-linear $F \sim x^{p?} + ?$ *(Power Law?)*

$$Volume(X) = \int_0^X \pi r^2 \, dx = \int_0^X \pi x(2R-x) \, dx$$

Potential Energy Geometry of Superballs and Related things



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Potential Energy Geometry of Superballs and Related things

(a)

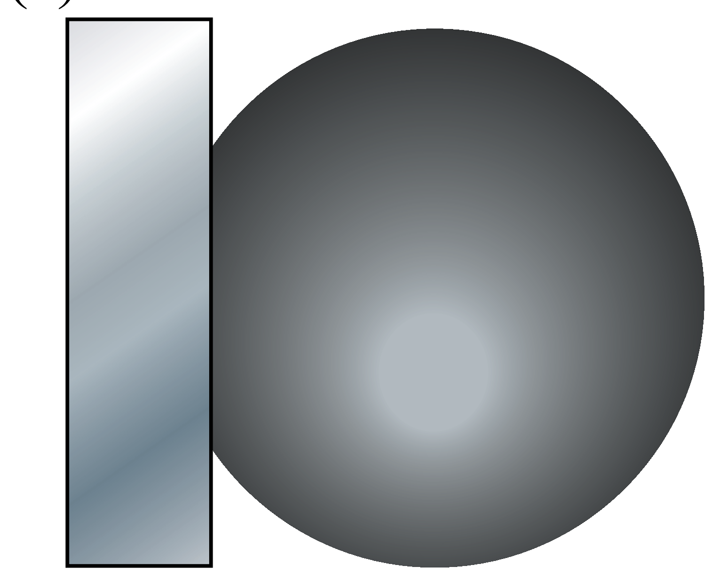

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Fig. 7.1
(modified)

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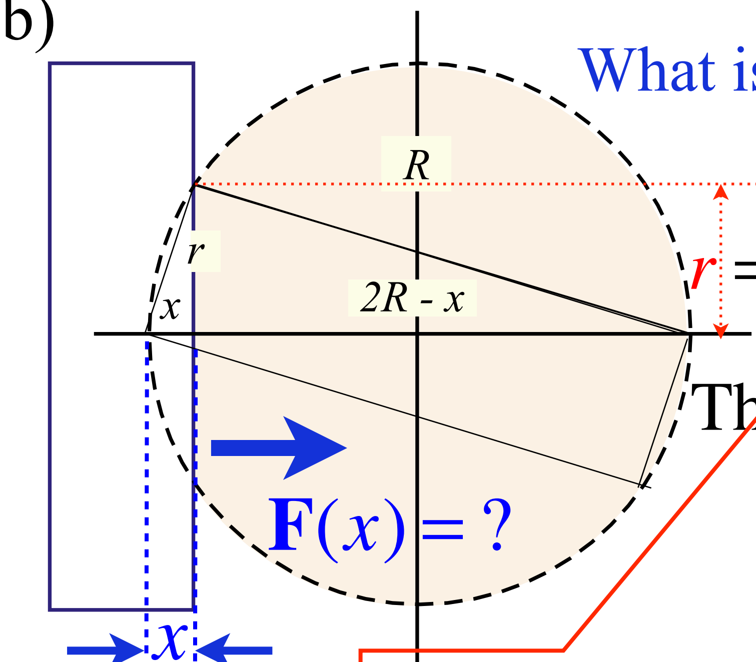

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It also depends on velocity $\dot{x} = \frac{dx}{dt}$. *Adiabatic* differs from *Isothermal* as shown by *"Project-Ball*"*

* *Am. J. Phys. 39, 656 (1971)*

(Discussed after p. 33)

Potential energy dynamics of Superballs and related things

Thales geometry and "Sagittal approximation" to force law

→ *Geometry and dynamics of single ball bounce*

→ *General Non-linear force (like superball-floor or ball-bearing-anvil)* *(Simulations)*

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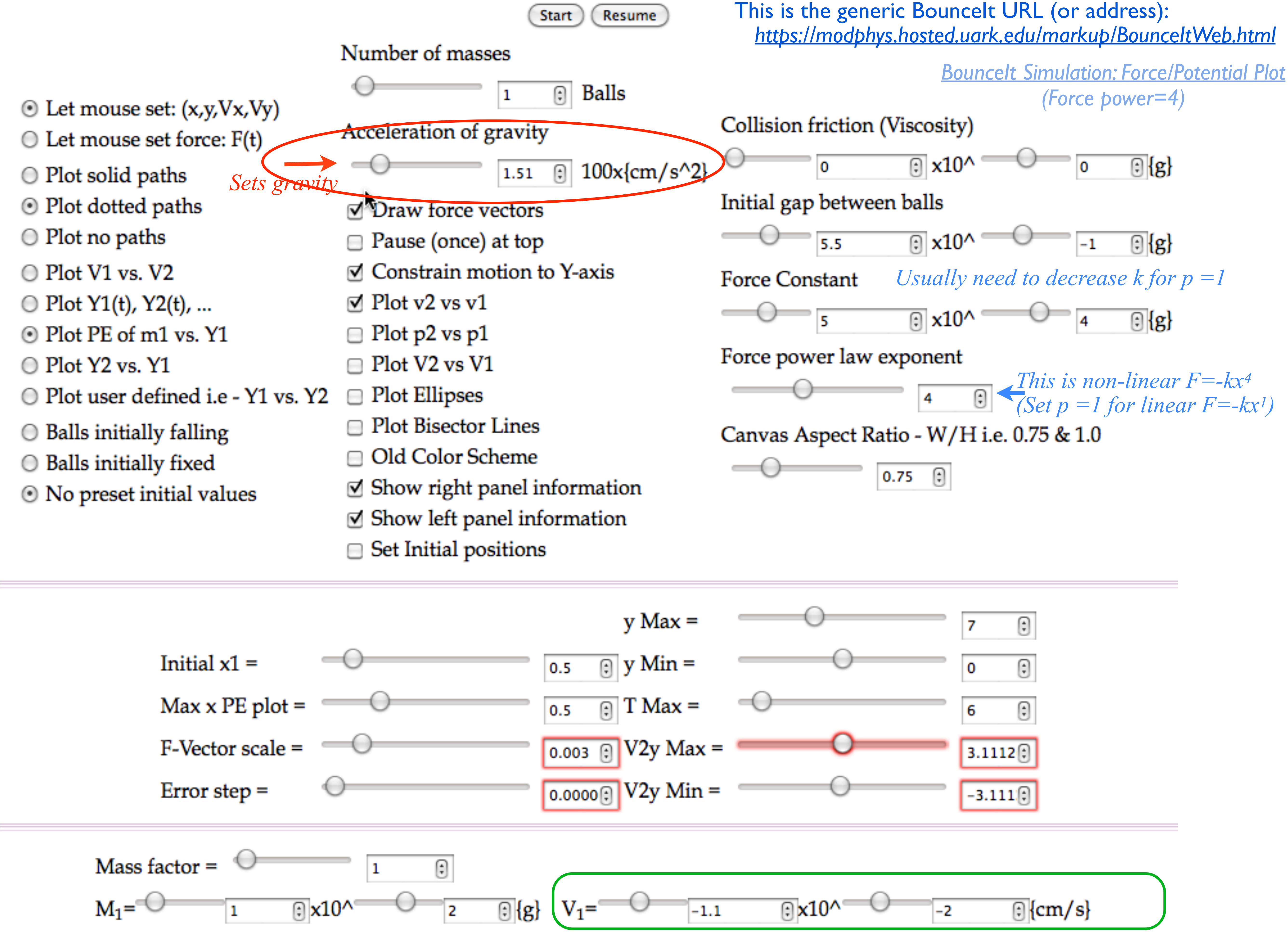
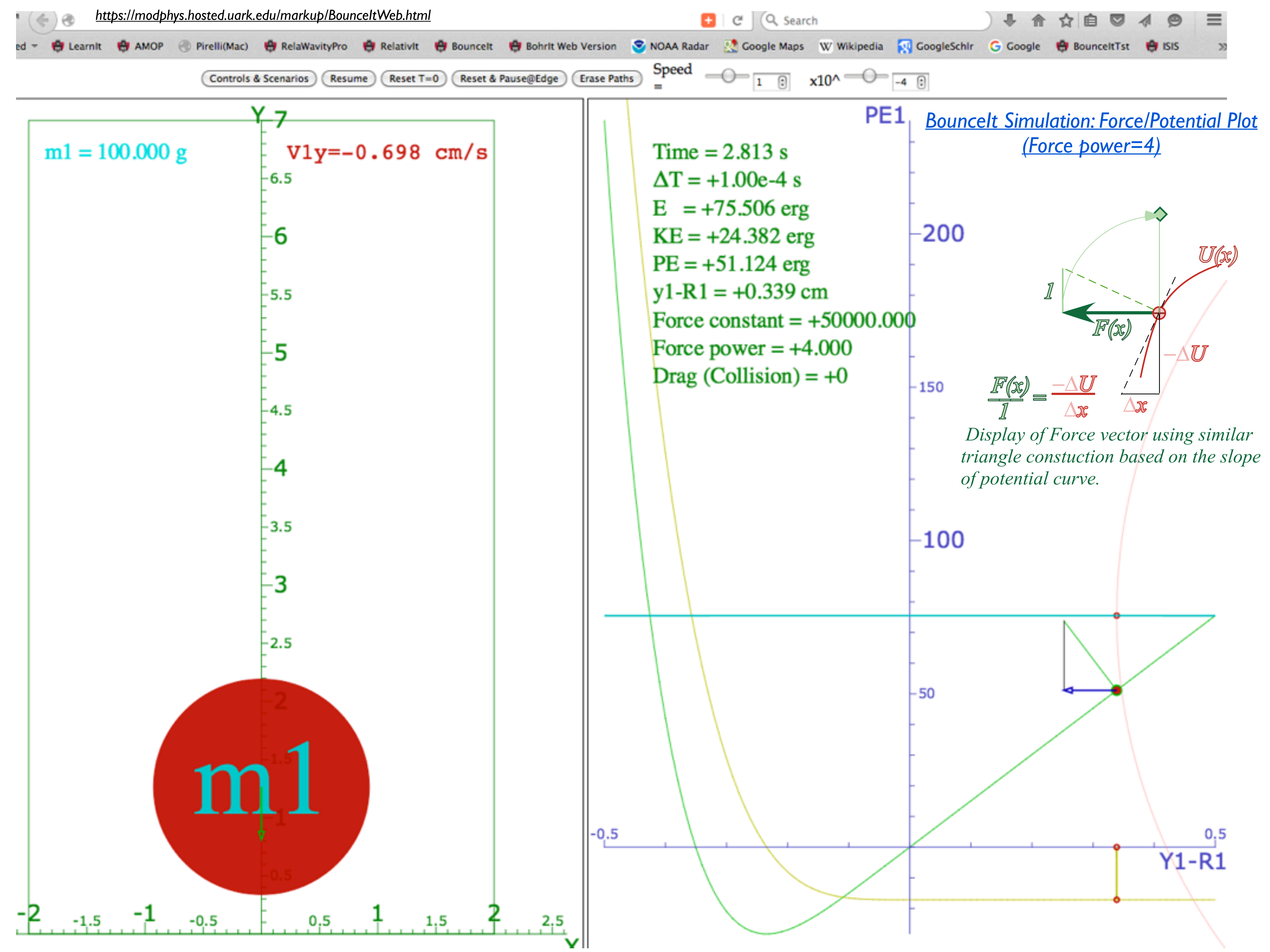
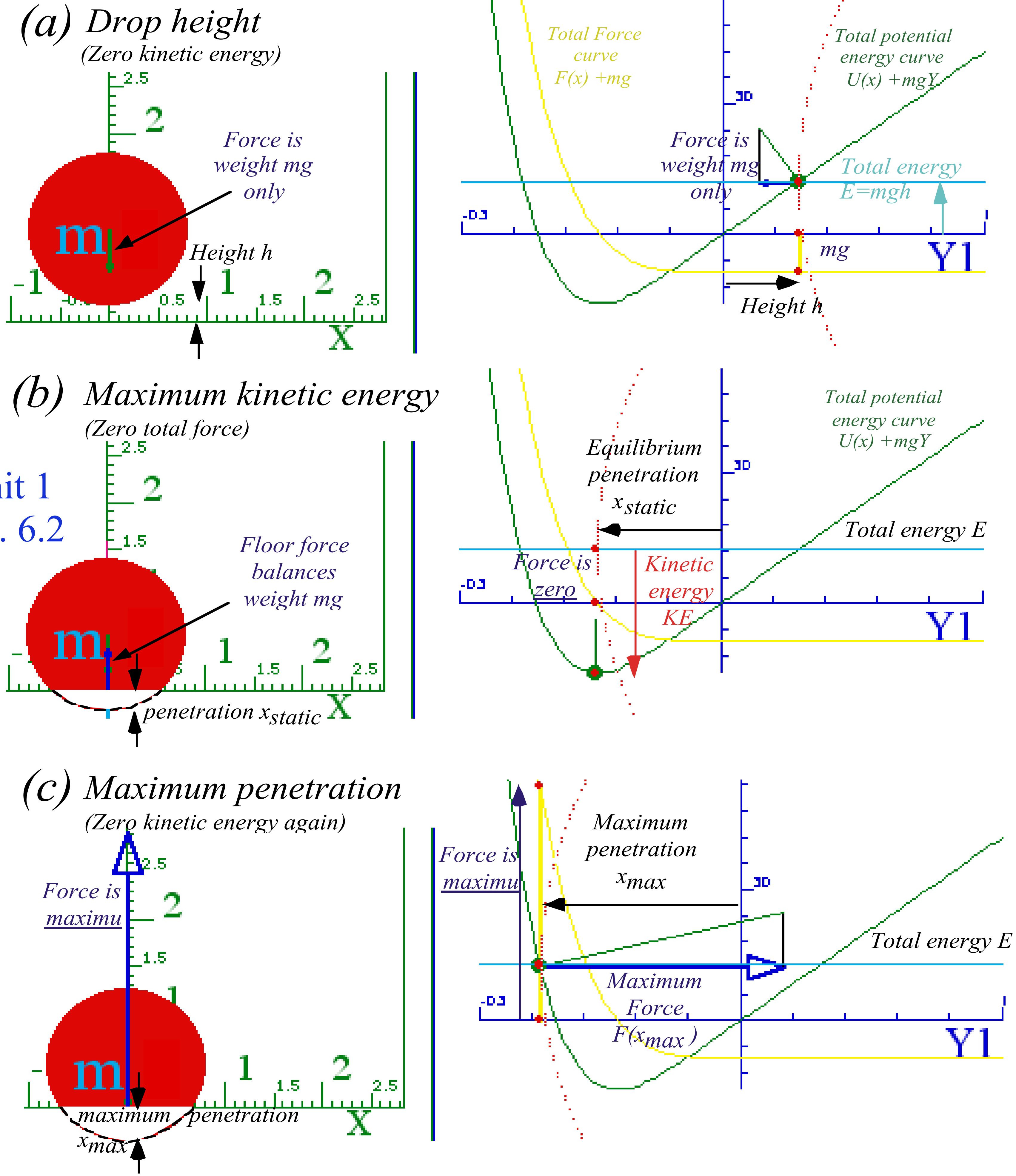
Main Control Panel



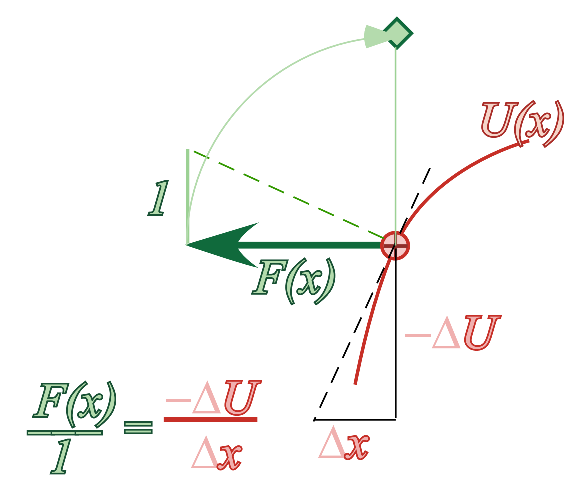

https://modphys.hosted.uark.edu/markup/BounceItWeb.html
Controls & Scenarios
Resume
Reset T=0
Reset & Pause@Edge
Erase Paths
Speed
x10^
m1 = 100.000 g
V1y=-0.698 cm/s
m1
PE1
BounceIt Simulation: Force/Potential Plot
(Force power=4)
Time = 2.813 s
ΔT = +1.00e-4 s
E = +75.506 erg
KE = +24.382 erg
PE = +51.124 erg
y1-R1 = +0.339 cm
Force constant = +50000.000
Force power = +4.000
Drag (Collision) = +0
U(x)
F(x)
−ΔU
Δx
F(x)/1 = −ΔU/Δx
Display of Force vector using similar triangle constuction based on the slope of potential curve.
Y1-R1

1990 BounceIt Mac simulations

Unit 1
Fig. 6.2



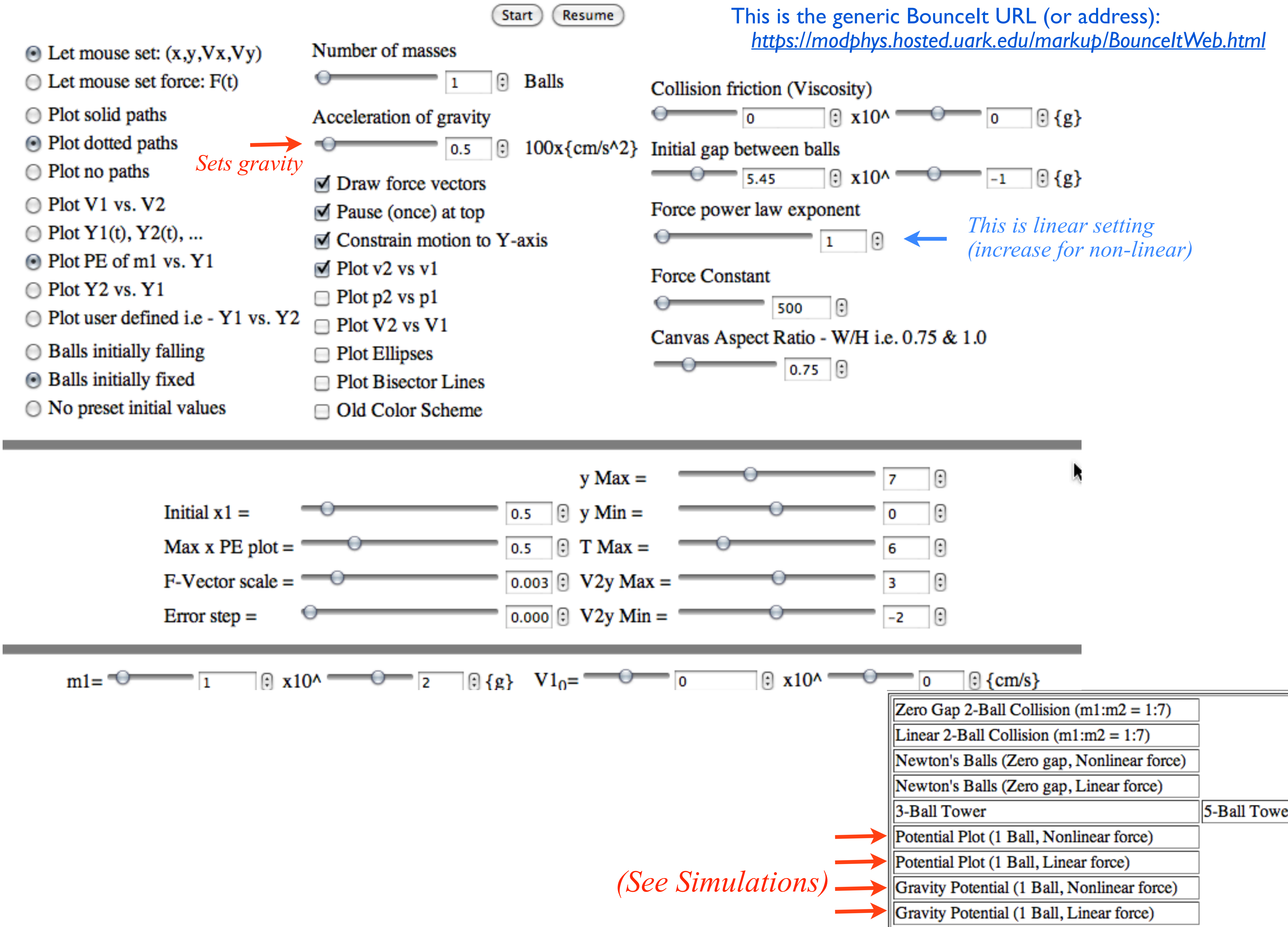
Details of each case follows
using newer *BounceIt Web simulations*

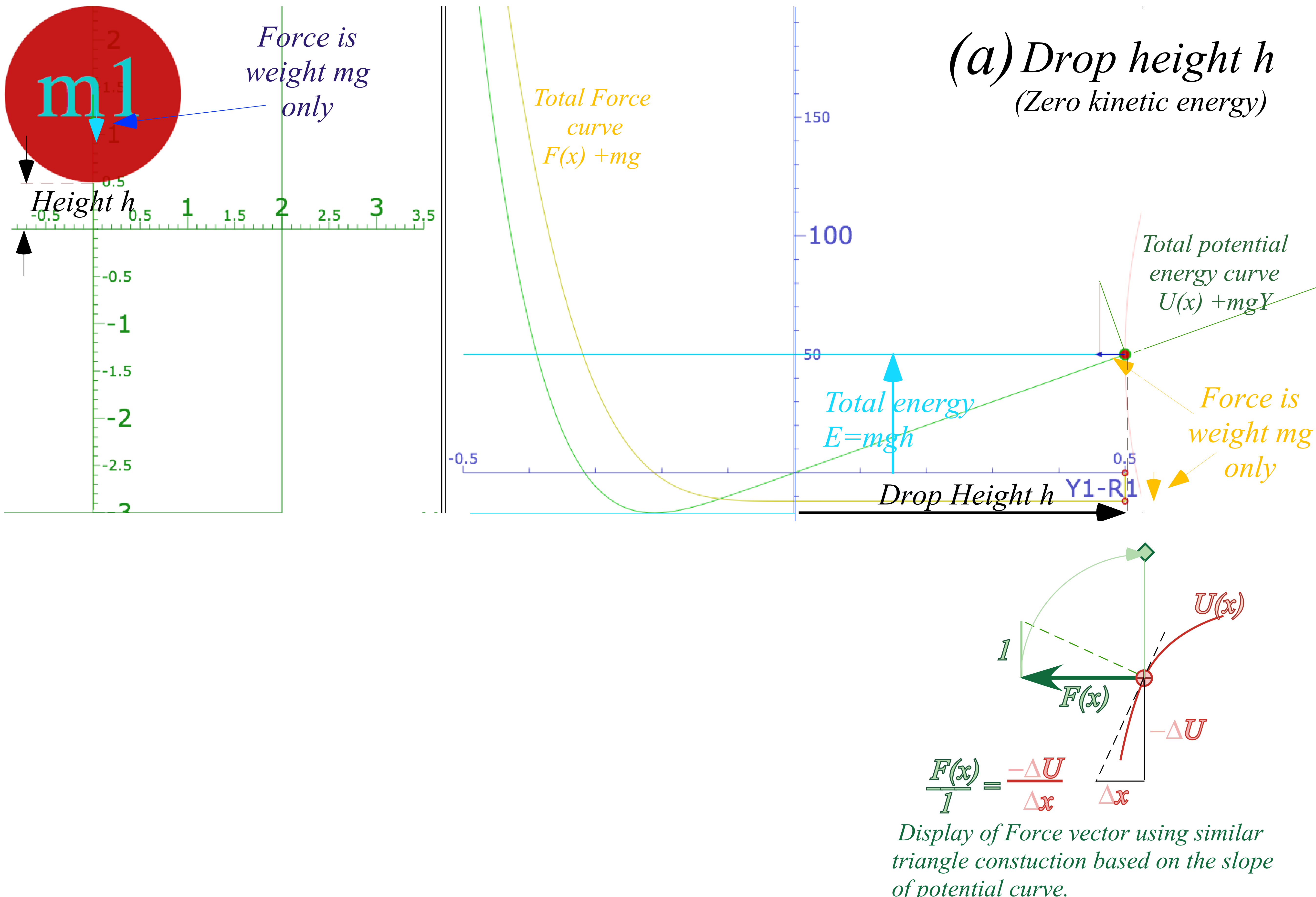


Display of Force vector using similar triangle constuction based on the slope of potential curve.

Main Control Panel

This is the generic BounceIt URL (or address): https://modphys.hosted.uark.edu/markup/BounceItWeb.html

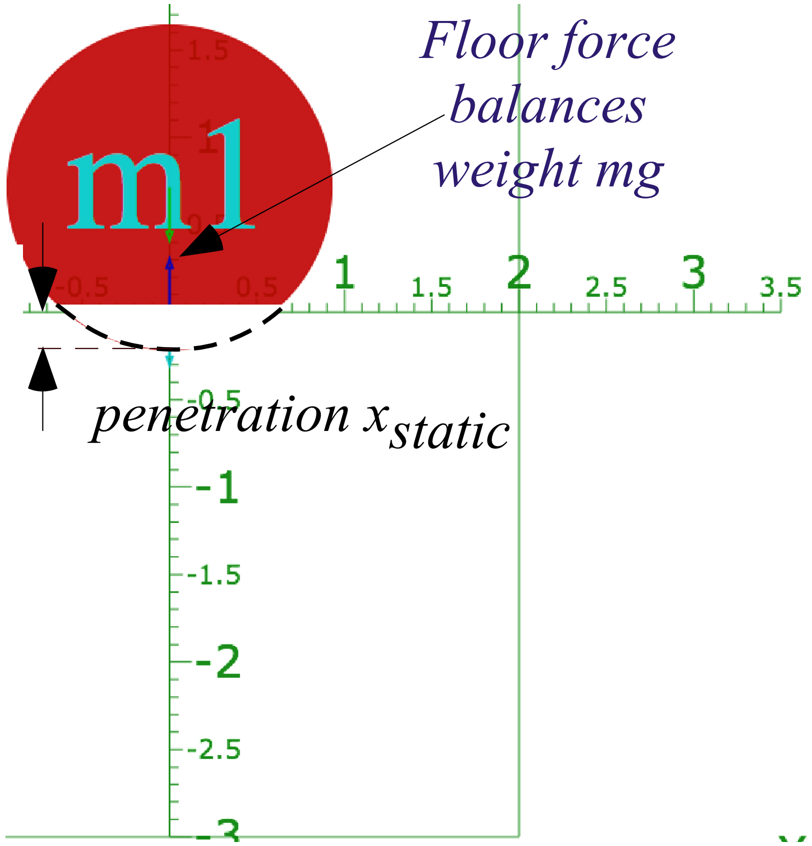


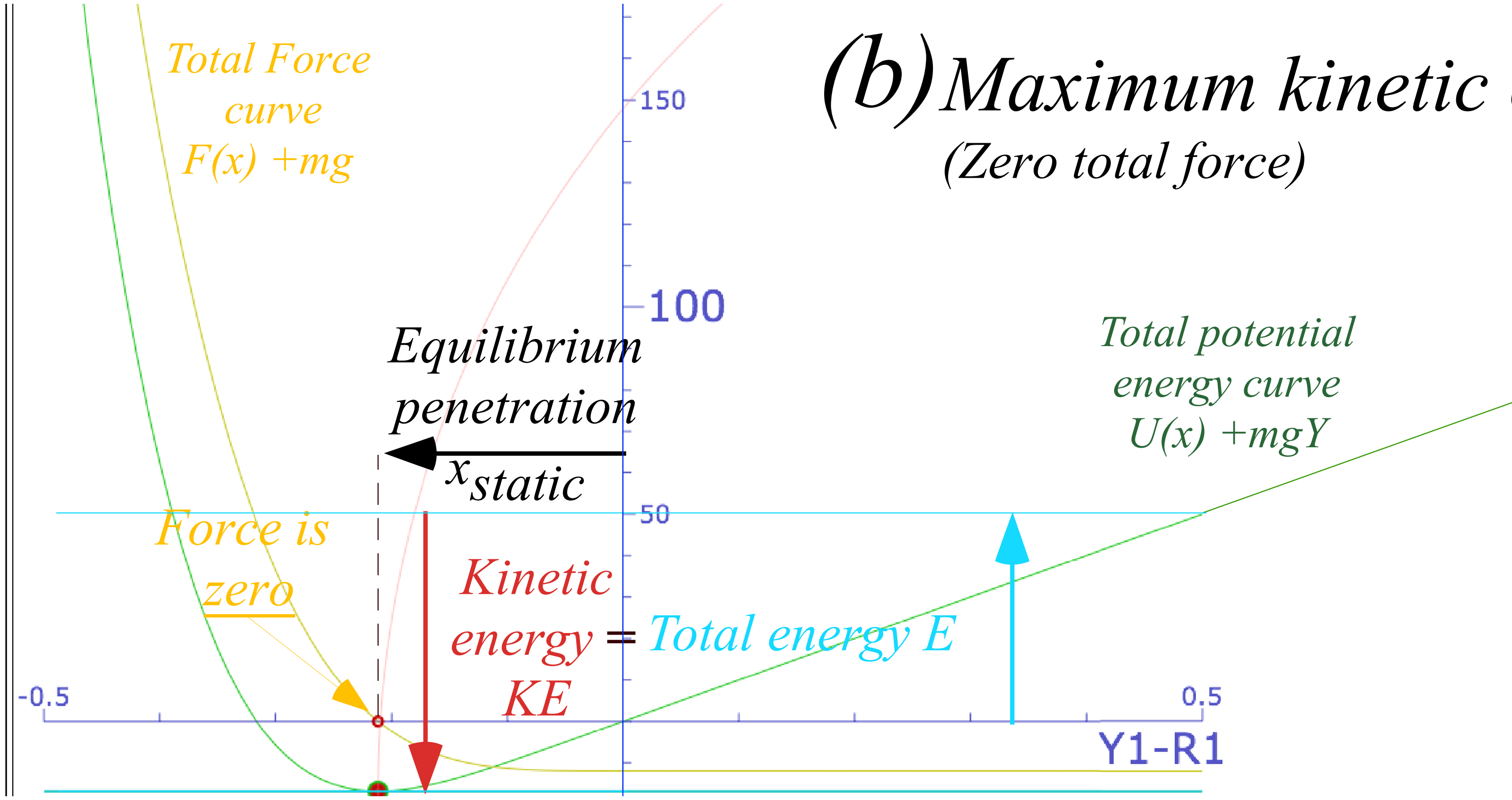


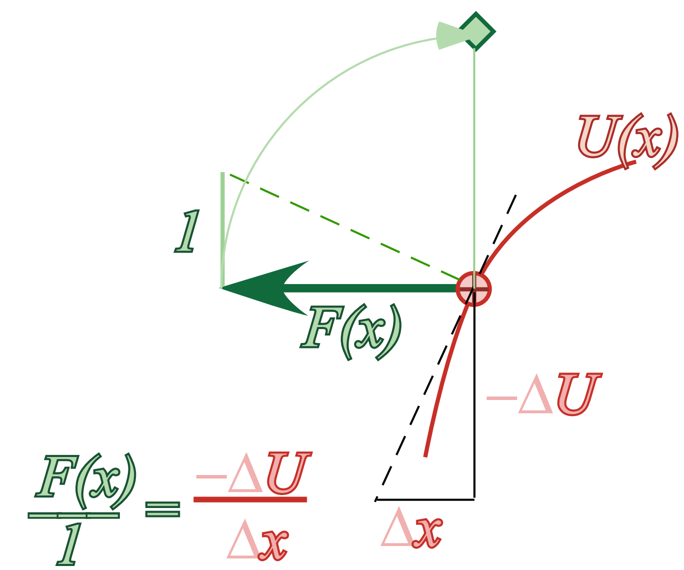
[BounceIt Simulation: Force/Potential Plot (Force power=4)]

(b) Maximum kinetic energy

(Zero total force)

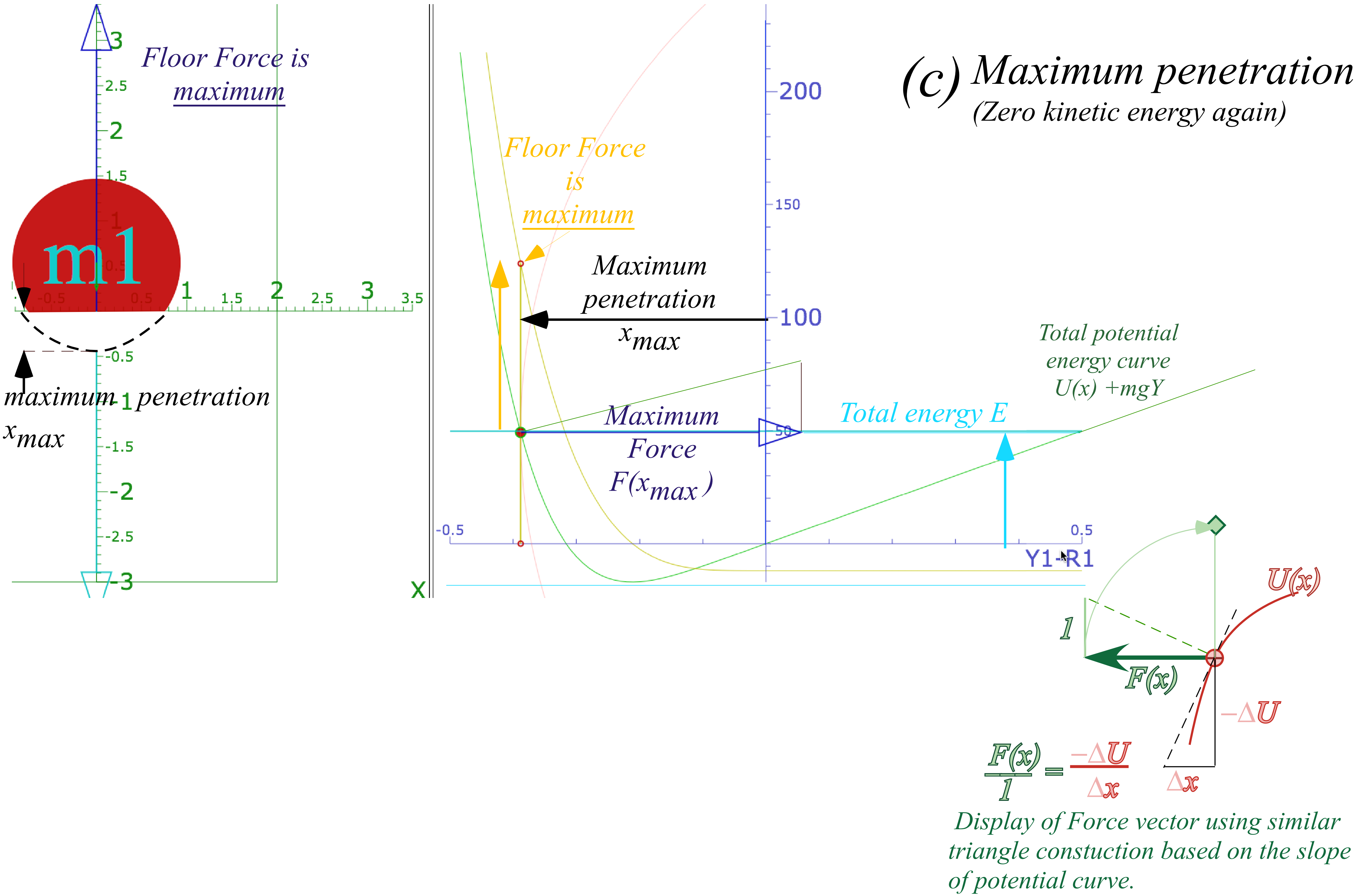




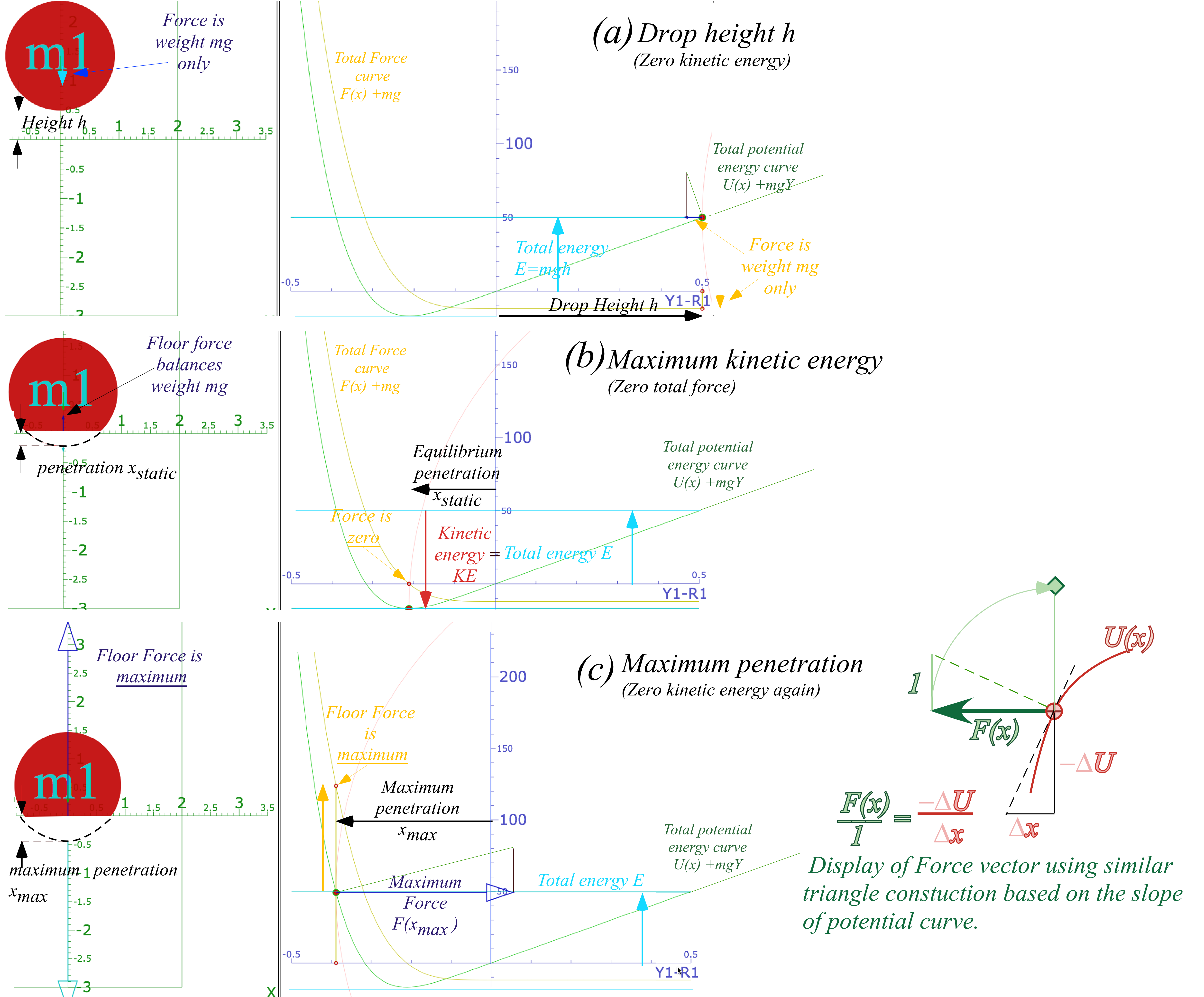


Display of Force vector using similar triangle constuction based on the slope of potential curve.

[BounceIt Simulation: Force/Potential Plot (Force power=4)]



(c) Maximum penetration
(Zero kinetic energy again)
Floor Force is maximum
m1
maximum penetration
x_{max}
Floor Force is maximum
Maximum penetration
x_{max}
Maximum Force
$F(x_{max})$
Total potential energy curve
U(x) +mgY
Total energy E
200
150
100
50
-0.5
0.5
Y1-R1
X
U(x)
1
F(x)
−ΔU
Δx
$\frac{F(x)}{1} = \frac{-\Delta U}{\Delta x}$
Display of Force vector using similar triangle constuction based on the slope of potential curve.



(a) Drop height h
(Zero kinetic energy)
Force is weight mg only
Height h
Total Force curve F(x) +mg
Total potential energy curve U(x) +mgY
Total energy E=mgh
Force is weight mg only
Drop Height h
Y1-R1
(b) Maximum kinetic energy
(Zero total force)
Floor force balances weight mg
penetration x static
Total Force curve F(x) +mg
Equilibrium penetration x static
Total potential energy curve U(x) +mgY
Force is zero
Kinetic energy KE = Total energy E
Y1-R1
(c) Maximum penetration
(Zero kinetic energy again)
Floor Force is maximum
maximum penetration x max
Floor Force is maximum
Maximum penetration x max
Total potential energy curve U(x) +mgY
Maximum Force F(x max)
Total energy E
Y1-R1
U(x)
1
F(x)
−ΔU
F(x)/1 = −ΔU/Δx
Δx
Display of Force vector using similar triangle constuction based on the slope of potential curve.

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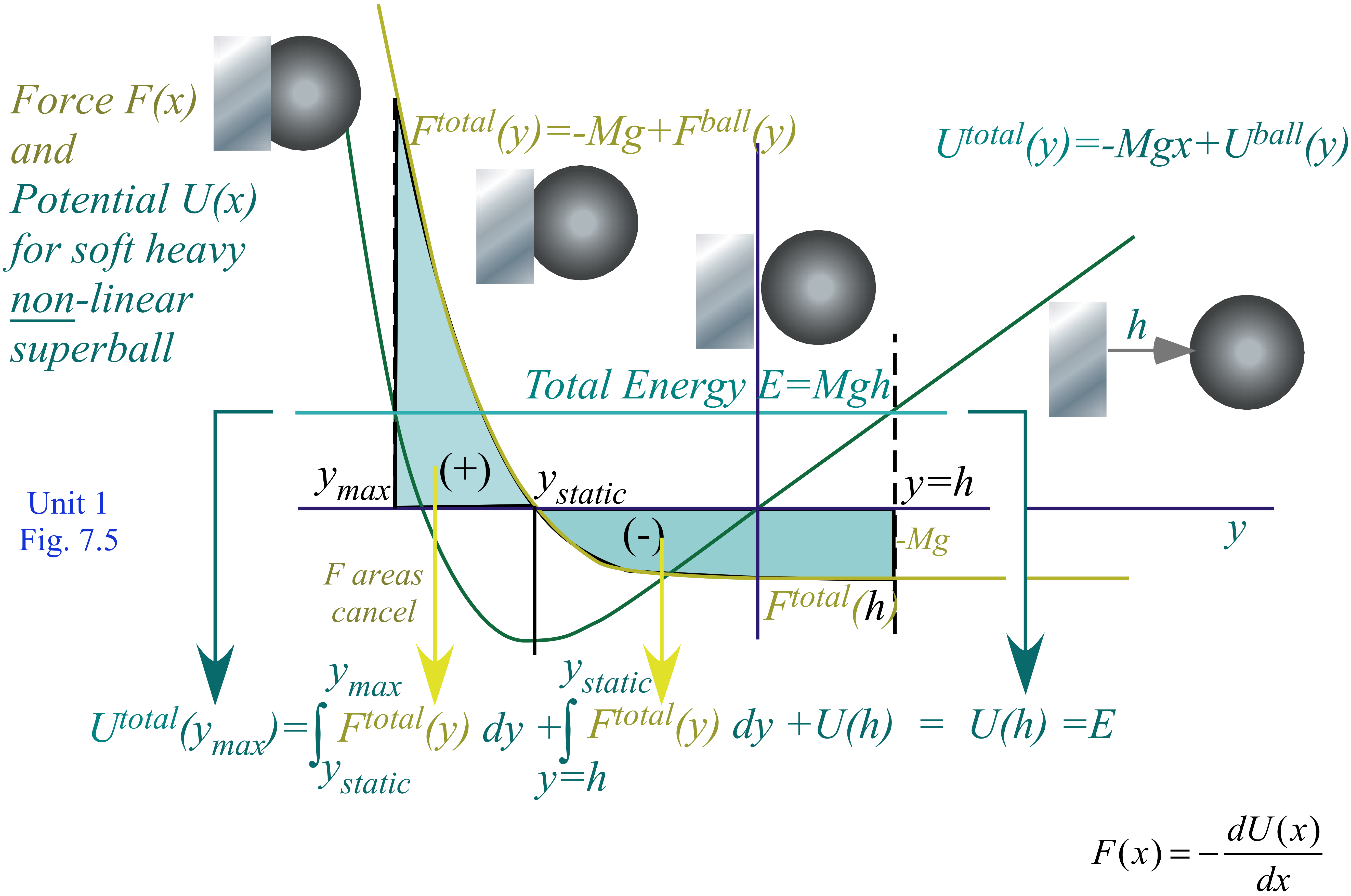
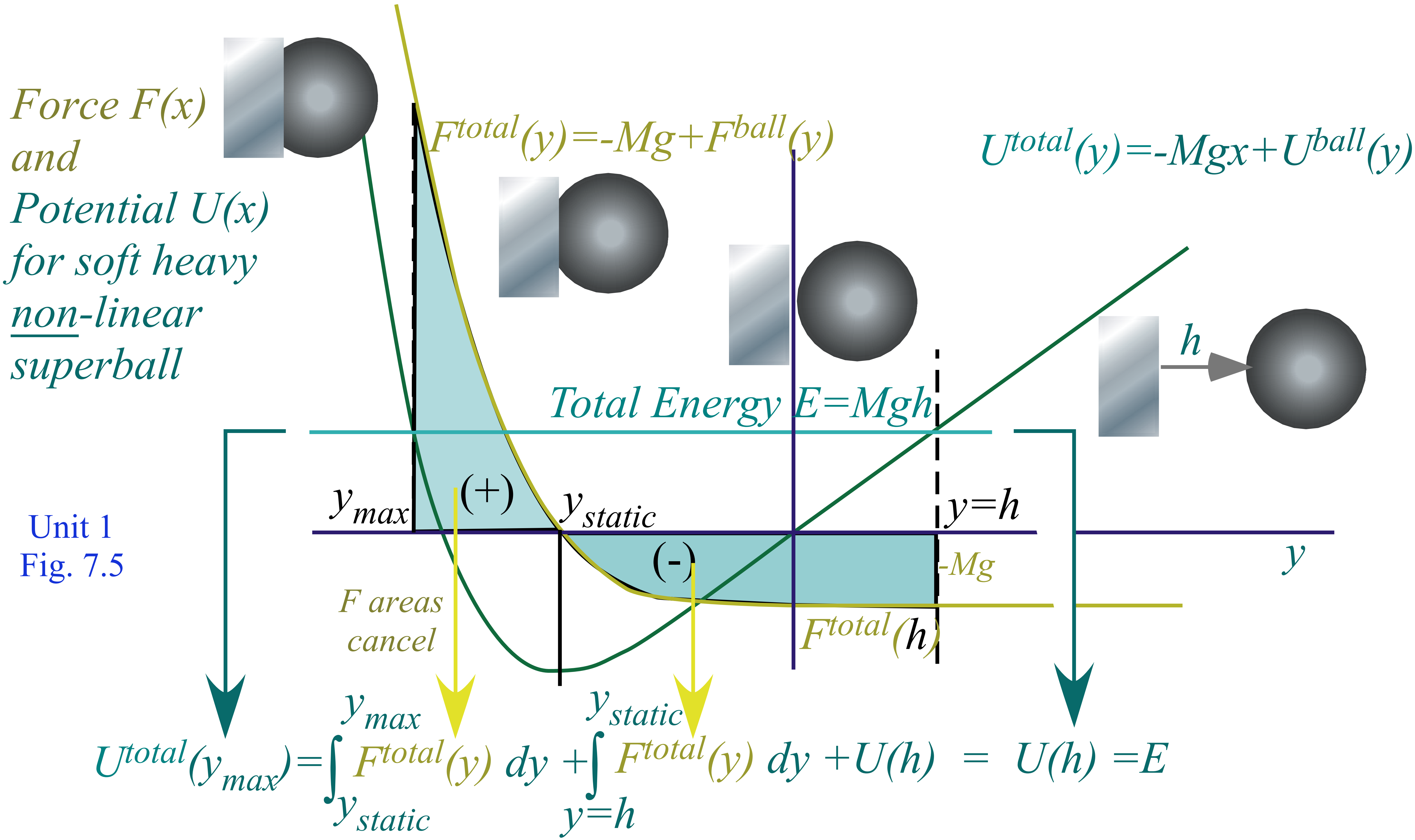
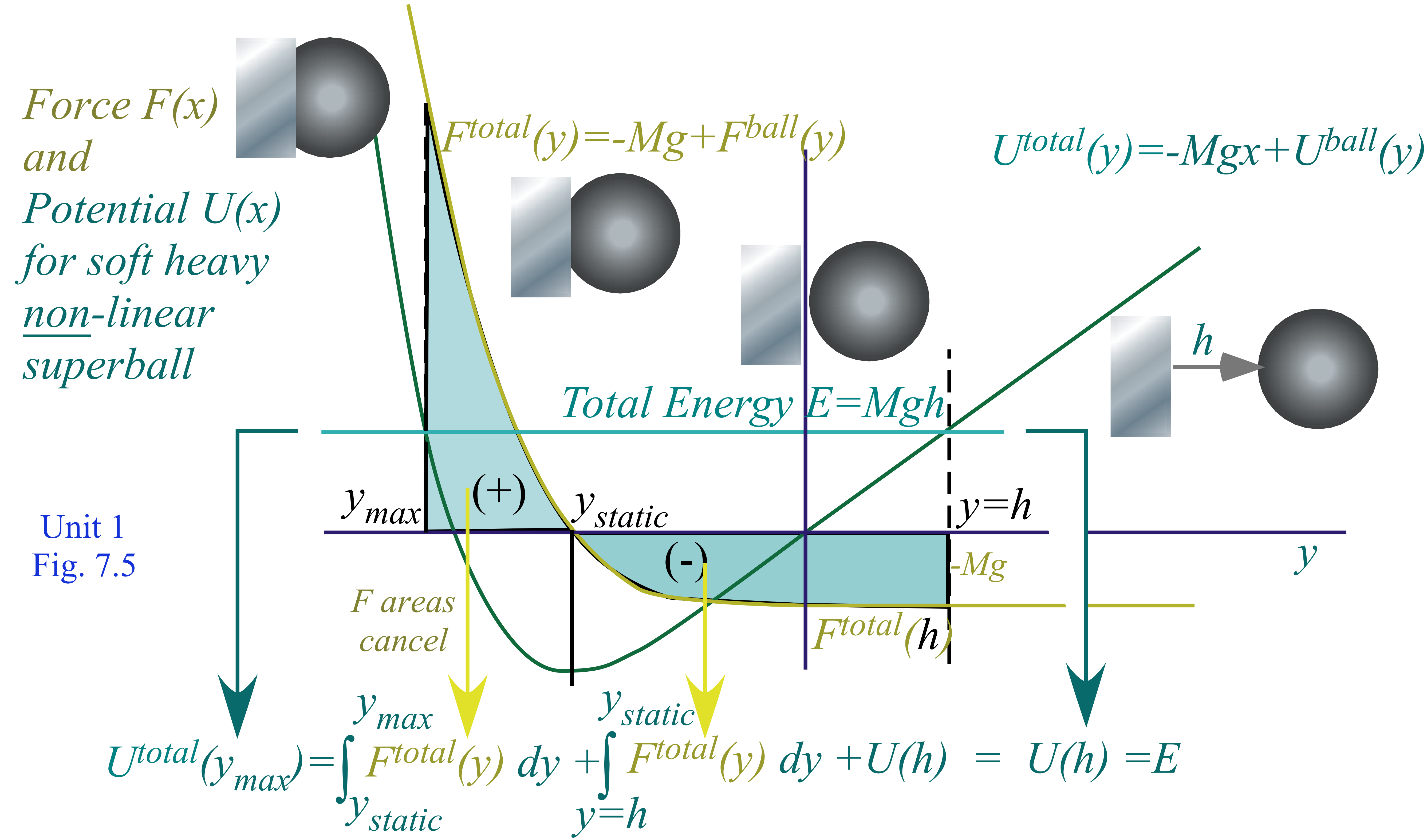


Force F(x) and Potential U(x) for soft heavy non-linear superball
$F^{total}(y)$=-Mg+$F^{ball}(y)$
$U^{total}(y)$=-Mgx+$U^{ball}(y)$
h
Total Energy E=Mgh
y_{max}
(+)
y_{static}
y=h
(-)
-Mg
y
F areas cancel
$F^{total}(h)$
Unit 1
Fig. 7.5
$U^{total}(y_{max})=\int_{y_{static}}^{y_{max}} F^{total}(y)\,dy + \int_{y=h}^{y_{static}} F^{total}(y)\,dy + U(h) = U(h) = E$
$F(x) = -\frac{dU(x)}{dx}$



$$Work = W = \int F(x)\,dx = Energy\ acquired = Area\ of\ F(x) = -U(x)$$

$$F(x) = -\frac{dU(x)}{dx}$$



$$Work = W = \int F(x)\,dx = Energy\ acquired = Area\ of\ F(x) = -U(x) \qquad F(x) = -\frac{dU(x)}{dx}$$

$$Impulse = P = \int F(t)\,dt = Momentum\ acquired = Area\ of\ F(t) = P(t) \qquad F(t) = \frac{dP(t)}{dt}$$

Potential energy dynamics of Superballs and related things

Thales geometry and "Sagittal approximation" to force law

Geometry and dynamics of single ball bounce

General Non-linear force (like superball-floor or ball-bearing-anvil)

Constant force F=-k (linear potential V=kx)

Some physics of dare-devil-diving 80 ft. into kidee pool *(Calculus)*

Linear force F=-kx (quadratic potential $V=\frac{1}{2}kx^2$ (like balloon))

Geometry and potential dynamics of 2-ball bounce

A parable of RumpCo. vs CrapCorp. (introducing 3-mass potential-driven dynamics)

A story of USC pre-meds visiting Whammo Manufacturing Co.

Geometry and dynamics of n-ball bounces

Analogy with shockwave and acoustical horn amplifier

Advantages of a geometric m_1, m_2, m_3,… series

A story of Stirling Colgate (Palmolive) and core-collapse supernovae

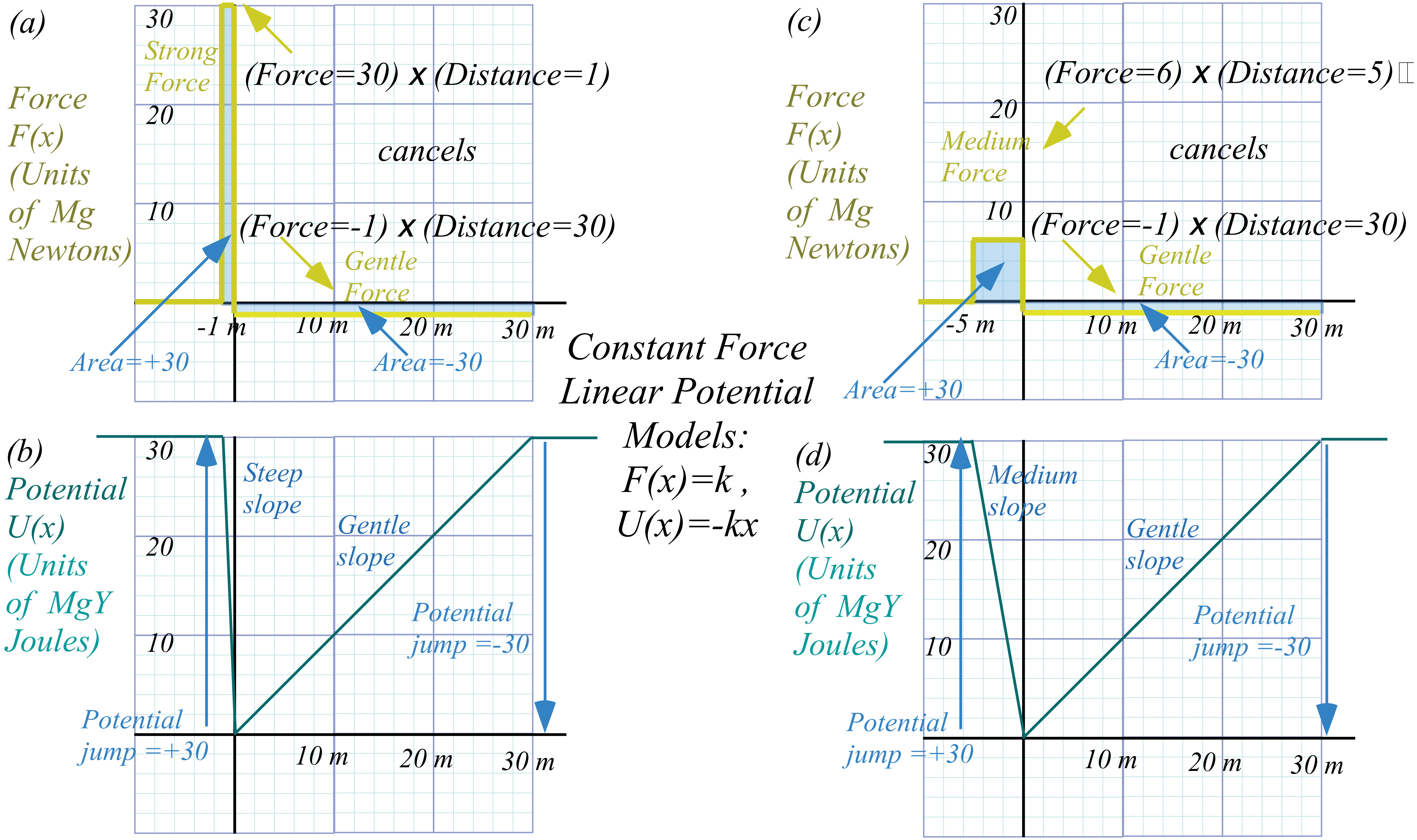
Many-body 1D collisions

Elastic examples: Western buckboard

Bouncing columns and Newton's cradle

Inelastic examples: "Zig-zag geometry" of freeway crashes

*Super-elastic examples: This really **is** "Rocket-Science"*



Unit 1
Fig. 7.3

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Potential energy dynamics of Superballs and related things

Thales geometry and "Sagittal approximation" to force law

→ *Geometry and dynamics of single ball bounce*

General Non-linear force (like superball-floor or ball-bearing-anvil)

Constant force F=-k (linear potential V=kx)

Some physics of dare-devil-diving 80 ft. into kidee pool

→ *Linear force F=-kx (quadratic potential $V={}^1/_2kx^2$ (like balloon))* *(Simulations)*

Geometry and potential dynamics of 2-ball bounce

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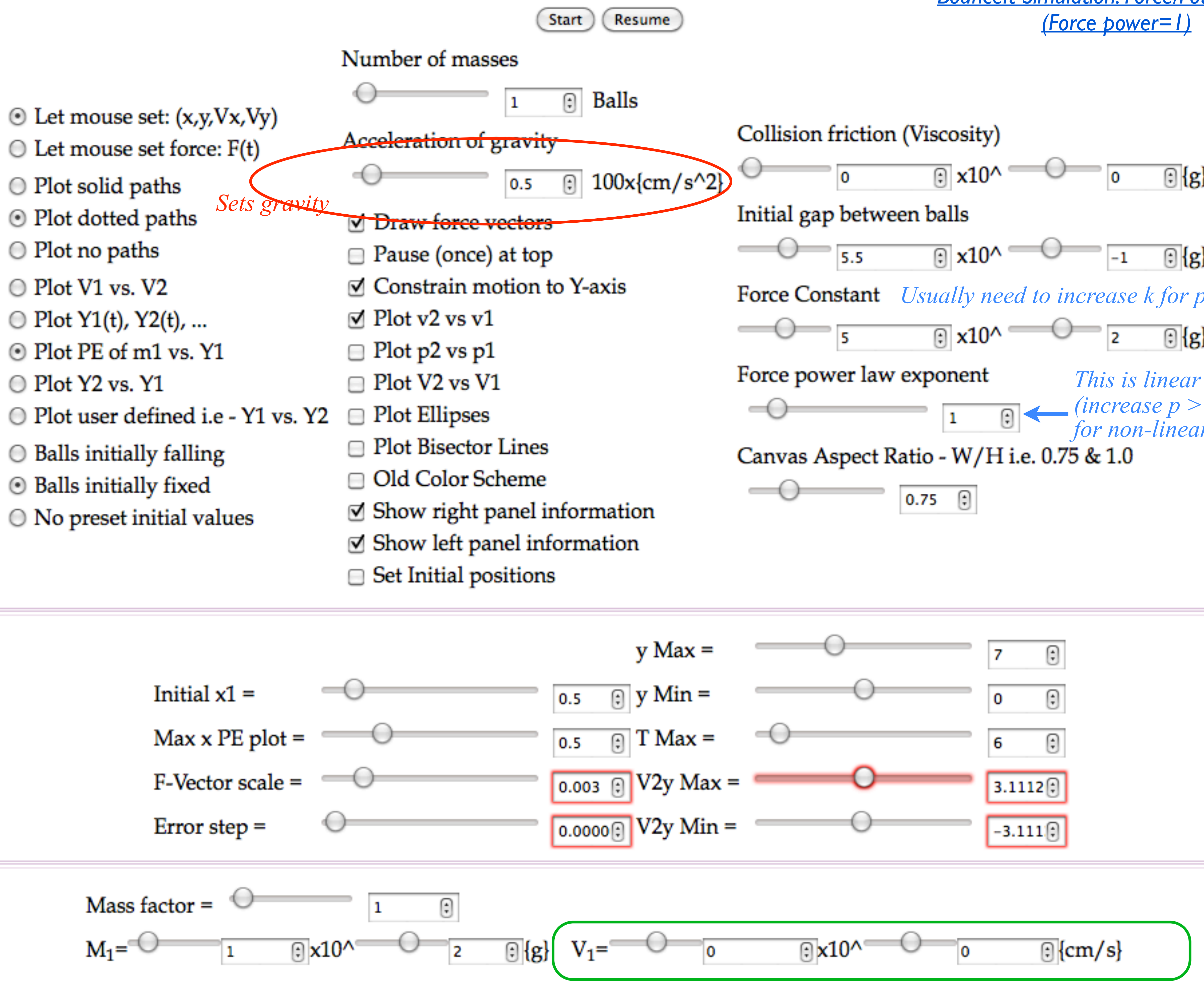
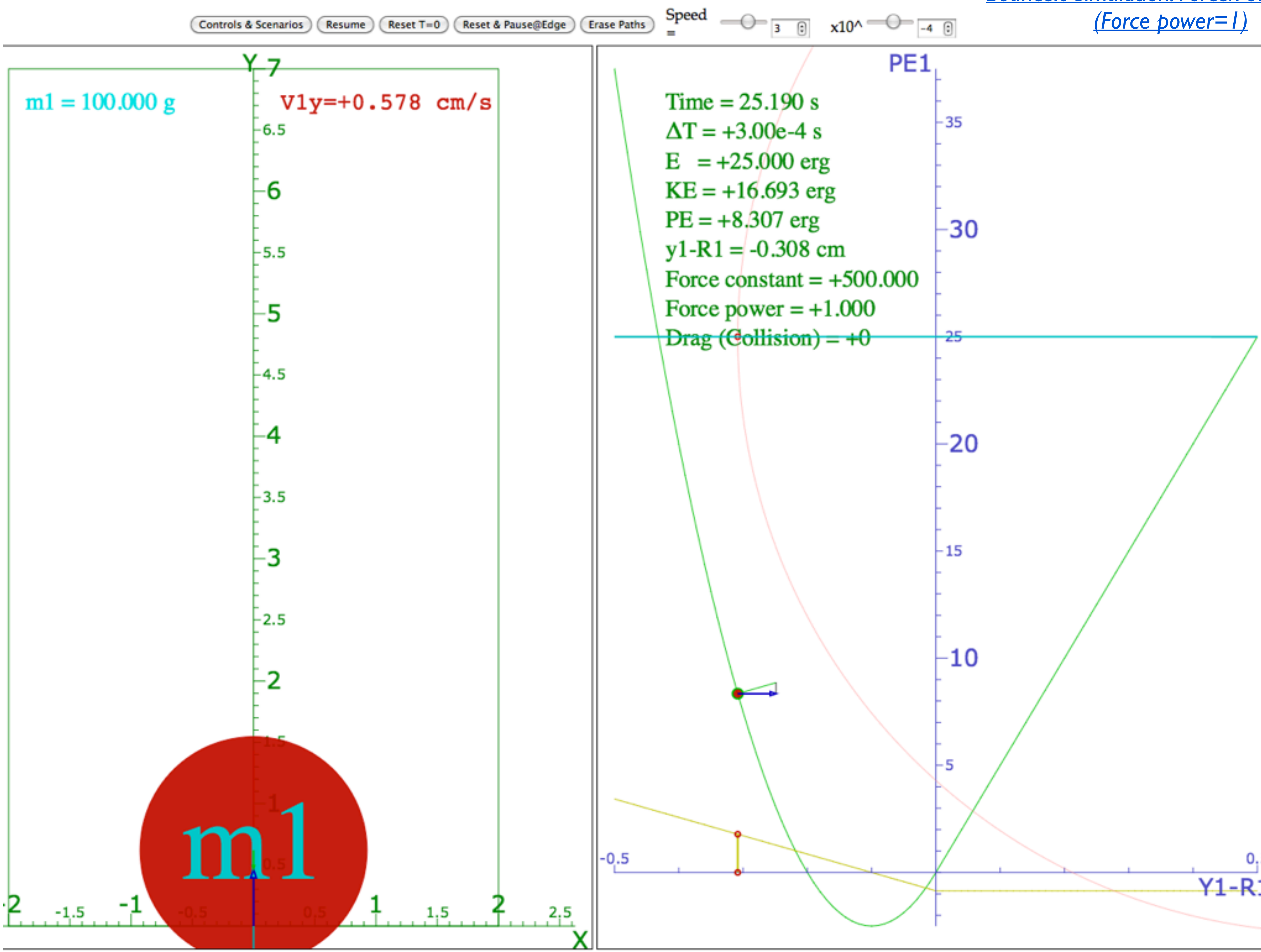
*Super-elastic examples: This really **is** "Rocket-Science"*

BounceIt Simulation: Force/Potential Plot (Force power=1)



[BounceIt Simulation: Force/Potential Plot (Force power=1)]



Potential energy dynamics of Superballs and related things

Thales geometry and "Sagittal approximation" to force law

Geometry and dynamics of single ball bounce

General Non-linear force (like superball-floor or ball-bearing-anvil)

Constant force F=-k (linear potential V=kx)

Some physics of dare-devil-diving 80 ft. into kidee pool

→ *Linear force F=-kx (quadratic potential $V=\frac{1}{2}kx^2$ (like balloon))* *(Calculations)*

Geometry and potential dynamics of 2-ball bounce

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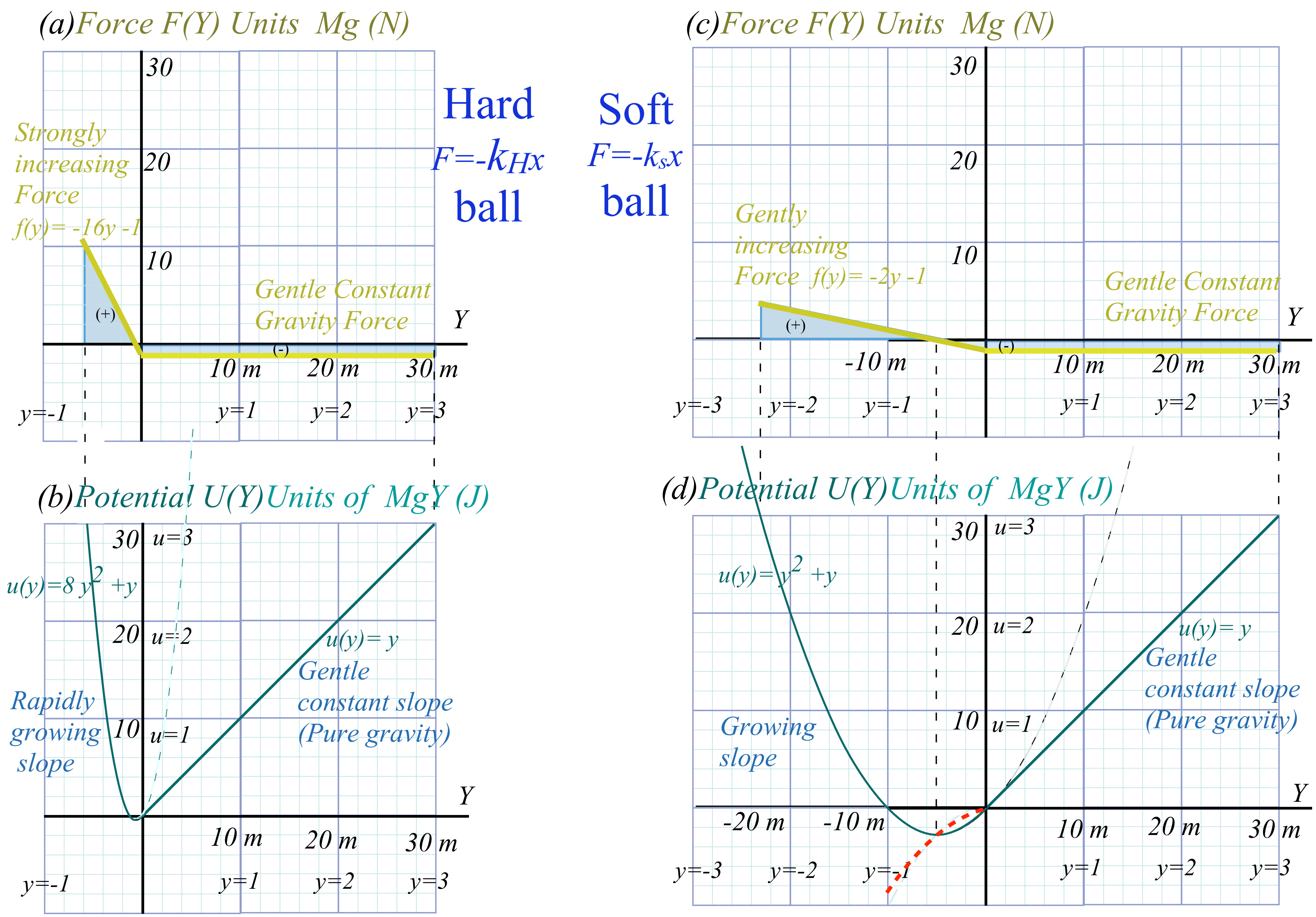
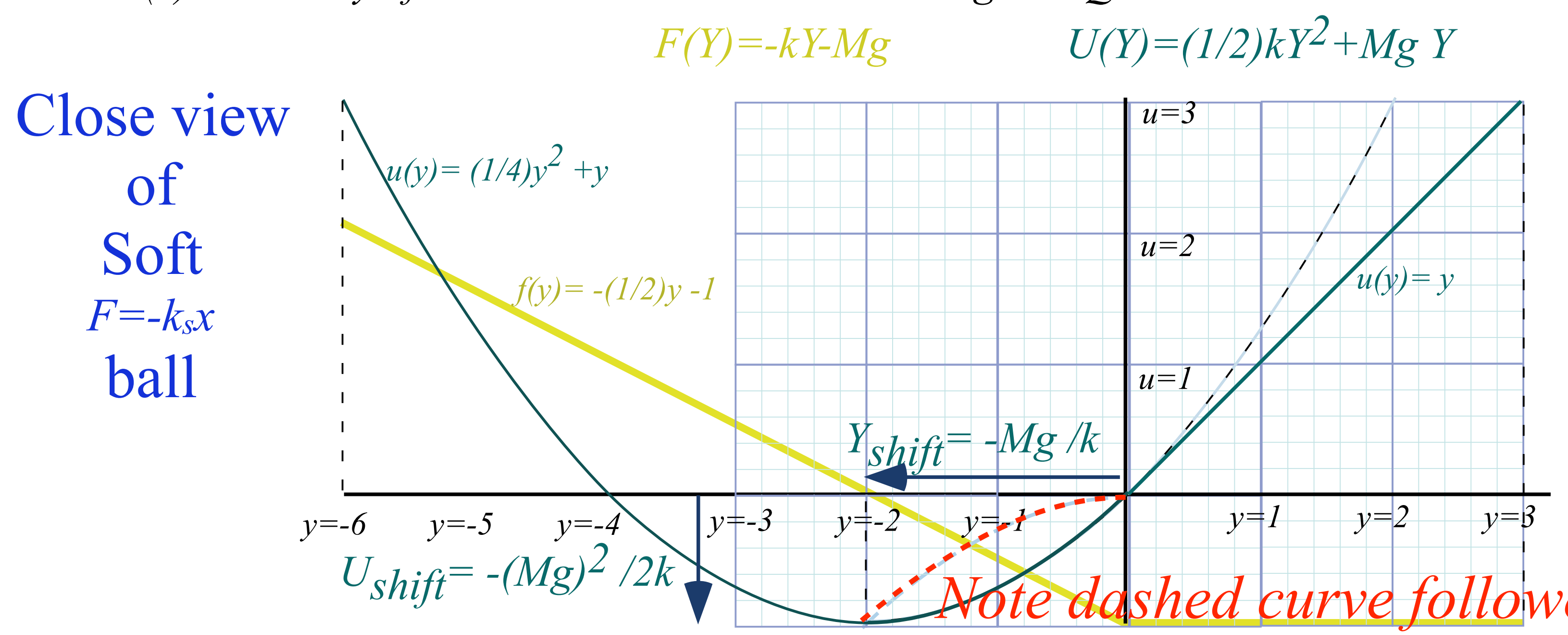


Unit 1
Fig. 7.4

(e) Geometry of Linear Force with Constant Mg and Quadratic Potential



Note dashed curve followed by PE minimum. Parabola? What?

$$F^{Total} = F^{grav} + F^{target} = \begin{cases} -Mg & (y \geq 0) \\ -Mg - ky & (y < 0) \end{cases}$$

$$U^{Total} = U^{grav} + U^{target} = \begin{cases} Mg\,y & (y \geq 0) \\ Mg\,y + \frac{1}{2}ky^2 & (y < 0) \end{cases}$$

Potential energy dynamics of Superballs and related things

Thales geometry and "Sagittal approximation" to force law

Geometry and dynamics of single ball bounce

General Non-linear force (like superball-floor or ball-bearing-anvil)

Constant force F=-k (linear potential V=kx)

Some physics of dare-devil-diving 80 ft. into kidee pool

Linear force F=-kx (quadratic potential $V=\frac{1}{2}kx^2$ (like balloon))

(Reviewing calculations and noticing "gap" effect)

Geometry and potential dynamics of 2-ball bounce

A parable of RumpCo. vs CrapCorp. (introducing 3-mass potential-driven dynamics)

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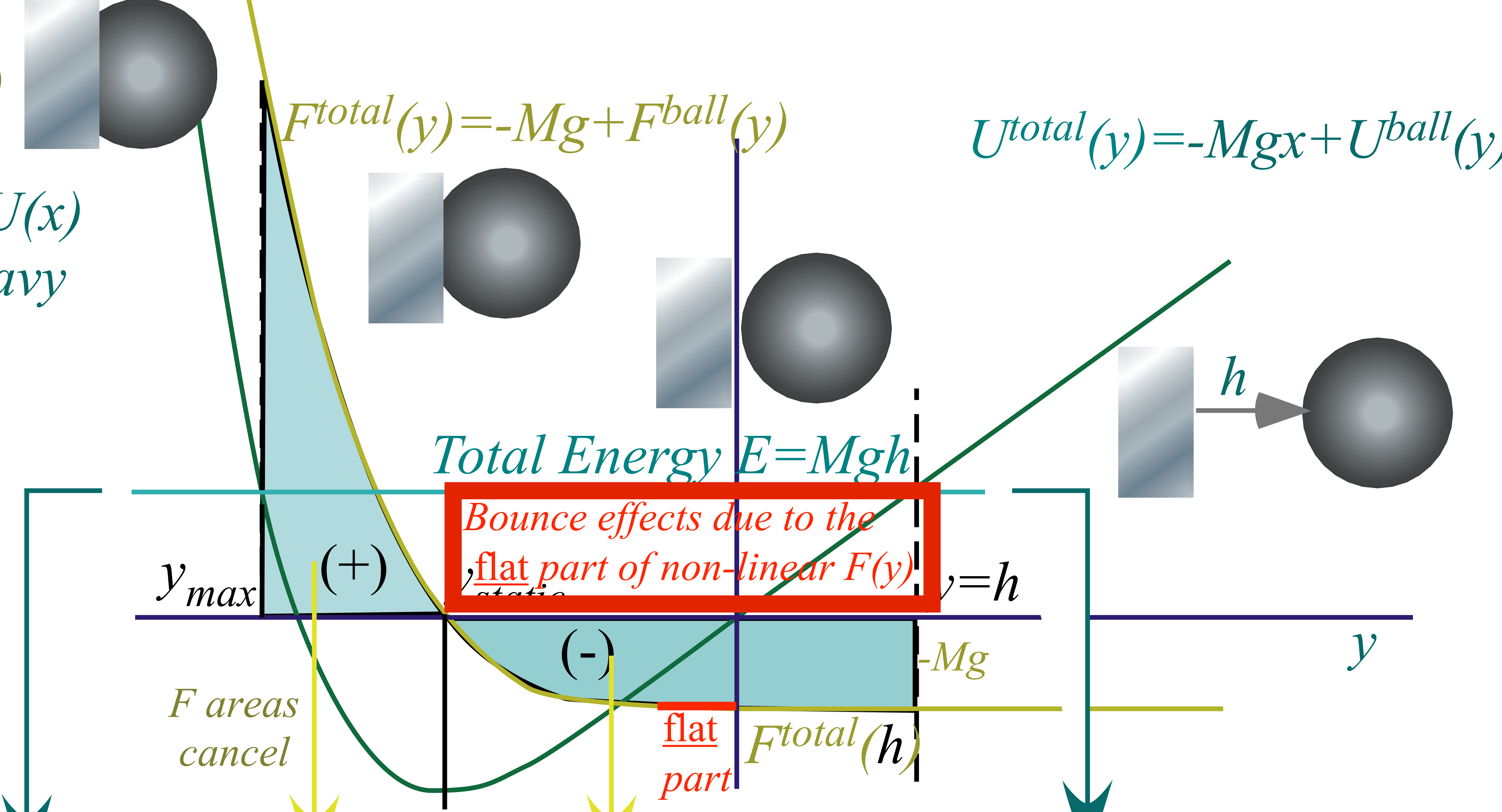
Inelastic examples: "Zig-zag geometry" of freeway crashes

*Super-elastic examples: This really **is** "Rocket-Science"*

Force F(x) and Potential U(x) for soft heavy non-linear superball



Unit 1
Fig. 7.5

$$U^{total}(y_{max})=\int_{y_{static}}^{y_{max}} F^{total}(y)\,dy+\int_{y=h}^{y_{static}} F^{total}(y)\,dy+U(h) \;=\; U(h)=E$$

$$Work = W = \int F(x)\,dx = Energy\ acquired = Area\ of\ F(x) = -U(x) \qquad F(x) = -\frac{dU(x)}{dx}$$

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Geometry and dynamics of single ball bounce

General Non-linear force (like superball-floor or ball-bearing-anvil)

Constant force F=-k (linear potential V=kx) *(Simulations)*

Some physics of dare-devil-diving 80 ft. into kidee pool

Linear force F=-kx (quadratic potential $V=\frac{1}{2}kx^2$ (like balloon))

Geometry and potential dynamics of 2-ball bounce

A parable of RumpCo. vs CrapCorp. (introducing 3-mass potential-driven dynamics)

Parable allegory for Los Alamos

Cheap&practical "seat-of-the pants" approach

Parable allegory for Livermore

Fancy&overpriced "political" approach

Advantages of a geometric m_1, m_2, m_3,... series

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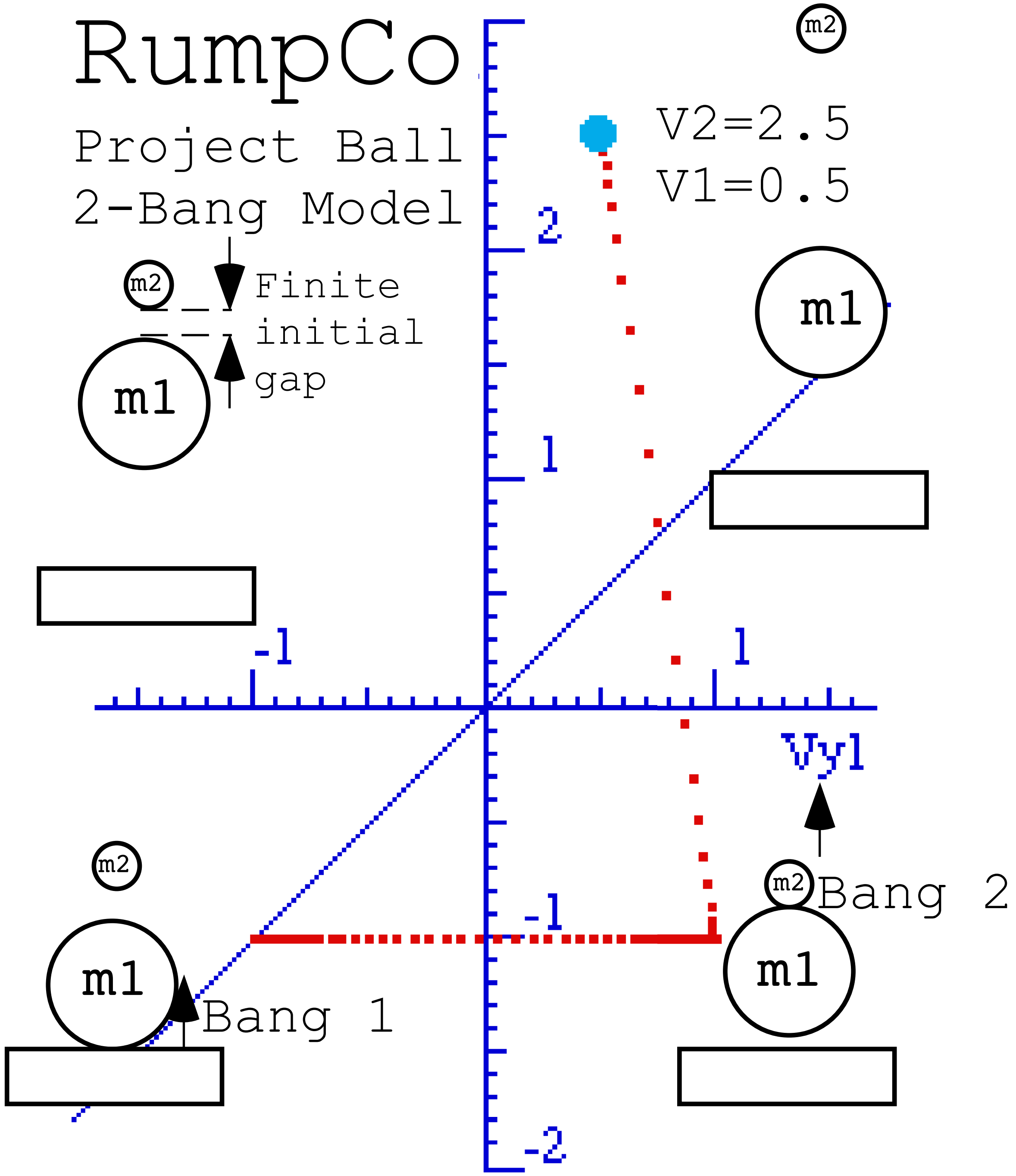


Velocity amplification
or "throw" factor =2.5

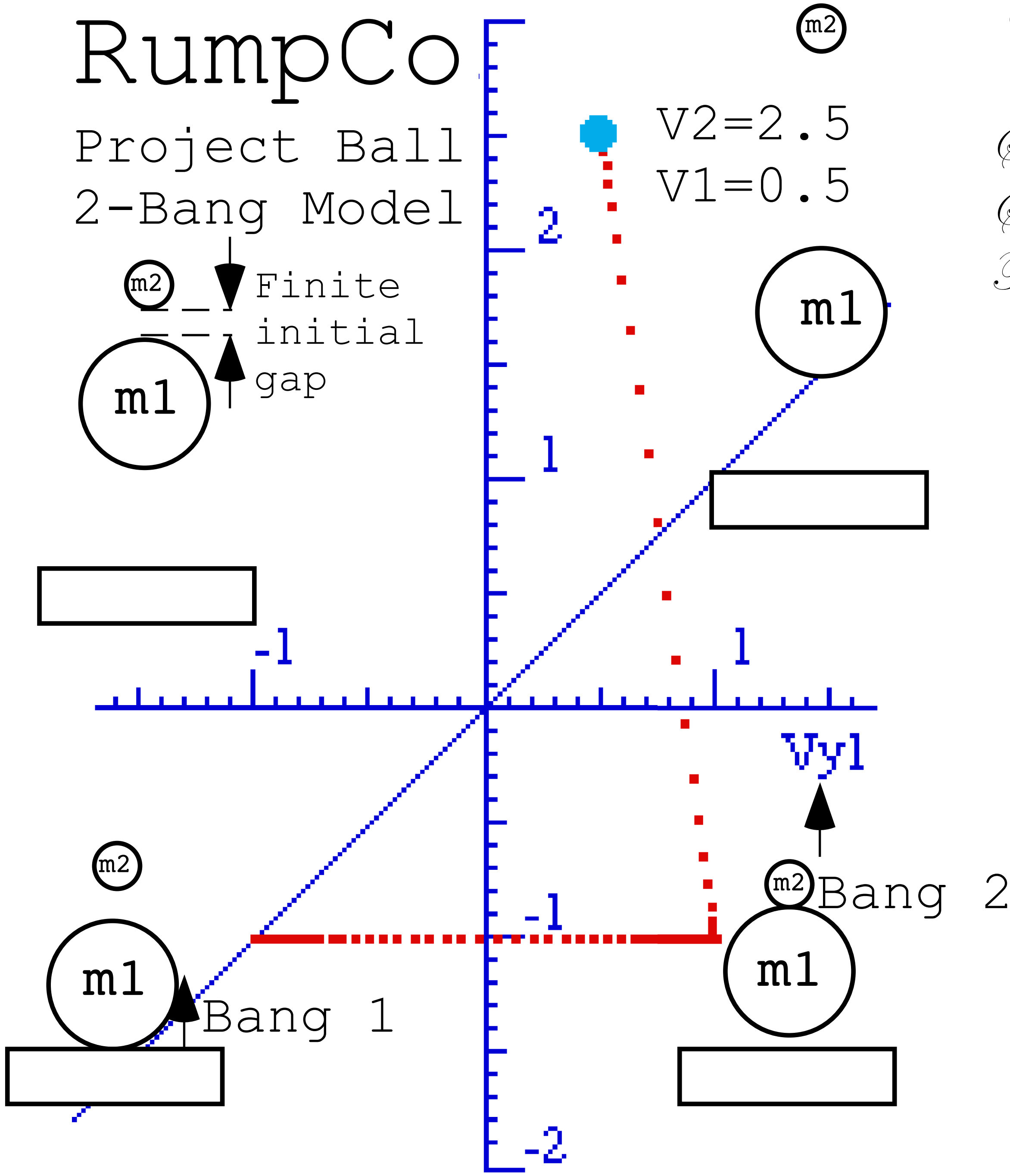
Unit 1
Fig. 7.6

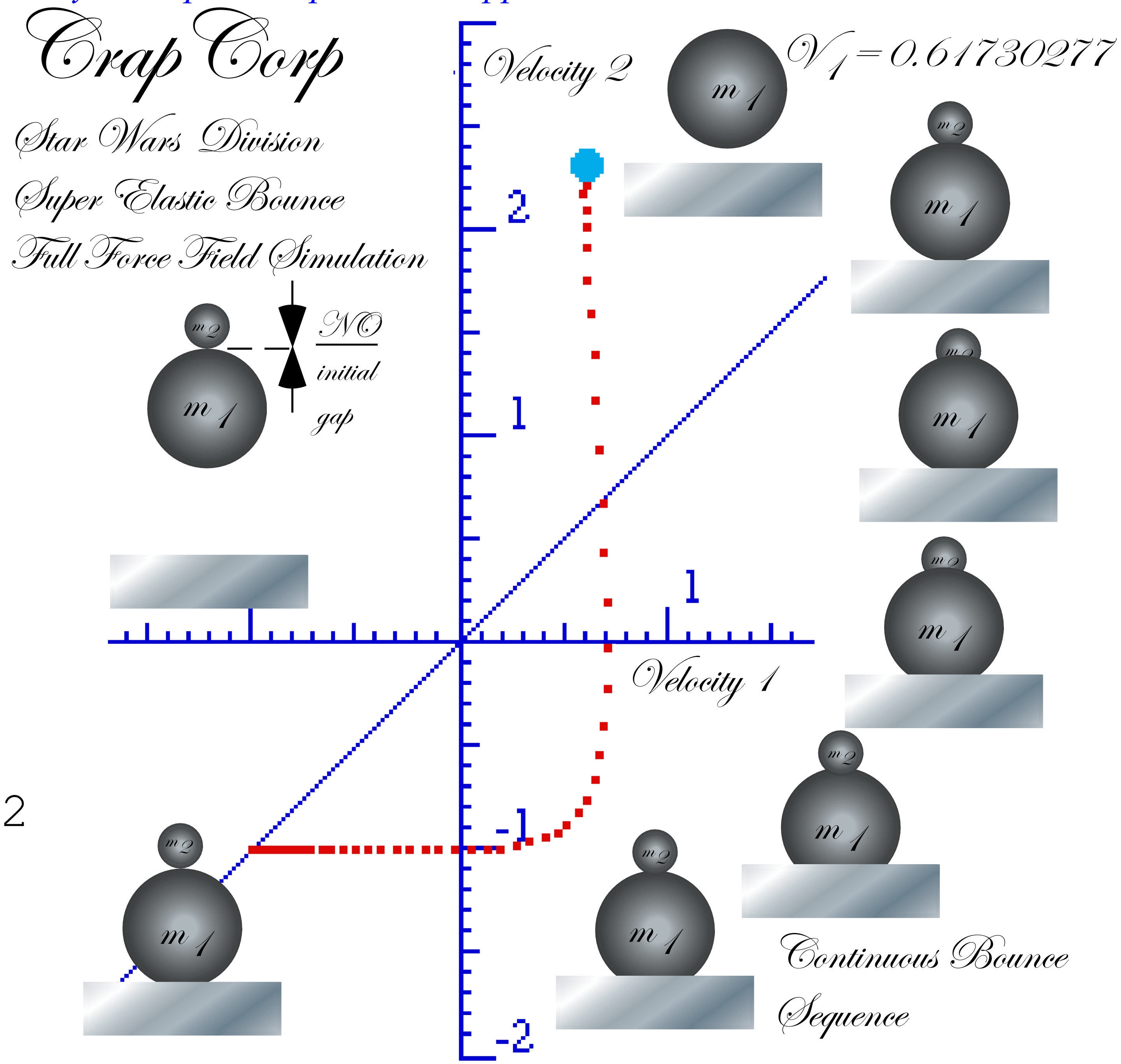
Parable allegory for Los Alamos

Cheap&practical "seat-of-the pants" approach

Parable allegory for Livermore

Fancy&overpriced "political" approach

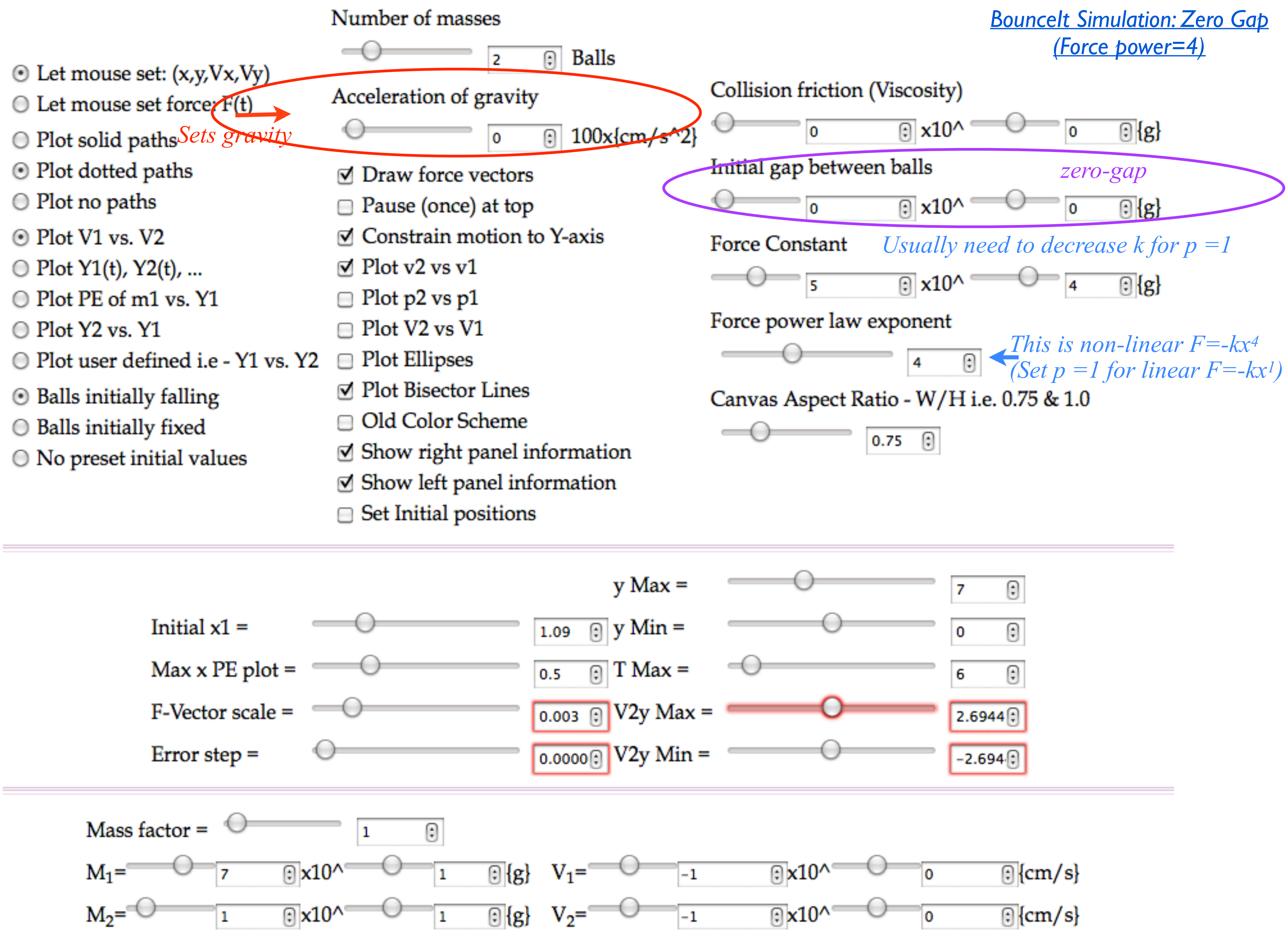


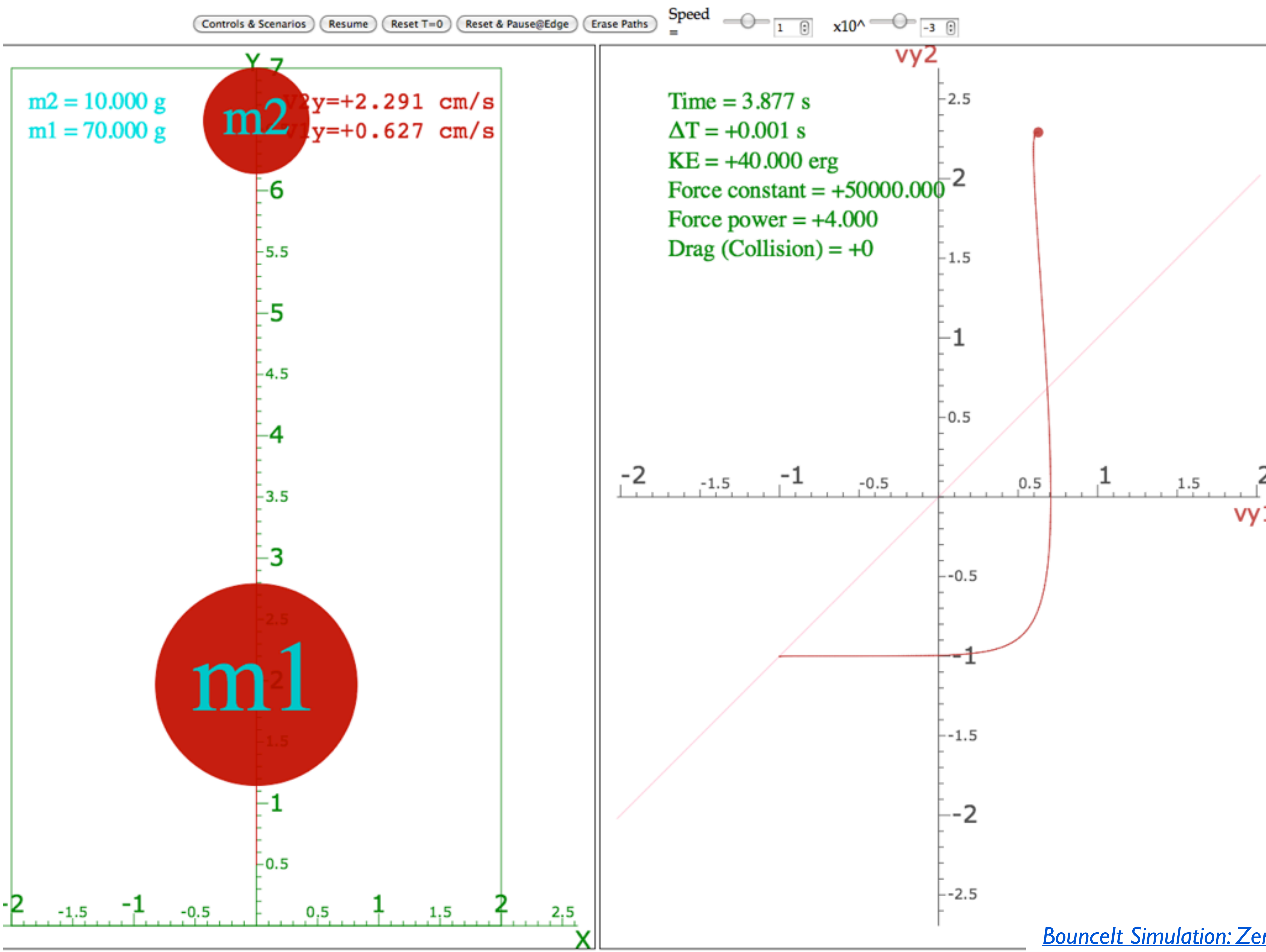


Velocity amplification
or "throw" factor =2.5

Velocity amplification
or "throw" factor =2.3
(about equal to RumpCo
finite gap experiment)

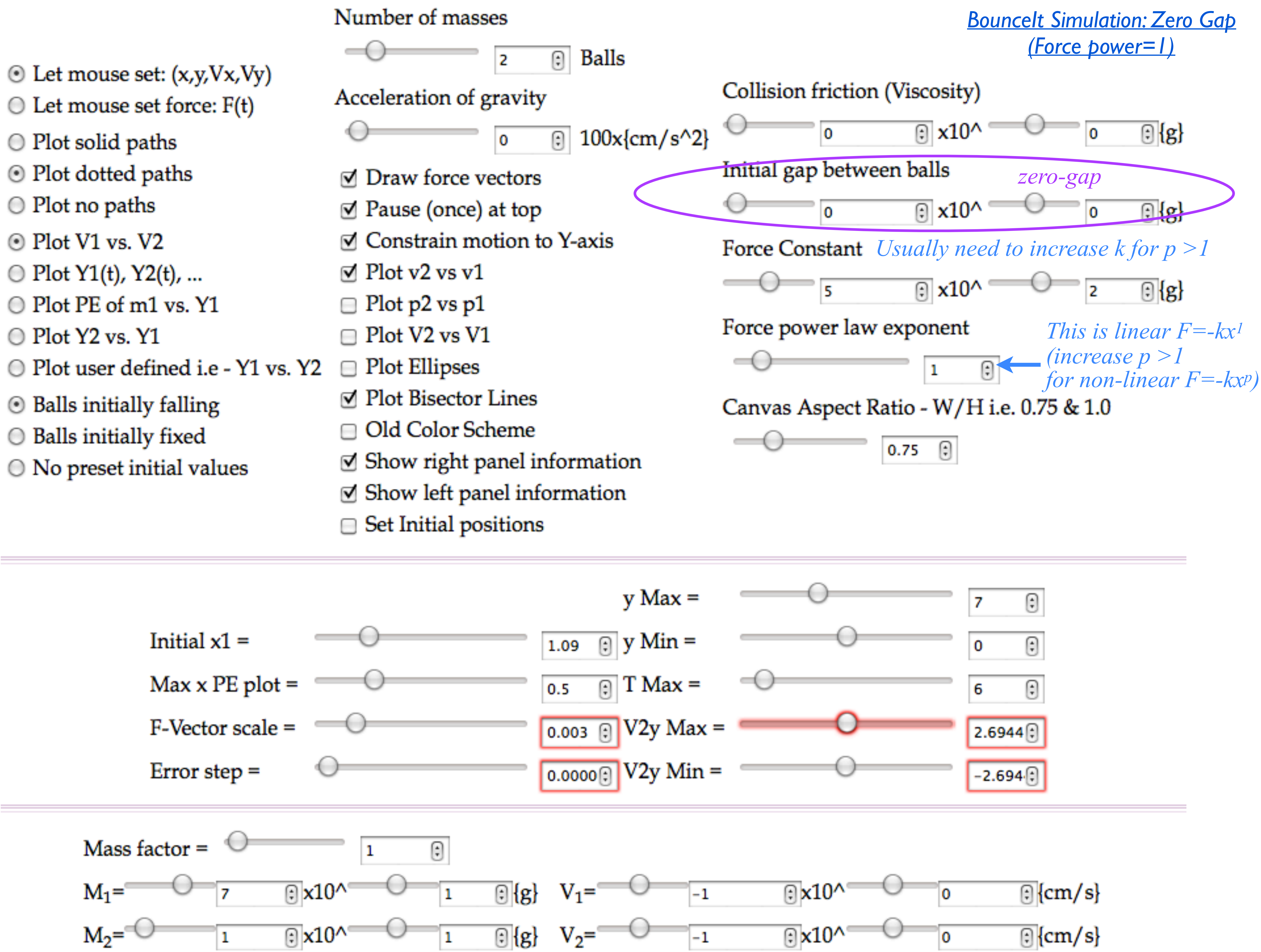
Unit 1
Fig. 7.6


BounceIt Simulation: Zero Gap
(Force power=4)
Number of masses
2
Balls
Let mouse set: (x,y,Vx,Vy)
Let mouse set force: F(t)
Plot solid paths
Sets gravity
Plot dotted paths
Plot no paths
Plot V1 vs. V2
Plot Y1(t), Y2(t), ...
Plot PE of m1 vs. Y1
Plot Y2 vs. Y1
Plot user defined i.e - Y1 vs. Y2
Balls initially falling
Balls initially fixed
No preset initial values
Acceleration of gravity
0
100x{cm/s^2}
Draw force vectors
Pause (once) at top
Constrain motion to Y-axis
Plot v2 vs v1
Plot p2 vs p1
Plot V2 vs V1
Plot Ellipses
Plot Bisector Lines
Old Color Scheme
Show right panel information
Show left panel information
Set Initial positions
Collision friction (Viscosity)
0
x10^
0
{g}
Initial gap between balls
zero-gap
0
x10^
0
{g}
Force Constant
Usually need to decrease k for p =1
5
x10^
4
{g}
Force power law exponent
4
This is non-linear F=-kx⁴
(Set p =1 for linear F=-kx¹)
Canvas Aspect Ratio - W/H i.e. 0.75 & 1.0
0.75
y Max =
7
Initial x1 =
1.09
y Min =
0
Max x PE plot =
0.5
T Max =
6
F-Vector scale =
0.003
V2y Max =
2.6944
Error step =
0.0000
V2y Min =
-2.694
Mass factor =
1
M₁=
7
x10^
1
{g}
V₁=
-1
x10^
0
{cm/s}
M₂=
1
x10^
1
{g}
V₂=
-1
x10^
0
{cm/s}

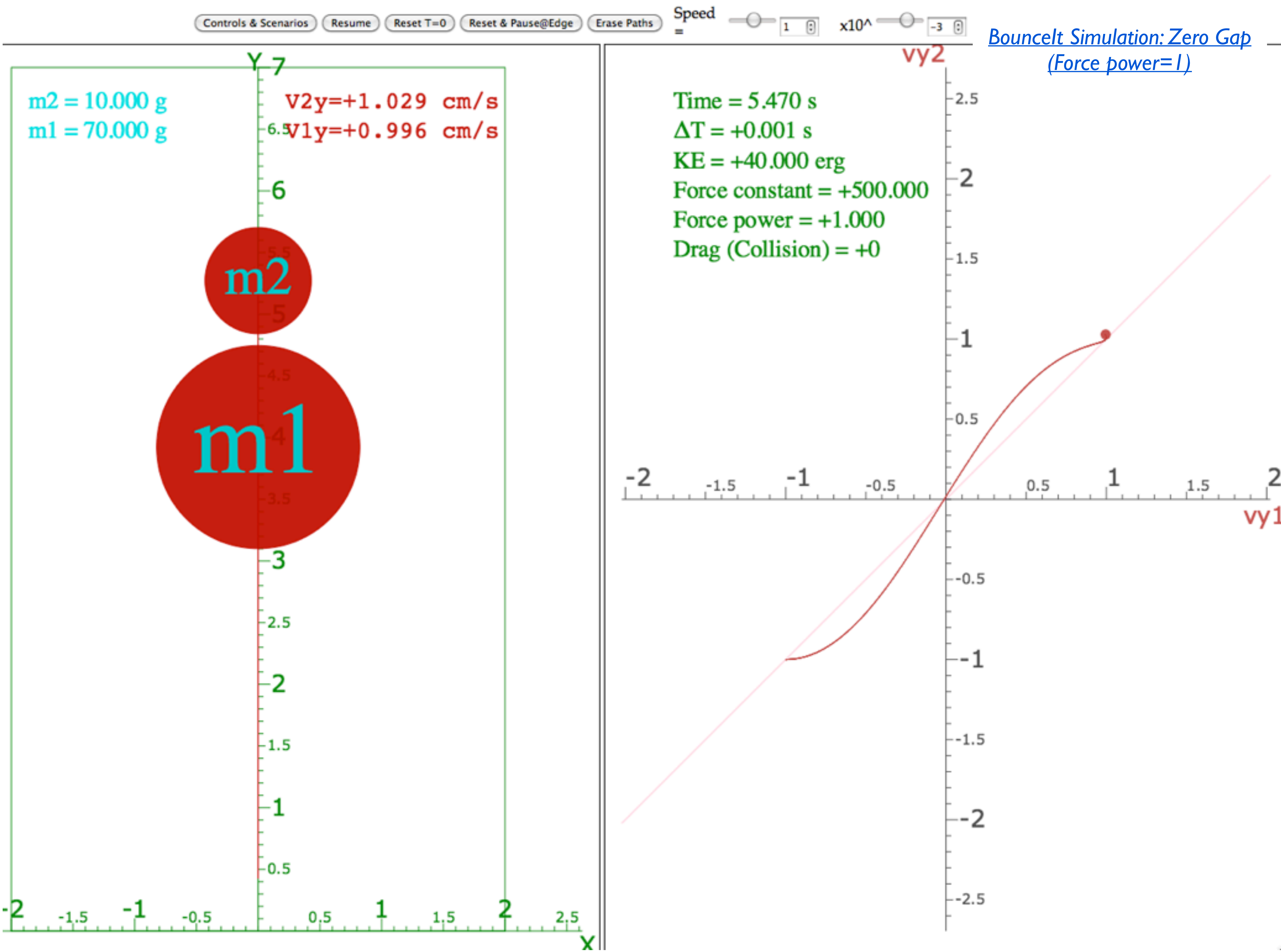


[BounceIt Simulation: Zero Gap (Force power=4)]

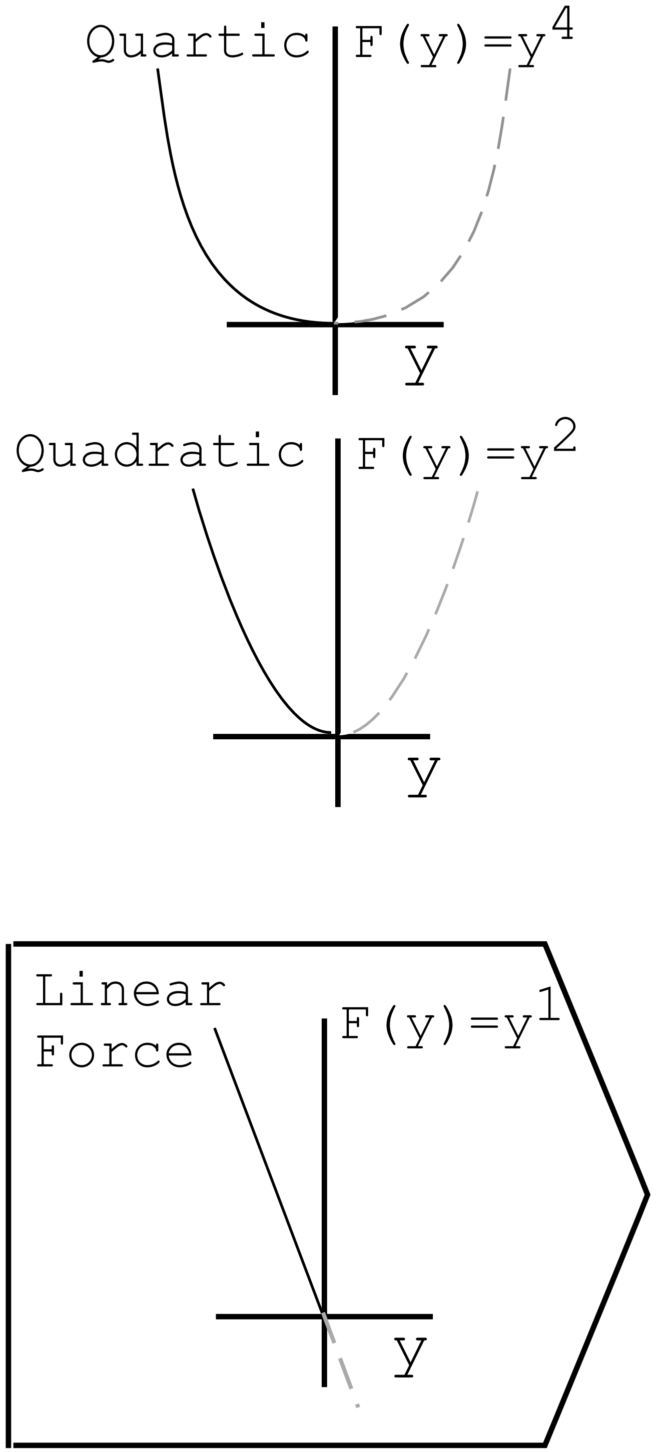
[BounceIt Simulation: Zero Gap (Force power=1)]

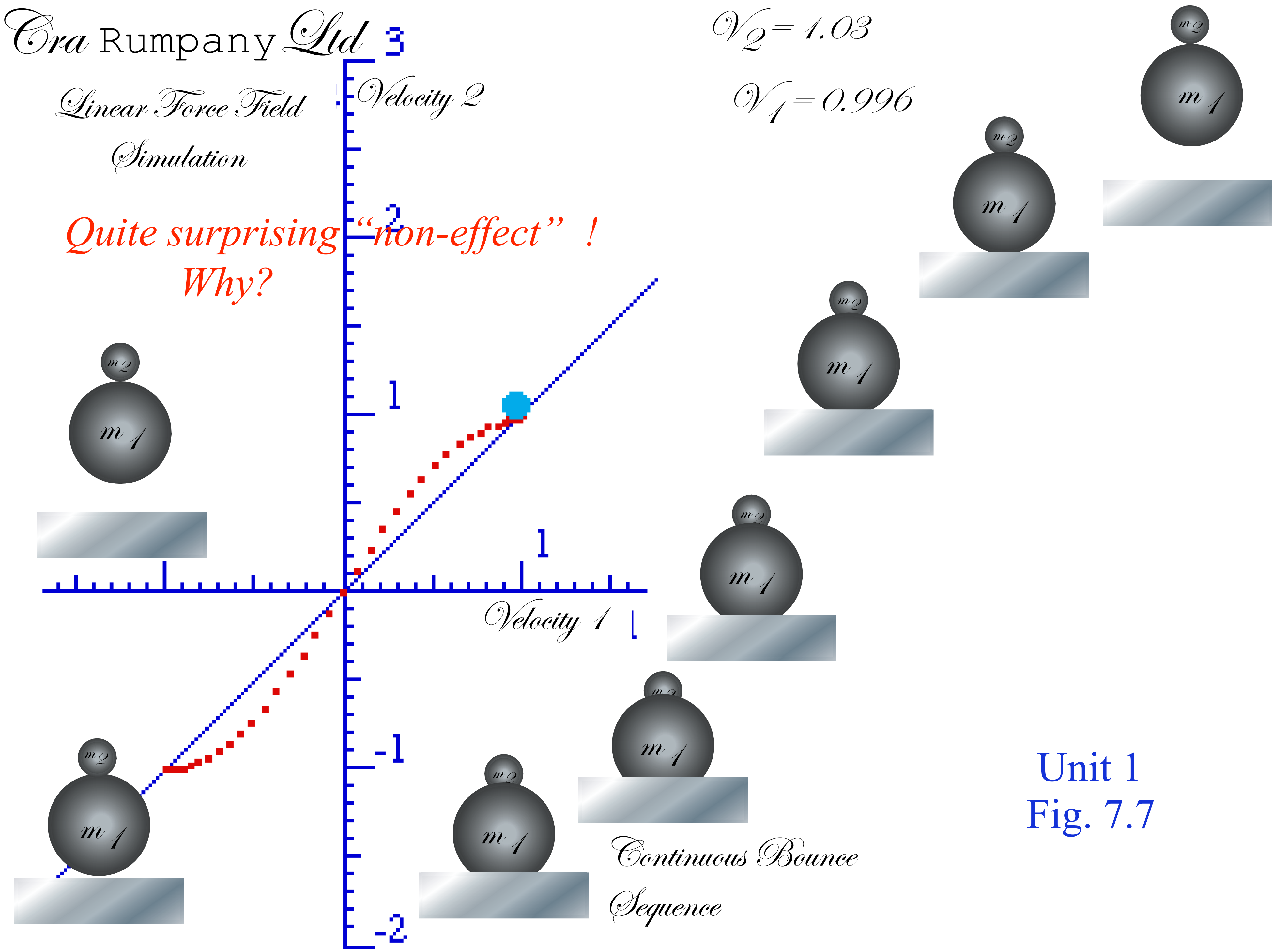


[BounceIt Simulation: Zero Gap (Force power=1)]



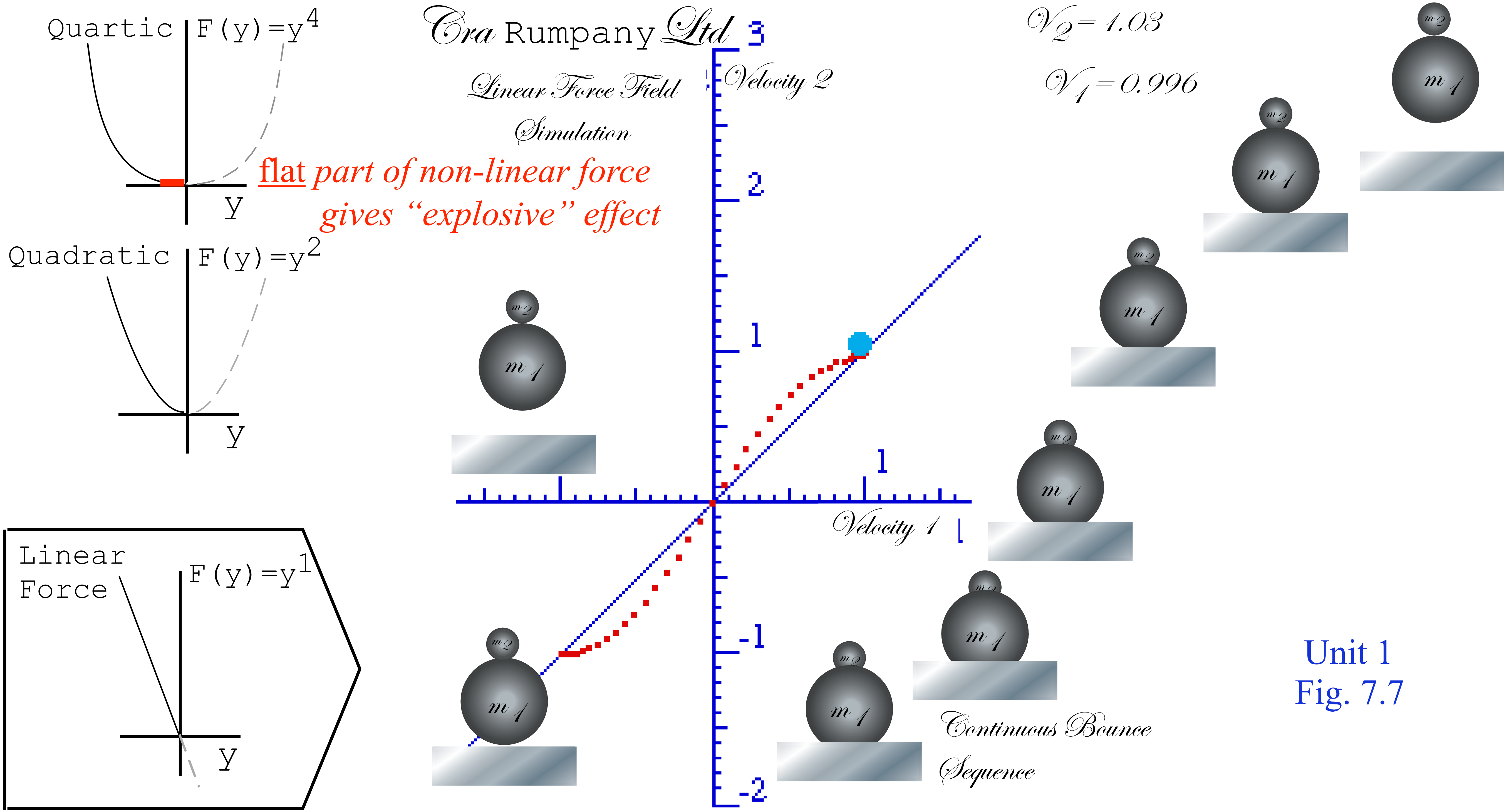
Cooperation between Los Alamos and Livermore yields insight to answer "What's going on?"





Unit 1
Fig. 7.7

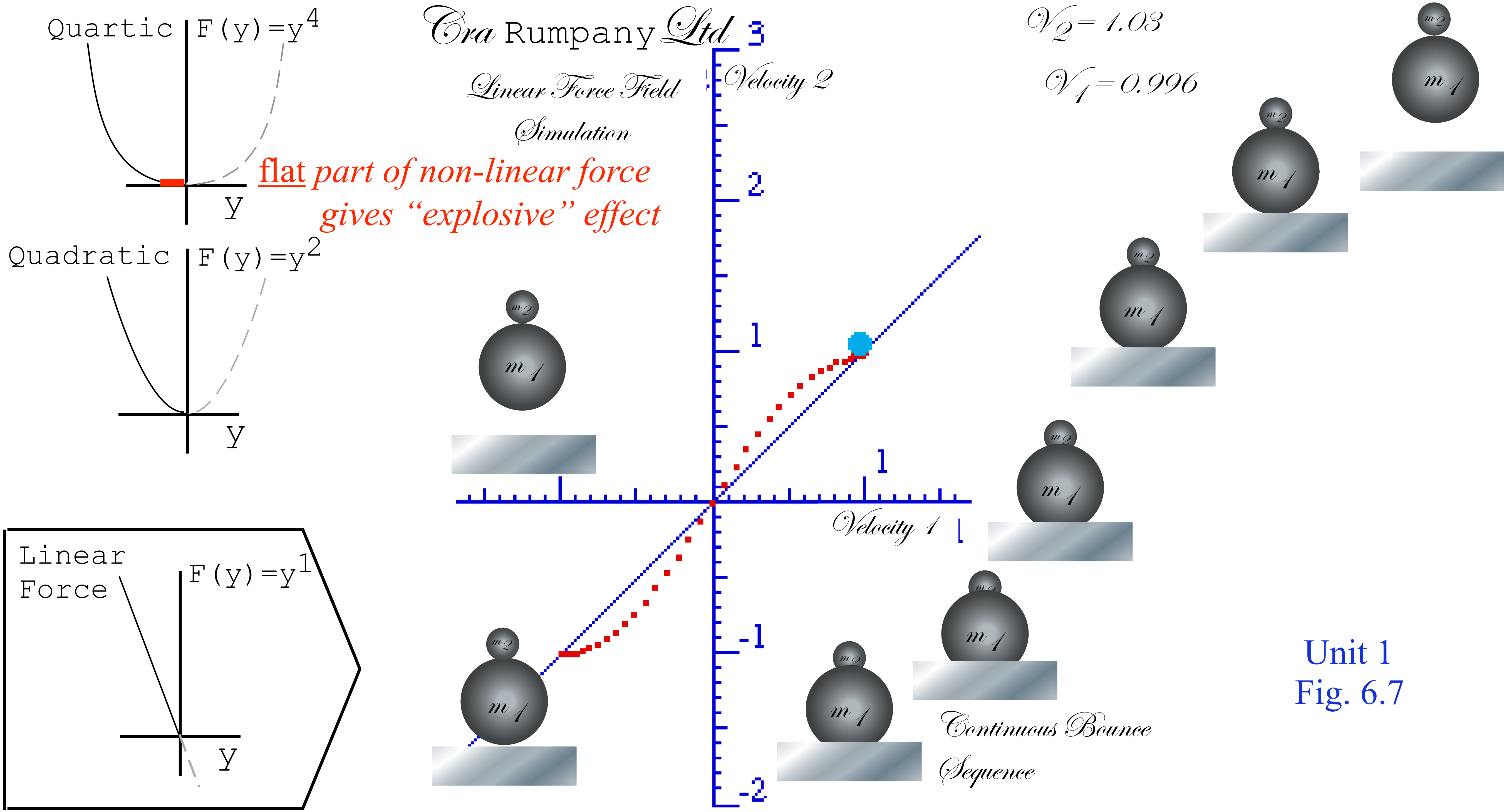
Cooperation between Los Alamos and Livermore yields insight to answer "What's going on?"



Unit 1
Fig. 7.7

Velocity amplification
or "throw" factor =1.03
(practically "no-throw")
for linear force F(y)= ky

Cooperation between Los Alamos and Livermore yields insight to answer "What's going on?"

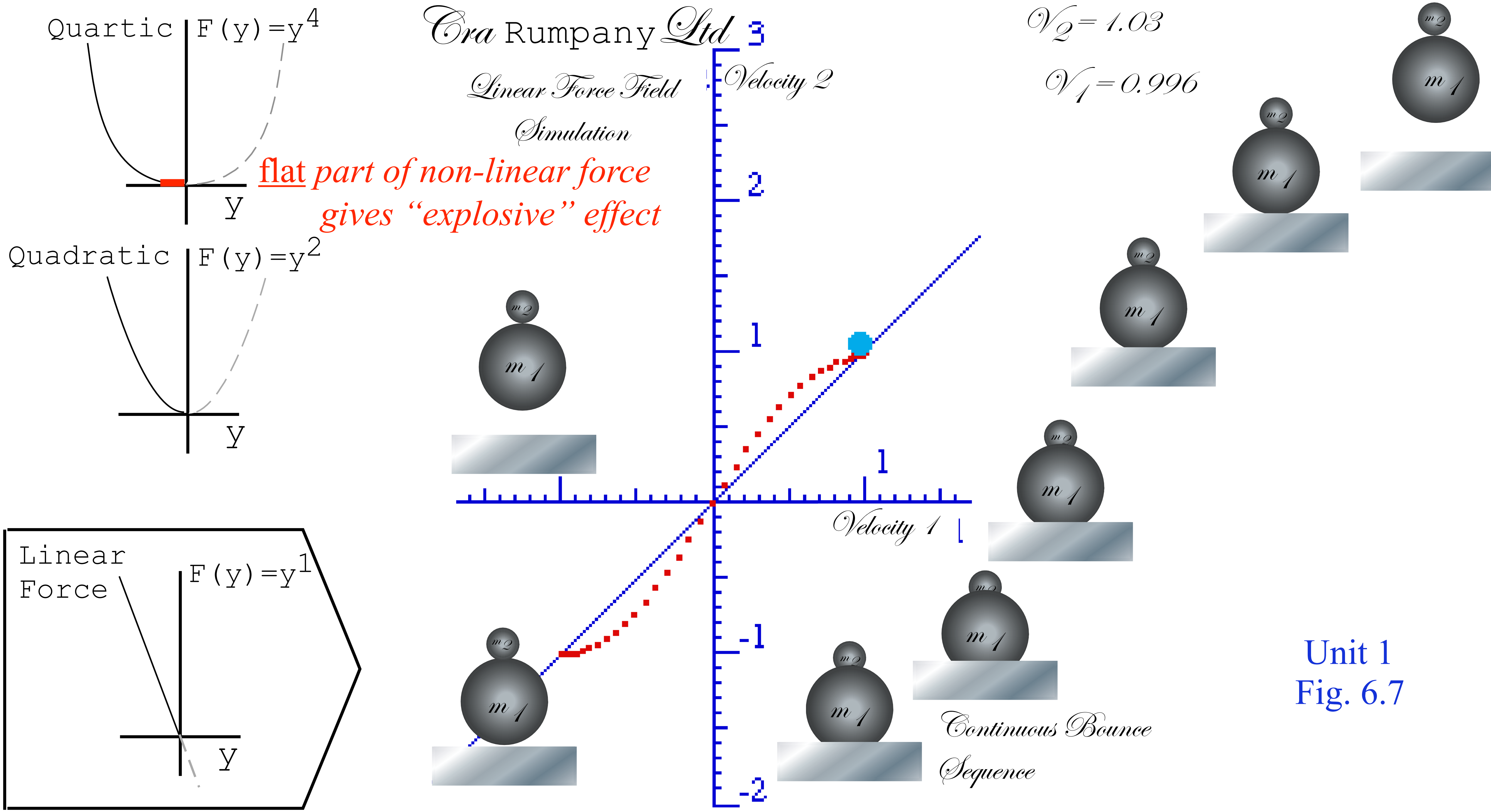


Unit 1
Fig. 6.7

Velocity amplification
or "throw" factor =1.03
(practically "no-throw")
for linear force F(y)= ky

Lesson: Fasten your seatbelt

Cooperation between Los Alamos and Livermore yields insight to answer "What's going on?"



Unit 1
Fig. 6.7

Velocity amplification
or "throw" factor =1.03
(practically "no-throw")
for linear force F(y)= ky

Lesson: Fasten your seatbelt
TIGHTLY!

Potential energy dynamics of Superballs and related things

Thales geometry and "Sagittal approximation" to force law

Geometry and dynamics of single ball bounce

(a) Constant force F=-k (linear potential V=kx)

Some physics of dare-devil-diving 80 ft. into kidee pool (Simulations)

(b) Linear force F=-kx (quadratic potential $V=\frac{1}{2}kx^2$ (like balloon))

(c) Non-linear force (like superball-floor or ball-bearing-anvil)

Geometry and potential dynamics of 2-ball bounce

A parable of RumpCo. vs CrapCorp. (introducing 3-mass potential-driven dynamics)

→ A story of USC pre-meds visiting Whammo Manufacturing Co. (Leads to Sagittal potential analysis of 2, 3, and 4 body towers)

Geometry and dynamics of n-ball bounces

Analogy with shockwave and acoustical horn amplifier

Advantages of a geometric $m_1, m_2, m_3, \ldots$ series

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Super-elastic examples: This really **is** "Rocket-Science"

A story of USC pre-meds visiting Whammo Manufacturing Co.

...and some results of "Project-Ball"

[Velocity Amplification in Collision Experiments Involving Superballs <Link>]

CLASS OF WILLIAM G. HARTER*
University of Southern California
Los Angeles, California 90007
(Received 25 September 1969; revised 25 September 1970)

If a pen is stuck in a hard rubber ball and dropped from a certain height, the pen may bounce to several times that height. The results of two such experiments, which can easily be duplicated in any undergraduate physics laboratory, are plotted for a range of mass ratios. A simple theoretical discussion which provides a qualitative understanding of the phenomenon is presented. A more complicated formulation which agrees very well with one of the experiments is also presented. The latter involves a simple analog computer program. Finally, an intriguing generalization of the phenomenon is considered.

INTRODUCTION

Shortly after the well-known Superball[1] appeared on the market, one of the authors quite accidentally discovered a surprising effect.[2] The point of a ball point pen is imbedded in the surface of a 3-in. diam Superball, and the pen and ball are dropped from a height of 4 or 5 ft so that the pen remains above the ball and perpendicular to a hard floor below. As the ball strikes the floor, the pen may be ejected so violently that it will strike the ceiling of the average room with considerable force. Furthermore, one can adjust the mass of the pen so that the ball remains completely at rest on the floor after ejecting the pen.

[1] Trade name of product by Whammo Manufacturing Co., San Gabriel, Calif.

* The members of the class of Dr. William G. Harter included: Calvin W. Gray, Jr., Robert C. Frickman, Brian P. Harney, Steven H. Hendrickson, Scott T. Jacks, David F. Judy, William D. Koltun, Sam C. Kaplan, Morton J. Kern, Edmund H. Kwan, Wayne E. Long, Michael E. Mason, William D. Moore, Willard W. Mosier, Gary P. Rudolf, Henry G. Rosenthal, William F. Skinner, Jay L. Stearn, Michael Weinberg, Mark Weiner, Frank J. Wilkinson, and David Willner.

ACKNOWLEDGMENT

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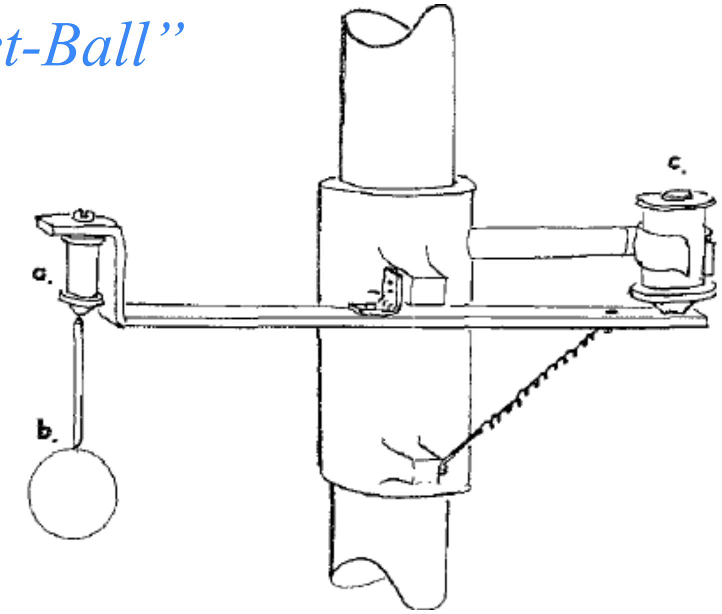

Class of W. G. Harter

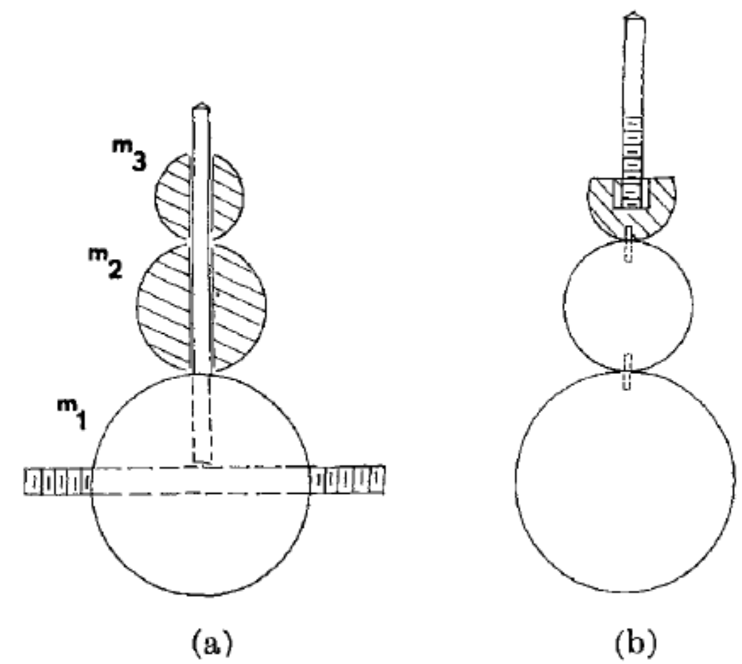

FIG. 14. Two designs for a multiple stage tower of balls. (a) Large number of balls can slide on a shaft. (b) Balls connected by small pins stand to lose appreciable amounts of binding energy.

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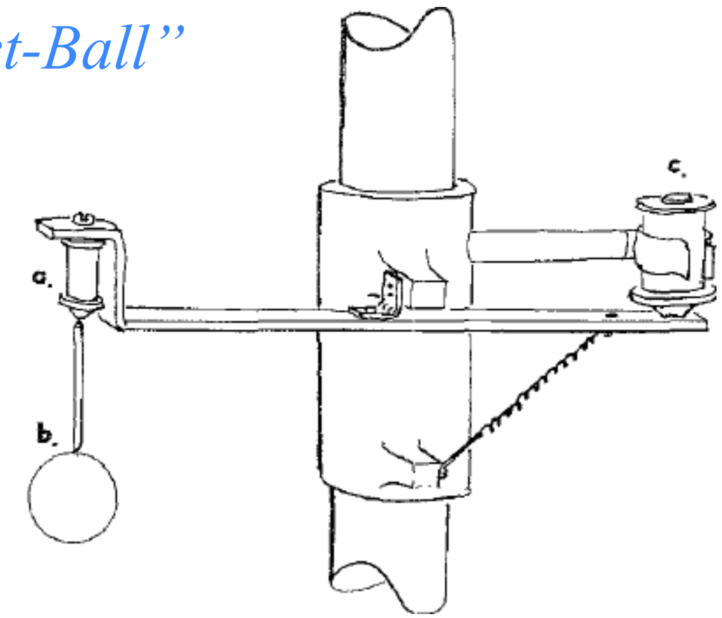

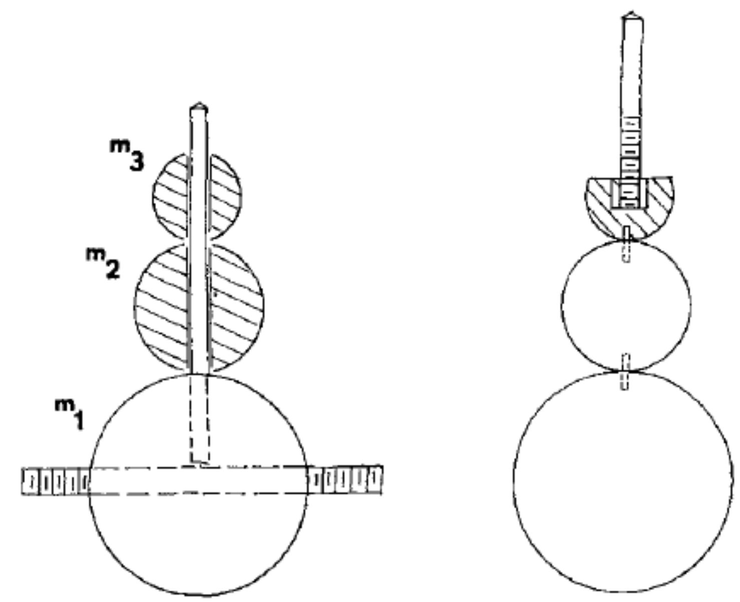

(a) (b)

FIG. 14. Two designs for a multiple stage tower of balls. (a) Large number of balls can slide on a shaft. (b) Balls connected by small pins stand to lose appreciable amounts of binding energy.

Basketball and Tennis Ball

Dropping a tennis ball on top of a basketball causes the tennis ball to bounce very high.

Source: 8.01 *Physics I: Classical Mechanics*, Fall 1999
Prof. Walter Lewin

Course Material Related to This Topic:

- Watch video clip from Lecture 17 (21:30 - 24:08)

http://ocw.mit.edu/high-school/physics/exam-prep/systems-of-particles-linear-momentum/impulse-and-momentum/

Much later....
Lots of profs try this out...
...including the unfortunate Harvard professor M. Tinkham...

(Still trying to find the video of the Tinkham incident...)

A story of USC pre-meds visiting Whammo Manufacturing Co.

...and some results of "Project-Ball"

After initial big NBC splash (Ray Dunkin Reports) in Fall 1968, USC mechanical engineers kindly measured super-ball force curves F(y) with their precision tensiometer and let us use their analog computer to calculate precise bounce heights.

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After this things began deteriorating in Old-Physics-Rm 69 (The Project-Ball-Room)

1. The fancy-pants computer theory did not jive with the fine drop-tower experiments.

2. USC B&G decided Rm 69 needed painting and kicked us out for a week.

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A call to Whammo Co. elicited interest in a big $$$$ product. Invited us to visit. Yay! $$$

A story of USC pre-meds visiting Whammo Manufacturing Co.

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Days later, finally, got a car convoy together so we all could visit San Gabriel plant.

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The engineering curves were <u>*isothermal*</u> *not* <u>*adiabatic*</u>*.*

Need latter. Can do latter by dropping dyed balls and measuring spot-size.

Measuring spot-size d gives energy vs. height.

Slope of E(x) gives force F(x) and G(x).

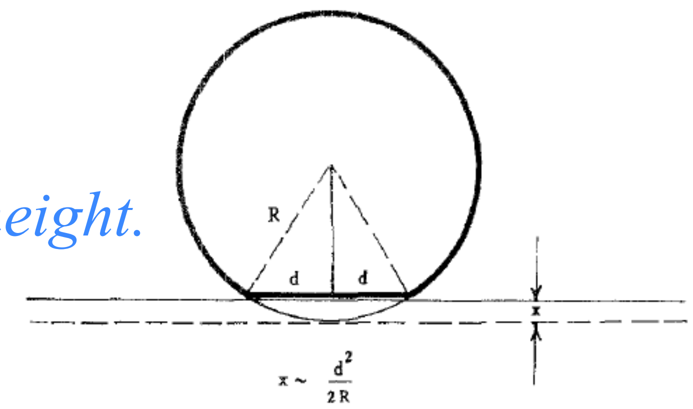
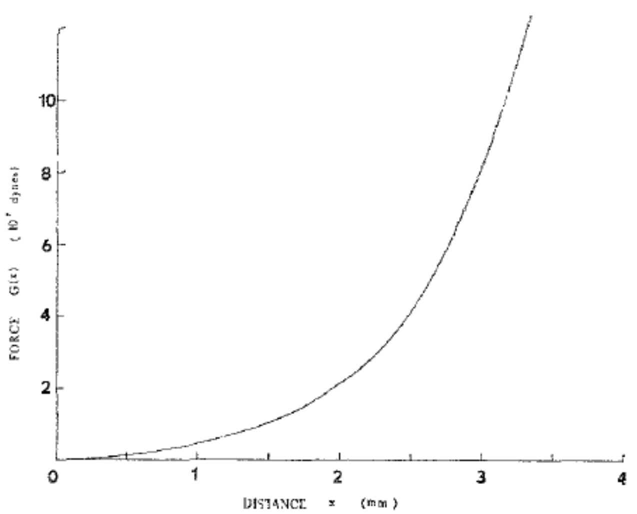


Fig. 10. Sagittal formula.

Collisions Involving Superballs



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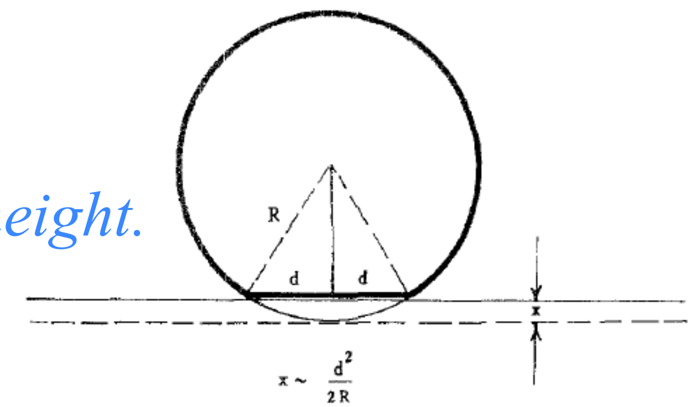

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If $F(x)$ and $G(x)$ were linear for all x, then the

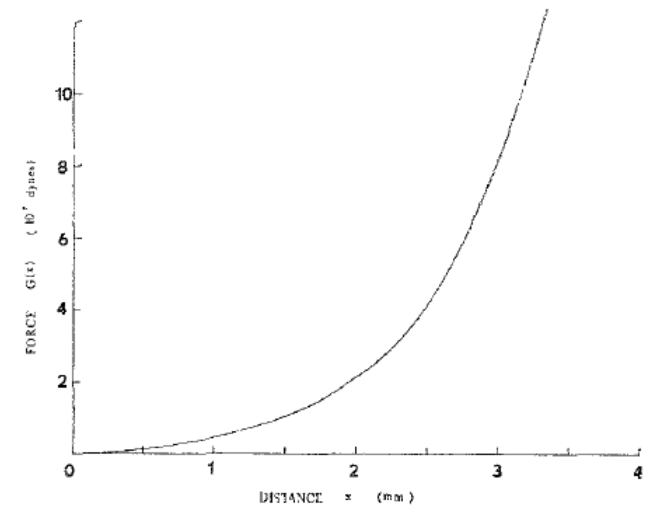

Fig. 12. Adiabatic force function $G(x)$.

Then fancy-pants computer theory can predict N-ball tower bounce

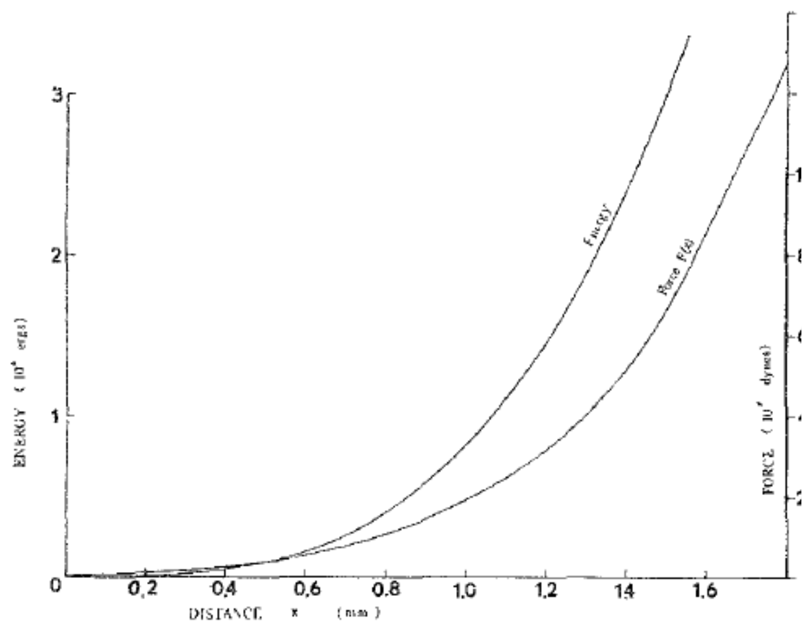

Fig. 11. Adiabatic force $F(x)$ and energy curves fc
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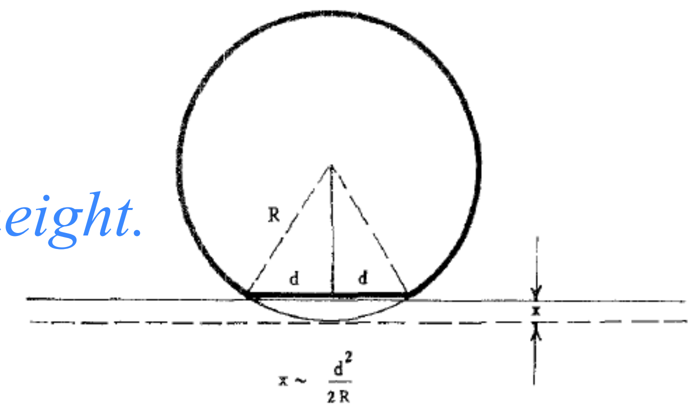

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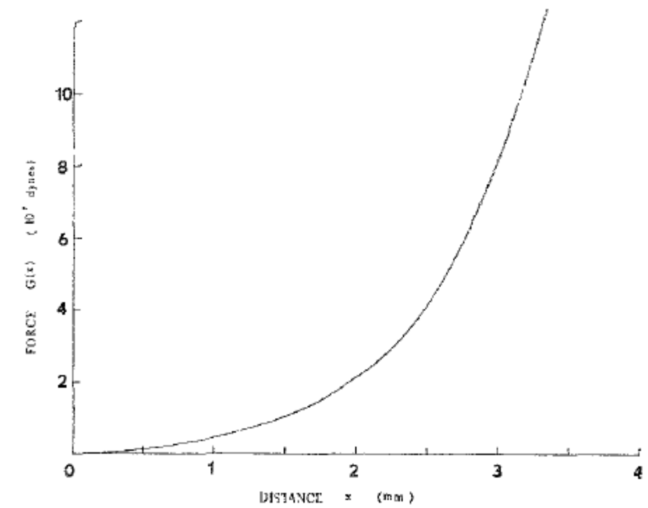

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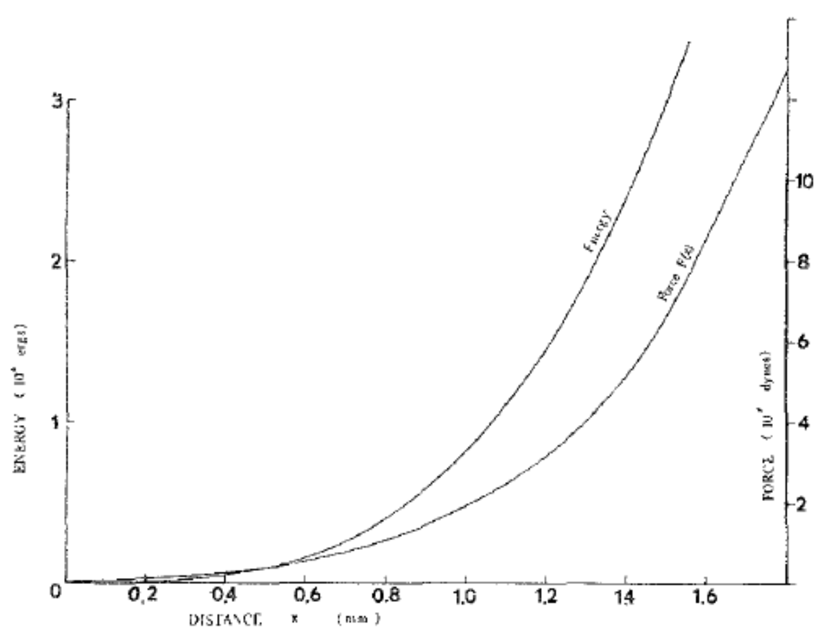
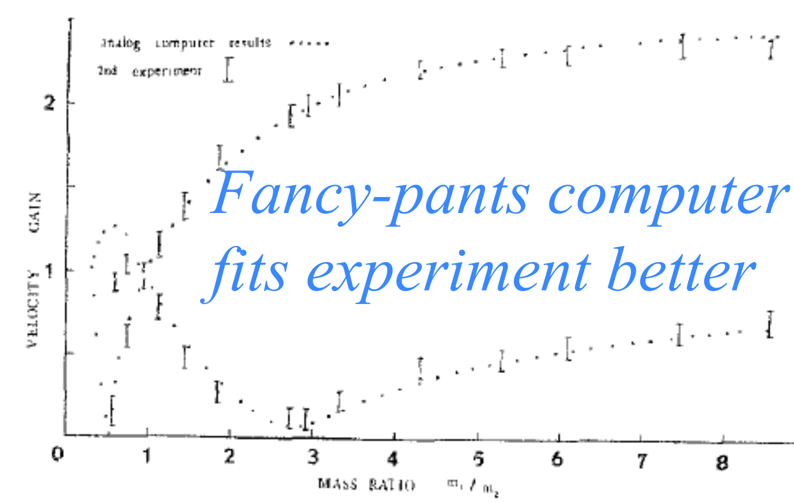


FIG. 11. Adiabatic force $F(x)$ and energy curves for Superball.

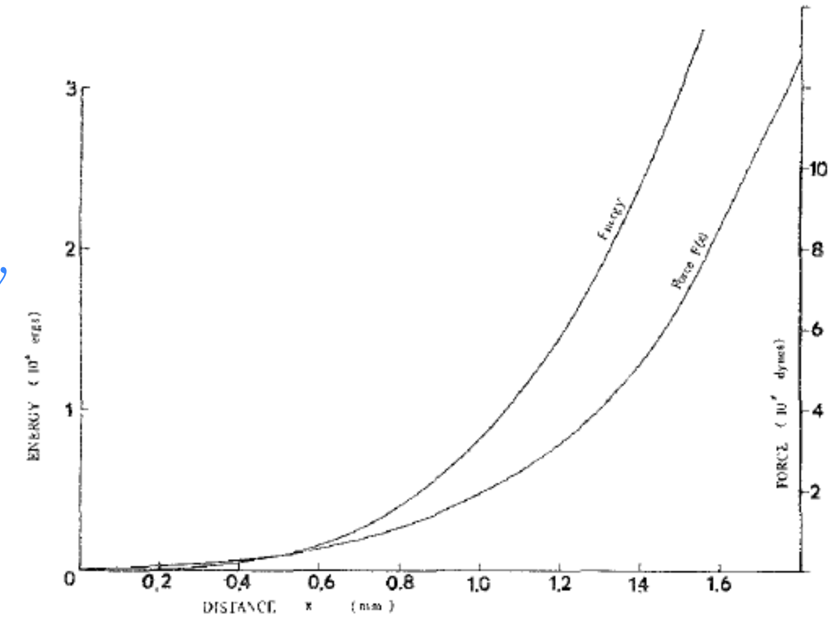
Functions $F(x)$ and $G(x)$ were then placed on the function generators of the analog computer.



Fancy-pants computer theory fits experiment better

FIG. 13. Comparison between analog computer gain curves and second experiment.

Then fancy-pants computer theory can predict N-ball tower bounces



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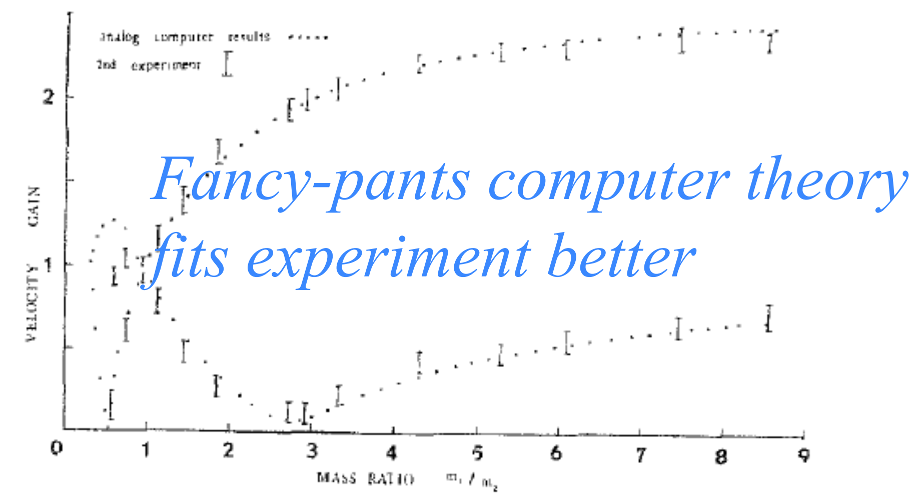

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FIG. 13. Comparison between analog computer gain curves and second experiment.

Here are some 3-ball tower bounce predictions

Class of W. G. Harter

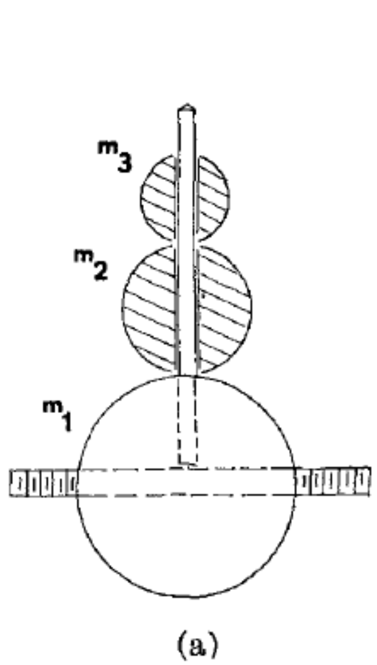

FIG. 14. Two designs for a multiple stage tower of balls. (a) Large number of balls can slide on a shaft. (b) Balls connected by small pins stand to lose appreciable amounts of binding energy.

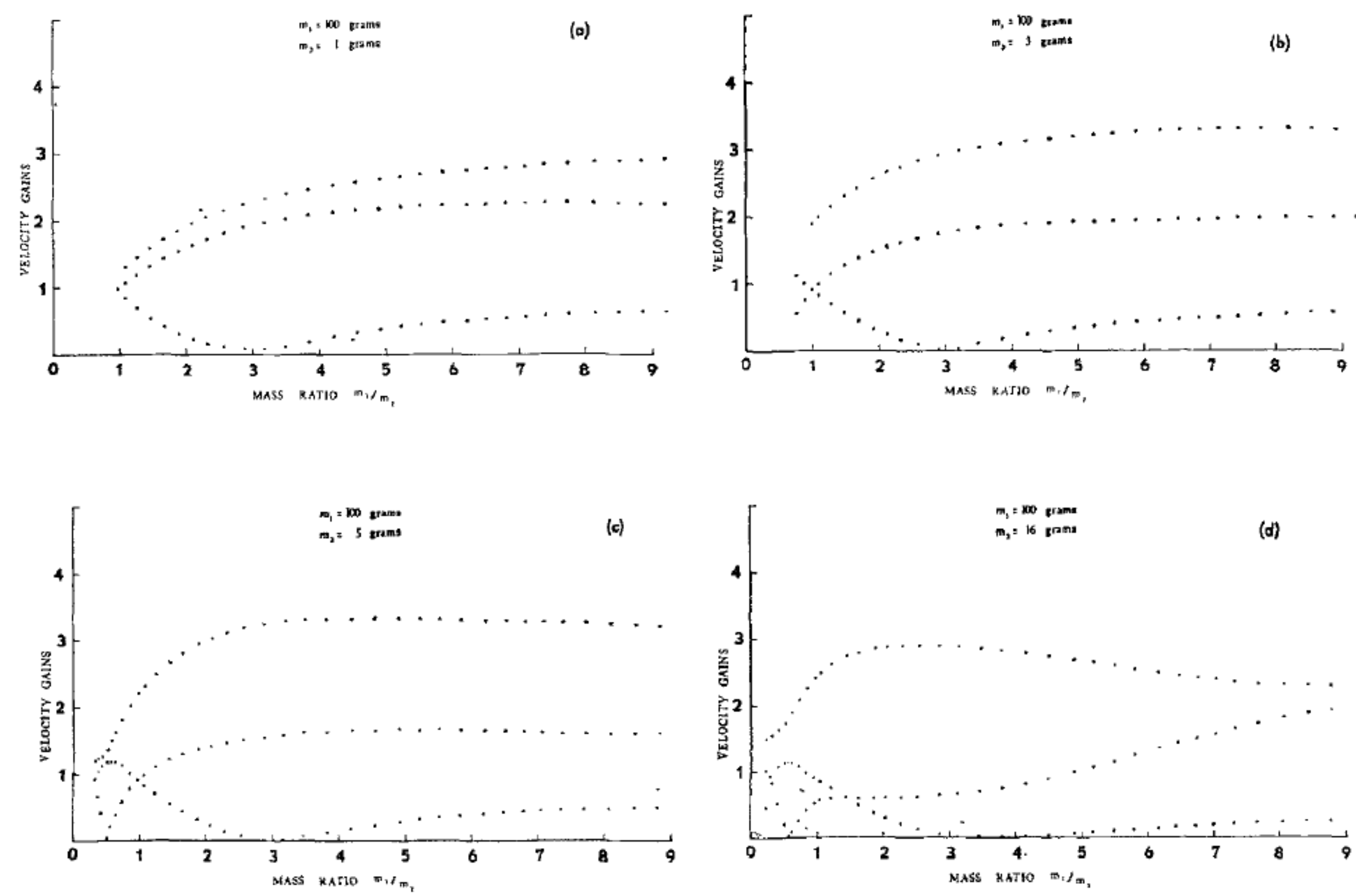

FIG. 15. (a)–(d) Analog computer output for velocity gains of three-ball system.

Potential energy dynamics of Superballs and related things

Thales geometry and "Sagittal approximation" to force law

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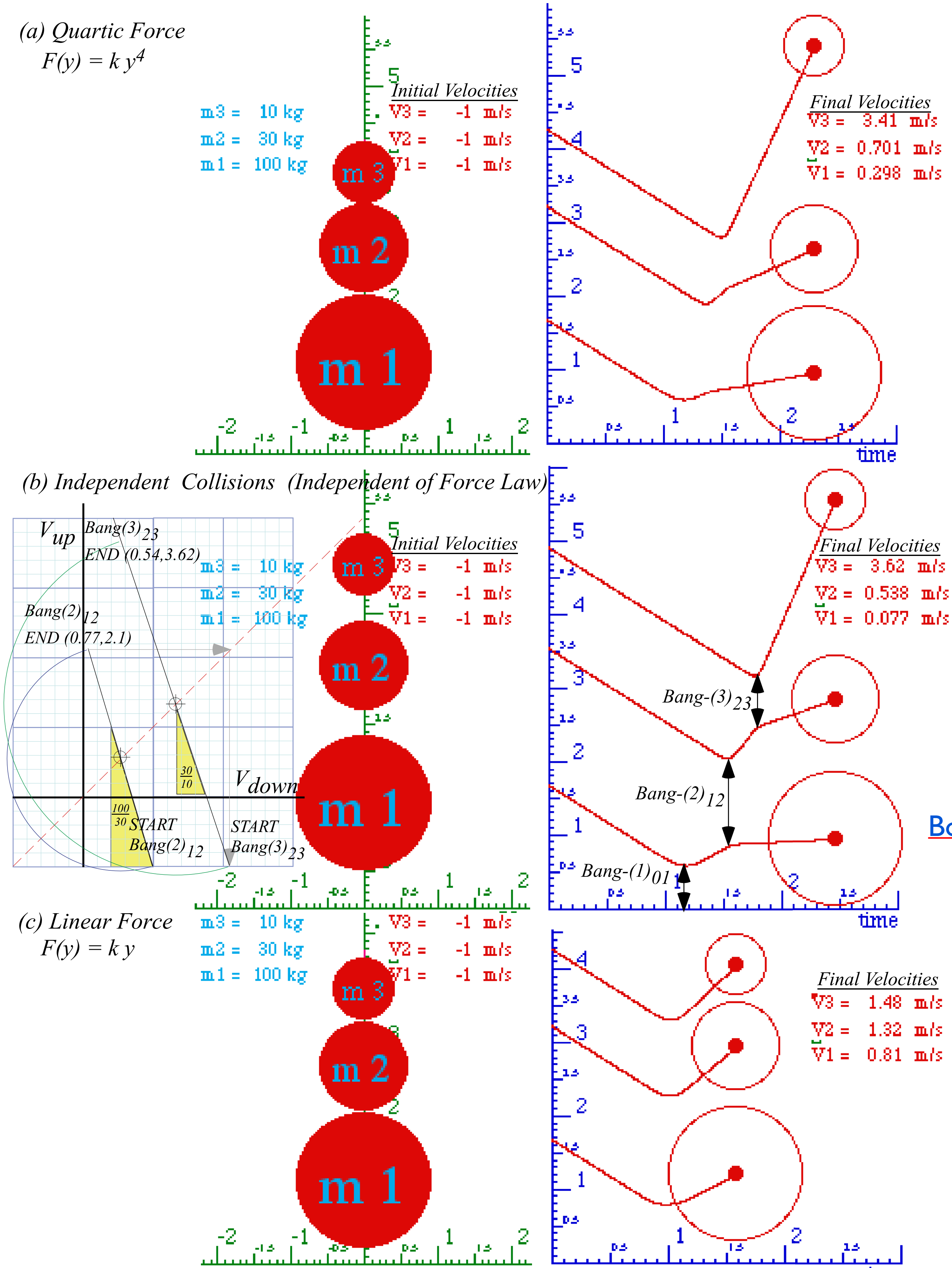
Inelastic examples: "Zig-zag geometry" of freeway crashes

*Super-elastic examples: This really **is** "Rocket-Science"*

Unit 1
Fig. 8.1a-c
Independent Bang Model
(IBM)
3-Body Geometry



[BounceIt Simulation: 3-Ball Tower w/ Quartic Force]

[BounceIt Simulation: 3-Ball (Gapped) Tower w/ Linear Force]

[BounceIt Simulation: 3-Ball Tower w/ Linear Force]

Unit 1
Fig. 8.1b
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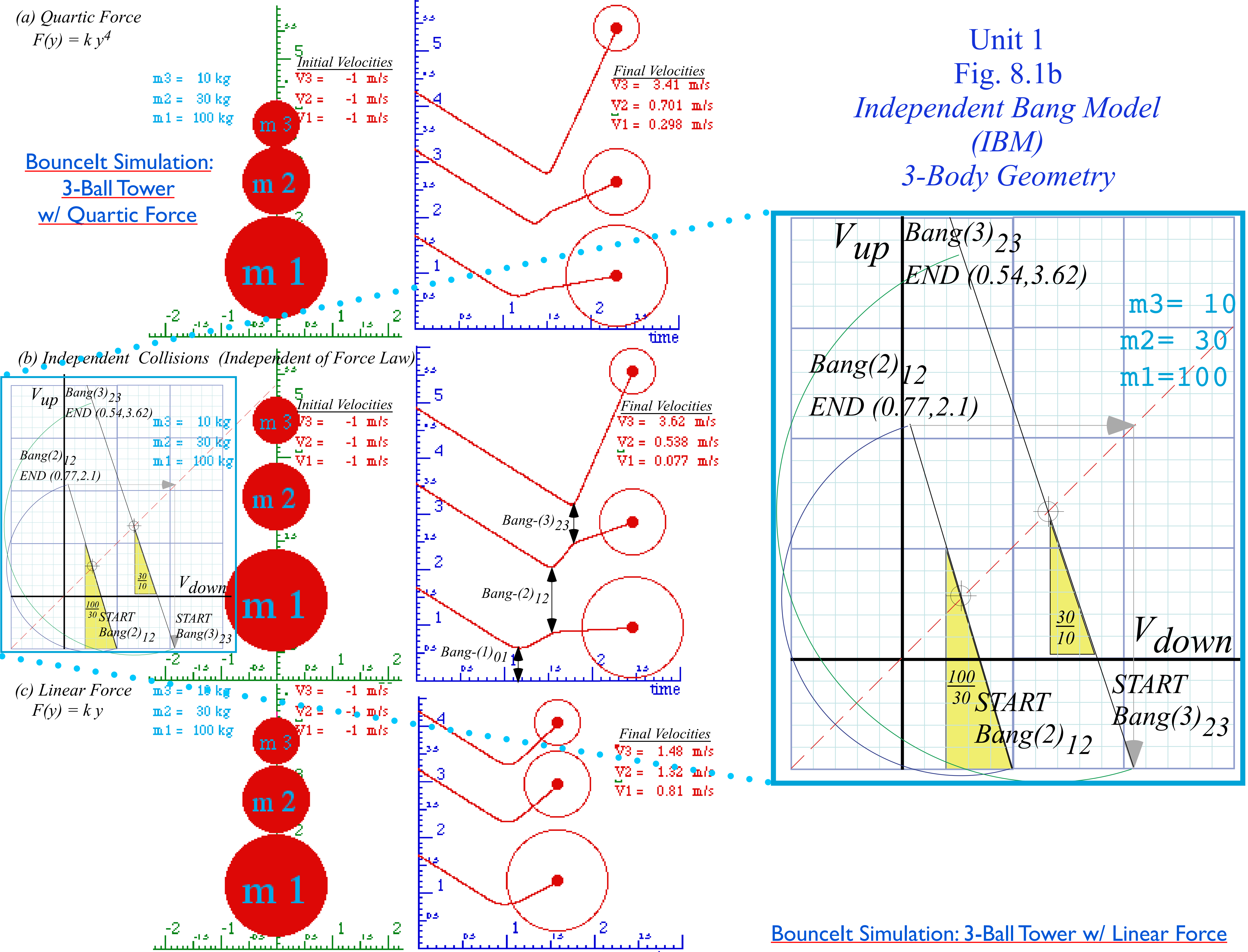
3-Body Geometry

BounceIt Simulation: 3-Ball Tower w/ Quartic Force



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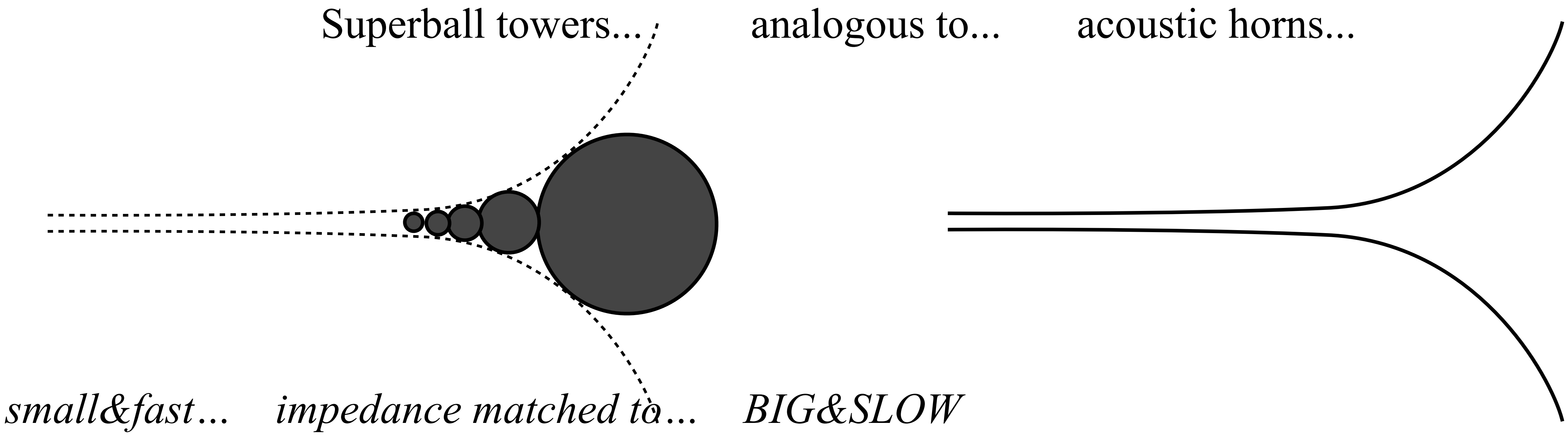

[6] J. B. Hart and R. B. Herrmann, Amer. J. Phys. **36**, 46 (1968).

1.8.3 *The optimal idler (An algebra/calculus problem)*

To get highest final v_3 of mass m_3 find optimum mass m_2 in terms of masses m_1 and m_3 that does that.

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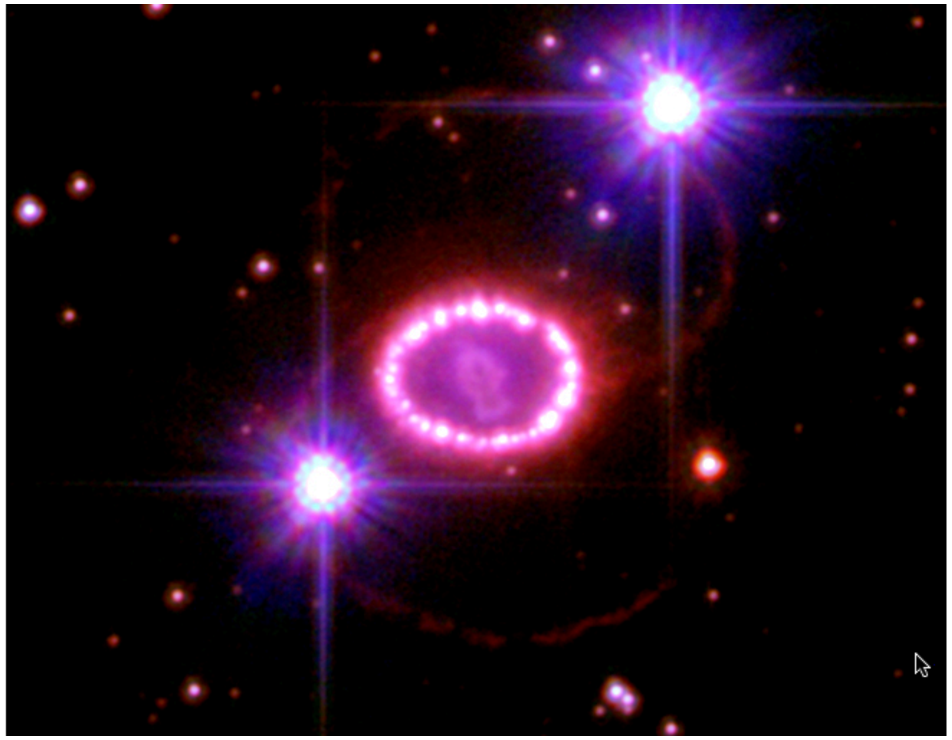
[Hubble Site: Supernova - SN 1987A](http://hubblesite.org/newscenter/archive/releases/2007/10/image/a/)

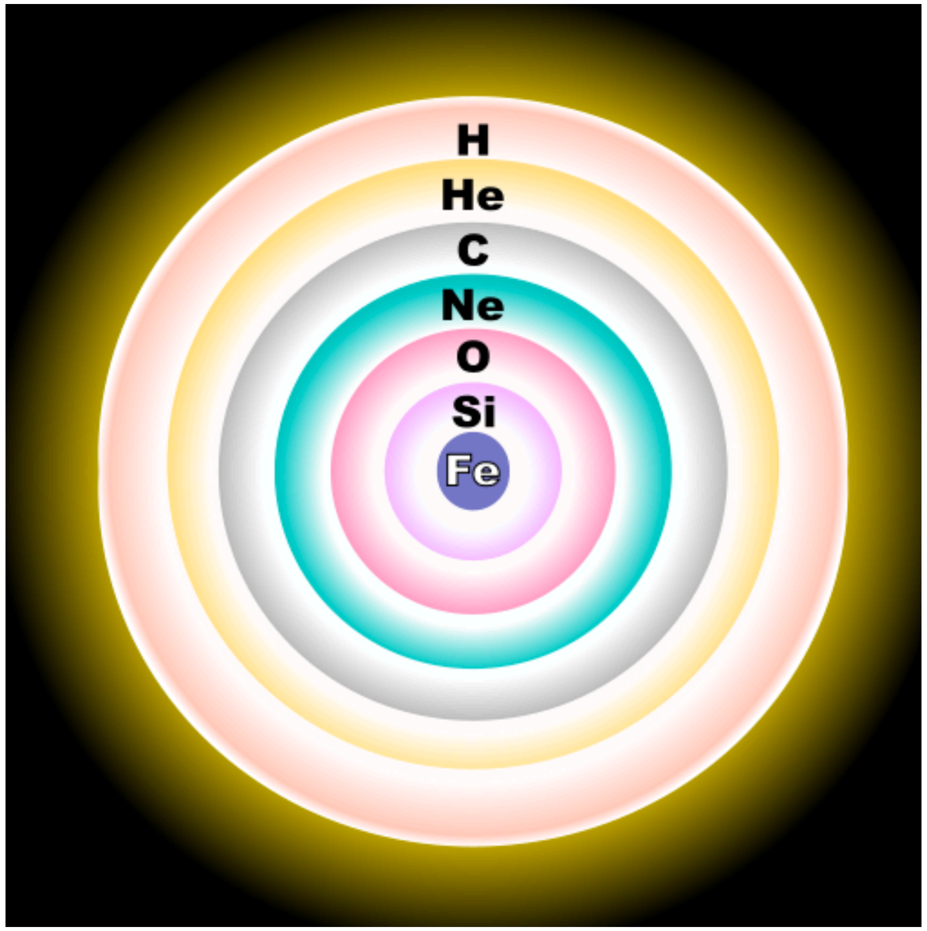


Source

http://hubblesite.org/newscenter/archive/releases/2007/10/image/a/

Author

NASA, ESA, P. Challis, and R. Kirshner (Harvard-Smithsonian Center for Astrophysics)



Core-burning nuclear fusion stages for a 25-solar mass star

Process	Main fuel	Main products	25 $M_\odot$ star[6]		
			Temperature (Kelvin)	Density (g/cm^3)	Duration
hydrogen burning	hydrogen	helium	7×10^7	10	10^7 years
triple-alpha process	helium	carbon, oxygen	2×10^8	2000	10^6 years
carbon burning process	carbon	Ne, Na, Mg, Al	8×10^8	10^6	10^3 years
neon burning process	neon	O, Mg	1.6×10^9	10^7	3 years
oxygen burning process	oxygen	Si, S, Ar, Ca	1.8×10^9	10^7	0.3 years
silicon burning process	silicon	nickel (decays into iron)	2.5×10^9	10^8	5 days

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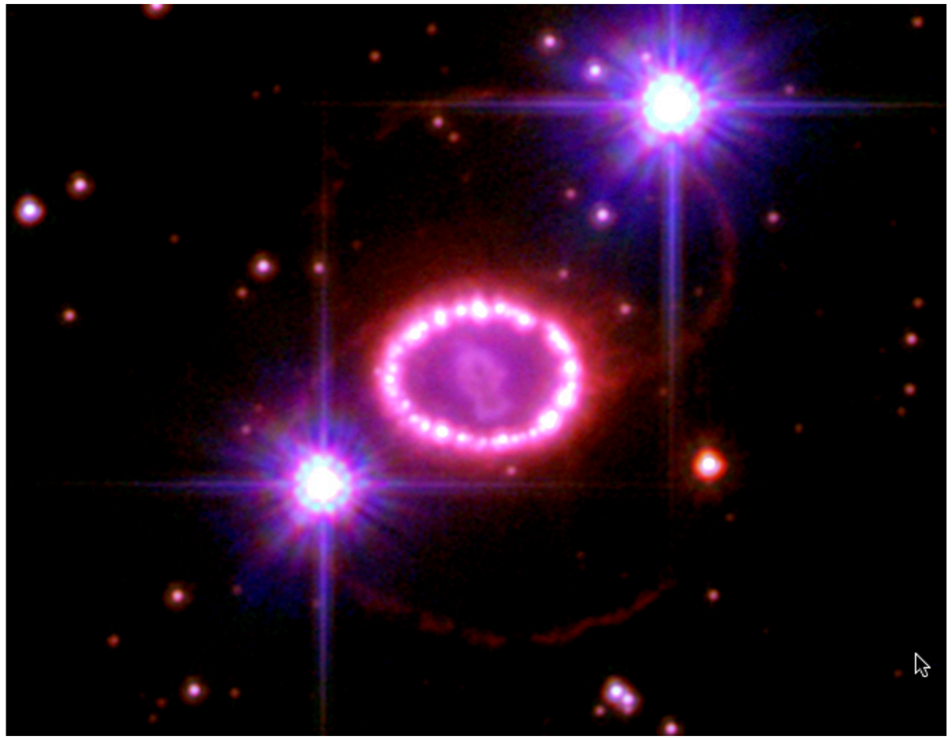

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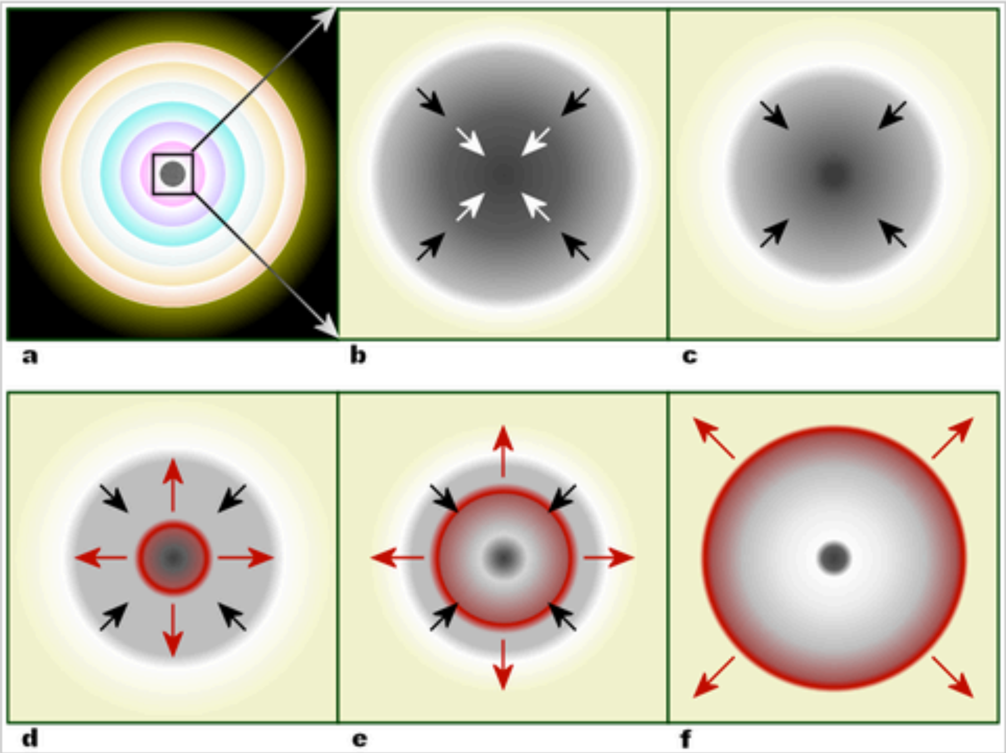
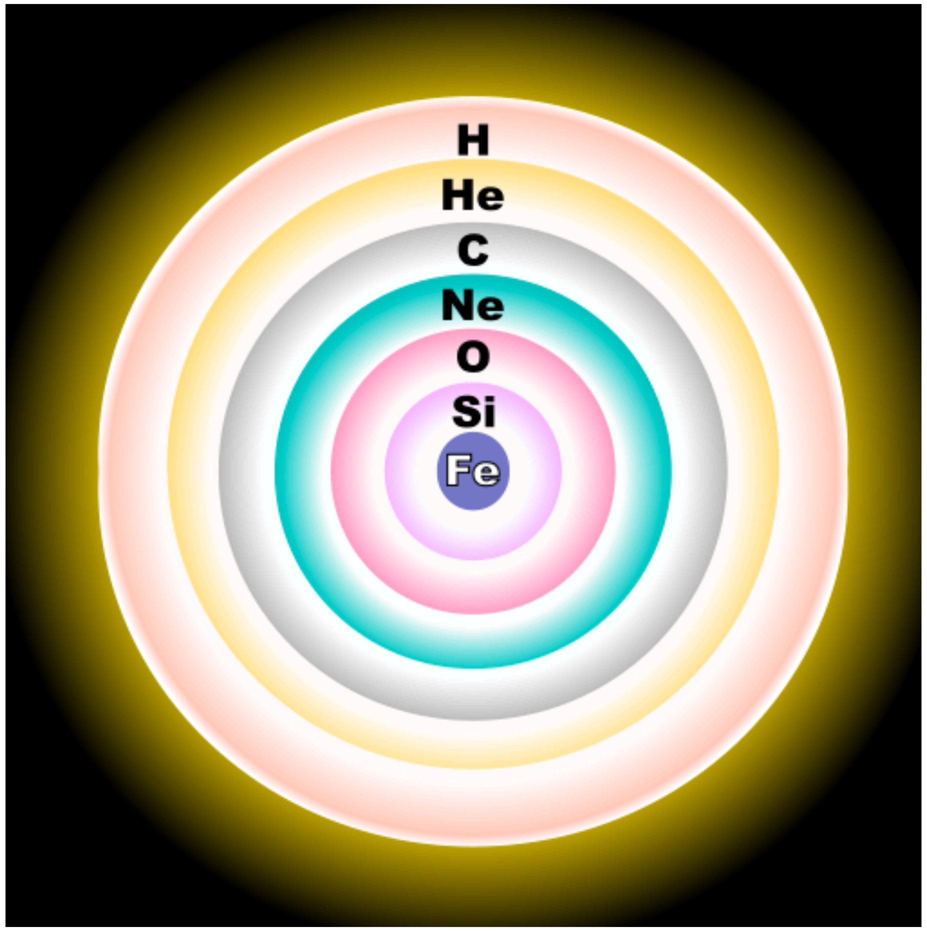


Within a massive, evolved star (a) the onion-layered shells of elements undergo fusion, forming a nickel-iron core (b) that reaches Chandrasekhar-mass and starts to collapse. The inner part of the core is compressed into neutrons (c), causing infalling material to bounce (d) and form an outward-propagating shock front (red). The shock starts to stall (e), but it is re-invigorated by neutrino interaction. The surrounding material is blasted away (f), leaving only a degenerate remnant.



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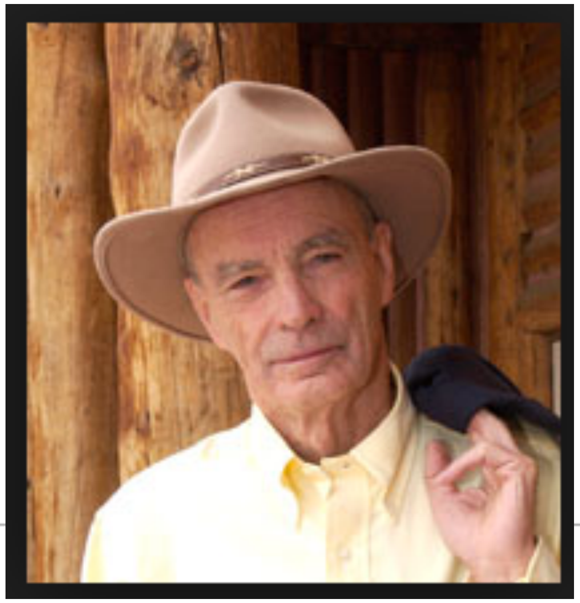
Stirling Colgate



From Wikipedia, the free encyclopedia

Stirling Auchincloss Colgate (November 14, 1925 – December 1, 2013) was an American physicist at Los Alamos National Laboratory and a professor emeritus of physics, past president at the New Mexico Institute of Mining and Technology (New Mexico Tech),[1] and an heir to the Colgate toothpaste family fortune.[2] He was America's premier[citation needed] diagnostician of thermonuclear weapons during the early years at the Lawrence Livermore National Laboratory in California. While much of his involvement with physics is still highly classified, he made many contributions in the open literature including physics education and astrophysics.[3] He was born in New York City in 1925, to Henry Auchincloss and Jeanette Thurber (née Pruyn) Colgate.[4]

..an amusing off-color aside
story of Stirling Colgate's NMIMT resignation...

(Not told in Wikipedia!)

Quote

- "*I was always enamored with explosives, and eventually I graduated to dynamite and then nuclear bombs*."

Patents

Multiple-collision accelerator assembly

US 5256071 A

Publication number	US5256071 A
Publication type	Grant
Application number	US 07/748,804
Publication date	Oct 26, 1993
Filing date	Aug 22, 1991
Priority date ⓘ	Aug 22, 1991
Fee status ⓘ	Paid
Inventors	Edward W. Hones, William G. Hones, Stirling A. Colgate
Original Assignee	Hones Edward W, Hones William G, Colgate Stirling A
Export Citation	BiBTeX, EndNote, RefMan

Patent Citations (3), Referenced by (4), Classifications (7), Legal Events (7)

External Links: USPTO, USPTO Assignment, Espacenet

ABSTRACT

A device comprising several highly elastic objects is presented whose purpose is to demonstrate an unobvious consequence of fundamental laws of physics--the acceleration of an object to high speed by multiple collisions among a series of heavier objects moving at slower speed. The objects, each of different mass, are arrayed in close proximity in order of decreasing mass with their centers lying along a straight line. This arrangement of the assembly of objects is maintained by a constraining element which permits the assembly axis to be oriented in any desired direction and permits the assembly to be moved or manipulated as a unit in any desired way without destroying the arrangement of objects. In the preferred embodiment the elastic objects are polybutadiene balls (12), the constraining element is an interior guide-pin (10) fastened in the largest ball and extending radially therefrom, on which the remaining balls can slide freely because of diametrical holes formed in them. In use this multiple-collision accelerator assembly is suspended in vertical orientation, with the largest ball downward, by holding the tip-end of the guide-pin which extends beyond the littlest ball. The assembly is then dropped onto a solid surface (14), the striking of which produces a sharp impulse that is transmitted from the largest ball, through the assembly, causing the littlest ball to be projected to a height many times that from which the assembly was dropped.

(Point allowing patent over previous 1973 proposal (4))

1st publication describing theory and experiment of this device 20 years before.

Velocity Amplification in Collision Experiments Involving Superballs

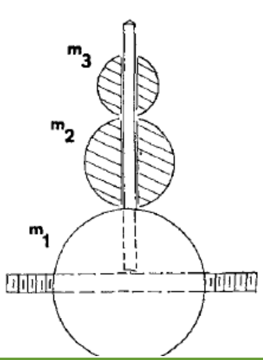
William G. Harter[1] (class of WGH)

— HIDE AFFILIATIONS

[1] University of Southern California, Los Angeles, California 90007

View the Scitation page for University of Southern California (USC).

Am. J. Phys. **39**, 656 (1971); http://dx.doi.org/10.1119/1.1986253



BUY: $30.00

(Now I have to pay APS for my own paper.)

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Product Code: AstroBlaster
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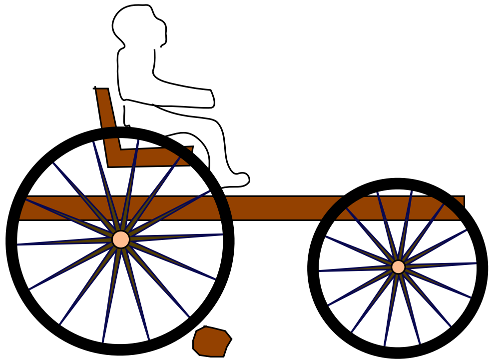
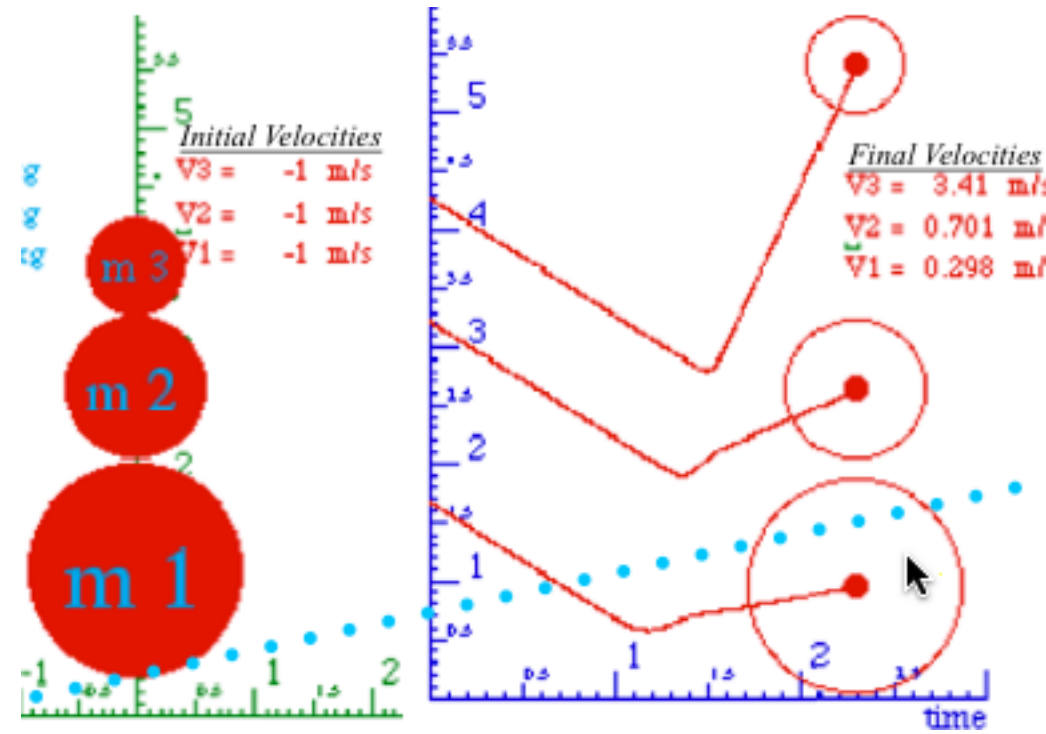
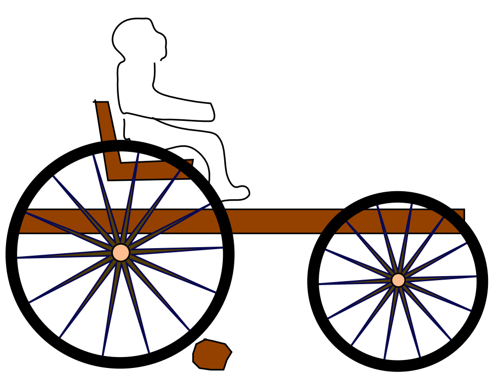
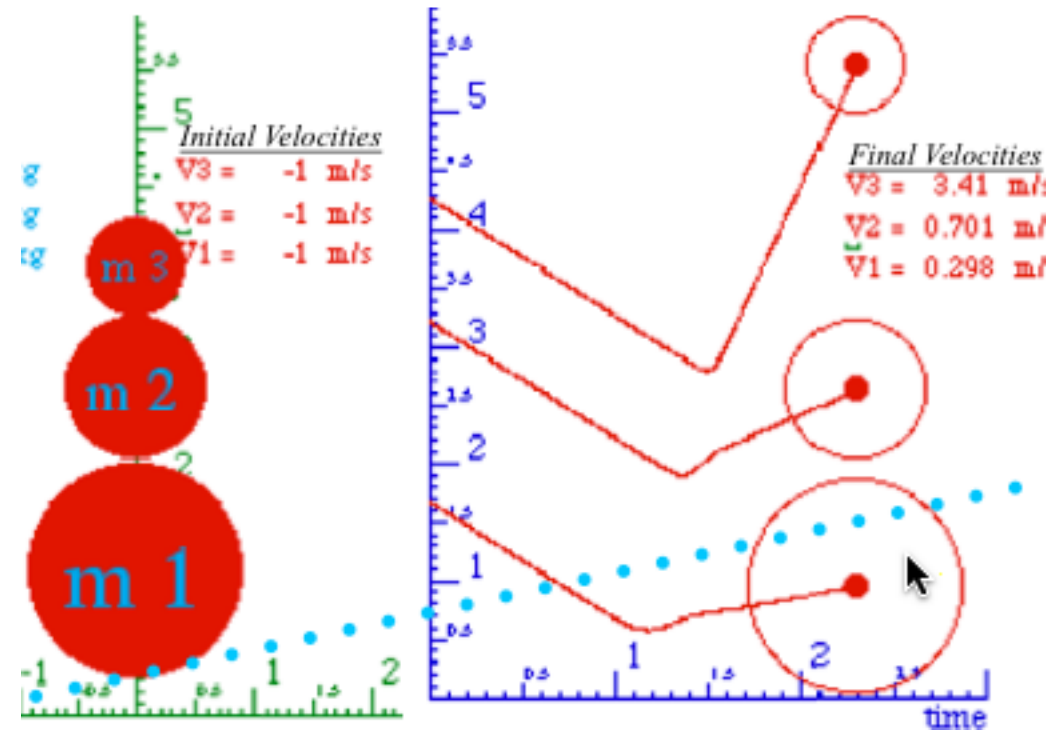


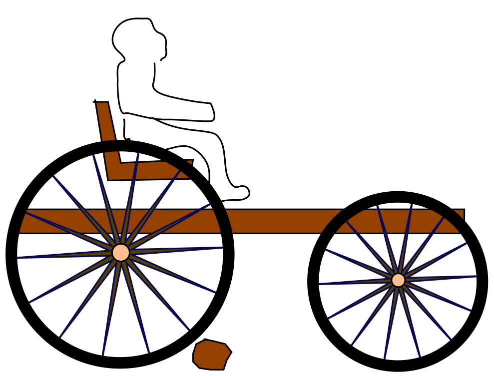


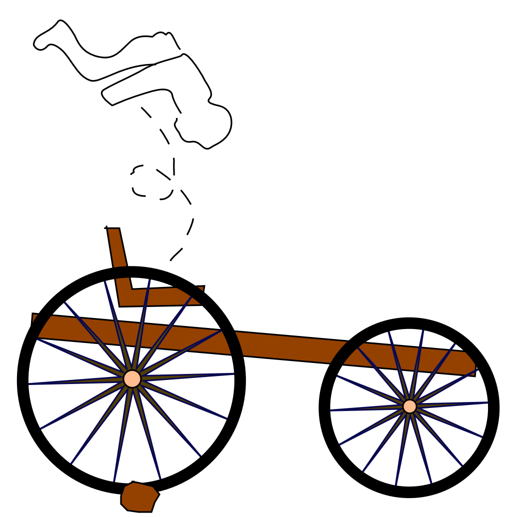
Western buckboard = *3-ball analogy*





Western buckboard = *3-ball analogy* *Disaster!*





Potential energy dynamics of Superballs and related things

Thales geometry and "Sagittal approximation" to force law

Geometry and dynamics of single ball bounce

(a) Constant force F=-k (linear potential V=kx)

Some physics of dare-devil-diving 80 ft. into kidee pool (Simulations)

(b) Linear force F=-kx (quadratic potential $V=\frac{1}{2}kx^2$ (like balloon))

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Many-body 1D collisions

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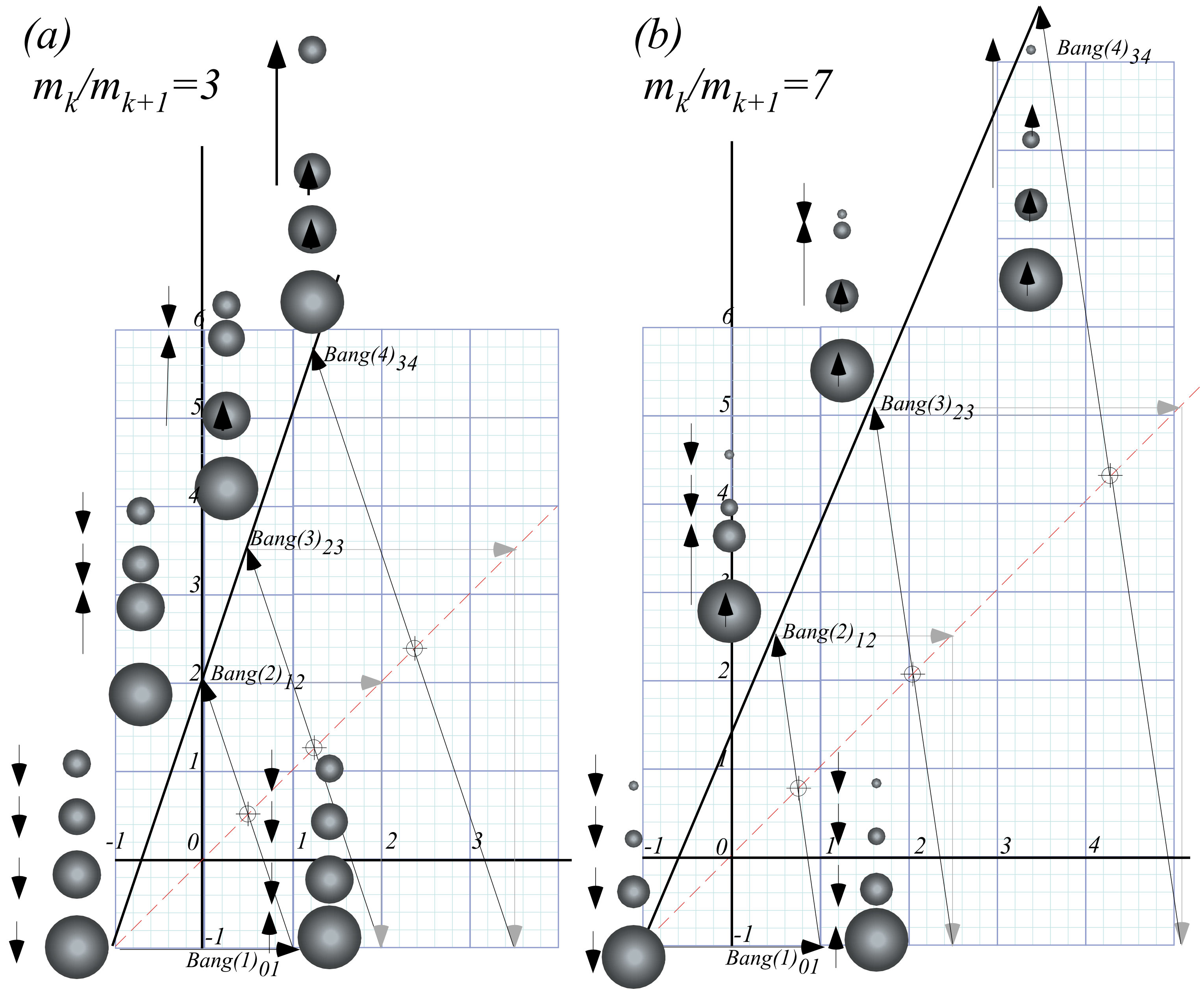
→ Bouncing columns and Newton's cradle

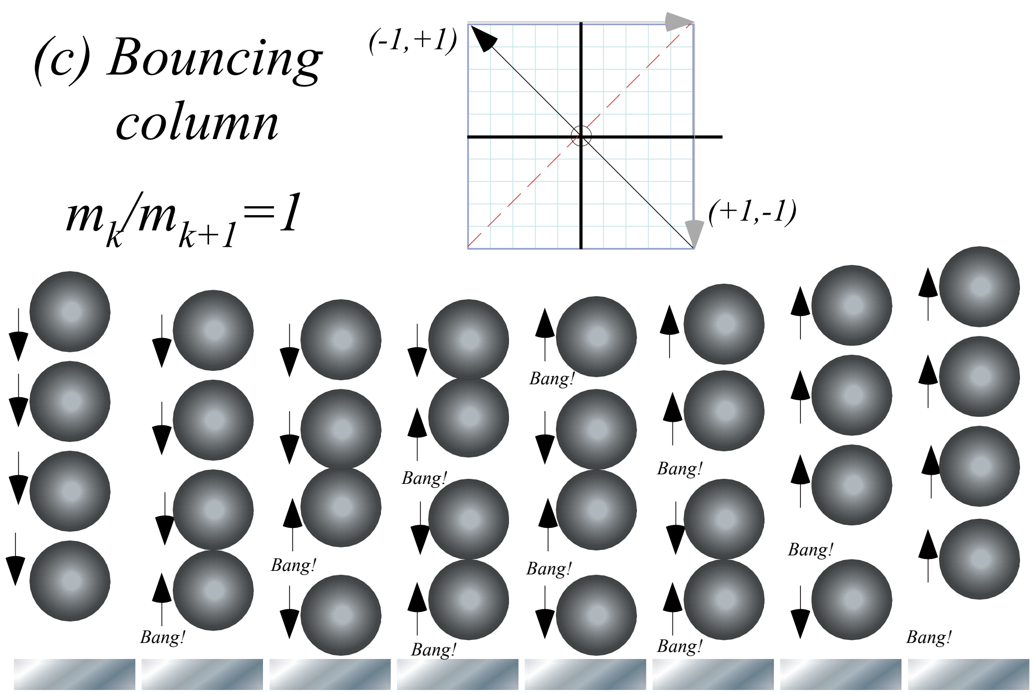
Inelastic examples: "Zig-zag geometry" of freeway crashes

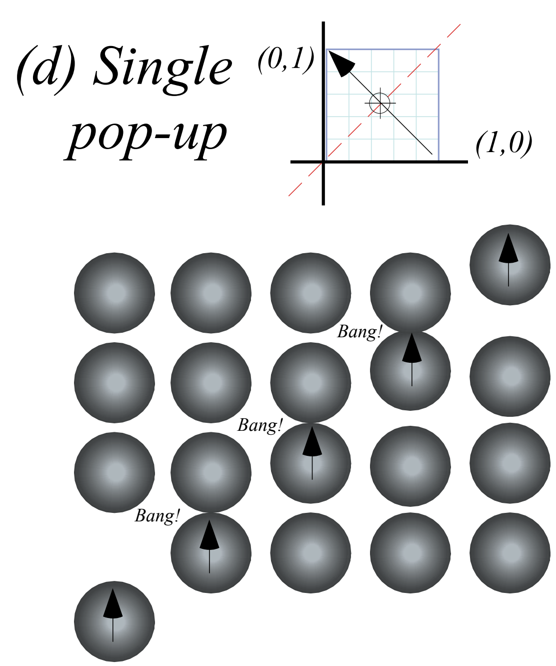
Super-elastic examples: This really **is** "Rocket-Science"

Unit 1
Fig. 8.2a-b
4-Body IBM Geometry
Fig. 8.2c-d
4-Equal-Body Geometry



[BounceIt Simulation: 4-Ball Tower w/ m_k/m_{k+1} = 3]





4-Equal-Body “Shockwave” or pulse wave Dynamics

Opposite of continuous wave dynamics introduced in Unit 2

[BounceIt Simulation: 4-Ball Tower w/ m_k/m_{k+1} = 1]

Potential energy dynamics of Superballs and related things

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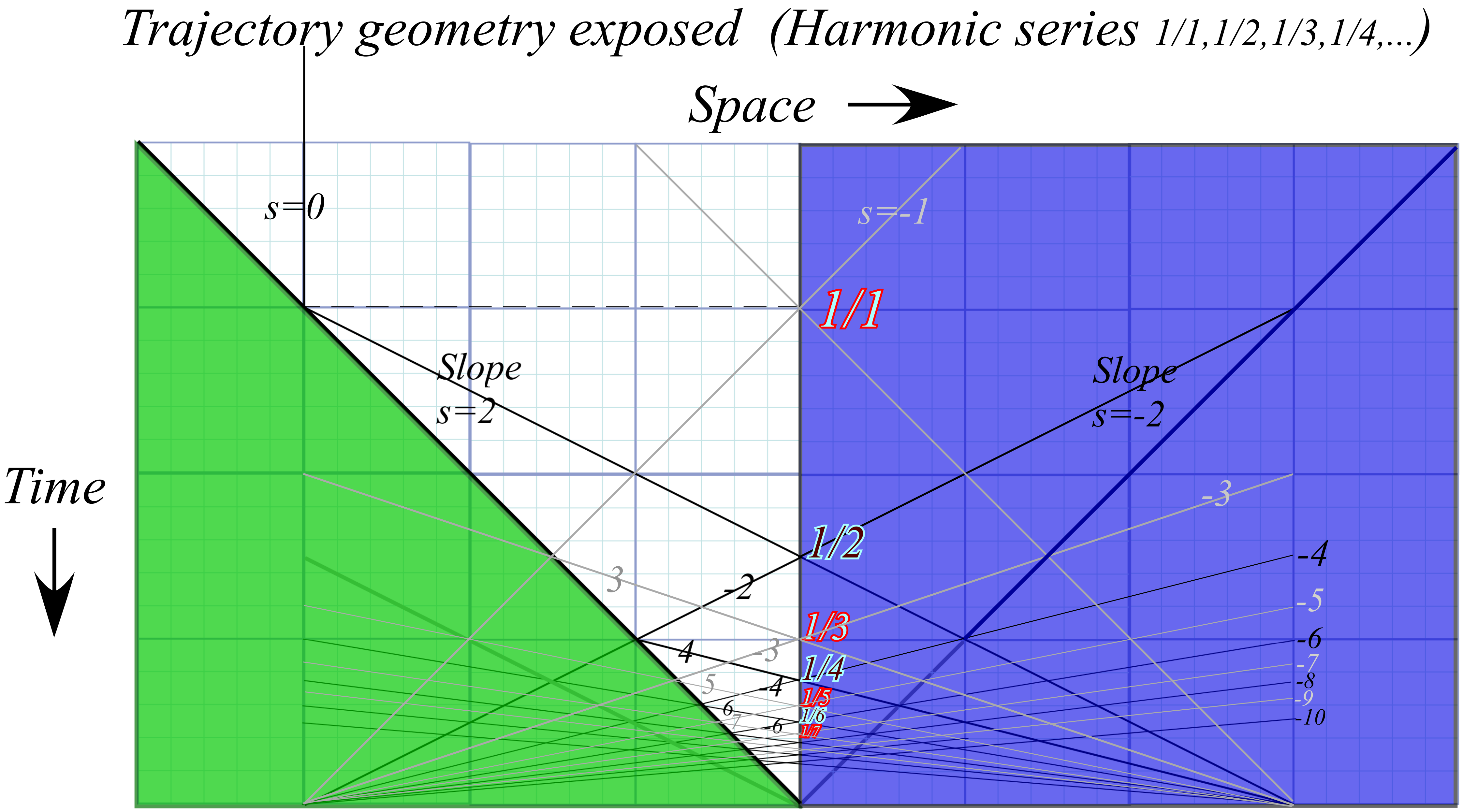
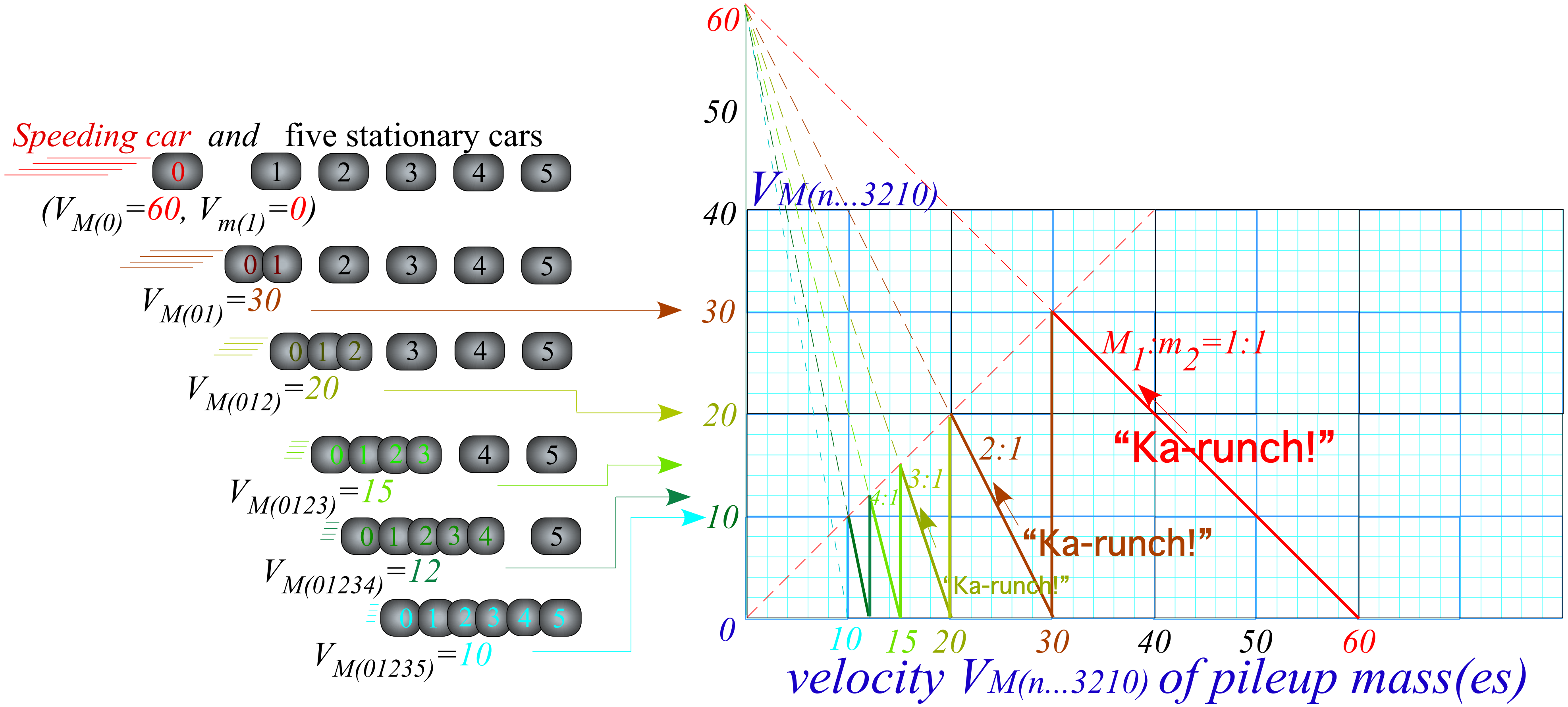
Super-elastic examples: This really **is** *"Rocket-Science"*

Inelastic examples: "Zig-zag geometry" of freeway crashes

First recall "zig-zag" fractions of "Monster Mash" in Lect. 4





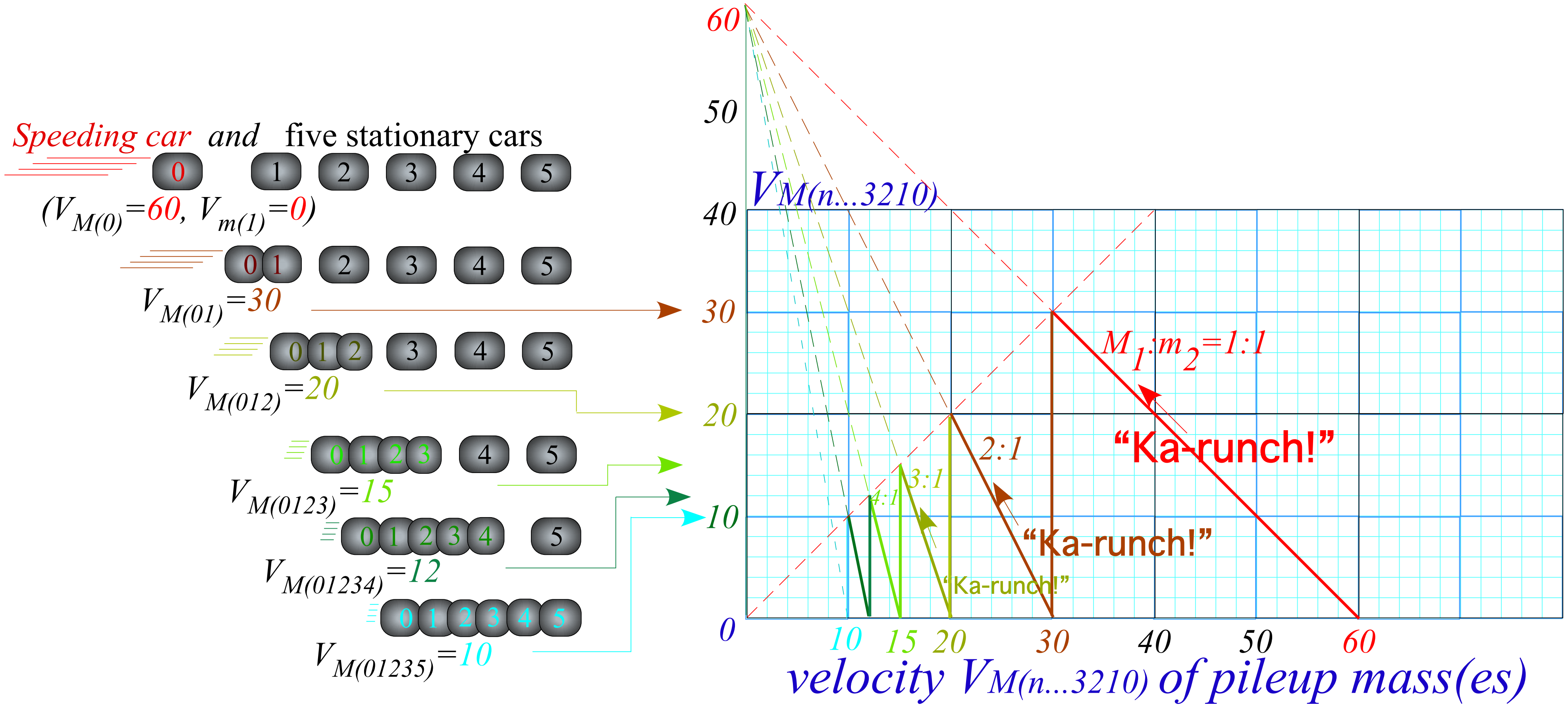
Unit 1
Fig. 8.5
Pile-up:
One *60mph* car hits five standing cars

[BounceIt Simulation: One ball hits 5 stationary balls (y vs x) and (x_i vs t)]

These graphs plot User determined quantities.
Choose and select from a context menu via right click on target axis, like the following set to V_{1x} and V_{2x}

[BounceIt Simulation: One ball hits 5 stationary balls (y vs x) and (V_{i+1x} vs V_{ix})]

Unit 1



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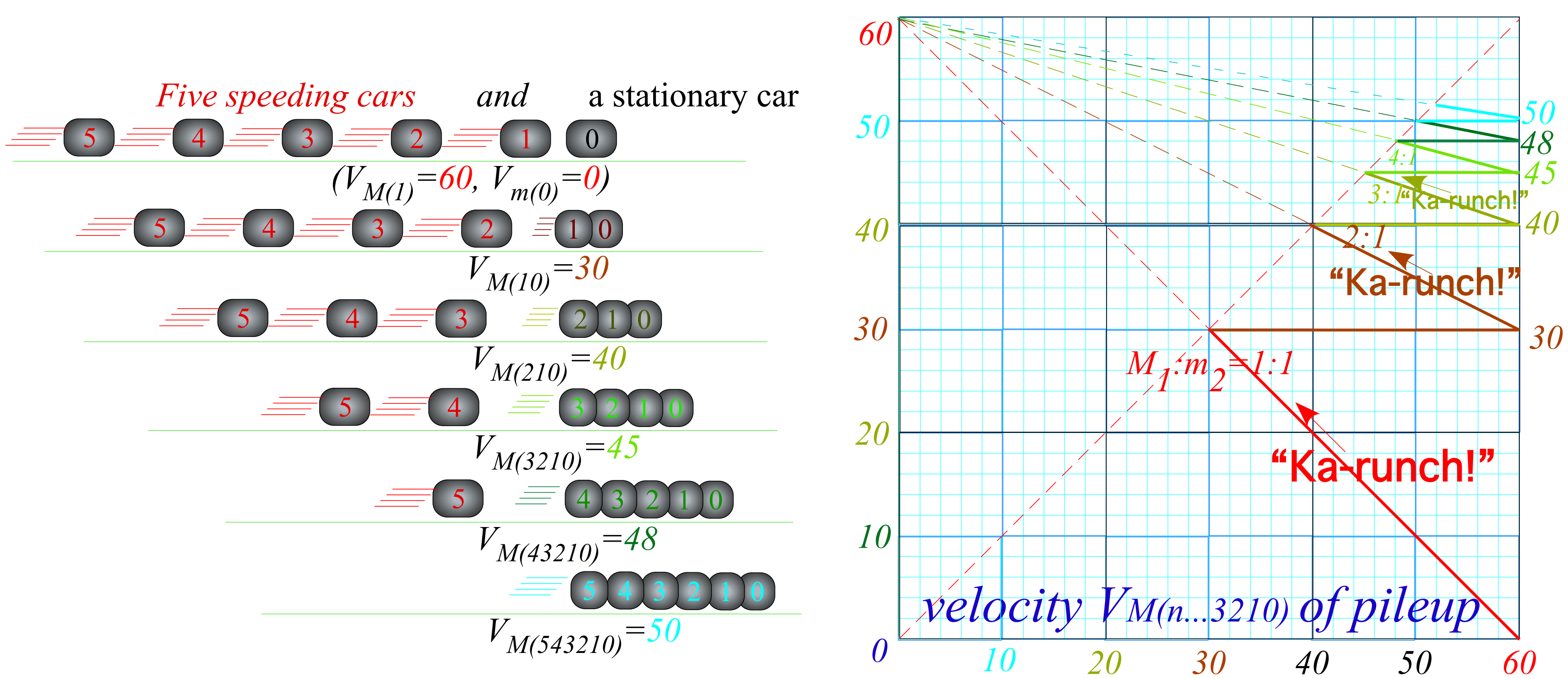
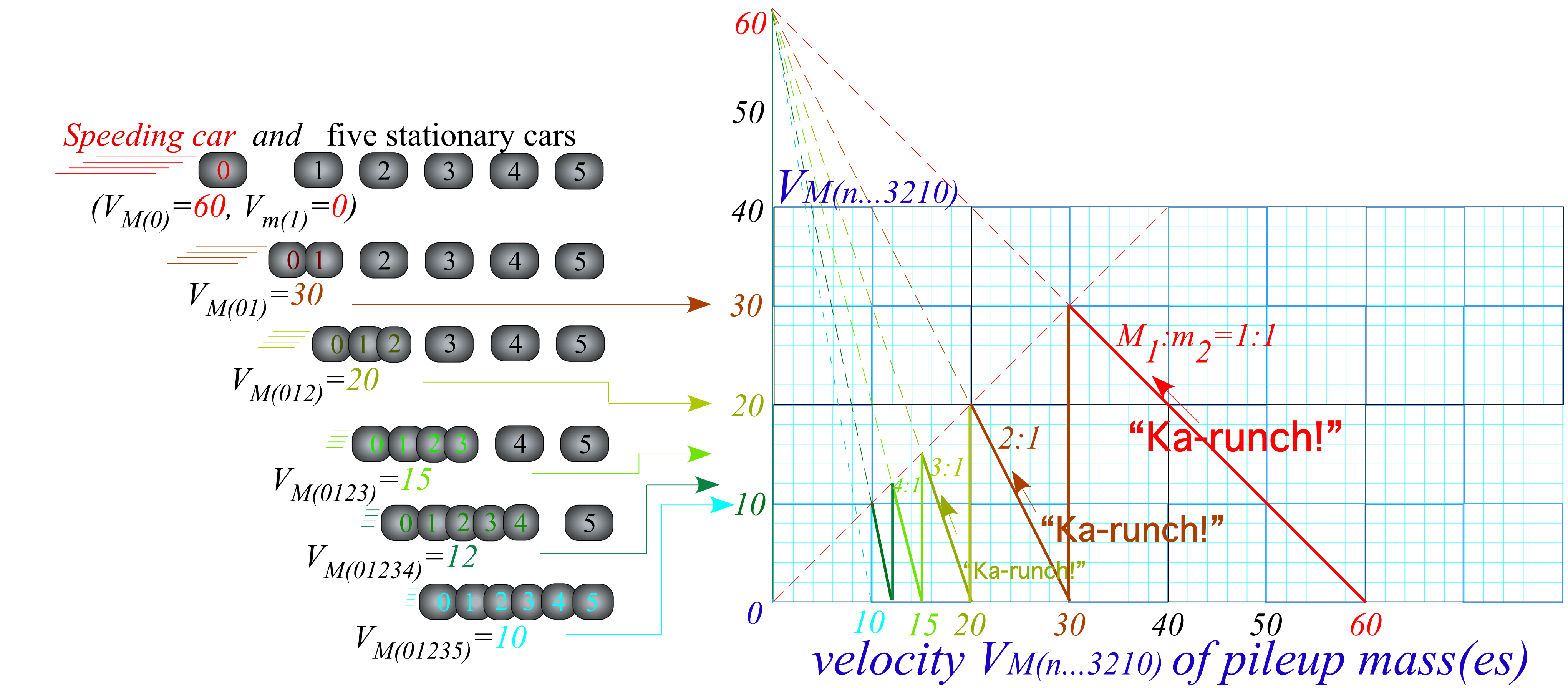


Fig. 8.6
Pile-up:
Five *60mph* cars
hit
one standing cars

[BounceIt Simulation: 5 balls hit 1 stationary ball (y vs x) and (v_{6x} vs v_{5x})]



Unit 1

Fig. 8.5
Pile-up:
One *60mph* car
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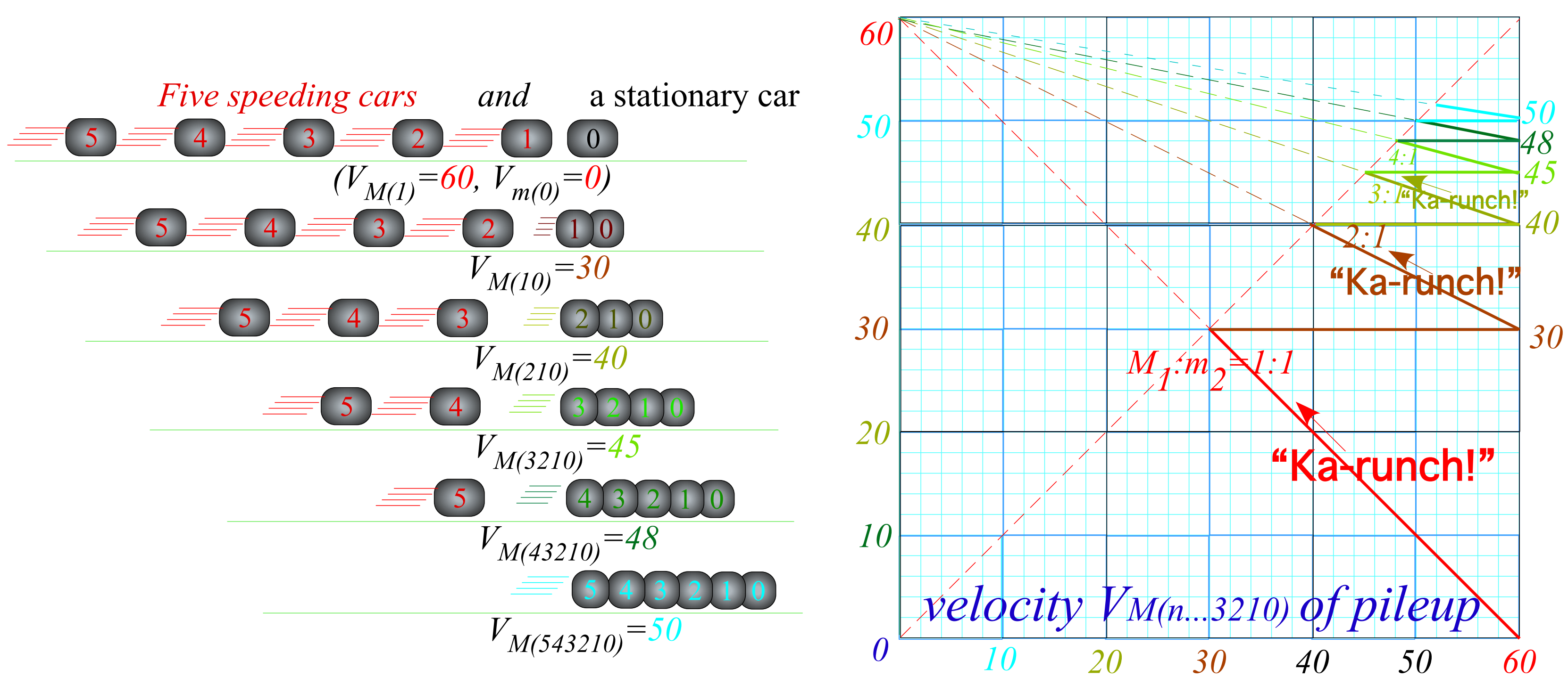

Fig. 8.6
Pile-up:
Five *60mph* cars
hit
one standing cars

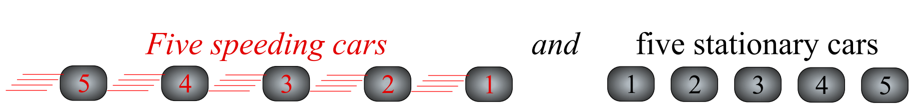

Fig. 8.7
Pile-up:
Five *60mph* cars
hit
five standing cars

(Fug-gedda-aboud-dit!!)

(Many possible scenarios depending on initial positions!)

[BounceIt Simulation: 5 balls hits 5 stationary balls (y vs x) and (x_i vs t)]

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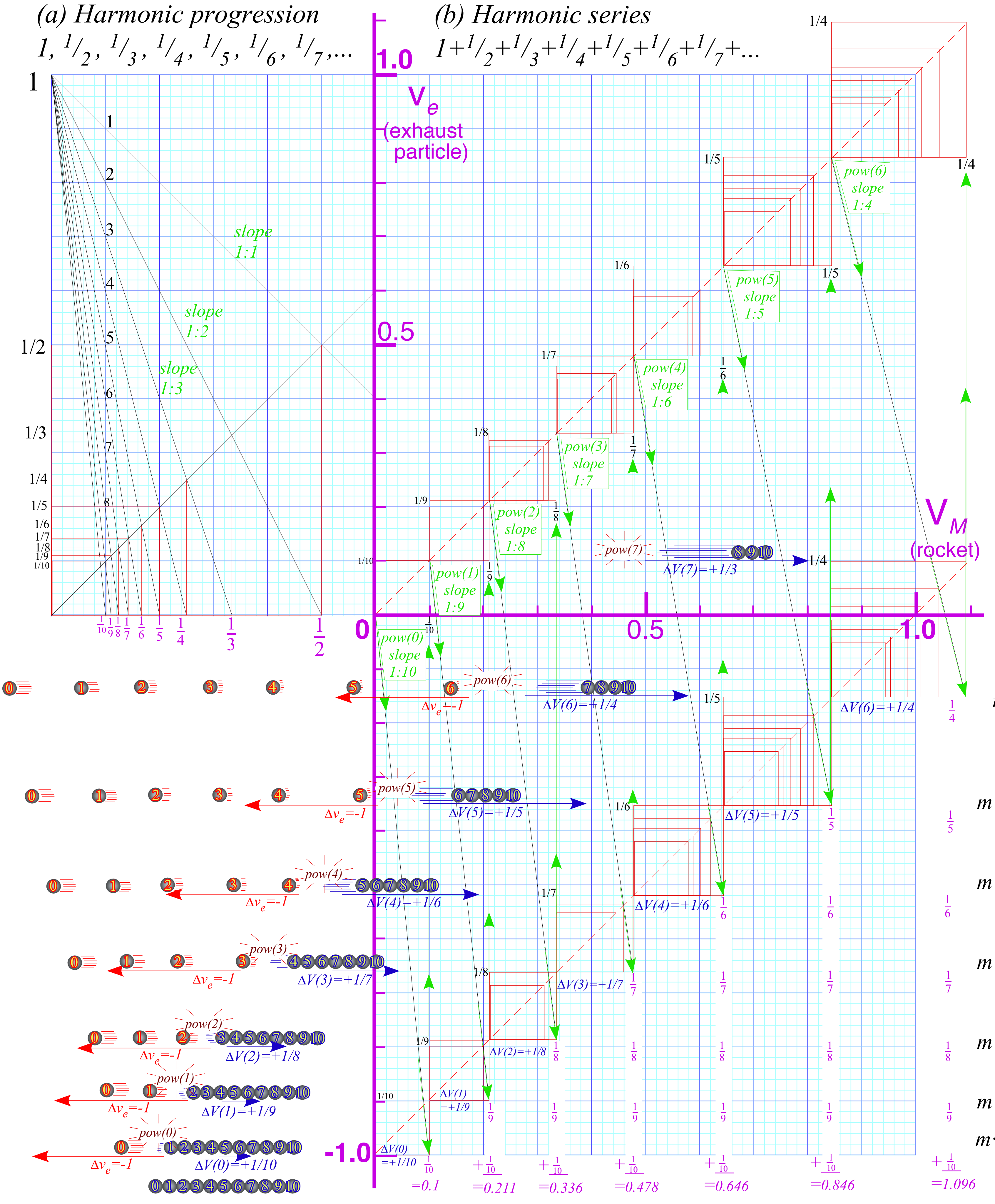
→ *Super-elastic examples: This really* ***is*** *"Rocket-Science"*

Unit 1
Fig. 8.8a-b

Rocket Science!



$m\cdot\Delta v_7+3m\cdot\Delta V_M(7)=0$

$m\cdot\Delta v_6+4m\cdot\Delta V_M(6)=0$

$m\cdot\Delta v_5+5m\cdot\Delta V_M(5)=0$

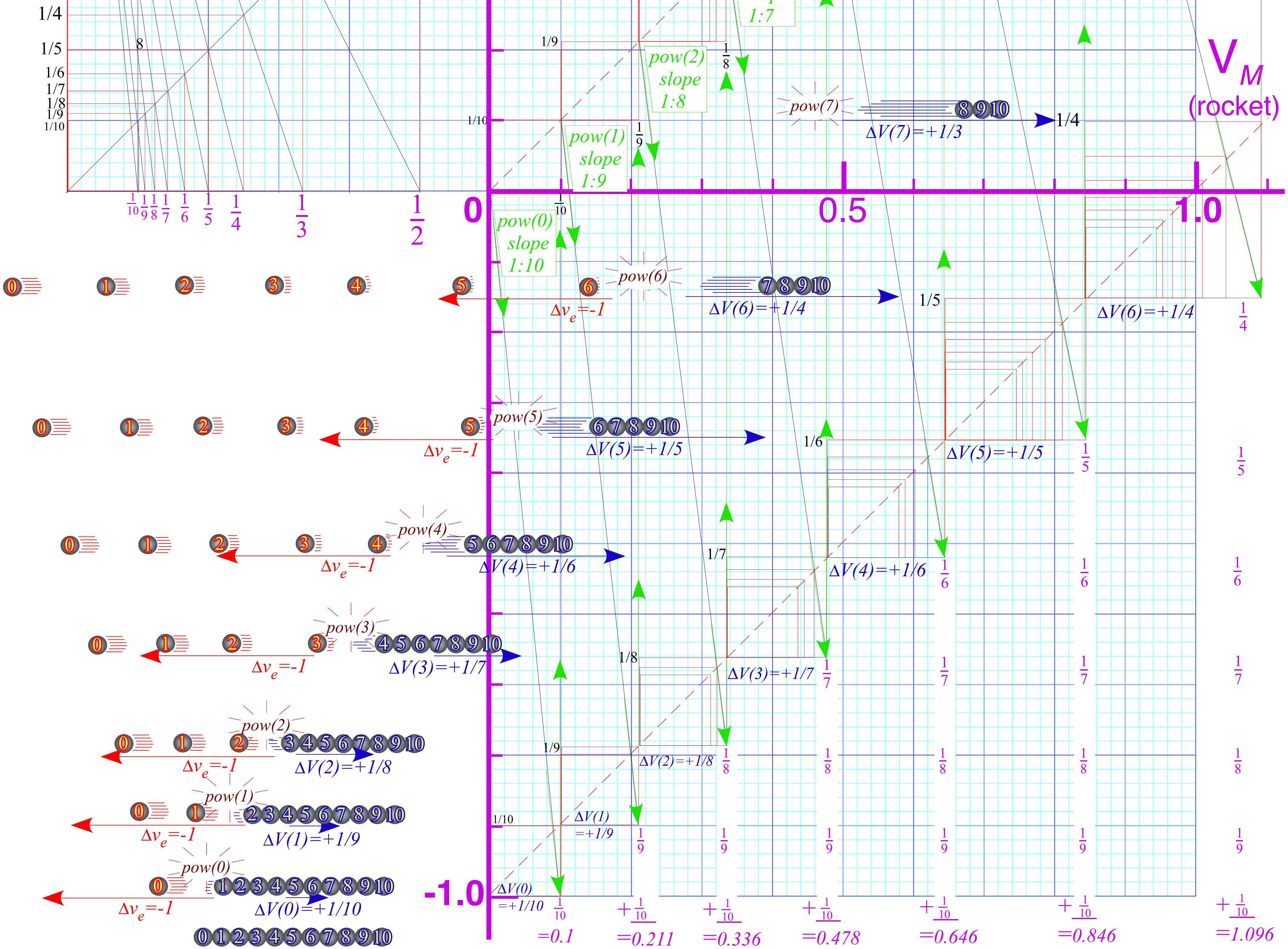
$m\cdot\Delta v_4+6m\cdot\Delta V_M(4)=0$

$m\cdot\Delta v_3+7m\cdot\Delta V_M(3)=0$

$m\cdot\Delta v_2+8m\cdot\Delta V_M(2)=0$

$m\cdot\Delta v_1+9m\cdot\Delta V_M(1)=0$

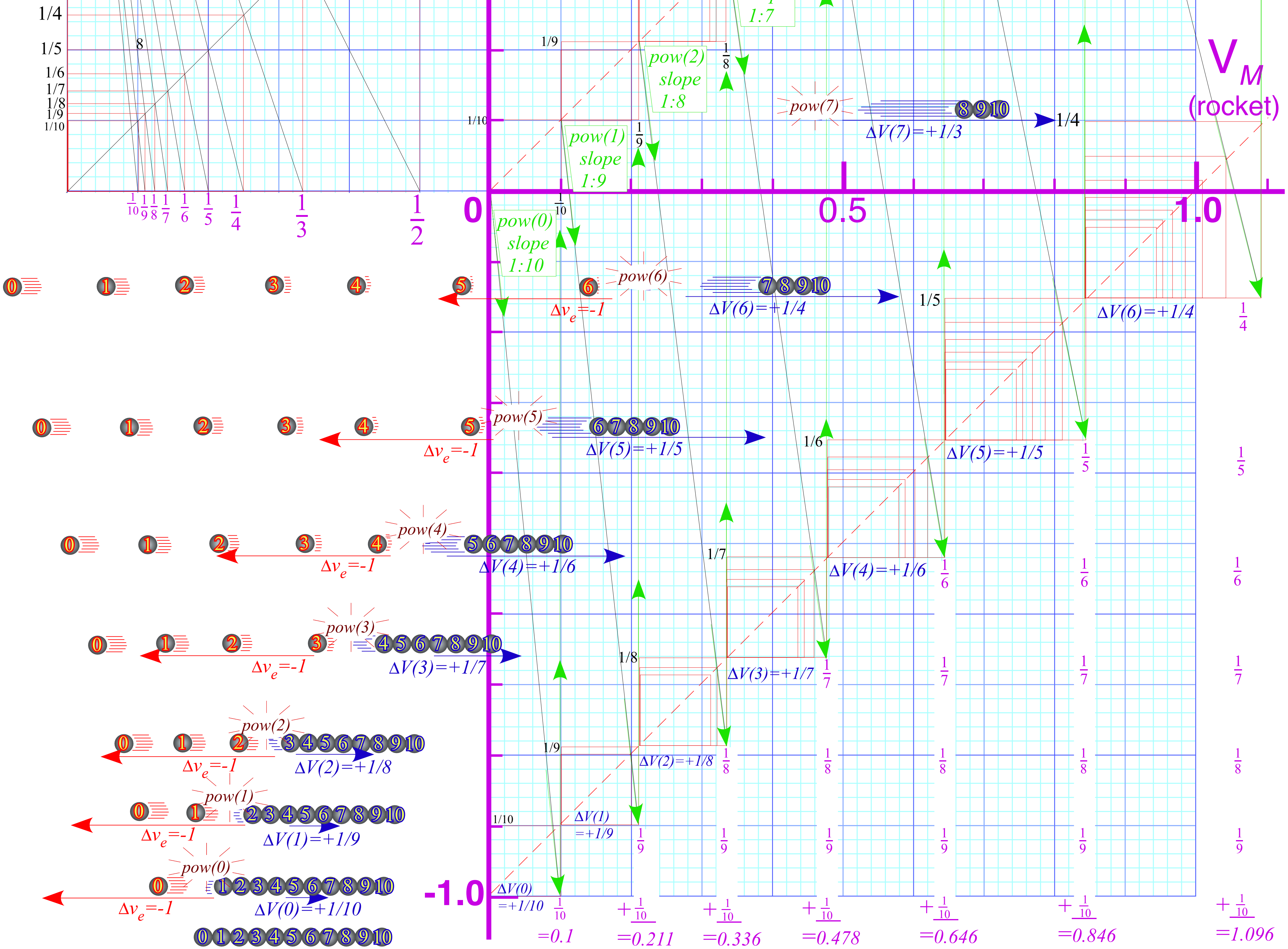
$m\cdot\Delta v_0+10m\cdot\Delta V_M(0)=0$



0th: $V(0)=1/10=0.1$ *1st*: $V(1)=1/10+1/9=0.211$ *2nd*: $V(2)=1/10+1/9+1/8=0.336$

3rd: $V(3)=V(2)+1/7=0.478$ *4th*: $V(4)=V(3)+1/6=0.646$ *5th*: $V(5)=V(4)+1/5=0.846$

6th: $V(6)=V(5)+1/4=1.096$ *7th*: $V(7)= V(6)+1/3=1.429$ *8th*: $V(8)=V(7)+1/2=1.929$



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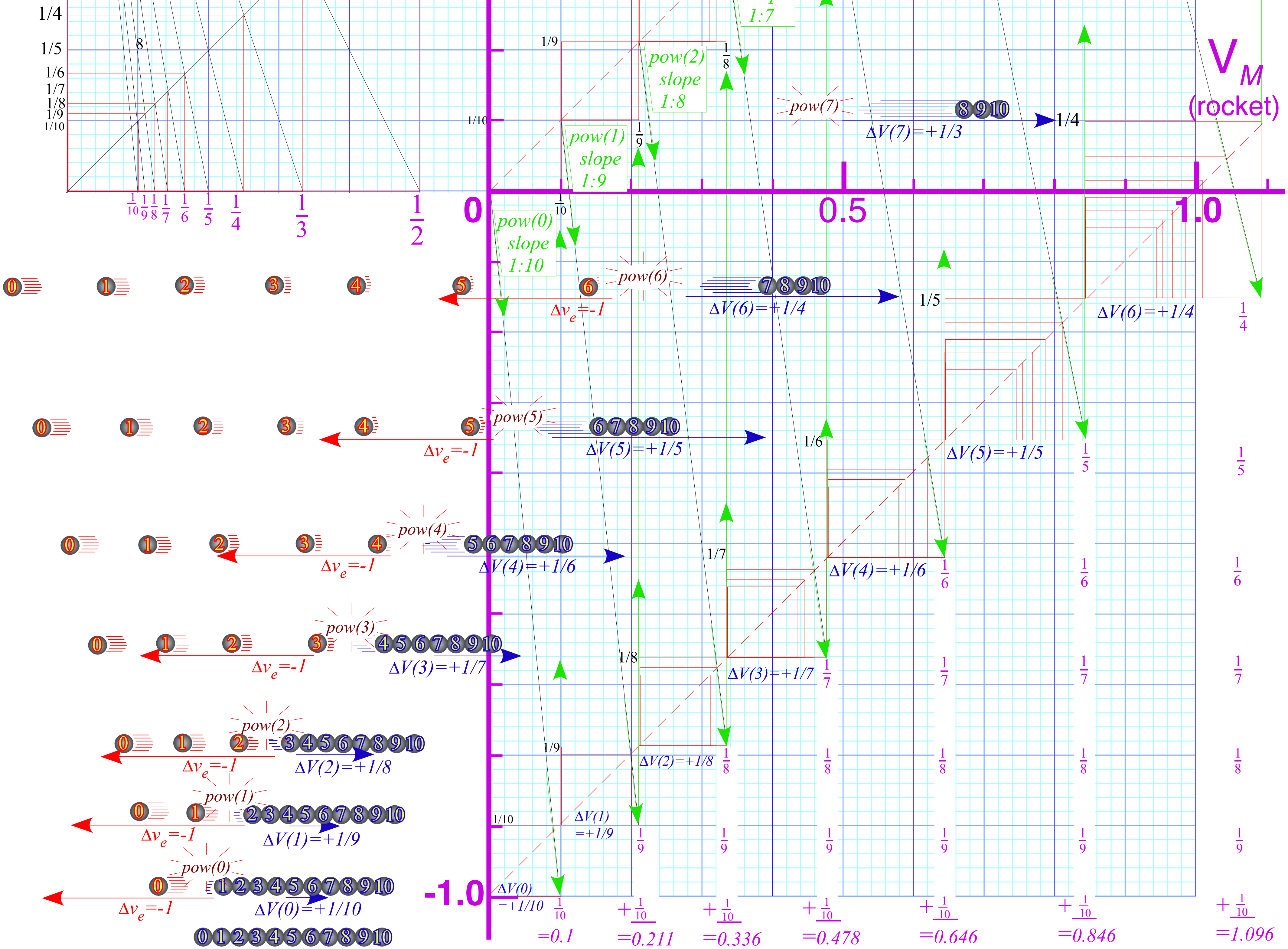
v_e known as **"Specific Impulse"**

By calculus: $M \cdot \Delta V = -v_e \cdot \Delta M$ or: $dV = -v_e \frac{dM}{M}$ Integrate: $\int_{V_{IN}}^{V_{FIN}} dV = -v_e \int_{M_{IN}}^{M_{FIN}} \frac{dM}{M}$



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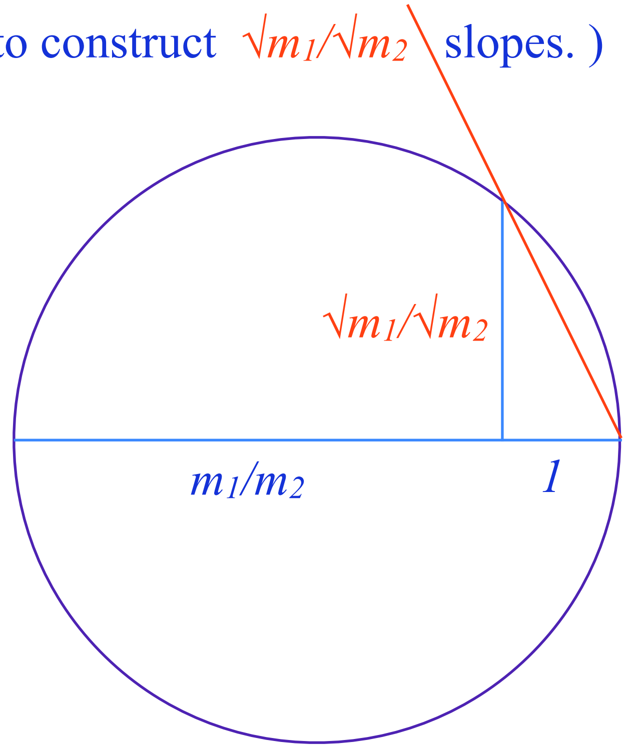
v_e *known as* **“Specific Impulse”**

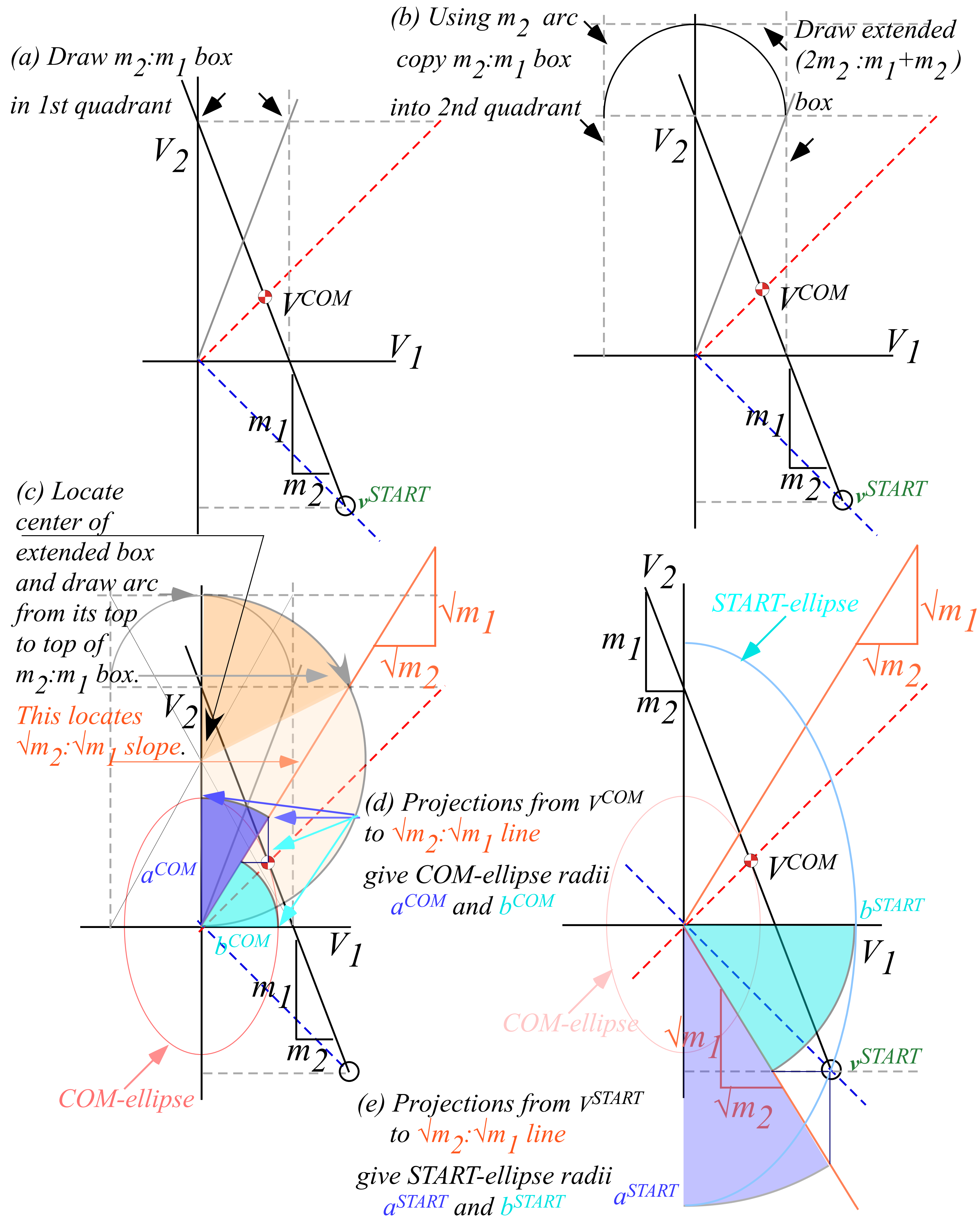
By calculus: $M\cdot\Delta V=-v_e\cdot\Delta M$ or: $dV=-v_e\dfrac{dM}{M}$ Integrate: $\int_{V_{IN}}^{V_{FIN}} dV=-v_e\int_{M_{IN}}^{M_{FIN}}\dfrac{dM}{M}$

The Rocket Equation: $V_{FIN}-V_{IN}=-v_e\left[\ln M_{FIN}-\ln M_{IN}\right]=v_e\left[\ln\dfrac{M_{IN}}{M_{FIN}}\right]$

A Thales construction for momentum-energy

(Made obsolete by Estrangian scaling to circular (V_1,V_2) plots. Still, one has to construct $\sqrt{m_1}/\sqrt{m_2}$ slopes.)





Unit 1
Fig. 8.4a-d

This is a detailed construction of the energy ellipse in a Largangian (v_1,v_2) plot given the initial (v_1,v_2).

The Estrangian (V_1,V_2) plot makes the (v_1,v_2) plot and this construction obsolete.

(Easier to just draw circle through initial (V_1,V_2).)

AMOP reference links *(Updated list given on **2nd** and 3rd pages of each class presentation)*

Web Resources - front page
UAF Physics UTube channel

Quantum Theory for the Computer Age
Principles of Symmetry, Dynamics, and Spectroscopy
Classical Mechanics with a Bang!
Modern Physics and its Classical Foundations

2014 AMOP
2017 Group Theory for QM
2018 AMOP

Representaions Of Multidimensional Symmetries In Networks - harter-jmp-1973

Alternative Basis for the Theory of Complex Spectra

- Alternative_Basis_for_the_Theory_of_Complex_Spectra_I_-_harter-pra-1973
- Alternative_Basis_for_the_Theory_of_Complex_Spectra_II_-_harter-patterson-pra-1976
- Alternative_Basis_for_the_Theory_of_Complex_Spectra_III_-_patterson-harter-pra-1977

Frame Transformation Relations And Multipole Transitions In Symmetric Polyatomic Molecules - RMP-1978
Asymptotic eigensolutions of fourth and sixth rank octahedral tensor operators - Harter-Patterson-JMP-1979
Rotational energy surfaces and high- J eigenvalue structure of polyatomic molecules - Harter - Patterson - 1984
Galloping waves and their relativistic properties - ajp-1985-Harter
Rovibrational Spectral Fine Structure Of Icosahedral Molecules - Cpl 1986 (Alt Scan)

Theory of hyperfine and superfine levels in symmetric polyatomic molecules.

I) Trigonal and tetrahedral molecules: Elementary spin-1/2 cases in vibronic ground states - PRA-1979-Harter-Patterson (Alt scan)
II) Elementary cases in octahedral hexafluoride molecules - Harter-PRA-1981 (Alt scan)

Rotation–vibration spectra of icosahedral molecules.

I) Icosahedral symmetry analysis and fine structure - harter-weeks-jcp-1989 (Alt scan)
II) Icosahedral symmetry, vibrational eigenfrequencies, and normal modes of buckminsterfullerene - weeks-harter-jcp-1989 (Alt scan)
III) Half-integral angular momentum - harter-reimer-jcp-1991

Rotation-vibration scalar coupling zeta coefficients and spectroscopic band shapes of buckminsterfullerene - Weeks-Harter-CPL-1991 (Alt scan)
Nuclear spin weights and gas phase spectral structure of 12C60 and 13C60 buckminsterfullerene -Harter-Reimer-Cpl-1992 - (Alt1, Alt2 Erratum)
Gas Phase Level Structure of C60 Buckyball and Derivatives Exhibiting Broken Icosahedral Symmetry - reimer-diss-1996
Fullerene symmetry reduction and rotational level fine structure/ the Buckyball isotopomer 12C 13C59 - jcp-Reimer-Harter-1997 (HiRez)
Wave Node Dynamics and Revival Symmetry in Quantum Rotors - harter - jms - 2001
Molecular Symmetry and Dynamics - Ch32-Springer Handbooks of Atomic, Molecular, and Optical Physics - Harter-2006

Resonance and Revivals

I) QUANTUM ROTOR AND INFINITE-WELL DYNAMICS - ISMSLi2012 (Talk) OSU knowledge Bank
II) Comparing Half-integer Spin and Integer Spin - Alva-ISMS-Ohio2013-R777 (Talks)
III) Quantum Resonant Beats and Revivals in the Morse Oscillators and Rotors - (2013-Li-Diss)

Resonance and Revivals in Quantum Rotors - Comparing Half-integer Spin and Integer Spin - Alva-ISMS-Ohio2013-R777 (Talk)
Molecular Eigensolution Symmetry Analysis and Fine Structure - IJMS-harter-mitchell-2013
Quantum Revivals of Morse Oscillators and Farey-Ford Geometry - Li-Harter-cpl-2013
QTCA Unit 10 Ch 30 - 2013
AMOP Ch 0 Space-Time Symmetry - 2019

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https://modphys.hosted.uark.edu/markup/AMOP_References.html
and eventually full on Apache-SOLR *Index and search for* nuanced, whole-site content/metadata level *searching.*

AMOP reference links *(Updated list given on 2nd and 3rd pages of each class presentation)*

(Int.J.Mol.Sci, 14, 714(2013) p.755-774 , *QTCA Unit 7 Ch. 23-26),* *(PSDS - Ch. 5, 7)*

Int.J.Mol.Sci, 14, 714(2013), QTCA Unit 8 Ch. 23-25, QTCA Unit 9 Ch. 26, PSDS Ch. 5, PSDS Ch. 7

Intro spin ½ coupling
Unit 8 Ch. 24 p3

H atom hyperfine-B-level crossing
Unit 8 Ch. 24 p15

Hyperf. theory Ch. 24 p48.

Hyperf. theory Ch. 24 p48.
Deeper theory ends p53

Intro 2p3p coupling
Unit 8 Ch. 24 p17.

Intro LS-jj coupling
Unit 8 Ch. 24 p22.

CG coupling derived (start)
Unit 8 Ch. 24 p39.

CG coupling derived (formula)
Unit 8 Ch. 24 p44.

Lande' g-factor
Unit 8 Ch. 24 p26.

Irrep Tensor building
Unit 8 Ch. 25 p5.

Irrep Tensor Tables
Unit 8 Ch. 25 p12.

Wigner-Eckart tensor Theorem.
Unit 8 Ch. 25 p17.

Tensors Applied to d,f-levels.
Unit 8 Ch. 25 p21.

Tensors Applied to high J levels.
Unit 8 Ch. 25 p63.

Intro 3-particle coupling.
Unit 8 Ch. 25 p28.

Intro 3,4-particle Young Tableaus
GrpThLect29 p42.

Young Tableau Magic Formulae
GrpThLect29 p46-48.

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AMOP reference links *(Updated list given on 2nd and **3rd** and **4th** pages of each class presentation)*

Predrag Cvitanovic's: Birdtrack Notation, Calculations, and Simplification

Chaos_Classical_and_Quantum_-_2018-Cvitanovic-ChaosBook
Group Theory - PUP_Lucy_Day_-_Diagrammatic_notation_-_Ch4
Simplification_Rules_for_Birdtrack_Operators_-_Alcock-Zeilinger-Weigert-zeilinger-jmp-2017
Group Theory - Birdtracks_Lies_and_Exceptional_Groups_-_Cvitanovic-2011
Simplification_rules_for_birdtrack_operators-_jmp-alcock-zeilinger-2017
Birdtracks for SU(N) - 2017-Keppeler

Frank Rioux's: UMA method of vibrational induction

Quantum_Mechanics_Group_Theory_and_C60_-_Frank_Rioux_-_Department_of_Chemistry_Saint_Johns_U
Symmetry_Analysis_for_H20-_H20GrpTheory-_Rioux
Quantum_Mechanics-Group_Theory_and_C60_-_JChemEd-Rioux-1994
Group_Theory_Problems-_Rioux-_SymmetryProblemsX
Comment_on_the_Vibrational_Analysis_for_C60_and_Other_Fullerenes_Rioux-RSP

Supplemental AMOP Techniques & Experiment

Many Correlation Tables are Molien Sequences - Klee (Draft 2016)
High-resolution_spectroscopy_and_global_analysis_of_CF4_rovibrational_bands_to_model_its_atmospheric_absorption-_carlos-Boudon-jqsrt-2017
Symmetry and Chirality - Continuous_Measures_-_Avnir
*

Special Topics & Colloquial References

r-process_nucleosynthesis_from_matter_ejected_in_binary_neutron_star_mergers-PhysRevD-Bovard-2017

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