Dynamics of Potentials and Force Fields
(Ch. 7 and part of Ch. 8 of Unit 1)

Potential energy dynamics of Superballs and related things

Thales geometry and “Sagittal approximation” to superball force law

Geometry and dynamics of single ball bounce

(a) Constant force \( F = -k \) (linear potential \( V = kx \))

Some physics of dare-devil diving 80 ft. into kidee pool

(b) Linear force \( F = -kx \) (quadratic potential \( V = \frac{1}{2}kx^2 \) (like balloon))

(c) Non-linear force (like superball-floor or ball-bearing-anvil)

Geometry and potential dynamics of 2-ball bounce

A parable of RumpCo. vs CrapCorp. (introducing 3-mass potential-driven dynamics)

A story of USC pre-meds visiting Whammo Manufacturing Co.

Geometry and dynamics of n-ball bounces

Analogy with shockwave and acoustical horn amplifier

Advantages of a geometric \( m_1, m_2, m_3, \ldots \) series

A story of Stirling Colgate (Palmolive) and core-collapse supernovae

Many-body 1D collisions

Elastic examples: Western buckboard

Bouncing columns and Newton’s cradle

Inelastic examples: “Zig-zag geometry” of freeway crashes

Super-elastic examples: This really is “Rocket-Science”
**This Lecture’s Reference Link Listing**

- Web Resources - front page
- UAF Physics UTube channel
- Classical Mechanics with a Bang!
- Quantum Theory for the Computer Age
- Principles of Symmetry, Dynamics, and Spectroscopy
- Modern Physics and its Classical Foundations
- 2017 Group Theory for QM
- 2018 Adv CM
- 2018 AMOP
- 2019 Advanced Mechanics

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**Lecture #5**

X2 paper: *Velocity Amplification in Collision Experiments Involving Superballs - Harter, et. al. 1971 (pdf)*

**MIT OpenCourseWare: High School/Physics/Impulse and Momentum**

**Hubble Site: Supernova - SN 1987A**

**BounceItIt Web Animation - Scenarios:**

- **Generic Scenario:** 2-Balls dropped no Gravity (7:1) - V vs V Plot (Power=4)
- 1-Ball dropped w/Gravity=0.5 w/Potential Plot: Power=1, Power=4
- 7:1 - V vs V Plot: Power=1
- 3-Ball Stack (10:3:1) w/Newton plot (y vs t) - Power=4
- 3-Ball Stack (10:3:1) w/Newton plot (y vs t) - Power=1
- 3-Ball Stack (10:3:1) w/Newton plot (y vs t) - Power=1 w/Gaps
- 4-Ball Stack (27:9:3:1) w/Newton plot (y vs t) - Power=4
- 4-Newton's Balls (1:1:1:1) w/Newtonian plot (y vs t) - Power=4 w/Gaps
- 6-Ball Totally Inelastic Collision (1:1:1:1:1:1) w/Gaps: Newtonian plot (t vs x), V6 vs V5 plot
- 5-Ball Totally Inelastic Pile-up w/ 5-Stationary-Balls - Minkowski plot (t vs x1) w/Gaps
- 1-Ball Totally Inelastic Pile-up w/ 5-Stationary-Balls - Vx2 vs Vx1 plot w/Gaps
Running Reference Link Listing

Web Resources - front page
UAF Physics UTube channel

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Modern Physics and its Classical Foundations

Prior to Lecture #5

BounceIt Web Animation - Scenarios:
- 49:1 y vs t
- 49:1 V2 vs V1
- 1:500:1 - 1D Gas Model w/ faux restorative force (Cool)
- 1:500:1 - 1D Gas (Warm)
- 1:500:1 - 1D Gas Model (Cool, Zoomed in)

Farey Sequence - Wolfram
Fractions - Ford-AMM-1938

Monsternbash BounceIt Animations:
- 1000:1 - V2 vs V1, 1000:1 with t vs x - Minkowski Plot
- Quantum Revivals of Morse Oscillators and Farey-Ford Geometry - Li-Harter-2013
- Quantum Revivals of Morse Oscillators and Farey-Ford Geometry - Li-Harter-cpl-2015
- Wave It Web Animation - Scenarios:
  - Quantum Carpet
  - Quantum Carpet wMBars
  - Quantum Carpet wMBars
  - Wave Node Dynamics and Revival Symmetry in Quantum Rotors - Harter-JMS-2001
  - Wave Node Dynamics and Revival Symmetry in Quantum Rotors - Harter-jms-2001 (Publ.)

BounceIt Dual plots
- m1:m2 = 3:1
  - v2 vs v1 and V2 vs V1: (v1, v2)=(1, 0.1), (v1, v2)=(1, 1)
  - Estrangian plot: (v1, v2)=(0, 1), (v1, v2)=(1, -1)
- m1:m2 = 4:1
  - v2 vs v1
  - Estrangian plot: (v1, v2)=(1, 0), (v1, v2)=(1, -1)
- m1:m2 = 100:1
  - V2 vs V1 Estrangian plot, y2 vs y1 plot
With g=0 and 70:10 mass ratio
- With non zero g, velocity dependent damping and mass ratio of 70:35
  - M1=49, M2=1 with Newtonian time plot
  - M1=49, M2=1 with V2 vs V1 plot
- Example with friction
  - Low force constant with drag displaying a Pass-thru, Fall-Thru, Bounce-Off
  - m1:m2 = 3:1 and (v1, v2) = (1, 0) Comparison with Estrangian

X2 paper: Velocity Amplification in Collision Experiments Involving Superballs - Harter et al. 1971 (pdf)
Car Collision Web Simulator: https://modphys.hosted.uark.edu/markup/CMMotionWeb.html
Superball Collision Web Simulator: https://modphys.hosted.uark.edu/markup/BounceItWeb.html; with Scenarios: 1007
BounceIt web simulation with g=0 and 70:10 mass ratio
- With non zero g, velocity dependent damping and mass ratio of 70:35
  - Elastic Collision Dual Panel Space vs Space: Space vs Time (Newton), Time vs. Space(Minkowski)
  - Inelastic Collision Dual Panel Space vs Space: Space vs Time (Newton), Time vs. Space(Minkowski)
Matrix Collision Simulator: M1=49, M2=1 V2 vs V1 plot <<Under Construction>>

More Advanced QM and classical references at the end of this Lecture
Potential energy dynamics of Superballs and related things

Thales geometry and “Sagittal approximation” to force law

Geometry and dynamics of single ball bounce

General Non-linear force (like superball-floor or ball-bearing-anvil)
Constant force $F=-k$ (linear potential $V=lx$)

Some physics of dare-devil-diving 80 ft. into kidee pool
Linear force $F=-kx$ (quadratic potential $V=1/2kx^2$ (like balloon))

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Bouncing columns and Newton’s cradle

Inelastic examples: “Zig-zag geometry” of freeway crashes
Super-elastic examples: This really is “Rocket-Science”
Potential Energy Geometry of Superballs and Related things

(a) Fig. 7.1 (modified)

What is superball bounce force law $F(x)$?

$F(x) = ?$

Thales' geometry and "Sagittal"* approx.

$r = \sqrt{x(2R-x)} \quad (\approx \sqrt{2Rx} \quad \text{for: } x \ll R)$

$x = \frac{r}{2R-x}$

* "bow"

Used for “thin-lense” optics
Potential Energy Geometry of Superballs and Related things

(a) Fig. 7.1 (modified)

(b) What is superball bounce force law $F(x)$?

If superball was a balloon its bounce force law would be linear $F = -k \cdot x$ (Hooke Law)

$$F_{balloon}(x) = P \cdot A = P \cdot \pi r^2$$

$$\approx P \cdot \pi 2Rx$$

$Thales' geometry and "Sagittal" approx.$

$x = \frac{r}{2R-x}$

$\approx \sqrt{2Rx}$ for $x \ll R$

$x < < R$
What is superball bounce force law $F(x)$?

If superball was a balloon its bounce force law would be linear $F = -k \cdot x$ (Hooke Law)

$$F_{balloon}(x) = P \cdot A = P \cdot \pi r^2$$

$$= P \cdot \pi 2Rx = P \cdot 2\pi Rx \approx P \cdot \pi 2Rx$$

$$= kx$$
Potential Energy Geometry of Superballs and Related things

If superball was a balloon its bounce force law would be linear $F = -k \cdot x$ (Hooke Law)

\[ F_{\text{balloon}}(x) = P \cdot A = P \cdot \pi r^2 \]

\[ \approx P \cdot \pi 2Rx = P \cdot 2\pi Rx \]

(Hooke spring constant $k$)

Instead superball force law depends on bulk volume modulus and is non-linear $F \sim x^p + ?$ (Power Law?)

\[ Volume(X) = \int_0^X \pi r^2 \, dx = \int_0^X \pi x(2R - x) \, dx \]
What is superball bounce force law $F(x)$?

If superball was a balloon, its bounce force law would be linear $F = -k \cdot x$ (Hooke Law)

$$F_{balloon}(x) = P \cdot A = P \cdot \pi r^2$$

$$\approx P \cdot \pi 2Rx = P \cdot 2\pi Rx$$

$$= kx$$

Instead superball force law depends on bulk volume modulus and is non-linear $F \sim x^p$ + ? (Power Law?)

$$Volume(X) = \int_0^X \pi r^2 \,dx = \int_0^X \pi x(2R-x) \,dx = \int_0^X 2\pi R x \,dx - \int_0^X \pi x^2 \,dx = R\pi X^2 - \frac{\pi X^3}{3} \approx \begin{cases} R\pi X^2 \quad \text{(for } X \ll R) \\ \frac{4}{3} \pi R^3 \quad \text{(for } X = 2R) \end{cases}$$
Potential Energy Geometry of Superballs and Related things

If superball was a balloon its bounce force law would be linear \( F = -k \cdot x \) (Hooke Law)

\[
F_{balloon}(x) = P \cdot A = P \cdot \pi r^2 \\
\approx P \cdot \pi 2Rx = P \cdot 2\pi Rx \\
= kx
\]

Instead superball force law depends on bulk volume modulus and is non-linear \( F \sim x^p \) ? \(^\dagger\) (Power Law?)

\[
Volume(X) = \int_0^X \pi r^2 \, dx = \int_0^X \pi x(2R-x) \, dx = \int_0^X 2R\pi x \, dx - \int_0^X \pi x^2 \, dx = R\pi X^2 - \frac{\pi X^3}{3} \approx \begin{cases} R\pi X^2 & (\text{for } X << R) \\ \frac{4}{3}\pi R^3 & (\text{for } X = 2R) \end{cases}
\]

It also depends on velocity \( \dot{x} = \frac{dx}{dt} \). Adiabatic differs from Isothermal as shown by “Project-Ball*”


(Discussed after p. 33)
Potential energy dynamics of Superballs and related things

Thales geometry and “Sagittal approximation” to force law

Geometry and dynamics of single ball bounce

General Non-linear force (like superball-floor or ball-bearing-anvil)

Constant force $F=-k$ (linear potential $V=kx$)

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Many-body 1D collisions

Elastic examples: Western buckboard

Bouncing columns and Newton’s cradle

Inelastic examples: “Zig-zag geometry” of freeway crashes

Super-elastic examples: This really is “Rocket-Science”
Sets gravity

This is non-linear $F = -kx^4$

Sometimes need to decrease $k$ for $p = 1$

This is the generic Bouncelt URL (or address):
https://modphys.hosted.uark.edu/markup/BounceltWeb.html
BounceIt Simulation: Force/Potential Plot

Force power = 4

\[ F = \left( \frac{\Delta y}{\Delta x} \right)^2 \]

\[ F(x) = -\frac{\Delta U}{\Delta x} \]

Display of Force vector using similar triangle construction based on the slope of potential curve.

Time = 2.813 s
\( \Delta T = +1.00 \times 10^{-4} \) s
E = +75.506 erg
KE = +24.382 erg
PE = +51.124 erg
y1-R1 = +0.339 cm
Force constant = +50000.00
Force power = +4.000
Drag (Collision) = +0

m1 = 100.000 g
V1y = -0.698 cm/s
(a) Drop height
(Zero kinetic energy)

1990 BounceIt Mac simulations

Details of each case follows using newer BounceIt Web simulations

Unit 1
Fig. 6.2

(b) Maximum kinetic energy
(Zero total force)

(c) Maximum penetration
(Zero kinetic energy again)

Display of Force vector using similar triangle construction based on the slope of potential curve.
Main Control Panel

- Let mouse set: \((x,y,V_x,V_y)\)
- Let mouse set force: \(F(t)\)
- Plot solid paths
- Plot dotted paths
- Plot no paths
- Plot \(Y_1(t), Y_2(t), \ldots\)
- Plot \(V_1, V_2, \ldots\)
- Plot user defined i.e - \(Y_1 vs. Y_2\)
- Balls initially falling
- Balls initially fixed
- No preset initial values

Sets gravity

Number of masses
- \(1\) Balls

Collision friction (Viscosity)
- \(0\) \(x10^\text{ }\)

Initial gap between balls
- \(5.45\) \(x10^\text{ }\)

Force power law exponent
- \(1\)

Force Constant
- \(500\)

Canvas Aspect Ratio - W/H i.e. 0.75 & 1.0
- \(0.75\)

Acceleration of gravity
- \(0.5\) \(100x\text{cm/s}^2\)

This is linear setting (increase for non-linear)

This is the generic Bouncelt URL (or address):
https://modphys.hosted.uark.edu/markup/BounceltWeb.html

(Simulations)
(a) Drop height $h$
(Zero kinetic energy)

Total Force curve $F(x) + mg$

Total potential energy curve $U(x) + mgY$

Display of Force vector using similar triangle construction based on the slope of potential curve.

BounceIt Simulation: Force/Potential Plot
(Force power=4)
BounceIt Simulation: Force/Potential Plot
(Force power=4)
Max penetration (Zero kinetic energy again)

Display of Force vector using similar triangle construction based on the slope of potential curve.
(a) Drop height \( h \)
(Zero kinetic energy)

(b) Maximum kinetic energy
(Zero total force)

(c) Maximum penetration
(Zero kinetic energy again)

Display of Force vector using similar triangle construction based on the slope of potential curve.
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Super-elastic examples: This really is “Rocket-Science”
Force $F(x)$ and Potential $U(x)$ for soft heavy non-linear superball

$F_{\text{total}}(y) = -Mg + F_{\text{ball}}(y)$

$U_{\text{total}}(y) = -Mgx + U_{\text{ball}}(y)$

Total Energy $E = Mg\,h$

Unit 1 Fig. 7.5

$U_{\text{total}}(y_{\text{max}}) = \int_{y_{\text{static}}}^{y_{\text{max}}} F_{\text{total}}(y) \, dy + \int_{y=h}^{y_{\text{static}}} F_{\text{total}}(y) \, dy + U(h) = U(h) = E$

$F(x) = -\frac{dU(x)}{dx}$
**Force F(x) and Potential U(x) for soft heavy non-linear superball**

Unit 1
Fig. 7.5

\[ F_{\text{total}}(y) = -Mg + F_{\text{ball}}(y) \]

\[ U_{\text{total}}(y) = -Mgx + U_{\text{ball}}(y) \]

**Total Energy**

\[ E = Mgh \]

\[ U_{\text{total}}(y_{\text{max}}) = \int_{y_{\text{static}}}^{y_{\text{max}}} F_{\text{total}}(y) \, dy + \int_{y=0}^{y=h} F_{\text{total}}(y) \, dy + U(h) = U(h) = E \]

**Work**

\[ W = W = \int F(x) \, dx = \text{Energy acquired} = \text{Area of } F(x) = -U(x) \]

\[ F(x) = -\frac{dU(x)}{dx} \]
Force $F(x)$ and Potential $U(x)$ for soft heavy non-linear superball

Unit 1 Fig. 7.5

$U_{\text{total}}(y) = -Mg + U_{\text{ball}}(y)$

$F_{\text{total}}(y) = -Mg + F_{\text{ball}}(y)$

Total Energy $E = Mgh$

$F_{\text{total}}(h)$

$U_{\text{total}}(y_{\text{max}}) = \int_{y_{\text{static}}}^{y_{\text{max}}} F_{\text{total}}(y) \, dy + \int_{y=h}^{y_{\text{static}}} F_{\text{total}}(y) \, dy + U(h) = U(h) = E$

Work $= W = \int F(x) \, dx = \text{Energy acquired} = \text{Area of } F(x) = -U(x)$

Impulse $= P = \int F(t) \, dt = \text{Momentum acquired} = \text{Area of } F(t) = P(t)$

$F(x) = -\frac{dU(x)}{dx}$

$F(t) = \frac{dP(t)}{dt}$
Potential energy dynamics of Superballs and related things

Thales geometry and “Sagittal approximation” to force law

Geometry and dynamics of single ball bounce

General Non-linear force (like superball-floor or ball-bearing-anvil)

Constant force $F=-k$ (linear potential $V=kx$)

Some physics of dare-devil-diving 80 ft. into kidee pool

Linear force $F=-kx$ (quadratic potential $V=\frac{1}{2}kx^2$ (like balloon))

Geometry and potential dynamics of 2-ball bounce

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Advantages of a geometric $m_1$, $m_2$, $m_3$, ... series

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Many-body 1D collisions

Elastic examples: Western buckboard

Bouncing columns and Newton’s cradle

Inelastic examples: “Zig-zag geometry” of freeway crashes

Super-elastic examples: This really is “Rocket-Science”
Work \( W = \int F(x) \, dx \) = Energy acquired = Area of \( F(x) = -U(x) \)

\[ F(x) = -\frac{dU(x)}{dx} \]

Impulse \( P = \int F(t) \, dt \) = Momentum acquired = Area of \( F(t) = P(t) \)

\[ F(t) = \frac{dP(t)}{dt} \]
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Super-elastic examples: This really is “Rocket-Science”
Sets gravity

This is linear $F = -kx$

Usually need to increase $k$ for $p > 1$

This is linear $F = -kx^p$

(increase $p > 1$ for non-linear $F = -kx^p$)
m1 = 100.000 g
Vly = +0.578 cm/s

Time = 25.190 s
ΔT = +3.00e-4 s
E = +25.000 erg
KE = +16.693 erg
PE = +8.307 erg
y1-R1 = -0.308 cm
Force constant = +500.000
Force power = +1.000
Drag (Collision) = +0
Potential energy dynamics of Superballs and related things

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Linear force $F=-kx$ (quadratic potential $V=\frac{1}{2}kx^2$ (like balloon)) (Calculations)

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Note dashed curve followed by PE minimum. Parabola? What?
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Parable allegory for Los Alamos
Cheap & practical “seat-of-the-pants” approach

Parable allegory for Livermore
Fancy & overpriced “political” approach

Advantages of a geometric $m_1, m_2, m_3, ...$ series
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Many-body 1D collisions
Elastic examples: Western buckboard
Bouncing columns and Newton’s cradle
Inelastic examples: “Zig-zag geometry” of freeway crashes
Super-elastic examples: This really is “Rocket-Science”
Velocity amplification or "throw" factor = 2.5

Parable allegory for Los Alamos
Cheap&practical "seat-of-the pants" approach

Unit 1
Fig. 7.6
Parable allegory for Los Alamos
Cheap & practical “seat-of-the-pants” approach

Parable allegory for Livermore
Fancy & overpriced “political” approach

**RumpCo**
Project Ball
2-Bang Model

- Finite initial gap
- 2 Bangs
- $V_2 = 2.5$
- $V_1 = 0.5$

**Crap Corp**
Star Wars Division
Super Elastic Bounce
Full Force Field Simulation

- Initial gap
- 2 Bangs
- $V_2 = 2.29472855$
- $V_1 = 0.61730277$

*Velocity amplification or “throw” factor = 2.5*

*Velocity amplification or “throw” factor = 2.3*

(about equal to RumpCo finite gap experiment)

Unit 1
Fig. 7.6
Sets gravity

This is non-linear $F=-kx^4$

Usually need to decrease $k$ for $p=1$

BounceIt Simulation: Zero Gap

(Force power=4)

Collision friction (Viscosity)

Initial gap between balls

Force Constant

Force power law exponent

Canvas Aspect Ratio - W/H i.e. 0.75 & 1.0

Balls initially falling

Balls initially fixed

No preset initial values

Let mouse set: $(x,y,Vx,Vy)$

Let mouse set force $F(t)$

Plot solid paths

Plot dotted paths

Plot $V1$ vs. $V2$

Plot $Y1(t)$, $Y2(t)$,

Plot PE of $m1$ vs. $Y1$

Plot $V2$ vs. $V1$

Plot user defined i.e - $Y1$ vs. $Y2$

Plot Ellipses

Plot Bisector Lines

Old Color Scheme

Show right panel information

Show left panel information

Set Initial positions

Number of masses

Acceleration of gravity

0.0000 $\times 10^4$

Balls

1.00

1.09

0.5

0.003

0.0000

$7.0 \times 10^8$

$6.0 \times 10^8$

$0.8 \times 10^8$

$-2.6944$

$2.6944$

$-1.0 \times 10^8$

$0.0 \times 10^8$

$-1.0 \times 10^8$

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$0.8 \times 10^8$

$0.8 \times 10^8$
BounceIt Simulation: Zero Gap

Force power = 4

\( m_2 = 10.000 \text{ g} \)
\( m_1 = 70.000 \text{ g} \)

\( v_y = +2.291 \text{ cm/s} \)
\( v_y = +0.627 \text{ cm/s} \)

Time = 3.877 s
\( \Delta T = +0.001 \text{ s} \)
KE = +40.000 erg
Force constant = +50000.000
Force power = +4.000
Drag (Collision) = +0

BounceIt Simulation: Zero Gap
(Force power=4)
This is linear $F=-kx$.

Usually need to increase $k$ for $p > 1$.

This is linear $F=-kx^p$ (increase $p > 1$ for non-linear $F=-kx^p$).
BounceIt Simulation: Zero Gap

\( m_2 = 10.000 \, \text{g} \)
\( m_1 = 70.000 \, \text{g} \)

\( v_{2y} = +1.029 \, \text{cm/s} \)
\( v_{1y} = +0.996 \, \text{cm/s} \)

Time = 5.470 s
\( \Delta T = +0.001 \, \text{s} \)
\( KE = +40.000 \, \text{erg} \)
\( \text{Force constant} = +500.000 \)
\( \text{Force power} = +1.000 \)
\( \text{Drag (Collision)} = +0 \)
Cooperation between Los Alamos and Livermore yields insight to answer “What’s going on?”

Quartic \( F(y) = y^4 \)
Quadratic \( F(y) = y^2 \)
Linear Force Field

Cra Rumpany Ltd

Quite surprising “non-effect”!
Why?

\[ V_2 = 1.03 \]
\[ V_1 = 0.996 \]

Unit 1
Fig. 7.7
Cooperation between Los Alamos and Livermore yields insight to answer “What’s going on?”

**Velocity amplification**

or “throw” factor = 1.03

(practically “no-throw”)

for linear force $F(y) = ky$
Cooperation between Los Alamos and Livermore yields insight to answer “What’s going on?”

- **Quartic** $F(y) = y^4$
- **Quadratic** $F(y) = y^2$
- **Linear Force** $F(y) = y^1$

**Flat part of non-linear force gives “explosive” effect**

**Velocity amplification or “throw” factor** $= 1.03$

(practically “no-throw”)

for linear force $F(y) = ky$

**Lesson:** Fasten your seatbelt
Cooperation between Los Alamos and Livermore yields insight to answer “What’s going on?”

**Linear Force Field**

- Quadratic: $F(y) = y^2$
- Linear: $F(y) = y^1$
- Quartic: $F(y) = y^4$

**Simulation**

- Flat part of non-linear force gives “explosive” effect
- Velocity amplification or “throw” factor $= 1.03$ (practically “no-throw”)

**Unit 1**

**Fig. 6.7**

- $V_2 = 1.03$
- $V_1 = 0.996$

**Lesson:** Fasten your seatbelt TIGHTLY!
Potential energy dynamics of Superballs and related things

Thales geometry and “Sagittal approximation” to force law

Geometry and dynamics of single ball bounce

(a) Constant force $F=-k$ (linear potential $V=kx$)

Some physics of dare-devil-diving 80 ft. into kiddee pool

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**Velocity Amplification in Collision Experiments Involving Superballs**

CLASS OF WILLIAM G. HARTER*
University of Southern California
Los Angeles, California 90007
(Received 25 September 1969; revised 25 September 1970)

If a pen is stuck in a hard rubber ball and dropped from a certain height, the pen may bounce to several times that height. The results of two such experiments, which can easily be duplicated in any undergraduate physics laboratory, are plotted for a range of mass ratios. A simple theoretical discussion which provides a qualitative understanding of the phenomenon is presented. A more complicated formulation which agrees very well with one of the experiments is also presented. The latter involves a simple analog computer program. Finally, an intriguing generalization of the phenomenon is considered.


**ACKNOWLEDGMENT**

We would like to thank John C. Fakan, John E. Heighway, and John II. Marburger for help during the initial and final stages of this project.

**INTRODUCTION**

Shortly after the well-known Superball\(^1\) appeared on the market, one of the authors quite accidentally discovered a surprising effect.\(^2\) The point of a ball point pen is imbedded in the surface of a 3-in. diam Superball, and the pen and ball are dropped from a height of 4 or 5 ft so that the pen remains above the ball and perpendicular to a hard floor below. As the ball strikes the floor, the pen may be ejected so violently that it will strike the ceiling of the average room with considerable force. Furthermore, one can adjust the mass of the pen so that the ball remains completely at rest on the floor after ejecting the pen.

\(^1\) Tradename of product by Whammo Manufacturing Co., San Gabriel, Calif.

---

**Fig. 14.** Two designs for a multiple stage tower of balls. (a) Large number of balls can slide on a shaft. (b) Balls connected by small pins stand to lose appreciable amounts of binding energy.
A story of USC pre-meds visiting Whammo Manufacturing Co.

Velocity Amplification in Collision Experiments Involving Superballs <Link>

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Much later....

Lots of profs try this out...

...including the unfortunate Harvard professor M. Tinkham...

(Still trying to find the video of the Tinkham incident...)

A story of USC pre-meds visiting Whammo Manufacturing Co.

...and some results of “Project-Ball”

After initial big NBC splash (Ray Dunkin Reports) in Fall 1968, USC mechanical engineers kindly measured super-ball force curves $F(y)$ with their precision tensiometer and let us use their analog computer to calculate precise bounce heights.
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The rest is history.
Little paint spots on floor show what was wrong with our fancy-pants computer theory
A story of USC pre-meds visiting Whammo Manufacturing Co.

...and some results of “Project-Ball”

The rest is history.
Little paint spots on floor show what was wrong with our fancy-pants computer theory.

The engineering curves were isothermal not adiabatic.
Need latter. Can do latter by dropping dyed balls and measuring spot-size.

Measuring spot-size $d$ gives energy vs. height.
Slope of $E(x)$ gives force $F(x)$ and $G(x)$.

Fig. 10. Sagittal formula.
A story of USC pre-meds visiting Whammo Manufacturing Co.

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Then fancy-pants computer theory can predict N-ball tower bounce
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Then Fancy-pants computer theory can predict N-ball tower bounce...
Then fancy-pants computer theory can predict N-ball tower bounces.

Here are some 3-ball tower bounce predictions

Class of W. G. Harter

Fig. 11. Adiabatic force \( F(x) \) and energy curves for Superball.

Fig. 13. Comparison between analog computer gain curves and second experiment.

Fancy-pants computer theory fits experiment better

Functions \( F(x) \) and \( G(x) \) were then placed on the function generators of the analog computer.

Fig. 14. Two designs for a multiple stage tower of balls. (a) Large number of balls can slide on a shaft. (b) Balls connected by small pins stand to lose appreciable amounts of binding energy.

Fig. 15. (a)–(d) Analog computer output for velocity gains of three-ball system.
Potential energy dynamics of Superballs and related things

Thales geometry and “Sagittal approximation” to force law

Geometry and dynamics of single ball bounce
(a) Constant force \( F = -k \) (linear potential \( V = kx \))
Some physics of dare-devil-diving 80 ft. into kidee pool
(b) Linear force \( F = -kx \) (quadratic potential \( V = \frac{1}{2}kx^2 \) (like balloon))
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Advantages of a geometric \( m_1, m_2, m_3, ... \) series
A story of Stirling Colgate (Palmolive) and core-collapse supernovae

Many-body 1D collisions
Elastic examples: Western buckboard
Bouncing columns and Newton’s cradle
Inelastic examples: “Zig-zag geometry” of freeway crashes
Super-elastic examples: This really is “Rocket-Science”
Unit 1

Fig. 8.1a-c
Independent Bang Model (IBM)
3-Body Geometry

(a) Quartic Force
\[ F(y) = ky^4 \]

\[
\begin{align*}
m_3 &= 10 \text{ kg} \\
m_2 &= 30 \text{ kg} \\
m_1 &= 100 \text{ kg}
\end{align*}
\]

Initial Velocities
\[
\begin{align*}
V_3 &= -1 \text{ m/s} \\
V_2 &= -1 \text{ m/s} \\
V_1 &= -1 \text{ m/s}
\end{align*}
\]

Final Velocities
\[
\begin{align*}
V_3 &= 3.41 \text{ m/s} \\
V_2 &= 0.701 \text{ m/s} \\
V_1 &= 0.298 \text{ m/s}
\end{align*}
\]

(b) Independent Collisions (Independent of Force Law)

BounceIt Simulation: 3-Ball Tower w/ Quartic Force

(c) Linear Force
\[ F(y) = ky \]

\[
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Final Velocities
\[
\begin{align*}
V_3 &= 3.52 \text{ m/s} \\
V_2 &= 0.538 \text{ m/s} \\
V_1 &= 0.077 \text{ m/s}
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\]

BounceIt Simulation: 3-Ball (Gapped) Tower w/ Linear Force
Unit 1
Fig. 8.1b
Independent Bang Model (IBM)
3-Body Geometry

<table>
<thead>
<tr>
<th>INITIAL VELOCITIES</th>
<th>FINAL VELOCITIES</th>
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<tbody>
<tr>
<td>( \mathbf{V}_1 = -1 \text{ m/s} )</td>
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BounceIt Simulation: 3-Ball Tower w/ Linear Force

\( m_1 = 100 \text{ kg} \)
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1.8.3 The optimal idler (An algebra/calculus problem)

To get highest final \( v_3 \) of mass \( m_3 \) find optimum mass \( m_2 \) in terms of masses \( m_1 \) and \( m_3 \) that does that.

*J. B. Hart and R. B. Herrmann, Amer. J. Phys. 36, 46 (1968).*
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Hubble Site: Supernova - SN 1987A

Source
http://hubblesite.org/newscenter/archive/releases/2007/10/image/a/

Author
NASA, ESA, P. Challis, and R. Kirshner (Harvard-Smithsonian Center for Astrophysics)

Core-burning nuclear fusion stages for a 25-solar mass star

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<tr>
<th>Process</th>
<th>Main fuel</th>
<th>Main products</th>
<th>Temperature (Kelvin)</th>
<th>Density (g/cm³)</th>
<th>Duration</th>
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<tr>
<td>hydrogen burning</td>
<td>hydrogen</td>
<td>helium</td>
<td>7×10⁷</td>
<td>10</td>
<td>10⁷ years</td>
</tr>
<tr>
<td>triple-alpha process</td>
<td>helium</td>
<td>carbon, oxygen</td>
<td>2×10⁸</td>
<td>2000</td>
<td>10⁶ years</td>
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<tr>
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<td>carbon</td>
<td>Ne, Na, Mg, Al</td>
<td>8×10⁸</td>
<td>10⁶</td>
<td>10³ years</td>
</tr>
<tr>
<td>neon burning process</td>
<td>neon</td>
<td>O, Mg</td>
<td>1.6×10⁹</td>
<td>10⁷</td>
<td>3 years</td>
</tr>
<tr>
<td>oxygen burning process</td>
<td>oxygen</td>
<td>Si, S, Ar, Ca</td>
<td>1.8×10⁹</td>
<td>10⁷</td>
<td>0.3 years</td>
</tr>
<tr>
<td>silicon burning process</td>
<td>silicon</td>
<td>nickel (decays into iron)</td>
<td>2.5×10⁹</td>
<td>10⁸</td>
<td>5 days</td>
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Source
http://hubblesite.org/newscenter/archive/releases/2007/10/image/a/

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<td>10⁷</td>
<td>0.3 years</td>
</tr>
<tr>
<td>silicon burning process</td>
<td>silicon</td>
<td>nickel (decays into iron)</td>
<td>2.5x10⁹</td>
<td>10⁸</td>
<td>5 days</td>
</tr>
</tbody>
</table>
Stirling Auchincloss Colgate (November 14, 1925 – December 1, 2013) was an American physicist at Los Alamos National Laboratory and a professor emeritus of physics, past president at the New Mexico Institute of Mining and Technology (New Mexico Tech),[1] and an heir to the Colgate toothpaste family fortune.[2] He was America's premier diagnostician of thermonuclear weapons during the early years at the Lawrence Livermore National Laboratory in California. While much of his involvement with physics is still highly classified, he made many contributions in the open literature including physics education and astrophysics.[3] He was born in New York City in 1925, to Henry Auchincloss and Jeanette Thurber (née Pruyn) Colgate.[4]

..an amusing off-color aside story of Stirling Colgate’s NMIMT resignation...

(Not told in Wikipedia!)

Quote

- "I was always enamored with explosives, and eventually I graduated to dynamite and then nuclear bombs."
Multiple-collision accelerator assembly

ABSTRACT

A device comprising several highly elastic objects is presented whose purpose is to demonstrate an unobvious consequence of fundamental laws of physics—the acceleration of an object to high speed by multiple collisions among a series of heavier objects moving at slower speed. The objects, each of different mass, are arrayed in close proximity in order of decreasing mass with their centers lying along a straight line. This arrangement of the assembly of objects is maintained by a constraining element which permits the assembly axis to be oriented in any desired direction and permits the assembly to be moved or manipulated as a unit in any desired way without destroying the arrangement of objects. In the preferred embodiment the elastic objects are polybutadiene balls (12), the constraining element is an interior guide-pin (10) fastened in the largest ball and extending radially therefrom, on which the remaining balls can slide freely because of diametrical holes formed in them. In use this multiple-collision accelerator assembly is suspended in vertical orientation, with the largest ball downward, by holding the tip-end of the guide-pin which extends beyond the littlest ball. The assembly is then dropped onto a solid surface (14), the striking of which produces a sharp impulse that is transmitted from the largest ball, through the assembly, causing the littlest ball to be projected to a height many times that from which the assembly was dropped.

1st publication describing theory and experiment of this device 20 years before.

Velocity Amplification in Collision Experiments Involving Superballs

William G. Harter\(^1\) (class of WGH)

---

HIDE AFFILIATIONS

\(^1\) University of Southern California, Los Angeles, California 90007

View the Scitation page for University of Southern California (USC).

Am. J. Phys. 39, 656 (1971); http://dx.doi.org/10.1119/1.1986253

(Point allowing patent over previous 1973 proposal (4))

Now I have to pay APS for my own paper.)
Potential energy dynamics of Superballs and related things

Thales geometry and “Sagittal approximation” to force law

Geometry and dynamics of single ball bounce

(a) Constant force $F = -k$ (linear potential $V = kx$)

Some physics of daredevil-diving 80 ft. into kidee pool

(b) Linear force $F = -kx$ (quadratic potential $V = \frac{1}{2}kx^2$ (like balloon))

(c) Non-linear force (like superball-floor or ball-bearing-anvil) (Simulations)

Geometry and potential dynamics of 2-ball bounce

A parable of RumpCo. vs CrapCorp. (introducing 3-mass potential-driven dynamics)

A story of USC pre-meds visiting Whammo Manufacturing Co.

Geometry and dynamics of n-ball bounces

Analogy with shockwave and acoustical horn amplifier

Advantages of a geometric $m_1$, $m_2$, $m_3$, ... series

A story of Stirling Colgate (Palmolive) and core-collapse supernovae (Leads to Sagittal potential analysis of 2, 3, and 4 body towers)

Many-body 1D collisions

Elastic examples: Western buckboard

Bouncing columns and Newton’s cradle

Inelastic examples: “Zig-zag geometry” of freeway crashes

Super-elastic examples: This really is “Rocket-Science”
Western buckboard = ?????
Western buckboard = ?????
Western buckboard = 3-ball analogy
Western buckboard  = 3-ball analogy Disaster!
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Unit 1
Fig. 8.2a-b
4-Body IBM Geometry
Fig. 8.2c-d
4-Equal-Body Geometry

BounceIt Simulation: 4-Ball Tower w/ $m_k/m_{k+1} = 3$

BounceIt Simulation: 4-Ball Tower w/ $m_k/m_{k+1} = 1$

4-Equal-Body
“Shockwave” or pulse wave Dynamics

Opposite of continuous wave dynamics introduced in Unit 2
Potential energy dynamics of Superballs and related things
Thales geometry and “Sagittal approximation” to force law
Geometry and dynamics of single ball bounce
(a) Constant force $F=-k$ (linear potential $V=kv$)
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(b) Linear force $F=-kv$ (quadratic potential $V=\frac{1}{2}kv^2$ (like balloon))
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Inelastic examples: “Zig-zag geometry” of freeway crashes
First recall “zig-zag” fractions of “Monster Mash” in Lect. 4

Trajectory geometry exposed (Harmonic series 1/1, 1/2, 1/3, 1/4, ...)

Space

Time
**Unit 1**

**Fig. 8.5**

Pile-up: One 60mph car hits five standing cars

---

**BounceIt Simulation:** One ball hits 5 stationary balls \((y \text{ vs } x)\) and \((x_i \text{ vs } t)\)

These graphs plot user determined quantities.
Choose and select from a context menu via right click on target axis, like the following set to \(V_{ix}\) and \(V_{2x}\)

**BounceIt Simulation:** One ball hits 5 stationary balls \((y \text{ vs } x)\) and \((V_{i+1x} \text{ vs } V_{ix})\)
BounceIt Simulation: 5 balls hit 1 stationary ball (y vs x) and (v6x vs v5x)
**Speeding car and five stationary cars**

\(\mathbf{V}_M(0) = 60, \ \mathbf{v}_m(1) = 0\)

\[\begin{align*}
\mathbf{V}_M(0) &= 30 \\
\mathbf{V}_M(12) &= 20 \\
\mathbf{V}_M(01) &= 15 \\
\mathbf{V}_M(01234) &= 12 \\
\mathbf{V}_M(01235) &= 10
\end{align*}\]

---

**Five speeding cars and a stationary car**

\(\mathbf{V}_M(1) = 60, \ \mathbf{v}_m(0) = 0\)

\[\begin{align*}
\mathbf{V}_M(1) &= 30 \\
\mathbf{V}_M(10) &= 40 \\
\mathbf{V}_M(21) &= 40 \\
\mathbf{V}_M(32) &= 45 \\
\mathbf{V}_M(4321) &= 48 \\
\mathbf{V}_M(54321) &= 50
\end{align*}\]

---

**Five speeding cars and five stationary cars**

\(\mathbf{V}_M(1) = 60, \ \mathbf{v}_m(0) = 0\)

\[\begin{align*}
\mathbf{V}_M(1) &= 30 \\
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\end{align*}\]

---

Unit 1

**Fig. 8.5**

Pile-up: One 60mph car hits five standing cars

---

**Fig. 8.6**

Pile-up: Five 60mph cars hit one standing cars

---

**Fig. 8.7**

Pile-up: Five 60mph cars hit five standing cars

---

*(Fug-gedda-aboud-dit!!)*

(Many possible scenarios depending on initial positions!)

_BounceIt Simulation: 5 balls hits 5 stationary balls (y vs x) and (x_i vs t)_
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Super-elastic examples: This really is “Rocket-Science”

(Simulations)

(Leads to Sagittal potential analysis of 2, 3, and 4 body towers)
(a) Harmonic progression
\[ 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \ldots \]

(b) Harmonic series
\[ 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \ldots \]

Unit 1
Fig. 8.8a-b

Rocket Science!

\[ m \Delta v_7 + 3m \Delta V_M(7) = 0 \]
\[ m \Delta v_6 + 4m \Delta V_M(6) = 0 \]
\[ m \Delta v_5 + 5m \Delta V_M(5) = 0 \]
\[ m \Delta v_4 + 6m \Delta V_M(4) = 0 \]
\[ m \Delta v_3 + 7m \Delta V_M(3) = 0 \]
\[ m \Delta v_2 + 8m \Delta V_M(2) = 0 \]
\[ m \Delta v_1 + 9m \Delta V_M(1) = 0 \]
\[ m \Delta v_0 + 10m \Delta V_M(0) = 0 \]

\[ \Delta v_0 = 1.096 \]
0th: $V(0)=1/10=0.1$

3rd: $V(3)=V(2)+1/7=0.478$

6th: $V(6)=V(5)+1/4=1.096$

1st: $V(1)=1/10+1/9=0.211$

4th: $V(4)=V(3)+1/6=0.646$

7th: $V(7)=V(6)+1/3=1.429$

2nd: $V(2)=1/10+1/9+1/8=0.336$

5th: $V(5)=V(4)+1/5=0.846$

8th: $V(8)=V(7)+1/2=1.929$
By calculus: \( M \cdot \Delta V = -v_e \cdot \Delta M \) or: \( dV = -v_e \frac{dM}{M} \) Integrate: \( \int_{V_{IN}}^{V_{FIN}} dV = -v_e \int_{M_{IN}}^{M_{FIN}} \frac{dM}{M} \)

\( v_e \) known as “Specific Impulse”
By calculus: $M \cdot \Delta V = -v_e \cdot \Delta M$ or $dV = -v_e \frac{dM}{M}$

Integrate: $\int_{V_{IN}}^{V_{FIN}} dV = -v_e \int_{M_{IN}}^{M_{FIN}} \frac{dM}{M}$

The Rocket Equation: $V_{FIN} - V_{IN} = -v_e \left[ \ln M_{FIN} - \ln M_{IN} \right] = v_e \left[ \ln \frac{M_{IN}}{M_{FIN}} \right]$
A Thales construction for momentum-energy

(Made obsolete by Estrangian scaling to circular \((V_1, V_2)\) plots. Still, one has to construct \(\sqrt{m_1/m_2}\) slopes.)
This is a detailed construction of the energy ellipse in a Lagrangian \((v_1,v_2)\) plot given the initial \((v_1,v_2)\).

The Estrangian \((V_1,V_2)\) plot makes the \((v_1,v_2)\) plot and this construction obsolete.

(Easier to just draw circle through initial \((V_1,V_2)\).)
**AMOP reference links** (Updated list given on 2nd and 3rd pages of each class presentation)

<table>
<thead>
<tr>
<th>Web Resources - front page</th>
<th>Quantum Theory for the Computer Age</th>
<th>2014 AMOP</th>
</tr>
</thead>
<tbody>
<tr>
<td>UAF Physics UTube channel</td>
<td>Principles of Symmetry, Dynamics, and Spectroscopy</td>
<td>2017 Group Theory for QM</td>
</tr>
<tr>
<td></td>
<td>Classical Mechanics with a Bang!</td>
<td>2018 AMOP</td>
</tr>
<tr>
<td></td>
<td>Modern Physics and its Classical Foundations</td>
<td></td>
</tr>
</tbody>
</table>

Representations Of Multidimensional Symmetries In Networks - harter-imp-1973

**Alternative Basis for the Theory of Complex Spectra**
- Alternative Basis for the Theory of Complex Spectra I - harter-pra-1973
- Alternative Basis for the Theory of Complex Spectra II - harter-patterson-pra-1976
- Alternative Basis for the Theory of Complex Spectra III - patterson-harter-pra-1977

Frame Transformation Relations And Multipole Transitions In Symmetric Polyatomic Molecules - RMP-1978

Asymptotic eigensolutions of fourth and sixth rank octahedral tensor operators - Harter-Patterson-JMP-1979

Rotational energy surfaces and high- J eigenvalue structure of polyatomic molecules - Harter - Patterson - 1984

Galloping waves and their relativistic properties - aip-1985-Harter

Rovibrational Spectral Fine Structure Of Icosahedral Molecules - Cpl 1986 (Alt Scan)

**Theory of hyperfine and superfine levels in symmetric polyatomic molecules.**
- I) Trigonal and tetrahedral molecules: Elementary spin-1/2 cases in vibronic ground states - PRA-1979-Harter-Patterson (Alt scan)
- II) Elementary cases in octahedral hexafluoride molecules - Harter-PRA-1981 (Alt scan)

**Rotation–vibration spectra of icosahedral molecules.**
- I) Icosahedral symmetry analysis and fine structure - harter-weeks-jcp-1989 (Alt scan)
- II) Icosahedral symmetry, vibrational eigenfrequencies, and normal modes of buckminsterfullerene - weeks-harter-jcp-1989 (Alt scan)
- III) Half-integral angular momentum - harter-reimer-jcp-1991


Nuclear spin weights and gas phase spectral structure of 12C60 and 13C60 buckminsterfullerene - Harter-Reimer-Cpl-1992 - (Alt1, Alt2 Erratum)

Gas Phase Level Structure of C60 Buckyball and Derivatives Exhibiting Broken Icosahedral Symmetry - reimer-diss-1996

Fullerene symmetry reduction and rotational level fine structure/ the Buckyball isotopomer 12C 13C59 - icp-Reimer-Harter-1997 (HiRez)

Wave Node Dynamics and Revival Symmetry in Quantum Rotors - harter - jms - 2001

Molecular Symmetry and Dynamics - Ch32-Springer Handbooks of Atomic, Molecular, and Optical Physics - Harter-2006

**Resonance and Revivals**
- I) QUANTUM ROTOR AND INFINITE-WELL DYNAMICS - ISMSLi2012 (Talk) OSU knowledge Bank
- II) Comparing Half-integer Spin and Integer Spin - Alva-ISMS-Ohio2013-R777 (Talks)
- III) Quantum Resonant Beats and Revivals in the Morse Oscillators and Rotors - (2013-Li-Diss)

Resonance and Revivals in Quantum Rotors - Comparing Half-integer Spin and Integer Spin - Alva-ISMS-Ohio2013-R777 (Talk)

Molecular Eigensolution Symmetry Analysis and Fine Structure - IJMS-harter-mitchell-2013

Quantum Revivals of Morse Oscillators and Farey-Ford Geometry - Li-Harter-cpl-2013

QTCA Unit 10 Ch 30 – 2013

AMOP Ch 0 Space-Time Symmetry - 2019

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AMOP reference links (Updated list given on 2nd and 3rd pages of each class presentation)

(Int.J.Mol.Sci, 14, 714(2013) p.755-774, QTCA Unit 7 Ch. 23-26), (PSDS - Ch. 5, 7)

Intro spin ½ coupling
Unit 8 Ch. 24 p3

H atom hyperfine-B-level crossing
Unit 8 Ch. 24 p15

Hyperf. theory Ch. 24 p48.

Intro 2p3p coupling
Unit 8 Ch. 24 p17.

Intro LS-jj coupling
Unit 8 Ch. 24 p22.

CG coupling derived (start)
Unit 8 Ch. 24 p39.

CG coupling derived (formula)
Unit 8 Ch. 24 p44.

Lande’g-factor
Unit 8 Ch. 24 p26.

Irrep Tensor building
Unit 8 Ch. 25 p5.

Irrep Tensor Tables
Unit 8 Ch. 25 p12.

Intro 3-particle coupling.
Unit 8 Ch. 25 p28.

Intro 3,4-particle Young Tableaus
GrpThLect29 p42.

Wigner-Eckart tensor Theorem.
Unit 8 Ch. 25 p17.

Tensors Applied to d,f-levels.
Unit 8 Ch. 25 p21.

Tensors Applied to high J levels.
Unit 8 Ch. 25 p63.

Deeper theory ends p53

Lande’g-factor
Unit 8 Ch. 24 p26.

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**Predrag Cvitanovic’s: Birdtrack Notation, Calculations, and Simplification**

- Chaos_Classical_and_Quantum - 2018-Cvitanovic-ChaosBook
- Group_Theory - PUP_Lucy_Day - Diagrammatic_notation - Ch4
- Group_Theory - Birdtracks_Lies_and_Exceptional_Groups - Cvitanovic-2011
- Simplification_rules_for_birdtrack_operators - imp-alcock-zeilinger-2017
- Birdtracks for SU(N) - 2017-Keppeler

**Frank Rioux’s: UMA method of vibrational induction**

- Quantum_Mechanics_Group_Theory_and_C60 - Frank_Rioux - Department_of_Chemistry_Saint_Johns_U
- Symmetry_Analysis_for_H2O - H2OGrpTheory - Rioux
- Group_Theory_Problems - Rioux - SymmetryProblemsX
- Comment_on_the_Vibrational_Analysis_for_C60_and_Other_Fullerenes_Rioux-RSP

**Supplemental AMOP Techniques & Experiment**

- Many Correlation Tables are Molien Sequences - Klee (Draft 2016)
- Symmetry and Chirality - Continuous_Measures - Avnir
  *

**Special Topics & Colloquial References**

- r-process_nucleosynthesis_from_matter_ejected_in_binary_neutron_star_mergers-PhysRevD-Bovard-2017

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