2019 CMwBang! site

Class YouTube Channel

Lecture 4
Mon. 9.09.2019

Kinetic Derivation of 1D Potentials and Force Fields

(Ch. 6, and Ch. 7 of Unit 1)

Review of $(V_1,V_2)\rightarrow(y_1,y_2)$ collision dynamics *High mass ratio M_1/m_2 =49*

Force "field" or "pressure" due to many small bounces

Force defined as momentum transfer rate

The 1D-Isothermal force field $F(y)$=const./y and the 1D-Adiabatic force field $F(y)$=const./y^3

Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)$=const./y^2

Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$

Example of 1D-Isothermal potential $U(y)$=const. ln(y)

"Monster Mash" classical segue to Heisenberg action relations

Example of very very large M_1 ball-wall(s) crushing a poor little m_2

How m_2 keeps its action

An interesting wave analogy: The "Tiny-Big-Bang" [Harter, J. Mol. Spec. 210, 166-182 (2001)]; [Harter, Li IMSS (2013)]

A lesson in geometry of fractions and fractals: Ford Circles and Farey Sums

[Lester. R. Ford, Am. Math. Monthly 45,586(1938)]; [John Farey, Phil. Mag.(1816) Wolfram]; [Li, Harter, Chem.Phys.Letters (2015) Elsevier]

[Li, Harter, Chem.Phys.Letters (2015) Local Copy]

Supplementary references and Interest items

[Web Resources - front page](#) | [Quantum Theory for the Computer Age](#) | [2017 Group Theory for QM](#)

[UAF Physics UTube channel](#) | [Principles of Symmetry, Dynamics, and Spectroscopy](#) | [2018 Adv CM](#)

[Classical Mechanics with a Bang!](#) | [2018 AMOP](#)

[Modern Physics and its Classical Foundations](#) | **[2019 Advanced Mechanics](#)**

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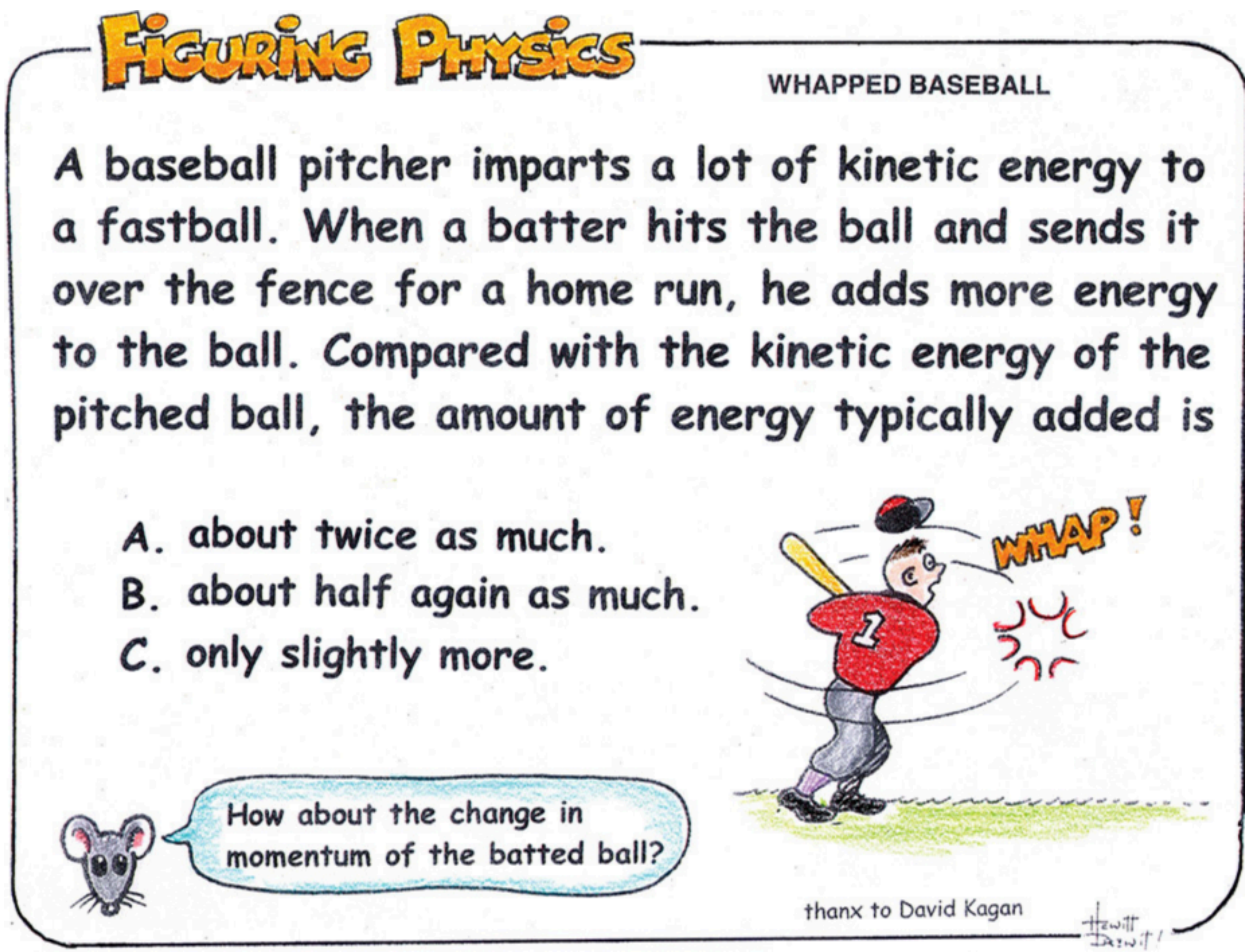
[AIP publications](#) | [AJP article on superball dynamics](#) | [AAPT summer reading](#)



(The answer to this month's "Figuring Physics" can be found at *TPT Online*, http://scitation.aip.org/upload/AAPT/TPT/Figuring/jan2017.pdf. The answer will also be printed in the February issue of *The Physics Teacher*. The answer to December's question appears on p. 54 of this issue.)

Running Reference Link Listing

Web Resources - front page
UAF Physics UTube channel

Quantum Theory for the Computer Age
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2017 Group Theory for QM
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Lecture #4

AAPT Summer Reading List
Scitation.org
HarterSoft Youtube Channel
BounceItIt Web Animation - Scenarios:
49:1 y vs t, *49:1 V2 vs V1*, *1:500:1 - 1D Gas Model w/ faux restorative force (Cool)*,
1:500:1 - 1D Gas (Warm), *1:500:1 - 1D Gas Model (Cool, Zoomed in)*,
Farey Sequence - Wolfram
Fractions - Ford-AMM-1938
Monstermash BounceItIt Animations:
1000:1 - V2 vs V1, *1000:1 with t vs x - Minkowski Plot*
Quantum Revivals of Morse Oscillators and Farey-Ford Geometry - Li-Harter-2013
Quantum_Revivals_of_Morse_Oscillators_and_Farey-Ford_Geometry - Li-Harter-cpl-2015
Quantum Revivals of Morse Oscillators and Farey-Ford Geometry - Harter-Li-CPL-2015 (Publ.)
Velocity_Amplification_in_Collision_Experiments_Involving_Superballs-Harter-1971
WaveIt Web Animation - Scenarios:
Quantum_Carpet, *Quantum_Carpet_wMBars*, *Quantum_Carpet_BCar*, *Quantum_Carpet_BCar_wMBars*
Wave Node Dynamics and Revival Symmetry in Quantum Rotors - Harter-JMS-2001
Wave Node Dynamics and Revival Symmetry in Quantum Rotors - Harter-jms-2001 (Publ.)

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Prior to Lecture #4

BounceIt **Superball Collision Web Simulations**

- *With g=0 and 70:10 mass ratio*
- *With non zero g, velocity dependent damping and mass ratio of 70:35*
- *M_1=49, M_2=1 with Newtonian time plot*
- *M_1=49, M_2=1 with V_2 vs V_1 plot*
- *Example with friction*
- *Low force constant with drag displaying a Pass-thru, Fall-Thru, Bounce-Off*
- m1:m2= 3:1 and (v1, v2) = (1, 0) Comparison with Estrangian

m_1:m_2 = 3:1 Dual plots

v_2 vs v_1 and V_2 vs V_1 m_1:m_2 = 3:1	*(v_1, v_2)=(1, 0.1)*	*(v_1, v_2)=(1, 0)*	
y_2 vs y_1 plots	*(v_1, v_2)=(1, 0.1)*	*(v_1, v_2)=(1, 0)*	*(v_1, v_2)=(1, -1)*
m_1:m_2 = 3:1 Estrangian plot *V_2 vs V_1*	*(v_1, v_2)=(0, 1)*		*(v_1, v_2)=(1, -1)*
m_1:m_2 = 4:1 (v_1, v_2)=(1, 0)	*v_2 vs v_1 plot*	*y_2 vs y_1 plot*	

m_1:m_2 = 100:1 (v_1, v_2)=(1, 0)

V_2 vs V_1 Estrangian plot

y2 vs y1 plot

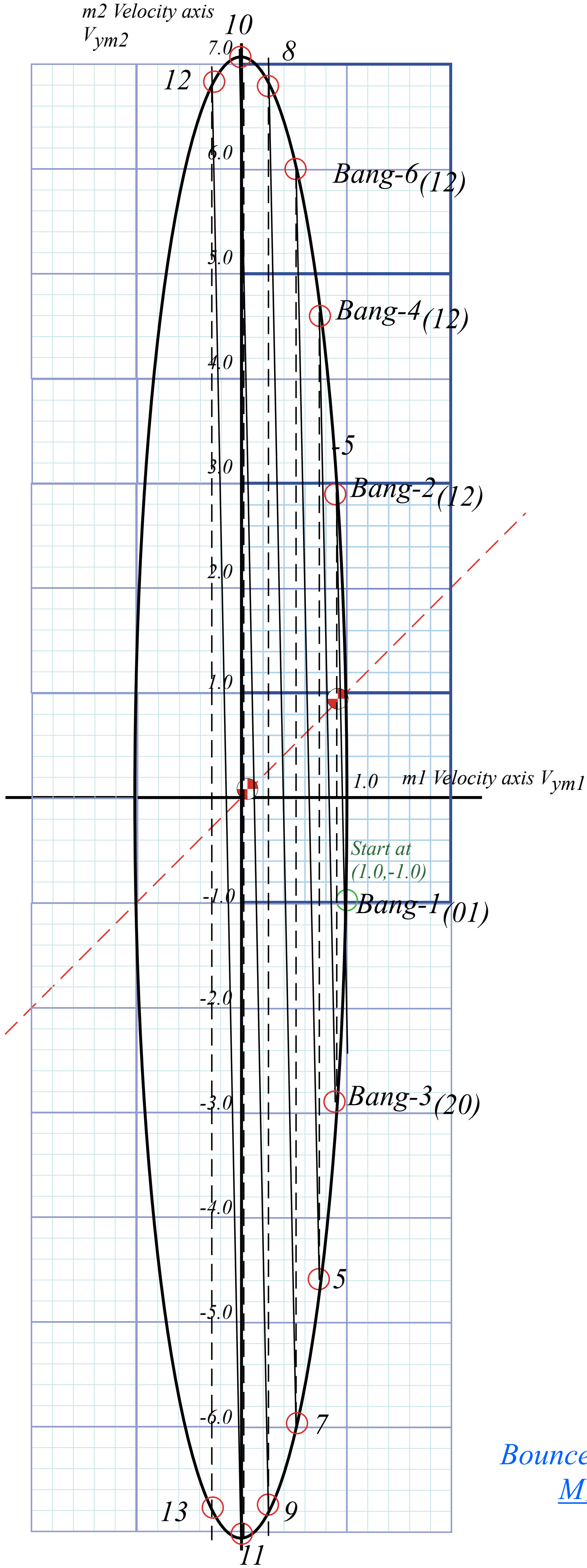
X2 paper: *Velocity Amplification in Collision Experiments Involving Superballs - Harter, et. al. 1971 (pdf)*
Car Collision Web Simulator: https://modphys.hosted.uark.edu/markup/CMMotionWeb.html
Superball Collision Web Simulator: https://modphys.hosted.uark.edu/markup/BounceItWeb.html; with Scenarios: 1007
BounceIt web simulation with g=0 and 70:10 mass ratio
With non zero g, velocity dependent damping and mass ratio of 70:35
Elastic Collision Dual Panel Space vs Space: Space vs Time (Newton) , Time vs. Space(Minkowski)
Inelastic Collision Dual Panel Space vs Space: Space vs Time (Newton), Time vs. Space(Minkowski)
Matrix Collision Simulator: M_1=49, M_2=1 V_2 vs V_1 plot <<*Under Construction*>>

More Advanced QM and classical references at the end of this Lecture

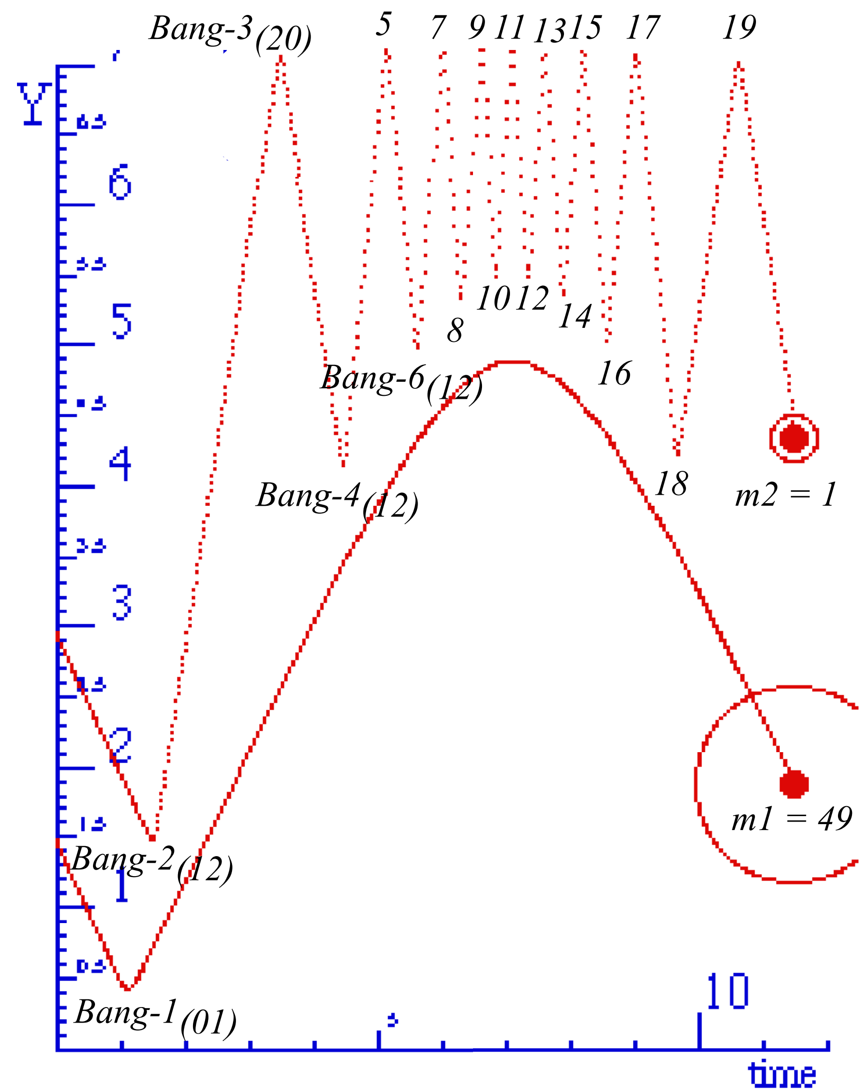
Review of $(V_1,V_2)\rightarrow(y_1,y_2)$ relations

→ *High mass ratio M_1/m_2 =49*

Geometric "Integration" (Converting Velocity data to Space-time trajectory)



Example with masses: m_1=49 and m_2=1



Kinetic Energy Ellipse

$$KE = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{49}{2} + \frac{1}{2} = 25$$

$$1 = \frac{v_1^2}{2KE/m_1} + \frac{v_2^2}{2KE/m_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Ellipse radius 1	Ellipse radius 2
$a_1 = \sqrt{2KE/m_1}$	$a_2 = \sqrt{2KE/m_2}$
$= \sqrt{2KE/49}$	$= \sqrt{2KE/1}$
$= \sqrt{50/49}$	$= \sqrt{50/1}$
$= 1.01$	$= 7.07$

Fig. 5.1
in Unit 1

BounceIt Superball Collision Web Simulator:
[M_1=49, M_2=1 with Newtonian time plot]

BounceIt Superball Collision Web Simulator:
[M_1=49, M_2=1 with V_2 vs V_1 plot]

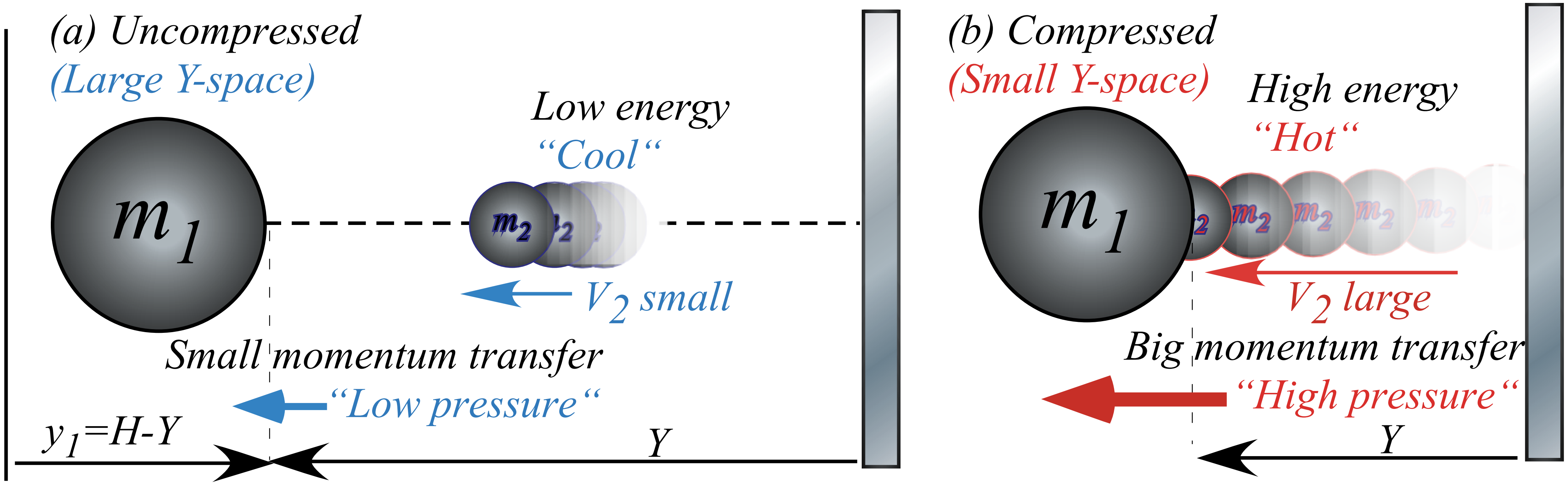
Force "field" or "pressure" due to many small bounces

Force defined as momentum transfer rate

The 1D-Isothermal force field $F(y)=const./y$ and the 1D-Adiabatic force field $F(y)=const./y^3$

Big mass-m_1 ball feeling "force-field" or "pressure" of small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

Unit 1
Fig. 6.1



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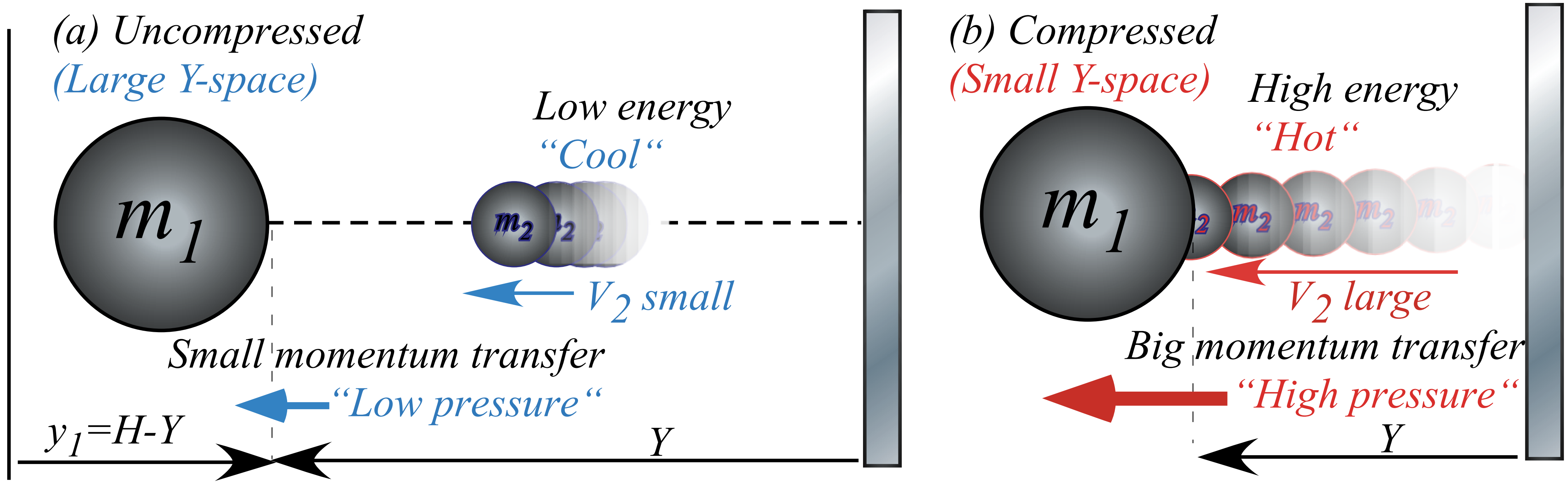
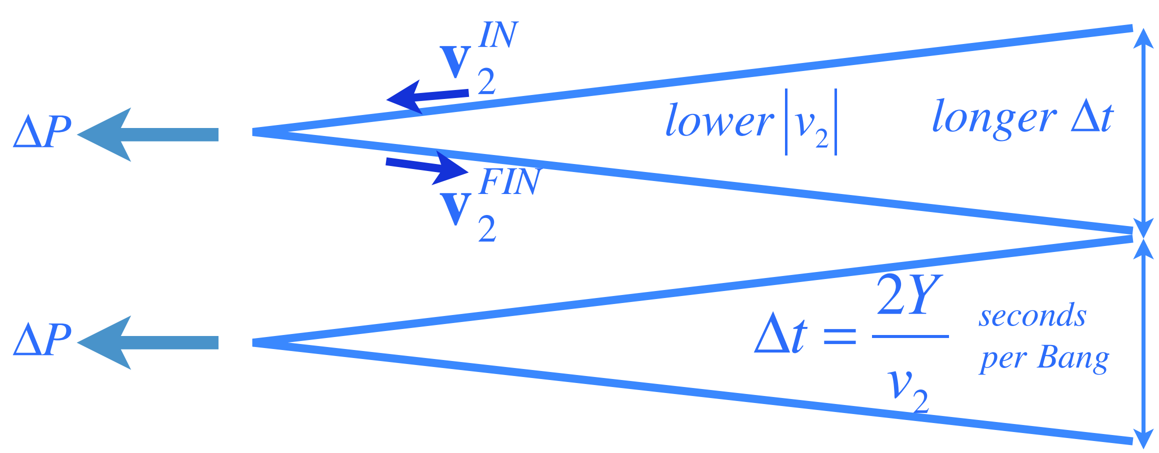
Unit 1
Fig. 6.1





Big mass-m_1 ball feeling "force-field" or "pressure" of small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

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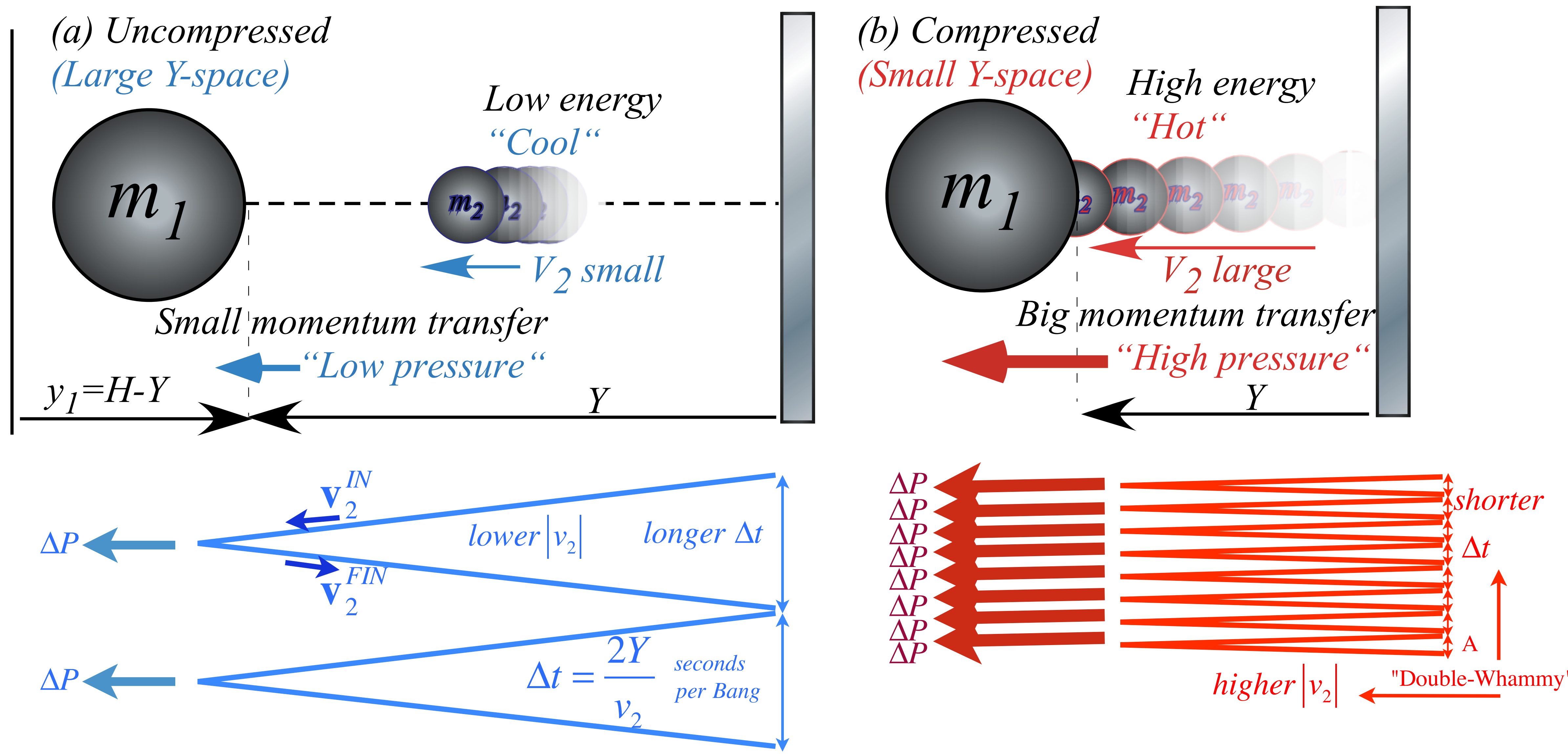

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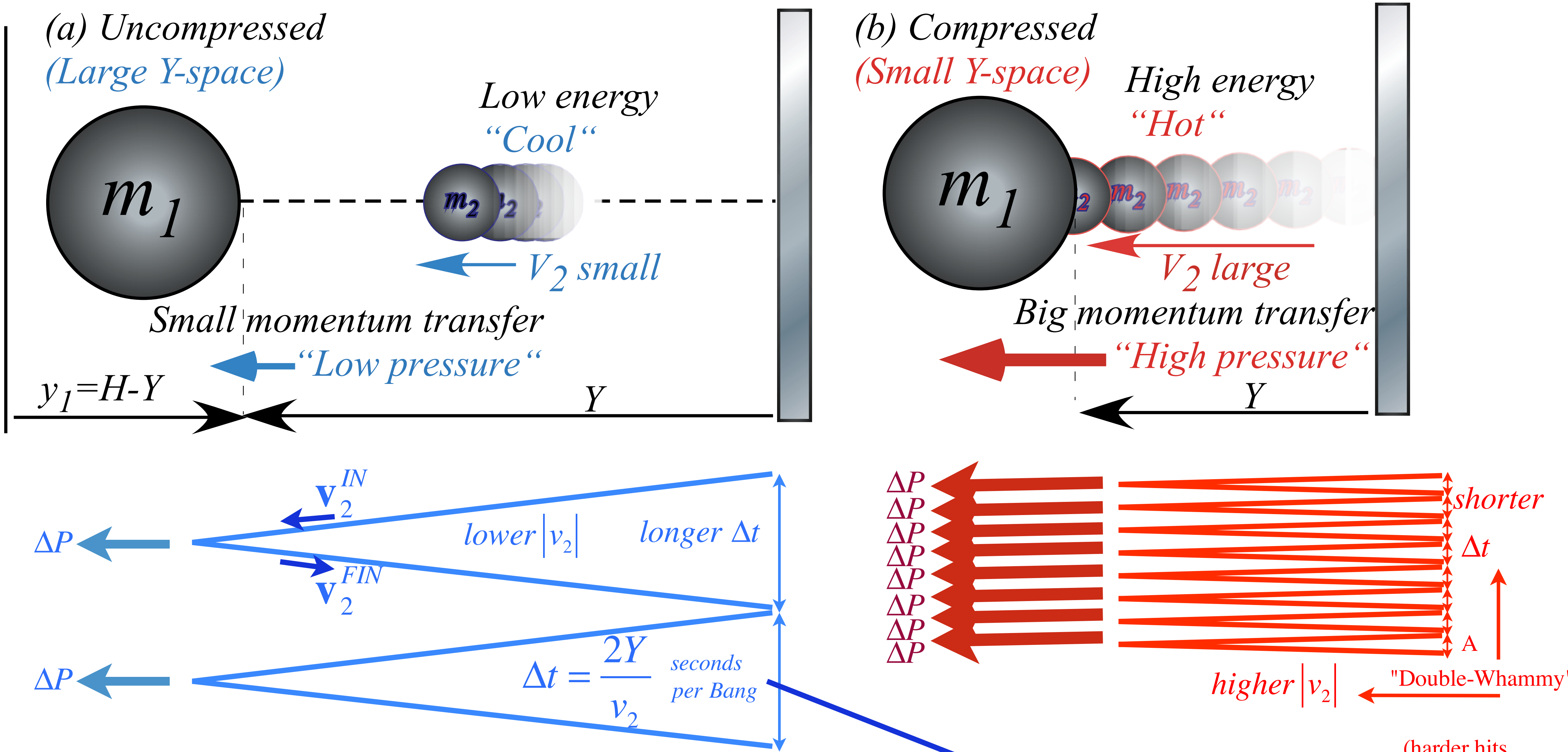


This introduction of Force...

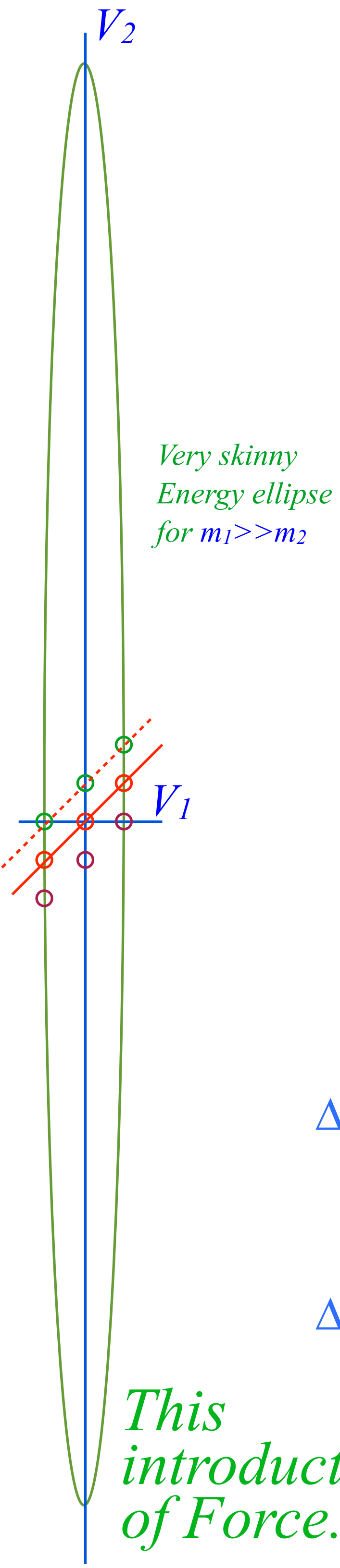
$$F = \frac{\Delta P}{\Delta t} \qquad \Delta P = m_2 v_2^{IN} - m_2 v_2^{FIN} \qquad \frac{1}{\Delta t} = \frac{v_2}{2Y}$$

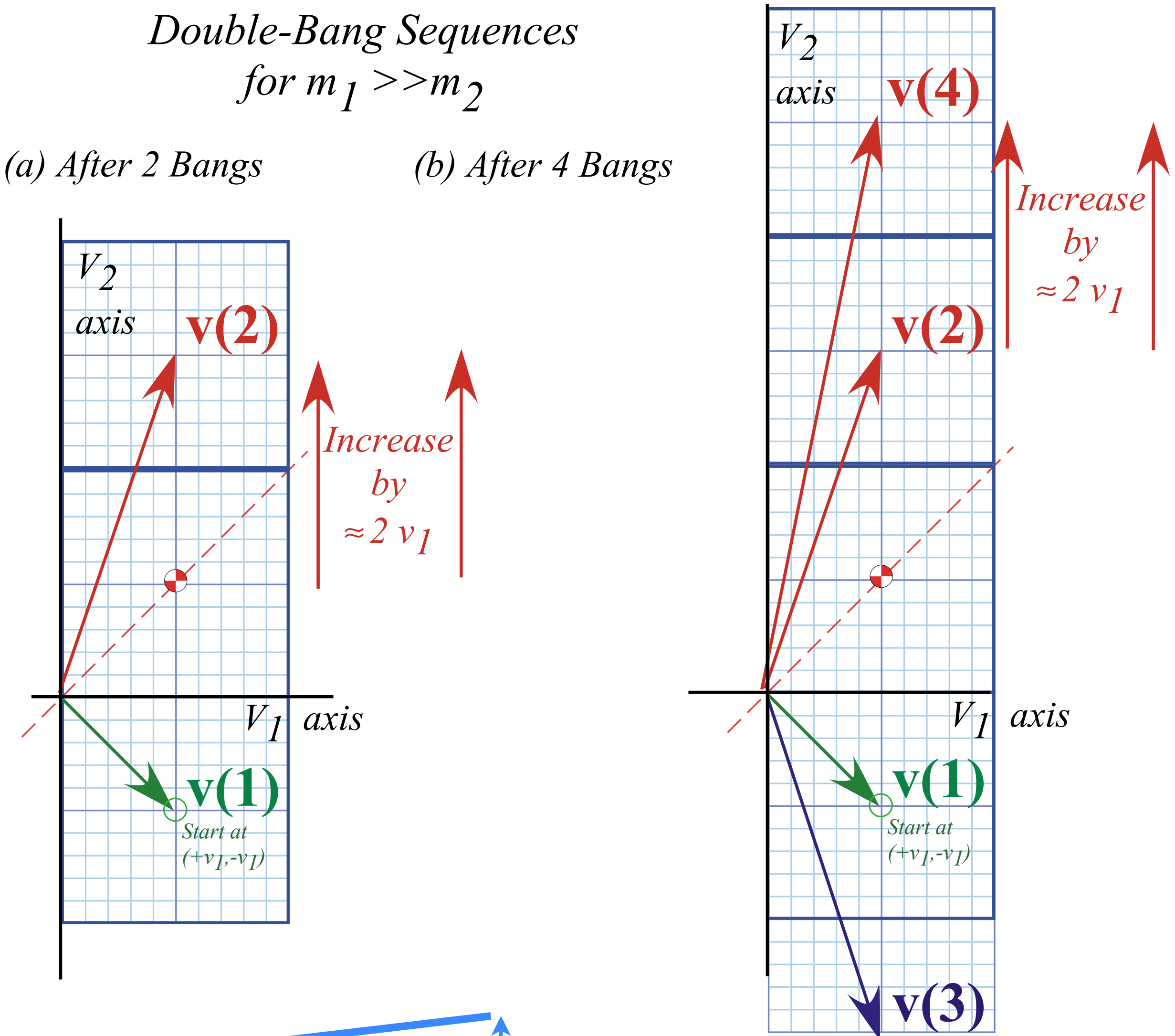
Force F on m_1 = (Momentum per sec.) = (Momentum per Bang)·(Bangs per second)

...is more of a <u>definition</u> than another <u>axiom</u>

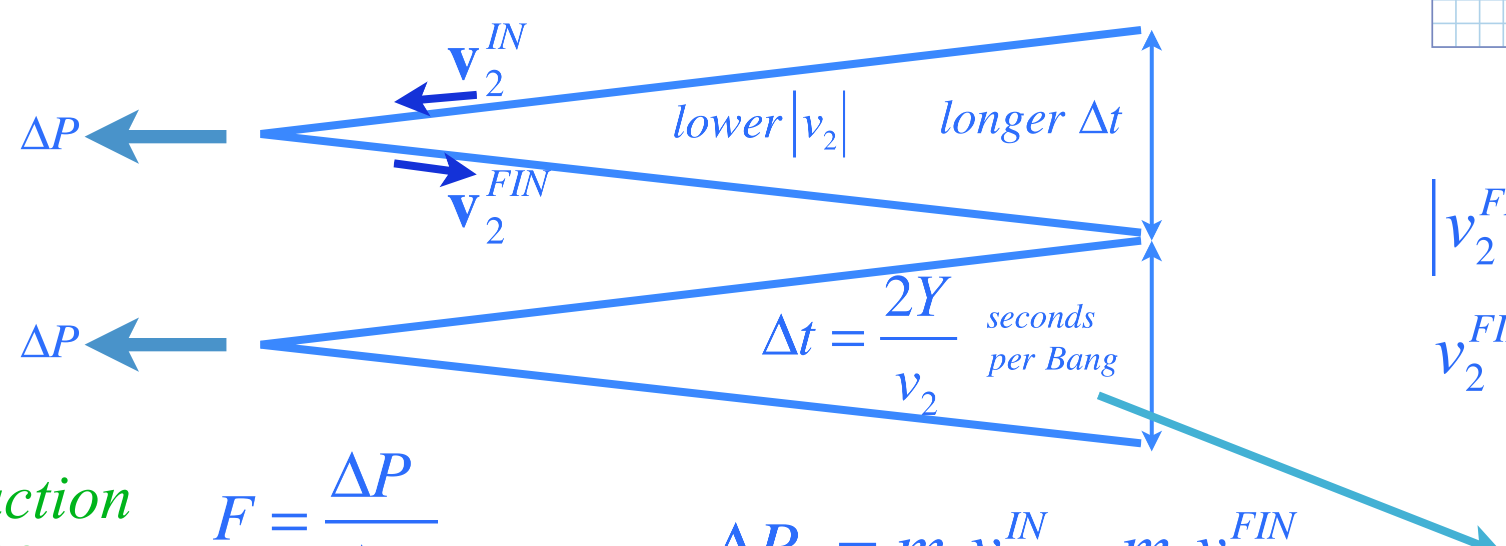
Quantum Planck-axiom $E=\hbar n\omega$ begins with Energy *not* momentum

Double-Bang Sequences for $m_1 >> m_2$





Unit 1
Fig. 6.2



$$\left|v_2^{FIN}\right| = \left|v_2^{IN}\right| + \left|2v_1\right| \quad \text{for: } m_1 >> m_2$$

$$v_2^{FIN} = -v_2^{IN} - 2v_1$$

This introduction of Force...

$$F = \frac{\Delta P}{\Delta t} \qquad \Delta P = m_2 v_2^{IN} - m_2 v_2^{FIN} \qquad \frac{1}{\Delta t} = \frac{v_2}{2Y}$$

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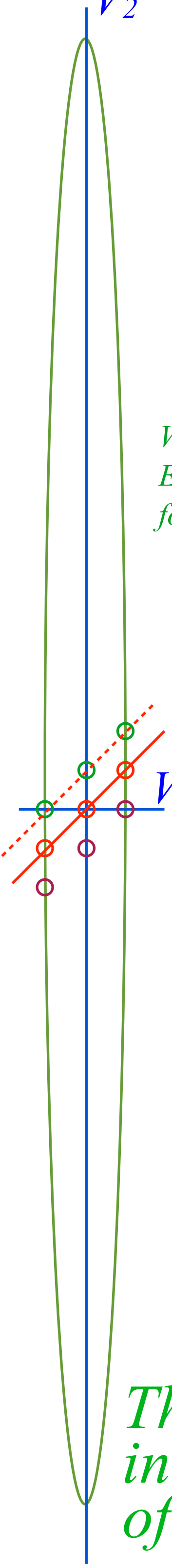
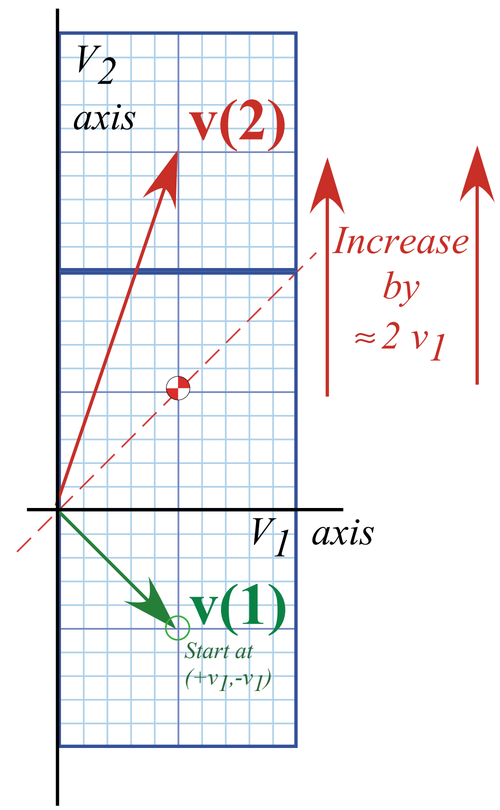
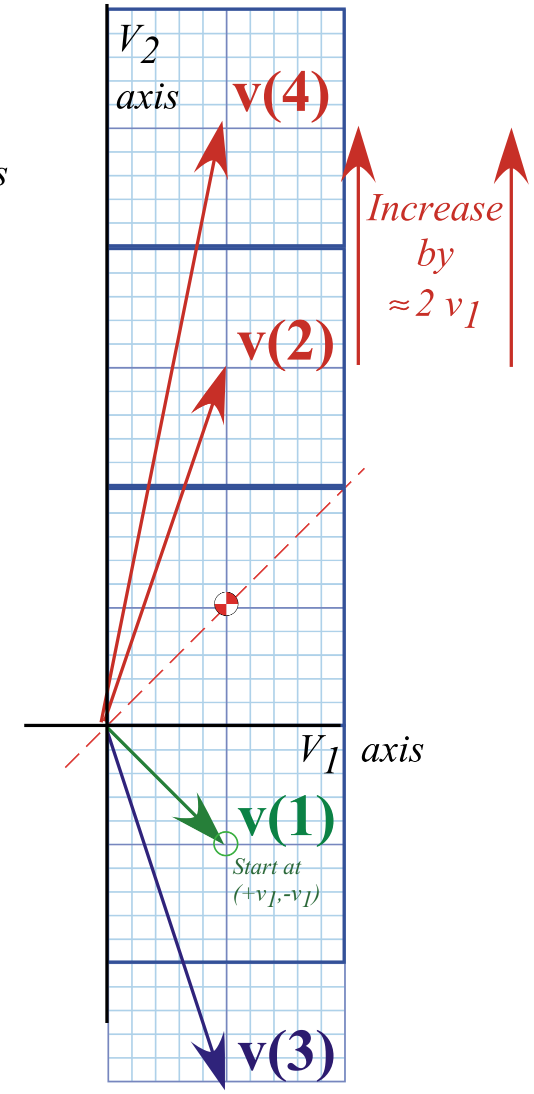


Double-Bang Sequences
for $m_1 >> m_2$

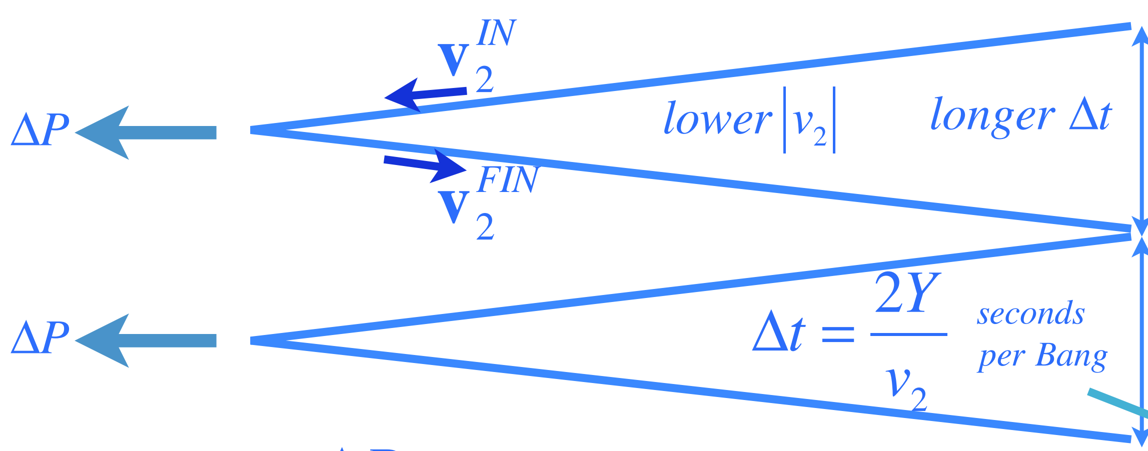
(a) After 2 Bangs



(b) After 4 Bangs



Unit 1
Fig. 6.2



$$|v_2^{FIN}| = |v_2^{IN}| + |2v_1| \quad \text{for: } m_1 >> m_2$$

$$v_2^{FIN} = -v_2^{IN} - 2v_1$$

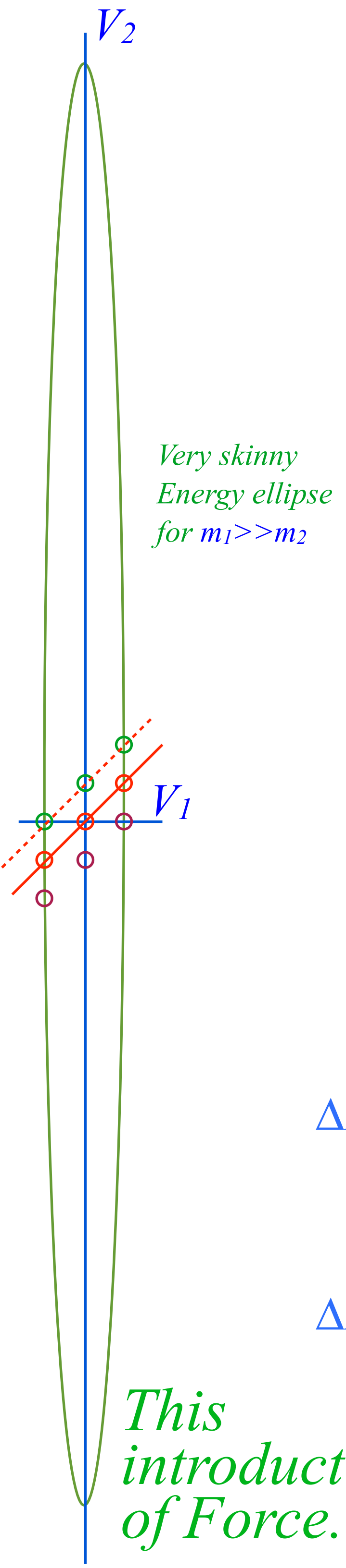
This introduction of Force...

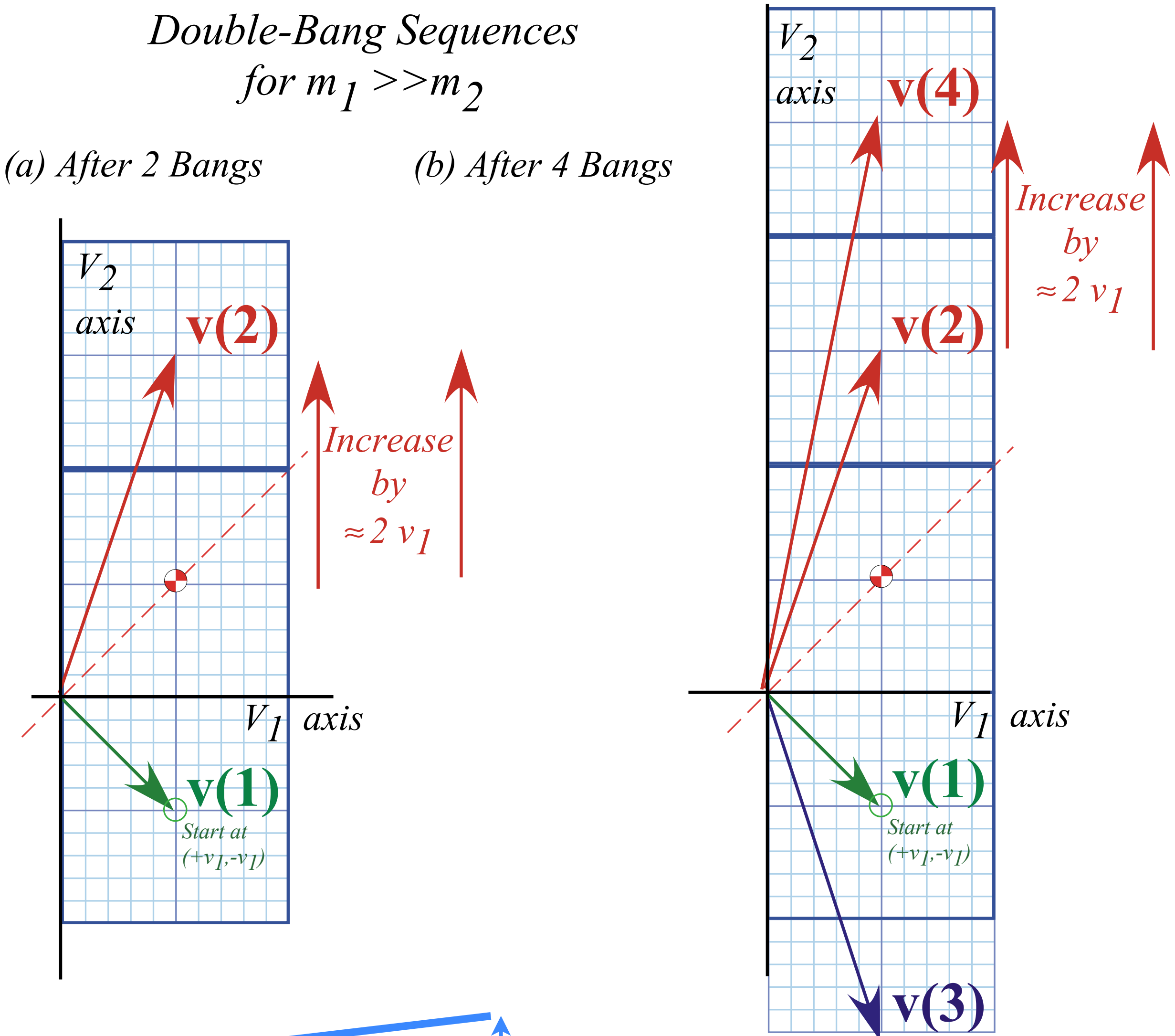
$$F = \frac{\Delta P}{\Delta t} \qquad \Delta P = m_2 v_2^{IN} - m_2 v_2^{FIN} \qquad \frac{1}{\Delta t} = \frac{v_2}{2Y}$$

Force F on m_1 = (Momentum per sec.) = (Momentum per Bang)·(Bangs per second)

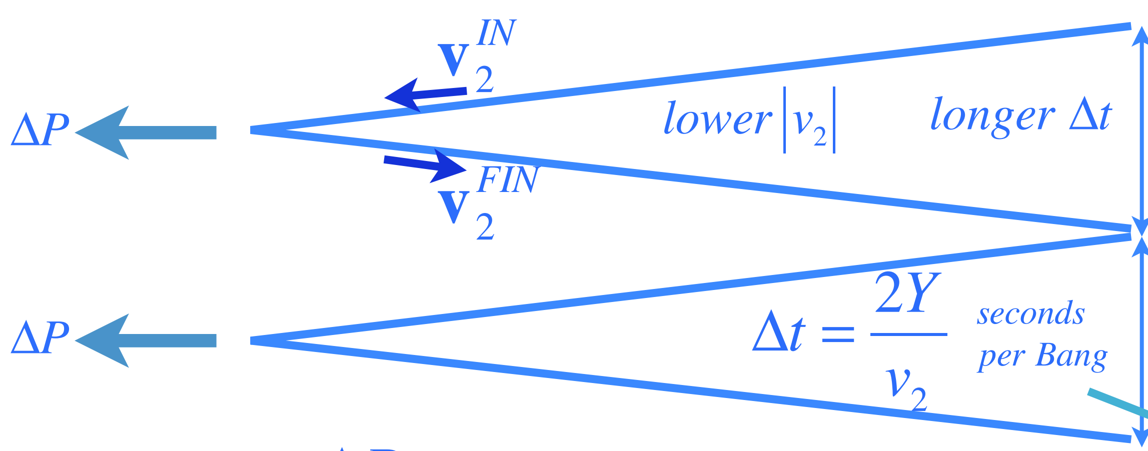
$$\Delta P = m_2 v_2^{IN} - m_2 (-v_2^{IN} - 2v_1) = 2m_2 v_2^{IN} + 2m_2 v_1$$

...is more of a <u>definition</u> than another <u>axiom</u>





Unit 1
Fig. 6.2



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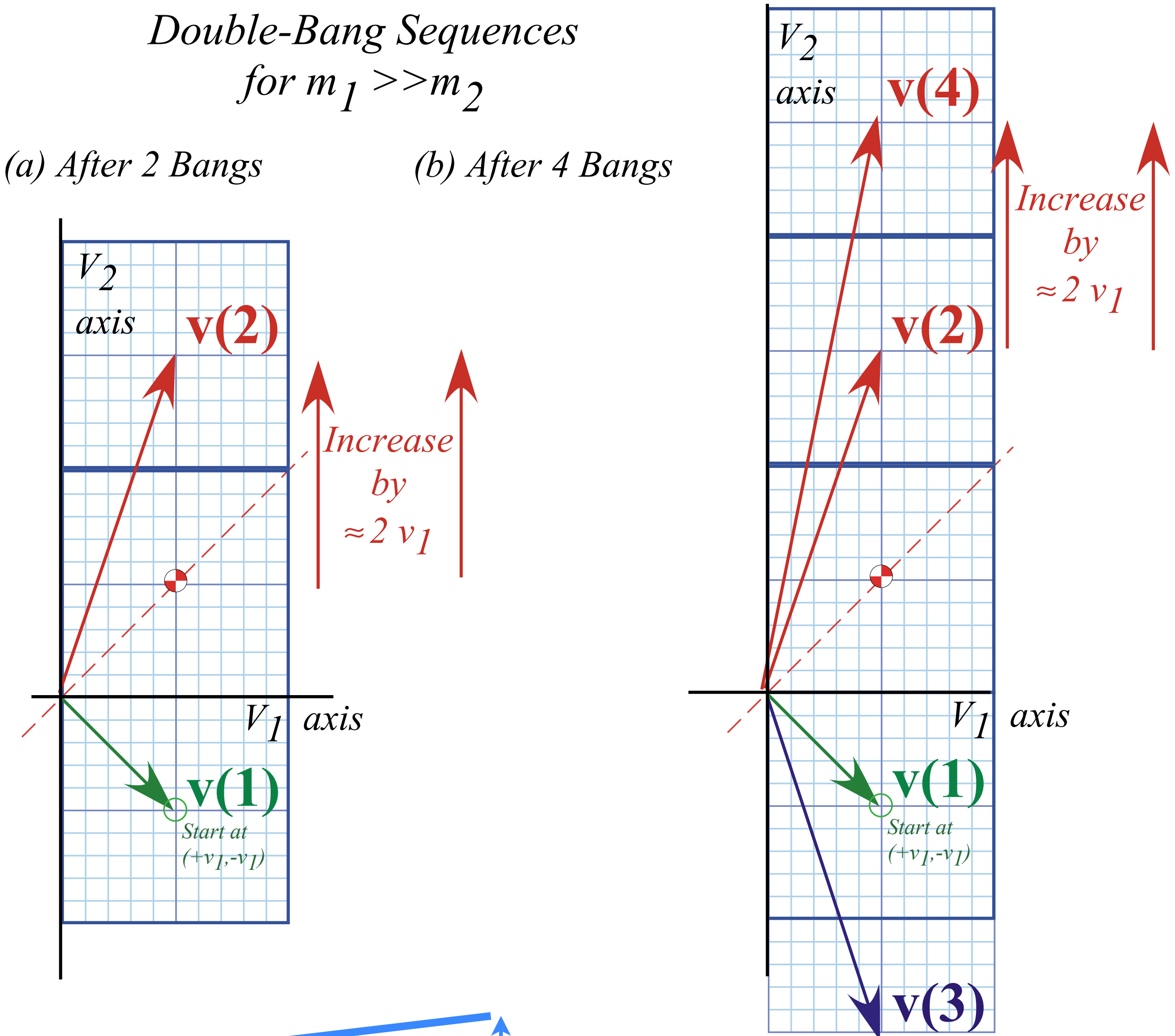
$$\Delta P = m_2 v_2^{IN} - m_2(-v_2^{IN} - 2v_1) = 2m_2 v_2^{IN} + 2m_2 v_1 \approx 2m_2 v_2^{IN}$$

...is more of a <u>definition</u> than another <u>axiom</u>

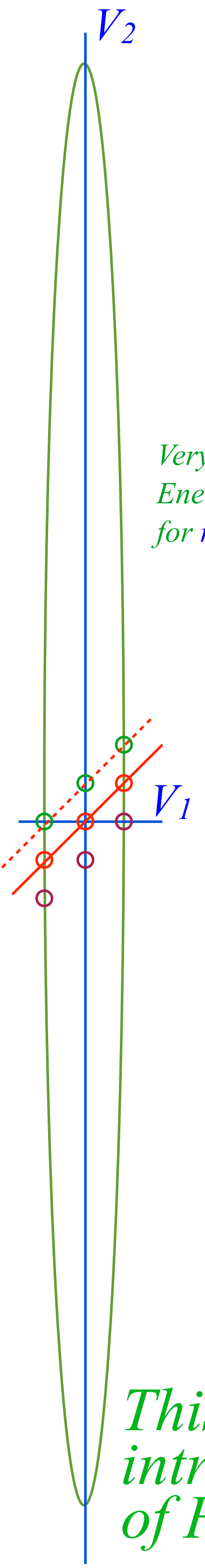
Assuming slow m_1 : $v_1 \ll v_2$

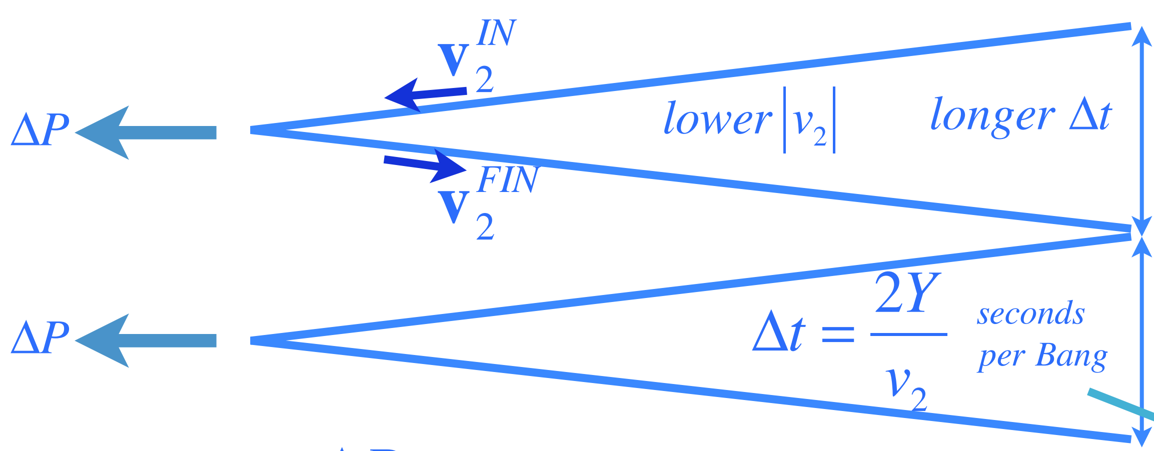
Double-Bang Sequences for $m_1 >> m_2$

(a) After 2 Bangs (b) After 4 Bangs



Unit 1
Fig. 6.2





$$|v_2^{FIN}| = |v_2^{IN}| + |2v_1| \quad \text{for: } m_1 >> m_2$$

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Force F on m_1 = (Momentum per sec.) = (Momentum per Bang)·(Bangs per second)

$$\Delta P = m_2 v_2^{IN} - m_2(-v_2^{IN} - 2v_1) = 2m_2 v_2^{IN} + 2m_2 v_1 \approx 2m_2 v_2^{IN}$$

...is more of a <u>definition</u> than another <u>axiom</u>

$$F = \frac{\Delta P}{\Delta t} = (\Delta P \approx 2m_2 v_2)\cdot\left(\frac{1}{\Delta t} \approx \frac{v_2}{2Y}\right) \approx \frac{m_2 v_2^2}{Y}$$

Assuming $\boxed{\text{slow}}$ m_1 : $v_1 \ll v_2$

$$F=\frac{\Delta P}{\Delta t}=\left(\Delta P\approx 2m_2v_2\right)\cdot\left(\frac{1}{\Delta t}\approx\frac{v_2}{2Y}\right)\approx\frac{m_2v_2^2}{Y}$$

Not a "Double-Whammy"...

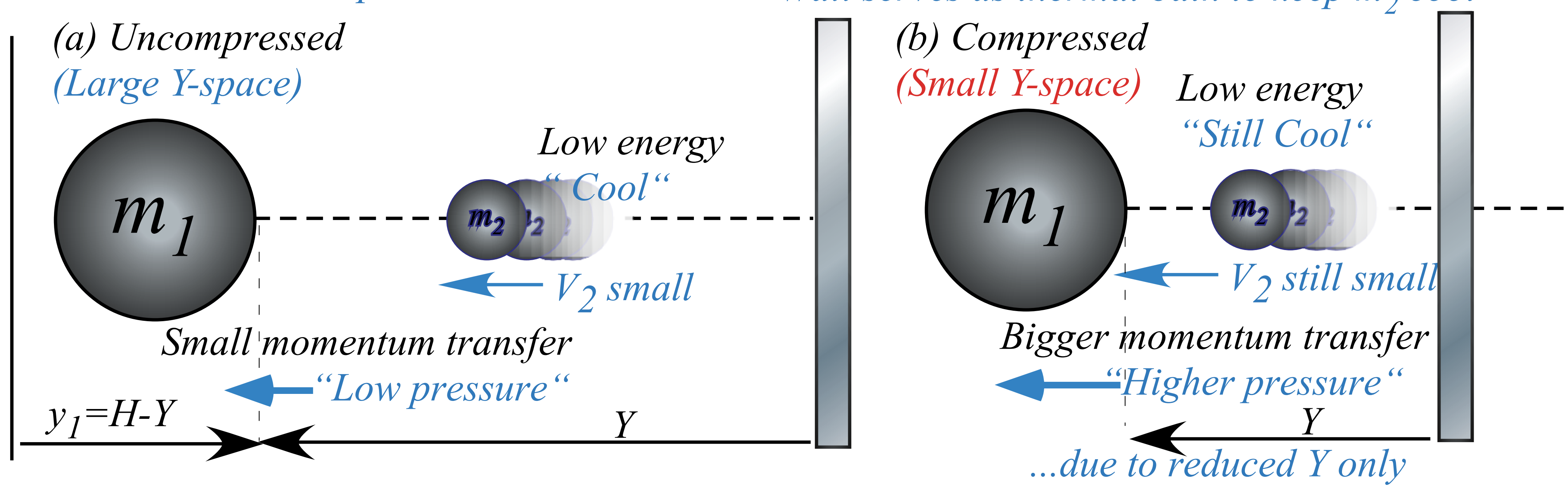
...only a "Single-Whammy"

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$\boxed{F=\frac{m_2v_2^2}{Y}=\frac{const.}{Y}}$$

Isothermal expansion or contraction:

Wall serves as thermal bath to keep m_2 cool



Force “field” or “pressure” due to many small bounces

Force defined as momentum transfer rate

The 1D-Isothermal force field $F(y)=const./y$ and the 1D-Adiabatic force field $F(y)=const./y^3$

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Not a "Double-Whammy"... ...only a "Single-Whammy"

1D-Isothermal Force Law (assume v_2 is constant for all Y): $\boxed{F=\frac{m_2v_2^2}{Y}=\frac{const.}{Y}}$

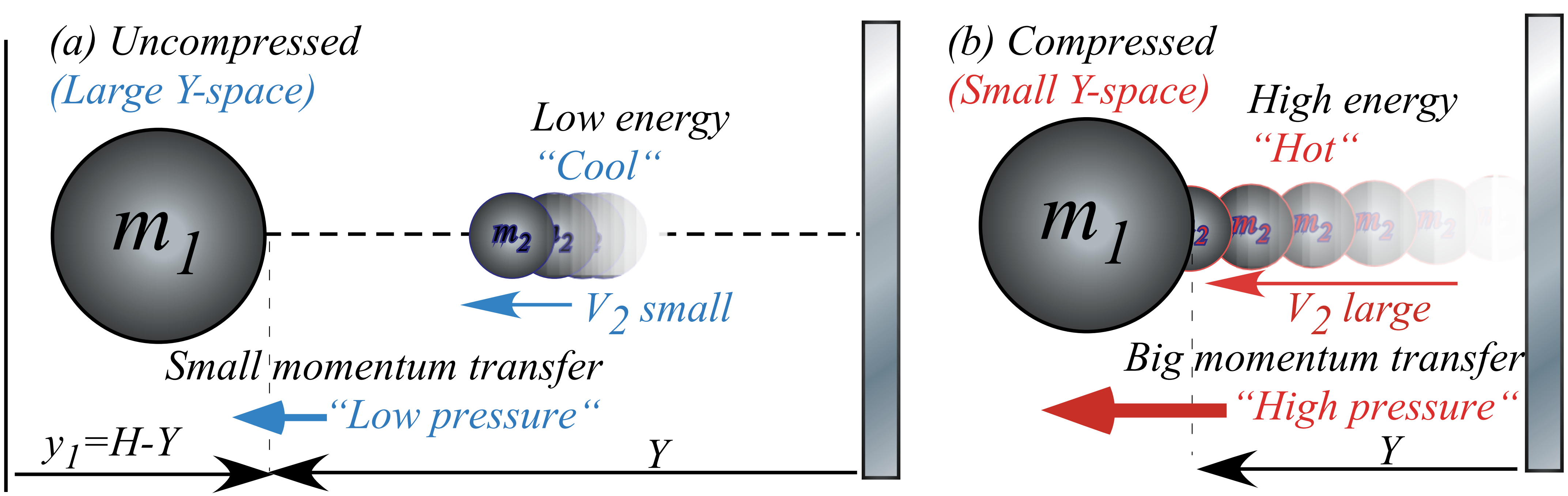
However, if ceiling is elastic, v_2 isn't constant if m_1 changes bounce range Y: $\frac{dy_1}{dt}\equiv v_1=-\frac{dY}{dt}$

When m_1 collides with m_2 it adds twice its velocity ($2v_1$) to v_2. This occurs at "bang-rate" $B=v_2/2Y$.

Here both v_2 and $Y=y_1$ may vary

$$\frac{dv_2}{dt}=2v_1B=2v_1\frac{v_2}{2Y}=-2\frac{dY}{dt}\frac{v_2}{2Y}$$

Wall not given time to give or take KE



A "Double-Whammy"

$$F = \frac{\Delta P}{\Delta t} = \left(\Delta P \approx 2m_2 v_2\right)\cdot\left(\frac{1}{\Delta t} \approx \frac{v_2}{2Y}\right) \approx \frac{m_2 v_2^2}{Y}$$

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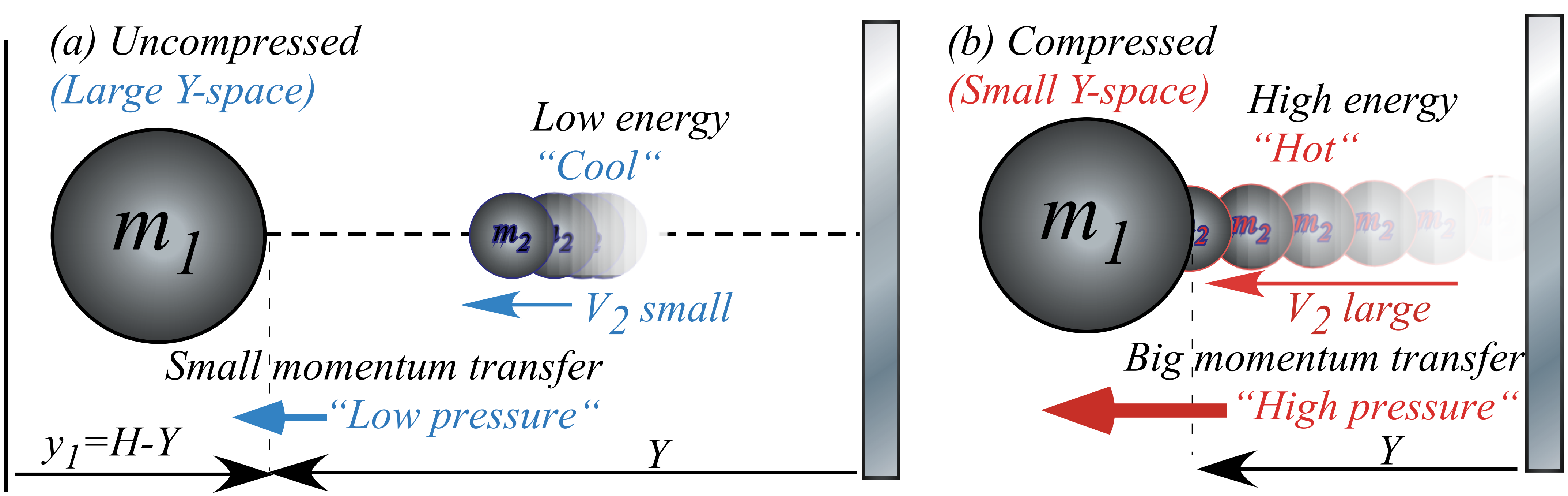
1D-Isothermal Force Law (assume v_2 is constant for all Y): $\boxed{F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}}$

However, if ceiling is elastic, v_2 isn't constant if m_1 changes bounce range Y: $\frac{dy_1}{dt} \equiv v_1 = -\frac{dY}{dt}$

When m_1 collides with m_2 it adds twice its velocity ($2v_1$) to v_2. This occurs at "bang-rate" $B=v_2/2Y$.

$$\frac{dv_2}{dt} = 2v_1 B = 2v_1 \frac{v_2}{2Y} = -2\frac{dY}{dt}\frac{v_2}{2Y} \qquad \text{simplifies to:} \quad \frac{dv_2}{v_2} = -\frac{dY}{Y}$$

Wall not given time to give or take KE



A "Double-Whammy"

$$F = \frac{\Delta P}{\Delta t} = \left(\Delta P \approx 2m_2 v_2\right)\cdot\left(\frac{1}{\Delta t} \approx \frac{v_2}{2Y}\right) \approx \frac{m_2 v_2^2}{Y}$$

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1D-Isothermal Force Law (assume v_2 is constant for all Y): $\boxed{F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}}$

However, if ceiling is elastic, v_2 isn't constant if m_1 changes bounce range Y: $\frac{dy_1}{dt} \equiv v_1 = -\frac{dY}{dt}$

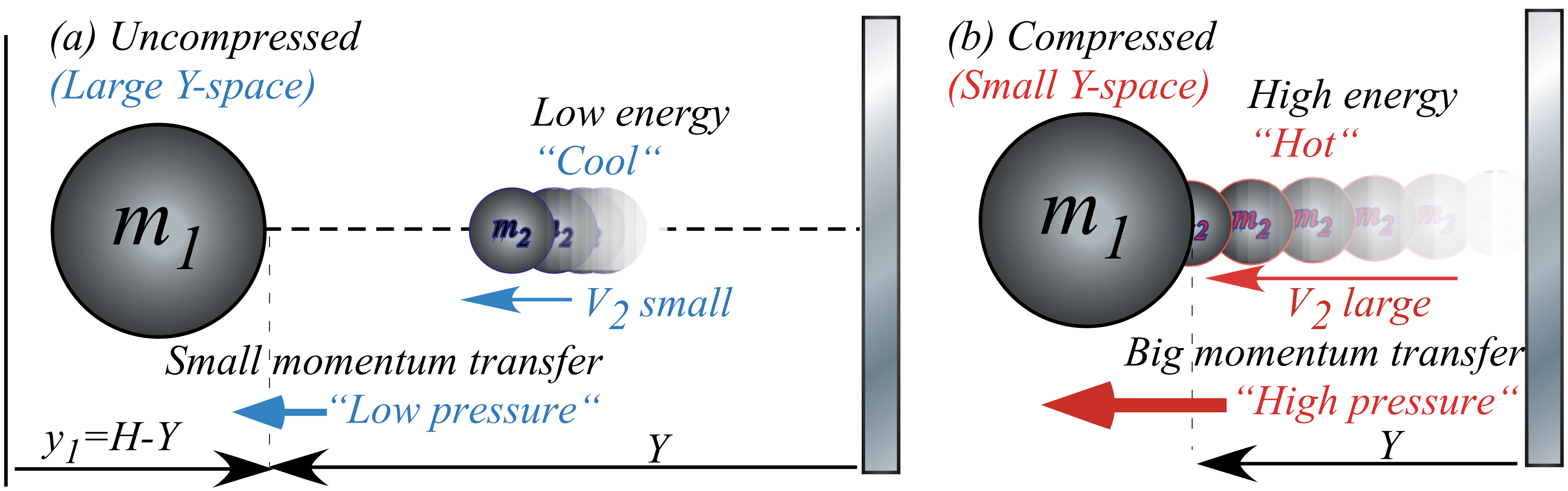
When m_1 collides with m_2 it adds twice its velocity ($2v_1$) to v_2. This occurs at "bang-rate" $B = v_2/2Y$.

$$\frac{dv_2}{dt} = 2v_1 B = 2v_1 \frac{v_2}{2Y} = -2\frac{dY}{dt}\frac{v_2}{2Y} \qquad \text{simplifies to:} \quad \frac{dv_2}{v_2} = -\frac{dY}{Y}$$

Differential equation results and has logarithmic integral. $\int \frac{dx}{x} = \ln x + C = \log_e x + \log_e e^C = \log_e(e^C x)$

$$\frac{dv_2}{v_2} = -\frac{dY}{Y} \quad \text{integrates to:} \quad \ln v_2 = -\ln Y + C \quad \text{or:} \quad \ln v_2 = \ln\frac{const.}{Y} \quad \text{or:} \quad v_2 = \frac{const.}{Y}$$

Wall not given time to give or take KE



A "Double-Whammy"

$$F = \frac{\Delta P}{\Delta t} = \left(\Delta P \approx 2m_2 v_2\right)\cdot\left(\frac{1}{\Delta t} \approx \frac{v_2}{2Y}\right) \approx \frac{m_2 v_2^2}{Y}$$

1D-Isothermal Force Law (assume v_2 is constant for all Y): $\boxed{F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}}$

Not a "Double-Whammy"... ...only a "Single-Whammy"

However, if ceiling is elastic, v_2 isn't constant if m_1 changes bounce range Y: $\frac{dy_1}{dt} \equiv v_1 = -\frac{dY}{dt}$

When m_1 collides with m_2 it adds twice its velocity ($2v_1$) to v_2. This occurs at "bang-rate" $B=v_2/2Y$.

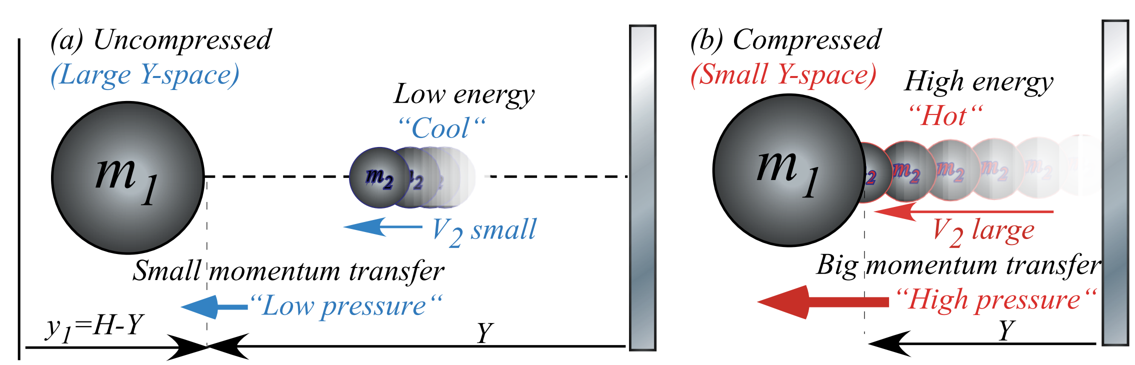
$$\frac{dv_2}{dt} = 2v_1 B = 2v_1 \frac{v_2}{2Y} = -2\frac{dY}{dt}\frac{v_2}{2Y} \qquad \text{simplifies to:} \quad \frac{dv_2}{v_2} = -\frac{dY}{Y}$$

Differential equation results and has logarithmic integral. $\int \frac{dx}{x} = \ln x + C = \log_e x + \log_e e^C = \log_e(e^C x)$

$$\frac{dv_2}{v_2} = -\frac{dY}{Y} \quad \text{integrates to:} \quad \ln v_2 = -\ln Y + C \quad \text{or:} \quad \ln v_2 = \ln\frac{const.}{Y} \quad \text{or:} \quad v_2 = \frac{const.}{Y}$$

Force law with this variable v_2 is called *adiabatic* or not-*diabatic* or not-gradual.

1D-Adiabatic Force Law (assume v_2 varies: $v_2 = \frac{const.}{Y} = \frac{v_2^{IN} Y(t=0)}{Y}$): $\boxed{F = \frac{m_2\left(v_2^{IN} Y(t=0)\right)^2}{Y^3} = \frac{const.}{Y^3}}$



A "Double-Whammy"

$$F=\frac{\Delta P}{\Delta t}=\left(\Delta P\approx 2m_2v_2\right)\cdot\left(\frac{1}{\Delta t}\approx\frac{v_2}{2Y}\right)\approx\frac{m_2v_2^2}{Y}$$

Not a "Double-Whammy"...
...only a "Single-Whammy"

1D-Isothermal Force Law (assume v_2 is constant for all Y): $\boxed{F=\frac{m_2v_2^2}{Y}=\frac{const.}{Y}}$

However, if ceiling is elastic, v_2 isn't constant if m_1 changes bounce range Y: $\frac{dy_1}{dt}\equiv v_1=-\frac{dY}{dt}$

When m_1 collides with m_2 it adds twice its velocity ($2v_1$) to v_2. This occurs at "bang-rate" $B=v_2/2Y$.

$$\frac{dv_2}{dt}=2v_1B=2v_1\frac{v_2}{2Y}=-2\frac{dY}{dt}\frac{v_2}{2Y} \qquad \text{simplifies to:} \quad \frac{dv_2}{v_2}=-\frac{dY}{Y}$$

Differential equation results and has logarithmic integral. $\int\frac{dx}{x}=\ln x+C=\log_e x+\log_e e^C=\log_e(e^Cx)$

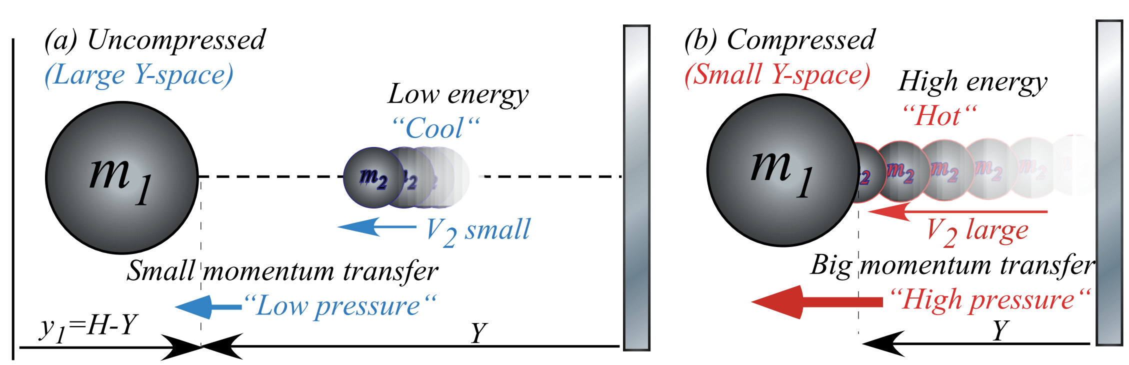
$$\frac{dv_2}{v_2}=-\frac{dY}{Y} \quad \text{integrates to: } \ln v_2=-\ln Y+C \quad \text{or:} \quad \ln v_2=\ln\frac{const.}{Y} \quad \text{or:} \quad v_2=\frac{const.}{Y}$$

$F=\frac{m_2v_2^2}{Y}$ *bcomes* $\frac{m_2(const.)^2}{YY^2}$

Force law with this variable v_2 is called *adiabatic* or not-*diabatic* or not-gradual.

1D-Adiabatic Force Law (assume v_2 varies: $v_2=\frac{const.}{Y}=\frac{v_2^{IN}Y(t=0)}{Y}$): $\boxed{F=\frac{m_2\left(v_2^{IN}Y(t=0)\right)^2}{Y^3}=\frac{const.}{Y^3}}$

See application on p.32
...or p.34



A "Double-Whammy"

Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)=const./y^2$

Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$

Example of 1D-Isothermal potential $U(y)=const.\ ln(y)$

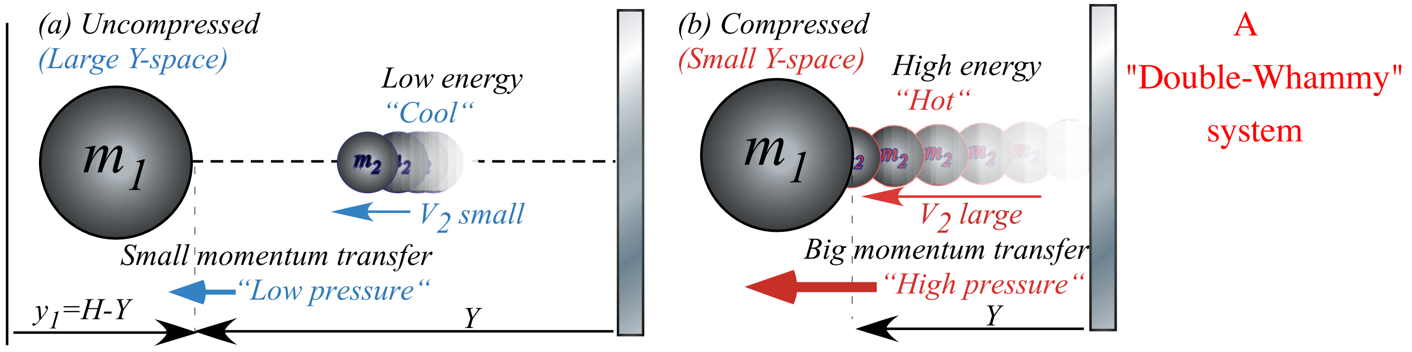
Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

In adiabatic case where $v_2 = \dfrac{const.}{Y}$ the total energy E is strictly conserved.

$$const. = E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \boxed{\frac{1}{2}m_1 v_1^2} + \boxed{\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2}$$

Define for big mass m_1: *Kinetic energy* $\boxed{KE(v_1)}$ vs $\boxed{\textit{Potential energy } PE(Y)=U(Y)}$

Potential energy $PE(Y)=U(Y)=\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$



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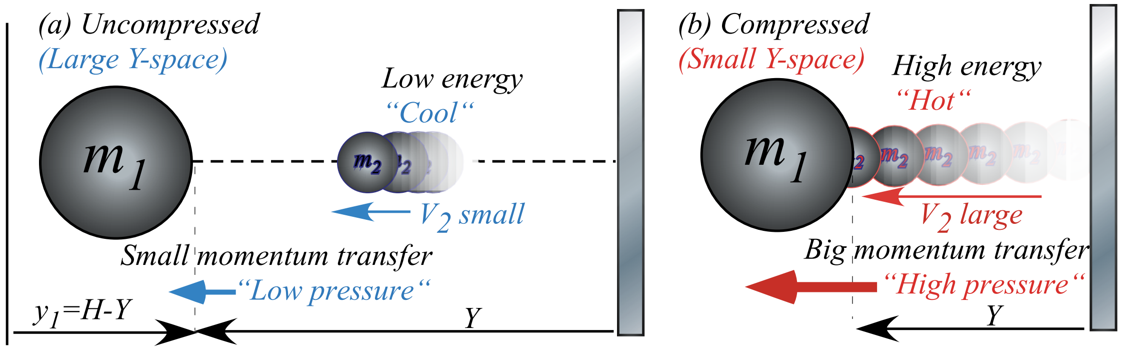

A
"Double-Whammy"
system

Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

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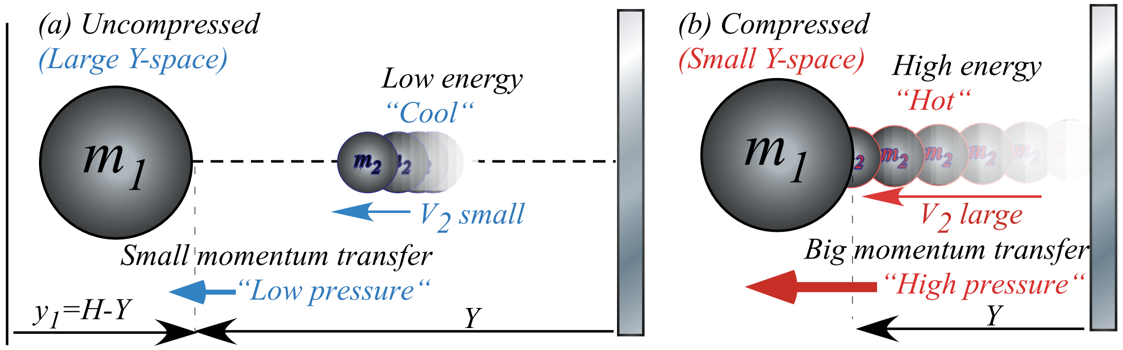
Q*?Another axiom?*



A "Double-Whammy" system

Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

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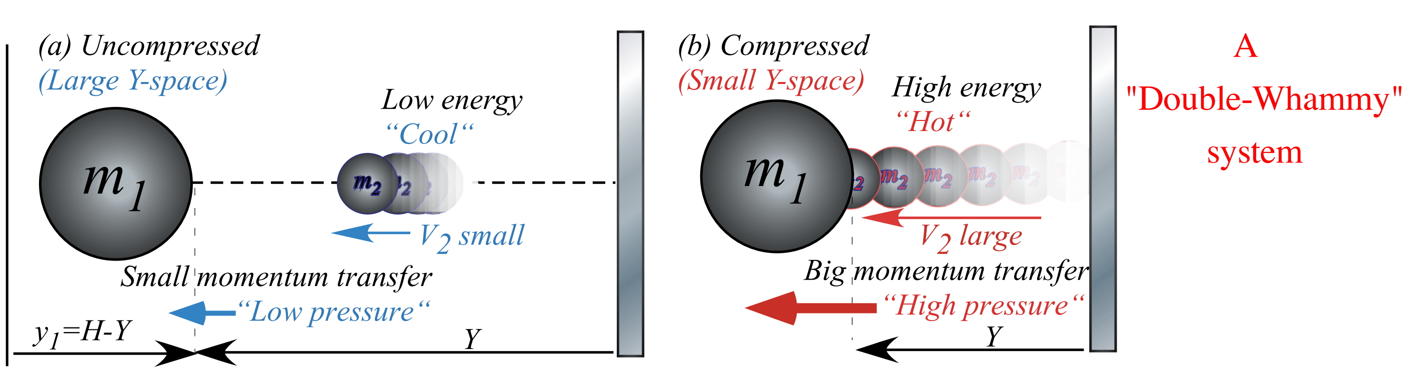
Q*?Another axiom?* A: No.



Big mass-m₁ ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

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Define for big mass m_1 : *Kinetic energy* $KE(v_1)$ vs *Potential energy* $PE(Y)=U(Y)$

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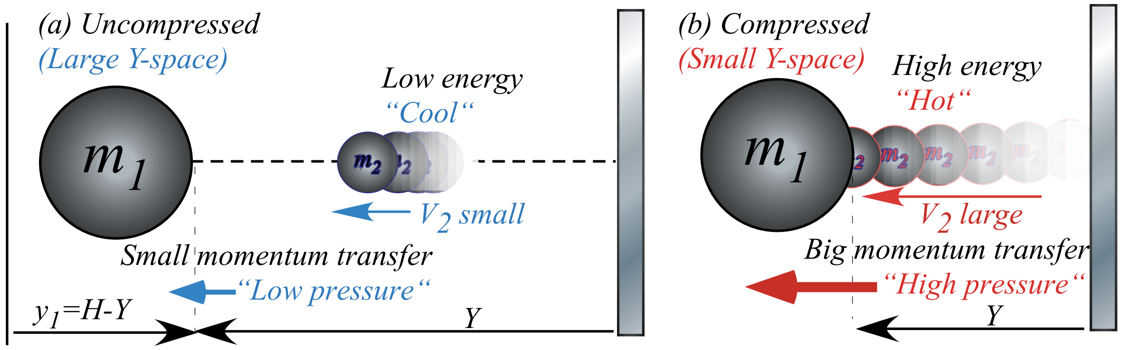
Q*?Another axiom?* A: No.

$$\int F\cdot dY = \int \frac{dp}{dt}\cdot dY = \int \frac{dY}{dt}\cdot dp = \int V\cdot dp = \int V\cdot d(mV) = m\frac{V^2}{2} + const = U$$

(Here: $V = v_2$)



A "Double-Whammy" system

Big mass-m_1 ball feeling "potential-field" or "gradient" due to small (m_2 << m_1) rapidly (v_2 >>v_1) bouncing ball

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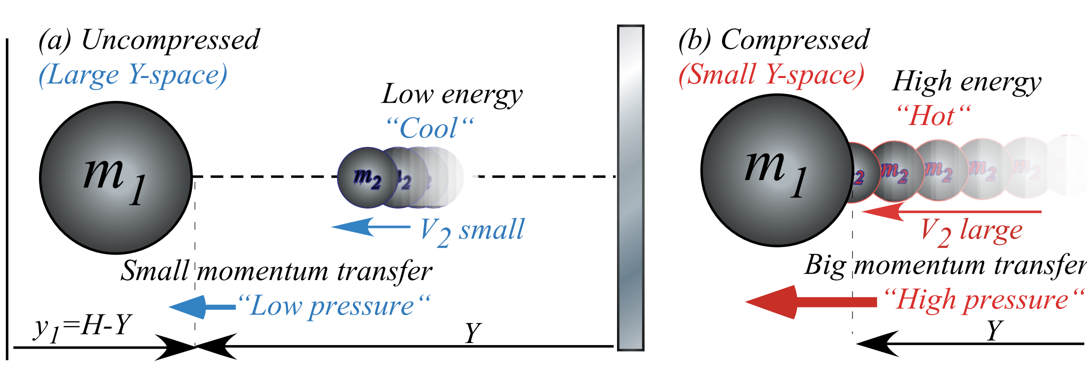
or else :

$$F\cdot\frac{dY}{dt} = \frac{dp}{dt}\cdot V = \frac{d(mV)}{dt}\cdot V = \frac{d(mV^2)/2}{dt} = \frac{dU}{dt} \qquad \text{(Here: } V = v_2\text{)}$$



A "Double-Whammy" system

Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)=const./y^2$

Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$

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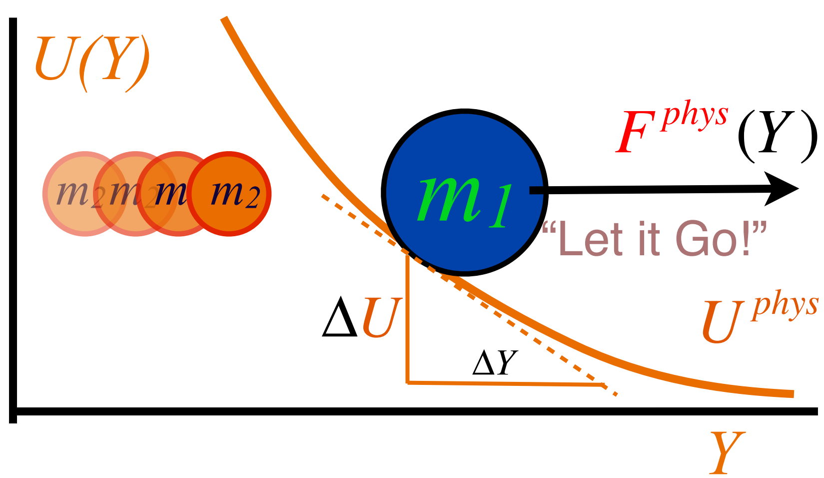
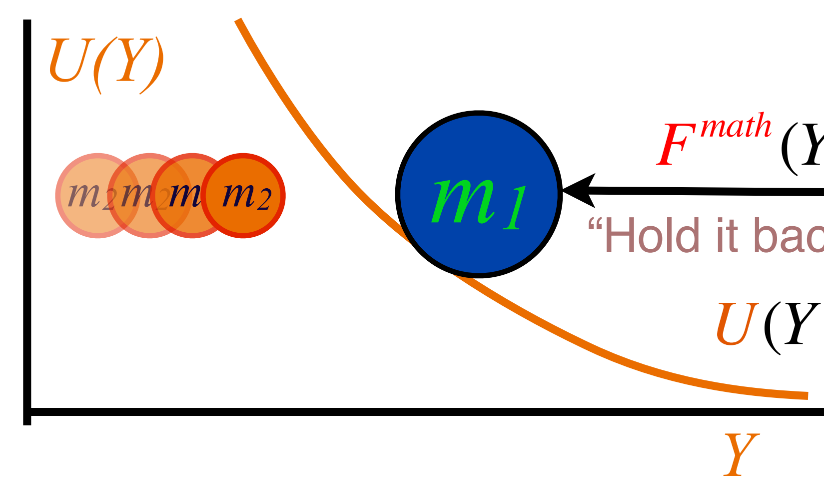
The "Physicist" View of Force



$$F^{phys}(Y) = -\frac{dU}{dY}$$

$$U^{phys}(Y) = -\int F^{phys}\, dY$$

The "Mathematician" View of Force



$$F^{math}(Y) = +\frac{dU}{dY}$$

$$U(Y) = +\int F^{phys}\, dY$$

Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

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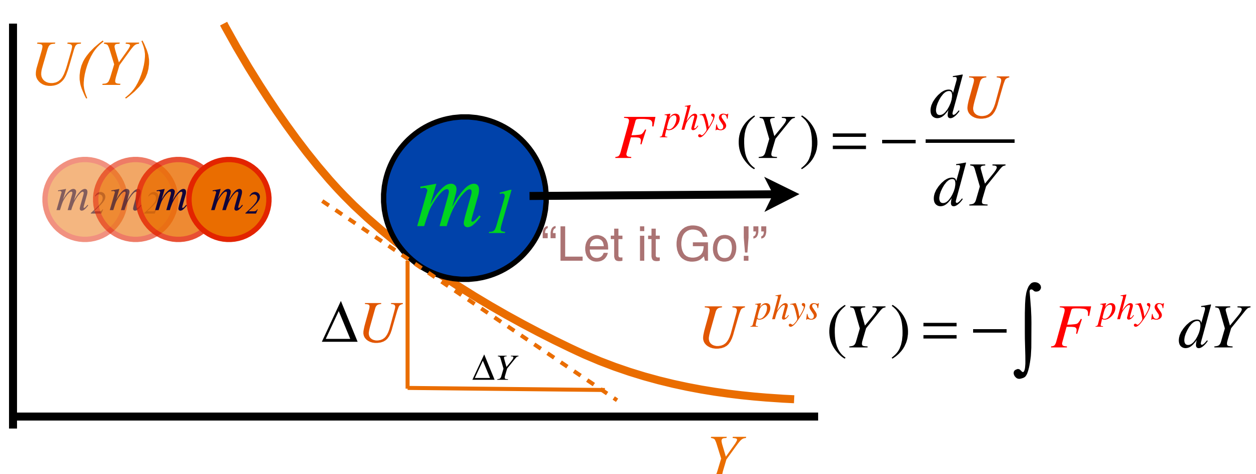
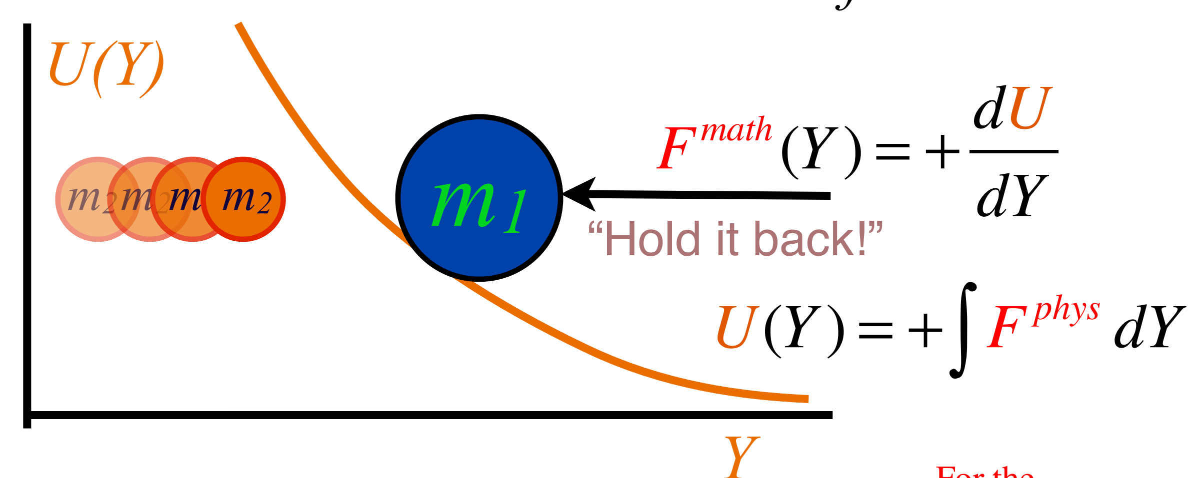
The "Physicist" View of Force



The "Mathematician" View of Force



(OK, But, is this consistent with the F=(const.)$^2/Y^3$ (on p.22)*?)*

For the "Double-Whammy" system

Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

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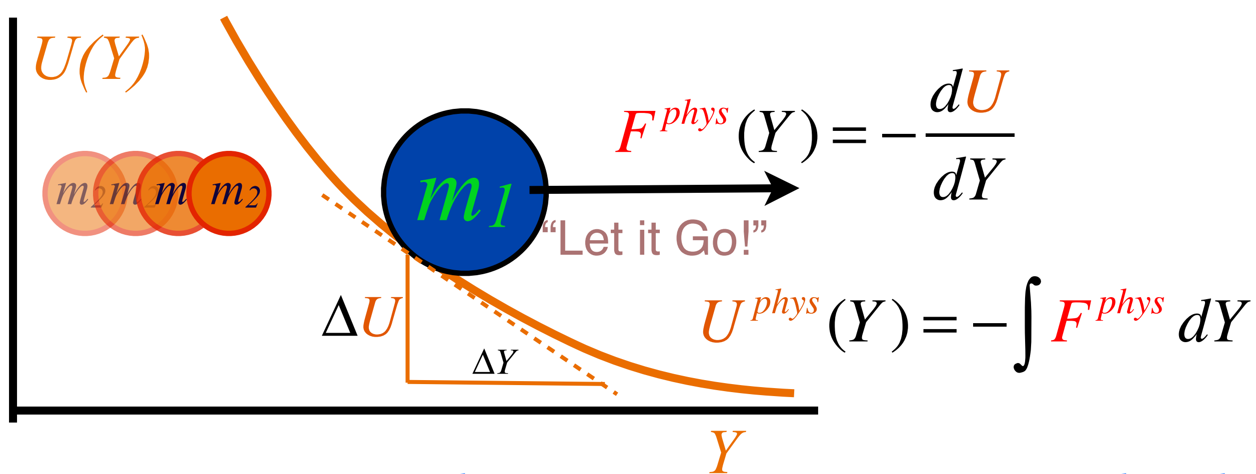
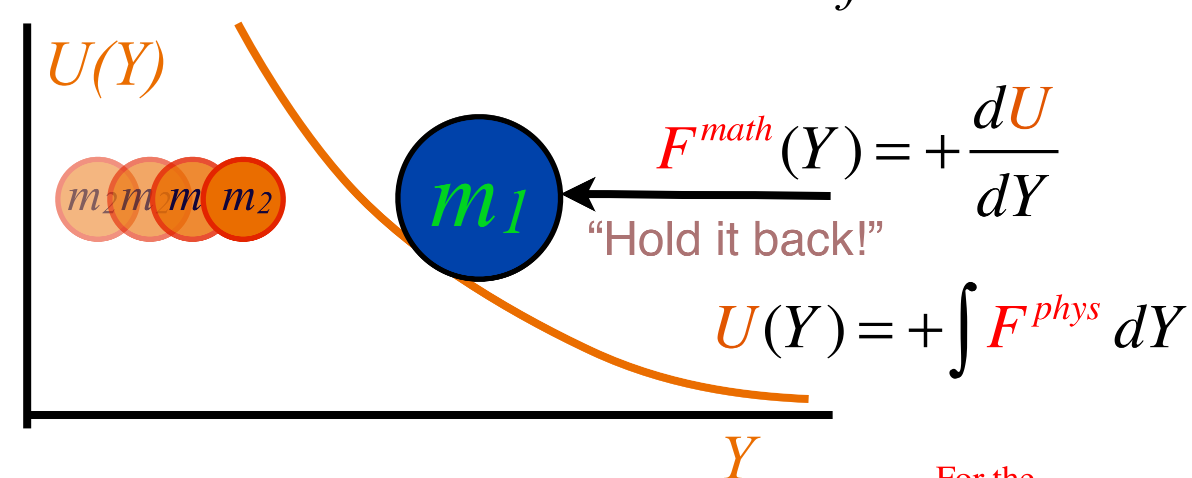
The "Physicist" View of Force



The "Mathematician" View of Force



(OK, But, is this consistent with the $F=(const.)^2/Y^3$ (on p.22)*?)* For the "Double-Whammy" system

$$F^{phys} = m_2 \frac{(const.)^2}{Y^3} \qquad \textit{consistent ? with ? :} \qquad F^{phys} = -\frac{\Delta U}{\Delta Y}$$

Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

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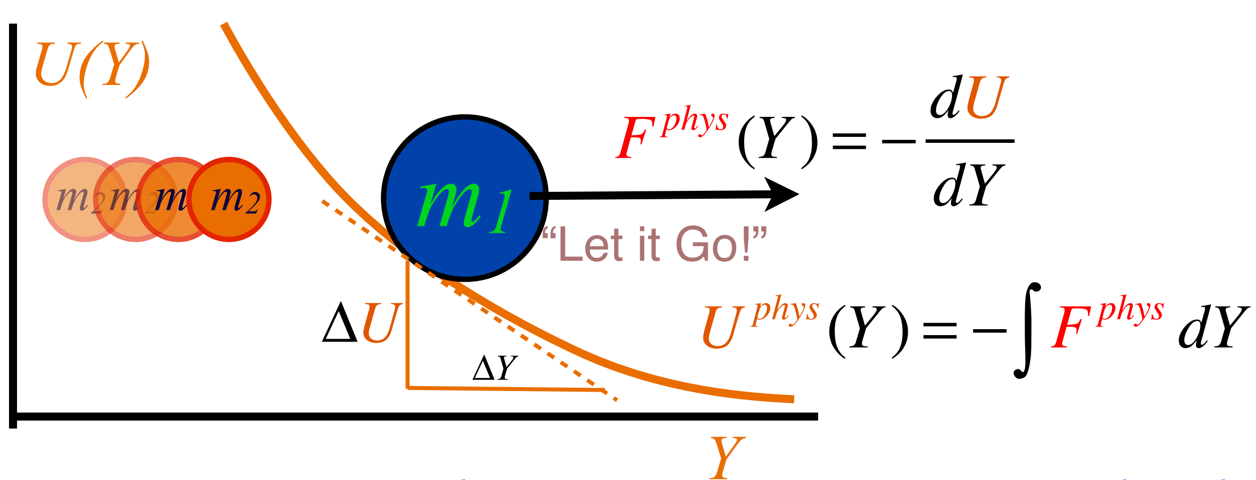
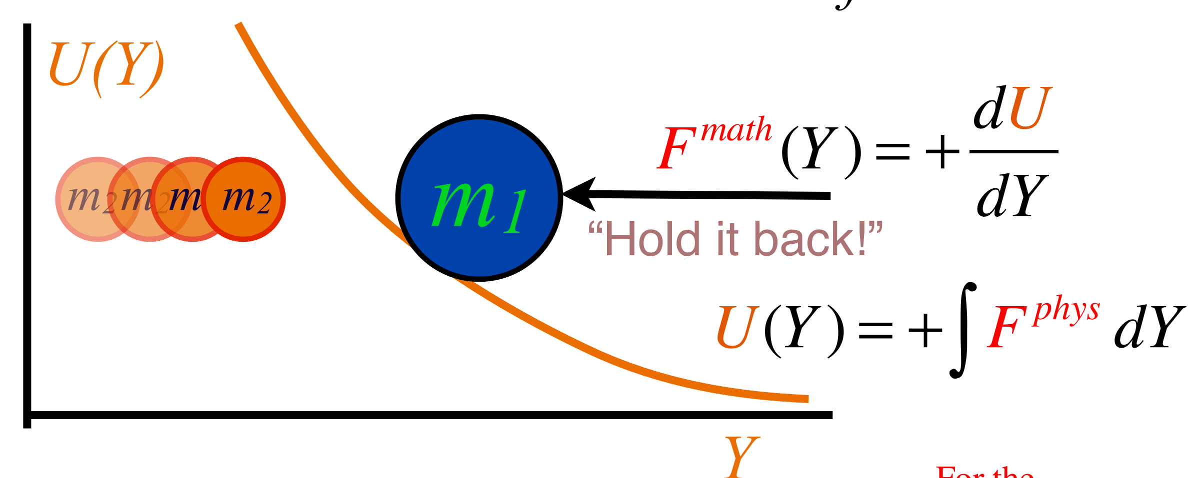
The "Physicist" View of Force



The "Mathematician" View of Force



(OK, But, is this consistent with the $F=(const.)^2/Y^3$ (on p.22) *?)* For the "Double-Whammy" system

$$F^{phys} = m_2\frac{(const.)^2}{Y^3}$$

consistent with :

$$F^{phys} = -\frac{\Delta U}{\Delta Y} = -\frac{d}{dY}\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2 = m_2\frac{(const.)^2}{Y^3}$$

Yes *(Hurrah!)*

Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)=const./y^2$

Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$

Example of 1D-Isothermal potential $U(y)=const.\ ln(y)$

1D-Isothermal Force Law *(assume v_2 is constant for all Y):* $\boxed{F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}}$

Not a "Double-Whammy"...

...only a "Single-Whammy"

$$F^{phys} = \frac{m_2 {v_2}^2}{Y} = -\frac{\Delta U}{\Delta Y} \qquad \textit{implies :} \qquad U(Y) = \int -F^{phys} dY = \int -\frac{m_2 {v_2}^2}{Y} dY = -m_2 {v_2}^2 \ln(Y)$$

1D-Isothermal Force Law (*assume* v_2 *is constant for all Y*): $\boxed{F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}}$

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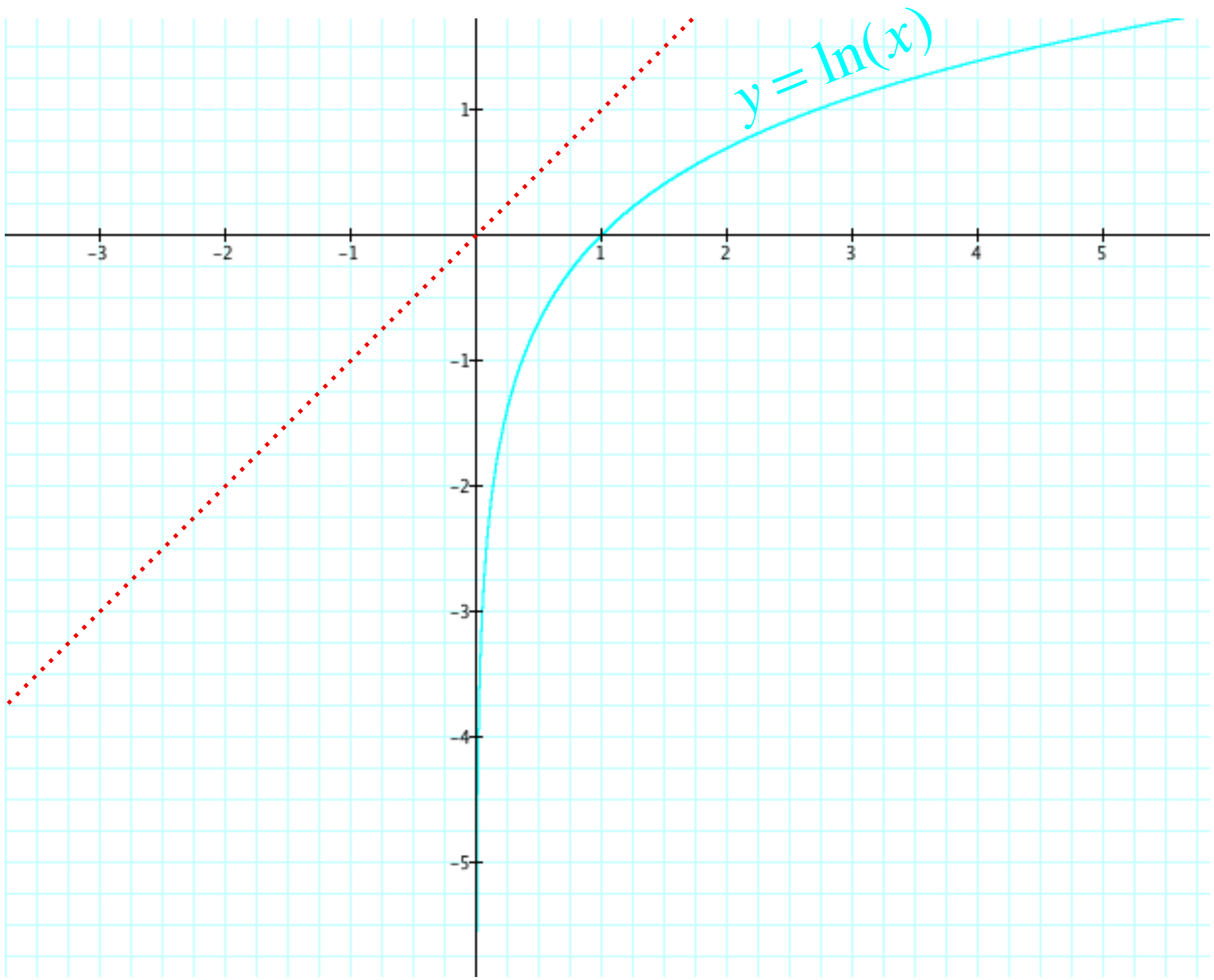

Notice how tightly
$ln(x)$ *hugs y-axis...*
It's the backside of exponential $y=e^x$...

1D-Isothermal Force Law (*assume* v_2 *is constant for all Y*): $\boxed{F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}}$

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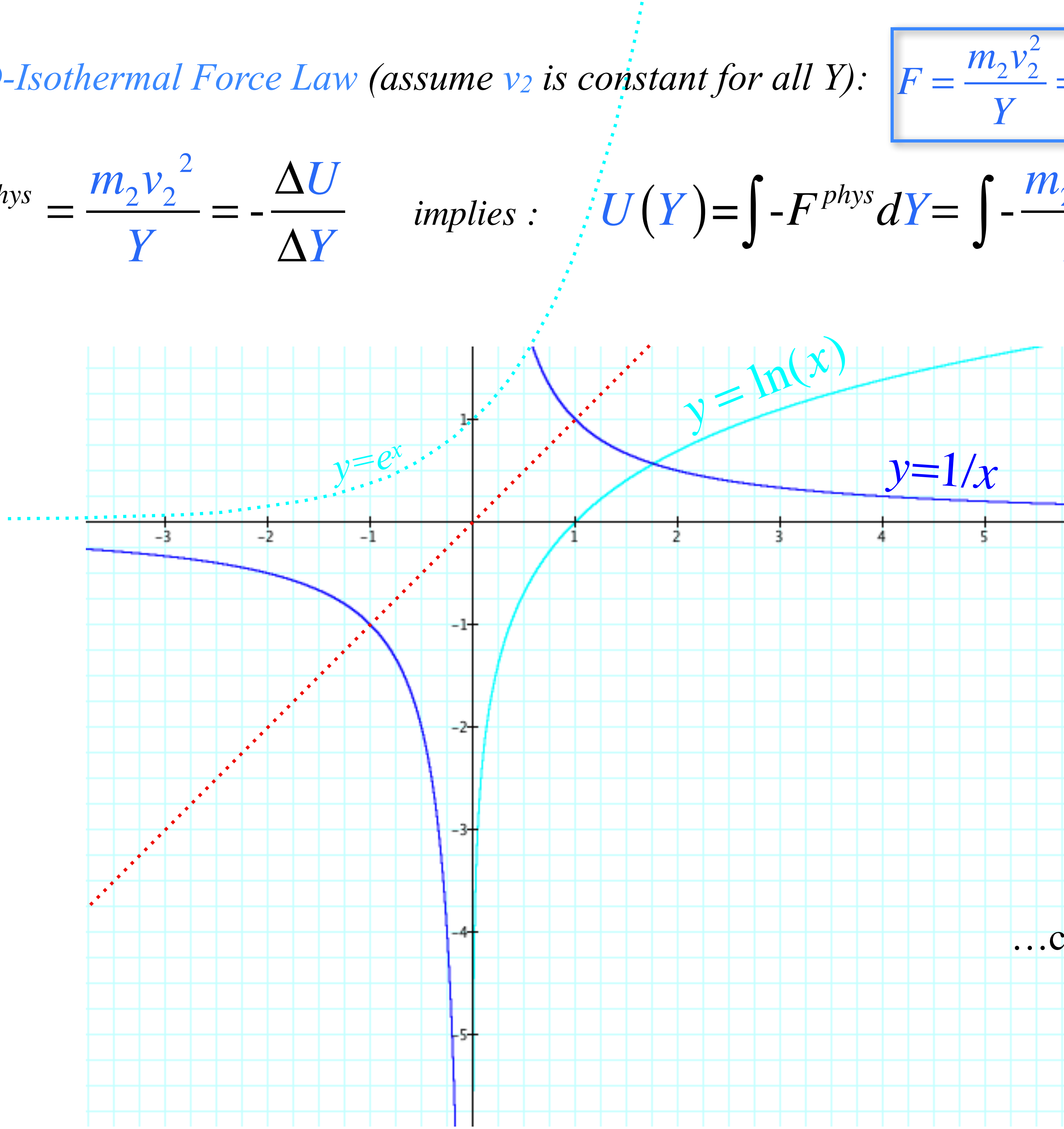

Notice how tightly *ln(x) hugs y-axis…*

It's the backside of exponential y=e^x…

…compared to *y=1/x or x=1/y*

1D-Isothermal Force Law (*assume* v_2 *is constant for all Y*): $\boxed{F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}}$

Not a "Double-Whammy"... ...only a "Single-Whammy"

$$F^{phys} = \frac{m_2 {v_2}^2}{Y} = -\frac{\Delta U}{\Delta Y} \quad implies: \quad U(Y) = \int -F^{phys} dY = \int -\frac{m_2 {v_2}^2}{Y} dY = -m_2 {v_2}^2 \ln(Y)$$

$$const. = E = \boxed{\frac{1}{2} m_1 {v_1}^2} + \boxed{U(Y) \quad where: U(Y) = -m_2 {v_2}^2 \ln(Y)}$$

Define for big mass m_1: *Kinetic energy* $\boxed{KE(v_1)}$ vs $\boxed{\textit{Potential energy } PE(Y) = U(Y)}$

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$$F^{phys}=\frac{m_2 {v_2}^2}{Y}=-\frac{\Delta U}{\Delta Y} \qquad implies: \qquad U(Y)=\int -F^{phys}dY=\int -\frac{m_2 {v_2}^2}{Y}dY=-m_2 {v_2}^2 \ln(Y)$$

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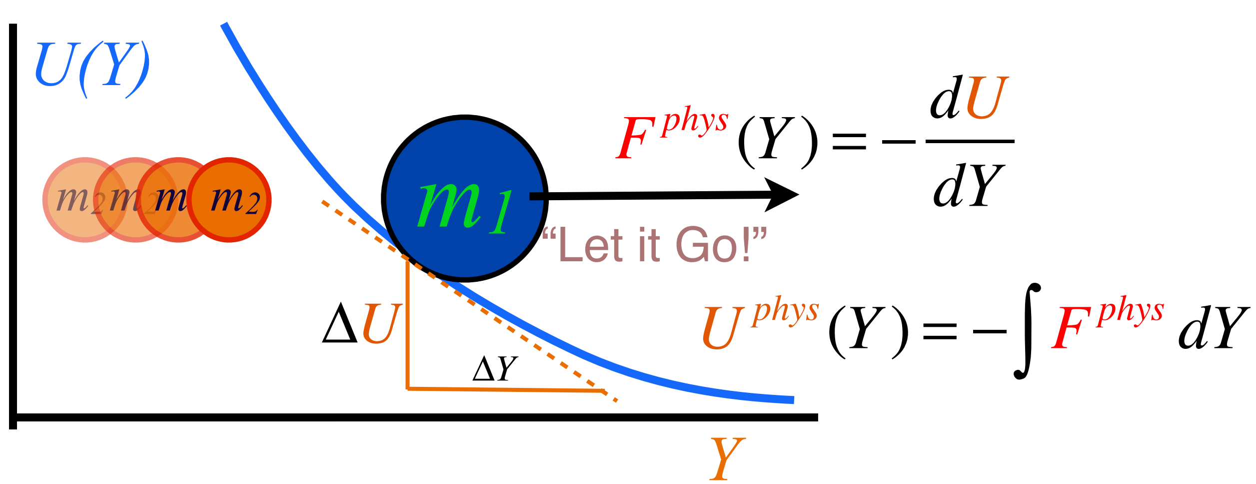
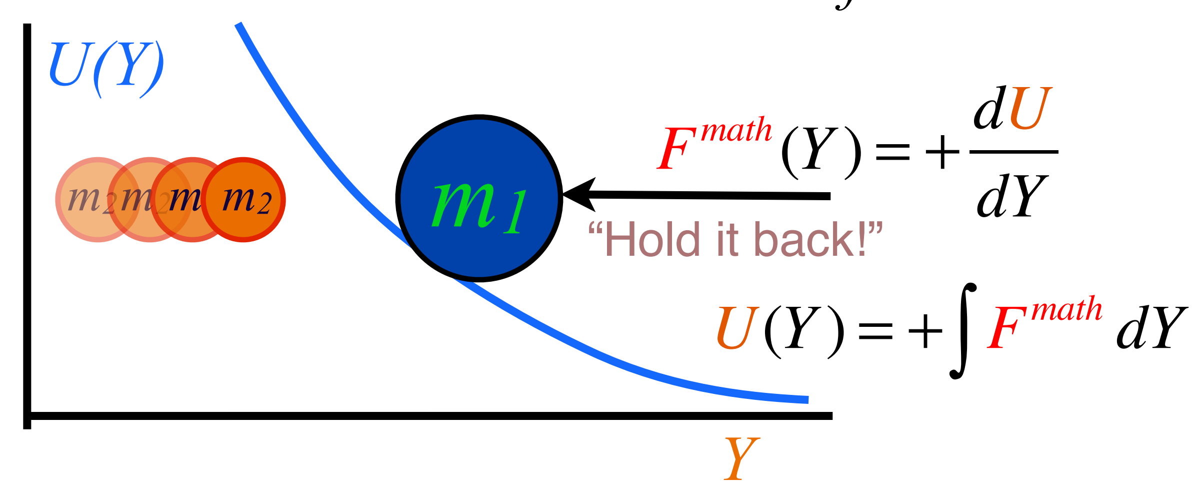
The "Physicist" View of Force



The "Mathematician" View of Force



1D-Isothermal Force Law (assume v_2 is constant for all Y): $\boxed{F=\frac{m_2 v_2^2}{Y}=\frac{const.}{Y}}$

Not a "Double-Whammy"... ...only a "Single-Whammy"

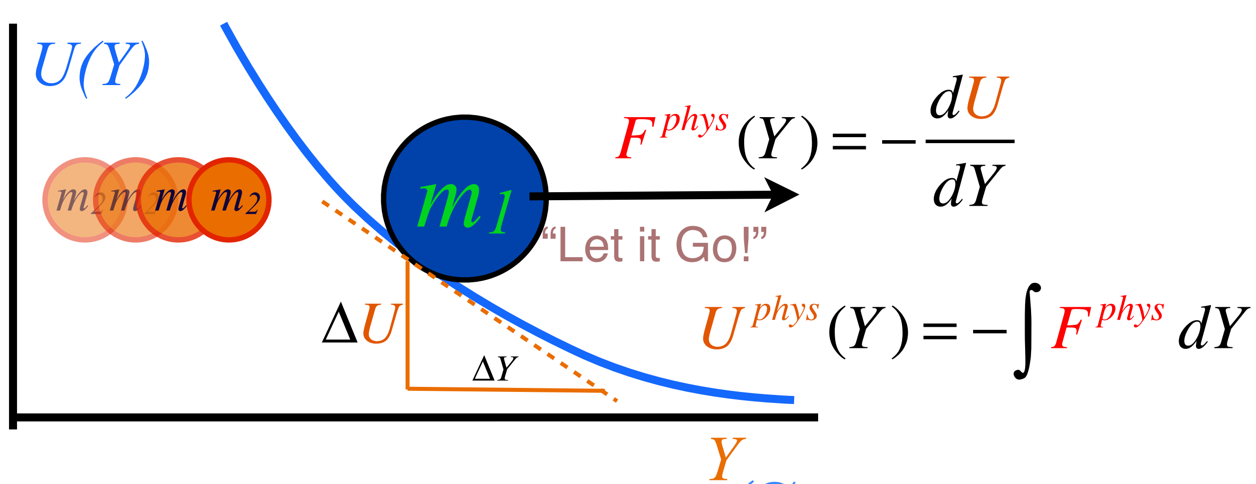
$$F^{phys}=\frac{m_2 {v_2}^2}{Y}=-\frac{\Delta U}{\Delta Y} \quad implies: \quad U(Y)=\int -F^{phys}dY=\int -\frac{m_2 {v_2}^2}{Y}dY=-m_2 {v_2}^2\ln(Y)$$

$$const.=E=\boxed{\frac{1}{2}m_1 {v_1}^2}+\boxed{U(Y) \quad where: U(Y)=-m_2 {v_2}^2\ln(Y)}$$

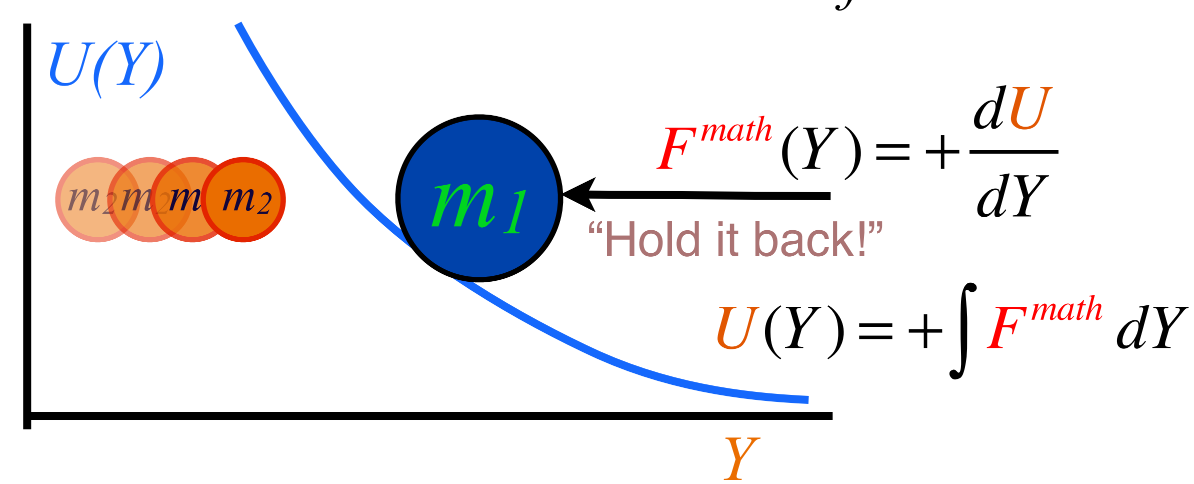
Define for big mass m_1: *Kinetic energy* $KE(v_1)$ vs *Potential energy* $PE(Y)=U(Y)$

Potential energy $PE(Y)=U(Y)=-m_2 {v_2}^2\ln(Y)$ relates to *Force* $F(Y)$ thru *Work relations* $F\cdot dY=\pm dU$

The "Physicist" View of Force



The "Mathematician" View of Force



(Same integral/differential relations)

$$F^{phys}=\frac{m_2 v_2^2}{Y}=\frac{const.}{Y} \quad consistent\ with: \quad F^{phys}=-\frac{\Delta U}{\Delta Y}=-\frac{d}{dY}\left(-const.\ln(Y)\right)=\frac{const.}{Y}$$

(Hurrah! again)

Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)=const./y^2$

Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$

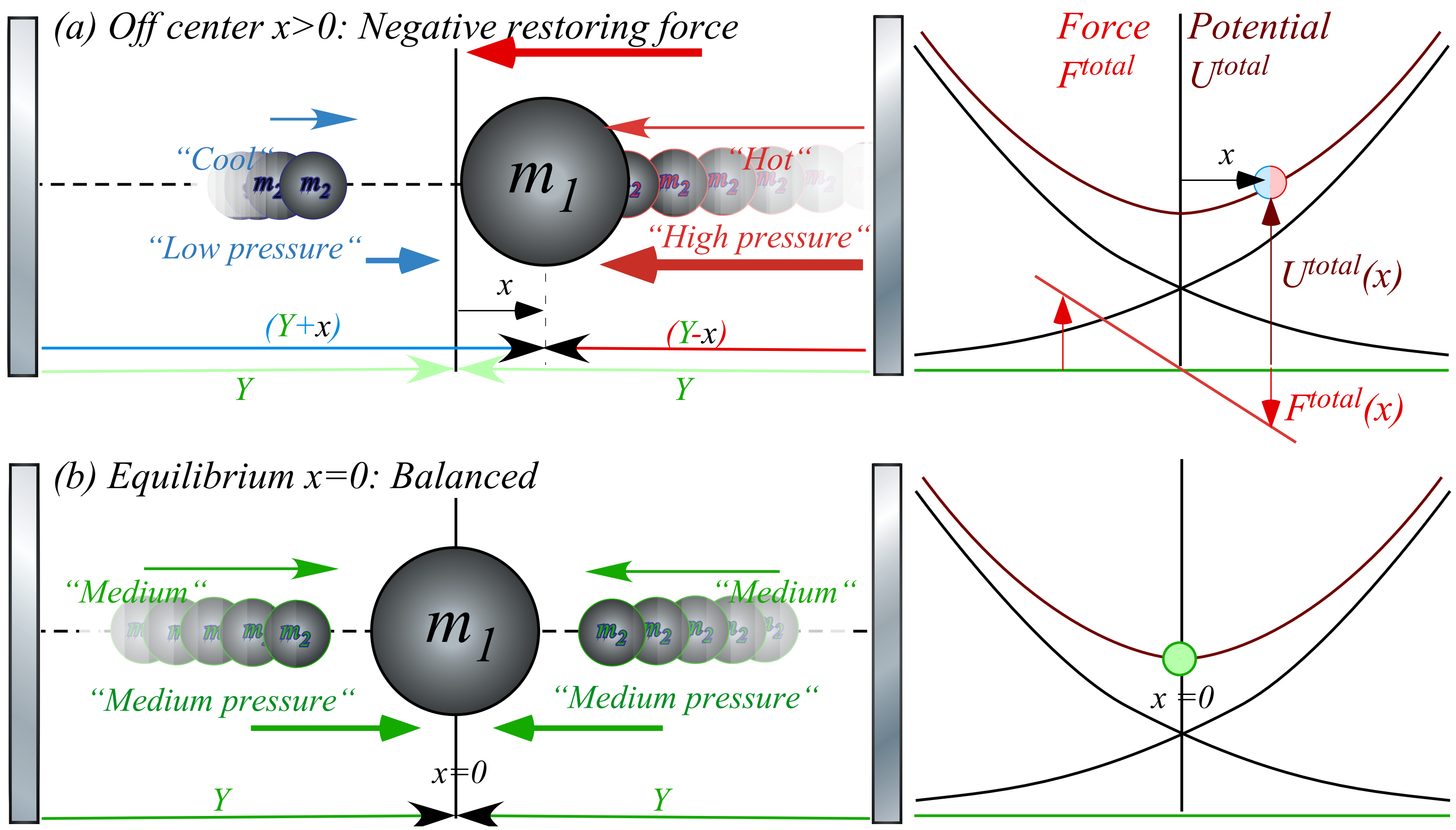
Example of 1D-Isothermal potential $U(y)=const.\ ln(y)$

Example of oscillator with opposing Isothermal potentials

Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y): $\boxed{F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}}$

$$F^{phys} = \frac{m_2 v_2^2}{Y} = -\frac{\Delta U}{\Delta Y} \quad \text{implies :} \quad U = -m_2 v_2^2 \ln(Y)$$



Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields

Harmonic oscillator term

Anharmonic oscillator terms...

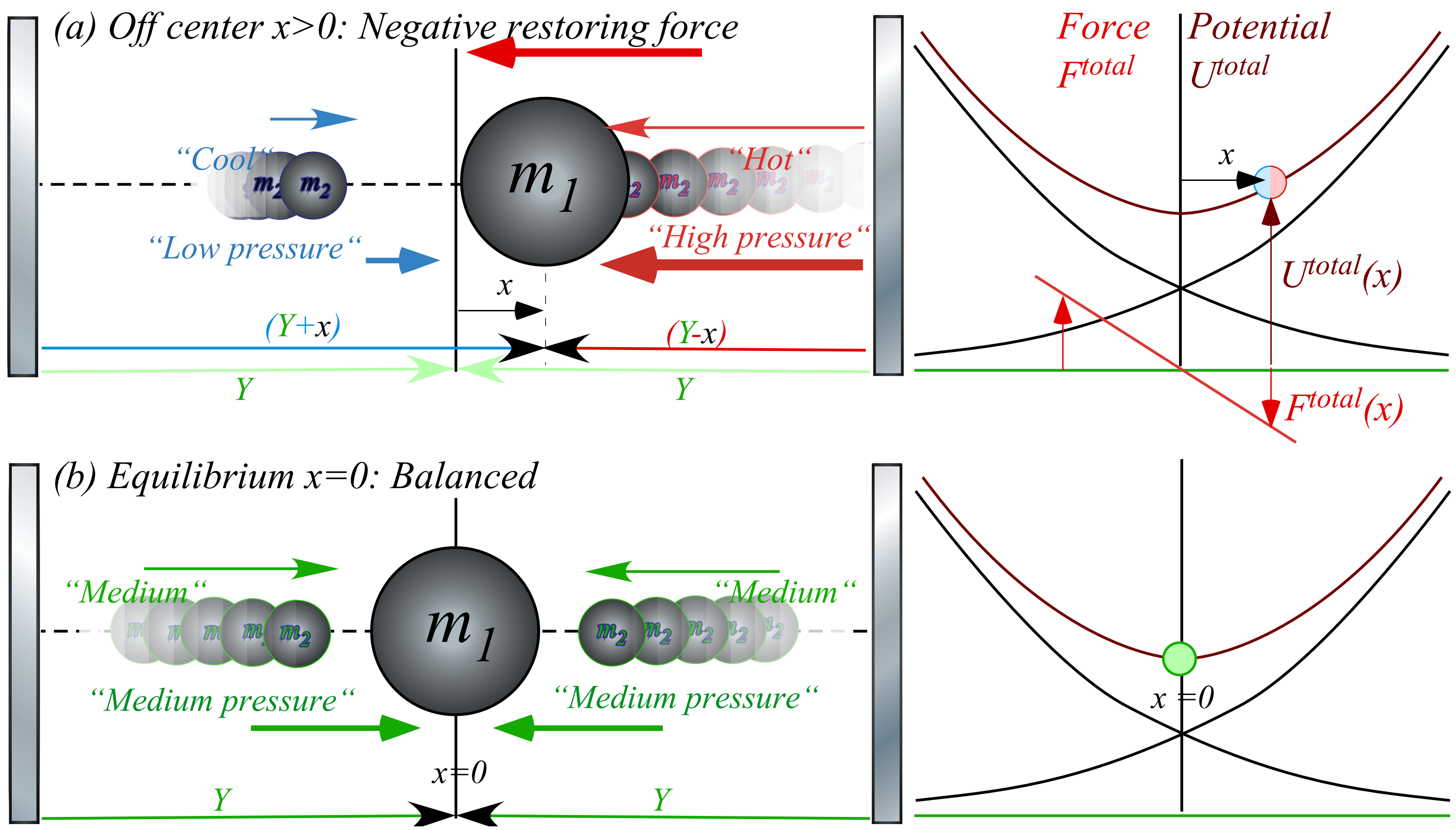
$$F^{total} = \frac{f}{1+x} - \frac{f}{1-x} = f\left[1 - x + x^2 - x^3 \ldots\right] - f\left[1 + x + x^2 + x^3 \ldots\right] = -2f\cdot x - 2f\cdot x^3 - \ldots$$

$$F^{HO} = -k\cdot x = -\frac{\partial U^{HO}}{\partial x} \qquad U^{HO} = \frac{1}{2}k\cdot x^2 = -\int F^{HO}\,dx \qquad \text{HO } \measuredangle\text{frequency: } \omega = \sqrt{\frac{k}{m_1}} = 2\pi\upsilon$$

Example of oscillator with opposing ***Isothermal potentials***

1D-Isothermal Force Law (assume v_2 is constant for all Y): $\boxed{F=\frac{m_2 v_2^2}{Y}=\frac{const.}{Y}}$

$$F^{phys}=\frac{m_2 v_2^2}{Y}=-\frac{\Delta U}{\Delta Y} \quad implies: \quad U=-m_2 v_2^2 \ln(Y)$$



Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields

$$F^{total}=\frac{f}{Y_0+x}-\frac{f}{Y_0-x}$$

Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y): $\boxed{F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}}$

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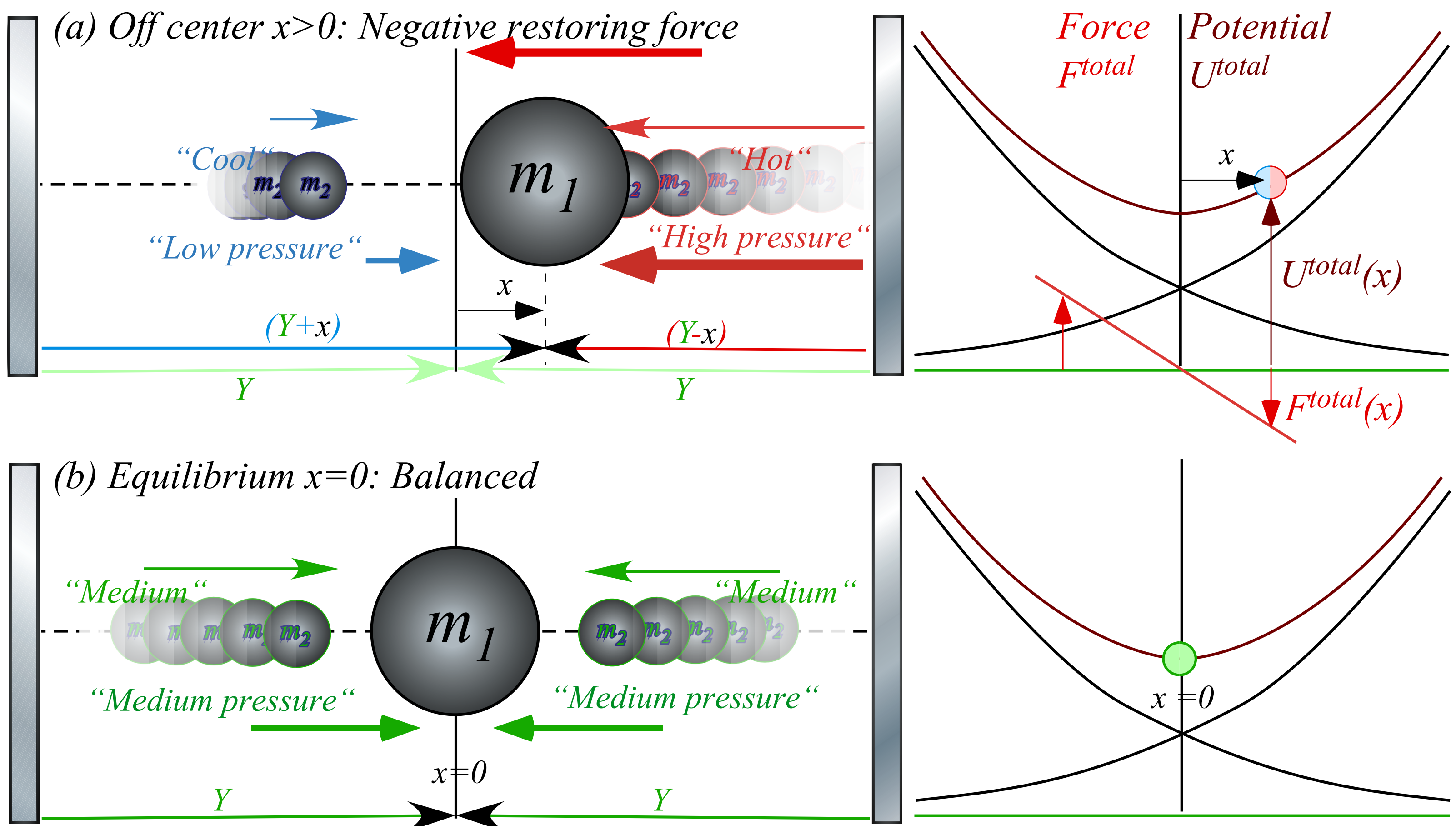

Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields

$$F^{total} = \frac{f}{Y_0 + x} - \frac{f}{Y_0 - x}$$

$$(Y_0 + x)^{-1} = Y_0^{-1} - xY_0^{-2} + x^2 Y_0^{-3} - x^3 Y_0^{-4} \ldots$$

$$(Y_0 + x)^n = Y_0^n + nY_0^{n-1}x + \frac{n(n-1)}{2} Y_0^{n-2}x^2 + \frac{n(n-1)(n-2)}{2\cdot 3} Y_0^{n-3}x^3 + \frac{n(n-1)(n-2)(n-3)}{2\cdot 3\cdot 4} Y_0^{n-4}x^4 \ldots$$

Binomial Theorem

Example of oscillator with opposing ***Isothermal potentials***

1D-Isothermal Force Law (assume v_2 is constant for all Y): $\boxed{F=\frac{m_2 v_2^2}{Y}=\frac{const.}{Y}}$

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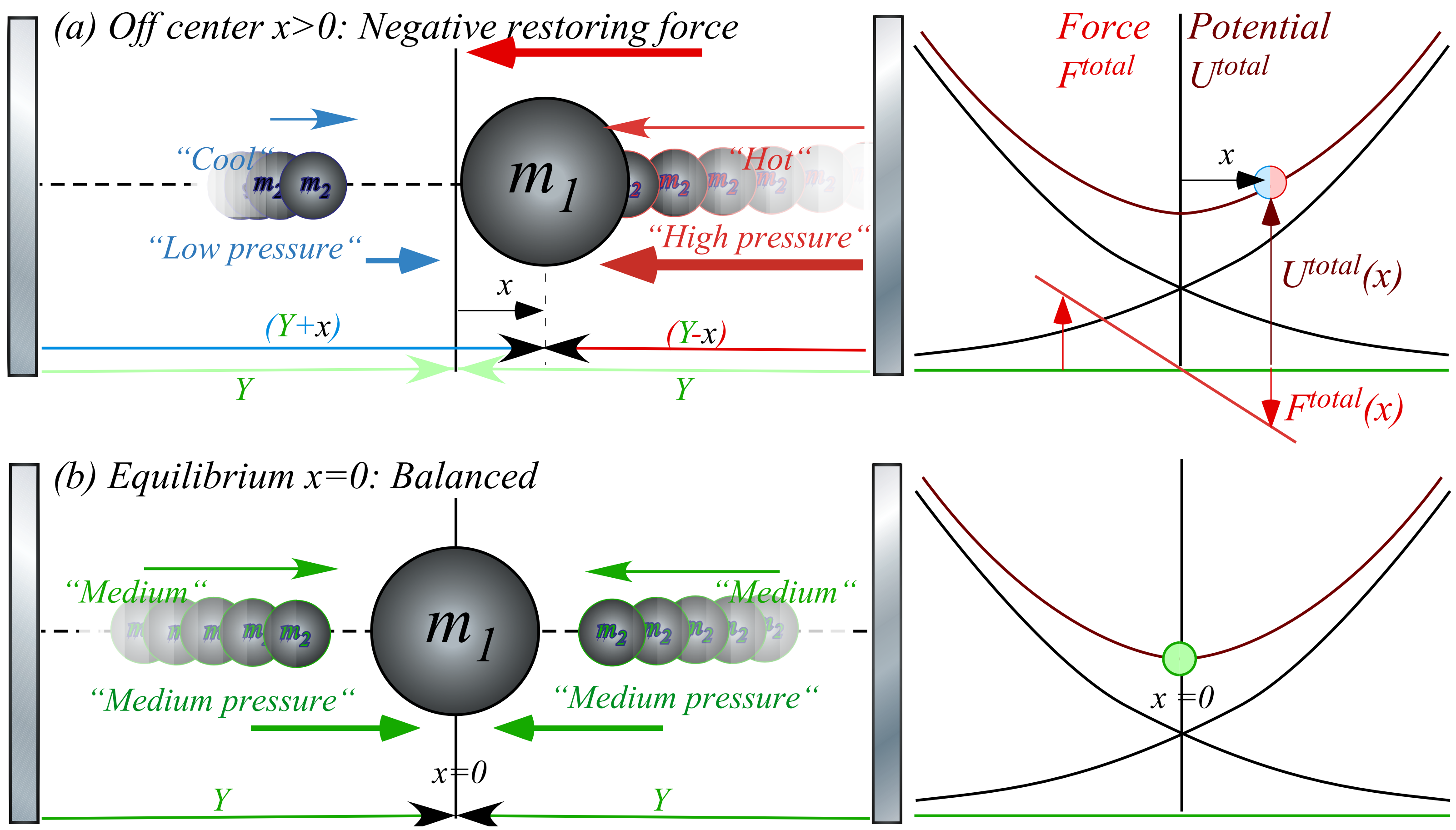

Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields

$$F^{total}=\frac{f}{Y_0+x}-\frac{f}{Y_0-x}=f\left[\frac{1}{Y_0}-\frac{x}{Y_0^2}+\frac{x^2}{Y_0^3}-\frac{x^3}{Y_0^4}+\cdots\right]-\ldots$$

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Example of oscillator with opposing Isothermal potentials

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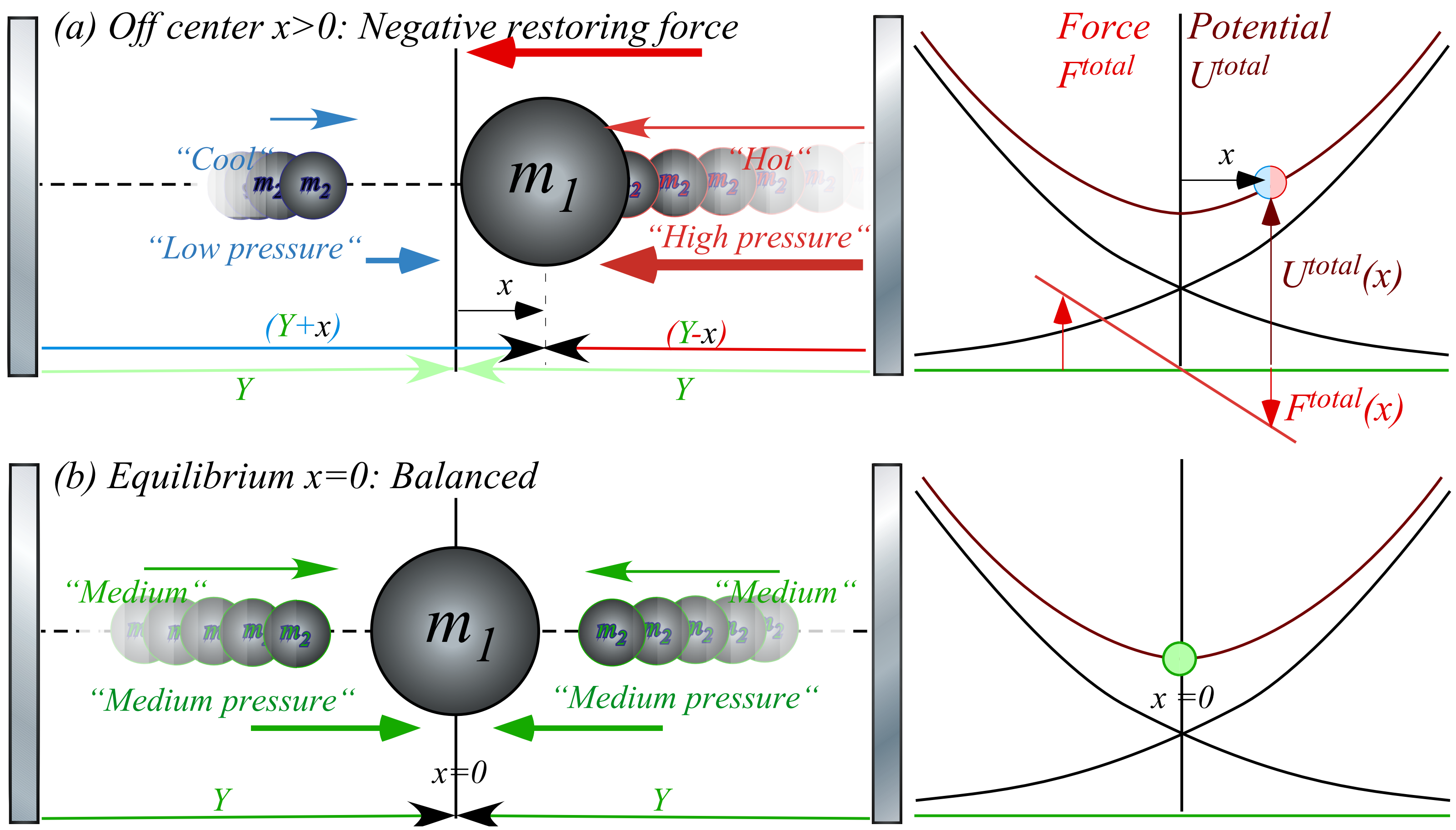

Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields

$$F^{total}=\frac{f}{Y_0+x}-\frac{f}{Y_0-x}=f\left[\frac{1}{Y_0}-\frac{x}{Y_0^2}+\frac{x^2}{Y_0^3}-\frac{x^3}{Y_0^4}+\cdots\right]-f\left[\frac{1}{Y_0}+\frac{x}{Y_0^2}+\frac{x^2}{Y_0^3}+\frac{x^3}{Y_0^4}+\cdots\right]$$

$$(Y_0+x)^{-1}=Y_0^{-1}-xY_0^{-2}+x^2Y_0^{-3}-x^3Y_0^{-4}\ldots \qquad (Y_0-x)^{-1}=Y_0^{-1}+xY_0^{-2}+x^2Y_0^{-3}+x^3Y_0^{-4}\ldots$$

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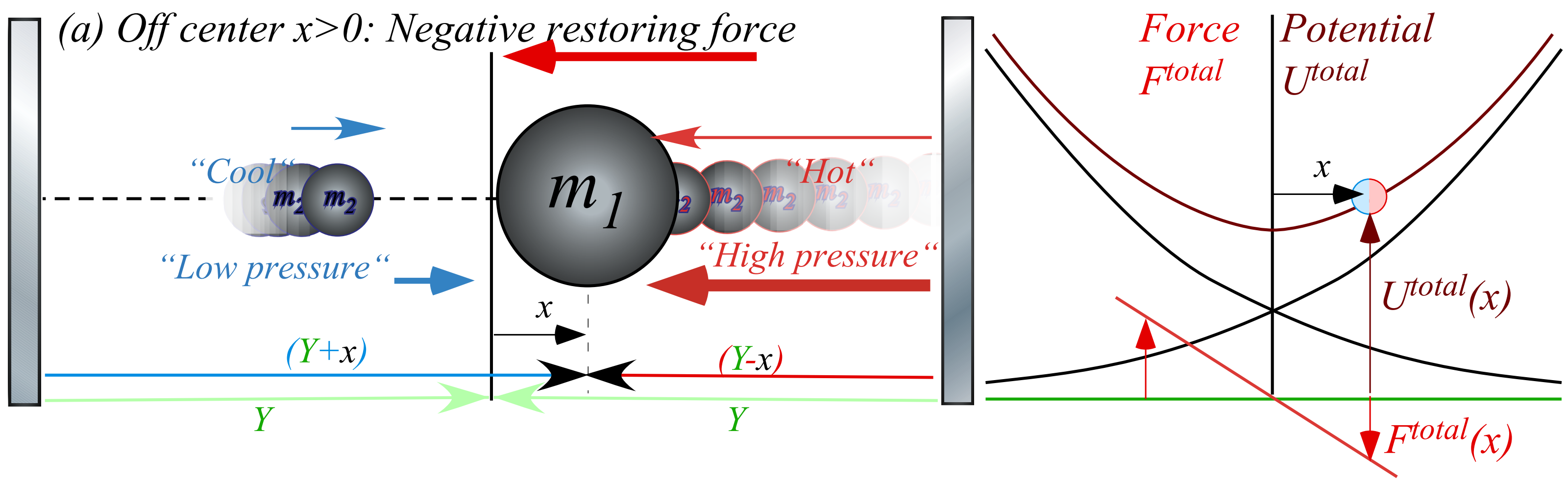
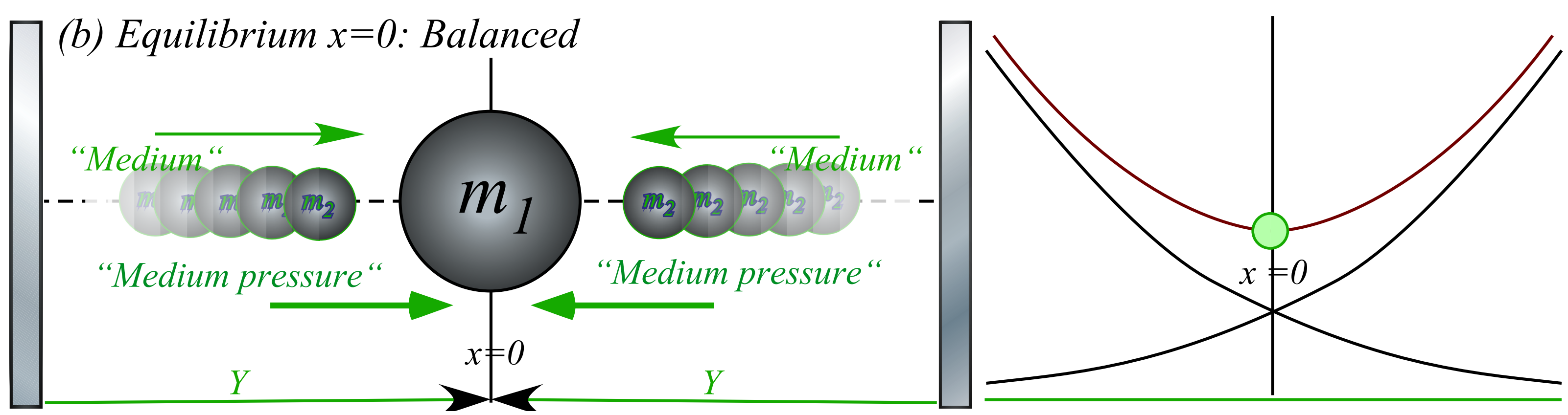
Unit 1
Fig. 6.2





Two opposing 1D-Isothermal Force fields

$$F^{total}=\frac{f}{Y_0+x}-\frac{f}{Y_0-x}=f\left[\cancel{\frac{1}{Y_0}}-\frac{x}{Y_0^2}+\cancel{\frac{x^2}{Y_0^3}}-\frac{x^3}{Y_0^4}+\cdots\right]-f\left[\cancel{\frac{1}{Y_0}}+\frac{x}{Y_0^2}+\cancel{\frac{x^2}{Y_0^3}}+\frac{x^3}{Y_0^4}+\cdots\right]$$

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Binomial Theorem

Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (*assume v_2 is constant for all Y*):
$$\boxed{F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}}$$

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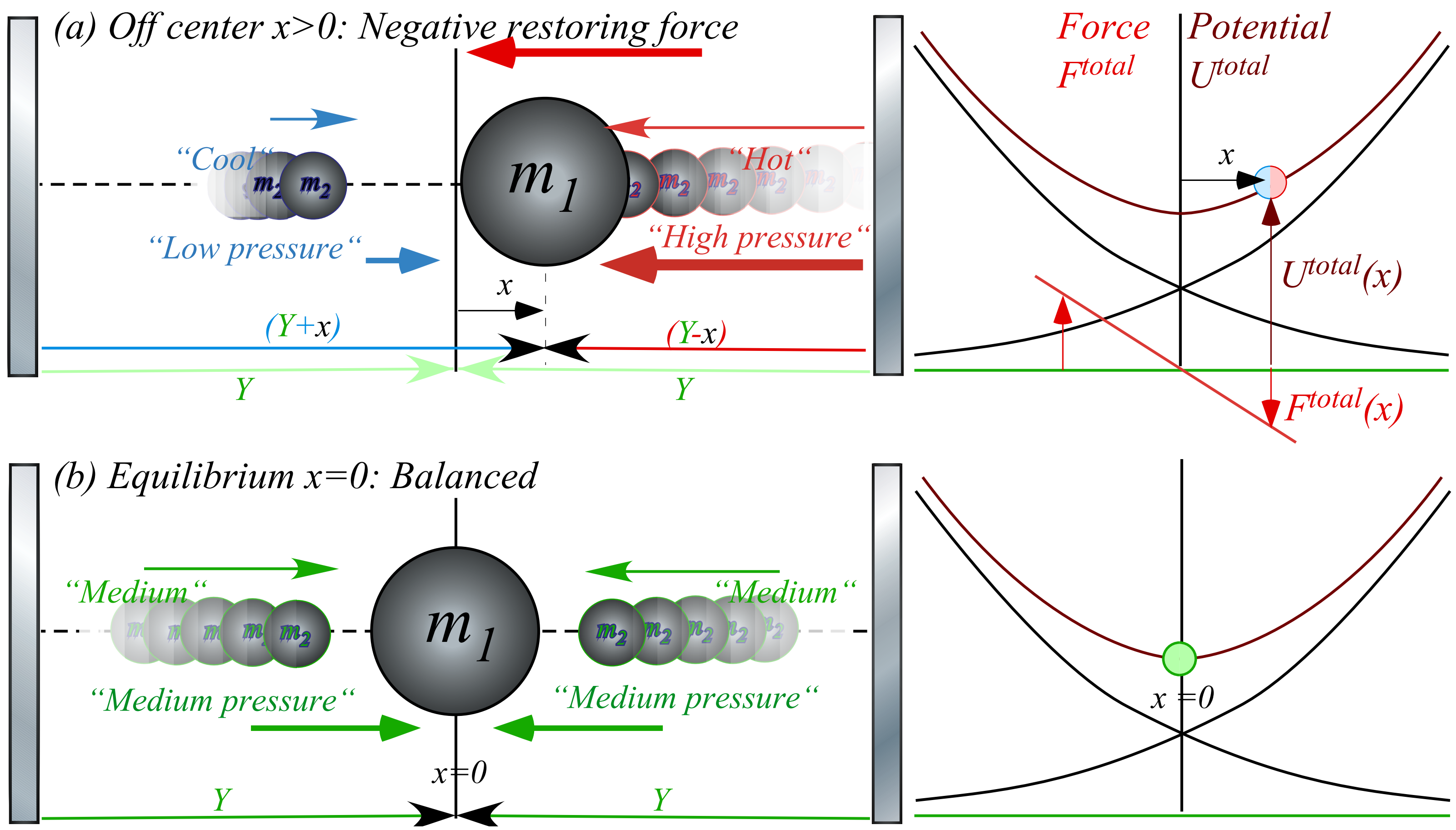

Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields approximate harmonic oscillator

Harmonic oscillator term

Anharmonic oscillator terms...

$$F^{total} = \frac{f}{Y_0+x} - \frac{f}{Y_0-x} = f\left[\cancel{\frac{1}{Y_0}} - \frac{x}{Y_0^2} + \cancel{\frac{x^2}{Y_0^3}} - \frac{x^3}{Y_0^4} + \cdots\right] - f\left[\cancel{\frac{1}{Y_0}} + \frac{x}{Y_0^2} + \cancel{\frac{x^2}{Y_0^3}} + \frac{x^3}{Y_0^4} + \cdots\right] = -2f\frac{x}{Y_0^2} - 2f\frac{x^3}{Y_0^4} - \ldots$$

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Binomial Theorem

Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y): $\boxed{F=\frac{m_2 v_2^2}{Y}=\frac{const.}{Y}}$

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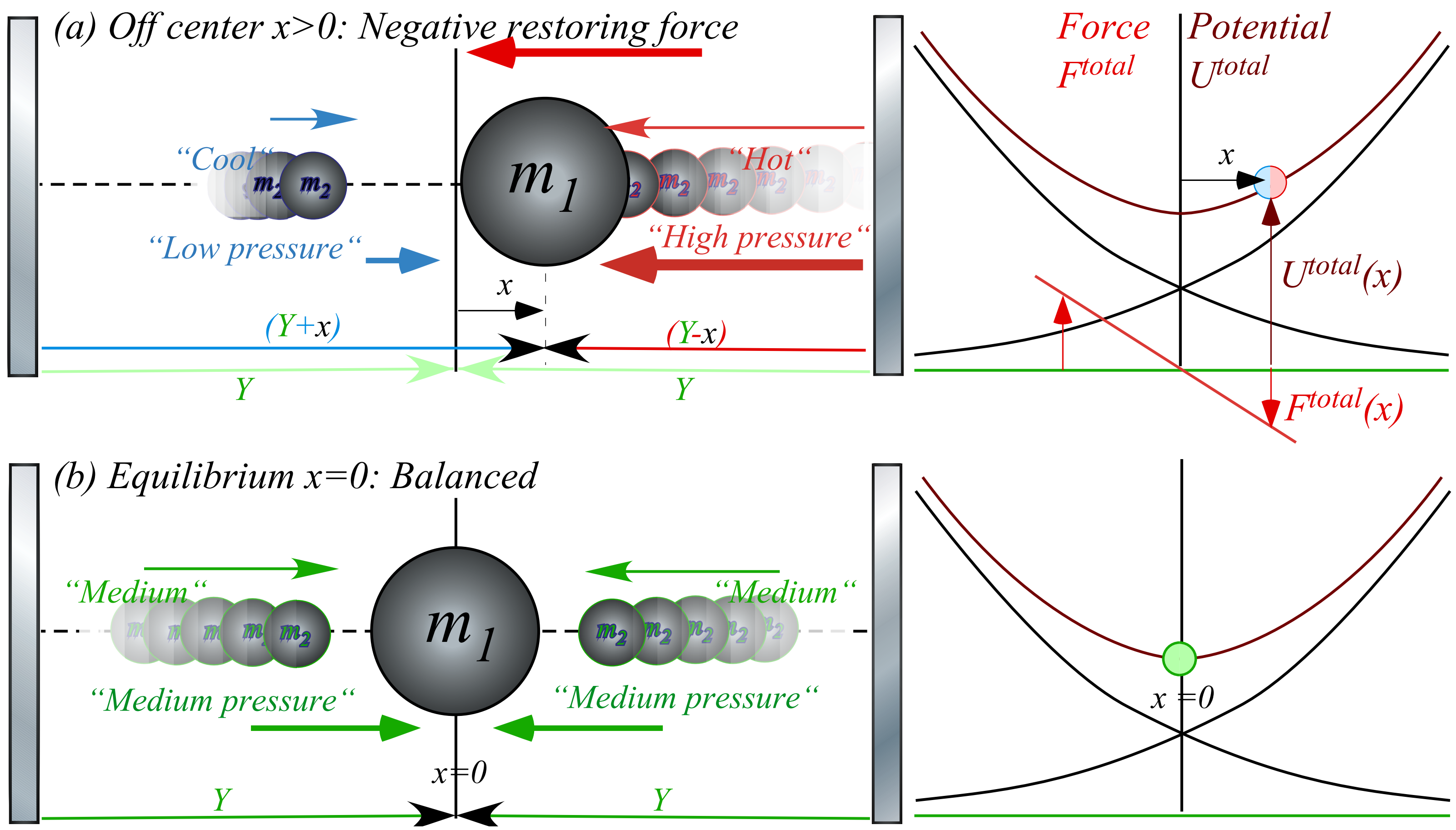

Unit 1
Fig. 6.2

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Harmonic oscillator force constant : $k=2f/Y_0^2=2m_2v_2^2/Y_0^2$

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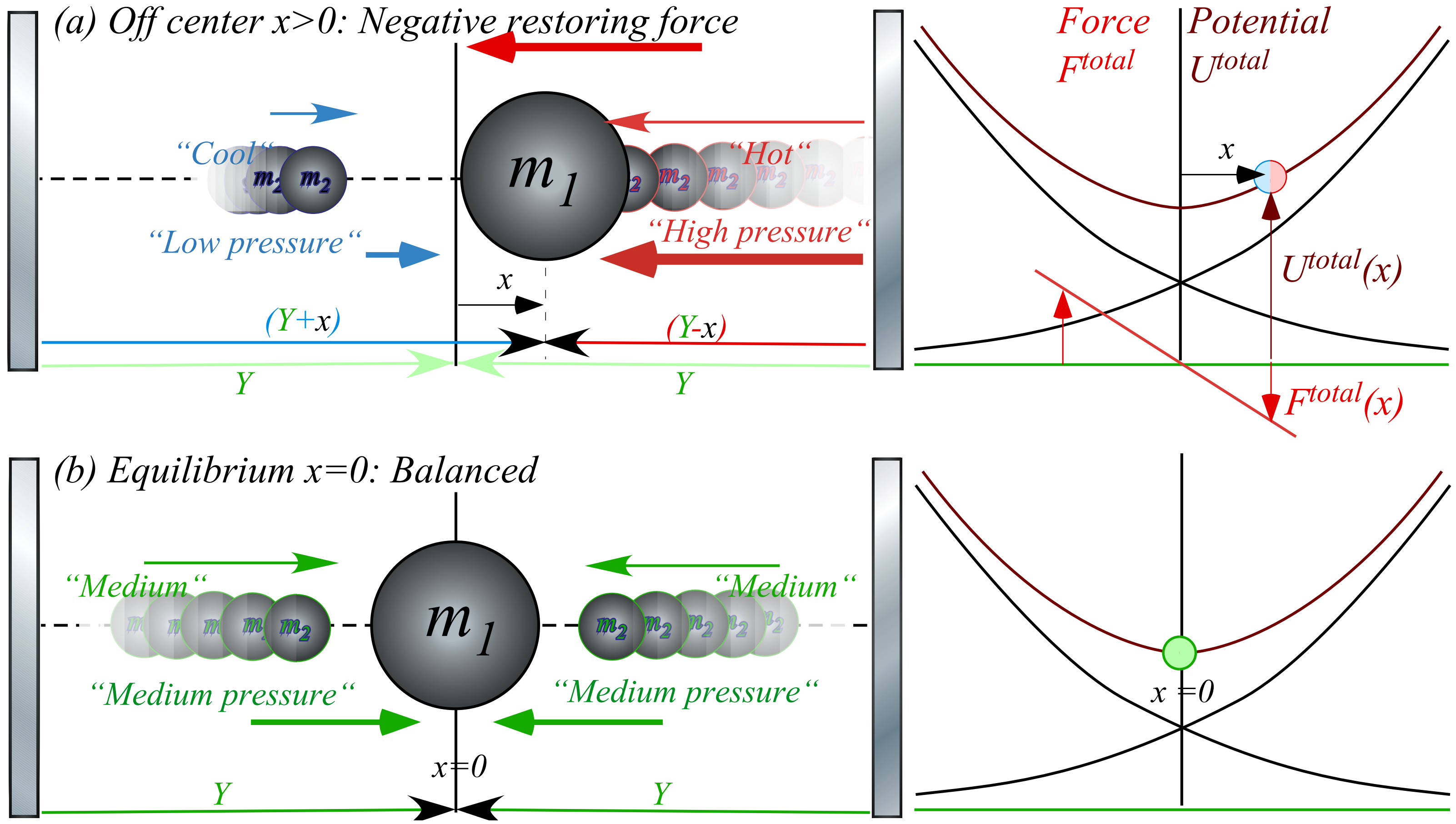

Unit 1
Fig. 6.2

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Harmonic Oscillator Force

$$F^{HO}=-k\cdot x=-\frac{\partial U^{HO}}{\partial x}$$

Example of oscillator with opposing Isothermal potentials

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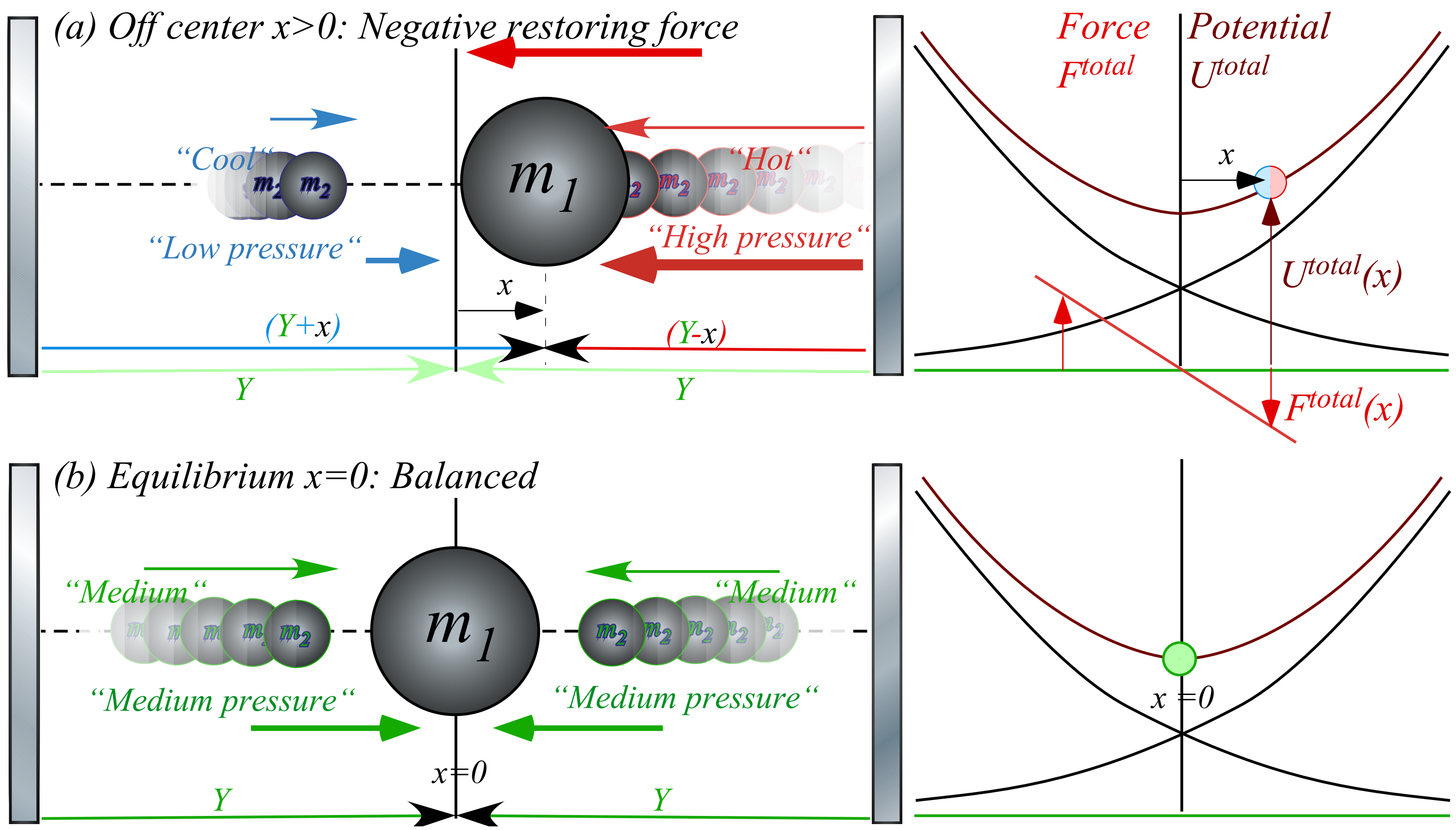

Unit 1
Fig. 6.2

Anharmonic oscillator terms...

Harmonic oscillator term

Two opposing 1D-Isothermal Force fields approximate harmonic oscillator

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Harmonic Oscillator Force

$$F^{HO}=-k\cdot x=-\frac{\partial U^{HO}}{\partial x}$$

Potential

$$U^{HO}=\frac{1}{2}k\cdot x^2=-\int F^{HO}\,dx$$

Example of oscillator with opposing ***Isothermal potentials***

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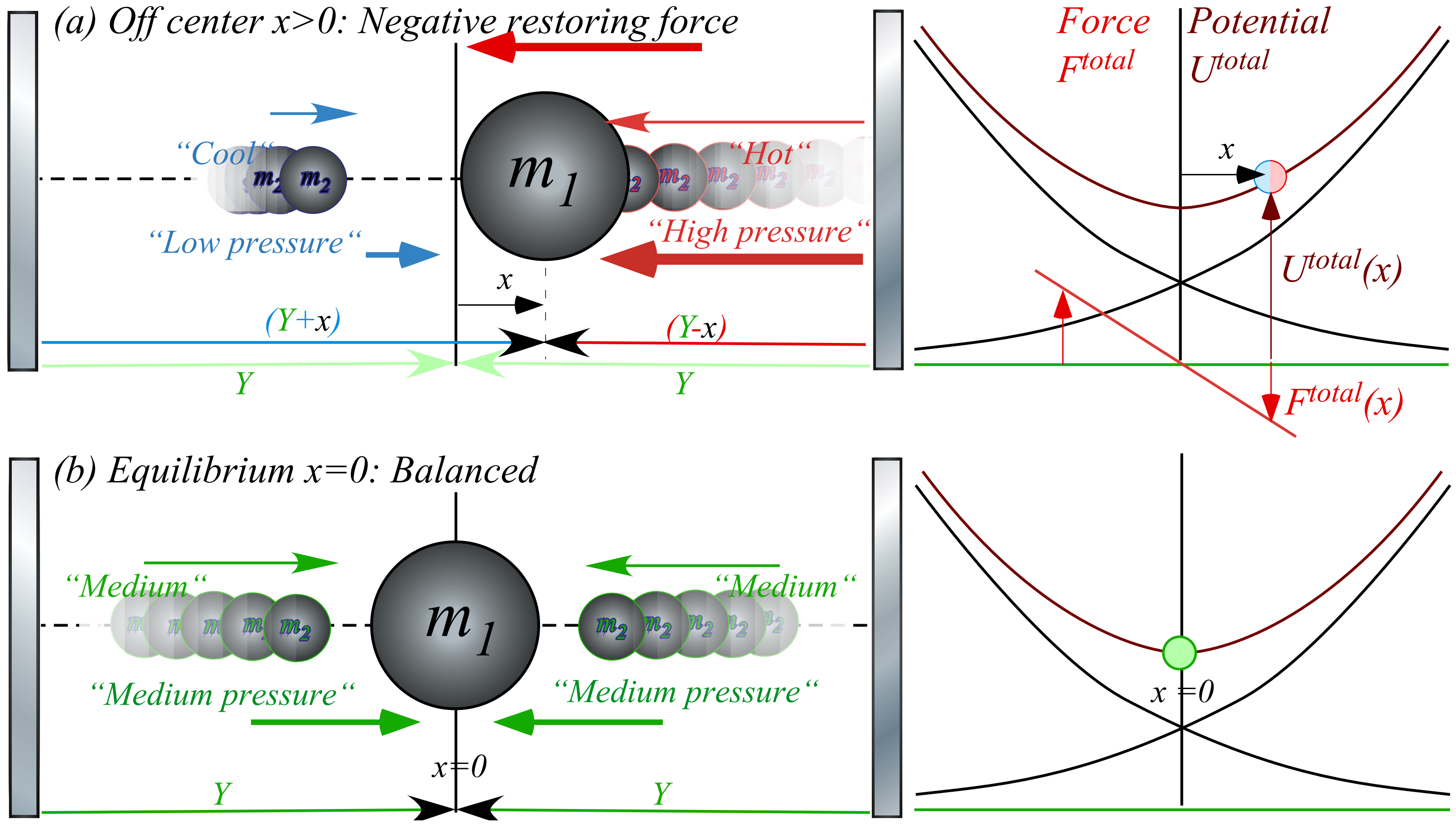

Unit 1
Fig. 6.2

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Harmonic Oscillator Force

$$F^{HO} = -k\cdot x = -\frac{\partial U^{HO}}{\partial x}$$

Potential

$$U^{HO} = \frac{1}{2}k\cdot x^2 = -\int F^{HO}\,dx$$

Frequency

HO $\measuredangle$frequency: $\omega = \sqrt{\frac{k}{m_1}} = \sqrt{\frac{2m_2}{m_1}}\frac{v_2}{Y_0} = 2\pi\upsilon$

What does *Harmonic* mean?

Given total energy $E = KE+PE = \frac{1}{2}mV^2 + \frac{1}{2}kY^2$

E is *same* function for *any* amplitude A of sine-oscillation where:

$Y = A\sin\omega t$ with velocity $V = A\omega\cos\omega t$

Because then: $E = \frac{1}{2}m\left(A\omega\cos\omega t\right)^2 + \frac{1}{2}k(A\sin\omega t)^2$

$$= \frac{1}{2}m\omega^2 A^2\left(\cos\omega t\right)^2 + \frac{1}{2}kA^2(\sin\omega t)^2$$

$$= \frac{1}{2}m\omega^2 A^2\left(\cos^2\omega t + \sin^2\omega t\right)^2 \quad \text{if: } m\omega^2 = k$$

$$= \frac{1}{2}m\omega^2 A^2 \qquad \text{if: } \omega = \sqrt{\frac{k}{m}}$$

What does *Harmonic* mean?

Given total energy $E = KE+PE = \frac{1}{2}mV^2 + \frac{1}{2}kY^2$

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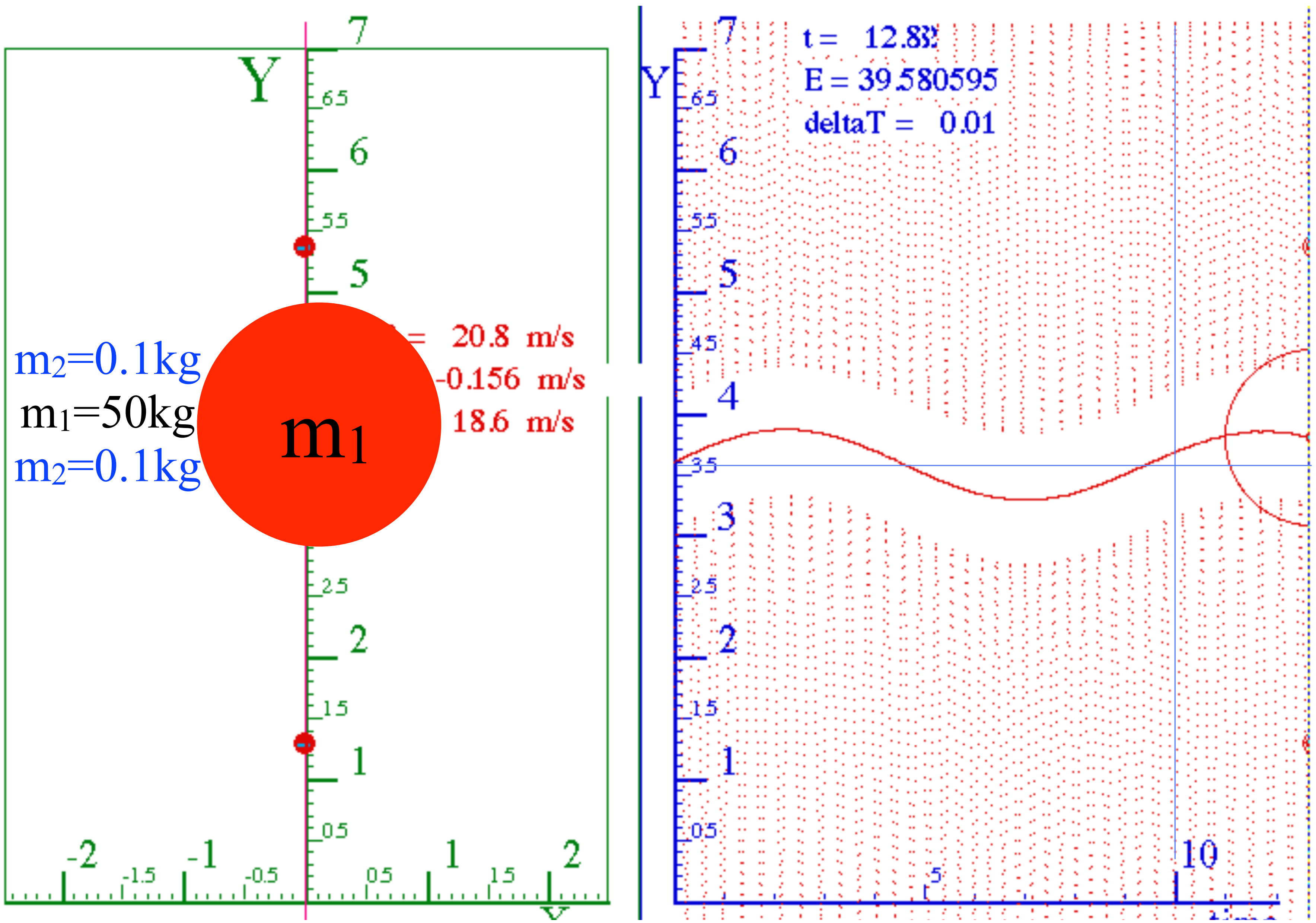
$$= \frac{1}{2}m\omega^2 A^2\left(\cos^2\omega t + \sin^2\omega t\right)^2 \quad \text{if: } m\omega^2 = k$$

$$= \frac{1}{2}m\omega^2 A^2 \quad \text{if: } \omega = \sqrt{\frac{k}{m}}$$

But, how does this square with *linear*-in-frequency Planck energy E=(const.)ω *?!?*

(More about that later in course.)

Switch
$m_1=m_3$
with
m_2
to match
formula



[BounceIt Superball Collision Web Simulator: 1:500:1 mass ratios (Small Amplitude)]

Unit 1
Fig. 6.3

Simulation of
the adiabatic case

Sample problem: *Compute isothermal frequency and/or period*

Frequency

HO ∡frequency: $\omega=\sqrt{\frac{k}{m_1}}=\sqrt{\frac{2m_2}{m_1}\frac{v_2}{Y_0}}=2\pi\upsilon$

Switch
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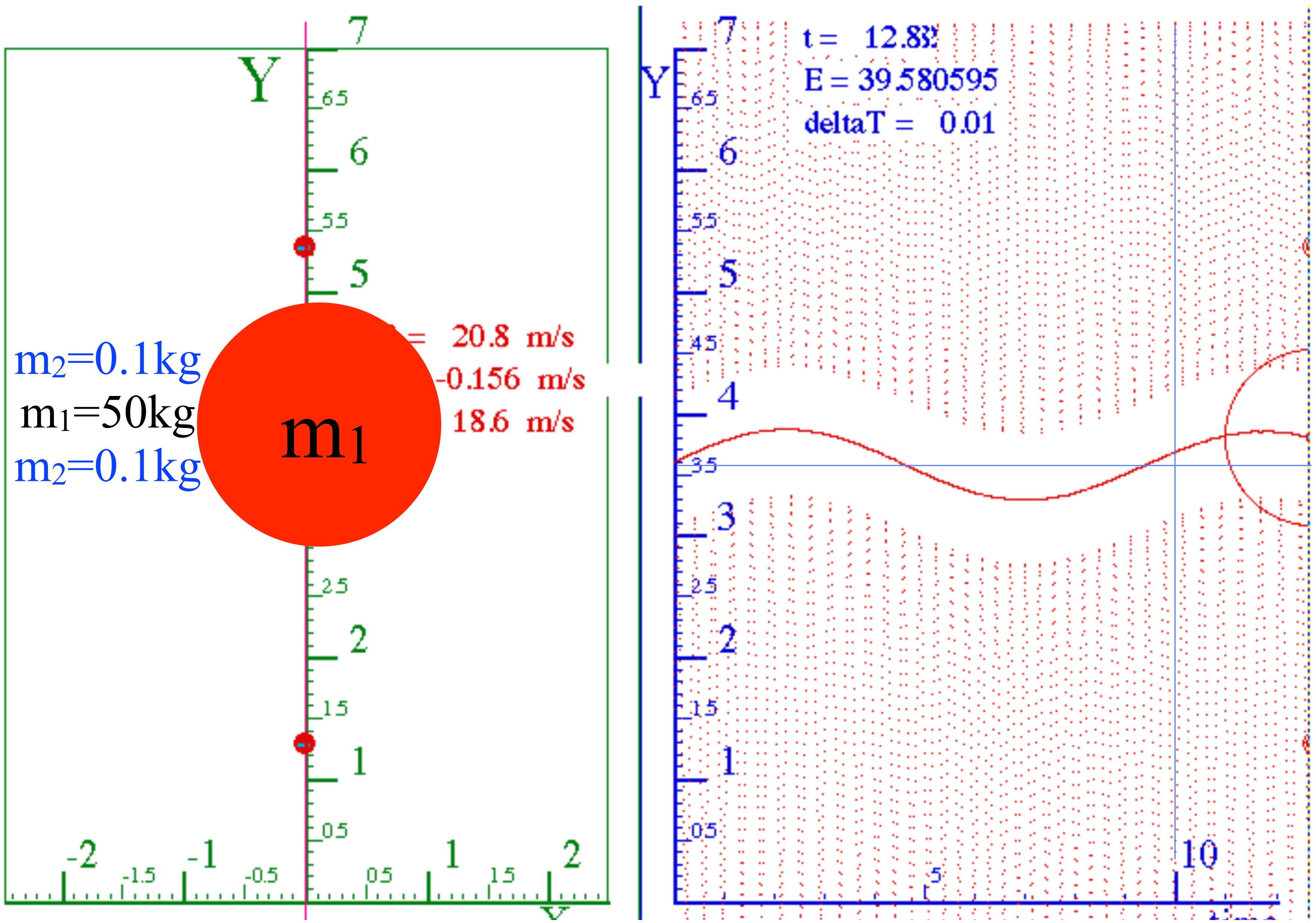


[BounceIt Superball Collision Web Simulator: 1:500:1 mass ratios (Small Amplitude)]

Unit 1
Fig. 6.3

Simulation of
the adiabatic case

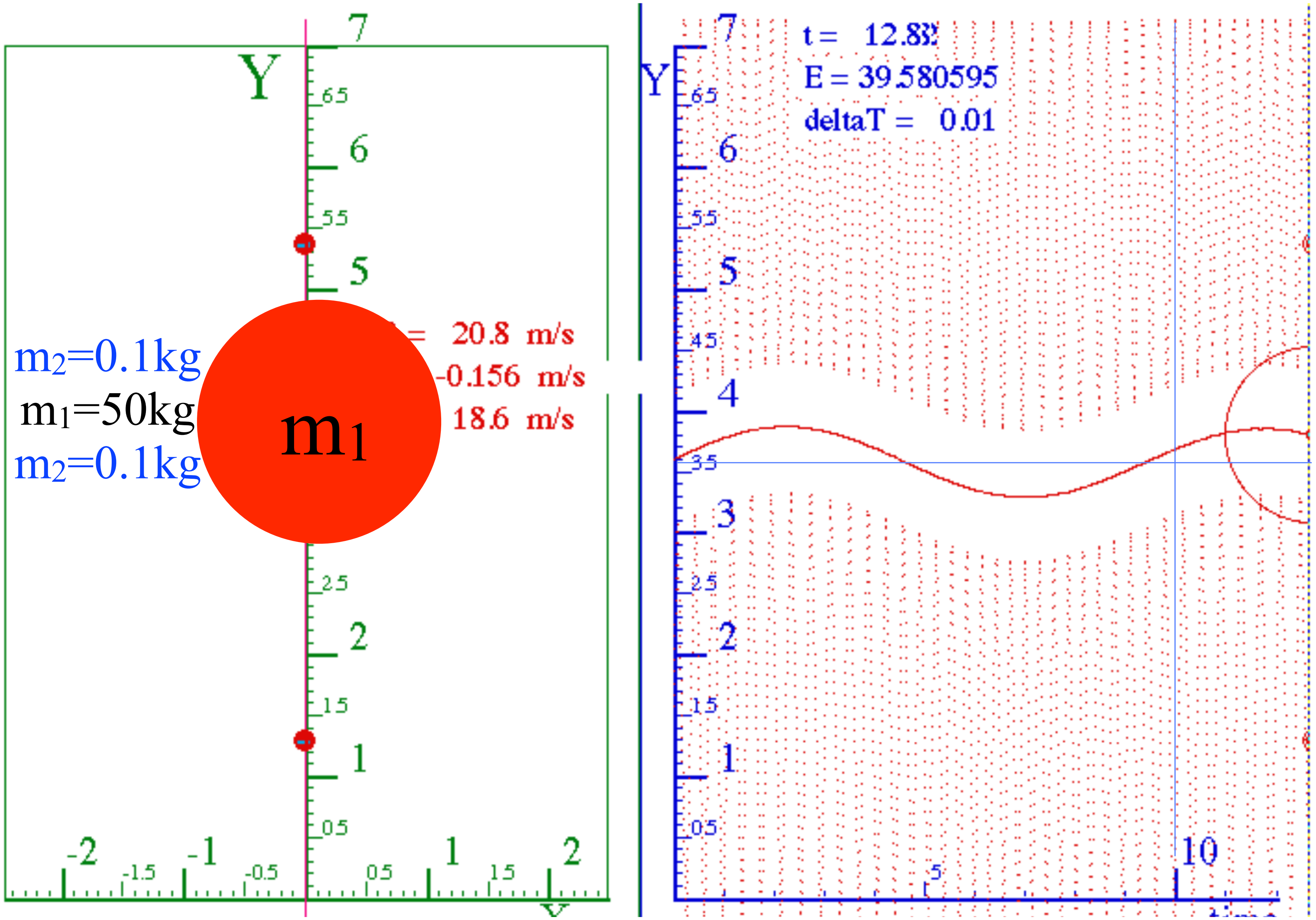
Sample problem: *Compute isothermal frequency and/or period*

$$\textit{Period}: \quad \tau = \frac{1}{\upsilon} = 2\pi\sqrt{\frac{m_1}{k}} = 2\pi\sqrt{\frac{m_1}{2m_2}\frac{Y_0}{v_2}}$$

Frequency

$$\text{HO } \measuredangle\text{frequency: } \omega = \sqrt{\frac{k}{m_1}} = \sqrt{\frac{2m_2}{m_1}\frac{v_2}{Y_0}} = 2\pi\upsilon$$

Switch $m_1=m_3$ with m_2 to match formula



Unit 1
Fig. 6.3

Simulation of the adiabatic case

[*BounceIt Superball Collision Web Simulator: 1:500:1 mass ratios (Small Amplitude)*]

Sample problem: *Compute isothermal period given* $m_1=50$, $m_2=0.1=m_3$, $v_2=20$, $Y_0=3.5$

Period : $\tau=\frac{1}{\upsilon}=2\pi\sqrt{\frac{m_1}{k}}=2\pi\sqrt{\frac{m_1}{2m_2}\frac{Y_0}{v_2}}$

Period :

$$\tau=2\pi\sqrt{\frac{m_1}{2m_2}\frac{Y_0}{v_2}}=6.28\sqrt{\frac{50}{2\cdot(0.1)}\frac{3.5}{20}}$$

$$=17.38$$

Frequency

HO $\measuredangle$frequency: $\omega=\sqrt{\frac{k}{m_1}}=\sqrt{\frac{2m_2}{m_1}\frac{v_2}{Y_0}}=2\pi\upsilon$

Switch
m_1=m_3
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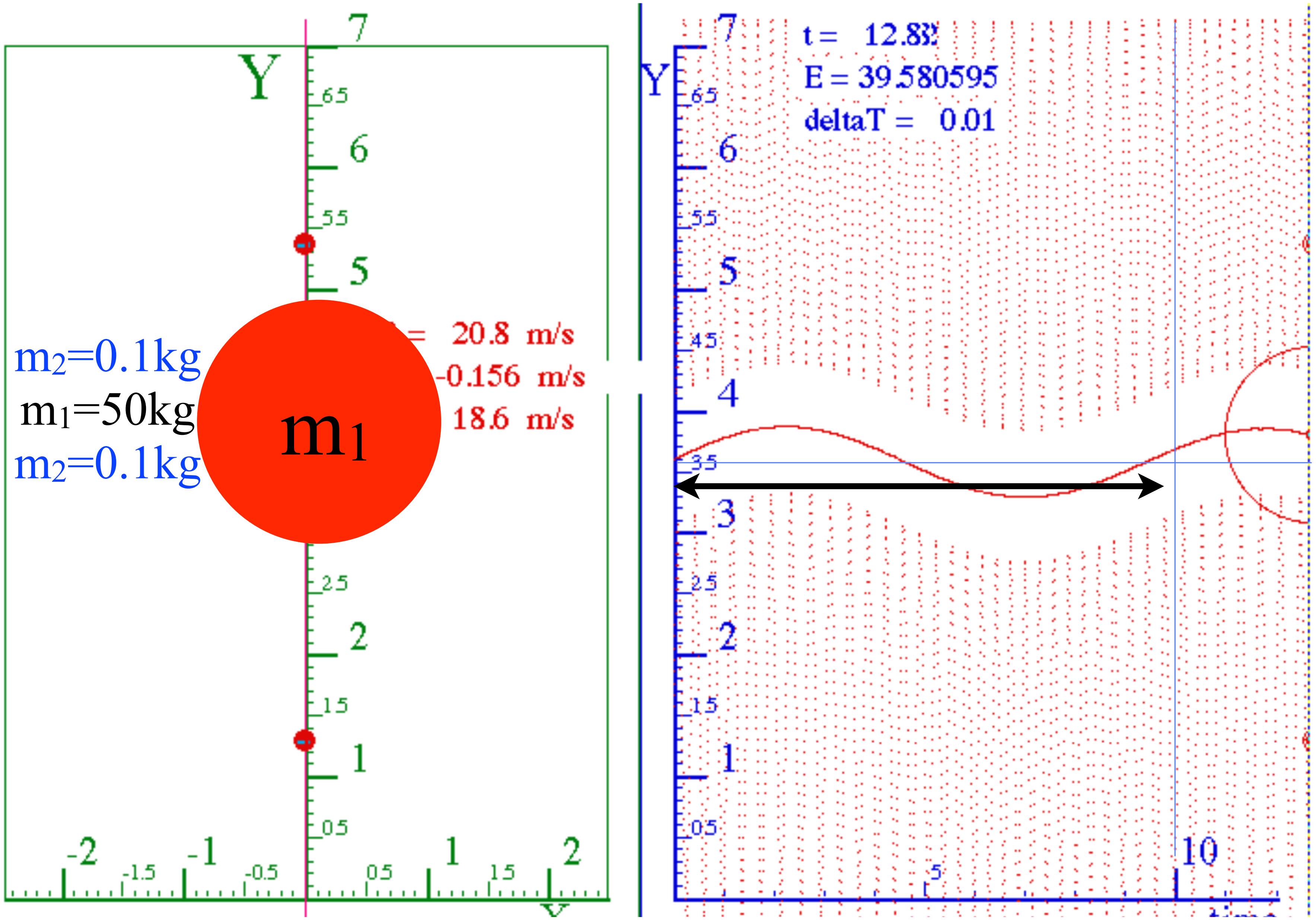

Simulation of
the adiabatic case

[*BounceIt Superball Collision Web Simulator: 1:500:1 mass ratios (Small Amplitude)*]

Sample problem: *Compute isothermal period given m_1=50, m_2=0.1=m_3, v_2=20, Y_0=3.5*

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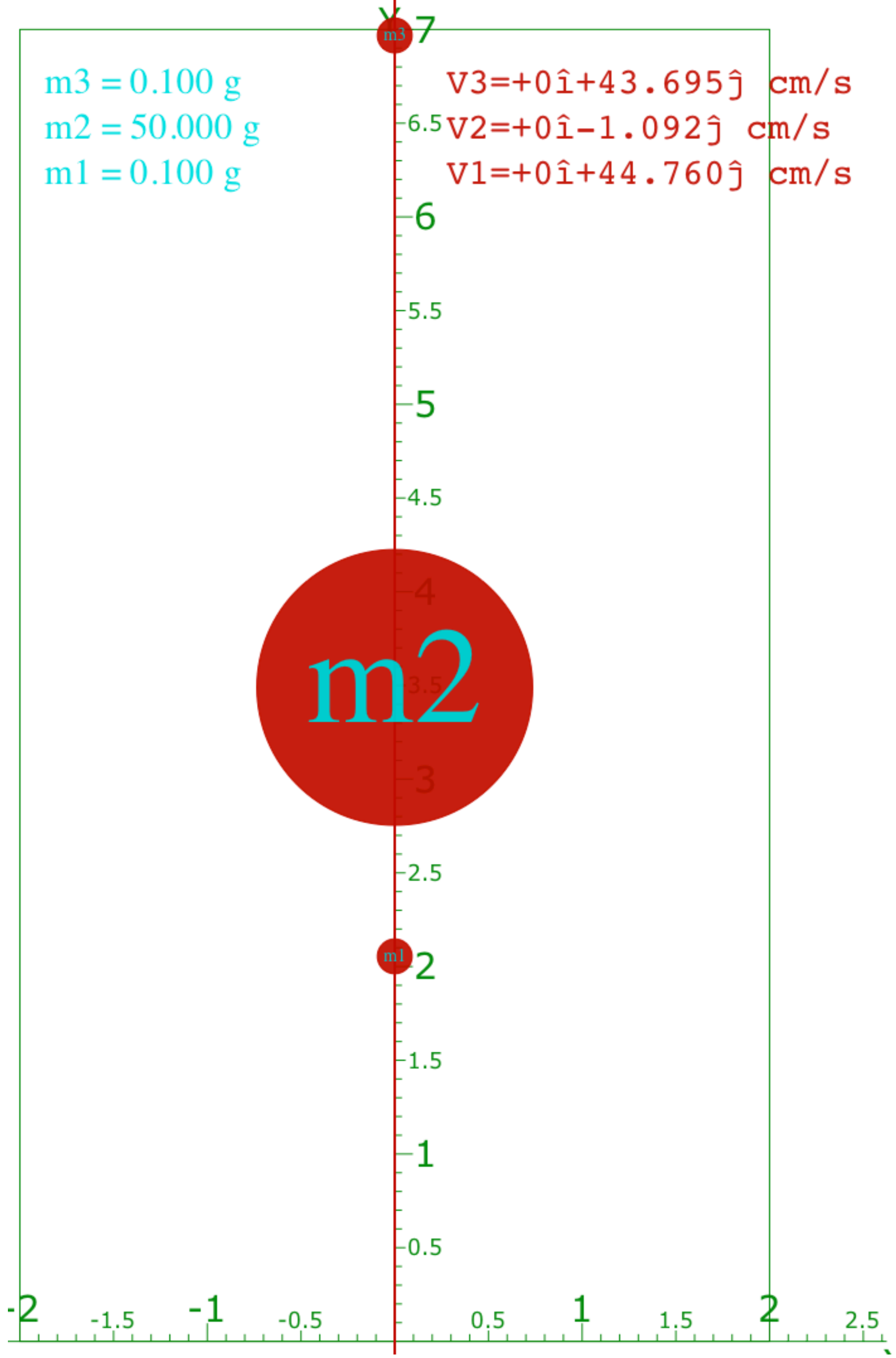
Period :

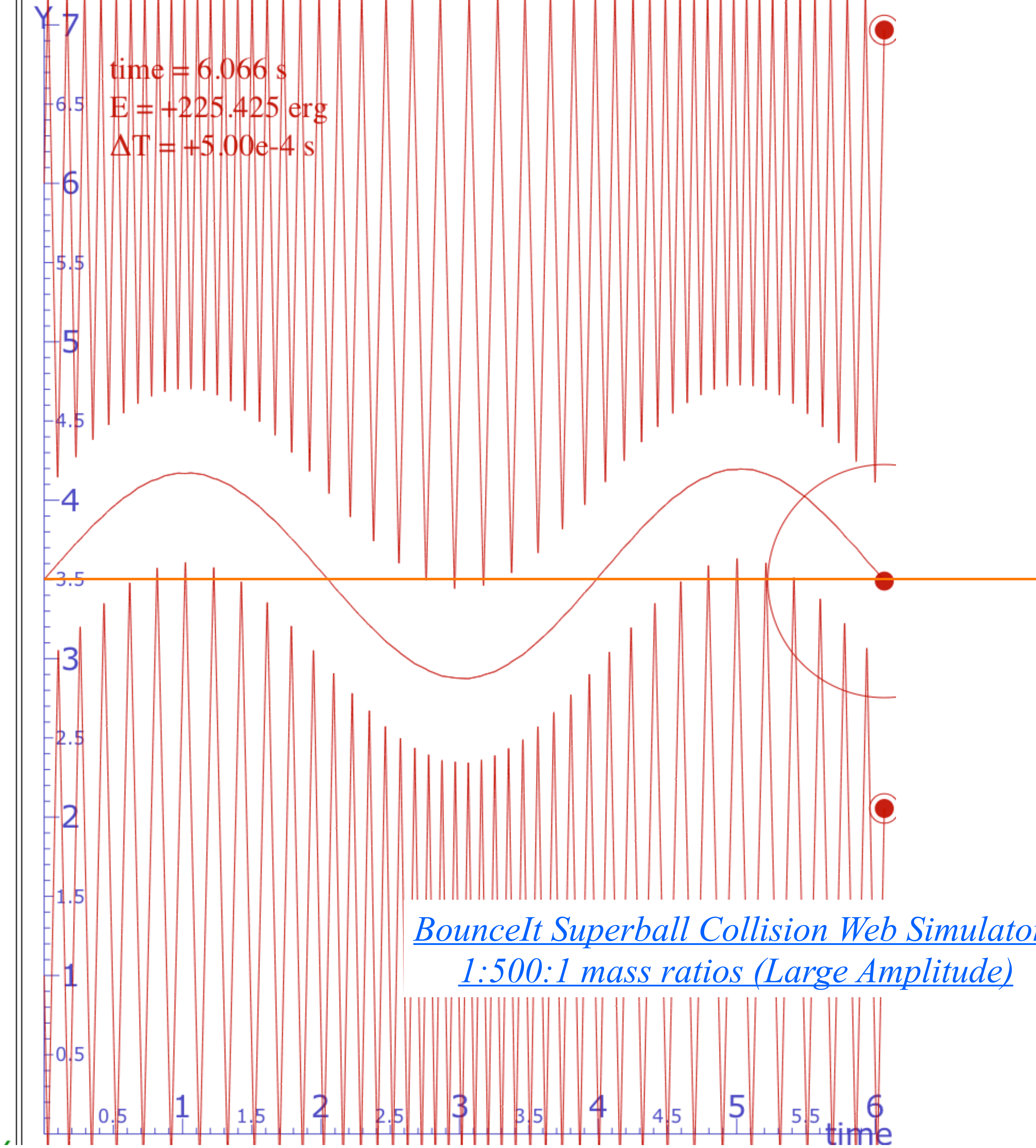
$$\tau = 2\pi\sqrt{\frac{m_1}{2m_2}\frac{Y_0}{v_2}} = 6.28\sqrt{\frac{50}{2\cdot(0.1)}\frac{3.5}{20}}$$

$=17.38$ *That's about √3 times too big!*

Frequency

HO ∡frequency: $\omega = \sqrt{\frac{k}{m_1}} = \sqrt{\frac{2m_2}{m_1}\frac{v_2}{Y_0}} = 2\pi\upsilon$





[*BounceIt Superball Collision Web Simulator: 1:500:1 mass ratios (Large Amplitude)*]

y Max = 7

Initial x1 = 0.75 y Min = 0

Max x PE plot = 0.5 T Max = 6

F-Vector scale = 0.003 V2y Max = 3

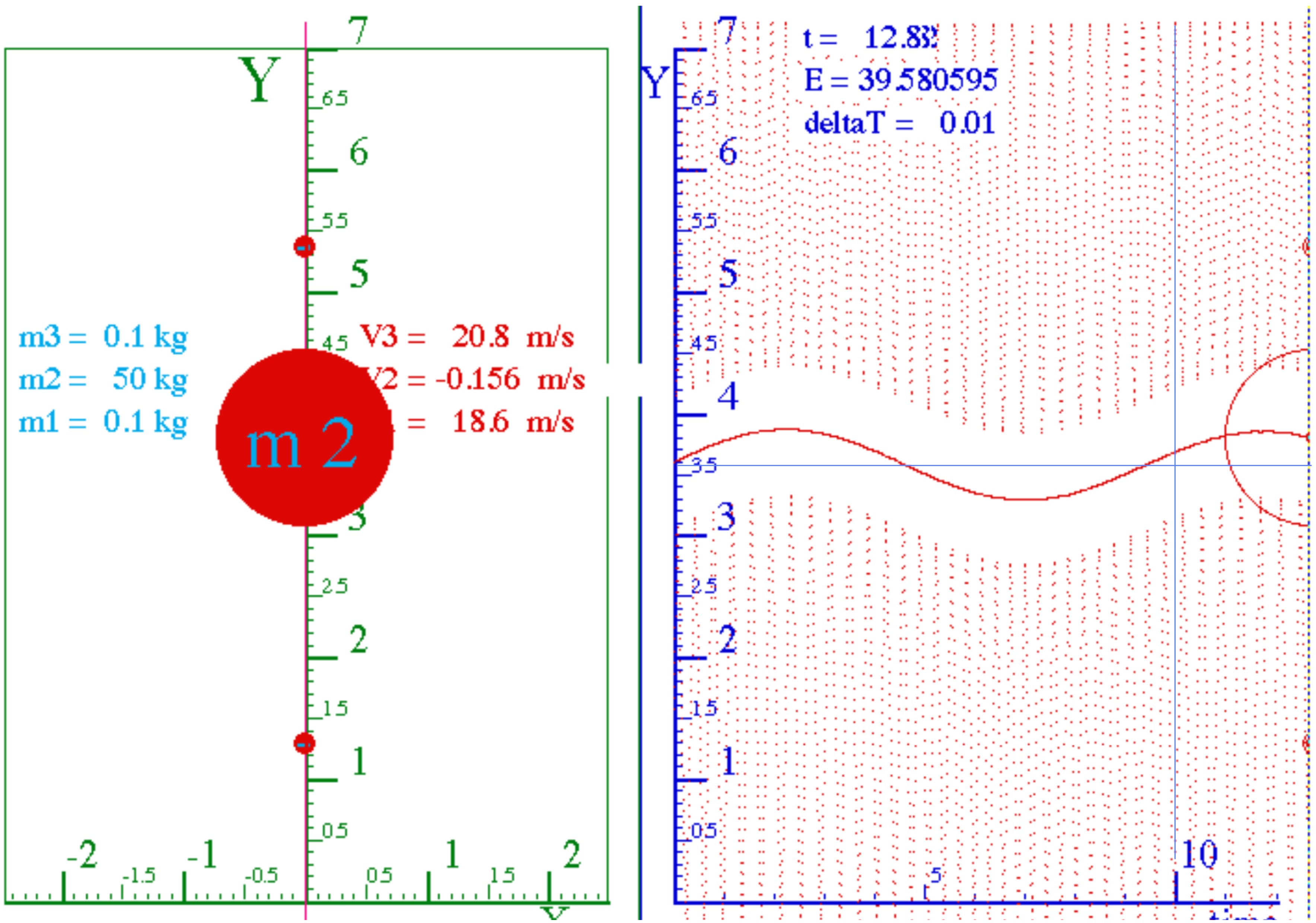
Error step = 0.000 V2y Min = -2

Adiabatic force scenarios	
	Quasi-harmonic oscillation (m1:m2 = 100:1)
	Quasi-harmonic oscillation (m1:m2 = 50:1)
	Quasi-harmonic oscillation (m1:m2 = 25:1)
	Large amplitude (m1:m2 = 100:1)

m1= 1 x10^ -1 {g} $X1_0$= 1 x10^ 0 {cm} $V1_0$= -4.5 x10^ 1 {cm/s}

m2= 5 x10^ 1 {g} $X2_0$= 3.5 x10^ 0 {cm} $V2_0$= 1 x10^ 0 {cm/s}

m3= 1 x10^ -1 {g} $X3_0$= 6 x10^ 0 {cm} $V3_0$= 4.5 x10^ 1 {cm/s}



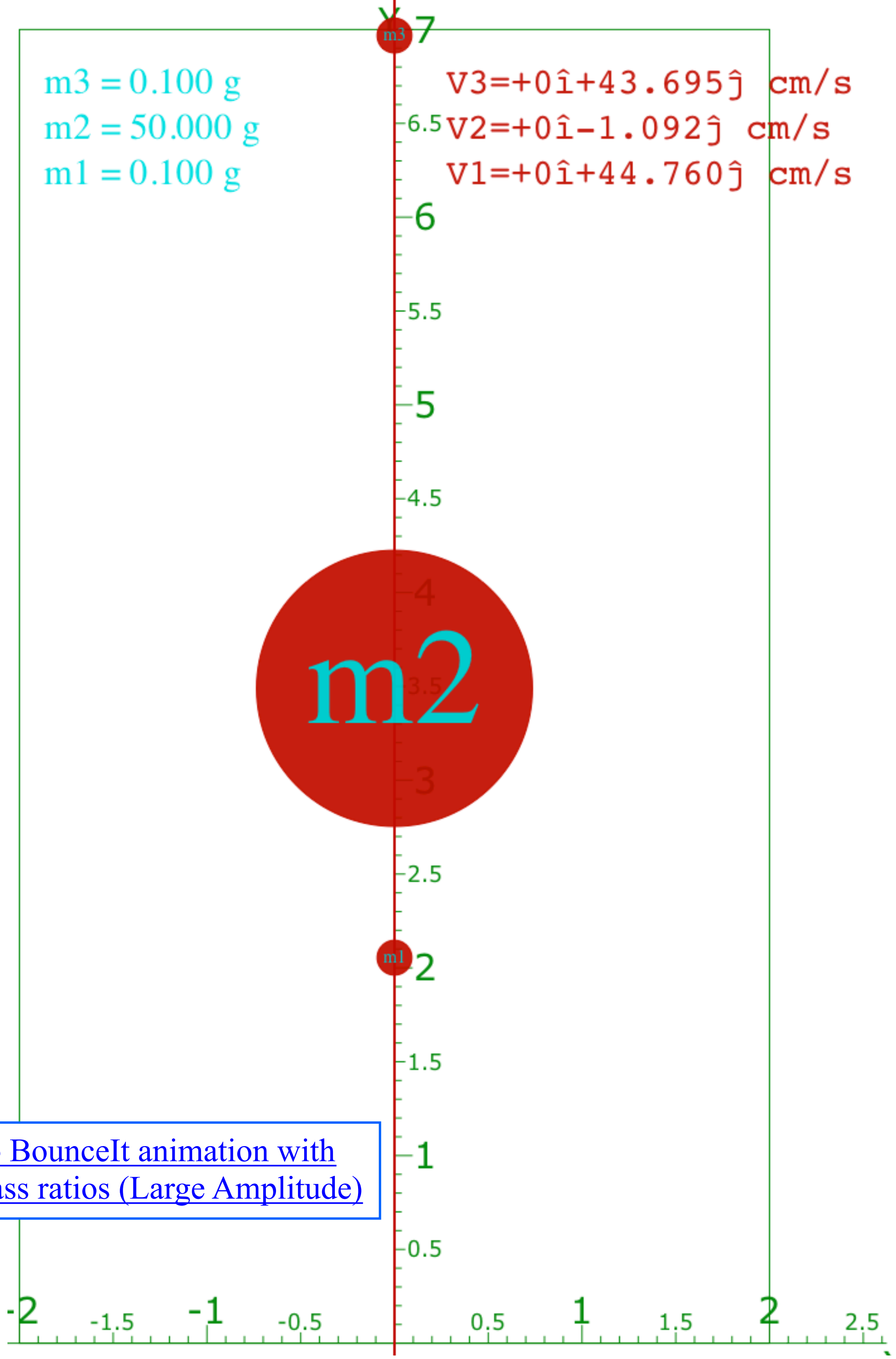
Unit 1
Fig. 6.3

Simulation of

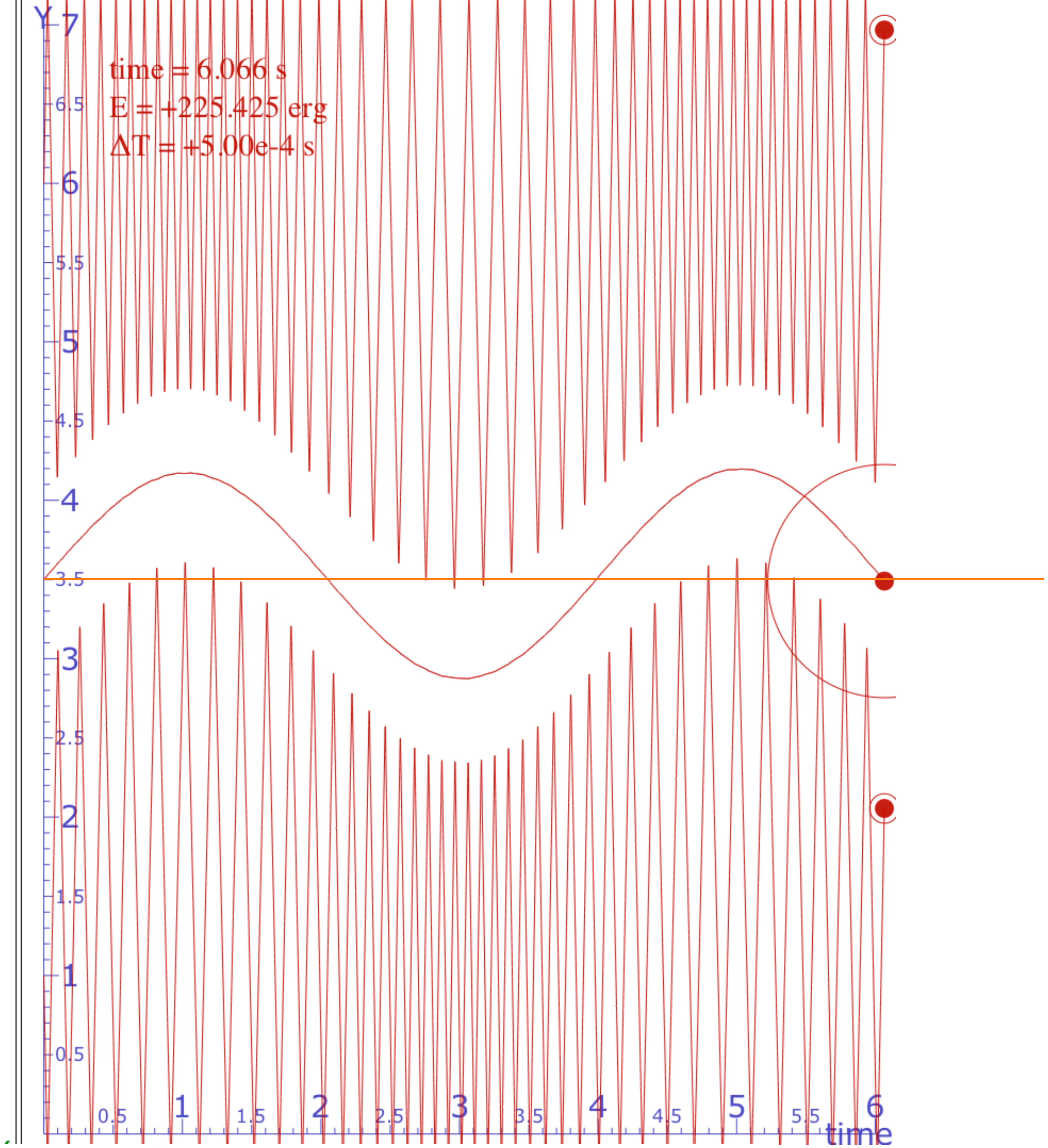
the adiabatic case

[* Link to BounceIt animation with 1:500:1 mass ratios (Small Amplitude)]

See Homework problem 1.6.5: *Compute frequency and/or period for both isoT and adiabatic cases*



[* Link to BounceIt animation with 1:500:1 mass ratios (Large Amplitude)]



y Max = 7

Initial x1 = 0.75
y Min = 0

Max x PE plot = 0.5
T Max = 6

F-Vector scale = 0.003
V2y Max = 3

Error step = 0.000
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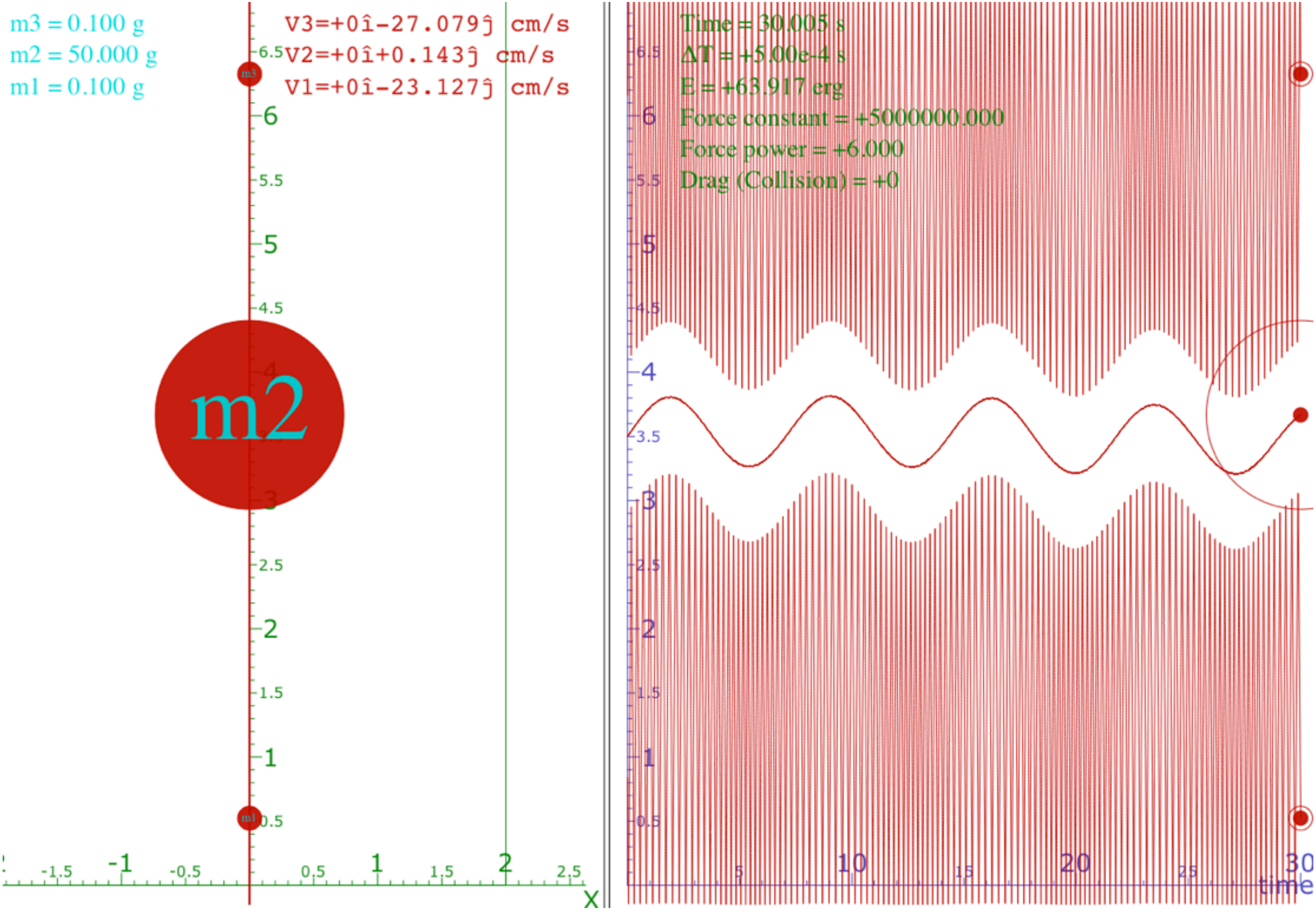
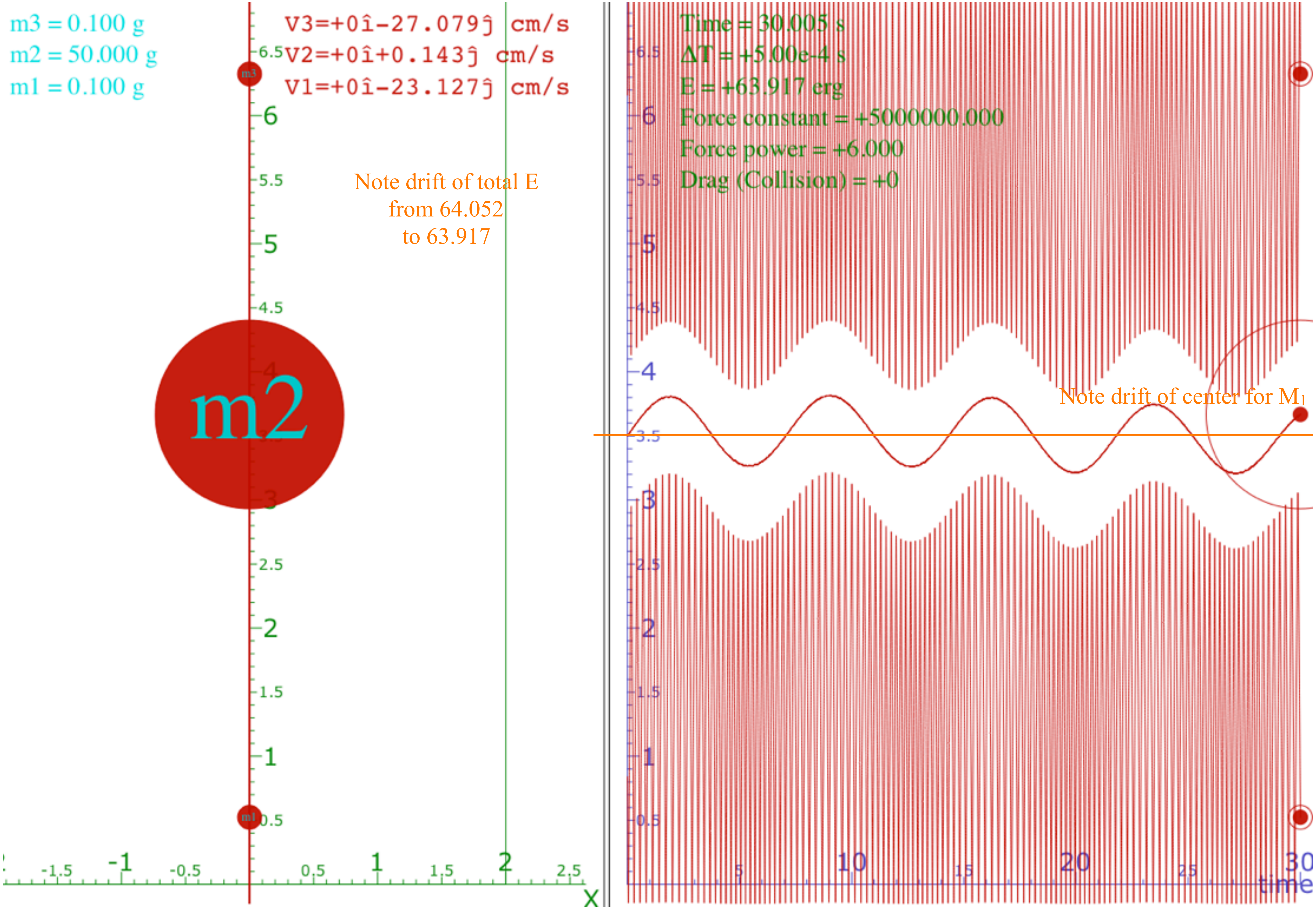
m2= 5 x10^ 1 {g} $X2_0$= 3.5 x10^ 0 {cm} $V2_0$= 1 x10^ 0 {cm/s}

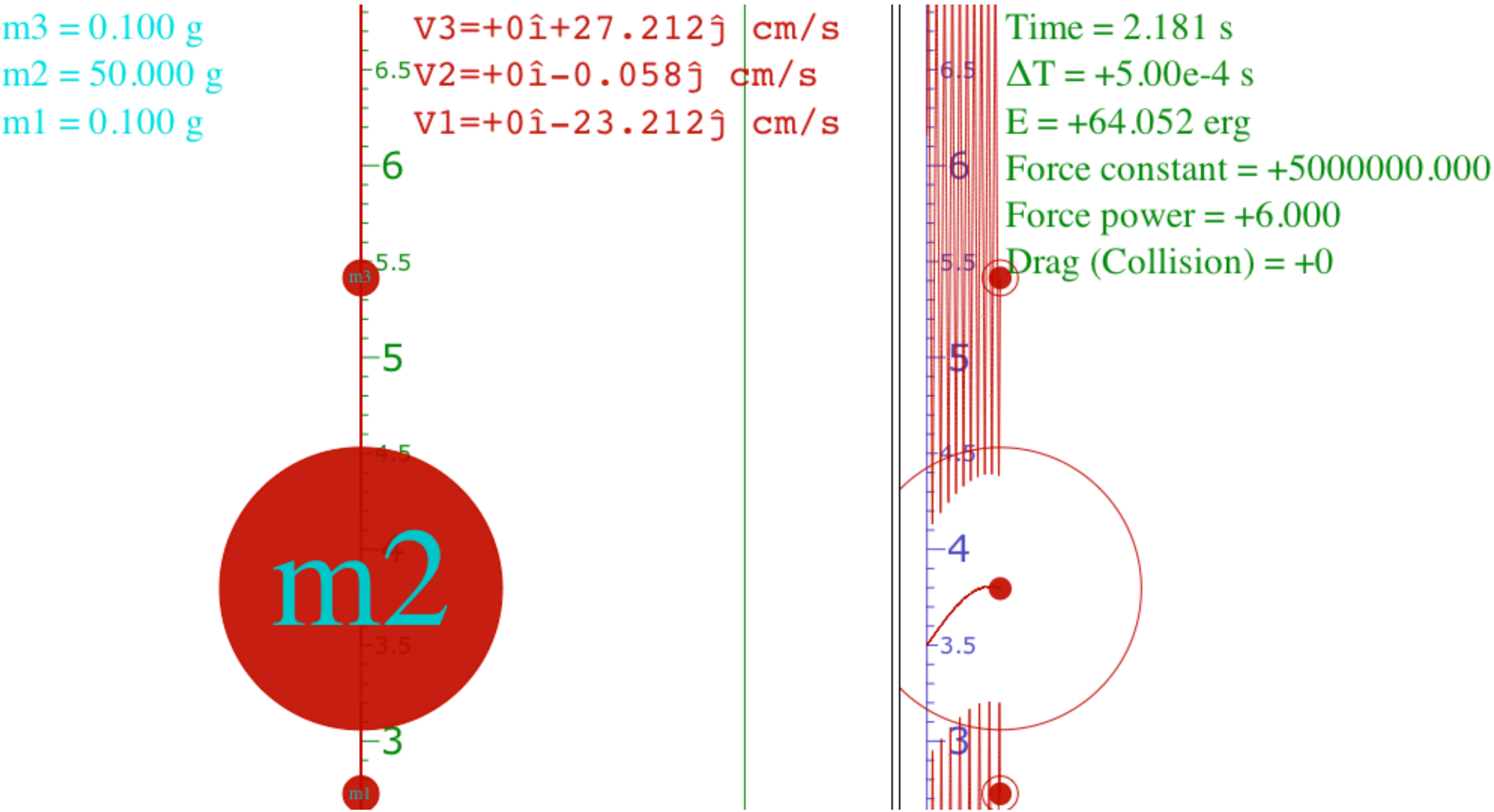
m3= 1 x10^ -1 {g} $X3_0$= 6 x10^ 0 {cm} $V3_0$= 4.5 x10^ 1 {cm/s}



[* Link to BounceIt animation with 1:500:1 mass ratios (Small Amplitude)]



[* Link to BounceIt animation with 1:500:1 mass ratios (Small Amplitude)]


m3 = 0.100 g
m2 = 50.000 g
m1 = 0.100 g
V3=+0î+27.212ĵ cm/s
V2=+0î-0.058ĵ cm/s
V1=+0î-23.212ĵ cm/s
Time = 2.181 s
ΔT = +5.00e-4 s
E = +64.052 erg
Force constant = +5000000.000
Force power = +6.000
Drag (Collision) = +0
m3
m2
m1
6.5
6
5.5
5
4.5
4
3.5
3

"Monster Mash" classical segue to Heisenberg action relations

Example of very very large M_1 ball-walls crushing a poor little m_2

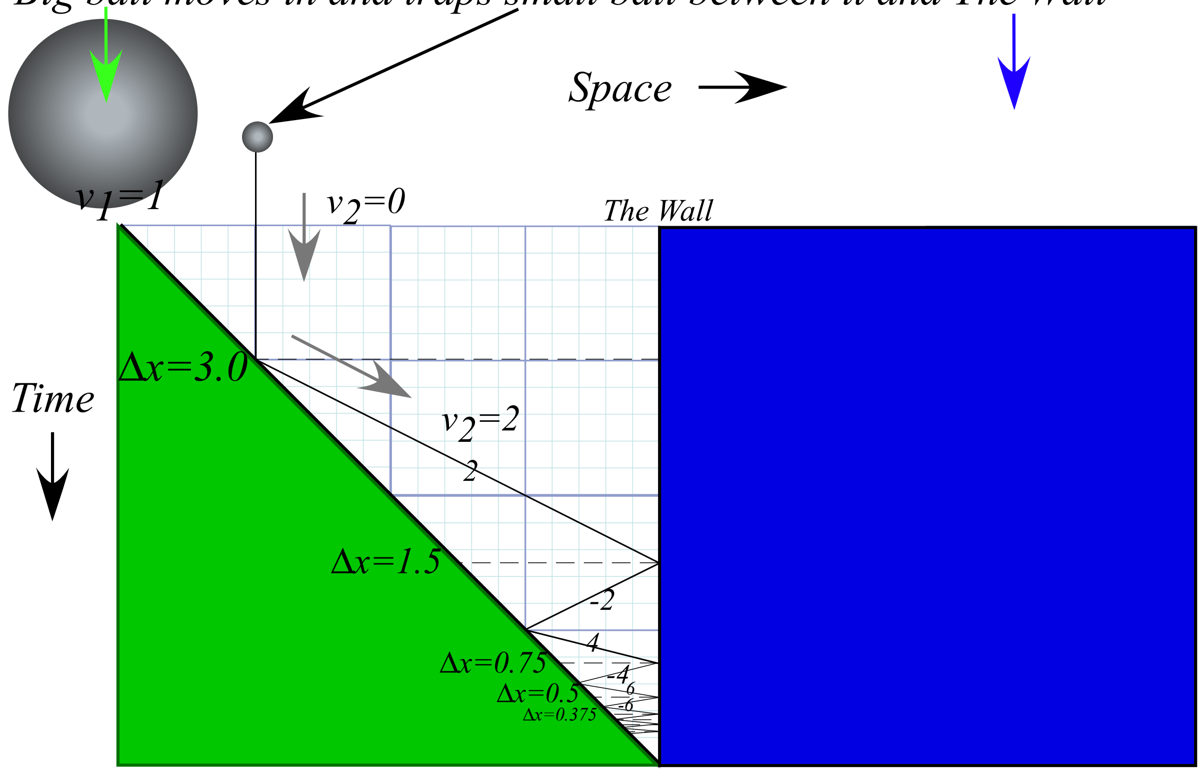
How m_2 keeps its action

An interesting wave analogy: The "Tiny-Big-Bang" [*Harter, J. Mol. Spec. 210, 166-182 (2001)*],[*Harter, Li IMSS (2012)*]

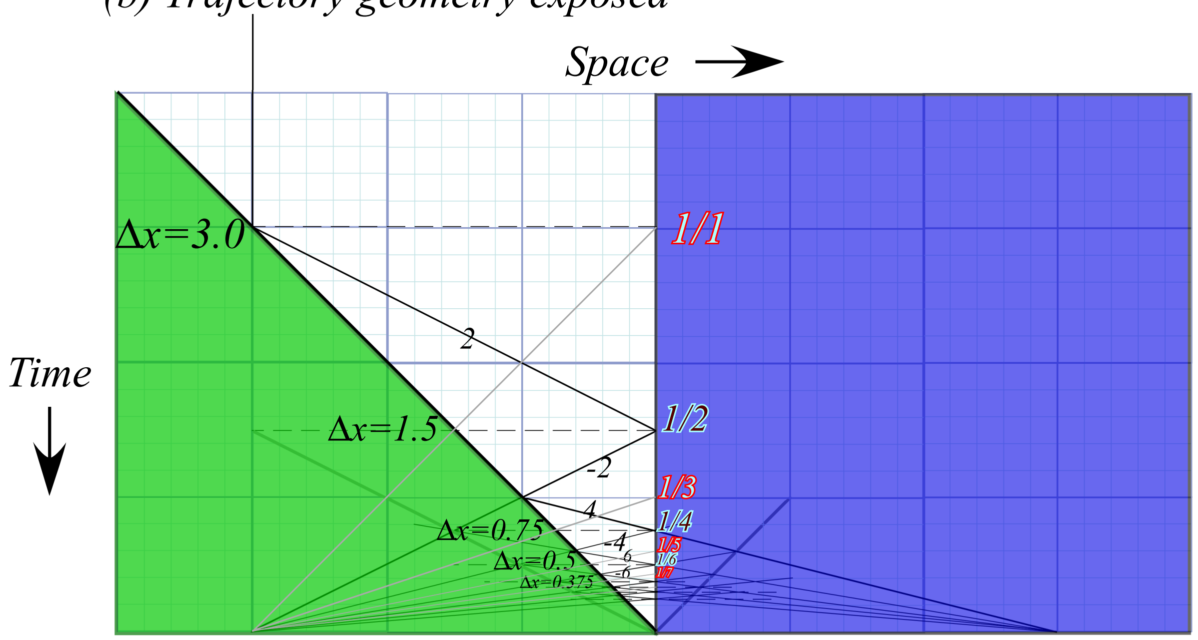
A lesson in geometry of fractions and fractals: Ford Circles and Farey Sums

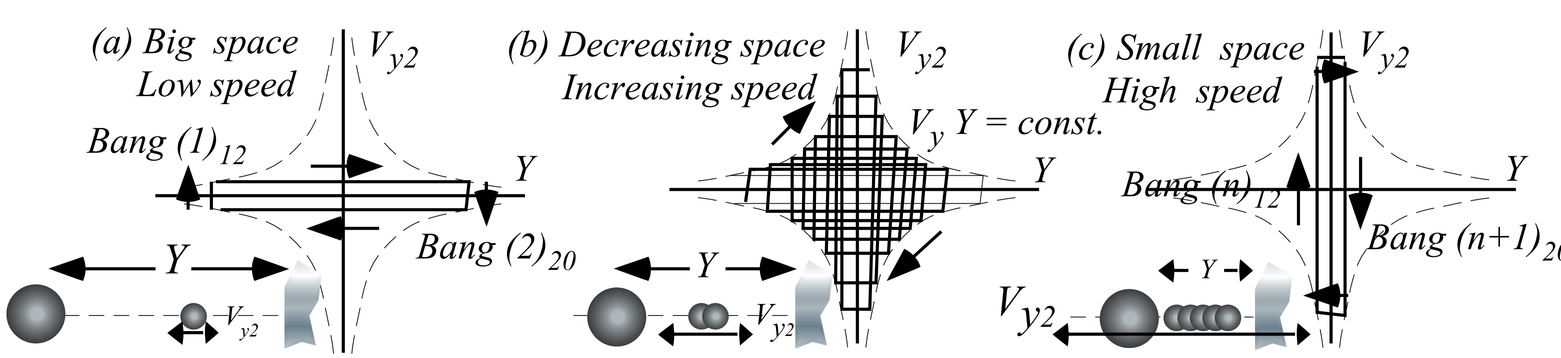
[*Lester. R. Ford, Am. Math. Monthly 45,586(1938)*] [*John Farey, Phil. Mag.(1816)*]

(a) Big ball moves in and traps small ball between it and The Wall



(b) Trajectory geometry exposed





The Classical “Monster Mash”

Classical introduction to

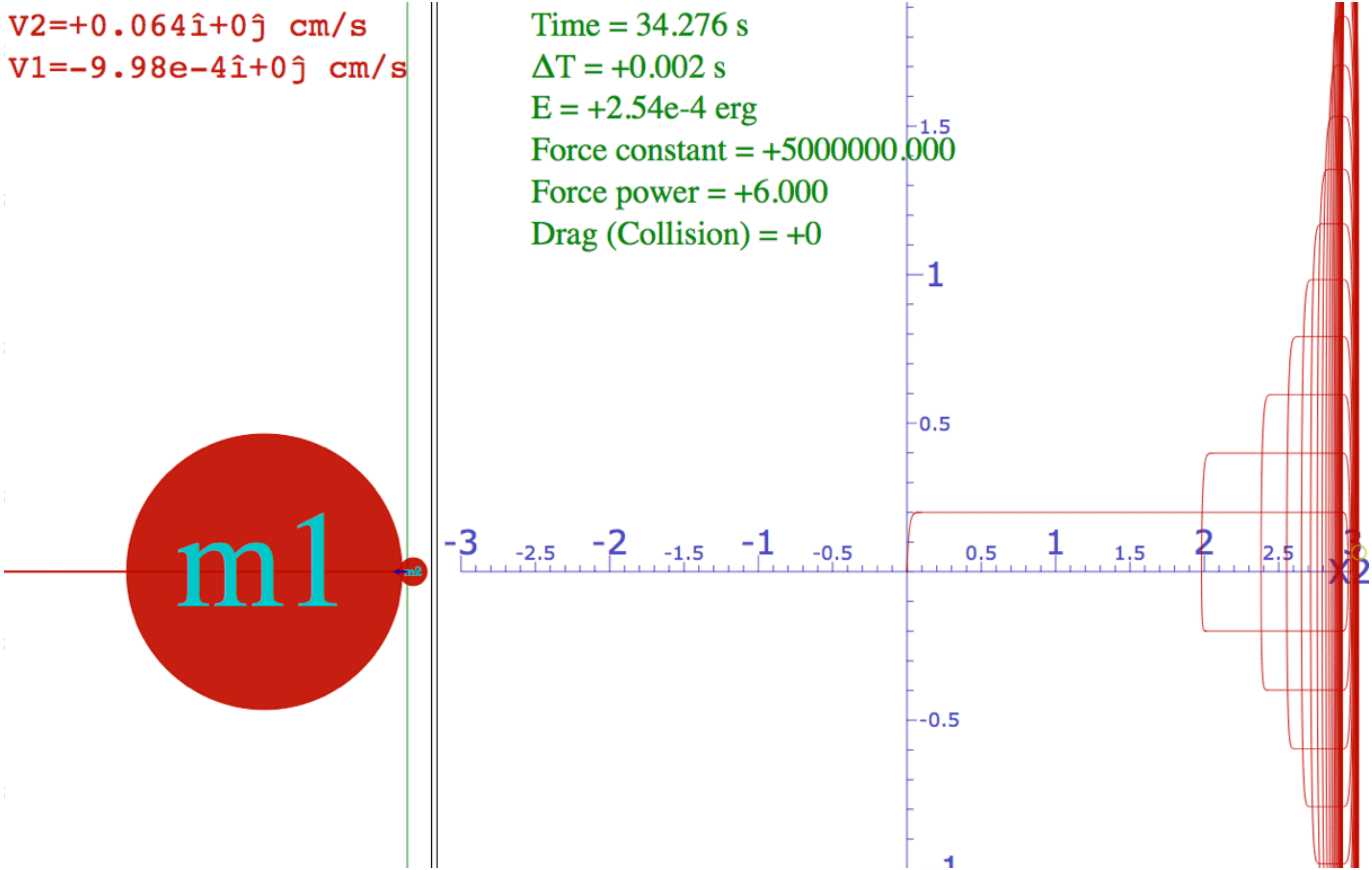
Heisenberg “Uncertainty” Relations

$$v_2 = \frac{const.}{Y} \quad \text{or:} \quad Y \cdot v_2 = const.$$

is analogous to: $\Delta x \cdot \Delta p = N \cdot \hbar$

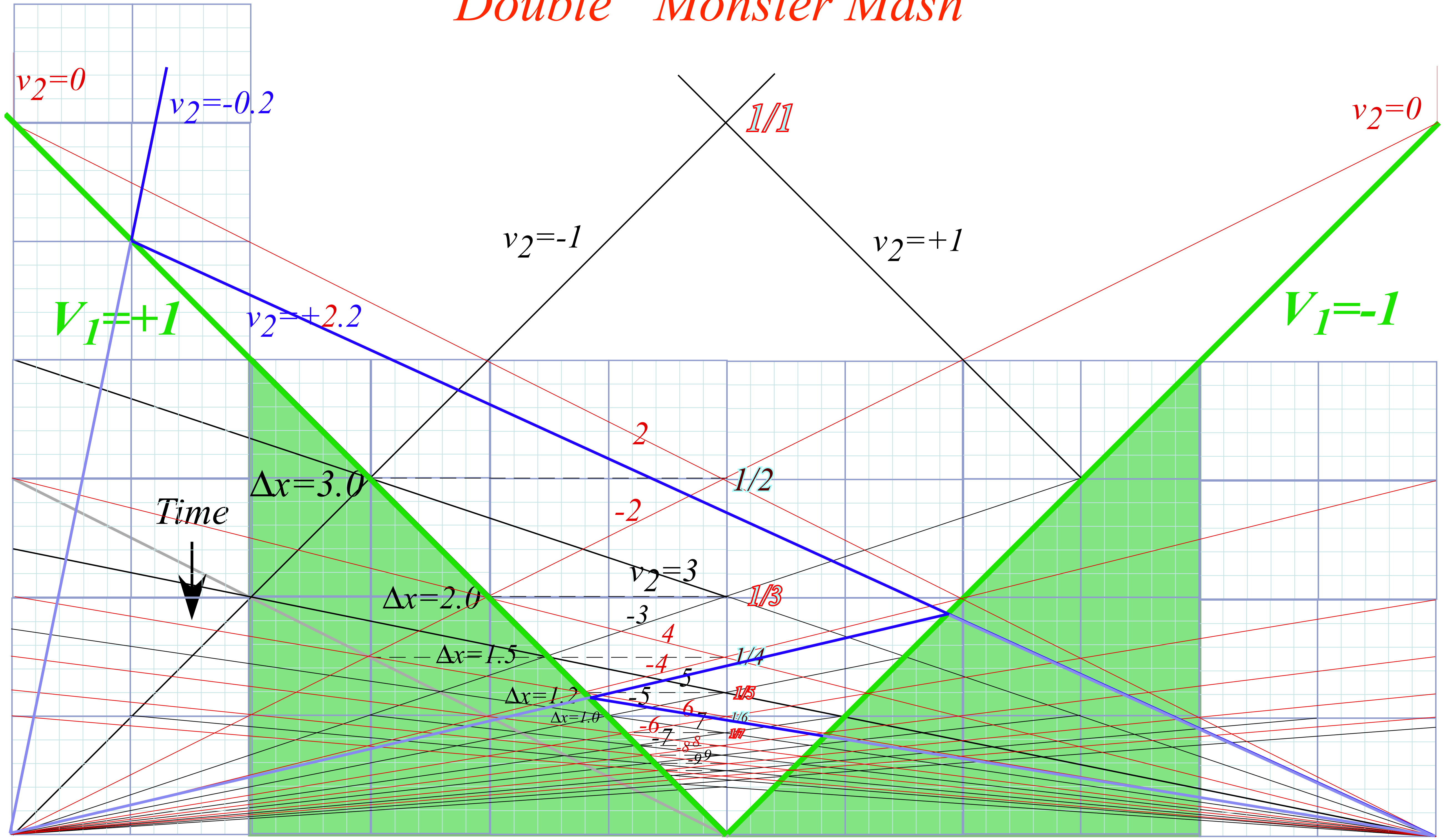
Unit 1
Fig. 6.4

* Link to BounceIt “*Monster Mash*” $x_2(t)$ animation
(Note: Time sense is inverted)



* [Link to BounceIt "*Monster Mash*" Vx_2 vs x_2 animation]

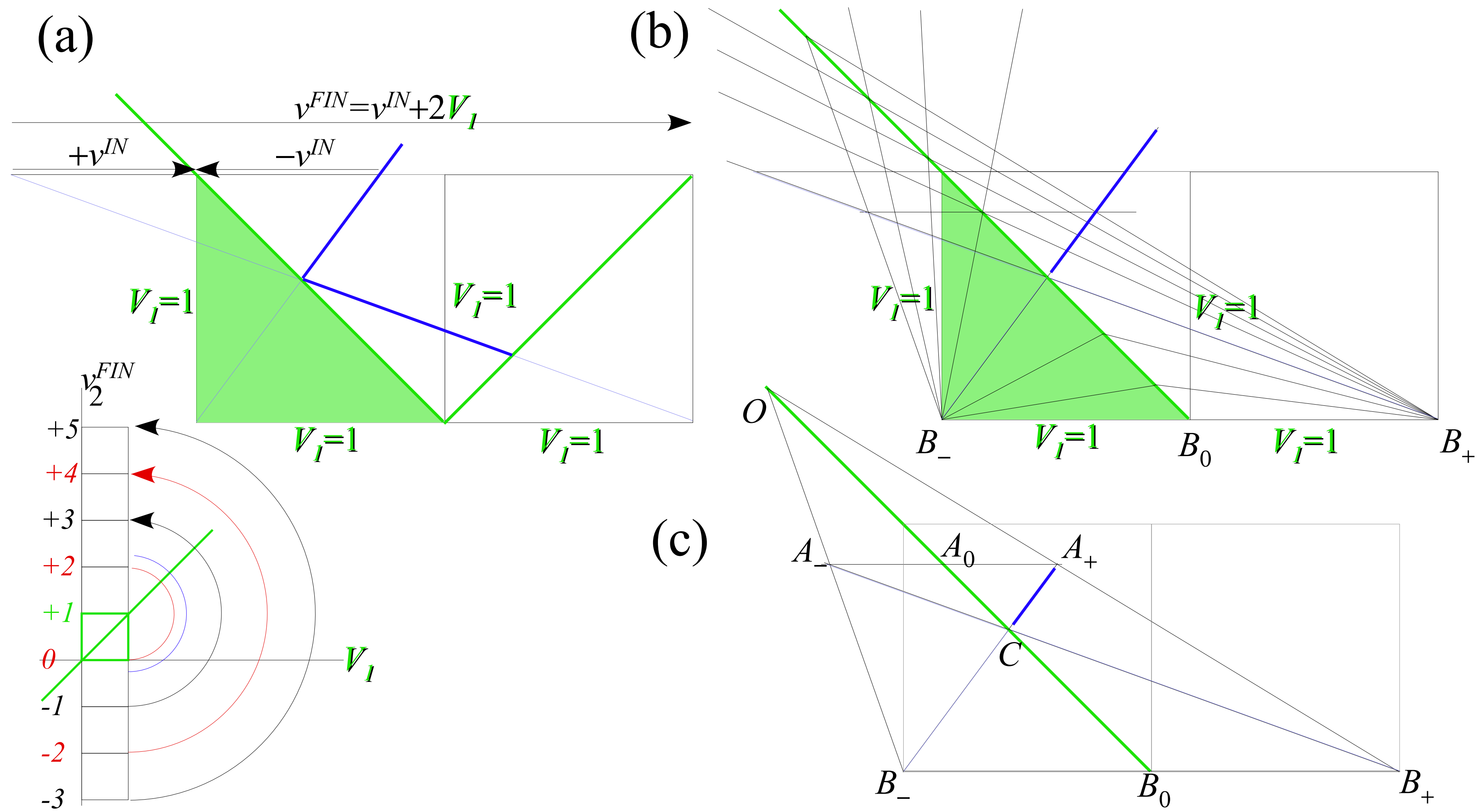
Double "Monster Mash"

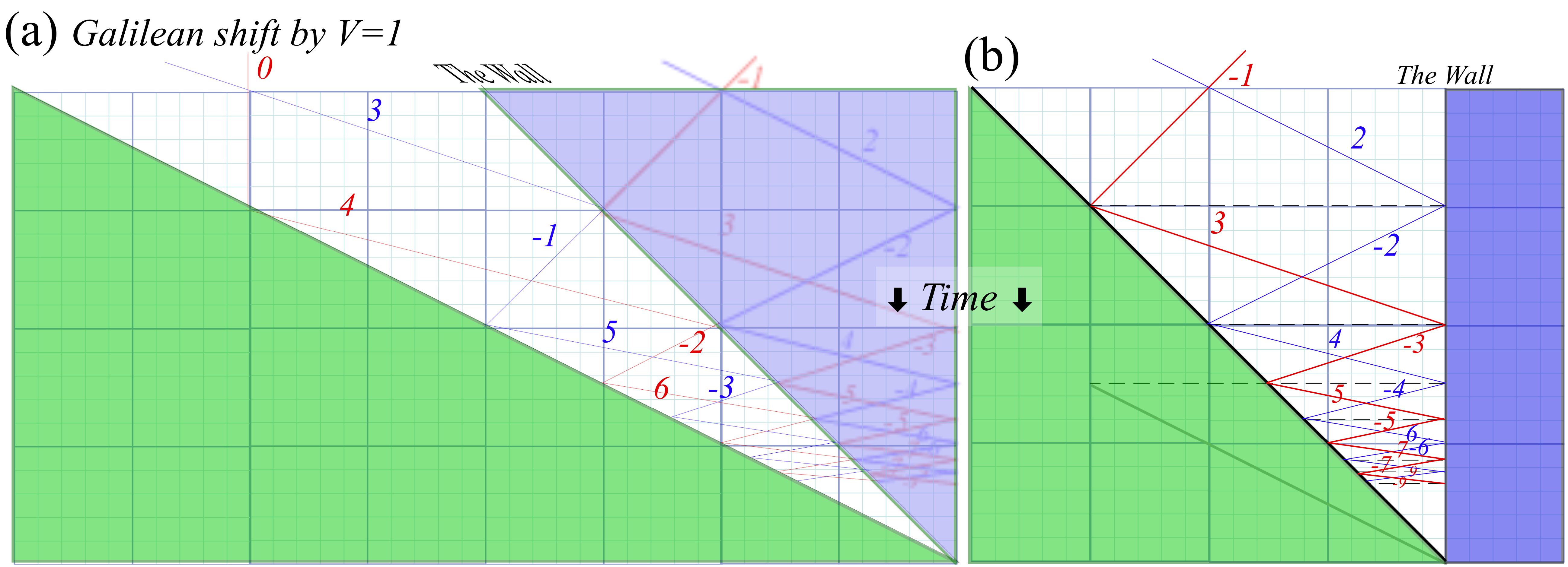


Unit 1
Fig. 6.5

See Homework problem 1.6.2: *Construct related spacetime case*

Unit 1
Fig. 6.6
and
Fig. 6.7





“Monster Mash” classical segue to Heisenberg action relations

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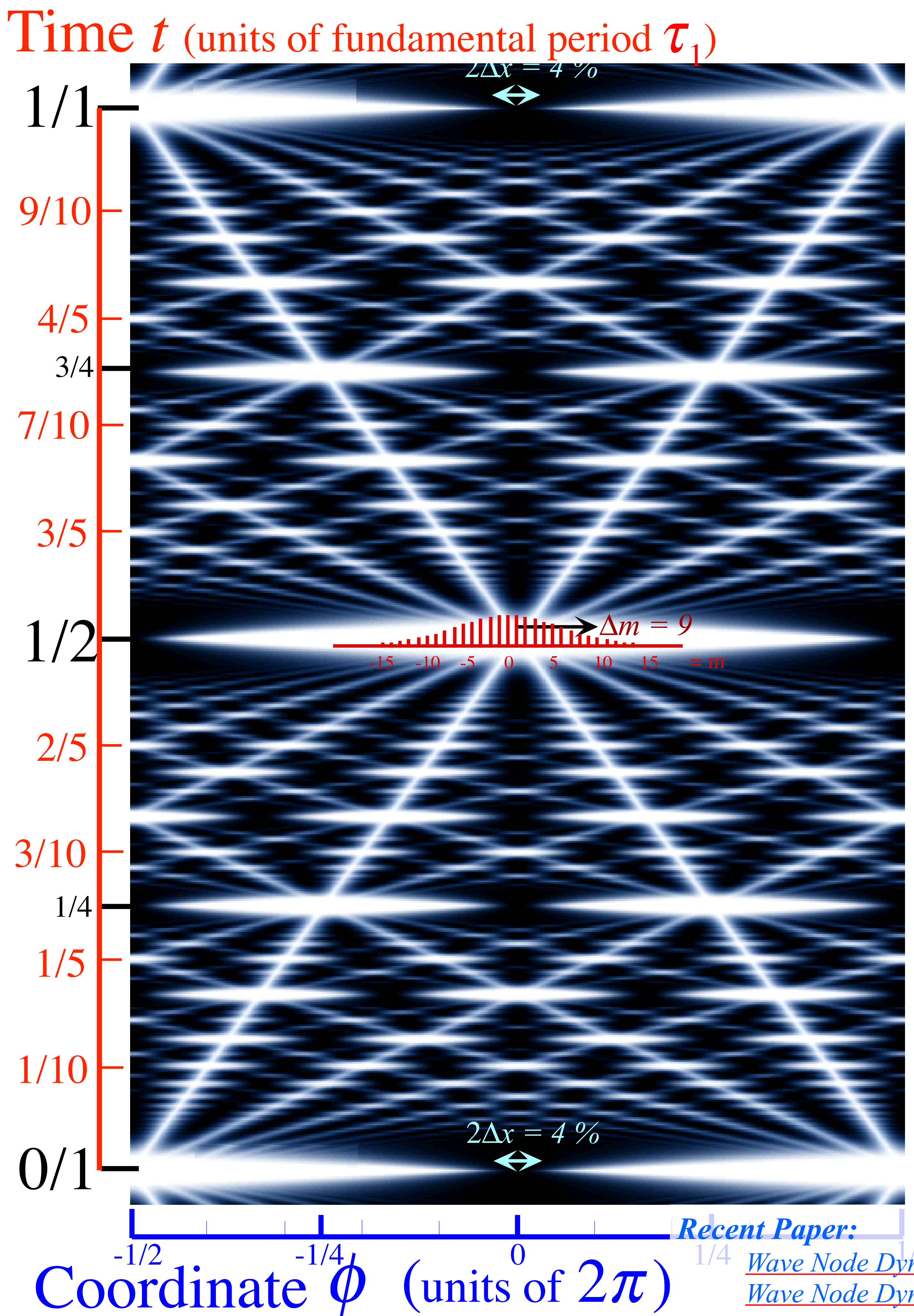

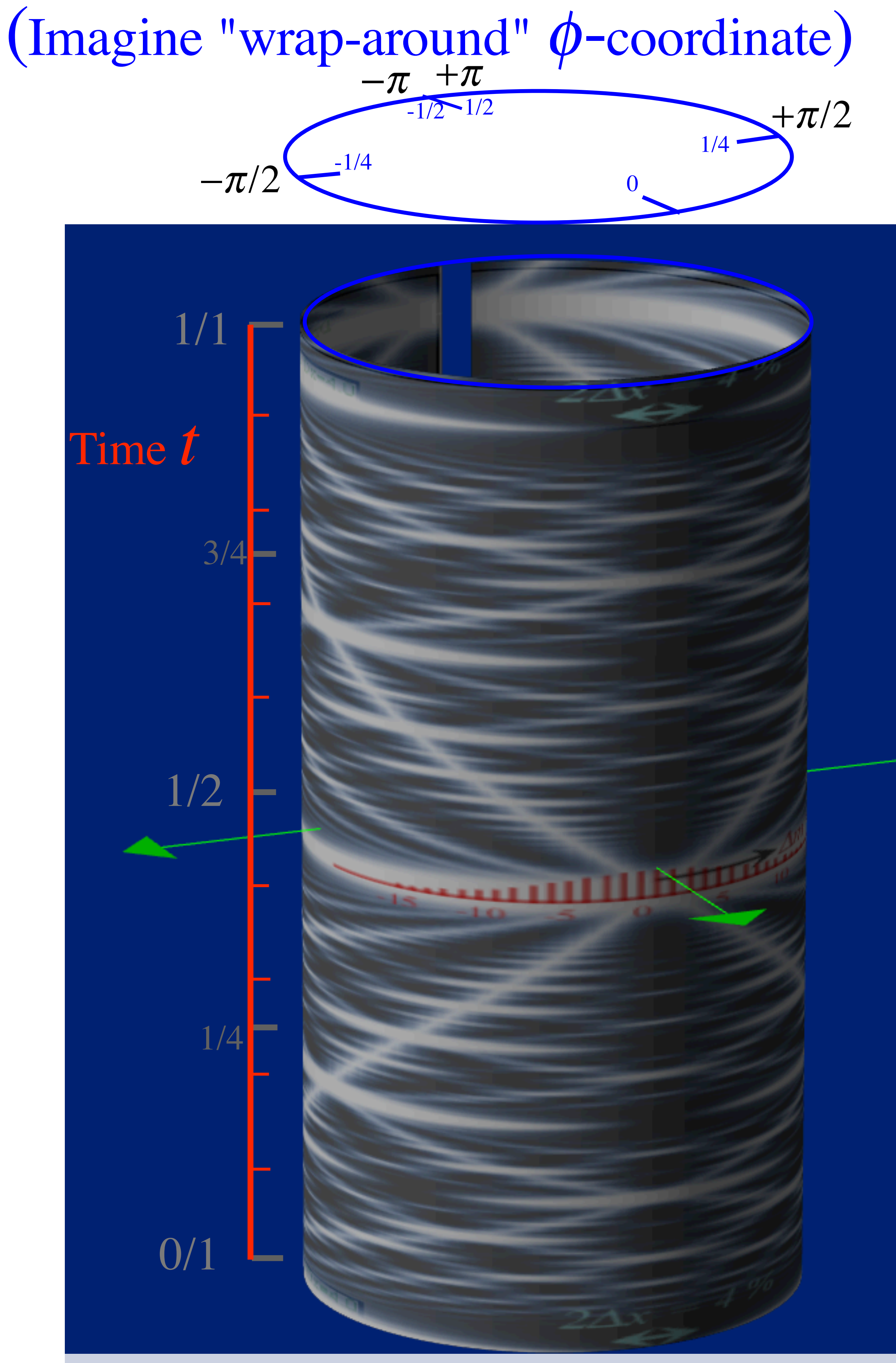

Recent Paper:

[Wave Node Dynamics and Revival Symmetry in Quantum Rotors - Harter-JMS-2001](#)

[Wave Node Dynamics and Revival Symmetry in Quantum Rotors - Harter-jms-2001 (Publ.)](#)

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[*Wave Node Dynamics and Revival Symmetry in Quantum Rotors - Harter-jms-2001 (Publ.)*]

Click here....

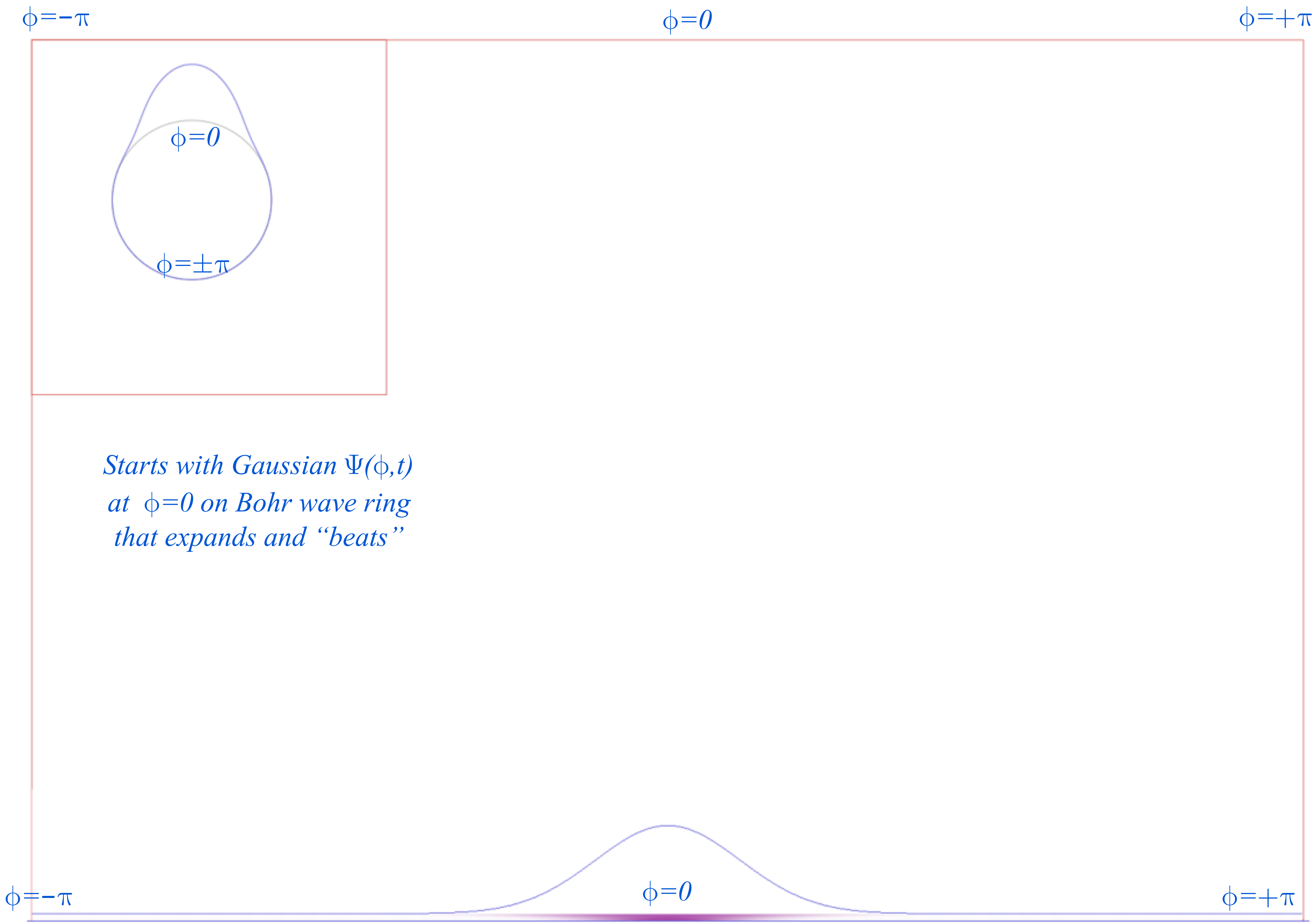
Launch | Fourier Control | Scenarios | Pause | Set T=0 | Zero Amps | T-Scale= 1

Twelve (n=12) oscillator
Twelve (n=12) oscillator
Twelve (n=12) oscillator
C(n) Character Table

..then here.... Quantum Carpet

WaveIt Web Animation - Scenarios:

Quantum_Carpet, Quantum_Carpet_wMBars, Quantum_Carpet_BCar, Quantum_Carpet_BCar_wMBars



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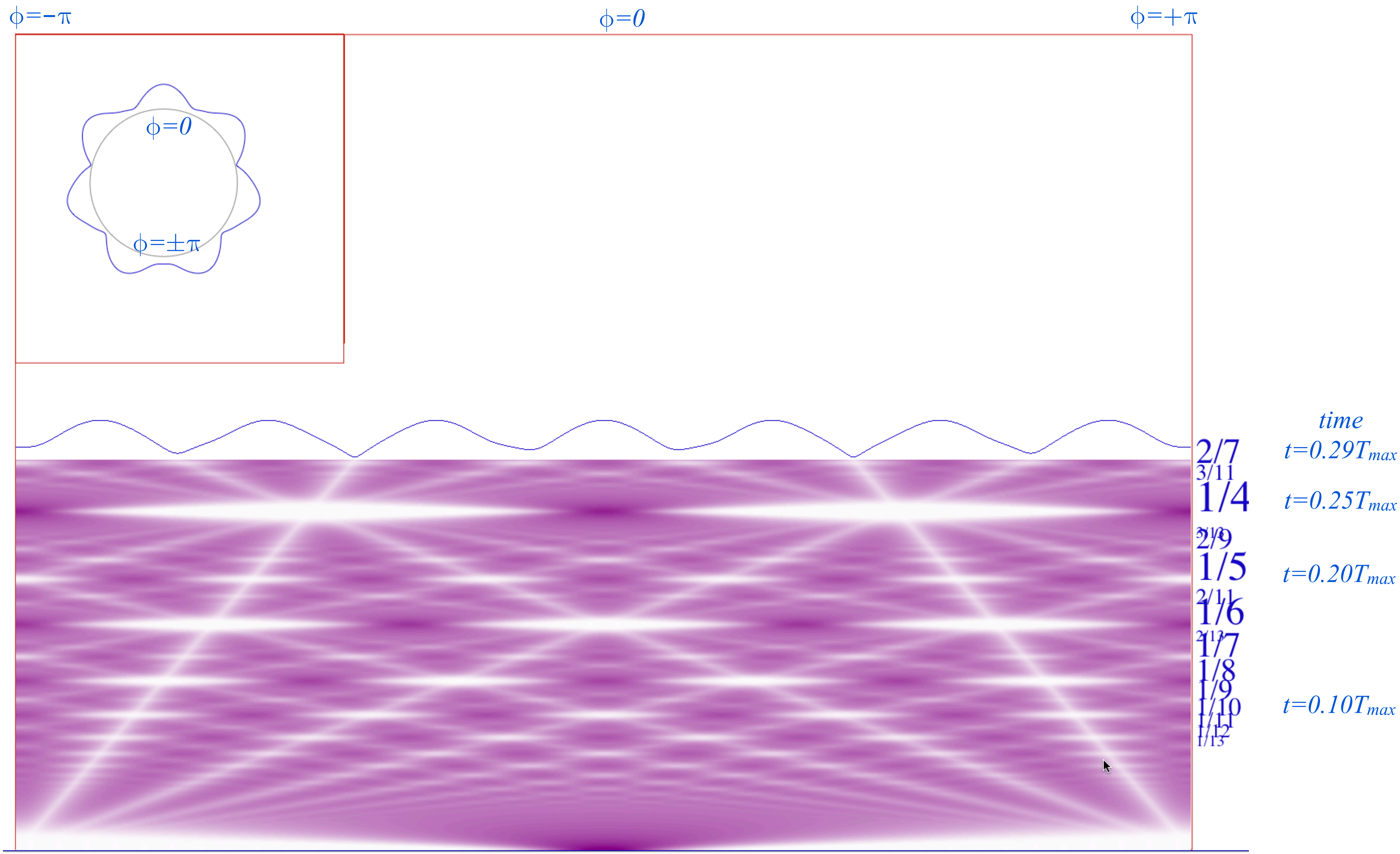

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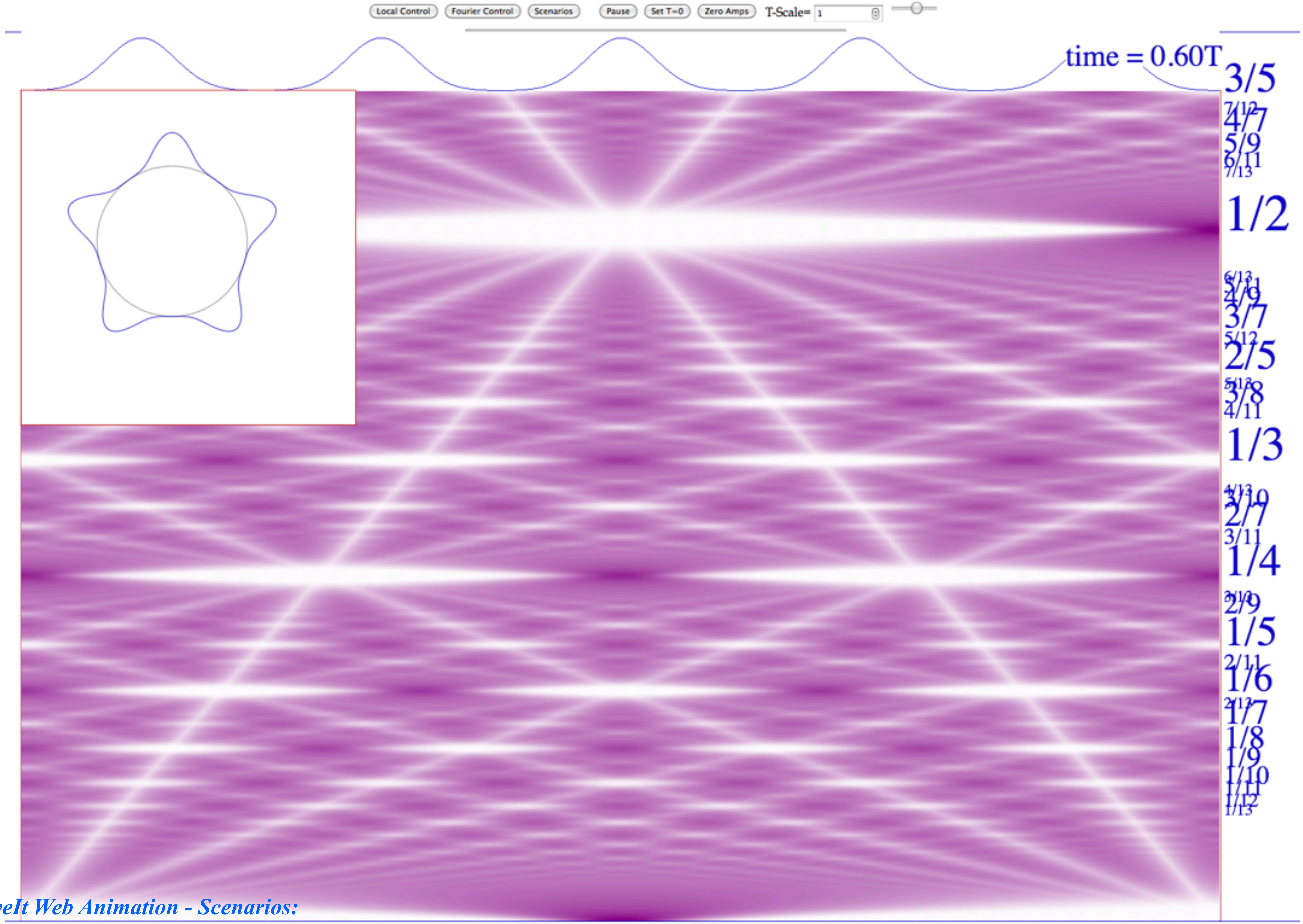


WaveIt Web Animation - Scenarios:

[Quantum_Carpet], [Quantum_Carpet_wMBars], [Quantum_Carpet_BCar], [Quantum_Carpet_BCar_wMBars]

Launch | Fourier Control | Scenarios | Pause | Set T=0 | Zero Amps | T-Scale= 1

Set this and then click here....

Type: Quantum Carpet

Time Behavior: Pause at End

Time Start (% Period) = 0

Time End (% Period)= 60

Del-x Width (% L) = 4

Excitation (Max n) = 20

Left (% L) = 0

Right (% L)= 100

n-Mean (% Max n)= 0

Peak1 Mean (% L)= 50

OverAll Scale = 1

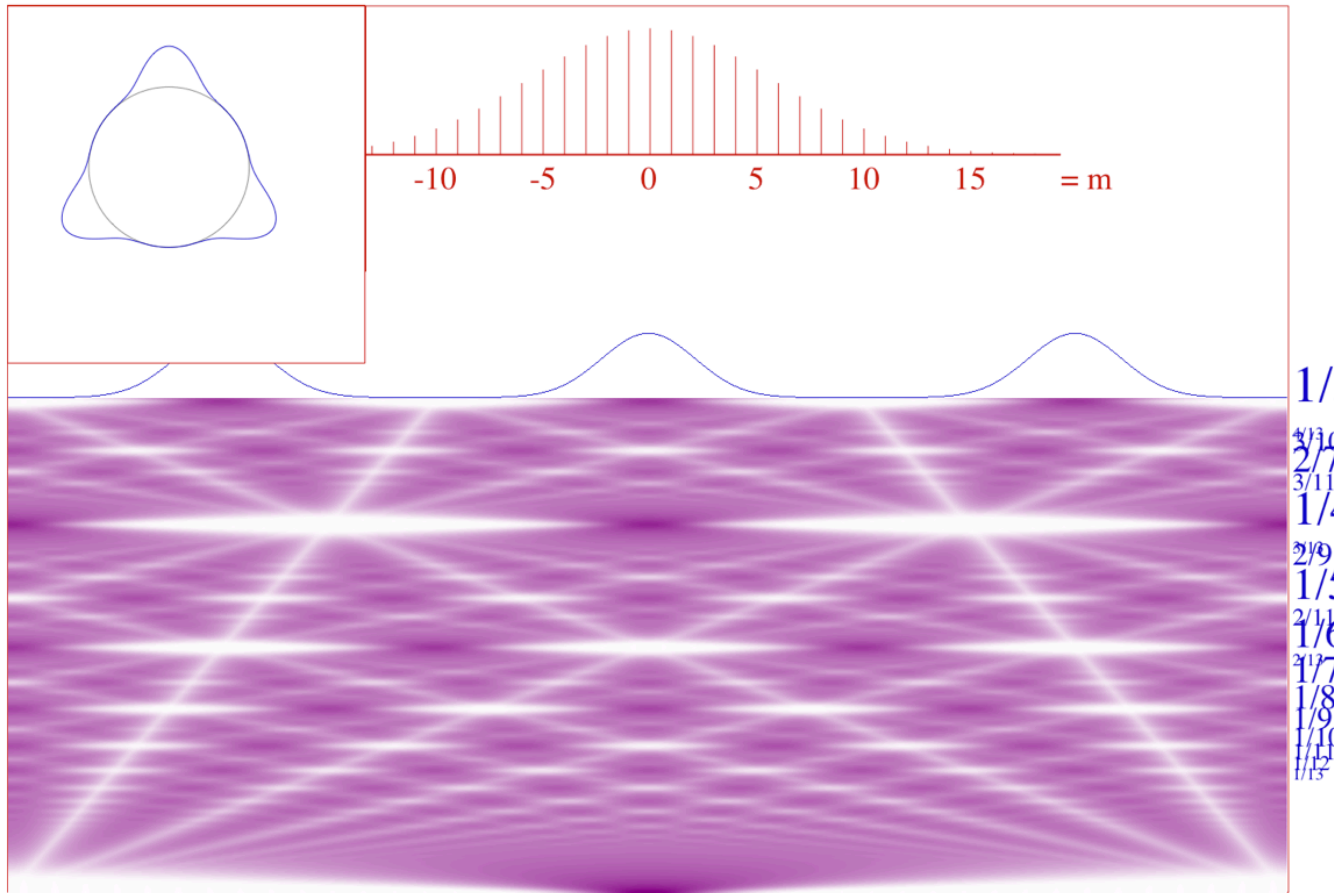
Peak2 Mean (% L)= 0

Peak2 Amp (% Peak1)= 0

Draw Ring ☑ m/n Labels ☑

m-Boxcar ☐

Draw m-Bars ☑ m-Bars Max = 30



Aspect Ratio {W/H} = 1.5

Red Level = 128

Green Level = 0

Blue Level = 128

Alpha Level = 1

Definition Level = 0.5

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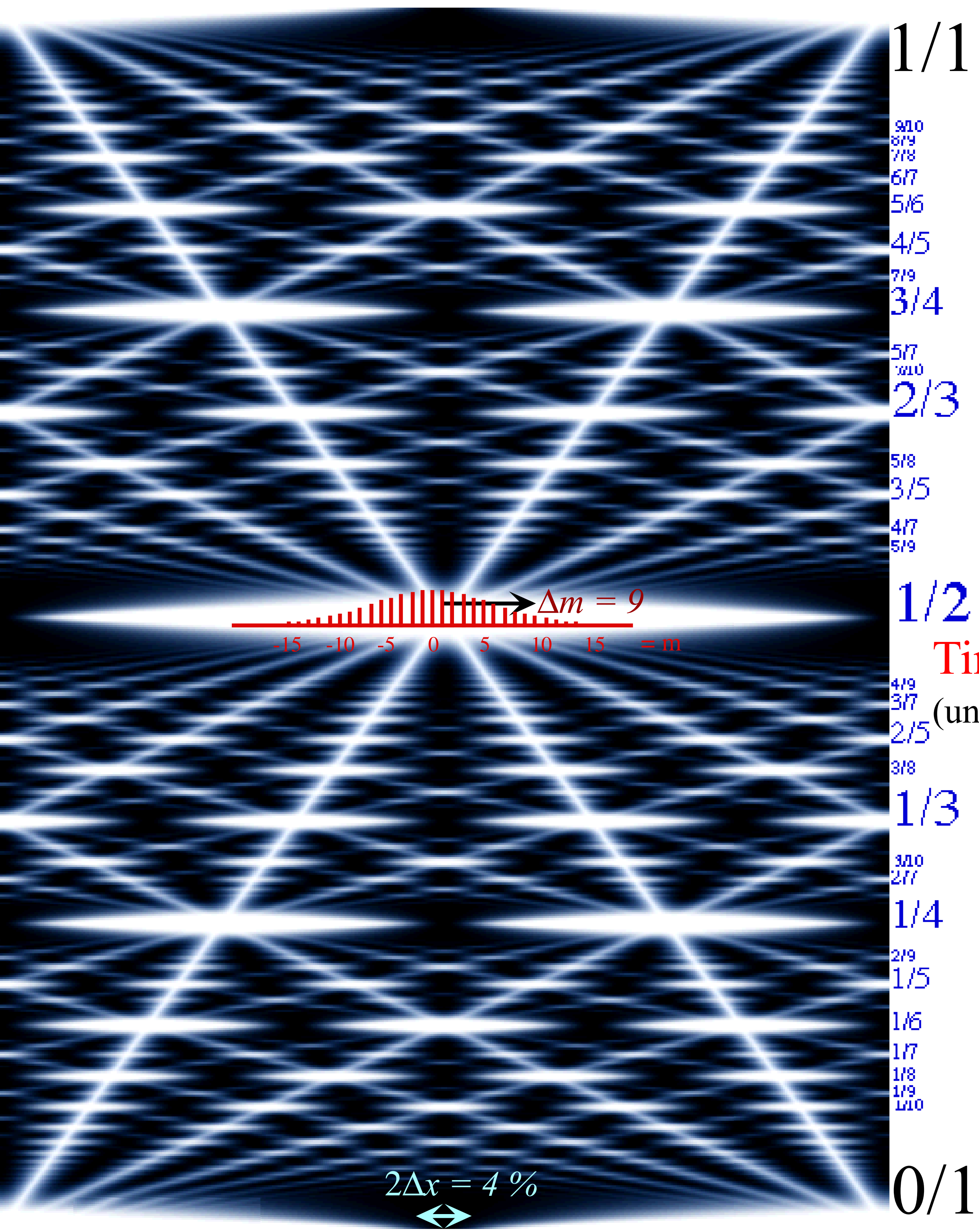
Wave Node Dynamics and Revival Symmetry in Quantum Rotors - Harter-jms-2001 (Publ.)

WaveIt Web Animation - Scenarios:

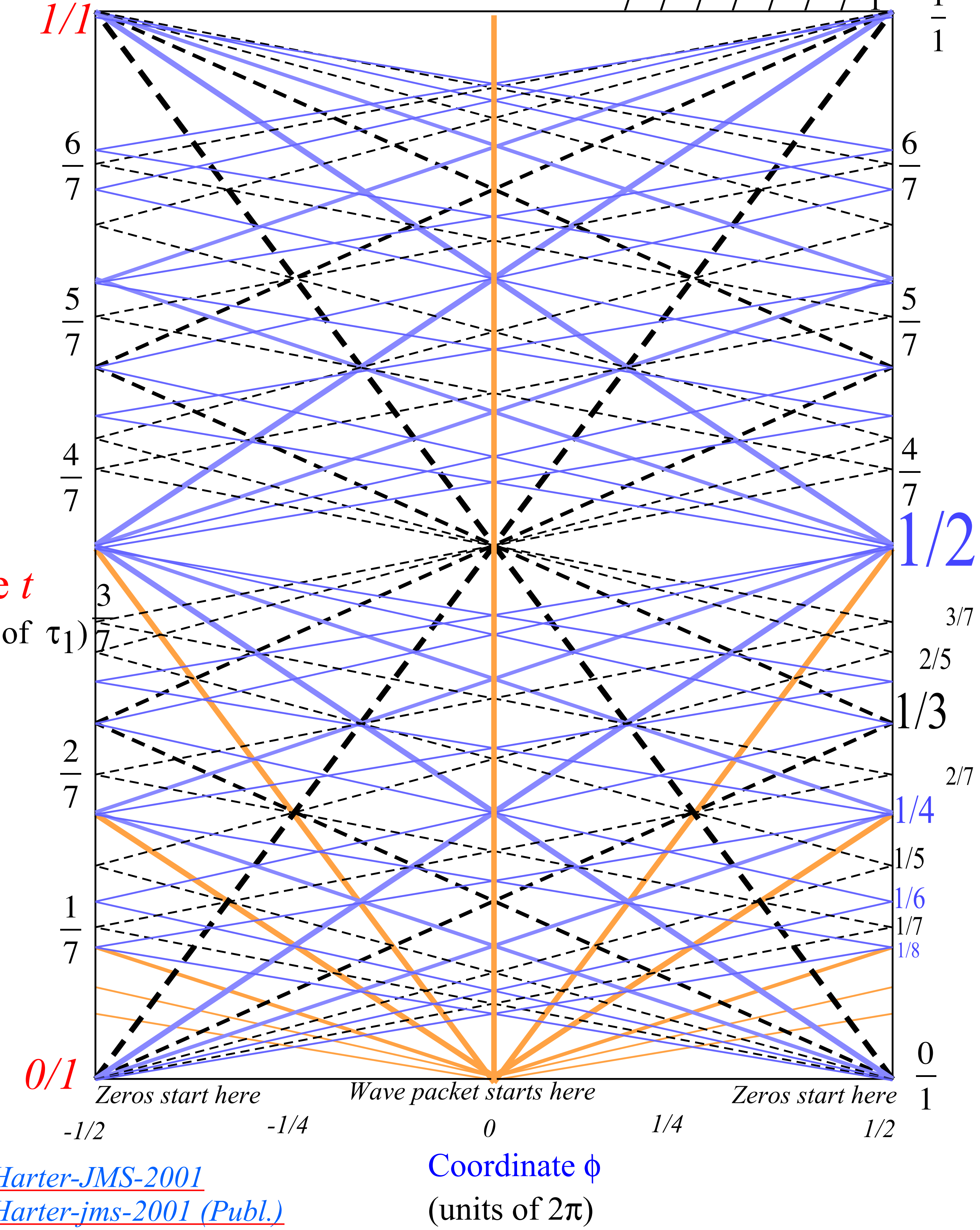
Quantum_Carpet, Quantum_Carpet_wMBars, Quantum_Carpet_BCar, Quantum_Carpet_BCar_wMBars

N-level-system and revival-beat wave dynamics

(9 or10-levels (0, ±1, ±2, ±3, ±4,..., ±9, ±10, ±11...) excited)



Zeros (clearly) and "particle-packets" (faintly) have paths labeled by fraction sequences like: $\frac{0}{7}, \frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}, \frac{1}{1}$



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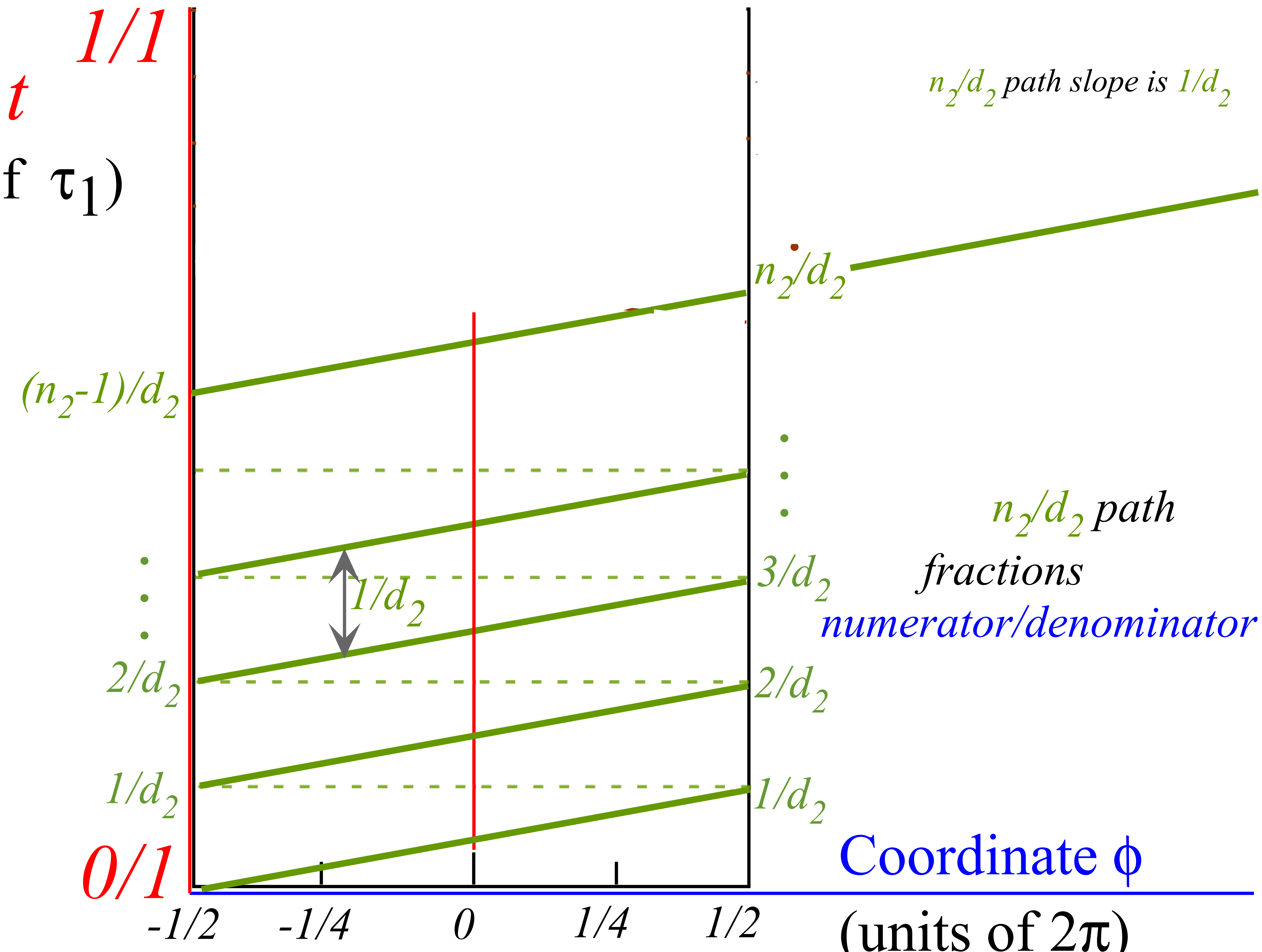
[*Wave Node Dynamics and Revival Symmetry in Quantum Rotors - Harter-jms-2001 (Publ.)*]

Farey Sum algebra of revival-beat wave dynamics

Label by *numerators N* and *denominators D* of rational fractions *N/D*



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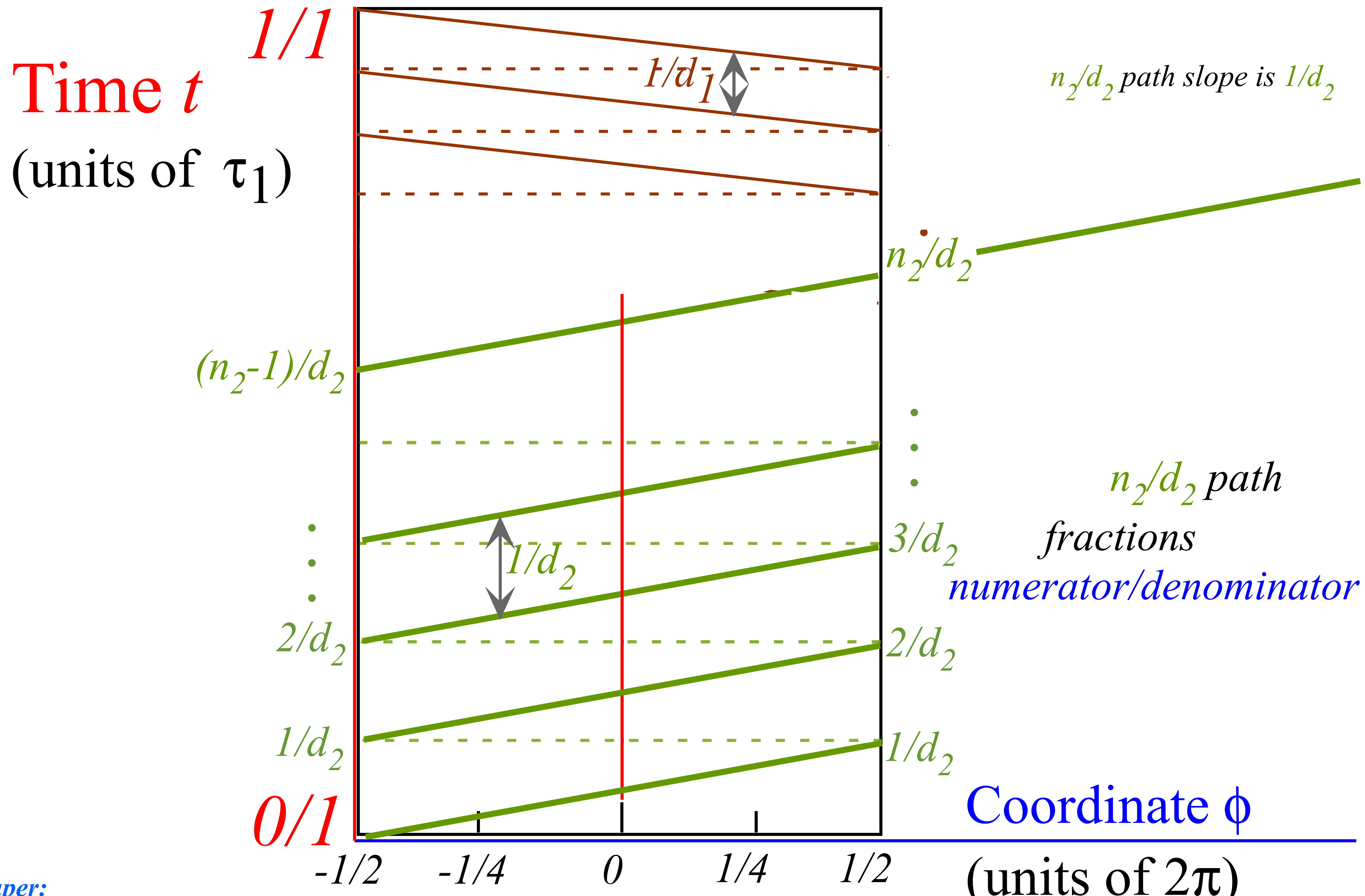

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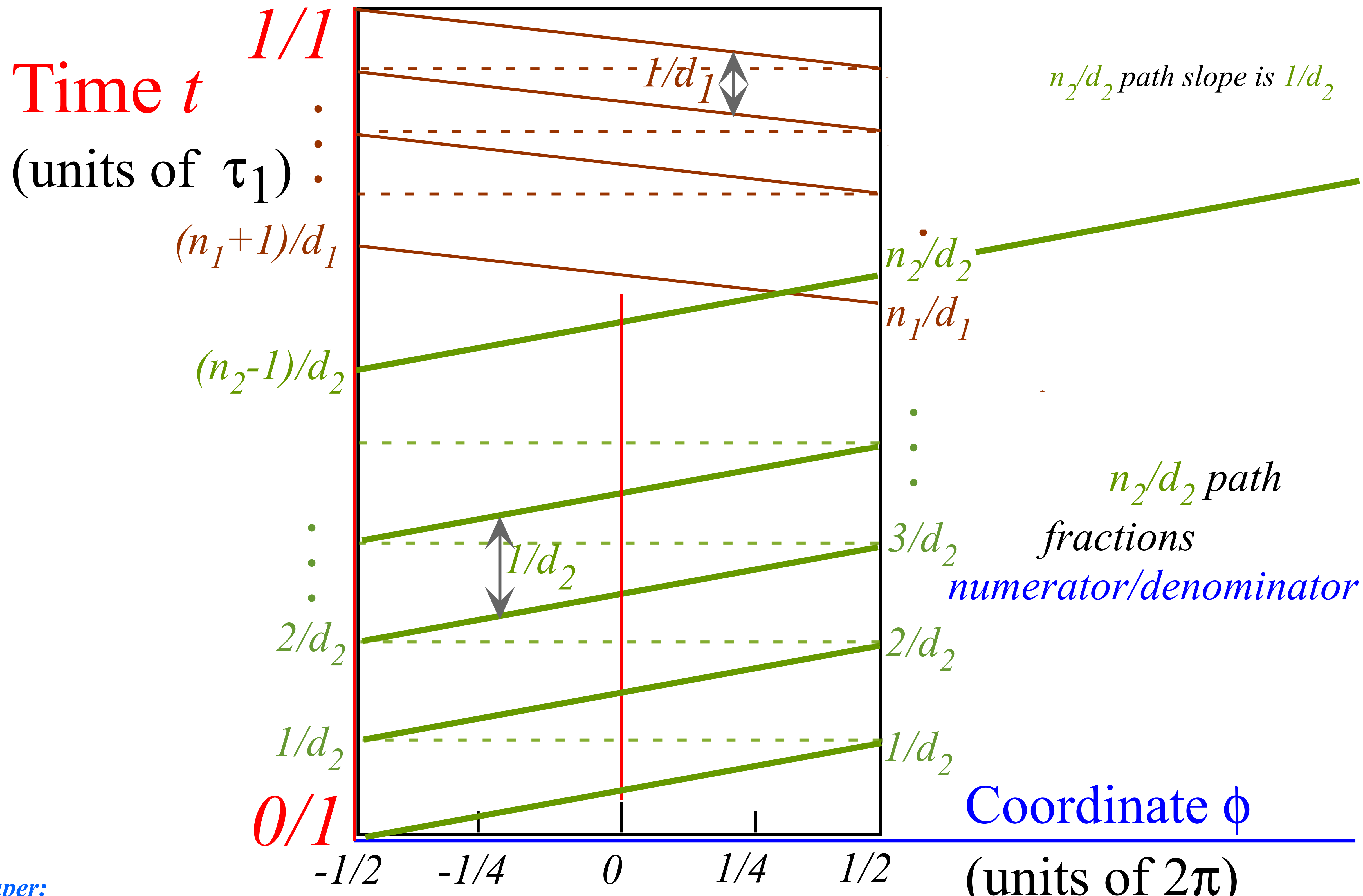

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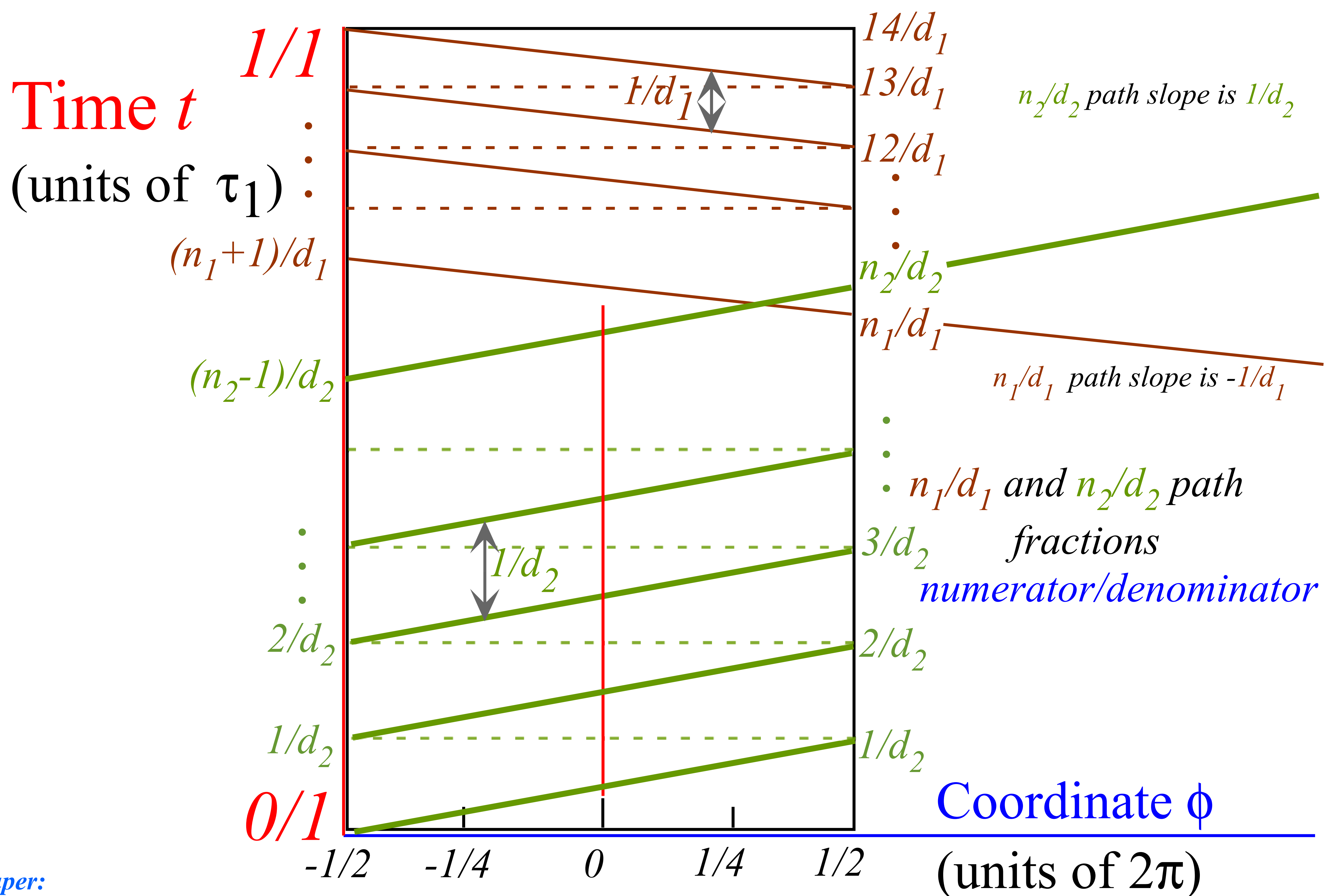

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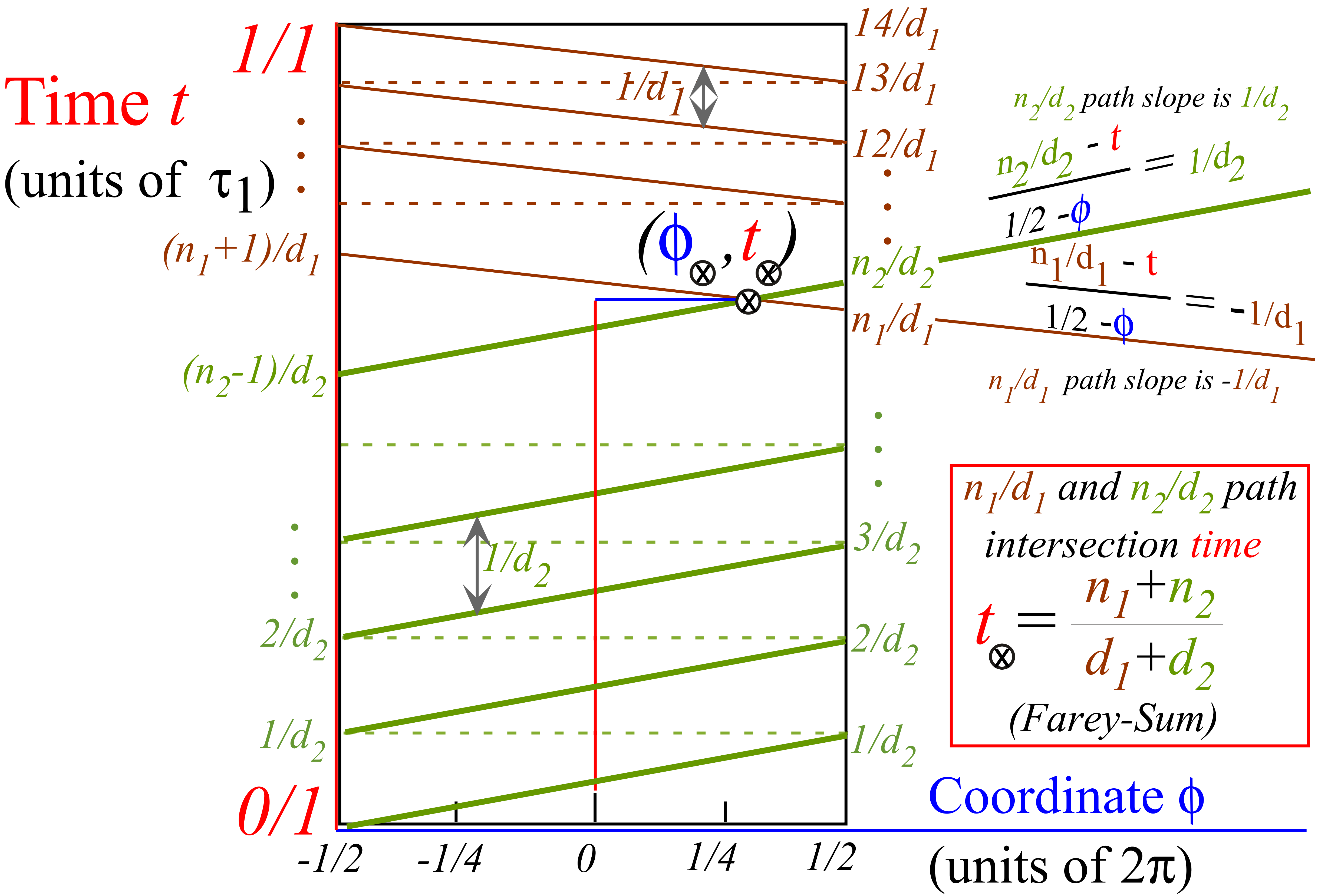


n_1/d_1 and n_2/d_2 path intersection time

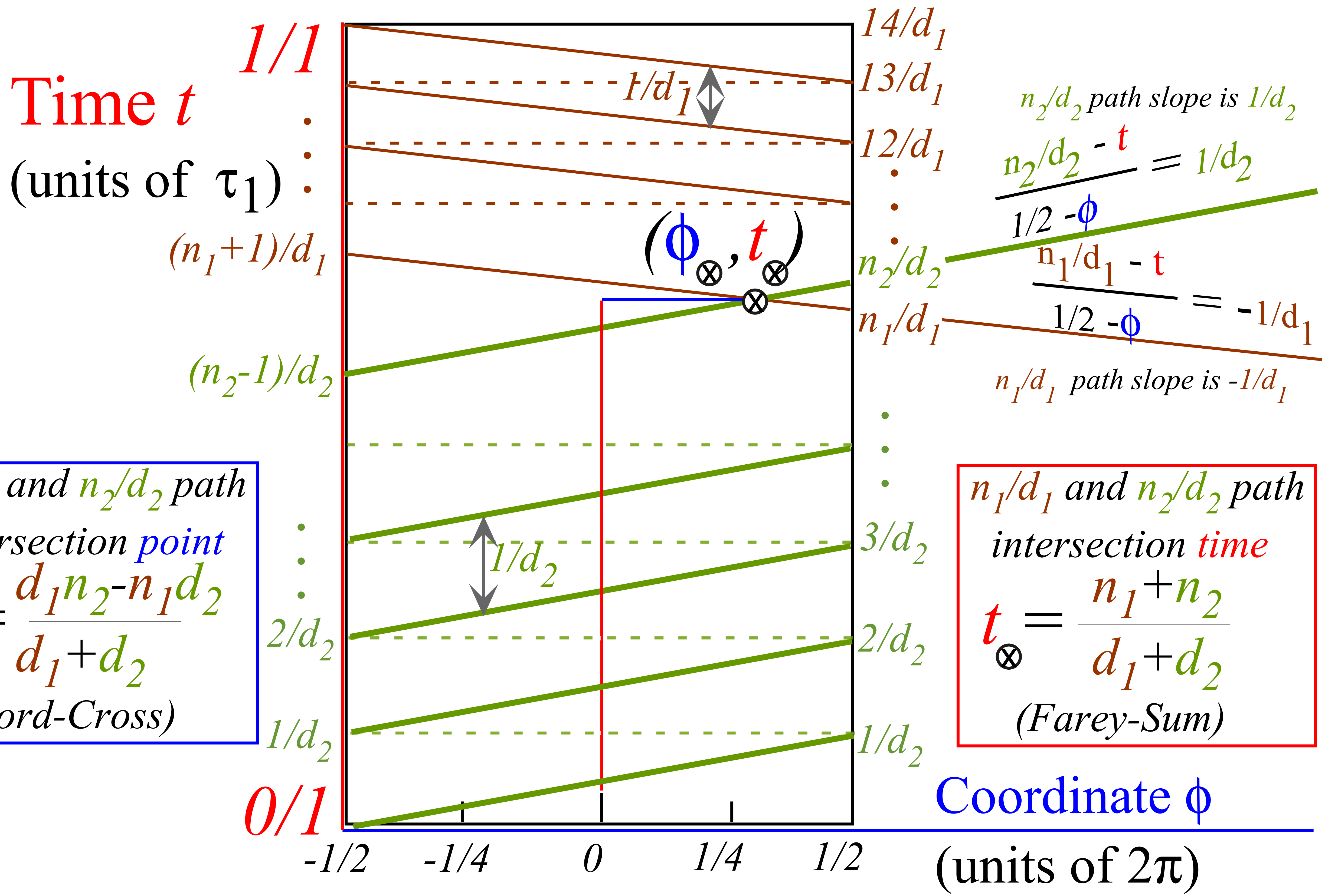
$$t_\otimes = \frac{n_1+n_2}{d_1+d_2}$$

(Farey-Sum)

[*John Farey, Phil. Mag.(1816)*]

Farey Sum algebra of revival-beat wave dynamics

Label by *numerators N* and *denominators D* of rational fractions *N/D*



n_1/d_1 and n_2/d_2 path intersection point

$$\phi_\otimes = \frac{d_1 n_2 - n_1 d_2}{d_1 + d_2}$$

(Ford-Cross)

n_1/d_1 and n_2/d_2 path intersection time

$$t_\otimes = \frac{n_1 + n_2}{d_1 + d_2}$$

(Farey-Sum)

[Lester. R. Ford, Am. Math. Monthly 45,586(1938)]

[John Farey, Phil. Mag.(1816)]

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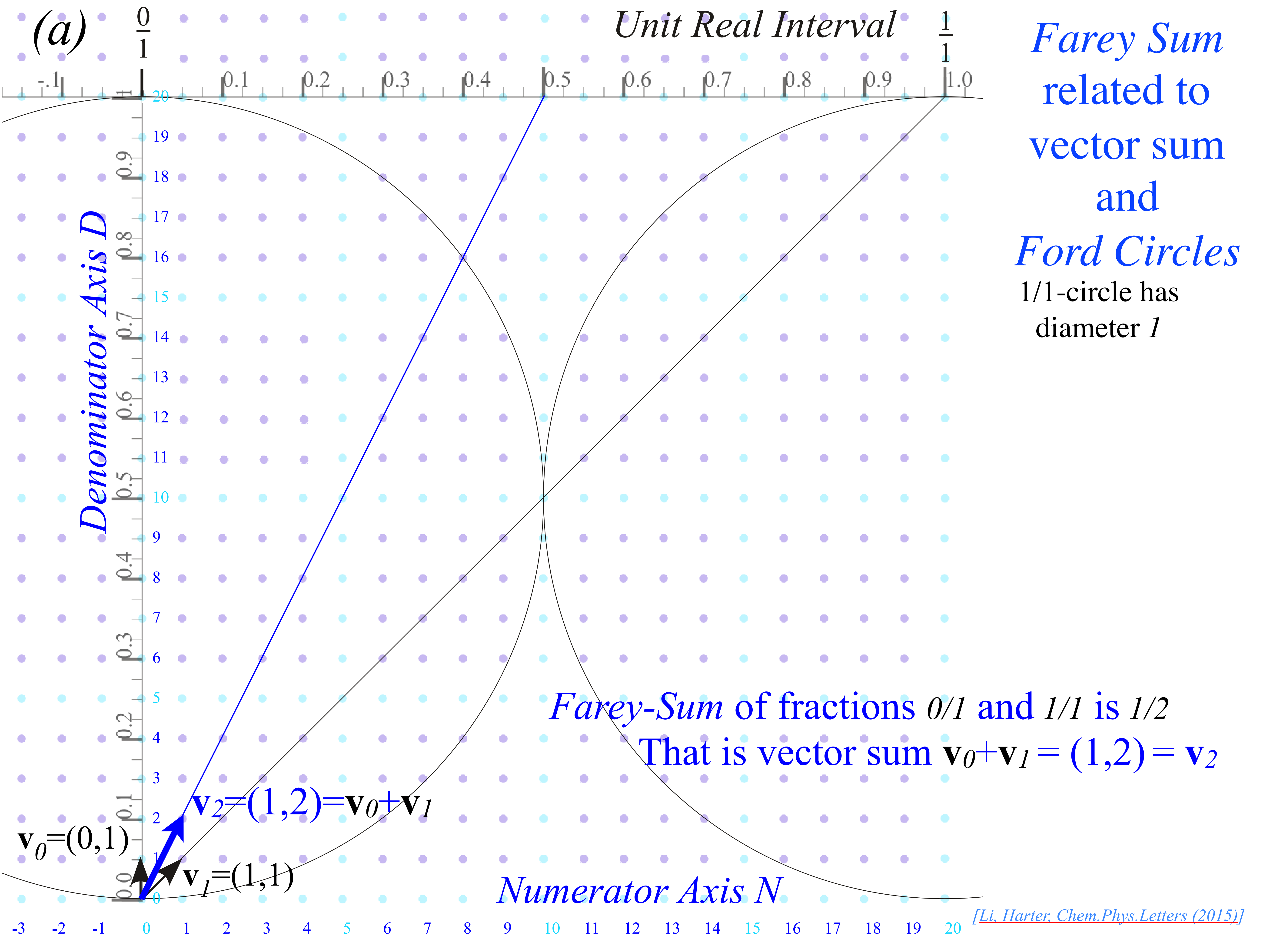
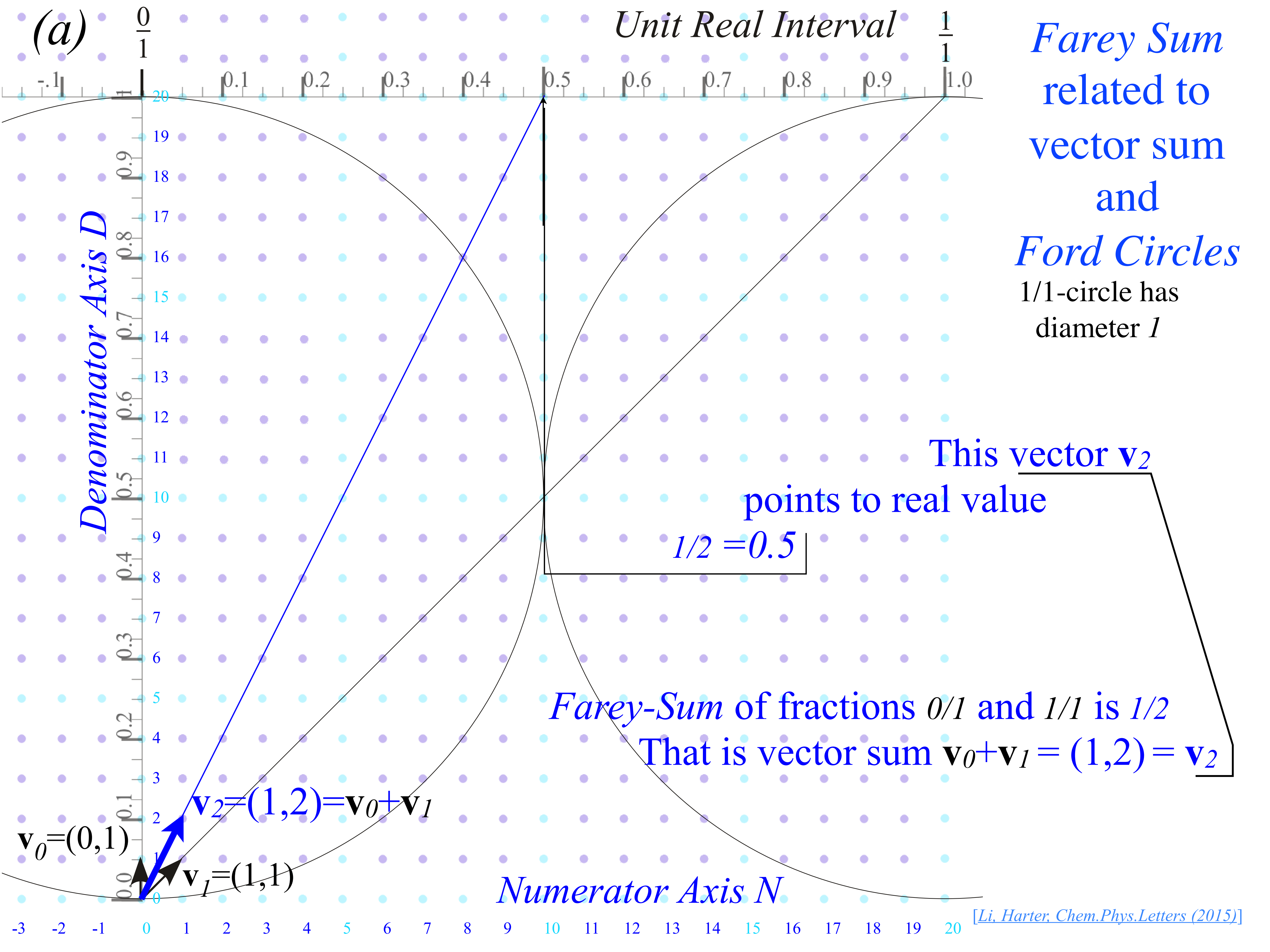
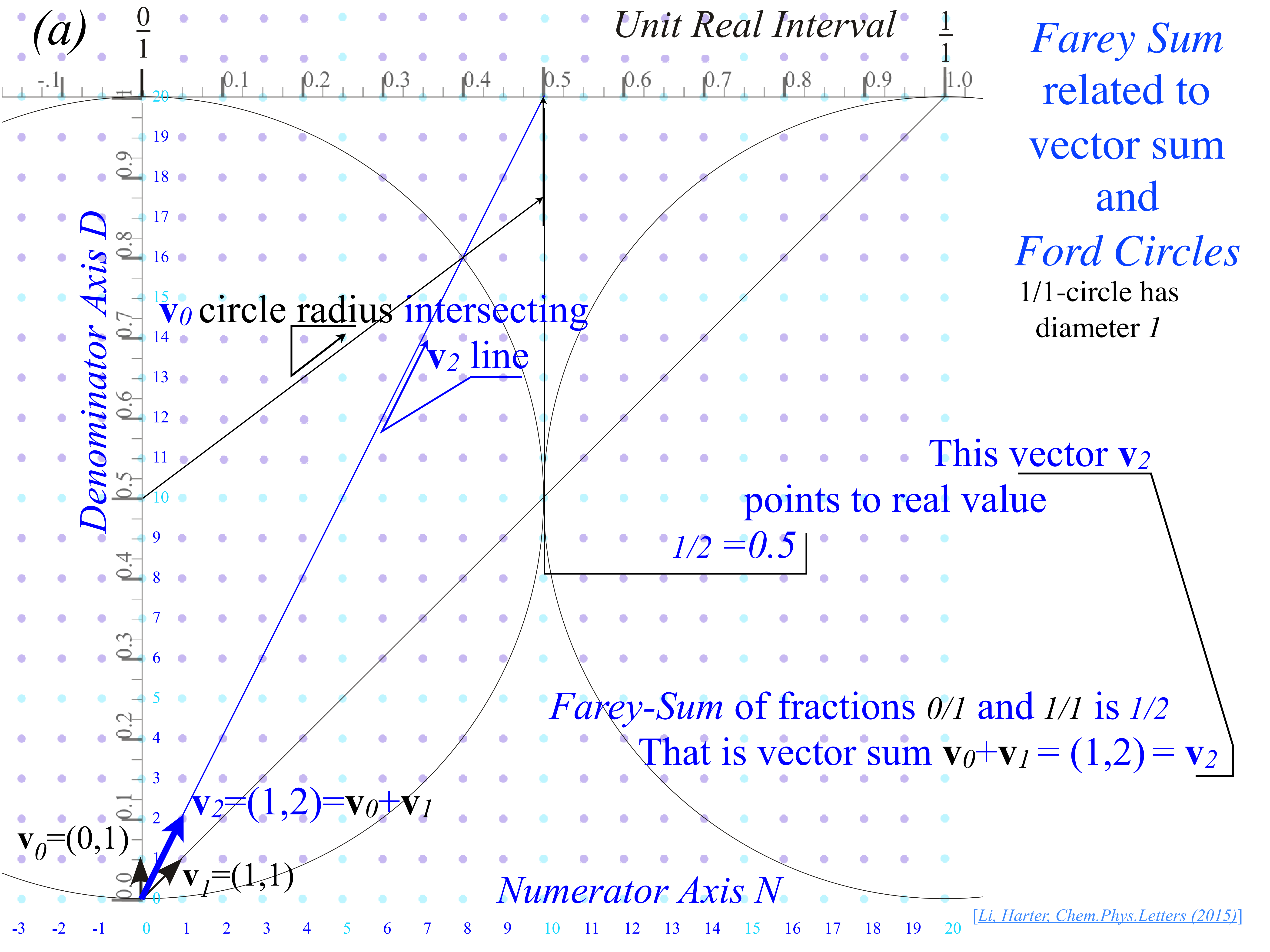


(a)
Unit Real Interval
Farey Sum related to vector sum and Ford Circles
1/1-circle has diameter 1
Denominator Axis D
Numerator Axis N
Farey-Sum of fractions 0/1 and 1/1 is 1/2
That is vector sum $\mathbf{v}_0+\mathbf{v}_1 = (1,2) = \mathbf{v}_2$
$\mathbf{v}_2=(1,2)=\mathbf{v}_0+\mathbf{v}_1$
$\mathbf{v}_0=(0,1)$
$\mathbf{v}_1=(1,1)$
[Li, Harter, Chem.Phys.Letters (2015)]


(a)
0/1
Unit Real Interval
1/1
Farey Sum related to vector sum and Ford Circles
1/1-circle has diameter 1
Denominator Axis D
This vector v2 points to real value 1/2 =0.5
Farey-Sum of fractions 0/1 and 1/1 is 1/2
That is vector sum v0+v1 = (1,2) = v2
v2=(1,2)=v0+v1
v0=(0,1)
v1=(1,1)
Numerator Axis N
[Li, Harter, Chem.Phys.Letters (2015)]


(a)
Unit Real Interval
Farey Sum related to vector sum and Ford Circles
1/1-circle has diameter 1
Denominator Axis D
Numerator Axis N
v0 circle radius intersecting v2 line
This vector v2 points to real value 1/2 =0.5
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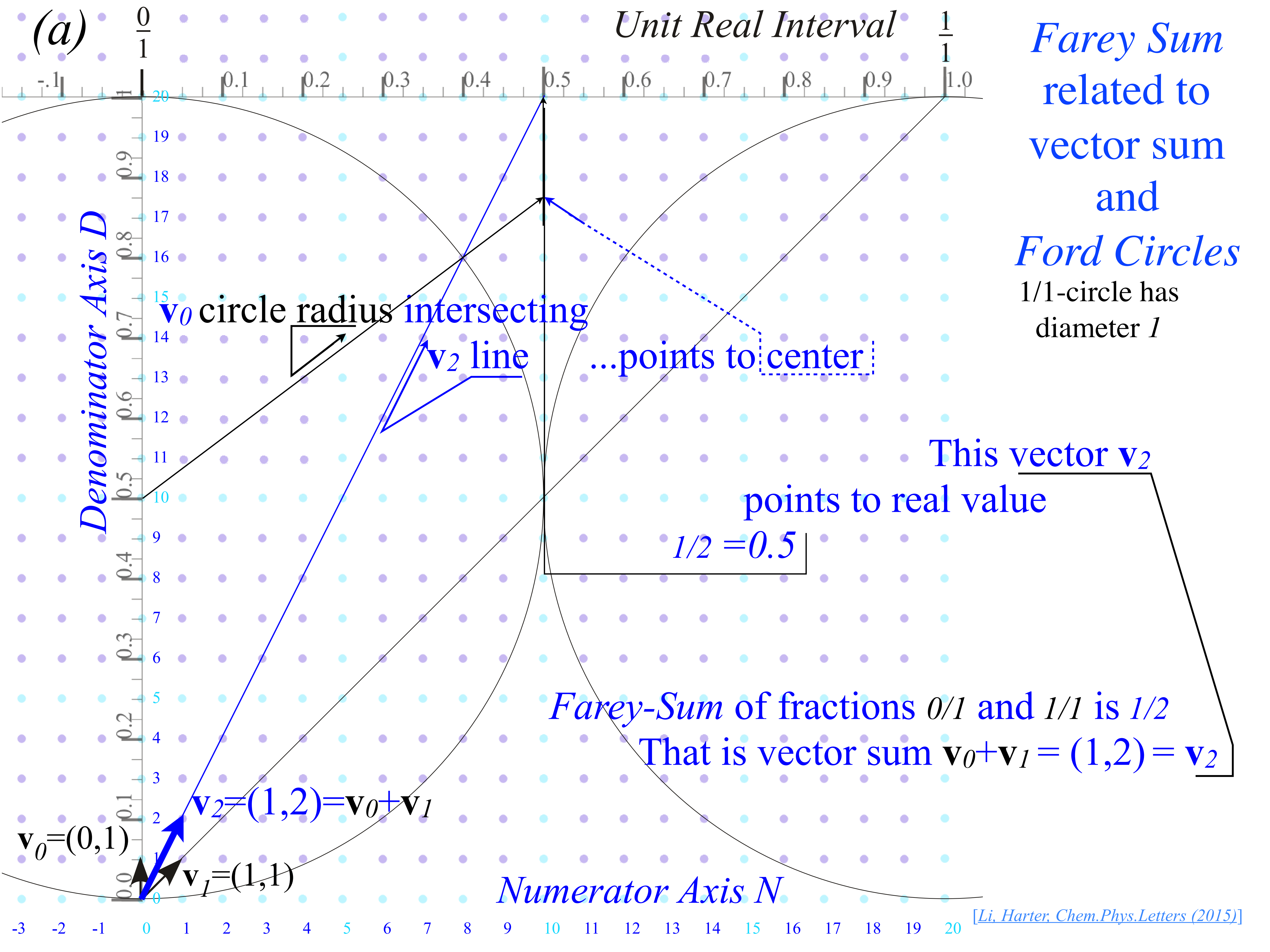

(a)
0/1
Unit Real Interval
1/1
Farey Sum related to vector sum and Ford Circles
1/1-circle has diameter 1
Denominator Axis D
$\mathbf{v}_0$ circle radius intersecting
$\mathbf{v}_2$ line
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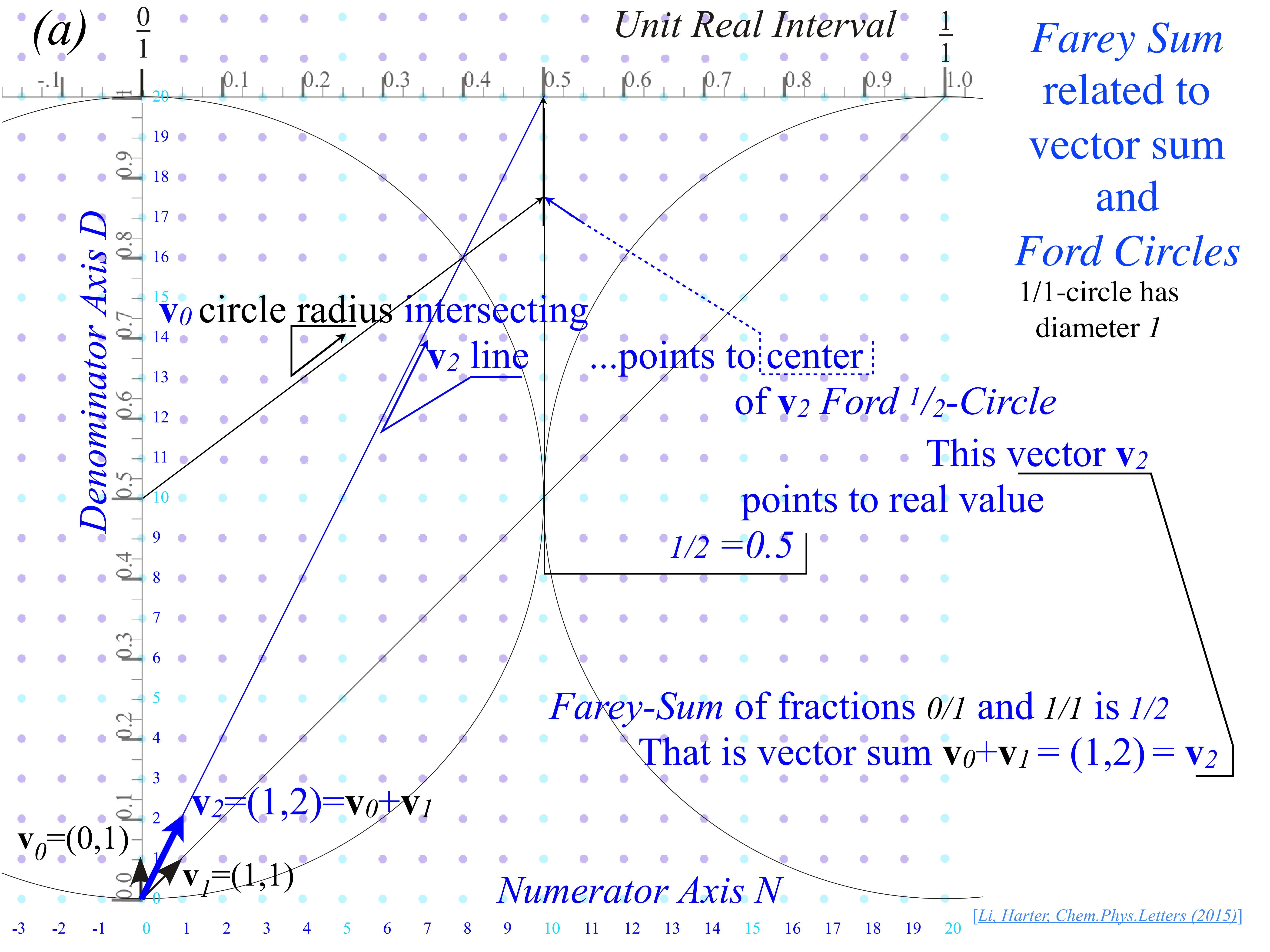
$\mathbf{v}_0=(0,1)$
$\mathbf{v}_1=(1,1)$
Numerator Axis N
[Li, Harter, Chem.Phys.Letters (2015)]



(a)
0/1
Unit Real Interval
1/1
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Denominator Axis D
v0 circle radius intersecting v2 line
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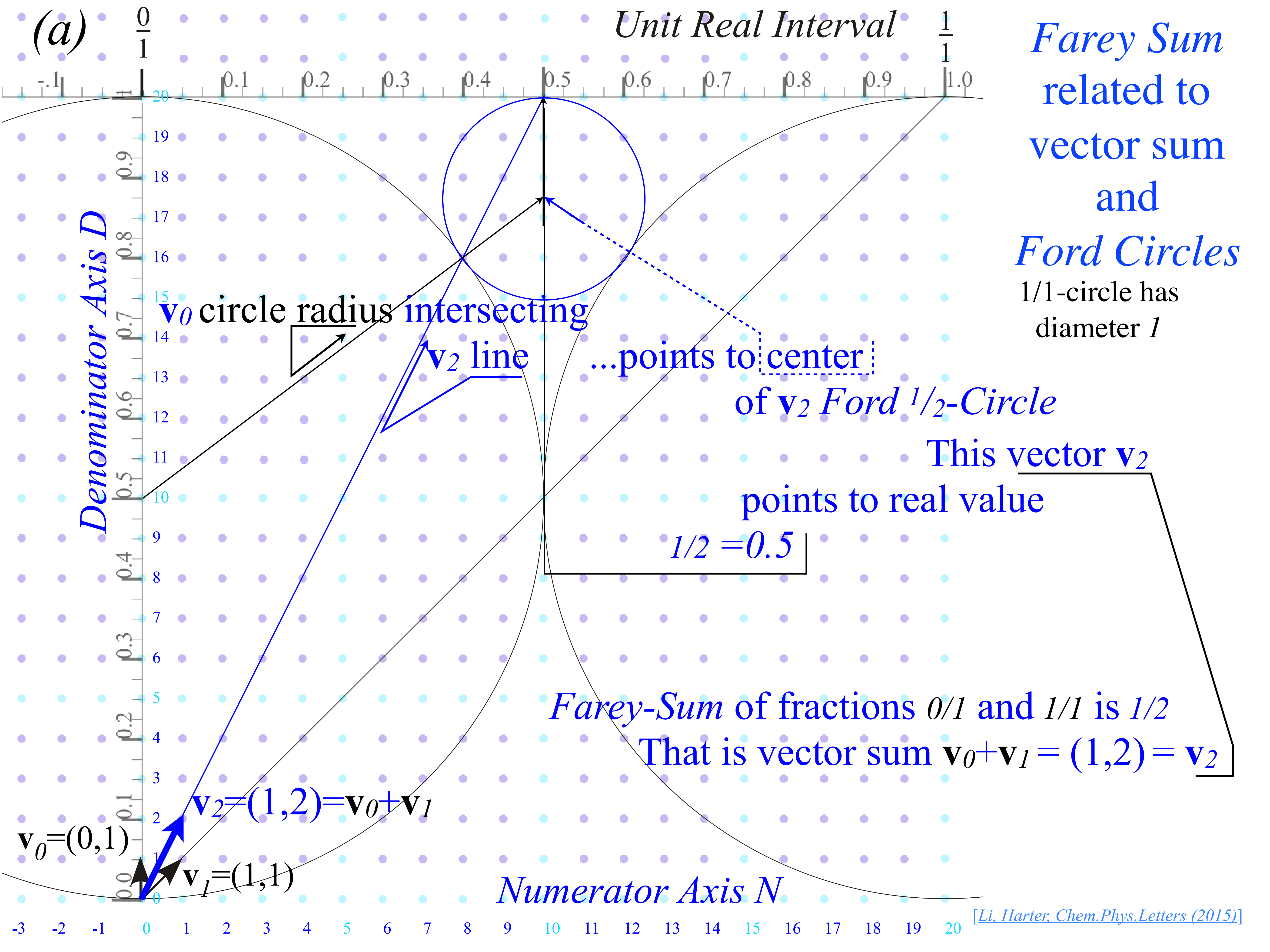

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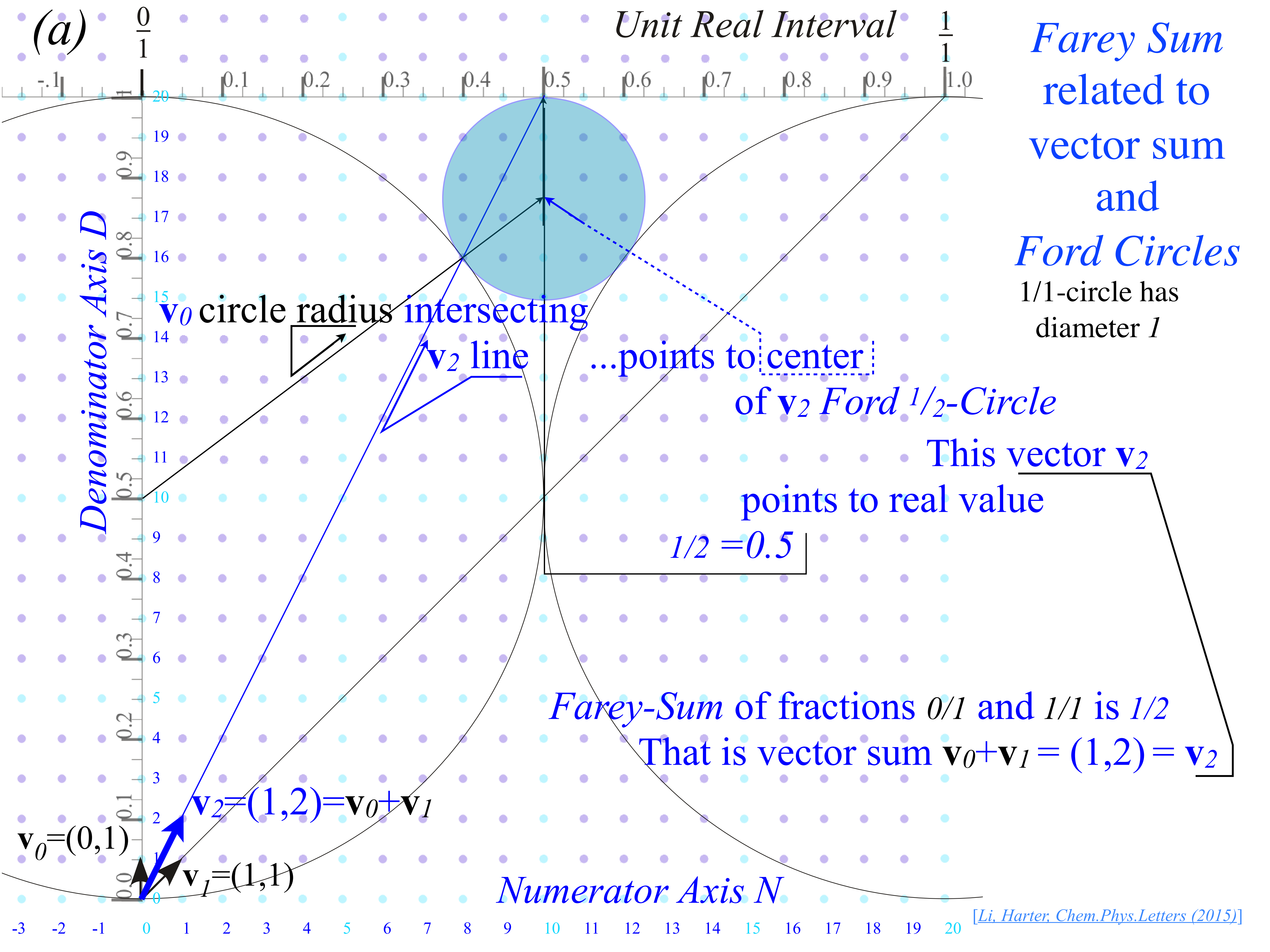
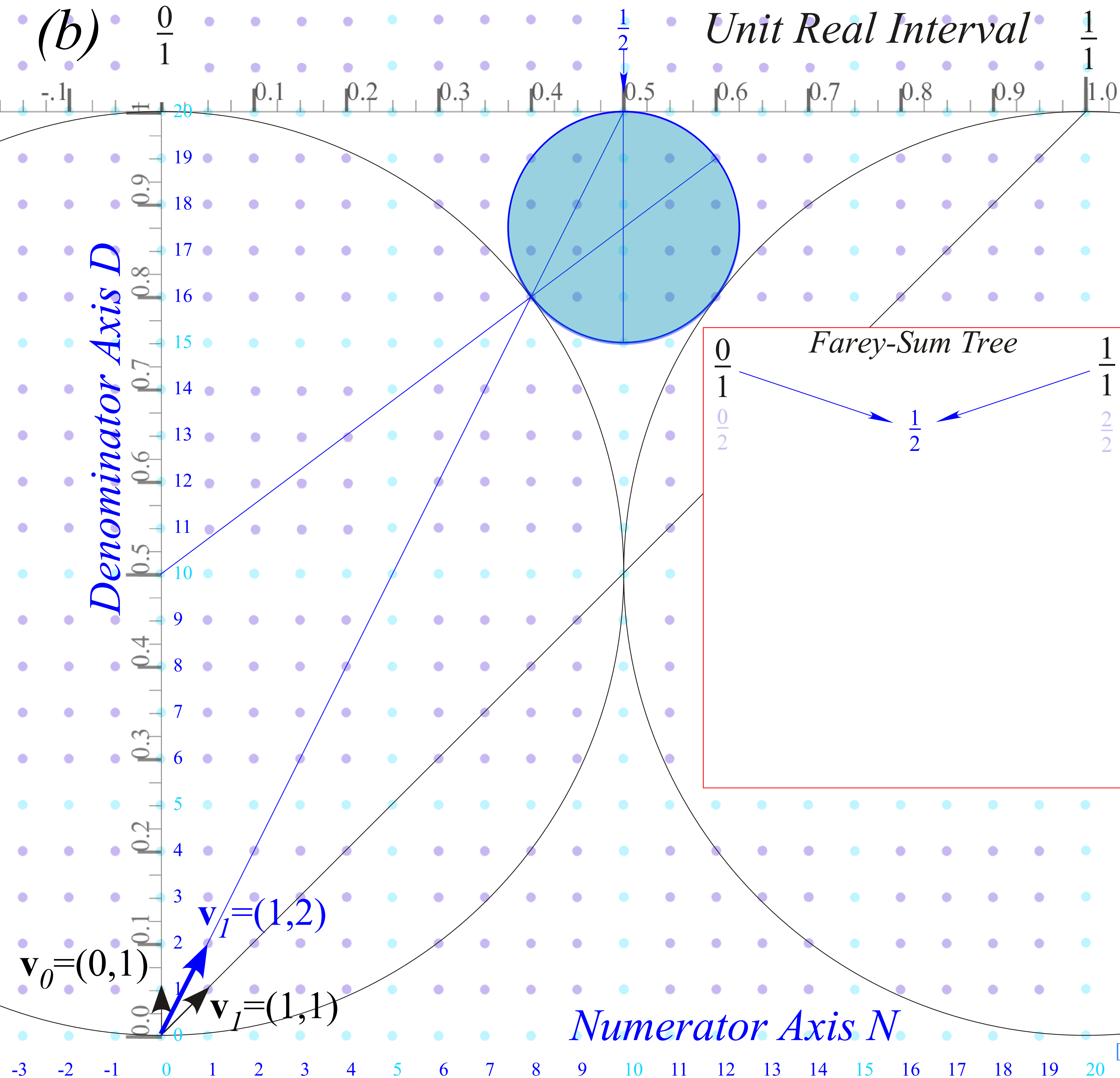
Numerator Axis N
[Li, Harter, Chem.Phys.Letters (2015)]


(a)
Unit Real Interval
Farey Sum related to vector sum and Ford Circles
1/1-circle has diameter 1
Denominator Axis D
Numerator Axis N
v0 circle radius intersecting
v2 line
...points to center
of v2 Ford 1/2-Circle
This vector v2
points to real value
1/2 =0.5
Farey-Sum of fractions 0/1 and 1/1 is 1/2
That is vector sum v0+v1 = (1,2) = v2
v2=(1,2)=v0+v1
v0=(0,1)
v1=(1,1)
[Li, Harter, Chem.Phys.Letters (2015)]

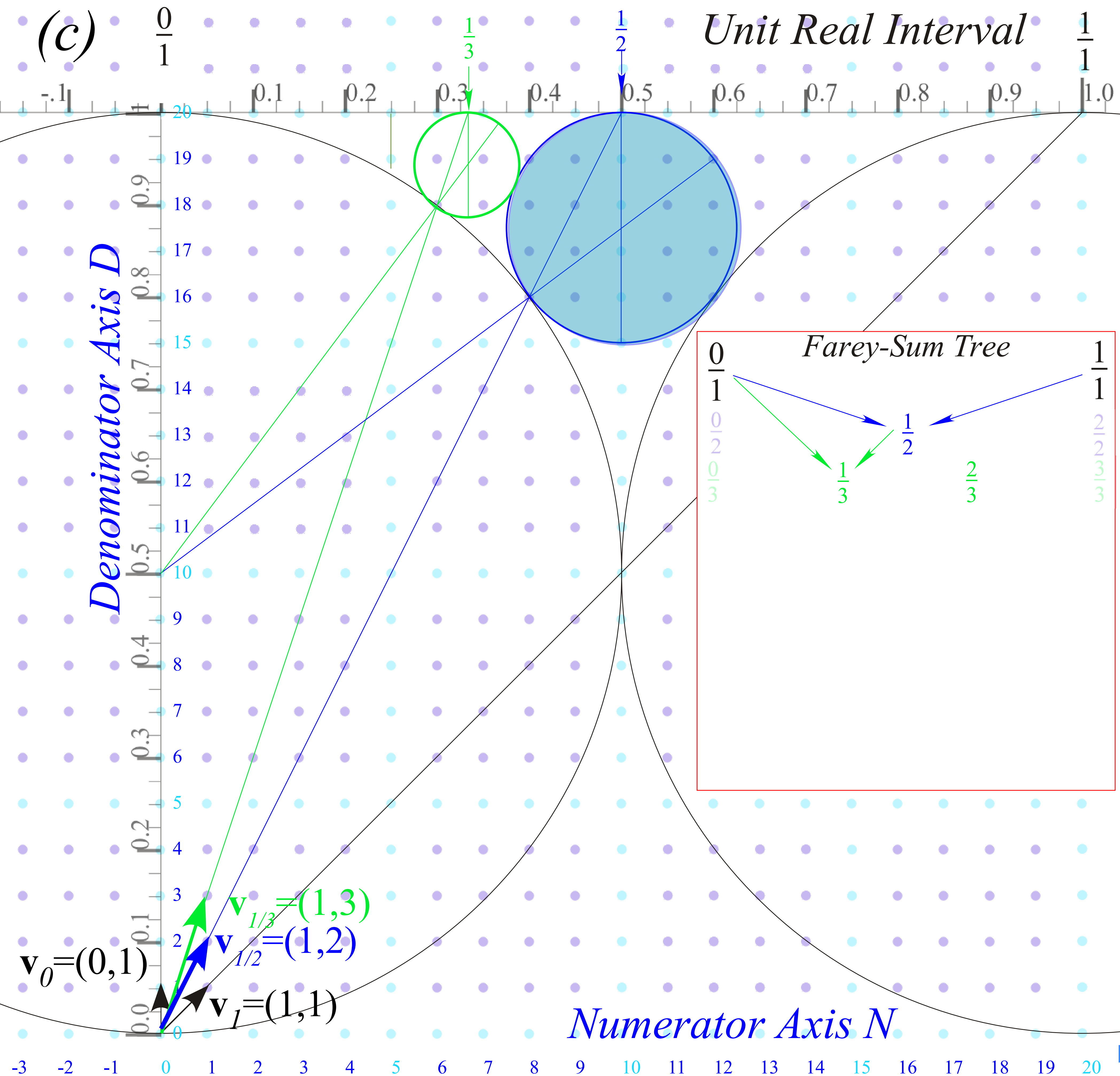


Farey Sum related to vector sum and *Ford Circles*

1/1-circle has diameter *1*

1/2-circle has diameter *1/2²=1/4*

[Li, Harter, Chem.Phys.Letters (2015)]

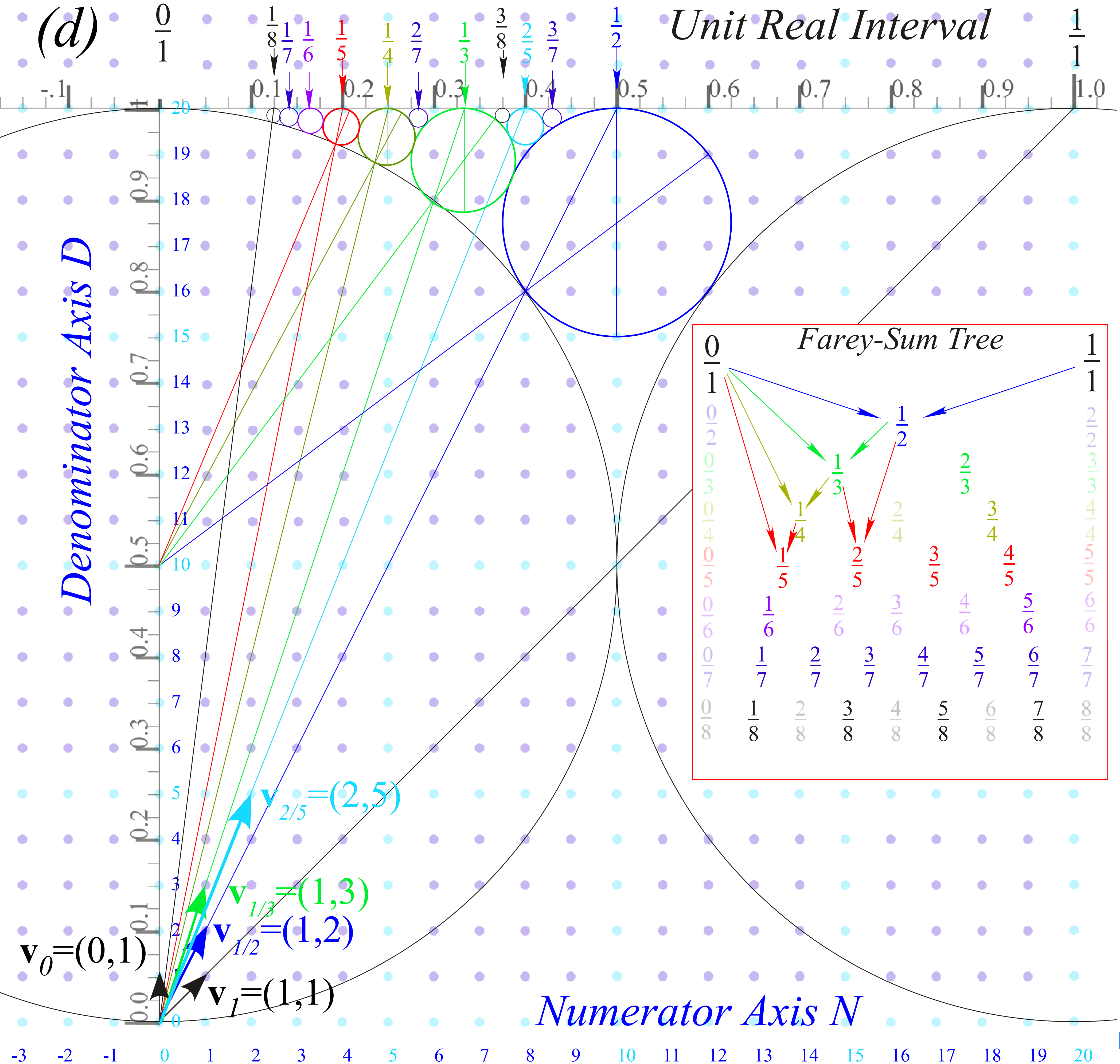


Farey Sum related to vector sum and *Ford Circles*

1/2-circle has diameter $1/2^2=1/4$

1/3-circles have diameter $1/3^2=1/9$

[Li, Harter, Chem.Phys.Letters (2015)]



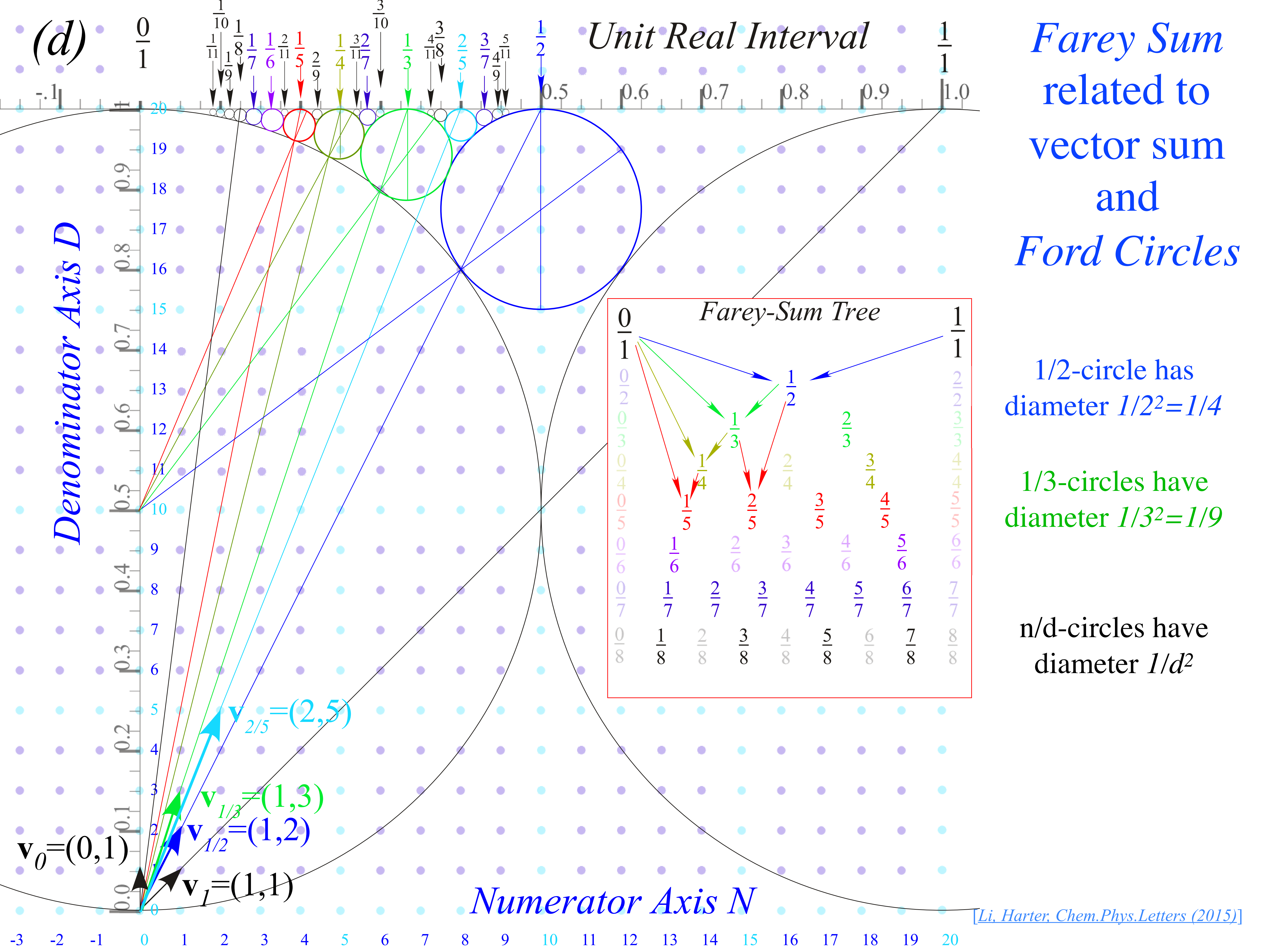
Farey Sum related to vector sum and *Ford Circles*

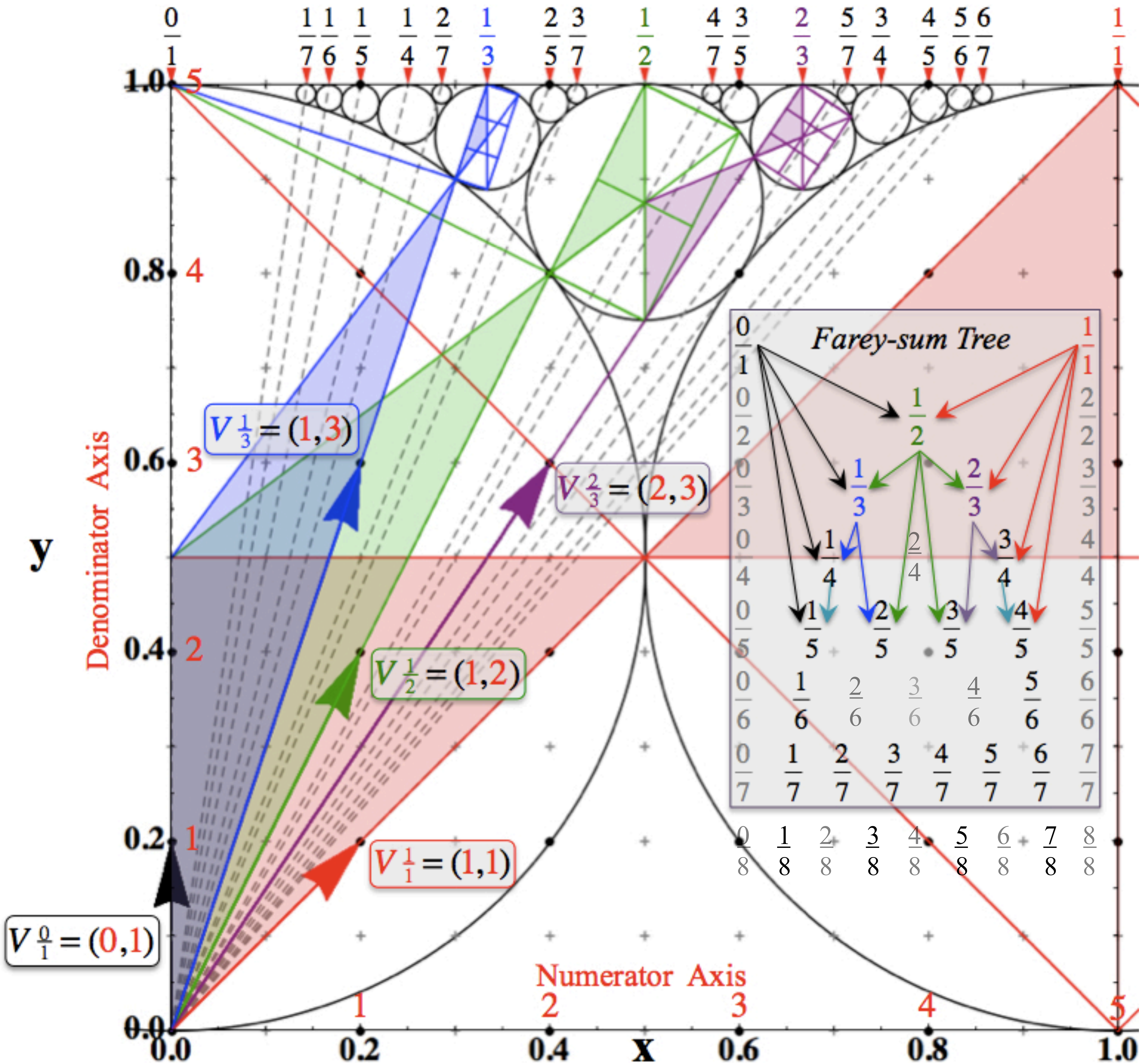
1/2-circle has diameter $1/2^2=1/4$

1/3-circles have diameter $1/3^2=1/9$

n/d-circles have diameter $1/d^2$

[Li, Harter, Chem.Phys.Letters (2015)]

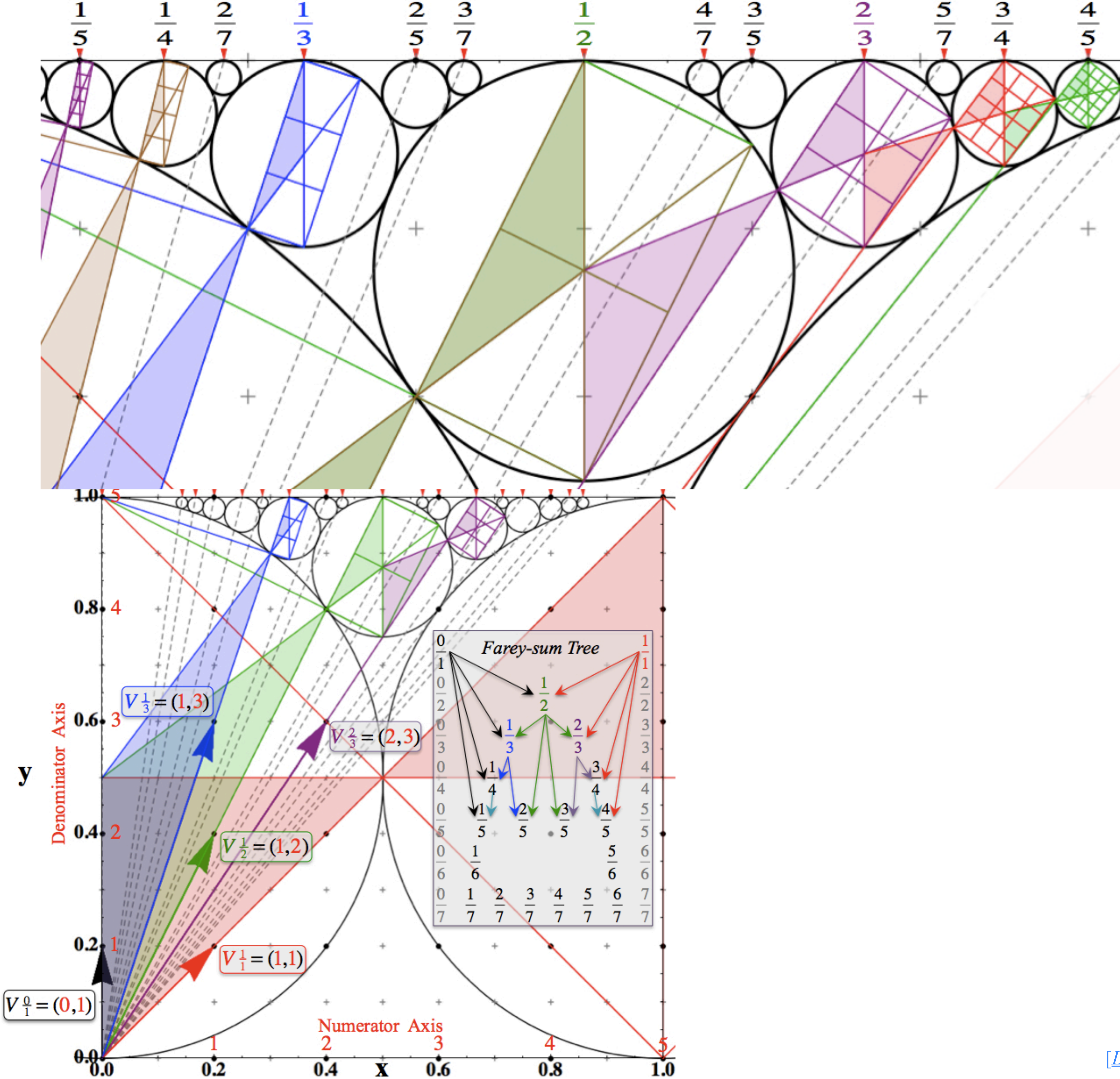

(d)
Unit Real Interval
Farey Sum related to vector sum and Ford Circles
Farey-Sum Tree
1/2-circle has diameter 1/2²=1/4
1/3-circles have diameter 1/3²=1/9
n/d-circles have diameter 1/d²
Denominator Axis D
Numerator Axis N
v_0=(0,1)
v_1=(1,1)
v_1/2=(1,2)
v_1/3=(1,3)
v_2/5=(2,5)
[Li, Harter, Chem.Phys.Letters (2015)]



Thales Rectangles

provide analytic geometry of fractal structure

[Li, Harter, Chem.Phys.Letters (2015)]



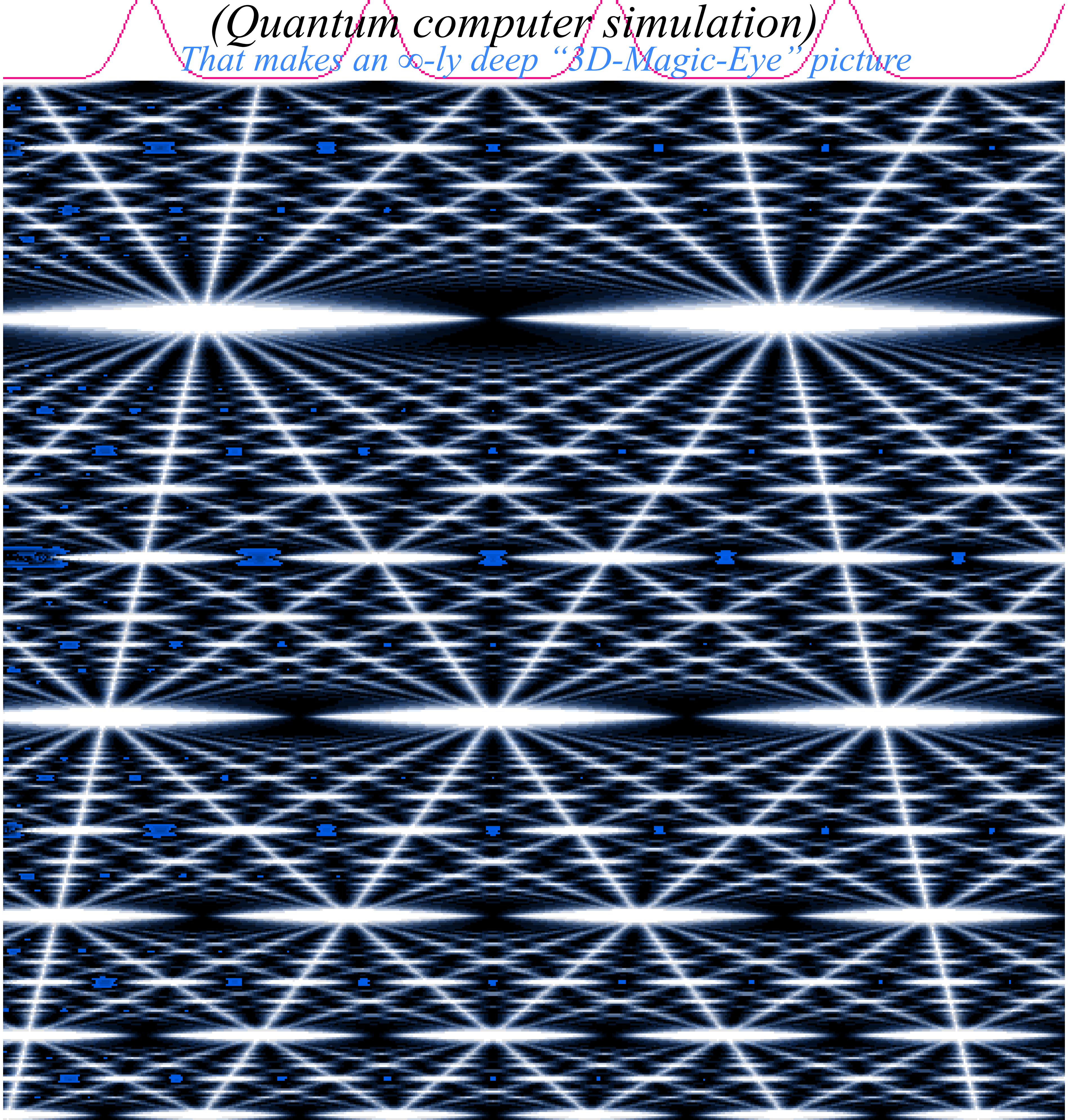
"Quantized" Thales Rectangles

provide analytic geometry of fractal structure

[Li, Harter, Chem.Phys.Letters (2015)]

(Quantum computer simulation)

That makes an ∞-ly deep "3D-Magic-Eye" picture



End of
Lecture 4

Geometric "Integration" (Converting Velocity data to Spacetime)

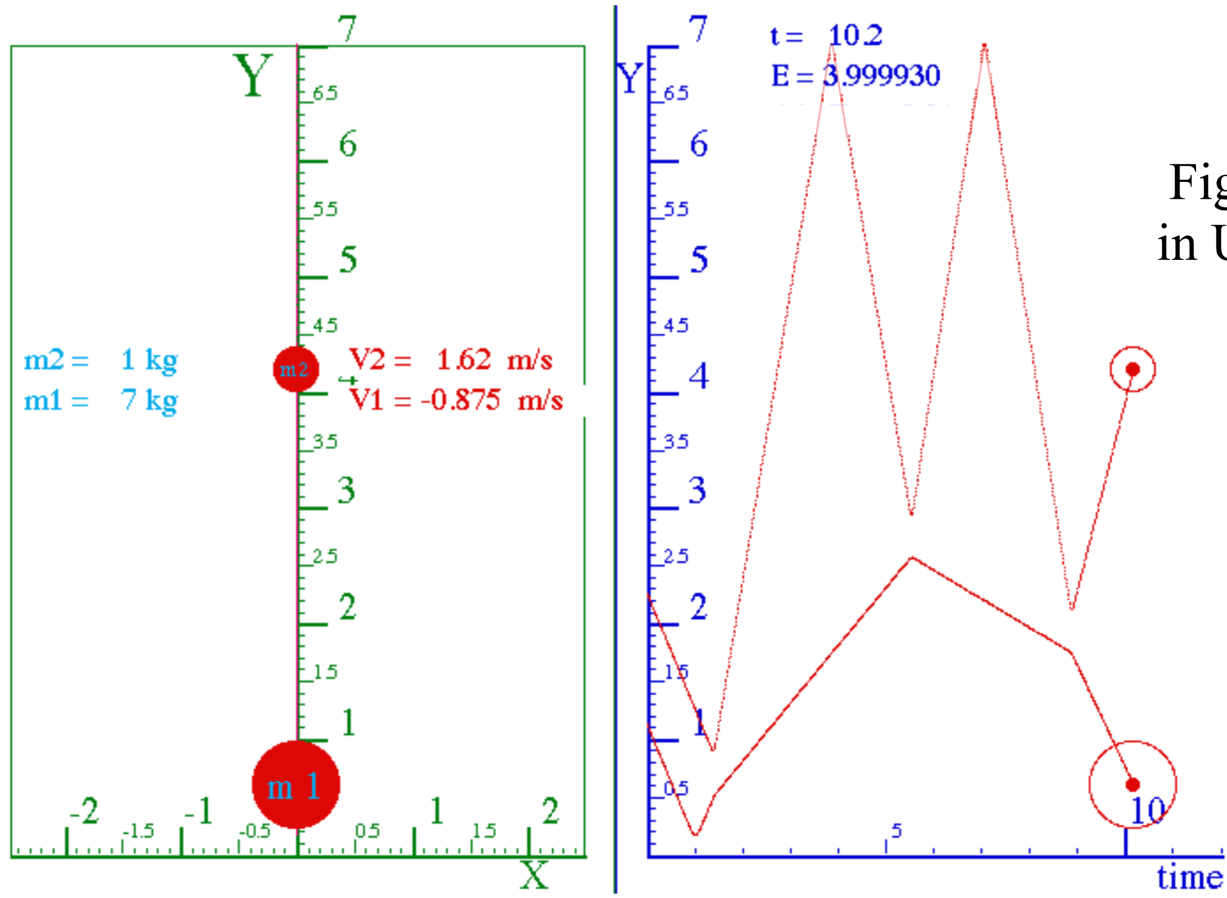


Fig. 4.8
in Unit 1

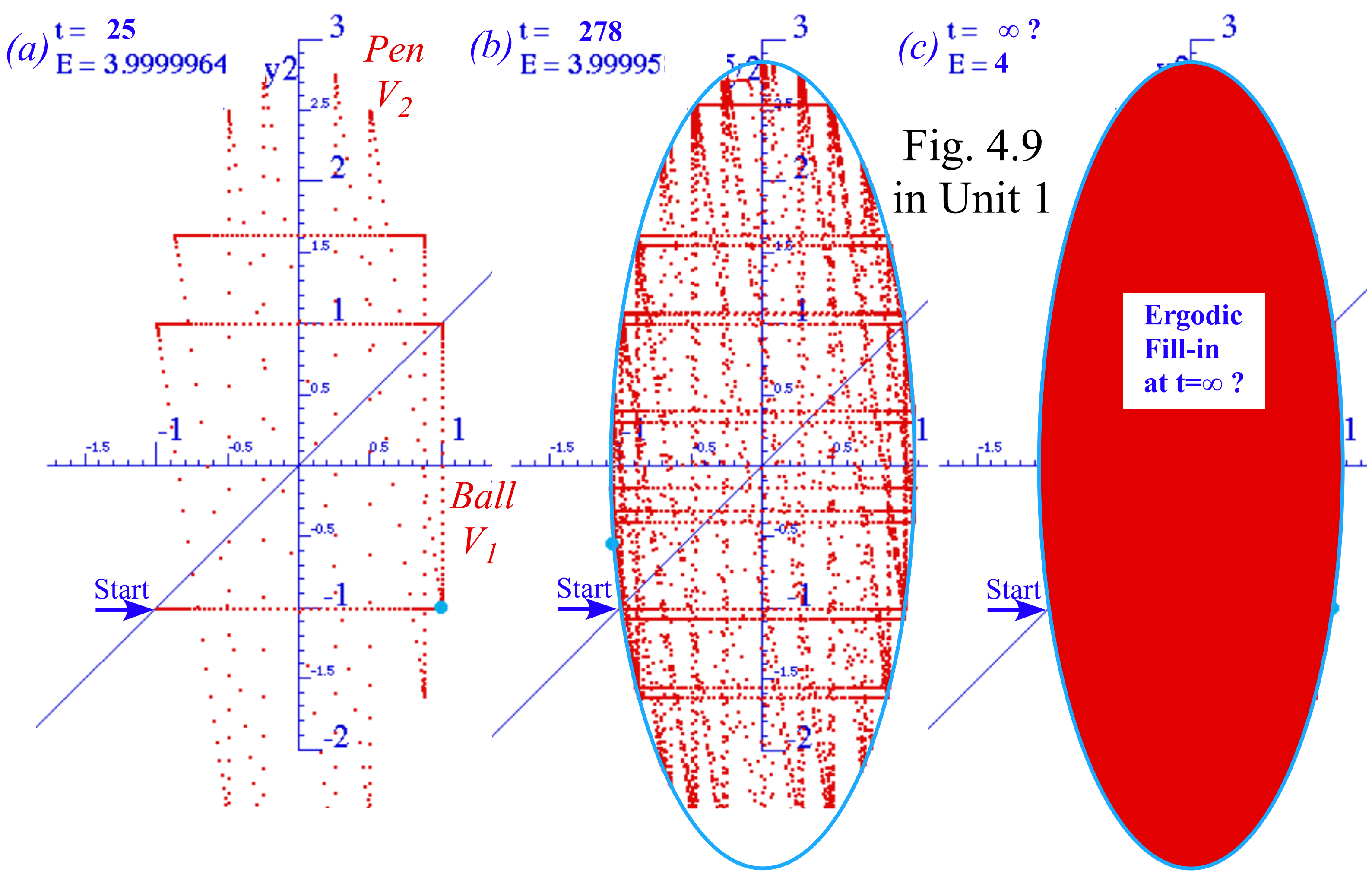
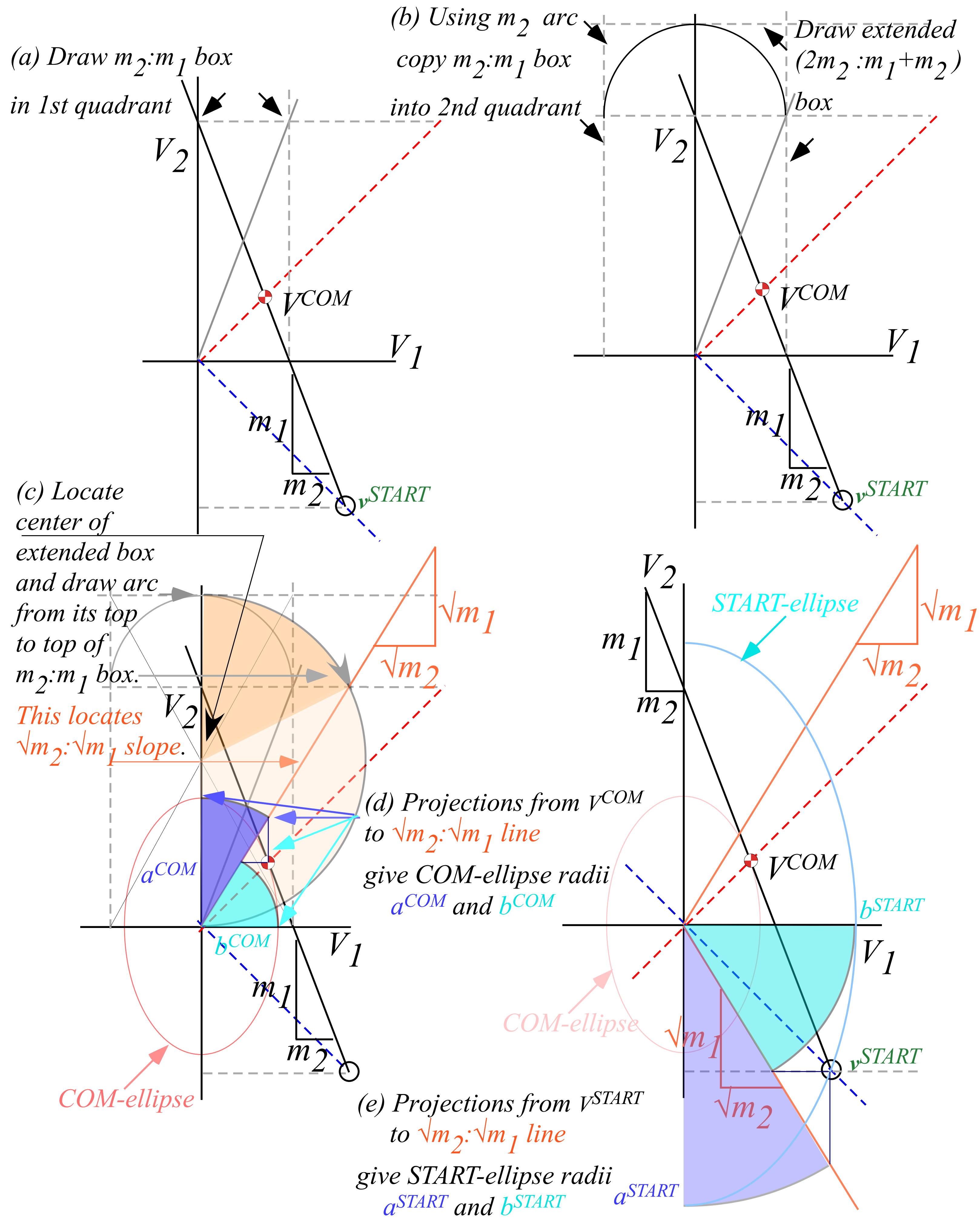


Fig. 4.9
in Unit 1



Unit 1
Fig. 8.4a-d

This is a construction of the energy ellipse in a Largangian (v_1,v_2) plot given the initial (v_1,v_2).

The Estrangian (V_1,V_2) plot makes the (v_1,v_2) plot and this construction obsolete.

(Easier to just draw circle through initial (V_1,V_2).)

Still, if you know a simpler construction, we'd like to hear about it!

AMOP reference links *(Updated list given on **2nd** and 3rd pages of each class presentation)*

Web Resources - front page
UAF Physics UTube channel

Quantum Theory for the Computer Age
Principles of Symmetry, Dynamics, and Spectroscopy
Classical Mechanics with a Bang!
Modern Physics and its Classical Foundations

2014 AMOP
2017 Group Theory for QM
2018 AMOP

Representaions Of Multidimensional Symmetries In Networks - harter-jmp-1973

Alternative Basis for the Theory of Complex Spectra

- Alternative_Basis_for_the_Theory_of_Complex_Spectra_I_-_harter-pra-1973
- Alternative_Basis_for_the_Theory_of_Complex_Spectra_II_-_harter-patterson-pra-1976
- Alternative_Basis_for_the_Theory_of_Complex_Spectra_III_-_patterson-harter-pra-1977

Frame Transformation Relations And Multipole Transitions In Symmetric Polyatomic Molecules - RMP-1978
Asymptotic eigensolutions of fourth and sixth rank octahedral tensor operators - Harter-Patterson-JMP-1979
Rotational energy surfaces and high- J eigenvalue structure of polyatomic molecules - Harter - Patterson - 1984
Galloping waves and their relativistic properties - ajp-1985-Harter
Rovibrational Spectral Fine Structure Of Icosahedral Molecules - Cpl 1986 (Alt Scan)

Theory of hyperfine and superfine levels in symmetric polyatomic molecules.

I) Trigonal and tetrahedral molecules: Elementary spin-1/2 cases in vibronic ground states - PRA-1979-Harter-Patterson (Alt scan)
II) Elementary cases in octahedral hexafluoride molecules - Harter-PRA-1981 (Alt scan)

Rotation–vibration spectra of icosahedral molecules.

I) Icosahedral symmetry analysis and fine structure - harter-weeks-jcp-1989 (Alt scan)
II) Icosahedral symmetry, vibrational eigenfrequencies, and normal modes of buckminsterfullerene - weeks-harter-jcp-1989 (Alt scan)
III) Half-integral angular momentum - harter-reimer-jcp-1991

Rotation-vibration scalar coupling zeta coefficients and spectroscopic band shapes of buckminsterfullerene - Weeks-Harter-CPL-1991 (Alt scan)
Nuclear spin weights and gas phase spectral structure of 12C60 and 13C60 buckminsterfullerene -Harter-Reimer-Cpl-1992 - (Alt1, Alt2 Erratum)
Gas Phase Level Structure of C60 Buckyball and Derivatives Exhibiting Broken Icosahedral Symmetry - reimer-diss-1996
Fullerene symmetry reduction and rotational level fine structure/ the Buckyball isotopomer 12C 13C59 - jcp-Reimer-Harter-1997 (HiRez)
Wave Node Dynamics and Revival Symmetry in Quantum Rotors - harter - jms - 2001
Molecular Symmetry and Dynamics - Ch32-Springer Handbooks of Atomic, Molecular, and Optical Physics - Harter-2006

Resonance and Revivals

I) QUANTUM ROTOR AND INFINITE-WELL DYNAMICS - ISMSLi2012 (Talk) OSU knowledge Bank
II) Comparing Half-integer Spin and Integer Spin - Alva-ISMS-Ohio2013-R777 (Talks)
III) Quantum Resonant Beats and Revivals in the Morse Oscillators and Rotors - (2013-Li-Diss)

Resonance and Revivals in Quantum Rotors - Comparing Half-integer Spin and Integer Spin - Alva-ISMS-Ohio2013-R777 (Talk)
Molecular Eigensolution Symmetry Analysis and Fine Structure - IJMS-harter-mitchell-2013
Quantum Revivals of Morse Oscillators and Farey-Ford Geometry - Li-Harter-cpl-2013
QTCA Unit 10 Ch 30 - 2013
AMOP Ch 0 Space-Time Symmetry - 2019

**In development - a web based A.M.O.P. oriented reference page, with thumbnail/previews, greater control over the information display, and eventually full on* Apache-SOLR *Index and search for* nuanced, whole-site content/metadata level *searching.*

AMOP reference links *(Updated list given on 2nd and **3rd** pages of each class presentation)*

(Int.J.Mol.Sci, 14, 714(2013) p.755-774 , QTCA Unit 7 Ch. 23-26), (PSDS - Ch. 5, 7)

Int.J.Mol.Sci, 14, 714(2013), QTCA Unit 8 Ch. 23-25, QTCA Unit 9 Ch. 26, PSDS Ch. 5, PSDS Ch. 7

Intro spin ½ coupling
Unit 8 Ch. 24 p3

H atom hyperfine-B-level crossing
Unit 8 Ch. 24 p15

Hyperf. theory Ch. 24 p48.

Hyperf. theory Ch. 24 p48.
Deeper theory ends p53

Intro 2p3p coupling
Unit 8 Ch. 24 p17.

Intro LS-jj coupling
Unit 8 Ch. 24 p22.

CG coupling derived (start)
Unit 8 Ch. 24 p39.

CG coupling derived (formula)
Unit 8 Ch. 24 p44.

Lande' g-factor
Unit 8 Ch. 24 p26.

Irrep Tensor building
Unit 8 Ch. 25 p5.

Irrep Tensor Tables
Unit 8 Ch. 25 p12.

Wigner-Eckart tensor Theorem.
Unit 8 Ch. 25 p17.

Tensors Applied to d,f-levels.
Unit 8 Ch. 25 p21.

Tensors Applied to high J levels.
Unit 8 Ch. 25 p63.

Intro 3-particle coupling.
Unit 8 Ch. 25 p28.

Intro 3,4-particle Young Tableaus
GrpThLect29 p42.

Young Tableau Magic Formulae
GrpThLect29 p46-48.

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AMOP reference links *(Updated list given on 2nd and **3rd** and **4th** pages of each class presentation)*

Predrag Cvitanovic's: Birdtrack Notation, Calculations, and Simplification

Chaos_Classical_and_Quantum_-_2018-Cvitanovic-ChaosBook
Group Theory - PUP_Lucy_Day_-_Diagrammatic_notation_-_Ch4
Simplification_Rules_for_Birdtrack_Operators_-_Alcock-Zeilinger-Weigert-zeilinger-jmp-2017
Group Theory - Birdtracks_Lies_and_Exceptional_Groups_-_Cvitanovic-2011
Simplification_rules_for_birdtrack_operators-_jmp-alcock-zeilinger-2017
Birdtracks for SU(N) - 2017-Keppeler

Frank Rioux's: UMA method of vibrational induction

Quantum_Mechanics_Group_Theory_and_C60_-_Frank_Rioux_-_Department_of_Chemistry_Saint_Johns_U
Symmetry_Analysis_for_H20-_H20GrpTheory-_Rioux
Quantum_Mechanics-Group_Theory_and_C60_-_JChemEd-Rioux-1994
Group_Theory_Problems-_Rioux-_SymmetryProblemsX
Comment_on_the_Vibrational_Analysis_for_C60_and_Other_Fullerenes_Rioux-RSP

Supplemental AMOP Techniques & Experiment

Many Correlation Tables are Molien Sequences - Klee (Draft 2016)
High-resolution_spectroscopy_and_global_analysis_of_CF4_rovibrational_bands_to_model_its_atmospheric_absorption-_carlos-Boudon-jqsrt-2017
Symmetry and Chirality - Continuous_Measures_-_Avnir

*

Special Topics & Colloquial References

r-process_nucleosynthesis_from_matter_ejected_in_binary_neutron_star_mergers-PhysRevD-Bovard-2017

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