Lecture 30
Wed. 12.11.2019

Relawavity : a novel introduction to relativistic mechanics III.

(CMwBang! Unit 8 , AMOP Ch.0,)

Review: Relawavity ρ functions and plots vs. ρ

Derive relawavity parameters and Minkowski coordinates for υ_R=2.5THz and υ_L=0.5THz

Derivation of relativistic quantum mechanics

- What's the matter with mass? Shining some light on the Elephant in the room
- Relativistic action and Lagrangian-Hamiltonian relations
  - Poincare' and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae

- Feynman diagram geometry
  - Compton recoil related to rocket velocity formula
  - Comparing 2nd-quantization "photon" number N and 1st-quantization wavenumber κ

Relawavity in accelerated frames

- Laser up-tuning by Alice and down-tuning by Carla makes *g*-acceleration grid
  - Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid
  - Animation of mechanics and metrology of constant-g grid

This Lecture's Reference Link Listing

Web Resources - front page
UAF Physics UTube channel

Quantum Theory for the Computer Age
Principles of Symmetry, Dynamics, and Spectroscopy
Classical Mechanics with a Bang!
Modern Physics and its Classical Foundations

2017 Group Theory for QM
2018 Adv CM
2018 AMOP
2019 Advanced Mechanics

Lecture #28-30

In reverse order

AMOP Chapter 0: Space-Time Symmetry
AMOP Detailed Development of RelaWavity
2018 Rochester Talk (Auxilary Slides)
Special Relativity and Quantum Theory by Ruler and Compass - Earlier, expanded draft
Ruler & Compass - Relawavity Exercise

Relativity Visualized - Epstein-ip-1985 for sale here @www.allbookstores.com

GuideIt Web Simulations: σ = 30°, σ = 60°

Pirelli Site - ***A Colorful Road to Relativity Using Occam's Razors and Evenson's Lasers***
- World of Clocks - Animations - 12 hr. clock, 24 hr. clock
- Phasor vs Thales(Pirelli Challenge) - phasors_2_3_zoom_anim.html

RelativIt Web Simulations
- Relativistic Events in Main Lighthouse's Space-Time Frame - *scenario=22*
- Relativistic Events in Ship's Space-Time Frame - *scenario=24*
- Epstein plot - *scenario=600*

BohrIt Web Simulations
- 2 CW ct vs x Plot (ck = ±2) - *scenario=-130022*
- *Multi-Panel 2 PW ct vs x Plot* - *scenario=30022*
- *1 CW ct vs x Plot (ck = -1)* - *scenario=-30001*
- *1 CW ct vs x Plot (ck = +4)* - *scenario=30004*
- 2 CW Minkowski Plot (ck = -1, +4) - *scenario=-30104*

CMwBang Text 2012 Unit 6 page=5
BounceIt Web App/Scenarios: 5002, 5003
CoulIt Web App/Scenarios:
TwoParticleCollision_LToR, TwoParticleCollision_LToR_CM, TwoParticleOrbit_Coulomb, TwoParticleOrbit_Coulomb_CM, TwoParticleOrbit_Hooke, TwoParticleOrbit_Hooke_CM

RelaWavity Web Simulations
- 2019 RelaWavity Portal Page
- Relations between Hypergeometric and Hypergeometric functions - *plotType=0,9&...*
- RelaWavity Web Simulation {Physical Terms - All Terms} - *plotType=4,8*

Keyboard of the Gods
- Per-Time vs Per-Space - *plotType=7,1*
- Dual Plot #1 - *plotType=7,2&bcStepInd=1*
- Dual Plot #2 - *plotType=7,2&bcStepInd=2*
- Dual Plot #3 - *plotType=7,2&bcStepInd=3*
- Dual Plot #7 - *plotType=7,2&bcStepInd=7*

- 16 Relativity Dimensions - *plotType=8,4*
- Relativistic Terms (Expanded Table) - *plotType=8,5*
- Minkowski graph (Multi-plot) - *plotType=8,8*
- Detailed Thales Geometry - *plotType=3,6*
- PerSpace - PerTime {All} - *plotType=3,6&minkGridPosCells=0*
- Expanded Relativistic Relations - *plotType=8,7*
- Wavefronts in Space-Space - *plotType=6,1*
- Spectral Ellipse {PerSpace-PerSpace} {β = u/c = 1/3} - *plotType=6,3&...*
- Spectral Ellipse {β = u/c = 3/4} - *plotType=6,3&...*

Select, exciting, and/or related Research
- Singular Motion of Asymetric Rotators AJP 44, 11 p1080 Harter-Kim-1976
- Molecular Eigensolution Symmetry Analysis and Fine Structure - Harter-IJMS-2013
- Lenz Vector and Orbital Analog Computers - AJP 44 p348 1976
- Some Geometric Aspects of Classical Coulomb Scattering AJP 40 4 p1852 1972
- How Molecules do Self-NMR - Harter-Mitchell-Columbus-2009
- Classical Mechanics with a Bang! - Asymmetric Top Demo
- Allbookstores.com - Compare for Heller's SemiClassical Way - 0691163731
- "My Bomerang Won't Come Back" (YouTube: Playlist)
- Rotating Solid Bodies in Microgravity (YouTube)
- Dancing T-handle in zero-g (YouTube)

In development, but close to role out.
More Advanced QM and classical references will *soon* be available through our: ***References Page***
Would be great to have our Apache SOLR *Search & Index system up for a bigger* Bang!*)*

Continued for 4 more pages↘

This Lecture's Reference Link Listing

Web Resources - front page | Quantum Theory for the Computer Age | 2017 Group Theory for QM
UAF Physics UTube channel | Principles of Symmetry, Dynamics, and Spectroscopy | 2018 Adv CM
Classical Mechanics with a Bang! | 2018 AMOP
Modern Physics and its Classical Foundations | **2019 Advanced Mechanics**

Lecture #22-27 *In reverse order*

CoulIt Web App Simulations: p19, p32, p72, p73, p92, R=-0.375, R=+0.5, Rutherford
OscillatorPE Web App: IHO Scenario 2, Coulomb Scenario 3
RelaWavity Web App/Simulator/Calculator: Elliptical - IHO orbits

JerkIt Web App: 2-, 2+, Amp50Omega147-, Amp50Omega296, Amp50Omega602, Gap(1)
MolVibes Web App: C3vN3
WaveIt Web App:
Dim = 3 w/Wave Components;
Static Char Table: 6, 12, 12(b), 16, 36, 256
Quantum Carpet with N=20: Gaussian, Boxcar
Quantum Revivals of Morse Oscillators and Farey-Ford Geometry - Li-Harter-CPL-2015
QTCA Unit_5 Ch14 2013
Lester. R. Ford, Am. Math. Monthly 45,586(1938)
John Farey, Phil. Mag.(1816) Wolfram
Harter, J. Mol. Spec. 210, 166-182 (2001)
Harter, Li IMSS (2013)
Li, Harter, Chem.Phys.Letters (2015)
Advanced Atomic and Molecular Optical Physics 2018 Class #9, pages: 5, 61
BoxIt Web Simulations
Pure A-Type A=4.9, B=0 ,C=0, & D=4.0
Pure B-Type: A=4.0, B=-0.2, C=0, & D=4.0
Pure C-Type A,D=4.055, B=0, C=0.1
Mixed AB-Type w/Cosine
Mixed AB Type A=4.0, BU2=0.866…, CU2=0, & D=1.0 w/Stokes & Freq rats
Mixed AB Type A=5.086 B=-0.27 C=0 D=2.024 w/Stokes plot
Mixed ABC Type A=4.833 B=0.2403 C=0.4162 D=4.277 w/Stokes plot
Recent mixed ABC Type A=0.325 B=0.375 C=0.825 D=0.05 w/Stokes plot

Select, exciting, and/or related Research
This Indestructible NASA Camera Revealed Hidden Patterns on Jupiter - seeker-yt-2019
What did NASA's New Horizons discover around Pluto? - Astrum-yt-2018
Synthetic_Chiral_Light_for_Efficient_Control_of_Chiral_Light–Matter_Interaction_-_Ayuso-np-2019

Classical Mechanics with a Bang! 2018
Lectures 8, 9, 23 page 93
Text Unit 6, page=27
ColorU2 for the Web - in development
Group Theory for Quantum Mechanics - 2017 Lectures: 6, 7, 8, and the combined 9-10
Quantum Theory for the Computer Age Unit 3 Ch.7-10, page=90
Spectral Decomposition with Repeated Eigenvalues - 2017 GTQM - Lecture 5

Web based 3D & XR (x∈ {A,M,V}, R=Reality) https://www.babylonjs.com/
Web based 3D graphics WebGL API (Graphics Layer modeled after OpenGL)

Recent In-House draft Articles:
Springer handbook on Molecular Symmetry and Dynamics - Ch_32 - Molecular Symmetry
AMOP Ch 0 Space-Time Symmetry - 2019
Seminar at Rochester Institute of Optics, Auxiliary slides, June 19, 2018

Quantum_Computing - (Current) State of the Art - Reimer-www-2019
Geometric Algebra- A Guided Tour through Space and Time - Reimer-www-2019
Wildlife Monitoring Identification and Behavioral Study - Section 1 - Reimer-www-2019
Wildlife Monitoring Identification and Behavioral Study - Section 2 - Reimer-www-2019

Quantum Computing (*QC*) and Geometric Algebra (*GA*) references:
Quantum_Supremacy_Using_a_Programmable_Superconducting_Processor_-_Arute-n-2019
Quantum Computing for Computer Scientists - Helwer-mr-yt-2018, Slides
Quantum Computing and Workforce, Curriculum, and App Devel - Roetteler-MS-2019

Quantum_Computing - (Current) State of the Art - Reimer-www-2019
Excerpts (Page 44-47 in *Preliminary Draft*) for a GA take on the Complex Numbers
Geometric Algebra- A Guided Tour through Space and Time - Reimer-www-2019
GA & QC references (Page 11-16 in Preliminary Draft)

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Web Resources - front page | Quantum Theory for the Computer Age | 2017 Group Theory for QM
UAF Physics UTube channel | Principles of Symmetry, Dynamics, and Spectroscopy | 2018 Adv CM
Classical Mechanics with a Bang! | 2018 AMOP
Modern Physics and its Classical Foundations | **2019 Advanced Mechanics**

Lectures #12 through #21

In reverse order

Wiki on Pafnuty Chebyshev
Nobelprize.org
2005 Physics Award

BoxIt Web Simulations:
A-Type w/Cosine, A-Type w/Freq ratios,
AB-Type w/Cosine, AB-Type 2:1 Freq ratio

OscillIt Web Simulations:
Default/Generic, Weakly Damped #18,
Forced : Way below resonance,On resonance
Way above resonance,Underdamped
Complex Response Plot

CoulIt Web Simulations:
Stark-Coulomb : Bound-state motion in parabolic coordinates
Molecular Ion : Bound-state motion in hyperbolic coordinates
Synchrotron Motion, Synchrotron Motion #2
Mechanical Analog to EM Motion (YouTube video)
iBall demo - Quasi-periodicity (YouTube video)

Trebuchet Web Simulations:
Default/Generic URL, Montezuma's Revenge, Seige of Kenilworth,
"Flinger",
Position Space (Course), Position Space (Fine)
Wacky Waving Solid Metal Arm Flailing Chaos Pendulum - Scooba_Steeve-yt-2015
Triple Double-Pendulum - Cohen-yt-2008
Punkin Chunkin - TheArmchairCritic-2011
Jersey Team Claims Title in Punkin Chunkin - sussexcountyonline-1999
Shooting range for medieval siege weapons. Anybody knows? - twcenter.net/forums
The Trebuchet - Chevedden-SciAm-1995
NOVA Builds a Trebuchet

Recent Articles of Interest:
A_Semi-Classical_Approach_to_the_Calculation_of_Highly_Excited_Rotational_Energies for … Asymmetric-Top_Molecules_-_Schmiedt-pccp-2017
Tunable and broadband coherent perfect absorption by ultrathin blk phos metasurfaces - Guo-josab-2019
Vortex Detection in Vector Fields Using Geometric Algebra - Pollock-aaca-2013.pdf

Pirelli Relativity Challenge (Introduction level) - *Visualizing Waves:*
Using Earth as a clock,
Tesla's AC Phasors ,
Phasors using complex numbers.
CM wBang Unit 1 - Chapter 10, pdf_page=135
Calculus of exponentials, logarithms, and complex fields,
RelaWavity Web Simulation - Unit Circle and Hyperbola (Mixed labeling)
Smith Chart, Invented by Phillip H. Smith (1905-1987)

Select, exciting, and related Research
Clifford_Algebra_And_The_Projective_Model_Of_Homogeneous_Metric_Spaces - Foundations - Sokolov-x-2013
Geometric Algebra 3 - Complex Numbers - MacDonald-yt-2015
Biquaternion -Complexified Quaternion- Roots of -1 - Sangwine-x-2015
An_Introduction_to_Clifford_Algebras_and_Spinors_-_Vaz-Rocha-op-2016
Unified View on Complex Numbers and Quaternions- Bongardt-wcmms-2015
Complex Functions and the Cauchy-Riemann Equations - complex2 - Friedman-columbia-2019
An_sp-hybridized_Molecular_Carbon_Allotrope-_cyclo-18-carbon_-_Kaiser-s-2019
An_Atomic-Scale_View_of_Cyclocarbon_Synthesis_-_Maier-s-2019
Discovery_Of_Topological_Weyl_Fermion_Lines_And_Drumhead_Surface_States_in_a_ Room_Temperature_Magnet_-_Belopolski-s-2019
"Weyl"ing_away_Time-reversal_Symmetry_-_Neto-s-2019
Non-Abelian_Band_Topology_in_Noninteracting_Metals_-_Wu-s-2019
What_Industry_Can_Teach_Academia_-_Mao-s-2019
RoVib-_quantum_state_resolution_of_the_C60_fullerene_-_Changala-Ye-s-2019 (Alt)
A_Degenerate_Fermi_Gas_of_Polar_molecules_-_DeMarco-s-2019

An assist from ***Physics Girl*** (YouTube Channel):
How to Make VORTEX RINGS in a Pool
Crazy pool vortex - pg-yt-2014
Fun with Vortex Rings in the Pool - pg-yt-2014

Running Reference Link Listing

Lectures #11 through #7

In reverse order

Eric J Heller Gallery:
Main portal, Consonance and Dissonance II, Bessel 21, Chladni

The Semiclassical Way to Molecular Spectroscopy - Heller-acs-1981
Quantum_dynamical_tunneling_in_bound_states_-_Davis-Heller-jcp-1981

Pendulum Web Simulation
Cycloidulum Web Simulation

Links to previous lecture: *Page=74, Page=75, Page=79*
Pendulum Web Sim
Cycloidulum Web Sim

***JerkIt* Web Simulations:** *Basic/Generic: Inverted, FVPlot*

CMwithBang Lecture 8, page=20
WWW.sciencenewsforstudents.org: Cassini - Saturnian polar vortex

"RelaWavity" Web Simulations:
2-CW laser wave, Lagrangian vs Hamiltonian,
Physical Terms Lagrangian L(u) vs Hamiltonian H(p)
CoulIt Web Simulation of the Volcanoes of Io
BohrIt Multi-Panel Plot:
Relativistically shifted Time-Space plots of 2 CW light waves

BoxIt Web Simulations:
Generic/Default
Most Basic A-Type
Basic A-Type w/reference lines
Basic A-Type A-Type with Potential energy
A-Type with Potential energy and Stokes Plot
A-Type w/3 time rates of change
A-Type w/3 time rates of change with Stokes Plot
B-Type (A=1.0, B=-0.05, C=0.0, D=1.0)

RelaWavity Web Elliptical Motion Simulations:
Orbits with b/a=0.125
Orbits with b/a=0.5
Orbits with b/a=0.7
Exegesis with b/a=0.125
Exegesis with b/a=0.5
Exegesis with b/a=0.7
Contact Ellipsometry

***CoulIt* Web Simulations:**
Basic/Generic
Exploding Starlet
Volcanoes of Io (Color Quantized)

***JerkIt* Web Simulations:**
Basic/Generic
Catcher in the Eye - IHO with Linear Hooke perturbation - Force-potential-Velocity Plot

***OscillatorPE* Web Simulation:**
Coulomb-Newton-Inverse_Square,
Hooke-Isotropic Harmonic,
Pendulum-Circular_Constraint

AMOP Ch 0 Space-Time Symmetry - 2019
Seminar at Rochester Institute of Optics, Aux. slides-2018

NASA Astronomy Picture of the Day -
Io: The Prometheus Plume *(Just Image)*
NASA Galileo - *Io's Alien Volcanoes*
New Horizons - *Volcanic Eruption Plume on Jupiter's moon IO*
NASA Galileo - A Hawaiian-Style Volcano on Io
Pirelli Site: Phasors animimation
CMwithBang Lecture #6, page=70 (9.10.18)

Select, exciting, and related Research & Articles of Interest:
Burning a hole in reality—design for a new laser may be powerful enough to pierce space-time - Sumner-KOS-2019
Trampoline mirror may push laser pulse through fabric of the Universe - Lee-ArsTechnica-2019
Achieving_Extreme_Light_Intensities_using_Optically_Curved_Relativistic_Plasma_Mirrors_-_Vincenti-prl-2019
A_Soft_Matter_Computer_for_Soft_Robots_-_Garrad-sr-2019
Correlated_Insulator_Behaviour_at_Half-Filling_in_Magic-Angle_Graphene_Superlattices_-_cao-n-2018
*Sorting_ultracold_atoms_in_a_three-dimensional_optical_lattice_in_a_*
realization_of_Maxwell's_Demon - Kumar-n-2018
Synthetic_three-dimensional_atomic_structures_assembled_atom_by_atom - Barredo-n-2018

Older ones:
Wave-particle_duality_of_C60_molecules - Arndt-ltn-1999
Optical_Vortex_Knots_-_One_Photon__At_A_Time - Tempone-Wiltshire-Sr-2018
Baryon_Deceleration_by_Strong_Chromofields_in_Ultrarelativistic_,
Nuclear_Collisions - Mishustin-PhysRevC-2007, APS Link & Abstract
Hadronic Molecules - Guo-x-2017
Hidden-charm_pentaquark_and_tetraquark_states - Chen-pr-2016

Running Reference Link Listing

Lectures #6 through #1

In reverse order

RelaWavity Web Simulation: Contact Ellipsometry
BoxIt Web Simulation: Elliptical Motion (A-Type)
CMwBang Course: Site Title Page
Pirelli Relativity Challenge: Describing Wave Motion With Complex Phasors
UAF Physics UTube channel

Velocity Amplification in Collision Experiments Involving Superballs - Harter, 1971
MIT OpenCourseWare: High School/Physics/Impulse and Momentum
Hubble Site: Supernova - SN 1987A

BounceItIt Web Animation - Scenarios:
49:1 y vs t, 49:1 V2 vs V1, 1:500:1 - 1D Gas Model w/ faux restorative force (Cool),
1:500:1 - 1D Gas (Warm), 1:500:1 - 1D Gas Model (Cool, Zoomed in),
Farey Sequence - Wolfram
Fractions - Ford-AMM-1938

Monstermash BounceItIt Animations:
1000:1 - V2 vs V1, 1000:1 with t vs x - Minkowski Plot
Quantum Revivals of Morse Oscillators and Farey-Ford Geometry - Li-Harter-2013
Quantum_Revivals_of_Morse_Oscillators_and_Farey-Ford_Geometry - Li-Harter-cpl-2015
Quant. Revivals of Morse Oscillators and Farey-Ford Geom. - Harter-Li-CPL-2015 (Publ.)
Velocity_Amplification_in_Collision_Experiments_Involving_Superballs-Harter-1971

WaveIt Web Animation - Scenarios:
Quantum_Carpet, Quantum_Carpet_wMBars,
Quantum_Carpet_BCar, Quantum_Carpet_BCar_wMBars
Wave Node Dynamics and Revival Symmetry in Quantum Rotors - Harter-JMS-2001
Wave Node Dynamics and Revival Symmetry in Quantum Rotors - Harter-jms-2001 (Publ.)

BounceIt Web Animation - Scenarios:
Generic Scenario: 2-Balls dropped no Gravity (7:1) - V vs V Plot (Power=4)
1-Ball dropped w/Gravity=0.5 w/Potential Plot: Power=1, Power=4
7:1 - V vs V Plot: Power=1
3-Ball Stack (10:3:1) w/Newton plot (y vs t) - Power=4
3-Ball Stack (10:3:1) w/Newton plot (y vs t) - Power=1
3-Ball Stack (10:3:1) w/Newton plot (y vs t) - Power=1 w/Gaps
4-Ball Stack (27:9:3:1) w/Newton plot (y vs t) - Power=4
4-Newton's Balls (1:1:1:1) w/Newtonian plot (y vs t) - Power=4 w/Gaps
6-Ball Totally Inelastic (1:1:1:1:1:1) w/Gaps: Newtonian plot (t vs x), V6 vs V5 plot
5-Ball Totally Inelastic Pile-up w/ 5-Stationary-Balls - Minkowski plot (t vs x1) w/Gaps
1-Ball Totally Inelastic Pile-up w/ 5-Stationary-Balls - Vx2 vs Vx1 plot w/Gaps

BounceIt Dual plots

$m_1:m_2 = 3:1$
v_2 vs v_1 and V_2 vs V_1, $(v_1, v_2)=(1, 0.1)$, $(v_1, v_2)=(1, 0)$
y_2 vs y_1 plots: $(v_1, v_2)=(1, 0.1)$, $(v_1, v_2)=(1, 0)$, $(v_1, v_2)=(1, -1)$
Estrangian plot V_2 vs V_1: $(v_1, v_2)=(0, 1)$, $(v_1, v_2)=(1, -1)$

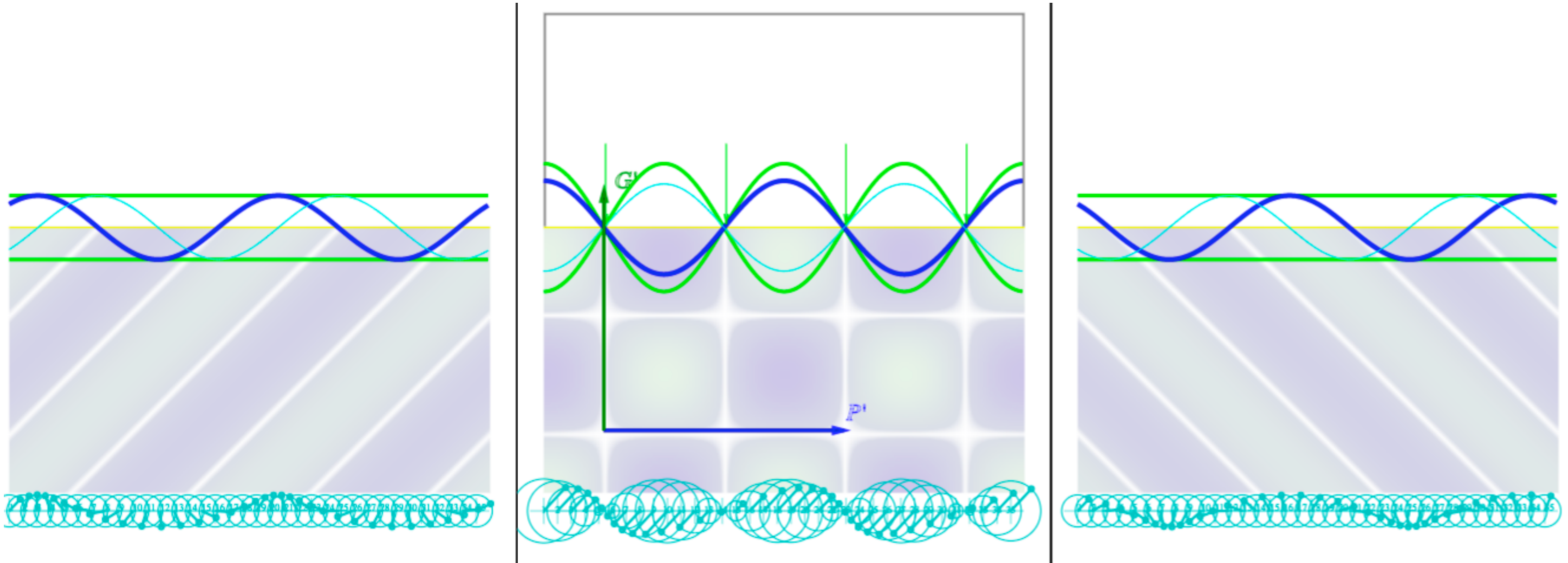
$m_1:m_2 = 4:1$
v2 vs v1, y2 vs y1

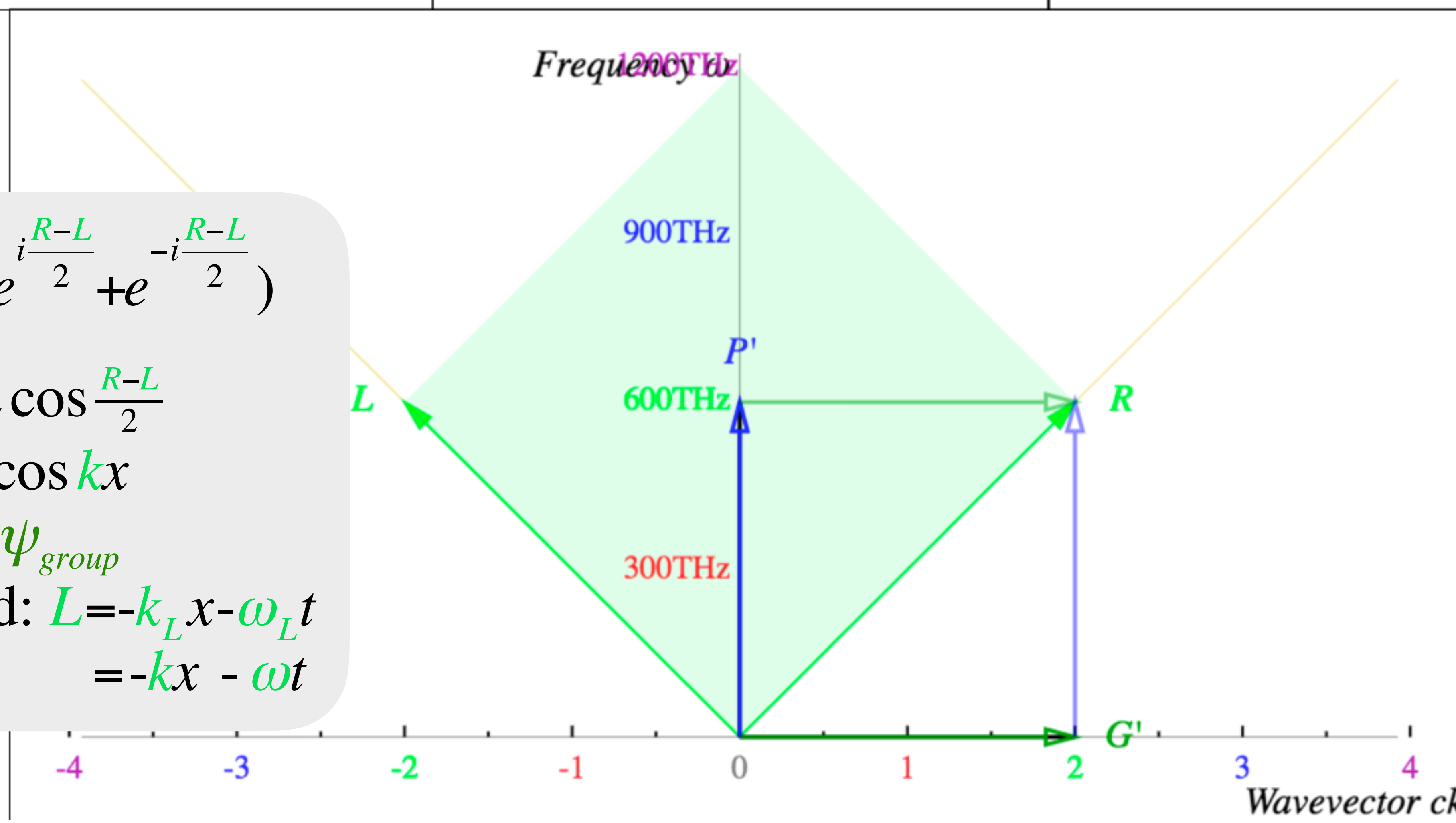
$m_1:m_2 = 100:1$, $(v_1, v_2)=(1, 0)$: V2 vs V1 Estrangian plot, y2 vs y1 plot

With g=0 and 70:10 mass ratio
With non zero g, velocity dependent damping and mass ratio of 70:35
M_1=49, M_2=1 with Newtonian time plot
M_1=49, M_2=1 with V_2 vs V_1 plot
Example with friction
Low force constant with drag displaying a Pass-thru, Fall-Thru, Bounce-Off
m1:m2= 3:1 and (v1, v2) = (1, 0) Comparison with Estrangian

AJP article on superball dynamics
AAPT Summer Reading List
Scitation.org - AIP publications
HarterSoft Youtube Channel

X2 paper: Velocity Amplification in Collision Experiments Involving Superballs - Harter, et. al. 1971 (pdf)
Car Collision Web Simulator: https://modphys.hosted.uark.edu/markup/CMMotionWeb.html
Superball Collision Web Simulator: https://modphys.hosted.uark.edu/markup/BounceItWeb.html; with Scenarios: 1007
BounceIt web simulation with g=0 and 70:10 mass ratio
With non zero g, velocity dependent damping and mass ratio of 70:35
Elastic Collision Dual Panel Space vs Space: Space vs Time (Newton), Time vs. Space(Minkowski)
Inelastic Collision Dual Panel Space vs Space: Space vs Time (Newton), Time vs. Space(Minkowski)
Matrix Collision Simulator: M_1=49, M_2=1 V_2 vs V_1 plot <<Under Construction>>





$$e^{iR}+e^{iL} = e^{i\frac{R+L}{2}}\left(e^{i\frac{R-L}{2}}+e^{-i\frac{R-L}{2}}\right)$$

$$= e^{i\frac{R+L}{2}}\, 2\cos\frac{R-L}{2}$$

$$= 2e^{-i\omega t}\cos kx$$

$$= \psi_{phase}\psi_{group}$$

$R = k_R x - \omega_R t$ and: $L = -k_L x - \omega_L t$

$= kx - \omega t$ $= -kx - \omega t$

Fig. 9 in text

BohrIt Web Simulation: ±600THz

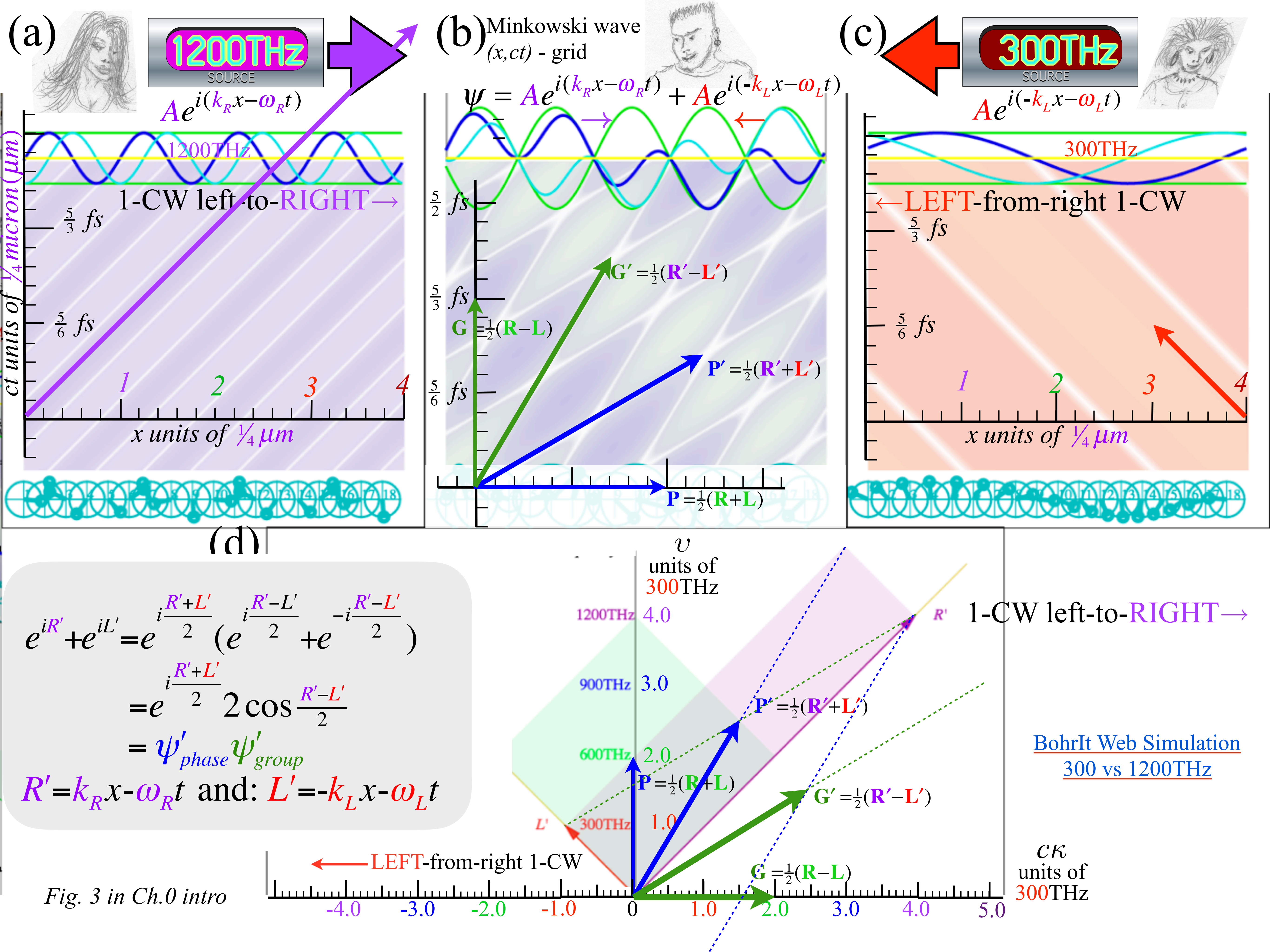
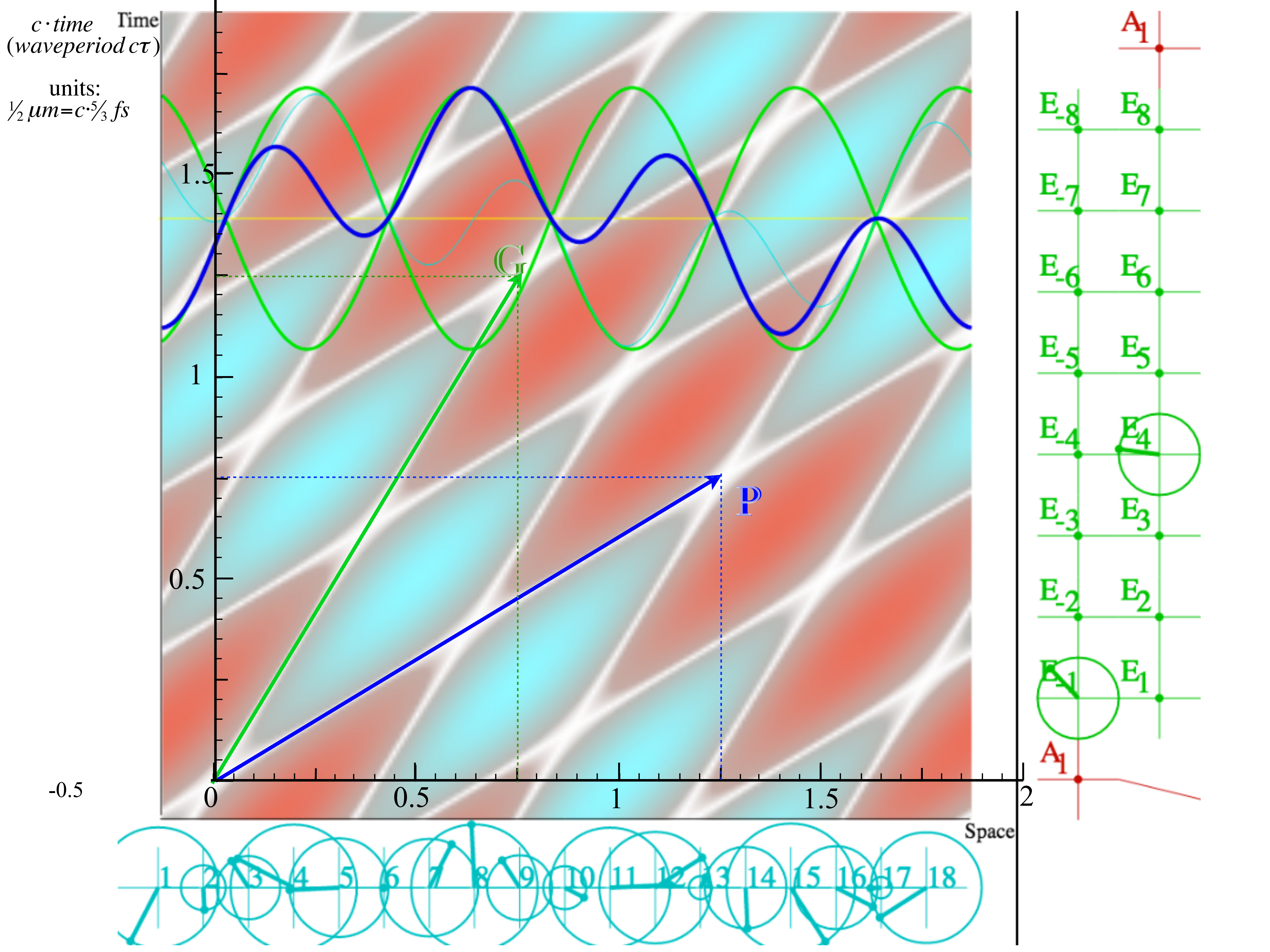


Fig. 3 in Ch.0 intro



c · time
(waveperiod cτ)
units:
½ μm=c·5⁄3 fs
Time
Space
G
P
1.5
1
0.5
-0.5
0
0.5
1
1.5
2
A1
E-8
E8
E-7
E7
E-6
E6
E-5
E5
E-4
E4
E-3
E3
E-2
E2
E-1
E1
A1
1
2
3
4
5
6
7
8
9
10
11
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13
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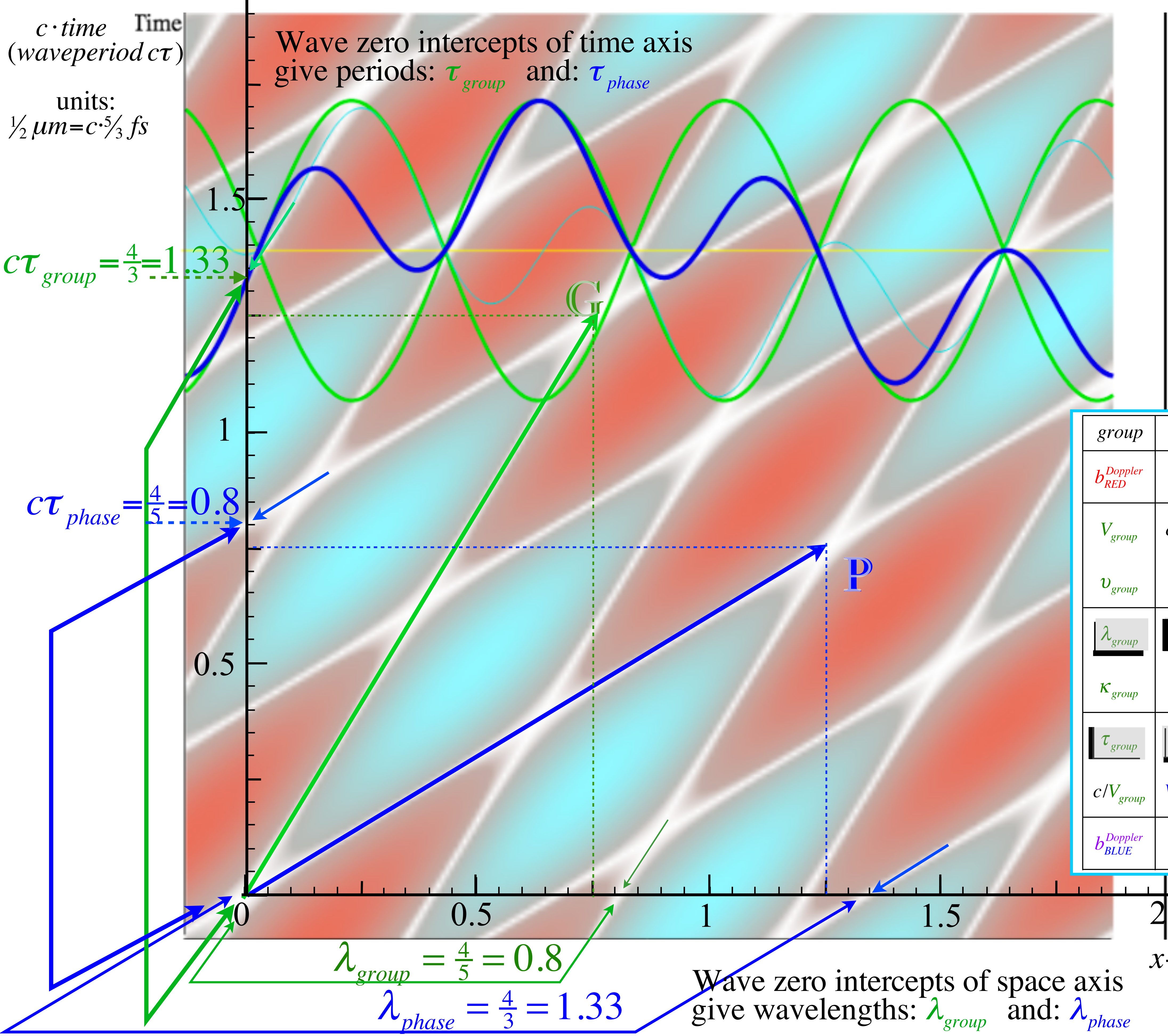
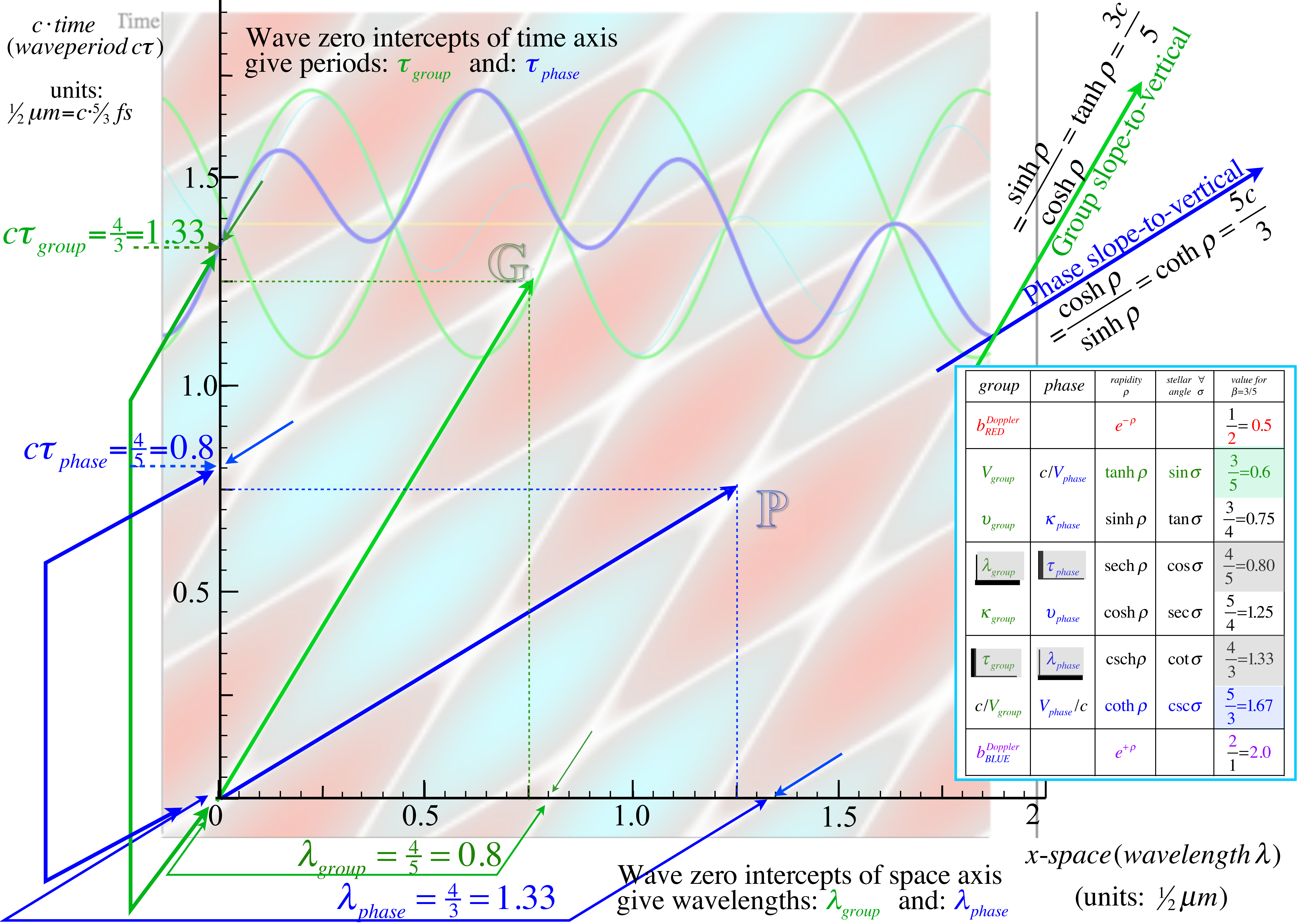
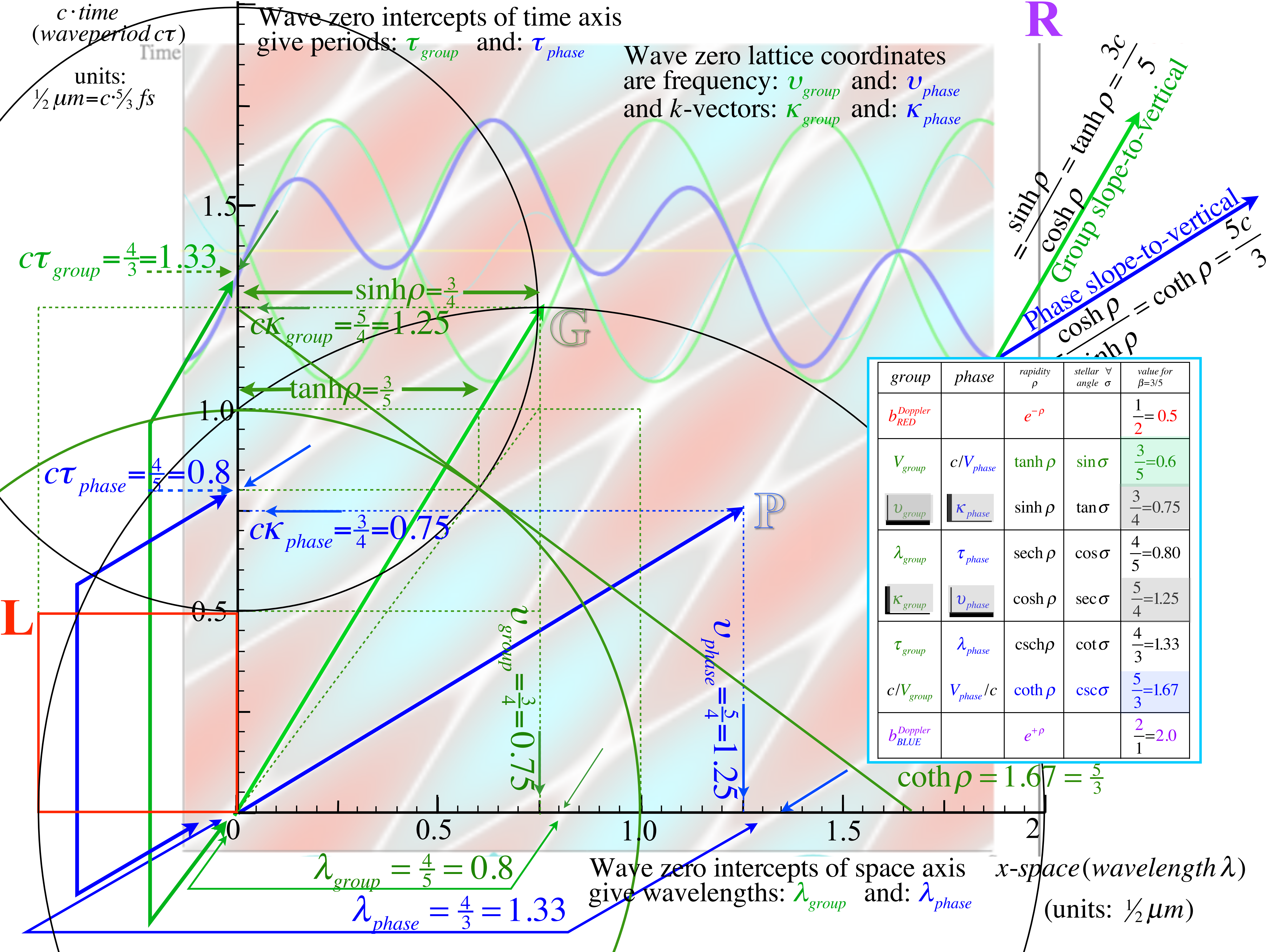


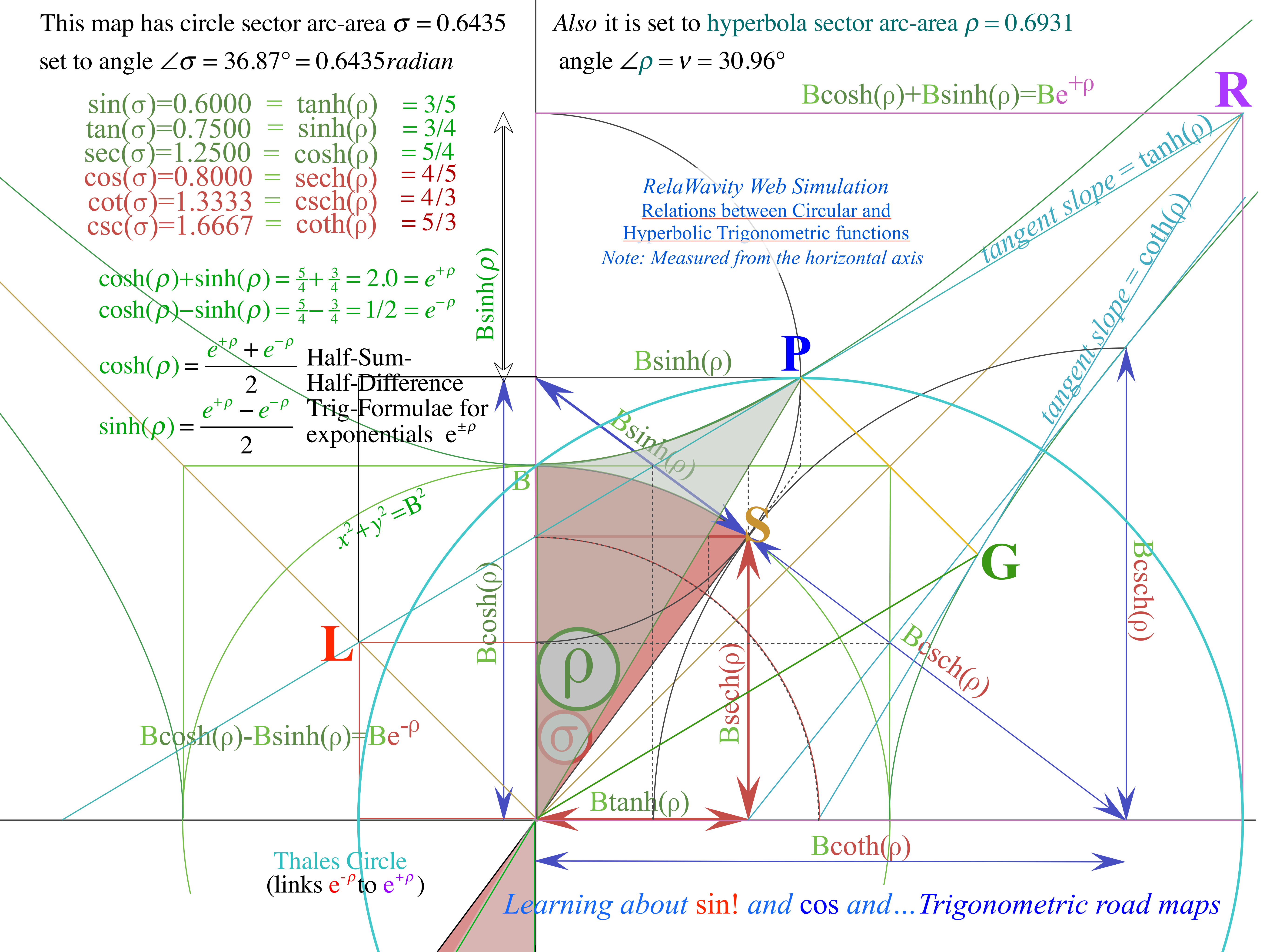
group	phase	rapidity ρ	stellar ∀ angle σ	value for $\beta=3/5$
$b^{Doppler}_{RED}$		$e^{-\rho}$		$\frac{1}{2}=0.5$
V_{group}	c/V_{phase}	$\tanh\rho$	$\sin\sigma$	$\frac{3}{5}=0.6$
υ_{group}	κ_{phase}	$\sinh\rho$	$\tan\sigma$	$\frac{3}{4}=0.75$
λ_{group}	τ_{phase}	$\text{sech}\,\rho$	$\cos\sigma$	$\frac{4}{5}=0.80$
κ_{group}	υ_{phase}	$\cosh\rho$	$\sec\sigma$	$\frac{5}{4}=1.25$
τ_{group}	λ_{phase}	$\text{csch}\,\rho$	$\cot\sigma$	$\frac{4}{3}=1.33$
c/V_{group}	V_{phase}/c	$\coth\rho$	$\csc\sigma$	$\frac{5}{3}=1.67$
$b^{Doppler}_{BLUE}$		$e^{+\rho}$		$\frac{2}{1}=2.0$



group	phase	rapidity ρ	stellar ∀ angle σ	value for $\beta=3/5$
$b_{RED}^{Doppler}$		$e^{-\rho}$		$\frac{1}{2} = 0.5$
V_{group}	c/V_{phase}	$\tanh\rho$	$\sin\sigma$	$\frac{3}{5}=0.6$
υ_{group}	κ_{phase}	$\sinh\rho$	$\tan\sigma$	$\frac{3}{4}=0.75$
λ_{group}	τ_{phase}	$\text{sech}\,\rho$	$\cos\sigma$	$\frac{4}{5}=0.80$
κ_{group}	υ_{phase}	$\cosh\rho$	$\sec\sigma$	$\frac{5}{4}=1.25$
τ_{group}	λ_{phase}	$\text{csch}\rho$	$\cot\sigma$	$\frac{4}{3}=1.33$
c/V_{group}	V_{phase}/c	$\coth\rho$	$\csc\sigma$	$\frac{5}{3}=1.67$
$b_{BLUE}^{Doppler}$		$e^{+\rho}$		$\frac{2}{1}=2.0$

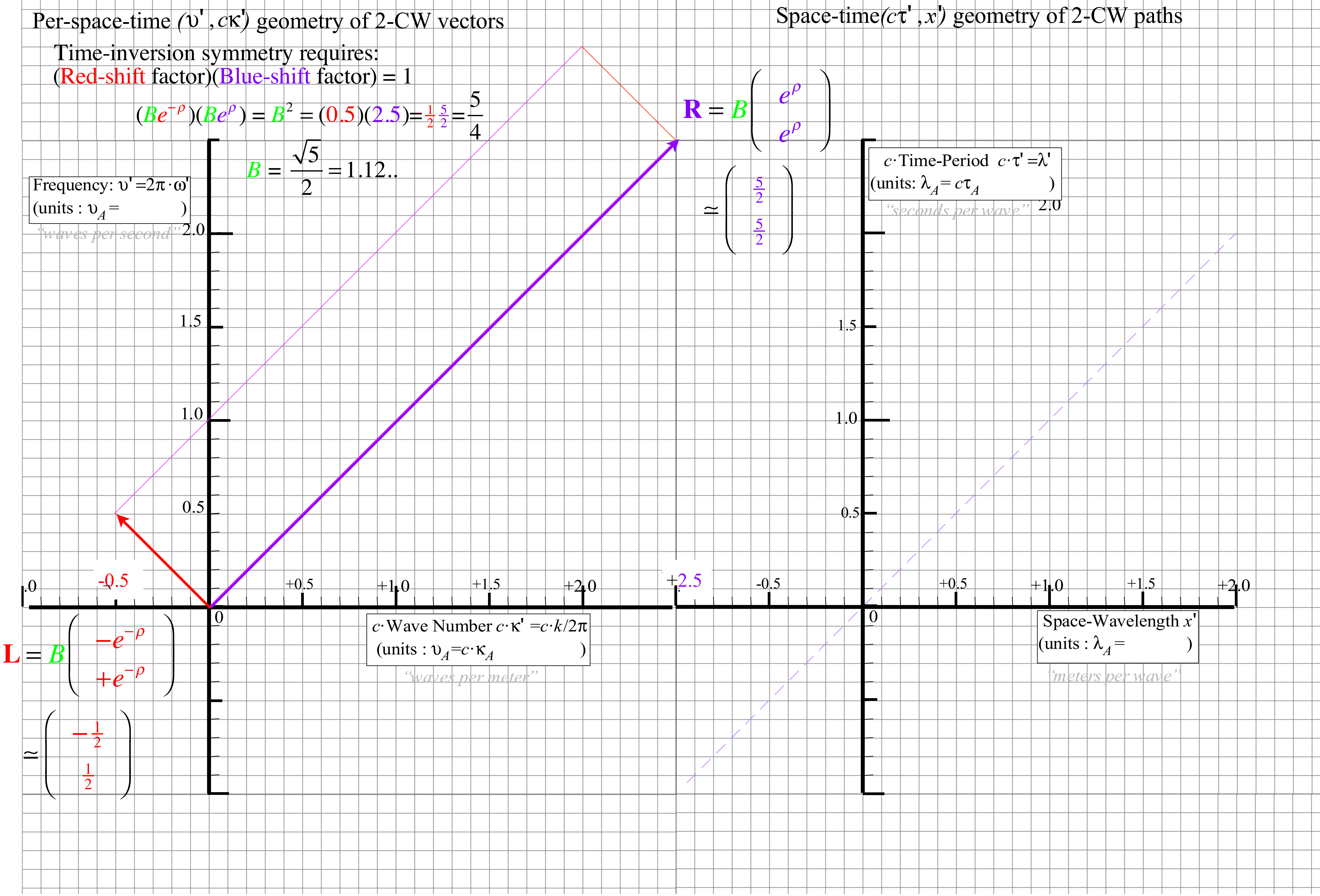


group	*phase*	*rapidity* ρ	*stellar* ∀ *angle* σ	*value for* $\beta=3/5$
$b_{RED}^{Doppler}$		$e^{-\rho}$		$\frac{1}{2}=0.5$
V_{group}	c/V_{phase}	$\tanh\rho$	$\sin\sigma$	$\frac{3}{5}=0.6$
υ_{group}	κ_{phase}	$\sinh\rho$	$\tan\sigma$	$\frac{3}{4}=0.75$
λ_{group}	τ_{phase}	$\text{sech}\,\rho$	$\cos\sigma$	$\frac{4}{5}=0.80$
κ_{group}	υ_{phase}	$\cosh\rho$	$\sec\sigma$	$\frac{5}{4}=1.25$
τ_{group}	λ_{phase}	$\text{csch}\,\rho$	$\cot\sigma$	$\frac{4}{3}=1.33$
c/V_{group}	V_{phase}/c	$\coth\rho$	$\csc\sigma$	$\frac{5}{3}=1.67$
$b_{BLUE}^{Doppler}$		$e^{+\rho}$		$\frac{2}{1}=2.0$

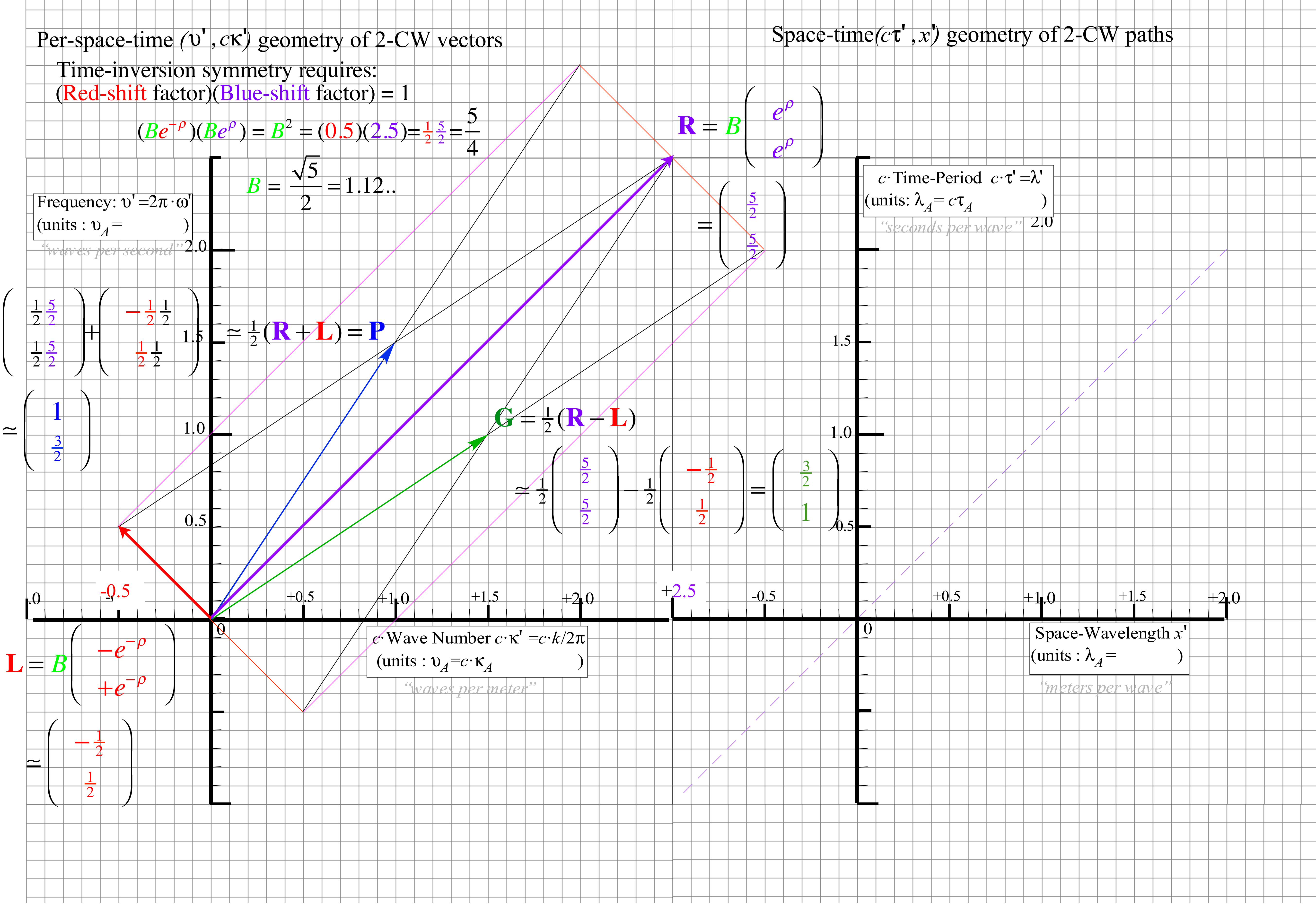


This map has circle sector arc-area σ = 0.6435
set to angle ∠σ = 36.87° = 0.6435 radian
Also it is set to hyperbola sector arc-area ρ = 0.6931
angle ∠ρ = ν = 30.96°
sin(σ)=0.6000 = tanh(ρ) = 3/5
tan(σ)=0.7500 = sinh(ρ) = 3/4
sec(σ)=1.2500 = cosh(ρ) = 5/4
cos(σ)=0.8000 = sech(ρ) = 4/5
cot(σ)=1.3333 = csch(ρ) = 4/3
csc(σ)=1.6667 = coth(ρ) = 5/3
cosh(ρ)+sinh(ρ) = 5/4 + 3/4 = 2.0 = e^{+ρ}
cosh(ρ)−sinh(ρ) = 5/4 − 3/4 = 1/2 = e^{−ρ}
cosh(ρ) = (e^{+ρ} + e^{−ρ})/2
sinh(ρ) = (e^{+ρ} − e^{−ρ})/2
Half-Sum-
Half-Difference
Trig-Formulae for
exponentials e^{±ρ}
Bcosh(ρ)+Bsinh(ρ)=Be^{+ρ}
R
RelaWavity Web Simulation
Relations between Circular and
Hyperbolic Trigonometric functions
Note: Measured from the horizontal axis
tangent slope = tanh(ρ)
tangent slope = coth(ρ)
Bsinh(ρ)
Bsinh(ρ)
Bsinh(ρ)
P
B
S
G
L
x^2+y^2=B^2
Bcosh(ρ)
Bcsch(ρ)
Bcsch(ρ)
Bsech(ρ)
ρ
σ
Bcosh(ρ)-Bsinh(ρ)=Be^{-ρ}
Btanh(ρ)
Bcoth(ρ)
Thales Circle
(links e^{-ρ} to e^{+ρ})
Learning about sin! and cos and…Trigonometric road maps

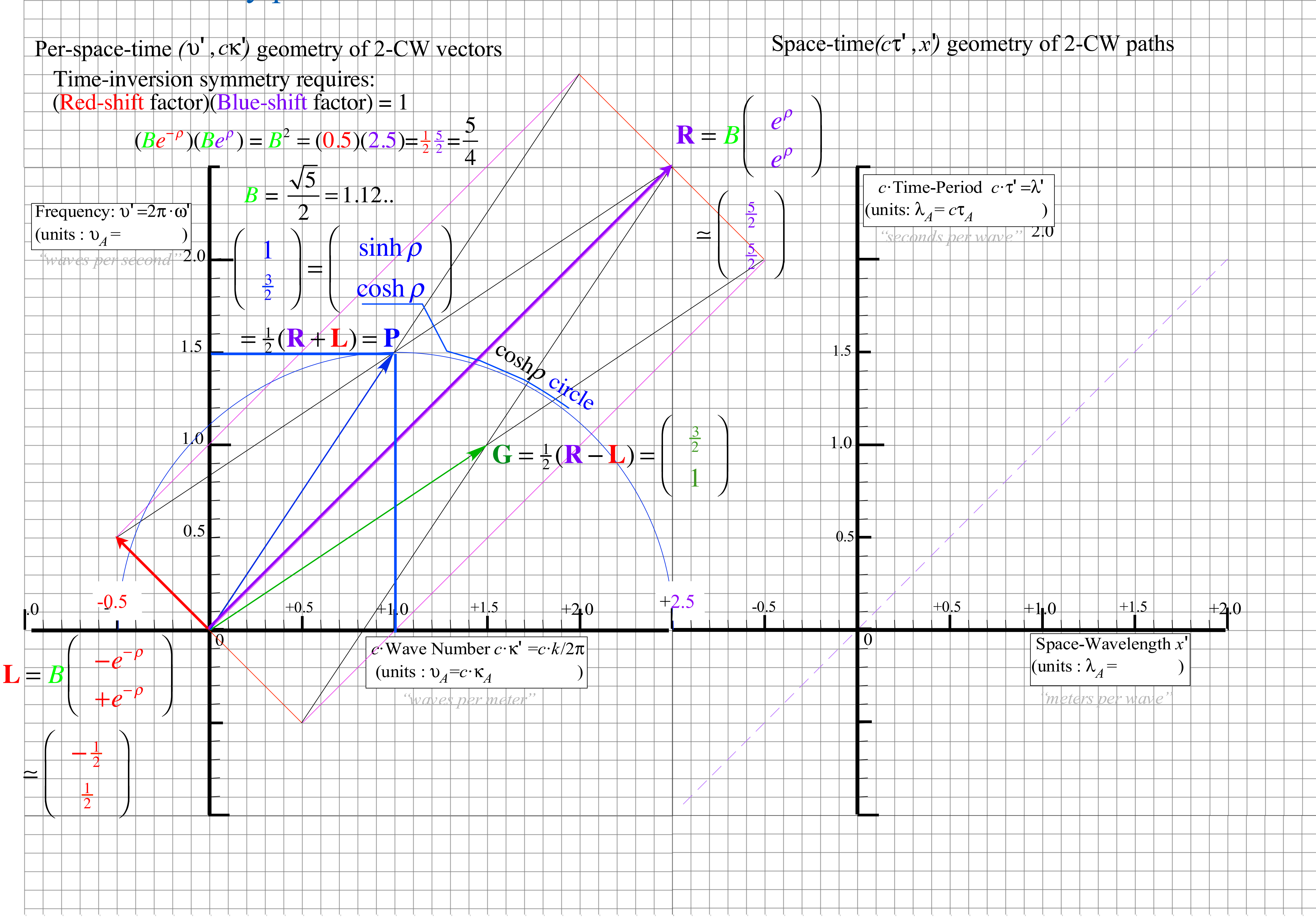
Derive relawavity parameters and Minkowski coordinates for υ_R=2.5THz and υ_L=0.5THz



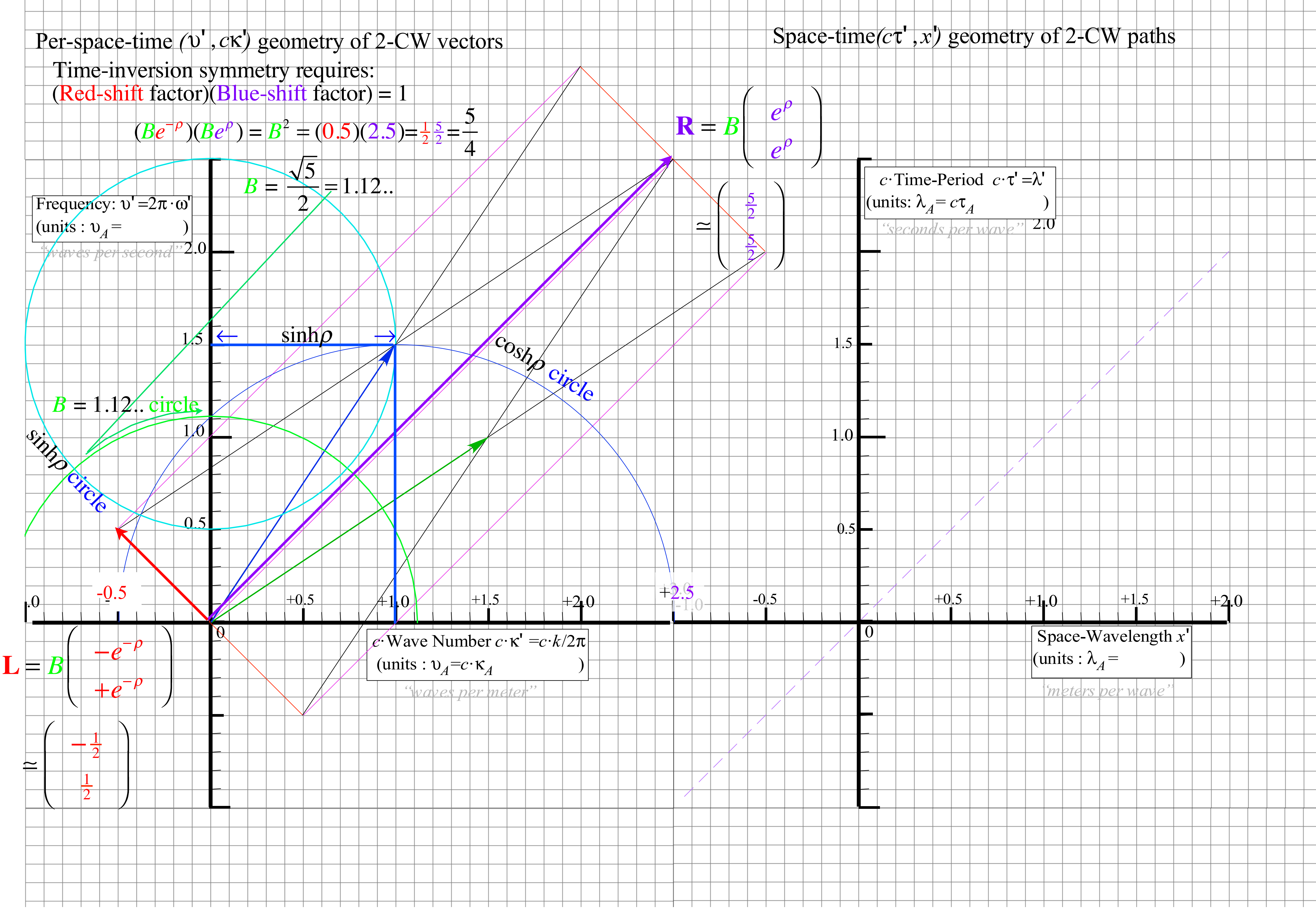
Derive relawavity parameters and Minkowski coordinates for υ_R=2.5THz and υ_L=0.5THz



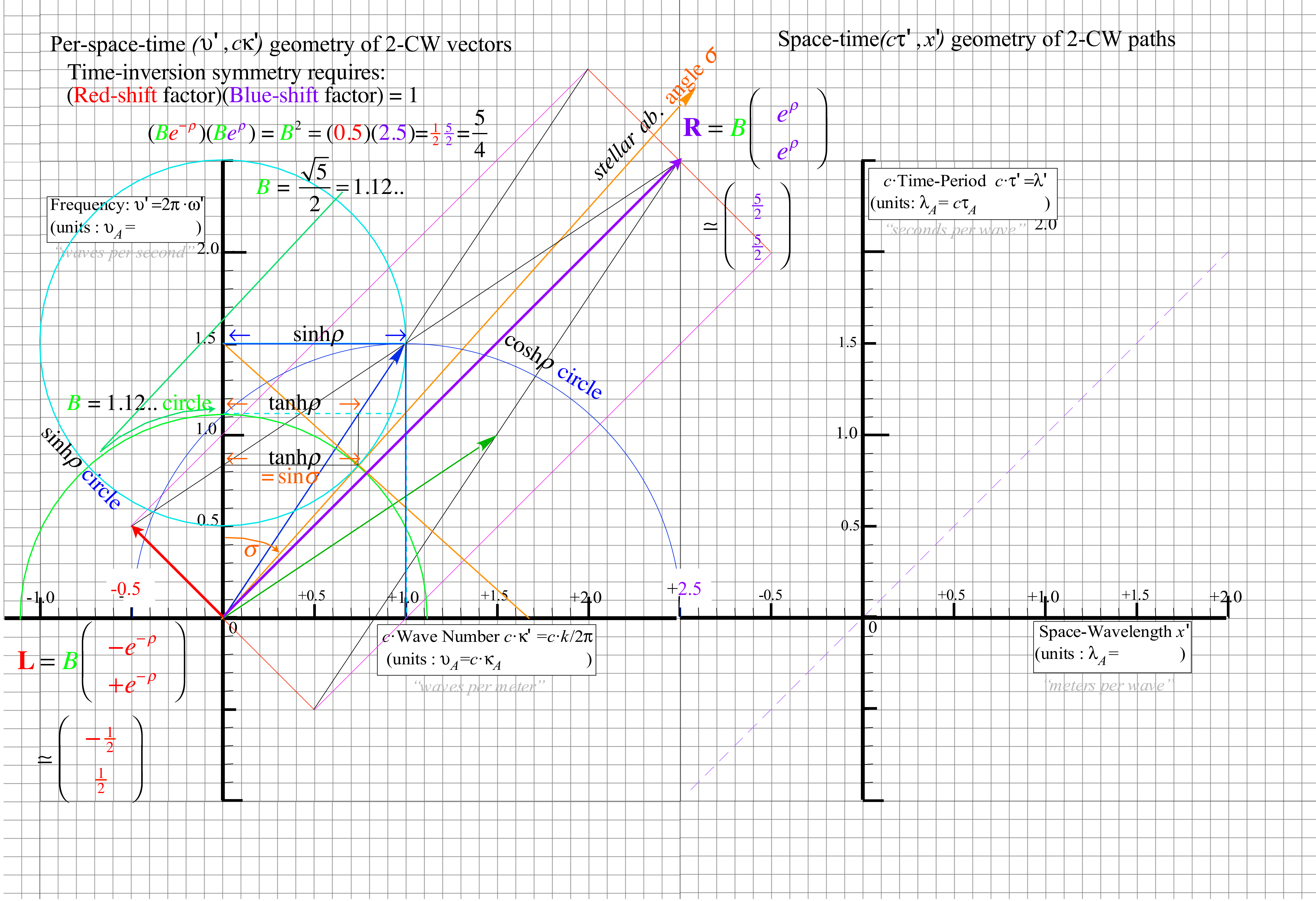
Derive relawavity parameters and Minkowski coordinates for υ_R=2.5THz and υ_L=0.5THz



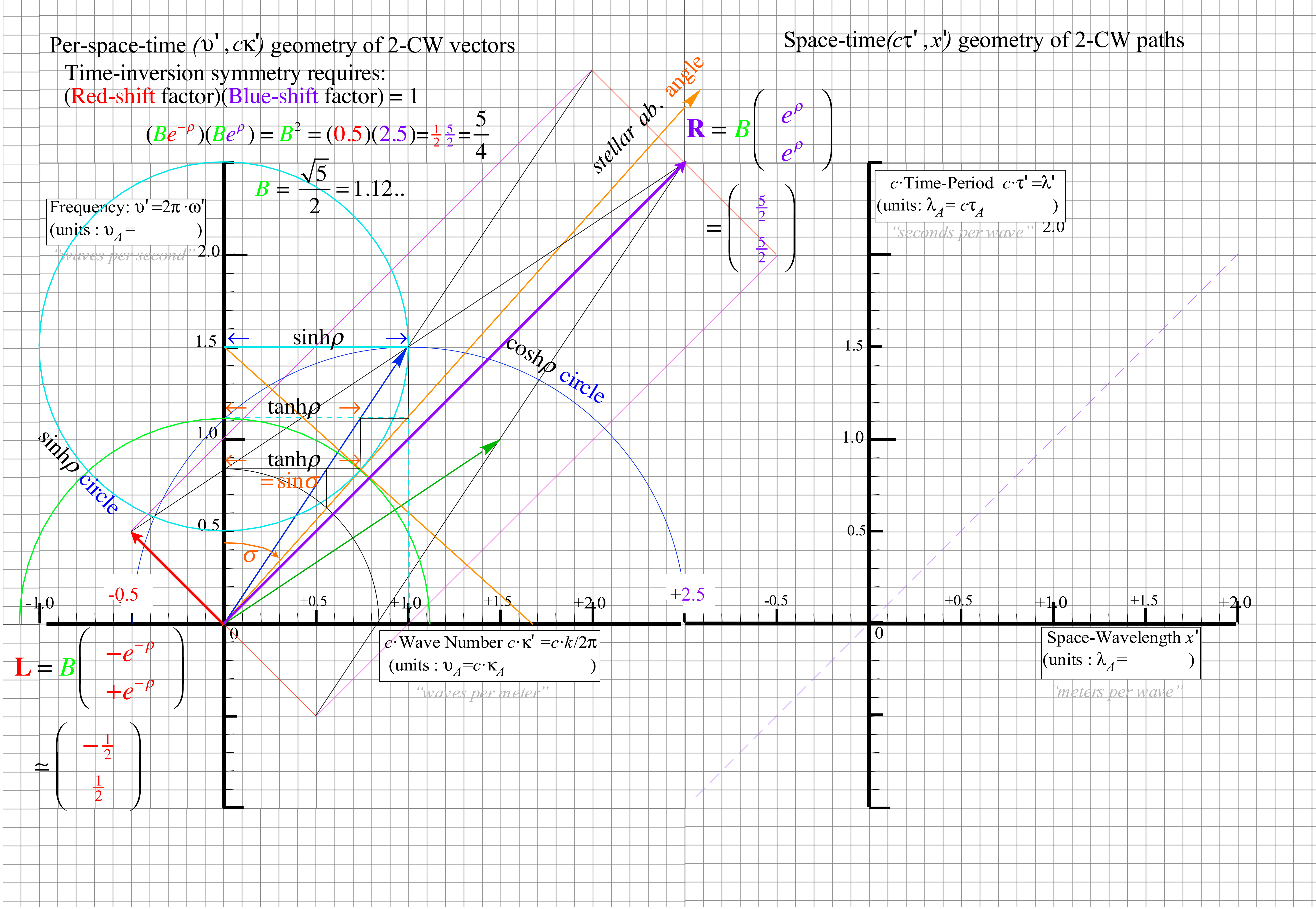
Derive relawavity parameters and Minkowski coordinates for υ_R=2.5THz and υ_L=0.5THz



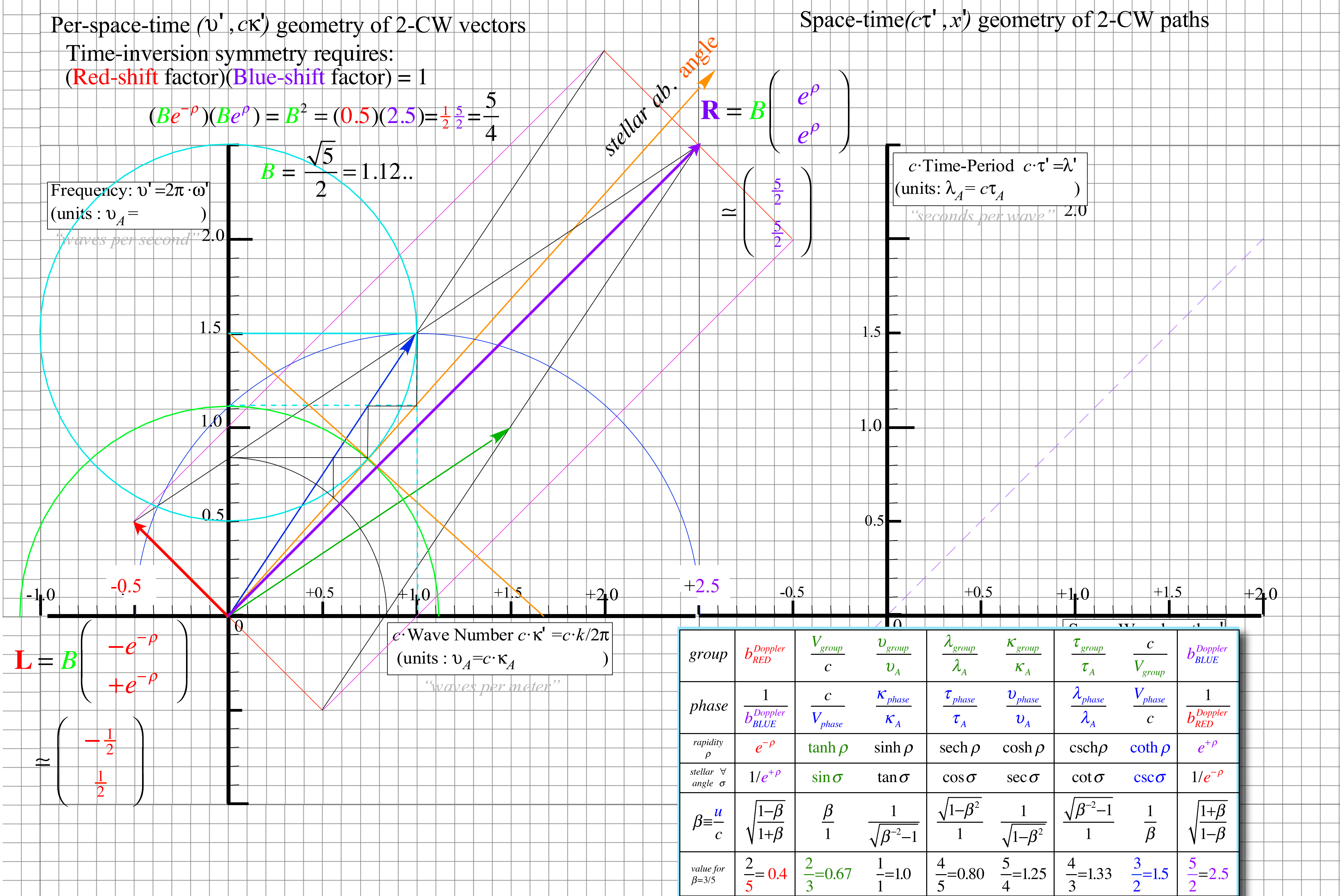
Derive relawavity parameters and Minkowski coordinates for υ_R=2.5THz and υ_L=0.5THz



Derive relawavity parameters and Minkowski coordinates for υ_R=2.5THz and υ_L=0.5THz

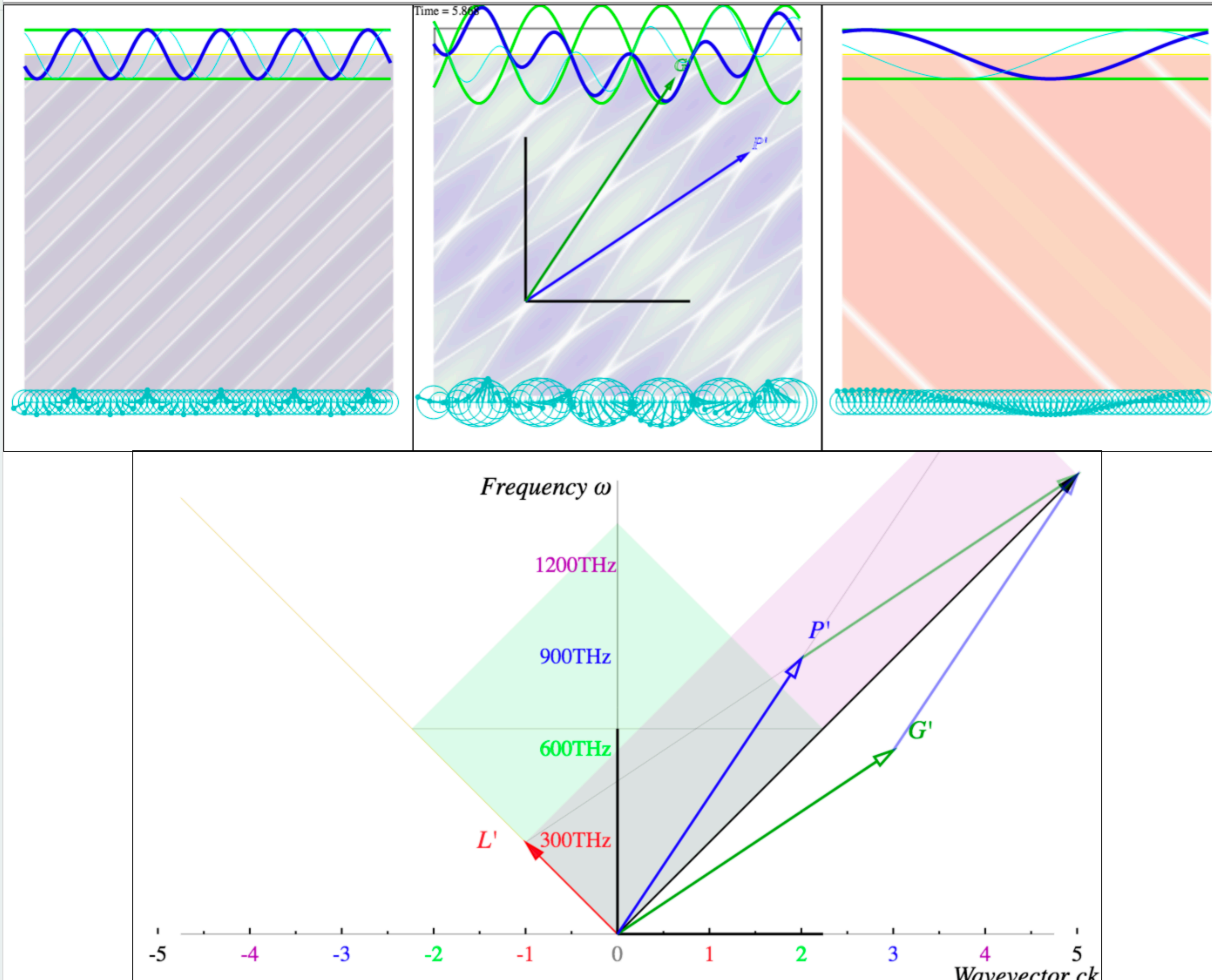


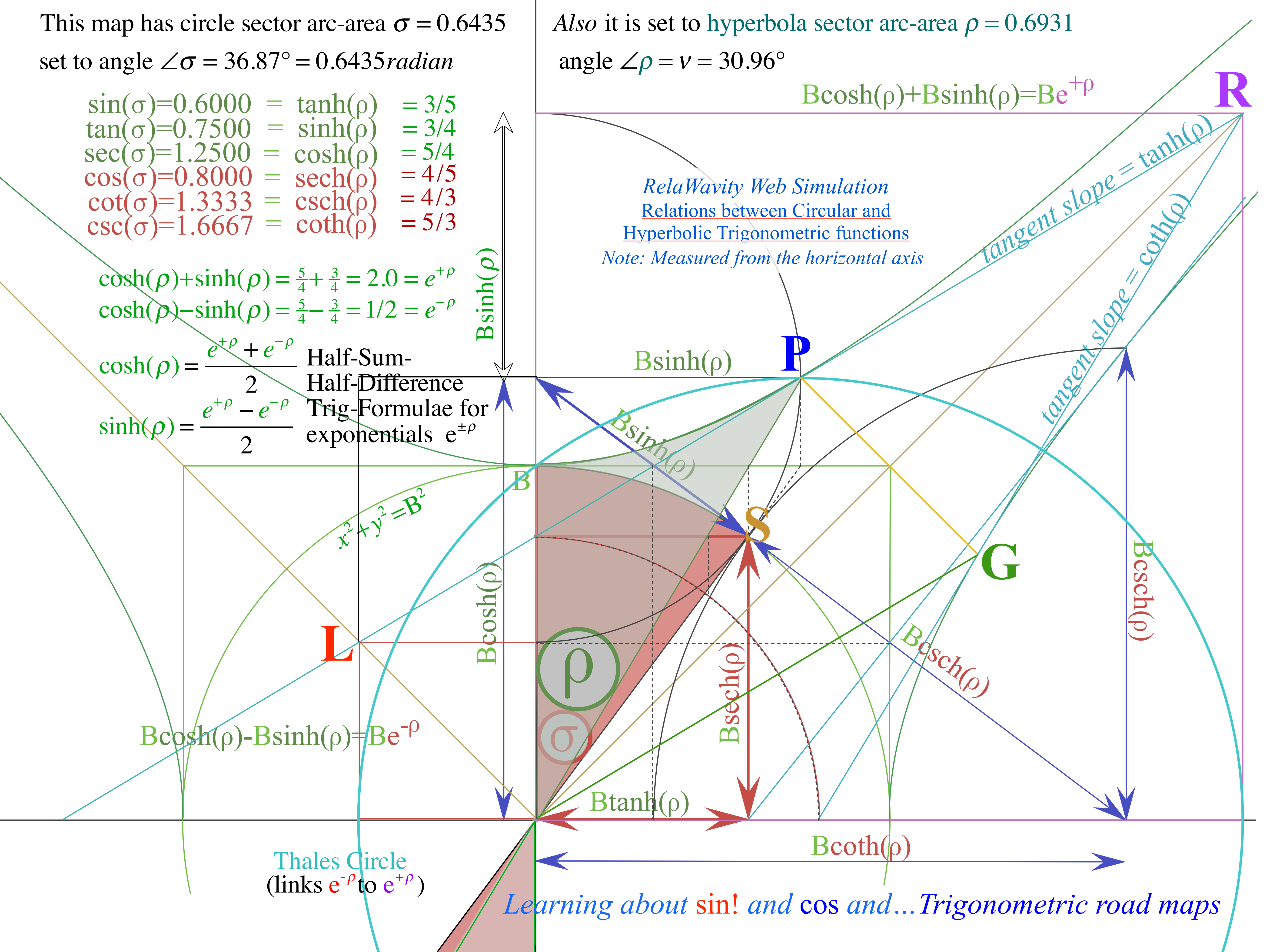
Derive relawavity parameters and Minkowski coordinates for υ_R=2.5THz and υ_L=0.5THz



group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{2}{5}$= 0.4	$\frac{2}{3}$=0.67	$\frac{1}{1}$=1.0	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{3}{2}$=1.5	$\frac{5}{2}$=2.5

(may contain errors)


Time = 5.868
G'
P'
Frequency ω
1200THz
900THz
600THz
300THz
P'
G'
L'
-5
-4
-3
-2
-1
0
1
2
3
4
5
Wavevector ck



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sec(σ)=1.2500 = cosh(ρ) = 5/4
cos(σ)=0.8000 = sech(ρ) = 4/5
cot(σ)=1.3333 = csch(ρ) = 4/3
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cosh(ρ)+sinh(ρ) = 5/4 + 3/4 = 2.0 = e^{+ρ}
cosh(ρ)−sinh(ρ) = 5/4 − 3/4 = 1/2 = e^{−ρ}
cosh(ρ) = (e^{+ρ} + e^{−ρ})/2
sinh(ρ) = (e^{+ρ} − e^{−ρ})/2
Half-Sum-
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RelaWavity Web Simulation
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Note: Measured from the horizontal axis
Bcosh(ρ)+Bsinh(ρ)=Be^{+ρ}
R
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tangent slope = coth(ρ)
Bsinh(ρ)
P
Bsinh(ρ)
Bsinh(ρ)
B
S
G
x²+y²=B²
L
Bcosh(ρ)
ρ
σ
Bsech(ρ)
Bcsch(ρ)
Bcsch(ρ)
Bcosh(ρ)-Bsinh(ρ)=Be^{-ρ}
Btanh(ρ)
Bcoth(ρ)
Thales Circle
(links e^{-ρ} to e^{+ρ})
Learning about sin! and cos and...Trigonometric road maps

Fig. 11 in text *Relawavity...*

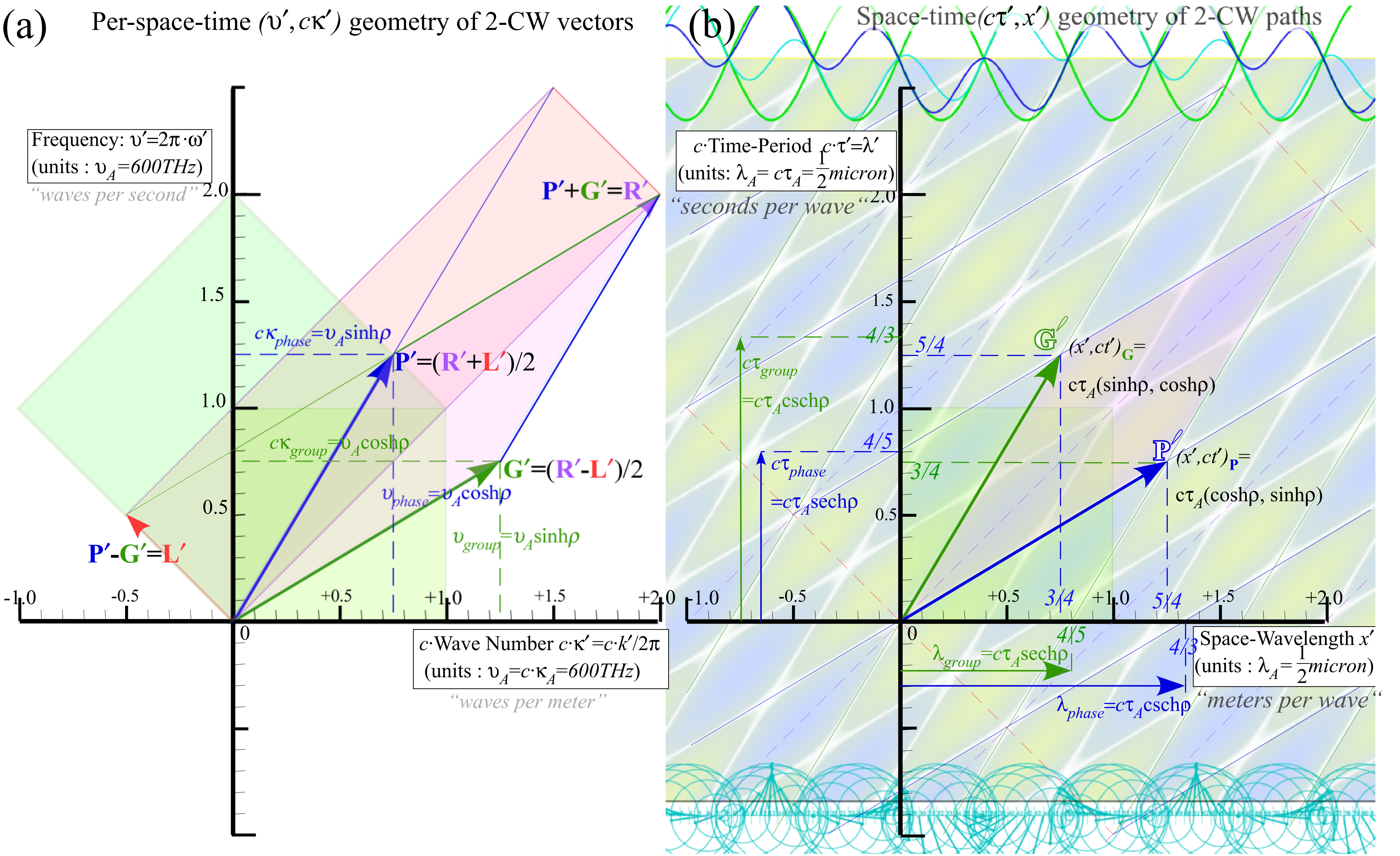
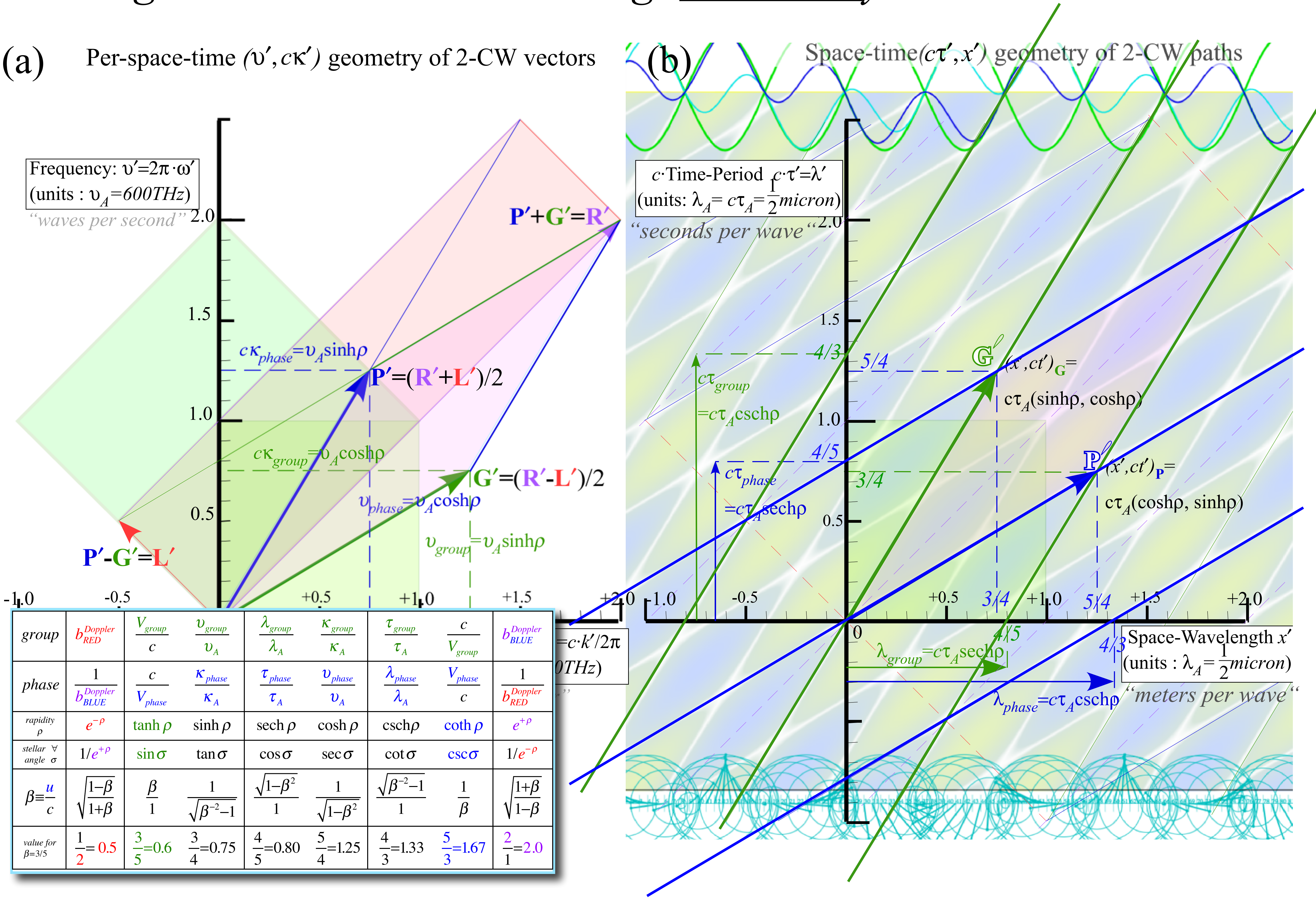


Fig. 4 in Ch.0 text introducing *Relawavity...*



group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Lecture 30
Wed. 12.05.2018

Review: Relawavity ρ functions and plots vs. ρ

Derive relawavity parameters and Minkowski coordinates for υ_R=2.5THz and υ_L=0.5THz

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity

Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry

"Occams Sword" and summary of 16 parameter functions of ρ and σ

Applications to optical waveguide, spherical waves, and accelerator radiation

➔ Derivation of relativistic quantum mechanics

What's the matter with mass? Shining some light on the Elephant in the room

Relativistic action and Lagrangian-Hamiltonian relations

Poincare' and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae

Feynman diagram geometry

Compton recoil related to rocket velocity formula

Comparing 2^{nd}-quantization "photon" number N and 1^{st}-quantization wavenumber κ

Relawavity in accelerated frames

Laser up-tuning by Alice and down-tuning by Carla makes *g*-acceleration grid

Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid

Animation of mechanics and metrology of constant-g grid

Using (some) wave parameters to develop relativistic quantum theory

$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c\text{)}$$

$$c\kappa_{phase} = B\sinh\rho \approx B\rho \quad \text{ (for } u \ll c\text{)}$$

At low speeds:

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2$$

$$\sinh\rho \approx \rho$$

$B = \upsilon_A$ sec.$^{-1}$

$B = \upsilon_A = c\,\kappa_A$ sec.$^{-1}$

$\frac{m.}{\text{sec.}} \frac{1}{m.}$

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

RelaWavity Web Simulation - Relativistic Terms (Short version)

Using (some) wave parameters to develop relativistic quantum theory

$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c)$$

$$c\kappa_{phase} = B\sinh\rho \approx B\rho \qquad \text{(for } u \ll c)$$

$$\frac{u}{c} = \tanh\rho \approx \rho \qquad \text{(for } u \ll c)$$

At low speeds:

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

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$B = \upsilon_A = c\kappa_A$ sec.$^{-1}$

$\frac{m.}{\text{sec.}} \frac{1}{m.}$

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

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$$B = \upsilon_A \quad \text{sec.}^{-1}$$

$$B = \upsilon_A = c\,\kappa_A \quad \text{sec.}^{-1}$$

$$\frac{m.}{\text{sec.}}\;\frac{1}{m.}$$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c)$$

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

$B = \upsilon_A$ sec.$^{-1}$

$B = \upsilon_A = c\,\kappa_A$ sec.$^{-1}$ $\frac{m.}{\text{sec.}}\frac{1}{m.}$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

time	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
space	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

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$B = \upsilon_A = c\kappa_A$ sec.$^{-1}$

$\frac{m.}{\text{sec.}} \cdot \frac{1}{m.}$

At low speeds:

$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy and momentum

Resembles: $const. + \frac{1}{2}Mu^2$

Resembles: Mu

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Using (some) wave parameters to develop relativistic quantum theory

$\upsilon_{phase} = B\cosh\rho \approx B + \frac{1}{2}B\rho^2$ (for $u \ll c$)

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$\frac{m.}{sec.}\frac{1}{m.}$

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So attach scale factor h to match units.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$

Resembles: $const. + \frac{1}{2}Mu^2$

(sec.)$^{-1}$

Resembles: Mu

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$

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phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
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$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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Lucky coincidences?? Cheap trick??

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phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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RelaWavity Web Simulation - Relativistic Terms (Short version)

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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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(old-fashioned notation)

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(old-fashioned notation)

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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Max Planck 1858-1947

Using (some) wave parameters to develop relativistic quantum theory

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phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$			
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For more, visit the Pirelli Challenge Site
Quantized amplitude

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Resolution and dirty secret: $\mathbf{E}$, N, and υ_{phase} are all frequencies!

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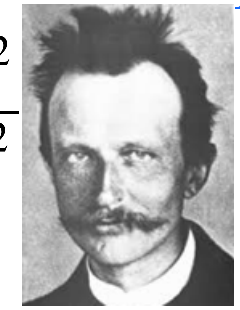

Max Planck
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$cp = \frac{Mcu}{\sqrt{1-u^2/c^2}}$

$\frac{1}{\sqrt{\beta^{-2}-1}} = \frac{\frac{u}{c}}{\sqrt{1-\frac{u^2}{c^2}}}$ (old-fashioned notation)

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value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Using (some) wave parameters to develop relativistic quantum theory

$\upsilon_{phase} = B\cosh\rho \approx B + \frac{1}{2}B\rho^2$ (for $u \ll c$)

$c\kappa_{phase} = B\sinh\rho \approx B\rho$ (for $u \ll c$)

$\frac{u}{c} = \tanh\rho \approx \rho$ (for $u \ll c$)

$\cosh\rho \approx 1 + \frac{1}{2}\rho^2 \approx 1 + \frac{1}{2}\frac{u^2}{c^2}$

$\sinh\rho \approx \rho \approx \frac{u}{c}$

Max Planck 1858-1947

Louis DeBroglie 1892-1987

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \Leftarrow \text{for } (u \ll c) \Rightarrow \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$ (The famous Mc^2 shows up here!)

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \Leftarrow \text{for } (u \ll c) \Rightarrow h\kappa_{phase} \approx \frac{hB}{c^2}u$$

So attach scale factor h *(or hN)* to match units.

$$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \Leftarrow \text{for } (u \ll c) \Rightarrow h\kappa_{phase} \approx Mu$$

~~Natural wave conspiracy~~ ~~Lucky coincidences??~~ ~~Cheap~~ Expensive trick??

...Try exact υ_{phase} and κ_{phase}...

Need to replace h with hN to match e.m. energy density $\varepsilon_0 \mathbf{E}^* \bullet \mathbf{E} = hN\upsilon_{phase}$

$$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$$

Planck (1900)

$$= \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1-u^2/c^2}}$$

Einstein (1905)

$$hc\kappa_{phase} = hB\sinh\rho = Mc^2\sinh\rho$$

$$\frac{1}{\sqrt{\beta^{-2}-1}} = \frac{\frac{u}{c}}{\sqrt{1-\frac{u^2}{c^2}}} \text{ (old-fashioned notation)}$$

$$cp = \frac{Muc}{\sqrt{1-u^2/c^2}}$$

$$\text{Momentum: } h\kappa_{phase} = p = \frac{Mu}{\sqrt{1-u^2/c^2}}$$

DeBroglie (1921)

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
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$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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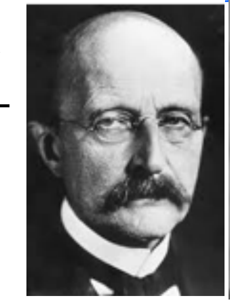

Max Planck 1858-1947

Louis DeBroglie 1892-1987

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$B = \upsilon_A = c\kappa_A$

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Planck (1900)

$$= \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1 - u^2/c^2}}$$

Einstein (1905)

This motivates the "particle" normalization $\int \Psi^*\Psi \, dV = N \quad \Psi = \sqrt{\frac{\varepsilon_0}{h\upsilon}}E$

$$hc\kappa_{phase} = hB\sinh\rho = Mc^2\sinh\rho$$

$$\frac{1}{\sqrt{\beta^{-2}-1}} = \frac{\frac{u}{c}}{\sqrt{1-\frac{u^2}{c^2}}} \text{ (old-fashioned notation)}$$

$$cp = \frac{Muc}{\sqrt{1-u^2/c^2}}$$

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group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$			
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$			
stellar ∀ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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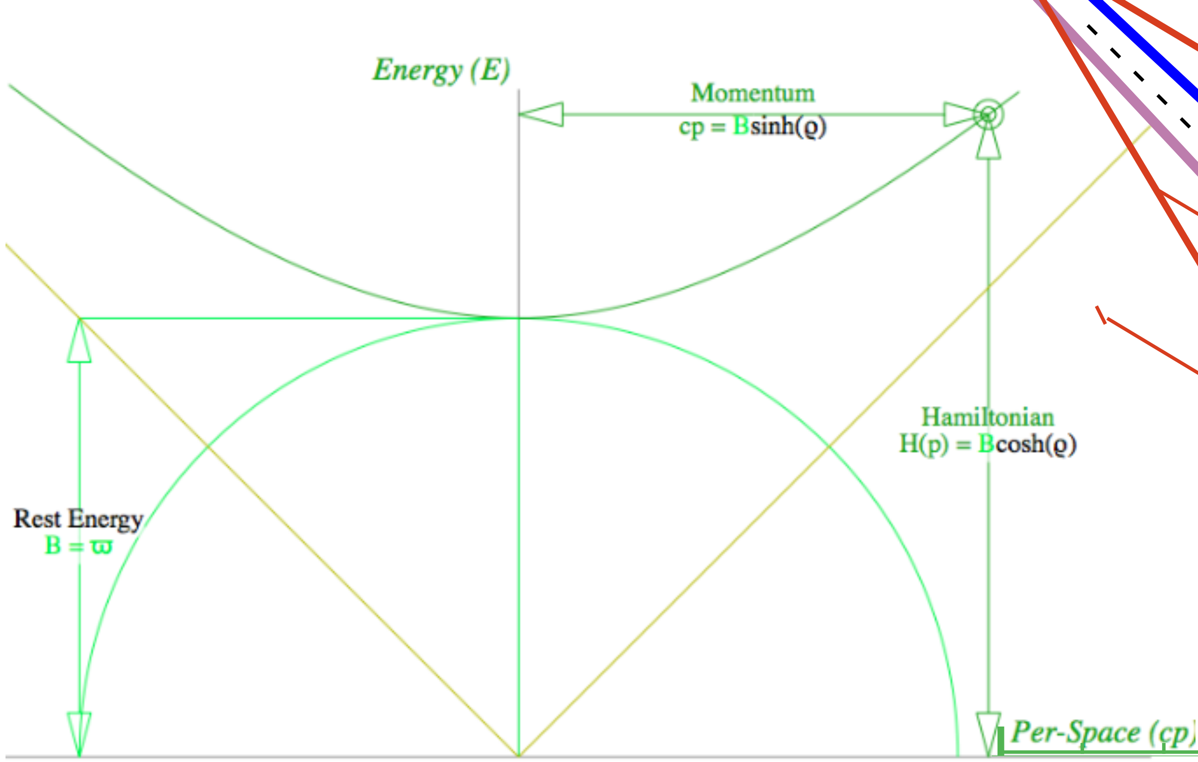
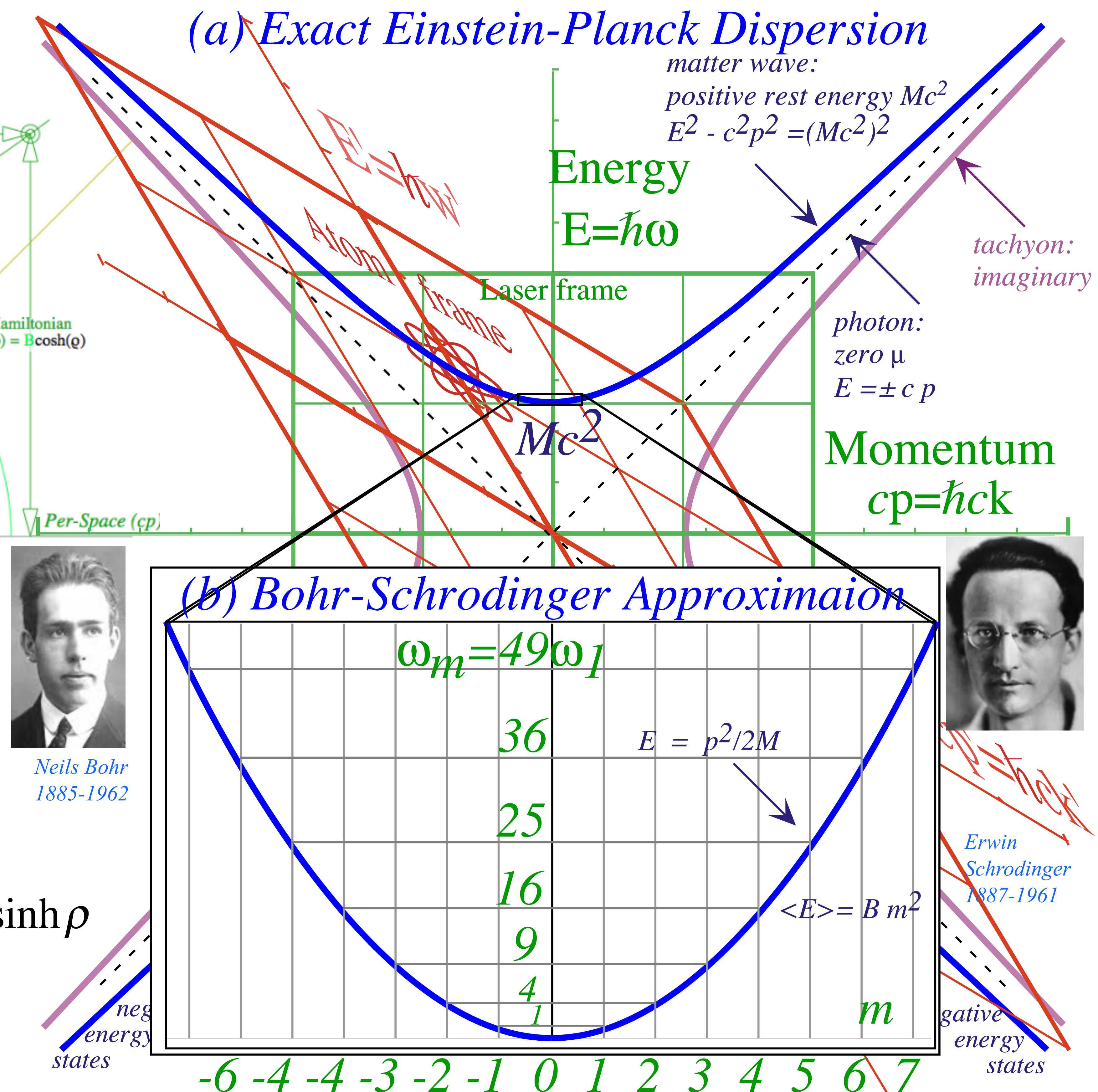
Using (some) wave coordinates for relativistic quantum theory





Neils Bohr
1885-1962

Erwin
Schrodinger
1887-1961

Mass (resting)

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Energy

$$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$$

Momentum

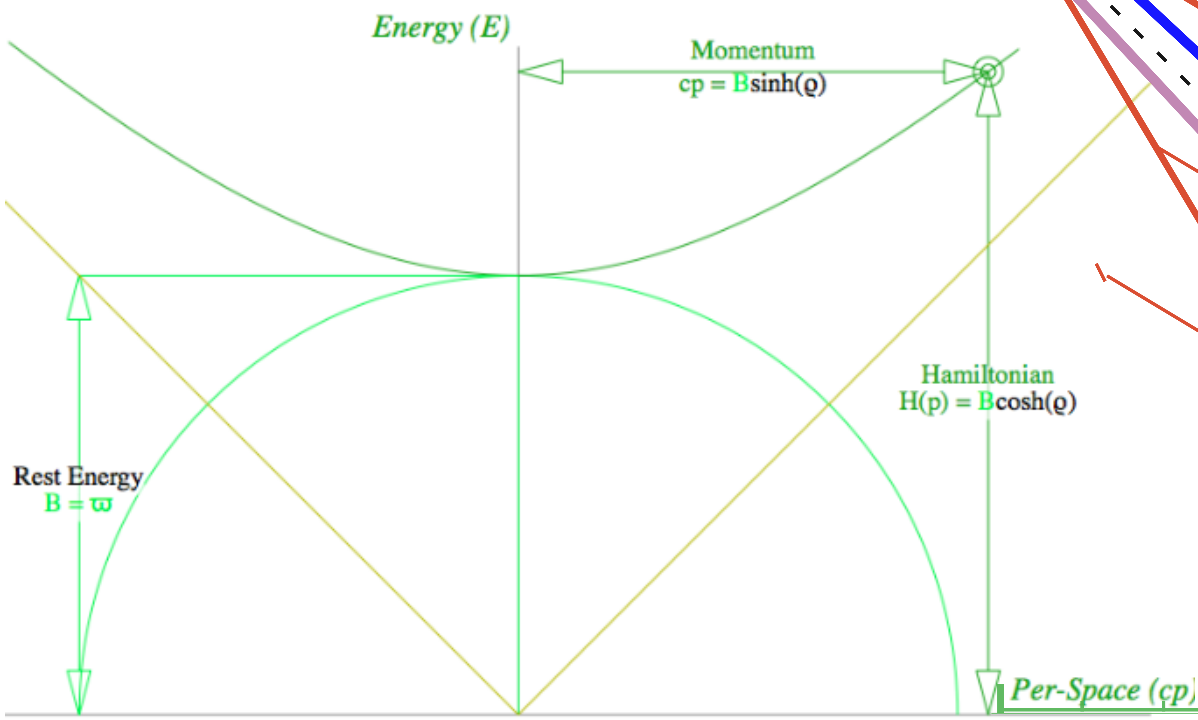
$$hc\kappa_{phase} = cp = hc\kappa_A \sinh\rho = h\upsilon_A \sinh\rho$$

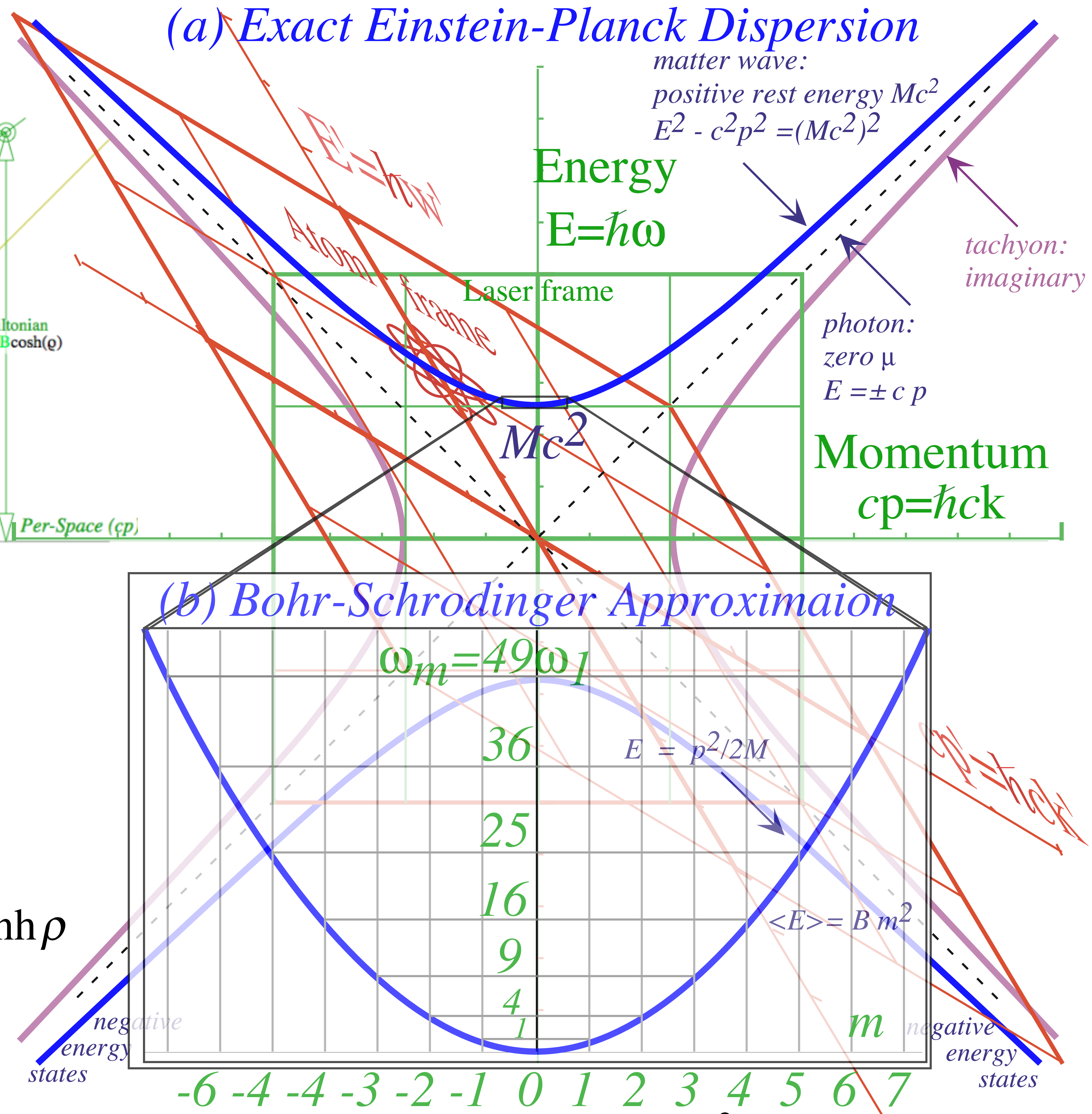
Energy versus Momentum

$$E^2 = \left(Mc^2\right)^2 \cosh^2\rho$$

$$= \left(Mc^2\right)^2\left(1+\sinh^2\rho\right) = \left(Mc^2\right)^2 + (cp)^2 \Rightarrow E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2} \approx Mc^2 + \frac{p^2}{2M}$$

Using (some) wave coordinates for relativistic quantum theory





Mass (*resting*)

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Energy

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low speed approximation

Relawavity variable tables

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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effects	$b_{RED}^{Doppler}$	V_{group}	*past-future asymmetry* (*off-diagonal Lorentz-transform*)	*x-contraction*(Lorentz) τ_{phase}*-contraction*	*t-dilation*(Einstein) υ_{phase}*-dilation* (*on-diagonal Lorentz-transform*)	*inverse asymmetry*	V_{phase}	$b_{BLUE}^{Doppler}$

Relativistic quantum mechanics variable tables

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
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functions		V_{group}= $c\tanh\rho$	*momentum* $cp=Mc^2\sinh\rho$	*-Lagrangian* $L=-Mc^2\mathrm{sech}\,\rho$	*Hamiltonian* $H=Mc^2\cosh\rho$	*DeBroglie* $\lambda=\alpha\,\mathrm{csch}\rho$	V_{phase} = $c\coth\rho$	

RelaWavity Web Simulation - Relativistic Terms (Expanded)

Lecture 30
Wed. 12.05.2018

Review: Relawavity ρ functions and plots vs. ρ

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity
- Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry
  - "Occams Sword" and summary of 16 parameter functions of ρ and σ
  - Applications to optical waveguide, spherical waves, and accelerator radiation

Derivation of relativistic quantum mechanics
- ➔ What's the matter with mass? Shining some light on the Elephant in the room
- Relativistic action and Lagrangian-Hamiltonian relations
  - Poincare' and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae
- Feynman diagram geometry
  - Compton recoil related to rocket velocity formula
  - Comparing 2nd-quantization "photon" number N and 1st-quantization wavenumber κ

Relawavity in accelerated frames
- Laser up-tuning by Alice and down-tuning by Carla makes *g*-acceleration grid
  - Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid
  - Animation of mechanics and metrology of constant-g grid

Definition(s) of mass for relativity/quantum

Given: Energy: $E = Mc^2 \cosh\rho$

$= h\upsilon_{phase}$

Rest Mass M_{rest} *(Einstein's mass)*

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Defines invariant hyperbola(s)

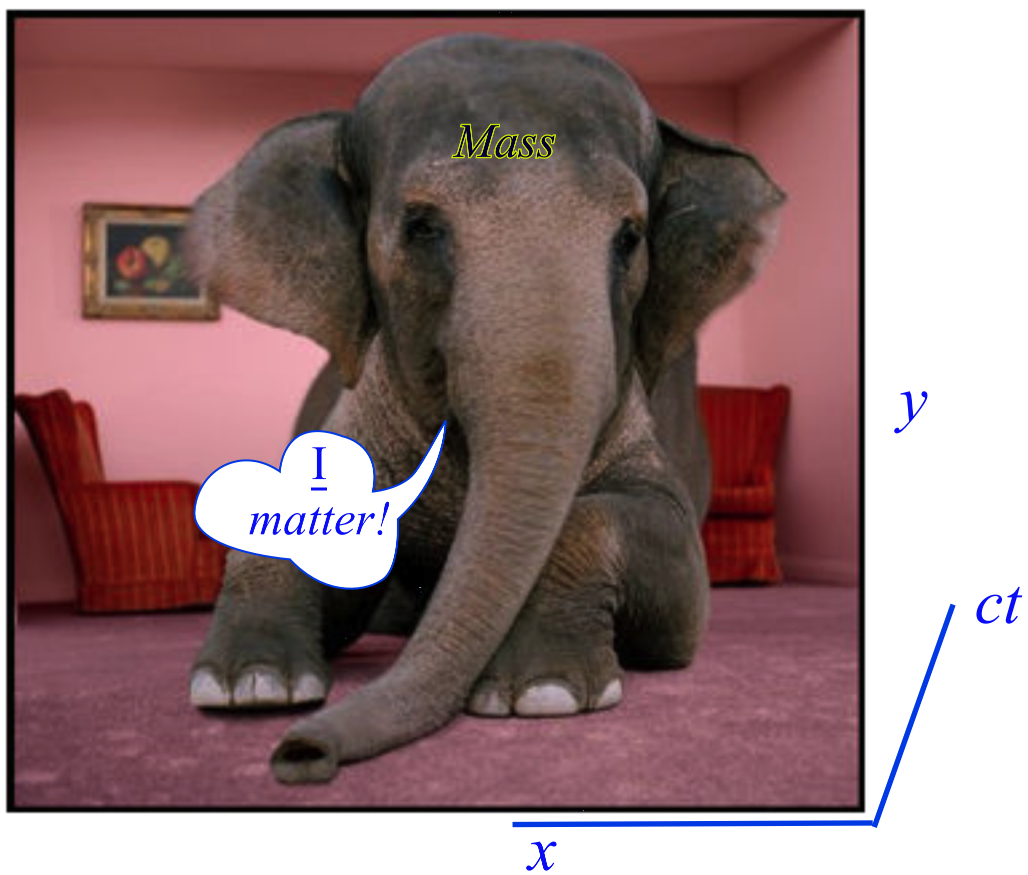
$$E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2}$$

momentum: $cp = Mc^2 \sinh\rho$

$= hc\kappa_{phase}$

velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

- ***What's the matter with Mass?***



Shining some light on the elephant in the spacetime room

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$$\frac{h\upsilon_{phase}}{c^2} = M_{rest}$$ Rest Mass

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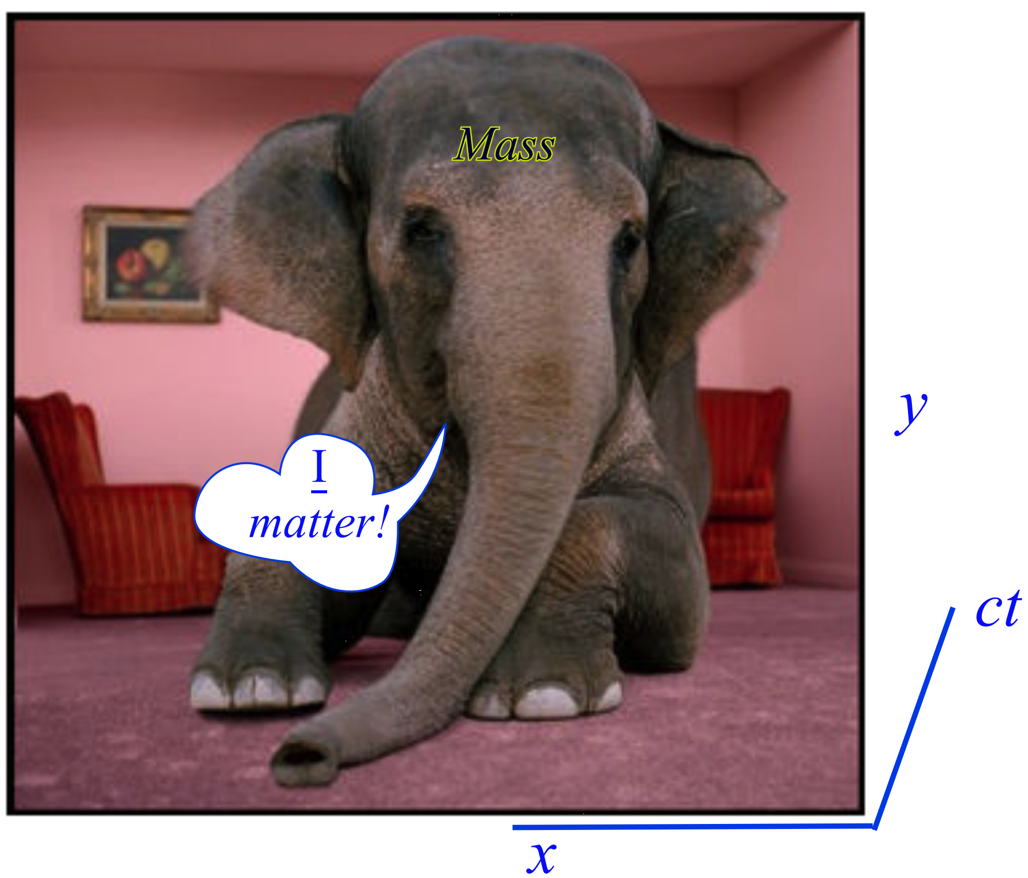

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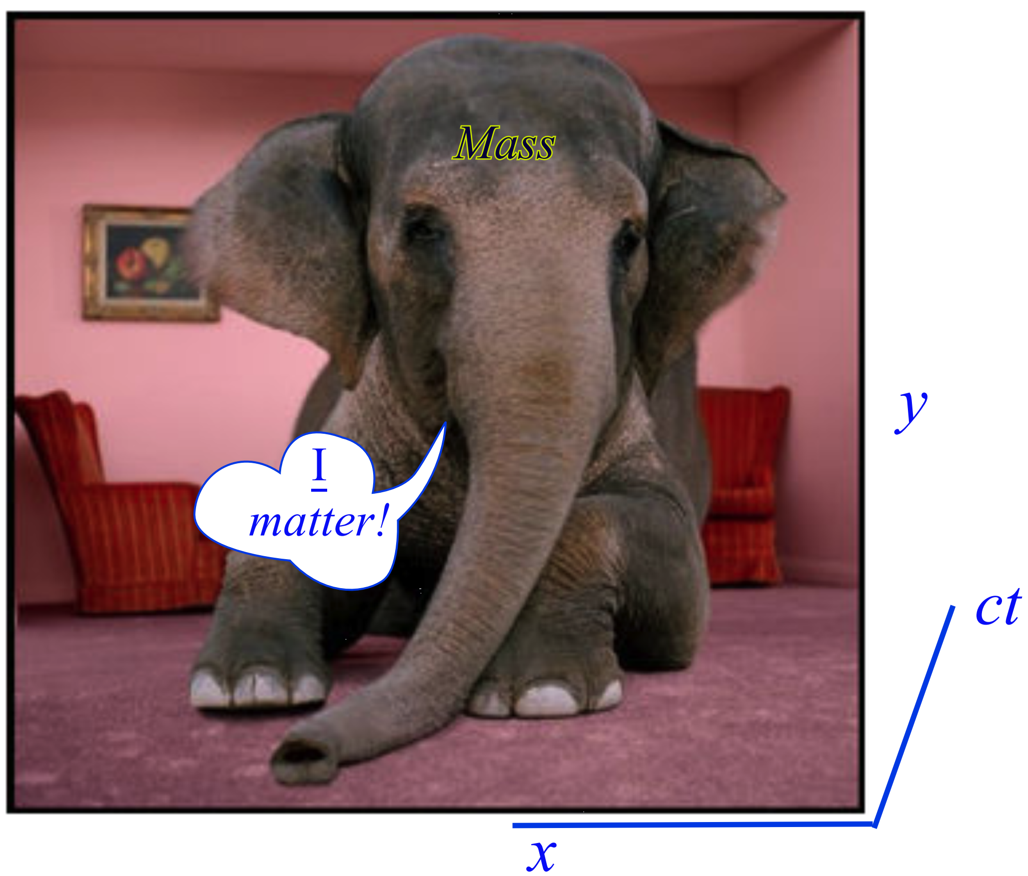
- *What's the matter with Mass?*



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Momentum Mass

Limiting cases:

$$M_{mom} \xrightarrow[u\to c]{} M_{rest}e^{\rho}/2$$

$$M_{mom} \xrightarrow[u\ll c]{} M_{rest}$$

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Effective Mass M_{eff} *(Newton's mass)* Defined by ratio *F/a=dp/du* of relativistic force to acceleration.

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Effective Mass M_{eff} (Newton's mass) Defined by ratio $F/a=dp/du$ of relativistic force to acceleration.

That is ratio of change $dp=Mc\cosh\rho\, d\rho$ in momentum to change $du=c\,\mathrm{sech}^2\rho\, d\rho$ in group velocity.

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Given: Energy: $E = Mc^2 \cosh\rho = h\upsilon_{phase}$

momentum: $cp = Mc^2 \sinh\rho = hc\kappa_{phase}$

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$$= M_{rest}\cosh\rho = \frac{M_{rest}}{\sqrt{1-u^2/c^2}} \quad \text{Momentum Mass}$$

Limiting cases:

$$M_{mom} \xrightarrow[u\to c]{} M_{rest} e^{\rho}/2$$

$$M_{mom} \xrightarrow[u\ll c]{} M_{rest}$$

Effective Mass M_{eff} (Newton's mass) Defined by ratio $F/a = dp/du$ of relativistic force to acceleration.

That is ratio of change $dp = Mc\cosh\rho\, d\rho$ in momentum to change $du = c\,\mathrm{sech}^2\rho\, d\rho$ in group velocity.

$$M_{eff} \equiv \frac{dp}{du} = M_{rest}\frac{c\cosh\rho}{c\,\mathrm{sech}^2\rho} = M_{rest}\cosh^3\rho$$

Definition(s) of mass for relativity/quantum

Given: Energy: $E = Mc^2 \cosh\rho$

$= h\upsilon_{phase}$

momentum: $cp = Mc^2 \sinh\rho$

$= hc\kappa_{phase}$

velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

Rest Mass M_{rest} *(Einstein's mass)*

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Defines invariant hyperbola(s)

$$E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2}$$

$$\frac{h\upsilon_{phase}}{c^2} = M_{rest}$$

Rest Mass

Momentum Mass M_{mom} *(Galileo's mass)* Defined by ratio *p/u* of relativistic momentum to group velocity.

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Effective Mass

Limiting cases:

$$M_{eff} \xrightarrow[u\to c]{} M_{rest}e^{3\rho}/2$$

$$M_{eff} \xrightarrow[u\ll c]{} M_{rest}$$

More common derivation using group velocity: $u \equiv V_{group} = \frac{d\omega}{dk} = \frac{d\upsilon}{d\kappa}$

$$M_{eff} \equiv \frac{dp}{du} = \frac{\hbar dk}{dV_{group}} = \frac{\hbar}{\frac{d}{dk}\frac{d\omega}{dk}} = \frac{\hbar}{\frac{d^2\omega}{dk^2}} = \frac{M_{rest}}{\left(1-u^2/c^2\right)^{3/2}}$$

Definition(s) of mass for relativity/quantum

Given: Energy: $E = Mc^2 \cosh\rho$
$= h\upsilon_{phase}$

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Group velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

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$$\frac{h\upsilon_{phase}}{c^2} = M_{rest}$$

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Momentum Mass

Limiting cases:

$$M_{mom} \xrightarrow[u\to c]{} M_{rest}e^{\rho}/2$$

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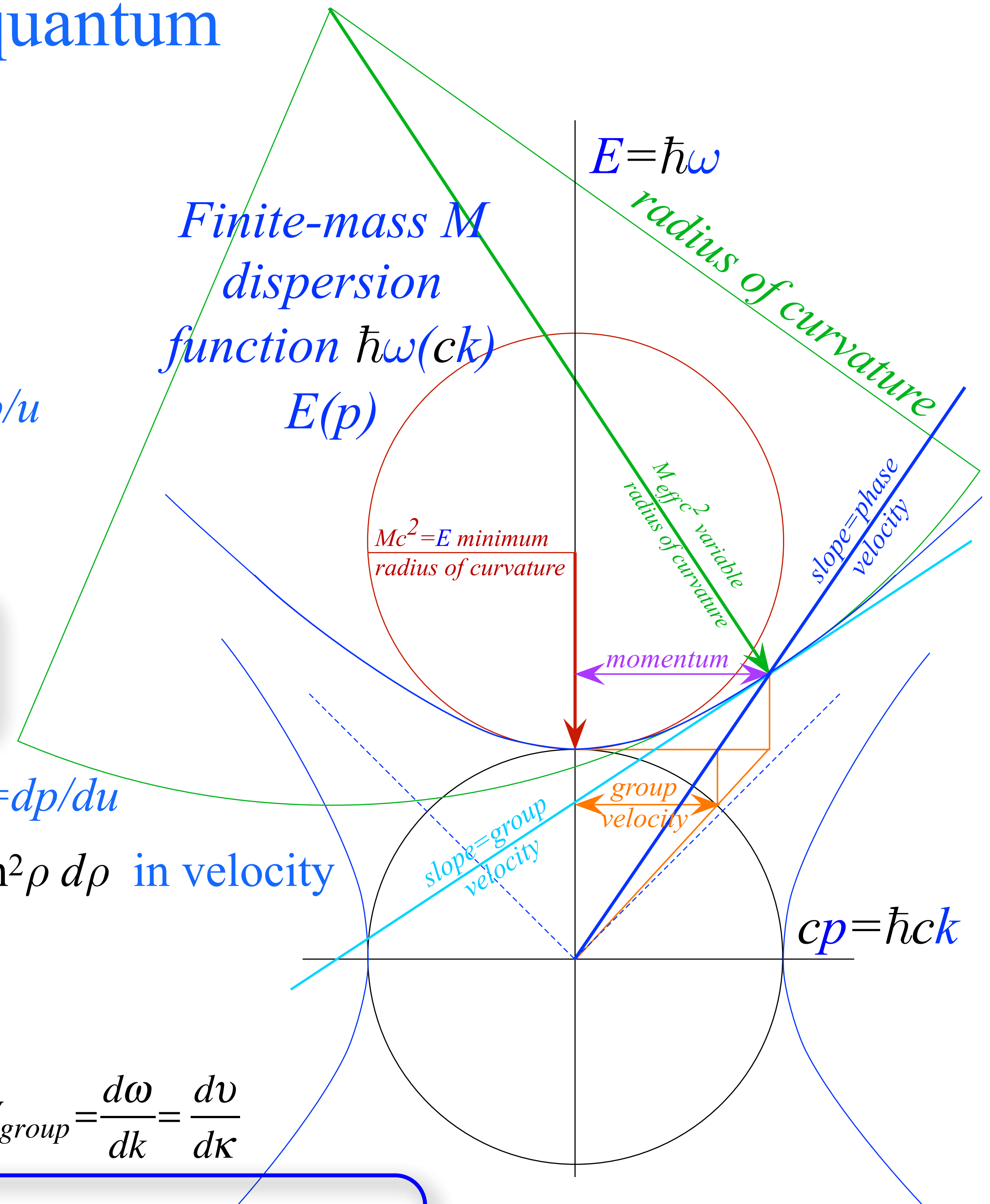
$$M_{eff} \equiv \frac{dp}{du} = \frac{\hbar dk}{dV_{group}} = \frac{\hbar}{\frac{d}{dk}\frac{d\omega}{dk}} = \frac{\hbar}{\frac{d^2\omega}{dk^2}} = \frac{M_{rest}}{\left(1 - u^2/c^2\right)^{3/2}} = M_{rest}\cosh^3\rho$$

Effective Mass

general wave formula to accompany $V_{group} = \frac{d\omega}{dk}$

Definition(s) of mass for relativity/quantum



Rest Mass M_{rest} *(Einstein's mass)*

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

$$\frac{h\upsilon_{phase}}{c^2} = M_{rest} = \frac{hc\kappa_{phase}}{c^2} \quad \text{Rest Mass}$$

Momentum Mass M_{mom} *(Galileo's mass)* Defined by *p/u*

$$M_{mom} \equiv \frac{p}{u} = \frac{M_{rest}\, c \sinh\rho}{c \tanh\rho}$$

$$= M_{rest} \cosh\rho = \frac{M_{rest}}{\sqrt{1-u^2/c^2}} \quad \text{Momentum Mass}$$

Effective Mass M_{eff} *(Newton's mass)* Defined by *F/a=dp/du*

That is ratio of $dp = Mc \cosh\rho\, d\rho$ to change $du = c\, \mathrm{sech}^2\rho\, d\rho$ in velocity

$$M_{eff} \equiv \frac{dp}{du} = M_{rest}\frac{c\cosh\rho}{c\,\mathrm{sech}^2\rho} = M_{rest}\cosh^3\rho \quad \text{Effective Mass}$$

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general wave formula to accompany $V_{group} = \frac{d\omega}{dk}$

Effective mass is proportional to the *radius of curvature* of $\omega(k)$ *dispersion*.

Definition(s) of mass for relativity/quantum

How much mass does a γ-photon have?

Rest Mass (a) γ-rest mass: $M^{\gamma}_{rest} = 0$,

Momentum Mass (b) γ-momentum mass: $M^{\gamma}_{mom} = \frac{p}{c} = \frac{h\kappa}{c} = \frac{h\upsilon}{c^2}$,

Effective Mass (c) γ-effective mass: $M^{\gamma}_{eff} = \infty$.

Newton complained about *his* "corpuscles" of light having *"fits"* (going *crazy*).

(All *this* would be evidence of *triple Schizophrenia*.)

$$M^{\gamma}_{mom} = \frac{h\upsilon}{c^2} = \upsilon(1.2 \cdot 10^{-51})kg \cdot s = 4.5 \cdot 10^{-36} kg \quad \text{(for: } \upsilon\text{=600THz)}$$

Pirelli site discussion of optical mass-energy

https://pirelli.hosted.uark.edu/html/light_energy_flux_1.html

Pirelli site discussion of Wave amplitude effects

https://pirelli.hosted.uark.edu/html/amplitude_probability_1.html

Lecture 30
Wed. 12.10.2019

Review: Relawavity ρ functions and plots vs. ρ

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity
Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry
"Occams Sword" and summary of 16 parameter functions of ρ and σ
Applications to optical waveguide, spherical waves, and accelerator radiation

Derivation of relativistic quantum mechanics
What's the matter with mass? Shining some light on the Elephant in the room
➔ Relativistic action and Lagrangian-Hamiltonian relations
Poincare' and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae
Feynman diagram geometry
Compton recoil related to rocket velocity formula
Comparing 2nd-quantization "photon" number N and 1st-quantization wavenumber κ

Relawavity in accelerated frames
Laser up-tuning by Alice and down-tuning by Carla makes *g*-acceleration grid
Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid
Animation of mechanics and metrology of constant-g grid

Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega' t'$ for wave of $k = k_{phase}$ and $\omega = \omega_{phase}$.

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega$$

$$\hbar \equiv \frac{h}{2\pi}$$

Prior wave relations

$$h\upsilon_A = Mc^2 = hc\kappa_A$$
$$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$$
$$hc\kappa_{phase} = cp = h\upsilon_A \sinh\rho$$

←linear Hz format

angular phasor→ format

$$\hbar\omega_A = Mc^2 = \hbar ck_A$$
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Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega' t'$ for wave of $k = k_{phase}$ and $\omega = \omega_{phase}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar\omega$ relation

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega$$

$$\hbar \equiv \frac{h}{2\pi}$$

$$p = \hbar k = Mc \sinh\rho \qquad E = \hbar\omega = Mc^2 \cosh\rho$$

	Prior wave relations	
$h\upsilon_A = Mc^2 = hc\kappa_A$		$\hbar\omega_A = Mc^2 = \hbar c k_A$
$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$	←linear Hz format angular phasor→ format	$\hbar\omega_{phase} = E = \hbar\omega_A \cosh\rho$
$hc\kappa_{phase} = cp = h\upsilon_A \sinh\rho$		$\hbar c k_{phase} = cp = \hbar\omega_A \sinh\rho$

$$\hbar \equiv \frac{h}{2\pi}$$

Relativistic action S and Lagrangian-Hamiltonian relations

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$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E \qquad \hbar \equiv \frac{h}{2\pi}$$

$$p = \hbar k = Mc \sinh\rho \qquad E = \hbar\omega = Mc^2 \cosh\rho$$

Prior wave relations

$h\upsilon_A = Mc^2 = hc\kappa_A$

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←linear Hz format

angular phasor format→

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Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar\omega$ relation to define *Hamiltonian* $H = E$

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L$$

Legendre transformation

$$p = \hbar k = Mc \sinh\rho \qquad E = \hbar\omega = Mc^2 \cosh\rho = H$$

Prior wave relations

←linear Hz format

$$h\upsilon_A = Mc^2 = hc\kappa_A$$
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angular phasor→ format

$$\hbar\omega_A = Mc^2 = \hbar ck_A$$
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$$\hbar \equiv \frac{h}{2\pi}$$

Relativistic action S and Lagrangian-Hamiltonian relations

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Legendre transformation

Use *Group velocity* : $u = \frac{dx}{dt} = c \tanh\rho$

$$p = \hbar k = Mc \sinh\rho \qquad E = \hbar\omega = Mc^2 \cosh\rho = H$$

Prior wave relations

$h\upsilon_A = Mc^2 = hc\kappa_A$

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Legendre transformation

Use *Group velocity* : $u=\frac{dx}{dt}=c\tanh\rho$

$$p = \hbar k = Mc\sinh\rho \qquad E = \hbar\omega = Mc^2\cosh\rho = H$$

$$L = pu - H = (Mc\sinh\rho)(c\tanh\rho) - Mc^2\cosh\rho$$

$h\upsilon_A = Mc^2 = hc\kappa_A$

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Legendre transformation

Use *Group velocity* : $u = \frac{dx}{dt} = c\tanh\rho$

$$p = \hbar k = Mc\sinh\rho \qquad E = \hbar\omega = Mc^2\cosh\rho = H$$

$$L = pu - H = (Mc\sinh\rho)(c\tanh\rho) - Mc^2\cosh\rho$$

$$= Mc^2\frac{\sinh^2\rho - \cosh^2\rho}{\cosh\rho} = -Mc^2\mathrm{sech}\rho$$

$$L \text{ is}: Mc^2\frac{-1}{\cosh\rho} = -Mc^2\mathrm{sech}\rho$$

Prior wave relations

←linear Hz format	angular phasor→ format
$h\upsilon_A = Mc^2 = hc\kappa_A$	$\hbar\omega_A = Mc^2 = \hbar ck_A$
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Relativistic action S and Lagrangian-Hamiltonian relations

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Legendre transformation

Use *Group velocity* : $u = \frac{dx}{dt} = c \tanh\rho$

$$p = \hbar k = Mc \sinh\rho \qquad E = \hbar\omega = Mc^2 \cosh\rho = H$$

$$L = pu - H = (Mc\sinh\rho)(c\tanh\rho) - Mc^2\cosh\rho$$

Note: $Mcu = Mc^2 \tanh\rho$

$$= Mc^2 \frac{\sinh^2\rho - \cosh^2\rho}{\cosh\rho} = -Mc^2 \operatorname{sech}\rho$$

Compare *Lagrangian L*

$$L = \hbar\dot{\Phi} = -Mc^2\sqrt{1 - \frac{u^2}{c^2}} = -Mc^2 \operatorname{sech}\rho$$

$h\upsilon_A = Mc^2 = hc\kappa_A$

$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$

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Prior wave relations

←linear Hz format   angular phasor format→

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Relativistic action S and Lagrangian-Hamiltonian relations

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$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L$$

Legendre transformation

Use *Group velocity* : $u = \frac{dx}{dt} = c \tanh\rho$

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Note: $Mcu = Mc^2 \tanh\rho$

$$= Mc^2 \frac{\sinh^2\rho - \cosh^2\rho}{\cosh\rho} = -Mc^2 \operatorname{sech}\rho$$

Compare *Lagrangian L*

$$L = \hbar\dot{\Phi} = -Mc^2\sqrt{1 - \frac{u^2}{c^2}} = -Mc^2 \operatorname{sech}\rho$$

with *Hamiltonian H=E*

$$H = \hbar\omega = Mc^2 \Big/ \sqrt{1 - \frac{u^2}{c^2}} = Mc^2\cosh\rho$$

$$h\upsilon_A = Mc^2 = hc\kappa_A$$
$$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$$
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Prior wave relations

←linear Hz format   angular phasor format→

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Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian* L using invariant wave phase $\Phi=kx-\omega t=k'x'-\omega't'$ for wave of $k=k_{phase}$ and $\omega=\omega_{phase}$.

Use DeBroglie-momentum $p=\hbar k$ relation and Planck-energy $E=\hbar\omega$ relation to define *Hamiltonian* $H=E$

$$L\equiv\hbar\frac{d\Phi}{dt}=\hbar k\frac{dx}{dt}-\hbar\omega=p\frac{dx}{dt}-E\equiv p\dot{x}-E\equiv pu-H=L$$

Legendre transformation

Use *Group velocity*: $u=\frac{dx}{dt}=c\tanh\rho$

$$p=\hbar k=Mc\sinh\rho \qquad E=\hbar\omega=Mc^2\cosh\rho=H$$

$$L=pu-H=(Mc\sinh\rho)(c\tanh\rho)-Mc^2\cosh\rho$$

$$=Mc^2\frac{\sinh^2\rho-\cosh^2\rho}{\cosh\rho}=-Mc^2\operatorname{sech}\rho$$

Note: $Mcu=Mc^2\tanh\rho$

Also: $cp=Mc^2\sinh\rho$

Compare *Lagrangian* L

$$L=\hbar\dot{\Phi}=-Mc^2\sqrt{1-\frac{u^2}{c^2}}=-Mc^2\operatorname{sech}\rho$$

with *Hamiltonian* $H=E$

$$H=\hbar\omega=Mc^2\Big/\sqrt{1-\frac{u^2}{c^2}}=Mc^2\cosh\rho$$

$$=Mc^2\sqrt{1+\sinh^2\rho}=Mc^2\sqrt{1+(cp)^2}$$

$\hbar=h/2\pi$

Prior wave relations

$h\upsilon_A=Mc^2=hc\kappa_A$
$h\upsilon_{phase}=E=h\upsilon_A\cosh\rho$
$hc\kappa_{phase}=cp=h\upsilon_A\sinh\rho$

←linear Hz format

angular phasor format→

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$$\frac{dS}{dt} = L \equiv \hbar\frac{d\Phi}{dt} = \hbar k\frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L$$

Legendre transformation

Use *Group velocity*: $u = \frac{dx}{dt} = c\tanh\rho$

$= c\sin\sigma$

$$p = \hbar k = Mc\sinh\rho \qquad E = \hbar\omega = Mc^2\cosh\rho = H$$

$$L = pu - H = (Mc\sinh\rho)(c\tanh\rho) - Mc^2\cosh\rho$$

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Note: $Mcu = Mc^2\tanh\rho$

$= Mc^2\sin\sigma$

Also: $cp = Mc^2\sinh\rho$

$= \hbar ck = Mc^2\tan\sigma$

Compare *Lagrangian L*

$$\dot{S} = L = \hbar\dot{\Phi} = -Mc^2\sqrt{1-\frac{u^2}{c^2}} = -Mc^2\operatorname{sech}\rho = -Mc^2\cos\sigma$$

with *Hamiltonian* $H = E$

$$H = \hbar\omega = Mc^2\Big/\sqrt{1-\frac{u^2}{c^2}} = Mc^2\cosh\rho = Mc^2\sec\sigma$$

$$= Mc^2\sqrt{1+\sinh^2\rho} = Mc^2\sqrt{1+(cp)^2}$$

Define *Action* $S = \hbar\Phi$

Including stellar angle σ

Prior wave relations

$h\upsilon_A = Mc^2 = hc\kappa_A$

$h\upsilon_{phase} = E = h\upsilon_A\cosh\rho$

$hc\kappa_{phase} = cp = h\upsilon_A\sinh\rho$

←linear Hz format

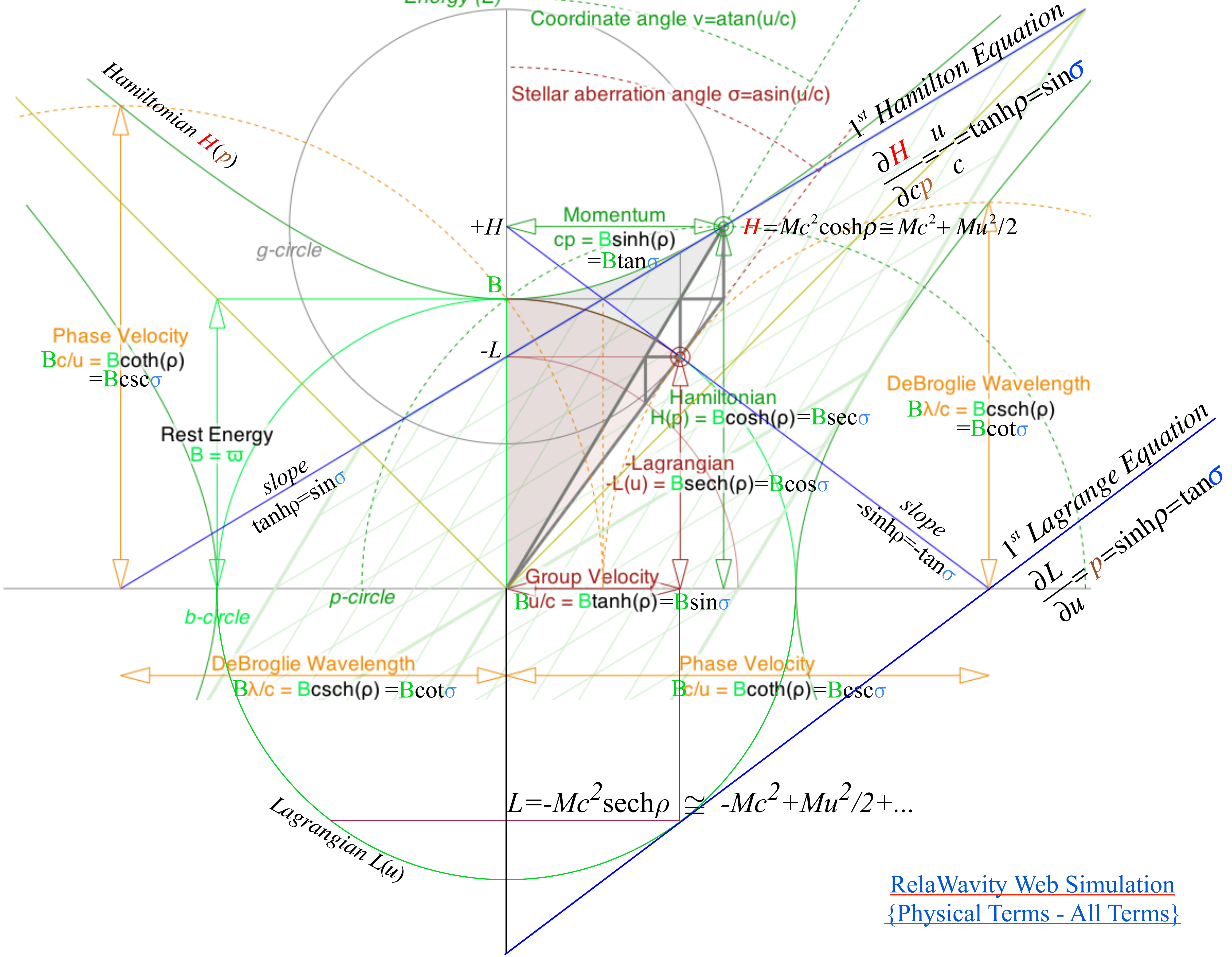
angular phasor format→

$\hbar\omega_A = Mc^2 = \hbar ck_A$

$\hbar\omega_{phase} = E = \hbar\omega_A\cosh\rho$

$\hbar ck_{phase} = cp = \hbar\omega_A\sinh\rho$

$$\hbar \equiv \frac{h}{2\pi}$$



RelaWavity Web Simulation
{Physical Terms - All Terms}

Lecture 30
Wed. 12.10.2019

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Define *Lagrangian L* using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega' t'$ for wave of $k = k_{phase}$ and $\omega = \omega_{phase}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar\omega$ relation to define *Hamiltonian* $H = E$

$$\frac{dS}{dt} = L \equiv \hbar\frac{d\Phi}{dt} = \hbar k\frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L$$

Legendre transformation

Compare *Lagrangian L*

$$\dot{S} = L = \hbar\dot{\Phi} = -Mc^2\sqrt{1-\frac{u^2}{c^2}} = -Mc^2\operatorname{sech}\rho = -Mc^2\cos\sigma$$

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Legendre transformation

$$dS \equiv L\,dt \equiv \hbar\, d\Phi = \hbar k\, dx - \hbar\omega\, dt = p\, dx - H\, dt$$

Poincare Invariant action differential

Compare *Lagrangian L*

$$\dot{S} = L = \hbar\dot{\Phi} = -Mc^2\sqrt{1-\frac{u^2}{c^2}} = -Mc^2\,\mathrm{sech}\rho = -Mc^2\cos\sigma$$

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Legendre transformation

Use *Group velocity* : $u = \frac{dx}{dt} = c \tanh\rho$

$$dS \equiv L\,dt \equiv \hbar\, d\Phi = \hbar k\, dx - \hbar\omega\, dt = p\,dx - H\,dt$$

Poincare Invariant action differential

$$\frac{\partial S}{\partial x} = p \qquad \frac{\partial S}{\partial t} = -H$$

Hamilton-Jacobi equations

Compare *Lagrangian L*

$$\dot{S} = L = \hbar\dot{\Phi} = -Mc^2\sqrt{1 - \frac{u^2}{c^2}} = -Mc^2 \operatorname{sech}\rho = -Mc^2 \cos\sigma$$

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$$\hbar\omega_{phase} = E = \hbar\omega_A \cosh\rho$$
$$\hbar c k_{phase} = cp = \hbar\omega_A \sinh\rho$$

$$\hbar \equiv \frac{h}{2\pi}$$

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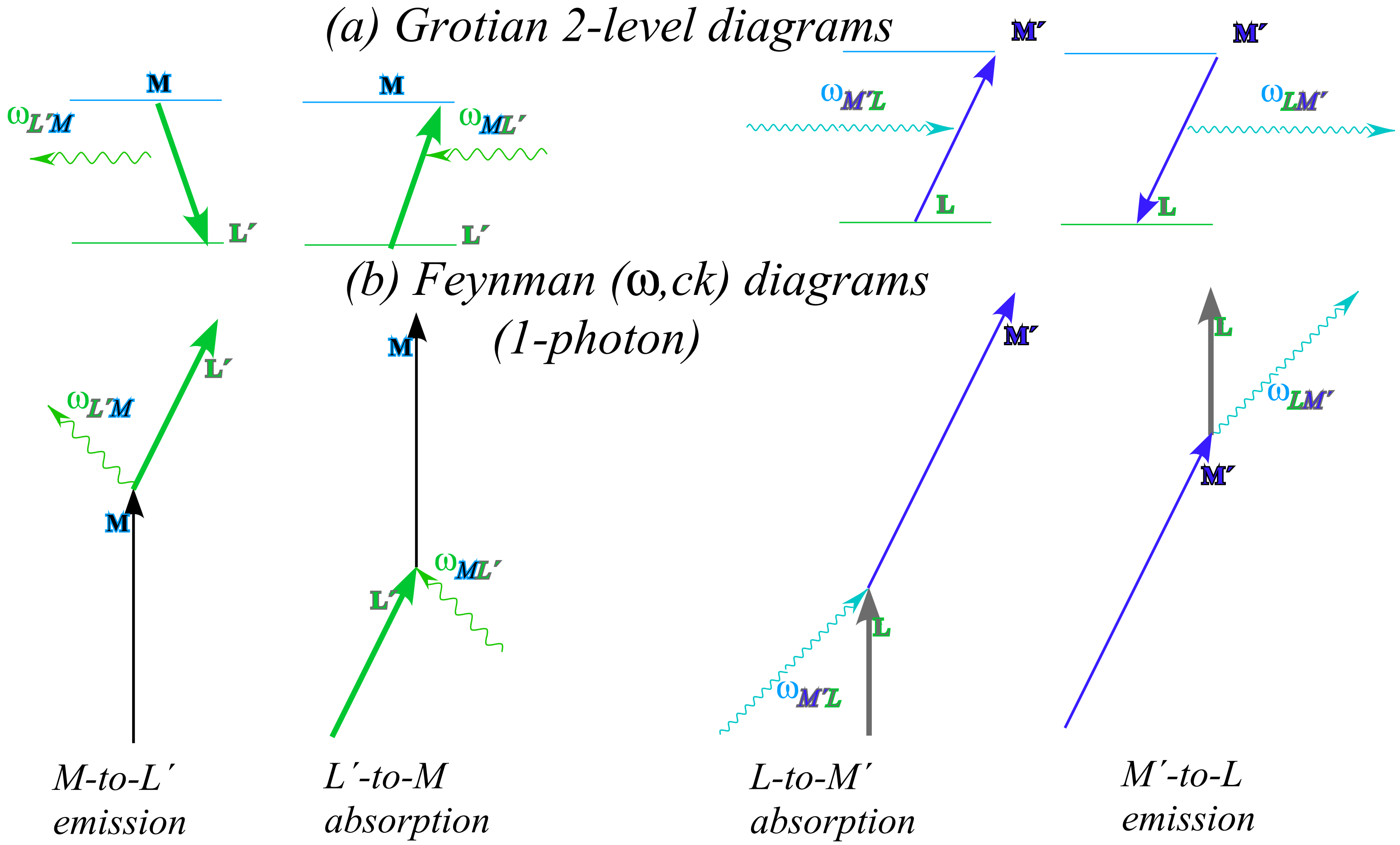
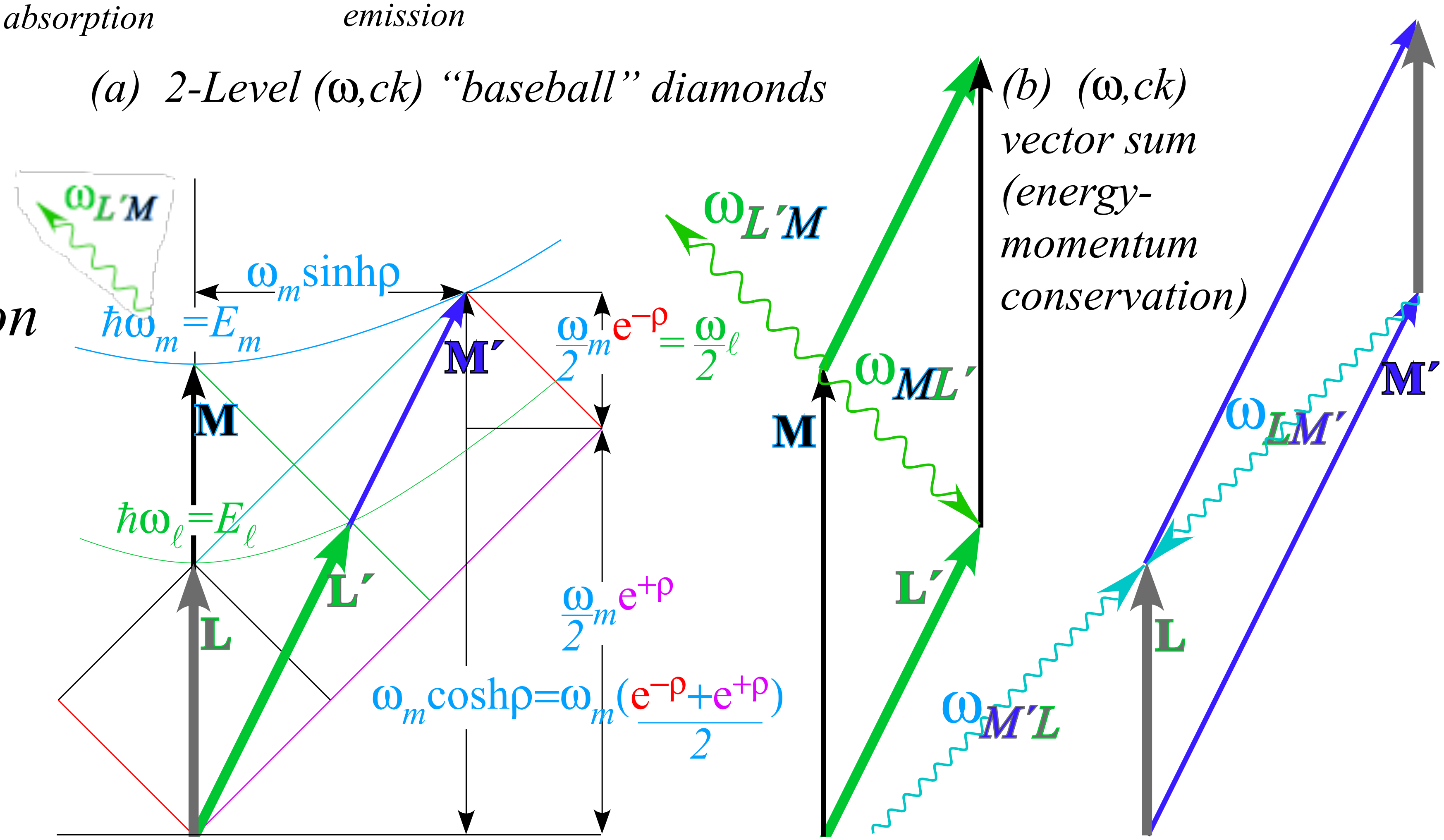
Relativistic optical transitions CMwBANG! Unit 8 Fig. 7.1 and 7.2



KEY: *m=mid-level*
ℓ=low-level

mid to *low*
m to *ℓ* transition gives photon



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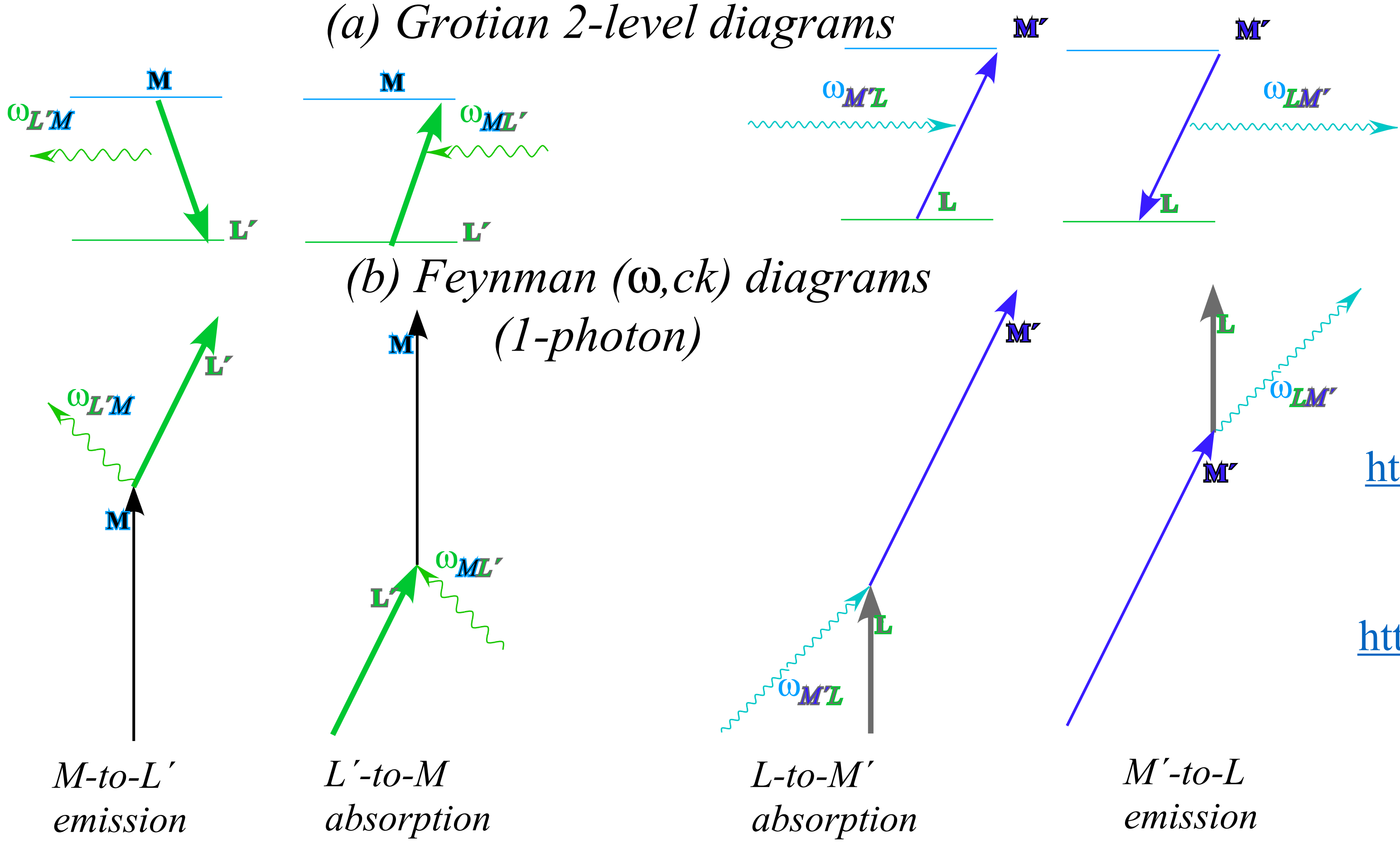


https://pirelli.hosted.uark.edu/html/poincare_inv_2.html

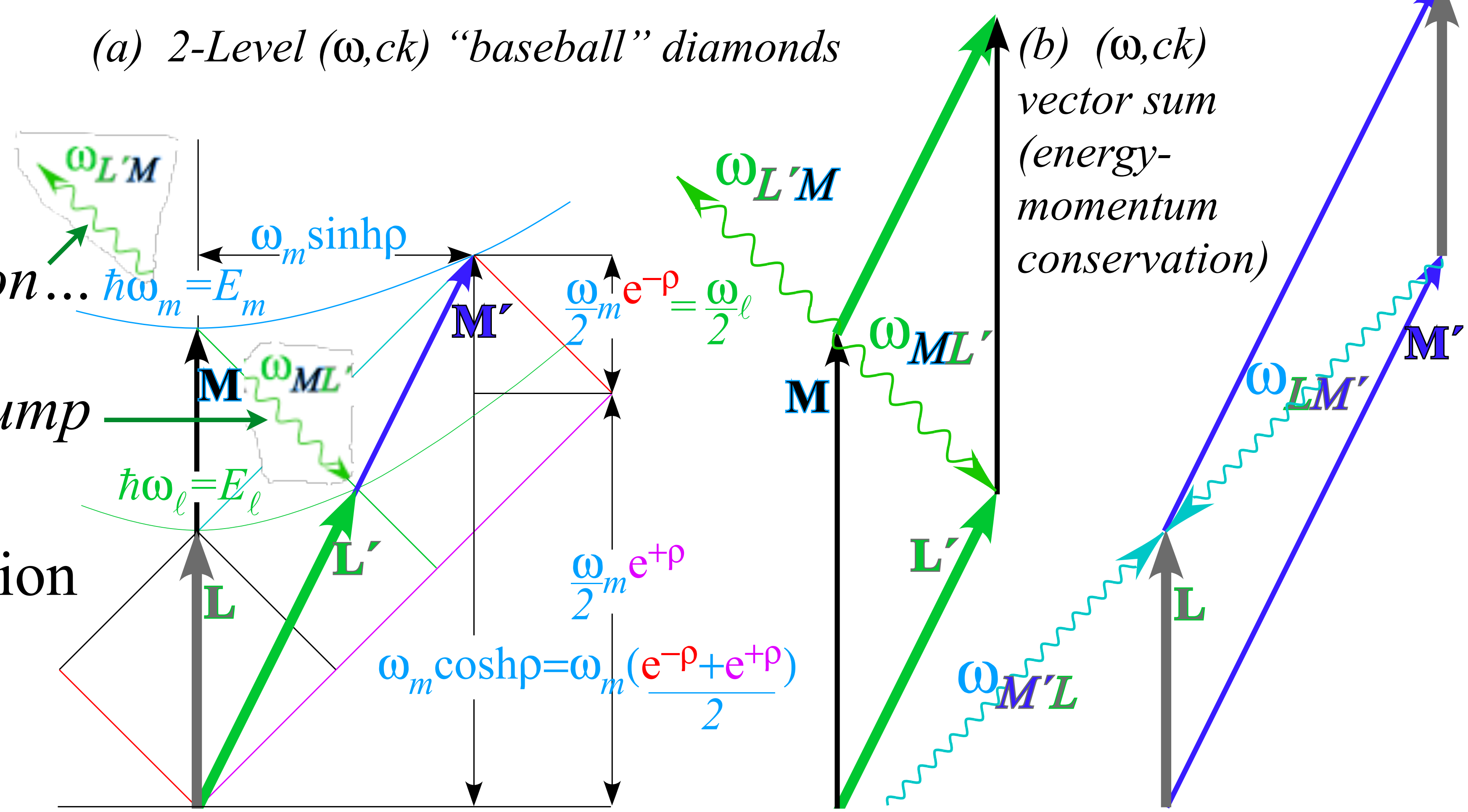
https://pirelli.hosted.uark.edu/html/compton_1.html

KEY: m=mid-level
ℓ=low-level

mid to low
m to ℓ transition gives photon…

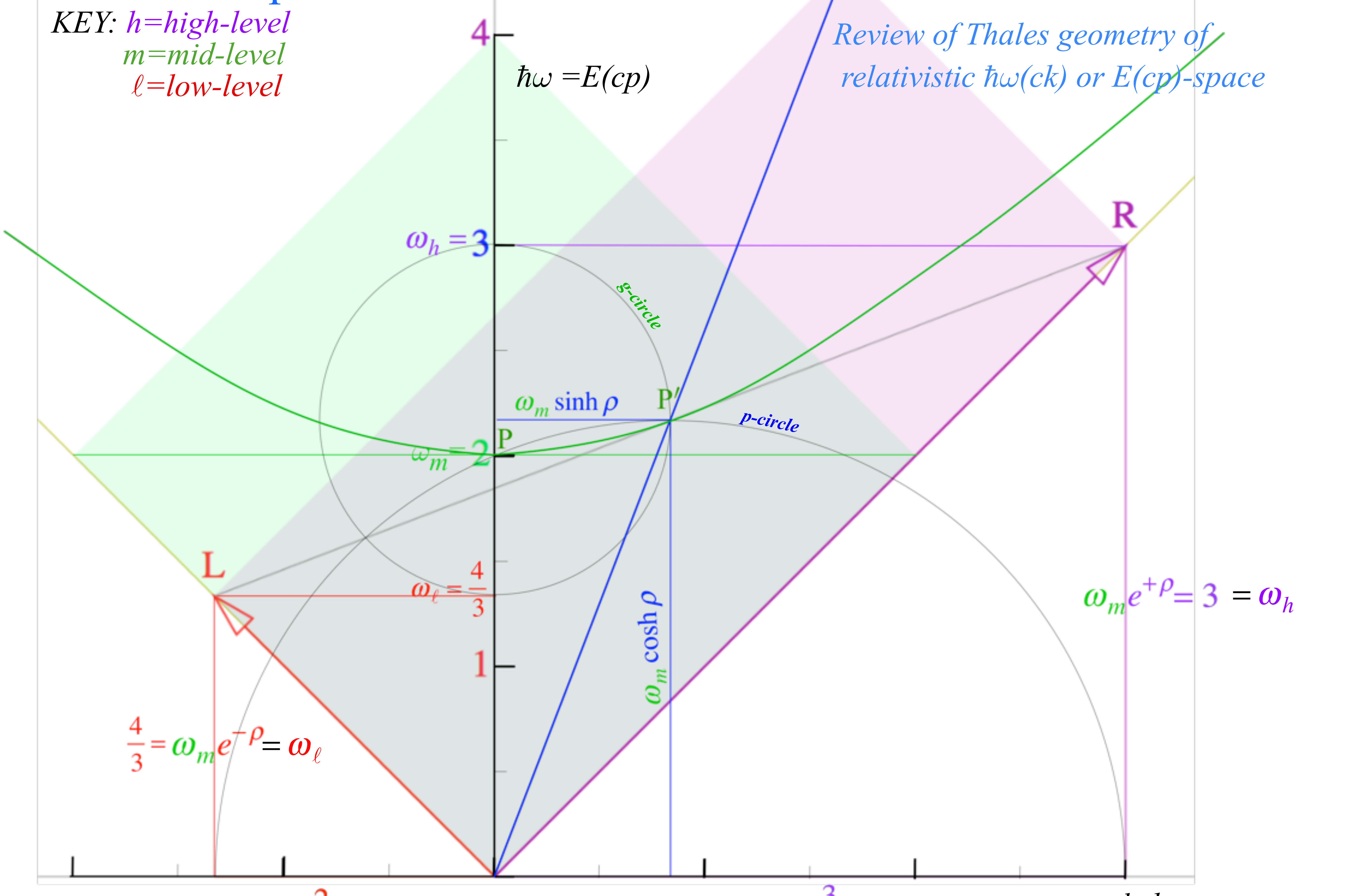
…and balancing atomic jump

(momentum-energy conservation
ia a wave coherence effect)



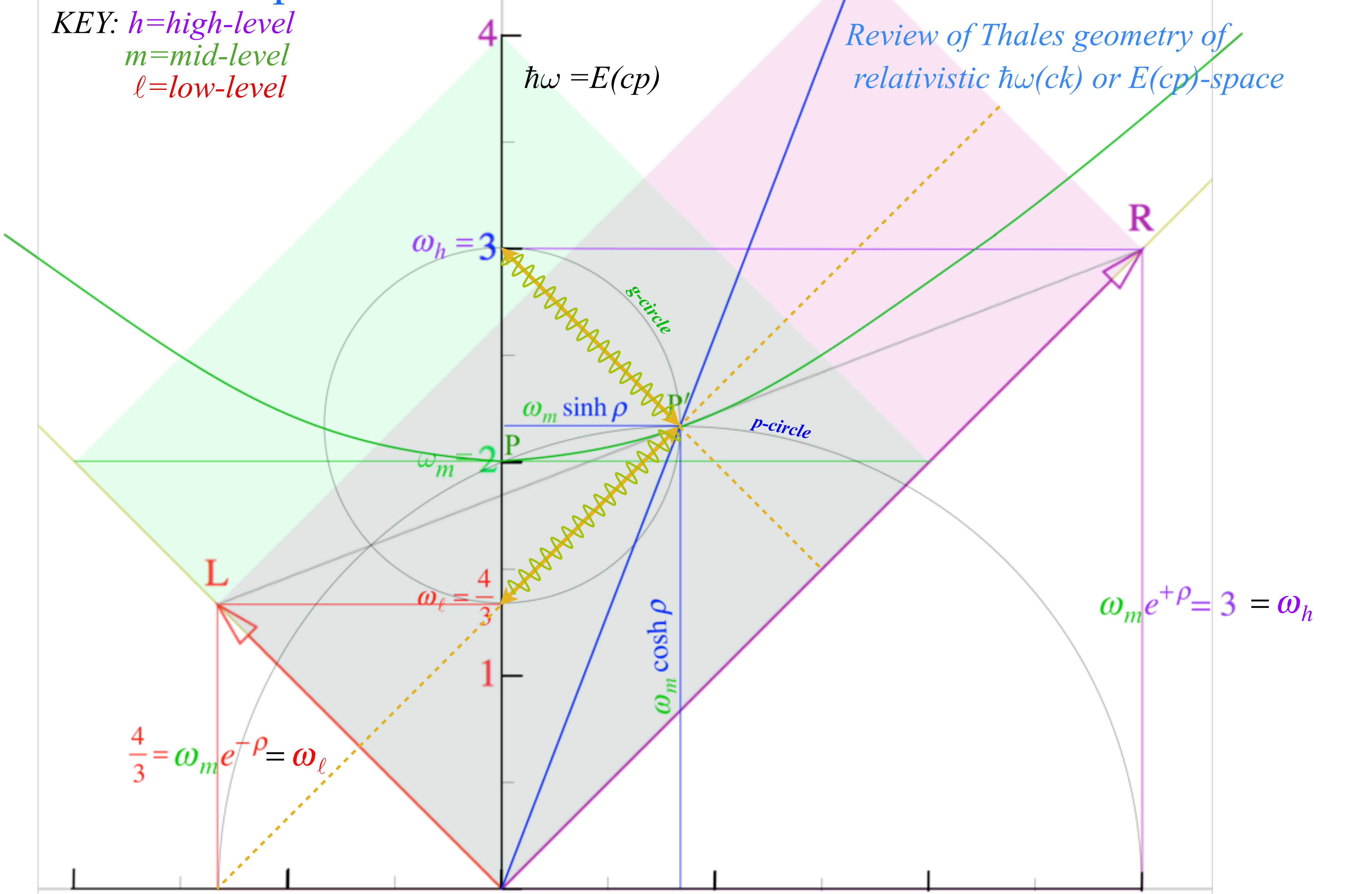
3-Level example

Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$



Doppler RED factor: $\frac{2}{3} = e^{-\rho}$ Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$



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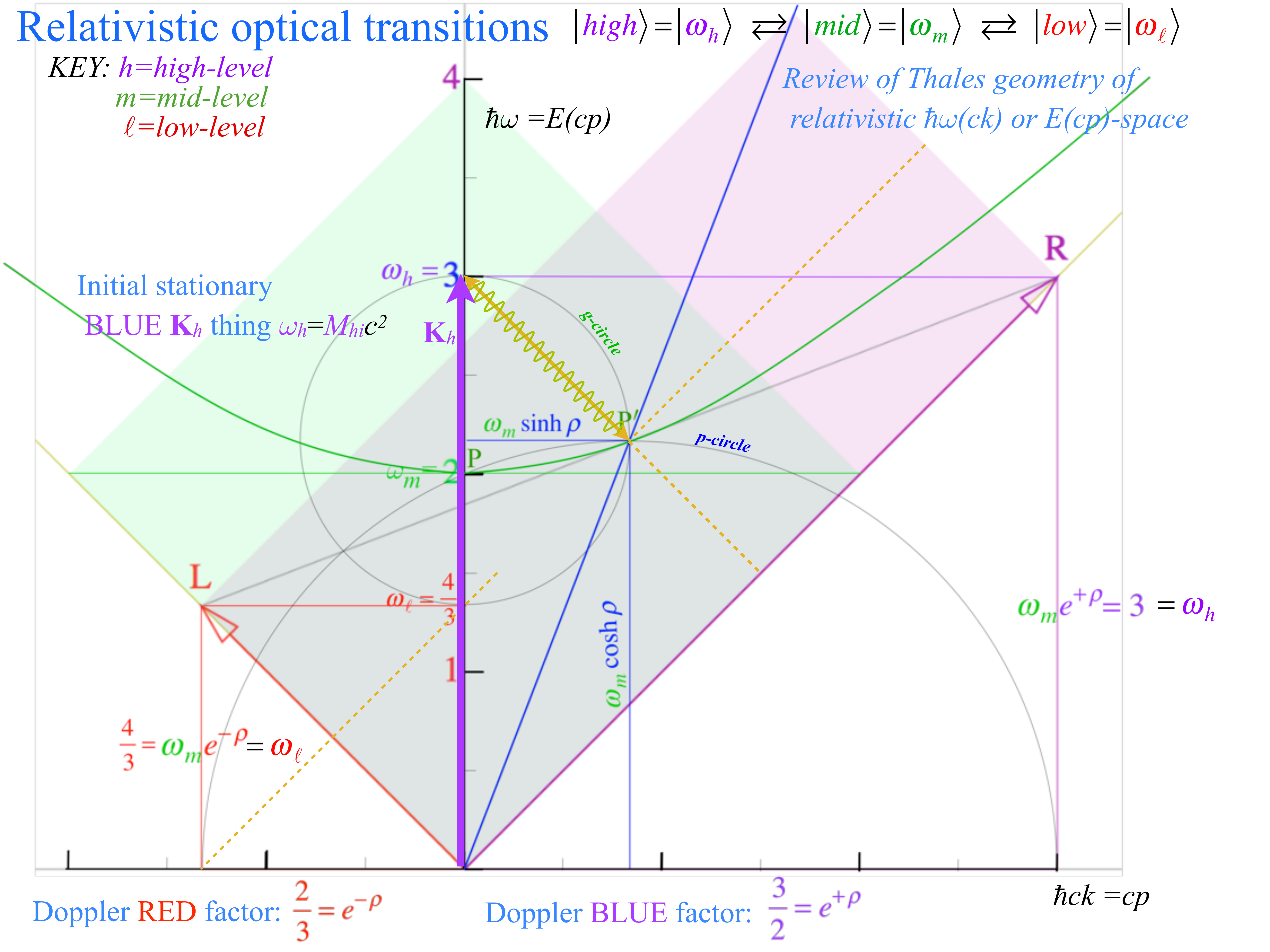

Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$
KEY: h=high-level
m=mid-level
ℓ=low-level
$\hbar\omega = E(cp)$
Review of Thales geometry of
relativistic $\hbar\omega(ck)$ or $E(cp)$-space
Initial stationary
BLUE $\mathbf{K}_h$ thing $\omega_h = M_{hi}c^2$
$\omega_h = 3$
$\mathbf{K}_h$
g-circle
p-circle
$\omega_m \sinh\rho$
P′
P
$\omega_m = 2$
R
L
$\omega_\ell = \frac{4}{3}$
$\omega_m \cosh\rho$
$\omega_m e^{+\rho} = 3 = \omega_h$
$\frac{4}{3} = \omega_m e^{-\rho} = \omega_\ell$
4
1
$\hbar ck = cp$
Doppler RED factor: $\frac{2}{3} = e^{-\rho}$
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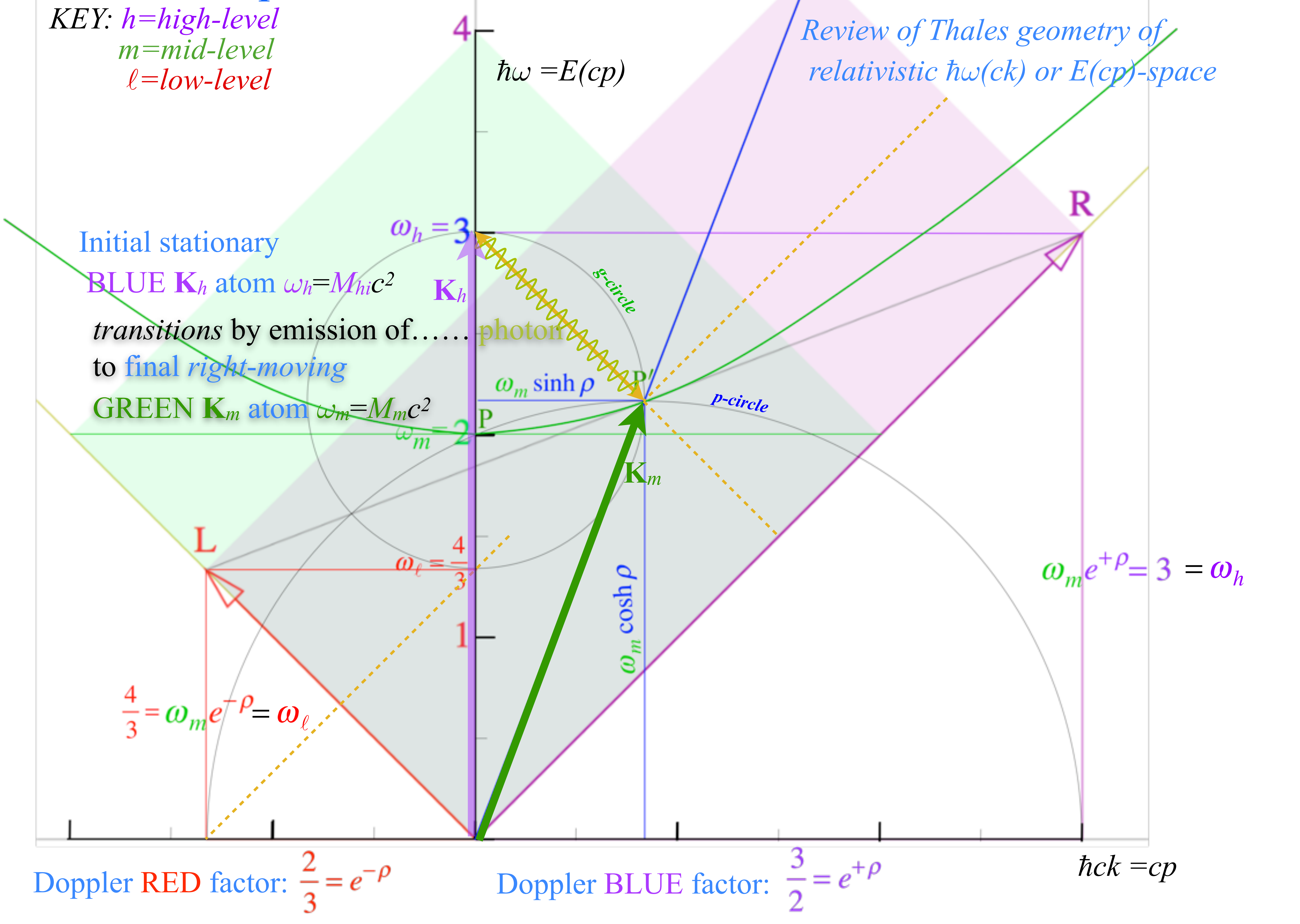
Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$



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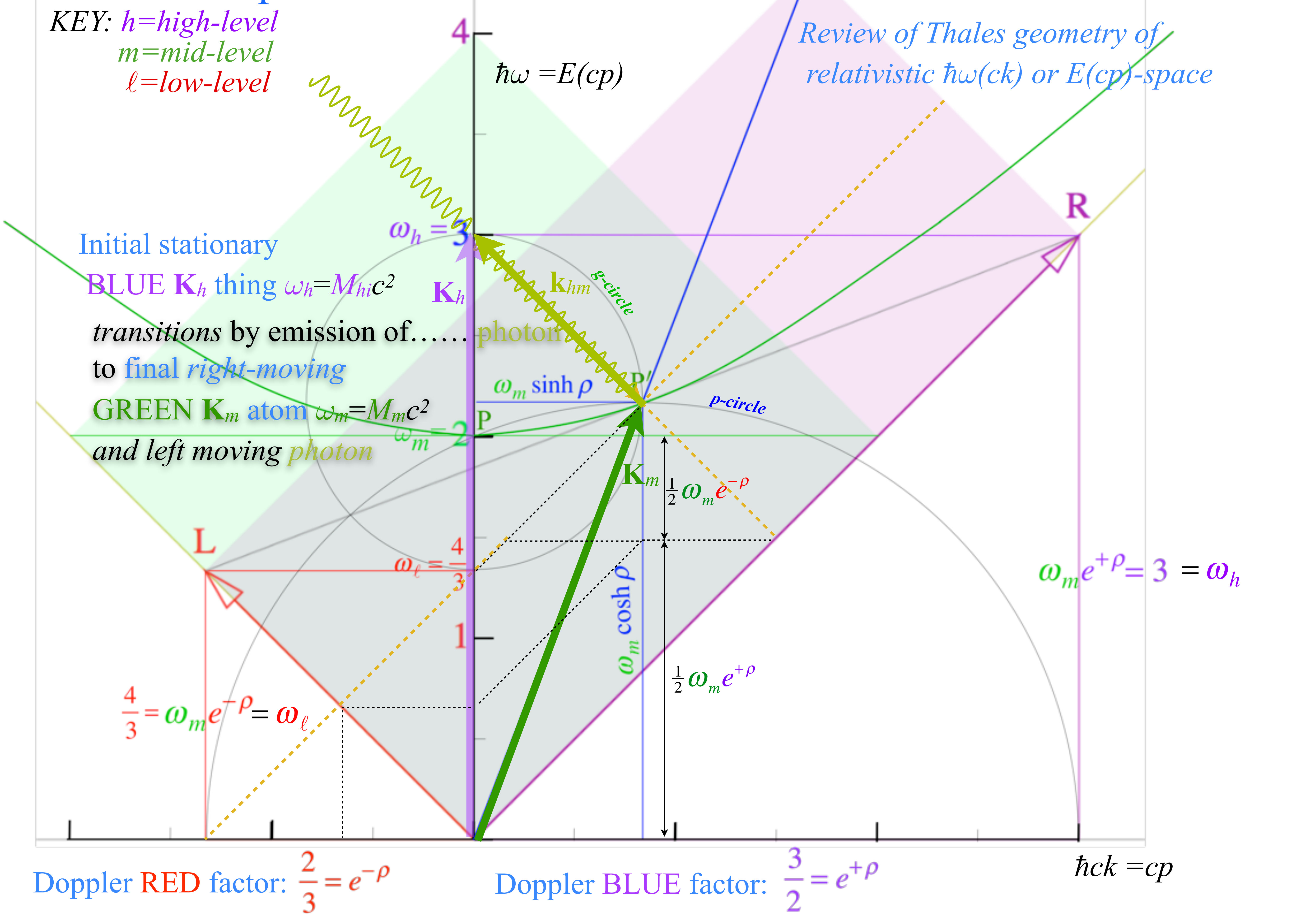

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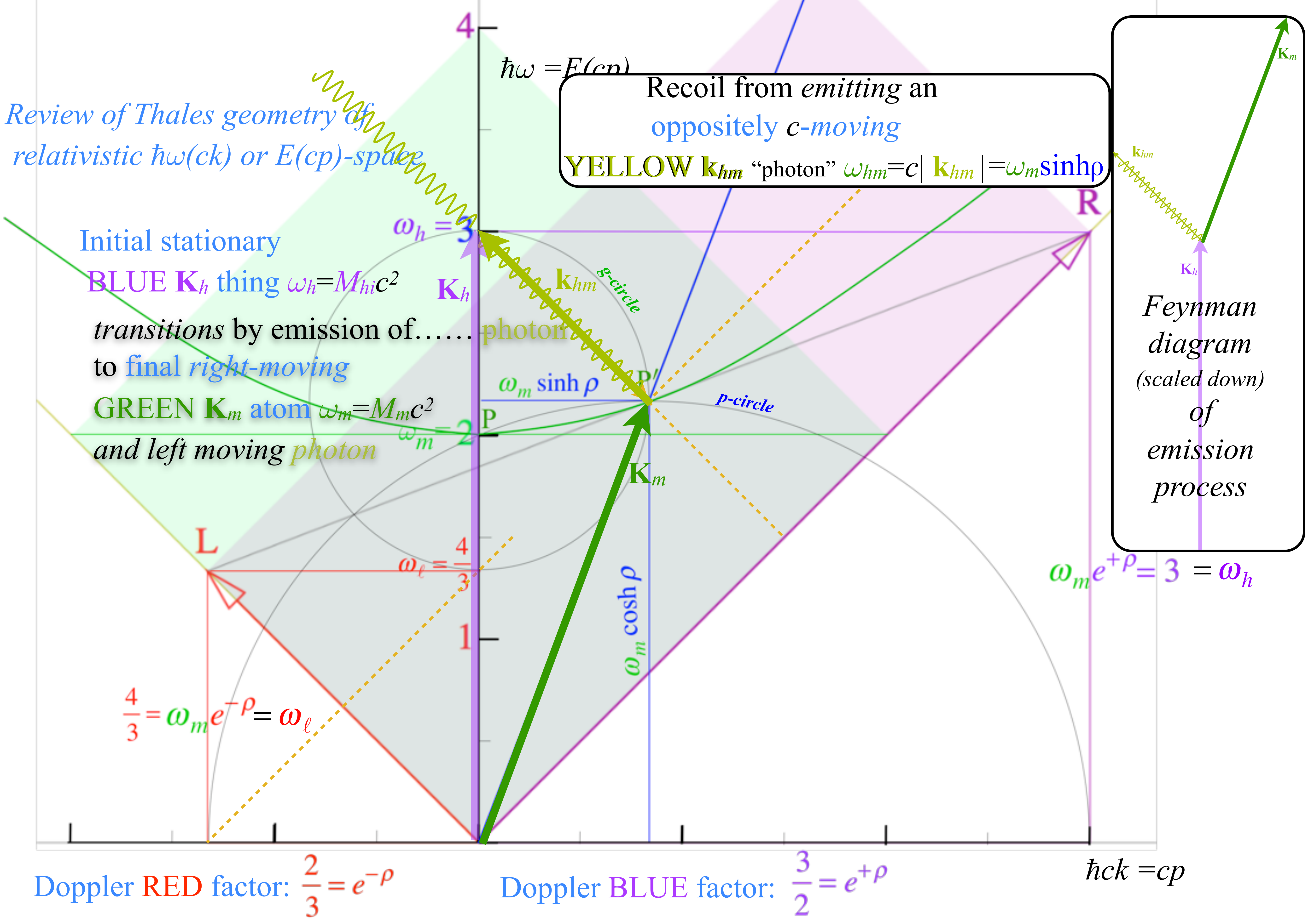

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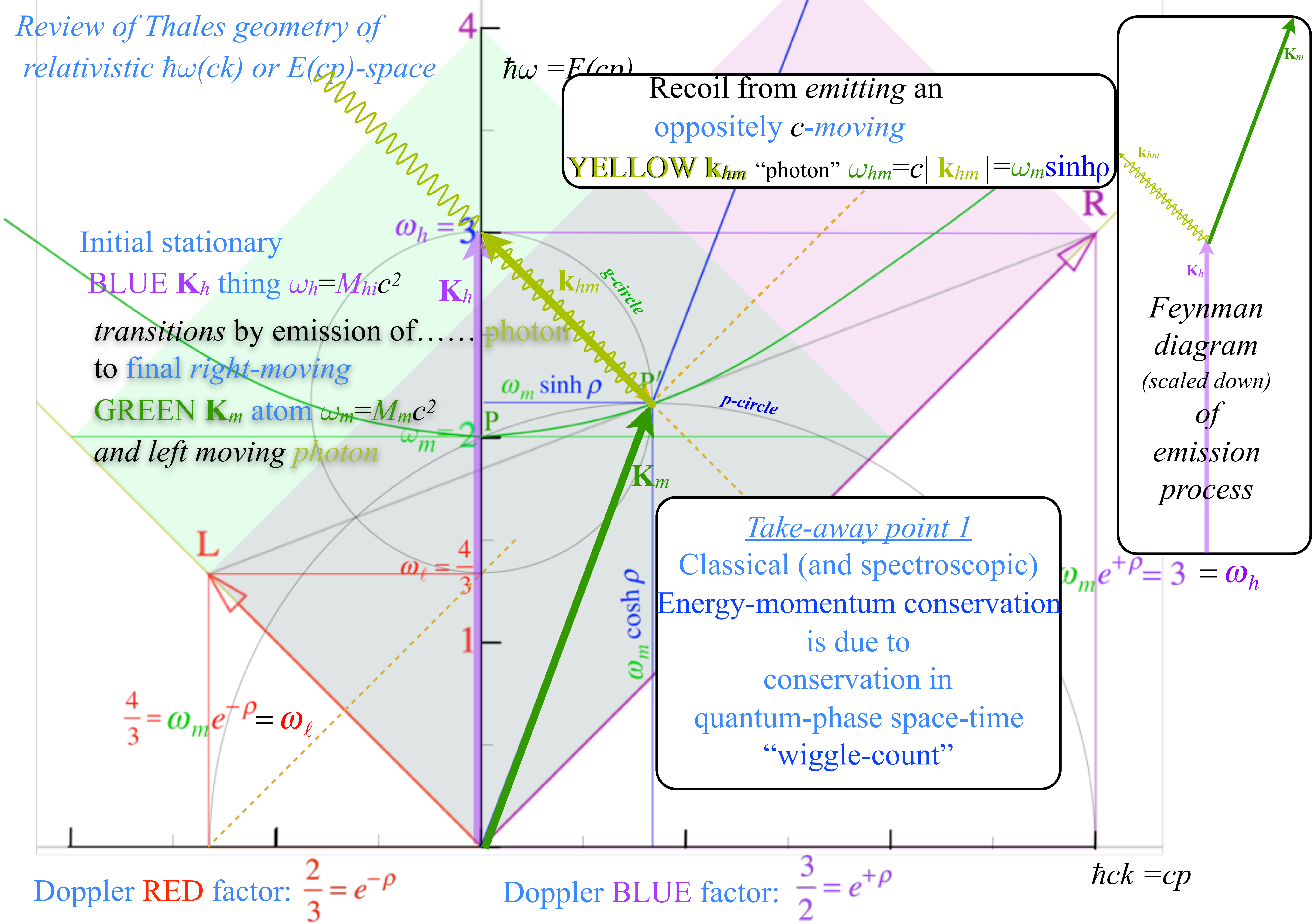

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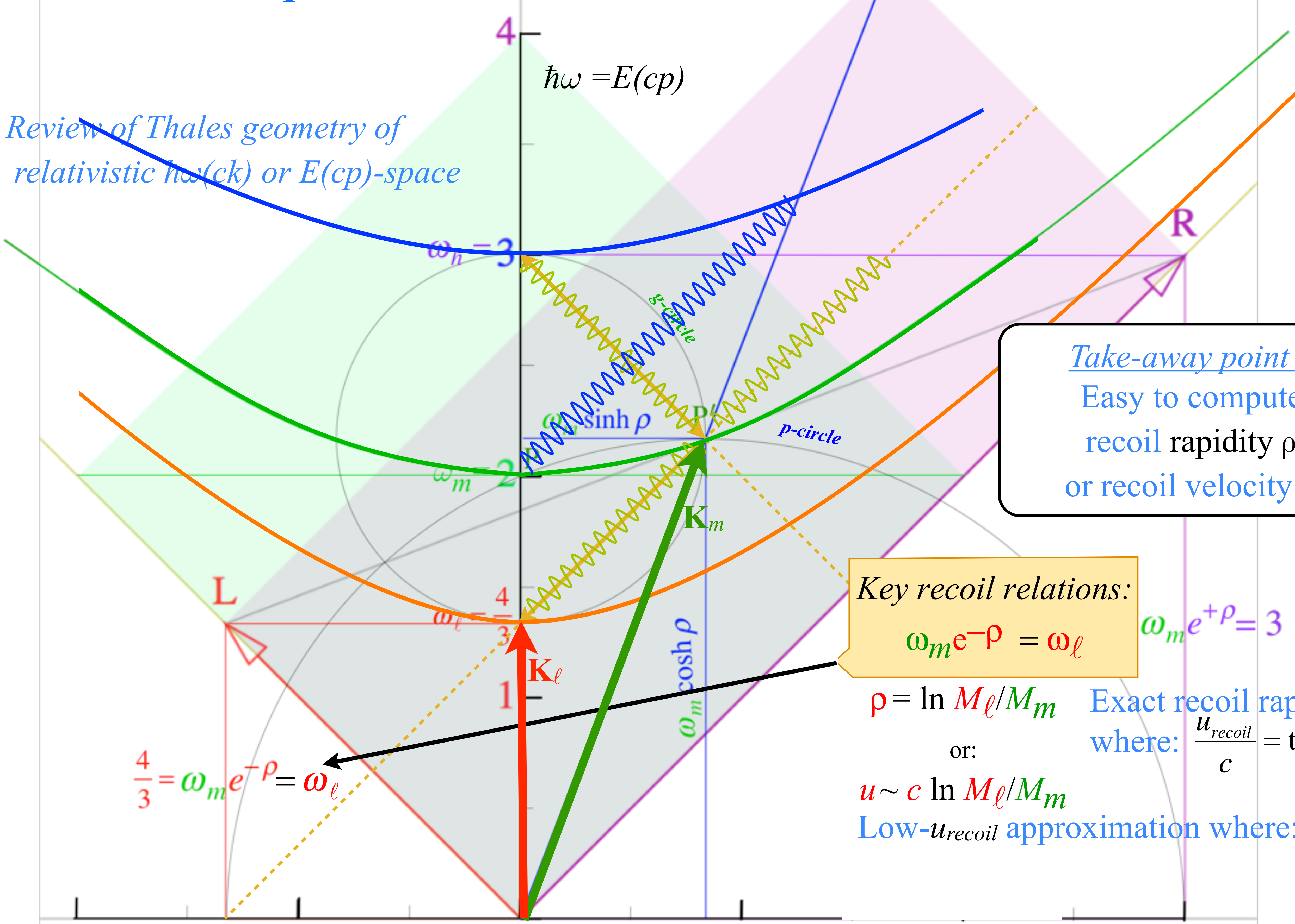
Review of Thales geometry of relativistic ħω(ck) or E(cp)-space



Take-away point 2
Easy to compute recoil rapidity ρ or recoil velocity u

Key recoil relations:

$\omega_m e^{-\rho} = \omega_\ell$

$\omega_m e^{+\rho} = 3 = \omega_h$

$\frac{4}{3} = \omega_m e^{-\rho} = \omega_\ell$

$\rho = \ln M_\ell / M_m$

or:

$u \sim c \ln M_\ell / M_m$

Exact recoil rapidity where: $\frac{u_{recoil}}{c} = \tanh \rho$

Low-u_{recoil} approximation where: $\rho \approx \frac{u_{recoil}}{c}$

Doppler RED factor: $\frac{2}{3} = e^{-\rho}$

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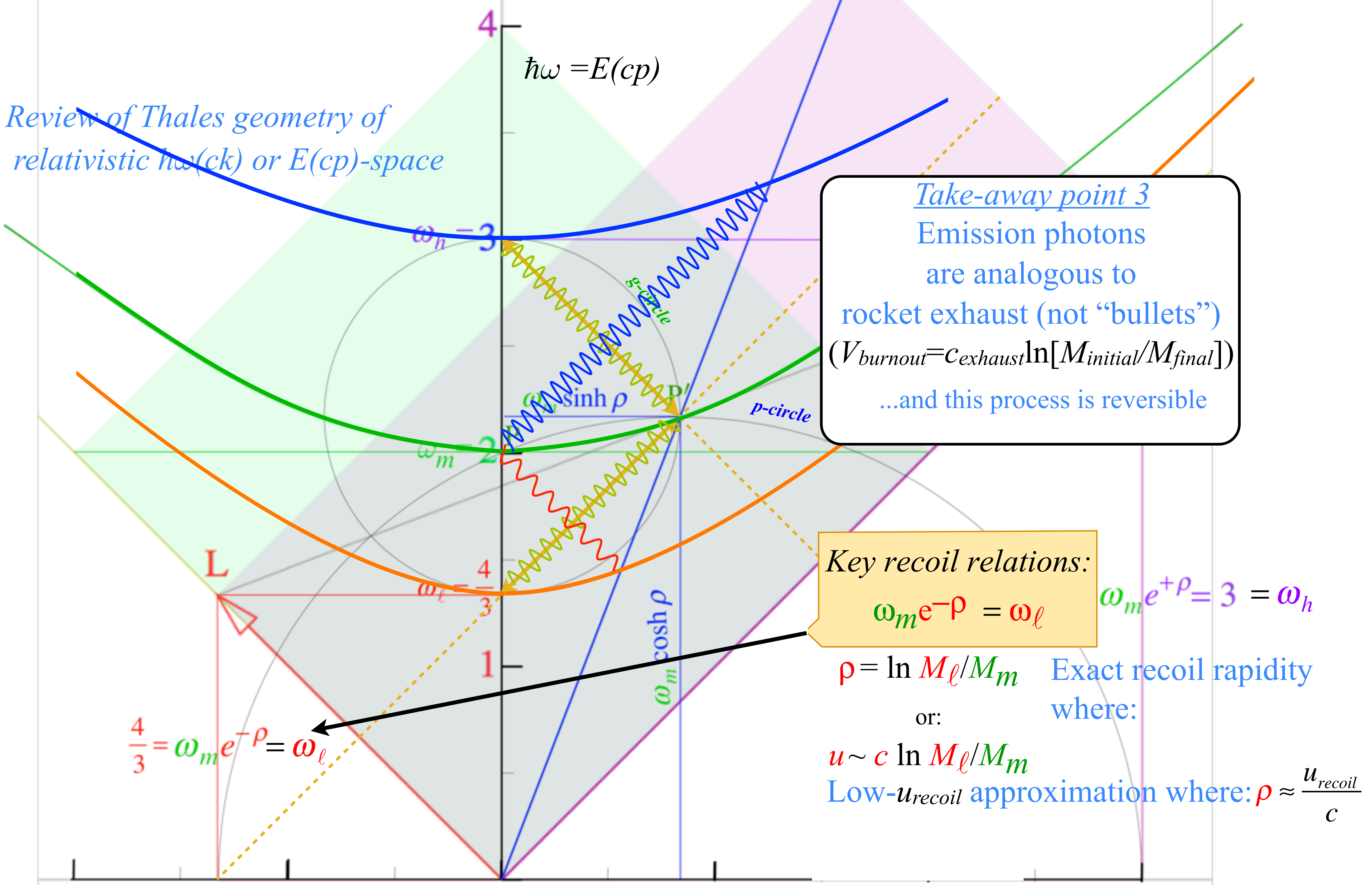

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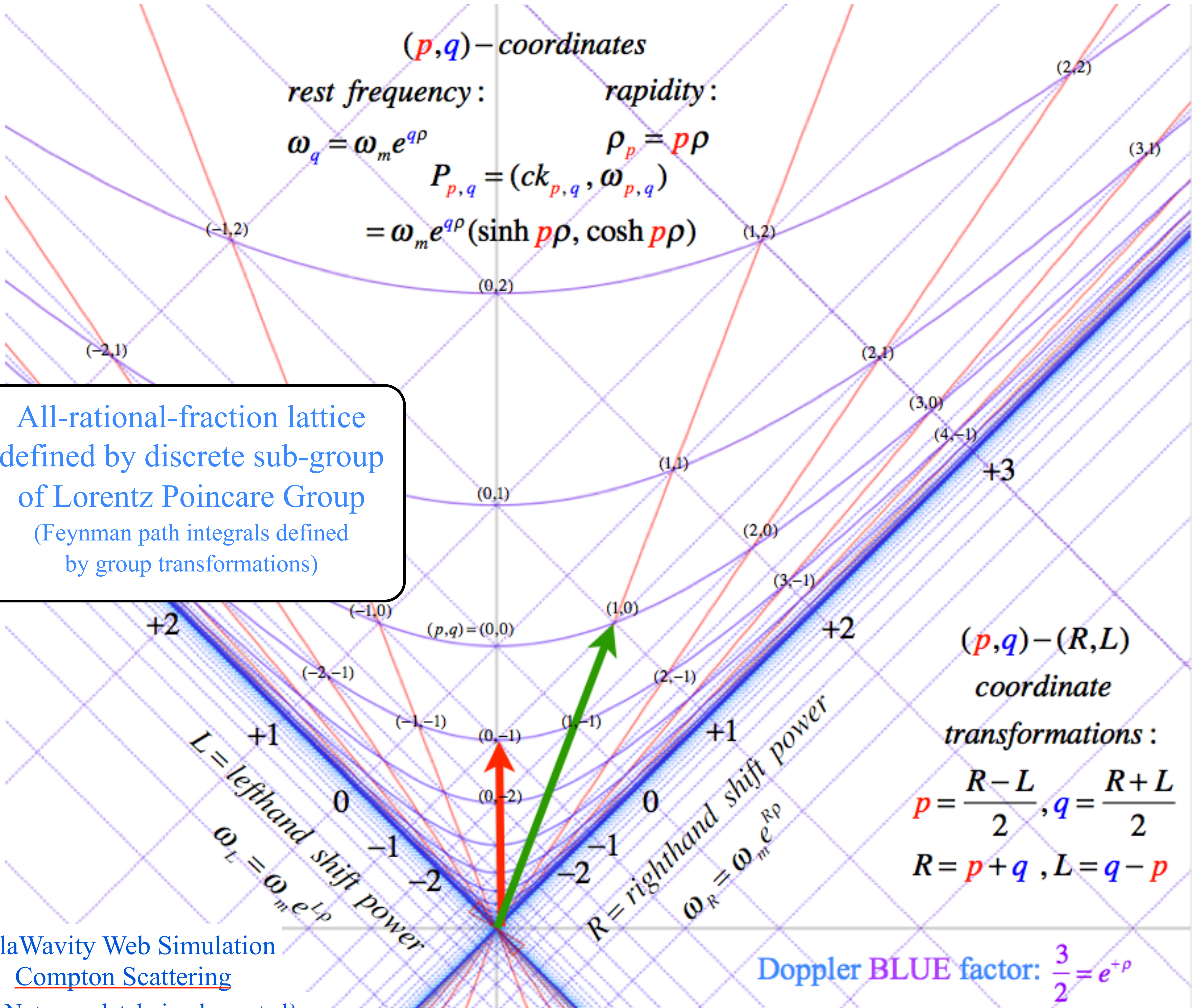

RelaWavity Web Simulation
Compton Scattering
Note: Not completely implemented}

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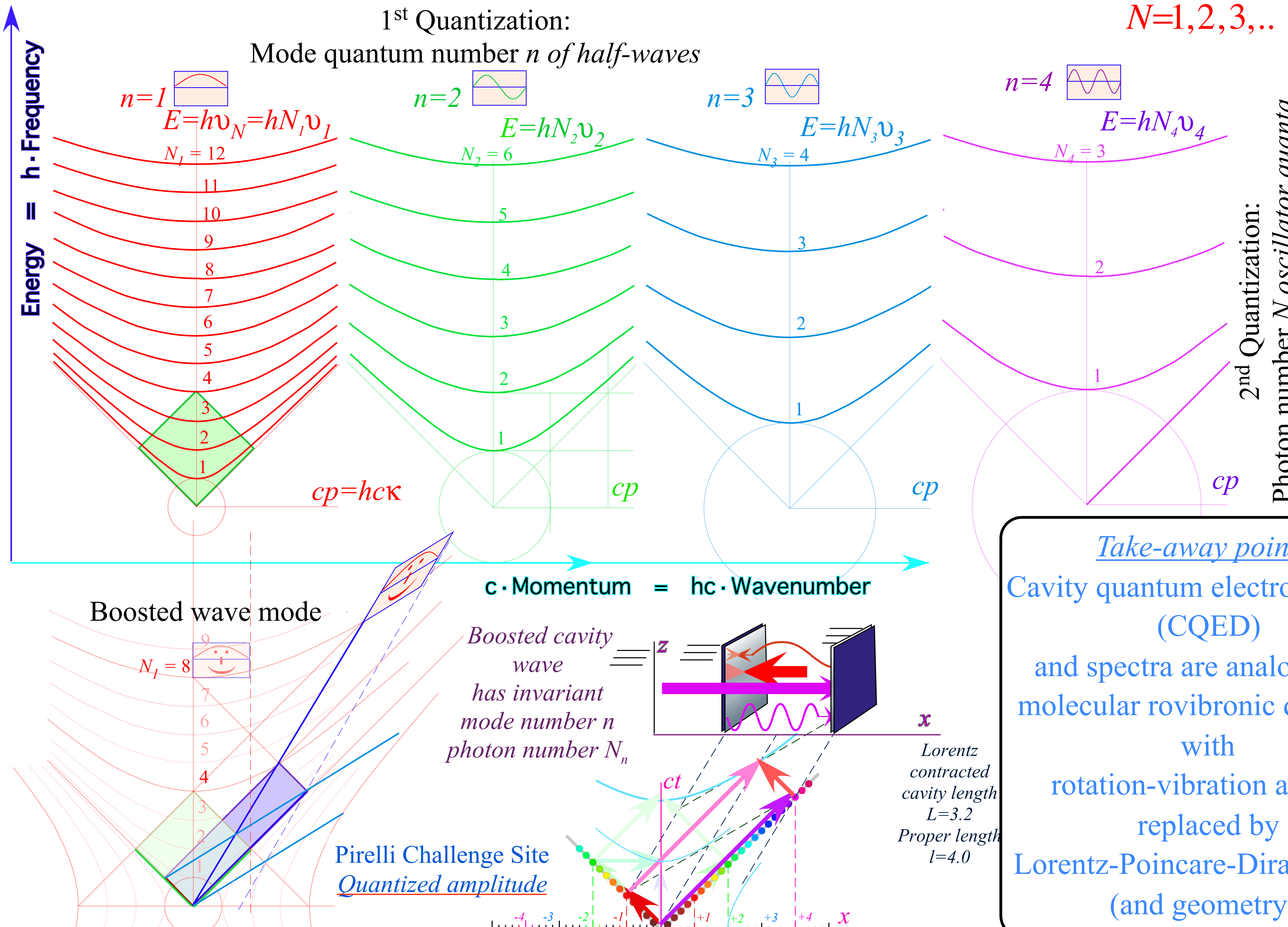
2nd Quantization:

$h\upsilon$ is actually $hN\upsilon$

$\left(h\upsilon_{phase} = E = h\upsilon_A \cosh\rho \right)$ *is actually* $\left(hN\upsilon_{phase} = E_N = hN\upsilon_A \cosh\rho \right.$ with *quantum* numbers$\left. \right)$

$N = 1, 2, 3, ..$



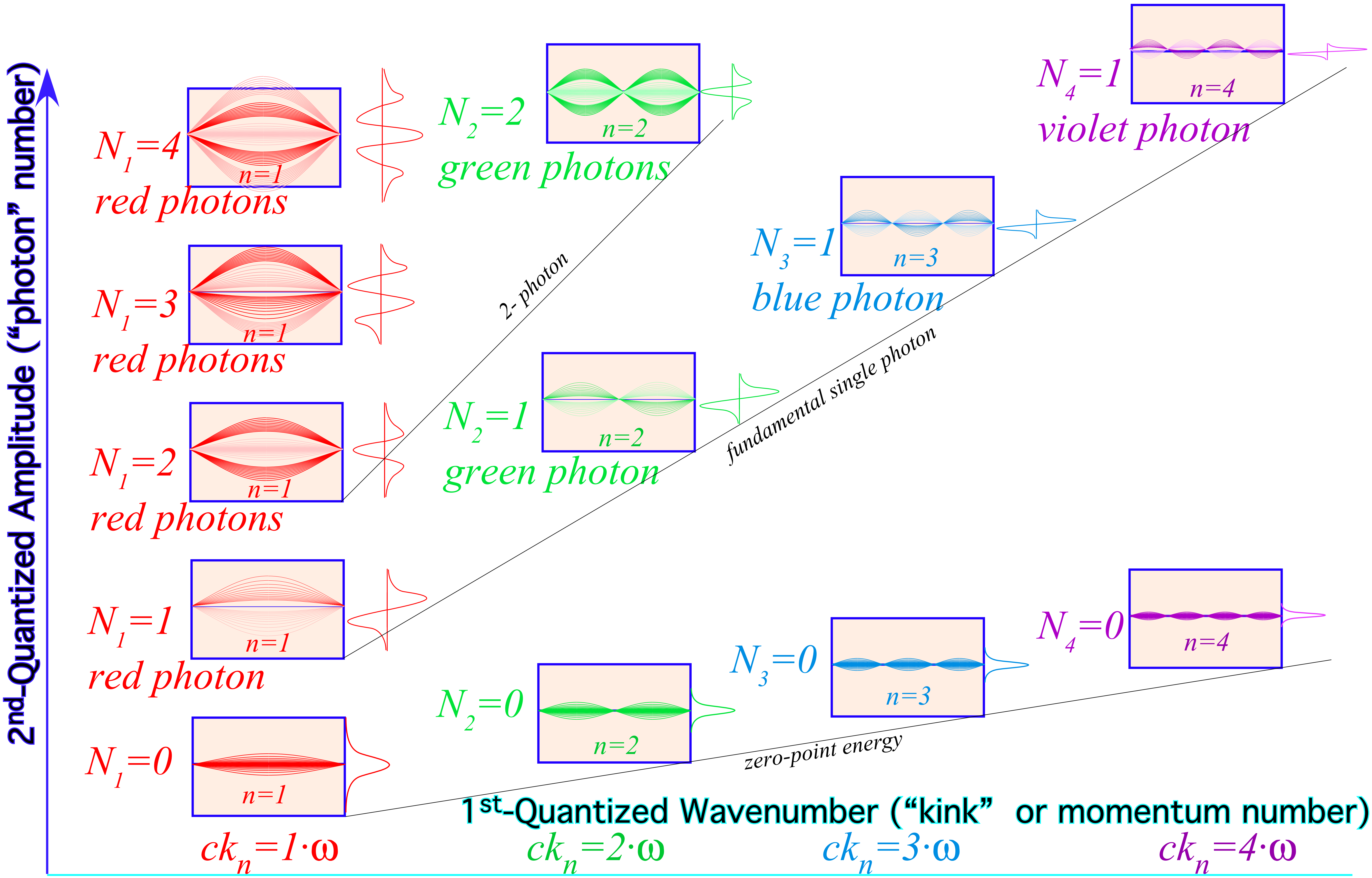
Take-away point 4

Cavity quantum electrodynamics (CQED) and spectra are analogous to molecular rovibronic dynamics with rotation-vibration algebra replaced by Lorentz-Poincare-Dirac algebra (and geometry!)

2nd Quantization:

$h\upsilon$ *is actually* $hN\upsilon$

$(h\upsilon_{phase}=E=h\upsilon_A\cosh\rho)$ *is actually* $(hN\upsilon_{phase}=E_N=hN\upsilon_A\cosh\rho \quad (N=1,2,..))$



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Acceleration by chirping laser pairs

Varying acceleration (General case)

From Lect. 35 ModPhys (2012)

Varying local acceleration $\rho = \rho(\tau)$

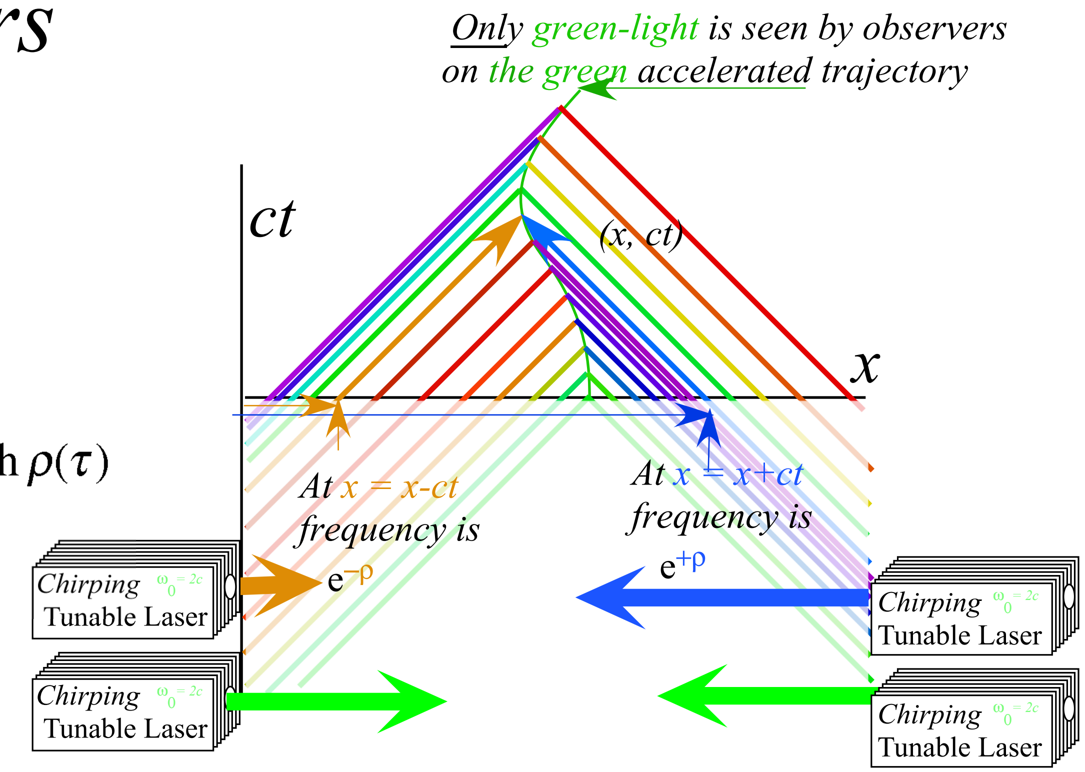
$$u = \frac{dx}{dt} = c\tanh(\tau)$$

$$\frac{dt}{d\tau} = \cosh\rho(\tau) \qquad \frac{dx}{d\tau} = \frac{dx}{dt}\frac{dt}{d\tau} = c\tanh\rho(\tau)\cosh\rho(\tau) = c\sinh\rho(\tau)$$

$$ct = c\int\cosh\rho(\tau)\,d\tau \qquad x = c\int\sinh\rho(\tau)\,d\tau$$

Constant local acceleration $\rho = \frac{g\tau}{c}$ "*Einstein Elevator*"

$$ct = c\int\cosh\frac{g\tau}{c}\,d\tau = \frac{c^2}{g}\sinh\frac{g\tau}{c} \qquad x = c\int\sinh\frac{g\tau}{c}\,d\tau = \frac{c^2}{g}\cosh\frac{g\tau}{c}$$



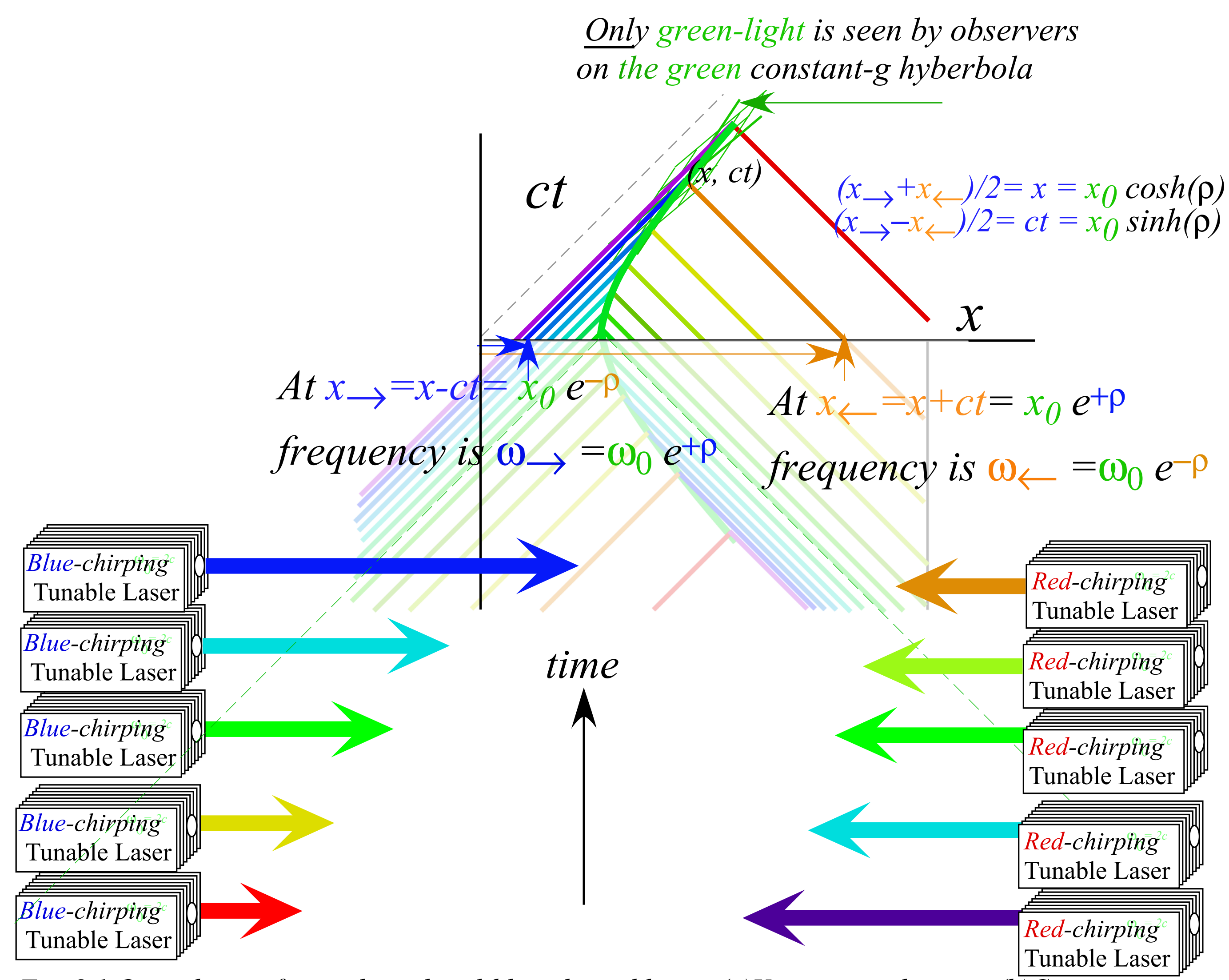
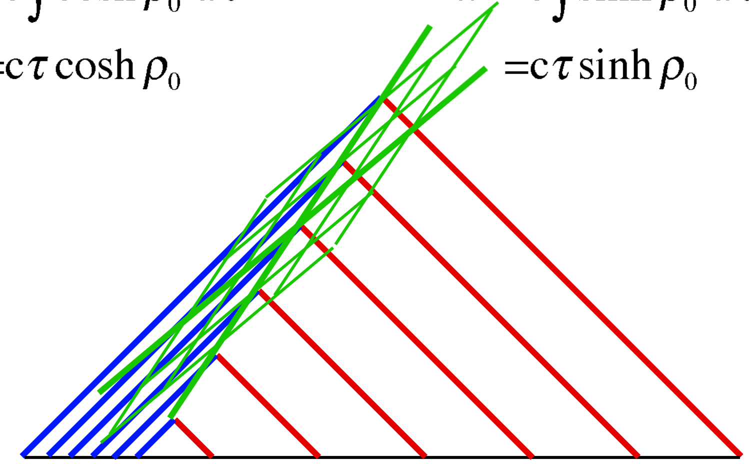


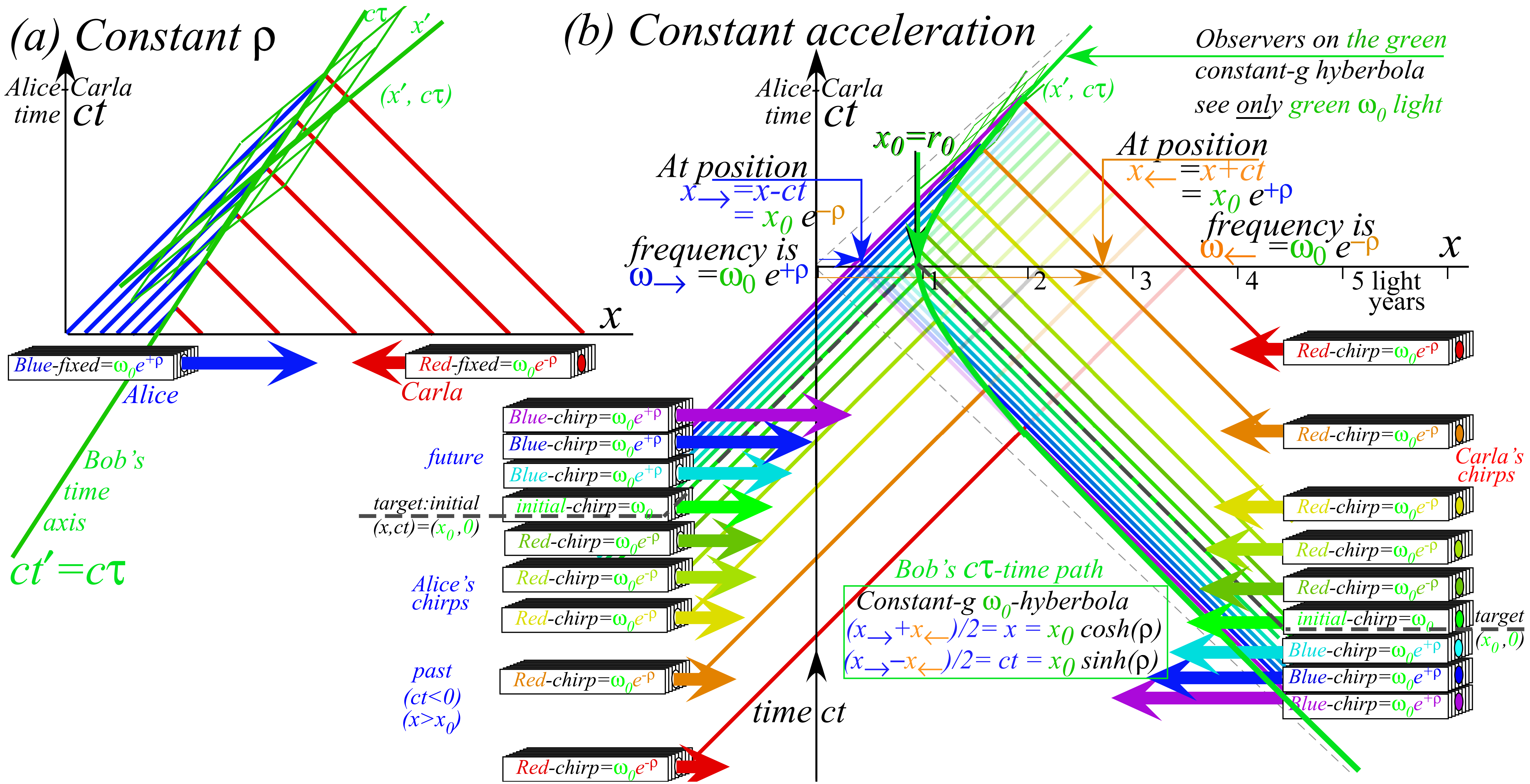
Fig. 8.1 Optical wave frames by red-and-blue-chirped lasers (a)Varying acceleration (b)Constant g

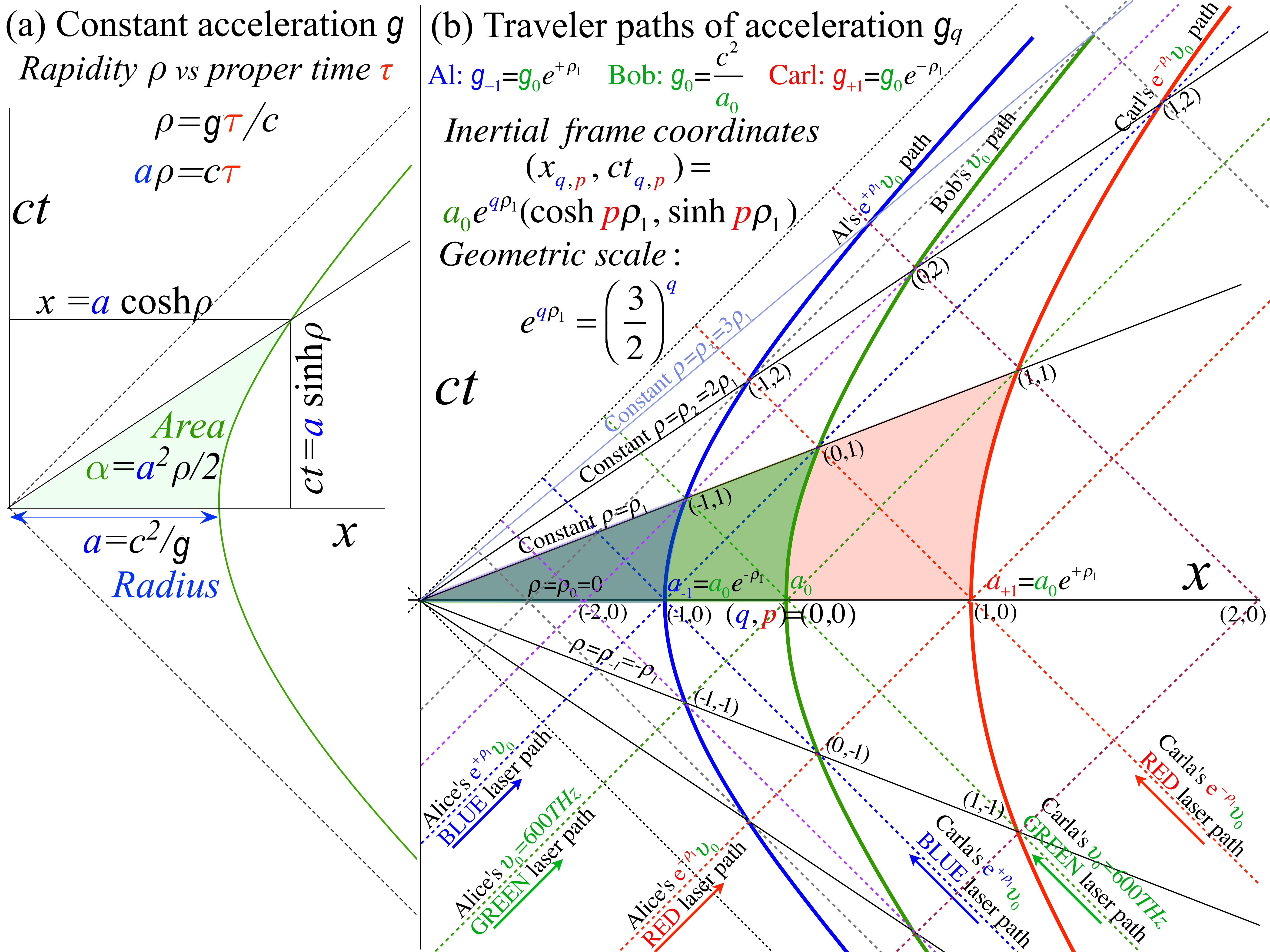
Previous examples involved constant velocity

Constant velocity $\rho = \rho_0 = const.$ "*Lorentztransformation*"

$$ct = c\int\cosh\rho_0\,d\tau = c\tau\cosh\rho_0 \qquad x = c\int\sinh\rho_0\,d\tau = c\tau\sinh\rho_0$$




(a) Constant ρ
Alice-Carla time ct
x
(x′, cτ)
Blue-fixed=ω0e+ρ
Red-fixed=ω0e-ρ
Alice
Carla
Bob's time axis
ct′=cτ
(b) Constant acceleration
Observers on the green constant-g hyberbola see only green ω0 light
At position x→=x-ct = x0 e−ρ
frequency is ω→ =ω0 e+ρ
At position x←=x+ct = x0 e+ρ
frequency is ω← =ω0 e−ρ
x0=r0
5 light years
future
target:initial (x,ct)=(x0,0)
Alice's chirps
past (ct<0) (x>x0)
Carla's chirps
Blue-chirp=ω0e+ρ
initial-chirp=ω0
Red-chirp=ω0e-ρ
Bob's cτ-time path
Constant-g ω0-hyberbola
(x→+x←)/2= x = x0 cosh(ρ)
(x→−x←)/2= ct = x0 sinh(ρ)
time ct
target (x0,0)



(a) Constant acceleration g
Rapidity ρ vs proper time τ
ρ=gτ/c
aρ=cτ
ct
x =a coshρ
ct=a sinhρ
Area
α=a²ρ/2
a=c²/g
Radius
x
(b) Traveler paths of acceleration g_q
Al: g_{-1}=g_0 e^{+ρ_1}
Bob: g_0=c²/a_0
Carl: g_{+1}=g_0 e^{-ρ_1}
Inertial frame coordinates
(x_{q,p}, ct_{q,p}) =
a_0 e^{qρ_1}(cosh pρ_1, sinh pρ_1)
Geometric scale:
e^{qρ_1} = (3/2)^q
ct
x
Al's e^{+ρ_1}υ_0 path
Bob's υ_0 path
Carl's e^{-ρ_1}υ_0 path
Constant ρ=ρ_3=3ρ_1
Constant ρ=ρ_2=2ρ_1
Constant ρ=ρ_1
ρ=ρ_0=0
ρ=ρ_{-1}=-ρ_1
a_{-1}=a_0 e^{-ρ_1}
a_0
a_{+1}=a_0 e^{+ρ_1}
(q,p)=(0,0)
(-2,0)
(-1,0)
(1,0)
(2,0)
(-1,1)
(0,1)
(1,1)
(-1,2)
(0,2)
(1,2)
(-1,-1)
(0,-1)
(1,-1)
Alice's e^{+ρ_1}υ_0 BLUE laser path
Alice's υ_0=600THz GREEN laser path
Alice's e^{-ρ_1}υ_0 RED laser path
Carla's e^{+ρ_1}υ_0 BLUE laser path
Carla's υ_0=600THz GREEN laser path
Carla's e^{-ρ_1}υ_0 RED laser path

Lecture 31
Thur. 12.10.2015

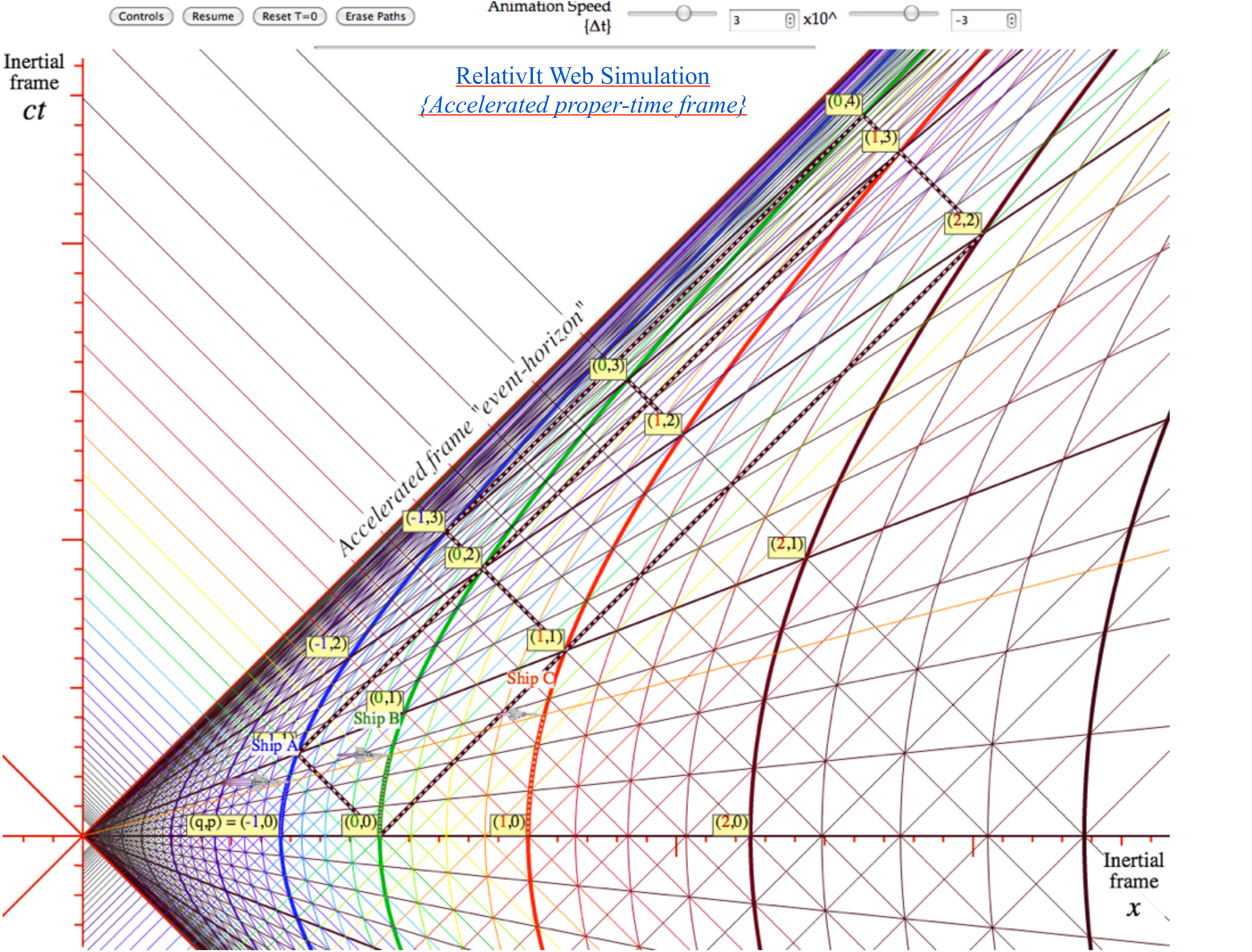
Review: Relawavity ρ functions Two famous ones Extremes and plot vs. ρ
Doppler jeopardy Geometric mean and Relativistic hyperbolas
Animation of $e^{\rho}=2$ spacetime and per-spacetime plots

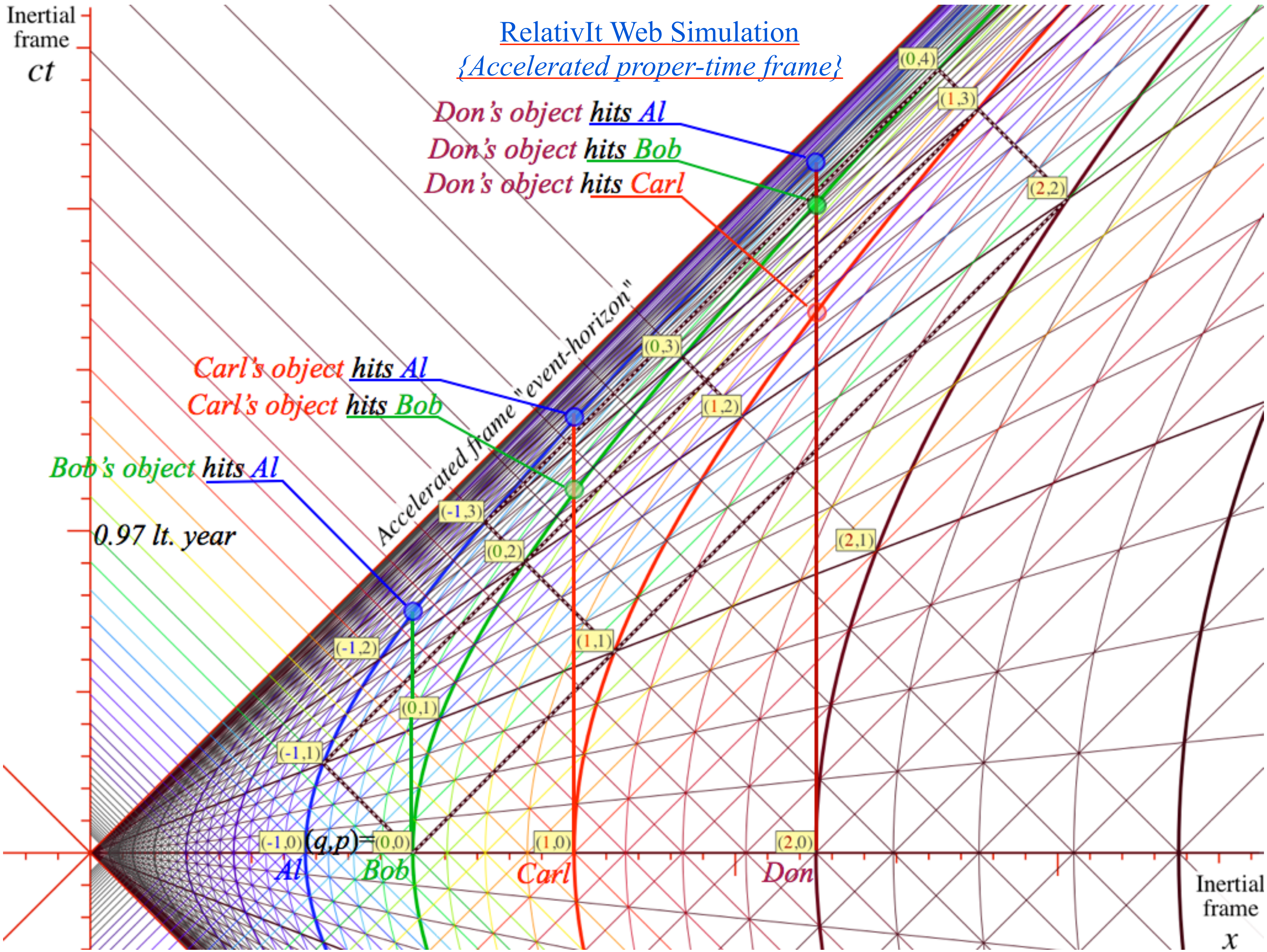
Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity
Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry
"Occams Sword" and summary of 16 parameter functions of ρ and σ
Applications to optical waveguide, spherical waves, and accelerator radiation
Learning about sin! *and* cos *and...*

Derivation of relativistic quantum mechanics
What's the matter with mass? Shining some light on the Elephant in the room
Relativistic action and Lagrangian-Hamiltonian relations
Poincare' and Hamilton-Jacobi equations

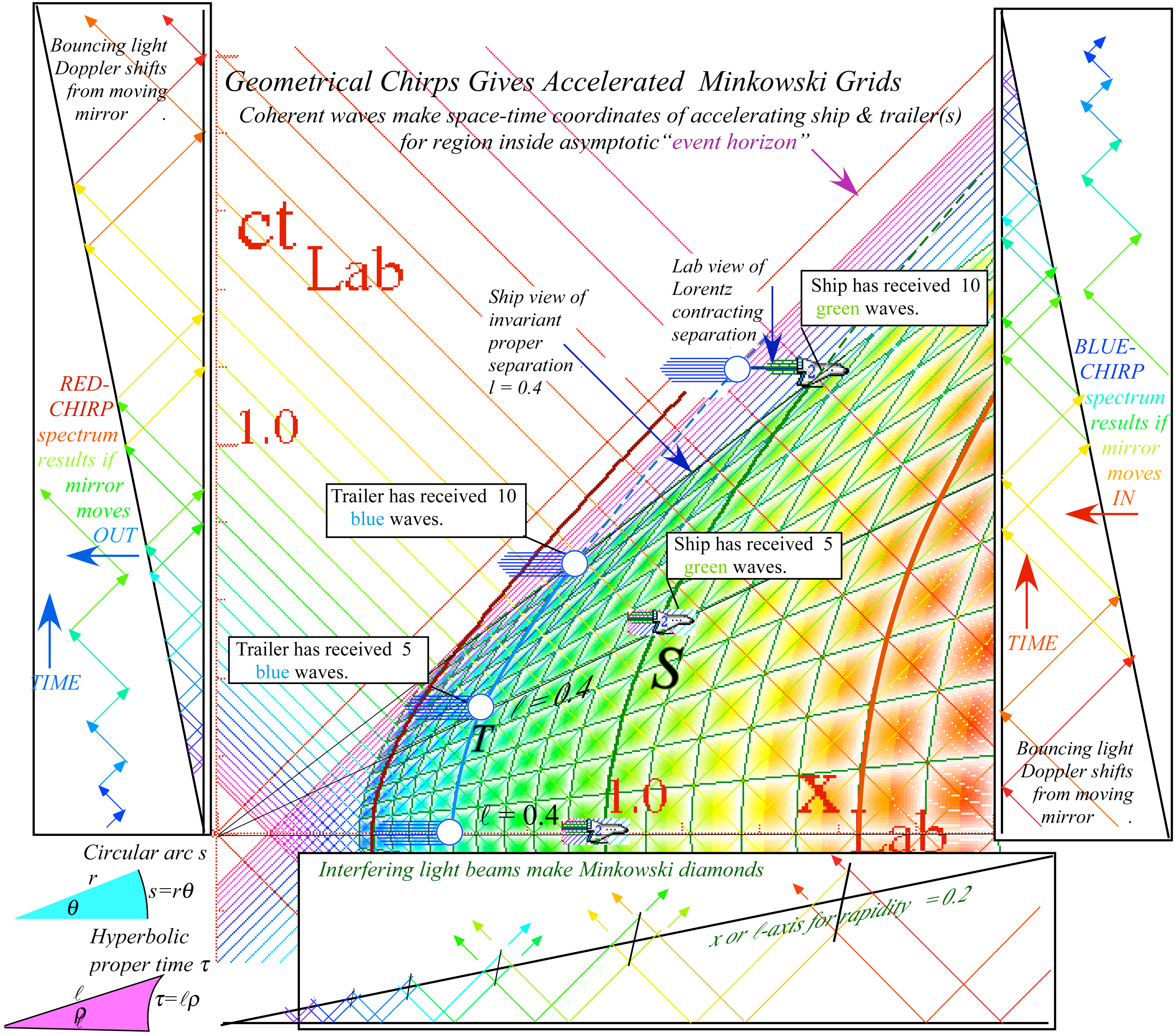
Relativistic optical transitions and Compton recoil formulae
Feynman diagram geometry
Compton recoil related to rocket velocity formula
Comparing 2^{nd}-quantization "photon" number N and 1^{st}-quantization wavenumber κ

Relawavity in accelerated frames
Laser up-tuning by Alice and down-tuning by Carla makes *g*-acceleration grid
Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid
➔ Animation of mechanics and metrology of constant-g grid


Controls
Resume
Reset T=0
Erase Paths
Animation Speed {Δt}
3
x10^
-3
RelativIt Web Simulation
{Accelerated proper-time frame}
Inertial frame ct
Accelerated frame "event-horizon"
(0,4)
(1,3)
(2,2)
(0,3)
(1,2)
(-1,3)
(0,2)
(2,1)
(-1,2)
(1,1)
Ship C
(0,1)
Ship B
Ship A
(q,p) = (-1,0)
(0,0)
(1,0)
(2,0)
Inertial frame x

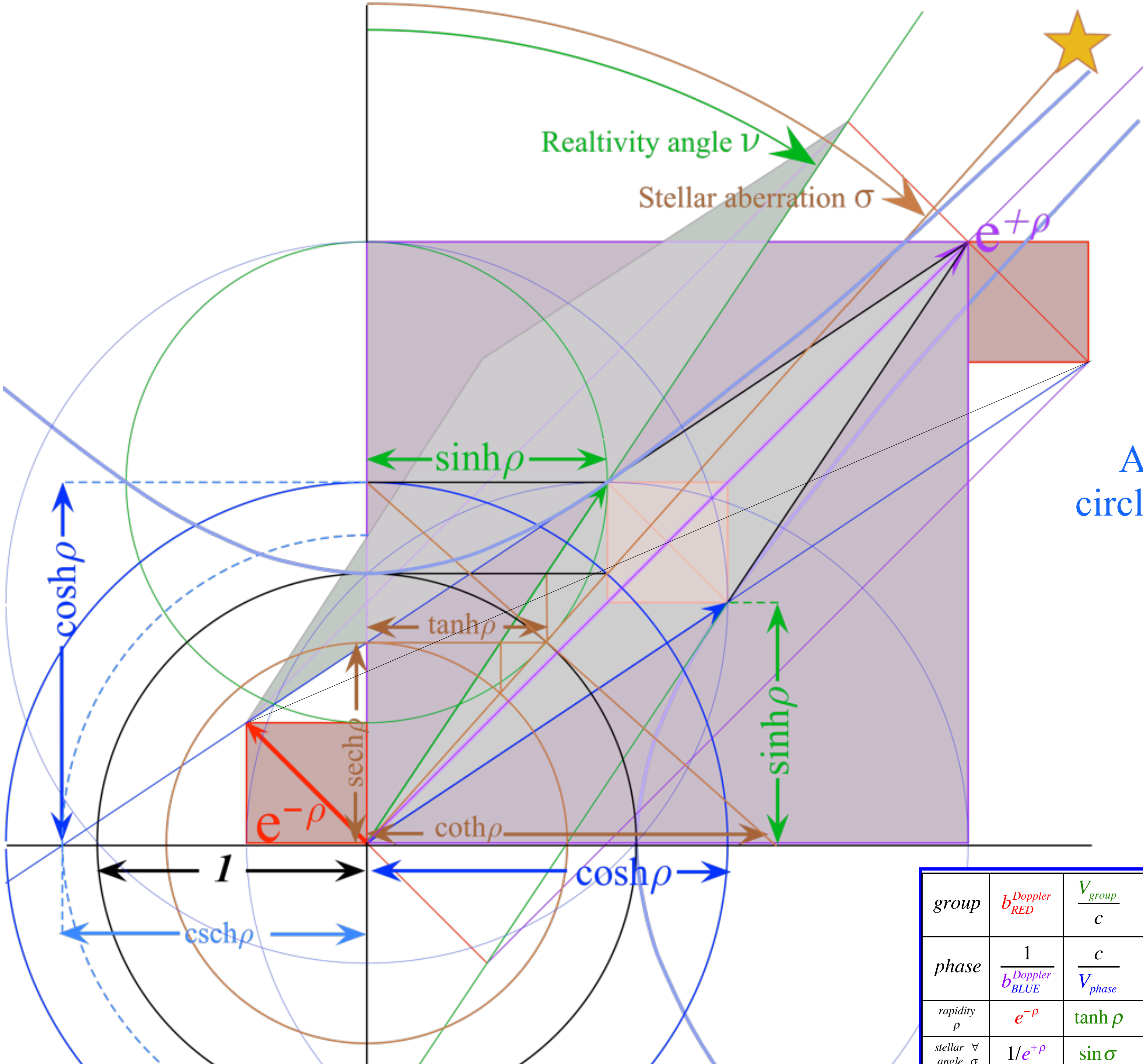


RelativIt Web Simulation
{Accelerated proper-time frame}
Inertial frame
ct
Don's object hits Al
Don's object hits Bob
Don's object hits Carl
Carl's object hits Al
Carl's object hits Bob
Bob's object hits Al
Accelerated frame "event-horizon"
0.97 lt. year
(0,4)
(1,3)
(2,2)
(0,3)
(1,2)
(-1,3)
(0,2)
(2,1)
(1,1)
(-1,2)
(0,1)
(-1,1)
(-1,0)
(q,p)=(0,0)
(1,0)
(2,0)
Al
Bob
Carl
Don
Inertial frame
x



From Lect. 35 ModPhys (2012)

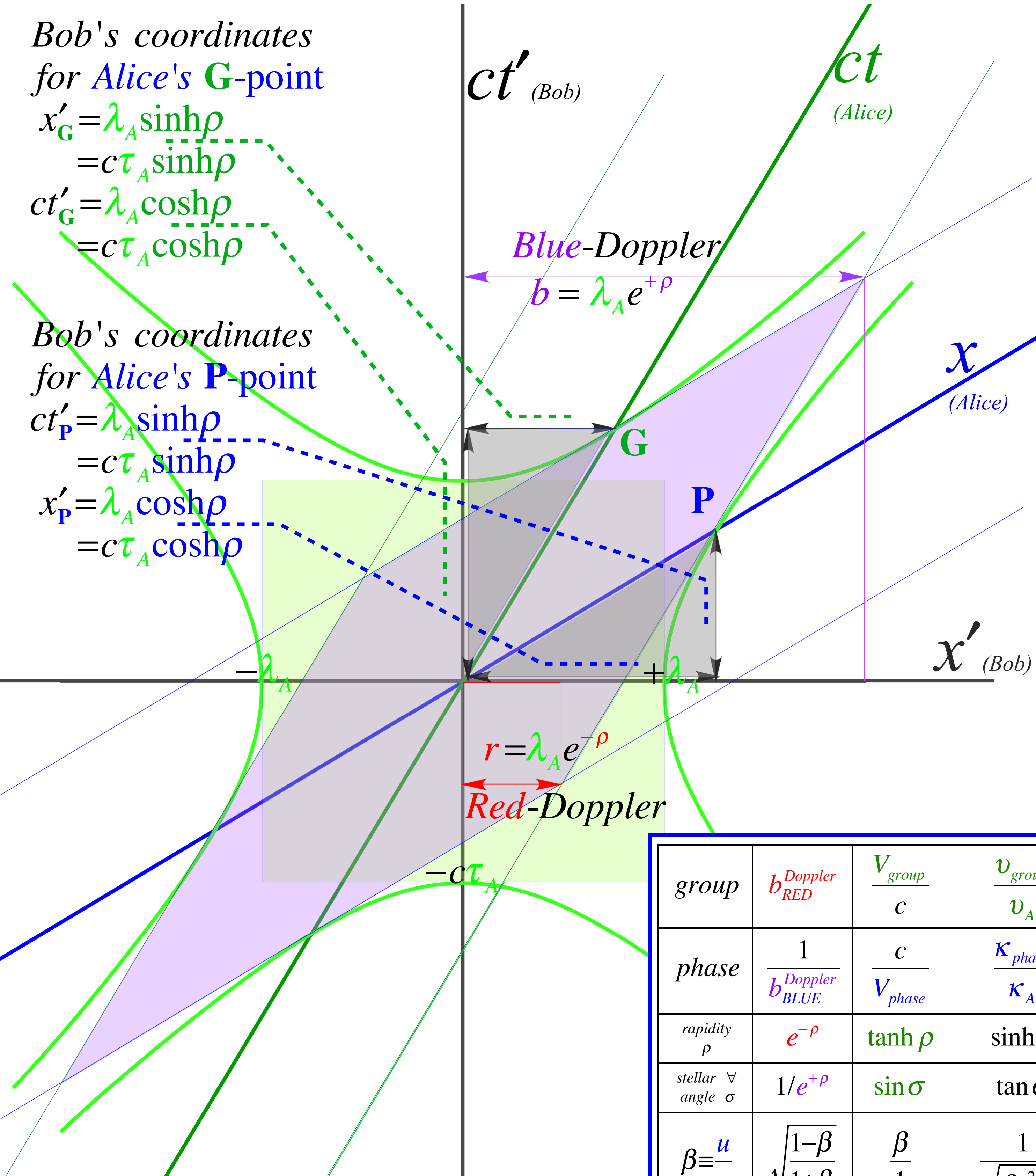
Fig. 8.2 Accelerated reference frames and their trajectories painted by chirped coherent light



A more compact circle-based geometry

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}= 0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

RelaWavity Web Simulation
Comprehensive dual plots with parameter table



Space-time parameters

$\lambda_{phase} = \lambda_A \operatorname{csch}\rho$

$\lambda_{group} = \lambda_A \operatorname{sech}\rho$

$c\tau_{phase} = c\tau_A \operatorname{sech}\rho$

$c\tau_{group} = c\tau_A \operatorname{csch}\rho$

Per-space-time parameters

$c\kappa_{phase} = c\kappa_A \sinh\rho$

$c\kappa_{group} = c\kappa_A \cosh\rho$

$\upsilon_{phase} = \upsilon_A \cosh\rho$

$\upsilon_{group} = \upsilon_A \sinh\rho$

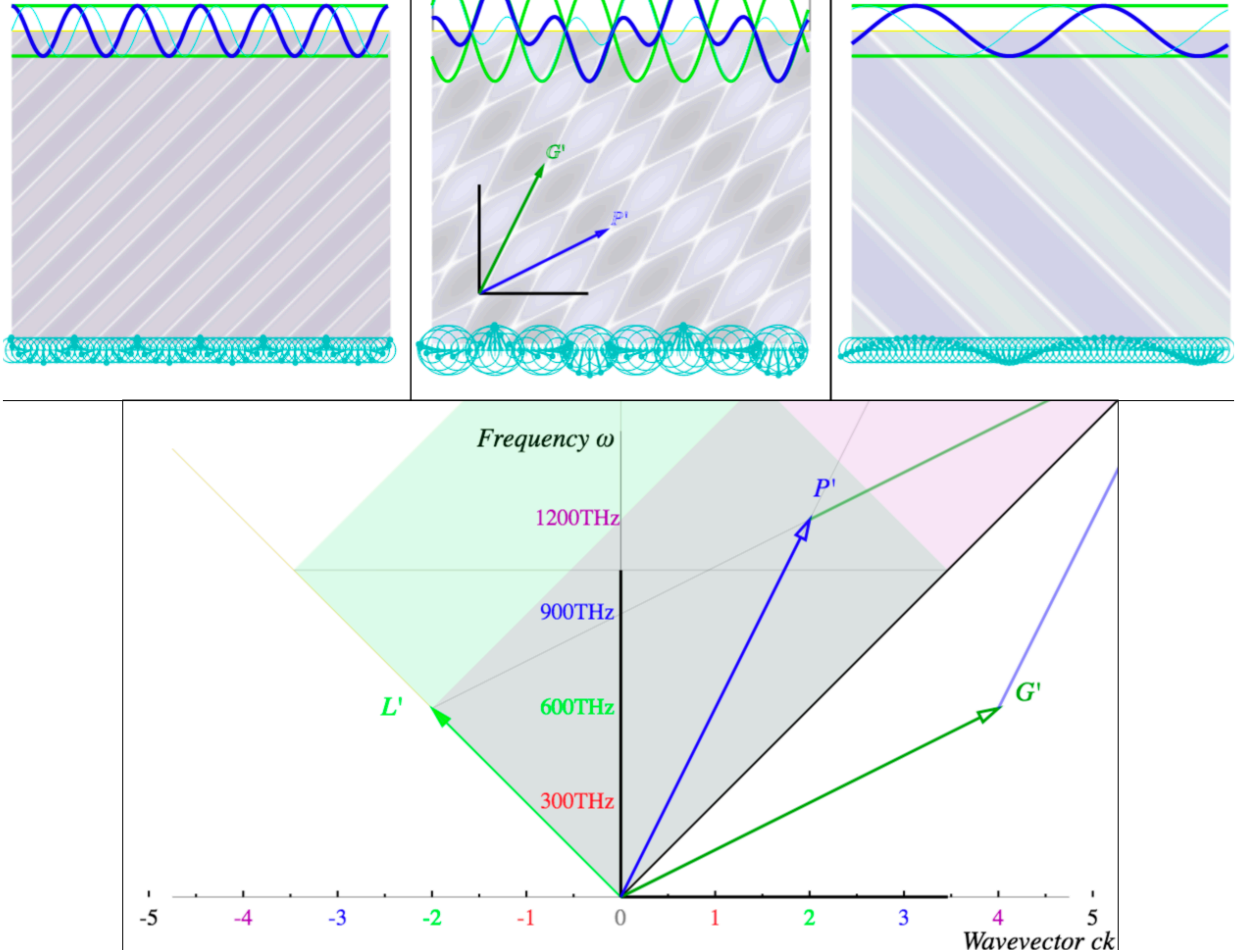
RelaWavity Web Simulations

[Comprehensive dual plots with parameter table]

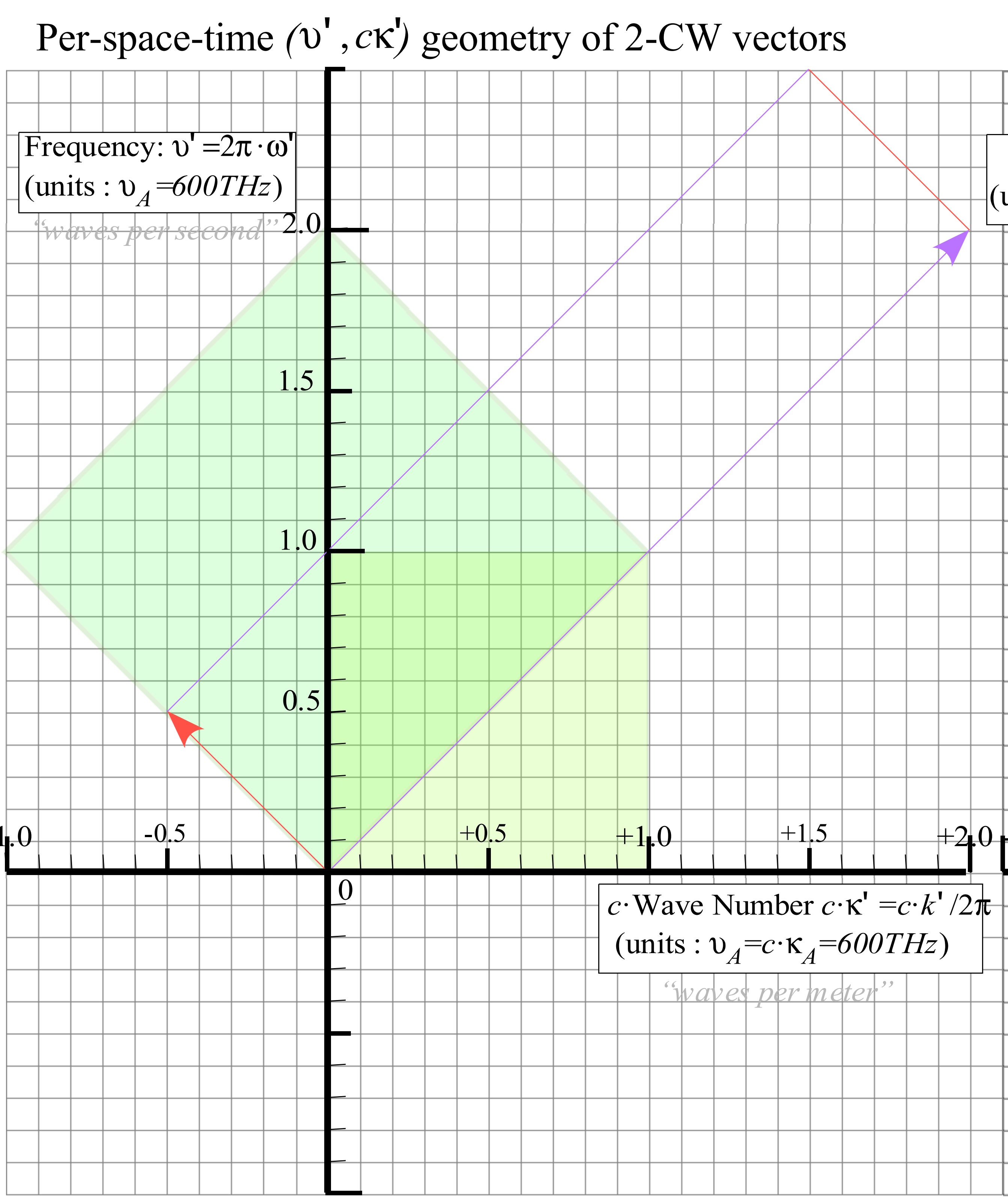
•

[ct' vs x' with parameter table]

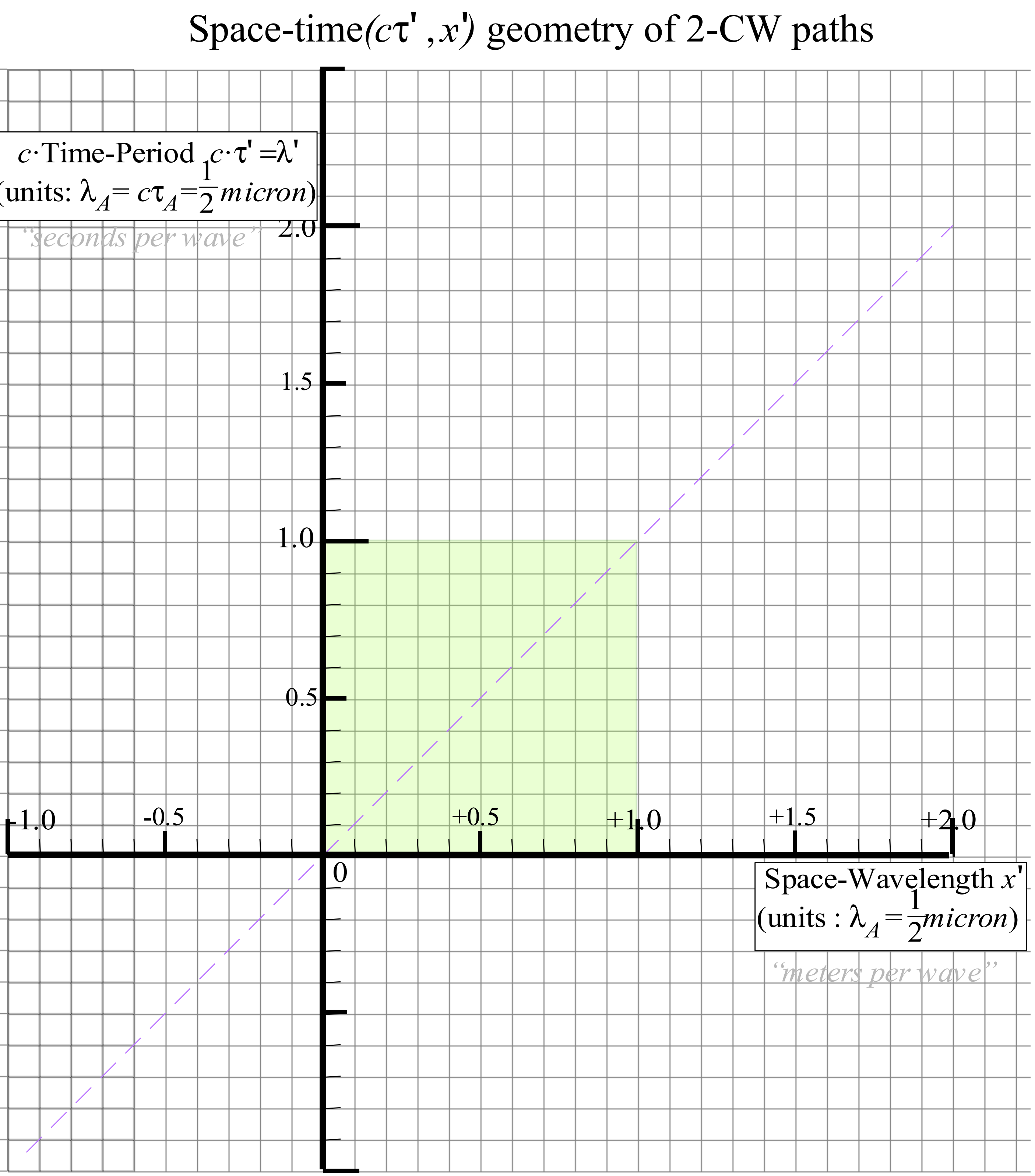
group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$
effects	$b^{Doppler}_{RED}$	V_{group}	past-future asymmetry (off-diagonal Lorentz-transform)	x-contraction (Lorentz) τ_{phase}-contraction	t-dilation (Einstein) υ_{phase}-dilation (on-diagonal Lorentz-transform)	inverse asymmetry	V_{phase}	$b^{Doppler}_{BLUE}$



G'
P'
Frequency ω
1200THz
900THz
600THz
300THz
P'
L'
G'
-5
-4
-3
-2
-1
0
1
2
3
4
5
Wavevector ck



Per-space-time (υ' , cκ') geometry of 2-CW vectors
Frequency: υ' =2π·ω'
(units : υ_A=600THz)
"waves per second"
2.0
1.5
1.0
0.5
-1.0
-0.5
+0.5
+1.0
+1.5
+2.0
0
c·Wave Number c·κ' =c·k' /2π
(units : υ_A=c·κ_A=600THz)
"waves per meter"



Space-time(cτ' , x') geometry of 2-CW paths
c·Time-Period c·τ' =λ'
(units: λ_A= cτ_A=1/2 micron)
"seconds per wave"
2.0
1.5
1.0
0.5
-1.0
-0.5
+0.5
+1.0
+1.5
+2.0
0
Space-Wavelength x'
(units : λ_A=1/2 micron)
"meters per wave"