Lecture 1
Mon. 8.26.2019

1st axioms and theorems of classical mechanics

(Ch. 1 thru Ch. 3 of Unit 1)

A preface comment: Text by Eric Heller (for Semester II of Advanced Mechanics)

Geometry of momentum conservation axiom (aka Occam's Razor)
- *Totally Inelastic "ka-runch" collisions* (begin 4:1 graph project)*
- *Perfectly Elastic "ka-bong" and Center Of Momentum (COM) symmetry**
  - *+Intro to weighty-averages and vector notation*
- *Comments on idealization in classical models*

Geometry of Galilean translation symmetry
- *45° shift in (V_1,V_2)-space*
- *Time reversal symmetry*
- *...of COM collisions*

Algebra,Geometry, and Physics of momentum conservation axiom
- *Vector algebra of collisions*
- *Matrix or tensor algebra of collisions*
- *Deriving Energy Conservation Theorem*

Numerical details of collision tensor algebra

After-class
Bull-session

⅔ of a *PhD* is *Ph*
that is Philosophy

Should physicists
worry about *philosophy*?
…about *economics*?
…about *current events*?
…about *politics*?…

Note - Many of the underlined phrases throughout these lecture files link to *Simulations* of those *specific, selected* cases viewed within your *default* Web browser

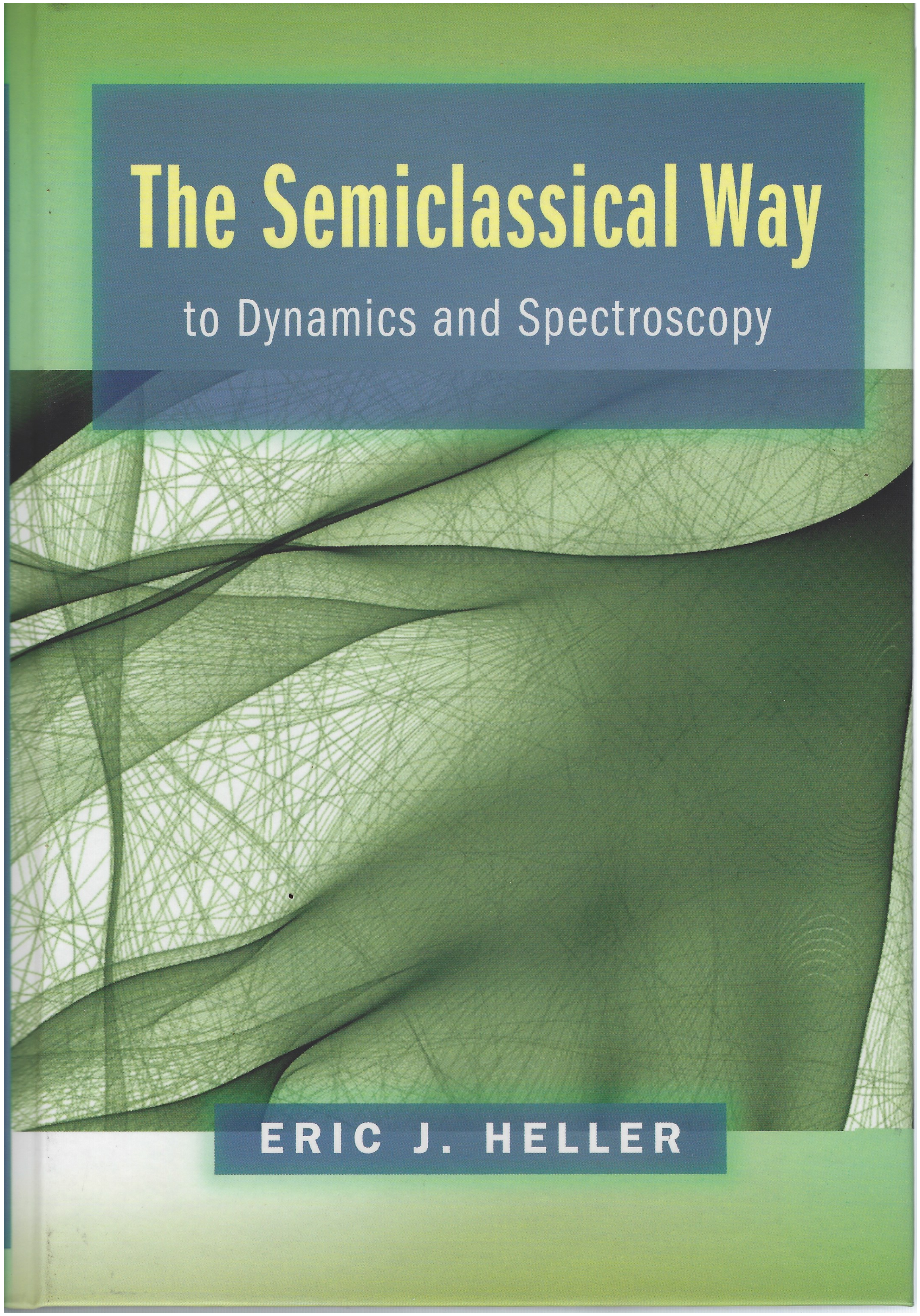
2019 UA PHYS 5103 Advanced Mechanics *Class web site* https://modphys.hosted.uark.edu/markup/CMwBang_TitlePage.html

*Launch Generic *Car Collision Web Simulator* https://modphys.hosted.uark.edu/markup/CMMotionWeb.html

*Launch Generic *Superball Collision Web Simulator* https://modphys.hosted.uark.edu/markup/BounceItWeb.html

*View X2 paper: *Velocity Amplification in Collision Experiments Involving Superballs - Harter, et. al. 1971 (pdf)*

A preface comment: Text by Rick Heller *that would be used for Semester II of Advanced Mechanics*



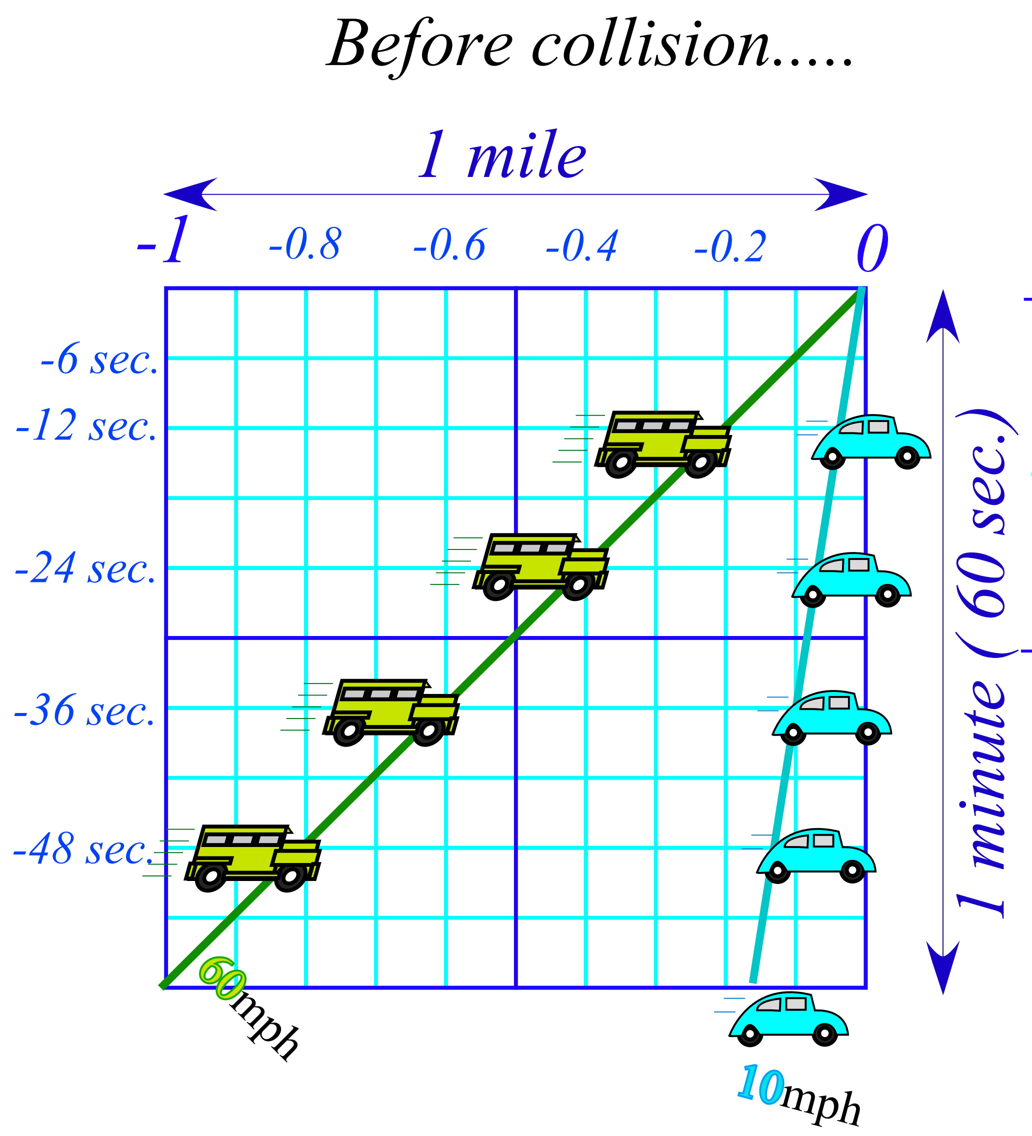
Expect Semester I to also reference gems of theory from this excellent text.

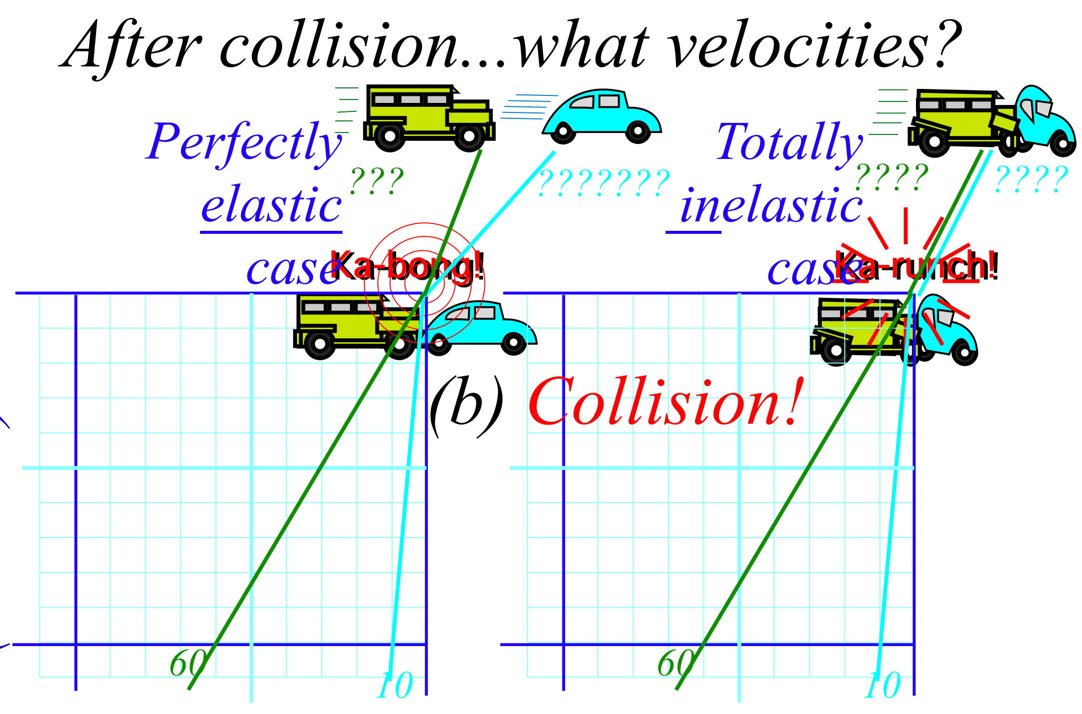
Allbookstore.com price comparison page *for new, used, rental, and digital copies*

Best new price was from

AbeBooks.com *Passion for books.*

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)





*Car Simulator links**

[Space vs Space Elastic]

[Space vs Space Inelastic]

[Elastic Collision Dual Panel Space vs Space and Space vs Time (Newton)]

[Inelastic Collision Dual Panel Space vs Space and Space vs Time (Newton)]

[Elastic Collision Dual Panel Space vs Space and Time vs. Space(Minkowski)]

[Inelastic Collision Dual Panel Space vs Space and Time vs. Space(Minkowski)]

Our web apps (and course site) are *FREE*, and available *world wide* through most connected modern web browsers!!
The links can be copy and pasted and emailed, so with a simple click one has a whole application suite at your fingertips - One *full* of **Physics**
Underlines usually denote active web links

**Car Collision simulator links:*

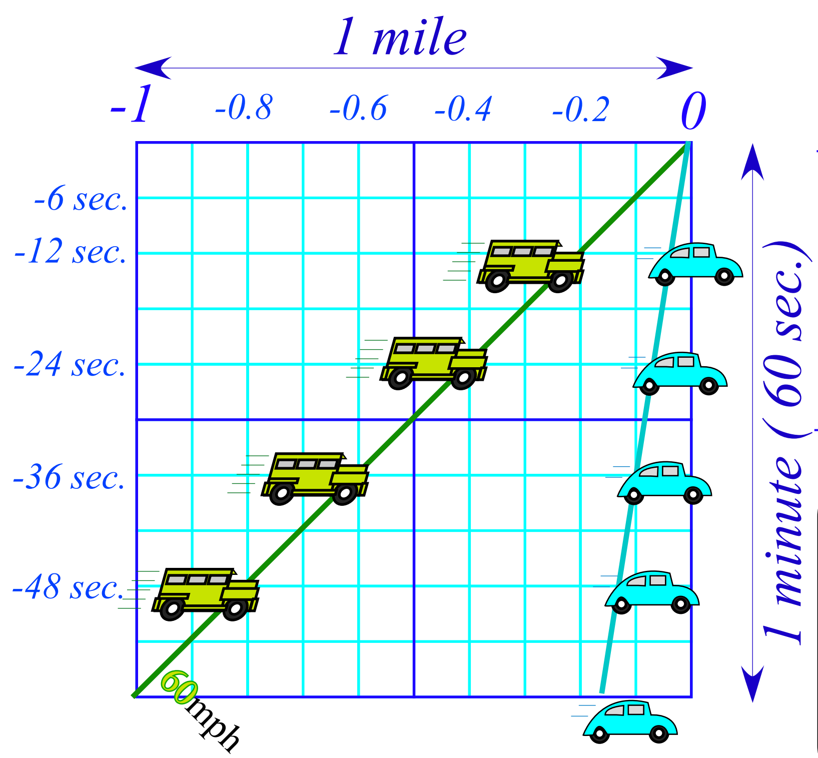
Modern syntax: https://modphys.hosted.uark.edu/markup/CMMotionWeb.html

Older Cavern *links are to redirects:* http://www.uark.edu/ua/modphys/markup/CMMotionWeb.html

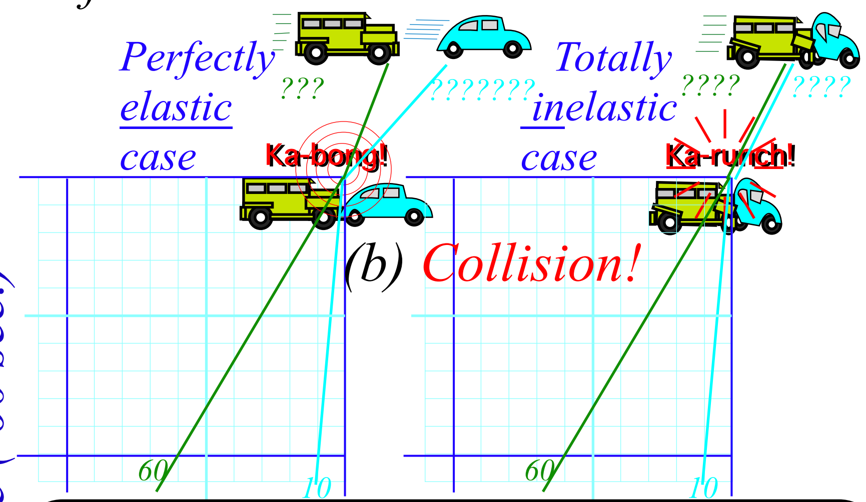
Links to "Testing *Area*": http://www.uark.edu/ua/modphys/testing/markup/CMMotionWeb.html

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Before collision.....



After collision...what velocities?



Conventional solution: Look up the usual momentum and energy formulas/axioms:

$\Sigma_i m V_i(initial) = \Sigma_i m V_i(final)$

$\Sigma_i m V^2_i(initial) = \Sigma_i m V^2_i(final)$

and solve...

Let's see if we can solve this *easily* with just ***one*** (*or one-and-a-half*) axiom(s)

Axiom-1: All mass or masses keep their total momentum until it is changed by some outsider.

**Launch Car Collision Web Simulator* https://modphys.hosted.uark.edu/markup/CMMotionWeb.html

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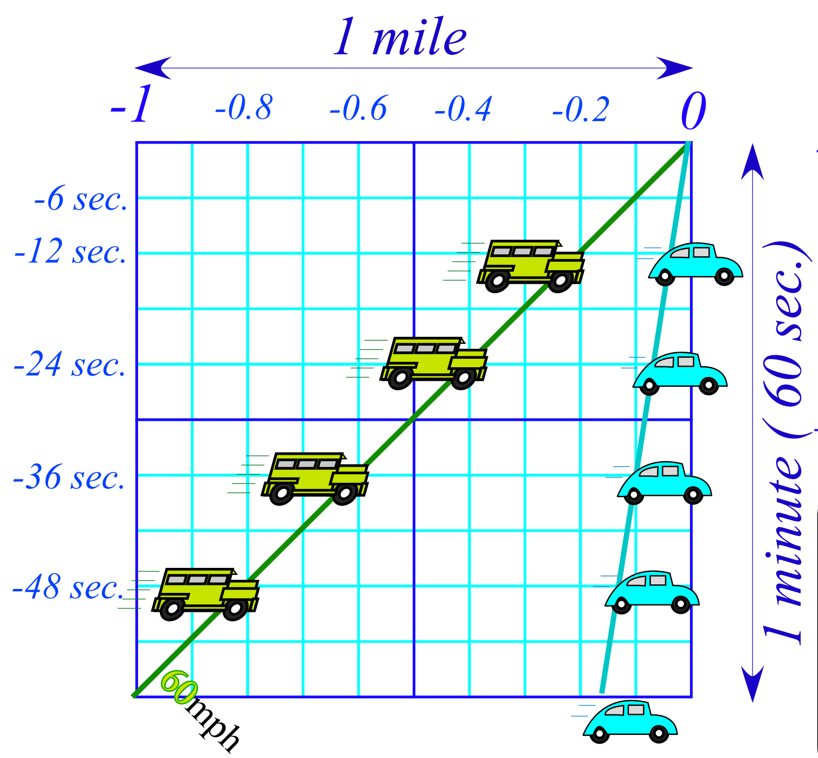

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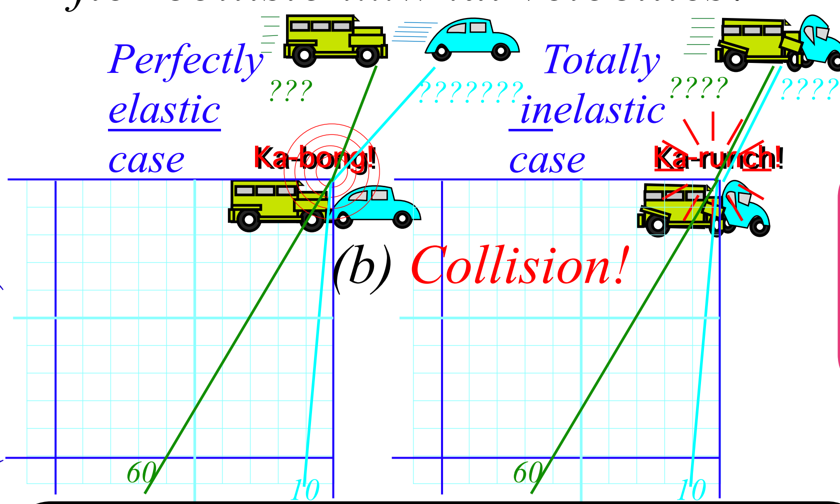

V_{SUV} and V_{VW} change violently but NOT ***total momentum***

$P_{Total} = M_{SUV} V_{SUV} + M_{VW} V_{VW}$

Conventional solution: Look up the usual momentum and energy formulas/axioms:

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but ***total momentum*** is constant
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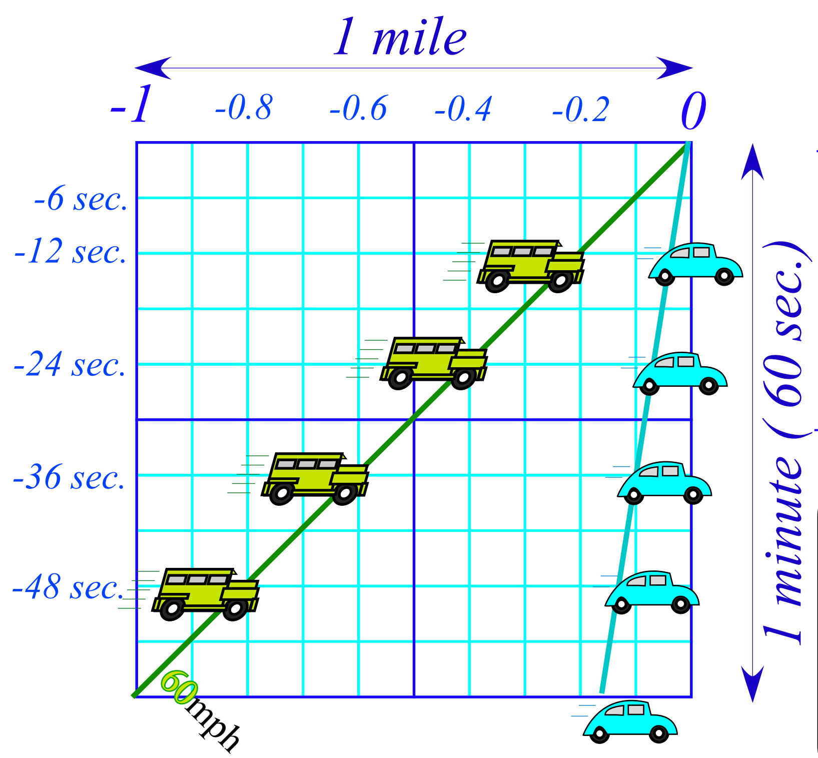
Before collision.....



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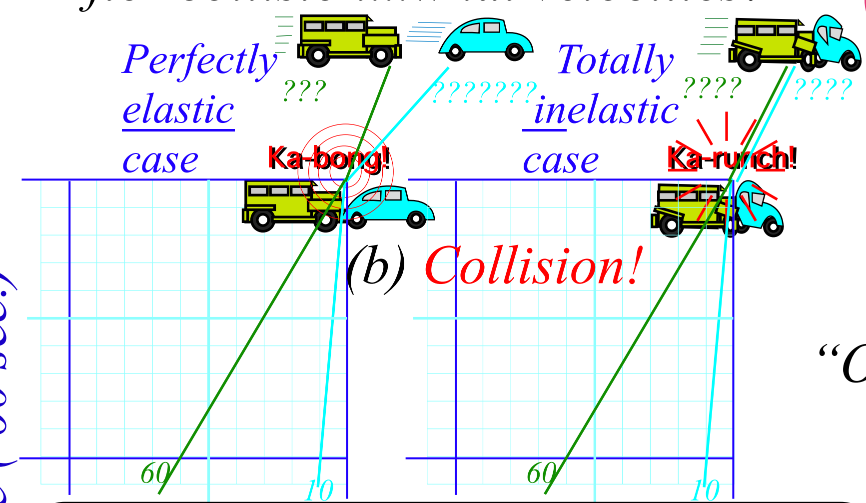

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and solve...

Inventor of
"Occam's Razor"

William of Ockham
1285-1349

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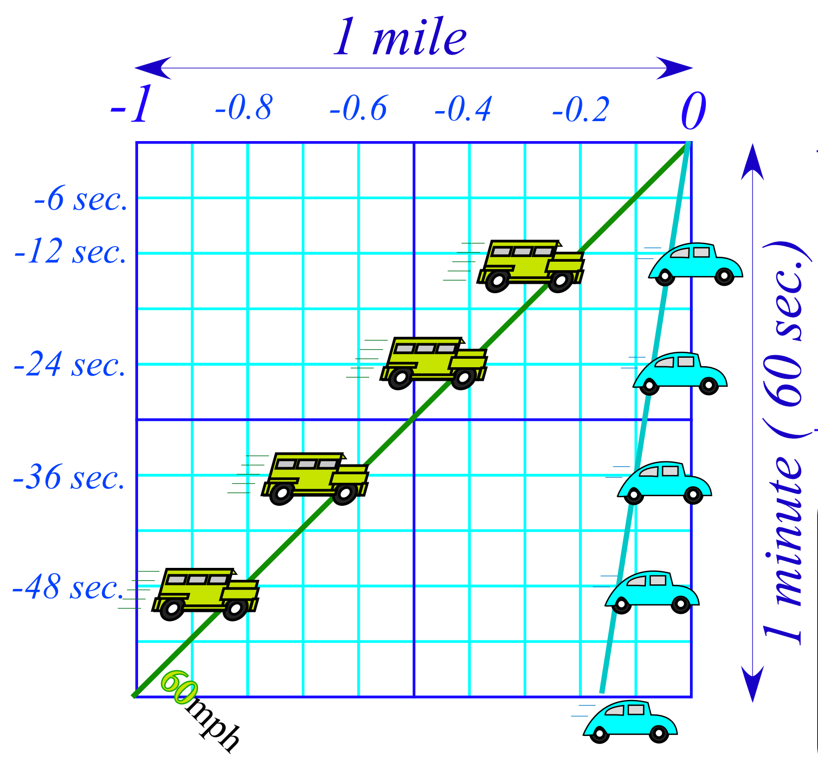

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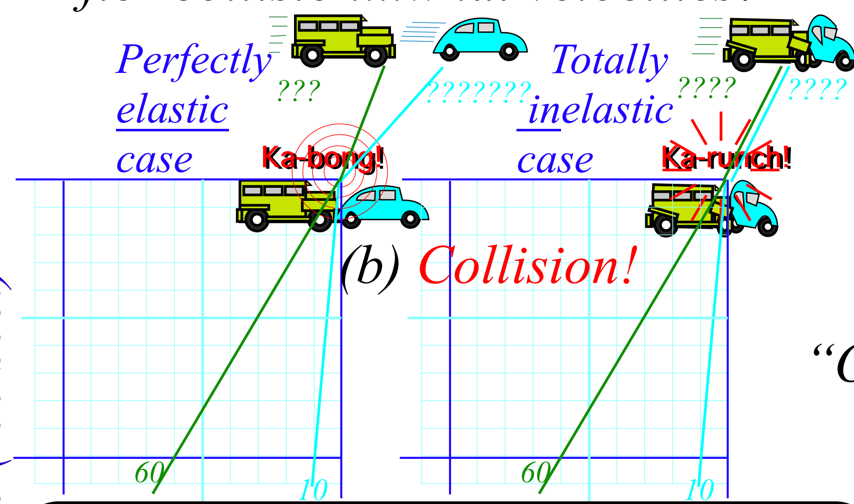

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"*Pluralitas non set poneda sine necessitate.*"
and has a number of interpretations:
1. Literally:"*Don't make pluralities of conjectures without necessity.*"
2. Logically: "*Assume less to prove more.*"
3. Practical coding advice: "*Keep it simple, make it powerful.*"

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V_{SUV} and V_{VW} change violently but ***total momentum*** is constant

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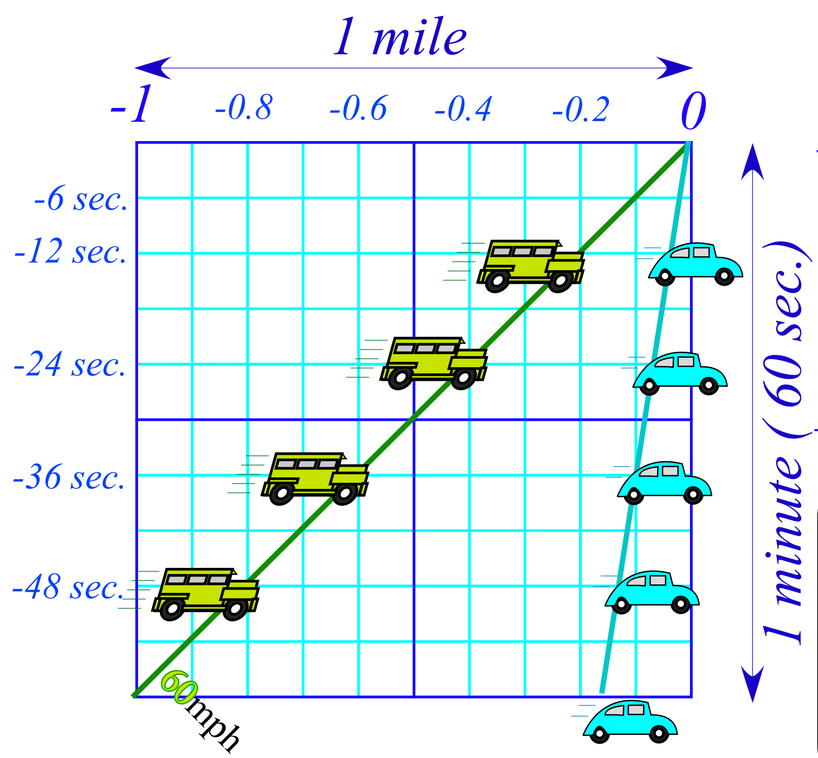

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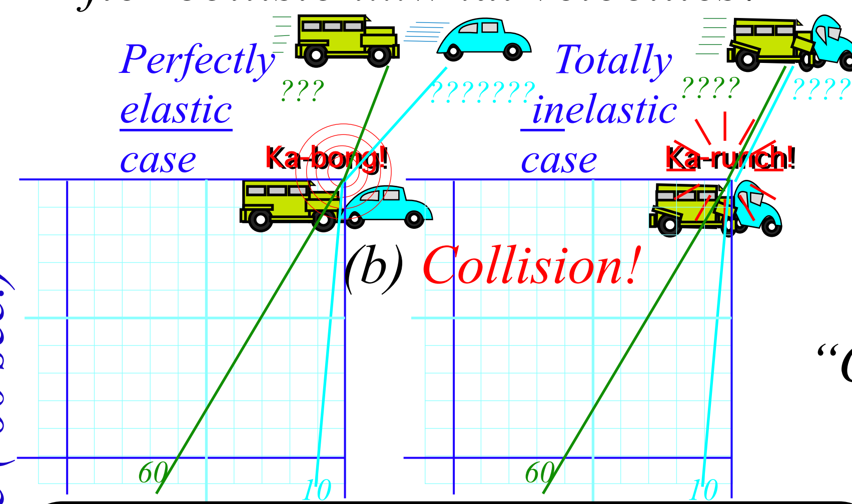

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BEFORE: (INITIAL or IN) *AFTER: (FINAL or FIN)*

$$M_{SUV}V^{IN}{}_{SUV} + M_{VW}V^{IN}{}_{VW} = M_{SUV}V^{FIN}{}_{SUV} + M_{VW}V^{FIN}{}_{VW} = constant$$

here :

$M_1 \equiv M_{SUV} = 4$

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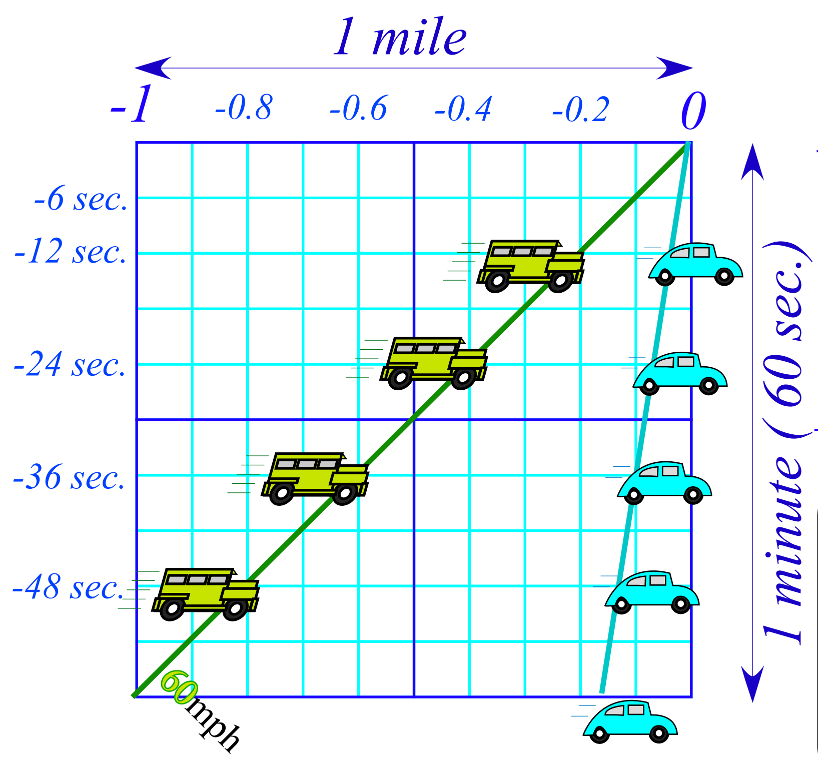

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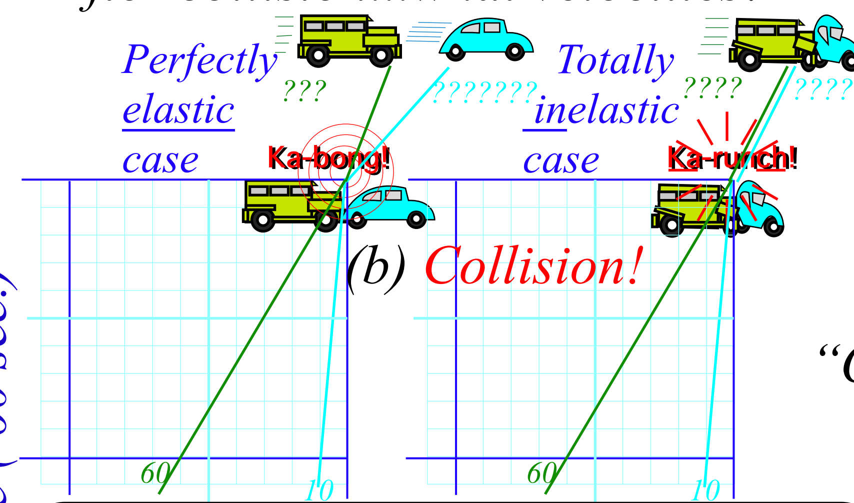

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here :

$$4 \cdot 60 \quad + \quad 1 \cdot 10 \quad = \quad 4 \cdot ? \quad + \quad 1 \cdot ?? \quad = 250$$

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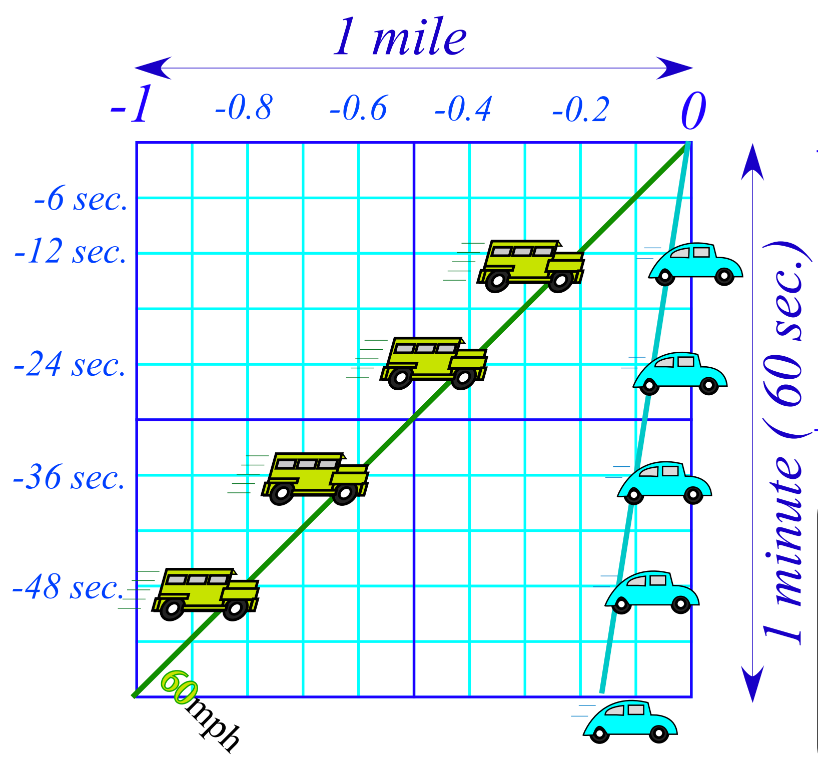

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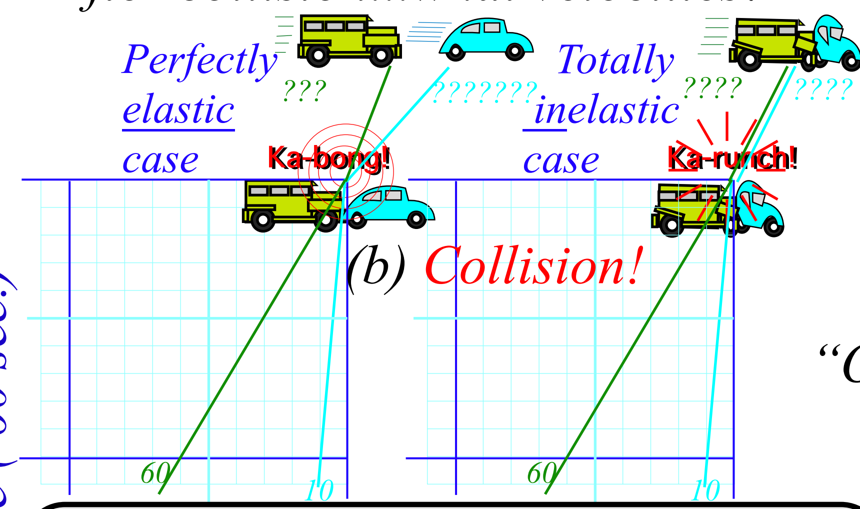

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here :

$$4 \cdot 60 + 1 \cdot 10 = 4 \cdot ? + 1 \cdot ?? = 250$$

$M_1 \equiv M_{SUV} = 4$

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It's a simple *Cartesian* equation

$$4 \cdot x + 1 \cdot y = 250$$

Rene Descartes
1596-1650

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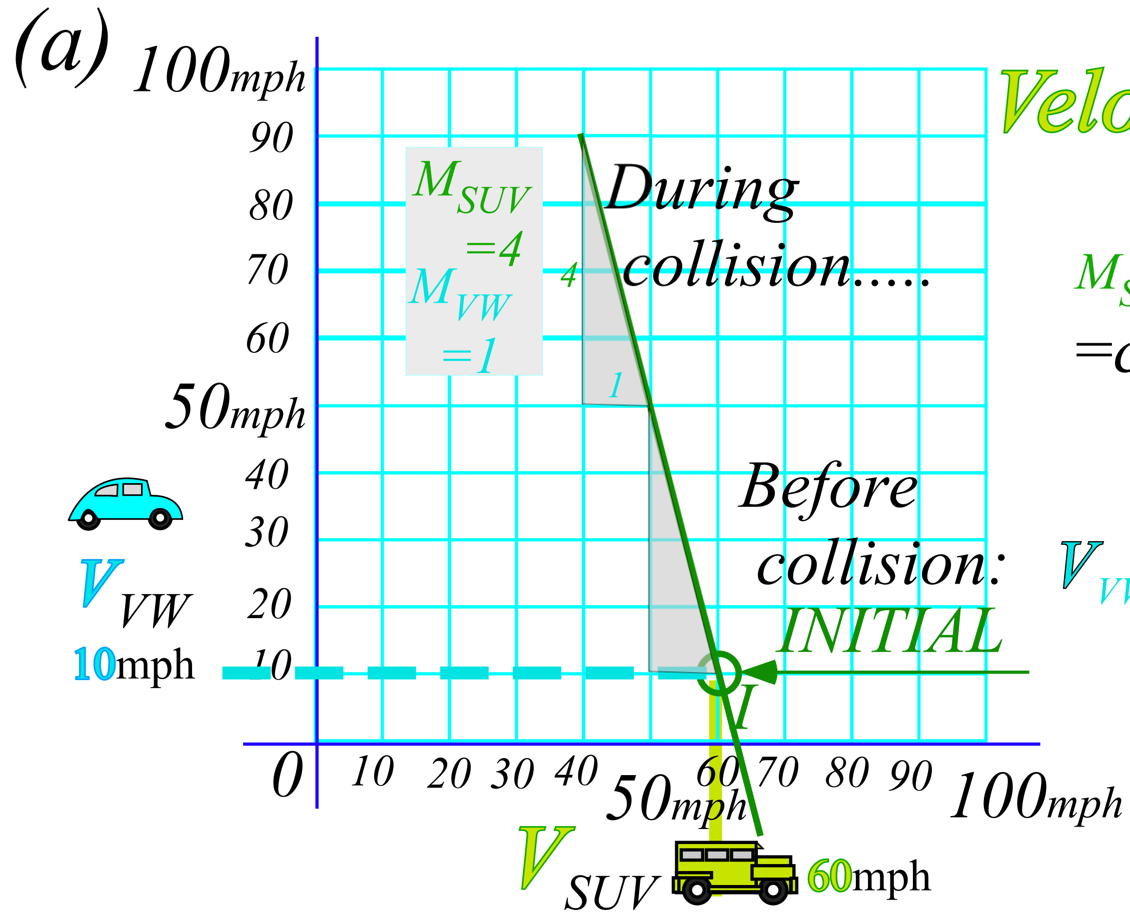


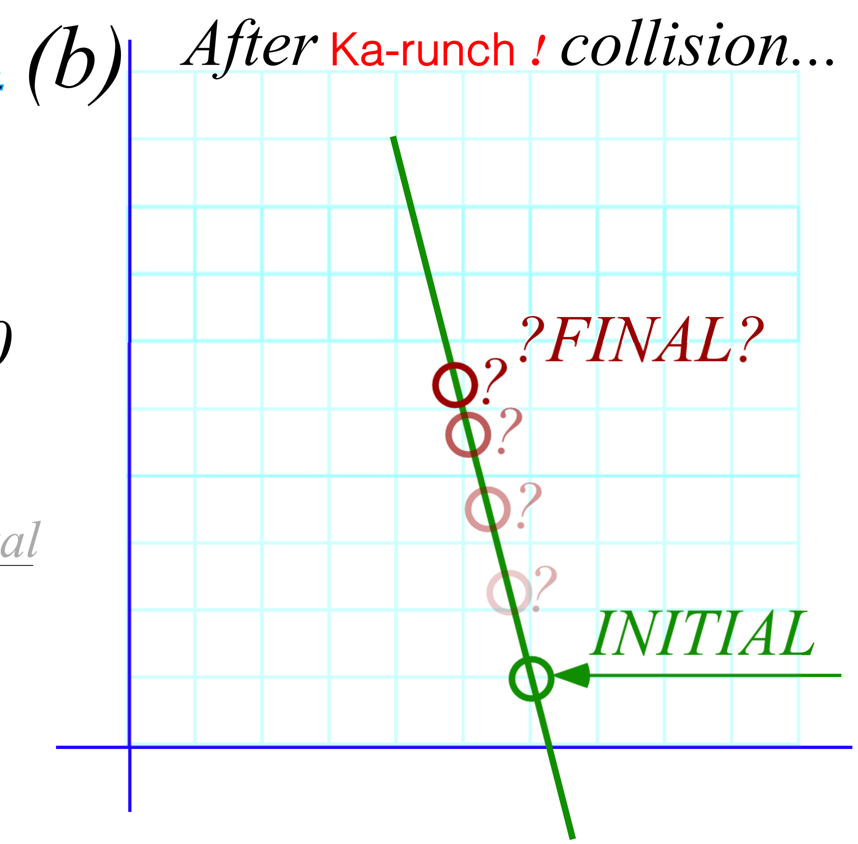
Velocity-velocity plot of Axiom-1:

$$M_{SUV}V_{SUV} + M_{VW}V_{VW}$$

$$= constant = P_{Total} = 250$$

$$V_{VW} = -\frac{M_{SUV}}{M_{VW}}V_{SUV} + \frac{P_{Total}}{M_{VW}}$$

$$= -4\ V_{SUV} + 250$$



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It's a simple *Cartesian* equation

$$4 \cdot x + 1 \cdot y = 250$$

...with a simple *Cartesian* line-plot.

Rene Descartes
1596-1650

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Geometry of momentum conservation axiom

- *Totally Inelastic "ka-runch" collisions*
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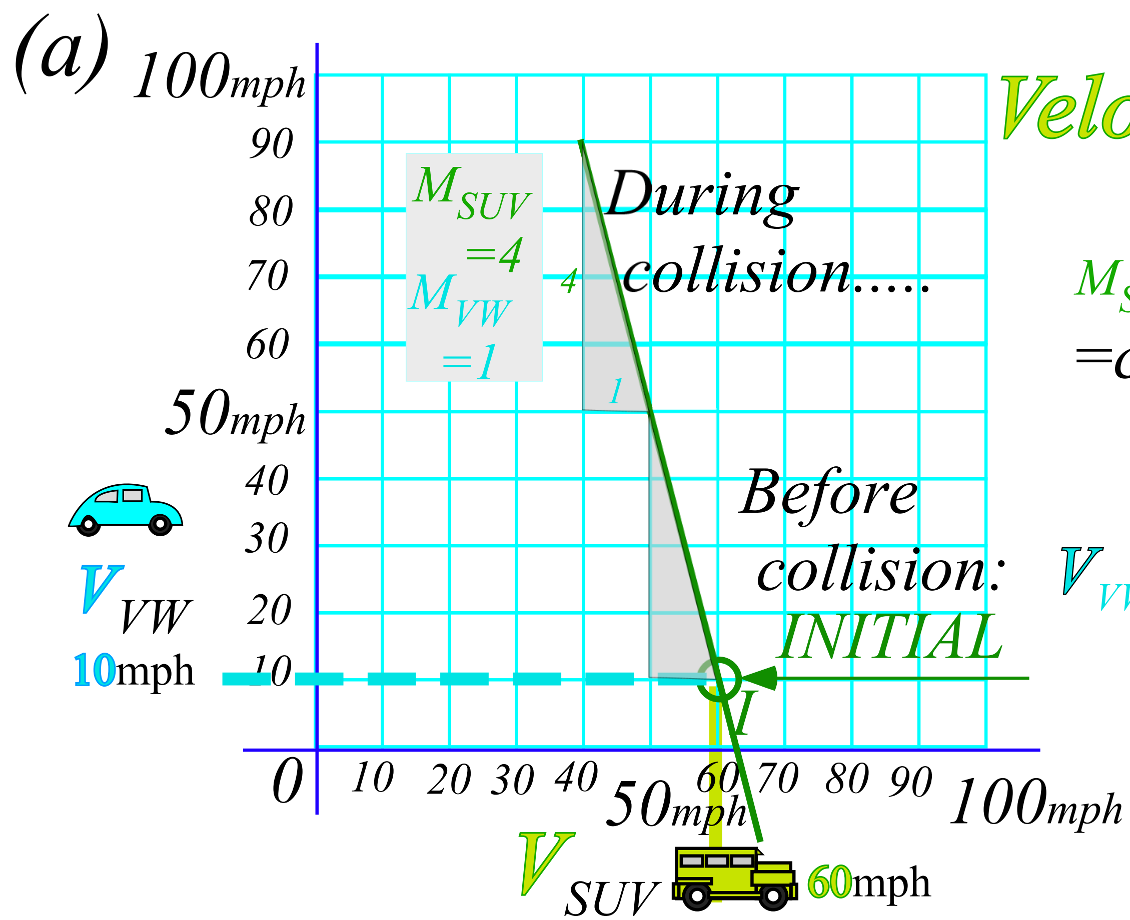

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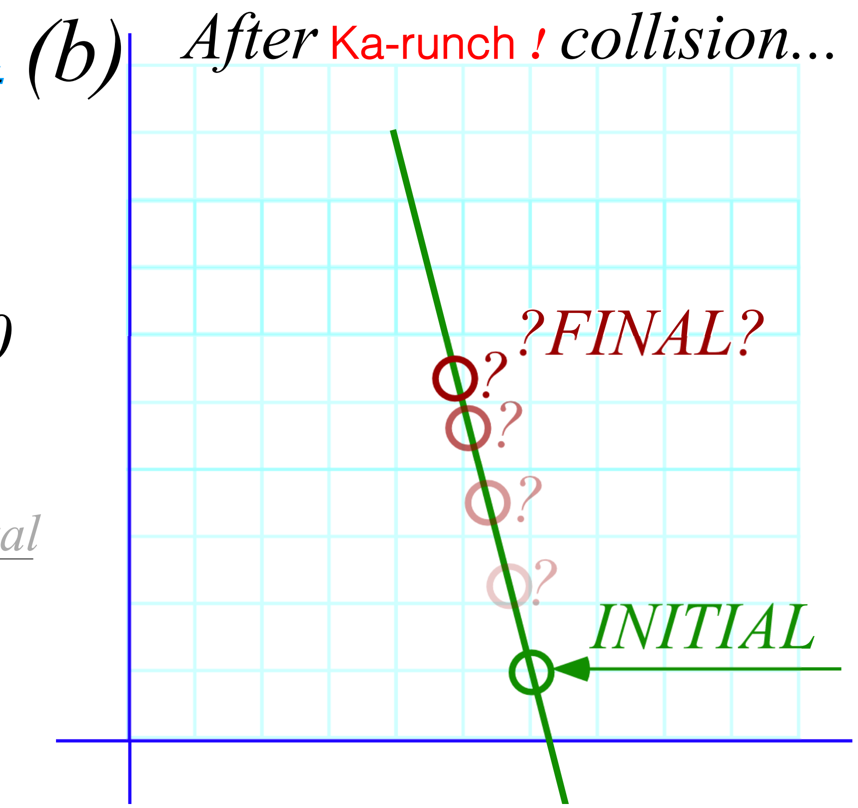

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$M_1 \equiv M_{SUV} = 4$

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It's a simple *Cartesian* equation...

$$4 \cdot x + 1 \cdot y = 250$$
$$y = 250 - 4 \cdot x$$

...with a simple *Cartesian* line-plot.

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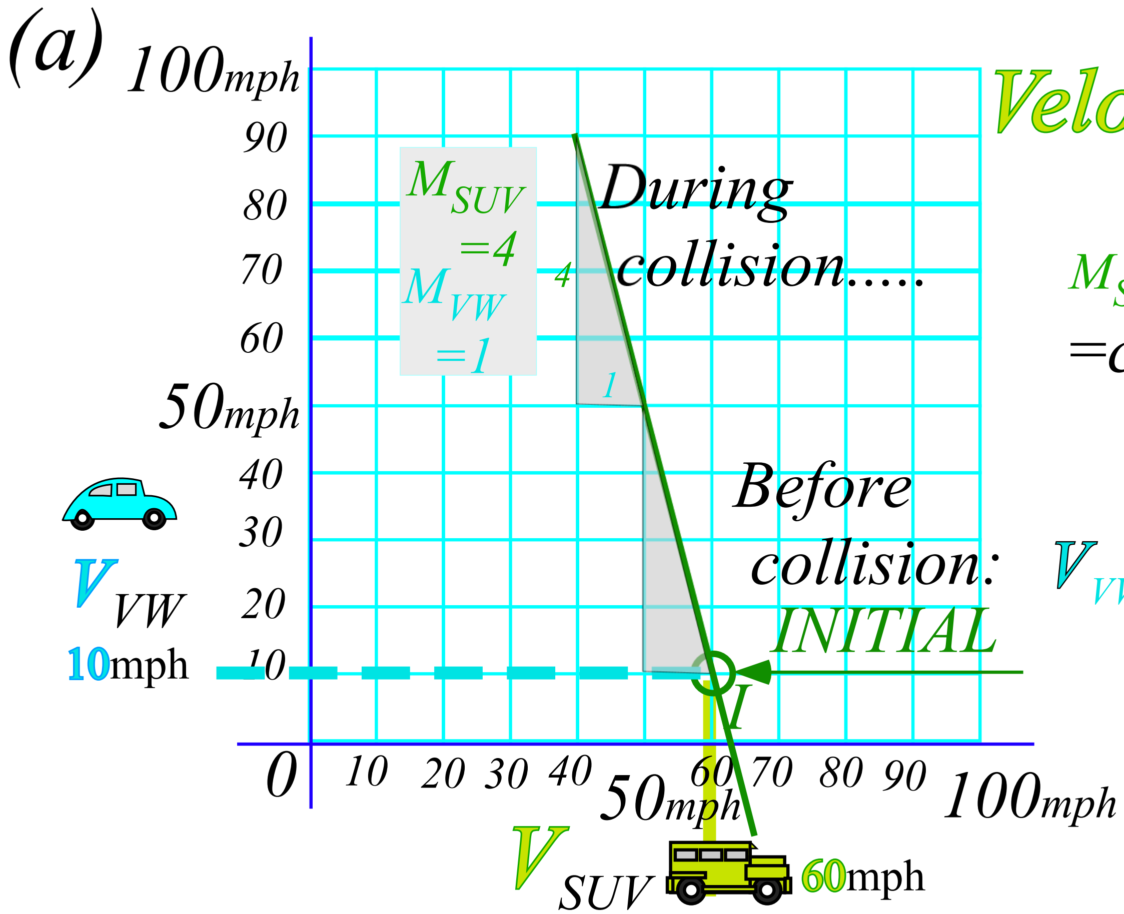
1596-1650



Velocity-velocity plot of Axiom-1:

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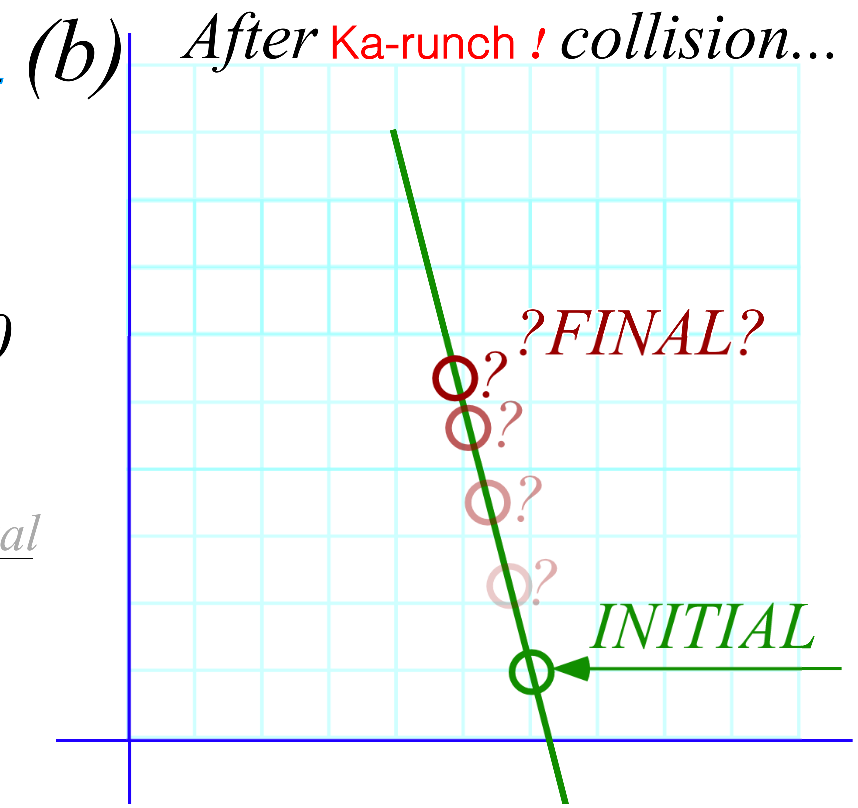
$$V_{VW}=-\frac{M_{SUV}}{M_{VW}}V_{SUV}+\frac{P_{Total}}{M_{VW}}$$

$$=-4\,V_{SUV}+250$$



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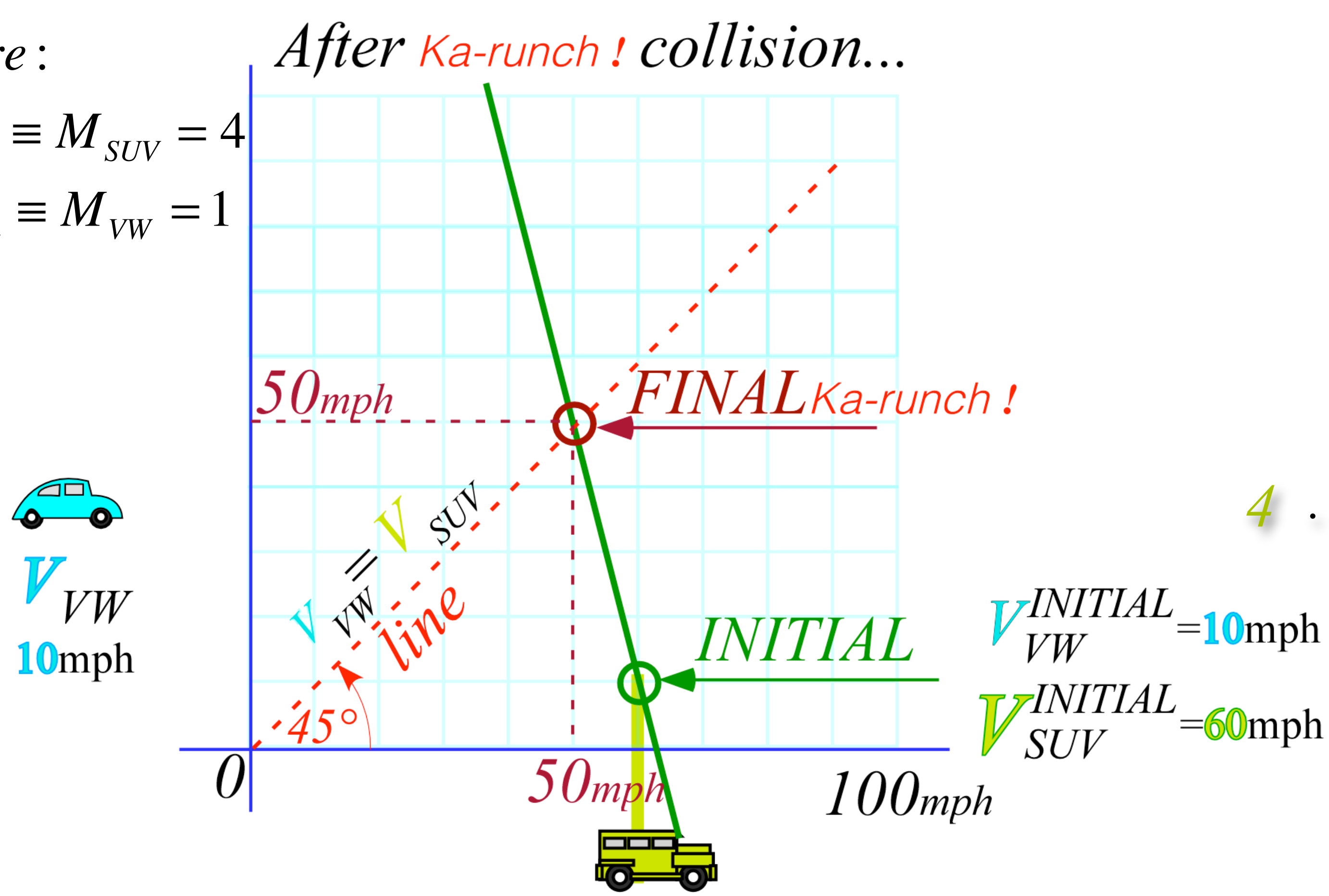

It's a simple *Cartesian* equation...

$$4\cdot x+1\cdot y=250$$

$$y=250-4\cdot x$$

$5\cdot x=250$ ...with $y=x=50$

...with a simple *Cartesian* line-plot.

Rene Descartes
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Geometry of momentum conservation axiom

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Perfectly Elastic "ka-bong" and Center Of Momentum (COM) symmetry

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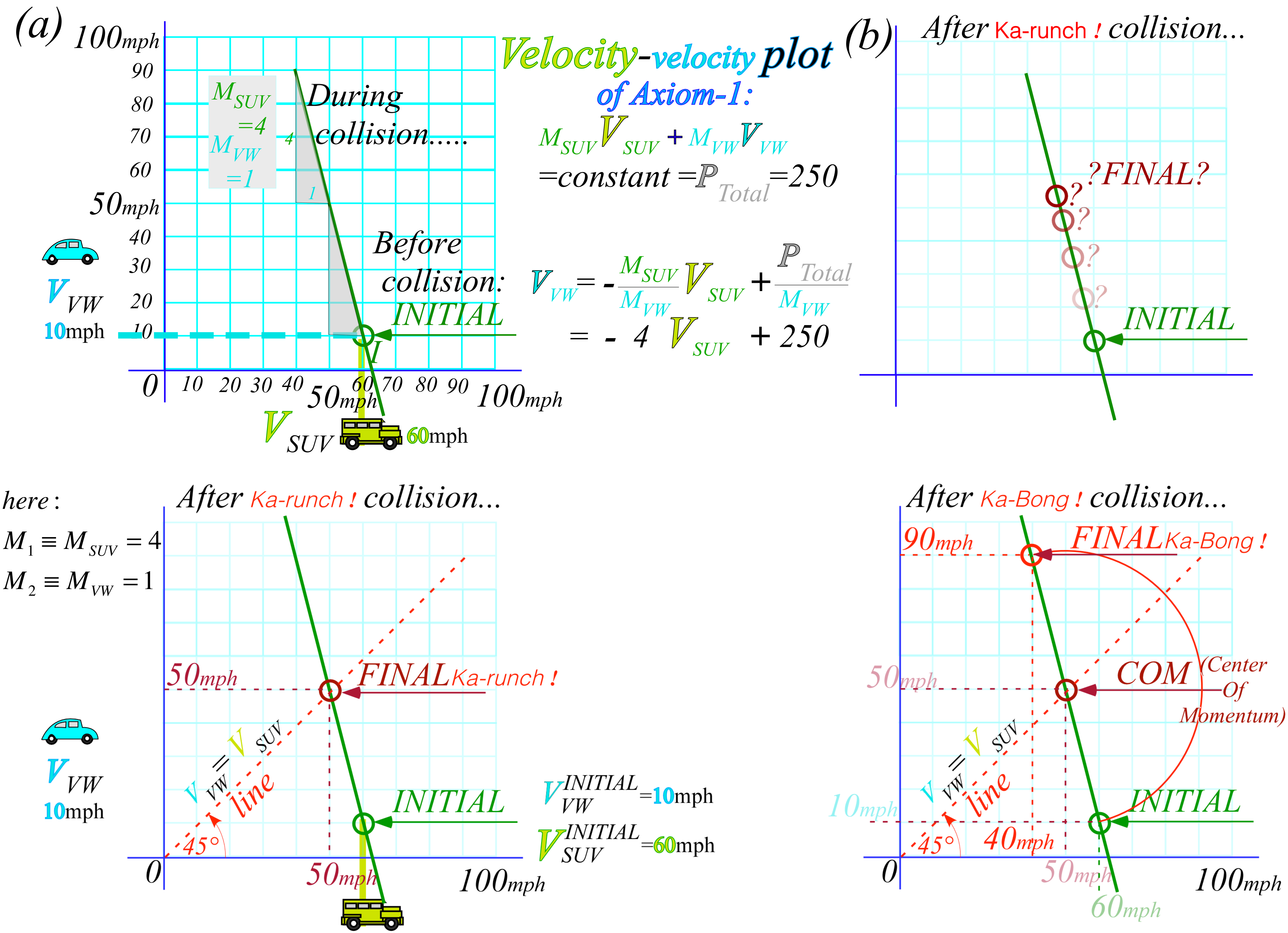

(a)
Velocity-velocity plot of Axiom-1:
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=constant $=P_{Total}$ =250
$V_{VW} = -\frac{M_{SUV}}{M_{VW}} V_{SUV} + \frac{P_{Total}}{M_{VW}}$
$= - 4 V_{SUV} + 250$
M_{SUV} =4
M_{VW} =1
During collision.....
Before collision:
INITIAL
V_{VW}
10mph
V_{SUV}
60mph
100mph
50mph
(b)
After Ka-runch ! collision...
?FINAL?
INITIAL
here:
$M_1 \equiv M_{SUV} = 4$
$M_2 \equiv M_{VW} = 1$
After Ka-runch ! collision...
50mph
FINAL Ka-runch !
$V_{VW} = V_{SUV}$ line
45°
INITIAL
$V_{VW}^{INITIAL}$ =10mph
$V_{SUV}^{INITIAL}$ =60mph
After Ka-Bong ! collision...
90mph
FINAL Ka-Bong !
COM (Center Of Momentum)
50mph
10mph
40mph
60mph
INITIAL

Geometry of momentum conservation axiom

Totally Inelastic "ka-runch" collisions

Perfectly Elastic "ka-bong" and Center Of Momentum (COM) symmetry

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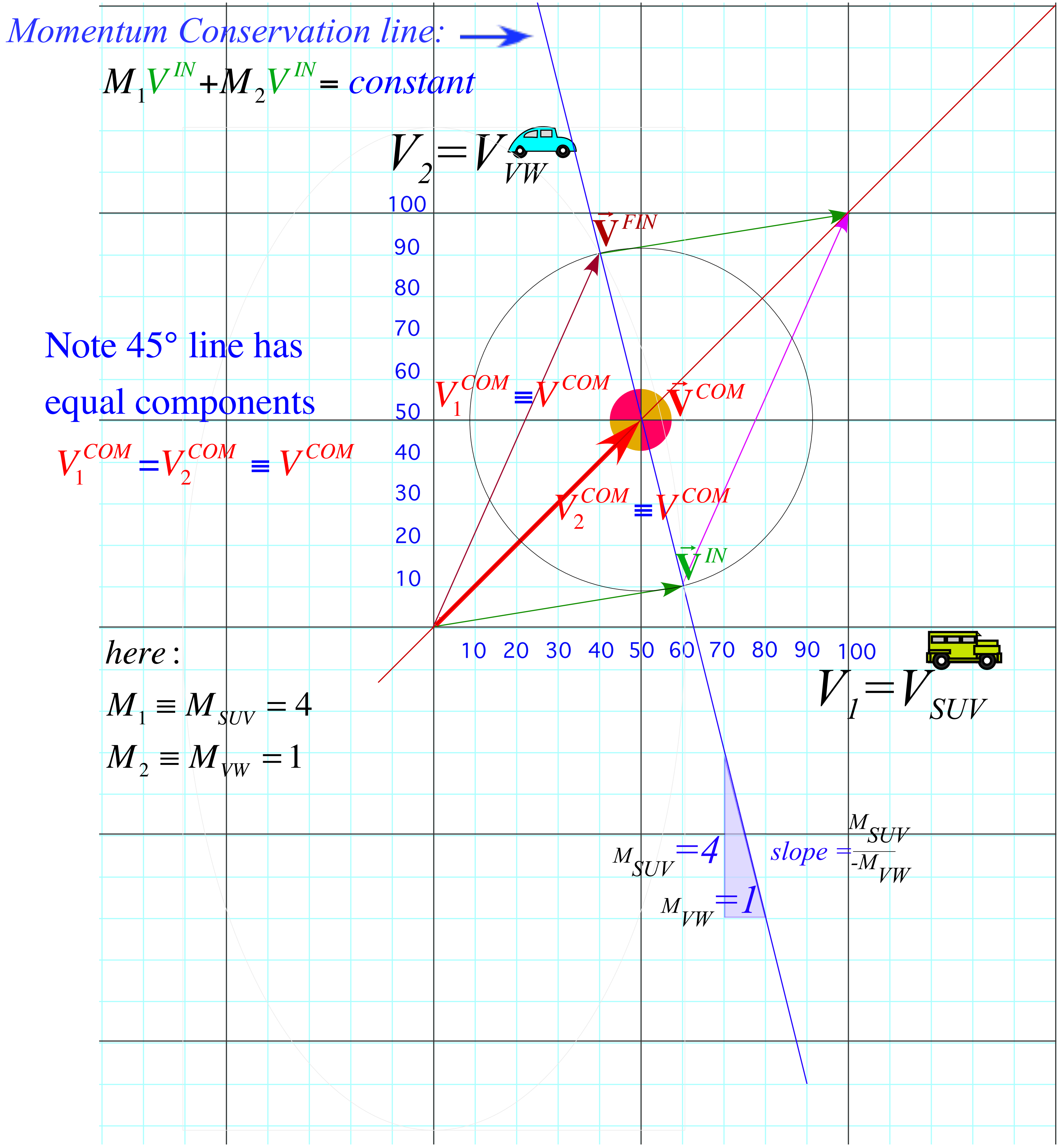
Comments on idealization in classical models

Geometry of Momentum Conservation **Axiom - 1**

$$(M_1+M_2)V^{COM} = M_1V_1^{IN}+M_2V_2^{IN} = M_1V_1^{COM}+M_2V_2^{COM} = M_1V_1^{FIN}+M_2V_2^{FIN} = M_{Total}V^{COM}$$



Geometry of Momentum Conservation **Axiom - 1**

$$(M_1+M_2)V^{COM} = M_1V_1^{IN}+M_2V_2^{IN} = M_1V_1^{COM}+M_2V_2^{COM} = M_1V_1^{FIN}+M_2V_2^{FIN} = M_{Total}V^{COM}$$

Momentum Conservation line: →

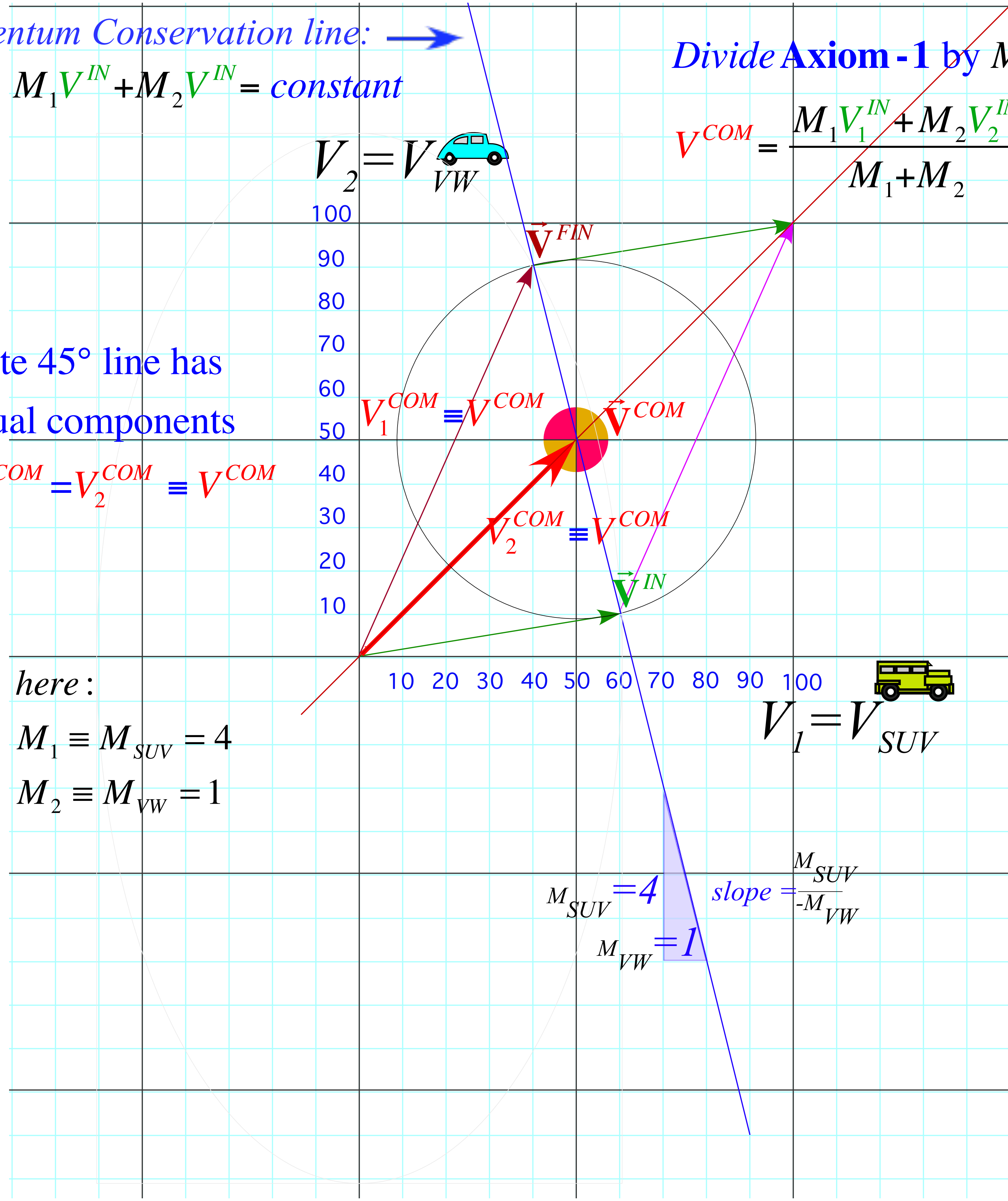
$$M_1V^{IN}+M_2V^{IN} = constant$$

Divide **Axiom - 1** by $M_{Total} = (M_1+M_2)$

$$V^{COM} = \frac{M_1V_1^{IN}+M_2V_2^{IN}}{M_1+M_2} = \frac{M_1V_1^{COM}+M_2V_2^{COM}}{M_1+M_2} = \frac{M_1V_1^{FIN}+M_2V_2^{FIN}}{M_1+M_2} = 50$$

Note 45° line has equal components

$$V_1^{COM} = V_2^{COM} \equiv V^{COM}$$



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$M_2 \equiv M_{VW} = 1$

Geometry of Momentum Conservation **Axiom - 1**

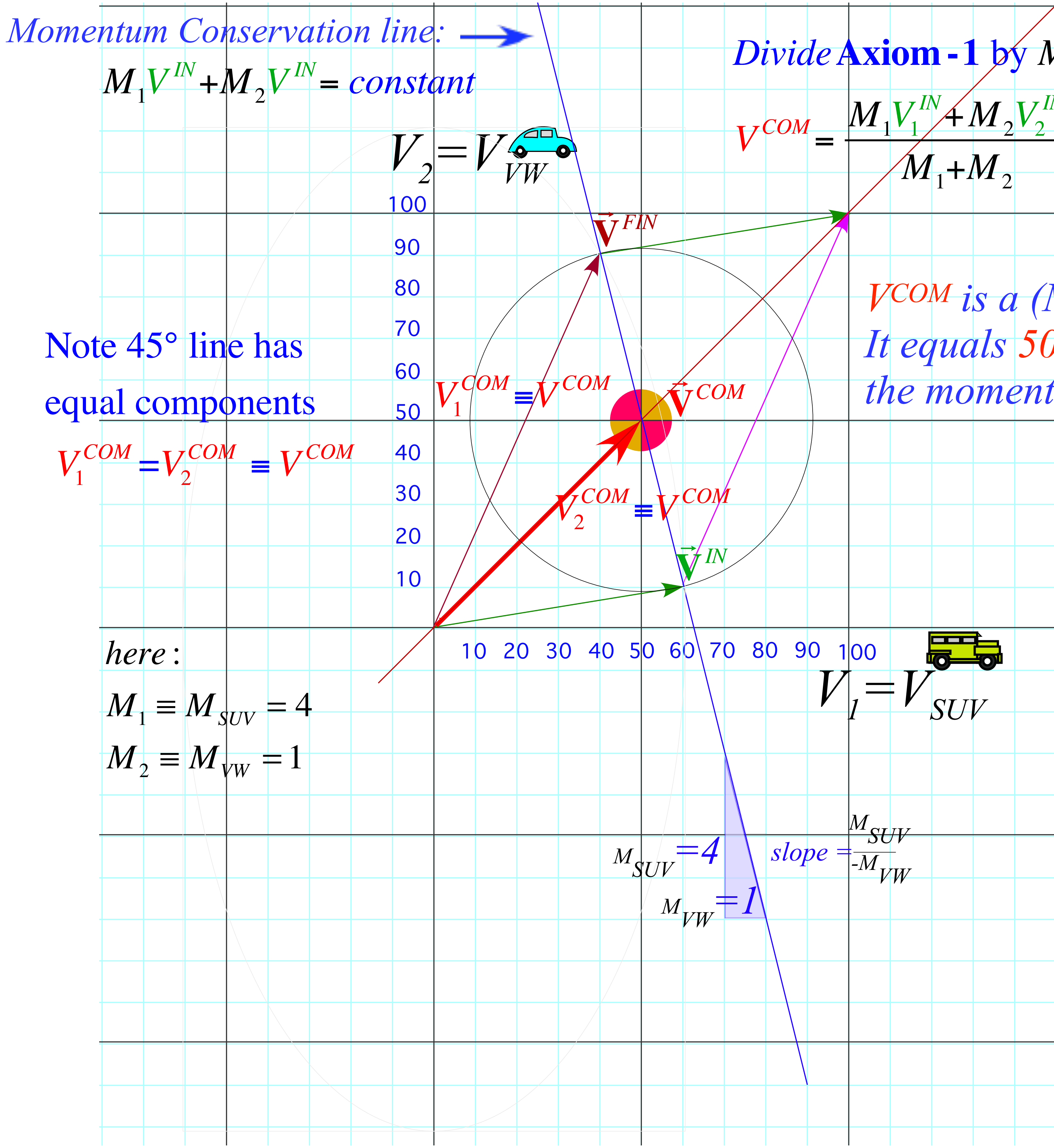
$$(M_1+M_2)V^{COM} = M_1V_1^{IN}+M_2V_2^{IN} = M_1V_1^{COM}+M_2V_2^{COM} = M_1V_1^{FIN}+M_2V_2^{FIN} = M_{Total}V^{COM}$$

Momentum Conservation line: →

$$M_1V^{IN}+M_2V^{IN} = constant$$

Divide **Axiom - 1** by $M_{Total} = (M_1+M_2)$

$$V^{COM} = \frac{M_1V_1^{IN}+M_2V_2^{IN}}{M_1+M_2} = \frac{M_1V_1^{COM}+M_2V_2^{COM}}{M_1+M_2} = \frac{M_1V_1^{FIN}+M_2V_2^{FIN}}{M_1+M_2} = 50$$



V^{COM} *is a* (M_1,M_2) ***Weighted Average*** *of* V_1 *and* V_2

It equals 50 for every point (V_1,V_2) *on the momentum line*

Geometry of momentum conservation axiom

Totally Inelastic "ka-runch" collisions

Perfectly Elastic "ka-bong" and Center Of Momentum (COM) symmetry

+Intro to weighty averages and vector notation

Comments on idealization in classical models

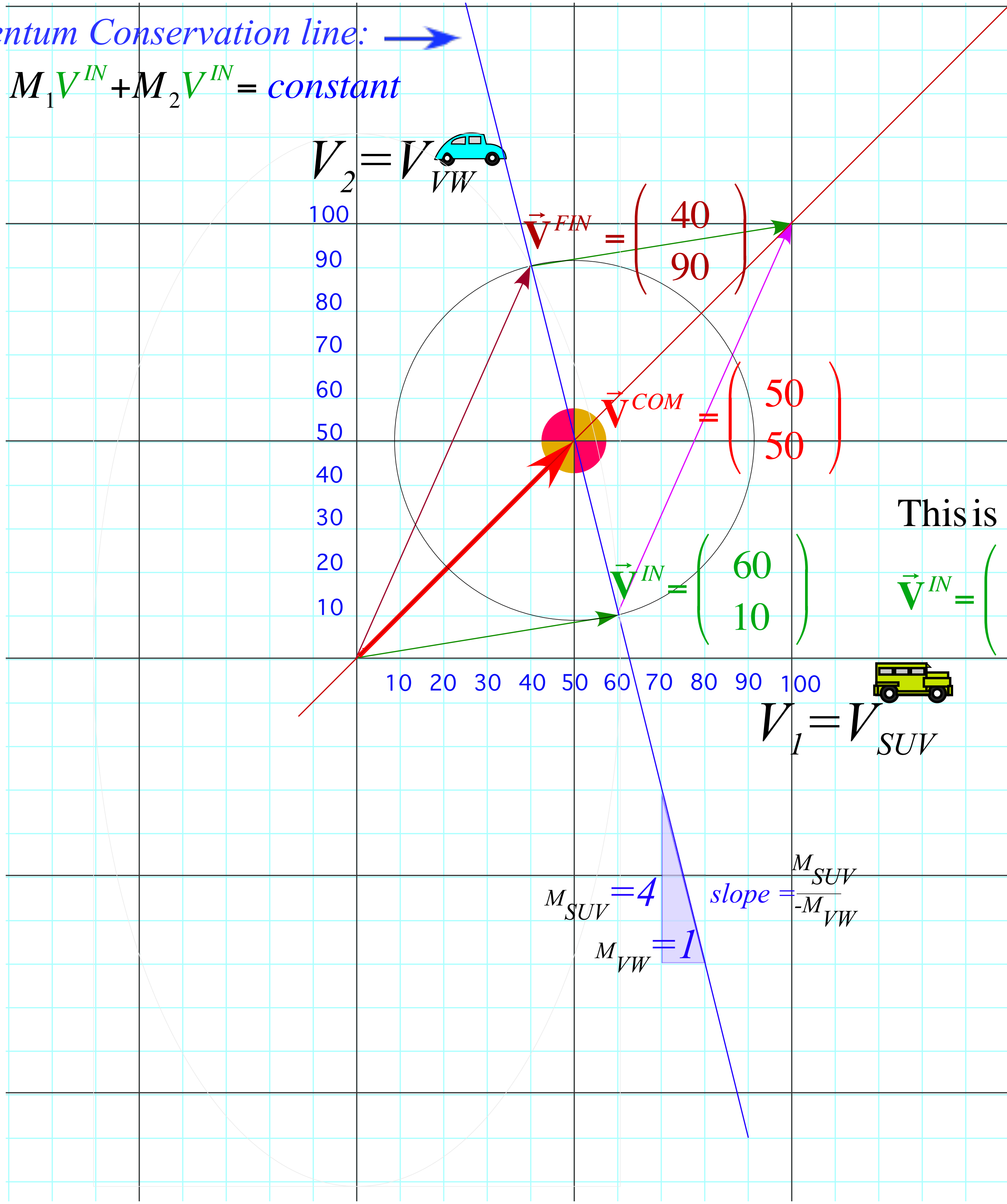
Geometry of Momentum Conservation **Axiom - 1**

$$(M_1+M_2)V^{COM} = M_1 V_1^{IN} + M_2 V_2^{IN} = M_1 V_1^{FIN} + M_2 V_2^{FIN}$$

Vector notation
...But not the x,y,.. components
Introducing SUV, VW .. components
...pretty WEIRD?!

Momentum Conservation line: →

$$M_1 V^{IN} + M_2 V^{IN} = \text{constant}$$



This is a *column-vector* (or *ket* $|IN\rangle$ in QM)

$$\vec{V}^{IN} = \begin{pmatrix} 60 \\ 10 \end{pmatrix}$$

Geometry of Momentum Conservation **Axiom - 1**

$$(M_1+M_2)V^{COM} = M_1 V_1^{IN} + M_2 V_2^{IN} = M_1 V_1^{FIN} + M_2 V_2^{FIN}$$

Momentum Conservation line: →

$$M_1 V^{IN} + M_2 V^{IN} = \textit{constant}$$

Vector notation

...But not the x,y,.. components

Introducing SUV, VW .. components

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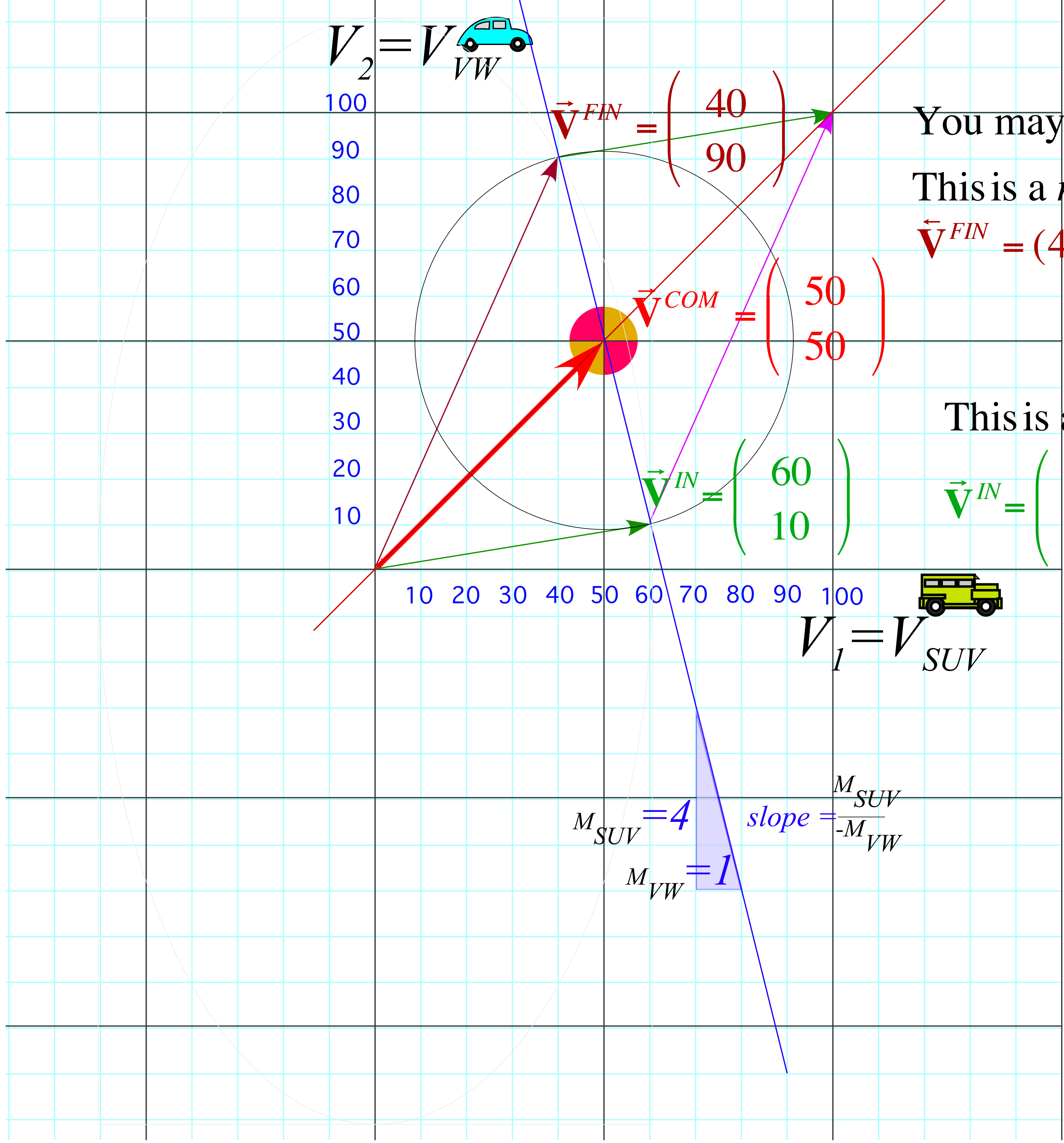


You may be used to giving coordinates as (x,y).

This is a *row-vector* (or *bra* $\langle FIN|$ in QM)

$$\vec{V}^{FIN} = (40,90)$$

This is a *column-vector* (or *ket* $|IN\rangle$ in QM)

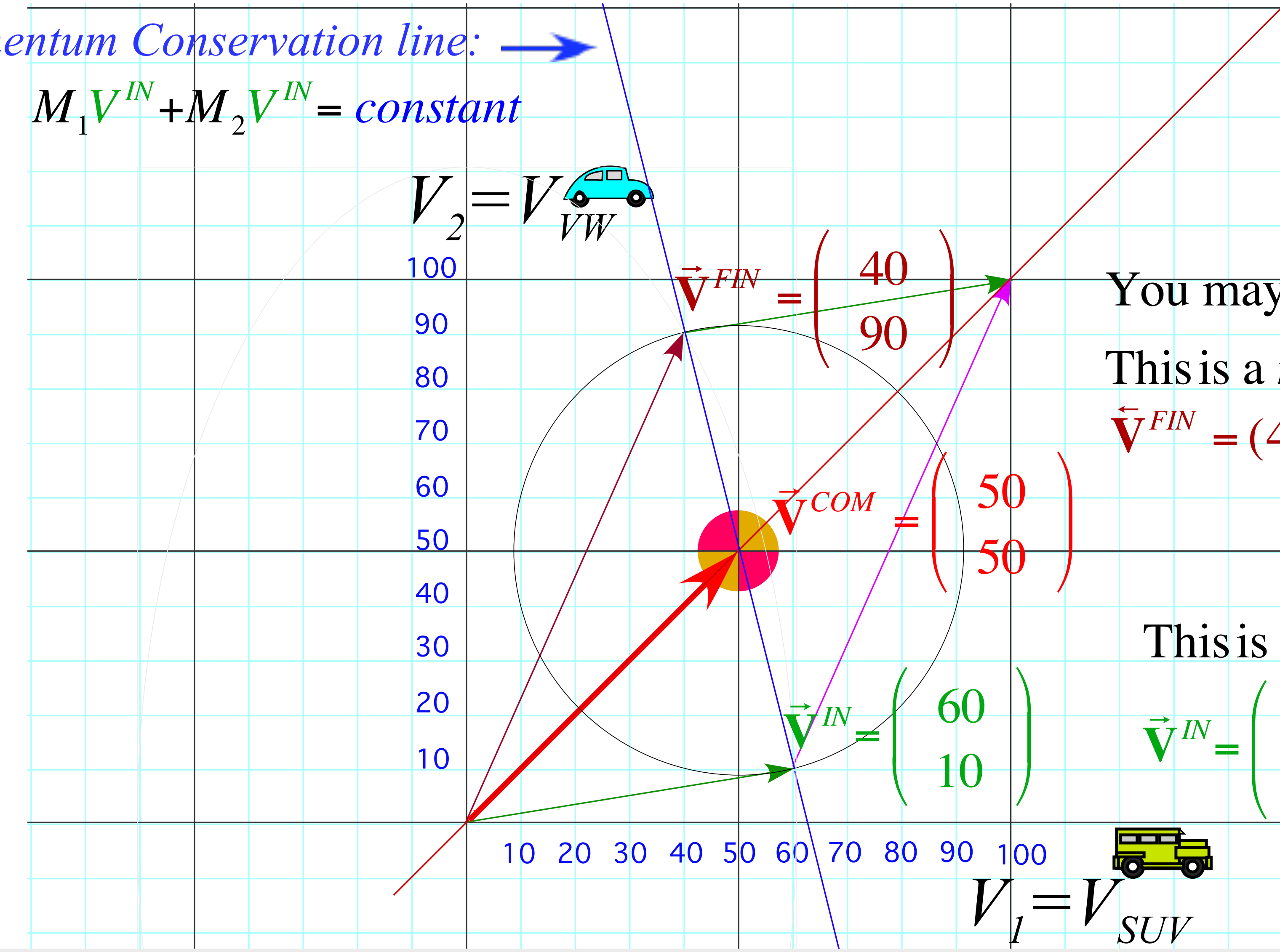
$$\vec{V}^{IN} = \begin{pmatrix} 60 \\ 10 \end{pmatrix}$$

Geometry of Momentum Conservation **Axiom - 1**

$$(M_1+M_2)V^{COM} = M_1 V_1^{IN} + M_2 V_2^{IN} = M_1 V_1^{FIN} + M_2 V_2^{FIN}$$

Momentum Conservation line:

$$M_1 V^{IN} + M_2 V^{IN} = constant$$



Vector notation

...But not the x,y,.. components

Introducing SUV, VW .. components

...pretty WEIRD?!

...well, not too weird, actually...

You may be used to giving coordinates as (x,y).

This is a *row-vector* (or *bra* $\langle FIN|$ in QM)

$\overleftarrow{\mathbf{V}}^{FIN} = (40,90)$

This is a *column-vector* (or *ket* $|IN\rangle$ in QM)

$$\vec{\mathbf{V}}^{IN} = \begin{pmatrix} 60 \\ 10 \end{pmatrix}$$

This is a *dot* product (or *scalar* product)

$$\overleftarrow{\mathbf{V}}^{FIN} \bullet \vec{\mathbf{V}}^{IN} = (40,90) \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix} = \langle FIN | IN \rangle = 40\ 60 + 90\ 10 = 2400 + 900 = 3300$$

of a *row-vector* $\overleftarrow{\mathbf{V}}^{FIN} = (40,90)$ (or *bra* $\langle FIN|$)

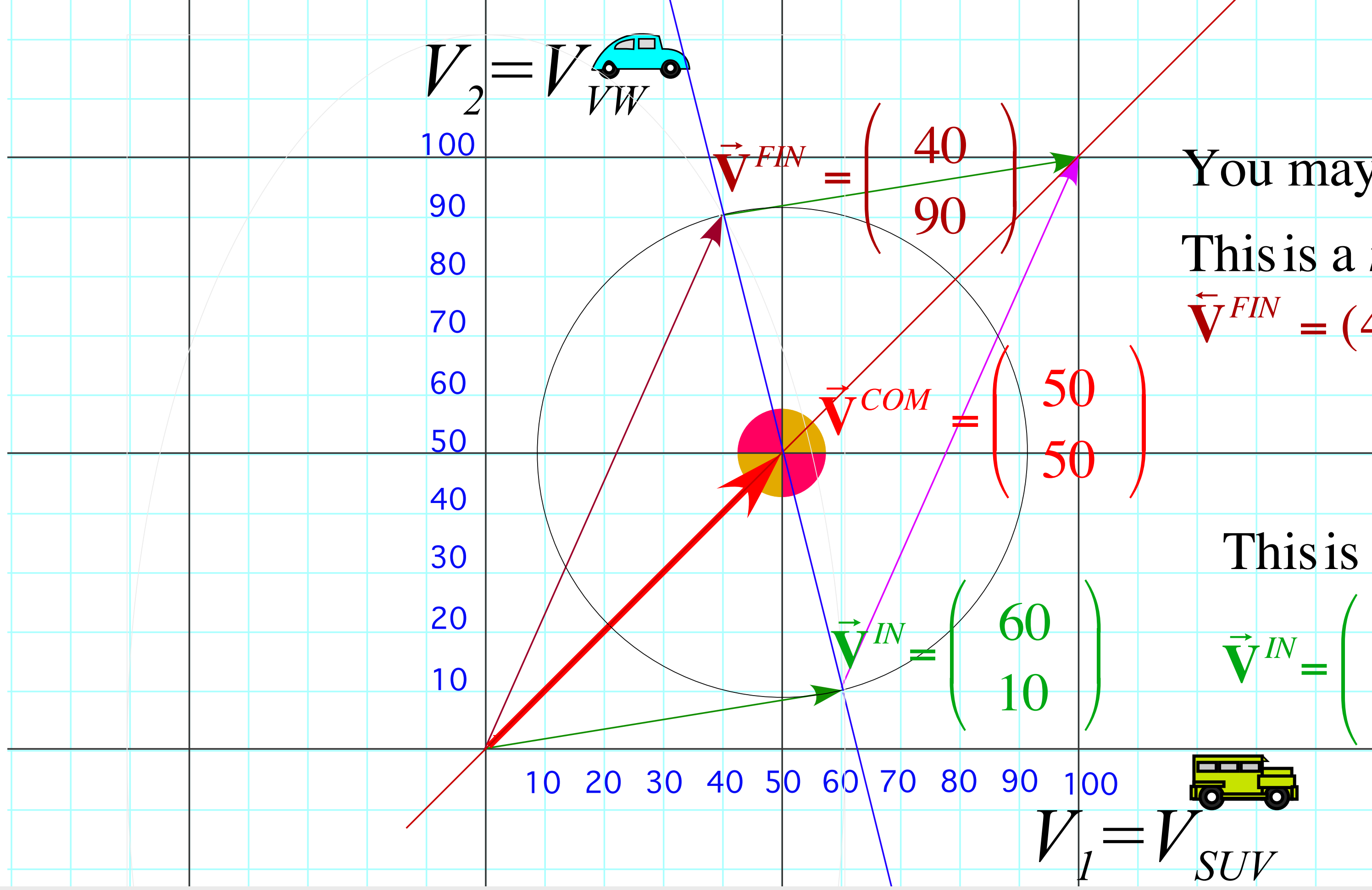
with *column-vector* $= \begin{pmatrix} 60 \\ 10 \end{pmatrix}$ (or *ket* $|IN\rangle$)

Geometry of Momentum Conservation **Axiom - 1**

$$(M_1+M_2)V^{COM} = M_1V_1^{IN} + M_2V_2^{IN} = M_1V_1^{FIN} + M_2V_2^{FIN}$$

Momentum Conservation line: →

$$M_1V^{IN} + M_2V^{IN} = constant$$



Vector notation

...But ***not*** *the x,y,.. components*

Introducing SUV, VW *.. components*

...pretty WEIRD?!

...well, not too weird, actually...

You may be used to giving coordinates as (x,y).

This is a *row-vector* (or *bra* $\langle FIN|$ in QM)

$\overleftarrow{\mathbf{V}}^{FIN} = (40,90)$

This is a *column-vector* (or *ket* $|IN\rangle$ in QM)

$$\vec{\mathbf{V}}^{IN} = \begin{pmatrix} 60 \\ 10 \end{pmatrix}$$

This is a *outer* product (or *tensor* product)

$$\vec{\mathbf{V}}^{IN} \otimes \overleftarrow{\mathbf{V}}^{FIN} = \begin{pmatrix} 60 \\ 10 \end{pmatrix} \otimes (40,90) = |IN\rangle\langle FIN| = \begin{pmatrix} 60\ 40 & 60\ 90 \\ 10\ 40 & 10\ 90 \end{pmatrix} = \begin{pmatrix} 2400 & 5400 \\ 400 & 900 \end{pmatrix}$$

of a *column-vector* $= \begin{pmatrix} 60 \\ 10 \end{pmatrix}$ (or *ket* $|IN\rangle$)

with a *row-vector* $\overleftarrow{\mathbf{V}}^{FIN} = (40,90)$ (or *bra* $\langle FIN|$)

Geometry of momentum conservation axiom

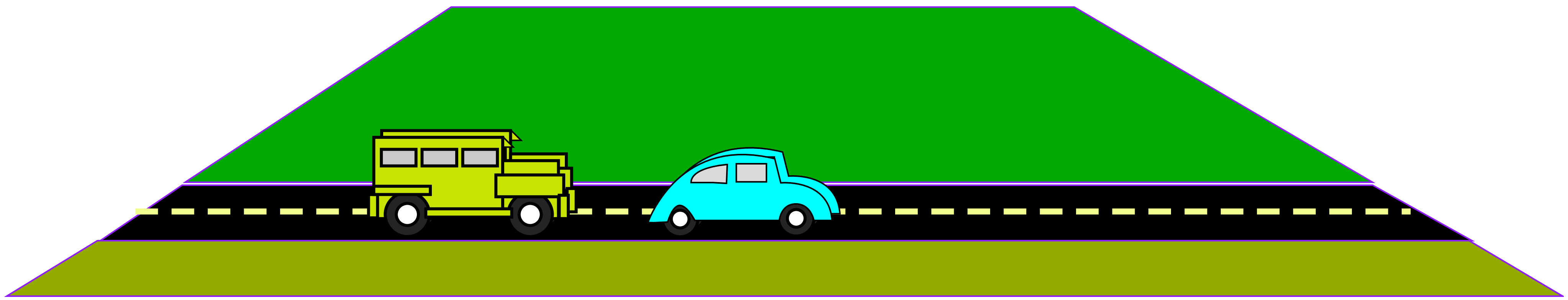
Totally Inelastic "ka-runch"collisions

Perfectly Elastic "ka-bong" and Center Of Momentum (COM) symmetry

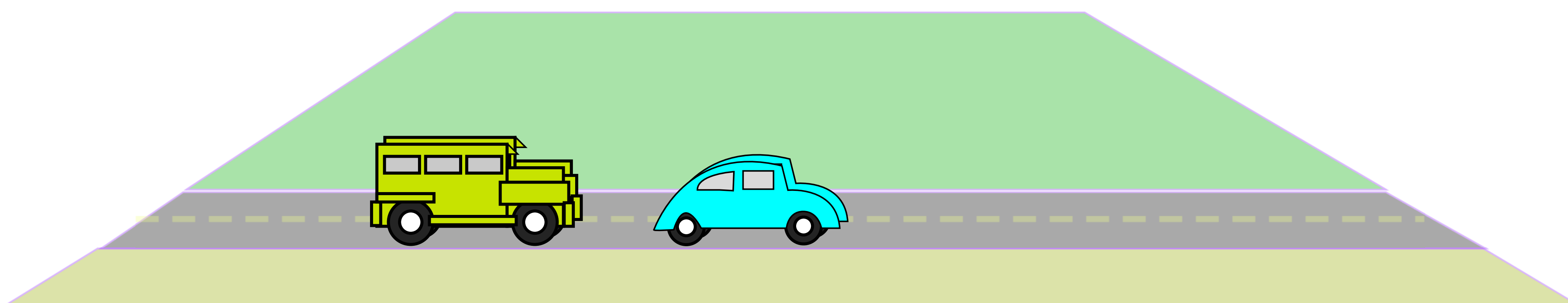
+Intro to weighty-averages and vector notation

→ *Comments on idealization in classical models* ←

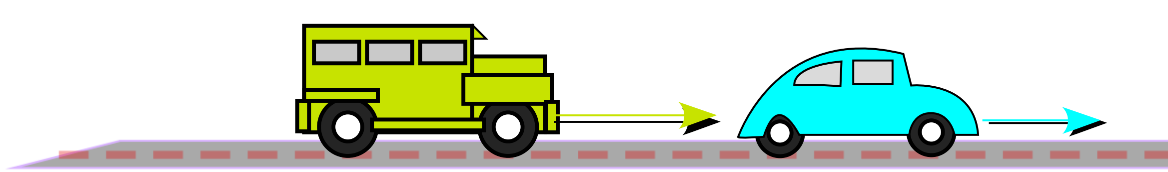
The SUV and VW *Idealized* thought experiments



Idealization 1. Ignore background.
(No rolling friction, air resistance, etc.)



Idealization 2. Make each 1-dimensional.
(Cars "constrained" to ride on frictionless rail)

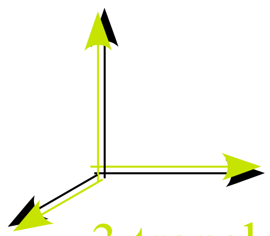


System now has
just *two* "dimensions"
or "*degrees-of-freedom*"

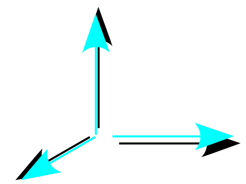
Landscape 1.1 Idealized model for collision model and thought experiments

Summary of Classical Mechanical Degrees of Freedom

Translation (Each body has 3 translational degrees of freedom) (Intoduced in Units 1 and 2)



3 translation
dimensions

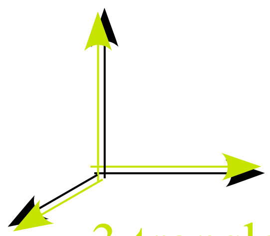


3 translation
dimensions

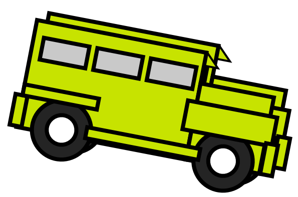
*6 translational
degrees of freedom
for SUV and VW.*

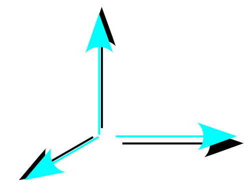
Summary of Classical Mechanical Degrees of Freedom

Translation (Each body has 3 translational degrees of freedom) (Intoduced in Units 1 and 2)

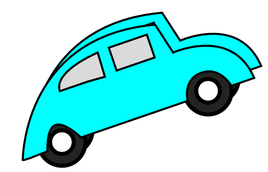


3 translation
dimensions



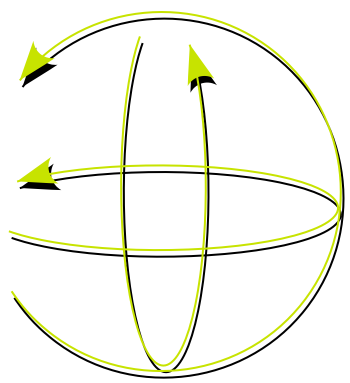


3 translation
dimensions



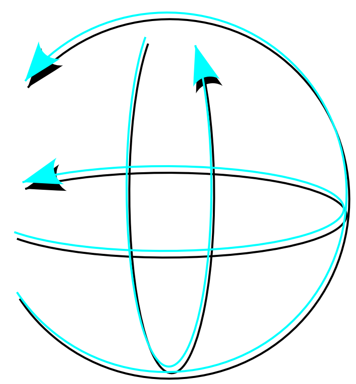
6 translational degrees of freedom for SUV and VW.

Rotation (Each body has 3 rotationaldegrees of freedom) (Intoduced in Units 3 and 7)



3 rotational
dimensions

*yaw-pitch-roll
Euler angles*



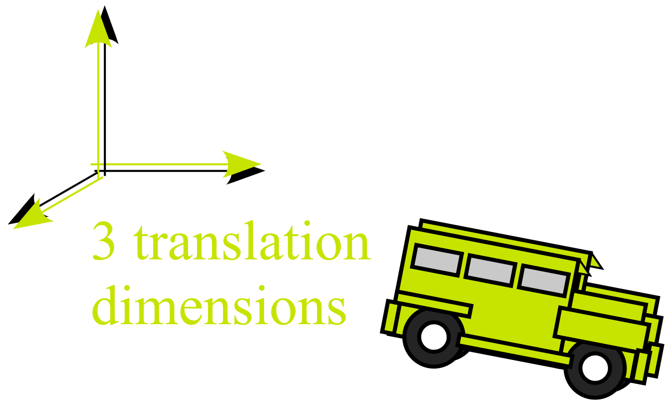
3 rotational
dimensions

*yaw-pitch-roll
Euler angles*

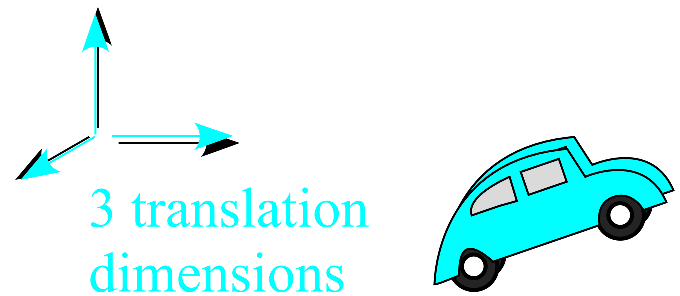
6 rotational degrees of freedom for SUV and VW.

Summary of Classical Mechanical Degrees of Freedom

Translation (Each body has 3 translational degrees of freedom) (Intoduced in Units 1 and 2)



3 translation dimensions



3 translation dimensions

6 translational degrees of freedom for SUV and VW.

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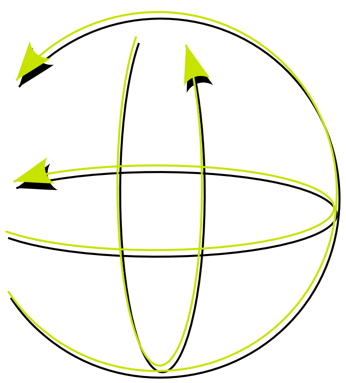
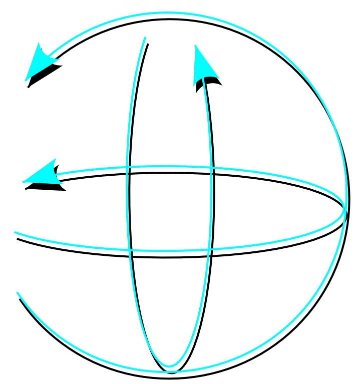


3 rotational dimensions

yaw-pitch-roll Euler angles



3 rotational dimensions

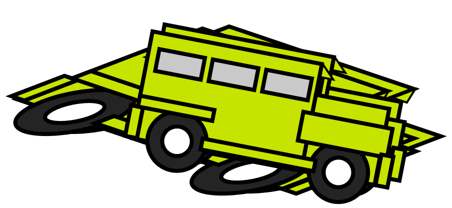
yaw-pitch-roll Euler angles

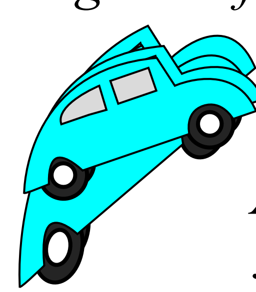
6 rotational degrees of freedom for SUV and VW.

SUV and VW system involves
12 rigid-body degrees of freedom

Vibration (Each body has <u>many</u> vibrational degrees of freedom) (Intoduced in Units 3-8)

Generalized Curvilinear Coordinates (GCC) introduced in Unit 1 Chapters 10 -12





An N-atom molecule has 3N-6 vibrational degrees of freedom

Landscape 1.2 Some idealized classical model degrees of freedom

Geometry of Galilean translation symmetry

45° shift in (V_1,V_2)-space

Time reversal symmetry

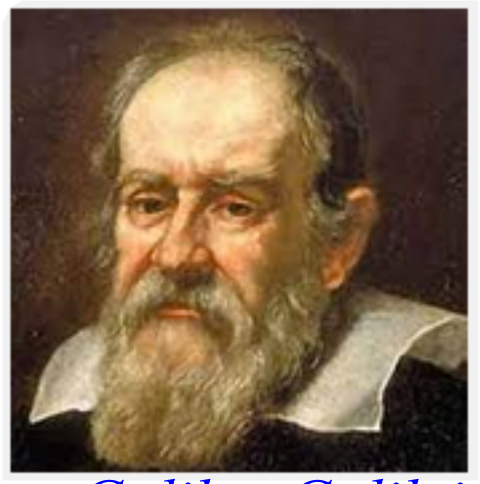
...of COM collisions

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Geometry of Galilean translation (A symmetry transformation)

If you increase your velocity by 50 mph,... *(In some direction x,y, or z...)*

*...the rest of the world appears to be 50 mph **slower*** *(In that direction...)*



Galileo Galilei
1564-1642

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

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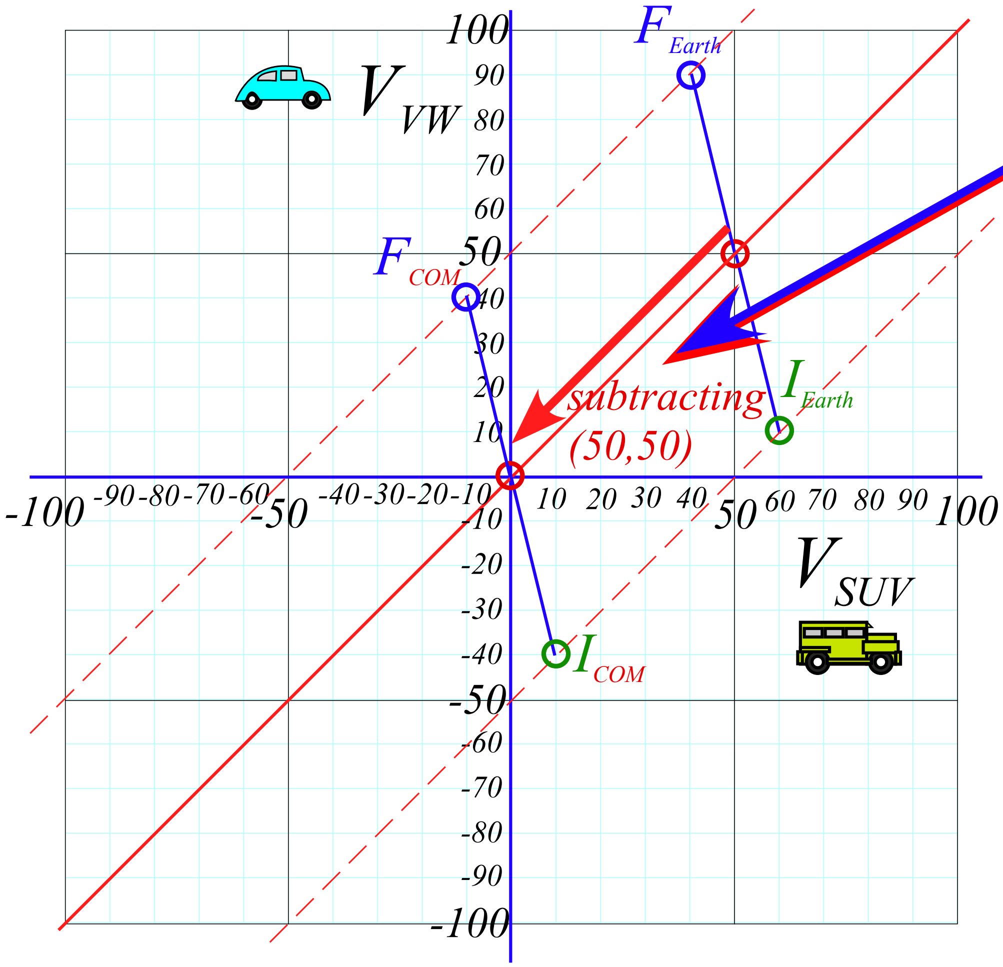
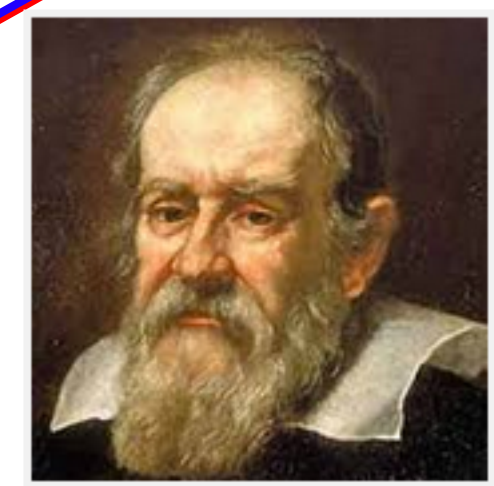
(a) Galileo transforms to COM frame





Galileo Galilei
1564-1642

Fig. 2.5a
in Unit 1

Q: Sir, does that include a light pulse?
A: Well, yeah…kinda…

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Geometry of Galilean translation (A symmetry transformation)

If you increase your velocity by 50 mph,... *(In some direction x,y, or z…)*

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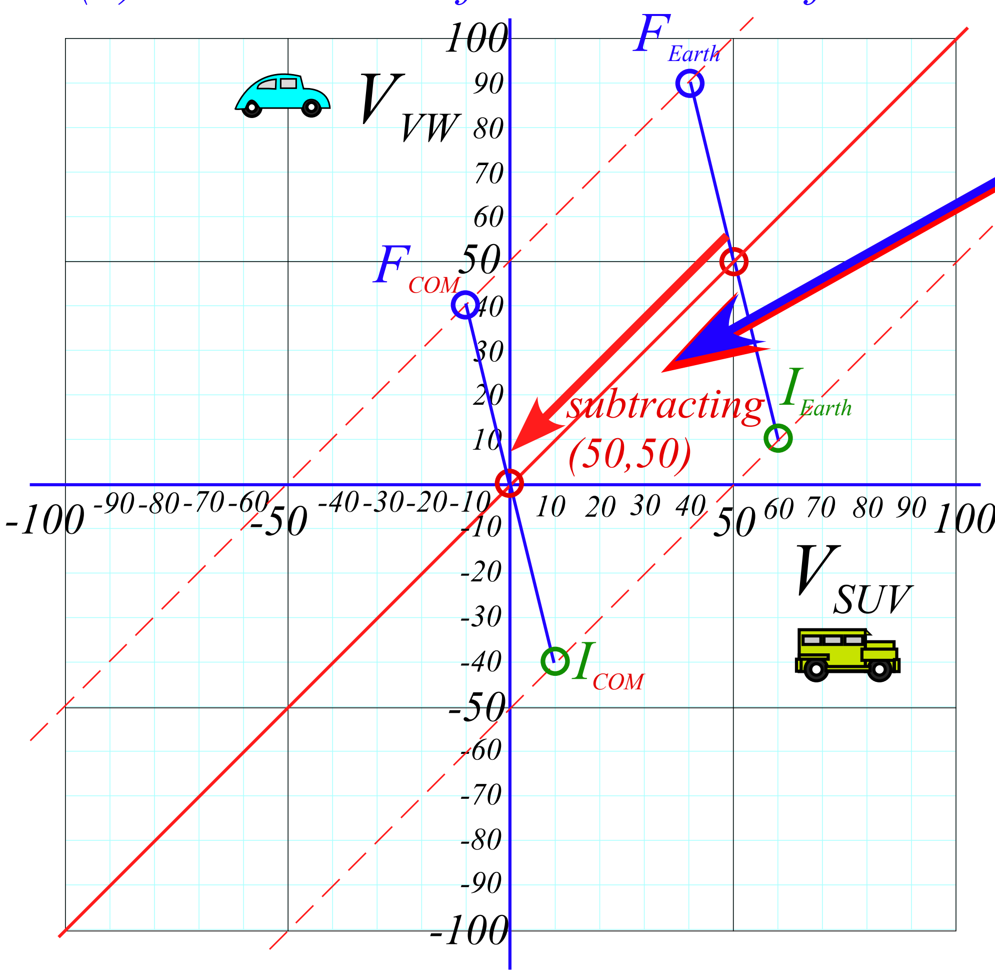


Fig. 2.5a
in Unit 1

(b) … and to five or six other reference frames

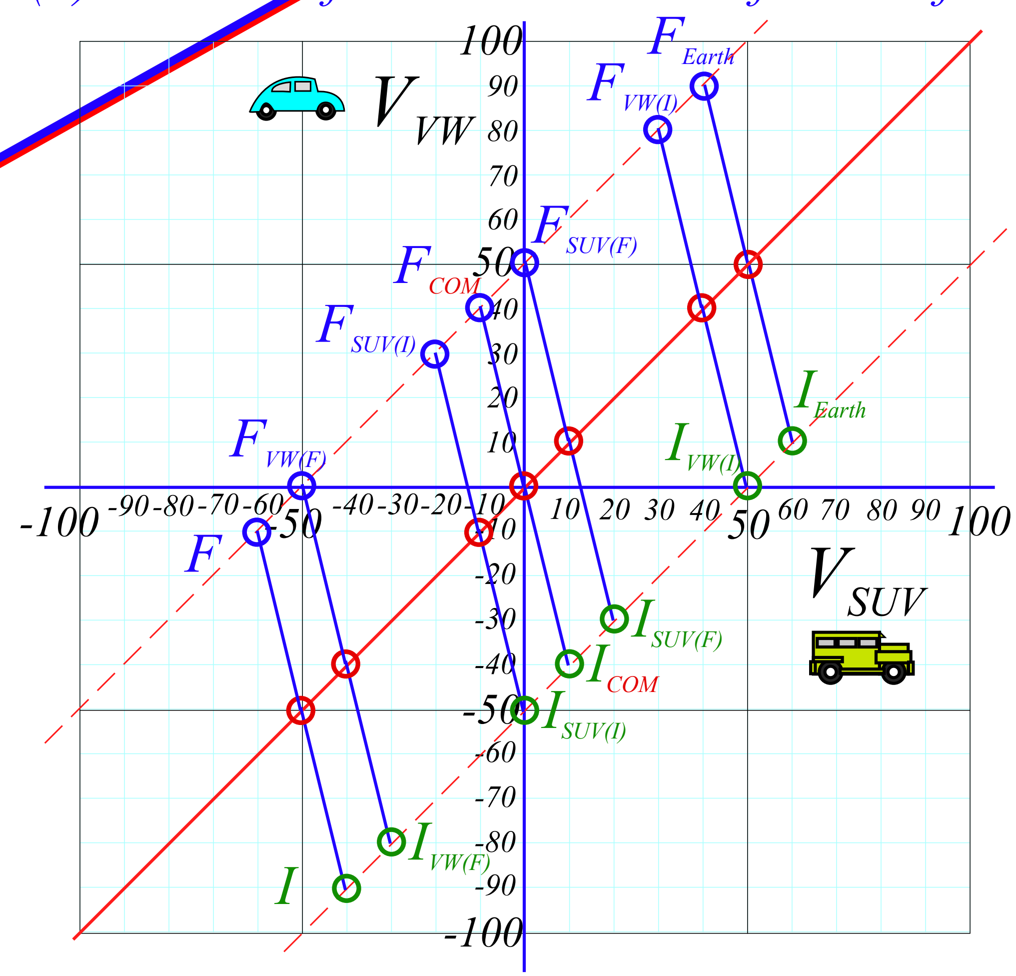
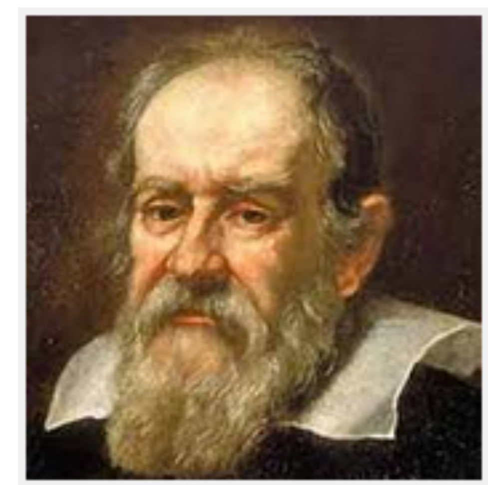


Fig. 2.5b
in Unit 1



Galileo Galilei
1564-1642

(Five of these have *0* for a velocity coord.)

Geometry of Galilean translation symmetry

45° shift in (V_1,V_2)-space

→ *Time reversal symmetry*

...of COM collisions

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Geometry of Galilean translation (A symmetry transformation)

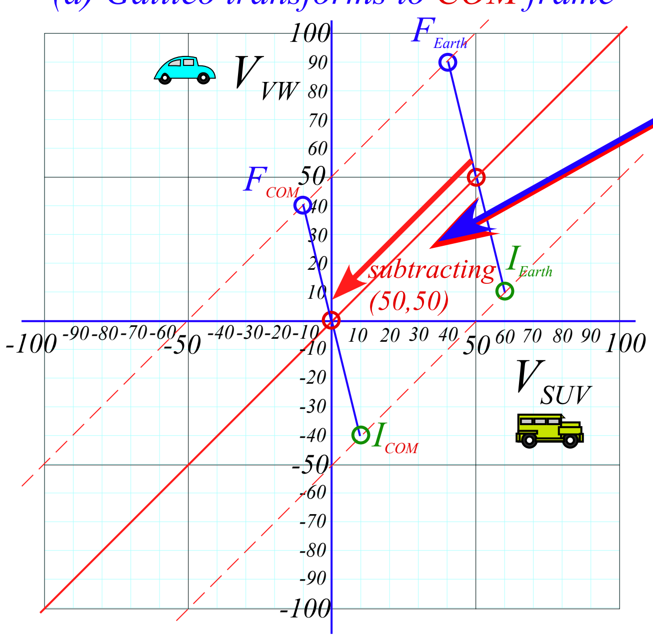
If you increase your velocity by 50 mph,...

...the rest of the world appears to be 50 mph slower

Final F and Initial I trade places...

Time-reversal (F-I) symmetry pairs (Four examples)

(a) Galileo transforms to COM frame



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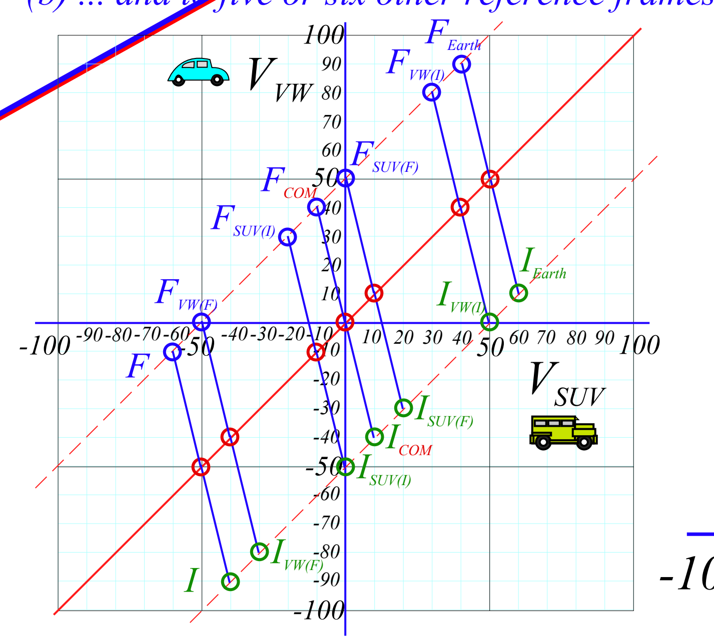
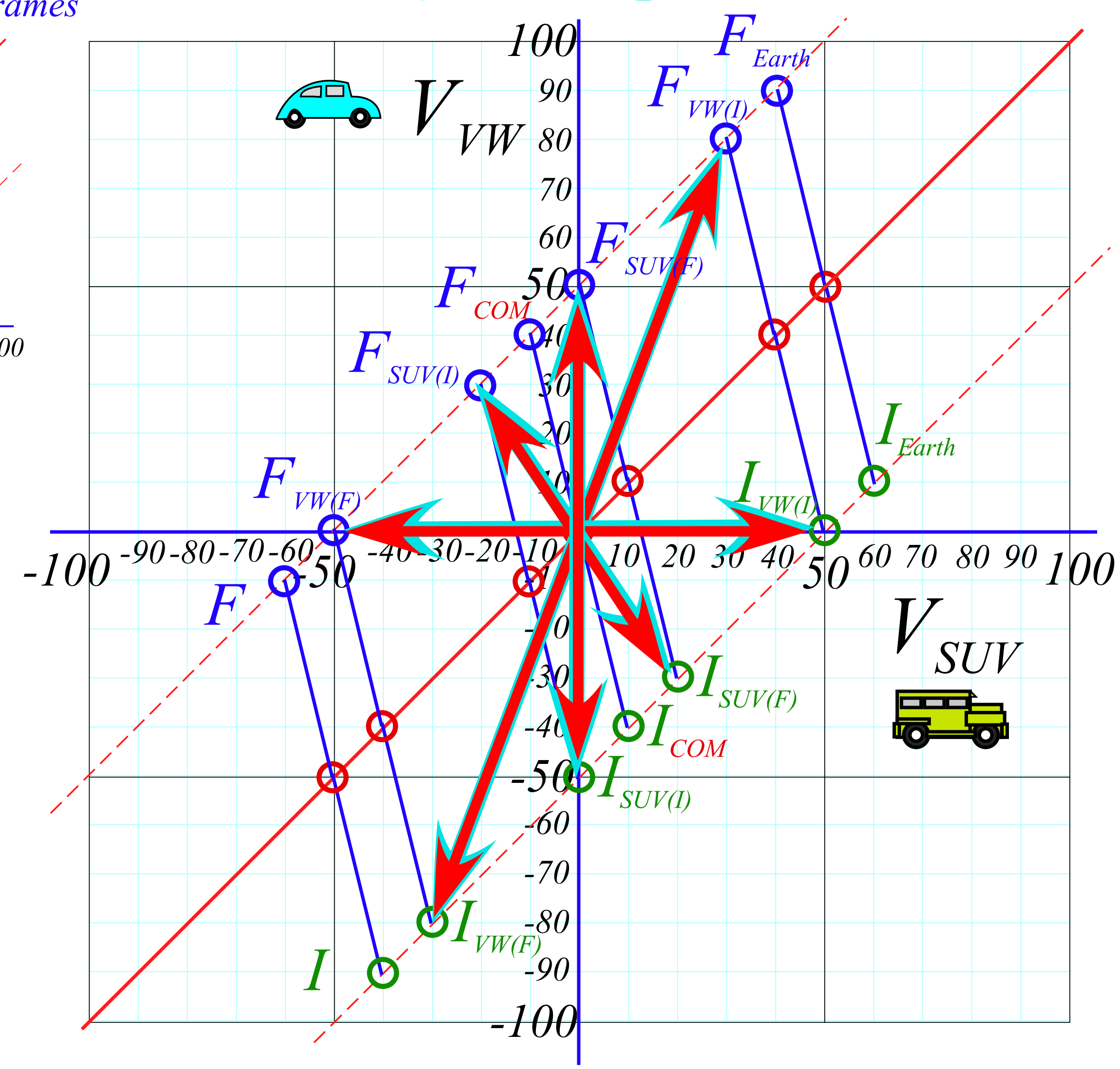


Fig. 2.5b
in Unit 1



Time-reversal means flip t to -t...
(Run a movie backwards)

A problem in ***space-time*** : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Geometry of Galilean translation (A symmetry transformation)

If you increase your velocity by 50 mph,...

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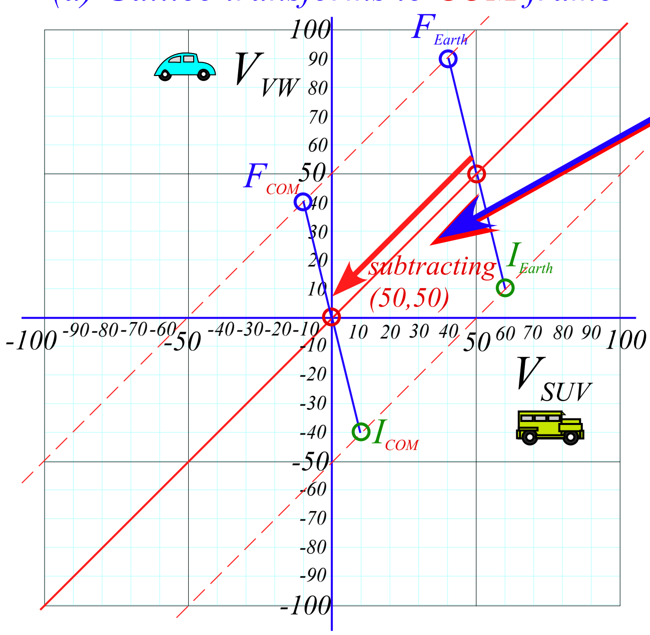

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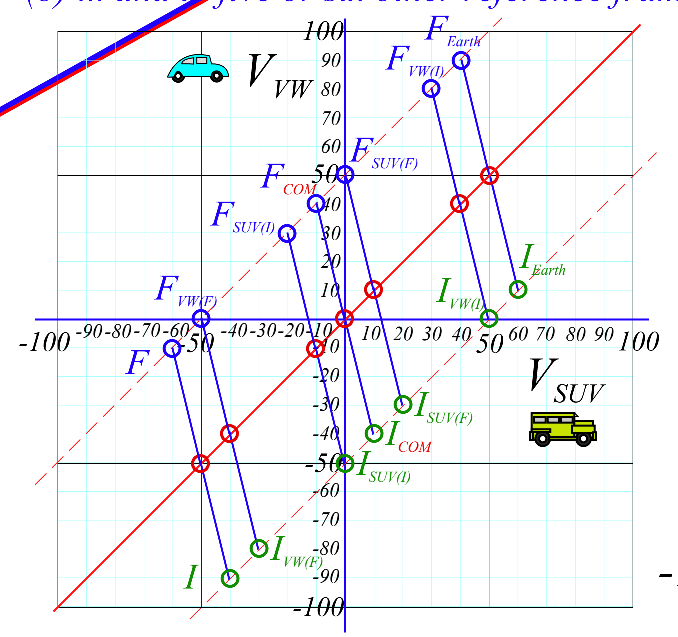
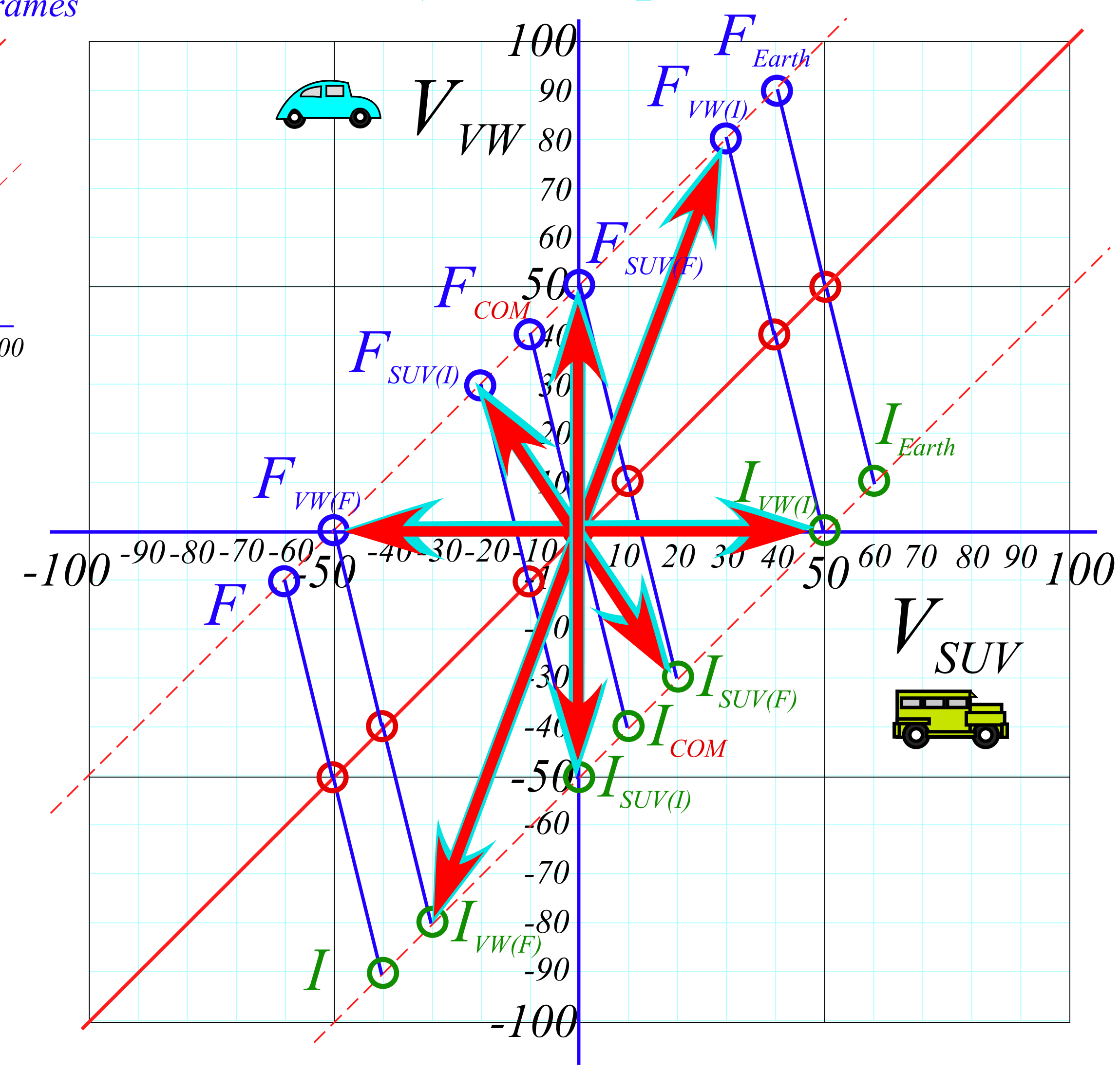


Fig. 2.5b
in Unit 1



Time-reversal means flip t to -t...
(Run a movie backwards)

That means you flip Velocity V to -V...
(Everything goes backwards)

Geometry of Galilean translation symmetry

45° shift in (V_1,V_2)-space

Time reversal symmetry

→ *...of COM collisions*

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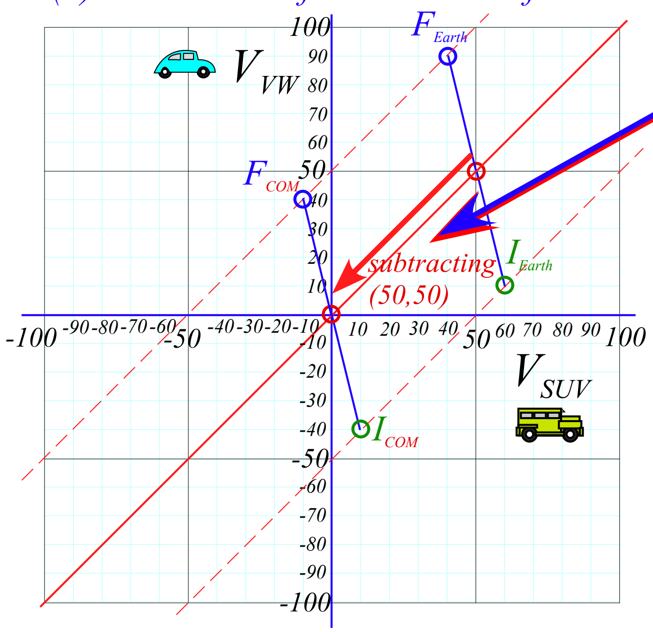

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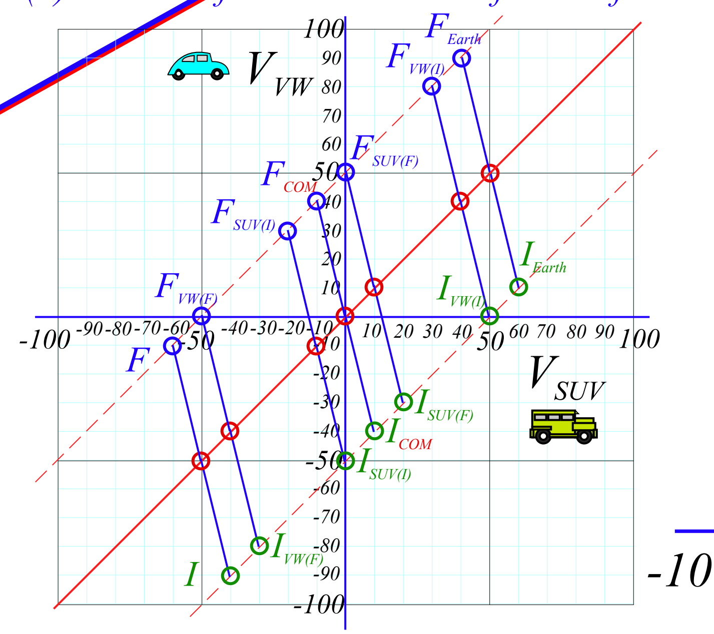
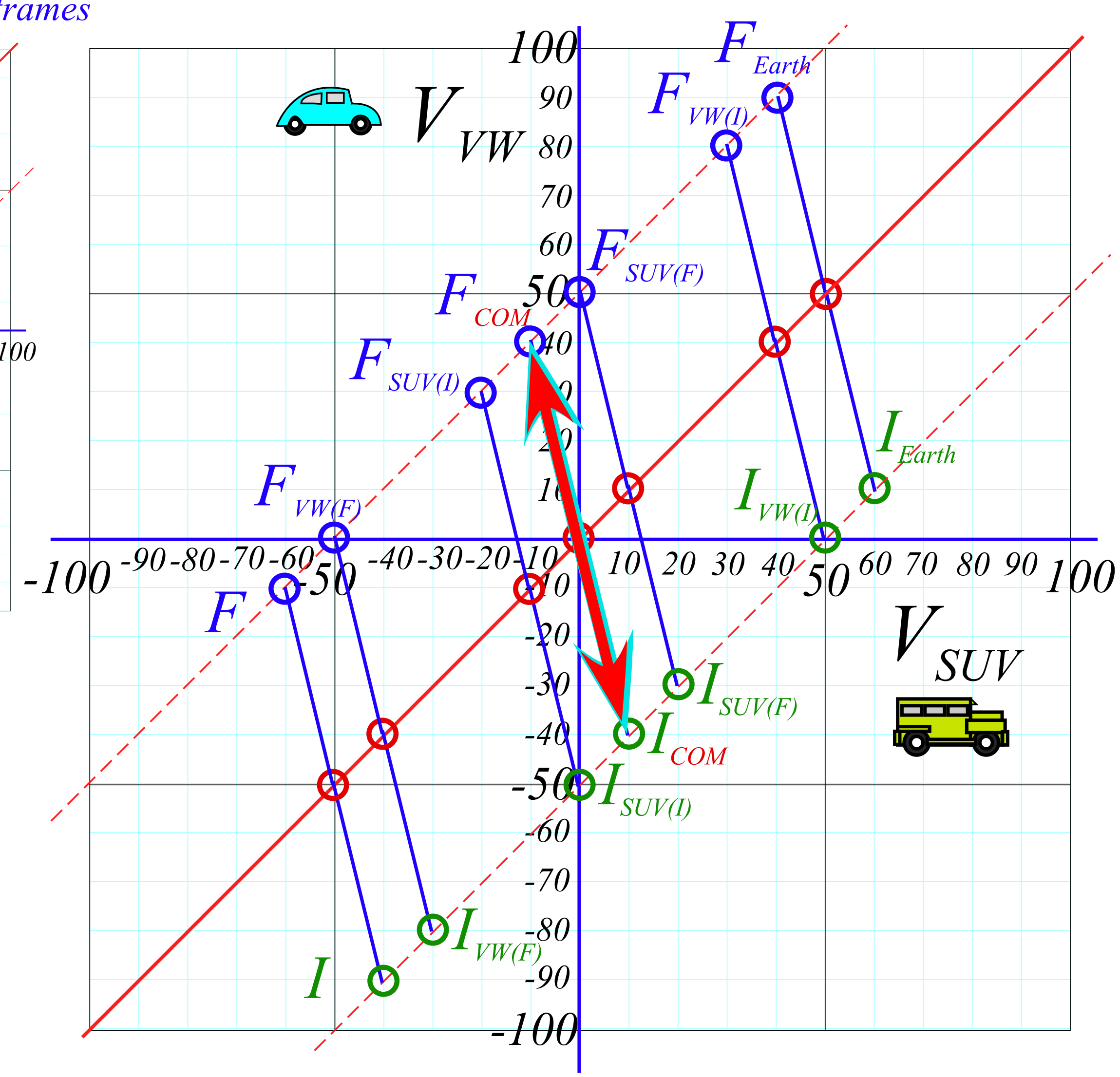


Fig. 2.5b
in Unit 1

THE COM Time-reversal symmetry pair (Just 1 case)



There is just one velocity frame in which the time-reversed collision looks just like the original collision

That is the
Center-of-Momentum
***(COM)**-frame*

Time-reversal means flip t to -t…
(Run a movie backwards)

That means you flip Velocity V to -V…
(Everything goes backwards)

Algebra, Geometry, and Physics of momentum conservation axiom

→ *Vector algebra of collisions*

Matrix or tensor algebra of collisions

Deriving Energy Conservation Theorem

Energy Ellipse geometry

Quick lesson on

Gibb's notation for

dot (•) product of matrix operator **M** and column vector $\mathbf{V}^{IN}$:

$$\overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \bullet \begin{pmatrix} x^{IN} \\ y^{IN} \end{pmatrix}$$

$$= \begin{pmatrix} Ax^{IN} + By^{IN} \\ Cx^{IN} + Dy^{IN} \end{pmatrix}$$

Algebra, Geometry, and Physics of momentum conservation axiom

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$$= \begin{pmatrix} Ax^{IN} + By^{IN} \\ Cx^{IN} + Dy^{IN} \end{pmatrix}$$

Quick lesson on
Dirac notation is
much simpler:
$M|IN\rangle$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \langle x|IN\rangle \\ \langle y|IN\rangle \end{pmatrix}$$

Algebra, Geometry, and Physics of momentum conservation axiom

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Matrix or tensor algebra of collisions
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Gibb's notation for
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$$\overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \bullet \begin{pmatrix} x^{IN} \\ y^{IN} \end{pmatrix}$$

$$= \begin{pmatrix} Ax^{IN} + By^{IN} \\ Cx^{IN} + Dy^{IN} \end{pmatrix}$$

Quick lesson on
Dirac notation is
much simpler:

$M|IN\rangle$ (...at first!)

$$\begin{pmatrix} \langle x|M|x\rangle & \langle x|M|y\rangle \\ \langle y|M|x\rangle & \langle y|M|y\rangle \end{pmatrix} \begin{pmatrix} \langle x|IN\rangle \\ \langle y|IN\rangle \end{pmatrix}$$

$$= \begin{pmatrix} \langle x|M|x\rangle\langle x|IN\rangle + \langle x|M|y\rangle\langle y|IN\rangle \\ \langle y|M|x\rangle\langle x|IN\rangle + \langle y|M|y\rangle\langle y|IN\rangle \end{pmatrix}$$

Algebra, Geometry, and Physics of momentum conservation axiom

→ *Vector algebra of collisions*
Matrix or tensor algebra of collisions
Deriving Energy Conservation Theorem
Energy Ellipse geometry

General **Inertia Tensor M** or inertia matrix of *2* coefficients $M_{11}=M_1$ and $M_{22}=M_2$ for 2 dimensions

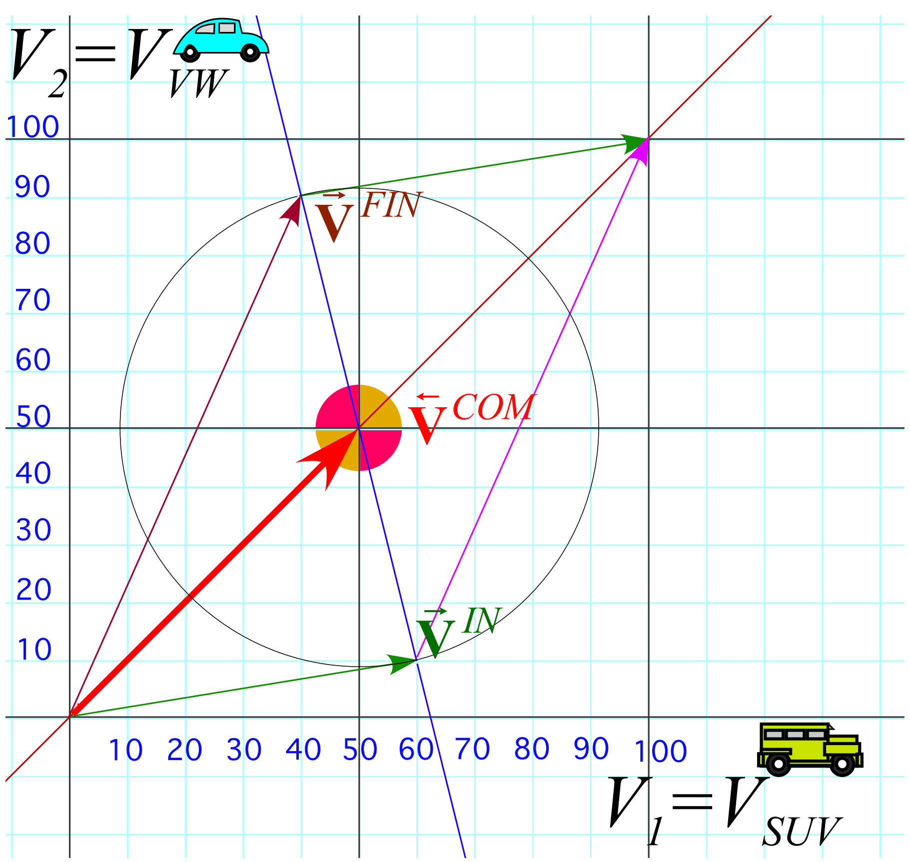
$$\left.\begin{array}{l} P_1 = M_1V_1 + M_{12}V_2 \\ P_2 = M_{21}V_1 + M_2V_2 \end{array}\right\} \quad denoted: \vec{\mathbf{P}} = \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}} \quad or: \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

Generalizing the definition of *momentum*...

With 45° diagonal $\mathbf{V}^{COM}$ so: . $V_1^{COM} = V_2^{COM} \equiv V^{COM}$

...and 3 ways to write *Axiom-1*

$$P_{Total} = M_1V_1^{IN} + M_2V_2^{IN} = M_1V_1^{FIN} + M_2V_2^{FIN} = M_1V^{COM} + M_2V^{COM} = (M_1+M_2)V^{COM} = M_{Total}V^{COM}$$



here:

$M_1 \equiv M_{SUV} = 4$

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With 45° diagonal $\mathbf{V}^{COM}$ so: . $V_1^{COM} = V_2^{COM} \equiv V^{COM}$ ...and 3 ways to write *Axiom-1*

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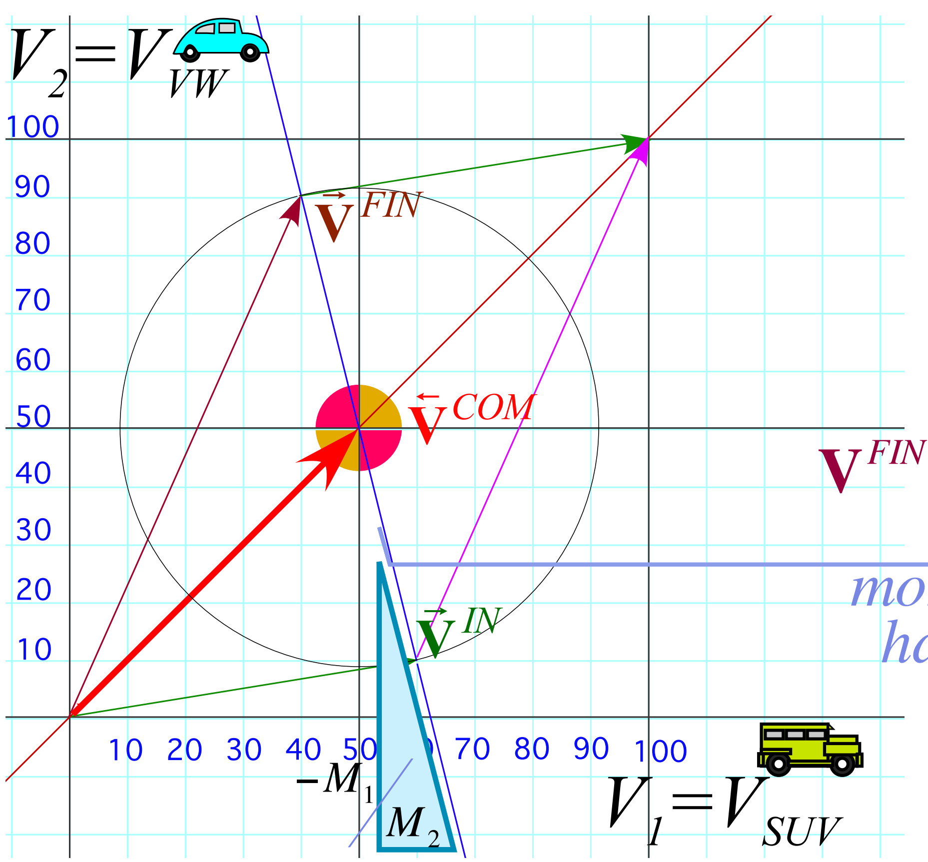

Points of vectors $\mathbf{V}^{FIN}$ and $\mathbf{V}^{COM}$ and $\mathbf{V}^{IN}$ all lie on *momentum - conservation line having slope* $-M_1 / M_1$

here:

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General Inertia Tensor M or inertia matrix of *3* coefficients M_{11}, M_{22} and $M_{12}=M_{21}$ for 2 dimensions

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Generalizing the definition of *momentum*...some more...

With 45° diagonal $\mathbf{V}^{COM}$ so: . $V_1^{COM} = V_2^{COM} \equiv V^{COM}$

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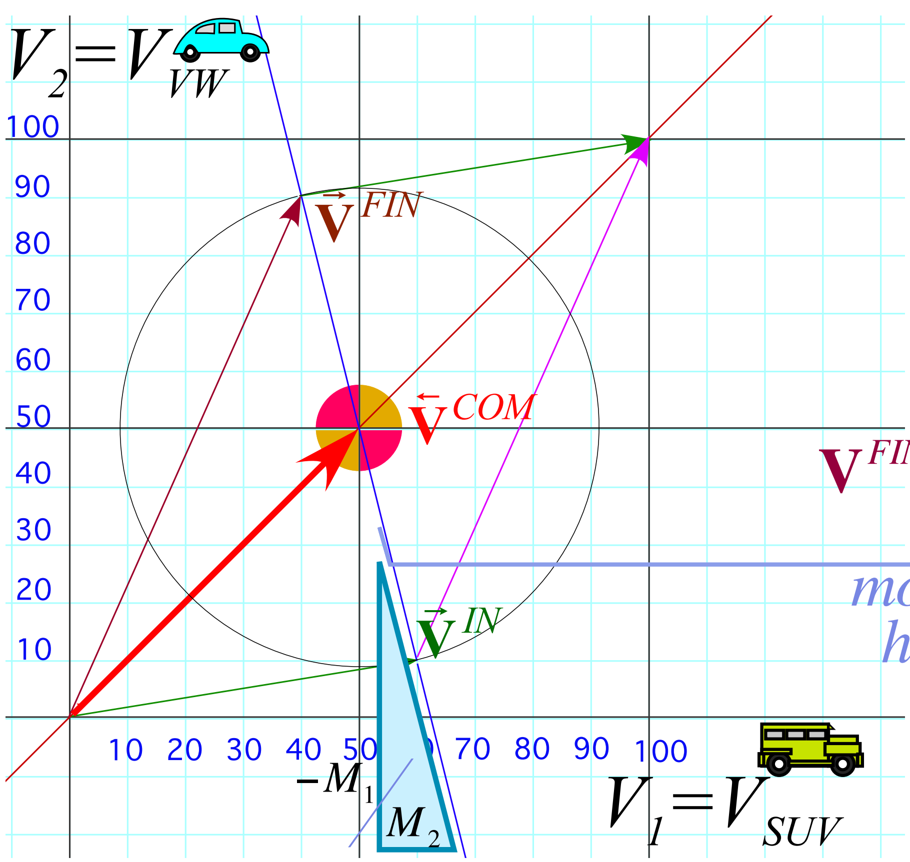
$$P_{Total} = M_1V_1^{IN} + M_2V_2^{IN} = M_1V_1^{FIN} + M_2V_2^{FIN} = M_1V^{COM} + M_2V^{COM} = (M_1+M_2)V^{COM} = M_{Total}V^{COM}$$

Here $M_{ij}=0$ for $i \neq j$



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General Inertia Tensor M or inertia matrix of $(n^2+n)/2$ coefficients $M_{jk} = M_{kj}$ for dimension $n=2, 3, \ldots$.

$$\left.\begin{array}{l} P_1 = M_{11}V_1 + M_{12}V_2 \cdots \\ P_2 = M_{21}V_1 + M_{22}V_2 \cdots \\ \vdots = \;\; \vdots \qquad\quad \vdots \;\; \ddots \end{array}\right\} \; denoted: \vec{\mathbf{P}} = \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}} \quad or: \begin{pmatrix} P_1 \\ P_2 \\ \vdots \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} & \cdots \\ M_{21} & M_{22} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ \vdots \end{pmatrix}$$

Generalizing the definition of *momentum*...some more...and more

With 45°diagonal $\vec{\mathbf{V}}^{COM} = (\vec{\mathbf{V}}^{FIN} + \vec{\mathbf{V}}^{IN})/2$ so: $V_1^{COM} = V_2^{COM} \equiv V^{COM}$ ...and 3 ways to write *Axiom-1*

$$P_{Total} = M_1 V_1^{IN} + M_2 V_2^{IN} = M_1 V_1^{FIN} + M_2 V_2^{FIN} = M_1 V^{COM} + M_2 V^{COM} = (M_1 + M_2) V^{COM} = M_{Total} V^{COM}$$

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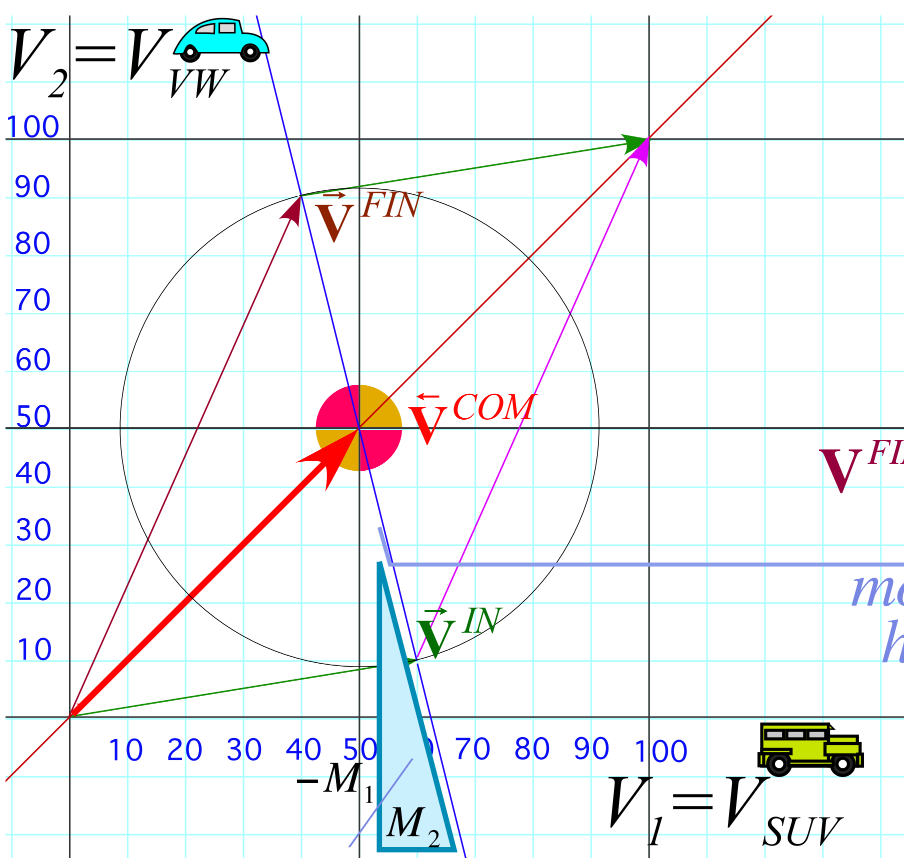

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Generalizing the definition of *momentum*...

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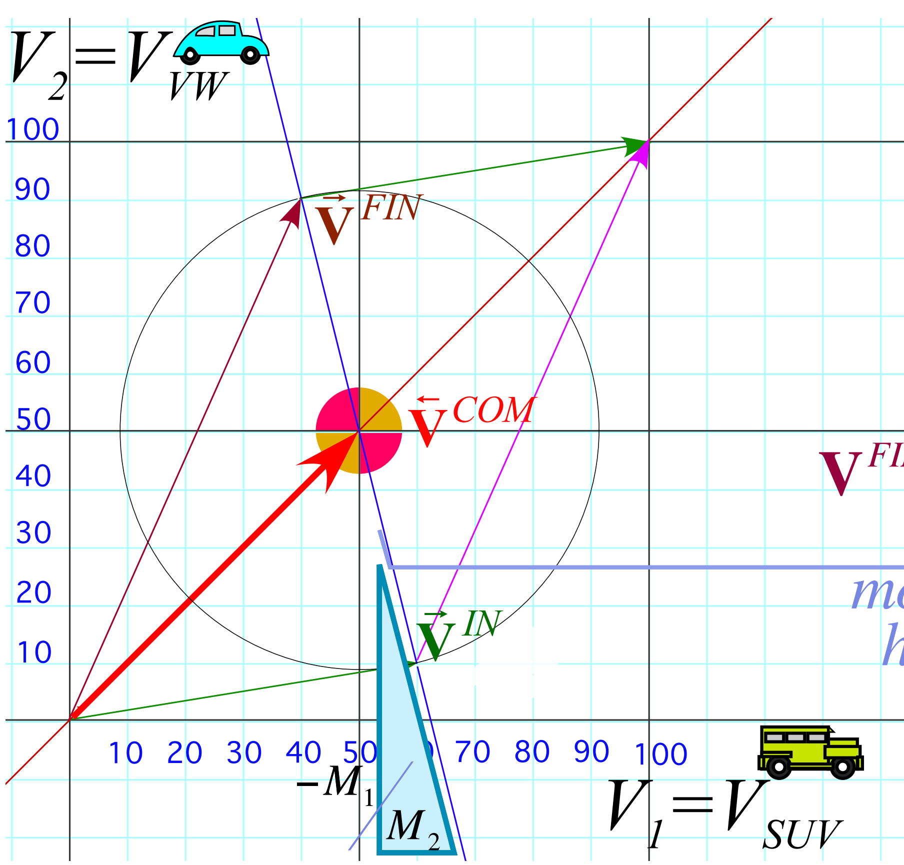

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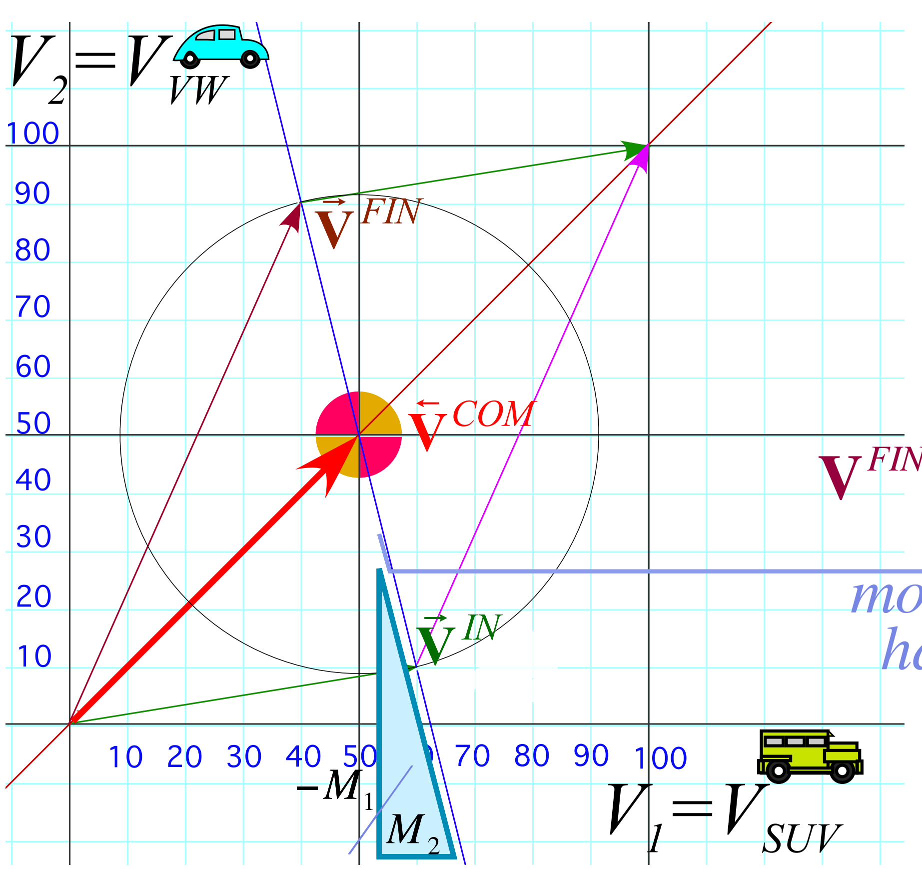

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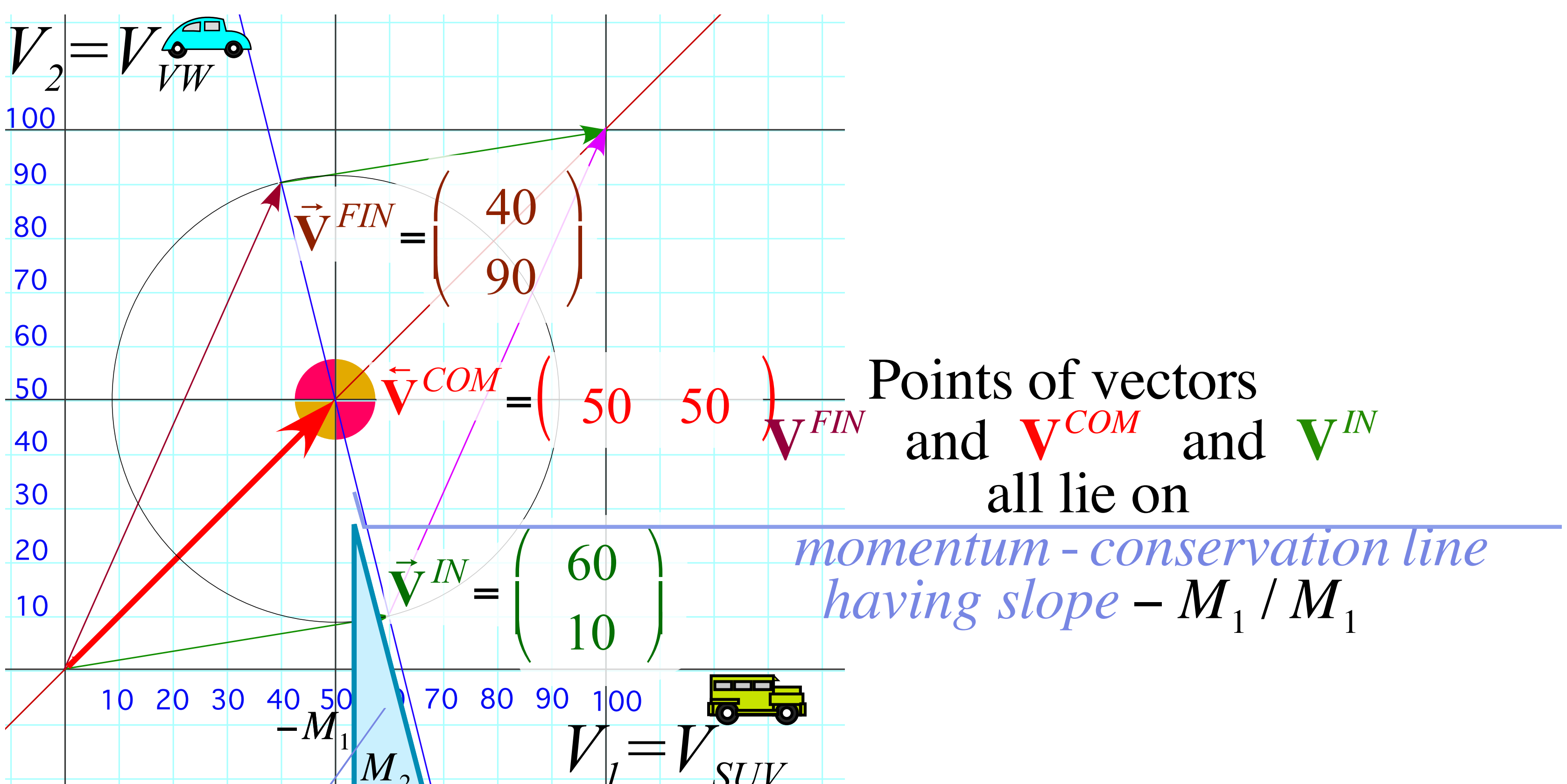
Write this out with the numbers used in Fig. 1.3 where $V^{COM} = 50$.

$$50P_{Total} = \begin{pmatrix} 50 & 50 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix} = \begin{pmatrix} 50 & 50 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix} = 50M_{Total}50 = 12{,}500$$



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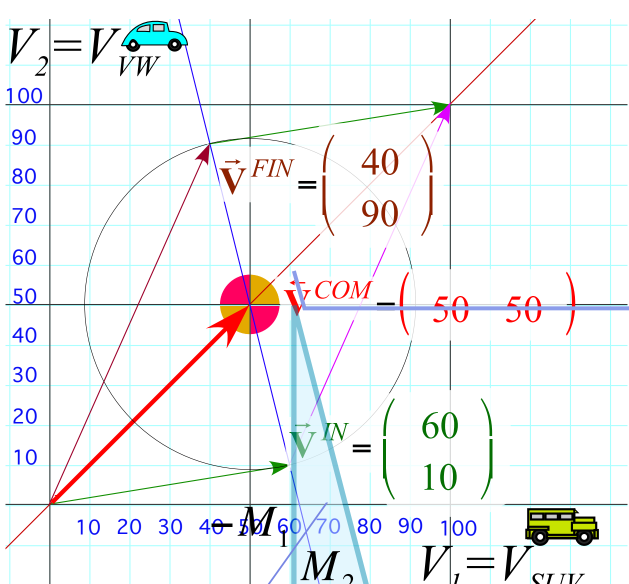
$$= \begin{pmatrix} 50 & 50 \end{pmatrix} \bullet \begin{pmatrix} 4\cdot 60 \\ 1\cdot 10 \end{pmatrix} \qquad = \begin{pmatrix} 50 & 50 \end{pmatrix} \bullet \begin{pmatrix} 4\cdot 40 \\ 1\cdot 90 \end{pmatrix} \qquad = 50M_{Total}50 = 12{,}500$$

$$50\cdot 250 = 50\cdot 4\cdot 60 + 50\cdot 1\cdot 10 \qquad = 50\cdot 4\cdot 40 + 50\cdot 1\cdot 90 \qquad = 50\cdot 5\cdot 50 = 12{,}500$$



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$$= \quad 100 \cdot 125 \quad = \quad 100 \cdot 125 \quad = 50 \cdot 250$$

$P_{Total} = 250$ is the same at IN, FIN, and COM. Now use T-symmetry: $\vec{\mathbf{V}}^{COM} = (\vec{\mathbf{V}}^{FIN} + \vec{\mathbf{V}}^{IN})/2$ ***(Axiom-2)***

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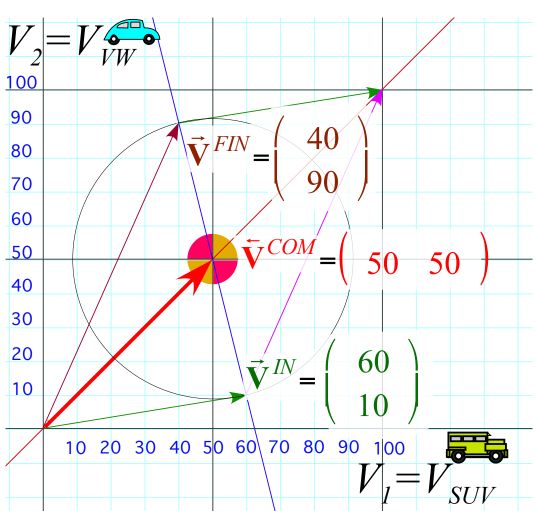

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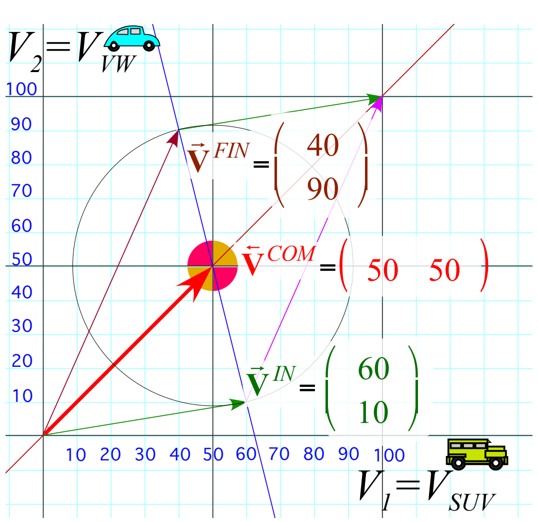
$$= \tfrac{1}{2}\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} + \tfrac{1}{2}\vec{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \tfrac{1}{2}\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN} + \tfrac{1}{2}\vec{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}$$



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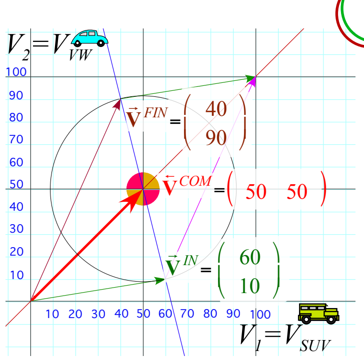
$$50P_{Total} = \begin{pmatrix} 50 & 50 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix} = \begin{pmatrix} 50 & 50 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix} = 50M_{Total}50 = 12{,}500$$

$$= 100 \cdot 125 \qquad = 100 \cdot 125 \qquad = 50 \cdot 250$$

$P_{Total} = 250$ is the same at IN, FIN, and COM. Now use *T*-symmetry: $\vec{\mathbf{V}}^{COM} = (\vec{\mathbf{V}}^{FIN} + \vec{\mathbf{V}}^{IN})/2$ ***(Axiom-2)***

$$V^{COM}P_{Total} = \frac{\vec{\mathbf{V}}^{FIN} + \vec{\mathbf{V}}^{IN}}{2} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \frac{\vec{\mathbf{V}}^{FIN} + \vec{\mathbf{V}}^{IN}}{2} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN} = \frac{\vec{\mathbf{V}}^{FIN} + \vec{\mathbf{V}}^{IN}}{2} \bullet \overleftrightarrow{\mathbf{M}} \bullet \frac{\vec{\mathbf{V}}^{FIN} + \vec{\mathbf{V}}^{IN}}{2}$$

$$= \tfrac{1}{2}\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} + \tfrac{1}{2}\vec{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \tfrac{1}{2}\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN} + \tfrac{1}{2}\vec{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}$$



Transpose symmetry ($M_{jk} = M_{kj}$) of **M**-matrix makes 'lopsided' FIN-IN-terms equal:

$$\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \vec{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}$$

$$\begin{pmatrix} 40 & 90 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix} = \begin{pmatrix} 60 & 10 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix}$$

$$= 100 \cdot 105 \qquad = 100 \cdot 105 \qquad = 10{,}500$$

General **Inertia Tensor M** or inertia matrix of 2 coefficients $M_{11}=M_1$ and $M_{22}=M_2$ for 2 dimensions

$$\left.\begin{aligned} P_1 &= M_1V_1 + M_{12}V_2 \\ P_2 &= M_{21}V_1 + M_2V_2 \end{aligned}\right\} \quad denoted: \vec{\mathbf{P}} = \overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}} \quad or: \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

Generalizing the definition of *momentum*...

Here $M_{ij}=0$ for $i \neq j$

With 45°diagonal $\vec{\mathbf{V}}^{COM}=(\vec{\mathbf{V}}^{FIN}+\vec{\mathbf{V}}^{IN})/2$ so: $V_1^{COM}=V_2^{COM}\equiv V^{COM}$ ...and 4 or 5 ways to write ***Axiom-1***

$$P_{Total} = M_1V_1^{IN}+M_2V_2^{IN} = M_1V_1^{FIN}+M_2V_2^{FIN} = M_1V^{COM}+M_2V^{COM} = (M_1+M_2)V^{COM} = M_{Total}V^{COM}$$

A product of total momentum P_{Total} and V^{COM} is expressed by *tensor quadratic forms* **v•M•u**

$$V^{COM}P_{Total} = \vec{\mathbf{V}}^{COM}\bullet\overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN} = \vec{\mathbf{V}}^{COM}\bullet\overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}}^{FIN} = \vec{\mathbf{V}}^{COM}\bullet\overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}}^{COM} = V^{COM}M_{Total}V^{COM}$$

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$$= 100\cdot125 \qquad = 100\cdot125 \qquad = 50\cdot250$$

$P_{Total}=250$ is the same at IN, FIN, and COM. Now use T-symmetry: $\vec{\mathbf{V}}^{COM}=(\vec{\mathbf{V}}^{FIN}+\vec{\mathbf{V}}^{IN})/2$ ***(Axiom-2)***

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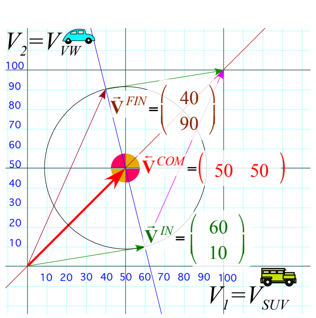
$$=\tfrac{1}{2}\vec{\mathbf{V}}^{FIN}\bullet\overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN}+\tfrac{1}{2}\vec{\mathbf{V}}^{IN}\bullet\overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN} = \tfrac{1}{2}\vec{\mathbf{V}}^{FIN}\bullet\overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}}^{FIN}+\tfrac{1}{2}\vec{\mathbf{V}}^{IN}\bullet\overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}}^{FIN}$$

$$V^{COM}P_{Total}-\tfrac{1}{2}\vec{\mathbf{V}}^{FIN}\bullet\overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN} = \tfrac{1}{2}\vec{\mathbf{V}}^{IN}\bullet\overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN} = \tfrac{1}{2}\vec{\mathbf{V}}^{FIN}\bullet\overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}}^{FIN}$$

FIN-IN-term is subtracted to give *Conservation of* ***Kinetic Energy***

$$KE = \tfrac{1}{2}\vec{\mathbf{V}}\bullet\overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}} = \tfrac{1}{2}M_1V_1^2+\tfrac{1}{2}M_2V_2^2$$



$$\vec{\mathbf{V}}^{FIN}\bullet\overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN} = \vec{\mathbf{V}}^{IN}\bullet\overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}}^{FIN}$$

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Generalizing the definition of *momentum*...

Here $M_{ij}=0$ for $i \neq j$

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50·250 -½·10,500

$$\tfrac{1}{2}\vec{\mathbf{V}}^{IN}\bullet\overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN} = \tfrac{1}{2}\vec{\mathbf{V}}^{FIN}\bullet\overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}}^{FIN}$$

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50·250 -½·10,500

12,500 - 5,250

$$\tfrac{1}{2}\vec{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \tfrac{1}{2}\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}$$

$$\tfrac{1}{2}\begin{pmatrix} 60 & 10 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix} = \tfrac{1}{2}\begin{pmatrix} 40 & 90 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix}$$

$$= \tfrac{1}{2}(60 \cdot 4 \cdot 60 + 10 \cdot 1 \cdot 10) = \tfrac{1}{2}(40 \cdot 4 \cdot 40 + 90 \cdot 1 \cdot 90)$$

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$$V^{COM}P_{Total} - \tfrac{1}{2}\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \tfrac{1}{2}\vec{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \tfrac{1}{2}\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}$$

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50·250 -½·10,500

12,500 - 5,250=7,250

$$\tfrac{1}{2}\vec{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \tfrac{1}{2}\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}$$

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$$= \tfrac{1}{2}(60 \cdot 4 \cdot 60 + 10 \cdot 1 \cdot 10) = \tfrac{1}{2}(40 \cdot 4 \cdot 40 + 90 \cdot 1 \cdot 90)$$

$$2 \cdot 3600 + 50 = 7250 = 2 \cdot 1600 + \tfrac{1}{2}8100 = 7250$$

Algebra,Geometry, and Physics of momentum conservation axiom

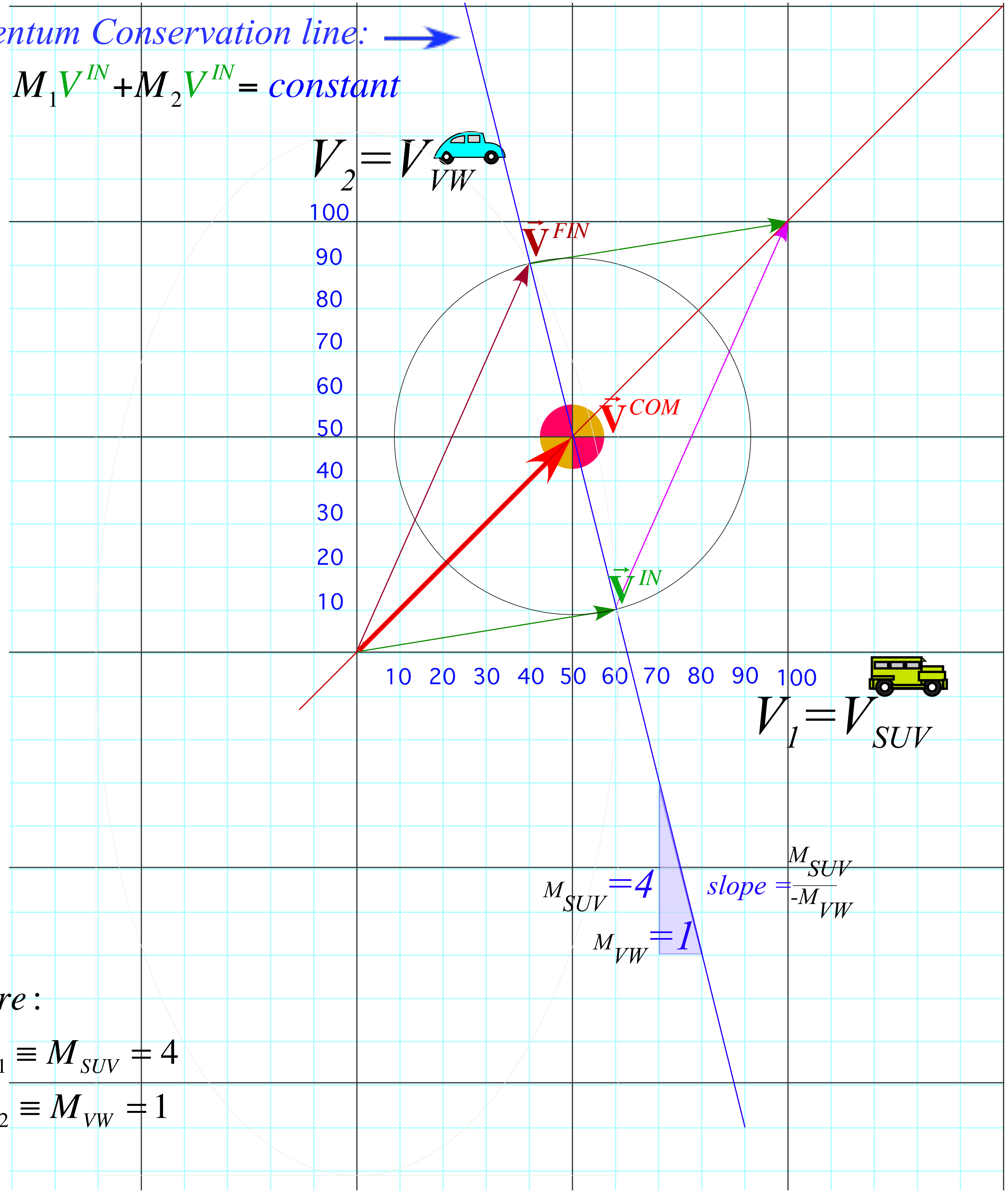
Vector algebra of collisions
Matrix or tensor algebra of collisions
Deriving Energy Conservation Theorem
→ *Energy Ellipse geometry*

Geometry of Momentum Conservation **Axiom - 1**

$(M_1+M_2)V^{COM} = M_1V_1^{IN} + M_2V_2^{IN} = M_1V_1^{FIN} + M_2V_2^{FIN}$

Momentum Conservation line: →

$M_1V^{IN} + M_2V^{IN} =$ *constant*



here :

$M_1 \equiv M_{SUV} = 4$

$M_2 \equiv M_{VW} = 1$

Collision *Web Simulator*
***Basic** elastic Collision*
Dual Panel
Space vs Space
and
V(VW) vs. V(SUV)

BounceIt
Superball Web Simulator
***Basic** elastic Collision*
Dual Panel
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Geometry of Momentum Conservation **Axiom - 1**

$(M_1+M_2)V^{COM}=M_1V_1^{IN}+M_2V_2^{IN}=M_1V_1^{FIN}+M_2V_2^{FIN}$

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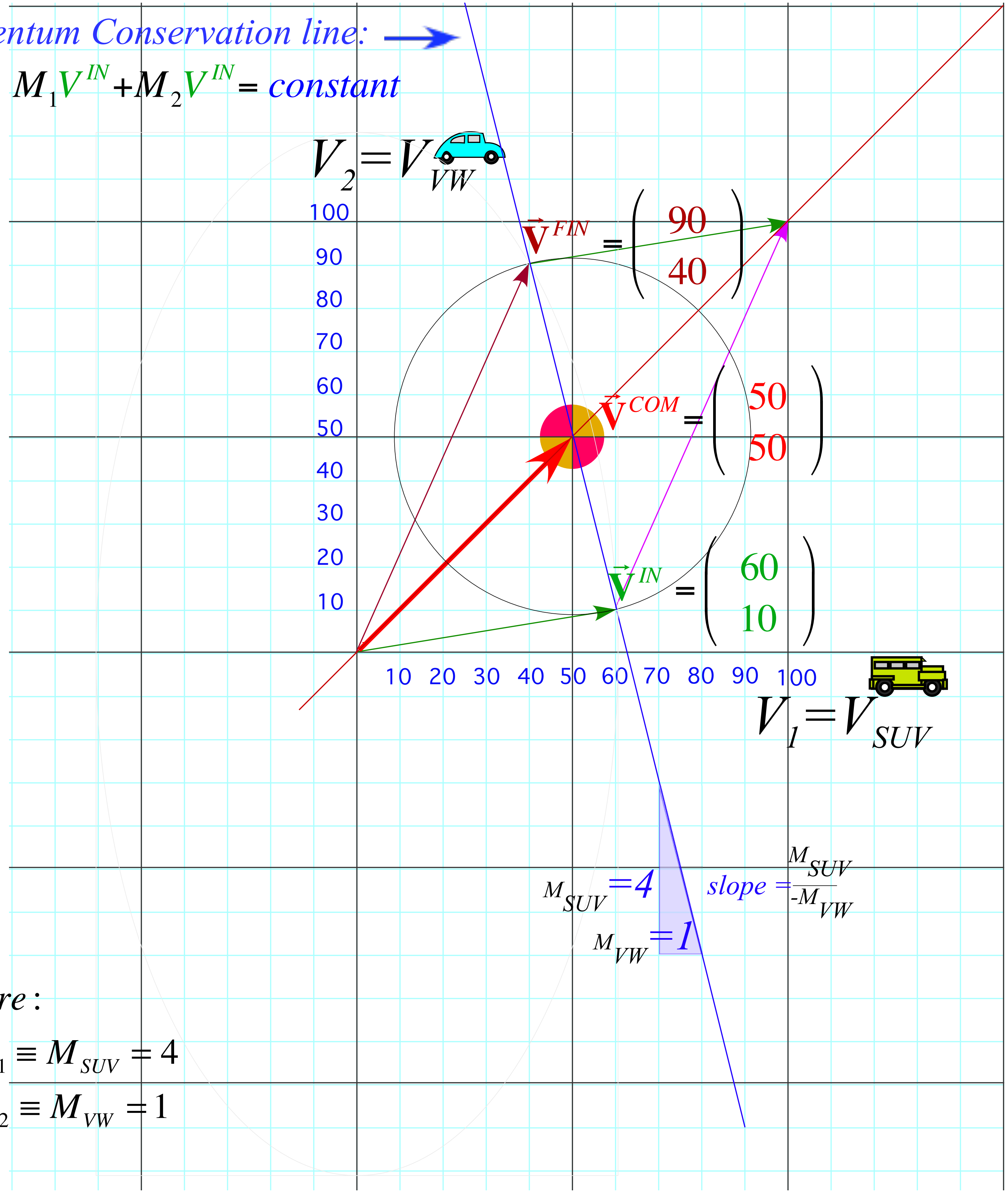

here:

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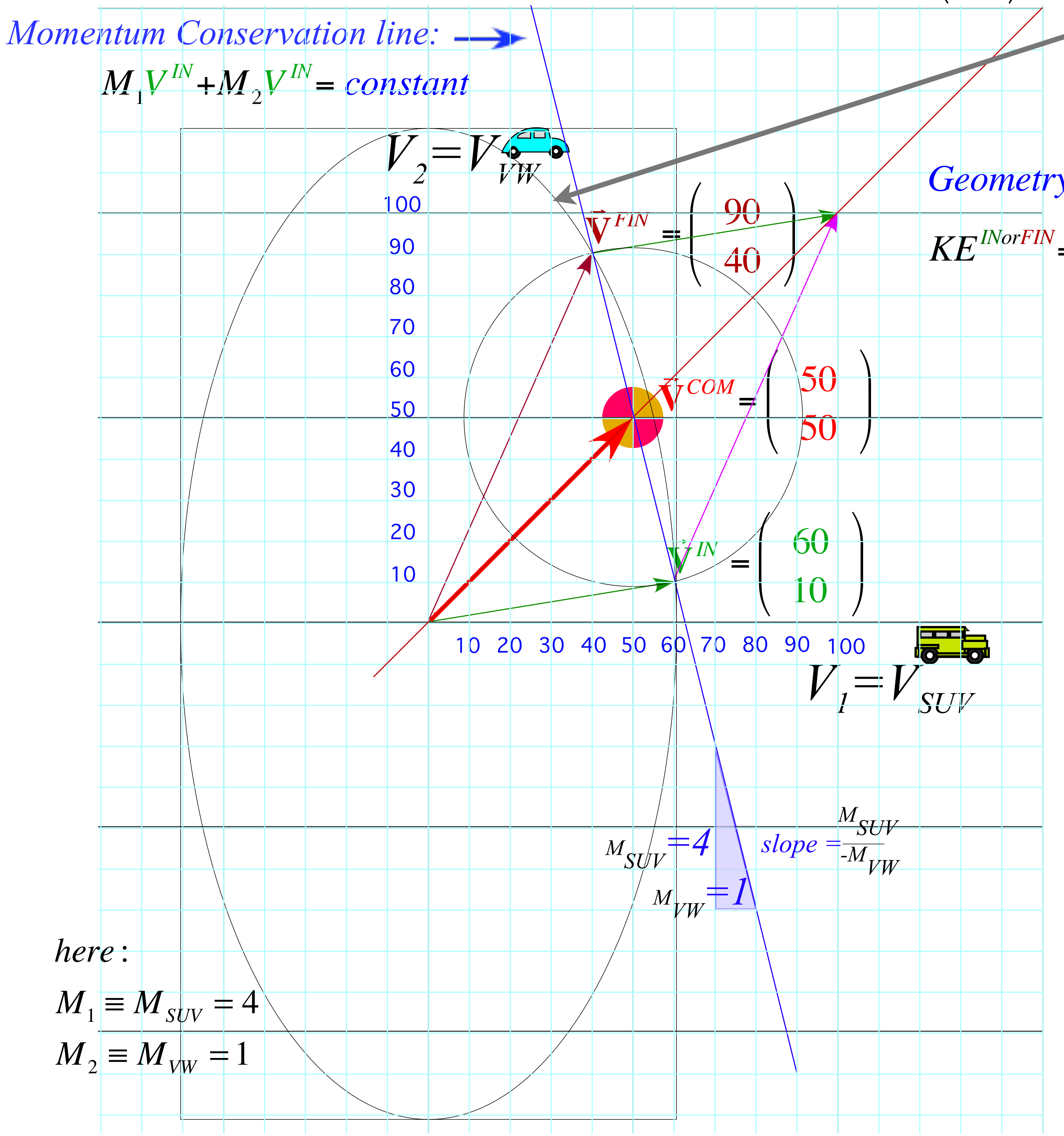
Geometry of Momentum Conservation **Axiom - 1**

$$(M_1+M_2)V^{COM}=M_1V_1^{IN}+M_2V_2^{IN}=M_1V_1^{FIN}+M_2V_2^{FIN}$$

Geometry of Kinetic Energy Conservation **Axiom-2/Theorem-1**

$$KE^{IN}=\tfrac{1}{2}M_1\left(V_1^{IN}\right)^2+\tfrac{1}{2}M_2\left(V_2^{IN}\right)^2=\tfrac{1}{2}M_1\left(V_1^{FIN}\right)^2+\tfrac{1}{2}M_2\left(V_2^{FIN}\right)^2=KE^{FIN}$$



Geometry of KE Conservation **Theorem - 1**

$$KE^{INorFIN}=\tfrac{1}{2}M_1\left(V_1\right)^2+\tfrac{1}{2}M_2\left(V_2\right)^2=\tfrac{1}{2}4(60)^2+\tfrac{1}{2}1(10)^2=7{,}250$$

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V(VW) vs. V(SUV)

Geometry of Momentum Conservation **Axiom - 1**

$$(M_1+M_2)V^{COM}=M_1V_1^{IN}+M_2V_2^{IN}=M_1V_1^{FIN}+M_2V_2^{FIN}$$

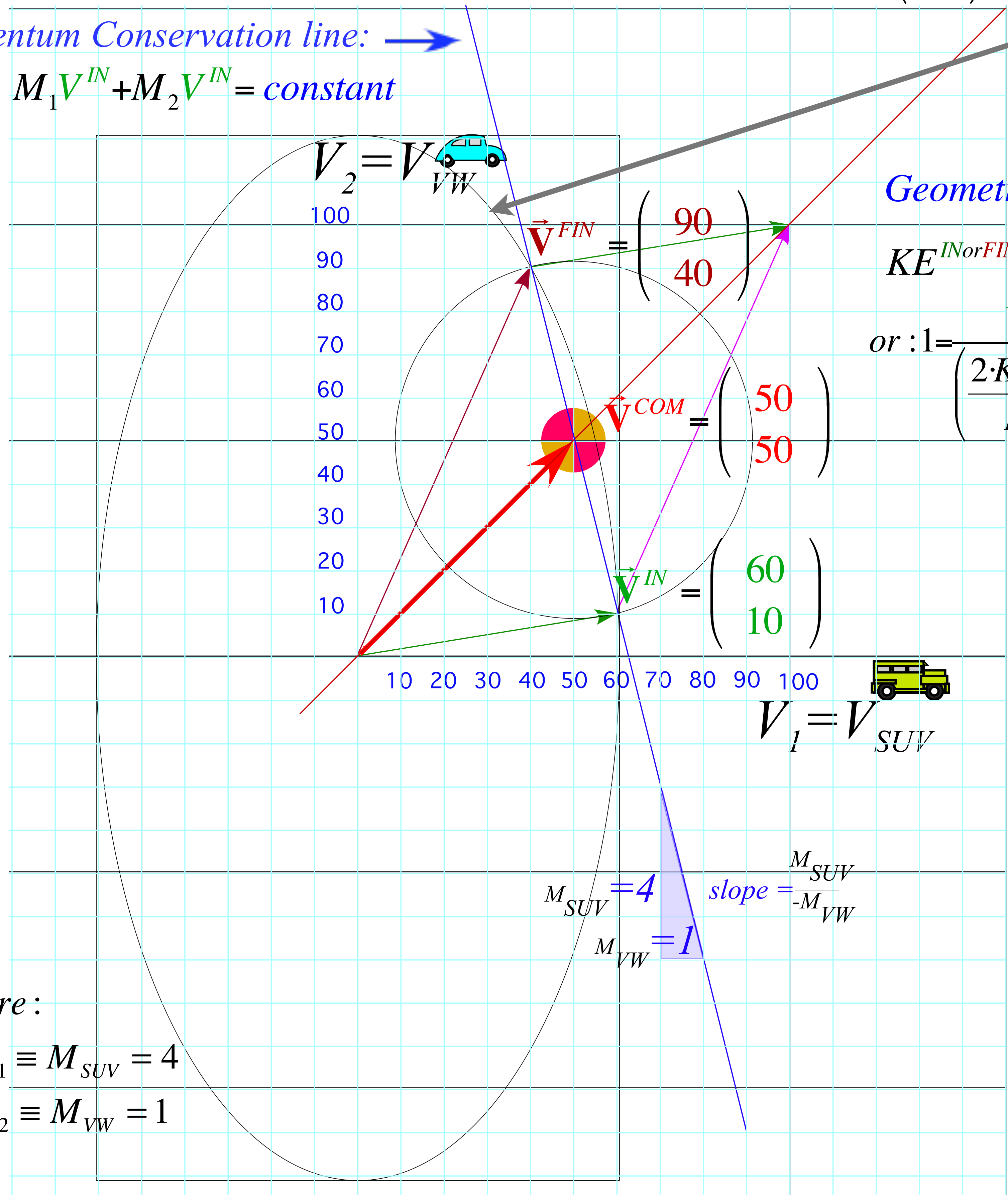
Geometry of Kinetic Energy Conservation **Axiom-2/Theorem-1**

$$KE^{IN}=\tfrac{1}{2}M_1\left(V_1^{IN}\right)^2+\tfrac{1}{2}M_2\left(V_2^{IN}\right)^2=\tfrac{1}{2}M_1\left(V_1^{FIN}\right)^2+\tfrac{1}{2}M_2\left(V_2^{FIN}\right)^2=KE^{FIN}$$

Momentum Conservation line:

$$M_1V^{IN}+M_2V^{IN}=\textit{constant}$$

KE Conservation ellipse:



here:

$M_1\equiv M_{SUV}=4$

$M_2\equiv M_{VW}=1$

Geometry of KE Conservation **Theorem - 1**

$$KE^{INorFIN}=\tfrac{1}{2}M_1\left(V_1\right)^2+\tfrac{1}{2}M_2\left(V_2\right)^2=\tfrac{1}{2}4(60)^2+\tfrac{1}{2}1(10)^2=7{,}250$$

$$or:1=\frac{V_1^2}{\left(\frac{2\cdot KE^{IFN}}{M_1}\right)}+\frac{V_2^2}{\left(\frac{2\cdot KE^{IFN}}{M_2}\right)}=\frac{V_1^2}{\left(\sqrt{\frac{2\cdot KE^{IFN}}{M_1}}\right)^2}+\frac{V_2^2}{\left(\sqrt{\frac{2\cdot KE^{IFN}}{M_2}}\right)^2}$$

Collision *Web Simulator*
[***Basic*** *elastic Collision*](#)
[*Dual Panel*](#)
[*Space vs Space*](#)
[*and*](#)
[*V(VW) vs. V(SUV)*](#)

BounceIt
Superball Web Simulator
[***Basic*** *elastic Collision*](#)
[*Dual Panel*](#)
[*Space vs Space*](#)
[*and*](#)
[*V(VW) vs. V(SUV)*](#)

Geometry of Momentum Conservation **Axiom - 1**

$$(M_1+M_2)V^{COM} = M_1V_1^{IN} + M_2V_2^{IN} = M_1V_1^{FIN} + M_2V_2^{FIN}$$

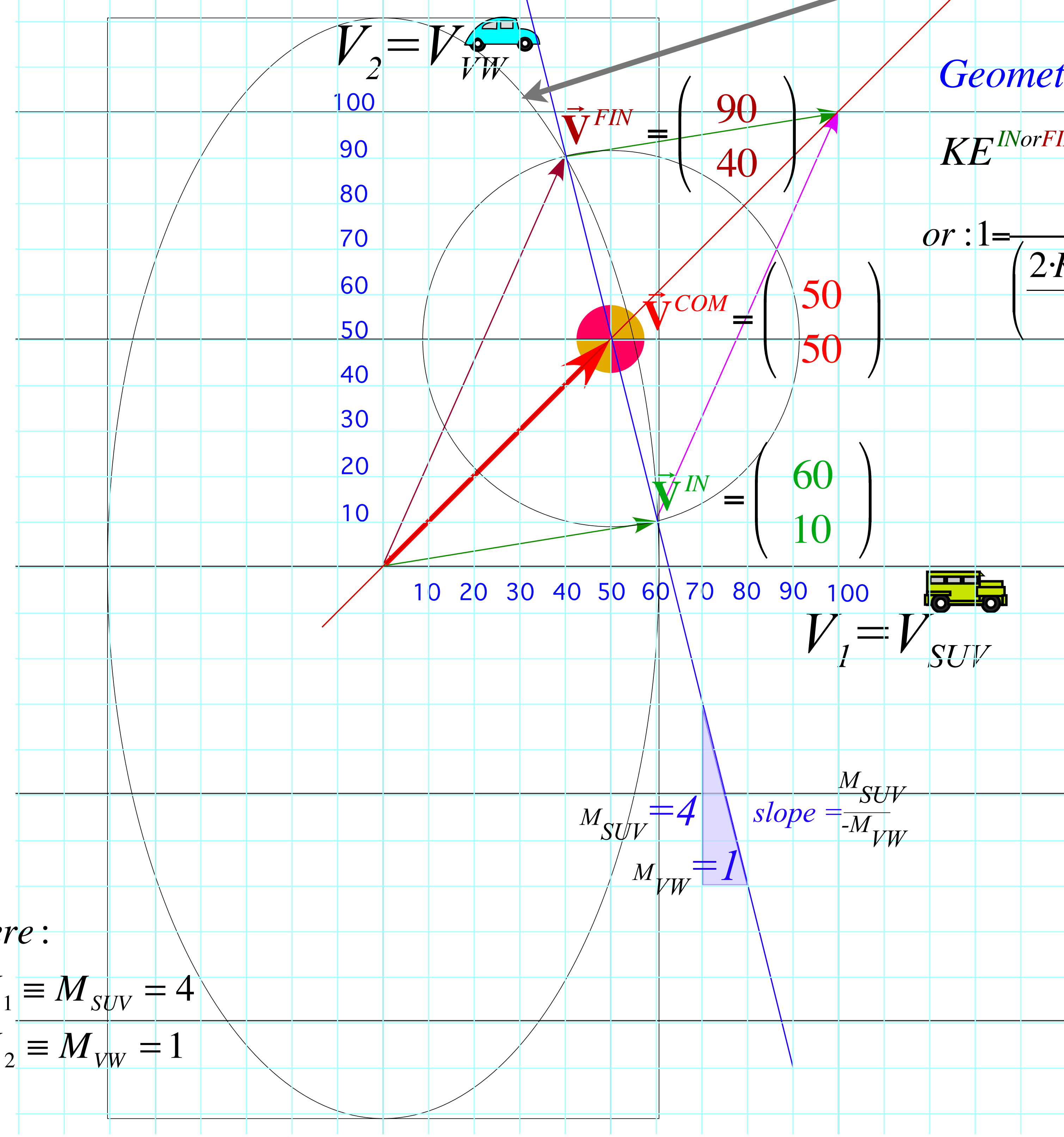
Geometry of Kinetic Energy Conservation **Axiom-2/Theorem-1**

$$KE^{IN} = \tfrac{1}{2}M_1\left(V_1^{IN}\right)^2 + \tfrac{1}{2}M_2\left(V_2^{IN}\right)^2 = \tfrac{1}{2}M_1\left(V_1^{FIN}\right)^2 + \tfrac{1}{2}M_2\left(V_2^{FIN}\right)^2 = KE^{FIN}$$

Momentum Conservation line: →

$$M_1V^{IN} + M_2V^{IN} = constant$$

KE Conservation ellipse:



here:

$M_1 \equiv M_{SUV} = 4$

$M_2 \equiv M_{VW} = 1$

Geometry of KE Conservation **Theorem - 1**

$$KE^{INorFIN} = \tfrac{1}{2}M_1\left(V_1\right)^2 + \tfrac{1}{2}M_2\left(V_2\right)^2 = \tfrac{1}{2}4\left(60\right)^2 + \tfrac{1}{2}1\left(10\right)^2 = 7{,}250$$

$$or: 1 = \frac{V_1^2}{\left(\frac{2\cdot KE^{IFN}}{M_1}\right)} + \frac{V_2^2}{\left(\frac{2\cdot KE^{IFN}}{M_2}\right)} = \frac{V_1^2}{\left(\sqrt{\frac{2\cdot KE^{IFN}}{M_1}}\right)^2} + \frac{V_2^2}{\left(\sqrt{\frac{2\cdot KE^{IFN}}{M_2}}\right)^2}$$

Has the algebraic form: $1 = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

Collision *Web Simulator*
***Basic** elastic Collision*
Dual Panel
Space vs Space
and
V(VW) vs. V(SUV)

BounceIt
Superball Web Simulator
***Basic** elastic Collision*
Dual Panel
Space vs Space
and
V(VW) vs. V(SUV)

Geometry of Momentum Conservation **Axiom - 1**

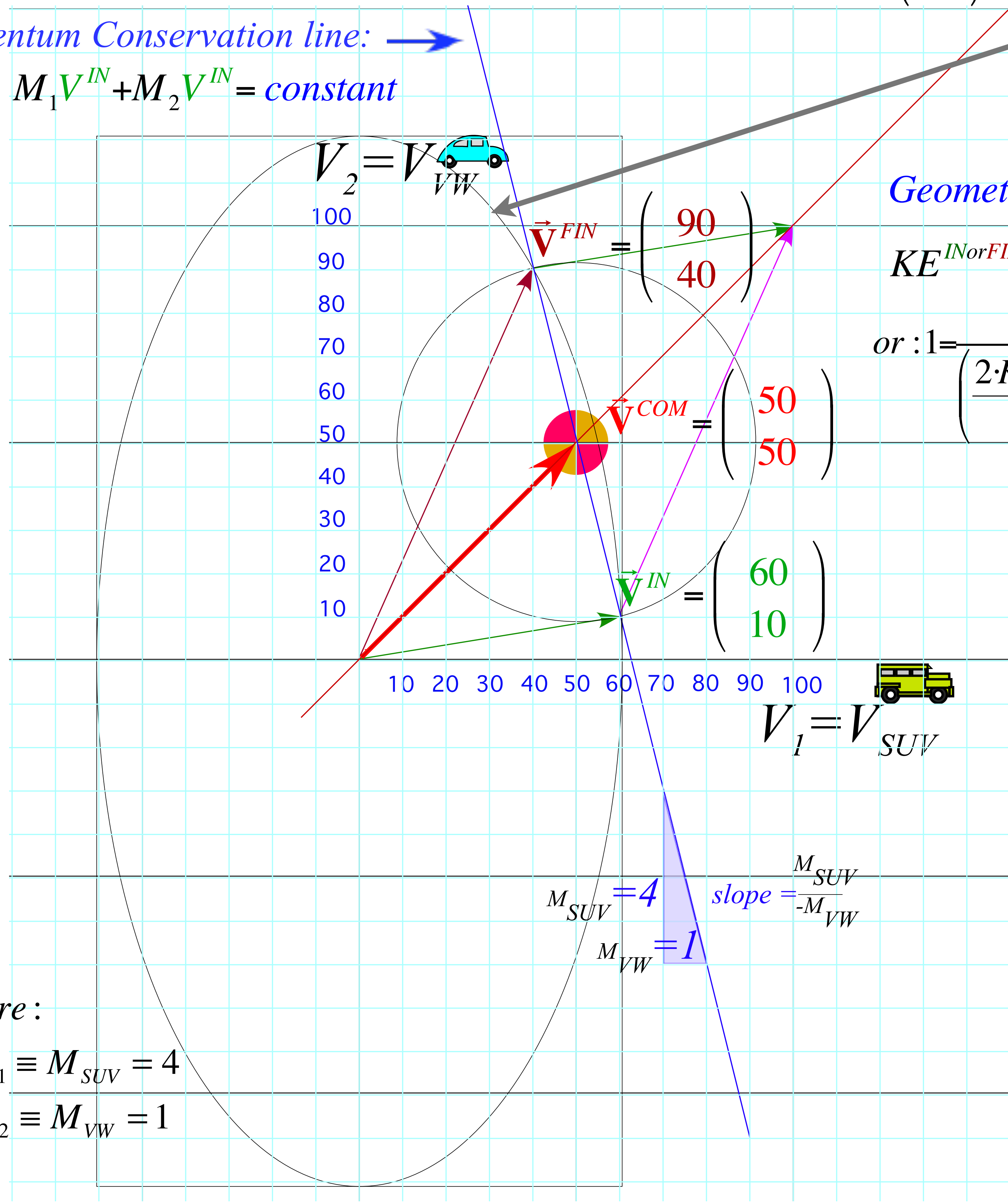
$$(M_1+M_2)V^{COM}=M_1V_1^{IN}+M_2V_2^{IN}=M_1V_1^{FIN}+M_2V_2^{FIN}$$

Geometry of Kinetic Energy Conservation **Axiom-2/Theorem-1**

$$KE^{IN}=\tfrac{1}{2}M_1\left(V_1^{IN}\right)^2+\tfrac{1}{2}M_2\left(V_2^{IN}\right)^2=\tfrac{1}{2}M_1\left(V_1^{FIN}\right)^2+\tfrac{1}{2}M_2\left(V_2^{FIN}\right)^2=KE^{FIN}$$

Momentum Conservation line:

$M_1V^{IN}+M_2V^{IN}=$ *constant*



here:

$M_1\equiv M_{SUV}=4$

$M_2\equiv M_{VW}=1$

Geometry of KE Conservation **Theorem - 1**

$$KE^{INorFIN}=\tfrac{1}{2}M_1\left(V_1\right)^2+\tfrac{1}{2}M_2\left(V_2\right)^2=\tfrac{1}{2}4(60)^2+\tfrac{1}{2}1(10)^2=7{,}250$$

$$or:1=\frac{V_1^2}{\left(\frac{2\cdot KE^{IFN}}{M_1}\right)}+\frac{V_2^2}{\left(\frac{2\cdot KE^{IFN}}{M_2}\right)}=\frac{V_1^2}{\left(\sqrt{\frac{2\cdot KE^{IFN}}{M_1}}\right)^2}+\frac{V_2^2}{\left(\sqrt{\frac{2\cdot KE^{IFN}}{M_2}}\right)^2}$$

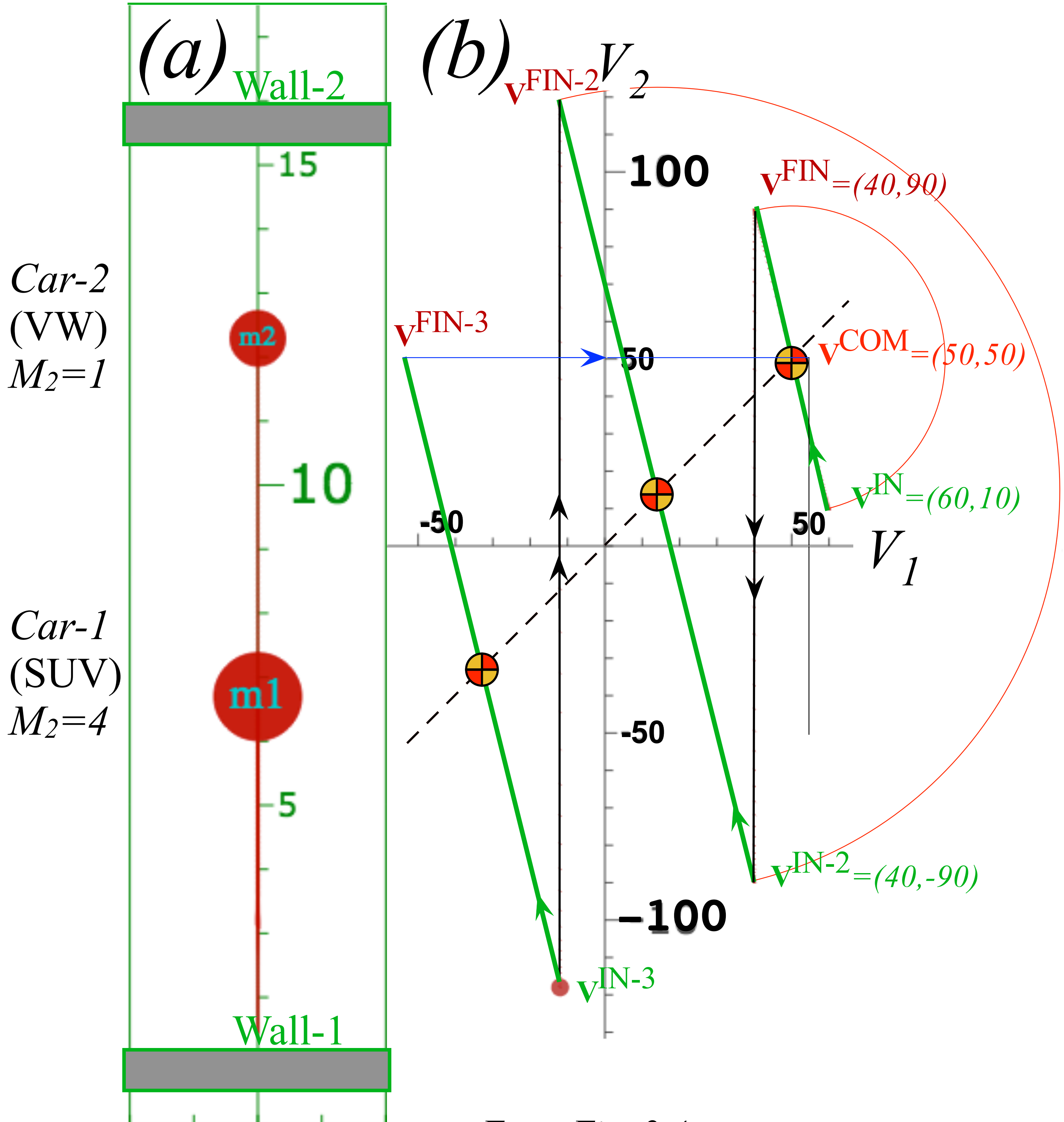
Has the algebraic form: $1=\frac{x^2}{a^2}+\frac{y^2}{b^2}$

$$elliptic\ radii: a=\sqrt{\frac{2KE^{INorFIN}}{M_1}} \quad b=\sqrt{\frac{2KE^{INorFIN}}{M_2}}$$

$$=\sqrt{\frac{2\cdot 7{,}250}{4}} \quad =\sqrt{\frac{2\cdot 7{,}250}{1}}$$

$$=60.21 \quad =120.42$$

BounceIt Simulation: frictionless 1D-track with elastic bumper cars bouncing between walls



From Fig. 2.4

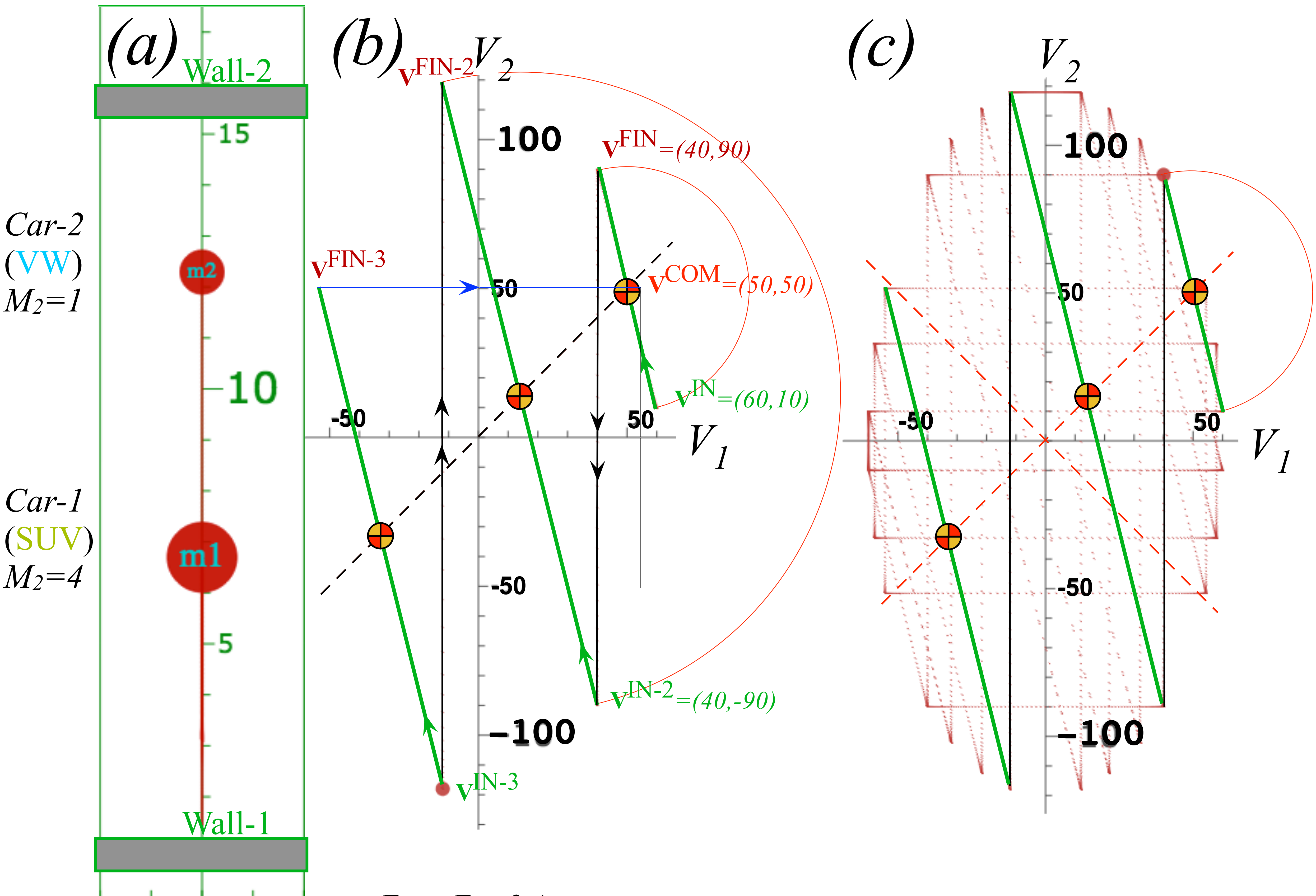
BounceIt
Superball Web Simulator

***Basic** elastic Collision*
Dual Panel
Space vs Space
and
V(VW) vs. V(SUV)

***Repeated** elastic Collisions*
Dual Panel
Space vs Space
and
V(VW) vs. V(SUV)

***Collision** Web Simulator*
***Basic** elastic Collision*
Dual Panel
Space vs Space
and
V(VW) vs. V(SUV)

BounceIt Simulation: frictionless 1D-track with elastic bumper cars bouncing between walls



From Fig. 2.4

Transpose symmetry ($M_{jk} = M_{kj}$) of the **M**-matrix implies: $\overleftarrow{\mathbf{V}}^{FIN} \cdot \overleftrightarrow{\mathbf{M}} \cdot \vec{\mathbf{V}}^{IN} = \overleftarrow{\mathbf{V}}^{IN} \cdot \overleftrightarrow{\mathbf{M}} \cdot \vec{\mathbf{V}}^{FIN}$

$$\begin{pmatrix} 40 & 90 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 60 \\ 10 \end{pmatrix} = \begin{pmatrix} 60 & 10 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 40 \\ 90 \end{pmatrix}$$

$$= 100 \cdot 105 = 100 \cdot 105 = 10{,}500$$

With ($M_{12} = 0 = M_{21}$) kinetic energy $KE^{INorFIN} = \frac{1}{2} \overleftarrow{\mathbf{V}} \cdot \overleftrightarrow{\mathbf{M}} \cdot \vec{\mathbf{V}}$ is the same at $\mathbf{V} = \mathbf{V}^{IN}$ and $\mathbf{V} = \mathbf{V}^{FIN}$.

$$V^{COM} P_{Total} - \frac{\overleftarrow{\mathbf{V}}^{FIN} \cdot \overleftrightarrow{\mathbf{M}} \cdot \vec{\mathbf{V}}^{IN}}{2} = \frac{\overleftarrow{\mathbf{V}}^{IN} \cdot \overleftrightarrow{\mathbf{M}} \cdot \vec{\mathbf{V}}^{IN}}{2} = \frac{\overleftarrow{\mathbf{V}}^{FIN} \cdot \overleftrightarrow{\mathbf{M}} \cdot \vec{\mathbf{V}}^{FIN}}{2} = KE^{INorFIN}$$

$$12{,}500 - \frac{10{,}500}{2} = \frac{\begin{pmatrix} 60 & 10 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 60 \\ 10 \end{pmatrix}}{2} = \frac{\begin{pmatrix} 40 & 90 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 40 \\ 90 \end{pmatrix}}{2} = KE^{INorFIN}$$

$$12{,}500 - 5{,}250 = 7{,}250 = 7{,}250$$

Transpose symmetry ($M_{jk} = M_{kj}$) of the **M**-matrix implies: $\overleftarrow{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \overleftarrow{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}$

$$\begin{pmatrix} 40 & 90 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix} = \begin{pmatrix} 60 & 10 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix}$$

$$= 100 \cdot 105 = 100 \cdot 105 = 10{,}500$$

With ($M_{12} = 0 = M_{21}$) kinetic energy $KE^{INorFIN} = \frac{1}{2} \overleftarrow{\mathbf{V}} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}$ is the same at $\mathbf{V} = \mathbf{V}^{IN}$ and $\mathbf{V} = \mathbf{V}^{FIN}$.

$$V^{COM} P_{Total} - \frac{\overleftarrow{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{2} = \frac{\overleftarrow{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{2} = \frac{\overleftarrow{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}}{2} = KE^{INorFIN}$$

$$12{,}500 - \frac{10{,}500}{2} = \frac{\begin{pmatrix} 60 & 10 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix}}{2} = \frac{\begin{pmatrix} 40 & 90 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix}}{2} = KE^{INorFIN}$$

$$12{,}500 - 5{,}250 = 7{,}250 = 7{,}250$$

Consider kinetic energy KE^{COM} when $\mathbf{V} = \mathbf{V}^{COM}$. It is reduced by *1,000* from 7,250 to 6,250.

$$KE^{COM} = \frac{1}{2} \overleftarrow{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{COM} = \frac{1}{2} \begin{pmatrix} 50 & 50 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 50 \\ 50 \end{pmatrix}$$

$$= \frac{1}{2}(50 \cdot 4 \cdot 50 + 50 \cdot 1 \cdot 50) = 2 \cdot 2500 + \frac{1}{2} \cdot 2500$$

$$= 5000 + \frac{1}{2} \cdot 1250 = 6{,}250$$

Geometry of Momentum Conservation Axiom-1

$$(M_1+M_2)V^{COM}=M_1V_1^{IN}+M_2V_2^{IN}=M_1V_1^{FIN}+M_2V_2^{FIN}$$

Geometry of Kinetic Energy Conservation Axiom-2/Theorem-1

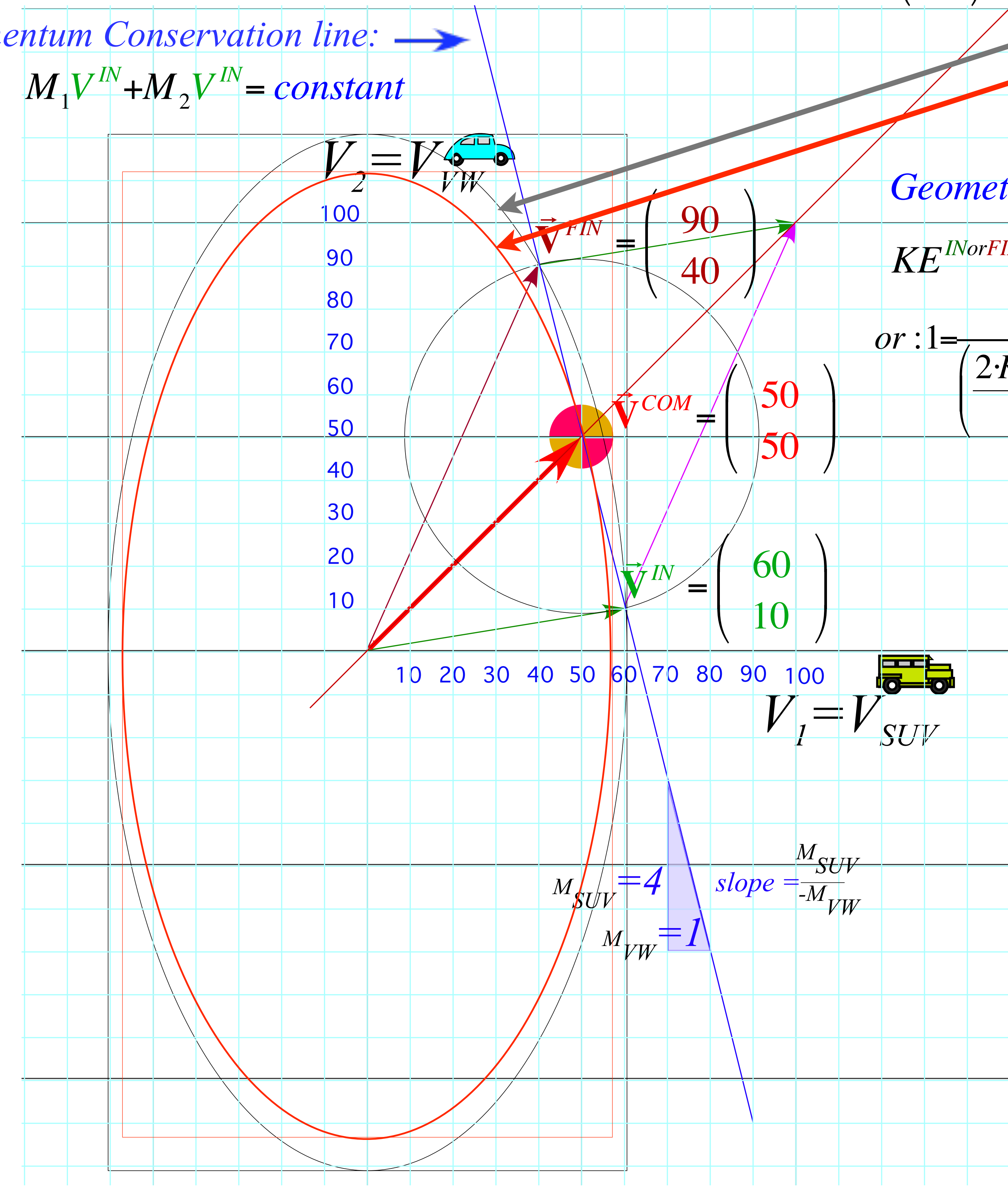
$$KE^{IN}=\tfrac{1}{2}M_1\left(V_1^{IN}\right)^2+\tfrac{1}{2}M_2\left(V_2^{IN}\right)^2=\tfrac{1}{2}M_1\left(V_1^{FIN}\right)^2+\tfrac{1}{2}M_2\left(V_2^{FIN}\right)^2=KE^{FIN}$$

Momentum Conservation line:

$M_1V^{IN}+M_2V^{IN}=$ constant

$KE^{INorFIN}$ Conservation ellipse:

KE^{COM} Ka-runch ellipse:



Geometry of KE Conservation Theorem-1

$$KE^{INorFIN}=\tfrac{1}{2}M_1\left(V_1\right)^2+\tfrac{1}{2}M_2\left(V_2\right)^2$$

$$or: 1=\frac{V_1^2}{\left(\frac{2\cdot KE^{IFN}}{M_1}\right)}+\frac{V_2^2}{\left(\frac{2\cdot KE^{IFN}}{M_2}\right)}=\frac{V_1^2}{\left(\sqrt{\frac{2\cdot KE^{IFN}}{M_1}}\right)^2}+\frac{V_2^2}{\left(\sqrt{\frac{2\cdot KE^{IFN}}{M_2}}\right)^2}$$

Has the algebraic form: $1=\frac{x^2}{a^2}+\frac{y^2}{b^2}$

Geometry of KE^{COM} at CenterOfMomentum

$$KE^{COM}=\tfrac{1}{2}M_1\left(V_1^{COM}\right)^2+\tfrac{1}{2}M_2\left(V_2^{COM}\right)^2$$

$$=\tfrac{1}{2}\left(M_1+M_2\right)\left(V^{COM}\right)^2$$

elliptic radii: $a^{COM}=\sqrt{\frac{KE^{COM}}{M_1}}=\sqrt{\frac{2\cdot 6{,}250}{4}}=55.90$ $\quad b^{COM}=\sqrt{\frac{KE^{COM}}{M_2}}=\sqrt{\frac{2\cdot 6{,}250}{1}}=111.80$

Transpose symmetry ($M_{jk} = M_{kj}$) of the **M**-matrix implies: $\bar{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \bar{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}$

$$\begin{pmatrix} 40 & 90 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 60 \\ 10 \end{pmatrix} = \begin{pmatrix} 60 & 10 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 40 \\ 90 \end{pmatrix}$$

$$= 100 \cdot 105 = 100 \cdot 105 = 10{,}500$$

With ($M_{12} = 0 = M_{21}$) kinetic energy $KE^{INorFIN} = \frac{1}{2} \bar{\mathbf{V}} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}$ is the same at $\mathbf{V} = \mathbf{V}^{IN}$ and $\mathbf{V} = \mathbf{V}^{FIN}$.

$$V^{COM} P_{Total} - \frac{\bar{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{2} = \frac{\bar{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{2} = \frac{\bar{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}}{2} = KE^{INorFIN}$$

$$12{,}500 - \frac{10{,}500}{2} = \frac{\begin{pmatrix} 60 & 10 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 60 \\ 10 \end{pmatrix}}{2} = \frac{\begin{pmatrix} 40 & 90 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 40 \\ 90 \end{pmatrix}}{2} = KE^{INorFIN}$$

$$12{,}500 - 5{,}250 = 7{,}250 = 7{,}250$$

Consider kinetic energy KE^{COM} when $\mathbf{V} = \mathbf{V}^{COM}$. It is reduced by *1,000* from 7,250 to 6,250.

$$KE^{COM} = V^{COM} P_{Total} - \frac{\bar{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{COM}}{2} = \frac{1}{2} V^{COM} P_{Total} = \frac{\bar{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{COM}}{2} = \frac{\bar{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4} + \frac{\bar{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4} = \frac{1}{2} KE^{INorFIN} + \frac{\bar{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4}$$

$$12{,}500 - \frac{12{,}500}{2} = 6{,}250 = 6{,}250 = 3{,}625 + 2{,}625 = 3{,}625 + 2{,}625$$

$$KE^{COM} = \frac{1}{2} \bar{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{COM} = \frac{1}{2} \begin{pmatrix} 50 & 50 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 50 \\ 50 \end{pmatrix}$$

$$= \frac{1}{2}(50 \cdot 4 \cdot 50 + 50 \cdot 1 \cdot 50) = 2 \cdot 2500 + \frac{1}{2} \cdot 2500$$

$$= 5000 + \frac{1}{2} \cdot 1250 = 6{,}250$$

Transpose symmetry ($M_{jk} = M_{kj}$) of the **M**-matrix implies: $\bar{\mathbf{V}}^{FIN} \cdot \overleftrightarrow{\mathbf{M}} \cdot \vec{\mathbf{V}}^{IN} = \bar{\mathbf{V}}^{IN} \cdot \overleftrightarrow{\mathbf{M}} \cdot \vec{\mathbf{V}}^{FIN}$

$$\begin{pmatrix} 40 & 90 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 60 \\ 10 \end{pmatrix} = \begin{pmatrix} 60 & 10 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 40 \\ 90 \end{pmatrix}$$

$$= 100 \cdot 105 = 100 \cdot 105 = 10{,}500$$

With ($M_{12} = 0 = M_{21}$) kinetic energy $KE^{INorFIN} = \frac{1}{2} \bar{\mathbf{V}} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}$ is the same at $\mathbf{V} = \mathbf{V}^{IN}$ and $\mathbf{V} = \mathbf{V}^{FIN}$.

$$V^{COM} P_{Total} - \frac{\bar{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{2} = \frac{\bar{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{2} = \frac{\bar{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}}{2} = KE^{INorFIN}$$

$$12{,}500 - \frac{10{,}500}{2} = \frac{\begin{pmatrix} 60 & 10 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 60 \\ 10 \end{pmatrix}}{2} = \frac{\begin{pmatrix} 40 & 90 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 40 \\ 90 \end{pmatrix}}{2} = KE^{INorFIN}$$

$$12{,}500 - 5{,}250 = 7{,}250 = 7{,}250$$

Consider kinetic energy KE^{COM} when $\mathbf{V} = \mathbf{V}^{COM}$. It is reduced by *1,000* from 7,250 to 6,250.

$$KE^{COM} = V^{COM} P_{Total} - \frac{\bar{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{COM}}{2} = \frac{1}{2} V^{COM} P_{Total} = \frac{\bar{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{COM}}{2} = \frac{\bar{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4} + \frac{\bar{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4} = \frac{1}{2} KE^{INorFIN} + \frac{\bar{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4}$$

$$12{,}500 - \frac{12{,}500}{2} = 6{,}250 = 6{,}250 = 3{,}625 + 2{,}625 = 3{,}625 + 2{,}625$$

$$KE^{COM} = \frac{1}{2} \bar{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{COM} = \frac{1}{2} \begin{pmatrix} 50 & 50 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 50 \\ 50 \end{pmatrix}$$

$$= \tfrac{1}{2}(50 \cdot 4 \cdot 50 + 50 \cdot 1 \cdot 50) = 2 \cdot 2500 + \tfrac{1}{2} \cdot 2500$$

$$= 5000 + \tfrac{1}{2} \cdot 1250 = 6{,}250$$

Introducing
Potential Energy=PE

Difference is inelastic "*ka-Runch*" $KE^{INorFIN}$-KE^{COM}. For elastic "*ka-Bong*" the *1,000* is PE^{COM} of compression.

$$KE^{INorFIN} - KE^{COM} = \frac{\bar{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4} - \frac{\bar{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4}$$

$$1{,}000 = 3{,}625 - 2{,}625$$

Transpose symmetry ($M_{jk} = M_{kj}$) of the **M**-matrix implies: $\vec{V}^{FIN}\bullet\overleftrightarrow{M}\bullet\vec{V}^{IN} = \vec{V}^{IN}\bullet\overleftrightarrow{M}\bullet\vec{V}^{FIN}$

$$\begin{pmatrix} 40 & 90 \end{pmatrix}\bullet\begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}\bullet\begin{pmatrix} 60 \\ 10 \end{pmatrix} = \begin{pmatrix} 60 & 10 \end{pmatrix}\bullet\begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}\bullet\begin{pmatrix} 40 \\ 90 \end{pmatrix}$$

$$= 100\cdot 105 = 100\cdot 105 = 10{,}500$$

With ($M_{12} = 0 = M_{21}$) kinetic energy $KE^{INorFIN} = \frac{1}{2}\vec{V}\bullet\overleftrightarrow{M}\bullet\vec{V}$ is the same at **V=V**^{IN} and **V=V**^{FIN}.

$$V^{COM}P_{Total} - \frac{\vec{V}^{FIN}\bullet\overleftrightarrow{M}\bullet\vec{V}^{IN}}{2} = \frac{\vec{V}^{IN}\bullet\overleftrightarrow{M}\bullet\vec{V}^{IN}}{2} = \frac{\vec{V}^{FIN}\bullet\overleftrightarrow{M}\bullet\vec{V}^{FIN}}{2} = KE^{INorFIN}$$

$$12{,}500 - \frac{10{,}500}{2} = \frac{\begin{pmatrix} 60 & 10 \end{pmatrix}\bullet\begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}\bullet\begin{pmatrix} 60 \\ 10 \end{pmatrix}}{2} = \frac{\begin{pmatrix} 40 & 90 \end{pmatrix}\bullet\begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}\bullet\begin{pmatrix} 40 \\ 90 \end{pmatrix}}{2} = KE^{INorFIN}$$

$$12{,}500 - 5{,}250 = 7{,}250 = 7{,}250$$

Consider kinetic energy KE^{COM} when **V=V**^{COM}. It is reduced by *1,000* from 7,250 to 6,250.

$$KE^{COM} = V^{COM}P_{Total} - \frac{\vec{V}^{COM}\bullet\overleftrightarrow{M}\bullet\vec{V}^{COM}}{2} = \frac{1}{2}V^{COM}P_{Total} = \frac{\vec{V}^{COM}\bullet\overleftrightarrow{M}\bullet\vec{V}^{COM}}{2} = \frac{\vec{V}^{IN}\bullet\overleftrightarrow{M}\bullet\vec{V}^{IN}}{4} + \frac{\vec{V}^{FIN}\bullet\overleftrightarrow{M}\bullet\vec{V}^{IN}}{4} = \frac{1}{2}KE^{INorFIN} + \frac{\vec{V}^{FIN}\bullet\overleftrightarrow{M}\bullet\vec{V}^{IN}}{4}$$

$$12{,}500 - \frac{12{,}500}{2} = 6{,}250 = 6{,}250 = 3{,}625 + 2{,}625 = 3{,}625 + 2{,}625$$

$$KE^{COM} = \frac{1}{2}\vec{V}^{COM}\bullet\overleftrightarrow{M}\bullet\vec{V}^{COM} = \frac{1}{2}\begin{pmatrix} 50 & 50 \end{pmatrix}\bullet\begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}\bullet\begin{pmatrix} 50 \\ 50 \end{pmatrix}$$

$$= \tfrac{1}{2}(50\cdot 4\cdot 50 + 50\cdot 1\cdot 50) = 2\cdot 2500 + \tfrac{1}{2}\cdot 2500$$

$$= 5000 + \tfrac{1}{2}\cdot 1250 = 6{,}250$$

Introducing
Potential Energy=PE

Difference is inelastic "*ka-Runch*" $KE^{INorFIN}$-KE^{COM}. For elastic "*ka-Bong*" the *1,000* is PE^{COM} of compression.

$$KE^{INorFIN} - KE^{COM} = \frac{\vec{V}^{IN}\bullet\overleftrightarrow{M}\bullet\vec{V}^{IN}}{4} - \frac{\vec{V}^{FIN}\bullet\overleftrightarrow{M}\bullet\vec{V}^{IN}}{4}$$

$$1{,}000 = 3{,}625 - 2{,}625$$

$$KE^{COM} = \frac{\vec{V}^{IN}\bullet\overleftrightarrow{M}\bullet\vec{V}^{IN}}{4} + \frac{\vec{V}^{FIN}\bullet\overleftrightarrow{M}\bullet\vec{V}^{IN}}{4}$$

$$6{,}250 = 3{,}625 + 2{,}625$$

Difference $KE^{INorFIN}$-KE^{COM} = *1,000* is the same in *all* frames including *COM*-frame where **V**^{COM}=**0**.

Geometry of Momentum Conservation **Axiom - 1**

$$(M_1+M_2)V^{COM}=M_1V_1^{IN}+M_2V_2^{IN}=M_1V_1^{FIN}+M_2V_2^{FIN}$$

Geometry of Kinetic Energy Conservation **Axiom-2/Theorem-1**

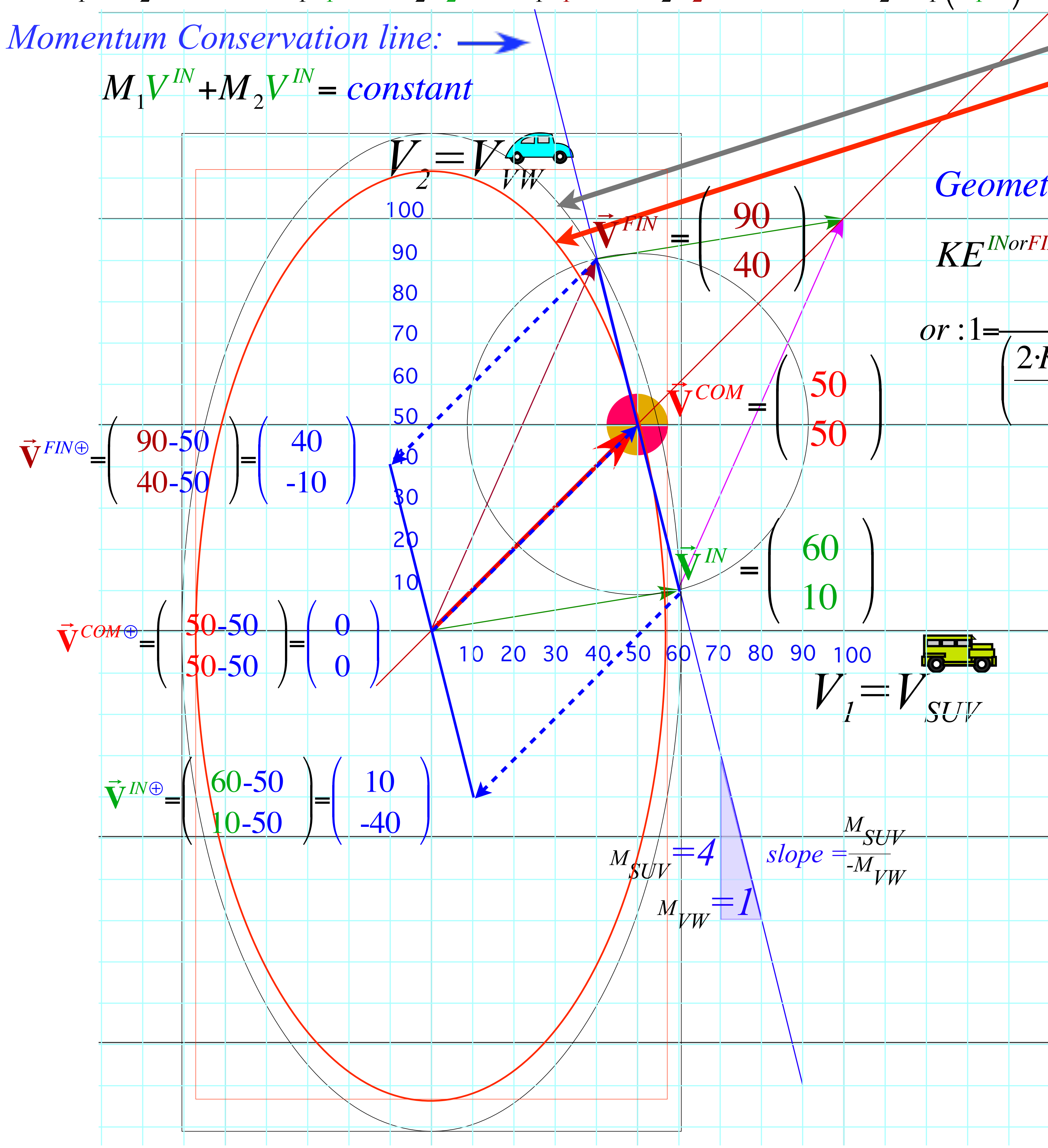
$$KE^{IN}=\tfrac{1}{2}M_1\left(V_1^{IN}\right)^2+\tfrac{1}{2}M_2\left(V_2^{IN}\right)^2=\tfrac{1}{2}M_1\left(V_1^{FIN}\right)^2+\tfrac{1}{2}M_2\left(V_2^{FIN}\right)^2=KE^{FIN}$$

Momentum Conservation line: →

$$M_1V^{IN}+M_2V^{IN}=\textit{constant}$$

$KE^{INorFIN}$ *Conservation ellipse:*

KE^{COM} *Ka-runch ellipse:*



Geometry of KE Conservation **Theorem - 1**

$$KE^{INorFIN}=\tfrac{1}{2}M_1\left(V_1\right)^2+\tfrac{1}{2}M_2\left(V_2\right)^2$$

$$or:1=\frac{V_1^2}{\left(\frac{2\cdot KE^{IFN}}{M_1}\right)}+\frac{V_2^2}{\left(\frac{2\cdot KE^{IFN}}{M_2}\right)}=\frac{V_1^2}{\left(\sqrt{\frac{2\cdot KE^{IFN}}{M_1}}\right)^2}+\frac{V_2^2}{\left(\sqrt{\frac{2\cdot KE^{IFN}}{M_2}}\right)^2}$$

Has the algebraic form: $1=\frac{x^2}{a^2}+\frac{y^2}{b^2}$

Geometry of KE^{COM} at CenterOfMomentum

$$KE^{COM}=\tfrac{1}{2}M_1\left(V_1^{COM}\right)^2+\tfrac{1}{2}M_2\left(V_2^{COM}\right)^2$$

$$=\tfrac{1}{2}\left(M_1+M_2\right)\left(V^{COM}\right)^2$$

elliptic radii: $a^{COM}=\sqrt{\frac{KE^{COM}}{M_1}}=\sqrt{\frac{2\cdot 6{,}250}{4}}=55.90$ $\quad b^{COM}=\sqrt{\frac{KE^{COM}}{M_2}}=\sqrt{\frac{2\cdot 6{,}250}{1}}=111.80$

Geometry of Momentum Conservation **Axiom-1**

$$(M_1+M_2)V^{COM}=M_1V_1^{IN}+M_2V_2^{IN}=M_1V_1^{FIN}+M_2V_2^{FIN}$$

Geometry of Kinetic Energy Conservation **Axiom-2/Theorem-1**

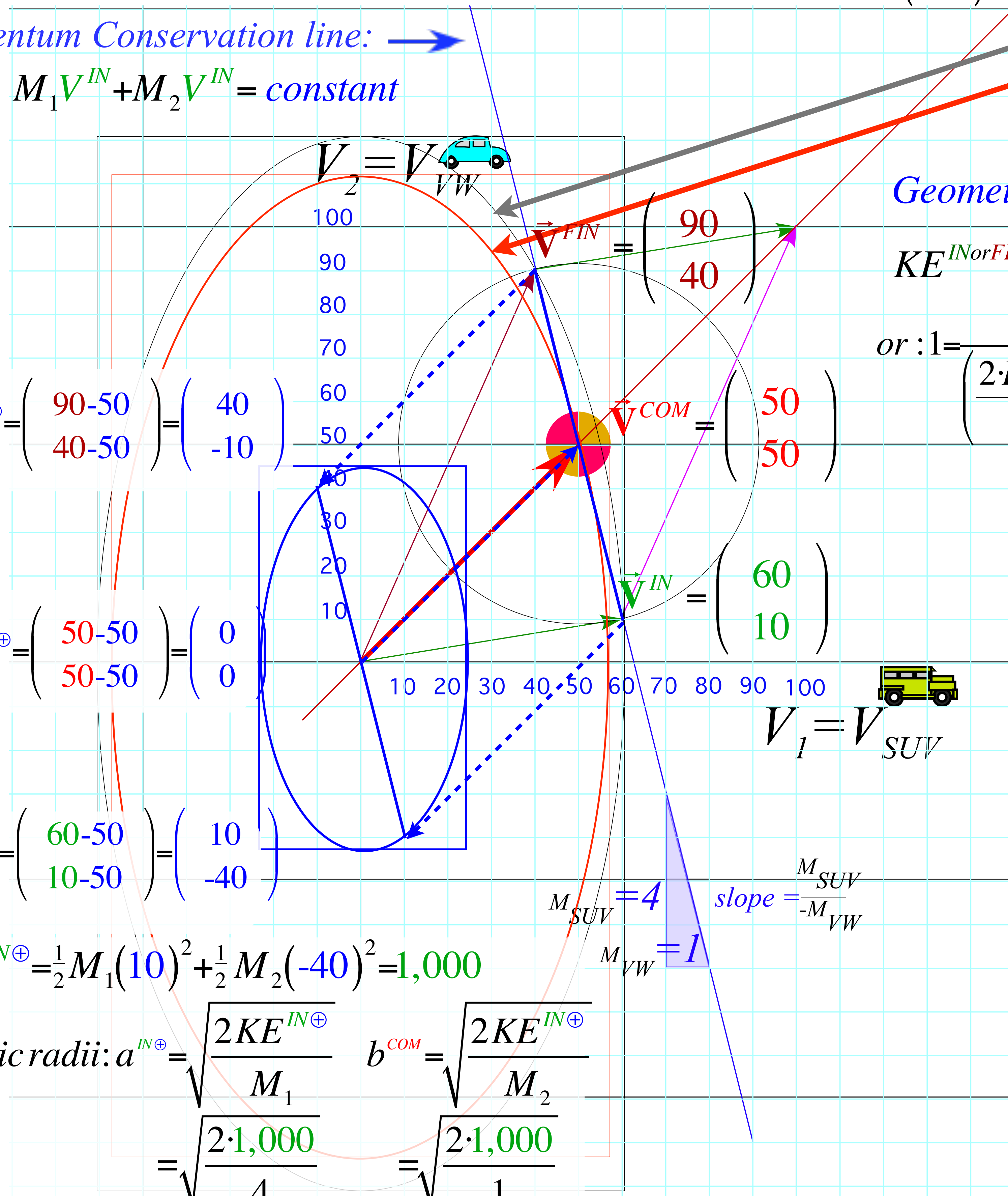
$$KE^{IN}=\tfrac{1}{2}M_1\left(V_1^{IN}\right)^2+\tfrac{1}{2}M_2\left(V_2^{IN}\right)^2=\tfrac{1}{2}M_1\left(V_1^{FIN}\right)^2+\tfrac{1}{2}M_2\left(V_2^{FIN}\right)^2=KE^{FIN}$$

Momentum Conservation line: →

$M_1V^{IN}+M_2V^{IN}=$ *constant*

$KE^{INorFIN}$ *Conservation ellipse:*

KE^{COM} *Ka-runch ellipse:*



$V_2=V_{VW}$

$V_1=V_{SUV}$

$\vec{\mathbf{V}}^{FIN}=\begin{pmatrix}90\\40\end{pmatrix}$

$\vec{\mathbf{V}}^{COM}=\begin{pmatrix}50\\50\end{pmatrix}$

$\vec{\mathbf{V}}^{IN}=\begin{pmatrix}60\\10\end{pmatrix}$

$\vec{\mathbf{V}}^{FIN\oplus}=\begin{pmatrix}90\text{-}50\\40\text{-}50\end{pmatrix}=\begin{pmatrix}40\\-10\end{pmatrix}$

$\vec{\mathbf{V}}^{COM\oplus}=\begin{pmatrix}50\text{-}50\\50\text{-}50\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$

$\vec{\mathbf{V}}^{IN\oplus}=\begin{pmatrix}60\text{-}50\\10\text{-}50\end{pmatrix}=\begin{pmatrix}10\\-40\end{pmatrix}$

$M_{SUV}=4$

$M_{VW}=1$

slope $=\dfrac{M_{SUV}}{-M_{VW}}$

$$KE^{IN\oplus}=\tfrac{1}{2}M_1(10)^2+\tfrac{1}{2}M_2(-40)^2=1{,}000$$

elliptic radii: $a^{IN\oplus}=\sqrt{\dfrac{2KE^{IN\oplus}}{M_1}}=\sqrt{\dfrac{2\cdot 1{,}000}{4}}=22.36$ $\quad b^{COM}=\sqrt{\dfrac{2KE^{IN\oplus}}{M_2}}=\sqrt{\dfrac{2\cdot 1{,}000}{1}}=44.72$

Geometry of KE Conservation **Theorem-1**

$$KE^{INorFIN}=\tfrac{1}{2}M_1\left(V_1\right)^2+\tfrac{1}{2}M_2\left(V_2\right)^2$$

$$or:1=\frac{V_1^2}{\left(\frac{2\cdot KE^{IFN}}{M_1}\right)}+\frac{V_2^2}{\left(\frac{2\cdot KE^{IFN}}{M_2}\right)}=\frac{V_1^2}{\left(\sqrt{\frac{2\cdot KE^{IFN}}{M_1}}\right)^2}+\frac{V_2^2}{\left(\sqrt{\frac{2\cdot KE^{IFN}}{M_2}}\right)^2}$$

Has the algebraic form: $1=\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}$

Geometry of KE^{COM} *at CenterOfMomentum*

$$KE^{COM}=\tfrac{1}{2}M_1\left(V_1^{COM}\right)^2+\tfrac{1}{2}M_2\left(V_2^{COM}\right)^2$$

$$=\tfrac{1}{2}\left(M_1+M_2\right)\left(V^{COM}\right)^2$$

elliptic radii: $a^{COM}=\sqrt{\dfrac{KE^{COM}}{M_1}}=\sqrt{\dfrac{2\cdot 6{,}250}{4}}=55.90$ $\quad b^{COM}=\sqrt{\dfrac{KE^{COM}}{M_2}}=\sqrt{\dfrac{2\cdot 6{,}250}{1}}=111.80$

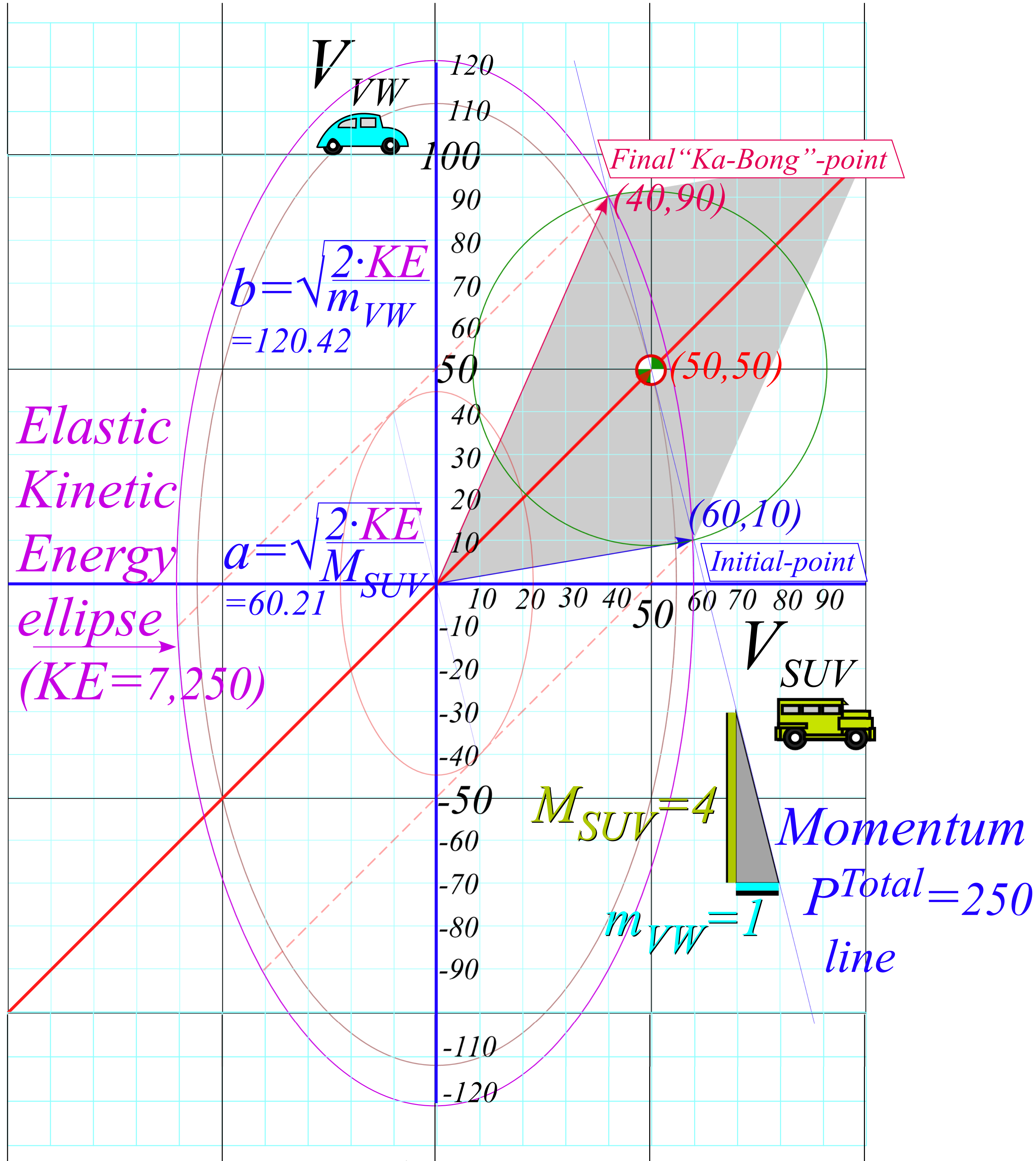


Fig. 3.1 a
in Unit 1

Fig. 3.1

Developing
Conservation-of-Momentum
The key axiom of mechanics
leading to
Conservation-of-Energy Theorem

If and only if…
*there is **T-Symmetry***

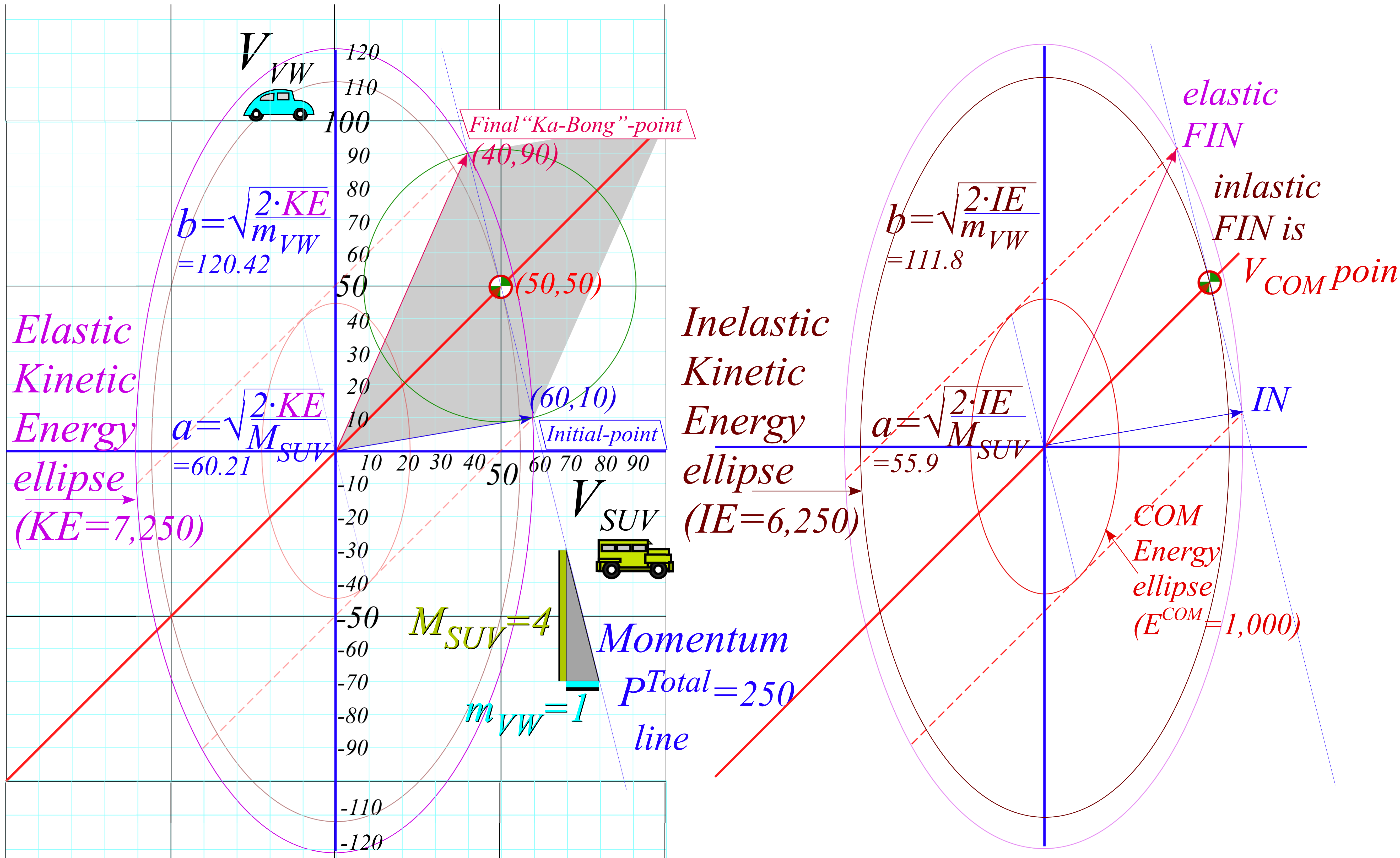


Fig. 3.1 a
in Unit 1

Fig. 3.1 b
in Unit 1

Fig. 3.1

As usual in physics, opposite extremes are easier to analyze than the generic "real(er) world" in between!

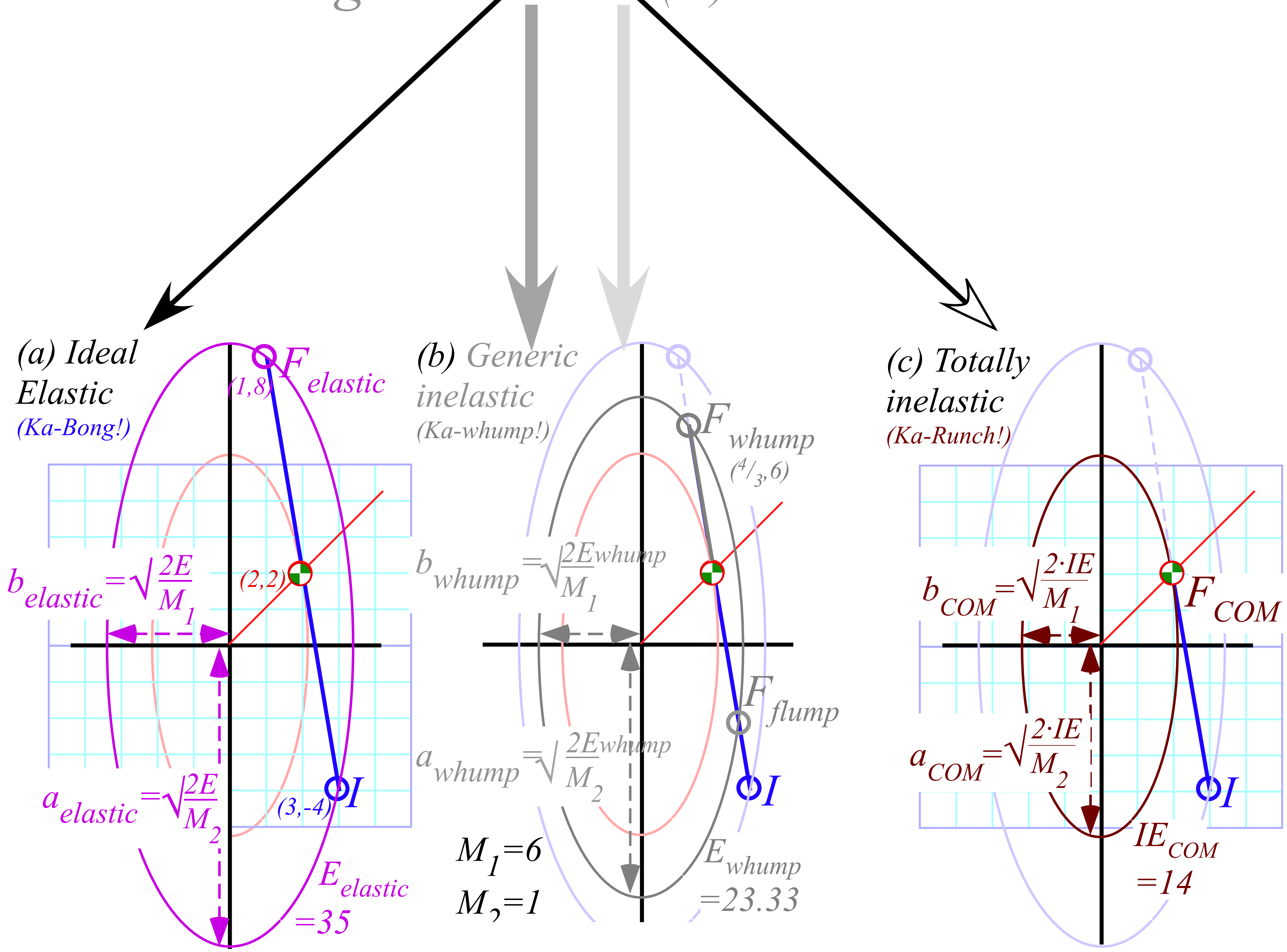


Fig. 2.3 (6-Ton SUV)

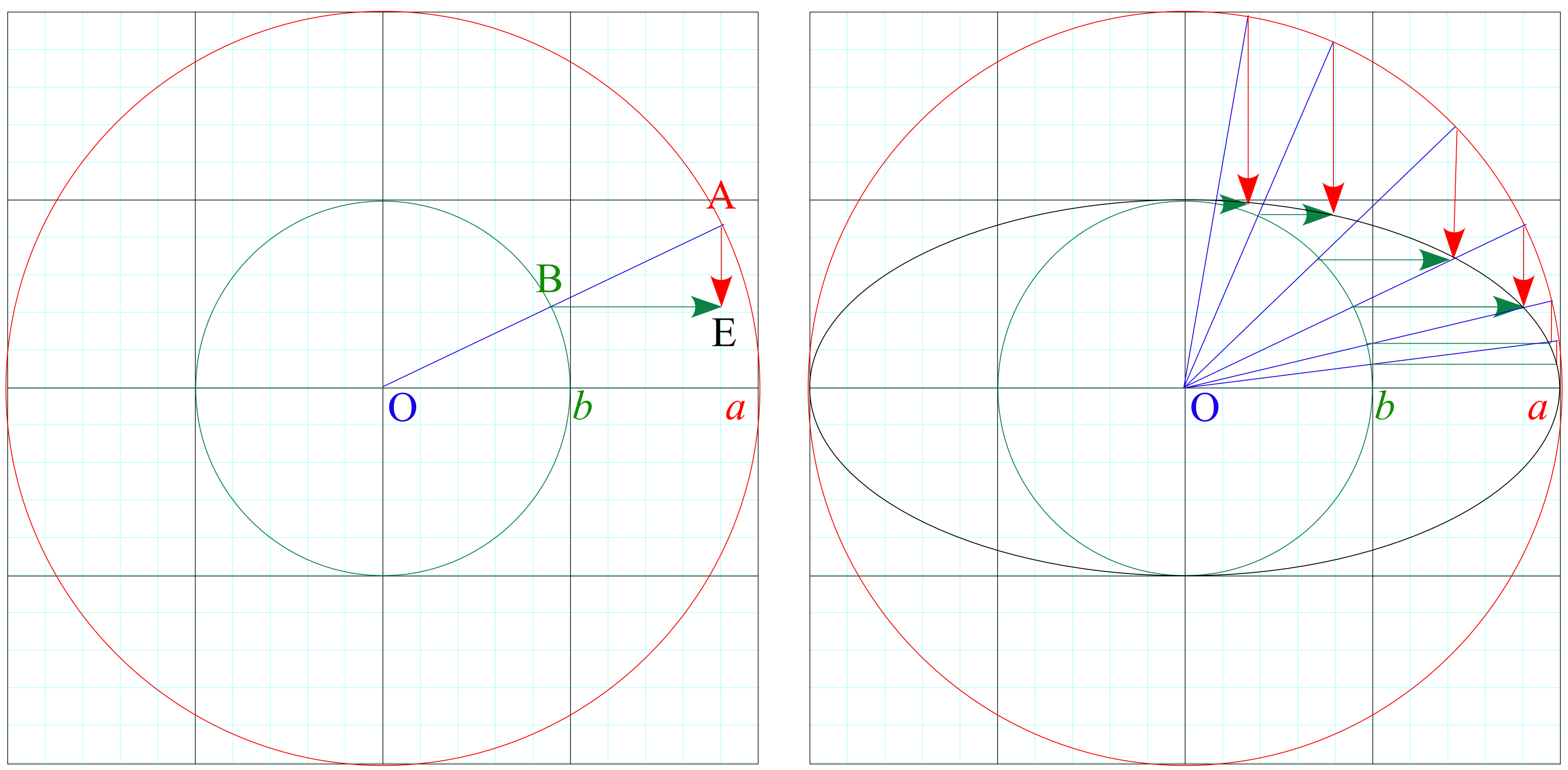
(During Bush II era an SUV with a mass of at least ***6 tons*** *allowed its owner to take a 100% write-off (up to $100,000) on Federal Income Tax.)*

Here ***T-Symmetry*** is best

Here ***T-Symmetry*** is less

Here ***T-Symmetry*** is least

Graph paper facilitates construction of energy ellipses given the two radii a and b in KE equation.
First step: draw concentric circles of radius a and b.
Then any radial line OBA "points" to point E on the ellipse.
Ellipse point E lies at the intersection of a vertical line AE thru radial intersection A with circle a and a horizontal line BE thru radial intersection B with circle b.
Graph grid helps locate E for a radius OBA, and usually there is no need to draw AE or BE.
You can pick x and find y or else *vice-versa*.



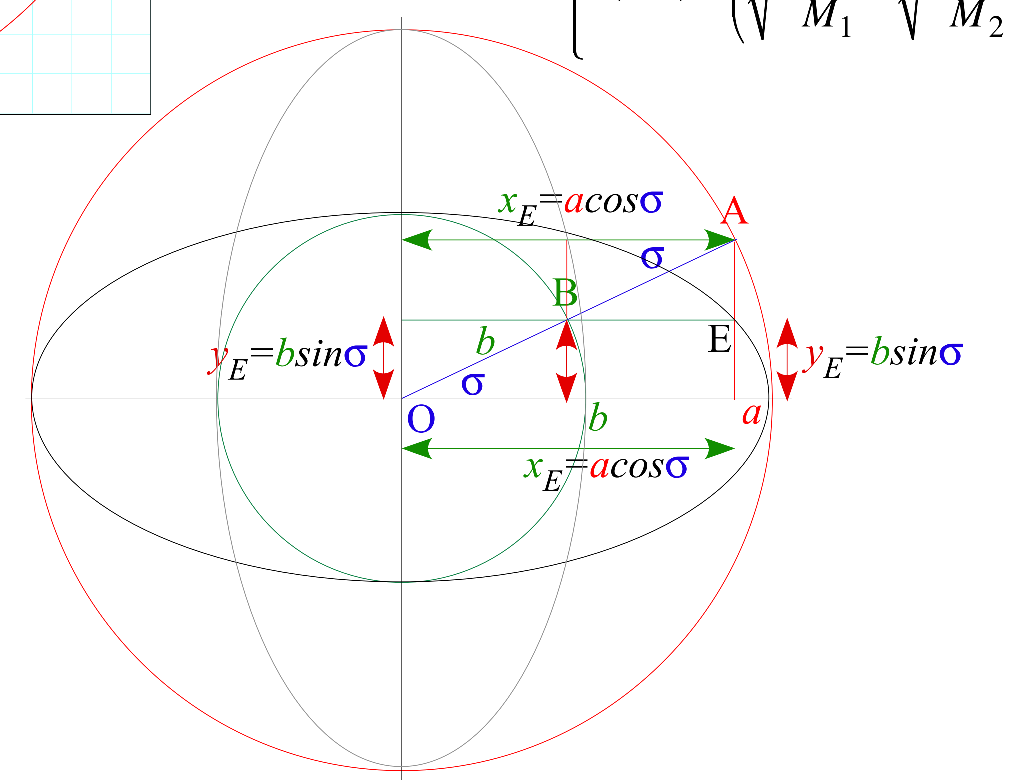
$$\frac{1}{2}M_1 \cdot V_1^{\,2} + \frac{1}{2}M_2 \cdot V_2^{\,2} = KE$$

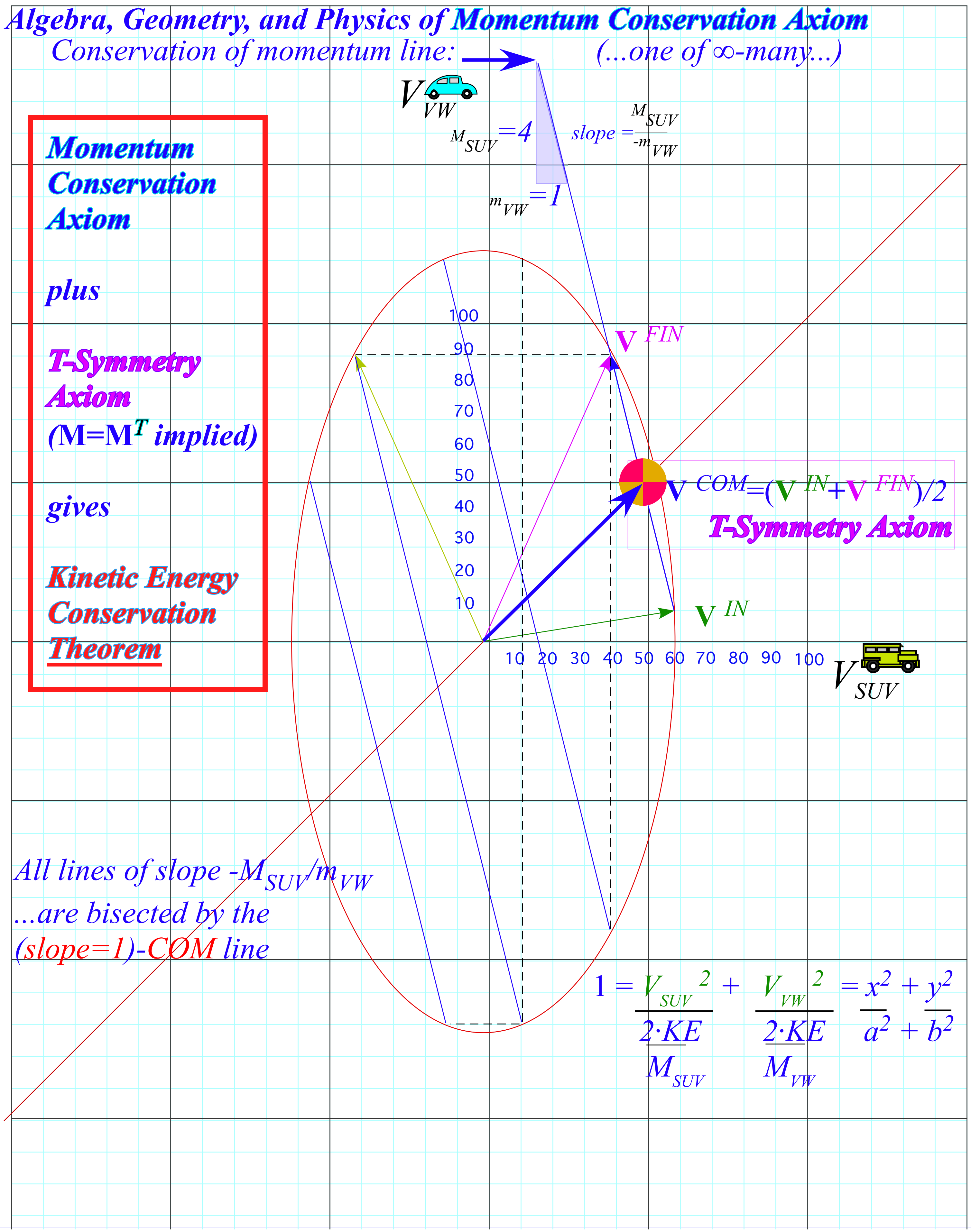
$$\frac{V_1^{\,2}}{\left(\frac{2 \cdot KE}{M_1}\right)} + \frac{V_2^{\,2}}{\left(\frac{2 \cdot KE}{M_2}\right)} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\left\{ \begin{array}{ll} (x,y) = & (V_1, V_2) \\ (a,b) = & \left(\sqrt{\frac{2 \cdot KE}{M_1}}, \sqrt{\frac{2 \cdot KE}{M_2}}\right) \end{array} \right.$$

Ellipse coordinates ($x_E = a \cdot \cos\sigma$, $y_E = b \cdot \sin\sigma$) are rescaled base and altitude
($x_r = r \cdot \cos\sigma$, $y_r = r \cdot \sin\sigma$) of Fig. 2.6.





Developing

Conservation-of-Momentum

The key axiom of mechanics

leading to

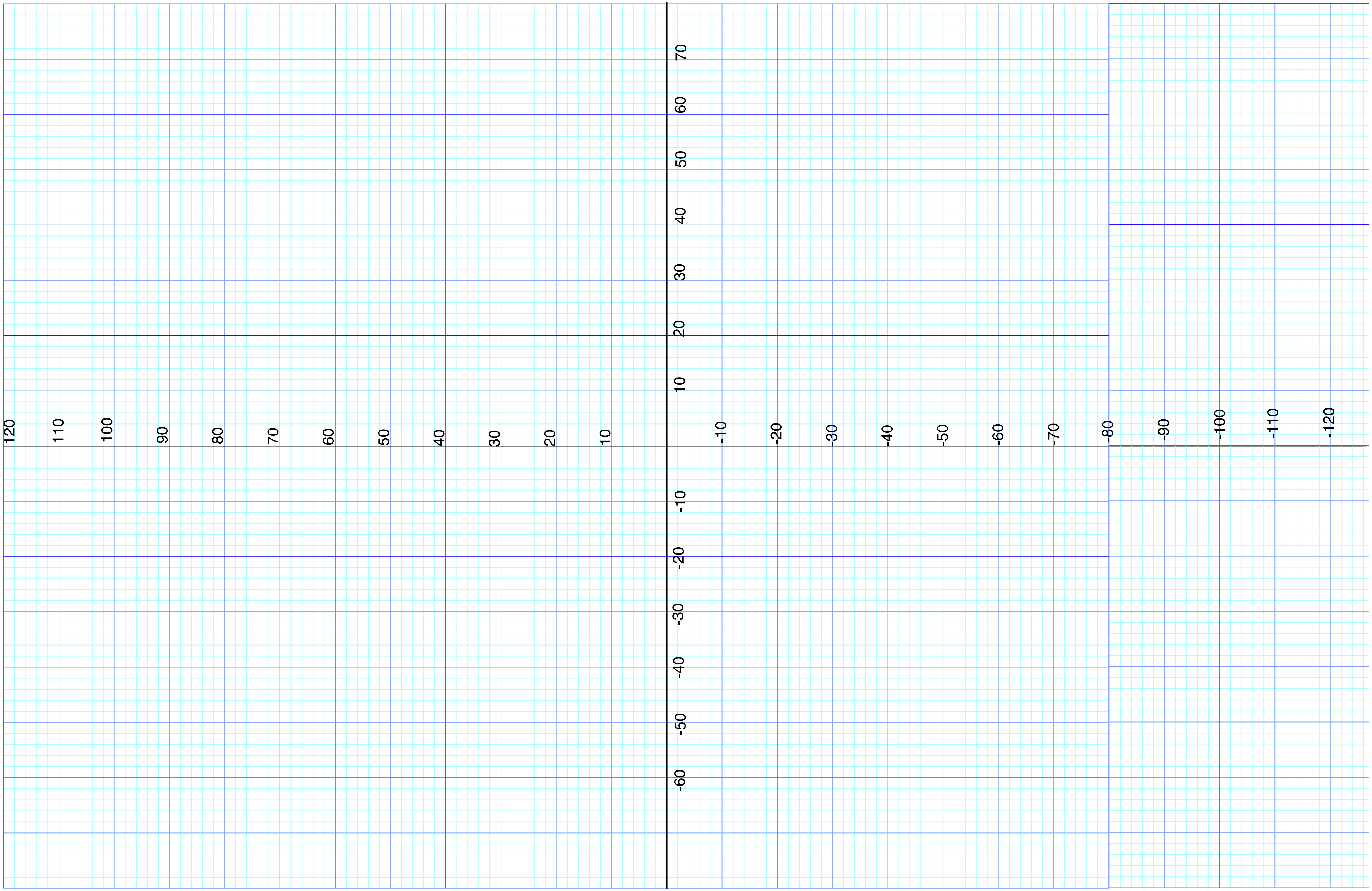
Conservation-of-Energy Theorem

If and only if...
there is T-Symmetry

$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM} - 1/2\,\mathbf{V}^{FIN}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}$
$= 1/2\,\mathbf{V}^{IN}\bullet\mathbf{M}\bullet\mathbf{V}^{IN} = 1/2\,\mathbf{V}^{FIN}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$

These are equations for energy conservation ellipse:

$$KE = {}^1/_2 M_{SUV} V_{SUV}^{\ 2} + {}^1/_2 M_{VW} V_{VW}^{\ 2}$$



120
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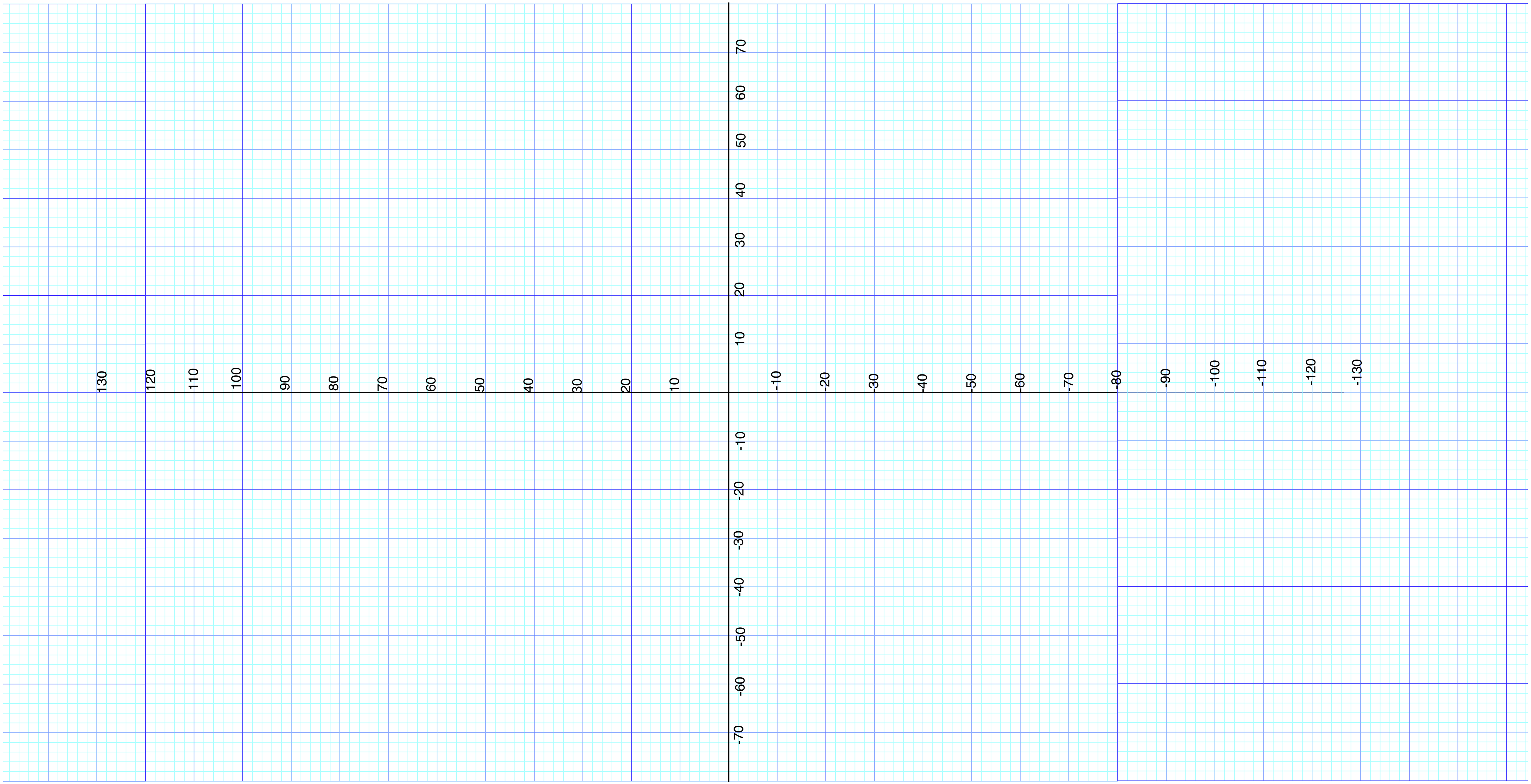
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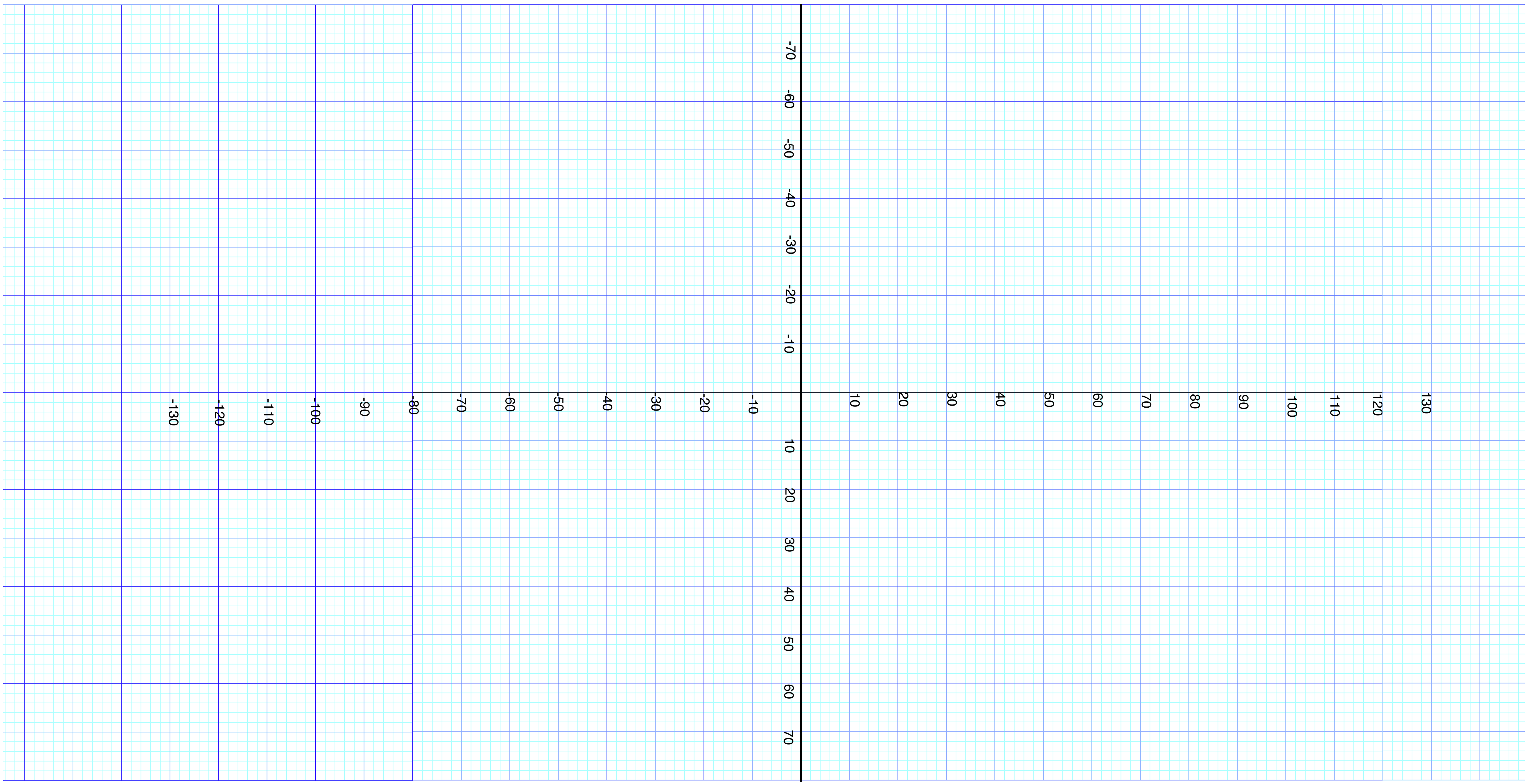
-110
-120
-130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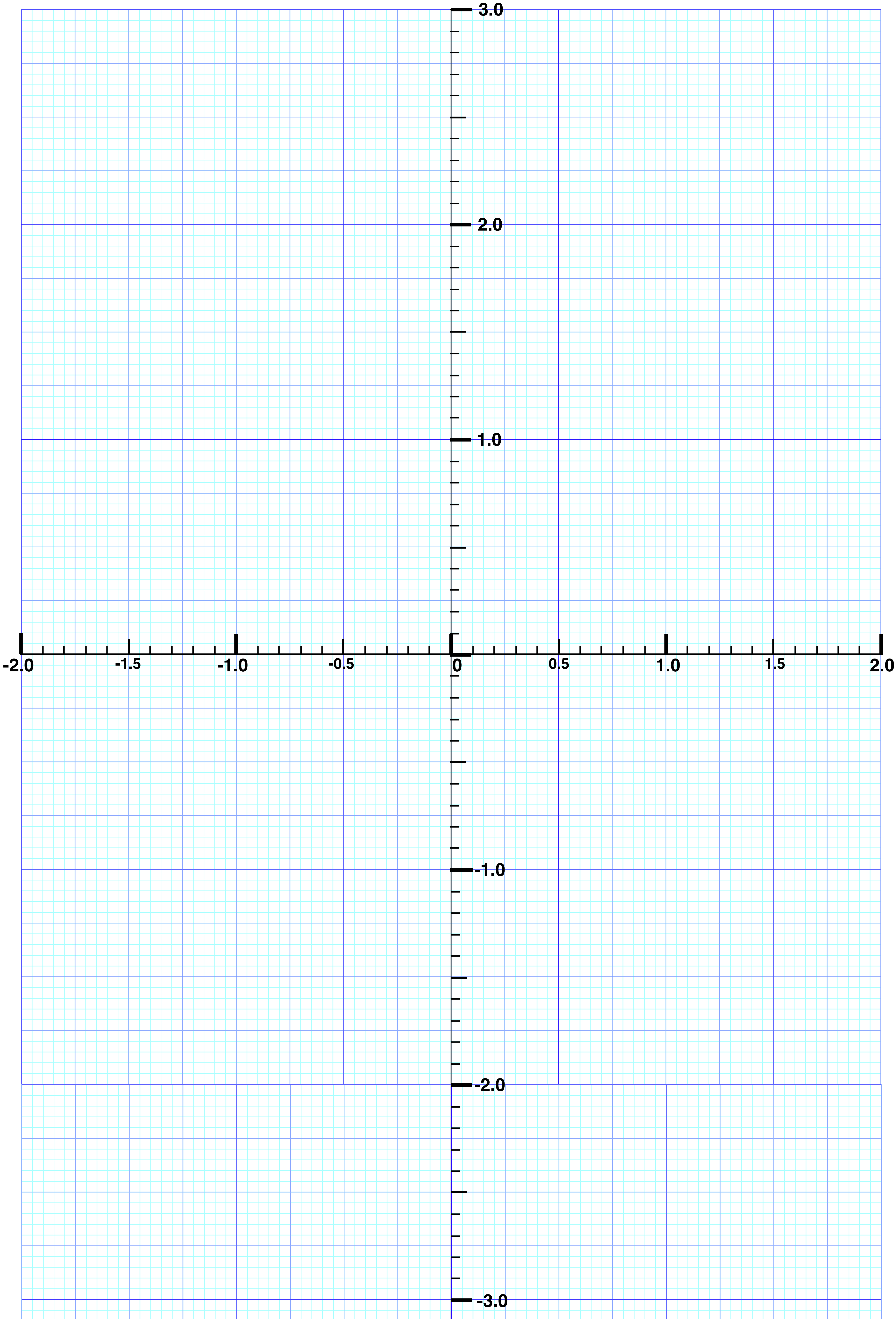

3.0
2.0
1.0
-2.0
-1.5
-1.0
-0.5
0
0.5
1.0
1.5
2.0
-1.0
-2.0
-3.0

Note "crunch" energy $_{Elastic}KE - _{inelastic}IE = 0.21$ is the same in all frames including COM-frame.

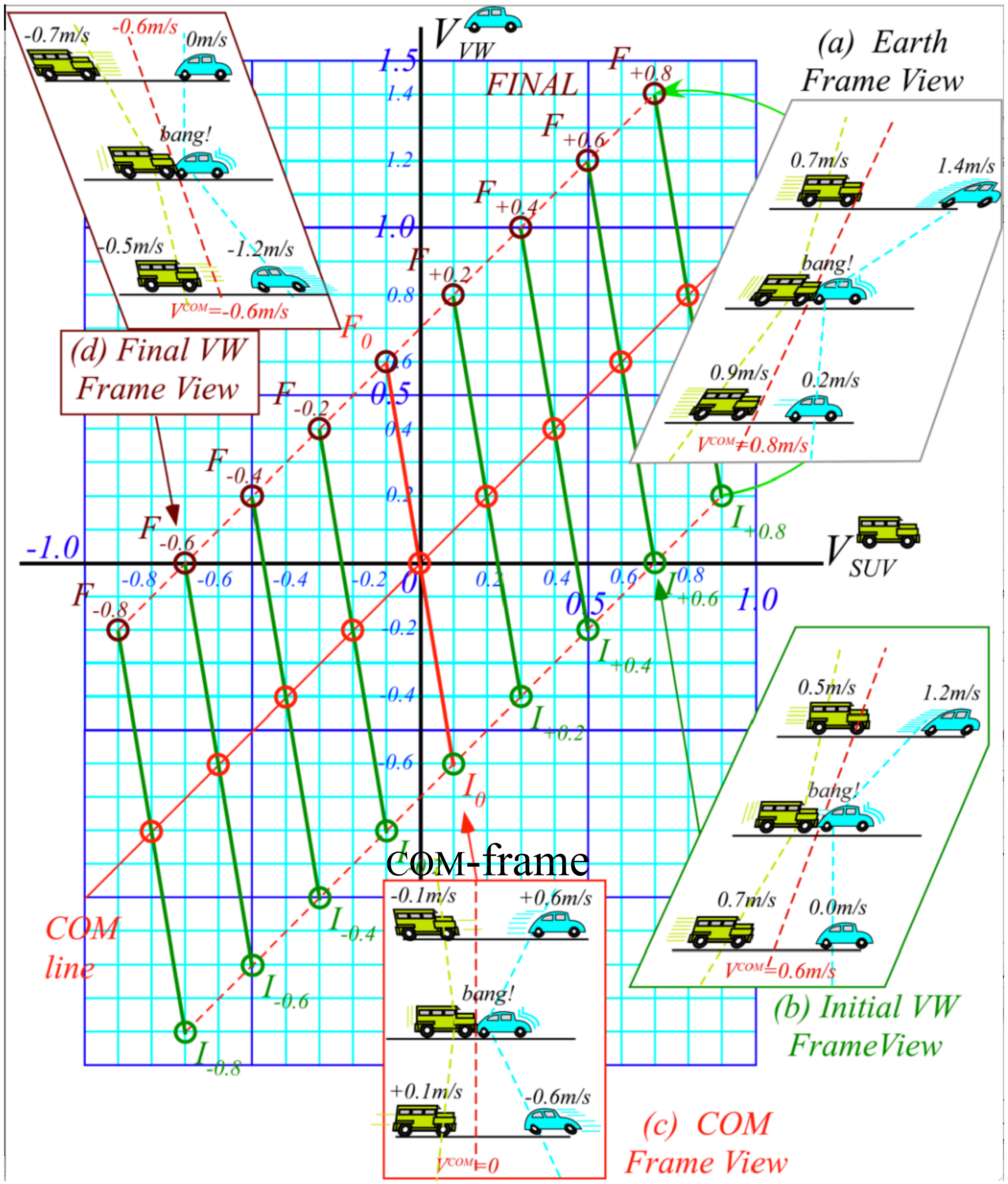

Fig. 3.4 Galilean Frame Views of collision like Fig. 2.5 or Fig. 3.1 with Bush (6:1) SUV.
(a) Earth frame view (b) Initial VW frame (VW initially fixed)
(c) COM frame view (d) Final VW frame (VW ends up fixed)

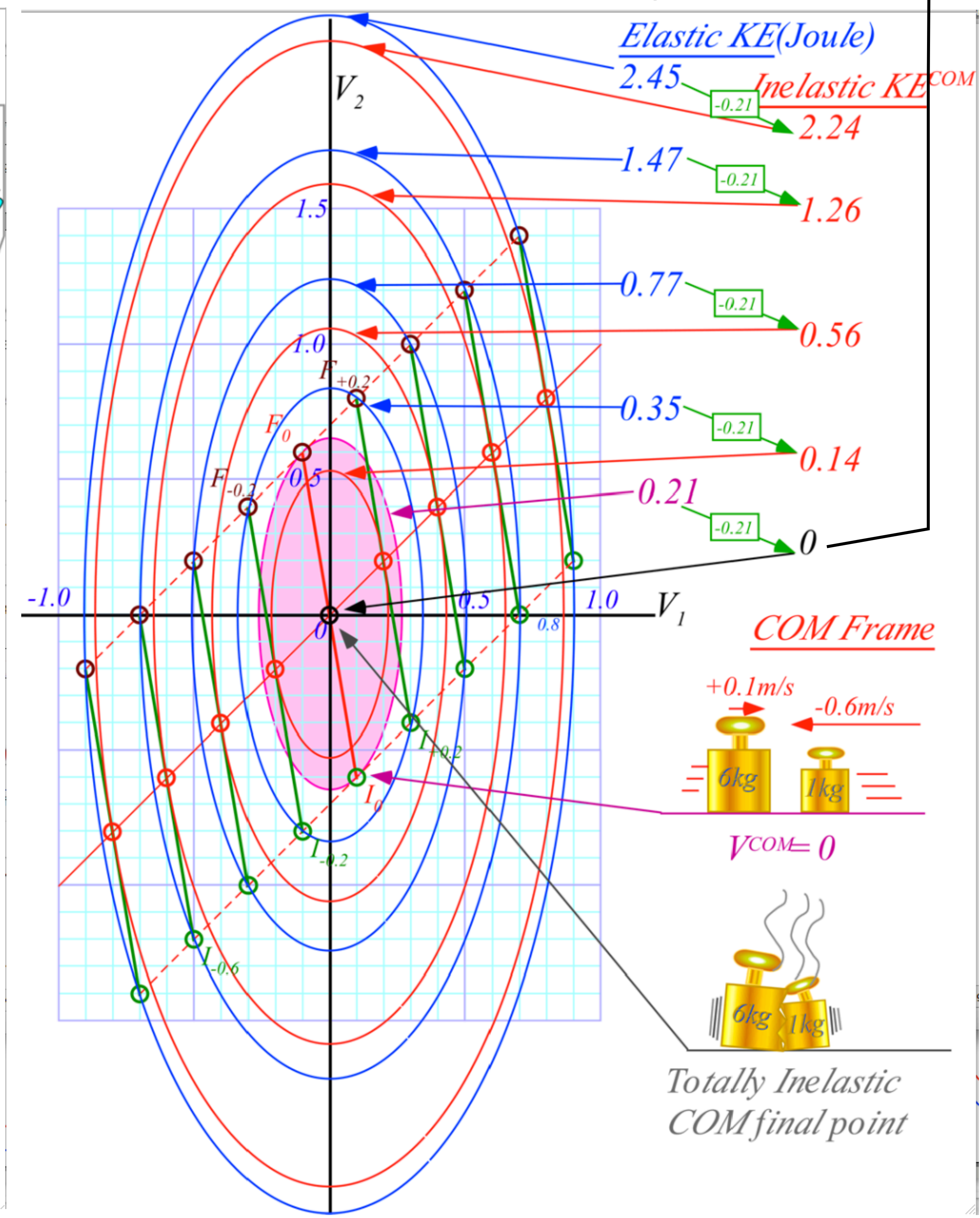

Fig. 3.5 Momentum (P=const.)-lines and energy (KE=const.)-ellipses appropriate for Fig. 3.4.

After-class Bull-session

⅔ of a *PhD* is *Ph*
that is *Philosophy*

Should physicists
worry about *philosophy*?
…about *economics*?
…about *current events*?
…about *politics*?…

After-class Bull-session

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Trumpian-authoritarian view of science and education:

B.S. (Bachelor of Science) — *Bull Sh*** *(hoax)*

M.S (Master of Science) — *More of the Same*

PhD (Doctor of Philosophy) — *Piled Higher and Deeper*

After-class Bull-session

⅔ of a *PhD* is *Ph*
that is *Philosophy*

Should physicists
worry about *philosophy*?
…about *economics*?
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…about *politics*?…

Excerpt of Preface to *CMwithBANG!* *(*What does *Liberal* mean*?)*

Humans attempts to develop the *CC* are so sporadic at first it is impossible to label its emergence. Traditionally one points to the *Seven Liberal Arts* as our break with pre-medieval superstition. The seven consisted of the *Trivium*: (*Grammar*, *Logic*, and *Rhetoric*), and the *Quadrivium*: (*Arithmetic*, *Geometry*, *Astronomy*, and *Music*). The term *Liberal* is interchangeable with *Liberating* and probably was used to designate a pathway to avoid slavery. It appears that the *Trivium* contains *drivers* of the creative results in the *Quadrivium*. Indeed the latter has grown to more like *Seven Thousand* Liberal Arts *and Sciences* in just a few centuries. It's an explosion! You'll have to excuse physics and chemistry for not making the first cut.

*(*What are alternatives to *Liberal?)*

After-class Bull-session

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…about *economics*?
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*(*What are famous historical alternatives to *Liberal?)*

Luther, on the other hand, was more anti-scholarly, at least with regard to Copernicus. His *LLL* attitude was less *Seven Liberal Arts* and more *Seven Deadly Sins*. These may also be divided into a *Trivium* and a *Quadrivium*, however now the latter (*Greed*, *Envy*, *Lust*, and *Gluttony*) are drivers of the former (*Pride*, *Wrath*, and *Sloth*), that is, *Pride* or *"Gloating"* if your *Greed*, *Envy*,..etc. yields success, or else *Wrath* or *"Rage"* if you are unsuccessful, followed by *Sloth* or *"Depression."* These are just drives and responses of *LLL* acquisition processes involving short-term ebb and flow of our small 3-to-5-ring molecules called *neurotransmitters*.

Should physicists worry about *philosophy*? …about *economics*? …about *current events*? …about *politics*?…

"All the News That's Fit to Print"

The New York Times

National Edition
Periodic clouds and sunshine. A few afternoon showers and thunderstorms west. Highs in the upper 70s to the 80s. Rain and thunder tonight. Details in SportsSunday, Page 10.

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Trump Tax Cut Pays Dividends For the G.O.P.

Donors Who Received Windfall Give Back

By JIM TANKERSLEY and MICHAEL TACKETT

WASHINGTON — Republicans are struggling to make the $1.5 trillion Trump tax cuts a winning

AP/ANDREW HARNIK

White House counsel Don McGahn listens during a Cabinet meeting Thursday. Fearing President Donald Trump was setting him up for blame, people close to him say, McGahn has been cooperating with the special counsel investigation to show he did nothing wrong.

TOP TRUMP AIDE GIVES MUELLER COVETED DETAILS

UNUSUAL COOPERATION

White House Counsel's Strategy Evolves Into Survival Tactic

By MICHAEL S. SCHMIDT and MAGGIE HABERMAN

WASHINGTON — The White House counsel, Donald F. McGahn II, has cooperated extensively in the special counsel investigation, sharing detailed accounts about

Pictured in yesterday's NYTimes are two Don's: Don McGahn, and Don Trump. Which is on the better path?