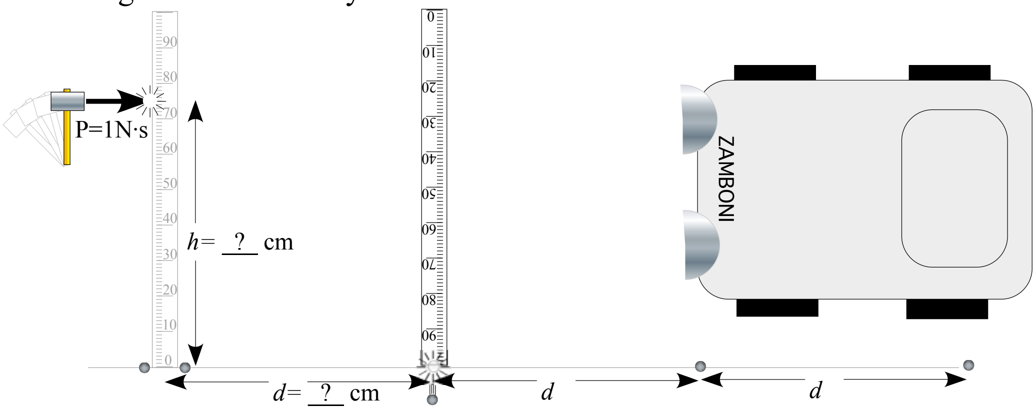
10/30/19 *Assignment 9 - CMwBANG!* Unit 2-2.9. Unit 3-3.8. - Lect.16-18 p.73-74 & p.102-110 Due Wed Nov. 6

An icy cycloid problem

Ex.1 (a) A *1kg*. meter stick lies on a smooth icy hockey rink surface with two marbles sitting at its end on either side of the 0.0cm mark. (See top-view figure) Assume frictionless ice rink.
A hammer give impulse $\mathbf{P}=(1\text{N}\cdot\text{s})\mathbf{e}_x$ to the 1kg stick at the h-cm. mark.
What height h is *least* likely to disturb the marbles.



(b) Now assume h-value from (a) and friction-free "icy" surface. At what distances d, $2d$, $3d$, … along x-axis should the 3^{rd}, 4^{th}, 5^{th},…marbles be placed so they are most likely to be knocked below the axis. Draw 6 equal time Δt interval snapshots of the stick as it flips by 180° and then to 360°. What is Δt for the 1kg stick?
(c) Compare path of stick if it struck with the same impulse at $h+10cm$. and if it struck at $h-10cm$.

Electromagnetic cycloids

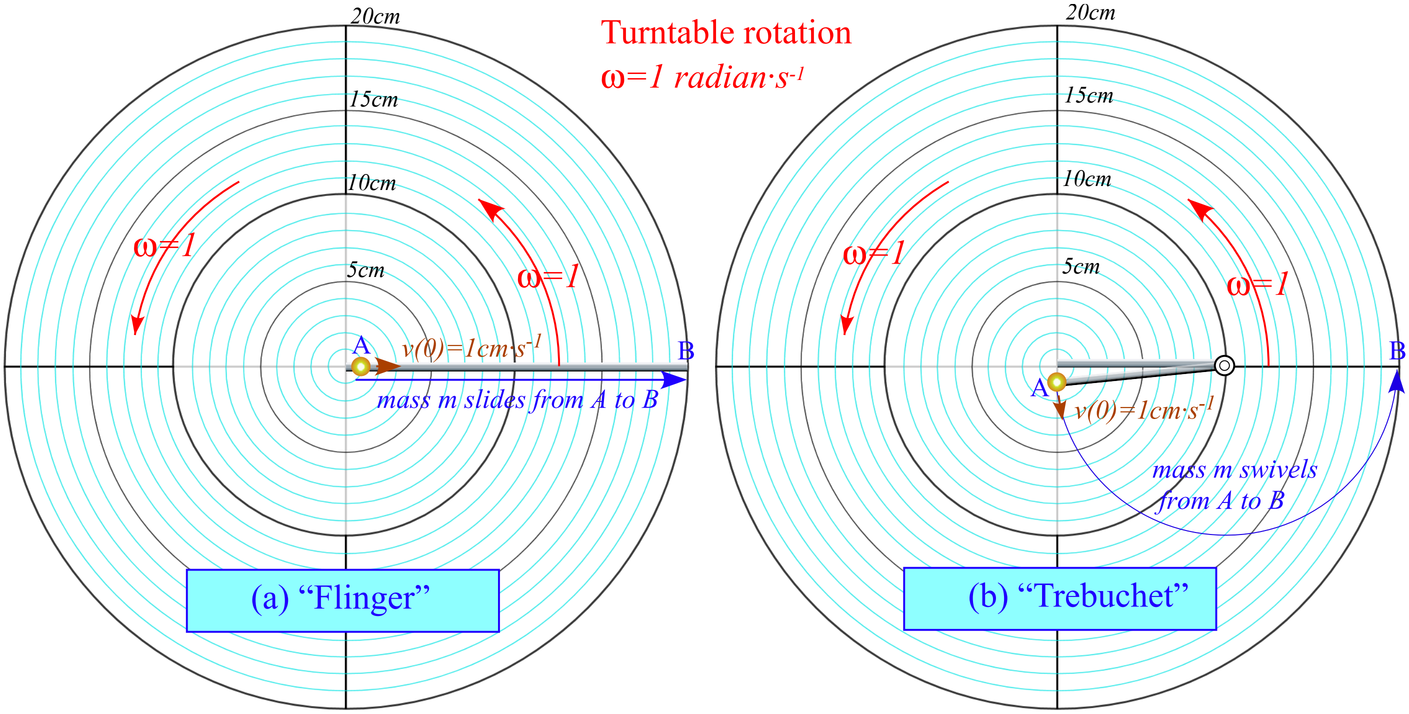
Ex.2 A unit mass $m=1$ *kg* and charge $Q=1$ *Coul.* (Dangerous!) starts at ($x=0=y$) on a frictionless (x,y)-surface in vertical Earth gravity (Say $g_y=-10m/s^2$) and in a strong z-axial magnetic field $\mathbf{B}_z=(0,0,B_z)$ normal to surface.
(a) What field B_z (in *Tesla*) has a mass with zero initial velocity $(v_x(0),v_y(0))=(0,0)$ follow a cycloid of 1 meter wheel diameter rolling along -x axis? What x-axis points does it hit? Are these hit points different for different $\mathbf{v}(0)$?
(b) What initial $\mathbf{v}(0)$ would cause the mass to fly a straight line along the -x-axis? … along the +x-axis?
(c) Describe and plot the resulting trajectory if instead the mass is thrown down with $(v_x(0),v_y(0))=(0,-2m/s)$.

Flinger vs. Trebuchet on turntable (geometric version)

Ex.3. Compare dynamics of mass *m* on a "Flinger" (Fig. (a)) to what it does on a "Trebuchet" (Fig. (b)).
Both begin at point A of radius $r(0)=1cm.$ from the center of a turntable rotating at $\omega=1(radian)s^{-1}$. Both have an initial speed of $v(0)=1cm\cdot s^{-1}$ and move from that point A to a final point B relative to turntable having radius $r(t_r)=20cm$ where we assume *m* is then released into the laboratory.

In Fig. (a) *m* slides $19cm$ along a rod of length $\ell=20cm$. (The $20cm$ rod is fixed to turntable.)

In Fig. (b) *m* swivels on a rod of length $\ell=10cm$ around a point fixed to turntable at $r=10cm$ radius.



(1a)Relative to turntable…
Find *m* release speed for "Flinger." _________

(2a)Relative to laboratory…
Find *m* release speed for "Flinger."_________

(3a)To scale†, sketch lab $\mathbf{v}(t_r)$ assuming release at B.
† Let $1cm$ be $1cm\cdot s^{-1}$.

(1b)Relative to turntable…
Find *m* release speed for "Trebuchet."__________

(2b)Relative to laboratory…
Find *m* release speed for "Trebuchet."__________

(3b)To scale†, sketch lab $\mathbf{v}(t_r)$ assuming release at B.
† Let $1cm$ be $1cm\cdot s^{-1}$.

How long does *m* take to go from A to release point B? _____sec.
Plot or (preferably) construct its orbit on a polar graph like Fig. (a) but in the lab-relative frame.
(4) Compare throwing turntable-relative and laboratory-relative performance (speed and direction) of the Flinger versus that of the Trebuchet.

Assignment 9 Solutions. **Exercise1** Hockey knock: An icy cycloid problem

A meter stick is lying flat on an ice rink with two marbles sitting at the lower end on either side of the 0.0cm mark on x-axis. (See figure) A hammer gives impulse **P**=*(1*N·s*)***e**$_x$ to the stick at the *h*-cm. mark.

What horizontal distance *h* is *least* likely to disturb the marbles. At what distances *d, 2d, 3d,* … along *x*-axis should the *3rd, 4th, 5th,…* marbles be placed so they are most likely to be knocked below the axis. (See figure above.)

Ex.1(a) Solution: Impulse *P* gives *x*-linear momentum $P=Mv_x$, linear velocity $v_x=P/M$ as well as angular momentum *L=(CM lever-arm)·P=(h-0.5)·P* , angular velocity $\omega_z=L/I$, where rot-inertia for stick (length *L=1/2* , mass *M*) is $\int_0^\ell \rho r^2\,dr = \frac{1}{3}\rho\ell^3 = \frac{1}{3}M\ell^2$ *=M/12*. (Same for two sticks end-to-end.) $I=M/12=1/12kg\cdot m^2=1/12kg\cdot m^2$.

That gives: $\omega_z=L/I=(h-0.5)\cdot P\cdot 12/M=(12h-6)\cdot P/M=(12h-6)\cdot v_x$.

To not disturb the marbles at $r_M=0.5$ we need $\omega_z\, r_M=(12h-6)\cdot v_x\cdot 0.5$ to cancel v_x.

So: $(12h-6)\cdot v_x\cdot 0.5 = v_x$, or $(6h-3)=1$, or <u>*h =2/3m=66.666…cm*</u>.

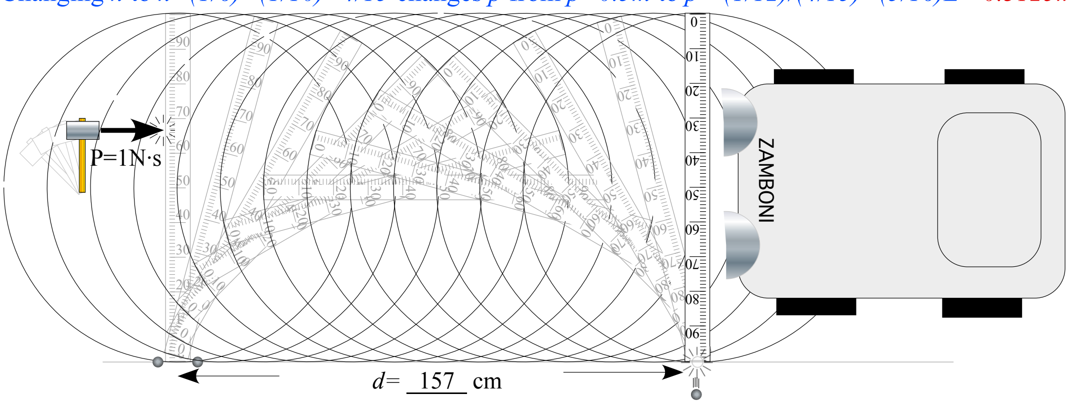
The meter stick rolls on *x-axis* like a wheel of radius *R=0.5m*. So <u>*d=π·R=π/2=1.57m*</u>. Time for 360°=$2\pi/\omega_z=2\pi/(12h-6)=\pi$ *sec*.

(b) Now assume *h*-value from (a) and friction-free "icy" surface. At what distances *d, 2d, 3d,* … along *x*-axis should the *3rd, 4th, 5th,…* marbles be placed so they are most likely to be knocked below the axis. Draw 6 equal time Δ*t* interval snapshots of the stick as it flips by 180° and then to 360°. What is Δ*t* for a 1kg stick? Δ*t= π/6 sec.* =0.532sec.

(c) Start with $hp=I/M=(1/12)L^2$ with *L=1m* and *h=1/6* above center at *0.5 m*. Lect. 18 p.13-14 or CMwBang! Unit 1 p.175-176.

Changing *h* to *h=(1/6) -(1/10)=1/15* changes *p* from *p=0.5m* to *p= (1/12)/(1/15)=(5/4)L =1.25m*. (much larger "p-wheel")

Changing *h* to *h=(1/6)+(1/10)=4/15* changes *p* from *p=0.5m* to *p= (1/12)/(4/15)=(5/16)L =0.3125m*. (smaller "p-wheel")



Solution to **Ex.2** *Electromagnetic cycloids* A vertical frictionless surface in Earth gravity (Say $g=10m/s^2$) guides a unit mass *m=1 kg* and charge *Q=1 Coul.* (Dangerous!) that is dropped from (*x=0=y*) in a strong magnetic **B**-field.

(a) How many Tesla of magnetic field **B** and in what direction would cause the mass to move to the left on a normal cycloid of one meter diameter? Where would it hit the horizontal *x*-axis? Primary hit points are independent of **v**(0).

Replace wheel radius $R_W=E/B^2=(eE_x/m)/(eB_z/m)^2$ in (2.8.24) with $R_W=(mg_x/m)/(QB_z/m)^2=m^2g_x/Q^2B_z^2=10/B^2$ where $R_W=0.5m$.

$$B_z=\sqrt{\frac{m^2g}{R_WQ^2}}=\sqrt{\frac{1kg\cdot 10ms^{-2}}{0.5m\cdot(1C)^2}}=\sqrt{20}Tesla=4.47Tesla$$

Normal $R_W=0.5m$ cycloid hits *x*-axis every $-2\pi R=-3.14m$.

(b) What initial speed and direction would cause the mass to fly straight-line along the x-axis?

Magnetic force mQv_xB_z supports weight *-mg* for finite negative velocity $v_x=g/QB_z=-10/1\cdot\sqrt{20}=-\sqrt{5}=-2.236m/s$…or *infinite* positive v_x!

(c) Describe and plot the resulting trajectory if the mass is thrown down with with a speed of *-2m/s*.

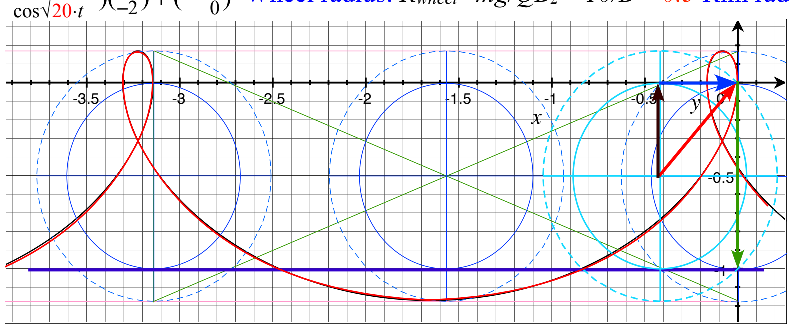
Use 2.(a) results with *(1). x(0)=0=y(0), (2).* $v_x(0)=0$, $v_y(0)=-2$, *(3).* $\varepsilon_x=0$, $\varepsilon_y=-g=-10$, and *(4).* $B=QB_z/m=B_z=\sqrt{20}$.

$$\begin{pmatrix}x(t)\\y(t)\end{pmatrix}=\begin{pmatrix}\cos B\cdot t & \sin B\cdot t\\ -\sin B\cdot t & \cos B\cdot t\end{pmatrix}\begin{pmatrix}-v_y(0)/B-\varepsilon_x/B^2\\ v_x(0)/B-\varepsilon_y/B^2\end{pmatrix}+\begin{pmatrix}\varepsilon_y t/B\\ -\varepsilon_x t/B\end{pmatrix}+\begin{pmatrix}x(0)+v_y(0)/B+\varepsilon_x/B^2\\ y(0)-v_x(0)/B+\varepsilon_y/B^2\end{pmatrix}$$

$$\begin{pmatrix}x(t)\\y(t)\end{pmatrix}=\begin{pmatrix}\cos B\cdot t & \sin B\cdot t\\ -\sin B\cdot t & \cos B\cdot t\end{pmatrix}\begin{pmatrix}2/B\\ 10/B^2\end{pmatrix}+\begin{pmatrix}-10t/B\\ 0\end{pmatrix}+\begin{pmatrix}-2/B\\ -10/B^2\end{pmatrix}=\begin{pmatrix}\cos\sqrt{20}\cdot t & \sin\sqrt{20}\cdot t\\ -\sin\sqrt{20}\cdot t & \cos\sqrt{20}\cdot t\end{pmatrix}\begin{pmatrix}2/\sqrt{20}\\ 1/2\end{pmatrix}+\begin{pmatrix}-t\sqrt{5}\\ 0\end{pmatrix}+\begin{pmatrix}-2\sqrt{20}\\ -1/2\end{pmatrix}$$

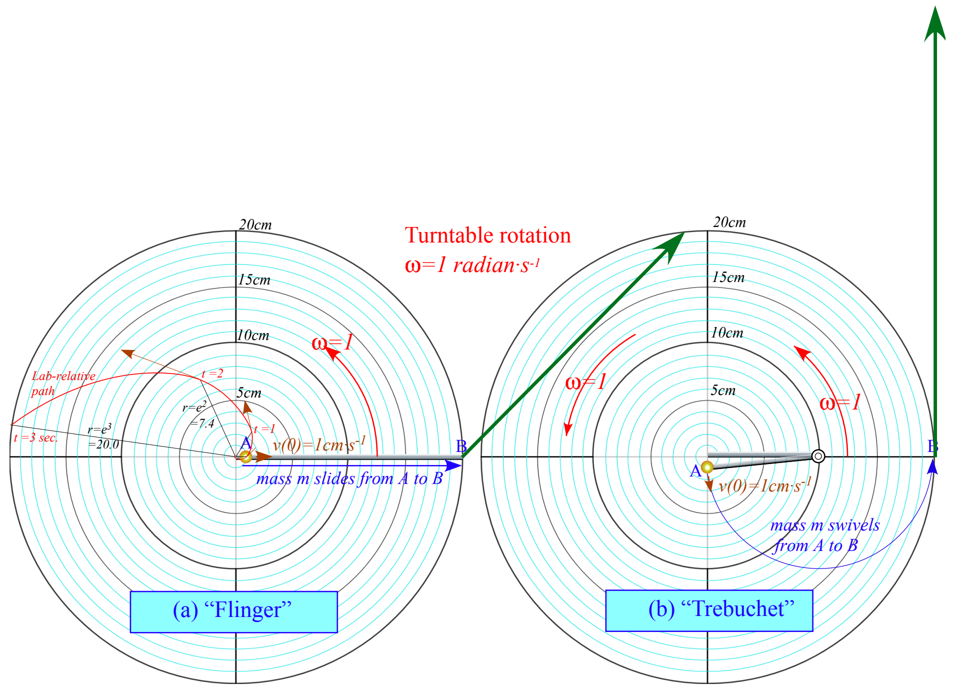
$$\begin{pmatrix}v_x(t)\\v_y(t)\end{pmatrix}=\begin{pmatrix}\cos\sqrt{20}\cdot t & \sin\sqrt{20}\cdot t\\ -\sin\sqrt{20}\cdot t & \cos\sqrt{20}\cdot t\end{pmatrix}\begin{pmatrix}\sqrt{5}\\ -2\end{pmatrix}+\begin{pmatrix}-\sqrt{5}\\ 0\end{pmatrix}$$

Wheel radius: $R_{wheel}=mg/QB_z^2=10/B^2=0.5$ Rim radius: $\sqrt{4/B^2+R_W^2}=\sqrt{0.2+0.25}=0.671$



Assignment 9(contd.) *Solutions to* **Ex.3.** *Flinger vs Trebuchet*

. . . .begin at point A $r(0)=1cm$. $v(0)=1cm\cdot s^{-1}$ relative to turntable at $\omega=1(\text{rad})s^{-1}$. . . Release at B $r(t_r)=20cm$.



Relative to turntable…

Find m release speed for "Flinger." 20cm·s^{-1}__

Relative to laboratory…

Find m release speed for "Flinger." 20√2cm·s^{-1}

To scale†, sketch lab $\mathbf{v}(t_r)$ assuming release at B.

† Let 1cm be 1cm·s^{-1}.

Relative to turntable…

Find m release speed for "Trebuchet." 20cm·s^{-1}__

Relative to laboratory…

Find m release speed for "Trebuchet." 40cm·s^{-1}__

To scale†, sketch lab $\mathbf{v}(t_r)$ assuming release at B.

† Let 1cm be 1cm·s^{-1}.

Assignment 9. **Ex.2.** *(contd) "Flinger" problems:*

How long does m take to go from A to release point B? __3__sec.

Plot or (preferably) construct its orbit on a polar graph like Fig. (a) but in the lab frame.

Turntable-relative Hamiltonian is a constant E. (Similar to harmonic oscillator H but with (-) sign.)

$$E = H = \frac{p^2}{2m} - \frac{1}{2}m\omega^2 r^2 = \frac{1}{2}m\left(\dot{r}^2 - r^2\right)$$

Solutions are real (as opposed to imaginary) exponentials.

$$r(t) = Ae^{\omega t} + Be^{-\omega t} = Ae^{t} + Be^{-t} \qquad \dot{r}(t) = A\omega e^{\omega t} - B\omega e^{-\omega t} = Ae^{t} - Be^{-t}$$

Given $r(0)=1$ and $\dot{r}(0)=1$ we settle on purely exponential solutions $r(t)=e^{t}=\dot{r}(t)$.

So turntable-relative velocity vector $\mathbf{v}=\dot{\mathbf{r}}$ is always equal to position vector $\mathbf{r}$.

To this we add the turntable velocity $\mathbf{v}_{table} = \omega\,\hat{\mathbf{e}}_z \times \mathbf{r}$ relative to lab whose magnitude also equals that of $\mathbf{r}$ but is perpendicular to it in the direction of rotation.

So lab-relative flinger velocity is always at 45° to table-relative flinger velocity and a factor of √2 larger.

As a result, the lab-relative flinger path shown below is a log-spiral that is always heading 45° to **r**.

Trebuchet lab final velocity is 2 times greater than table relative final velocity so it beats the flinger!