## An icy cycloid problem

Ex. 1 (a) A 1 kg . meter stick lies on a smooth icy hockey rink surface with two marbles sitting at its end on either side of the 0.0 cm mark. (See top-view figure) Assume frictionless ice rink.
A hammer give impulse $\mathbf{P}=(1 \mathrm{~N} \cdot \mathrm{~s}) \mathbf{e}_{\mathbf{x}}$ to the 1 kg stick at the $h-\mathrm{cm}$. mark.
What height $h$ is least likely to disturb the marbles.

(b) Now assume $h$-value from (a) and friction-free "icy" surface. At what distances $d, 2 d, 3 d, \ldots$ along $x$-axis should the $3^{r d}, 4^{\text {th }}, 5^{\text {th }}, \ldots$ marbles be placed so they are most likely to be knocked below the axis. Draw 6 equal time $\Delta t$ interval snapshots of the stick as it flips by $180^{\circ}$ and then to $360^{\circ}$. What is $\Delta t$ for the 1 kg stick?
(c) Compare path of stick if it struck with the same impulse at $h+10 \mathrm{~cm}$. and if it struck at $h-10 \mathrm{~cm}$.

## Electromagnetic cycloids

Ex. 2 A unit mass $m=1 \mathrm{~kg}$ and charge $Q=1$ Coul. (Dangerous!) starts at ( $x=0=y$ ) on a frictionless $(x, y)$-surface in vertical Earth gravity (Say $g_{y}=-10 \mathrm{~m} / \mathrm{s}^{2}$ ) and in a strong $z$-axial magnetic field $\mathbf{B}_{z}=\left(0,0, B_{z}\right)$ normal to surface.
(a) What field $B_{z}$ (in Tesla) has a mass with zero initial velocity $\left(v_{x}(0), v_{y}(0)\right)=(0,0)$ follow a cycloid of 1 meter wheel diameter rolling along $-x$ axis? What $x$-axis points does it hit? Are these hit points different for different $\mathbf{v}(0)$ ?
(b) What initial $\mathbf{v}(0)$ would cause the mass to fly a straight line along the $-x$-axis? $\ldots$ along the $+x$-axis?
(c) Describe and plot the resulting trajectory if instead the mass is thrown down with $\left(v_{x}(0), v_{y}(0)\right)=(0,-2 \mathrm{~m} / \mathrm{s})$.

Flinger vs. Trebuchet on turntable (geometric version)
Ex.3. Compare dynamics of mass $m$ on a "Flinger" (Fig. (a)) to what it does on a "Trebuchet" (Fig. (b)).
Both begin at point A of radius $r(0)=1 \mathrm{~cm}$. from the center of a turntable rotating at $\omega=1$ (radian) $s^{-1}$. Both have an initial speed of $v(0)=1 \mathrm{~cm} \cdot \mathrm{~s}^{-1}$ and move from that point A to a final point B relative to turntable having radius $r\left(t_{r}\right)=20 \mathrm{~cm}$ where we assume $m$ is then released into the laboratory.

In Fig. (a) $m$ slides 19 cm along a rod of length $\ell=20 \mathrm{~cm}$. In Fig. (b) $m$ swivels on a rod of length $\ell=10 \mathrm{~cm}$ (The 20 cm rod is fixed to turntable.) around a point fixed to turntable at $r=10 \mathrm{~cm}$ radius.

(1a)Relative to turntable...
Find $m$ release speed for "Flinger." $\qquad$
(2a)Relative to laboratory...
Find $m$ release speed for "Flinger." $\qquad$
(3a)To scale ${ }^{\dagger}$, sketch lab $\mathbf{v}\left(t_{r}\right)$ assuming release at B. (3b)To scale ${ }^{\dagger}$, sketch $\underline{\mathrm{lab}} \mathbf{v}\left(t_{r}\right)$ assuming release at B.
$\dagger$ Let 1 cm be $1 \mathrm{~cm} \cdot \mathrm{~s}^{-1}$.
How long does $m$ take to go from A to release point B?
$\dagger$ Let 1 cm be $1 \mathrm{~cm} \cdot \mathrm{~s}^{-1}$.
$\qquad$ sec.
Plot or (preferably) construct its orbit on a polar graph like Fig. (a) but in the lab-relative frame.
(4) Compare throwing turntable-relative and laboratory-relative performance (speed and direction) of the Flinger versus that of the Trebuchet.

