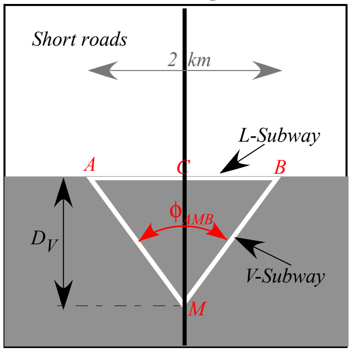
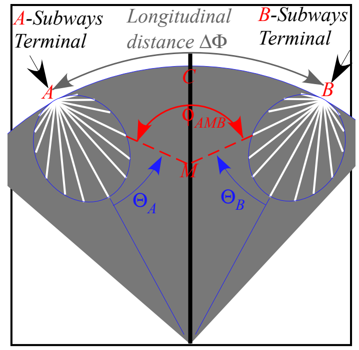
Oct. 2 2019 - Assignment 6 - due Wed Oct. 09 - Mainly Chapters 9 , 11, and 12. Lect. 11 Name____________





Geometric Optimization Exercises

Fig. 1 Local friction-free subways on Sophomore Physics-Earth (SPE). Fig. 2 Global friction-free subways on SPE.

1. a. Friction-free local subway path *ACB* in Fig. 1 is laser-beam straight and normal to line thru Earth center only at turning point *C*. How long does it take for subway car starting with $v(0)=0$ at point *A* to arrive at point *B*?

b. Assume *constant* gravity $g=9.8m/s^2$ for friction-free local subway path *AMB* in Fig. 1 where turning vertex *M* is negotiated ideally with no loss of energy (or life!). Derive depth D_V of point *M* and angle ϕ_{AMB} that gives the quickest trip from *A* to *B* ($AC=1km=CB$) thru *M* and derive the *AMB* time of travel τ_{AMB}=______ *min*.

c. Cycloid (Lect. 11 p.33-42) gives *absolute* minimum τ_{AB}. Derive its (x,y) formula and plot using attached cycloid graph paper. Over it plot the optimal V-shaped path *AMB* of 1b to help compare the two.

d. Use Lagrange equation of circle angle ϕ to find cycloid travel time τ_{AB}=______*min* and compare it to τ_{AMB}.

2. Assume *Isotropic Harmonic Oscillator* IHO gravity in Fig. 2 with acceleration $g=9.8m/s^2$ at surface dropping to $g=0$ at Earth-center $C_\oplus$ (bottom-center of Fig.2). The objective (as in Ex. 1) is to find path *AMB* and angle ϕ_{AMB} having least time of travel τ_{AMB} between Terminal points *A* and *B* separated by great circle longitude angle $\Delta\Phi_{AB}$.

Before solving main objective consider some alternative routes whose travel times should be easy to derive.

a. Direct straight line route from *A* to *B*: $\tau_{A\,direct\,toB}$=______ *min*. Relate to SPE half-orbit period $\tau_\odot/2$

b. Straight line segment routes *A* to *C* then *C* to *B*: $\tau_{AtoCtoB}$=______ *min*. " "

c. Direct *A* to Earth-center $C_\oplus$ then $C_\oplus$ back up to *B*: $\tau_{AtoC_\oplus toB}$ =______ *min*. " "

3. To solve main objective of Ex.2, imagine subway cars from terminal *A* or *B* leave their terminal at time $t_0=0$ and fall along straight tunnels (white lines) to positions at later time t_1 indicated by points on blue oval in Fig. 2. Ovals *A* and *B* expand equally until a touch-time t_T when they are tangent to each other and to vertical line *CM*.

a. That touch-time t_T is related to total minimum travel time τ_{AMB}. How? (Recall τ_{AMB} for Ex. 1.)

b. Derive polar $r(\Theta,t)$ equation for "ovals" relative to Terminal origin. What Thales geometric form is it?

c. Relate optimizing angle ϕ_{AMB} to angle $\Delta\Phi_{AB}$ of longitudinal *A* to *B* separation. Plot geometry for $\Delta\Phi=\frac{\pi}{2}$.
It helps to define a slope angle α between optimal subway path and terminal vertical radial line.

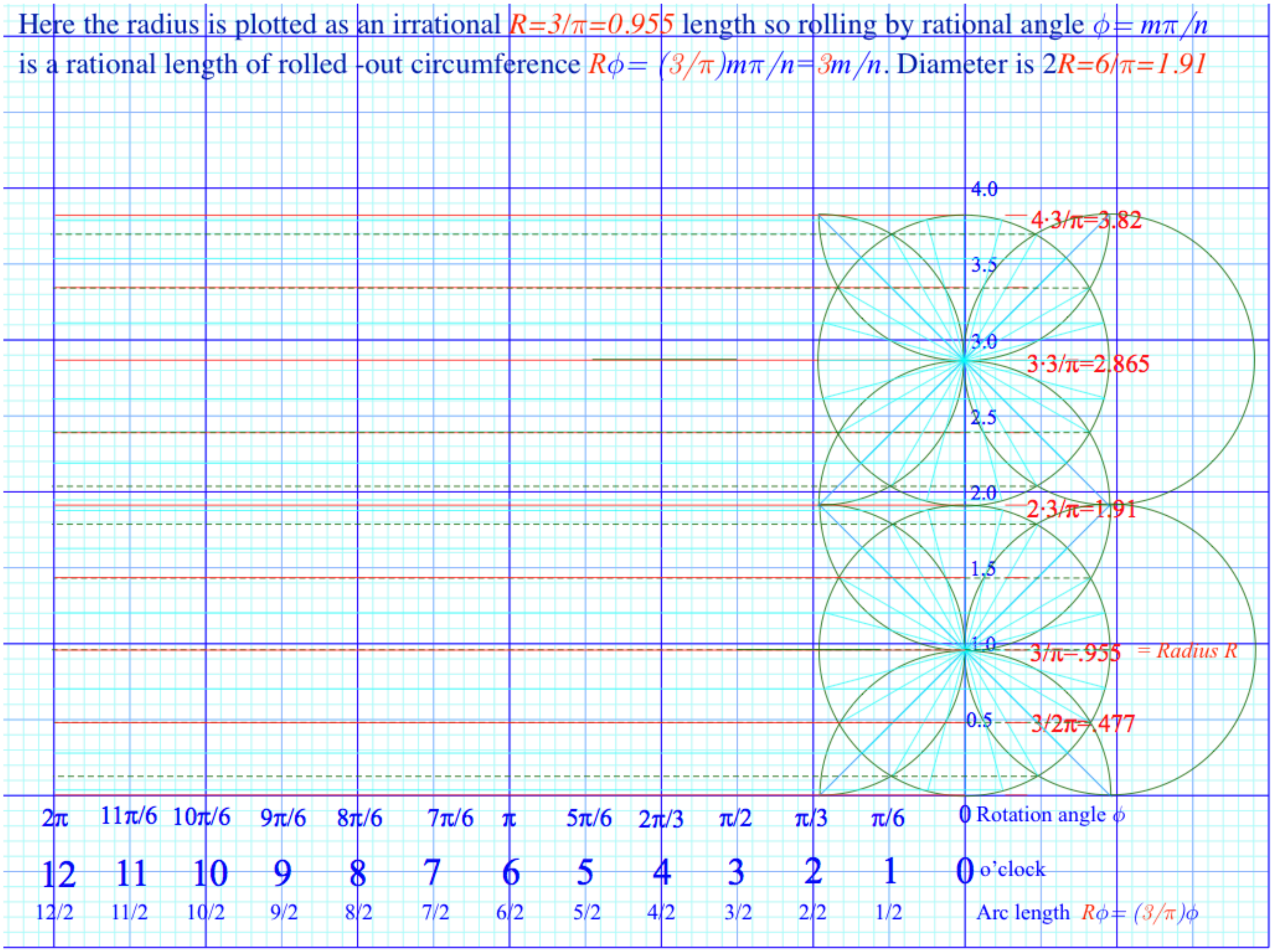
d. A terminal-launched SPE circular orbit serves as a clock hand that quantifies growing "oval" size.
Include this in your geometric plot to quantify optimal travel time τ_{AMB} and plot car positions vs. t.
Show car positions at fractions $\frac{1}{12},\frac{2}{12},\frac{3}{12},\ldots$of half-orbit period $\tau_\odot/2$ to help relate to travel time τ_{AMB}.

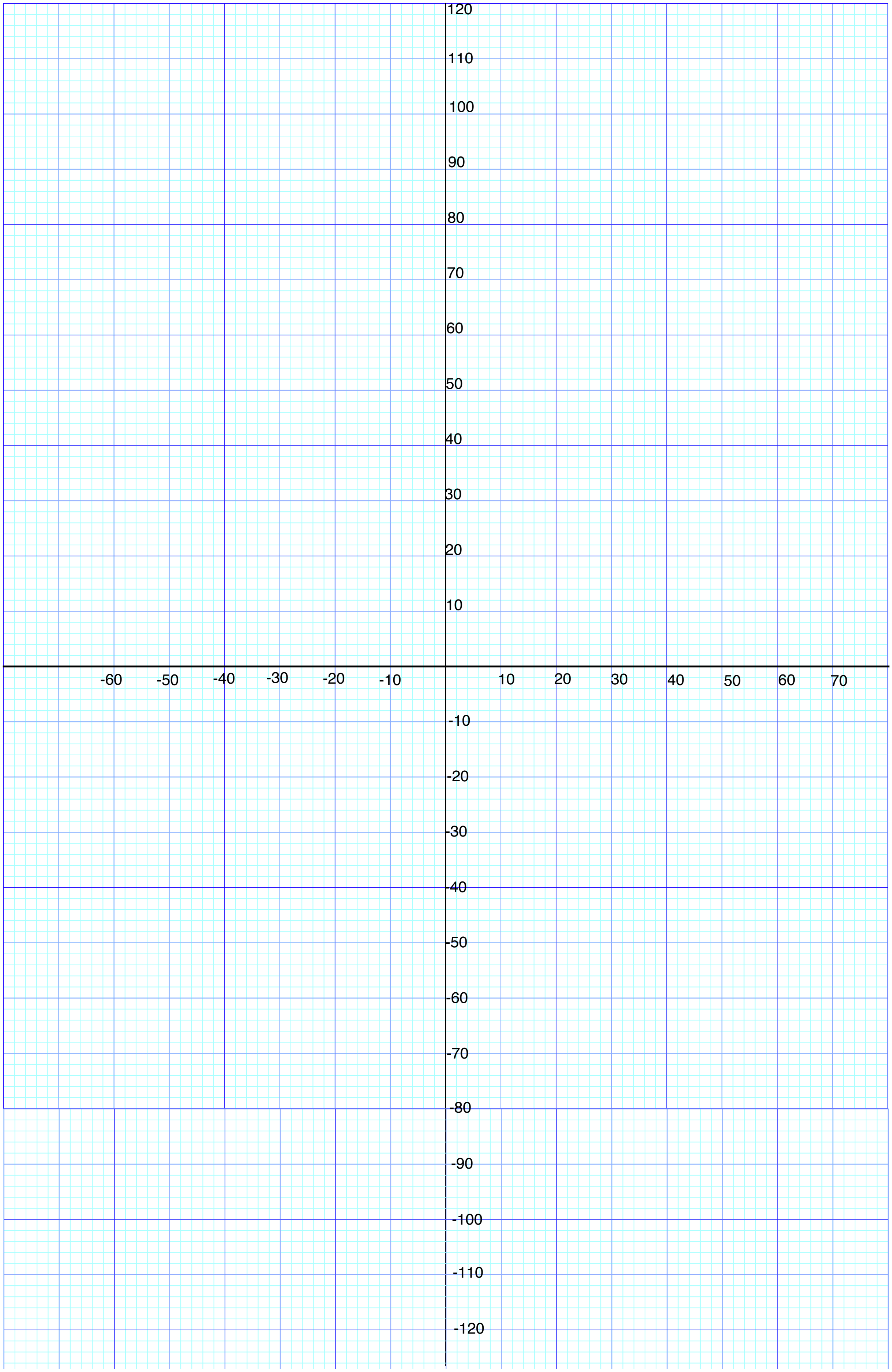
e. Verify geometry c and d in extreme cases of distance that is small ($\Delta\Phi\ll\frac{\pi}{2}$ like Ex.1) or large ($\Delta\Phi=\pi$).

For Exercises 1c and 1d:

Huygen's cycloidal-pendulum construction graph (See Lect. 11p.33-42).

Motion may be calculated using Lagrange's 1D equation using single angle variable ϕ and angular velocity $\dot{\phi}$.
In *mks* units the circumference $2\pi R$ of the circle (and width *ACB* of cycloid) should be 2*km* as shown in Fig. 1.

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